Liquidity Regulation, Bail-ins and Bailouts*

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June 8, 2018

Abstract: A major regulatory innovation in the wake of the 2008 financial crisis was the introduction of liquidity requirements for banks in the Basel framework. While the philosophy of such requirements is intuitive, policymakers received little guidance from academia on a number of specific questions such as the measure of the liquidity buffer, its possible decomposition into multiple tiers (as is done for capital requirements), the treatment of interbank exposures or of the securitization of legacy assets, or the recognition of central-bank-eligible assets as part of the buffer. More fundamentally, the consistency between liquidity and solvency regulations has been under-researched. Accordingly, the introduction of layers of "bail-inable" liabilities to complement the Basel III framework occurred without conceptual framework for thinking about liquidity and solvency regulation. The paper develops such a framework, integrates the asset and liability sides into an overall design of prudential regulation and assesses the regulatory reform.

Keywords: Liquidity weighted assets (LWA), prudential regulation, bailouts, bail-ins, fire sales, baseload and peakload liquidity, central bank liquidity support, central bank eligible assets.

JEL numbers: D82, G21, M48.

*The authors are grateful to Paul-Henri Moisson for excellent research assistance. This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement n° 669217 - ERC MARKLIM). We thank Jeremy Stein and participants at the "New Frontiers in Banking" conference (Rome) and seminars at AFSE, Banque Nationale de Belgique, Berkeley and CUHK (Shenzhen) for helpful comments. The Toulouse School of Economics receives research support from various private and public institutions, including the Bank of France and the Central Bank of Luxembourg.

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1 Introduction

1.1 Overview of the paper

This paper develops a theory of the optimal prudential control of a financial institution's liquidity management. To withstand temporary cash needs, a financial institution engaged in maturity transformation (a "bank") can avail itself of a variety of instruments. First, on the asset side of the balance sheet, it can resell or securitize a range of assets with different yields and degrees of liquidity. Some assets command a low yield but trade at par if resold early; others deliver a higher return when held to maturity, but trade at a discount when liquidated early, thereby worsening the solvency of the bank as a result of its attempt to address its liquidity shortage. Second, the bank (or its regulator) can restore its impaired solvency through the dilution of equity holders, the conversion of hybrid securities into equity, or by bailing in junior debtholders, secured creditors or corporate depositors for instance. Finally, the bank can benefit from public assistance from the central bank or the Treasury.

Even though maturity transformation and illiquidity played a central role in the 2008 financial crisis and provided the impetus for the recent regulatory reforms under Basel III, our theoretical understanding of how liquidity should be regulated is scant. Answers to basic questions such as the aggregation of liquid positions are still elusive. In contrast, an intense debate has been raging in regulatory circles over the last ten years. This paper provides a comprehensive theoretical framework in which these reforms can start being assessed.

Section 2 studies the socially optimal liquidity mismatch and portfolio when asset returns are exogenous. It first demonstrates that it is socially suboptimal for the bank's buffer to cover extreme risks; rather, the state should step in and provide open bank assistance in those circumstances. Second, and analogous to optimal investment portfolios in a world of uncertain demands (Boiteux 1949), banks optimally hoard a liquid-assets portfolio composed of "baseload liquidity" and of "peakload liquidity". Baseload liquidity consists of low-yield, highly liquid assets that are used frequently for liquidity coverage. Peakload liquidity needs are covered through higher-yield, higher-haircut assets; these assets constitute idle liquidity in normal times, but bring a liquidity service in rougher times.

Under laissez-faire, banks underinvest in liquidity as they anticipate that the state will (rationally) come to their rescue when in dire straits. Liquidity coverage must therefore be regulated. We derive conditions under which the optimal micro-prudential regulation involves an LWA (liquidity-weighted assets)[1] requirement. That is, a bank is asked to hoard a given amount of total liquidity, where the measure of liquidity assigns weights

[1]This terminology is inspired by the notion of RWA (risk-weighted assets) operative in the prudential regulation of solvency since Basel I (1988).

on liquid assets based on their haircuts if they are not held to maturity. Constrained by this floor on liquidity, banks then choose the optimal structure of their liquid portfolio between baseload and peakload liquid assets. That the composition need not be regulated remains true when the supply of highly liquid assets is scarce: the LWA requirement then induces banks to rely less on these highly liquid, but costly assets to maintain their liquidity level. Section 2 then considers how liquidity requirements change as (a) legacy asset securitization, through private markets or central banks, can contribute to limiting bailouts, and (b) banks enter mutual exposures. We first show how the possibility of securization of legacy assets can be incorporated into the liquidity ratio through an appropriate measurement of adverse selection. We then analyse how liquidity pooling and the concomitant cross-exposures affect liquidity requirements when the regulator is able to assess the directionality of positions and therefore the scope for cross-insurance.

In practice, banking regulations (Basel III), while specifying discounts for assets eligible for meeting the liquidity requirements, also adopt a directive approach by specifying minimum percentages of specific classes of assets (e.g. government bonds). Section 3 enriches the framework by allowing for endogenous resale discounts. The literature has emphasized that the limited depth of resale markets generates a fire-sales externality. We micro-found fire-sale discounts and revisit the conventional wisdom in a world with and without bailouts. The novelty of our analysis here is two-fold; first, it derives the implications of fire sales for the regulation of liquidity ratios and the need for specifying a minimum fraction of level-1 liquid assets; second, it sheds light on the ongoing debate as to whether central-bank-eligible assets should be counted as high-quality liquid assets.

To concentrate on the asset side of the liquidity equation, Sections 2 and 3 relied on an "exogenous bail-inability" assumption. This in the past was natural in the context of financial crises, as equity-holders, but not the other claimholders, have traditionally absorbed losses. To study bail-inability, Section 4 relaxes this assumption and assumes that different classes of investors, with different risk appetites, can be offered claims that are explicitly bail-inable under prespecified circumstances. Symmetrically with the asset side, where liquidity provision follows a pecking order, optimal liquidity provision on the liability side rests on a priority order. Low-risk-aversion investors sort themselves out so by acquiring liabilities (to be interpreted as outside equity, hybrid claims, subordinated debt . . .) that are bailed in first, second, and so on. Some liabilities- insured deposits- are safe (never bailed in). We provide a condition under which the structure of liquidity can be delegated to the bank through an LWA requirement in which all bail-inable securities receive identical weight 1 (and, as earlier, liquid assets receive a weight that reflects their haircut in case of early sale).

Section 5 provides a detailed comparison between theoretical suggestions and the actual practice as reflected by international agreements. Section 6 concludes with a summary of the main results and some alleys for future research. The rest of this introduction is devoted to a brief overview of regulatory reforms and of related contributions.

1.2 Overview of recent regulatory reforms

Basel III introduced two liquidity ratios: a short-term ratio, the Liquidity Coverage Ratio (LCR), looking at a one-month horizon, and a longer-term ratio, the Net Stable Funding Ratio (NSFR), looking at a one-year horizon. In our simple model with a single liquidity shock, we cannot distinguish between the two horizons, so we concentrate here on the LCR, which tries "to ensure that a bank has an adequate stock of unencumbered high quality liquid assets (HQLA) which consists of cash or assets that can be converted into cash at little or no loss of value in private markets to meet its liquidity needs for a 30 calendar day liquidity stress scenario".[2] The LCR is the ratio of the value of the stock of HQLA divided by the total net cash outflows, over the next 30 calendar days.

The numerator of the LCR is the stock of HQLA. These assets should be liquid in markets during a time of stress. They are comprised of level-1 and level-2 assets. The former include cash, central bank reserves, and certain marketable securities backed by sovereigns and central banks, among others. They are considered as the most liquid and can in principle constitute the whole HQLA stock. Level-2 assets are comprised of level-2A assets (certain government securities, covered bonds and corporate debt securities) and level-2B assets (lower rated corporate bonds, residential mortgage backed securities and some equities). Level-2 assets may not account for more than 40% of a bank's HQLA stock, and level-2B assets may not account for more than 15% of this stock.

The denominator of the LCR is the total net cash outflows (outflows minus inflows, which are capped at 75% of total outflows) in the specified stress scenario for the subsequent 30 calendar days. Outflows (resp. inflows) are calculated by multiplying the outstanding balances of various categories or types of liabilities and off-balance sheet commitments (resp. contractual receivables) by the rates at which they are expected to run off or be drawn down (resp. flow in). For liabilities without fixed horizons, evidence from the 2007-2008 financial crisis has led the Basel Committee to choose much higher outflow parameters for wholesale deposits relative to retail deposits.

Besides liquidity ratios, Basel III strengthened the Basel II solvency requirements. It focused on the "quality of capital", namely "common equity tier 1" which was significantly raised while the rest of tier-1, as well as tier-2 capital, was left as in Basel II. This choice reflected the fact that in the crisis, after the Lehman bankruptcy and out of fears for financial instability, public authorities inflicted losses almost exclusively on common equity-holders. This being said, before Basel III was finalized, the Financial Stability Board decided, for the biggest banks worldwide (G-SIB's, or Global Systemically Important Banks), to add to Basel III solvency requirements, additional loss-absorbency in order to better protect taxpayers. In effect, this meant coming back to a kind of tier-2 capital, while being much more explicit about the 'bail-inability', i.e. loss-absorbency, of these claims: they contribute to TLAC, for Total Loss Absorbing Capacity (the EU

[2] See Basel Committee on Banking Supervision (2013).

has decided to extend the idea to all its banks and not only G-SIB's, under the acronym MREL, for Minimum Requirement on own funds and Eligible Liabilities).

1.3 Relationship to the literature

Our paper relates to a small and nascent literature on liquidity regulation. Diamond and Kashyap (2016), in the tradition of Allen-Gale (1997), Bhattacharya-Gale (1987), Ennis-Keister (2006) and Farhi et al (2009), allow banks facing the threat of a run to invest in liquid assets, and study whether privately optimal choices are socially optimal. Diamond and Kashyap make an interesting distinction between liquidity positions that cover only fundamental withdrawals (which correspond to the Net Stable Funding Ratio) and those that hold additional liquid assets beyond those needed for the fundamental withdrawals (this broader notion is interpreted as a kind of Liquidity Coverage Ratio): Liquidity that covers fundamental needs may or may not make the deposits "run free", which is what depositors demand. Their contribution and ours have different objectives. We are mostly concerned with fundamental liquidity crises (see however the discussion of the Bagehot doctrine in Section 3) and put less emphasis on the above-mentioned distinction. Also, Diamond and Kashyap emphasize ex-ante asymmetric information between the bank and depositors regarding the level of liquidity needed, while we assume regulators base their optimal regulation on an accurate observation of liquidity positions (otherwise we would need to introduce a menu of regulatory contracts). By contrast, their model has only one liquid asset and so the focus is on liquidity level rather than structure. Also the possible inefficiency in their model comes from runs, while in ours it is related both to involuntary liquidity support and to fire sales.

Our paper more generally connects to the debate about whether jumpstarting frozen markets is costly for the state. This debate was prominent in 2008, with economists on both sides of it. Some argued that buying undervalued assets does not create a put on taxpayer money (in accordance with the Bagehot doctrine), while others viewed such interventions as costly in expectation. In the latter camp, the theoretical contributions of Philippon-Skreta (2012) and Tirole (2012) argued that, when asset resales are voluntary and plagued with adverse selection, state interventions to jumpstart markets are costly for public finances as long as the state is not better informed than the market,[3] but nonetheless are worthwhile. Section 3.3 of this paper, which involves resale discounts but no adverse selection, argues that it is the uncertainty about the resale market's depth that drives the need for costly interventions.

Sections 2 and 3 stress the regulatory demand for the hoarding of liquid assets by banks. The emphasis here differs from most of the existing literature in two ways. First,

[3]In these papers, the government optimally purchases the worst assets and lets the cleaned-up market rebound. The intervention would be different if the government held privy information. Biglaiser et al (2018) study the used car market in which a knowledgeable car dealer (the counterpart of our government) is able to intermediate and certify the best cars, leaving the worst cars to the non-intermediated market.

in Section 2, liquidity requirements are motivated solely by an externality on public finances (as in Farhi-Tirole 2012) rather than by a fire-sales externality among banks (Shleifer-Vishny 1992, Allen-Gale 1994, Lorenzoni 2008, Stein 2012, Kurlat 2016; see also Perotti and Suarez 2011 as well as its discussion by von Thadden 2011, who make the assumption that higher aggregate short-term borrowing reduces the value of banks). Second, the existing literature has focused on liquidity levels, rather than on liquidity structure, a key concept for the design of regulation.[4]

In the process of studying liquidity regulation, we will encounter a number of perhaps-unexpected links with electricity regulation. Banking and electricity regulations indeed share a number of characteristics. First, and related to Section 2, uncertain demand (liquidity needs, electricity loads) raises the issue of optimal liquidity management or plant dispatch, with a resulting pecking order (in the power industry, the low-marginal-cost/high-investment-cost plants are dispatched first): see Boiteux (1949) and Leautier (2018) for treatments in the context of the electricity industry. Second, and related to Section 3, market participants may exercise negative externalities among each other. Market integrity is a public good; in power markets, insufficient investments in peakload capacity may lead to network collapses, hurting everyone (Joskow-Tirole 2007); similarly, insufficient liquidity provision may lead to fire sales, to the detriment of the entire banking sector. Third, and related to Section 4, electricity customers have different demands for the certainty with which they are served, giving rise to priority servicing (Wilson 1993), akin to the priority structure for bail-ins in Section 4. Finally, both exhibit ex-post interventions to protect politically sensitive clients (SMEs and retail depositors for banks, retail customers for electricity markets), although these interventions have different characteristics (mostly bailouts for banks and price ceilings in electricity markets).

The paper integrates the asset and liability sides into an overall design for liquidity surveillance. Brunnermeier and Pedersen (2009)'s work on market and funding liquidity analyzes how the funding of asset buyers impacts the volatility of asset prices, and how margins demanded on buyers' positions (which determine their funding liquidity) can vary suddenly and affect market liquidity. So like in our Section 3, resale prices are contingent of the financial muscle of buyers. Brunnermeier and Pedersen show that margins increase with market illiquidity when financiers cannot distinguish fundamental shocks from liquidity shocks and fundamentals have time-varying volatility, and that liquidity crises simultaneously affect many securities, mostly risky high-margin securities, resulting in commonality of liquidity and flight to quality.

[4]Note also that the policy debate on prudential reforms has led to various qualitative and quantitative assessments. Basel III liquidity ratios are hard to assess, since we are in the early days of their introduction. This being said, early assessments (see Basel Committee on Banking Supervision 2016) indicate very moderate effects, if any, of an LCR introduction on the level of private lending (the European Banking Authority conducted detailed Quantitative Impact Studies on the subject, see EBA 2013, 2014; it is also consistent with Banerjee-Mio 2015, which looks at the impact of an LCR-like ratio introduced earlier in the UK). Note that our fixed-investment model shuts down this liquidity-lending channel, while our appendix considers it by allowing bank investment to vary in size.

The focus of Brunnermeier and Pedersen and more generally of the vast literature on the limits to arbitrage however is rather different from ours. They offer a rich description of the dynamics of asset markets, which we do not, but their interest does not lie in the regulation of liquidity, and externalities are within-industry externalities rather than externalities on third parties (taxpayers, depositors). Furthermore, their notion of funding liquidity refers to the ability of buyers to raise funds rather than the seller's/bank's ability to bail in its existing claimholders.

Finally, our modeling of the liability side adds a new perspective to the vast corporate finance literature. The existing literature exhibits a dichotomy between "inside equity", held by active agents in need of incentives, and the set of claims held by passive investors and therefore satisfying no incentive purpose, and to which the Modigliani-Miller theorem applies.[5] The literature has often been criticized for focusing on inside equity and ignoring the role of outside equity and junior claims as a cushion protecting debtholders. By positing that different classes of investors have different risk preferences and introducing optimal bail-inability, this paper attempts to address this widespread criticism and to capture the notion of "cushion".

2 Exogenous resale discounts

2.1 Model

There are three dates, $t = 0, 1, 2$, and no discounting between the periods.

Banks. A representative banking entrepreneur at date 0 has a fixed-size investment opportunity, whose cost we normalize at 1. The entrepreneur has a large enough endowment at date 0 and can use it to finance the investment as well as liquid assets: There is no date-0 credit rationing.

At date 1, the entrepreneur has no endowment[6] and faces a liquidity need ρ, distributed at date 0 according to the density $f(\rho)$ and cumulative distribution $F(\rho)$ with full support $[0, \bar{\rho}]$. For simplicity, the shock ρ is the same for all banks, which rules out risk sharing among them.[7] To pursue the project at scale $j \in [0, 1]$, the entrepreneur must find cash ρj. Only the banking entrepreneur can pursue the project.[8]

[5]An exception is our 1994 paper, in which control rights are exerted by outside investors and so the financial structure serve to discipline the managers (with, optimally, a relative congruence between management and shareholders and a transfer of control to more conservative debtholders when the firm underperforms).

[6]The entrepreneur has no date-2 endowment either, or if he has one, he cannot pledge it.

[7]See Section 2.5 for an extension to imperfectly correlated shocks and cross-exposures.

[8]The assumption that the banker is indispensable implies that he does not lose his job in case of bailout or bail-in. Even in the absence of indispensability, managerial turnover costs would make firing the banker time-inconsistent given the absence of adverse selection in the model. Finally, banker removal would not prevent the bank from under-hoarding liquidity.

"Date 2" represents the future. In particular, meeting the date-1 liquidity shock through the resale of liquid assets, the bail-in of specific liabilities or public sector assistance can be interpreted as "restarting the bank in a healthy state".

To first focus on the asset side of liquidity hoarding, we assume "exogenous bail-inability". One possibility is that all income pledgeable to investors accrues to very risk-averse depositors, who cannot be bailed-in at all.[9] Another possibility is that no date-2 income can be pledged to investors. Equivalently, we could assume that continuation at date 1 creates pledgeable income ρ_0 and that date-0 investors are risk neutral, so they can be costlessly[10] bailed in at date 1; in this case the actual liquidity shock is $\rho - \rho_0$, since the issuing of new securities following on the bail-in yields ρ_0. So, with a renormalization, the analysis in Sections 2 and 3 is unaltered. We study bail-ins in Section 4 and will therefore assume by necessity that $\rho_0 > 0$ in that section.

At date 2, the banking entrepreneur receives benefit bj from the project. The project will be assumed worth undertaking (this will be the case if b is sufficiently large).

Liquidity. Besides the illiquid investment just described, the bank can hoard assets at date 0 from two classes:[11] $i = 1, 2$. One unit of asset i costs $p_i = 1/r_i$ at date 0 and delivers 1 if held to maturity, i.e. to date 2; so r_i is the rate of return obtained when holding the asset to maturity. If sold at date 1, asset i delivers only $\theta_i \leq 1$ to the bank. This captures the idea that assets yield a higher return to the bank if held to maturity. Without loss of generality, assume that $\theta_1 = 1$ (asset 1 will be called "level-1 liquidity") and $\theta_2 = \theta < 1$ (asset 2 is only partly liquid and is called "level-2 liquidity"). For instance, managing the asset between dates 1 and 2 generates a private benefit b_i, with $b_1 = 0$ and $b_2 = 1 - \theta_2$.[12] Let ℓ_1 and ℓ_2 denote the quantities of the two types of liquid assets acquired by the bank. These choices are observable by the state.

State. The state/regulator[13] puts no weight on the bankers' welfare.[14] It can provide open bank assistance at date 1. If then faces a shadow cost λ. One can think of λ as a political cost of bailout. Alternatively, λ may reflect a shadow cost of public funds.[15] That is, a \$1 emergency support requires using distortionary taxation and imposes cost

[9]Section 4 will provide a condition for this special case to obtain.

[10]In the sense that bail-ins involve no deadweight loss. Of course, they will be willing to pay less for the corresponding security if they are bailed in with a higher probability.

[11]See the Appendix for the generalization to n asset classes and to a continuous investment.

[12]Alternatively, level-2 assets could be subject to adverse selection and therefore a discount when resold at date 1. On this, see also Section 2.4.

[13]In this paper, we will view the state as a consolidated player. This simplifying assumption will resurface when we discuss liquidity assistance and bailouts. According to the official doctrine, bailouts come from the Treasury while the central bank supplies liquidity without incurring losses (the Bagehot doctrine). In practice, the distinction is of course murkier, especially as central banks have ventured into non-conventional policies and assumed potential important risks (perhaps for political economy reasons as central banks are independent: they can decide rapidly and their actions receive a bit less media attention).

[14]More generally, the state could value the bankers' welfare as long as it puts a lower weight on them than on the rest of society.

[15]For a discussion of this assumption, see chapter 5 in Holmström-Tirole (2011).

$\$(1+\lambda)$ on the rest of society. By contrast redeeming cash to consumers at date 1 has no specific value (is valued one-for-one).

The state has a strong stake in the bank's continuation. While it does not directly internalize the banker's welfare, a lower continuation scale translates into a credit crunch, lower activity and unemployment. The state values continuation scale j at βj.[16] Letting τ_0 denote the tax (subsidy if negative) levied on the bank at date 0, the state's date-1 objective function is therefore:

$$\tau_0 + \beta j - (1+\lambda)T,$$

where $T \geq 0$ is the support brought at date 1 to the bank. For expositional conciseness, we suppose that β is sufficiently large that a soft budget constraint operates over the entire range of shocks:

$$\beta \geq (1+\lambda)\bar{\rho}. \tag{1}$$

Condition (1) implies that we can restrict attention to allocations in which the bank always continues.

We assume that:

Assumption 1

(i) Liquid assets are costly: $1 > r_i$ *for all* i*.*

(ii) No asset is dominated: $r_2 > r_1 > \theta r_2$*.*

(iii) Bailouts are costlier than baseload liquidity hoarding: $1+\lambda > \frac{1}{\theta r_2} > \frac{1}{r_1}$*.*

The implication of the costly-assets Assumption 1 (i) is that a banker will not want to invest in liquid assets unless such assets are needed to face the liquidity need. Assumption 1 (ii) makes the choice of liquidity composition non-trivial. And Assumption 1 (iii) means that a perfectly anticipated shock requires liquidity coverage from a social perspective and that even level-2 liquid assets are competitive for securing such baseload liquidity (even though they are dominated by level-1 assets for this purpose).

Laissez-faire. By "laissez-faire", we mean a situation in which the state does not interact at date 0 with the bank, but may intervene to rescue it when push comes to shove at date 1. This situation is similar to that of investment banks and AIG prior to 2008 ("date 0") and in 2008 ("date 1"). Under *laissez-faire*, $\tau_0 = 0$. The state, which observes at date 1 the realization ρ of the liquidity shock, offers support $T \geq 0$ to the bank. Because it does not internalize the banker's welfare, the state first counts on internal liquidity and attempts at minimizing its support. In the absence of internal liquidity, the state fully covers the liquidity shock ($T = \rho$). The bank hoards no liquidity ($\ell_1 = \ell_2 = 0$)

[16] See e.g. Farhi-Tirole (2012) for micro-foundations of this reduced-form internalization.

and is bailed out with probability 1. The bank's utility (net of its initial endowment) is $U^{LF} = b - 1$, and expected welfare is $W^{LF} = \beta - (1 + \lambda)E[\rho]$.

Prudential regulation. At date 0, the state offers a regulatory contract $\{\tau_0, \ell_1, \ell_2, x_1(\cdot), x_2(\cdot)\}$ specifying a payment at date 0, amounts of level 1 and 2 liquid assets that the bank needs to acquire, and the fractions $x_1(\rho)$ and $x_2(\rho)$ in $[0, 1]$ of these assets that are liquidated at date 1 for shock ρ.

We can make alternative assumptions about the bank's date-0 reservation utility $\underline{U}$, that is its utility when rejecting the contract. If the regulator can withdraw the banking license (which requires being able to prevent migration to the shadow banking sector), then $\underline{U} = 0$. If the bank can keep performing banking activities in the shadow banking sector, then $\underline{U} = U^{LF} = b - 1$. A third possibility is that the state is mandated to ensure that the taxpayer breaks even, but cannot go beyond levying a fair insurance premium at date 0. These three alternatives yield the same policy with regards to liquidity hoarding and utilization and differ only with respect to the level of τ_0.[17] For concreteness, we assume throughout the paper that $\underline{U} = 0$.

2.2 Optimal liquidity regulation

At date 0, the bank hoards liquid assets ℓ_1 and ℓ_2. Let $x_i(\rho) \in [0, 1]$ denote the fraction of assets of type i that are resold at date 1 when the liquidity shock is ρ. Because there is no point redeeming cash to consumers at date 1, even if an asset exhibits no resale discount, one necessarily has:

$$\rho - \Sigma_i \ell_i \theta_i x_i(\rho) \geq 0. \tag{2}$$

Using the fact that the bank will continue in all states of nature, we maximize social welfare:

$$\max_{\{\tau_0, \ell_1, \ell_2, x_1(\cdot), x_2(\cdot)\}} \left\{ \tau_0 + \beta - \int_0^{\bar{\rho}} (1 + \lambda)[\rho - \Sigma_i \ell_i \theta_i x_i(\rho)] dF(\rho) \right\}$$

s.t. condition (2)[18] and the banker's participation constraint:

$$b + \int_0^{\bar{\rho}} \Sigma_i \ell_i [1 - x_i(\rho)] dF(\rho) \geq 1 + \Sigma_i p_i \ell_i + \tau_0.$$

Substituting the participation constraint into the objective function, the state solves:

$$\max_{\{\ell_1, \ell_2, x_1(\cdot), x_2(\cdot)\}} \left\{ \int_0^{\bar{\rho}} \Sigma_i \ell_i \left[1 - x_i(\rho) + x_i(\rho)(1 + \lambda)\theta_i\right] dF(\rho) - \Sigma_i p_i \ell_i \right\}$$

[17] See Section 4 for a caveat.

[18] Alternatively we could ignore condition (2) and rewrite the maximand as

$$\tau_0 + \beta - \int_0^{\bar{\rho}} \left[(1 + \lambda) \max\{0, \rho - \Sigma_i \ell_i \theta_i x_i(\rho)\} + \min\{0, \rho - \Sigma_i \ell_i \theta_i x_i(\rho)\}\right] dF(\rho).$$

subject to (2). Letting $\nu(\rho)dF(\rho)$ denote the shadow price of constraint (2), the resale decisions are governed by:

$$x_i(\rho) = 1 \quad \text{iff} \quad [1 + \lambda - \nu(\rho)]\theta_i - 1 \geq 0.$$

This defines a pecking order: $x_2(\rho) = 1 \Rightarrow x_1(\rho) = 1$. At date 1, the bank optimally depletes first its most liquid asset ($\theta_1 = 1$) and then only its less liquid one ($\theta_2 = \theta < 1$). Let ρ^* denote the maximal shock that the bank can withstand without any support from the state:

$$\rho^* \equiv \ell_1 + \ell_2\theta. \tag{3}$$

Let $\widehat{\rho}$ denote the shock at which the less liquid assets start being put on the market:

$$\widehat{\rho} \equiv \ell_1. \tag{4}$$

To obtain the optimal levels of liquidity, we rewrite the program in terms of choice variables ρ^* and $\widehat{\rho}$ (ℓ_1 and ℓ_2 are then given by conditions (3) and (4)). The new optimization program is

$$\begin{aligned}\max_{\{\rho^* \geq \widehat{\rho}\}} \Bigg\{ & -\frac{\widehat{\rho}}{r_1} - \frac{\rho^* - \widehat{\rho}}{\theta r_2} + \int_0^{\widehat{\rho}} \left[\widehat{\rho} - \rho + \frac{\rho^* - \widehat{\rho}}{\theta}\right] dF(\rho) + \int_{\widehat{\rho}}^{\rho^*} \left[\frac{\rho^* - \rho}{\theta}\right] dF(\rho) \\ & - \int_{\rho^*}^{\bar{\rho}} (1+\lambda)(\rho - \rho^*) dF(\rho) \Bigg\}. \end{aligned} \tag{I}$$

The first two, negative terms represent the date-0 cost of buying liquid assets in quantities ℓ_1 and ℓ_2 while the last term is the shadow cost of expected bailouts that occur when the liquidity shock more than exhausts all liquid assets. The first two integrals measure the value of unused liquid assets when meeting the shock requires only level-1 assets, enabling less liquid ones to be enjoyed at their full, undiscounted value (first integral), and when it requires also the liquidation of less liquid ones (second integral).

Proposition 1 *(Optimal regulation of liquidity with fixed returns)*

(i) The optimal regulatory contract always induces hoarding of level-1 liquidity ($\ell_1^* > 0$). Hoarding of level-2 liquidity ($\ell_2^* > 0$) occurs if and only if θ exceeds some threshold θ^*. Level-1 liquidity then satisfies:

$$\frac{1-\theta}{\theta} F(\ell_1^*) = \frac{1}{\theta r_2} - \frac{1}{r_1},$$

while level-2 liquidity is given by:

$$F(\ell_1^* + \ell_2^*\theta) + [1 - F(\ell_1^* + \ell_2^*\theta)](1+\lambda)\theta = \frac{1}{r_2}.$$

If $\theta < \theta^*$, only level-1 liquidity is hoarded, and is given by:

$$F(\ell_1^*) + [1 - F(\ell_1^*)](1+\lambda) = \frac{1}{r_1}.$$

(ii) The optimal regulation of liquidity obeys a peak-load principle. Level-1 liquidity serves as baseload liquidity and is sold first. For larger shocks, less liquid assets, the peak liquidity, are liquidated. Very large shocks are always met through public liquidity ($\rho^* < \bar{\rho}$). The thresholds are given by {(3), (4), (5), (6)} below.

(iii) The regulator need not intrude into the bank's choice of composition of liquid assets. Indeed an optimal policy consists in setting a liquidity-weighted asset (LWA) required level:

$$\ell_1 + \ell_2\theta \geq \rho^*,$$

Proof: (i) and (ii) It is straightforward to derive the optimal policy. Suppose that it is optimal to hoard some level-2 assets. At the optimum the social planner must be indifferent to the substitution of 1 of a unit of level-1 liquidity for $1/\theta$ units of level-2 liquidity (both deliver \$1 at date 1 if sold). Because these marginal units are held to maturity with probability $F(\widehat{\rho})$,

$$-\frac{1}{r_1} + F(\widehat{\rho}) = -\frac{1}{\theta r_2} + \frac{1}{\theta}F(\widehat{\rho}),$$

or

$$\left(\frac{1-\theta}{\theta}\right) F(\widehat{\rho}) = \frac{1}{\theta r_2} - \frac{1}{r_1}. \tag{5}$$

As expected, the more illiquid asset 2 is (θ decreases), the more the bank resorts to asset 1 ($\widehat{\rho}$ increases).

The maximization with respect to ρ^* requires that the marginal cost of securing one extra unit of liquidity through peak liquidity, $1/\theta r_2$, equals the marginal benefit, which comes from economizing the shadow cost of public funds when $\rho > \rho^*$ and from earning the full value of the unused liquidity when $\rho \leq \rho^*$:

$$[F(\rho^*) + [1 - F(\rho^*)](1+\lambda)\theta]\, r_2 = 1. \tag{6}$$

Condition (6) defines a unique solution $0 < \rho^* < \bar{\rho}$ provided that $1 + \lambda > 1/(r_2\theta)$, as we assumed. Intuitively, if liquid assets covered the highest possible liquidity shock, the last unit would be used with almost nil probability. Because hoarding it for long-term purposes has negative NPV, it is socially optimal to let the extreme shocks be covered by the state.

Let (ℓ_1^*, ℓ_2^*) denote the solution to {(5), (6)} once the values of $(\widehat{\rho}, \rho^*)$ are substituted for using {(3), (4)}.

A necessary and sufficient condition for a positive investment in asset 2 is that $\rho^* > \widehat{\rho}$, or $\theta \geq \theta^* \in ((1-r_2)r_1/(1-r_1)r_2, r_1/r_2)$ where[19]

$$1+\lambda = \frac{\frac{1}{r_1} - \frac{1}{r_2}}{\theta^*\left(\frac{1}{r_1}-1\right) - \left(\frac{1}{r_2}-1\right)}. \tag{7}$$

(iii) If the regulator imposes a liquidity-weighted-asset floor for the bank ($\ell_1 + \ell_2\theta \geq \rho^*$), then the bank selects the socially optimal portfolio of liquid assets because the bailout cost,

$$\int_{\rho^*}^{\bar{\rho}} (1+\lambda)(\rho-\rho^*)dF(\rho)$$

depends only on ρ^* and not how shocks below ρ^* are financed by the bank. There is no need to monitor the composition of the liquidity buffer.

To check this more formally, the reader will solve the representative bank's problem:

$$\max_{\{\ell_1,\ell_2\}} \left\{ -\frac{\ell_1}{r_1} - \frac{\ell_2}{r_2} + \int_0^{\ell_1} [(\ell_1-\rho)+\ell_2]\, dF(\rho) + \int_{\ell_1}^{\rho^*} \left[\ell_2 - \frac{\rho-\ell_1}{\theta}\right] dF(\rho) \right\}$$

s.t.

$$\ell_1 + \ell_2\theta \geq \rho^* \tag{II}$$

and check that $\ell_1 = \ell_1^*$ and $\ell_2 = \ell_2^*$.

■

Comparative statics. Consider the impact of date-0 news about likely shocks. The distribution of shocks is $F(\rho;\gamma)$, where γ shifts the distribution in the first-order stochastic dominance sense ($F_\gamma < 0$). Let $\widehat{\rho}(\gamma)$ and $\rho^*(\gamma)$ denote the desired cutoffs. In the case of a uniform shift ($F(\rho-\gamma)$), conditions (5) implies that $\widehat{\rho}(\gamma) = \widehat{\rho}(0) + \gamma$: An increase in the expected liquidity shock implies an increase in level-1 liquidity of the same magnitude.

[19] An alternative way of obtaining (7) goes as follows: suppose that the bank hoards only the level-1 liquid asset. The optimal program is then:

$$\max_{\{\rho^*\}} \left\{ -\frac{\rho^*}{r_1} + \int_0^{\rho^*} (\rho^*-\rho)dF(\rho) - \int_{\rho^*}^{\bar{\rho}} (1+\lambda)(\rho-\rho^*)dF(\rho) \right\}.$$

This yields

$$-\frac{1}{r_1} + F(\rho^*) + [1-F(\rho^*)](1+\lambda) = 0.$$

Intuitively, buying 1 unit of asset 1 will yield 1 at date 1. The value of this extra unit of liquidity is 1 when $\rho \leq \rho^*$ and $1+\lambda$ when $\rho > \rho^*$. Would one want to buy, on top of this amount of asset 1, $1/\theta$ units of asset 2, yielding 1 more in case it is resold at date 1? The social value of this small amount of asset 2 is positive if and only if:

$$-\frac{1}{\theta r_2} + F(\rho^*)\frac{1}{\theta} + [1-F(\rho^*)](1+\lambda) \geq 0.$$

Using the first condition, it is optimal to use asset 2 iff (7) is satisfied.

Condition (6) shows that $\rho^*(\gamma) = \rho^*(0) + \gamma$ as well. This implies that level-2 liquidity, $\ell_2\theta = \rho^*(\gamma) - \widehat{\rho}(\gamma)$ is invariant. More generally, one has

Corollary 1

(i) Consider a first-order-stochastic-dominance shift in the distribution of liquidity shocks $F(\rho;\gamma)$ with $F_\gamma < 0$. Then level-1 and overall liquidity, ℓ_1^* and ρ^*, increase, while the impact on level-2 liquidity ℓ_2^* is in general ambiguous.

(ii) A uniform upward shift γ in the distribution of shocks, $F(\rho-\gamma)$, requires an increase in level-1 liquidity of the same magnitude, and no change in level-2 liquidity.

Remark: Monitoring of reinvestment and implementation. We have assumed that the state can contract on the use of hoarded liquidity to reinvest. If it cannot do so, then the bank will reinvest spontaneously when $\rho \leq \rho^*$ provided that

$$b \geq \ell_1^* + \ell_2^*$$

(with $\widehat{\rho} = \rho^*$ when $\theta \leq \theta^*$, as we have seen).

For $\rho > \rho^*$, the state must offer assistance for the difference between ρ and $\ell_1 + \theta\ell_2$.[20]

Arbitrary number of liquid assets. As mentioned earlier, the Appendix studies the n-asset case (as well as a continuous investment). The analysis confirms the existence of a pecking order: Assets with the highest liquidity θ_i are disposed of in priority.

State-contingent liquidations. In the analysis, assets are either fully liquid in the sense that they will always be resold before any public money is spent, or fully illiquid and therefore are never resold if they are held for whatever reason. Were the cost of bailouts λ random at date 0 and realized at date 1, this dichotomy would still hold contingent on the realization of λ, but not overall: some highly illiquid assets would be resold only for high realizations of λ.

2.3 Impact of the supply of safe assets on prudential supervision

We have so far assumed that level-1 liquid assets are available in unlimited quantities, so their return does not depend on the regulatory demand. However, there may be a scarcity of such assets either at the world level (especially if regulatory rules for banks, insurance companies and pension funds encourage their use by regulated institutions), or at the country level if there is a home bias in the banks' portfolio.

[20] This remark in a sense assumes weaker power for the state, which can no longer seize the liquid assets to force the bank to reinvest. The implicit timing is: At date 0, the regulator offers $\{\tau_0, \ell_1, \ell_2\}$ under the threat of revoking the banking license; at date 1, the regulator offers $\{\tau_1, x_1, x_2\}$ where $\tau_1 \leq 0$ is state assistance; the regulator can still withdraw the banking license if the offer is not accepted, but not seize ℓ_1 and ℓ_2. Weak enforcement might be worth studying in the future.

Level-1 liquidity is mostly covered by Sovereign bonds or similar securities. The design of the Liquidity Coverage Ratio by the Basel Committee raised the issue of whether a one-size-fits-all approach makes sense in a world in which Australia's debt-to-GDP ratio is 20% and Japan's 250%.[21] Does our model shed light on this debate? Of course this debate is meaningful only if cross-country diversification is not perfect (Australian banks do not hoard substantial amounts of Japanese bonds and hedge against the Yen/A$ exchange rate fluctuations), and so domestic liquidity matters. In practice banks hoard primarily domestic bonds; we will take this fact for granted and will not investigate its causes.

Suppose that there is a fixed supply L_1 of level-1 assets in the economy, and that the price $p_1 = 1/r_1$ of these assets is an increasing function of banks' demand for this asset:[22] $p_1 = P_1(L_1 - \ell_1) = P_1(L_1 - \widehat{\rho})$ with $P_1' < 0$ and $P_1(+\infty) = 1$ where, recall, $\widehat{\rho} = \ell_1$ denotes the maximum shock that is covered solely by level-1 liquidity. One may have in mind that (a) L_1 is driven by the supply of public goods and the public deficit and (b) banks compete with other parties for safe assets.[23] We further assume that $P_1'' \geq 0$.

Program (I) is unchanged, except that the return on level-1 assets depends on prudential requirements, and so the first term in the maximand writes: $-\widehat{\rho}/r_1 = -P_1(L_1 - \widehat{\rho})\widehat{\rho}$. Monotone comparative statics imply that $\widehat{\rho}$ is an increasing function of L_1. By contrast, ρ^*, given by (6), is invariant to the supply of level-1 assets, unless this supply is so abundant and thus the return r_1 so high that level-2 assets are no longer used.[24]

Proposition 2 *(supply of safe, liquid assets)*

(i) Provided that the supply of level-1 liquid assets is not so abundant that level-2 liquid assets are no longer used for prudential purposes, the amount of level-1 assets at the optimal prudential regulation grows with the supply of the assets. Total liquidity is invariant to the supply of level-1 assets, which means that level-1 and level-2 assets substitute one-for-one for each other as level-1 liquidity adjusts to the supply.

(ii) A country with abundant net supply uses only level-1 liquid assets, and total liquidity expands with the supply as liquidity is very cheap.

[21] Specifically, banks from countries with low sovereign debt levels are allowed to fill part of their level-1 buffer with 'Committed Liquidity Facilities' (CLF) obtained from their Central Bank at 'market rates'.

[22] We assume that higher level-1 liquidity requirements reduce the net supply of such assets and thus raise their price. When the Sovereign's solvency is a concern, a doom-loop mechanism could also be at play, as in e.g. Farhi-Tirole (2018).

[23] For simplicity, we will not incorporate the utility of these third parties into social welfare. Note also that we assume that L_1 is a legacy stock of liquid assets. It would be straightforward to extend the model to allow for date-0 issuance of public debt. The analysis would then incorporate the public sector's endogenous cost of issuing debt.

[24] For L_1 sufficiently large, r_1, which goes to 1 as the supply of level-1 liquidity goes to infinity, will be such that the RHS of condition (7) (which also converges to 1) is smaller than the left-hand side for all $\theta \leq 1$. And so the level-2 asset is no longer used. The asymptotic value of ρ^* when L_1 goes to infinity is $\bar{\rho}$. Conversely for L_1 small, level-1 assets are expensive and are used only if $P_1(L_1) < 1/\theta r_2$; if $P_1(L_1) \geq 1/\theta r_2$, the bank hoards no level-1 assets when their supply is small, as depicted in Figure 1. Finally, we could also study the situation in which the supply of level-2 liquid assets is itself limited.

Figure 1 depicts the optimal policy. A few comments are in order.

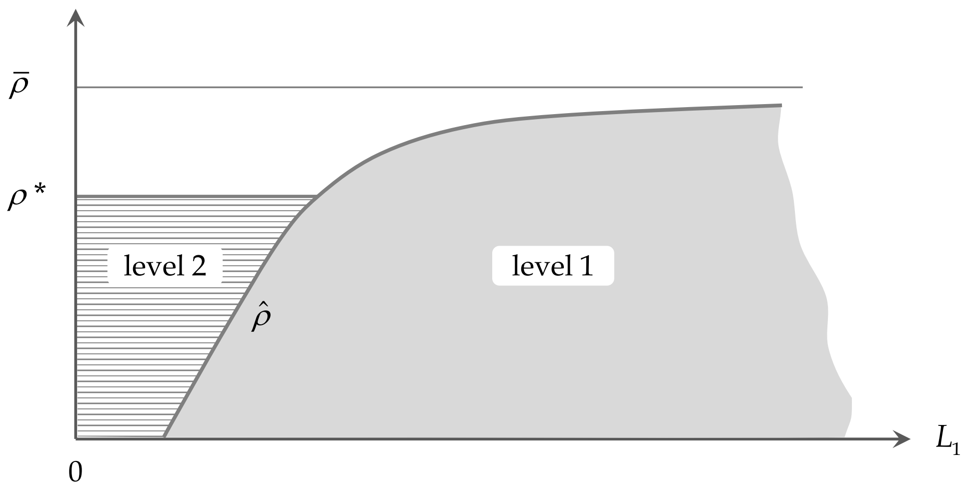

Figure 1: Adaptation to the supply of safe assets.

Delegation. The liquidity composition can again be left to banks; put differently, only total liquidity needs to be regulated in the sense of an LWA requirement.

International diversification. Suppose that banks invest in international liquidity rather than just domestic liquidity. What are the implications? Total liquidity is now $L_1 = \Sigma_k L_1^k$, where L_1^k is country k's level-1 liquidity. Low-supply/low-interest-rate countries benefit from the relative expansion of level-1 liquidity and hoard less level-2 liquidity than under autarky; the reverse holds for high-supply/high-interest-rate countries. Again, an analogy with electricity markets can be illuminating. Consumers and industrial users located in cheap electricity regions usually oppose the construction of new transmission lines connecting regions or countries with more expensive electricity, as they know that the price of electricity will rise in their own region or country. Here, banks in high level-1 supply countries will be hurt if banks in other countries start diversifying.

Safe sovereign debt? Finally, we have assumed that Sovereign bonds remain safe as their supply expands. The possibility that this is not the case has received substantial attention in the international finance literature, and there is no point rehashing the latter's insights. Let us just qualify Proposition 2 by recalling that it holds only as long as an increase in the supply does not substantially affect the probability of Sovereign default.

2.4 Transforming illiquid assets into liquid ones through securitization

Banks routinely securitize or sell legacy loans to meet liquidity needs. For example, Irani et al (2017) find that banks that are less capitalized or face higher unexpected capital requirements stemming from Basel III reduce loan retention, especially loans with higher capital requirements, at times when capital is scarce.

We now make a distinction between liquid assets, which are assets that the bank would not hoard (as is the case in our model with perfectly foreseeable bailouts) or would under-hoard (as would be the case more generally), and illiquid assets that the bank holds voluntarily as part of its normal activity and which can be transformed into liquid assets, possibly with central bank support.

Suppose that the representative bank holds at date 0 an amount ℓ_0 of legacy assets. Only the bank knows the (date-2) return of its legacy asset. For the regulator and the market,

$$r_0 = \begin{cases} 1 & \text{with probability } \theta_0 \\ 0 & \text{with probability } 1 - \theta_0. \end{cases}$$

The notation θ_0 is not fortuitous since, as we will see, θ_0 will also index the liquidity of a high-quality legacy asset. We assume that

$$\frac{1}{1+\lambda} < \theta_0 < \theta_2.$$

The first inequality will make it optimal to securitize the legacy asset when the bank has disposed of all its liquid assets. The second inequality states that the legacy asset is less liquid than the liquid assets.

Definition. A legacy asset is said to be "less prone to adverse selection" when θ_0 increases.

Intuitively, one would feel that adverse selection is low when θ_0 is close to 0 or to 1, so there is little uncertainty about the asset's true value. However when θ_0 is close to 0, the value of the asset for liquidity provision is low. The above definition will turn out to be appropriate. Actually, it is easy to show that with multiple legacy assets the pecking order requires securitizing first the ones with the lowest levels of adverse selection according to the definition.

Ignoring the value $\ell_0 r_0$ of the legacy asset, each bank's reservation utility is $\underline{U}$, where the exact value of $\underline{U}$ depends, as we have seen, on whether the bank loses its banking license or operates as a shadow bank if it rejects the regulatory contract. So if $U(r_0)$ denotes the utility of a bank with legacy asset r_0 and $1 - x_0(\rho)$ denotes the skin in the game kept by the high-quality-legacy-asset banks, the participation and incentive constraints[25] are:

$$U(1) \geq \ell_0 + \underline{U}$$

and

$$U(0) \geq U(1) - \left[\int_0^{\bar{\rho}} [1 - x_0(\rho)]dF(\rho)\right] \ell_0.$$

The following proposition is demonstrated in the Appendix.

[25]The incentive constraint reflects the fact that the low-legacy-asset-value type can mimick the high-value type and that the two types receive a different utility only when the high type keeps skin in the game (the average skin in the game is $E[1 - x_0(\rho)]$).

Proposition 3 *(securitization)*

(i) When the bank owns legacy assets, on whose value it has private information, such assets can serve as supplements to the liquid assets for the purpose of liquidity coverage.

(ii) The purchase of liquid assets decreases with the level of legacy assets: Provided that $(1+\lambda)\theta_0 > 1$*, total weighted liquidity* ρ^* *decreases with* ℓ_0*.*

(iii) With multiple legacy assets, the pecking order requires securitizing the ones less prone to adverse selection first.

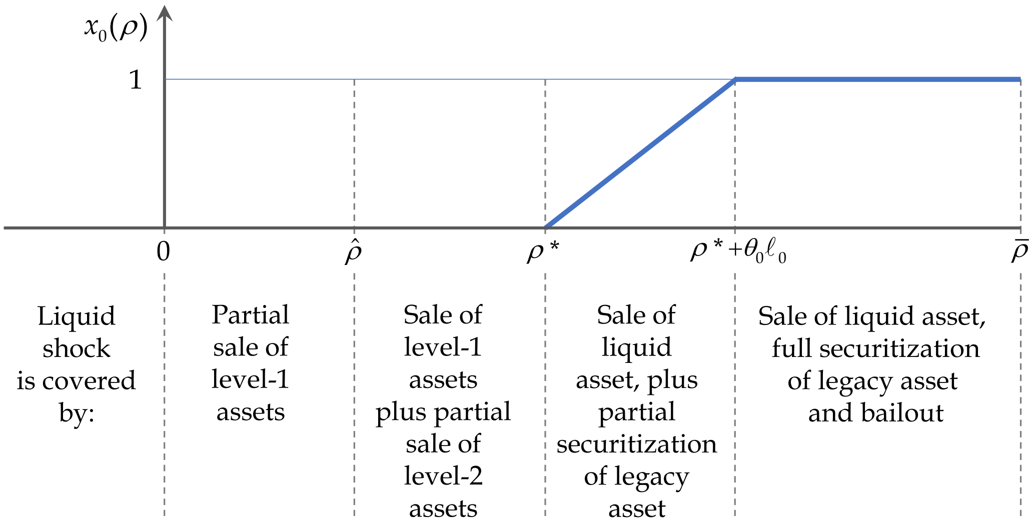

Figure 2: Pecking order with legacy assets

When $\rho > \rho^*$, so that liquid assets do not suffice to cover the shock, the regulator can first force the bank to go to the private market place and obtain haircut $1-\theta_0$ (for a high-quality legacy asset). However, if $\rho > \rho^* + \theta_0\ell_0$, the only option for the state is to offer liquidity assistance by taking the legacy asset at a non-market rate.

2.5 Treatment of liquidity pooling

Banks share liquidity in multiple ways: derivatives, credit default swaps, interbank loans,[26] etc. Should this shared liquidity be counted as meeting the liquidity coverage requirement?

[26] Donaldson and Piacentino (2018) argue that the desire to share liquidity may explain why banks maintain off-setting long term debts without netting them. Their idea is that the absence of netting may be a mutual insurance device. If one bank is short of cash to meet a liquidity shock, it can sell its claim on the other bank and simultaneously issue senior debt, which de facto dilutes the other bank in case of default (which is not the case for the cash-rich bank).

An imperfect correlation of liquidity shocks across banks creates scope for liquidity sharing. In this section, we take the best case for such pooling; we assume that banking cross-exposures are legitimate in that they are used for hedging rather than gambling. Needless to say, the case for a favorable regulatory treatment of interbank exposures is much weaker when the supervisor is unable to assess the directionality of risk taking in interbank arrangements.

To study liquidity pooling while allowing for macroeconomic shocks, we generalize the model in the following way. There is an even number of ex-ante identical banks. Each bank's liquidity shock is drawn from the same distribution $F(\rho)$ on $[0, \bar{\rho}]$ as earlier. For expositional simplicity, we assume that the distribution $F(\rho)$ is symmetrical around $\bar{\rho}/2$.

What differs is that the banks are no longer perfectly correlated. With probability x, banks face a common shock ρ (so $x = 1$ in the previous treatment); with probability $1-x$, even-numbered banks face liquidity shock ρ while odd-numbered face liquidity shock $\bar{\rho} - \rho$. The symmetry assumption implies that the marginal distribution is unchanged:

$$\frac{1}{2}F(\rho) + \frac{1}{2}[1 - F(\bar{\rho} - \rho)] = F(\rho).$$

In the absence of liquidity pooling, the optimal regulation specifies the same pair $\{\ell_1^*, \ell_2^*\}$ as in Proposition 1. Let $\{\ell_1^{**}, \ell_2^{**}\}$ denote the optimal levels when even-numbered banks pool their liquidity with odd-numbered banks, so liquidity pooling is indeed used for hedging purposes. The following proposition is proved in the Appendix.

Proposition 4 *(prudential treatment of interbank exposures)*

Provided that liquidity pooling is used to provide hedges,

(i) Liquidity requirements should be relaxed: $\ell_1^{**} + \theta\ell_2^{**} < \ell_1^* + \theta\ell_2^*$. *The lower the correlation, the lower the liquidity requirement.*

(ii) The liquidity requirement can be decentralized through an LWA requirement.

Intuitively, cross-exposures make it more likely that baseload liquidity will be employed, raising their attractiveness. Peak liquidity by contrast is less useful, as some of the high shocks are covered through the mutual insurance arrangement.

2.6 Adding capital requirements

The model focuses on a pure illiquidity risk. As in Wooford (1990), the banker has sufficient wealth and faces no credit rationing at the initial investment stage ("date 0"). Alternatively, we could have followed Holmström-Tirole (1998) and Farhi-Tirole (2102) and assumed that the banker is credit constrained at date 0. In that case, laissez-faire would result in too high a scale of operations (too much leverage), assuming the latter is variable, and too low a level of liquidity (indeed none under our assumption of widespread bailouts), and the optimal contract between the state and the banker would reflect a trade-off between scale and liquidity.

The key results in this slightly more complex model are unchanged. The added complexity would deliver a few additional insights. For example, a higher liquidity buffer would typically reduce, as it does in reality, the cost of funding (e.g. of bail-inable securities). It would also better integrate liquidity requirements and capital requirements. As we noted in the literature review, a strength of our approach is that it departs from the traditional inside-equity/managerial-incentives approach and incorporates the notion of outside equity (broadly construed as bail-inable securities) as a cushion.

3 Shallow markets and macroprudential aspects

Let us now assume that the date-1 market for asset 2 has limited absorption capacity. It may be that the marginal buyer has a decreasing valuation for the asset (Shleifer-Vishny 1992, Lorenzoni 2008, Stein 2012), that potential buyers have limited cash (Allen-Gale 1994), or that buyers with lower ability to distinguish assets and therefore more concerned about adversely-selected purchases become the marginal buyers (Kurlat 2016).[27]

To provide micro-foundations for the depth of markets, suppose that asset 2 can be managed by alternative buyers. Each buyer can manage one asset. A mass α of potential buyers has wealth a_H each at date 1, while there is an unbounded number of buyers who have no wealth: $a_L = 0$. To guarantee the existence of haircuts (illiquidity), we assume that $a_H < b_2$, where b_2, recall, is the haircut when selling a level-2 asset to a cashless buyer. A bidder with wealth a can bid up to $a + (1 - b_2)$; that is, he can bring equity a and lever up by pledging the liquid asset's pledgeable income $1 - b_2$. Let $x_2(\rho)$ denote the fraction of type-2 assets that are put up for sale in state of nature ρ. Because the pledgeable income can always serve to finance the purchase (by leveraging up) but does not exhaust total value, then the resale price depends on whether the marginal buyer has low or high net wealth, and is equal to $\theta_k \equiv 1-(b_2-a_k)$, where $k = H$ if $\ell_2 x_2(\rho) \leq \alpha$ and $k = L$ if $\ell_2 x_2(\rho) > \alpha$. So, regardless of how much is offered for sale, there is a discount as potential buyers do not have enough wealth to pay for the non-pledgeable income; but this discount can be high or low depending on how much is put up for sale. And while the asset sells at a discount, it reaches its highest value (strictly or weakly) in private hands. The model of section 2 is a special case of this model, with α sufficiently large ($\alpha \geq \ell_2^*$) and $\theta = \theta_H$.

The buyers' profit is higher in the high-discount situation than in the low-discount one. This raises the issue of how the buyers' welfare is weighted in the social welfare

[27]What matters for what follows is a) the existence of a resale discount, and b) the dependence of this discount on the volume of securitized assets.

function. For notational simplicity, we assume that the social planner puts no weight on buyer welfare.[28]

The Bagehot doctrine. Note also that in the absence of public intervention there can be multiple equilibria for a given amount of liquidity (ℓ_1, ℓ_2). This occurs whenever the banks are keen on continuing even if that means selling at a big discount, but the shock is not so high as to preclude a low-discount equilibrium: $\ell_2 \geq (\rho - \ell_1)/\theta_L$ (banks can continue if they sell all liquid assets, even at a steep discount on type-2 assets) and the marginal buyer is an a_H (resp. a_L)–buyer when the price is θ_H (resp. θ_L)

$$\alpha\theta_L \leq \rho - \ell_1 \leq \alpha\theta_H. \tag{8}$$

Note that in this case, the state can enforce the "good equilibrium" (the low-discount one), by setting a floor, i.e. by standing ready to purchase assets at price (slightly smaller than) θ_H. In equilibrium the state purchases no assets, but it prevents fire sales through the backstop. This can be viewed as an illustration of the Bagehot doctrine, that directs central banks to bring liquidity without incurring losses.

Proposition 5 *(shallow markets and the Bagehot doctrine)*

Under condition (8), multiple equilibria with different degrees of liquidity coexist, that are Pareto comparable (from the point of view of banks and the State). The State can enforce the preferred equilibrium by standing ready to purchase liquid assets at a price floor.

In the following, we will assume that the low-discount equilibrium prevails when (8) is satisfied; so coordination failures are not the rationale for regulation in the remainder of this section.

3.1 No-bailout benchmark

As a benchmark for our main result for this section, let us first verify that the classic result obtained in the literature carries over to this setting, namely that illiquidity carries with it a negative externality when resale markets are not deep: When a bank puts up for sale a higher quantity of assets, the concomitant price reduction reduces the liquidity available to other sellers of that asset; consequently, regulations that force banks to hoard more level-1 liquidity at date 0 or prevent them from reselling too much of their assets at date 1 improve welfare.

[28]This assumption is much stronger than needed: Even if buyers received the same weight as consumers, their obtaining a higher discount forces banks to hoard more of the costly liquidity, altering the net transfer to the state and generating a social loss.

Suppose that, as is assumed in the literature, there is no bailout (which would be the case in our model if β were small enough). Assume further that bankers are keen enough on continuing so that they choose to hoard enough liquidity to withstand all shocks.[29]

When are there fire-sale externalities? The quantity of asset 1 is implicitly given by condition (5) for $\theta = \theta_H$. Call it $\ell_1^* \equiv \widehat{\rho}$. For a shock slightly greater than ℓ_1^*, very little of the type-2 asset is resold, and so the discount is small (equal to $1 - \theta_H$). Suppose more generally that asset 2 is always resold at the high price, that is, at θ_H. Then ℓ_2 is given by:

$$\ell_1^* + \ell_2\theta_H = \bar{\rho} \tag{9}$$

as bankers are keen on continuing. If the resulting amount of level-2 assets satisfies $\ell_2 > \alpha$, i.e. if $\ell_1^* + \alpha\theta_H < \bar{\rho}$, then necessarily there are fire sales at shocks close to the highest shock. Consider the shock $\rho^\dagger$ such that

$$\ell_1^* + \alpha\theta_H = \rho^\dagger \tag{10}$$

then at $\rho = \rho^\dagger + \varepsilon$, the equilibrium in the resale market necessarily involves a steep discount. Suppose that all banks increase their level of type-1 liquidity a bit above ℓ_1^* and adjust their type-2 liquidity downward so as to keep condition (9) satisfied. Fixing the state-contingent discount, banks incur only a second-order loss from the envelope theorem. But the small-discount region expands, and so the banks' welfare increases. More formally, consider liquidity vectors $(\ell_1, L_2(\ell_1))$ that allow the bank to cover all shocks:

$$\ell_1 + \theta_L L_2(\ell_1) \equiv \bar{\rho}.$$

[29]This is the case if b is sufficiently large. More precisely, for this to be the case, suppose that the bank accumulates just enough liquidity to withstand shocks up to $\rho^* < \bar{\rho}$. Let the banker hoard a bit more of asset 2. At worst, asset 2's resale price is θ_L. So to have 1 more unit of liquidity it suffices to buy an extra $1/\theta_L$ units of asset 2 at date 0. If

$$-\frac{1}{\theta_L r_2} + F(\rho^*)\frac{1}{\theta_L} + bf(\rho^*) > 0;$$

then at the margin holding more liquidity is desirable. A sufficient condition for this condition to obtain everywhere and therefore for continuation to be deterministic is

$$b \min_{\{\rho\}}\{f(\rho)\} \geq \frac{1}{\theta_L r_2}.$$

For such vectors, the bank's utility under laissez-faire is

$$\begin{aligned} U(\ell_1|\ell_1^a) &\equiv b - \frac{\ell_1}{r_1} - \frac{L_2(\ell_1)}{r_2} + \int_0^{\ell_1} [\ell_1 + L_2(\ell_1) - \rho]\, dF(\rho) \\ &+ \int_{\ell_1}^{\rho^\dagger(\ell_1^a)} \left[L_2(\ell_1) - \frac{\rho - \ell_1}{\theta_H} \right] dF(\rho) \\ &+ \int_{\rho^\dagger(\ell_1^a)}^{\bar{\rho}} \left[L_2(\ell_1) - \frac{\rho - \ell_1}{\theta_L} \right] dF(\rho), \end{aligned}$$

where $\rho^\dagger(\ell_1^a)$ is the market breakdown cutoff given the choice ℓ_1^a of the representative bank:

$$\rho^\dagger(\ell_1^a) \equiv \ell_1^a + \alpha\theta_H.$$

We know that

$$U(\ell_1^a|\ell_1^a) = \max_{\{\ell_1\}} U(\ell_1|\ell_1^a).$$

Using the envelope theorem, we have

$$\left. \frac{dU(\ell_1|\ell_1)}{d\ell_1} \right|_{\ell_1 = \ell_1^*} = f(\rho^\dagger(\ell_1^*)) \left[\rho^\dagger(\ell_1^*) - \ell_1^* \right] \left[\frac{1}{\theta_L} - \frac{1}{\theta_H} \right] > 0.$$

Proposition 6 *(absence of bailout)*

In the absence of bailout (β is small), there can be fire sales (this is in particular the case if b is large and $\ell_1^* + \alpha\theta_H < \bar{\rho}$). The regulator must monitor the composition of the banks' liquidity: On top of setting a global LWA requirement, the regulator must impose a minimum level of level-1 liquidity.

This analysis confirms the conventional wisdom that in a no-bailout economy, there are inefficient fire sales in the absence of regulation. *The new feature relative to the literature is not that there is under-hoarding of liquidity, but rather that the banks choose the wrong kind of liquidity:* Banks hoard too much level-2 and too little level-1 liquidity.

Let us note the different nature of the government's intervention relative to the regulation of liquidity in Section 2. In Section 2, the banks exerted a negative externality on public finances, and regulation (together with a transfer) achieved a Pareto improvement among banks and the state. Here there is no such externality as there is no bailout. Rather, the externality is among banks; their welfare is reduced by an inappropriate composition of liquidity. The banks can indeed achieve the outcome described in Proposition 4 by themselves through self-regulation; alternatively, the regulator can propose to regulate the banks provided that they all play the game and do not try to free ride on the others' provision of the public good (the absence of fire sales).

3.2 Economy with bailouts

Assume now that condition (1) is satisfied. And let (ℓ_1^*, ℓ_2^*) denote the optimal liquidity management (satisfying conditions (3) through (6)) when $\theta = \theta_H$. We maintain Assumption 1 and so $1 + \lambda > 1/\theta_H r_2 > 1/r_1$. The following proposition is demonstrated in the Appendix.

Proposition 7 *(bailouts)*

Assume that there are systematic bailouts (condition (1) is satisfied). Then,

(i) If the resale market for level-2 liquidity is deep in the sense that $\ell_1^* + \alpha\theta_H \geq \rho^*$, there are no fire-sales externalities; and the decision concerning the composition of the liquidity portfolio can be delegated to the bank through a LWA requirement: No minimum level-1-liquidity requirement needs to be imposed.

(ii) If the resale market for level-2 liquidity is shallow ($\ell_1^* + \alpha\theta_H < \rho^*$), there exists $\underline{\theta}_L > 1/(1+\lambda)r_2$ such that

(a) if $\theta_L \leq \underline{\theta}_L$, then again there are no fire-sales. The optimal liquidity portfolio (ℓ_1, ℓ_2) satisfies $\ell_1 > \ell_1^*$ and $\ell_2 = \alpha < \ell_2^*$. Bailouts are more frequent than when the market for level-2 liquidity is deep. In contrast with the case of a deep market, an LWA requirement does not suffice: individual LWA-constrained banks would select too little baseload liquidity ($\ell_1 = \ell_1^*$) and too much peakload liquidity ($\ell_2 > \alpha$), generating fire sales. The two wedges between private and social interests (involuntary bailouts and fire sales) require two instruments: an LWA requirement and a minimum-level-1-liquidity requirement.

(b) If $\theta_L > \underline{\theta}_L$, then the social optimum involves some fire sales as well as the use of public funds. There exist $\widehat{\rho} \leq \rho^* \leq \mathring{\rho}$ such that the pecking order for liquidity use is level-1 liquidity for $\rho \in [0, \widehat{\rho})$, complemented by level-2 liquidity with a low discount on $[\widehat{\rho}, \rho^*]$ and high discount on $[\rho^*, \mathring{\rho}]$. An LWA requirement based on the worst-case scenario ($\ell_1 + \ell_2\theta_L \geq \mathring{\rho}$) does not suffice to implement the social optimum given that the latter involves fire sales: The structure, and not only the level of liquidity must be monitored.

3.3 Stress periods and the stabilization function of central bank eligibility

Suppose next that the depth of the resale market is ex ante unknown, and so is the future haircut. Intuitively, the state can in case of stress in the resale market stabilize the price of level-2 assets by offering to take a fraction of level-2 liquidity out of the market. The state will thereby incur a loss, but this loss is still preferable to either not using level-2 assets at all or tolerating a fire sale. We provide an informal treatment of this idea.

With probability s ("s" for "stress"), the resale market for level-2 assets is shallow in that $\alpha = \alpha_-$, while with probability $1 - s$, it is deep: $\alpha = \alpha_+ > \alpha_-$. Which obtains is revealed only at date 1. For computational simplicity let us assume that s is arbitrarily close to 0 (the analysis holds more generally if s lies below a threshold). Letting (as in Proposition 7 (i)) ℓ_1^* and ρ^* denote the level-1 liquidity and the cutoff that prevail when it is known that the resale market for level-2 assets is deep (i.e. $\ell_1^* + \alpha_+\theta_H \geq \rho^*$), then this policy is still optimal when the probability of stress is arbitrarily close to 0. However if $\ell_1^* + \alpha_-\theta_H < \rho^*$, the liquidation of level-2 assets in a situation of stress creates fire sales. Assuming that the cost of such fire sales exceeds that of public funds (in that $1 + \lambda < 1/\theta_L$), the optimal policy in case of stress involves making level-2 assets central bank eligible (CBE) with a low haircut $1 - \theta_H$, so that an amount α_- of level-2 assets be resold in the marketplace at low haircut as well. The rest is supported by the central bank, which accepts level-2 assets at a loss since its own valuation for them is θ_L.[30]

Proposition 8 *(stabilization through CBE)*

Suppose that fire sales are not a concern when the depth of the market is known at date 0 (the conditions for this are given in Proposition 7 (i)). Introduce a small probability of a stress situation in which level-2 liquidity would be sold at fire-sale prices for a range of liquidity shocks. Then the optimal policy in the stress scenario when $1 + \lambda < 1/\theta_L$ *consists in making level-2 assets central bank eligible, so as to keep the haircut at its no-stress level,* $1 - \theta_H$.

We here encounter a rationale for central bank eligibility, in which there is here no distinction between the liquid assets entering the liquidity ratio and the central bank eligible assets. The role of eligibility is solely to prevent fire sales. Two important remarks to conclude this section. First, central bank eligible assets are here assets that count toward the liquidity coverage requirement.[31] Second, unlike in the case where the central bank pledges liquidity support so as to eliminate the poor-coordination equilibrium (see the beginning of Section 3), the Bagehot doctrine no longer applies; the state accepts an expected loss in order to avoid fire sales.

[30] Assuming authorities cannot manage the asset themselves and have to hire a professional manager.

[31] Note that an addition to the LCR (see Basel Committee of Banking Supervision 2014) which is in line with Proposition 8 consists in 'Restricted Committed Liquidity Facilities' (RCLF): banks from all countries, including those which do not qualify for 'standard' CLF (i.e. have a 'sufficient' amount of sovereign debt) can still fill part of their level-2 buffer with RCLF. This Central-Bank funding is designed to be 'quite expensive' in 'normal times' but can become cheaper up to a limit in 'stress times' (a notion to be internationally monitored ex-post). Note that such a provision (not yet activated until now to our knowledge) represents an interesting partial move from quantity regulation towards price regulation, to follow the distinction highlighted in Stein (2013).

4 The liability side of liquidity: Bail-ins

When designing liquidity requirements, practitioners worry about shocks on both the asset and liability sides of the balance sheet.[32] The sudden stop in wholesale deposit roll-over, a potential hazard for uninsured deposits, is one of the events that require liquidity coverage; by contrast insured deposits are deemed much more stable. The availability of insured deposits varies across countries and with policies regarding competing investments in safe vehicles (such as life insurance funds in euros in France). Introducing heterogeneous investor preferences allows us to start investigating the impact of such macroeconomic differences across countries.

We also want to shed light on the ongoing debate regarding what should be "bail-inable" or not. Historically only equity has been bailed in the aftermath of bank distress. In the wake of the 2008 crisis, governments and regulators have insisted on the need to force more junior claims, such as hybrid securities (preferred equity, Cocos), long and medium term subordinated debt, corporate deposits, or uninsured retail deposits, share losses. We observe that the logic for making multiple classes of junior claimholders share risk is not to make them accountable, as these claimholders have no control rights in this model; rather, bail-ins can economize on bailouts. And we use the heterogeneity in risk appetite to build a priority order on claims.

To study the liability side of liquidity and the possibility of bail-ins, we introduce some pledgeable income: the bank delivers ρ_0 to investors at date 2 in case of date-1 continuation; so claims on this pledgeable income can be issued by banks. The investor side is an overlapping-generations one: Date-0 investors consume at date 1, and so must sell their assets to new investors who are "born" at date 1.[33] The following assumption, which we discuss below, will simplify the exposition considerably:

Assumption 2

(i) No shadow banking: A bank's banking license can be withdrawn if it rejects the regulatory contract.

(ii) Date-1 security demand: Date-1 investors are ordinary risk-neutral investors, willing to pay 1 at date 1 for 1 unit of expected date-2 income.

(iii) Equal treatment: All investors receive the same weight (1) in the social welfare function.

Let us comment on these assumptions. Assumption (i) prevents shadow banks from competing with regulated banks for retail deposits. While this assumption earlier involved

[32] Or to use Brunnermeier and Pedersen (2009)'s terminology, they look at both market and funding liquidity.

[33] Alternatively we could have used the Diamond-Dybvig (1983) model. The important ingredient is that some date-0 investors need their money at date 1.

no loss of generality, now it does. Farhi and Tirole (2017) show that the shadow banking sector may cleverly use financial engineering so as to create quasi-deposits and compete with regulated banks for depositors. Interestingly, the resulting put on taxpayer money makes asset valuations clientele-dependent. Allowing shadow banking therefore would complicate the computation of banks' reservation utility, which here can be taken to be equal to 0.

Assumption (ii) implies that the design of liabilities and of bail-inability requirements reflects solely the preferences of date-0 investors, and does not take into account the possibility that security design could enhance date-1 investors' willingness to pay for the pledgeable income. Another implication of this assumption is that it does not matter whether the pledgeable income ρ_0 is non-risky or rather is just an expectation of a random payoff.

Assumption (iii) implies that, when designing safe assets, the social planner does not benefit from extracting the rent of the most risk-averse investors. Otherwise, further distortions would be desirable so as to reduce investors' rents.

We will denote assets by $i \in \mathcal{I} \equiv \{1, \ldots, I\}$ (subscripts) and investors by $j \in \mathcal{J} = \{1, \ldots, J\}$ (superscripts). Assets yield 1 per unit if held to maturity, are priced $p_i = 1/r_i$ and are ranked by their liquidity $1 \geq \theta_1 > \theta_2 > \cdots > \theta_I$. As earlier, we can assume without loss of generality that if $r_{i'} > r_i$, then $\theta_{i'} r_{i'} < \theta_i r_i$ (no asset is dominated).

There is at date 0 a mass $\bar{\ell}^j$ of investors of type j (we normalize the mass of banks to be 1). Each is willing to pay $1/\theta^j$ for the right to 1 unit of income at date 1, where

$$1 \geq \theta^1 > \theta^2 > \cdots > \theta^J.$$[34]

The interpretation is that type-j investors have a date-1 project that they value at $1/\theta^j$ and can be undertaken only if they have 1 unit of cash at date 1.[35] The banking system can issue liabilities that are subscribed by type-j investors in an amount $\ell^j \in [0, \bar{\ell}^j]$. Let

$$\Theta \equiv \left\{ \theta_1, \ldots, \theta_I, \theta^1, \ldots, \theta^J, \frac{1}{1+\lambda} \right\}.$$

Proposition 9 *(bail-ins and the regulation of liquidity)*

The optimal regulatory scheme exhibits the following features:

(i) Liabilities targeted to type-j investors are bail-inable if $j \in J^1$ and non-bail-inable (insured) if $j \in J^2$, where

$$J^1 \equiv \{j | \theta^j (1+\lambda) > 1\} \text{ and } J^2 \equiv J \backslash J^1.$$

[34] We can see the model of Section 2 as a special case with a single type of date-0 investors, which are either infinitely risk-averse, or risk neutral (in which case ρ becomes $\rho - \rho_0$).

[35] An investor thus can have two or more incarnations: for example as a type-j investor for the first unit of date-1 income and a type-1 investor for the income beyond one unit.

(ii) Provided that some securities are bail-inable, there is no rationing of insured liabilities: $\ell^j = \bar{\ell}^j$ for $j \in J^2$.

(iii) Pecking order: Liquid assets are resold and liabilities bailed in according to their value of θ: The highest θ item in Θ (either a liquid asset or a liability) is used to cover small liquidity shocks, and so forth until some ρ^* beyond which all bail-inable liabilities are wiped out and all liquid assets are sold, and the shortfall in liquidity is made up through public funds.

(iv) The optimum can be decentralized through a LWA requirement, in which bail-inable securities all receive weight 1:

$$\sum_{i \in \mathcal{I}} \theta_i \ell_i + \sum_{j \in J^1} \ell^j \geq \rho^*.$$

Proposition 9 (iii) is reminiscent of "priority servicing" in electricity retail markets (Wilson 1993). Note also that bailing in liabilities involves no free lunch. As long as holders of liability j care about the certainty of being paid back ($\theta^j < 1$), bailing in the security creates a deadweight loss, that needs to be compared with the concomitant reduction in liquid assets or the reduced likelihood of a bailout.

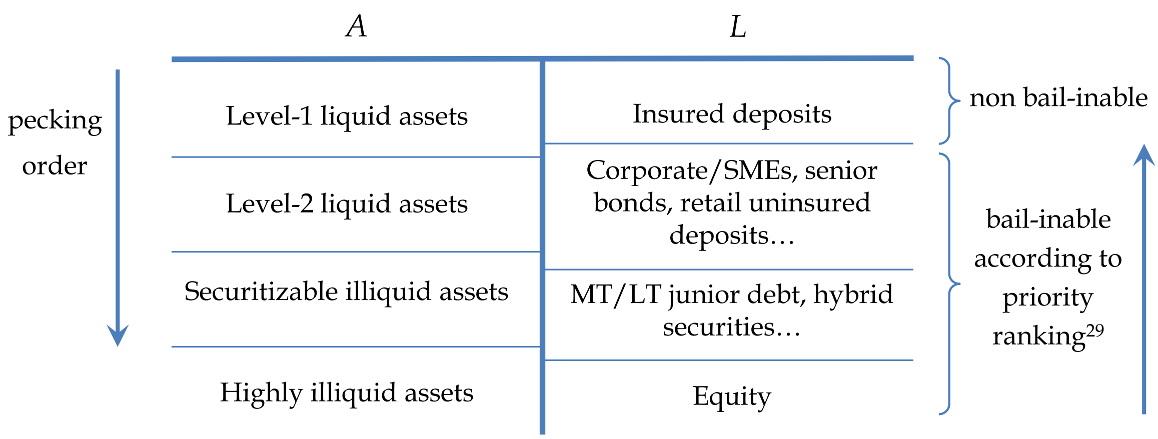

Figure 3: Regulation of funding and market liquidity

Pricing. Can the allocation described in Proposition 9 be implemented through a price system when the types of investors are not observable? Let $z^1 \leq z^2 \leq \cdots < z^{j^*} < z^{j^*+1} = \cdots = z^J = 1$ denote the probabilities of repayment for the various types of investors (with $J^1 = \{1, \ldots, j^*\}$). This monotonicity is a consequence of incentive compatibility. The utility of investors of type j, U^j, must obey the "adjacent incentive constraints":

$$U^j \geq U^{j-1} + z^{j-1} \left(\frac{1}{\theta^j} - \frac{1}{\theta^{j-1}} \right).$$

[29]See however the remark at the end of this section.

To implement the allocation, one can pick prices for the various liabilities:

$$p^1 = \frac{z^1}{\theta^1},\ p^2 - p^1 = \frac{z^2 - z^1}{\theta^2},\ p^3 - p^2 = \frac{z^3 - z^2}{\theta^3}, \text{ etc.}$$

The reader will check that these necessary adjacent constraints are also sufficient: The overall allocation is incentive compatible.

An illustration: One liquid asset, one bail-inable security, insured deposits

To establish a parallel with the analysis of Section 2, let us assume that there is a liquid asset ($\theta_1 = 1$) with return r_1 and price $p_1 = 1/r_1$ and two types of investors; type-1 investors are bail-inable ($\theta^1(1+\lambda) > 1$) while type-2 investors are not ($\theta^2(1+\lambda) < 1$). For expositional simplicity, assume that $\rho_0 > \bar{\ell}^2$. We further assume that the demand for bail-inable liabilities is large ($\bar{\ell}^1 = \infty$ say).[30]

Proposition 10 *(illustration)*

Suppose that the bank hoards liquid assets ($\ell_1 > 0$). A necessary and sufficient condition for this to obtain is:

$$\frac{F(\rho_0)}{\theta^1} + (1+\lambda)[1 - F(\rho_0)] \geq \frac{1}{r_1}.$$

Then the amount of liquid assets (ℓ_1) decreases one-for-one with the pledgeable income (ρ_0) and increases one-for-one with the level of insured deposits ($\bar{\ell}^2$), implying that liquid assets decrease one-for with bail-inable securities ($\rho_0 - \bar{\ell}^2$).

Proof. Because the resale of perfectly liquid assets comes first in the pecking order, small liquidity shocks should be covered by asset resales.

(a) Suppose, first, that $\ell_1 > 0$. Let $\widehat{\rho} \equiv \ell_1 - \bar{\ell}^2 > -\bar{\ell}^2$ and $\rho^* \equiv \widehat{\rho} + \rho_0$, so shocks between $\widehat{\rho}$ and ρ^* will be covered by partial bail-in.

The social planner solves

$$\max \left\{ -\frac{\widehat{\rho} + \bar{\ell}^2}{r_1} + \frac{\int_0^{\widehat{\rho}+\rho_0} [(\widehat{\rho} + \rho_0) - \rho] dF(\rho)}{\theta^1} - (1+\lambda) \int_{\widehat{\rho}+\rho_0}^{\bar{\rho}} [\rho - (\widehat{\rho} + \rho_0)] dF(\rho) \right\}$$

and so optimizing over $\widehat{\rho}$,

$$\frac{F(\widehat{\rho} + \rho_0)}{\theta^1} + (1+\lambda)[1 - F(\widehat{\rho} + \rho_0)] = \frac{1}{r_1}. \tag{11}$$

This solution requires that $\widehat{\rho} \geq 0$ or

$$\frac{F(\rho_0)}{\theta^1} + (1+\lambda)[1 - F(\rho_0)] \geq \frac{1}{r_1}. \tag{12}$$

[30] It would be interesting to study the impact of a shortage in bail-inable liabilities.

(b) If ρ_0 is sufficiently large and so (12) is violated, then $\ell_1 = 0$ and liquidity is provided solely through the bail-in of liabilities. ■

Remark (wholesale deposits). Regulatory liquidity ratios, such as the LCR, presuppose that most or all wholesale deposits with maturity shorter than the horizon of the liquidity ratio will vanish overnight in case of adverse news. Such wholesale deposits in effect are akin to retail deposits in that their exit option de facto gives them insurance against bad news.[31] Proposition 10 then suggests that the level of liquid assets should grow one-for-one with the amount of wholesale deposits.

5 Comparison with international regulation

As discussed in the introduction, the design of the liquidity coverage ratio (LCR) obeys the following principles:[32]

1) The stock of HQLA should exceed the total net cash outflows over the next 30 calendar days.

2) *Asset side*: assets are considered to be HQLA if they can be easily and immediately converted into cash at little or no loss of value.

   - They should be unencumbered.[33]
   - HQLA (except level-2B assets) should ideally be eligible at central banks for intraday liquidity needs and overnight liquidity facilities, but central bank eligibility does not by itself constitute the basis for the categorization of an asset as HQLA.
   - Level-1 assets (banknotes, central bank reserves, claims on safe sovereigns and multilateral organizations. . .) face no haircut. Level-2A assets (claims on sovereigns and multilateral organizations receiving a 20% credit risk weight under Basel II, high-grade corporate securities and covered bonds not issued by another financial institution. . .) and level-2B assets (RMBS, relatively safe corporate debt, diversified and not-currency-risk-exposed equity. . .) face haircuts that may depend on the type of security ($\theta_{2A} = 0.85$ and $\theta_{2B} \in [0.50, 0.75]$).

[31] To see this, imagine that prior to date 1 (say at "date 1^-"), the realization of ρ is revealed and wholesale depositors can refuse to roll over their deposits and can cash in. They are then senior to other securities even though they legally have a junior claim.

[32] This account of course summarizes only the main lines. For more detail, see Basel Committee on Banking Supervision (2013), on which this summary builds.

[33] "Unencumbered" means free of legal, regulatory, contractual or other restrictions on the ability of the bank to liquidate, sell, transfer, or assign the asset. An asset in the stock should not be pledged (either explicitly or implicitly) to secure, collateralize or credit-enhance any transaction, nor be designated to cover operational costs (such as rents and salaries).

- Minimal levels are specified: At least 60% of level 1 assets after haircuts have been applied. Level-2B assets should not account for more than 15% in the 40% level-2 amount.

3) *Supply of assets*: Jurisdictions that have an insufficient supply of level-1 assets (or both level-1 and level 2 assets) in their domestic currency to meet the aggregate demand of banks may qualify for alternative treatment. Three options are contemplated for the alternative treatment: (i) banks can access contractual committed liquidity facilities (with a maturity over 30 days) provided by the relevant central bank for a fee; (ii) banks can hold HQLA in a currency that does not match the currency of the associated liquidity risk, (iii) banks that evidence a shortfall of HQLA in the domestic currency are allowed to hold additional level-2A assets in the stock (these additional assets facing $\theta = 0.80$ instead of 0.85).

4) *Liability side*: Total expected cash outflows are calculated by multiplying the outstanding balances of various categories of liabilities and off-balance sheet commitments by the rates at which they are expected to run off or be drawn down. Retail deposits, considered stable, usually receive a run-off factor of 5%. The unsecured wholesale funding run-off factor is 100% with some exceptions.[34] Secured liabilities receive a run-off factor that depends on the quality of the collateral,[35] so there is in effect some netting with the asset side. Finally credit lines granted by the bank are also allocated drawdown factors.

Propositions 1, 2, 3, 7 and 8 indicate that the LCR design fits well with the theory on the asset side:

- HQLA are weighted by their liquidity discount;
- central bank eligibility per se does not make an asset a HQLA, but CBEA is a plus for HQLA qualification;
- minimum percentages of higher quality assets are specified;
- substitution of level-2 assets for level-1 assets is allowed in case of shortage of the latter.

Proposition 4 suggests a reduction in liquidity coverage when banks grant each other insurance. No such provision is made under the LCR. This disparity may be associated with a prudent approach in view of the caveat in Proposition 4 that supervisors can check that liquidity pooling is used for hedging purposes.

[34] E.g. deposits of SMEs or of non-financial corporates or central banks, which receive run-off factors intermediate between 5% and 100%.

[35] E.g. 0% if the assets are level-1 assets, 15% if they are level 2A, 50% if they are level 2B. The default factor is otherwise 100%.

The fit with theory is weaker on the liability side. To see why let us compare the theoretical recommendation:

$$\sum_{i \in I} \theta_i \ell_i + \sum_{j \in J^1} \ell^j \geq \rho^*,$$

with the formulation of the LCR

$$\sum_{i \in I} \theta_i \ell_i \geq \sum_{j \in J} \theta^j \ell^j$$

where $\theta^j \in [0, 1]$ is the run-off factor.

Simplifying, suppose that there are three types of liabilities: Retail deposits, wholesale deposits, and bail-inable liabilities;[36] and that retail deposits ℓ^r as well as bail-inable liabilities ℓ^b are assumed fully stable ($\theta^r, \theta^b \approx 0$), while wholesale deposits ℓ^w are fully unstable ($\theta^w = 1$). The LCR requirement writes:

$$\sum_{i \in I} \theta_i l_i \geq \ell^w,$$

while the formula in Proposition 9 implies that

$$\sum_{i \in I} \theta_i \ell_i + \rho_0 - \ell^r - \ell^w \geq \rho^* \;\Leftrightarrow\; \sum_{i \in I} \theta_i \ell_i \geq [\rho^* - \rho_0] + [\ell^r + \ell^w].$$

This comparison illustrates both the strength and the incompleteness of the theoretical approach. Theory says little about the actual value of the wedge $\rho^* - \rho_0$ on the right-hand side of the last inequality. In fact, equation (6) shows that this wedge (logically) depends on the distribution F of the shock, on the cost and haircut of level-2 liquidity, and on the cost of bailing out banks. It is hard to calibrate this wedge without either empirical work or trial and error.

By contrast, theory indicates that treating retail and wholesale deposits in diametrically opposite ways may not be warranted. Retail and wholesale deposits share the property that one cannot count on diluting them to meet liquidity shocks that cannot be met through HQLA sales, as they are explicitly or implicitly insured.

The narrative behind the LCR logic is that even a small concern about the bank's health will induce a run by legally uninsured short-term wholesale deposits. This run will create difficulties for the bank if it cannot find alternative funding liquidity; the concomitant liquidation of illiquid assets will lead to losses for the bank. By contrast, retail deposits are stable.

Basel III embodies a fundamental tension between the (understandable) unwillingness to insure large depositors and the fact that, to consent to low rates, these depositors

[36] Wholesale deposits are theoretically bail-inable, but their quick-exit option, as we observed, de facto makes them unbail-inable. Non bail-inable liabilities can be lumped together with wholesale deposits.

insist on highly liquid deposits, giving them the option to run fast if they are worried and making them de facto insured. Our analysis concurs with the LCR design on the wholesale-deposits front: Regulation should require banks to be able to handle even a 100% outflow solely by selling liquid assets.

But what about insured retail deposits, where the LCR view is that there will not be massive outflows, because of deposit insurance? When the bank faces disappointing net cash inflows, it has to obtain the money from liquid asset sales, bail-ins or bailouts. Our key point is that, since neither insured retail deposits nor short-term wholesale deposits can credibly be bailed in, the regulator's choice is identical for both: either more liquid assets or more bailouts. The benefit of our perspective is that it explicitly links liquidity and solvency.

6 Summary, policy implications and future research

Broad conceptual framework. The paper provides a theoretical framework to study liquidity regulation and its interaction with solvency concerns. The model exhibits two classic wedges between private and social interests. First, time inconsistency implies that an illiquid bank may receive support more often than would be warranted, i.e. not just in exceptional circumstances. Second, endogenously limited arbitrage creates scope for fire sales externalities. These two ingredients, as we show, enable us to get insights into the design of liquidity coverage surveillance.

Optimal liquidity management, from both a private and a social perspectives, obeys the logic of investment to meet a random load in electricity. Low-yield, highly-liquid assets ("level-1 liquidity") serve as baseload to meet ordinary liquidity shocks; higher-yield, less-liquid assets ("level-2 liquidity") complement the former to address larger liquidity shocks; finally, exceptional liquidity shocks require state liquidity support. Under laissez-faire, though, banks hoard insufficient liquidity and may pick the wrong kind of liquidity under regulation.

We investigated whether the optimal liquidity requirement takes the form of a simple liquidity-weighted-asset (LWA) requirement in which assets receive weights that reflect the discount that they would incur in a resale at the horizon of the liquidity coverage ratio, or to the contrary the supervisor should interfere in the composition of the liquid portfolio. We showed that mingling with the composition of the portfolio is unnecessary in the absence of fire sales externalities. In the presence of fire sales externalities, the supervisor must also ensure that the bank hoards enough high-quality (level-1) liquid assets. The optimal surveillance of liquidity coverage therefore thus mirrors the practice for capital adequacy (made up of a requirement on the ratio of capital to risk-weighted assets and a floor on tier-1 capital).

When we enriched the model to endogenize loss-absorbency by the various claimholders of the bank, we derived an overall sequence of asset sales and bail-in of the various claims issued by the bank before the bailout occurs. We also derived implications as to the treatment of wholesale deposits and to the differentiation across countries of liquidity requirements.

Policy implications. The paper provides the following tentative conclusions on the Basel III reforms (needless to say, further work, including along the lines discussed below, will shed more light on their suitability):

a) On the positive side, we brought theoretical support for the existence of a liquidity ratio, for a liquidity-weighted approach, and, provided that there is a risk of fire sales, for a minimum level-1-liquidity requirement. We also explained the rationale for defining bail-inable securities, a rationale that is not grounded in accountability (a criterion which would restrict bail-inability to equity).

   In the debate on the use of central-bank-eligible assets toward meeting the liquidity requirement, we came on the side of those objecting to this practice and so concurred with the Basel III approach. The logic of our argument is that liquid assets are an *individual* contribution toward bailout avoidance. By contrast, CBEAs are meant to address *asset market breakdowns*, and are driven by coordination or across-banks-externality problems. We thus view the two regulatory interventions as serving different purposes.

   We also found that if capital markets are segmented along country lines, liquidity requirements should depend on local availability of high-quality liquid assets, and so one size does not fit all.

b) Our analysis suggests a reduction in liquidity coverage when banks grant each other liquidity insurance. This conclusion stands in contrast with the design of the LCR, which does not account for interbank exposures. As we discussed, though, "interbank exposures" in practice need not be synonymous with "mutual insurance", even though in theory they should. So caution may require to not reduce the liquidity requirement unless joint stress tests creates some reliable measurement of the correlation between the banks or else one of the banks is strong enough to reliably honour the insurance scheme.

c) On the negative side, we argued against the differentiated treatment of retail and wholesale deposits in the LCR requirement, on the grounds that they are (explicitly for the former, de facto for the latter) both insured

Future research. Needless to say, this is only a first step toward a better understanding of liquidity regulation. Beyond the extensions discussed in the Appendix, let us mention a few important alleys for future research.

First, we only touch on cross-border liquidity management. We looked at the polar cases in which either level-1 liquid assets are available in unlimited quantity,[37] or banks' liquid portfolios exhibit an extreme home bias. In practice, there is often some cross-country investment but a limited amount of high-quality stores of value, which raises a new research line concerning countries' provision of a global public good, the availability of worldwide liquidity.

Second, while a three-period model suffices to capture basic insights about the needed liquidity in level and structure, liquidity is a dynamic concept. For example, a classic regulatory debate concerns whether liquidity should be allowed to be depleted following a shock or to the contrary should be replenished along the way (this is Goodhart's celebrated "last taxi at the station that cannot take a customer so as not to leave the taxi line empty" metaphor[38]). Relatedly, within Basel III, liquidity ratios have recently been designed at various horizons (one month for the Liquidity Coverage ratio, one year for the Net Stable Funding ratio), with little knowledge of the proper tiling of the various ratios.

Besides shedding light on these regulatory debates, the study of dynamics is theoretically interesting for multiple reasons: a) News about liquidity needs may accrue over time (think about the accrual of information about non-performing loans);[39] b) Giving time to liquidate the assets may lower the market discount; c) Finally, liquidity may be reallocated among banks, so that the mutual insurance question studied in Section 2.5 has an intertemporal as well as a cross-state nature. A multistage perspective could help us discuss a liability's withdrawal speed and its impact on liquidation speed and therefore bailouts. Outflow rates would not only be horizon-dependent, but also could depend on the type of liability even within the class of bail-inable liabilities.

We leave these fascinating topics for future research.

[37]Level-2 assets as well, but the limited number of buyers in Section 3 put a de facto limit on their use.

[38]Goodhart (2008) argued that liquidity which is never used is wasteful (on this and other potential unintended consequences of liquidity requirements, see also Dewatripont 2014). There are very few models on the dynamic management of liquidity. In Biais et al (2010), liquidity is constantly replenished through downsizing when facing adverse news.

[39]On this issue, see the recent paper by Santos and Suarez (2018), which considers a continuous-time model where authorities gradually learn news about the health of a bank, and where the stock of HQLAs can be chosen so as to allow them to 'buy the right amount of time' before having to decide whether or not to support the bank.

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Appendix

Extension: continuous investment and continuum of asset classes

Suppose now that investment i belongs to $[0, +\infty)$ and exhibits constant returns to scale. The liquidity shock ρ is still drawn from $F(\cdot)$ on $[0, \bar{\rho}]$. By reinvesting ρj, with $0 \le j \le i$, the bank can continue at scale j, delivering payoff bj, with $b > 1$, to the banker, and benefit βj to the state. As in Farhi-Tirole (2012), we will need a condition on β so that the state does not want to fund the bank at date 0 (so as to increase investment), but is willing to support it at date 1. The banker has initial endowment a.

There is a continuum of liquid assets, indexed by the return $r \in (\underline{r}, 1)$ obtained when the asset is kept to maturity. The date-0 price of asset r is $p(r) = 1/r$. Let $\theta(r)$ denote the unit proceeds when reselling the asset at date 1, with $\theta(r) \le 1$. To ensure that no asset is dominated, we assume that

$$(r\theta(r))' < 0.$$

We assume further that

$$\beta \ge (1+\lambda)\bar{\rho}$$

so that under laissez-faire, the banker is always bailed out and therefore does not buy liquid assets (so $i = a$). The banker's laissez-faire utility is ba.

Under regulation, the key properties of the two-asset-classes model carry over: Shocks below some threshold ρ^* are covered by hoarded liquid assets, and the shortfall for $\rho > \rho^*$ is covered through public funds; furthermore, the use of liquid assets obeys the pecking-order principle, so that more liquid, lower-return assets are sold first.

Let $r(\rho)$ denote the class of assets that are resold if and only if the realized shock $\tilde{\rho}$ exceeds ρ, and let $\ell(r(\rho))$ denote the amount of such assets per unit of investment. $r(\rho)$ is increasing in ρ. Liquidity coverage requires that:

$$\left[\int_0^{\rho} \ell(r(\tilde{\rho}))\theta(r(\tilde{\rho}))d\tilde{\rho}\right] i = \rho i \qquad \text{for all } \rho \le \rho^*,$$

yielding

$$\ell(r(\rho))\theta(r(\rho)) = 1. \tag{A.1}$$

For a given ρ^*, let us define $L(\rho^*)$ as the net cost of hoarding liquidity per unit of investment:

$$L(\rho^*) \equiv \int_0^{\rho^*} \left[\frac{\ell(r(\rho))}{r(\rho)} - \ell(r(\rho))[1 - F(\rho)]\right] d\rho \tag{A.2}$$

(recall that with probability $F(\rho)$, $\ell(r(\rho))$ is not used at date 1).

Minimizing $L(\rho^*)$ subject to (A.1) amounts to choosing a function $r(\cdot)$ that solves

$$\min_{\{r(\cdot)\}} \left\{ \int_0^{\rho^*} \frac{1}{\theta(r(\rho))} \left[\frac{1}{r(\rho)} - [1 - F(\rho)] \right] d\rho \right\}.$$

Whenever $r(\cdot)$ is non-constant, it satisfies:

$$-\frac{\theta'(r(\rho))}{\theta(r(\rho))} - \frac{1}{r(\rho) - r^2(\rho)[1 - F(\rho)]} = 0.$$

Social welfare is obtained by solving

$$\max_{\{\rho^*\}} \left\{ \tau_0 + \beta i - (1 + \lambda) \left[\int_{\rho^*}^{\bar{\rho}} (\rho - \rho^*) dF(\rho) \right] i \right\}$$

s.t.

$$a = L(\rho^*)i + i + \tau_0.$$

Or:

$$\max_{\{\rho^*\}} \left\{ a + \left[\beta - L(\rho^*) - (1 + \lambda) \int_{\rho^*}^{\bar{\rho}} (\rho - \rho^*) dF(\rho) - 1 \right] i \right\}. \tag{A.3}$$

So, using (A.1) and (A.2)

$$\frac{\frac{1}{r(\rho^*)} - [1 - F(\rho^*)]}{\theta(r(\rho^*))} = (1 + \lambda)[1 - F(\rho^*)]$$

Because of constant returns to scale, we need to ensure that β is not so large (while still exceeding $(1 + \lambda)\bar{\rho}$) as to vindicate an infinite investment: The term in brackets in (A.3) must be non-positive.[40]

If the banker's gross reservation utility is $\underline{U} = a$ (the banking license can be withdrawn), then $i = 0$. If the banker's reservation utility is $\underline{U} = ba$ (the banker can operate as a shadow bank), then $bi \geq ba$ or $i = a$.

[40]To show that this condition is not inconsistent with the condition $\beta \geq (1 + \lambda)\bar{\rho}$, consider for instance the bimodal distribution $\rho = 0$ with probability $1 - \alpha$ and $\rho = \bar{\rho}$ with probability α. The optimal policy involves no liquidity hoarding if $\min_{\{r\}} \left\{ \frac{1}{r\theta(r)} - \frac{1-\alpha}{\theta(r)} \right\} > (1 + \lambda)\alpha$. To obtain ex-post bailouts but no ex-ante subsidies, β must satisfy

$$(1 + \lambda)\bar{\rho} < \beta < 1 + (1 + \lambda)\alpha\bar{\rho}$$

which is feasible if

$$1 > (1 + \lambda)(1 - \alpha)\bar{\rho}.$$

Other extensions

The model can relatively easily be extended in several directions, at the cost of heavier notation and formulas.

- *Wider support for the liquidity shock.* The parameter ρ could have full support $[0, +\infty)$. Then the bank is not always rescued. The cut-off for the liquidity shock beyond which the banks are no longer rescued then depends on β. By contrast, β plays no role at the margin in the analysis below.

- *Moral hazard.* We could allow the distribution $F(\cdot)$ of shocks to be affected by an ex-ante choice made by the bank.[41] Liquidity regulation would then also address moral hazard, and not only unwanted liquidity provision and fire sales, like in this paper.

Proof of Proposition 3

As usual, one can write social welfare as the difference between total surplus and rent. The expected rent is $\theta_0 U(1) + (1-\theta_0)U(0)$. Total surplus can be written as:

$$E\Big[\Sigma_{i=1,2}\ell_i[1-x_i(\rho)] + \theta_0\ell_0[1-x_0(\rho)] \\ -(1+\lambda)\max\big\{\rho - \Sigma_{i=0}^2\ell_i\theta_i x_i(\rho), 0\big\} - \min\big\{\rho - \Sigma_{i=0}^2\ell_i\theta_i x_i(\rho), 0\big\}\Big].$$

Substracting the expected rent using the incentive and the participation constraints, social welfare can be written as:

$$W = E\Big[\Sigma_0^2\ell_i[1-x_i(\rho)] - (1+\lambda)\max\big\{\rho - \Sigma_0^2\ell_i\theta_i x_i(\rho), 0\big\} \\ - \min\big\{\rho - \Sigma_0^2\ell_i\theta_i x_i(\rho), 0\big\}\Big] - \Sigma_1^2\frac{\ell_i}{r_i} - \ell_0 - \underline{U}.$$

In the region in which there is not enough liquidity ($\rho > \Sigma_0^2\ell_i\theta_i$), then $x_i(\rho)$ is indeed equal to 1 as long as $(1+\lambda)\theta_i > 1$.

In the no-bailout region, in which $\rho = \Sigma_0^2\ell_i\theta_i x_i(\rho)$, then the pecking order applies as $\theta_1 = 1 > \theta_2 > \theta_0$, and so the illiquid legacy asset is used last.

[41] As in Dewatripont-Tirole (2012).

And so

$$\begin{aligned}
W = & -\frac{\widehat{\rho}}{r_1} - \frac{\rho^* - \widehat{\rho}}{\theta r_2} + \int_0^{\widehat{\rho}} \left[\widehat{\rho} - \rho + \frac{\rho^* - \widehat{\rho}}{\theta} + \ell_0\right] dF(\rho) \\
& + \int_{\widehat{\rho}}^{\rho^*} \left[\frac{\rho^* - \rho}{\theta} + \ell_0\right] dF(\rho) + \int_{\rho^*}^{\rho^* + \theta_0 \ell_0} \left[\ell_0 - \frac{\rho - \rho^*}{\theta_0}\right] dF(\rho) \\
& - \int_{\rho^* + \theta_0 \ell_0}^{\bar{\rho}} (1 + \lambda)(\rho - \rho^* - \theta_0 \ell_0) dF(\rho) - \ell_0 - \underline{U}.
\end{aligned}$$

The first-order condition with respect to $\widehat{\rho}$ is unchanged relative to the case $\ell_0 = 0$: Condition (5) still prevails. By contrast, the first-order condition with respect to ρ^* becomes:

$$\frac{F(\rho^*)}{\theta} + \frac{F(\rho^* + \theta_0 \ell_0) - F(\rho^*)}{\theta_0} + [1 - F(\rho^* + \theta_0 \ell_0)](1 + \lambda) = \frac{1}{\theta r_2}.$$

This condition coincides with condition (6) when $(1 + \lambda)\theta_0 = 1$, but not if $(1 + \lambda)\theta_0 > 1$. Furthermore, $\frac{\partial^2 W}{\partial \rho^* \partial \ell_0} < 0$ and so ρ^* decreases with ℓ_0.

■

Proof of Proposition 4

With probability x, shocks are correlated and the liquidity coverage constraint is, as earlier,

$$\rho - \Sigma_i \ell_i \theta_i x_i(\rho) \geq 0.$$

For conciseness, we will focus on the case in which $\rho^* \geq \bar{\rho}/2$. That is, liquidity coverage ensures that there is probability at least 1/2 (given the symmetry of F) that there is no need for public intervention, a reasonable assumption. This is guaranteed if λ is large enough or liquid assets are not too costly. With probability $1 - x$, the liquidity coverage constraint is

$$\frac{\bar{\rho}}{2} - \Sigma_i \ell_i \theta_i x_i(\rho) \geq 0.$$

Program (I) becomes Program (I′):

$$
\begin{aligned}
\max W = & -\frac{\widehat{\rho}}{r_1} - \frac{\rho^* - \widehat{\rho}}{\theta r_2} \\
& + x\Bigg[\int_0^{\widehat{\rho}} \left(\widehat{\rho} - \rho + \frac{\rho^* - \widehat{\rho}}{\theta} \right) dF(\rho) + \int_{\widehat{\rho}}^{\rho^*} \left(\frac{\rho^* - \rho}{\theta} \right) dF(\rho) \\
& \qquad - \int_{\rho^*}^{\bar{\rho}} (1+\lambda)(\rho - \rho^*) dF(\rho) \Bigg] \\
& + (1-x) \begin{cases} \dfrac{\rho^* - \frac{\bar{\rho}}{2}}{\theta} & \text{if } \widehat{\rho} \leq \dfrac{\bar{\rho}}{2} \quad \text{(case (A))} \\ \widehat{\rho} - \dfrac{\bar{\rho}}{2} + \dfrac{\rho^* - \widehat{\rho}}{\theta} & \text{if } \widehat{\rho} \geq \dfrac{\bar{\rho}}{2} \quad \text{(case (B))} \end{cases}
\end{aligned}
$$

Regardless of whether level-1 liquidity suffices to cover the liquidity shock in the insurance state (case (A)) or not (case (B)), total liquidity is given by

$$F(\rho^*) + [1 - F(\rho^*)](1+\lambda)\theta = \frac{1}{x r_2}$$

and so

$$\frac{\partial \rho^*}{\partial x} > 0.$$

Level-1 liquidity is driven by

$$
\begin{aligned}
\left.\frac{\partial W}{\partial \widehat{\rho}}\right|_{(A)} &= x\left(\frac{1-\theta}{\theta}\right) F(\widehat{\rho}) - \left(\frac{1}{\theta r_2} - \frac{1}{r_1}\right) \\
&= \left.\frac{\partial W}{\partial \widehat{\rho}}\right|_{(B)} + (1-x)\left(\frac{1-\theta}{\theta}\right).
\end{aligned}
$$

So if

$$x\left(\frac{1-\theta}{\theta}\right) F\left(\frac{\bar{\rho}}{2}\right) - \left(\frac{1}{\theta r_2} - \frac{1}{r_1}\right) < 0, \text{ then } \widehat{\rho} > \bar{\rho}/2.$$

Similarly, if

$$x\left(\frac{1-\theta}{\theta}\right) F\left(\frac{\bar{\rho}}{2}\right) - \left(\frac{1}{\theta r_2} - \frac{1}{r_1}\right) - (1-x)\left(\frac{1-\theta}{\theta}\right) > 0, \text{ then } \widehat{\rho} < \bar{\rho}/2.$$

Finally, if neither obtains, then $\widehat{\rho} = \bar{\rho}/2$.

Note also that

$$\left.\frac{\partial^2 W}{\partial \widehat{\rho} \partial x}\right|_{(B)} > 0 > \left.\frac{\partial^2 W}{\partial \widehat{\rho} \partial x}\right|_{(A)}.$$

So $\widehat{\rho}$ increases (decreases) with x in case (B) (case (A)). Intuitively, the liquidity in excess of $\bar{\rho}/2$ is wasted in the insurance state, making the case for the higher-yield level-2 liquidity more compelling.

The decentralization result has the same logic as earlier: The externality on public finances is addressed entirely through the liquidity requirement ρ^*. Given this, the bank will by itself opt for the least-cost way of generating this level of total liquidity.

$\blacksquare$

Proof of Proposition 7

(i) Suppose that the regulator requires that $\ell_1 + \ell_2\theta_H \geq \rho^*$. Then each bank selects the same portfolio (ℓ_1^*, ℓ_2^*) as in Section 2. And there will not be fire sales as $\ell_2^* \leq \alpha$.

(ii) Let us now focus on the interesting case in which $\ell_1^* + \alpha\theta_H < \rho^*$ (shallow market). Consider first the date-1 pecking order. The date-1 opportunity cost of 1 unit of liquidity is 1 when using level-1 liquidity, $1+\lambda$ when using public funds, and $1/\theta(\rho)$ when using level-2 liquidity where $\theta(\rho) \in \{\theta_L, \theta_H\}$ measures the state-contingent marketability/liquidity of level-2 assets.

(a) If $1 + \lambda < 1/\theta_L$, fire sales are always socially undesirable. And so there is no point hoarding level-2 liquidity $\ell_2 > \alpha$, as excess level-2 liquidity will never be used. The solution is therefore derived as in Section 2, except that the constraint $\ell_2 \leq \alpha$ is binding, and so:

$$\ell_1 + \alpha\theta_H = \rho^*, \tag{A.4}$$

which defines a function $\rho^*(\ell_1)$. We still have

$$\widehat{\rho} \equiv \ell_1. \tag{A.5}$$

The social planner's program is:

$$\max_{\{\ell_1\}} \Bigg\{ -\frac{\ell_1}{r_1} - \frac{\alpha}{r_2} + \int_0^{\ell_1} [(\ell_1 - \rho) + \alpha]\, dF(\rho) + \int_{\ell_1}^{\rho^*(\ell_1)} \left[\alpha - \frac{\rho - \ell_1}{\theta_H}\right] dF(\rho) - \int_{\rho^*(\ell_1)}^{\bar{\rho}} (1+\lambda)\,[\rho - \rho^*(\ell_1)]\, dF(\rho) \Bigg\}.$$

The first-order condition is such that the marginal benefit of level-1 liquidity is equal to the marginal cost (1)

$$\left[F(\widehat{\rho}) + [F(\rho^*) - F(\widehat{\rho})]\left(\frac{1}{\theta_H}\right) + [1 - F(\rho^*)]\,(1+\lambda)\right] r_1 = 1. \tag{A.6}$$

Substituting (A.4) and (A.5) into (A.6) yields the optimal level of level-1 liquidity.

Next, we show that $\ell_1 > \ell_1^*$ and $\ell_2 < \ell_2^*$. Suppose, a contrario, that $\ell_1 \leq \ell_1^*$. Condition (A.6), which is also satisfied by the unconstrained solution of Section 2 (to see this, add up (5) and (6)), together with $1 + \lambda > 1/\theta_H$, imply that ρ^* is higher than in Section 2 and so

$$\begin{aligned} & \ell_1 + \ell_2\theta_H \geq \ell_1^* + \ell_2^*\theta_H \\ \Rightarrow \quad & \ell_2 - \ell_2^* \geq \frac{(\ell_1^* - \ell_1)}{\theta_H} \geq 0 \\ \Rightarrow \quad & \ell_2 \geq \ell_2^* > \alpha, \end{aligned}$$

a contradiction. So, $\ell_1 > \ell_1^*$ and $\ell_2 < \ell_2^*$. Furthermore, bailouts are more frequent (ρ^* is smaller) than when the level-2-liquidity market is deep.

It is also easy to see that the optimal solution cannot be decentralized through a mere LWA requirement. Otherwise, the representative bank would solve program (II) and thus pick $\ell_1 = \ell_1^*$. A minimum-level-1 requirement in complement of the LWA is needed to achieve the optimum.

(b) Finally, suppose that $(1 + \lambda)\theta_L > 1$. The date-1 pecking order is then: (1) level-1 liquidity, (2) level-2 liquidity at a low discount, (3) level-2 liquidity at a high discount, (4) public funds (since $1 < 1/\theta_H < 1/\theta_L < 1 + \lambda$). Let the cutoffs be $\widehat{\rho} < \rho^* < \mathring{\rho}$ with $\widehat{\rho} \equiv \ell_1$, $\rho^* \equiv \ell_1 + \alpha\theta_H$ and $\mathring{\rho} \equiv \ell_1 + \ell_2\theta_L$ (which imply $\ell_2 > \alpha$).

The social planner solves:

$$\max_{\{\ell_1, \ell_2\}} \Bigg\{ -\frac{\ell_1}{r_1} - \frac{\ell_2}{r_2} + \int_0^{\widehat{\rho}} [\ell_1 - \rho + \ell_2] dF(\rho) + \int_{\widehat{\rho}}^{\rho^*} \left[\ell_2 - \frac{\rho - \ell_1}{\theta_H}\right] dF(\rho)$$
$$+ \int_{\rho^*}^{\mathring{\rho}} \left[\ell_2 - \frac{\rho - \ell_1}{\theta_L}\right] dF(\rho) - \int_{\mathring{\rho}}^{\bar{\rho}} (1 + \lambda)(\rho - \mathring{\rho}) dF(\rho) \Bigg\}.$$

Assuming $\mathring{\rho} > \rho^*$ (otherwise the solution is the same as in (a)), the maximization with respect to ℓ_2 yields

$$-1 + \left[F(\mathring{\rho}) + \left[1 - F(\mathring{\rho})\right](1 + \lambda)\theta_L\right] r_2 = 0.$$

A necessary condition for such a $\mathring{\rho}$ to exist (ignoring the constraint that $\mathring{\rho} \geq \rho^*$) is that $(1 + \lambda)\theta_L r_2 > 1$, which is stronger than $(1 + \lambda)\theta_L > 1$.

Note that the first α units of level-2 liquidity incur a capital loss $\alpha(\theta_H - \theta_L)$ when $\rho > \rho^*$, and so a necessary condition for this regime to be feasible is that $\mathring{\rho} > \rho^*$ or

$$\ell_2 > \frac{\theta_H}{\theta_L}\alpha.$$