Managing Diversity to Attract Talent†

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Abstract

How do homophily and talent shape the dynamics of collegial organizations? This paper shows that to avoid depriving itself of its talent pool, an organization's majority may leverage the minority's own homophily by voluntarily engaging in affirmative action. This occurs when the organization, while currently unattractive to the minority, still retains enough quality and diversity to allow a turnaround. A small decrease in the initial quality or diversity of the organization may induce a vicious path instead of a virtuous one. We consider both quality-based and diversity-based interventions and characterize when they improve or reduce welfare.

Keywords: Organizations, diversity, affirmative action, homophily, virtuous and vicious paths, tipping points.

JEL Codes: D7, D02, M5.

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1 Introduction

Humans have a natural inclination to engage with others who share similar identities.[1] Homophily is the proximate cause of discrimination against minorities, undermining the recruitment and promotion of the most talented. Yet in practice, while occurrences of in-group favoritism are well documented, many diversity initiatives arise spontaneously in universities, corporations, and other organizations. From a theoretical perspective, this observation creates a puzzle: Why would an incumbent majority prefering new members to be talented and to belong to the in-group, ever recruit a minority candidate when an at-least-as-talented majority candidate would be eager to join the organization?

This paper shows that majority-run organizations may voluntarily engage in affirmative action to leverage the minority's own homophily preferences, and thereby build or restore the organization's attractiveness to talented would-be members.[2] The paper characterizes when this happens, the consequences for organizational dynamics, and the complementarity or substitutability with public interventions to promote either diversity or quality.

Specifically, we build a simple dynamic model of an organization's recruitments when members and candidates are far-sighted and driven by two motives: quality and homophily. At each instant, a flow of departing members is matched by a flow of incoming recruits selected via majority voting by the existing members.

Individuals have a two-dimensional type: they belong to one of two *horizontal* groups – capturing gender, religion, ethnicity, background, politics, scientific field, etc. –, and are either talented or ordinary (*vertical* dimension). A talented hire brings extra utility (knowledge, prestige, budgets, etc.) to *all* other members of the organization, while homophily benefits accrue *only to members of the hire's in-group*. So, while a higher quality (average talent) for a given horizontal composition makes the organization more attractive to both majority and minority candidates, a stronger homogeneity makes the organization more attractive to majority candidates, and less attractive to minority ones.

To avoid trivial dynamics in which it is a dominant strategy for the majority to recruit only in-group, we assume that quality benefits (per recruit) exceed homophily ones, and that talent is in short supply. Lastly, we assume that talented candidates have higher outside options than ordinary ones.

We show that when individuals are forward-looking, the majority may favor a minor-

[1] See, e.g., Verbrugge (1977), McPherson et al (2001), Currarini et al (2009).

[2] Importantly, we do not assume any direct, causal impact of a workforce's diversity on organizational performance, unlike, e.g., Granovetter (1973) and Page (2008).

ity candidate over an equally talented – or, in a minor generalization of our model, over an even more talented – majority one. Intuitively, such a hiring strategy improves the overall environment for all minority members and may later on make the organization attractive even to the most talented ones. But the effort to make the organization more diverse must be sustained over time, i.e., be time-consistent. We study Markov Perfect equilibria: Strategies are functions of the organization's current quality and diversity. We characterize the environments in which organizational dynamics involve voluntary affirmative action (AA), with the organization's majority picking minority candidates over equally-talented majority candidates.

Strikingly, while a higher initial quality always leads to a (weakly) higher steady-state welfare (defined as aggregate payoff), a lower initial diversity can lead to a higher or a lower steady-state welfare. Indeed, starting from a state in which the organization fails to attract any talented candidate, reducing diversity may help the organization bring back on board talented majority candidates, improving the organization's quality. A higher initial homogeneity improves, resp. harms, the organization's steady-state quality only if the organization's initial quality is low and the outside option large, resp. only if the organization's initial quality is high.

Looking at trajectories, there exist both *virtuous* paths, along which the quality of the organization's new hires improves over time (or remains constant at the maximum level), and *vicious* paths, along which it does not.[3] Along a virtous path, the organization's quality and homogeneity can be nonmonotone over time.

Section 2 provides the main intuitions for a one-shot-appointment campaign, in which part (or all) of the membership is renewed at once. It derives the three conditions – necessity, effectiveness and desirability of AA to attract minority talent – under which voluntary AA arises. The same conditions still play a central role under gradual renewal, together with a fourth one that captures the time-consistency requirement that arises only in an ongoing hiring process.

Section 3 indeed looks at a continuous renewal process, in which hiring policies face a time-consistency problem. We first exhibit an MPE. In this MPE, the minority candidates behave in a myopic fashion: they accept offers if and only if their current flow payoff in the organization exceeds their flow (reservation) utility outside it. The equilibrium features a region of states (in the quality-diversity space) in which the organization engages in "full" AA (adopting the hiring policy that the minority members would myopi-

[3]We label the latter as "vicious" path to contrast them with the "virtuous" paths. As will be clear shortly, there exist tipping points, from which virtuous and vicious paths drift away.

cally choose for themselves), until it becomes diverse enough to attract talented minority candidates, at which point it reduces AA to keep the latter just indifferent between accepting their offers and declining them – and the myopic acceptance strategy is thus the talented minority candidates' best response.

We then envision a process in which the majority draws and announces a recruiting plan in a forward-looking way. A recruiting plan is *feasible* if it respects the candidates' participation constraints. We say that a recruiting plan is *credible* if at any state along its trajectory, the organization's current majority would not want to incur a (even negligible) cost to re-adjust to another feasible recruiting plan – intuitively, it would not want to change course and switch to another continuation equilibrium.[4] We show that the unique credible recruitment plan is the MPE that we earlier described.

The organization does not always engage in AA by itself. A social planner may therefore be tempted to intervene, depending on the available information, to induce the organization to aim for higher diversity and/or higher quality. Section 4.1 studies mandated diversity in the form of a minimal diversity threshold. By preventing the organization from converging to homogeneity, mandated diversity makes standard favoritism less appealing to the majority, and thus reduces its opportunity cost of voluntary AA. The intervention may thus give the organization the "little push" it needs to engage in voluntary AA. Mandated diversity and voluntary AA are then complements. However, when the organization is highly reluctant to engage in voluntary AA and actually struggles to attract talented majority candidates, mandated diversity can prevent the organization from becoming homogeneous enough to attract talented majority candidates. The intervention then traps the organization in a region in which it fails to attract any talent.

Consequently, mandated diversity increases aggregate welfare if and only if (a) the principal is sufficiently patient and puts a sufficiently high weight on the organization's quality (or diversity), and (b) the organization's initial state is sufficiently close to, but not in the region where voluntary AA emerges under *laissez-faire*, i.e. if its initial quality and diversity are sufficiently high. If these conditions fail, the intervention is either neutral or strictly harmful – trapping the organization on a low-diversity/low-quality path. Crucially, the impact of mandated affirmative action goes beyond the (temporarily binding) constraint it imposes on recruitments: by reducing the long-term value of in-group favoritism, it changes (current and future) candidates' prospects and the majority's op-

[4]In fact, our requirement is even weaker as we restrict the organization to switch only to *dominant* recruitment paths (if any), i.e, paths that achieve the maximum feasible payoff at any state along their trajectory.

timal strategy, yielding markedly different trajectories.

Because diversity and quality covary, demanding a high performance may conceivably boost both. Section 4.2 considers a quality floor, arising either from a profitability imperative, or as a principal can use assessment exercises and figure out (at least periodically) the current quality of the organization, committing to dismantle it if its quality falls below a threshold. We show that such (myopic) quality floors have contrasted effects depending on the organization's initial quality. If the organization only needs a "little push" to engage in voluntary AA and its initial quality is sufficiently high, a quality floor can induce the organization to opt for voluntary AA (aiming for high quality) to avoid hitting the floor and being dismantled – quality floors and voluntary AA are then complements. By contrast, if the organization's initial quality is below a threshold, a quality floor dooms the organization to disappear in a finite time regardless of its recruitment strategy. Hence, the organization does not live long enough to reap the benefits of AA: a quality floor then makes the organization switch from voluntary AA to standard favoritism. Consequently, the desirability for a principal of imposing a quality floor echoes the one of imposing a cap on majority size. In both cases, failure to take into account organizational dynamics can make the intervention backfire.

Related literature. Our paper is related to the vast literature on hiring discrimination, and most particularly to two papers, Cai et al (2018) and Moisson-Tirole (2024),[5] that study the interplay of talent and homophily motives in recruitments, but with an emphasis on *control* concerns, whereas this paper focuses on *attractivity* concerns. This paper makes at least four contributions. Firstly, to the best of our knowledge, our paper is the first to show that voluntary affirmative action may result from a concern for not being sufficiently attractive to minority employees,[6] and that it can be sustained over time. Secondly, the presence of an outside option focuses the analysis on mutual expectations about the entire future recruiting policy and its credibility. A technical contribution of our paper is to formalize the credibility issue and to show that, under this requirement, the a priori complex game admits a simple and intuitive solution. Thirdly, our paper describes the existence of non-monotone trajectories for an organization's quality and diversity over time, converging to one of several steady states that can be Pareto-compared

[5]We refer to the literature review therein for additional references.

[6]While the claim that organizations *should* voluntarily engage in affirmative action may be familiar from the management literature, we are not aware of any formal argument relying on homophily alone. For, as noted above, voluntary AA arises in our paper without assuming a direct causal impact of workforce diversity on organizational quality.

– hence, our paper describes both *virtuous* and *vicious* paths. Fourthly, and again to the best of our knowledge, our paper is the first to show that diversity or quality requirements can be either complements or substitutes to voluntary affirmative action, and characterize when each case arises.

In its emphasis on "virtuous" and "vicious" paths, the paper is most closely related to Board et al. (2025). The latter contribution derives rich dynamics from the reasonable assumption that talented people are better at identifying new talents. It shows that high-skill firms post high wages, screen applicants first, extract talent from the applicant pool, and exert a negative compositional externality on low-skill, low-wage firms. Consequently, talent is a source of sustainable competitive advantage. Their trajectories have a different origin from ours: a stronger organization makes more informed hiring choices. Their analysis focuses on the vertical (quality) dimension. In our paper, talented minority (and perhaps also talented majority) candidates turn down an organization that lacks diversity and/or talent.

Our paper shares its focus on the consequences of current cooptation decisions for the dynamics of organizations with several papers, notably Roberts (2015), Barberà et al (2001), Bai-Lagunoff (2011), Acemoglu et al (2012), and more recently, Gieczewski (2021). Our analysis applies readily to firms, clubs, and any organization with the ability to restrict its membership – on a more macro level, it can also apply to countries choosing an immigration policy. Through its focus on open organizations (e.g., cities, trade unions), Gieczewski's (2021) analysis is a complement to ours. Besides making the recruitment decision the focus of our analysis, our paper also adds a vertical dimension to the horizontal differentiation one (to make affirmative action a question of interest even if welfare is solely efficiency-based) and endogenizes the minority candidates' myopic acceptance decision – in contrast, forward-looking majority candidates optimally do not behave myopically.[7]

While in the literature on dynamic political decision-making, the ruling group tends

[7] Gieczewski (2021) assumes agents behave myopically as they are free to join or leave the organization at any point in time. By contrast, in our environment, the agents' participation constraints involve intertemporal considerations. As we will explain, our insights go through if agents are free to leave the organization at any point in time. But the difference with Gieczewski (2021) remains because agents can only join the organization if invited to.

As in Gieczewski (2021), in Bénabou (1993) and Baccara and Yariv (2013, 2016) individuals choose whether joining an "organization" ("community", or "group”) without control by an owner or a vote by the members. Put differently, the organization is a (key) strategic player in our environment, whereas it is passive in those environments. Another significant difference between their environments and ours is that while in theirs, preferences are one-dimensional, they are two-dimensional in ours – a vertical dimension with preference alignment (as in Bénabou, 1993) and a horizontal one with homophily (as in Baccara and Yariv, 2013, 2016).

to resist change in order to avoid “slippery slopes”, in our setting by contrast, due to preference alignment along the vertical dimension and preference misalignment along the horizontal dimension, it can deliberately engage in change, thus triggering virtuous paths.[8]

Lastly, Fershtman and Pavan (2021) study the search for and evaluation of candidates, and examine "soft affirmative action", defined as a policy that increases the relative percentage of minority candidates in the candidate pool. Strikingly, such policies may backfire if the evaluation of minority candidates is noisier than that of majority ones, even if the former are at least as qualified and as valuable as the latter. Our paper assumes away search and evaluation frictions, yet assumes homophily preferences, thus endogenizing the value to current (and future) members of recruiting a given candidate. As a consequence, increasing the share of talented minority candidates can actually switch the organization’s trajectory, putting it on a virtuous path towards high-diversity/high-quality instead of a decline towards zero-diversity/medium-quality.

2 One-time renewal of membership

This Section introduces several key assumptions, motivated through a static version of our model. Together with a time-consistency condition, these assumptions will still govern the dynamics of gradual renewal.

The organization has a unit mass of members. Individuals (members or candidates) have a publicly observable two-dimensional type with a *vertical* component and a *horizontal* one. The vertical type captures ability or talent and takes one of two possible values $\{0, s\}$, where $s > 0$ is the incremental contribution of a talented individual to each other member’s payoff (the contribution of an ordinary member is normalized at 0). The horizontal type stands for race/gender/tastes/opinions/etc., and can take two values $\{1, 2\}$. The organization’s *majority* and *minority* are defined with respect to horizontal types. A member of a given horizontal type exerts an externality $b > 0$ on members of the same type, and 0 on members of the opposite type, regardless of their talent. To make

[8] In Besley et al (2025), an organization incurs short-term costs when implementing socially-motivated policies, but doing so increases the motivation of its (current) workers, who pass their motivation to new hires via workplace socialization. However, new hires are assumed myopic and choose their motivation level once in the organization (they are ex ante identical). By contrast, in our environment, new hires are forward-looking, their payoffs depend on their own and their current and future colleagues’ fixed and observable attributes (talent and horizontal group), and the key strategic decision of candidates is whether to accept or reject the organization’s offers, if any. Accordingly, our environment allows us to ask different questions and study different policy interventions, such as mandated diversity and quality floors.

things interesting, we focus on the case where quality matters more than homophily: $s > b$.[9]

A single individual – the "recruiter" – has a given horizontal type (say, 1) and gets to form the organization from scratch by making offers to candidates.[10] Job offers are observable by all.

The organization is characterized by the couple (M, S), where $M \in [1/2, 1]$ is the majority's size (the majority group will be the recruiter's group) and $S \in [0, 1]$ the share of talented members.[11] For a given post-recruitment state (M, S), the payoffs of a majority and a minority member are thus respectively

$$Ss + Mb \qquad \text{and} \qquad Ss + (1 - M)b.$$

Candidates. There is an excess supply of ordinary candidates of each group, as well as a mass $x < 1/2$ of talented candidates from each group (talent is scarce).[12] Talented and ordinary candidates differ in their outside options. Talented candidates can obtain a payoff $u > 0$ outside the organization, whereas ordinary ones can only obtain a zero payoff. Hence, ordinary candidates always accept their offers, whereas a talented candidate accepts an offer if and only if their utility from joining the organization is greater than or equal to their outside option, that is if and only if expecting a post-recruitment state (M, S),

$$Ss + Mb \geq u, \qquad \text{resp.} \qquad Ss + (1 - M)b \geq u$$

for a talented majority, resp. minority candidate. We refer to these inequalities as the *participation constraints* of talented candidates. As $s > b$, a talented candidate is more likely to accept a recruitment offer if other talented candidates, regardless of their group, accept theirs (there is strategic complementarity among the talented). Consequently, we focus on equilibria in which there is no *coordination failure* within and across groups of talented candidates: talented candidates coordinate on accepting their offers if when

[9] When $s < b$, the majority recruits only in-group candidates.

[10] While the case of building from scratch (or equivalently of full renewal) conveys all key intuitions, we study in Online Appendix F the general, static environment in which a fraction $X \in (0, 1]$ of incumbent members (randomly drawn among current members) exogenously quits the organization. Departing members are replaced by new hires, keeping the mass (1) of members constant. Recruitments are then decided by remaining members according to a (simple) majority vote. Lemma 1 below follows from the general analysis in Online Appendix F.

[11] By symmetry, it is optimal for the recruiter to pick her horizontal type as the majority type.

[12] Analogous insights obtain in the case $x > 1/2$ (excess of talented candidates), with the subcase $x \geq 1$ leading to homogamic recruitments alone.

doing so, their participation constraints are satisfied.

A high-quality-yet-highly-homogeneous organization is unattractive to talented minority candidates if $2xs + xb < u$.[13] Then, attracting talented minority candidates requires recruiting *un*talented minority candidates, which we refer to as voluntary *affirmative action* (AA).[14]

Assumption 1 (**Necessity of AA for the organization to be attractive to talented minority candidates in the static model**). *Affirmative action is necessary for the organization to be attractive to talented minority candidates if*

$$2xs + xb \; < \; u. \tag{N}$$

Specifically, attracting talented minority candidates along talented majority ones requires recruiting at least a mass $(u - 2xs - xb)/b$ of ordinary minority candidates to meet the participation constraint of talented minority candidates.

Such a recruitment objective is thus *effective* only if it does not require the minority to become the majority (as then talented candidates from the former majority would not get on board): $x + \frac{u-2xs-xb}{b} \leq \frac{1}{2}$. The next condition follows.

Assumption 2 (**Effectiveness of AA in the static model**). *Affirmative action is effective in attracting talented minority (and majority) candidates only if*

$$u \; \leq \; 2xs + \frac{b}{2}. \tag{E}$$

Suppose therefore that $2xs + xb < u \leq 2xs + b/2$, so that affirmative action is both necessary and effective to attract talented minority candidates. The recruiter chooses the organization's new recruits by comparing the (highest) payoff conditional on attracting talented candidates of both groups, $2xs+(2xs+b-u)$, and the (highest) payoff conditional on attracting only talented majority candidates, $xs + b$. The next condition follows.

Assumption 3 (**Desirability of AA in the static model**). *Given* (N) *and* (E),

[13] If not (i.e., $2xs + xb \geq u$), then the recruiter can achieve their first-best, recruiting all talented candidates and filling the remaining positions with ordinary candidates from its in-group.

[14] As $s > b$, recruiting a talented minority candidate brings a higher payoff to majority members than recruiting an ordinary majority candidate, and due to the strategic complementarity among the talented, recruiting a talented candidate (of any horizontal type) loosens the participation contraints of all talented candidates more than recruiting an ordinary candidate (of any horizontal type) does. Consequently, the majority tries to recruit all the talented candidates it can, and fills the remaining positions with ordinary (majority or minority) candidates.

engaging in affirmative action to attract talented minority candidates benefits the recruiter if and only if

$$u \ < \ 3xs. \tag{D}$$

Lemma 1 (One-shot renewal). *Supppose conditions* (N), (E), (D) *hold. Then, a recruiter able to choose all organization members in a one-shot campaign engages in voluntary affirmative action.*

Conditions (N), (E) and (D) capture respectively the necessity, effectiveness and desirability of affirmative action when all positions (except the recruiter's) are renewed simultaneously. To focus on the (most interesting) case in which voluntary affirmative action arises, we henceforth assume that conditions $(N),(E),(D)$ hold.

To study the case when the organization is not formed from scratch, but can renew only a fraction of positions at a time, we now introduce our dynamic model.

3 Gradual renewal

3.1 Model

Time is continuous and the horizon infinite: $t \in (-\infty, +\infty)$. As in the static model, the organization has a unit mass of members and individuals have a publicly observable two-dimensional type with a *vertical* component and a *horizontal* one, both with binary values. The vertical type captures talent and takes one of two possible values $\{0, \tilde{s}\}$, where $\tilde{s}dt > 0$ is the incremental contribution of a talented individual to each other member's payoff in the time interval $[t, t+dt)$. The horizontal type can take two values $\{1, 2\}$. A member of a given horizontal type exerts an externality $\tilde{b}dt > 0$ during the time interval $[t, t+dt)$ on members of the same type, but not on members of the opposite type, regardless of their talent. To keep things interesting, we focus (as we did in Section 2) on the case where quality matters more than homophily: $\tilde{s} > \tilde{b}$. Let $r > 0$ be the (pure) rate of time preference.

The organization's current state at a given time t is characterized by the couple (M, S) where $M \in [1/2, 1]$ is the majority's size and $S \in [0, 1]$ the share of talented members at time t. The payoffs during $[t, t+dt)$ of a majority and a minority member are thus respectively

$$[S\tilde{s} + M\tilde{b}]dt \qquad \text{and} \qquad [S\tilde{s} + (1-M)\tilde{b}]dt.$$

Between times t and $t + dt$, a fraction χdt of incumbent members exogenously exit, and χdt new members are recruited according to a (simple) majority vote. At any time t, we rule out coordination failures within the date-t majority, and thus assume that it behaves as a single player.

Between times t and $t + dt$, there is an excess supply of ordinary candidates of each group, as well as $x\chi dt$ potential talented candidates from each group ("potential" meaning that talented candidates may prefer their outside option to applying). To make our insights more salient, we assume $x < 1/2$ (talent is scarce).

Candidates can enter the organization when they arrive (and only then) and have a death rate equal to χ inside or outside the organization. Hence, their discount rate is $r+\chi$.[15] [As will be clear shortly, our assumption of exogenous departures is for simplicity; our insights can be easily adapted if agents can freely quit the organization at any time (see Section 3.2).]

Talented and ordinary candidates differ in their outside option. Talented candidates obtain a flow payoff $\tilde{u}dt$ outside the organization, whereas ordinary ones obtain no flow payoff outside the organization. A talented candidate thus accepts an offer if and only if their utility, i.e., the discounted sum of their future flow payoffs, is greater than or equal to that with the outside option $\tilde{u}/(r + \chi)$, whereas an ordinary candidate always accepts an offer.

While an agent's effective discount factor is $r + \chi$, the (quality or homophily) benefits that a new member brings to a given current member must be discounted by $r + 2\chi$ since the flow probability that either the current member or the new member exits the organization is $2\chi dt$. Accordingly, we define the expected intertemporal contributions of a new member: $s \equiv \tilde{s}/(r + 2\chi)$, $b \equiv \tilde{b}/(r + 2\chi)$. To keep surpluses comparable, we similarly define $u \equiv \tilde{u}/(r + 2\chi)$.

Strategies. We restrict attention to (pure) type-symmetric Markov strategies: hence, any two candidates (or resp. members) sharing the same two-dimensional type share the

[15] Our analysis actually holds with the two alternative interpretations: either each individual leaves the game (foregoing membership and outside-option payoffs) with probability χdt in a time interval $[t, t + dt)$, or each member leaves the organization with probability χdt in a time interval $[t, t + dt)$ and henceforth receives the outside-option flow payoff if talented, and nothing if ordinary.

same strategy, and strategies depend on the history of play only via the organization's current state (M, S).

Candidates' acceptance strategies. As we noted, ordinary candidates have no outside option and thus accept any offer they receive. So, candidates' acceptance strategies are described by the acceptance strategies of talented candidates $\boldsymbol{a} \equiv (a_1, a_2)$, where $a_i(M, S) \in \{0, 1\}$ denotes the acceptance decision of a group-i talented candidate as a function of the organization's current state (M, S), with $a_i(M, S) = 1$ if the candidate accepts a recruitment offer (which the organization may or may not make), and $a_i(M, S) = 0$ if the candidate rejects it.

For future reference, we define the *myopic acceptance strategy* for talented minority candidates as the strategy specifying acceptance of an offer in state (M, S) $(a_2(M, S) = 1)$ if and only if $Ss + (1 - M)b \geq u$. In other words, with the myopic acceptance strategy, talented minority candidates accept the organization's offer if and only if the organization's *current* state yields a flow payoff weakly higher than the flow outside option.

Majority members' recruitment strategies. As standard, we assume that each majority member votes as if they were pivotal. Hence, let $\sigma_1(M, S) \in [0, 1]$, resp. $\sigma_2(M, S) \in [0, 1]$, denote the fraction of talented majority candidates, resp. minority ones, to which the organization makes a recruitment offer in state (M, S), and let $\sigma_0(M, S) \in [0, 1]$ denote the fraction of remaining slots (with mass $\big(1 - x(\sigma_1 a_1 + \sigma_2 a_2)\big)$), that are allocated to ordinary majority candidates (who always accept any recruitment offer). So, $\sigma_0(M, S) < 1$ indicates some *voluntary affirmative action* (the majority selects ordinary out-group candidates over equally ordinary in-group ones). Let $\boldsymbol{\sigma} \equiv (\sigma_0, \sigma_1, \sigma_2)$ denote the organization's (state-contingent) hiring strategy.

Section 3.2 shows the existence, and describe the properties of a pure-strategy, type-symmetric Markov Perfect equilibrium in which talented minority candidates follow the myopic acceptance strategy. We refer to such an equilibrium as a *myopic-minority equilibrium.* Motivating our equilibrium selection, we will later show that the myopic-minority equilibrium we describe in Section 3.2 is the unique equilibrium that satisfies a natural *credibility* criterion (Section 3.3).

3.2 Existence of a myopic-minority equilibrium

Assumptions $(N), (E), (D)$, introduced in the one-shot renewal model of Section 2, transcribe into conditions on steady states, implying that a steady-state absence of affirmative action is bound to put off talented minority candidates (N), that there exists a steady state at which a feasible affirmative action strategy makes the organization attractive to talented minority candidates (E), and that this steady state is preferred by the majority to a steady state in which the organization is homogeneous and attracts only talented majority candidates (D).

Yet, in contrast with the one-shot renewal case, voluntary AA must now be time-consistent to arise when members are gradually renewed:

Assumption 4 (Time-consistency of AA). *Affirmative action can gradually improve the organization's attractiveness to talented minority candidates:*

$$u < xs + (1-x)b. \tag{TC}$$

Condition (TC) is specific to the dynamic model. It states that recruiting ordinary minority candidates along talented majority ones raises the minority candidates' net flow payoff. When it is violated, an organization that currently offers a negative net flow payoff to talented minority candidates, remains unattractive to such candidates even if it sustains a strong form of AA, unless it hires some of them. However, talented minorities accept an offer when the current flow payoff is lower than $\tilde{u}$ only if they are confident that they will later on receive a "rent" (a flow payoff above the reservation flow payoff) in compensation. Yet, under a credibility requirement for the majority's recruitment strategy, the majority later on has no incentive to deliver these future rents to talented minority members, who then must behave myopically and reject the offer. In this sense, (TC) is indeed a time-consistency condition: Because the promise of compensating current losses with future largesse is not credible, if (TC) is violated, then strong AA cannot make the organization ever attractive to talented minorities.

3.2.1 Attracting talented minority candidates

For the sake of exposition, we first investigate the case $u \leq b/2$, in which, provided that the majority does not switch (which is the case in equilibrium), talented majority candidates are always willing to join the organization – indeed, a majority member then secures a flow payoff of at least $\tilde{b}/2$ from homophily alone –, and thus $a_1(M,S) = 1$ at any state (M,S).[16]

Fixing the minority's myopic acceptance strategy, the majority's optimization be-

[16]In addition, $u \leq b/2$ implies that conditions (E) and (TC) hold.

comes an optimal control problem. We show in Appendix A that the dynamics of the majority's (unique) best-reply exhibit four regions. For future reference, we refer to this strategy as the *canonical recruitment strategy.*

Region 1 ($Ss + (1 - M)b > u$): Standard majority favoritism. When talented minority candidates (myopically) strictly prefer accepting their offers, the majority favors its own candidates in the ordinary group ($\sigma_0(M, S) = 1$). Dynamics are given by

$$\frac{dS}{dt} = \chi\Big[-S + 2x\Big], \qquad \text{and} \qquad \frac{dM}{dt} = \chi\Big[-M + (1 - x)\Big].$$

Region 2 ($Ss+(1-M)b = u$): Mild affirmative action to keep talented minority candidates on board. When talented minority candidates are myopically indifferent between accepting an offer or choosing the outside option, the majority selects candidates so as to maintain talented minority candidates' indifference:

$$\frac{d}{dt}\left(Ss + (1 - M)b\right) = 0 \qquad \Longleftrightarrow \qquad \sigma_0(M, S) = \frac{2xs + (1 - x)b - u}{(1 - 2x)b} \equiv \sigma_0^\dagger.$$

Conditions (N) and (E) imply that $\sigma_0^\dagger \in (0, 1)$. Dynamics are given by

$$\frac{dS}{dt} = \chi\Big[-S + 2x\Big], \qquad \text{and} \qquad \frac{dM}{dt} = \chi\Big[-M + x + (1 - 2x)\sigma_0^\dagger\Big].$$

Region 3 ($u > Ss+(1-M)b \geq v$ for some $v \in [0, u)$): Strong affirmative action to make the organization eventually attractive to talented minority candidates. Talented minority candidates turn down current offers. To make the organization eventually attractive to them, the majority selects $\sigma_0(M, S) = 0$, i.e., all ordinary recruits are minority candidates. Dynamics are given by

$$\frac{dS}{dt} = \chi\Big[-S + x\Big], \qquad \text{and} \qquad \frac{dM}{dt} = \chi\Big[-M + x\Big].$$

Region 4 ($Ss + (1 - M)b < v$): Giving up on talented minority candidates. The majority selects only majority candidates, as the "investment cost" to make the organization sufficiently attractive to talented minority candidates is too large. Dynamics are given by

$$\frac{dS}{dt} = \chi\Big[-S + x\Big], \qquad \text{and} \qquad \frac{dM}{dt} = \chi\Big[-M + 1\Big].$$

Under this recruitment strategy, talented minority candidates cannot improve upon

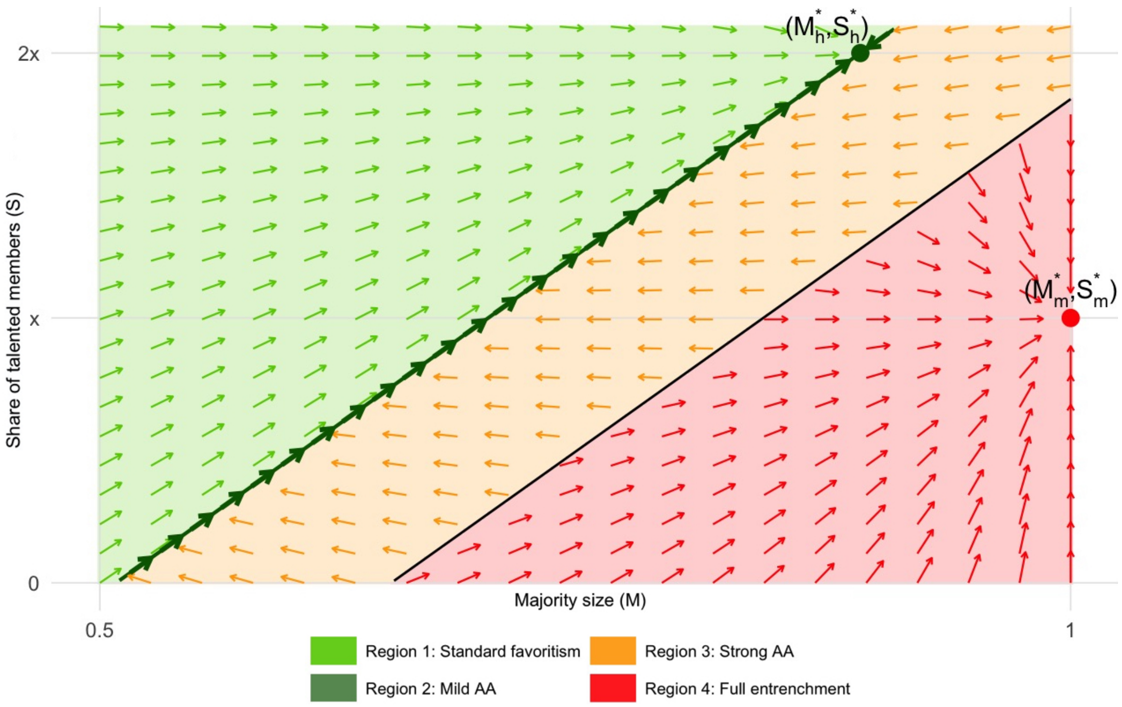

Figure 1: Phase diagram of the organization's majority size M and quality S.[17]

the myopic acceptance rule: in region 2, their flow utility remains equal to $\tilde{u}$; in regions 3 and 4, it is less than $\tilde{u}$ and will be at most $\tilde{u}$ in the future; in region 1, it is above $\tilde{u}$ and decreases down to $\tilde{u}$ upon reaching region 2. Hence, the canonical recruitment strategy for the majority and the myopic acceptance strategy for talented minority candidates form an equilibrium.

Let us flesh out the description of this equilibrium (see Figure 1). There exist two steady states, located in regions 2 and 4 respectively: a high-diversity/high-quality steady state (M_h^*, S_h^*), with $M_h^* = 1 - (u - 2xs)/b \in (1/2, 1 - x)$ and $S_h^* = 2x$, and a zero-diversity/medium-quality steady state $(M_m^*, S_m^*) = (1, x)$. Both majority and minority members have higher payoffs in the high-diversity/high-quality steady state (M_h^*, S_h^*). The organization's path is uniquely determined by its initial state. Starting from regions 1 or 3, the organization always reaches region 2 in a finite time, and then converges to (M_h^*, S_h^*), whereas starting in region 4, the organization converges to (M_m^*, S_m^*).

Strikingly, strong and immediate affirmative action is optimal in region 3: if an organization currently unattractive to talented minority candidates engages in affirmative action, it does so from date 0 onwards, and "at full scale", recruiting in fact the *minor-*

[17] For generality, Figure 1 allows S to exceed $2x$. For instance, there might have been a more favorable supply of talent prior to date 0.

ity's preferred candidates (talented candidates and ordinary minority candidates),[18] until reaching region 2 in a finite time. Region 3 is thus the set of states (M, S) at which the organization is currently unattractive to talented minority candidates and such that the majority's "homophily sacrifice" is worth the investment. Specifically, region 3 is the set of states (M, S) with $Ss + (1 - M)b < u$ such that

$$\left(\frac{Mb - Ss + x(s-b)}{xs + (1-x)b - u}\right)^{r+\chi} \leq \left(\frac{3xs + (1-x)b - u}{(1-x)b}\right)^{\chi}, \qquad (1)$$

where condition (D) implies that the RHS is strictly higher than 1 and increases with the turnover rate χ, while condition (TC) implies that the LHS is also strictly higher than 1 for any (M, S) with $Ss - Mb < u - b$, and increases with the discount rate, $r + \chi$. Intuitively, the RHS captures the long-term benefits from voluntary AA (the faster the turnover, the closer such benefits), while the LHS captures its costs (the further away from region 2 the initial state (M, S) or the higher the agents' *im*patience $r + \chi$, the higher such costs).

As a consequence, the boundary between regions 3 and 4 is an increasing line in the plane (M, S), with slope b/s, and thus parallel to region 2. Moreover, under conditions (N), (E), (D), (TC), region 4 is non-empty as it includes the state $(1, x)$,[19] and it is absorbing: When the organization starts in region 4, it converges towards the zero-diversity/medium-quality steady state $(M_m^*, S_m^*) = (1, x)$.

Proposition 1 (Voluntary affirmative action when $u \leq b/2$). *Suppose conditions $(N), (E), (D), (TC)$ hold, and $u \leq b/2$ (so that talented majority candidates are always on board). There exists a unique myopic-minority equilibrium. In this equilibrium (which includes regions 1 to 4), the organization's trajectory in the state space (M, S) is uniquely determined by its initial state, and converges to one of two steady-states – either a high-diversity/high-quality one, or a zero-diversity/medium-quality one. Moreover,*

(i) ***Transitory voluntary AA:*** *There exists a range of states (region 3) in which the*

[18] As noted before, condition (TC), which holds for $u \leq b/2$, implies that strong affirmative action steers the organization closer to regions 1 and 2 (if it exists) and away from region 4.

[19] Intuitively, the non-emptiness of region 4 stems from the double role of the turnover rate, χ, which determines both the renewal rate of one's colleagues, but also one's own time horizon within the organization. As a thought experiment, if members failed to take into account the latter (hence, discounting future payoffs at rate r instead of $r + \chi$), then the frontier between regions 3 and 4 would be given by

$$\left(\frac{Mb - Ss + x(s-b)}{xs + (1-x)b - u}\right)^{r} \leq \left(\frac{3xs + (1-x)b - u}{(1-x)b}\right)^{\chi},$$

which, for χ sufficiently high (a sufficiently fast turnover), would be satisfied for any (M, S) such that $Ss - Mb < u - b$, i.e. region 4 would be empty.

majority engages in full-scale voluntary AA ($\sigma_0(M,S)=0$) to get talented minority candidates on board in the future, and reaches region 2 in a finite time.

(ii) ***Long-run voluntary AA:*** *Affirmative action is never at full scale in the long-run ($\sigma_0(M,S)>0$).*[20]

(iii) ***Tipping points:*** *There exist tipping points (forming a line with slope b/s) from which a lower initial quality S_0, or a larger initial majority size M_0, generates a vicious path (instead of the virtuous path selected at the tipping point), and a lower steady-state utility for both majority and minority members.*

Remark: Weak and strong affirmative action. The organization's voluntary AA in regions 2 and 3 can be considered as a *weak* form of AA (breaking vertical ties between majority and minority ordinary candidates in favor of the latter, but not promoting a strictly less talented minority candidate against a strictly more talented majority candidate). However, under analogous conditions, strong voluntary AA would exist in equilibrium even if the majority's ordinary candidates were (slightly) more productive than their minority counterparts.

Remark: Coalitions of talented minority candidates. The equilibrium described in Section 3.2 is robust to deviations by coalitions of talented minority candidates (keeping as given the strategies of majority members and candidates) in the sense that no coalition of (current and future) talented minority candidates can deviate and secure a weakly higher payoff for all coalition members. Notwithstanding, such minority-candidates coalition-proofness is not sufficient to yield AA as an equilibrium outcome. For instance, a *distrust* equilibrium, robust to deviations by coalitions of talented minority candidates, can exist in which talented minority candidates expect the organization never to engage in AA and the organization never does (see Online Appendix I).

Comparative statics. As turnover (χ) increases, the organization's speed along trajectories increases. As a result, a higher turnover widens the set of initial states for which voluntary affirmative action arises (region 3 expands) and from which the high-diversity/high-quality steady state is reached. Likewise, a higher patience (a lower r) makes the prospect of a higher future quality more attractive, and thus widens the set of

[20]While affirmative action in the long-run remains strictly positive ($\sigma_0(M,S)<1$) in our main case of interest ($2xs+xb<u$), it is nil in the case $u \leq 2xs+xb$ (which we sidelined).

initial states for which voluntary affirmative action arises.[21]

Corollary 1 (**Comparative statics: Turnover and patience**). *Suppose conditions* $(N), (E), (D), (TC)$ *hold, and that* $u \leq b/2$. *A faster turnover or a greater patience foster voluntary affirmative action and improve the organization's long-run quality and diversity.*

However, in the limit, the cases $\chi \to +\infty$ (infinite turnover) and $r \to 0$ (infinite patience) in the dynamic framework do not coincide with the one-shot static framework of Section 2: in the continuous-renewal framework, voluntary affirmative action never obtains for all states (M, S) such that $Ss - Mb < u - b$, i.e., region 4 never vanishes. Indeed, in the dynamic framework, a current member eventually leaves the organization at a strictly positive rate. In fact, a member's discount rate is the sum of their (pure) time-preference rate, r, and the rate at which they leave the organization, i.e., the turnover rate χ. So in particular, the higher the turnover, the more *impatient* the agents, and even with a zero rate of time-preference ($r = 0$), agents remain finitely patient as long as there is a strictly positive turnover ($\chi > 0$).

The double social dividend of minority education/training. Suppose that due to either educational discrimination or social norms (e.g., underrepresentation of the minority in certain professions), the fraction of talented majority candidates, x, and the one of talented minority candidates, y, satisfy: $y < x < 1/2$ with y sufficiently close to x.[22] The same arguments as before yield the existence of a myopic-minority equilibrium with the same four regions as above.

An (unanticipated) increase in the share of talented minority candidates y (e.g., by reducing educational discrimination or changing a social norm) has no impact on the boundaries of regions 1 and 2 (as long as conditions (N), (E), (D) still hold): Since these regions are determined by talented minority candidates' *myopic* acceptance, they depend only on the current state and parameters s, b, u, and do not depend on parameters that affect only future payoffs, such as the shares of talented candidates, x, y, and the discount

[21] That patience fosters voluntary AA may be contrasted with the result in Moisson-Tirole (2024) that patience fosters entrenchment and majority favoritism (see Proposition 2 therein). The focus in Moisson-Tirole (2024) is on control over the organization's (future) recruitments, while here it is on insufficient attractiveness to talented (minority) candidates. When control is at stake (and control lies in numbers), a higher patience heightens the importance of preserving the majority, thereby inducing stronger majority favoritism. By contrast, when control is not at stake, a higher patience heightens the importance of long-run quality, making transitory homophily sacrifices worthwhile.

[22] Namely, y is such that conditions $(N), (E), (D)$ still hold (condition (TC) remains unchanged): resp. $(x + y)s + yb < u$, $u \leq (x + y)s + b/2$ and $u < (x + 2y)s$.

and turnover rates, r, χ.

By contrast, an (unanticipated) increase in the share of talented minority candidates y has two mutually reinforcing effects on the organization's willingness to engage in voluntary AA: A higher y increases the majority's (and minority's) quality payoff in regions 1 and 2; moreover, as $s > b$, it relaxes the minority's participation constraint in region 2, allowing the majority to reduce voluntary AA in region 2 ($\sigma_0^\dagger$ increases with the share of talented minority candidates). Consequently, an (unanticipated) increase in the share of talented minority candidates expands region 3 and reduces region 4. It can thus make the organization switch from majority favoritism to voluntary AA, converging to the high-diversity/high-quality steady state instead of the zero-diversity/medium-quality steady state.[23]

Corollary 2 (**The double social dividend of minority education**). *Reducing discrimination in education and training can yield a double dividend: it increases the number of talented minority candidates and members (intensive margin), and takes the organization on a higher-diversity and higher-quality path by voluntarily embracing affirmative action (extensive margin).*

Extensions: Voluntary departures and lenient layoff regulation. If members can freely quit the organization at any time (*voluntary departures)*, the above strategies still form an equilibrium, except that talented minority members immediately leave the organization in regions 3 and 4. Their departure causes a drop in the organization's quality and diversity. The organization thus falls further away from region 2, before either engaging in voluntary AA (if it remained in region 3) or majority favoritism (if it fell in region 4).

If a majority of voters can decide to fire any member (*lenient layoff regulation*), then, when in region 3 *and* in some states of region 4 (close to region 3), talented majority members would now like to engage in AA, laying off ordinary majority members and immediately replacing them with ordinary minority members, so as to reach region 2 instantaneously. By contrast, when in region 3, ordinary majority members would still prefer AA if they knew their position were safe, but may now oppose it as they may expect

[23]Similarly, mentoring by current members may improve the quality of candidates within the same horizontal group, as in Athey et al (2000). Hence, the larger a group's size within the organization, the larger the fraction of talented candidates of that group. In particular, if the minority gains more from mentoring than the majority (e.g., if mentoring gains follow a "learning curve"), then a minority-mentoring program can make the organization switch from majority favoritism to voluntary AA, i.e., from a zero-diversity/medium-quality path to a high-diversity/high-quality path.

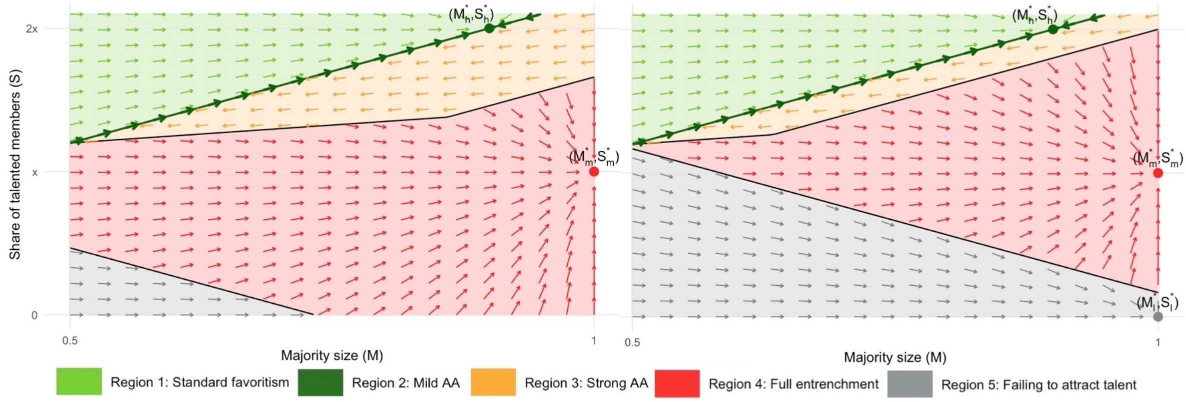

Figure 2: Phase diagram of the organization's majority size M and quality S when $u > xs + b/2$, for high (resp. low) χ/r on the left (resp. right).

being laid off along the way.[24] Hence, strikingly, when layoffs are possible, a misalignment arises *within the majority*: talented majority members now favor AA (paired with the dismissal of ordinary majority members) for a larger set of initial states, while ordinary minority members now oppose AA for a larger set of initial states.

By contrast, when in region 4 and further away from region 3, both talented and ordinary majority members would like to lay off ordinary minority members (immediately replacing them with ordinary majority members) so as to move closer to region 4's steady-state, (M_m^*, S_m^*), as fast as possible: preferences within the majority remain aligned, at the expense of ordinary minority members.

3.2.2 Failing to attract talented majority candidates

When $u > b/2$, the willingness of talented majority candidates to join the organization can no longer be taken for granted. In a myopic-minority equilibrium, a fifth region can exist in which the organization fails to attract talented candidates, and moves towards homogeneity and zero quality ($M_l^* = 1$, $S_l^* = 0$), until possibly leaving the region. This fifth region emerges notably when the outside option is sufficiently high, and when the turnover-to-impatience ratio (χ/r) is sufficiently low. Intuitively, the organization then fails to renew its composition fast enough, so that the future benefits from voluntary AA remain too distant in time, while its sacrifices last for too long.

Interestingly, when $u > b/2$, a stronger initial homogeneity (larger initial majority size M_0) can enhance the organization's steady-state quality by allowing the organization to attract talented majority candidates and converge towards the steady state (M_m^*, S_m^*)

[24] In region 3, talented majority members and talented and ordinary minority members all favor laying off (some) ordinary majority members.

instead of the zero-diversity/zero-quality steady state (M_l^*, S_l^*). Moreover, for a given initial state in region 5, the organization may escape the region and start a virtuous path by becoming sufficiently homogeneous to attract talented majority members thanks to homophily prospects: it can do so only if the outside option (u) is not too large, and for an intermediate u, only if the turnover-to-impatience ratio (χ/r) is not too large. Otherwise, region 5 is absorbing and the organization is trapped on a path towards zero-diversity and zero-quality. Figure 2 provides an illustration.[25,26]

Proposition 2 (**Virtuous and vicious paths when** $u > b/2$)**.** *Suppose conditions* $(N), (E), (D), (TC)$ *hold, and* $u > b/2$ *(so that the acceptance of talented majority candidates cannot be taken for granted). There exists a myopic-minority equilibrium that achieves the upper bound on a majority member's payoff from any state (M,S). There are at most five regions in the space* (M, S)*: regions 1 to 4 as described earlier, and region 5 where no talented candidates join the organization and full majority favoritism prevails. Moreover,*

(i) ***Failing to attract majority talent:*** *The organization fails to attract talent for some initial states if the turnover-to-impatience ratio* (χ/r) *is low or the outside option (u) high.*

(ii) ***Tipping points:*** *In addition to the tipping points described earlier, there can now exist tipping points from which a larger initial majority size* M_0 *generates a higher steady-state quality for the organization and a higher utility for both majority and minority members (if any). A higher initial homogeneity improves, resp. harms,*

[25] Specifically, the boundary between regions 4 and 5 is given by the talented majority candidates being indifferent between rejecting their offers, or accepting them (incurring a short-term loss for a later rent converging to $x\tilde{s} + \tilde{b} - \tilde{u} > 0$). Appendix B shows that, in the equilibrium we study, the boundary between regions 4 and 5 is given by the set of states (M, S) with

$$Ss + Mb = u - \frac{\chi}{r+\chi}(xs + b - u),$$

and that regions 5 and 3 have no shared boundary. Since trajectories are straight lines, region 5 is absorbing if and only if the state $(1, 0)$ lies strictly below this boundary, i.e., if and only if

$$u > b - \frac{\chi}{r+2\chi}xs.$$

Otherwise, any trajectory that starts initially in region 5 leaves it in finite time.

[26] In contrast to the cases $u \leq b/2$ and $u \geq xs + b/2$ in which region 4 is absorbing, when $b/2 < u < xs + b/2$, some trajectories starting in region 4 reach region 3 in a finite time, and from there region 2, thus engaging the organization on virtuous paths along which diversity first decreases, then increases, before decreasing again, as the organization progresses towards (M_h^*, S_h^*). However, in all cases, any trajectory starting in region 5 (if any) never reaches region 3, and converges either to (M_l^*, S_l^*) or (eventually) (M_m^*, S_m^*) (see Appendix B).

the organization's steady-state quality only if the organization's initial quality is low, resp. high.

Maybe surprisingly, while for $u \leq b/2$, a larger initial majority size always generates a (weakly) lower steady-state quality for the organization and steady-state utility for all members, the opposite may hold if $u > b/2$. Indeed, when region 5 is non-empty and the organization's initial quality is low, a higher initial homogeneity may allow the organization to start in region 4 rather than in region 5, converging to the steady state (M_m^*, S_m^*) instead of (M_l^*, S_l^*). By contrast, when the organization's initial quality is high, a higher initial homogeneity risks making the organization start in region 4 rather than in region 3, converging to (M_m^*, S_m^*) instead of (M_h^*, S_h^*) (as in the case $u \leq b/2$).

Remark: Group-specific outside options. Our analysis can be extended to environments in which talented candidates have different outside options depending on their horizontal group: u_1, u_2. In particular, when the outside option of talented majority candidates, say type 1, is higher than the one of talented minority candidates ($u_1 > u_2$), region 5 may have a shared boundary with regions 1, 2 and 3 (when they still exist). When the gap between u_1 and u_2 is sufficiently wide, new regions may exist in which talented majority candidates decline the organization's offers, while talented minority candidates accept theirs.

3.3 Equilibrium selection: Credible recruitment plans

To vindicate our focus on the equilibrium described in Section 3.2, we now argue that it is the unique equilibrium to satisfy a *credibility* criterion. A *recruitment plan*, $\boldsymbol{\rho} = \{\boldsymbol{\sigma}(M,S), \boldsymbol{a}(M,S)\}_{(M,S)}$, describes hiring and acceptance strategies, $\boldsymbol{\sigma}$ and $\boldsymbol{a}$ respectively. Starting at an arbitrary state (M_t, S_t), this recruitment plan defines in the state space a trajectory $\mathcal{T}^{\boldsymbol{\rho}}(M_t, S_t)$, a set of points for $\tau \geq t$, starting at (M_t, S_t). Given two recruitment plans $\boldsymbol{\rho}, \boldsymbol{\rho}'$, we say that $\boldsymbol{\rho} \underset{(M,S)}{\neq} \boldsymbol{\rho}'$ ("$\boldsymbol{\rho}$ and $\boldsymbol{\rho}'$ differ at state (M,S)") if the trajectories induced by two recruitment plans starting from (M,S) differ over a set of time indices with strictly positive measure.

Definition (**Feasible recruitment plan**). *A recruitment plan $\boldsymbol{\rho}$ is feasible from an initial state (M_t, S_t) if obedience constraints are satisfied: letting for all $\tau \geq t$, $V_i^{\boldsymbol{\rho}}(M_\tau, S_\tau)$ denote the present discounted payoff of a date-τ member of type $i \in \{1,2\}$ under recruitment plan $\boldsymbol{\rho}$, then $a_i(M_\tau, S_\tau) = 0$ if $V_i^{\boldsymbol{\rho}}(M_\tau, S_\tau) < \tilde{u}/(r+\chi)$ and $a_i(M_\tau, S_\tau) = 1$ if*

$V_i^{\boldsymbol{\rho}}(M_\tau, S_\tau) > \tilde{u}/(r+\chi)$.

Let $\mathcal{F}_0$ denote the set (over all possible initial states) of feasible recruitment plans.

Definition (**Dominant recruitment plan**). *Starting at an arbitrary state* (M_t, S_t)*, a feasible recruitment plan* $\boldsymbol{\rho}$ *is dominant if for all states* $(M_\tau, S_\tau) \in \mathcal{T}^{\boldsymbol{\rho}}(M_t, S_t)$*, any other feasible recruitment plan* $\boldsymbol{\rho}' \in \mathcal{F}_0$ *satisfies* $V_1^{\boldsymbol{\rho}}(M_\tau, S_\tau) > V_1^{\boldsymbol{\rho}'}(M_\tau, S_\tau)$ *whenever* $\boldsymbol{\rho}' \underset{(M_\tau,S_\tau)}{\neq} \boldsymbol{\rho}$.

That is, along a dominant trajectory, the majority would not want to change course and announce a new feasible recruitment plan at date τ, even if the latter were believed to hold forever after by current and future candidates.

Definition (**Non-credible recruitment plans**). *Starting at state* (M_t, S_t)*, a feasible recruitment plan* $\boldsymbol{\rho}$ *is not credible if and only if for some* $\tau \geq t$*, there exists a dominant recruitment plan* $\boldsymbol{\rho}' \underset{(M_\tau,S_\tau)}{\neq} \boldsymbol{\rho}$ *at* $(M_\tau, S_\tau) \in \mathcal{T}^{\boldsymbol{\rho}}(M_t, S_t)$ *with* $V_1^{\boldsymbol{\rho}'}(M_\tau, S_\tau) > V_1^{\boldsymbol{\rho}}(M_\tau, S_\tau)$.

Intuitively, a non-credible recruitment plan would be abandoned by the organization whenever it would cross the trajectory of a *dominant* recruitment plan, as the current majority would then like to switch to the latter.

Let $\mathcal{D}_0(M_t, S_t)$ denote the set of non-credible recruitment plans starting from a state (M_t, S_t). We let $\mathcal{F}_1 = \mathcal{F}_0 \setminus \left(\cup_{(M_t,S_t)} \mathcal{D}_0(M_t, S_t) \right)$ denote the set of feasible plans once the non-credible ones are eliminated. We then iterate: letting $\mathcal{D}_1(M_t, S_t)$ denote the set of recruitment plans starting from (M_t, S_t) that are not credible among the set of recruitment plans $\mathcal{F}_1$, we construct $\mathcal{F}_2 = \mathcal{F}_1 \setminus \left(\cup_{(M_t,S_t)} \mathcal{D}_1(M_t, S_t) \right)$, etc.

Definition (**Credible recruitment plans**). *Credible recruitment plans are given by* $\mathcal{F}_\infty$.

Importantly, there always exist a credible recruitment plan (i.e, $\mathcal{F}_\infty \neq \emptyset$), as there remains at least one credible recruitment plan from each initial state.[27]

We refer to the myopic-minority equilibrium strategies described in Section 3.2 as the *canonical strategies.* Appendix C first shows that the *canonical recruitment plan* induced by these strategies is dominant from any state $(M, S) \in R_2$ and hence, any credible recruitment plan must coincide with this recruitment plan at any state $(M, S) \in R_2$. Next, to attract talented minority candidates when the current state would give them a

[27]In fact, since a dominant recruitment plan may not exist, it may be that all feasible recruitment plans are credible. Hence, our credibility requirement is a weak criterion, which makes our uniqueness result below even stronger.

flow payoff strictly lower than the outside option, the majority needs to promise them future rents, which requires crossing the indifference line $\{(M, S) \mid Ss + (1 - M)b = u\} = R_2$. However, such promises are not credible, as whenever the organization reaches the indifference line and therefore attracts the talented minorities, the majority would want to change course and remain on (or below) the indifference line.

Since the canonical recruitment plan induced by the canonical strategies is dominant starting from region 2, it does not belong to $\mathcal{D}_0(M, S)$ for any $(M, S) \in R_2$. Moreover, from any state $(M, S) \notin R_2$, for a recruitment plan $\boldsymbol{\rho}$ other than canonical one starting at (M, S) to be dominant and outperform the canonical recruitment plan, the trajectory $\mathcal{T}^{\boldsymbol{\rho}}(M, S)$ must reach R_2 at some finite time,[28] after which the recruitment plan $\boldsymbol{\rho}$ should coincide with the canonical recruitment plan. Yet, fixing the recruitment plan over R_2 and the coordination of future talented candidates, the canonical recruitment plan is the generically[29] unique optimal recruitment plan for the majority from any state $(M, S) \notin R_2$. Hence, the canonical recruitment plan belongs to $\mathcal{F}_1$, and it is dominant within $\mathcal{F}_1$ from any initial state.

Proposition 3 (**Credible recruitment plans and canonical strategies**). *Assume conditions* (*N*)*,* (*E*)*,* (*D*)*,* (*TC*) *hold. The canonical recruitment plan is the generically unique credible recruitment plan.*

Remark: Credibility and renegotiation-proofness. Our credibility criterion can be compared with the renegotiation-proofness criterion introduced by Farrell and Maskin (1989). While the latter requires the renegotiation to (weakly) benefit all players, the credibility criterion enables a single key player – here, the current majority of members – to "choose" the continuation equilibrium, regardless of what other players would favor. Hence, while the renegotiation-proofness criterion thus implies a notion of cooperativeness and equal standing among players, the credibility criterion fits environments in which players are naturally asymmetric, and in which a collective renegotiation is unrealistic.

[28]Indeed, remaining in region 1 forever yields a strictly lower majority size after some finite time, while remaining in the union of regions 3, 4 and 5 forever implies that the majority can do not better than standard favoritism.

[29]Any two credible recruitment plans can differ only on the shared boundaries of regions 3 and 4, and 4 and 5. Selecting the Pareto-dominating equilibrium on such boundaries would yield that the canonical recruitment plan is *the* unique credible recruitment plan.

4 Policy

For each policy intervention we will consider, the existence of a (generically) unique *credible* equilibrium follows from the same arguments as under laissez-faire.[30] We refer to this equilibrium as the *constrained canonical equilibrium.* In particular, it exhibits (generically) at most the same five regions as the canonical equilibrium described in Section 3.2: At any state (M, S), the equilibrium dynamics are described by those of one of the five regions – except when policy constraints are immediately binding.

The principal's objective is the net present value of all members' flow payoffs (possibly with positive externalities from the organization's quality):

$$W \equiv \int_0^{+\infty} e^{-r_P t}\left(qS_t\tilde{s} + \left[M_t^2 + (1-M_t)^2\right]\tilde{b}\right)dt$$

where $r_P > 0$ is the principal's discount rate and $(q-1) \geq 0$ captures quality externalities outside the organization (if any).

4.1 Mandated diversity and voluntary AA

When should diversity requirements be imposed on the organization? Suppose a principal – e.g., a government, a university's dean or a firm's executive – observes members' and candidates' horizontal types, and can force the organization's majority size to remain (weakly) below a cap $\overline{M} \in [M_h^*, 1)$.

Proposition 4 (**Mandated diversity**)**.** *Suppose that* $(N), (E), (D), (TC)$ *hold. In the* $\overline{M}$*-constrained canonical equilibrium relative to laissez-faire,*

(i) ***Crowding in voluntary AA:*** *Region 3 expands at the expense of region 4. The cap* $\overline{M}$ *makes standard favoritism less appealing and can induce the organization to engage in voluntary AA.*

(ii) ***Discouraging talented majority candidates:*** *Region 5 expands at the expense of region 4. By making standard favoritism less appealing, the cap* $\overline{M}$ *can make the organization unattractive to talented majority candidates.*

[30]Indeed, as shown in the proof of Proposition 3 (see Appendix C), a necessary condition for credibility is that in equilibrium, talented minority candidates accept their offers myopically (accepting in state (M, S) if and only if $Ss + (1-M)b \geq u$). We then show that there exists a (generically) unique equilibrium that achieves an upper bound on the majority's payoff conditional on myopic acceptance by talented minority candidates.

(iii) ***Breaking virtuous trajectories:*** *When in the absence of cap, a virtuous path involves increasing homogeneity, the cap* $\overline{M}$ *can trap the organization in region 5, forcing it into a vicious path.*

Importantly, the impact of mandated diversity goes beyond the immediate constraint on the majority size. Indeed, mandated diversity in the form of a cap on majority size reduces the value of in-group favoritism, thereby improving the relative gains from voluntary AA, while also reducing the organization's ability to leverage homogeneity in order to attract talented majority candidates. Consequently, depending on the organization's initial state, mandated diversity can either improve long-run quality – if the organization's initial state is in region 4 and close to region 3 –, or decrease it – if the organization's initial state is in region 5 or in region 4 and close to region 5.

Therefore, the principal benefits from a cap on majority size if and only if (a) it is sufficiently patient and puts a sufficiently high weight (q) on the organization's quality, and (b) the organization's initial state is sufficiently close to the region where voluntary affirmative action emerges under *laissez-faire* and its initial quality is sufficiently high (see Online Appendix J).

4.2 Quality floors and voluntary AA

This Section considers a quality floor rather than a majority cap. Such a floor may arise for at least two reasons. The first is that low quality implies low profitability. For given production costs, a low quality generates losses and the possibility that the organization may be terminated. In the second interpretation, a principal can use assessment exercises and figure out (at least periodically) the current quality of the organization – in contrast, it may not be able to figure out its horizontal diversity. How does a quality floor impact the organization's willingness to engage in affirmative action?

For simplicity, we consider the extreme case of a *myopic* quality floor: $S_t \geq \underline{S}$ for all $t \geq 0$, for some minimal quality $\underline{S}$. If the organization's quality falls below the quality floor at some date t, the organization disappears and its members return to the outside option (with flow $\tilde{u}$ for the talented).[31] To make things interesting, we assume that $\underline{S} \in (x, 2x)$.[32] Then, the organization is bound to be terminated in a finite time unless it

[31] As mentioned earlier, it does not matter for our analysis whether χ is interpreted as the turnover rate for organization members, or as the "death" rate for individuals both inside and outside the organization (see footnote 15).

[32] If $\underline{S} \leq x$, the dynamics at any state (M, S) with $S > \underline{S}$ do not change with respect to laissez-faire. Indeed, in region 5, all talented candidates declined their offers even without the quality floor (and

manages to get talented minority candidates on board.

Such a quality floor has contrasted effects depending on the organization's initial quality. When the latter is high, region 3 expands at the expense of region 4: converging to the medium-quality steady state of region 4 leads the organization to doom in a finite time, while voluntary affirmative action enables it to survive by converging to the high-quality steady state of region 2, provided that the organization's initial quality is sufficiently high that it can reach region 2 before hitting the quality threshold. By contrast, when the organization's initial quality is low, a myopic quality floor (locally) shrinks region 3 to the benefit of region 4: then, regardless of its recruitment strategy, the organization is doomed to disappear in a finite time and thus does not live long enough to reap the benefits of affirmative action (reaching region 2).[33]

Lastly, in region 5 (when it includes states above $S = \underline{S}$), a myopic quality floor may break virtuous paths that would have taken the organization outside of region 5 and into region 4 – intuitively, the organization hits the floor before becoming homogeneous enough to attract talented majority candidates. Hence, region 5 may expand at the expense of region 4.

Proposition 5 (**Quality floors and voluntary AA**). *Suppose that* (N)*,* (E)*,* (D)*,* (TC) *hold. Suppose that the* $\underline{S}$*-constrained canonical equilibrium is selected. A myopic quality floor (*$\underline{S} \in (x, 2x)$*) triggers voluntary AA instead of standard favoritism when the organization's initial quality is high, but triggers standard favoritism instead of voluntary AA when the organization's initial quality is low.*

The desirability for a principal of imposing a quality floor (committing to dismantle the organization if its quality falls below) echoes the one of imposing a cap on majority size. Namely, the principal benefits from setting a quality floor if and only if (a) it is sufficiently patient and puts a sufficiently high weight on the organization's quality, and (b) the organization's initial state is sufficiently close to the region where voluntary affirmative action emerges under *laissez-faire* and its initial quality is sufficiently high. In both cases, failure to take into account organizational dynamics can make the intervention backfire.

so the organization cannot change its course given the quality floor), whereas in the other regions, the organization's quality moves away from the quality floor (recruiting at least majority talented candidates), and thus the floor is never binding.

[33] A myopic quality floor does not affect the dynamics in regions 1 and 2.

5 Concluding remarks

This paper delivers a rich set of testable implications regarding organizational recruitment dynamics. It shows that when the organization is currently unattractive to talented minority candidates but not too much so, it voluntarily engages in affirmative action to get talented minority candidates on board in the future, thereby creating a virtuous path. While diversity decreases along any vicious path, it is either decreasing or non-monotonic along virtuous paths – e.g., increasing then decreasing, or decreasing then increasing then decreasing again.

The impact of diversity-based or quality-based external interventions can go well beyond the immediate constraints they impose when binding. Such interventions have expectational effects and can either generate a virtuous path, delivering higher long-term welfare when the organization's initial quality is sufficiently high, or backfire and trigger a vicious path when the organization's initial quality is lower.

The paper's main contributions have been (1) to derive and select an intertemporal recruitment equilibrium in which the organization and the majority and minority candidates all behave with foresight, and to show that (2) affirmative action can be spontaneous, and (3) depending on the organization's initial state, mandated diversity or a quality floor may either push the organization on a virtuous path and yield higher long-term welfare, or generate a vicious and detrimental path. We leave it to future research to design policy interventions that are consistent with available regulatory information and will foster meritocratic recruitment without the perverse effects unveiled in this paper.

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Appendix

Omitted proofs, together with complements, can be found in the Online Appendix.

A Proof of Proposition 1

We use throughout the proofs the following simple result: let X_t follow $dX_t/dt = \chi(-X_t + X^*)$. Then, for $t \geq 0$, $X_t = (X_0 - X^*)e^{-\chi t} + X^*$, and the present-discounted value of the flow X_t is a convex combination of the initial value and the steady state value:

$$\int_0^{\infty} e^{-(r+\chi)t} X_t dt = \frac{1}{r+\chi}\left(\frac{(r+\chi)X_0 + \chi X^*}{r+2\chi}\right).$$

The next Lemma draws a useful implication of this result.

Lemma A.1. *Suppose there exists an absorbing region $\mathcal{R} \subset [1/2, 1] \times [0, 1]$ over which the majority picks a constant strategy $(\sigma_0, \sigma_1, \sigma_2)(M, S) \equiv (\sigma_0^*, 1, 1)$ for all $(M, S) \in \mathcal{R}$, and all talented candidates accept their offers ($a_1(M, S) = a_2(M, S) = 1$). Consider an initial state $(M_0, S_0) \in \mathcal{R}$, and let $V_1(M_0, S_0) = \int_0^{\infty} e^{-(r+\chi)t}(M_t\tilde{b} + S_t\tilde{s})dt$ denote the continuation value function for a majority member. Then, $V_1(M_0, S_0) = M_0 b + S_0 s + \frac{\chi}{r+\chi}\Big([x + (1-2x)\sigma_0^*]b + 2xs\Big)$, and thus $\frac{\partial V_1}{\partial M_0}(M_0, S_0) = b$, and $\frac{\partial V_1}{\partial S_0}(M_0, S_0) = s$.*

To derive the majority's best-response to talented minority candidates accepting their offers myopically and talented majority candidates always accepting their offers, we begin by looking for an upper bound on the majority's present discounted payoff. We build on two intuitions. Firstly, we study the relaxed problem in which the initial majority group retains control over recruitments even if its size decreases below 1/2.[34] Secondly, we restrict attention to recruitment strategies such that $\sigma_1(M, S) = \sigma_2(M, S) = 1$ for all states (M, S). Intuitively, since $s > b$ (and thus all agents prefer a talented out-group recruit to an untalented in-group one), recruiting talented candidates instead of untalented ones not only improves the majority's current payoff, but also relaxes the participation constraints of both majority and minority talented candidates in the future.

[34] Intuitively, this constraint is not binding in equilibrium because, the organization being large and facing a large pool of applicants, the current majority can always replicate (symmetrically) what the rival majority would do. Hence, relinquishing control would only change the horizontal type of talented candidates whose participation constraint is binding. To derive an upper bound, we further assume that talented majority candidates always accept their offers, even if the majority size decreases below 1/2.

This restriction can be shown to be without loss of generality by considering the full control problem for the majority (over $\sigma_0, \sigma_1, \sigma_2$). Solving for the optimal controls σ_1 and σ_2 is straightforward. Consequently, we henceforth restrict attention to the recruitment strategy for ordinary candidates, σ_0, assuming that the majority offers to recruit all talented candidates.

Hence, for any initial state (M_0, S_0), we derive an upper bound on the majority's payoff by considering the following relaxed program:

$$\max_{\sigma_0} \int_0^{+\infty} e^{-(r+\chi)t} \Big[S_t \tilde{s} + M_t \tilde{b} \Big] dt \tag{P}$$

subject to

(i) if $S_t s + (1 - M_t) b \geq u$,

$$\frac{dM_t}{dt} = \chi \Big[- M_t + x + (1 - 2x)\sigma_0 \Big], \quad \text{and} \quad \frac{dS_t}{dt} = \chi \Big[- S_t + 2x \Big] \tag{2}$$

(ii) if $S_t s + (1 - M_t) b < u$,

$$\frac{dM_t}{dt} = \chi \Big[- M_t + x + (1 - x)\sigma_0 \Big], \quad \text{and} \quad \frac{dS_t}{dt} = \chi \Big[- S_t + x \Big] \tag{3}$$

We show that the canonical recruitment strategy σ_0, defined as

$$\sigma_0(M, S) = \begin{cases} 1 & \text{if } Ss - Mb > u - b, \\ \sigma_0^\dagger & \text{if } Ss - Mb = u - b, \\ 0 & \text{if } Ss + Mb < u - b \text{ and } (M, S) \text{ satisfies (1)}, \\ 1 & \text{if } Ss + Mb < u - b \text{ and } (M, S) \text{ violates (1)}, \end{cases} \tag{4}$$

is the unique solution to the majority's relaxed problem (P) for any initial state. [The uniqueness of the optimal strategy holds almost everywhere (all equalities regarding optimal controls hold a.e.).] Since the induced trajectory actually satisfies the majority-size constraint we ignored ($M_t \geq 1/2$ for all $t \geq 0$, and $\frac{dM_t}{dt} \geq 0$ whenever $M_t = 1/2$), the canonical recruitment strategy σ_0 is the majority's best-response to the talented minority candidates' myopic acceptance strategy.

Proposition A.2. *Suppose conditions $(N), (E), (D), (TC)$ hold and $u \leq b/2$. For any initial state, the unique solution to the majority's relaxed problem (P) is the canonical* *recruitment strategy σ_0 defined by (4).*

Proof. Let "region 1", "region 2" and "region 3∪4" denote the sets $R_1 \equiv \{(M,S)|Ss - Mb > u - b\}$, $R_2 \equiv \{(M,S)|Ss - Mb = u - b\}$ and $R_{3\cup4} \equiv \{(M,S)|Ss - Mb < u - b\}$.

Region 2 (R_2). Let us first argue that a necessary condition for a strategy σ_0 to solve (P) is that R_2 be absorbing. Consider an initial state $(M_0, S_0) \in R_2$. Any strategy σ_0 inducing a trajectory leaving region R_2 for region $R_{3\cup4}$ at date 0 is strictly dominated by the canonical recruitment strategy (with $\sigma_0(M,S) = \sigma_0^\dagger$ for all $(M,S) \in R_2$). Indeed, at any time $t > 0$ such that (M_t, S_t) is still in $R_{3\cup4}$ (without having reached R_2 again),

$$\begin{aligned} S_t s + M_t b &\leq (S_0 - x)se^{-\chi t} + xs + (M_0 - 1)be^{-\chi t} \\ &< (S_0 - x)se^{-\chi t} + xs + (M_0 - 1)be^{-\chi t} + (3xs - u)(1 - e^{-\chi t}) \\ &= (S_0 - 2x)se^{-\chi t} + 2xs + [M_0 - x - (1-2x)\sigma_0^\dagger]be^{-\chi t} + [1 - x - (1-2x)\sigma_0^\dagger]b. \end{aligned}$$

where the second inequality stems from condition (D).

Similarly, starting from an initial state $(M_0, S_0) \in R_2$ and leaving R_2 for R_1 at date 0 yields that at any time $t > 0$ such that (M_t, S_t) is still in R_1 (without having reached R_2 again),

$$\begin{aligned} S_t s + M_t b &< S_t s + (S_t s + b - u) \\ &\leq 2(S_0 - 2x)se^{-\chi t} + 4xs + b - u \\ &= (S_0 - 2x)se^{-\chi t} + 2xs + [M_0 - x - (1-2x)\sigma_0^\dagger]be^{-\chi t} + [1 - x - (1-2x)\sigma_0^\dagger]b \end{aligned}$$

as in R_1 the majority can secure at best the same quality dynamics, but cannot avoid a strictly lower majority size than if it had stayed in R_2.

In both cases, the deviating trajectory reaches R_2 again (if ever) on a lower or equal iso-payoff curve with respect to the canonical recruitment strategy. Indeed, the latter ensures that the maximum share of talented candidates is recruited ($2x$), so that the aggregate quality achieved by the deviating trajectory remains always weakly below the canonical one, and thus by definition of R_2 (states such that $Ss - Mb = u - b$), the deviating trajectory's majority size when it reaches R_2 again (if ever) is weakly lower than the canonical trajectory's one.

Therefore, any solution to (P) is such that R_2 is absorbing. Then, as described in the text, the optimal strategy over R_2 is the constant strategy $\sigma_0(M_t, S_t) = \sigma_0^\dagger$ for all

$(M_t, S_t) \in R_2$, as it ensures that the organization remains in R_2, while leaving zero rent to talented minority candidates and members.

Region 1 (R_1). As R_2 is absorbing, the optimal strategy in R_1 is given by the constant strategy $\sigma_0(M_t, S_t) = 1$ for all $(M_t, S_t) \in R_1$. Indeed, for an initial state $(M_0, S_0) \in R_1$, let us compare the trajectory induced by the constant strategy $\sigma_0(M_t, S_t) = 1$ over R_1 to the trajectory induced by another strategy $\tilde{\sigma}_0$ over R_1, letting $T < +\infty$ and $\tilde{T} \leq +\infty$ be the respective times at which they reach R_2.[35] Then, as R_2 is absorbing with any optimal trajectory and as the two trajectories thus share the same quality dynamics at all times $t \geq 0$, $M_{t|\sigma_0} \geq M_{t|\tilde{\sigma}_0}$ at all times $t \geq 0$, strictly so following any non-empty open interval $(t_0, t_1) \subset [0, T]$ on which $\tilde{\sigma}_0(M_t, S_t) < 1$ for a.a. $t \in (t_0, t_1)$.

Region $3 \cup 4$ **($R_{3\cup 4}$).** We first consider trajectories such that R_2 is reached in a finite time $T < +\infty$, and derive the optimal controls and finite time for R_2 to be reached. We denote by $\sigma_0^\sharp$ the control (function of time t) corresponding to the strategy σ_0 (function of state (M, S)). We then compare the (optimal) value of reaching R_2 in a finite time to the (optimal) value of never reaching it. The cutoff condition (1) will draw the line between regions 3 and 4 (resp. R_3 and R_4).

(i) Let $T < +\infty$ be the "final time" at which the organization ends in R_2. To avoid the discontinuity in the state dynamics when reaching R_2, we consider the (further) relaxed problem in which, starting from an initial state in $R_{3\cup 4}$, the organization may reach $R_1 \cup R_2$ (states (M, S) with $Ss - Mb \geq u - b$) prior to date T, while still failing to attract talented minority candidates at such states. The solutions will turn out to reach $R_1 \cup R_2$ for the first time at T.

Conditional on reaching R_2 in finite time T (and picking the optimal constant control $\sigma_0^\dagger$ thereafter), the majority's problem writes as

$$\max_{\sigma_0^\sharp, T} \left(\int_0^T e^{-(r+\chi)t} \Big[\tilde{s}S(t) + \tilde{b}M(t) \Big] dt + e^{-(r+\chi)T} V_1(M(T), S(T)) \right)$$

where for all $t \in [0, T]$, $S(t) = (S_0 - x)e^{-\chi t} + x$, subject to the final-time constraint:

$$S(T)s - M(T)b = u - b, \tag{5}$$

[35] Any optimal trajectory starting in R_1 reaches R_2 in a finite time. Indeed, if an optimal path never leaves R_1, the optimal strategy along that path is the constant strategy $\sigma_0(M_t, S_t) = 1$, which leads to R_2 in a finite time, a contradiction.

and the state dynamics for M:

$$\frac{dM}{dt} = \chi\Big[-M + x + (1-x)\sigma_0^\sharp\Big]. \tag{6}$$

Remark: Sufficient condition for optimality. The function $(\sigma_0^\sharp, M) \mapsto \chi[-M + x + (1-x)\sigma_0^\sharp]$ (from the state dynamics (6)) is affine. The integrand in the majority's objective $(\tilde{s}S + \tilde{b}M)$ is an affine (and thus convex) function of M (and does not have the control $\sigma_0^\sharp$ as an argument), and by Lemma A.1, $V_1(M(T), S(T))$ is an affine (and thus convex) function of $M(T)$. As a consequence, any Pontryagin extremal $\sigma_0^\sharp$ is a solution to the majority's problem.

The Hamiltonian of the problem writes as

$$H(t, \sigma_0^\sharp, M, p) \equiv e^{-(r+\chi)t}\Big[S(t)\tilde{s} + M(t)\tilde{b}\Big] + \chi p(t)\Big[-M(t) + x + (1-x)\sigma_0^\sharp(t)\Big].$$

Letting $\psi \in \mathbb{R}$ be the multiplier for the final-time constraint (5), the costate equations are given by:

$$\begin{cases} -\dfrac{dp}{dt} = \tilde{b}e^{-(r+\chi)t} - \chi p, \\ p(T) = e^{-(r+\chi)T}\dfrac{\partial V}{\partial M}(M(T), S(T)) - \psi b. \end{cases}$$

Therefore, by integration, for all $t \in [0, T]$,

$$p(t) = be^{-(r+\chi)t} - \psi b e^{-\chi(T-t)}.$$

The Hamiltonian's partial derivative with respect to the control thus writes as

$$\frac{\partial H}{\partial \sigma_0^\sharp} = \chi(1-x)\Big[be^{-(r+\chi)t} - \psi b e^{-\chi(T-t)}\Big] \tag{7}$$

If $\psi \leq 0$, Pontryagin's maximum principle with final-point constraint and variable horizon yields that the optimal control must satisfy $\sigma_0^\sharp(t) = 1$ for a.a. $t \in [0, T]$. However, with $\sigma_0^\sharp(t) = 1$ for a.a. $t \in [0, T]$ and an initial state (M_0, S_0) such that $S_0 s - M_0 b < u - b$, the organization never reaches R_2, i.e., at all future times $t > 0$, $S(t)s - M(t)b < u - b$, as $\sigma_0^\sharp(t) = 1$ implies that, when in a neighborhood of R_2, the trajectory moves away from R_2.[36] Therefore, $\psi > 0$, and thus by (7), $\partial H/\partial \sigma_0^\sharp$ strictly decreases with $t \in (0, T)$. Moreover, by the final-time constraint (5), there exists $\mathfrak{T} < T$ such that for all $t \in (\mathfrak{T}, T]$, $\frac{\partial H}{\partial \sigma_0^\sharp}(t) < 0$.[37] Hence, any solution $\sigma_0^\sharp$ is such that there exists $\mathfrak{T}$ with $0 \leq \mathfrak{T} < T$ such that

$$\sigma_0^\sharp(t) = \begin{cases} 1 & \text{for a.a. } t < \mathfrak{T}, \\ 0 & \text{for a.a. } t \in (\mathfrak{T}, T), \end{cases} \tag{9}$$

This implies that any optimal trajectory reaches R_2 for the *first* time at time T, i.e., that $S(t)s - M(t)b < u - b$ for any $t \in (0, T)$, since (i) as noted above, $\sigma_0^\sharp(t) = 1$ implies that, when in a neighborhood of R_2, the trajectory moves *away from* R_2, and (ii) $\sigma_0^\sharp(t) = 0$ implies that, when in a neighborhood of R_2, the trajectory moves *closer to* R_2 if $S_t s - M_t b < u - b$, and *away from* R_2 if $S_t s - M_t b > u - b$.[38]

[36]Formally, for any state (M, S) such that $Ss - Mb = u - b + \varepsilon$ with $\varepsilon < 0$, for $\sigma_0^\sharp(t) = 1$, condition (N) implies that:

$$\frac{d}{dt}(S(t)s - M(t)b) = \chi\big[-(S(t)s - M(t)b) + xs - b\big] = \chi\big[xs - u + \varepsilon\big]s < 0.$$

[37]As the majority chooses the final time T, for any Pontryagin extremal, the sum of the Hamiltonian and the partial derivative of the final cost with respect to the final time, evaluated at the final time T, must be nil:

$$\begin{aligned} & e^{-(r+\chi)T}\big[\tilde{s}S(T) + \tilde{b}M(T)\big] + \chi p(T)\big[-M(T) + x + (1-x)\sigma_0^\sharp(T)\big] \\ & \quad = (r+\chi)e^{-(r+\chi)T}V_1(M(T), S(T)), \end{aligned}$$

and thus, using Lemma A.1 and the final-time constraint (5),

$$e^{-(r+\chi)T}\big[3xs + (1-x)(1-\sigma_0^\sharp(T))b - u\big] = \psi\big[xs + (1-x)(1-\sigma_0^\sharp(T))b - u\big] \tag{8}$$

Condition (D) implies that the LHS is always strictly positive, and thus, as $\psi > 0$,

$$xs + (1-x)(1-\sigma_0^\sharp(T))b - u > 0,$$

which implies that, if a solution $\sigma_0^\sharp$ exists, then $\sigma_0^\sharp(T) < 1$ (as $u > xs$ by condition (E)). For a solution to exist in the case $u > b/2$, it must be that $u < xs + (1-x)b$ (condition (TC)). As a consequence, for any $t < T$,

$$\begin{aligned} \frac{\partial H}{\partial \sigma_0^\sharp}(t) &= \chi(1-x)be^{-(r+\chi)t}\left[1 - e^{-(r+2\chi)(T-t)}\left(\frac{3xs - u + (1-x)(1-\sigma_0^\sharp(T))b}{xs - u + (1-x)(1-\sigma_0^\sharp(T))b}\right)\right] \\ &\leq \chi(1-x)be^{-(r+\chi)t}\left[1 - e^{-(r+2\chi)(T-t)}\left(\frac{3xs + (1-x)b - u}{xs + (1-x)b - u}\right)\right] \end{aligned}$$

by monotonicity with respect to $\sigma_0^\sharp(T)$. Hence, by continuity with respect to t, there exists $\mathfrak{T} < T$ such that for all $t \in (\mathfrak{T}, T]$, $\frac{\partial H}{\partial \sigma_0^\sharp}(t) < 0$.

[38]Formally, for any state (M, S) such that $Ss - Mb = u - b + \varepsilon$ with $\varepsilon \in \mathbb{R}$ and $|\varepsilon|$ sufficiently small, for $\sigma_0^\sharp(t) = 0$, the inequality $u \leq b/2$ (and condition (TC) in the case $u > b/2$) implies

$$\frac{d}{dt}(S_t s - M_t b) = \chi\big[-(S_t s - M_t b) + xs - xb\big] = \chi\big[xs + (1-x)b - u + \varepsilon\big]s > 0.$$

Lastly, let us consider the subset of $R_{3\cup 4}$ composed of states $(M, S) \in R_{3\cup 4}$ such that

$$\left(\frac{Mb - Ss + x(s-b)}{xs + (1-x)b - u}\right)^{\frac{r+\chi}{\chi}} \leq \frac{3xs + (1-x)b - u}{(1-x)b} \tag{1}$$

We refer to this subset as "region 3", R_3, and refer to its complement in region $3 \cup 4$, i.e. the set of states $(M, S) \in R_{3\cup 4}$ violating (1), as "region 4", R_4.

We compute in Online Appendix G the majority's value function from reaching R_2 with the control $\sigma_0^\sharp$ defined in (9) as a function of $\mathfrak{T}$. We show that, for any $\mathfrak{T}$ such that $(M(\mathfrak{T}), S(\mathfrak{T}))$ satisfies (1), the majority's value function at time 0 decreases with $\mathfrak{T}$. Hence, for any optimal trajectory, $(M(\mathfrak{T}), S(\mathfrak{T})) \in R_3$ implies that $\mathfrak{T} = 0$. As a consequence, from any state $(M, S) \in R_{3\cup 4}$ satisfying (1), the optimal control satisfies either (a) $\mathfrak{T} = 0$, and thus $\sigma_0^\sharp(t) = 0$ for all $t \in (0, T)$, or (b) $\mathfrak{T} > 0$ and $(M(\mathfrak{T}), S(\mathfrak{T})) \in R_4$.

To complete the proof, it remains to show that from any initial state in R_4, the constant control $\sigma_0^\sharp(t) = 0$ until reaching R_2 cannot be optimal. This will imply that from any initial state in R_3, the constant control $\sigma_0^\sharp(t) = 0$ is optimal until R_2 is reached (i.e., $\mathfrak{T} = 0$ with $\mathfrak{T}$ defined in (9)).

(ii) Let us compare the value of reaching R_2 in a finite time with the constant control $\sigma_0^\sharp(t) = 0$, to the value of never reaching it. Conditionally on never reaching R_2, the optimal control is the constant control $\sigma_0^\sharp(t) = 1$. For a given initial state (M_0, S_0), the former value is higher than the latter one if and only if:

$$\int_0^\infty e^{-(r+\chi)t} u_t^{(4)} dt \leq \int_0^T e^{-(r+\chi)t} u_t^{(3)} dt + \int_T^\infty e^{-(r+\chi)t} u_t^{(2)} dt \tag{10}$$

where $T < \infty$ is the time at which R_2 is reached with $\sigma_0^* = 0$ starting from the initial state (M_0, S_0), and is thus given by

$$T \equiv \frac{1}{\chi} \ln\left[\frac{M_0 b - S_0 s + x(s-b)}{xs + (1-x)b - u}\right] \geq 0,$$

and where

$$\begin{cases} u_t^{(4)} = [M_0 b + S_0 s - xs - b]e^{-\chi t} + xs + b, & \text{for } t \geq 0, \\ u_t^{(3)} = [M_0 b + S_0 s - xs - xb]e^{-\chi t} + xs + xb, & \text{for } t \in [0, T], \\ u_t^{(2)} = \left[u_T^{(3)} - 2xs - xb - (1-2x)\sigma_0^\dagger b\right]e^{-\chi(t-T)} + 2xs + xb + (1-2x)\sigma_0^\dagger b, & \text{for } t \geq T. \end{cases}$$

Condition (1) obtains by rearranging (10). Consequently, the optimal control $\sigma_0^\sharp$ over R_3 is the constant control $\sigma_0^\sharp(t) = 0$, while the optimal control $\sigma_0^\sharp$ over R_4 is the constant control $\sigma_0^\sharp(t) = 1$. □

Remark: Equilibrium preference. Both majority and minority members prefer the steady state (M_h^*, S_h^*) to the steady state (M_m^*, S_m^*) as $3xs > u$ (by condition (D)) for majority members, and $u > xs$ (by condition (N)) for minority members.

B Proof of Proposition 2 ($u > b/2$)

The incumbent majority retains control when $M_t = 1/2$ if and only if $dM_t/dt \geq 0$, and immediately loses control if $dM_t/dt < 0$. As the organization is large and faces a large pool of candidates (with unlimited supply of ordinary candidates), the majority can always keep control if it wants to. In fact, losing control cannot be strictly optimal for the incumbent majority. Indeed, suppose by contradiction that at some state (M_t, S_t) with $M_t = 1/2$, it is strictly optimal for the current majority to lose control ($dM_t/dt < 0$). Then, the new majority gets control at state (M_t, S_t). By symmetry, it is then strictly optimal for the new majority to lose control too, and thus $dM_t/dt = 0$, a contradiction. We thus restrict attention without loss of generality to recruitment strategies for the incumbent majority such that $dM/dt \geq 0$ whenever $M = 1/2$.

Our analysis in the case $u \leq b/2$ remains valid over $R_1 \equiv \{(M, S)|Ss - Mb > u - b\}$ and $R_2 \equiv \{(M, S)|Ss - Mb = u - b\}$ (see Appendix A) and thus in a myopic-minority equilibrium, the majority chooses the same strategy as before over these two regions:

- (Region 1) $\sigma_0(M, S) = 1$ whenever $Ss - Mb > u - b$
- (Region 2) $\sigma_0(M, S) = \sigma_0^\dagger$ whenever $Ss - Mb = u - b$.

Let "region $3 \cup 4 \cup 5$" denote the set $R_{3\cup4\cup5} \equiv \{(M, S)|Ss - Mb < u - b\}$ (formerly, $R_{3\cup4}$). When $u > b/2$, R_2 intersects the vertical axis ($M = 1/2$) at state $\left(1/2, \frac{u-b/2}{s}\right)$, strictly above the horizontal axis ($S = 0$). Hence, the analysis for regions 3 and 4 differs from the previous case ($u \leq b/2$) as some full-AA trajectories (with constant strategy $\sigma_0(M, S) = 0$) starting from initial states $(M_0, S_0) \in R_{3\cup4\cup5}$ now reach the vertical axis before reaching R_2. The analysis thus depends on whether upon reaching the vertical axis, the organization can later reach R_2, i.e., whether $u - b/2 < xs$.

Lastly, we look for an upper bound on the majority's payoff. Hence, we look for a (myopic-minority) equilibrium in which talented majority candidates coordinate on accepting the organization's offers – as will be clear shortly.

B.1 Case $u \geq xs + b/2$

Let $u - b/2 \geq xs$. Consider an initial state $(M_0, S_0) \in R_{3\cup4\cup5}$. Suppose first that talented majority candidates accept the organization's offers from time 0 onward. As long as the trajectory remains in $R_{3\cup4\cup5}$, the organization's quality thus converges towards $S_m^* = x$. However, when $u - b/2 \geq xs$, R_2 lies above this quality level: for any $(M, S) \in R_2$, $Ss \geq u - (1 - M)b \geq u - b/2 \geq xs$.

As a consequence, starting from $(M_0, S_0) \in R_{3\cup4\cup5}$, any trajectory that reaches the vertical axis strictly below R_2 cannot subsequently reach R_2. Hence, starting from $(M_0, S_0) \in R_{3\cup4\cup5}$, there exists a trajectory reaching R_2 in a finite time if and only if the full-AA trajectory (with constant strategy $\sigma_0(M, S) = 0$) reaches R_2 in a finite time. The set of such states is thus the set of states $(M, S) \in R_{3\cup4\cup5}$ such that

$$(1 - 2x)\Big(Ss - xs\Big) \geq 2(M - x)\left(u - \frac{b}{2} - xs\right). \tag{11}$$

We denote the set of states $(M, S) \in R_{3\cup4\cup5}$ satisfying (11) as *region* A (initial states from which affirmative action can be effective).[39] As $u > xs + b/2$, the line defined by (11) with equality is upward-sloping in the plane (M, S).

Within region A, *conditional on talented majority candidates accepting the organization's offers*, the previous analysis applies. Specifically, from an initial state (M, S) in region A, the majority's optimal strategy is the full-AA strategy, $\sigma_0(M, S) = 0$, until R_2 is reached if and only if (1) holds, and the majority's optimal strategy is the standard-favoritism strategy, $\sigma_0(M, S) = 1$, otherwise.

In the complement of region A within $R_{3\cup4\cup5}$ ("*region* A^c"), i.e., the set of states $(M, S) \in R_{3\cup4\cup5}$ violating (11), the organization cannot reach R_2, and thus, as the majority cannot hope to ever bring talented minority candidates back on board, its optimal

[39] As $u > b/2$, the state $(1/2, 0)$ does not belong to region A. By contrast, the state $(1/2, 2x)$ belongs to region A as $2xs + b/2 \geq u$ (condition (E)).

strategy is the standard-favoritism strategy, $\sigma_0(M,S)=1$.

Region 5. When $u > b/2$, the participation constraint of talented majority candidates cannot be taken for granted. A fifth region, which we refer to as "region 5", $R_5 \subset R_{3\cup4\cup5}$, may then exist where the organization fails to recruit such candidates – and so, any talented candidate at all. As we look for an upper bound on the majority's payoff, we look for an equilibrium in which R_5 is minimal.

Condition (TC) implies that at R_4's (candidate) steady state (M_m^*, S_m^*), $M_m^* b + S_m^* s = b + xs > u$. Hence, the participation constraint of talented majority candidates is satisfied at this steady-state. As noted in the text, R_4 is thus non-empty as it always includes (M_m^*, S_m^*). Moreover, starting from an initial state (M,S) such that $Mb + Ss \geq u$, if talented majority candidates coordinate on joining the organization at any future time $t \geq 0$, majority members can secure a payoff at least as high as the one they would get from standard favoritism:

$$\int_0^\infty e^{-(r+\chi)t}\left((M-1)\tilde{b}e^{-\chi t} + \tilde{b} + (S-x)\tilde{s}e^{-\chi t} + x\tilde{s}\right)dt > \int_0^\infty e^{-(r+\chi)t}\tilde{u}dt.$$

Hence, talented majority members' decision to join is strictly optimal at any initial state (M,S) such that $Mb + Ss \geq u$. Therefore, R_5 lies strictly below the line $\Gamma \equiv \{(M,S) \,|\, Mb + Ss = u\}$.[40]

By construction, line Γ reaches the vertical axis ($M = 1/2$) at the same state as R_2, namely $\left(1/2, \frac{u-b/2}{s}\right)$, and since R_5 lies strictly below Γ, this implies that R_5 has no shared boundary with R_2.[41] Moreover, R_5 has no shared boundary with R_3 since R_5 lies strictly below the (downward-sloping) line Γ, while R_3 lies above the (upward-sloping) full-AA trajectory reaching R_2 at its intersection with the vertical axis, and the two lines intersect on the vertical axis (at state $(1/2, \frac{u-b/2}{s})$).[42]

Therefore, R_3 is the set of states $(M,S) \in R_{3\cup4\cup5}$ satisfying (11) and (1),[43] and R_5 is given by the set of states strictly below its frontier with R_4. Specifically, the frontier

[40]Note that Γ represents the acceptance threshold for *myopic* talented majority candidates.

[41]This argument holds regardless of whether $u - b/2 \lesseqgtr xs$.

[42]Formally, for $u \geq xs + b/2$, there exists no state (M,S) that satisfies $Mb + Ss < u$ and (11). Indeed, if there exists such a state, then

$$\left(\frac{1}{2} - x\right)(u - Mb - xs) > (M - x)\left(u - \frac{b}{2} - xs\right),$$

which implies that $(1/2 - M)(u - xs) > 0$, a contradiction.

[43]Hence, from any state below R_3 and close to R_3's boundary with R_4, the trajectory induced by standard favoritism ($\sigma_0(M,S) = 1$) moves away from R_3.

between R_5 and R_4 is thus given by the set of states $(M,S) \in R_{3\cup 4\cup 5}$ such that current talented majority candidates are indifferent between rejecting the organization's offers and accepting them, conditional on future majority candidates accepting their offers and the organization applying standard favoritism at all future times:

$$\int_0^\infty e^{-(r+\chi)t}\left((S-x)\tilde{s}e^{-\chi t} + x\tilde{s} + (M-1)\tilde{b}e^{-\chi t} + \tilde{b}\right)dt \;=\; \int_0^\infty e^{-(r+\chi)t}\tilde{u}dt$$

$$\iff \qquad Ss + Mb \;=\; u - \frac{\chi}{r+\chi}(xs+b-u). \tag{12}$$

The frontier between R_5 and R_4 is thus a downward sloping line in the plane (M,S),[44] and R_5 is the set of states (M,S) such that[45]

$$Ss + Mb \;<\; u - \frac{\chi}{r+\chi}(xs+b-u). \tag{13}$$

As a consequence, since the boundary between R_5 and R_4 has a downward slope in the plane (M,S), R_5 is non-empty if and only if the initial state $(1/2,0)$ satisfies (13), i.e., if and only if

$$u - \frac{b}{2} \;>\; \frac{\chi}{r+2\chi}\left(xs + \frac{b}{2}\right). \tag{14}$$

and thus, as $u - b/2 \geq xs$, R_5 is non-empty for any χ/r sufficiently low, and for any χ/r if $u - b/2 > \frac{xs+b/2}{2}$.

Region R_4 is thus given by $R_4 = R_{3\cup 4\cup 5} \setminus (R_3 \cup R_5)$. As for any state $(M,S) \in R_4$, close to the boundary with R_5 (resp. R_3), the trajectory induced by standard favoritism drives the organization away from R_5 (resp. R_3), R_4 is absorbing – as in the case $u \leq b/2$.

Since R_5 is given by the set of states strictly below its boundary with R_4, and since R_4 is absorbing, any equilibrium trajectory starting in R_5 never reaches R_2, i.e., the organization never attracts talented minority candidates. Therefore, the optimal strategy for the majority in R_5 is standard favoritism ($\sigma_0(M,S) = 1$). The state dynamics in R_5 are then given by

$$\frac{dM}{dt} = \chi(-M+1), \qquad \text{and} \qquad \frac{dS}{dt} = -\chi S,$$

[44] This argument holds regardless of whether $u - b/2 \geq xs$.

[45] Note that, by (TC), (12) implies $Ss + Mb < u$.

The induced trajectories are thus straight lines in the plane (M, S), converging to $(M_l^*, S_l^*) = (1, 0)$. Consequently, R_5 is absorbing (with (M_l^*, S_l^*) its steady state) if and only if the state $(1, 0)$ lies in the interior of R_5, i.e., if and only if

$$u > b + \frac{\chi}{r + 2\chi} xs. \tag{15}$$

By contrast, if the state $(1, 0)$ lies outside R_5, then from any initial state in R_5, the organization leaves R_5 (and thus reaches R_4) in a finite time. A higher homogeneity then makes the organization (eventually) attractive to talented majority candidates, allowing it to converge to the zero-diversity/medium-quality steady state, (M_m^*, S_m^*).

B.2 Case $b/2 < u < xs + b/2$

Let $u \in (b/2, b/2 + xs)$. Consider any initial state $(M_0, S_0) \in R_{3\cup4\cup5}$. Suppose that talented majority candidates accept the organization's offers from time 0 onward. Analogous arguments to those of the case $u \leq b/2$ (see Appendix A[46]) yield that, conditional on reaching R_2 in a finite time, all optimal trajectories are of the form "standard favoritism, then full AA", i.e. there exists $\mathfrak{T} \geq 0$ such that the optimal control satisfies $\sigma_0^\sharp(M(t), S(t)) = 1$ for all $t < \mathfrak{T}$, and $\sigma_0^\sharp(M(t), S(t)) = 0$ for all $t \in (\mathfrak{T}, T)$, with T the time at which the trajectory reaches R_2. Moreover, we showed that in the case $u \geq b/2$, if the "turning point" $(M(\mathfrak{T}), S(\mathfrak{T}))$ satisfies (1), then the majority's payoff strictly decreases with $\mathfrak{T}$, and thus that trajectories in R_3 start with full AA ($\mathfrak{T} = 0$, no standard favoritism phase). The argument in the case $u \leq b/2$ relies on the fact that any trajectory eventually reaching R_2 necessarily reaches R_2 before the vertical axis (i.e., never reaches the vertical axis). This same property holds in the case $u \geq xs + b/2$. By contrast, for $b/2 < u < xs+b/2$, there exist states from which the organization could reach the vertical axis ($M = 1/2$) before reaching R_2. From such states, the optimal trajectory is subject to the additional constraint: $dM/dt = 0$ when $M = 1/2$. Analogous computations yield that: (i) for states from which full AA takes the organization straight to R_2 (not reaching the vertical axis before), the optimal strategy is full AA ($\sigma_0(M, S) = 0$), as in the case $u \leq b/2$; (ii) for states from which full AA takes the organization to the vertical axis before R_2, the optimal trajectory is the trajectory along which the organization first selects $\sigma_0(M, S) = 1$, until it reaches the state from which choosing $\sigma_0(M, S) = 0$ thereafter becomes necessary and sufficient to reach R_2 at its intersection with the vertical axis

[46]The argument goes through as $xs + (1 - x)b - u > 0$ (condition (TC)).

(state $(1/2, \frac{u-b/2}{s})$).[47]

Specifically, consider the set of states $(M, S) \in R_{3\cup4\cup5}$ from which full AA ($\sigma_0(M, S) = 0$) leads to R_2 exactly at its intersection with the vertical axis (state $(1/2, \frac{u-b/2}{s})$), i.e., such that

$$\left(\frac{1}{2} - x\right)(Ss - xs) + \left(xs + \frac{b}{2} - u\right)(M - x) = 0. \tag{16}$$

Hence, as $xs + b/2 > u$, (16) defines a downward-sloping line in the space (M, S), which we refer to as $\underline{\Gamma}$.

For any state $(M, S) \in R_{3\cup4\cup5}$ satisfying (16) with a (weak) positive inequality (on or above line $\underline{\Gamma}$), i.e., satisfying

$$\left(\frac{1}{2} - x\right)(Ss - xs) + \left(xs + \frac{b}{2} - u\right)(M - x) \geq 0, \tag{17}$$

the same analysis as in the case $u \leq b/2$ applies. Hence, from such a state the organization chooses full AA ($\sigma_0(M, S) = 0$) if and only if

$$\left(\frac{M_0 b - S_0 s + xs - xb)}{xs + (1-x)b - u}\right)^{\frac{r+\chi}{\chi}} \leq \frac{3xs + (1-x)b - u}{(1-x)b}, \tag{1}$$

and standard favoritism otherwise ($\sigma_0(M, S) = 1$).

By contrast, for any state $(M, S) \in R_{3\cup4\cup5}$ violating (17) (strictly below line $\underline{\Gamma}$), the optimal trajectory features standard favoritism ($\sigma_0(M, S) = 1$) until it reaches line $\underline{\Gamma}$, and then switches to full AA ($\sigma_0(M, S) = 0$) if (1) holds at that state, and continues with standard favoritism otherwise.

R_3 is thus the set of states $(M, S) \in R_{3\cup4\cup5}$ satisfying (17) and (1). The set of states $(M, S) \in R_{3\cup4\cup5}$ from which the optimal trajectory eventually reaches R_2 is given by the set of states (M, S) in the following triangle: below R_2, to the left of the line given by (1) with equality, and above the standard-favoritism trajectory that goes through the intersection of the line given by (1) with equality and line $\underline{\Gamma}$. In other words, it is the set

[47] Intuitively, fix the state on R_2 at which the trajectory reaches R_2. Since all trajectories share the same quality dynamics (recruiting talented majority candidates), the optimal strategy is the one that maximizes homophily benefits along the way: engaging in standard favoritism as long as it can, then turning back and racing to reach R_2 on time. As for the optimal state at which the organization should reach R_2, the same logic as in the case $u \leq b/2$ applies: when to the left of the line given by (1) with equality, it is optimal to reach R_2 as soon as possible. Hence, for states from which full AA takes the organization to the vertical axis before R_2, this implies reaching R_2 at its intersection with the vertical axis (its "lowest" point in the space (M, S)).

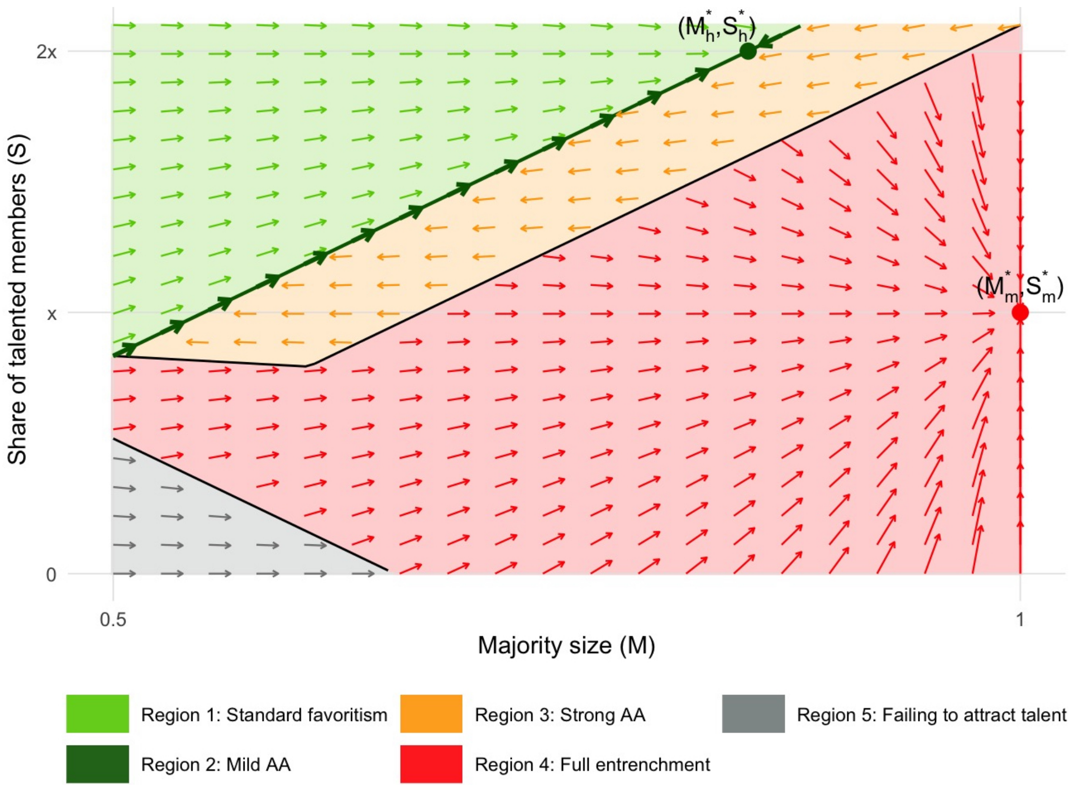

Figure 3: Phase diagram for $u \in (b/2, xs + b/2)$.

of states satisfying $Ss - Mb < u - b$, (1) and

$$S \geq \tfrac{x-S'}{1-M'}M + \left(S' - \tfrac{x-S'}{1-M'}M'\right), \tag{18}$$

where (M', S') is the state at which the line given by (1) with equality and line $\underline{\Gamma}$ intersect:

$$\begin{cases} S's = xs - \left(xs + \dfrac{b}{2} - u\right)\left(\dfrac{3xs + (1-x)b - u}{(1-x)b}\right)^{\frac{\chi}{r+\chi}}, \\ M' = x + \left(\dfrac{1}{2} - x\right)\left(\dfrac{3xs + (1-x)b - u}{(1-x)b}\right)^{\frac{\chi}{r+\chi}}. \end{cases} \tag{19}$$

Within this set, R_3 is the subset of states satisfying (17), while the subset of states violating (17) belongs to R_4.[48]

Region 5. As before, we look for an upper bound on the majority's payoff, and thus for an equilibrium in which R_5 is minimal. Hence, R_5 remains given by the same condition as in the case $u - b/2 > xs$ (i.e., (12)) if and only if starting from a state on the line

[48]In contrast with the cases $u \leq b/2$ and $u > xs + b/2$, when $b/2 < u < xs + b/2$, there exists states below R_3 such that the trajectory induced by standard favoritism ($\sigma_0(M,S) = 1$) from those states reaches R_3 in a finite time (see Figure 3). Consequently, R_4 is not absorbing.

defined by (12) with equality, the trajectory induced by standard favoritism (with talented majority candidates accepting their offers) never reaches R_3. This holds if and only if the state $\left(1/2, \frac{u-b/2}{s} - \frac{\chi}{r+\chi}\frac{xs+b-u}{s}\right)$ (at which the line defined by (12) crosses the vertical axis ($M = 1/2$)) violates (18), i.e.

$$u - \frac{b}{2} + \frac{\chi}{r+\chi}(u - xs - b) < \frac{x - S'}{1 - M'}\frac{s}{2} + S's - \frac{x - S'}{1 - M'}M's.$$

Using (19), and rearranging yields that the above inequality is equivalent to

$$\frac{2(1-x)\Big[xs + \frac{b}{2} - u + \frac{\chi}{r+\chi}(xs + b - u)\Big]}{2(1-x)\Big[xs + \frac{b}{2} - u\Big] + (1-2x)\frac{\chi}{r+\chi}\Big[xs + b - u\Big]} > \left(\frac{3xs + (1-x)b - u}{(1-x)b}\right)^{\frac{\chi}{r+\chi}}.$$

The LHS, resp. the RHS, is a strictly increasing and strictly concave, resp. strictly increasing and strictly convex, function of $\chi/(r+\chi)$. Moreover, the respective limits of the LHS and the RHS are both equal to 1 as $\chi/(r+\chi)$ goes to 0. Hence, the inequality holds for all $\chi/r > 0$ if and only if it holds in the limit as $\chi/(r+\chi)$ goes to 1, i.e., if and only if

$$\frac{2(1-x)\Big[xs + \frac{b}{2} - u + (xs + b - u)\Big]}{2(1-x)\Big[xs + \frac{b}{2} - u\Big] + (1-2x)\Big[xs + b - u\Big]} > \frac{3xs + (1-x)b - u}{(1-x)b},$$

which is equivalent to

$$xs + b - u > \frac{3xs - u}{(1-x)b}\left(2(1-x)\Big[xs + \tfrac{b}{2} - u\Big] + (1-2x)\Big[xs + b - u\Big]\right).$$

The LHS, resp. the RHS, is strictly increasing and linear in $(xs - u)$, resp. strictly increasing and strictly convex in $(xs - u)$. By condition (D), $xs - u > -2xs$, and for $xs - u = -2xs$, the RHS is equal to zero, while the LHS is strictly positive. The inequality thus holds for all $(xs - u)$ if and only if it holds for $xs - u = -b/2$ (as by assumption, $u < xs + b/2$ in this case). Hence, it holds if and only if

$$1 > \frac{(2xs - \frac{b}{2})(1 - 2x)}{(1-x)b},$$

which is satisfied as $xs/b < (\frac{1}{2} - x)$.[49]

Consequently, the same analysis as in the case $u \geq xs + b/2$ holds regarding the properties of R_5 (in particular, whether it is non-empty, and/or absorbing).

[49]Indeed, by condition (N) and in this case, $2xs + xb < u < xs + b/2$, and thus $xs < (\frac{1}{2} - x)b$.

C Proof of Proposition 3

Let us first show that the recruitment plan induced by the canonical strategies is dominant from any state $(M, S) \in R_2$. For an initial state (M_t, S_t), the present-discounted payoff at date t of a member of type $i \in \{1, 2\}$ is given by

$$V_1^{\rho}(M_t, S_t) = \int_t^{+\infty} e^{-(r+\chi)(\tau-t)}[S_\tau \tilde{s} + M_\tau \tilde{b}]d\tau,$$
$$V_2^{\rho}(M_t, S_t) = \int_t^{+\infty} e^{-(r+\chi)(\tau-t)}[S_\tau \tilde{s} + (1 - M_\tau)\tilde{b}]d\tau.$$

Let $\bar{V}(M_t, S_t)$ denote the following upper bound on the "total payoff" at state (M_t, S_t), $V_1^{\rho}(M_t, S_t) + V_2^{\rho}(M_t, S_t)$, starting at (M_t, S_t):

$$\bar{V}(M_t, S_t) \equiv \int_t^{+\infty} e^{-(r+\chi)(\tau-t)} \left([2(S_t - 2x)e^{-\chi\tau} + 4x]\tilde{s} + \tilde{b} \right) d\tau.$$

The canonical strategies described in Section 3.2 yield a total payoff $V_1^c(M_t, S_t) + V_2^c(M_t, S_t) = \bar{V}(M_t, S_t)$ starting from any state (M_t, S_t) in $R_1 \cup R_2$. Moreover, in that equilibrium, starting from $(M_t, S_t) \in R_2$, $V_2^c(M_t, S_t) = \tilde{u}/(r+\chi)$, so that the participation constraint of talented minority candidates is binding, and hence

$$V_1^c(M_t, S_t) = \int_t^{+\infty} e^{-(r+\chi)(\tau-t)} \left([2(S_t - 2x)e^{-\chi\tau} + 4x]\tilde{s} + \tilde{b} - \tilde{u} \right) d\tau.$$

Let us note that condition (D) implies that $4xs + b - u > xs + b$, and thus, that for any $(M_t, S_t) \in R_2$ (i.e., with $S_t s - M_t b = u - b$) and $t' > t$,

$$\int_t^{t'} e^{-(r+\chi)(\tau-t)} \left([2(S_t - 2x)e^{-\chi\tau} + 4x]\tilde{s} + \tilde{b} - \tilde{u} \right) d\tau$$
$$> \int_t^{t'} e^{-(r+\chi)(\tau-t)} \left([(S_t - x)e^{-\chi\tau} + x]\tilde{s} + [(M_t - 1)e^{-\chi\tau} + 1]\tilde{b} \right) d\tau.$$

When the participation constraint of talented minority candidates is violated at (M_t, S_t) ($V_2^{\rho}(M_t, S_t) < \tilde{u}/(r + \chi)$), two cases can arise. Either $V_2(M_{t'}, S_{t'}) < \tilde{u}/(r + \chi)$ at all states $(M_{t'}, S_{t'})_{t' \geq t}$, and thus talented minority candidates always decline their offers from (M_t, S_t) onwards, in which case

$$V_1^{\rho}(M_t, S_t) \leq \int_t^{+\infty} e^{-(r+\chi)(\tau-t)} \left([(S_t - x)e^{-\chi\tau} + x]\tilde{s} + [(M_t - 1)e^{-\chi\tau} + 1]\tilde{b} \right) d\tau$$
$$< \int_t^{+\infty} e^{-(r+\chi)(\tau-t)} \left([2(S_t - 2x)e^{-\chi\tau} + 4x]\tilde{s} + \tilde{b} - \tilde{u} \right) d\tau = V_1^c(M_t, S_t).$$

Or there exists $\delta \in (0, +\infty)$ such that talented minority candidates decline their offers at all states $(M_{t'}, S_{t'})_{t \leq t' < t+\delta}$, but accept theirs at $(M_{t+\delta}, S_{t+\delta})$ (hence, with $V_2^{\rho}(M_{t+\delta}, S_{t+\delta}) = \tilde{u}/(r+\chi)$), which implies that

$$V_1^{\rho}(M_t, S_t) < \int_t^{t+\delta} e^{-(r+\chi)(\tau-t)} \left([(2S_t - 2x)e^{-\chi\tau} + 4x]\tilde{s} + \tilde{b} - \tilde{u} \right) d\tau$$
$$+ e^{-(r+\chi)\delta} V_1^{\rho}(M_{t+\delta}, S_{t+\delta})$$
$$< \int_t^{t+\delta} e^{-(r+\chi)(\tau-t)} \left([2(S_t - 2x)e^{-\chi\tau} + 4x]\tilde{s} + \tilde{b} - \tilde{u} \right) d\tau$$
$$+ e^{-(r+\chi)\delta} [\overline{V}(M_{t+\delta}, S_{t+\delta}) - V_2^{\rho}(M_{t+\delta}, S_{t+\delta})]$$
$$= \int_t^{+\infty} e^{-(r+\chi)(\tau-t)} \left([2(S_t - 2x)e^{-\chi\tau} + 4x]\tilde{s} + \tilde{b} - \tilde{u} \right) d\tau = V_1^c(M_t, S_t).$$

As a consequence, at any state (M_t, S_t) in region 2, any credible recruitment plan must guarantee a payoff to the majority at least equal to

$$\int_t^{+\infty} e^{-(r+\chi)(\tau-t)} \left([2(S_t - 2x)e^{-\chi\tau} + 4x]\tilde{s} + \tilde{b} - \tilde{u} \right) d\tau,$$

and this requires that talented minority candidates accept their offers at all states $(M_{t'}, S_{t'})_{t' \geq t}$, and have a zero rent at (M_t, S_t), i.e. $V_2^{\rho}(M_t, S_t) = \tilde{u}/(r+\chi)$.

Therefore, the recruitment plan induced by the canonical strategies ("canonical recruitment plan") is dominant from any state $(M, S) \in R_2$. Hence, for any recruitment plan in $\mathcal{F}_1$, $a_2(M_t, S_t) = 0$ at any state in region $3 \cup 4 \cup 5$ ($S_t s - M_t b < u - b$) as a talented minority candidate's flow payoff is strictly lower than $\tilde{u}$ until the trajectory reaches R_2, if ever, at which point their value function is equal to $\tilde{u}/(r+\chi)$. Similarly, for any recruitment plan in $\mathcal{F}_1$, $a_2(M_t, S_t) = 1$ at any state in region 1 ($S_t s - M_t b > u - b$) as a talented minority candidate's flow payoff is strictly higher than $\tilde{u}$ until the trajectory reaches R_2, if ever, at which point their value function is equal to $\tilde{u}/(r+\chi)$. Hence, in any recruitment plan in $\mathcal{F}_1$, and thus in any credible recruitment plan in $\mathcal{F}_\infty \subseteq \mathcal{F}_1$, talented minority candidates follow the myopic acceptance strategy.

Consequently, using the same argument as in the proof of Proposition 1 (Appendix

A), in any credible recruitment plan and in any dominant recruitment plan (if any), R_2 is absorbing.

Hence, in any credible recruitment plan and in any dominant recruitment plan (if any), talented minority candidates accept, resp. reject their offers in $R_{1\cup 2}$ (states (M, S) with $Ss - Mb \geq u - b$), resp. $R_{3\cup 4\cup 5}$ (states (M, S) with $Ss - Mb < u - b$). As the canonical strategies are the (generically) unique strategies that achieve an upper bound on a majority member's payoff conditionally on talented minority candidates following such an acceptance strategy, the canonical recruitment plan is dominant from any state (M, S), and is the (generically) unique dominant recruitment plan from any state (M, S). Therefore, the canonical recruitment plan is the generically unique credible recruitment plan.

D Proof of Proposition 4

The result follows from analogous arguments to the ones used in the proofs of Propositions 1 and 2 (see Appendices A and B). The only difference lies in the cap on the majority size $\overline{M} \in (M_h^*, 1)$.

The cap does not affect the trajectories induced by the canonical strategies in R_2 and R_3 as such trajectories converge to majority sizes $M_h^* < \overline{M}$ and $x < \overline{M}$, respectively. Similarly, the cap does not affect trajectories induced by the canonical strategy in R_1 whenever $M < \overline{M}$, and only affects them when $M = \overline{M}$ by imposing that the majority size remains constant, equal to the cap $\overline{M}$, while quality converges to $S_h^* = 2x$.

By contrast, trajectories induced by the canonical strategies in R_4 and R_5 are affected by the cap as they converge to majority size $1 > \overline{M}$ (full homogeneity). Upon hitting the cap, such trajectories thus feature a constant majority size, equal to the cap $\overline{M}$, while quality converges to $S_m^* = x$ and $S_l^* = 0$ respectively. As $\overline{M} < M_m^* = M_l^* = 1$, the cap thus reduces the (present discounted) payoff for a majority member from the canonical trajectories in R_4 and R_5.

The result thus obtains by considering which trajectories yield the highest payoffs for a majority member, and whether talented candidates get a higher payoff by accepting an offer than with the outside option. In particular, in a neighborhood of the frontier between R_3 and R_4 under laissez-faire, the trajectories that yield the two highest payoffs for a majority member are: the one induced by standard favoritism with talented majority candidates accepting their offers (canonical trajectory in R_4), and the one induced by full

AA (canonical trajectory in R_3) until R_2 is reached. As the cap reduces the payoff from standard favoritism, it expands the set of states from which full AA until reaching R_2 yields the highest payoff. In addition, in a neighborhood of the frontier between R_4 and R_5 under laissez-faire, the payoff for talented majority candidates from accepting an offer given a trajectory induced by standard favoritism (inside R_4) is close to the outside option. As the cap reduces the payoff from standard favoritism (and thus trajectories inside R_4), it expands the set of states at which talented majority candidates strictly prefer the outside option. Hence, R_3 and R_5 expand at the expense of R_4.

To illustrate Proposition 4, we derive in Online Appendix J.1 explicit formulas for the boundaries between $R_3 - R_4$ and $R_4 - R_5$ for a specific set of parameters (see Figure 4 therein).

E Proof of Proposition 5

The result follows from analogous arguments to the ones used in the proofs of Propositions 1 and 2 (see Appendices A and B). The only difference lies in the lower bound on aggregate quality $\underline{S} \in (x, 2x)$, which implies in particular that any trajectory along which talented minority candidates decline their offers *at all times* $t \geq 0$ leads the organization to be dismantled in finite time. Hence, along any such trajectory, the present discounted payoff of members is strictly lower than under laissez-faire ($\underline{S} = 0$).

Specifically, R_3 expands in a neighborhood of the frontier between R_3 and R_4 under laissez-faire whenever the trajectory induced by the full-AA strategy reaches R_2 before hitting the quality floor. For simplicity, let us first focus on the case $u \leq b/2$. For a given initial state (M_0, S_0), the full-AA trajectory reaches R_2 at time T, where

$$T \equiv \frac{1}{\chi} \ln \left[\frac{M_0 b - S_0 s + x(s-b)}{xs + (1-x)b - u} \right] \geq 0,$$

and thus with a quality

$$S_T = (S_0 - x) \frac{xs + (1-x)b - u}{M_0 b - S_0 s + x(s-b)} + x.$$

Hence, $S_T \geq \underline{S}$ if and only if

$$\frac{S_0 - x}{\underline{S} - x} > \frac{M_0 b - S_0 s + x(s-b)}{xs + (1-x)b - u}. \tag{20}$$

Consequently, for any state (M_0, S_0) such that $M_0 b - S_0 s > b - u$ and satisfying (20), the present discounted payoff of a majority member from full AA until reaching R_2 then mild AA with $\sigma_0(M_t, S_t) = \sigma_0^\dagger$, remains the same as absent the quality floor, whereas the present discounted payoff from standard favoritism decreases to

$$\int_0^{\underline{T}} e^{-(r+\chi)t}\Big[(S_0 - x)\tilde{s}e^{-\chi t} + x\tilde{s} + (M_0 - 1)\tilde{b}e^{-\chi t} + \tilde{b}\Big]dt \\ < \int_0^{+\infty} e^{-(r+\chi)t}\Big[(S_0 - x)\tilde{s}e^{-\chi t} + x\tilde{s} + (M_0 - 1)\tilde{b}e^{-\chi t} + \tilde{b}\Big]dt,$$

where $\underline{T}$ is the time at which the organization hits the quality floor, $\underline{S}$, and is dismantled:

$$\underline{T} = \frac{1}{\chi}\ln\left(\frac{S_0 - x}{\underline{S} - x}\right).$$

By contrast, for any state (M_0, S_0) such that $M_0 b - S_0 s > b - u$ violating (20), the present discounted payoff of a majority member from full AA becomes

$$\int_0^{\underline{T}} e^{-(r+\chi)t}\Big[(S_0 - x)\tilde{s}e^{-\chi t} + x\tilde{s} + (M_0 - x)\tilde{b}e^{-\chi t} + x\tilde{b}\Big]dt \\ < \int_0^{\underline{T}} e^{-(r+\chi)t}\Big[(S_0 - x)\tilde{s}e^{-\chi t} + x\tilde{s} + (M_0 - 1)\tilde{b}e^{-\chi t} + \tilde{b}\Big]dt,$$

as such a trajectory hits the quality floor at the same time $\underline{T}$ than the trajectory induced by standard favoritism.

Analogous computations apply in the case $u > b/2$ (see Appendix B).[50] Moreover, R_5 lies below the myopic-acceptance threshold for talented majority candidates, $Mb+Ss < u$, and thus R_5 cannot shrink with a quality floor with respect to laissez-faire. On the opposite, as the quality floor implies that along trajectories in R_4, the organization is dismantled in finite time, R_5 expands at the expense of R_4 (still remaining below the myopic-acceptance threshold for talented majority candidates).

[50] The case $b/2 < u < xs + b/2$ is the only case in which, under laissez-faire, a part of the boundary between R_3 and R_4 is not given by (1) with equality (see Appendix B.2). It is then given instead by the full-AA trajectory that reaches R_2 at its intersection with the vertical axis. It thus lies below the horizontal line $S = x$ (as $u < xs + b/2$), and thus below the quality floor $\underline{S} > x$. Hence, this part of the boundary disappears with the quality floor.

Online Appendix

F Complements on the one-time renewal model

Let us generalize the one-time renewal model introduced in Section 2. The game now begins with a fraction $X \in (0,1]$ of incumbent members exogenously quitting the organization (randomly drawn among current members). Departing members are replaced by new hires, keeping the mass of members constant (equal to 1). Recruitments are decided by remaining members according to a majority vote. We assume that offers are made only to the majority's preferred candidates (no coordination failures within the majority). In addition, the mass x of talented candidates from each group now satisfies $x < X/2$. All the other assumptions remain unchanged. Section 2 thus corresponds to the case $X = 1$.

We rule out coordination failures within and across groups of talented candidates. As a consequence, the organization's majority can choose the composition of the fraction X of members to be renewed, subject to the participation constraint of each type of talented candidates. We look for an equilibrium maximizing a majority member's payoff.

Proposition F.1 (One-time renewal). *Supppose conditions* (N)*,* (E)*,* (D) *hold. Let* $X \in (0,1]$*, and let* (M_0, S_0) *denote the organization's initial state. There exist cutoffs* u_a*,* u_b*,* u_c*, and* S_A*, with* $u_a > u > u_b$ *and* $S_A s < u - b/2$*, such that in equilibrium, there exist at most four regions:*

(i) ***Standard majority favoritism.*** *The organization attracts all talented candidates and otherwise favors in-group candidates if*

$$S_0 s + (1 - M_0)b \geq u_a.$$

(ii) ***Affirmative action to attract talented minority candidates.*** *The organization attracts all talented candidates and engages in AA if*

$$S_0 \geq S_A \qquad \text{and} \qquad u_b \leq S_0 s + (1 - M_0)b < u_a.$$

(iii) ***Giving up on talented minority candidates.*** *The organization attracts only*

majority talented candidates and does not engage in AA if

$$S_0 < S_A \quad or \quad S_0 s + (1 - M_0)b < u_b, \qquad and \qquad S_0 s + M_0 b \geq u_c.$$

(iv) ***Failing to attract talent.*** *The organization fails to attract any talented candidate and recruits only ordinary majority candidates if*

$$S_0 < S_A \quad or \quad S_0 s + (1 - M_0)b < u_b, \qquad and \qquad S_0 s + M_0 b < u_c.$$

Let us refer to a *myopic* candidate in this static environment as a candidate who would believe to be the only new recruit (as if $X = 0$). Since $u_a > u > u_b$ (see below), the organization engages in voluntary AA from some states above and from some states below the "myopic participation constraint" of talented minority candidates, i.e., the line defined by $S_0 s + (1 - M_0)b = u$. The threshold u_a increases with X, while the thresholds u_b and S_A decrease with X: the larger the share of members to be renewed (X), the larger the set of states from which the organization engages in voluntary AA.

Proof. Since $M \geq 1/2$ by construction, if the participation constraint of talented minority candidates is met, then so is the one of talented majority candidates. Hence, there exist at most four cases ("regions" in the space (M_0, S_0)):

(i) **Standard majority favoritism.** The organization attracts all talented candidates and otherwise favors in-group candidates if

$$(1 - X)[S_0 s + (1 - M_0)b] + X[2xs + xb] \geq u.$$

When the above inequality is violated, the organization chooses whether to engage in voluntary AA by recruiting a sufficient share $\tilde{\sigma} \in [0, 1 - 2x]$ of its new recruits among ordinary minority candidates so as to lure in talented minority candidates:

$$(1 - X)[S_0 s + (1 - M_0)b] + X[2xs + (x + \tilde{\sigma})b] = u,$$

or give up on talented minority candidates and recruit only majority candidates (talented if possible). Such a recruitment objective is *effective* only if

$$\tilde{\sigma} \leq 1 - 2x \qquad \text{and} \qquad (1 - X)(1 - M_0) + X(x + \tilde{\sigma}) \leq \frac{1}{2},$$

i.e., only if

$$u \ \leq\ (1-X)S_0 s + X(2xs) + \min\left((1-X)(1-M_0)b + X(1-x)b,\ \frac{b}{2}\right) \qquad (21)$$

Ignoring the participation constraint of talented majority candidates,[51] such a recruitment objective is *desirable*, i.e., the organization's majority prefers engaging in voluntary AA to attract talented minority candidates if and only if

$$2xs + (1-x-\tilde{\sigma})b \geq xs + b,$$

i.e., if and only if

$$u \leq (1-X)[S_0 s + (1-M_0)b] + X[3xs].$$

Hence, the three remaining cases are:

(ii) **Affirmative action to attract talented minority candidates.** The organization attracts all talented candidates and engages in affirmative action if (21) holds and

$$(1-X)[S_0 s + (1-M_0)b] + X[3xs] \ \geq\ u\ >\ (1-X)[S_0 s + (1-M_0)b] + X[2xs + xb].$$

(iii) **Giving up on talented minority candidates.** The organization attracts only majority talented candidates and does not engage in affirmative action if either (21) does not hold or

$$(1-X)[S_0 s + M_0 b] + X[xs + b] \ \geq\ u\ >\ (1-X)[S_0 s + (1-M_0)b] + X[3xs].$$

(iv) **Failing to attract talent.** The organization fails to attract any talented candidate and recruits only ordinary majority candidates if

$$u\ >\ (1-X)[S_0 s + M_0 b] + X[xs + b].$$

[51] As noted above, if the participation constraint of talented minority candidates holds, so does the one of talented majority candidates.

Proposition F.1 follows by letting

$$
\begin{aligned}
u_a &\equiv u + \frac{X}{1-X}(u - 2xs - xb), \\
u_b &\equiv \max\left(u - \frac{X}{1-X}(2xs + (1-x)b - u);\ u - \frac{X}{1-X}(3xs - u)\right), \\
u_c &\equiv u + \frac{X}{1-X}(u - xs - b), \\
S_A &\equiv \frac{1}{s}\left(u - \frac{b}{2} - \frac{X}{1-X}\Big(2xs + \frac{b}{2} - u\Big)\right).
\end{aligned}
$$

□

G Complements on the proof of Proposition 1

We establish a claim made in the proof of Proposition A.2: for any initial state in $R_{3\cup 4}$ and any optimal control $\sigma_0^\sharp$ conditional on reaching R_2 in finite time, let $\mathfrak{T}$ be the time defined by (9), i.e., at which $\sigma_0^\sharp$ switches from standard favoritism to full AA. Then, as long as the state reached at time $\mathfrak{T}$, $(M(\mathfrak{T}), S(\mathfrak{T}))$, satisfies (1), the majority's value function at time 0 is a decreasing function of $\mathfrak{T}$. Hence, for any optimal trajectory, $(M(\mathfrak{T}), S(\mathfrak{T})) \in R_3$ implies that $\mathfrak{T} = 0$.

For a given initial state $(M_0, S_0) \in R_3$ and a given $\mathfrak{T} \geq 0$, the time $T(\mathfrak{T})$ at which the organization reaches R_2 is given by

$$
\Big[(S_0 - x)s - (M_0 - 1)b\Big]e^{-\chi T} - (1-x)be^{-\chi(T-\mathfrak{T})} = -\Big[xs + (1-x)b - u\Big]. \tag{22}
$$

The majority's objective as a function of $\mathfrak{T}$ thus writes as (up to a factor $1/(r+2\chi)$)

$$
\begin{aligned}
&\int_0^{\mathfrak{T}} e^{-(r+\chi)t}\left(\Big[(S_0 - x)s + (M_0 - 1)b\Big]e^{-\chi t} + xs + b\right)dt \\
&+ \int_{\mathfrak{T}}^{T(\mathfrak{T})} e^{-(r+\chi)t}\left(\Big[(S(\mathfrak{T}) - x)s + (M(\mathfrak{T}) - x)b\Big]e^{-\chi(t-\mathfrak{T})} + xs + xb\right)dt \\
&+ \int_{T(\mathfrak{T})}^{\infty} e^{-(r+\chi)t}\Bigg(\Big[(S(T(\mathfrak{T})) - 2x)s + (M(T(\mathfrak{T})) - x - (1-2x)\sigma_0^\dagger)b\Big]e^{-\chi(t-T(\mathfrak{T}))} \\
&\qquad\qquad + 2xs + \Big[x + (1-2x)\sigma_0^\dagger\Big]b\Bigg)dt.
\end{aligned} \tag{23}
$$

Let us show that for any $\mathfrak{T}$ such that $(M(\mathfrak{T}), S(\mathfrak{T}))$ satisfies (1), this objective decreases with $\mathfrak{T}$. The derivative of the majority's objective (23) with respect to $\mathfrak{T}$ is proportional to

$$e^{-(r+\chi)\mathfrak{T}}\left(xs+(1-x)b-u-\Big[3xs+(1-x)b-u\Big]e^{-(r+2\chi)(T(\mathfrak{T})-\mathfrak{T})}\right),$$

and, by (22), the term between large parentheses is equal to

$$\begin{aligned}
&xs+(1-x)b-u-\Big[3xs+(1-x)b-u\Big]\left(\frac{xs+(1-x)b-u}{(1-x)be^{\chi\mathfrak{T}}-(S_0-x)s+(M_0-1)b}\right)^{\frac{r+2\chi}{\chi}}e^{(r+2\chi)\mathfrak{T}}\\
&=xs+(1-x)b-u-\Big[3xs+(1-x)b-u\Big]\left(\frac{xs+(1-x)b-u}{(1-x)b-(S_0-x)se^{-\chi\mathfrak{T}}+(M_0-1)be^{-\chi\mathfrak{T}}}\right)^{\frac{r+2\chi}{\chi}}\\
&\leq xs+(1-x)b-u-\Big[3xs+(1-x)b-u\Big]\left(\frac{(1-x)b}{3xs+(1-x)b-u}\right)^{\frac{r+2\chi}{r+\chi}}\\
&<0,
\end{aligned}$$

where the first inequality follows from $(M(\mathfrak{T}),S(\mathfrak{T}))$ satisfying (1), and the second inequality from the observation that under conditions $(N),(D),(TC)$,[52]

$$\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right)^{\frac{r+2\chi}{r+\chi}}<\frac{3xs+(1-x)b-u}{xs+(1-x)b-u}. \tag{24}$$

Consequently, for any $\mathfrak{T}$ such that $(M(\mathfrak{T}),S(\mathfrak{T}))$ satisfies (1), the majority's objective decreases with $\mathfrak{T}$.

H Complements on Section 3.2: Comparative statics

H.1 Proof of Corollary 1

The comparative statics follow from the analysis. Indeed, for $u\leq b/2$ and under conditions (N), (E), (D) and (TC), the four regions described in the text exist and are non-empty. The organization converges to the steady-state (M_h^*,S_h^*) from any initial

[52] Indeed, one then has that:

$$\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right)^{\frac{r+2\chi}{r+\chi}}<\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right)^{2},$$

and

$$\frac{\big[3xs+(1-x)b-u\big]\big[xs+(1-x)b-u\big]}{\big[(1-x)b\big]^2}=1-\frac{(3xs-u)(u-xs)}{\big[(1-x)b\big]^2}-\frac{2(u-2xs)}{(1-x)b}<1.$$

state in regions 1, 2 or 3, and converges to the steady-state (M_m^*, S_m^*) from any initial state in region 4. Lastly, region 2 – and thus also region 1 – depend neither on χ nor on r, while a higher χ or a lower r loosen condition (1), and thus widen region 3 and reduce region 4.[53]

H.2 Comparative statics when $u > b/2$

A higher initial homogeneity strictly improves the organization's steady-state quality if and only if it pushes the organization out of R_5 and inside R_4 when R_5 is absorbing. Conversely, a lower initial homogeneity strictly improves the organization's steady-state quality if and only if it pushes the organization out of R_4 and inside R_3, or in the case $b/2 < u < xs + b/2$, within R_4 towards initial states from which the organization reaches R_3 in a finite time.[54]

The analysis in Appendices B.1-B.2 shows that R_5 always lies below R_3 and below the subset of R_4 from which R_3 is reached in a finite time, and that the boundary between R_5 and R_4 is downward-sloping. Therefore, a higher initial homogeneity may strictly improve, resp. strictly deteriorate, the organization's steady-state quality only if the organization's initial quality is low (and R_5 non-empty and absorbing), resp. only if the organization's initial quality is high.

I Complements on Section 3.3

As mentioned in the text, a *distrust* equilibrium, robust to deviations by coalitions of talented minority candidates, can exist in which talented minority candidates expect the organization never to engage in affirmative action and the organization never does.

For $u \leq b/2$ and under conditions (N), (E), (D) and (TC), this equilibrium exists

[53] For $2xs + xb < u < \min(3xs, b/2)$,

$$\frac{d}{d\chi}\left(\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right)^{\frac{\chi}{r+\chi}}\right) = \frac{r}{(r+\chi)^2}\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right)^{\frac{\chi}{r+\chi}} \ln\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right) > 0,$$

$$\frac{d}{dr}\left(\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right)^{\frac{\chi}{r+\chi}}\right) = -\frac{\chi}{(r+\chi)^2}\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right)^{\frac{\chi}{r+\chi}} \ln\left(\frac{3xs+(1-x)b-u}{(1-x)b}\right) < 0.$$

[54] When $b/2 < u < xs + b/2$, some trajectories starting in R_4 reach R_3 in a finite time (see Appendix B.2). The set of such initial states lies below a downward-sloping line. Hence, a lower initial homogeneity can make the organization start in this subregion of R_4, eventually converging to (M_h^*, S_h^*), instead of starting in the rest of R_4, where it would converge to (M_m^*, S_m^*).

only if

$$\frac{r+\chi}{r+2\chi}(3xs) + \frac{\chi}{r+2\chi}(xs) < u.$$

In this equilibrium, talented minority candidates accept the organization's offers only at states (M, S) such that

$$Ss + (1-M)b \geq u + \frac{\chi}{r+\chi}(u - xs) \quad (> u),$$

while in the equilibrium of Proposition 1, they do so whenever $Ss + (1-M)b \geq u$.

More generally, for $u \leq b/2$ and under conditions (N), (E), (D) and (TC), there exist at most two (generic) equilibria, robust to coalitions of talented candidates, in which talented minority candidates accept their offers at (M, S) if and only if $Ss+(1-M)b \geq u'$, for some $u' \geq 0$:

(i) the canonical equilibrium, which corresponds to $u' = u$,

(ii) the distrust equilibrium, which corresponds to $u' = u + \frac{\chi}{r+\chi}(u - xs) > u$, and exists only if $\frac{r+\chi}{r+2\chi}(3xs) + \frac{\chi}{r+2\chi}(xs) < u$.

Indeed, using the same arguments as in the proof of Proposition 1 (see Appendix A), three cases must be distinguished depending on u':

(i) if $u' < u$, there exists a non-empty region of states (M, S) (analogous to region 3), close to $\{(M, S)|Ss+(1-M)b = u'\}$ at which the organization engages in voluntary AA until it reaches $\{(M, S)|Ss + (1-M)b = u'\}$, and it subsequently maintains AA so as to remain in $\{(M, S)|Ss + (1-M)b = u'\}$ at all future dates. But then the set $\{(M, S)|Ss + (1-M)b = u'\}$ is absorbing and talented minority candidates thus strictly prefer rejecting their offers when $Ss + (1-M)b = u'$, a contradiction.

(ii) if $u < u' < 3xs$, there again exists a non-empty region of states (M, S) (analogous to region 3), close to $\{(M, S)|Ss+(1-M)b = u'\}$ at which the organization engages in voluntary AA until it reaches $\{(M, S)|Ss + (1-M)b = u'\}$, and it subsequently maintains AA so as to remain in $\{(M, S)|Ss + (1-M)b = u'\}$ at all future dates. But then talented minority candidates strictly prefer accepting their offers at states (M, S) with $Ss + (1-M)b = u' - \varepsilon$ and $\varepsilon > 0$ small, a contradiction.

(iii) lastly, if $3xs < u'$,the organization never engages in voluntary AA and instead adopts standard favoritism. Hence, talented minority candidates strictly prefer accepting their offers whenever $Ss+(1-M)b > u+\frac{\chi}{r+\chi}(u-xs)$. Then, the (steady-state) relative benefit for the majority from having talented minority candidates on board thanks to AA is equal to $3xs - u' = 3xs - u - \frac{\chi}{r+\chi}(u - xs)$. It is strictly negative, i.e., the majority strictly prefers not to engage in AA if and only if[55]

$$u > \tfrac{r+\chi}{r+2\chi}(3xs) + \tfrac{\chi}{r+2\chi}(xs).$$

J Complements on Section 4.1

J.1 Complement on Proposition 4

To illustrate Proposition 4, we derive explicit formulas for the boundaries between $R_3 - R_4$ and $R_4 - R_5$ for a specific set of parameters (see Figure 4). We focus on the case $u > xs + b/2$. With (constant) recruitment strategies $\sigma_1(M,S) = \sigma_2(M,S) = \sigma_0(M,S) = 1$ and (constant) acceptance strategies $a_1(M,S) = 1$, $a_2(M,S) = 0$ for talented candidates, the majority's net-present-value payoff at time 0 is equal to

$$\begin{aligned}
&\int_0^T e^{-(r+\chi)t}\left[(M_0\tilde{b} + S_0\tilde{s} - x\tilde{s} - \tilde{b})e^{-\chi t} + x\tilde{s} + \tilde{b}\right]dt \\
&\qquad\qquad + \int_T^\infty e^{-(r+\chi)t}\left[\overline{M}\tilde{b} + (S_0\tilde{s} - x\tilde{s})e^{-\chi t} + x\tilde{s}\right]dt \\
&= \frac{1}{r+\chi}\Big[(r+\chi)S_0 s + \chi xs\Big] + \frac{1}{r+\chi}\Big[(r+\chi)M_0 + \chi b - \chi(1-\overline{M})be^{-(r+2\chi)T}\Big]
\end{aligned}$$

where $T = \frac{1}{\chi}\ln(\frac{1-M_0}{1-\overline{M}})$. This value strictly decreases with $\overline{M}$, i.e. the lower the upper bound on the majority size $\overline{M}$, the lower the majority's payoff from following the standard-favoritism strategy.

Adapting condition (12), the frontier between R_4 and R_5 now writes as

$$S_0 s + M_0 b \;=\; u + \frac{\chi}{r+\chi}(u - xs - b) + \frac{\chi}{r+\chi}(1-\overline{M})\left(\frac{1-\overline{M}}{1-M_0}\right)^{\frac{r+2\chi}{\chi}} b.$$

as long as (i) any trajectory starting from that boundary and moving inside R_4 remains in R_4 at all times, and (ii) any trajectory starting from that boundary and moving inside

[55] By contrast, in the canonical equilibrium, this relative benefit is equal to $3xs - u$, and it is strictly positive by condition (D).

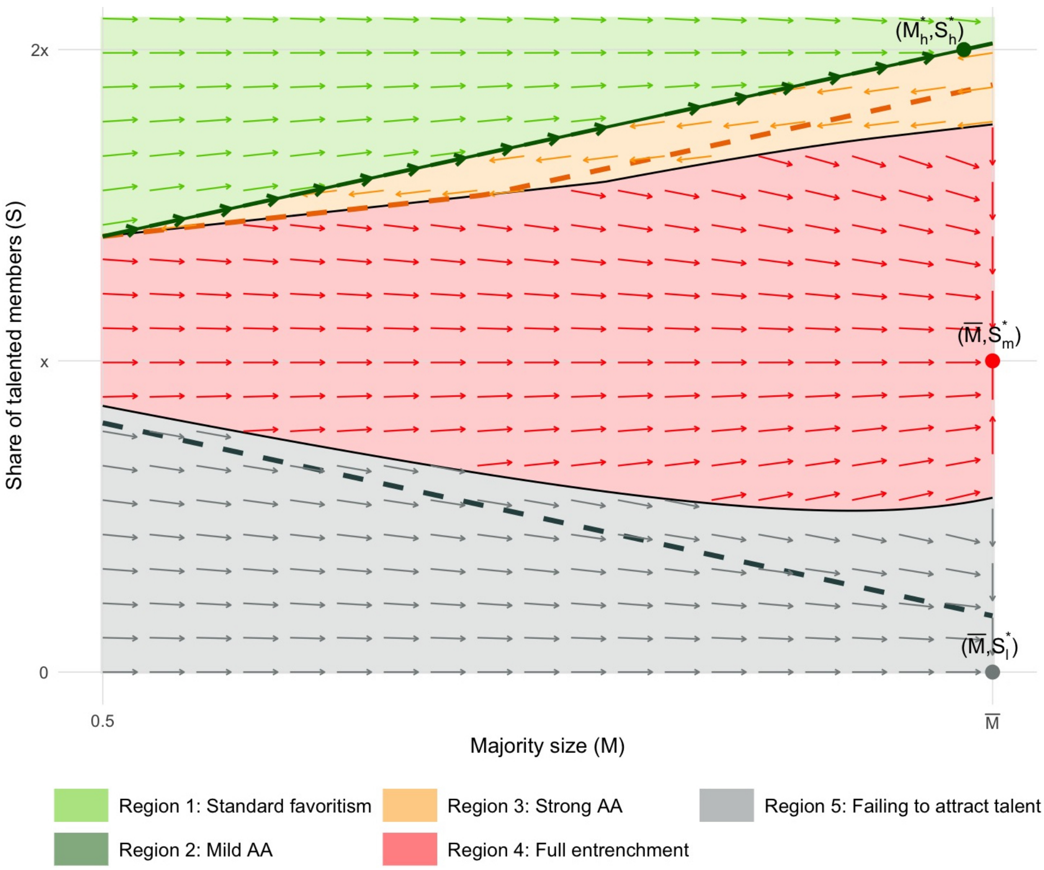

Figure 4: Phase diagram for $\overline{M} \in (M_h^*, 1)$. Thick dotted curves indicate the boundaries of regions 3&4 and 4&5 under laissez-faire.

R_5 remains in R_5 at all times. For these two conditions to be satisfied, we thus assume that

$$xs + \overline{M}b > u, \qquad \text{and} \qquad u > \frac{\chi}{r + 2\chi} xs + \overline{M}b.$$

Similarly, adapting condition (1) with the cap $\overline{M}$, the majority is indifferent between the trajectory induced by full AA until reaching R_2, then mild AA to (M_h^*, S_h^*), and the trajectory induced by standard favoritism (with talented majority candidates accepting their offers, but talented minority ones declining theirs) if and only if:

$$\left(\frac{Mb - Ss + x(s-b)}{xs + (1-x)b - u}\right)^{\frac{r+\chi}{\chi}} - \frac{3xs + (1-x)b - u}{(1-x)b - (1-\overline{M})\left(\frac{1-\overline{M}}{1-M}\right)^{\frac{r+2\chi}{\chi}} b} = 0 \qquad (25)$$

The majority strictly prefers the trajectory induced by full AA until reaching R_2 if the LHS is strictly negative, resp. the one induced by standard favoritism if the LHS is strictly positive.

Equation (25) thus determines the boundary between R_3 and R_4 only if such strategies are time-consistent, i.e., only if (i) any trajectory starting from that boundary and moving inside R_3 remains in R_3 at all times until reaching R_2, and (ii) any trajectory starting from that boundary and moving inside R_4 remains in R_4 at all times. However, there always exists a neighborhood of $\overline{M}$ (with $M < \overline{M}$) such that (i) is violated. To see this, note that for any (M, S) satisfying (25) and with the dynamics induced by the full AA strategy, the derivative of the RHS in (25) with respect to time has the same sign as

$$-(r+\chi)(1-x)b\left[1-\left(\frac{1-\overline{M}}{1-M}\right)^{\frac{r+3\chi}{\chi}}\right]+\chi(M-x)\left(\frac{1-\overline{M}}{1-M}\right)^{\frac{r+3\chi}{\chi}}b,$$

which is a strictly increasing function of M, strictly positive for $M = \overline{M}$.

Hence, let $\overline{M}' < \overline{M}$ be such that

$$-(r+\chi)(1-x)b\left[1-\left(\frac{1-\overline{M}}{1-\overline{M}'}\right)^{\frac{r+3\chi}{\chi}}\right]+\chi(\overline{M}'-x)\left(\frac{1-\overline{M}}{1-\overline{M}'}\right)^{\frac{r+3\chi}{\chi}}b=0,$$

and let $\overline{S}'$ be the quality determined by (25) with $M = \overline{M}'$. For $M \geq \overline{M}'$, the boundary between R_3 and R_4 is then given by the set of states such that the trajectory starting from such states and induced by the full AA strategy reaches majority size $M = \overline{M}'$ with quality $S = \overline{S}'$.

Therefore, the boundary between R_3 and R_4 is given by the set of states (M, S) with $Ss - Mb < u - b$ such that either

- $M < \overline{M}'$ and (M, S) satisfies (25), or
- $M \geq \overline{M}'$ and $S - x = \dfrac{\overline{S}' - x}{\overline{M}' - x}(M - x)$.

J.2 The desirability of mandated affirmative action

Corollary J.1. ***(The desirability of mandated affirmative action)*** *Suppose that* $(N), (E), (D), (TC)$ *hold. Let* $\overline{M} \in (M_h^*, 1)$*. Suppose that the* $\overline{M}$*-constrained canonical equilibrium is selected. Three cases arise following the principal's implementation of a cap on majority size:*

(i) If the intervention does not change the region in which the organization's initial state lies, then the intervention strictly decreases W *if the majority-size cap binds after a finite time, and does not affect* W *otherwise.*

(ii) If the intervention creates a virtuous path (the initial state lies in region 3 instead of region 4), it strictly increases W if and only if the principal is sufficiently patient and puts a sufficiently high weight on the organization's quality,[56] *and it decreases W otherwise.*

(iii) If the intervention creates a vicious path (the initial state lies in region 5 instead of region 4), it strictly decreases W.

Proof. The result follows from the characterization of the intervention's outcomes in Proposition 4, noting that the principal's flow objective,

$$qS_t\tilde{s} + \left[M_t^2 + (1 - M_t)^2\right]\tilde{b}$$

strictly increases with $S_t \in (0, 1)$ and $M_t \in (1/2, 1)$. □

[56] More specifically, if and only if the principal's discount rate r_P satisfies $r_P < \overline{r}(q)$, where $\overline{r}(q)$ strictly increases with q, with $\overline{r}(1) < r$ and $\lim_{q \to +\infty} \overline{r}(q) = +\infty$.