Missed Sales and the Pricing of Ancillary Goods: Supplementary Material

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This document contains additional results for the manuscript *"Missed sales and the Pricing of Ancillary Goods."* All numbered items (i.e., sections, propositions, and equations) in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text. Please refer to the main text for notation and definitions.

Section S1 extends the baseline model from the manuscript to an environment where the consumer demand for the basic good is elastic. Section S2 contains the proofs omitted in Section S1.

S1. Pricing when the demand for the basic good is elastic

In order to isolate the effects of missed sales and price transparency, the baseline model assumed that consumers have an inelastic demand for the basic good. If, however, the demand for the basic good is elastic, the ancillary good price may be used for price discrimination. This new effect comes in addition to the forces discussed in the text. The goal of this supplementary material is to illustrate how our analysis extends to this richer setting.

To simplify matters, suppose consumers value the basic good by either v_l or v_h, with probabilities θ_l and θ_h. We let $\triangle \equiv v_h - v_l > 0$. As in the baseline model, if a consumer bypasses the ancillary good, she incurs a nuisance cost $b \sim G$, independent of v_k, $k \in \{l, h\}$, with support on $\mathbb{R}_+$. The shopping cost s is set to zero.

The firm chooses a basic price p for the basic good and a price τ for the ancillary one. The marginal cost of production of the basic good is c and of the ancillary good is γ. To avoid trivialities, we assume that $c < v_l$ (so that efficiency calls for selling the basic good to some low-valuation consumers). Proceeding analogously to Section 2, consider consumers with valuation v_k, where

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$k \in \{l, h\}$. Such a consumer buys the basic good alone if $v_k - b \geq p$ and $\tau > b$. She buys both goods if $v_k \geq p + \tau$ and $b \geq \tau$. She buys nothing if $v_k - b < p$ and $v_k < p + \tau$. Accordingly, missed sales occur among k-valuation consumers if and only if $v_k < p + \tau$.

Proceeding as in Section 3, the next proposition describes the firm's pricing behavior. We denote by (p^*, τ^*) the profit-maximizing price pair.

Proposition S1 ***(optimal firm behavior)*** *In the unique equilibrium, the ancillary good is offered by the firm if and only if the bundle is viable for high-valuation consumers: $\gamma \leq v_h - c$. In this case, there exist thresholds γ_1 and γ_2 such that:*

1. ***Below-cost pricing I:*** *If $\gamma < \gamma_1 \leq v_l - c$, then τ^* uniquely solves*

$$\tau - \gamma = -r_G^{-1}(\tau), \tag{S1}$$

   and $p^* = v_l - \tau^*$. No missed sales occur for consumers of either low or high valuations.

2. ***Above-cost pricing:*** *If $\gamma \in [\gamma_1, \gamma_2)$, the ancillary good price is above marginal cost ($\tau^* > \gamma$), and missed sales occur among low-valuation consumers only.*

3. ***Below-cost pricing II:*** *If $\gamma \in [\gamma_2, v_h - c]$, the ancillary good price is below marginal cost ($\tau^* \leq \gamma$), and missed sales occur among low-valuation consumers only.*

4. ***Missed sales:*** *If $\gamma > v_h - c$, the firm does not offer the ancillary good, and missed sales occur among low and high-valuation consumers.*

When the marginal cost of the ancillary good is low, it is not optimal for the firm to incur missed sales with consumers of either low or high valuation. The optimal price pair is then analogous to the one described of Proposition 1 (after evaluating the no-missed-sales condition at $v = v_l$).

Under heterogeneity on the consumers' valuations for the basic good, inducing missed sales among those with low (but not high) valuation may be in the firm's interest. To see why, suppose $\gamma = v_l - c$. In this case, there is no profit to be obtained from selling the ancillary good to low-valuation consumers. By contrast, it is profitable to sell the ancillary good to high-valuation consumers (as $\gamma < v_h - c$). One might then presume that setting τ according to (S1) and choosing p to make the no-missed-sales condition binding for high-valuation consumers (i.e., $p = v_h - \tau$) is optimal for the firm. This is however not true, as the price pair under consideration generates too few sales to low-valuation consumers. Indeed, when only low-valuation consumers incur missed sales, the firm has an incentive to reduce the basic price (which is its only instrument to increase sales to low-valuation consumers), while simultaneously increasing the ancillary good price (so as to extract more rents from high-valuation consumers). This price-discriminatory motive may lead to above-cost pricing on the ancillary good. This occurs only when the marginal cost γ belongs to the intermediate range

$[\gamma_1, \gamma_2)$, where $\gamma_1 \leq v_l - c$ and $\gamma_2 \leq v_h - c$. For instance, when G in the uniform distribution in $[0, 1]$, this range in non-empty provided $\theta_h \theta_l^{-1} + 1 < 2\triangle$.

When the marginal cost of the ancillary good is sufficiently high, the ancillary good is again priced below marginal cost. To see why, first note that, by the same arguments in Proposition 1, the firm is willing to offer the ancillary good if and only if the bundle is viable for some consumer type, i.e, $\gamma \leq v_h - c$. For γ close to this level, the no-missed-sale condition binds for high-valuation consumers, and obtaining a positive margin on the basic good requires subsidizing the consumption of the ancillary one. As such, when the marginal cost γ is sufficiently high, the logic of Proposition 1 is again valid.

The baseline model, by ruling out heterogeneity on v, shuts down the price-discrimination role of ancillary good prices, which are always below cost. This serves to isolate the impact of missed sales on the firm's pricing decisions, and highlight the role of incomplete price information in generating above-cost pricing (hold-ups). In combination with Propositions 1 and 4, Proposition S1 then reveals that only missed sales can generate below-cost pricing on the ancillary good, but that above-cost pricing can be explained by either incomplete price information (hold-ups) or price discrimination (echoing for example Chen-Ross 1993).

The next result reveals that the regulatory implications obtained in the baseline model remain valid in the presence of heterogeneity on the consumer's valuation for the basic good. Because in this richer setting the ancillary good price may lie either above or below marginal cost, optimal regulation calls for combining the cap and floor requirements considered in Section 3 and 4. Accordingly, we study regulation mandating the ancillary good to be priced exactly at marginal cost. As before, our welfare measure combines consumer surplus and the firm profit.

Before stating the result, note that, under the restriction that $\tau = \gamma$, the firm obtains a profit equal to $v_l - c - \gamma$ if no missed sales occur. Denote by $\Pi^{ms}(\gamma, c)$ the maximal firm's profit that results from incurring missed sales.

Proposition S2 ***(marginal-cost pricing)*** *Consider a regulation that requires the ancillary good price to be set at marginal cost ($\tau = \gamma$), and assume that $\gamma \leq v_l - c - \Pi^{ms}(c)$. This regulation strictly increases welfare. Relative to laissez-faire, the firm is worse off and consumers are strictly better off.*

The condition is Proposition S2 is analogous to the one in Proposition 3, requiring that γ is not high enough to generate missed sales when the ancillary good is priced at marginal cost. As in the baseline model, this is a weak condition satisfied in most applications of interest (where the marginal cost of production of the ancillary good is a small fraction of the gains from trade generated by the basic good). Under this condition, regulation makes the firm weakly worse off, as banning loss-making sales constrains its pricing choices. Consumer surplus however increases, as regulation eliminates the price-balancing distortion. As in the baseline model, regulation generates a strict welfare gain by correcting *externalities across participants and non-participants* (who are those consumers who buy both goods, or only the basic good, respectively).

One can further extend the model considered here to introduce an ancillary good supplier with market power. As in the baseline model, cost absorption by the firm (due to concerns about missed sales) leads to supplier to jack up its wholesale price. Regulation in this case, beyond eliminating the price-balancing distortion, reduces the equilibrium wholesale price due to the accountability effect discussed in Section 3.

It is also straightforward (but tedious) to combine heterogeneity on consumer's valuations with positive shopping costs and imperfect price information. In this case, the ancillary good may be priced above cost due to either (or both) hold-ups and price discrimination. Mandating price transparency addresses hold-ups, but does not eliminate above-cost pricing due to price discrimination, nor prevents the inefficiencies produced by give-aways. Optimal regulation then calls for the simultaneous imposition of price transparency (which "opens the eyes" of possibly naïve consumers) and marginal-cost pricing on the ancillary good.

S2. Proofs

Proof of Proposition S1. The solution to the firm's problem belongs to either one of 3 cases.
Case 1: no missed sales occur among any type.

The firm's problem is

$$\max_{p,\tau} \{p - c + (1 - G(\tau))(\tau - \gamma)\}$$

subject to $v_l \geq p + \tau$. This constraint has to bind, in which case the problem is

$$\max_{\tau} \{v_l - \tau - c + (1 - G(\tau))(\tau - \gamma)\}.$$

The optimal ancillary good price then satisfies (S1), and $p = v_l - \tau^*(\gamma)$.

It is convenient to define

$$\Pi_1(\gamma, c) \equiv \max_{p} \{p - c + (1 - G(v_l - p))(v_l - p - \gamma)\} \qquad \text{s.t.} \qquad v_l = p + \tau$$

$$= \max_{p} \{v_l - c - \gamma + G(v_l - p)(p - (v_l - \gamma))\}.$$

When $v_l = c + \gamma$, then $\Pi_1(v_l - c, c) = \max_p \{G(v_l - p))(p - c)\}$. By the envelope theorem,

$$\frac{\partial \Pi_1(\gamma, c)}{\partial \gamma} = -(1 - G(\tau^*)).$$

Case 2: missed sale occur among low types but not among high types.

The firm's problem is to

$$\max_{p,\tau} \{\theta_h(p - c + (1 - G(\tau))(\tau - \gamma)) + \theta_l G(v_l - p)(p - c)\}$$

subject to $v_h \geq p + \tau$. We divide this case into two sub-cases.

Sub-case 2a: The optimum is such that $v_h > p + \tau$. It is convenient to define

$$\Pi_{2a}(\gamma, c) \equiv \max_{p,\tau} \{\theta_h(p - c + (1 - G(\tau))(\tau - \gamma)) + \theta_l G(v_l - p)(p - c)\}.$$

In this case, $\tau^* = \tau^m(\gamma)$ and p^* solves

$$\theta_h - \theta_l g(v_l - p)(p - c) + \theta_l G(v_l - p) = 0, \tag{S2}$$

or, equivalently,

$$p - c = \frac{\theta_h}{\theta_l} \frac{1}{g(v_l - p)} + r_G^{-1}(v_l - p).$$

By the envelope theorem,

$$\frac{\partial \Pi_{2a}(\gamma, c)}{\partial \gamma} = -\theta_h(1 - G(\tau^m(\gamma))).$$

Sub-casease 2b: The optimum is such that $v_h = p + \tau$. In this case, the firm's problem is

$$\Pi_{2b}(\gamma, c) \equiv \max_{\tau} \{\theta_h(v_h - \tau - c + (1 - G(\tau))(\tau - \gamma)) + \theta_l G(\tau - \triangle)(v_h - \tau - c)\}$$

$$= \max_p \left\{ \theta_h (p - c + (1 - G(v_h - p))(v_h - p - \gamma)) + \theta_l G(v_l - p)(p - c) \right\}.$$

By the envelope theorem,

$$\frac{\partial \Pi_{2b}(\gamma, c)}{\partial \gamma} = -\theta_h (1 - G(\tau^*)).$$

The first-order condition is then

$$\theta_h(-G(\tau) - g(\tau)(\tau - \gamma)) + \theta_l \left(g(\tau - \triangle)(v_h - \tau - c) - G(\tau - \triangle) \right) = 0.$$

Let $\Pi_2(\gamma, c) \equiv \max \{\Pi_{2a}(\gamma, c), \Pi_{2b}(\gamma, c)\}$. Notice that

$$\Pi_2(\gamma, c) \geq \Pi_{2b}(\gamma, c) = \max_p \left\{ \theta_h (v_h - c - \gamma + G(v_h - p))(p - (v_h - \gamma)) + \theta_l G(v_l - p)(p - c) \right\}.$$

When $\gamma = v_l - c$,

$$\begin{aligned}
\Pi_2(v_l - c, c) \geq \Pi_{2b}(v_l - c, c) &= \max_p \left\{ \theta_h (v_h - v_l + G(v_h - p))(p - c - (v_h - v_l)) + \theta_l G(v_l - p)(p - c) \right\} \\
&= \max_p \left\{ \theta_h (\triangle(1 - G(v_h - p)) + G(v_h - p)(p - c)) + \theta_l G(v_l - p)(p - c) \right\} \\
&\geq \max_p \left\{ \theta_h \triangle(1 - G(v_h - p)) + G(v_l - p)(p - c) \right\} \geq \Pi_1(v_l - c, c).
\end{aligned}$$

Because $\left|\frac{\partial \Pi_2(\gamma,c)}{\partial \gamma}\right| < \left|\frac{\partial \Pi_1(\gamma,c)}{\partial \gamma}\right|$, this reveals that $\Pi_2(\gamma, c) < \Pi_1(\gamma, c)$ if and only if $\gamma < \gamma_1$ for some $\gamma_1 < v_l - c$.

Case 3: missed sale occur among both types.

The firm's problem is

$$\Pi_3(c) \equiv \max_p \left\{ \theta_h G(v_h - p)(p - c) + \theta_l G(v_l - p)(p - c) \right\}.$$

Notice that $\Pi_3(c) > \Pi_2(\gamma, c)$ only if $\Pi_2(\gamma, c) = \Pi_{2b}(\gamma, c)$. Direct inspection reveals that $\Pi_{2b}(v_h - c, c) = \Pi_3(c)$, what implies that $\Pi_3(c) > \Pi_2(\gamma, c)$ if and only if $\gamma > v_h - c$.

Finally, note that $\gamma_1 < \gamma_2$ if and only if $\tau^m(\gamma) + p^* < v_h$, where p^* is given by equation (S2). If $G = U[0, 1]$, this is the case provided $\frac{1}{2}\left[\frac{\theta_h}{\theta_l} + 1\right] < \triangle$. Depending on parameters, we might have that $\gamma_1 = \gamma_2$, in which case the ancillary good price is always below cost. ■

Proof of Proposition S2. If no missed sales occur, welfare is

$$W = \theta_h v_h + \theta_l v_l - c - \int_0^\tau b g(b) db - (1 - G(\tau))\gamma,$$

which, by the same argument as in Corollary 4, is maximal at $\tau = \gamma$.

If missed occur among low-valuation consumers only, the firm's profit is

$$\Pi_l^{ms}(\gamma, c) \equiv \max_p \left\{ \theta_h (p - c) + \theta_l G(v_l - p)(p - c) \right\} \qquad \text{s.t.} \qquad p \leq v_h - \gamma.$$

In turn, if missed occur among low and high-valuation consumers, the firm's profit is

$$\Pi_{lh}^{ms}(\gamma, c) \equiv \max_p \left\{ \theta_h G(v_h - p)(p - c) + \theta_l G(v_l - p)(p - c) \right\}.$$

The maximal profit that results from incurring missed sales is then

$$\Pi^{ms}(\gamma, c) \equiv \max\{\Pi_l^{ms}(\gamma, c), \Pi_{lh}^{ms}(\gamma, c)\}.$$

If $\gamma \leq v_l - c - \Pi^{ms}(c)$, regulation leads to no missed sales and, by construction, sets the ancillary good price at its welfare-maximizing level. This implies that welfare necessarily increases. The firm is (weakly) worse off, as its pricing choices are constrained, whereas consumers are strictly better off. $\blacksquare$