Knowing your Lemon before you Dump It[*]

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Abstract

In many games of interest (e.g., trade, entry, leadership, warfare, and partnership environments), one player (the leader) covertly acquires information about the state of Nature before choosing whether to engage with another player (the follower). The friendliness of the follower's reaction depends on his beliefs about what motivated the leader's choice to engage. We provide necessary and sufficient conditions for the leader's value of acquiring more information to increase with the follower's expectations. We then derive the economic implications of this characterization, focusing on three closely related topics (cognitive traps, disclosure, and cognitive styles), and drawing policy implications.

Keywords: Endogenous adverse selection, expectation conformity, generalized lemons problem, cognitive traps, optimal policy.

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1 Introduction

Many strategic situations of interest can be thought of as Stackelberg games in which one player, player L (the leader, "she") chooses between an "adverse-selection-sensitive" action and an "adverse-selection-insensitive" one. The reaction of the other player, player F (the follower, "he") to the adverse-selection-sensitive action depends on his beliefs about what motivated L's choice of action. For example, player L may represent a seller choosing between offering to trade with a buyer (the adverse-selection-sensitive action) and opting out of the negotiations, as in Akerlof's (1970) lemons model. More generally, player F may still act following the adverse-selection-insensitive action. For example, the latter action may represent the seller's decision to disclose hard information proving unambiguously what the seller knows about the value of the asset. In this case, the decision to disclose hard information is adverse-selection-insensitive because the price offered by the buyer (the follower's reaction) is invariant to his beliefs about what motivated the seller's decision to disclose. The key assumption is that information that makes player L eager to engage with player F by choosing the adverse-selection-sensitive action (for example, by choosing not to disclose what she knows) makes player F react in an unfriendlier manner. Notable examples of such situations include, in addition to Akerlof's (1970) lemons model, many entry and partnership games that are central to the Industrial Organization, Finance, and Organization Economics literatures.

We enrich this classic model by allowing player L to covertly acquire information about the state of Nature before making her engagement decision. We are particularly interested in understanding how player L's information choice depends on player F's expectations (the relationship between the two naturally reflecting how strategic considerations shape the value of information in the class of games under consideration). We identify sufficient and/or necessary conditions for *expectation conformity* (EC) to hold in these games, namely for player L to find it more valuable to acquire more information when player F expects her to do so. Besides being of independent interest, EC plays a major role in equilibrium analysis and has important economic implications. In particular, interactions or markets may switch behavior abruptly; for instance, asset markets can tip from a pattern in which the assets receive little scrutiny to one in which they are heavily scrutinized by participants. EC also shapes the benefits of disclosure of hard information and plays a key role in the possibility that the players end up in a cognitive trap where they suffer from the information they are expected to acquire.

Section 2 defines a broad class of generalized lemons environments, in which one of the players acquires information covertly and then decides whether or not to engage with another player (i.e., chooses between an adverse-selection-sensitive action and an adverse-selection-insensitive one); as shown in the online Supplement, a number of familiar games can be reinterpreted within this framework.

Section 3 introduces the notion of EC. To put flesh on the characterization, we compare information structures through the mean-preserving-spread (MPS) order, or the more refined rotations order. The MPS order says that the distribution over the posterior mean under a more informative structure is a mean-preserving spread of the corresponding distribution under a less informative structure, which is always the case when the former distribution is obtained through an experiment that Blackwell-dominates the one generating the latter distribution. The rotations order is a strengthening of the MPS order that obtains, for instance, under non-directed search, that is when player L invests in the

probability of learning the state of Nature (equivalently, the value of the interaction with the other player). For more general environments, it is a property of the family of distributions over player F's posterior mean (we give examples with uniform, Pareto and Exponential distributions).

The analysis delivers a sufficient (and, under further assumptions, necessary) condition for such games to satisfy EC. This condition says that more cognition (equivalently, the choice by player L of a Blackwell-more-informative experiment) (a) aggravates the adverse selection problem, in a well-defined sense, which makes F's reaction less friendly to player L, and (b) that an unfriendlier reaction by F in turn raises L's incentive to acquire a more informative experiment; or that both conditions are simultaneously reversed. The condition for EC is easier to check than verifying directly that EC prevails. It obtains, for example, when, holding player F's reaction fixed, an increase in cognition reduces the probability of trade, both when such a probability is computed by player L, given her actual choice of cognition, and by player F, given the level of cognition he anticipates from player L.

In the cognition-augmented lemons game under non-directed search where the leader is a seller of an asset and the buyer a representative of a competitive market, as in Akerlof's model, EC holds when the gains from trade are large, but not for low gains. This is because large gains from trade induce the buyer to offer a high price that the seller finds it optimal to accept when uninformed. More cognition by the seller (equivalently, a higher probability of the seller learning the true value of the asset) then reduces the probability of trade by making the seller engage selectively when informed. Cognition thus unambiguously aggravates adverse selection, inducing the buyer to lower the price. This in turn raises the cost for the seller of parting with the asset when its value is high, raising the seller's value of acquiring more information. Hence, EC holds in this case. When, instead, the gains from trade are small, the price offered by the buyer is low, which makes the seller unwilling to trade when uninformed. Because the seller engages only when informed, an increase in cognition has no effect on the severity of the adverse selection problem and hence on the price offered by the buyer. EC thus does not obtain for low gains from trade.

The paper then derives the economic implications of this characterization in Section 4, focusing on three closely-related topics: cognitive traps, disclosure, and cognitive styles. In generalized lemons games, under the key condition for EC mentioned above (namely, that more cognition reduces the probability of trade), the information-acquiring player is worse off in a high-information-intensity equilibrium than in a low-information-intensity one. This cognitive trap is due to the unfriendlier reaction of the follower in response to the exacerbated adverse selection problem under the leader's more informative structure. Importantly, player L may be trapped even when information is free. We then modify the game by assuming that the information acquired is semi-hard instead of soft in that the information-acquiring player can disclose evidence proving that she devoted external resources to the issue no less than the amount disclosed. Such disclosure turns out to be mostly irrelevant. The intuition is related to the cognitive-trap phenomenon: Disclosure serves to demonstrate that one is knowledgeable, which in the end is not profitable. Along a similar vein, it is shown that it is optimal for the leader to pose as an "informational puppy dog," i.e., to convince the other player that she is dumb or busy, that is, that her cost to acquire information is high

Section 5 discusses how the results change in economies in which the lemons assumption is replaced by its anti-lemon counterpart (that is, states in which the leader is most eager to engage are those in

which the follower's response is most favorable to the leader). The condition for EC is flipped. EC obtains when more cognition induces the follower to respond in a friendlier manner and the marginal benefit of cognition increases (instead of decreases) with the friendliness of the follower's reaction; or both conditions are simultaneously reversed.

Section 6 contains policy analysis. It identifies conditions under which subsidies/taxes to trade are welfare enhancing as well as conditions under which the endogeneity of information calls for larger policy interventions. For example, in the Akerlof's model, subsidies to trade are optimal when (a) the cost of public funds is small, (b) cognition aggravates the adverse selection problem, and (c) subsidies reduce cognition. Furthermore, relative to the case where cognition is exogenous, the optimal level of the subsidy is larger. This is because subsidies come with a *double dividend* under endogenous information: In addition to inducing player L to engage more often, they discourage player L from acquiring information, with the second effect further contributing to a reduction in the adverse selection problem and hence to an increase in trade.

Section 7 discusses the robustness of the key insights to alternative (more flexible) information-acquisition technologies: The results qualify in what sense the key conditions for EC extend to these alternative settings. Section 8 concludes. Omitted proofs are in the Appendix at the end of the document or in the online Supplement. The latter also contains various examples of generalized lemons and anti-lemons problems, and discusses the connection to other covert investment games.

Related Literature. The paper is related to various strands of the literature. The first one is the literature on the lemons problem under alternative information structures. Kartik and Zhong (2019) consider a bilateral trading environment with interdependent values and characterize the combination of consumer and producer surplus that can be sustained in equilibrium under any possible information structure. The analysis parallels the one in Bergemann, Brooks and Morris (2015) but in a setting with interdependent payoffs. Related are also Levin (2001), Kessler (2001), and Bar-Isaac et al. (2018). These papers, as Kartik and Zhong (2019), study how payoffs, the volume of trade, and the efficiency of bargaining outcomes vary with the information structure in variants of the Akerlof's model. In contrast, we study (a) how the acquisition of information is shaped by other players' expectations, (b) how the latter expectations depend on the information acquisition technology and the effect of information on the severity of the adverse selection problem, (c) how players may end up in a cognitive trap, and (d) how policy interventions can alleviate the inefficiencies associated with the asymmetry and endogeneity of information.

Dang (2008) and Thereze (2022) also endogenize the information structure in the Akerlof's model. However, the focus of the analysis in these papers is different. Dang (2008) derives conditions under which no information is acquired in equilibrium as well as conditions under which the player acquiring information receives positive surplus despite not having bargaining power at the negotiation stage. Thereze (2022), instead, considers a competitive adverse selection market in which the buyers' information also affects the sellers' costs (as in health markets), and investigates how the elasticity of the demand and the market equilibrium are affected by a change in the cost of information. In Thereze (2022), the buyers acquire information after seeing the prices asked by the sellers. In contrast, in our model, as in Ravid, Roesler, and Szentes (2022), information acquisition takes place prior to observing

the prices.[1]

A fairly vast literature studies information acquisition in bargaining games with private values. See for example Ravid (2020), and Ravid, Roesler, and Szentes (2022) and the references therein. The first paper considers a repeated bargaining setting with a rationally-inattentive buyer. The second paper investigates the properties of the equilibrium when the cost of the buyer's information vanishes in a one-shot ultimatum-bargaining game. Our paper, instead, considers games with interdependent payoffs (as in the lemons problem). It investigates how the information acquired in equilibrium is shaped by the effect of information on the severity of the adverse selection problem. It shows how EC is intrinsically related to the possibility of cognitive traps whereby the information-acquiring player is worse off in a high-information equilibrium than in a low-information one, with these traps emerging even when information is free and the other player is competitive and hence obtains no surplus in equilibrium (as in Akerlof's original model).

Pavan and Tirole (2022a) shares with the present paper the interest in how the possibility to disclose verifiable/hard information affects equilibrium outcomes in settings with interdependent payoffs. That paper focuses on the welfare effects of mandatory disclosure laws. The present paper, instead, focuses on the effects of cognition on the severity of the adverse selection problem and on policy interventions aimed at alleviating such a severity. Expectation conformity is also studied in Pavan and Tirole (2022b). The analysis in that paper is not specific to settings with adverse selection and none of the results in the present paper have counterparts in that paper.

Finally, the discussion of how benevolent governments can improve the efficiency of markets affected by adverse selection is related to Philippon and Skreta (2012) and Tirole (2012). The sellers' information in those papers is exogenous. Instead, the present paper studies how different governments' programs influence the acquisition of information. See also Colombo, Femminis and Pavan (2022) for how governments can incentivize information acquisition in economies with investment complementarities, and Pavan, Sundaresan, and Vives (2022) for how governments can influence information acquisition in financial markets.

2   Framework

2.1   Description

Consider the following game between two players, a "leader" (she) and a "follower" (he).

   (a) Actions and timing

Player L (the "leader") first covertly selects an information structure. After updating her beliefs about the state of Nature upon observing the realization of the selected information structure (equivalently, of the selected experiment), player L then chooses between two actions, $a = 0$ and $a = 1$. Player F (the "follower"), after observing player L's action a but not L's choice of an information structure and its realization, then chooses his reaction to the leader's action. As we explain below, player F's reaction to $a = 0$ plays no role in the analysis and hence we do not formally describe it. His reaction to $a = 1$,

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[1] See also Cremer and Khalil (1992), and Cremer, Khalil, and Rochet (1998) for earlier work on information acquisition in other contractual settings.

instead, will be denoted by $r \in \mathbb{R}$. We normalize player F's action so that a higher r stands for a friendlier response: player L's utility is increasing in r.

(b) Information/cognition

Prior to choosing a, player L acquires information about the state of Nature. The state of Nature, say the car's quality in the lemons model, is denoted by $\omega \in (-\infty, +\infty)$, and is commonly believed to be drawn from a distribution G with prior mean ω_0. We will assume that the two players' preferences are affine in ω, so they care only about the posterior mean m of the state. An experiment, indexed by $\rho \in \mathbb{R}_+$, will be taken to be the choice of a cumulative distribution function $G(\cdot; \rho)$ of the induced posterior mean m, satisfying the martingale property $\int_{-\infty}^{+\infty} m\,dG(m; \rho) = \omega_0$ for all ρ.[2] We will assume that the set of experiments (equivalently, of distributions, $G(\cdot; \rho)$) player L can choose from has the cardinality of the continuum, and then denote such a set by $[0, \bar{\rho})$, with $\bar{\rho} \in \mathbb{R}_+$. To ease the exposition, we also assume that the distributions are ordered in such a way that, for any $m \in \mathbb{R}$, the function $G(m; \cdot)$ is differentiable in ρ and then denote by $G_\rho(m; \rho)$ the partial derivative of $G(m; \rho)$ with respect to ρ. We will also assume that $G_\rho(\cdot; \rho)$ is integrable in m. As the analysis below will clarify, these assumptions permit us to describe some of the key conditions in a concise form. None of the key results hinge on these differentiability assumptions. However, many of the relevant conditions are heavier when the derivatives are replaced with differentials across information structures. For most of the results, we will also assume that higher ρ index distributions $G(\cdot; \rho)$ generated by Blackwell-more-informative experiments. With this in mind, we will often refer to a higher ρ as a "more-informative experiment" and interpret the choice of a higher ρ as the result of the player "engaging in more cognition".

The cost of acquiring information will be denoted by $C(\rho)$. When higher ρ index Blackwell-more-informative experiments, we will assume that C is non-decreasing, differentiable, and weakly convex.[3]

(c) Preferences

Follower. Action $a = 1$ is "adverse-selection-sensitive," in the sense that player F's reaction to $a = 1$ depends on his beliefs about what information privately held by player L motivated L to engage. Consider a fictitious game in which L's cognition is exogenously fixed at $\rho^\dagger$. We assume that, for any $\rho^\dagger$, the equilibrium is unique and denote by $a^*(\cdot; \rho^\dagger)$ and $r(\rho^\dagger)$, respectively, L's engagement strategy and F's reaction to $a = 1$ in the unique equilibrium of the $\rho^\dagger$-game. The function function $a^*(\cdot; \rho^\dagger)$ specifies, for each posterior mean m, the probability $a^*(m; \rho^\dagger) \in [0, 1]$ that player L engages when her posterior mean is m. In the game in which cognition is endogenous, we assume that, when F expects L to select cognition $\rho^\dagger$, he also expects L to engage according to $a^*(\cdot; \rho^\dagger)$. We then denote by $\hat{G}(\cdot; \rho^\dagger)$ the cumulative distribution function describing F's beliefs over L's posterior mean m, when expecting L to select cognition $\rho^\dagger$ and engaging according to $a^*(\cdot; \rho^\dagger)$, after observing $a = 1$.[4] Given $\hat{G}(\cdot; \rho^\dagger)$, F

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[2] Note that the support of $G(\cdot; \rho)$ can be a strict subset of $\mathbb{R}$.

[3] We will use the differentiability assumption to write the optimality conditions in concise form.

[4] We are interested in situations in which, after choosing cognition $\rho^\dagger$, L engages with positive probability. In this case, when expecting cognition $\rho^\dagger$, player F, after observing $a = 1$, updates his beliefs $G(\cdot; \rho^\dagger)$ about m using Bayes rule and the engagement strategy $a^*(\cdot; \rho^\dagger)$. Also, in some of the applications of interest, it may be more natural to think of L as engaging after observing F's action r. Our results apply to some of these setting as well. For example, in the Akerlof's model where F stands for a competitive buyer, whether player L (the seller) observes the price offered by F before deciding to put the asset on sale, or puts the asset on sale anticipating the price offered by the competitive buyer is inconsequential

maximizes his expected payoff $\mathbb{E}_{\hat{G}(\cdot;\,\rho^\dagger)}[u_F(1,\,r,\,m)]$ by means of an action $r \in \mathbb{R}$, where $u_F(1,\,r,\,m)$ is F's payoff when L engages (i.e., selects $a = 1$), F's reaction is r, and L's posterior mean is m.[5]

By contrast, action $a = 0$ is "adverse-selection-insensitive." In some applications, such as Akerlof's lemons example below, action $a = 0$ involves no decision for the follower. More generally, we assume that the follower's reaction to $a = 0$ is independent of his beliefs about $\rho^\dagger$. This is the case, for instance, when $a = 0$ corresponds to the decision by player L to disclose hard information proving that the state (or L's posterior belief) is m, making F's conjecture about L's cognition irrelevant.[6]

Leader. Player L's payoff differential between $a = 1$ and $a = 0$ depends on the friendliness r of F's reaction and on player L's posterior mean m. Let $u_L(0,\,m)$ denote L's payoff when choosing $a = 0$. As just discussed, this payoff may depend on F's reaction. However, because the latter is invariant in F's expectations over L's cognition, we can omit it to ease the notation and interpret $u_L(0,m)$ as L's payoff in state m given F's reaction to $a = 0$. Similarly let $u_L(1,\,r,\,m)$ denote L's payoff when choosing $a = 1$ and then denote by

$$\delta_L(r,\,m) \equiv u_L(1,\,r,\,m) - u_L(0,\,m)$$

L's payoff differential between $a = 1$ and $a = 0$, when F's reaction to $a = 1$ is r and L's posterior mean is m.

Assumption 1 (***leader's preferences***). *Player L's payoff differential, δ_L, is Lipschitz continuous and twice continuously differentiable in each argument, strictly increasing in r, strictly decreasing in m, and such that the marginal impact of a friendlier reaction is weakly increasing in L's posterior mean: for any (r,m),*

$$\frac{\partial^2}{\partial r \partial m}\delta_L(r,m) \geq 0. \tag{1}$$

That δ_L is increasing in r reflects the normalization that a higher r represents a friendlier reaction, favoring $a = 1$. That δ_L is decreasing in m implies that a lower m favors $a = 1$. The strict monotonicity of δ_L in m in turn implies that, no matter the actual choice of cognition ρ, L optimally chooses to engage if and only if m falls below some cutoff $m^*(r)$ that depends on F's reaction r, with the cutoff $m^*(r)$ solving $\delta_L(r, m^*(r)) = 0$ and hence strictly increasing in r. Clearly, in any equilibrium in which L's actual cognition is ρ, the cognition $\rho^\dagger$ expected by F coincides with L's actual cognition ρ, and F's reaction is $r(\rho)$, where recall that $r(\rho)$ is F's equilibrium reaction in a fictitious game in which cognition is exogenously fixed at ρ.

Condition (1) in Assumption 1 says that L's marginal benefit of a friendlier reaction by F is larger in states in which L's payoff from engaging is lower. The condition will be used to determine whether cognition becomes more or less attractive to player L when player F behaves in a friendlier way (see the proof of Part (iii) of Proposition 1 below).

Let player F anticipate cognition $\rho^\dagger$ by player L. Out-of-equilibrium, $\rho^\dagger$ can differ from L's actual cognition ρ, because cognition is covert. However, suppose for a moment that cognition is exogenous and equal to $\rho^\dagger$. Because player F's payoff is quasilinear in ω, his reaction $r(\rho^\dagger)$ depends on the distribution

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because player F's reaction is predictable at the time player L engages.

[5]The assumption that L's and F's payoffs are affine in ω implies that $u_F(1,\,r,\,m)$ is also F's ex-post payoff when the state is $\omega = m$.

[6]See example (c) in the online Supplement.

$\hat{G}(\cdot; \rho^\dagger)$ describing his beliefs over L's posterior mean m only through the mean $\mathbb{E}_{\hat{G}(\cdot; \rho^\dagger)}[m]$ of $\hat{G}(\cdot; \rho^\dagger)$. Furthermore, as explained above, when L's cognition is exogenously fixed at $\rho^\dagger$, in equilibrium, player L's engagement strategy $a^*(\cdot; \rho^\dagger)$ takes the form of a cutoff rule, i.e., L optimally chooses $a = 1$ if and only if $m \leq m^*$, in which case $\mathbb{E}_{\hat{G}(\cdot; \rho^\dagger)}[m] = M^-(m^*; \rho^\dagger)$, where, for any $(m^*, \rho^\dagger)$,

$$M^-(m^*; \rho^\dagger) \equiv \mathbb{E}_{G(\cdot; \rho^\dagger)}[m | m \leq m^*] = m^* - \frac{\int_{-\infty}^{m^*} G(m; \rho^\dagger) dm}{G(m^*; \rho^\dagger)}$$

denotes the truncated mean of the distribution $G(\cdot; \rho^\dagger)$ of m, under cognition $\rho^\dagger$. An increase in M^- can then be viewed as a reduction of the adverse selection problem.

Assumption 2 (lemons). *The friendliness of player F's reaction to an increase in player L's cognition depends positively on the effect of L's cognition on the severity of the adverse selection problem:[7]*

$$\frac{dr(\rho^\dagger)}{d\rho^\dagger} \overset{sgn}{=} \frac{\partial}{\partial \rho^\dagger} M^-\left(m^*(r(\rho^\dagger)); \rho^\dagger\right). \tag{2}$$

Remark [Relative adverse-selection sensitivity]. As explained above, we assume that action $a = 0$ is "adverse-selection-insensitive." However, we expect most of the results to extend to settings in which F's reaction to $a = 0$ also depends on F's beliefs about ρ and m, but with a lower sensitivity to these variables than F's reaction to $a = 1$. The following example illustrates the type of applications that this more general setting can capture. Player L is an employee who can choose between a high- and a low-powered incentive scheme (for brevity, HPIS and LPIS). Action $a = 0$ corresponds to the decision to choose HPIS, whereas $a = 1$ corresponds to the decision to choose LPIS. Let y_a denote the employee's "skin in the game," e.g., the amount of shares of the firm held, with $0 \leq y_1 < y_0 \leq 1$. Player F is a (competitive) employer whose payoff is $\kappa + (1 - y_a)(e_a + m) - r_a$, where κ is a constant, e_a is the effort optimally exerted by the employee (at increasing and convex private cost $\psi(e)$) after choosing action $a \in \{0, 1\}$, and r_a is a fixed wage paid by F to L on top of the money paid through the incentive payment y_a. Hence, in this application, there are two reactions by player F, r_1 and r_0, and each may depend on $\rho^\dagger$. Let $U_L(a, r_a, m)$ and $U_F(a, r_a, m)$ denote the two players' payoffs when the leader takes action a, the follower reacts with action r_a, and L's posterior mean is m. Then $U_L(a, r_a, m) \equiv \max_e\{r_a + y_a(e + m) - \psi(e)\}$ and $U_F(a, r_a, m) \equiv \kappa + (1 - y_a)(e_a + m) - r_a$. Let $r \equiv r_1 - r_0$ and $K_0 \equiv y_1 e_1 - \psi(e_1) - y_0 e_0 + \psi(e_0)$. Then,

$$\delta_L(r, m) \equiv U_L(1, r_1, m) - U_L(0, r_0, m) = r - (y_0 - y_1)m + K_0.$$

For any r, the engagement threshold is then given by $m^*(r) = (r + K_0)/(y_0 - y_1)$. Let $z \equiv (1 - $

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[7]Consistently with what anticipated above, to ease the exposition, we assume that $r(\cdot)$ and $M^-\left(m^*(r(\rho^\dagger)); \cdot\right)$ are differentiable in $\rho^\dagger$ and denote by $\frac{\partial}{\partial \rho^\dagger} M^-\left(m^*(r(\rho^\dagger)); \rho^\dagger\right)$ the partial derivative of $M^-(m^*; \rho^\dagger)$ with respect to $\rho^\dagger$, holding m^* fixed at $m^* = m^*(r(\rho^\dagger))$, where $m^*(r(\rho^\dagger))$ is the engagement threshold for L's equilibrium strategy $a^*(\cdot; \rho^\dagger)$ in the fictitious game in which L's cognition is exogenously fixed at $\rho^\dagger$. These differentiability assumptions permit us to write Condition (2) in concise terms. The key property behind Assumption (2) is that, for any $\rho, \rho^\dagger \in \mathbb{R}_+$, $r(\rho) - r(\rho^\dagger) \overset{sgn}{=} M^-\left(m^*(r(\rho^\dagger)); \rho\right) - M^-\left(m^*(r(\rho^\dagger)); \rho^\dagger\right)$.

$y_0)/(1 - y_1) < 1$ and $K_1 \equiv (1 - y_1)e_1 - (1 - y_0)e_0$ and, for any m^* and $\rho^\dagger$, denote by $M^+(m^*; \rho^\dagger) \equiv \mathbb{E}_{G(\cdot;\rho^\dagger)}[m|m > m^*]$ the expected value of m under the distribution $G(\cdot; \rho^\dagger)$, conditional on m exceeding m^*. Because F is competitive, for any $\rho^\dagger$, $r(\rho^\dagger)$ is then given by the solution to

$$r = K_1 + (1 - y_1) \left[M^-(m^*(r); \rho^\dagger) - zM^+(m^*(r); \rho^\dagger) \right].$$

In our model, $z = 0$. Our results extend to this type of settings provided that (1) δ_L depends only on r and m and satisfies Assumption 1 above (as in this example), (2) z is small so that action $a = 0$ is relatively less "adverse-selection-sensitive" than $a = 1$, and (c) Assumption 2 holds with $M^- - zM^+$ instead of M^- (which is the case, for example, when m is drawn from a Uniform or a Pareto distribution).

2.2 Examples

The Stackelberg game described above (and its key assumptions, 1 and 2) may look somewhat abstract. In this subsection, we show how Akerlof's lemons problem, augmented by the seller's endogenous covert information acquisition, maps into the general framework described above, and then briefly discuss other examples developed in the online Supplement.

Akerlof's model. In Akerlof's (1970) model, player L is a seller of an asset (e.g., a used car). She can sell the good in the market ($a = 1$) or keep it for herself for own consumption ($a = 0$). Player F is a representative of a set of competitive buyers who choose a price r equal to the expected value of the good conditional on the good being put in the market. Suppose that the players' gross values for the good are m for the seller and $m + \Delta$ for the representative buyer, where Δ parametrizes the gains from trade, with $\Delta \in (0, \sup\{supp(G)\} - \omega_0)$, where $supp(G)$ is the support of G.[8] Then, $r(\rho^\dagger)$ is the price offered by the competitive buyer when the seller's cognition is exogenously fixed at $\rho^\dagger$ and is given by the solution to the following equation

$$r = \mathbb{E}_{G(\cdot;\rho^\dagger)}[m + \Delta|m \leq r] = M^-(r; \rho^\dagger) + \Delta, \tag{3}$$

reflecting the fact that the cutoff $m^*(r)$ for L's equilibrium engagement strategy $a^*(\cdot; \rho^\dagger)$ is equal to r. Consistently with what explained above, we assume that the solution to (3) is unique, which is the case, for example, when $G(\cdot; \rho^\dagger)$ is absolutely continuous with density $g(\cdot; \rho^\dagger)$, and the inverse hazard rate $G(\cdot; \rho^\dagger)/g(\cdot; \rho^\dagger)$ of the distribution of m for cognition $\rho^\dagger$ is increasing in m.[9] Assumption 2 is then satisfied. So is Assumption 1, given that, in this application, $\delta_L(r, m) = r - m$.

Turning to the case in which the seller's cognition is endogenous, we then have that L's optimal choice of ρ when L anticipates a reaction r by F is given by

$$\max_\rho \{ G(r; \rho)r + \int_r^\infty m \, dG(m; \rho) - C(\rho) \}.$$

When C and G are differentiable in ρ and the above objective function for player L satisfies the appro-

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[8]When $\Delta \geq \sup\{supp(G)\} - \omega_0$, there is no adverse selection; the competitive buyer offers $\omega_0 + \Delta$ and the seller sells no matter her posterior mean. This case is not interesting.

[9]Then $\partial M^-(r; \rho^\dagger)/\partial m^* \in (0, 1)$. See An (1998).

priate quasi-concavity conditions (we will maintain these assumptions throughout the entire paper when referring to this example), the optimal level of ρ is then given by the following first-order condition[10]

$$-\int_{r}^{+\infty} G_{\rho}(m;\rho)dm = C'(\rho). \tag{4}$$

Other examples. The general model above also admits as a special case a different version of the Akerlof model in which the buyer, instead of being competitive, has full bargaining power. This version is the interdependent-value counterpart of the game considered in Ravid, Roesler, and Szentes (2022). In the Supplement, we show how a number of other games of interest fit into the framework introduced above. In the first example, a government engages in asset repurchases so as to jump-start a frozen market. In the second example, the good is divisible (a share in a project); the owner benefits from the synergies resulting from taking an associate in the project, but is hesitant about sharing the proceeds if she knows the project is highly profitable. In the third example, the seller may have hard information about the quality of the good and chooses whether to keep the evidence secret (which amounts to engaging in this example) or disclose it to the buyer (not engaging). The fourth example describes herding with interdependent payoffs; for example, by entering a market, a firm may encourage a rival to follow suit. The fifth example is a marriage game, in which covenants smooth the hardship of a subsequent divorce, but also signal bad prospects about the marriage. Some of these examples naturally feature a non-linear δ_L function which explains the generality introduced above.[11] We refer the reader to the Supplement for the details.

3 Expectation conformity

We now investigate how L's choice of cognition (i.e., of an information structure) is influenced by F's expectations and how the latter in turn depend on whether L's cognition aggravates adverse selection. Adverse selection is here captured by the truncated mean $M^-(m^*;\rho^\dagger)$. Consistently with what discussed above, we will simplify teh notation by assuming that $M^-(m^*;\rho^\dagger)$ is differentiable in ρ.

Definition 1 (impact of cognition on adverse selection). Starting from cognition $\rho^\dagger$, an increase in cognition by player L

- aggravates adverse selection if $\frac{\partial}{\partial\rho^\dagger}M^-(m^*(r(\rho^\dagger));\rho^\dagger) < 0$

- alleviates adverse selection if $\frac{\partial}{\partial\rho^\dagger}M^-(m^*(r(\rho^\dagger));\rho^\dagger) > 0$.

Simple computations show that, for any cognition $\rho^\dagger$ and truncation point m^*,

$$\frac{\partial}{\partial\rho^\dagger}M^-(m^*;\rho^\dagger) \overset{\text{sgn}}{=} A(m^*;\rho^\dagger) \tag{5}$$

---

[10]Note that the FOC for ρ can also be written as $\int_{-\infty}^{r} G_{\rho}(m;\rho)dm = C'(\rho)$. This is because $\int_{-\infty}^{+\infty} mdG(m;\rho)$ is invariant in ρ, implying that $\int_{-\infty}^{+\infty} G_{\rho}(m;\rho)dm = 0$.

[11]As explained below, a non-linear δ_L function also brings additional effects to the analysis, for example by making L's value for information depend, among other things, on the induced volatility of m.

where

$$A(m^*; \rho^\dagger) \equiv \left[m^* - M^-(m^*; \rho^\dagger) \right] G_\rho(m^*; \rho^\dagger) - \int_{-\infty}^{m^*} G_\rho(m; \rho^\dagger) dm. \tag{6}$$

The first term of A captures the direct effect of a change in the probability that player L engages on player F's expectation of the state. Because $m^* \geq M^-(m^*; \rho^\dagger)$, an increase in cognition alleviates adverse selection when it increases the chances that player L engages (i.e., when $G_\rho(m^*; \rho^\dagger) > 0$), whereas it aggravates it when it reduces the probability of such an event (i.e., when $G_\rho(m^*; \rho^\dagger) < 0$). The second term, of A, $\int_{-\infty}^{m^*} G_\rho(m; \rho^\dagger) dm$, in turn is related to the effect of cognition on the dispersion of L's posterior mean m. When more cognition induces more dispersion in the sense of second-order stochastic dominance (which is always the case when higher ρ index distributions $G(\cdot; \rho)$ generated by Blackwell-more-informative experiments), this second effect unambiguously contributes to an aggravation of the adverse selection problem. Hereafter, we will refer to

$$\mathrm{A}(\rho^\dagger) \equiv A(m^*(r(\rho^\dagger)); \rho^\dagger) \tag{7}$$

as the "*adverse-selection effect*" of an increase of cognition at $\rho^\dagger$. Note that, under Assumption 2, when $A(\rho^\dagger) > 0$ (alternatively, $A(\rho^\dagger) < 0$), starting from $\rho^\dagger$ a small increase in the cognition expected by player F triggers a friendlier (alternatively, an unfriendlier) reaction by player F.

Now recall that L's ex-ante payoff, gross of the cognitive cost, from choosing cognition ρ when expecting a reaction r by F to her decision to engage is equal to

$$\Pi(\rho; r) \equiv \sup_{a(\cdot)} \left\{ U_L(0) + \int_{-\infty}^{+\infty} a(m)\, \delta_L(r, m) dG(m; \rho) \right\}$$

where $U_L(0) \equiv \int_{-\infty}^{+\infty} u_L(0, m) dG(m)$ is L's ex-ante expected payoff when she never engages, and $a(m)$ represents the probability that L engages when her posterior mean is m.[12]

Then let

$$B(\rho; \rho^\dagger) \equiv -\frac{\partial^2 \Pi(\rho; r(\rho^\dagger))}{\partial \rho \partial r}$$

denote the effect of a reduction in the friendliness of F's reaction, starting from $r = r(\rho^\dagger)$, on L's marginal value of information, evaluated at cognition ρ. Hereafter, we will refer to $B(\rho; \rho^\dagger)$ as the "*benefit of friendlier reactions effect*".

Definition 2 (cognition incentive effect of unfriendly reactions). Given $(\rho, \rho^\dagger)$, a reduction in the friendliness of player F's reaction starting from $r = r(\rho^\dagger)$, raises (alternatively, lowers) player L's incentive to invest in cognition at ρ if $B(\rho; \rho^\dagger) > 0$ (alternatively, if $B(\rho; \rho^\dagger) < 0$).

Using the envelope theorem along with the fact that, for any ρ, the optimal engagement strategy for L when F anticipates cognition $\rho^\dagger$, is to engage if and only if $m \leq m^*(r(\rho^\dagger))$, and integrating by parts,

---

[12]Note that, because $u_L(0, m)$ is affine in m, $\int_{-\infty}^{+\infty} u_L(0, m) dG(m; \rho) = \int_{-\infty}^{+\infty} u_L(0, m) dG(m)$ for any ρ, implying that $U_L(0)$ is invariant in ρ.

we have that

$$
\begin{aligned}
B(\rho; \rho^\dagger) \;=\; & -\frac{\partial \delta_L(r(\rho^\dagger), m^*(r(\rho^\dagger)))}{\partial r} G_\rho\Big(m^*(r(\rho^\dagger)); \rho\Big) \\
& + \int_{-\infty}^{m^*(r(\rho^\dagger))} \frac{\partial^2 \delta_L(r(\rho^\dagger), m)}{\partial r \partial m} G_\rho(m; \rho) dm.
\end{aligned}
\tag{8}
$$

Because δ_L is increasing in r, the sign of the first term of $B(\rho; \rho^\dagger)$ is determined by whether an increase in cognition increases or reduces the chances that player L engages. Under Assumption 1, the marginal benefit of a friendlier reaction by player F are increasing in the posterior mean m. As a result, the second term of $B(\rho; \rho^\dagger)$ is always positive when higher cognition by player L indexes a mean preserving spread of the induced posterior mean.

Next, let $V_L(\rho; \rho^\dagger) \equiv \Pi(\rho; r(\rho^\dagger))$ denote the maximal payoff that player L can obtain by engaging in cognition ρ when player F expects cognition $\rho^\dagger$.

Definition 3 (expectation conformity). Expectation conformity (EC) holds at $(\rho, \rho^\dagger)$ if and only if

$$
\frac{\partial^2 V_L(\rho; \rho^\dagger)}{\partial \rho \partial \rho^\dagger} > 0.
$$

Hence, EC is a local property that says that the marginal value to player L from expanding her cognition at ρ is higher when player F, anticipating cognition $\rho^\dagger$, expects a higher cognition. When there is an interval $[\rho_1, \rho_2]$ such that the property holds for all $\rho, \rho^\dagger \in [\rho_1, \rho_2]$, the gross value to player L from raising cognition from ρ_1 to ρ_2 is higher when player F expects her to engage in cognition ρ_2 than when he expects her to engage in cognition ρ_1: $V_L(\rho_2; \rho_2) - V_L(\rho_1; \rho_2) > V_L(\rho_2; \rho_1) - V_L(\rho_1; \rho_1)$. In this sense, EC captures a certain type of complementarity between actual and anticipated cognition. Below we relate this property to the determinacy of equilibria and a few other phenomena of interest.

In a number of applications, information structures are ordered by their informativeness. Say that the distribution $G(\cdot; \rho)$ is obtained by observing the realization $z \in Z$ of some experiment $q : \Omega \to \Delta(Z)$, where Z is a measurable space of signal realizations. Then if higher ρ index distributions (over the posterior mean) obtained from Blackwell-more-informative experiments, the family of distributions $G(\cdot; \rho)$ must be consistent with the *mean-preserving-spread* (MPS) order.[13]

Assumption 3 (MPS). *Player L's set of feasible information structures is consistent with the MPS order if, for any ρ and $\rho' > \rho$, any $m^* \in \mathbb{R}$, $\int_{-\infty}^{m^*} G(m; \rho') dm \geq \int_{-\infty}^{m^*} G(m; \rho) dm$, with $\int_{-\infty}^{+\infty} G(m; \rho') dm = \int_{-\infty}^{+\infty} G(m; \rho) dm$.*

Consistently with what assumed above, when invoking Assumption 3, we will maintain that the set of information structures is ordered in such a way that $G(m; \rho)$ is differentiable in ρ, for any $m \in \mathbb{R}$. Assumption 3 then boils down to the requirement that, for any $m^* \in \mathbb{R}$ and ρ, $\int_{-\infty}^{m^*} G_\rho(m; \rho) dm \geq 0$, with $\int_{-\infty}^{+\infty} G_\rho(m; \rho) dm = 0$. Most of our results below assume that information structures are consistent with the MPS order. Some of them assume a strengthening of such an order whereby the spreads correspond to "rotations."

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[13]An experiment q'' is Blackwell more informative than another experiment q' if observing the realization of q'' is equivalent to observing the realization of q' along with the realization of some other experiment $t : \Omega \to \Delta(Z)$.

Definition 4 (rotations). Player L's set of possible information structures are "rotations" (or "simple mean-preserving spreads" or experiments consistent with the "single-crossing property") if, for any ρ, there exists a rotation point m_ρ such that $G_\rho(m; \rho) \geq 0$ for $-\infty < m \leq m_\rho$ and $G_\rho(m; \rho) \leq 0$ for $m_\rho \leq m < +\infty$ (with some inequalities strict).

A simple mean-preserving spread is a mean-preserving spread, but the converse is not true. For example, a combination of two rotations need not be a rotation, unless they have the same rotation point. As is well known, however, any mean-preserving spread can be obtained through a sequence of simple mean-preserving spreads.

Non-directed search. Assume that information collection follows the standard non-directed search technology. Then,

$$G(m; \rho) = \begin{cases} \rho G(m) & \text{for } m < \omega_0 \\ \rho G(m) + 1 - \rho & \text{for } m \geq \omega_0. \end{cases}$$

That is, L learns the true state with probability $\rho \in [0, 1]$ and nothing with probability $1 - \rho$. In this example, the rotation point is thus equal to the prior mean ω_0. Figure 1 below illustrates the idea for the special case in which G is uniform.
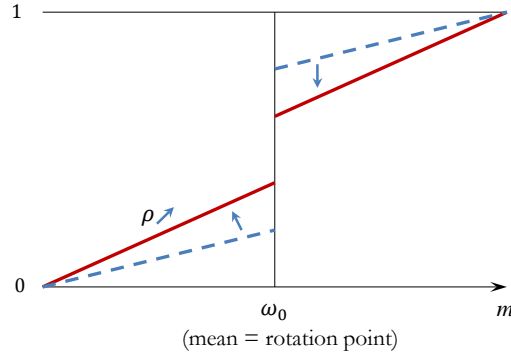

Figure 1: Cumulative distribution function $G(m; \rho)$ for non-directed search

Other examples of rotations include a normally distributed state ω together with a signal that is normally distributed around the true state (ρ is then the precision of this signal), and the family of Pareto, Exponential, and Uniform distributions in Proposition 1 below. See Diamond and Stiglitz (1974) and Johnston and Myatt (2006) for a broader discussion of rotations and their properties.

Proposition 1 (*expectation conformity*). *Suppose that Assumptions 1, 2, and 3 hold.*
(i) EC holds at $(\rho, \rho^\dagger)$ if and only if the adverse selection effect and the benefit of a friendlier reaction effect are of opposite sign: $A(\rho^\dagger)B(\rho; \rho^\dagger) < 0$.
(ii) Cognition always aggravates adverse selection at $\rho^\dagger$ (i.e., $A(\rho^\dagger) < 0$) when the distribution $G(\cdot; \rho)$ from which m is drawn is Uniform, Pareto, or Exponential. For other distributions, a sufficient condition for cognition to aggravate adverse selection at $\rho^\dagger$ is that $G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger) < 0$.
(iii) Starting from $r(\rho^\dagger)$, a reduction in the friendliness of player F's reaction raises player L's incentive for cognition at ρ (i.e., $B(\rho; \rho^\dagger) > 0$) if $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$.

(iv) Therefore a sufficient condition for EC at $(\rho, \rho^\dagger)$ is that

$$\max\left\{G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger), G_\rho(m^*(r(\rho^\dagger)); \rho)\right\} < 0. \tag{9}$$

(v) Suppose that, for any m^*, $M^-(m^*; \rho)$ is decreasing in ρ (as for the Uniform, Pareto, and Exponential distributions), implying that, for any $\rho^\dagger$, $A(\rho^\dagger) < 0$. If $\partial^2 \delta_L(r, m)/\partial r \partial m = 0$, as is the Akerlof's model described above, then $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$ is a necessary and sufficient condition for EC at $(\rho, \rho^\dagger)$.[14] When the distributions $G(\cdot; \rho)$ are rotations, in the sense of Definition 4, $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$ if and only if $m^*(r(\rho^\dagger))$ is to the right of the rotation point m_ρ.

Proof. (i) By the chain rule and the definitions of the V_L and B functions, we have that

$$\frac{\partial^2 V_L(\rho; \rho^\dagger)}{\partial \rho \partial \rho^\dagger} = -B(\rho; \rho^\dagger) \frac{dr(\rho^\dagger)}{d\rho^\dagger}.$$

Assumption 2, together with Conditions (5), (6), and (7) imply that $dr(\rho^\dagger)/d\rho^\dagger$ is of the same sign as $A(\rho^\dagger)$. EC thus holds at $(\rho, \rho^\dagger)$ if, and only if, $A(\rho^\dagger)$ and $B(\rho; \rho^\dagger)$ are of opposite sign.

(ii) Using Condition (5), we have that, for any $m^* \in \mathbb{R}$ and $\rho^\dagger$, the sign of $\partial M^-(m^*; \rho^\dagger)/\partial \rho^\dagger$ is given by the sign of $A(m^*; \rho^\dagger)$, with $A(m^*; \rho^\dagger)$ as defined in (6). Because a higher ρ indexes a mean-preserving spread, the second term of (6) is always negative. Hence, starting from $\rho^\dagger$, cognition always aggravates adverse selection (that is, $A(\rho^\dagger) < 0$) when the first term of (6) is also negative, which is the case when $G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger) < 0$. Note, however, that this condition is sufficient but not necessary for $A(\rho^\dagger) < 0$. For a number of distributions, $\partial M^-(m^*(r(\rho^\dagger)); \rho^\dagger)/\partial \rho^\dagger < 0$ regardless of the sign of $G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger)$. These distributions include the Uniform, Pareto, and Exponential distributions, as shown below.

- *Uniform distribution*: m is drawn uniformly from $[\underline{m}(\rho), \bar{m}(\rho)]$, with $\underline{m}(\rho)$ decreasing in ρ and satisfying $\underline{m}(\rho) \leq \omega_0$ for all ρ, and $\bar{m}(\rho) = 2\omega_0 - \underline{m}(\rho)$ for all ρ (mean preservation). Then for any $m \in [\underline{m}(\rho), \bar{m}(\rho)]$, $G(m; \rho) = (m - \underline{m}(\rho))/[2(\omega_0 - \underline{m}(\rho))]$. This family of distributions is thus consistent with the rotation order of Definition 4, with rotation point $m_\rho = \omega_0$ for all ρ. Furthermore, for any $m^* \in [\underline{m}(\rho), \bar{m}(\rho)]$,

$$M^-(m^*; \rho) = \frac{m^* + \underline{m}(\rho)}{2}$$

  which is decreasing in ρ.

- *Pareto distribution*: m is drawn from $[\underline{m}(\rho), +\infty)$ according to the survival function $1 - G(m; \rho) = (\underline{m}(\rho)/m)^{\alpha(\rho)}$, with $\underline{m}(\rho)$ decreasing in ρ and $\alpha(\rho) = \omega_0/(\omega_0 - \underline{m}(\rho))$ for all ρ.[15] This family of distributions too is consistent with the rotation order of Definition 4. For each ρ, the rotation

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[14]See also Examples (a), (c), and (d) in the Supplement for alternative games in which $\partial^2 \delta_L(r, m)/\partial r \partial m = 0$.

[15]Note that the function $\alpha(\rho)$ is constructed so that, for any ρ, given $\underline{m}(\rho)$, $\mathbb{E}_{G(\cdot; \rho)}[m; \rho] = \frac{\alpha(\rho)\underline{m}(\rho)}{\alpha(\rho) - 1} = \omega_0$ (mean preservation).

point is $m_\rho = \underline{m}(\rho) \exp\left(\left(\omega_0 - \underline{m}(\rho)\right)/\underline{m}(\rho)\right)$. Furthermore, for any $m^* > \underline{m}(\rho)$,

$$M^-(m^*; \rho) = \omega_0 \frac{1 - \left(\frac{\underline{m}(\rho)}{m^*}\right)^{\alpha(\rho)-1}}{1 - \left(\frac{\underline{m}(\rho)}{m^*}\right)^{\alpha(\rho)}}$$

which is decreasing in ρ.

- *Exponential distribution*: m is drawn from $[\underline{m}(\rho), +\infty)$ according to the survival function $1 - G(m; \rho) = e^{-\lambda(\rho)(m - \underline{m}(\rho))}$, with $\underline{m}(\rho)$ decreasing in ρ and $\lambda(\rho) = 1/(\omega_0 - \underline{m}(\rho))$ for all ρ.[16] One can verify that an increase in ρ induces a rotation of $G(m; \rho)$ in the sense of Definition 4, with rotation point $m_\rho = \omega_0$ for all ρ. Furthermore, for any $m^* > \underline{m}(\rho)$,

$$M^-(m^*; \rho) = \omega_0 - \frac{\left(m^* - \underline{m}(\rho)\right) e^{-\lambda(\rho)(m^* - \underline{m}(\rho))}}{1 - e^{-\lambda(\rho)(m^* - \underline{m}(\rho))}}$$

which is decreasing in ρ.

(iii)   Recall that, starting from $r = r(\rho^\dagger)$, a reduction in the friendliness of F's reaction raises the incentive for cognition at ρ if and only if $B(\rho; \rho^\dagger) > 0$, with $B(\rho; \rho^\dagger)$ satisfying Condition (8). Note that the second term in the right-hand side of (8) is positive because, by virtue of Assumption 3, ρ is a mean-preserving-spread index and $\partial^2 \delta_L / \partial r \partial m$ is positive (by virtue of Assumption 1) and constant in m (by virtue of the assumption that δ_L is affine in m). Because δ_L is increasing in r by virtue of Assumption 1, the first term in the right-hand-side of (8) is positive provided that $G_\rho\left(m^*\left(r(\rho^\dagger)\right); \rho\right) < 0$. Hence, starting from $r = r(\rho^\dagger)$, a reduction in the friendliness of F's reaction raises the incentive for cognition at ρ if $G_\rho\left(m^*\left(r(\rho^\dagger)\right); \rho\right) < 0$.

(iv)   The result follow from parts (i)-(iii) in the proposition.

(v)   The result follows from parts (i)-(iii) in the proposition along with the fact that, in this case, the second term in the right-hand-side of (8) is zero. Because δ_L is increasing in r, we thus have that

$$B(\rho; \rho^\dagger) \overset{sgn}{=} -G_\rho\left(m^*\left(r(\rho^\dagger)\right); \rho\right).$$

Hence, $B(\rho; \rho^\dagger) > 0$ if and only if $G_\rho\left(m^*\left(r(\rho^\dagger)\right); \rho\right) < 0$.   ∎

Hence, EC holds at $(\rho, \rho^\dagger)$ when, fixing player F's reaction at $r(\rho^\dagger)$, an increase in cognition by player L decreases the probability that L engages, both when such an increase is evaluated from player L's perspective (i.e., starting from cognition ρ) and when evaluated from player F's perspective (i.e., starting from cognition $\rho^\dagger$)—formally, when Condition (9) holds. This is because, from F's perspective, that player L engages less often (formally, that $G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger) < 0$) implies an aggravation in the perceived adverse selection problem, which induces player F to respond in an unfriendlier manner (part (ii) in the proposition). That player F responds in an unfriendlier manner, together with the fact that $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$, in turn implies a higher marginal value for player L to increase her cognition

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[16]Again, the function $\lambda(\rho)$ is constructed so that, for any ρ, given $\underline{m}(\rho)$, $\mathbb{E}_{G(\cdot; \rho)}[m; \rho] = \underline{m}(\rho) + \frac{1}{\lambda(\rho)} = \omega_0$ (mean preservation).

starting from ρ (part (iii) in the proposition). Jointly, the above two properties (captured by Condition (9) in the proposition) thus imply that, when player F expects more cognition from player L (starting from $\rho^\dagger$), the benefit for player L to expand her cognition (starting from ρ) is higher. That is, EC holds at $(\rho, \rho^\dagger)$. Importantly, Condition (9) is sufficient for EC but not necessary. For example, when the family of distributions from which the posterior mean is drawn in Uniform, Pareto, or Exponential, more cognition always aggravates adverse selection, implying that EC holds at $(\rho, \rho^\dagger)$ if $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$ irrespectively of whether $G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger) < 0$. Furthermore, the sufficiency of Condition (9) hinges on the information structures being consistent with the MPS order. The result is thus perhaps less obvious than what it may look like.

Furthermore, EC holds at $(\rho, \rho^\dagger)$ also when $A(\rho^\dagger) > 0$ and $B(\rho; \rho^\dagger) < 0$, that is, when more cognition by player L (starting from $\rho^\dagger$) induces player F to respond in a friendlier way because it alleviates adverse selection, and a friendlier reaction by player F raises player L's marginal value for cognition (starting from ρ).

Finally, the last part of the proposition establishes that, when cognition always aggravates adverse selection and L's payoff is separable in m and r, as in Akerlof's model, that cognition reduces the probability of engagement starting from ρ (i.e., that $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$) is not only sufficient for EC at $(\rho, \rho^\dagger)$, but also necessary.

As we document in the next section, EC is at the core of various economic phenomena. Before doing so, we first illustrate how EC naturally emerges in Akerlof's model under non-directed search.

3.1 Example: Cognition-augmented Akerlof's model under non-directed search

Under non-directed search, the rotation point is the prior mean. Proposition 1, when applied to the Akerlof's model of Subsection 2.2, thus says that EC holds at $(\rho, \rho^\dagger)$ whenever the engagement threshold $m^* = r(\rho^\dagger)$ is to the right of the prior mean, that is, when the price offered by the competitive buyer is sufficiently high. In other words, EC arises when the gains from trade (in the example parametrized by Δ) are large, and it never occurs when they are small.

To gather some intuition, recall that, in Akerlof's model, the seller puts her car up for sale when her value for the car is small (i.e., when the posterior mean is below a threshold m^* that coincides with the price $r(\rho^\dagger)$ offered by the buyer). Naturally, when the gains from trade Δ are large, the price offered by the buyer is also large, in which case $r(\rho^\dagger)$ exceeds the rotation point, which coincides with the prior mean ω_0 of the car's value for the seller. Economically, what this implies is that the seller finds it optimal to enter the market both when she is uninformed and when she learns that her value for the car, ω, is below the price $r(\rho^\dagger)$. Starting from such a situation, an increase in the cognition expected from the seller by the buyer reduces the quality of the car perceived by the buyer after seeing that the car is on sale. Faced with an exacerbated adverse selection problem, the buyer then reduces the price offered. But then it becomes even more important for the seller to learn the value of the car, that is, to increase her cognition starting from ρ. So EC naturally holds for $(\rho, \rho^\dagger)$ in this case.[17]

While the mechanism just described is fairly natural, it is important to appreciate that it need not

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[17]Consistently with the result in Proposition 1, note that, when $r(\rho^\dagger) > \omega_0$, Condition (9) always holds (see Figure 1).

always be in place. In fact, EC fails to obtain in this model when the gains from trade are positive but small. To see this, note that, when Δ is small, because of the adverse selection problem, the price offered by the buyer may well be lower than the ex-ante prior mean of the asset, meaning that $r(\rho^\dagger) < \omega_0$. Anticipating such a low price, the seller enters the market only if she receives information that reveals that $\omega \leq r(\rho^\dagger)$. The buyer then understands that the expected value of the car conditional on the seller putting it in on the market is the same independently of the seller's cognition: $M^-(r(\rho^\dagger); \rho^\dagger) = \int_{-\infty}^{r(\rho^\dagger)} \omega dG(\omega)/G(r(\rho^\dagger))$, which, given the price $r(\rho^\dagger)$, is invariant in the cognition $\rho^\dagger$. When this is this case, an increase in the cognition $\rho^\dagger$ expected from the seller by the buyer does not affect the price offered by the buyer, and hence does not increase L's incentives to search. We thus have the following result:

Corollary 1 (**lemons under non-direct search**). *In the cognition-augmented Akerlof's model under non-directed search, EC holds at $(\rho, \rho^\dagger)$ if and only if the gains from trade Δ are sufficiently large (namely, if and only if the unique solution $r(\rho^\dagger)$ to $r = M^-(r; \rho^\dagger) + \Delta$ exceeds the prior mean ω_0).*

3.2 Gains from engagement

The example in the previous subsection suggests that EC is more likely to obtain when the gains from engagement for player L are large. The next result shows that this is true more generally.

Proposition 2 (**gains from engagement**). *Suppose that Assumptions 1 and 2 hold and that information structures take the form of rotations, as in Definition 4. Further assume that player L's payoff differential from playing $a = 1$ instead of $a = 0$ is $\delta_L(m, r) = \bar{\delta}_L(m, r) + \theta$, where $\bar{\delta}_L(m, r)$ is an arbitrary function satisfying Assumption 1, and $\theta \in \mathbb{R}$. For all $(\rho, \rho^\dagger)$, there exists $\theta^*(\rho, \rho^\dagger)$ such that, for all $\theta \geq \theta^*(\rho, \rho^\dagger)$, EC holds at $(\rho, \rho^\dagger)$: EC is more likely, the larger the gains from engagement.*

Proof. For any (r, θ), let $m^*(r; \theta)$ denote the optimal cut-off below which player L engages when the gains from engagement are parametrized by θ and F's reaction is r. For any $(\rho^\dagger, \theta)$, then let $r(\rho^\dagger; \theta)$ denote player F's response when player L's cognition is exogenously fixed at $\rho^\dagger$ and the gains from engagement are parametrized by θ. Observe that, under Assumption 1, given r, the engagement threshold $m^*(r; \theta)$, which is implicitly defined by the solution to $\bar{\delta}_L(r, m) + \theta = 0$, is strictly increasing in θ. Also observe that, under Assumption 2, given $\rho^\dagger$, $r(\rho^\dagger; \theta)$ is increasing in θ; this is because, fixing r and $\rho^\dagger$, a higher θ implies a higher engagement point $m^*(r; \theta)$, and hence a higher truncated mean $M^-(m^*(r; \theta); \rho^\dagger)$ which in turn implies a higher equilibrium response $r(\rho^\dagger; \theta)$ by virtue of Assumption 2. Because, for any θ, $m^*(r; \theta)$ is also increasing in r, we conclude that, for any $\rho^\dagger$, and any $\theta'' > \theta'$,

$$m^*(r(\rho^\dagger; \theta''); \theta'') \geq m^*(r(\rho^\dagger; \theta'); \theta'). \tag{10}$$

Now take any $(\rho, \rho^\dagger, \theta')$ such that

$$\max\left\{ G_\rho(m^*(r(\rho^\dagger; \theta'), \theta'); \rho^\dagger), G_\rho(m^*(r(\rho^\dagger; \theta'), \theta'); \rho) \right\} < 0. \tag{11}$$

Proposition 1 (part (iv)) implies that, when $\theta = \theta'$, EC holds at $(\rho, \rho^\dagger)$. That information structures are rotations in turn implies that $\max\{m_\rho, m_{\rho^\dagger}\} \leq m^*(r(\rho^\dagger; \theta'), \theta')$, which along with Condition (10), implies that $\max\{m_\rho, m_{\rho^\dagger}\} \leq m^*(r(\rho^\dagger; \theta''), \theta'')$ and hence that

$$\max\left\{G_\rho(m^*(r(\rho^\dagger; \theta''), \theta''); \rho^\dagger), G_\rho(m^*(r(\rho^\dagger; \theta''), \theta''); \rho)\right\} < 0. \tag{12}$$

Hence, when EC holds at $(\rho, \rho^\dagger)$, it also holds at $(\rho, \rho^\dagger)$. ∎

Proposition 2 says that higher gains from engagement reinforce EC. On the other hand, holding player F's reaction fixed, larger gains from engagement reduce the marginal benefit from cognition under the sufficient condition for EC identified in Proposition 1:

$$\frac{\partial^2}{\partial\theta\partial\rho}\left[\int_{-\infty}^{m^*(r(\rho^\dagger;\theta),\theta)} [\bar\delta_L(r(\rho^\dagger; \theta), m) + \theta] dG(m; \rho)\right] = G_\rho(m^*(r(\rho^\dagger; \theta), \theta); \rho) \leq 0.$$

The reason for this last result is the following: Holding player F's reaction fixed, cognition reduces the probability that player L engages, which is costly when the gains from engagement are large. This property helps clarify that it is only because of the adverse selection problem that larger gains from engagement contribute to EC. They make player F respond to an increase in the anticipated cognition by player L by reducing r more sharply, which in turn raises player L's value of cognition.

4 Cognitive Traps, Disclosure, and Cognitive Styles

We now turn to three phenomena that are intrinsically related to EC in the type of situations described above, cognitive traps, disclosure, and cognitive styles.

4.1 Cognitive traps

Assume that ρ captures the intensity of cognition, so that $C(\rho)$ is weakly increasing in ρ.

Proposition 3 (*cognitive traps*). *Suppose that Assumptions 1 and 2 hold and that ρ_1 and ρ_2 are both equilibrium cognitive levels, with $\rho_1 < \rho_2$. If, for any $\rho^\dagger \in [\rho_1, \rho_2]$, $A(\rho^\dagger) < 0$ (which is the case for example when Assumption 3 also holds so that information structures are consistent with the MPS order and $G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger) < 0$ for all $\rho^\dagger \in [\rho_1, \rho_2]$, meaning that cognition reduces the probability of trade, or when the distributions are Uniform, Pareto, or Exponential), then player L is better off in the low-cognition equilibrium ρ_1. Conversely, when for any $\rho^\dagger \in [\rho_1, \rho_2]$, $A(\rho^\dagger) > 0$, player L is better off in the high-cognition equilibrium ρ_2.*

Proof: Under Assumptions 1 and 2, for any $\rho^\dagger \in [\rho_1, \rho_2]$, $dr(\rho^\dagger)/d\rho^\dagger \overset{\text{sgn}}{=} A(\rho^\dagger)$. For any given r, player L's welfare is given by

$$\mathcal{V}(r) = \sup_\rho \left\{U_L(0) + \int_{-\infty}^{m^*(r)} \delta_L(r, m) dG(m; \rho) - C(\rho)\right\}.$$

The envelope theorem, along with the property that $\delta_L(r, m)$ is increasing in r under Assumption 1,

imply that $d\mathcal{V}(r)/dr > 0$. The result then follows from the fact that $r(\rho_2) < r(\rho_1)$ when $A(\rho^\dagger) < 0$ for all $\rho^\dagger \in [\rho_1, \rho_2]$, whereas $r(\rho_2) > r(\rho_1)$ when $A(\rho^\dagger) > 0$ for all $\rho^\dagger \in [\rho_1, \rho_2]$. ∎

Cognitive traps do not result just from the fact that, when $C(\rho)$ is increasing, in a high-cognition equilibrium, player L spends more resources in cognition. In fact, at the margin, player L's gain from a more informative structure is equal to the increase in the cost of information acquisition. Rather, cognitive traps occur because player F, anticipating an exacerbated adverse selection problem when expecting player L to invest more in cognition, reacts in an unfriendlier way, which not only forces player L to acquire more information, vindicating player F's expectation, but hurts player L.

To illustrate, consider again the Akerlof's model under non-direct search of the previous section. The equilibrium cognitive levels ρ and the corresponding prices $r(\rho)$ are given by the solutions to Conditions (3) and (4). For example, when G is Uniform over $[0, 1]$, the cost of cognition is $C(\rho) = \rho^2/20$, and $\Delta = 0.25$, there are two equilibria in which the price exceeds the prior mean $\omega_0 = 0.5$. In the first equilibrium $\rho_1 \approx 0.48$ and $r(\rho_1) \approx 0.69$; in the second equilibrium, $\rho_2 \approx 0.88$ and $r(\rho_2) \approx 0.58$. Because, for any $m^* > \omega_0$, $G(m^*; \rho^\dagger)$ is decreasing in $\rho^\dagger$, cognition always aggravates adverse selection at $\rho^\dagger$ when $r(\rho^\dagger) > \omega_0$. In this example, $r(\rho^\dagger) > \omega_0$ for all $\rho^\dagger \in [\rho_1, \rho_2]$, implying that $A(\rho^\dagger) < 0$ for all $\rho^\dagger \in [\rho_1, \rho_2]$. Hence, the conditions in the previous proposition apply. The seller is better off in the low-cognition equilibrium ρ_1 than in the high-cognition equilibrium ρ_2.

The result in the previous proposition contrasts with what one obtains in markets without adverse selection. To see this, consider a setting in which player F is a monopolistic seller maximizing expected profits $p - c(\omega)$ by means of a take-it-or-leave-it offer p, whereas player L is a monopsonistic buyer choosing how much information ρ to acquire about her gross value $v = -\omega$ for the seller's product and whether or not to accept the seller's offer of $r = -p$ so as to maximize her net payoff $v - p - C(\rho)$. When the seller's cost c is invariant in ω, this model corresponds to the private-value setting of Ravid, Roesler, and Szentes (2022). In such a setting, when information is free and the buyer can choose any mean-preserving contraction $G(\cdot; \rho)$ of the prior distribution G from which ω is drawn at no cost (so that $C(\rho) = 0$ for all ρ, with higher ρ denoting mean-preserving spreads), there are multiple equilibria. All equilibria are Pareto ranked, with each player's payoff maximized in the equilibrium in which the buyer fully learns the state. When, instead, payoffs are interdependent with c decreasing with ω, (e.g., $c(\omega) = -\omega - \Delta$ for all ω), the result in Proposition 3 suggests that, when more cognition by the buyer aggravates the adverse selection problem, which is consistent with the distributions $G(\cdot; \rho)$ being mean-preserving contractions of G, then equilibria in which the buyer acquires more information are equilibria in which the buyer is necessarily worse off, no matter the cost of information, and hence a fortiori when $C(\rho) = 0$ for all ρ. The reason for the difference is precisely the negative effect that more cognition exerts on the severity of the adverse selection problem, which induces the seller to ask for a higher price.

Next, consider markets in which player F is competitive, as in Akerlof's original lemons model. The result in the previous proposition implies that, when cognition aggravates adverse selection, in the presence of multiple equilibria, the equilibria are Pareto ranked, with the buyer's payoff constant across equilibria and with the seller's payoff decreasing in the equilibrium cognitive level.

The result in Proposition 3 calls for government interventions aimed at discouraging the players from acquiring information. We discuss some of these interventions in Section 6. Here, instead, we

want to emphasize that cognitive traps are intrinsically related to EC. Recall that EC relates to the benefit of cognition in strategic settings. It does not depend on the cost of information. When the sufficient conditions for EC of Proposition 1 hold, one can identify cost functionals for which multiple equilibria arise.[18] The same conditions then imply that player L is worse-off in the more cognitive-intense equilibria.

4.2 Disclosure and Cognitive Style

So far we have assumed that information acquisition is covert. Suppose that it is indeed covert, but that some form of disclosure prior to F's action is feasible: player L can disclose how much attention or external resources she devoted to the issue (but not what she actually learnt).[19] Namely, given her actual cognition ρ, she can prove that her cognition is above any level $\hat{\rho} \leq \rho$. The disclosed information is hard. For any $\hat{\rho} \in \mathbb{R}_+$, let the "$\hat{\rho}$-constrained cognitive game" be the no-disclosure game with modified cost function $\hat{C}(\rho; \hat{\rho}) = C(\rho)$ if $\rho \geq \hat{\rho}$ and $\hat{C}(\rho; \hat{\rho}) = +\infty$ if $\rho < \hat{\rho}$. Let $E(\hat{\rho})$ denote the set of equilibrium cognitive levels of the $\hat{\rho}$-constrained cognitive game and assume that $E(\hat{\rho})$ is non-empty for all $\hat{\rho} \in \mathbb{R}_+$. We say that the function $e(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is a selection if, for any $\hat{\rho} \in \mathbb{R}_+$, $e(\hat{\rho}) \in E(\hat{\rho})$.

Definition 5 (**monotone selections**). The selection $e(\cdot)$ is *monotone* if for all $\hat{\rho}$ and $\hat{\rho}'$, with $\hat{\rho} < \hat{\rho}'$, $e(\hat{\rho}) \leq e(\hat{\rho}')$.

In words, the selection is monotone if, when L is constrained to choose among cognitive levels that exceed a certain threshold, in equilibrium, as the threshold increases, L selects a higher cognitive level. Note that, because $E(\hat{\rho}) \cap \{\rho \mid \rho \geq \hat{\rho}'\} \subseteq E(\hat{\rho}')$, it is always possible to construct monotone selections.

Definition 6 (**regularity**). Take any equilibrium of the primitive game with disclosure. The equilibrium is *regular* if the selection $e(\cdot)$ describing player L's cognitive choice following any possible disclosure $\hat{\rho} \in \mathbb{R}_+$ is monotone and $e(0)$ is an equilibrium of the no-disclosure game.

Clearly, in any pure-strategy equilibrium of the game with disclosure, L selects a unique level of cognition on path. In this case, regularity imposes restrictions on the off-path behavior of the two players. Namely, for any pure-strategy equilibrium of the game with disclosure in which player L's equilibrium cognition is ρ^*, let $\hat{\rho}(\rho^*)$ denote the information L discloses on path. The equilibrium being regular implies, among other things, that, if L were to disclose any $\hat{\rho} < \hat{\rho}(\rho^*)$ (alternatively, any $\hat{\rho} > \hat{\rho}(\rho^*)$), in the continuation game, she would then select a cognition weakly below ρ^* (alternatively, weakly above ρ^*).

Let $\bar{\rho}$ be the highest cognitive level supported by a pure-strategy equilibrium of the game without disclosure. It is easy to see that, without the above refinement, the game with disclosure may admit

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[18]Namely, suppose that Assumptions 1, 2, and 3 hold and that there exist ρ_1 and ρ_2, with $\rho_2 > \rho_1$, such that $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$ for all $\rho^\dagger, \rho \in [\rho_1, \rho_2]$. There exist monotone cost functionals $C(\cdot)$ such that ρ_1 and ρ_2 are both equilibrium cognitive levels. Furthermore, under any such cost functionals, player L is better off in the low-cognition equilibrium ρ_1 than in the high-cognition equilibrium ρ_2.

[19]Shishkin (2022) studies an evidence acquisition game. He shows that, when the probability of obtaining information is small, the Sender's optimal policy has a pass/fail structure and reveals only whether the quality is above or below a threshold. The game considered in this section differs in two respects: First, the acquired information is soft; second, the acquisition is either overt or "semi-overt" in that the intensity of information acquisition can be disclosed but not the actual information obtained.

pure-strategy equilibria supporting cognitive levels ρ^* strictly above $\bar{\rho}$. For example, suppose that cognition always aggravates adverse selection (i.e., $A(\rho^\dagger) < 0$ for all $\rho^\dagger$, as in the case of Uniform, Pareto, or Exponential distributions). These equilibria can be sustained by a strategy for player L according to which, on path, L discloses $\hat{\rho}(\rho^*) = \rho^* > \bar{\rho}$. Off path, after disclosing any $\hat{\rho} < \rho^*$, L selects a cognitive level above ρ^* anticipating a low reaction by player F, supported by the expectation of a large cognition by player L aggravating the adverse selection problem. In other words, without the refinement, there is not enough connection between the equilibrium cognitive levels of the game with and without disclosure.

Proposition 4 (*disclosure*). *Assume that Assumptions 1 and 2 hold and that $A(\rho^\dagger) < 0$ for all $\rho^\dagger$, implying that cognition always aggravates adverse selection.*

- *Any pure-strategy equilibrium cognitive level of the game in which disclosure is not feasible is also an equilibrium cognitive level in the disclosure game.*

- *Conversely, the largest and smallest cognitive levels sustained by pure-strategy regular equilibria of the disclosure game are also equilibrium cognitive levels in the game without disclosure.*

Proof. (i) The logic is similar to the one behind Proposition 3. Consider a pure-strategy equilibrium of the game without disclosure in which cognition is equal to ρ^*. To see that ρ^* can also be supported in a pure-strategy equilibrium of the game with disclosure, for any $\hat{\rho} \in \mathbb{R}_+$, let $e(\hat{\rho})$ denote the level of cognition selected by L when disclosing $\hat{\rho}$. Consider the following strategy for L in the game with disclosure. For any $\hat{\rho} \leq \rho^*$, $e(\hat{\rho}) = \rho^*$, whereas for any $\hat{\rho} > \rho^*$, $e(\hat{\rho}) \geq \hat{\rho}$ (the precise value is not important). Under Assumption 2, that $A(\rho^\dagger) < 0$ for all $\rho^\dagger$ implies that F's reaction $r(\rho^\dagger)$ is non-increasing in the cognition $\rho^\dagger$ anticipated by player F. Hence, for any $\hat{\rho} > \rho^*$, F's reaction is $r(e(\hat{\rho})) \leq r^* \equiv r(\rho^*)$, whereas, for any $\hat{\rho} \leq \rho^*$, F's reaction is $r(e(\hat{\rho})) = r^*$. These properties imply that

$$\sup_{\{\rho,\hat{\rho}\}} \left\{ \int_{-\infty}^{m^*(r(e(\hat{\rho})))} \delta_L(r(e(\hat{\rho})), m) dG(m; \rho) - C(\rho) \right\} = \int_{-\infty}^{m^*(r^*)} \delta_L(r^*, m) dG(m; \rho^*) - C(\rho^*),$$

where the equality follow from the fact that ρ^* is an equilibrium of the no-disclosure game along with the fact that L's payoff is non-decreasing in F's reaction by Assumption 1.

(ii) Conversely, let ρ^* be a cognitive level supported by a regular equilibrium of the disclosure game (with associated disclosure $\hat{\rho}(\rho^*) \leq \rho^*$ and reaction $r^* \equiv r(\rho^*)$). Suppose that $\rho^* < \underline{\rho}$, where $\underline{\rho}$ is the lowest equilibrium cognitive level of the no-disclosure game. That the equilibrium supporting ρ^* is regular, along with the fact that $r(\cdot)$ is non-increasing in $\rho^\dagger$ (by virtue of the assumption that $A(\rho^\dagger) < 0$ for all $\rho^\dagger$) implies that, for any $\hat{\rho} < \hat{\rho}(\rho^*)$, $e(\hat{\rho}) = \rho^*$ and hence $r(e(\hat{\rho})) = r^*$—for, otherwise, L has a profitable deviation—and that, for any $\hat{\rho} > \hat{\rho}(\rho^*)$, $e(\hat{\rho}) \geq \rho^*$ and hence $r(e(\hat{\rho})) \leq r^*$. Hence, given any actual choice of cognition ρ, the most profitable disclosure for player L induces a reaction r^*. This means that, under the reaction r^*, the payoff that L obtains by selecting ρ^* is weakly higher than the payoff that she obtains by selecting any other cognitive level ρ. Therefore, cognition ρ^* can also be sustained in the no-disclosure game, a contradiction. Similar arguments imply that the highest cognitive level that

can be sustained in any regular equilibrium of the disclosure game is $\bar{\rho}$, where $\bar{\rho}$ is the largest cognitive equilibrium level in the no-disclosure game. ∎

Under Assumption 2, more cognition by player L, by aggravating the adverse selection problem, reduces the friendliness of F's reaction. Player L then never gains from proving that her cognition is large, when higher disclosures are interpreted as informative of higher actual cognition. When, in addition, the marginal benefit of extra cognition to player L decreases with the friendliness of F's reaction (which, by virtue of part (iii) of Proposition 1, is the case when cognition reduces the chances that L engages, i.e., when, given $\rho^{\dagger}$, for any ρ, $G_{\rho}(m^*(r(\rho^{\dagger})); \rho) < 0$), player L benefits from aligning her cognitive choice with player F's expectations (that is, EC holds) as in the game without disclosure.

In the same vein, one can consider the possibility of *transparency*, namely a commitment to reveal the exact amount of cognitive resources selected. In this case, $\hat{\rho} = \rho$ for any ρ (overt information acquisition). Clearly, player L is better off committing ex ante to transparency than retaining the possibility to disclose information voluntary ex post (the case just studied). She is also better off under transparency than in the game with complete absence of any disclosure. More interestingly, when F's reaction is non-increasing in $\rho^{\dagger}$ (which is the case when cognition always aggravates adverse selection, i.e., when $A(\rho^{\dagger}) < 0$ for all $\rho^{\dagger}$), under transparency, in equilibrium, player L may choose a cognitive level $\rho^* \leq \underline{\rho}$ that is lower than the lowest equilibrium cognitive level in the no-disclosure game. Similar conclusions obtain when player L cannot reveal her cognition perfectly, but can prove that it is below some level $\hat{\rho}$ of her choosing, for example by proving that she is unable to undertake more than a certain number of informative tests. In such situations, equilibria may exist in which player L proves that her cognition is below the lowest equilibrium level of the no disclosure game.

Another focus of comparative statics concerns player L's *cognitive style*. We provide here only an informal account. Continue to assume that cognition aggravates adverse selection, but now suppose that the cost of cognition $C(\rho; \xi)$ depends on a parameter ξ, interpreted as cognitive ability. A higher-ability player L has a lower marginal cost of cognition: for any ξ, $C(0; \xi) = 0$ and $C_{\rho}(0; \xi) = 0$, whereas for any $\rho > 0$, $C_{\rho}(\rho; \xi) > 0$, and $C_{\rho\rho}(\rho; \xi) > 0$, with $C_{\rho}(\rho; \xi)$ decreasing in ξ. Under the conditions for EC of Proposition 1, as player L's ability increases, the equilibrium cognition also increases (in case of multiple equilibria, in the sense of monotone comparative statics, that is, the lowest and highest cognitive levels of the equilibrium set corresponding to ability ξ increase with ξ). Put it differently, player L's ability, while directly beneficial, indirectly hurts her as player F becomes more wary of adverse selection. This suggests that, if player L has side opportunities to signal cognitive ability, she will want to adopt a dumbed-down profile.

Suppose indeed that player L can be bright (ξ_H) or dumb (ξ_L). A bright person can demonstrate she is bright (and can always mimic a dumb one), but the reverse is impossible. The set of equilibrium cognitive levels is monotonically increasing in the posterior probability that $\xi = \xi_H$. Let us assume a monotone selection in this equilibrium set: Player F's action r is decreasing in the probability that she assigns to $\xi = \xi_H$ (a property automatically satisfied if the equilibrium is unique, for any possible belief). Then if we add a disclosure game prior to the cognitive game in which player L can disclose she is bright if this is indeed the case, the equilibrium is a pooling one, in which the bright player L does not disclose her brightness. Conversely, player L will disclose, if she can, that she is overloaded

with work (assume that she cannot prove that she has a low workload), and therefore that her marginal cost of cognition is high. In either case, player L poses as an "informational puppy dog" (in the sense of Fudenberg and Tirole (1984)).

5 Anti-lemons

The analysis in the previous sections can be adapted to environments that do not satisfy Assumption 2 above. To see this, suppose that an increase in anticipated cognition increases the friendliness of F's reaction, instead of reducing it, when it aggravates the adverse selection problem, that is, when it reduces the truncated mean. Assumption 2 is then reversed and replaced by the following assumption:

Assumption 2′ (anti-lemons). *The friendliness of player F's reaction to an increase in player L's cognition depends negatively on the effect of L's cognition on the truncated mean:*

$$\frac{dr(\rho^\dagger)}{d\rho^\dagger} \overset{\text{sgn}}{=} -\frac{\partial}{\partial \rho^\dagger} \, \mathrm{M}^- \big(\mathrm{m}^*(\mathrm{r}(\rho^\dagger)); \, \rho^\dagger \big).$$

Hence, under Assumption 2', starting from $\rho^\dagger$, an increase in the cognition that player F expects from player L increases the friendliness of player F's reaction if and only if, holding the engagement threshold fixed at $m^*(r(\rho^\dagger))$, an increase in cognition at $\rho^\dagger$ reduces the truncated mean $M^- \big(m^*(r(\rho^\dagger)); \, \rho^\dagger \big)$ — the opposite of what assumed in the analysis above. The following example illustrates:

Spencian signaling. An agent (player L) has disutility of effort ω for studying which is negatively correlated with the agent's productivity $\theta = a - b\omega$ from working on the relevant job after leaving school. The labor market is populated by competitive employers (player F) offering the agent a wage r equal to the agent's expected productivity. Normalizing L's payoff from not engaging to zero, we the have that, in this model, $\delta_L(r, m) = r - m - p$, where p is the cost of enrolling in the school program under consideration (say, an MBA). Hence, the agent enrolls if and only if the cost of studying is low, that is, $m < m^*(r) = r - p$, with r satisfying $r = a - bM^-(m^*(r); \rho^\dagger)$.

In the Supplement, we discuss other settings satisfying Assumption 2' above. The first one is a market where player L is an entrepreneur starting a project under the anticipation that, with positive probability, she may need to liquidate the assets before the latter deliver the cash flows; the decision to start the project ($a = 1$) then carries good information for the buyers who purchase the assets in case of liquidation. The second one is a warfare game in which a country (L) must decide whether to start a fight against another country ($a = 1$) or refrain from doing so ($a = 0$); the decision to engage signals L's confidence that the chances $\omega \in [0, 1]$ of F winning the battle in case F does not surrender are low and hence triggers a friendlier response by F. The third one is a variant of Hermalin (1998)'s leadership model in which the leader is the a founder of a company who benefits from persuading another key player that the project will fail with low probability (captured by ω) and hence that it is profitable for the other player to get on board.

Under Assumption 2′, EC at $(\rho, \rho^\dagger)$ requires that $\mathrm{A}(\rho^\dagger)\mathrm{B}(\rho; \rho^\dagger) > 0$ (the opposite of the condition in

Proposition 1). Because $A(\rho^\dagger) < 0 < B(\rho; \rho^\dagger)$ when

$$\max\left\{G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger), G_\rho(m^*(r(\rho^\dagger)); \rho)\right\} \leq 0,$$

that is, when more cognition reduces the probability that player L engages, no matter whether evaluated from L's perspective (i.e., starting from cognition ρ) or F's perspective (that is, starting from cognition $\rho^\dagger$), under the key condition for EC in Proposition 1, EC never arises when Assumption 2 is replaced with Assumption 2'. This is because, when $G_\rho(m^*(r(\rho^\dagger)), \rho^\dagger) \leq 0$, an increase in the cognition expected from L by F aggravates the adverse selection problem, but, in the anti-lemons case, this triggers a friendlier reaction by player F. In turn, because the marginal value of cognition decreases with the friendliness of player F's reaction when $G_\rho(m^*(r(\rho^\dagger)), \rho) \leq 0$, an increase in the cognition $\rho^\dagger$ anticipated by player F reduces the value for L to expand her cognition starting from ρ. Hence EC never arises under the key condition for EC in Proposition 1.

The following result summarizes the relationship between expectations and incentives for higher cognition in the anti-lemons case (the proof follows from the arguments above):

Proposition 5 (*expectation conformity – anti-lemons*). *Suppose that Assumptions 1, 2', and 3 hold and that cognition aggravates adverse selection at $\rho^\dagger$, i.e., $A(\rho^\dagger) < 0$ (recall that the last property holds when information structures are Uniform, Pareto, or Exponential, or, more generally, when $G_\rho\left(m^*(r(\rho^\dagger)); \rho^\dagger\right) < 0$). Then EC holds at $(\rho, \rho^\dagger)$ only if*

$$G_\rho\left(m^*(r(\rho^\dagger)); \rho\right) > 0.$$

Furthermore, $G_\rho\left(m^*(r(\rho^\dagger)); \rho\right) > 0$ is both necessary and sufficient for EC at $(\rho, \rho^\dagger)$ if $\partial^2\delta_L(m, r)/\partial m \partial r = 0$ for all m and r (as in the Spencian model above).[20].

Hence, in the case of rotations, EC holds at $(\rho, \rho^\dagger)$ when

$$m_{\rho^\dagger} < m^*(r(\rho^\dagger)) < m_\rho,$$

that is, when the engagement threshold is between the rotation points $m_{\rho^\dagger}$ and m_ρ of players F and L, respectively. This condition is quite stringent. For example, it is never satisfied under non-direct search, for, in that case, $m_{\rho^\dagger} = m_\rho = \omega_0$.

Naturally, many of the results in the previous sections are reversed when Assumption 2 (lemons) is replaced with Assumption 2' (anti-lemons). For example, disclosure can be effective, and player L may want to appear an "inoffensive cognitive fat cat" (in the sense of Fudenberg and Tirole (1984)) in the anti-lemons case.

6 Policy Interventions

We now investigate how a benevolent government can improve over the laissez-faire equilibrium by subsidizing (alternatively, taxing) trade. We start with a fairly general analysis geared at shedding

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[20]*For other anti-lemon settings in which δ_L is linear, see Examples (f) and (h) in the Supplement.*

light on (1) what forces contribute to the optimality of subsidies (alternatively, taxes), and (2) how the endogeneity of information calls for larger (alternatively, smaller) interventions. We then apply the insights to the Akerlof's model of Subsection 2.2 (the analysis also delivers interesting implications for Example (a) in the Supplement where the government designs an asset purchase scheme to increase the efficiency of trade in a market affected by adverse selection).

6.1 Optimality of subsidizing/taxing engagement

Let $\delta_F(r, m) \equiv u_F(1, r, m) - u_F(0, m)$ denote the follower's payoff from responding with a reaction r to the leader's choice of engaging, when L's posterior mean (equivalently, the state) is m, net of her payoff in case L does not engage. Let s denote the subsidy (tax if $s < 0$) the government promises to pay to player L in case of engagement.[21] Abusing notation, for any r and s, we let $m^*(r, s)$ denote the optimal engagement threshold for the leader when the follower's reaction is r and the subsidy is s, with m^* implicitly defined by the solution to $\delta_L(r, m^*) + s = 0$. Let $\rho^*(s)$ and $r^*(s)$ denote, respectively, the leader's equilibrium cognition and the follower's equilibrium response in the continuation game that starts after the planner announces a subsidy equal of s. Throughout, we assume that, for any s, $\rho^*(s)$ and $r^*(s)$ are unique, Lipschitz continuous, and differentiable. Likewise, we assume that the payoff functions $\delta_L(r, m)$ and $\delta_F(r, m)$ and the distributions $G(m; \rho)$ are differentiable and Lipschitz-continuous. In addition to facilitating the description of the relevant optimality conditions, these properties validate a certain envelope theorem that we use in the Appendix to establish the results in this section.

For simplicity, assume that player F is a representative of a competitive market in which case, for any s, given the leader's cognition $\rho^*(s)$, the follower's reaction $r^*(s)$ satisfies

$$\int_{-\infty}^{m^*(r^*(s),s)} \delta_F(r^*(s), m) dG(m; \rho^*(s)) = 0.$$

Many of the insights below extend to settings in which player F's expected payoff is different from zero and the planner cares about F's payoff. The exposition, however, is heavier and hence we focus here on the case where F is a competitive player.

For any s, total welfare is given by (up to scalars that are irrelevant for the analysis)

$$W(s) \equiv \int_{-\infty}^{m^*(r^*(s),s)} \left(\delta_L(r^*(s), m) + s \right) dG(m; \rho^*(s)) - C(\rho^*(s)) - (1 + \lambda) s G(m^*(r(s), s); \rho^*(s)),$$

where $\lambda \geq 0$ is the unit cost of public funds (linked to the deadweight loss of non-uniform taxation). The first two terms represent the leader's payoff, whereas the last term represents the cost of the program to the government. Hereafter, we assume that W is strictly quasi-concave.

Proposition 6 (*social value of subsidizing/taxing trade*). *Suppose Assumption 1 holds. In the lemons case (i.e., when Assumption 2 also holds), there exists a threshold $K > 0$ such that a strictly pos-*

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[21]More generally, both the decision to engage and that of not engage can be subject to taxes and subsidies. For example, in the Akerlof's model, the decision to hold on a car or a security can be taxed. Hence, in the analysis below, s should be interpreted as the differential in the subsidy/tax when player L engages relative to when she does not engage.

itive subsidy is optimal if $\frac{d}{ds}M^-(m^*(r^*(0),s);\rho^*(s))\big|_{s=0} > K$, whereas a tax on engagement is optimal when the above inequality is reversed. When, instead, Assumption 2' holds (anti-lemons), there exists a threshold $K < 0$ such that a strictly positive subsidy is optimal if $\frac{d}{ds}M^-(m^*(r^*(0),s);\rho^*(s))\big|_{s=0} < K$, whereas a tax on engagement is optimal when $\frac{d}{ds}M^-(m^*(r^*(0),s);\rho^*(s))\big|_{s=0} > K$.

Hence, whether subsiding trade is preferable to taxing it depends on whether the government faces a lemon or an anti-lemon problem and whether, fixing F's reaction at $r^*(0)$, subsidizing (alternatively, taxing) trade has a strong enough effect on the adverse selection problem to offset the cost of the program. Note that

$$\frac{d}{ds}M^-(m^*(r^*(0),s);\rho^*(s))\big|_{s=0} = \frac{\partial}{\partial m^*}M^-(m^*(r^*(0),0);\rho^*(0))\,\frac{\partial m^*(r^*(0),s)}{\partial s}\bigg|_{s=0}$$

$$+\frac{\partial}{\partial\rho}M^-(m^*(r^*(0),0);\rho^*(0))\frac{d\rho^*(0)}{ds}.$$

When information is endogenous, there are two channels through which a subsidy alleviates (alternatively, aggravates) the adverse selection problem. The first one is through its effect on the leader's engagement, as captured by the threshold m^*. The second one is through its effect on the leader's cognition, ρ^*. A higher subsidy always increases the engagement threshold m^*. Because, for any ρ, M^- is increasing in m^*, the first effect always contributes to alleviating adverse selection. Hence, through this channel, the planner induces a friendlier reaction by player F when Assumption 2 holds (lemons), and a more adversarial one when, instead, Assumption 2' holds (anti-lemons). The second effect, instead, can be either positive or negative, depending on whether cognition aggravates or alleviates adverse selection (which is linked to EC) and whether a positive subsidy increases or decreases the leader's equilibrium cognitive level.

The threshold K, whose formula is in the Appendix, depends on the primitives of the environment. For example, in the Akerlof's model, $K = \lambda$, i.e., the threshold coincides with the unit cost of public funds, and the following conditions jointly imply that subsidizing trade is optimal:

1. $\frac{\partial}{\partial m^*}M^-(m^*(r^*(0),0);\rho^*(0)) > \lambda$;
2. $\frac{\partial}{\partial\rho}M^-(m^*(r^*(0),0);\rho^*(0)) < 0$;
3. $d\rho^*(0)/ds < 0$.

The first condition is satisfied when the unit cost of public funds, λ, is small. When information is exogenous, this condition is jointly necessary and sufficient for a positive subsidy on trade to be optimal. When, instead, information is endogenous, the other two conditions also play a key role. The second condition says that cognition aggravates adverse selection. From Proposition 1, we know that this condition always holds when information structures are consistent with the MPS order and $G_\rho(m^*(r^*(0),0);\rho^*(0)) < 0$, i.e., cognition reduces the probability the seller engages by putting the asset on sale.[22] Condition 3, in turn holds when, in addition to $G_\rho(m^*(r^*(0),0);\rho^*(0)) < 0$ (which, as shown in Proposition 1, also implies that the benefit of expanding cognition decreases with the friendliness of the follower's reaction), the comparative statics of the equilibrium have the same monotonicity as those

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[22] Also recall that cognition always aggravates adverse selection, no matter whether it reduces or increases the probability of trade, when the distributions from which the mean m is drawn are Uniform, Pareto, or Exponential.

of the best responses.[23]

6.2 Effects of endogeneity of information on optimal policy

Next we turn to the effects of the endogeneity of information on the optimal policy. Let s^* denote the optimal policy when information is endogenous. Now suppose information is exogenous and equal to $\rho = \rho^*(s^*)$, where $\rho^*(s^*)$ is the equilibrium cognition when information is endogenous and the subsidy is equal to s^*. For any s, let $\hat{r}(s)$ denote the follower's equilibrium reaction when the subsidy is equal to s and information is exogenous and equal to $\rho^*(s^*)$. Clearly, for $s = s^*$, $\hat{r}(s^*) = r^*(s^*)$, where $r^*(s^*)$ is the equilibrium reaction when information is endogenous. Let $W^\#(s)$ denote welfare when information is exogenous and equal to $\rho^*(s^*)$. Hereafter, we assume that $W^\#(s)$ is strictly quasi-concave and then denote by s^{**} the level of the policy that maximizes $W^\#(s)$. Finally, for any (r, s), let

$$\hat{W}(r, s) \equiv \int_{-\infty}^{m^*(r,s)} (\delta_L(r, m) + s)\, dG(m; \rho^*(s^*)) - C(\rho^*(s^*)) - (1 + \lambda)sG(m^*(r, s); \rho^*(s^*))$$

denote the level of welfare that is attained when information is exogenous and equal to $\rho = \rho^*(s^*)$, the follower's reaction is r, the subsidy is s, and the leader engages if and only if $m < m^*(r, s)$.[24]

We then have the following result:

Lemma 1 (*effect of endogeneity of information on optimal policy*). *The endogeneity of the leader's information calls for larger policy interventions (i.e., $s^* > s^{**}$) if*

$$\left(\frac{d\hat{r}(s^*)}{ds} - \frac{dr^*(s^*)}{ds}\right) \frac{\partial \hat{W}(r^*(s^*), s^*)}{\partial r} + (1 + \lambda)s^* G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*))\frac{d\rho^*(s^*)}{ds} < 0,$$

whereas the opposite is true (i.e., $s^* < s^{**}$) if the above inequality is reversed.

The result is intuitive. The endogeneity of the leader's information calls for larger policy interventions when (a) $\partial \hat{W}(r^*(s^*), s^*)/\partial r > 0$, meaning that the social value of increasing the follower's reaction beyond $r^*(s^*)$ is positive, accounting for the fact that a friendlier reaction induces more engagement which in turn comes with a larger cost to the government (due to the deadweight-loss of non-uniform taxation), (b) an increase in the subsidy, starting from s^*, triggers a larger response by the follower when information is endogenous than when it is exogenous, i.e., $dr^*(s^*)/ds > d\hat{r}(s^*)/ds$, and (c) the extra cost

$$(1 + \lambda)s^* G_\rho(m^*(r(s^*), s^*); \rho^*(s^*))\frac{d\rho^*(s^*)}{ds} \tag{13}$$

that the government incurs to fund the program due to the endogeneity of information is small. Note that, when $s^* > 0$, $G_\rho(m^*(r(s^*), s^*); \rho^*(s^*)) < 0$ (which, under the MPS order, is the key condition for EC identified in Proposition 1), and $d\rho^*(s^*)/ds < 0$, the term in (13) is positive: the government

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[23]Note that, holding cognition fixed at $\rho^*(0)$, an increase in the subsidy, starting from $s = 0$, always increases the friendliness of the follower's reaction. Likewise, holding r fixed at $r^*(0)$, an increase in the subsidy, starting from $s = 0$, always reduces the leader's cognition when $G_\rho(m^*(r^*(0), 0); \rho^*(0)) < 0$.

[24]Clearly, $W^\#(s) = \hat{W}(\hat{r}(s), s)$.

expects to pay s^* more often when the leader reduces her cognition in response to a larger subsidy. As a result, this last effect contributes to a lower level of the optimal policy when information is endogenous.

Next note that the optimality of s^* when information is endogenous reveals that

$$\frac{dr^*(s^*)}{ds}\frac{\partial \hat{W}(r^*(s^*),s^*)}{\partial r} = (1+\lambda)s^* g(m^*(r^*(s^*),s^*);\rho^*(s^*))\frac{\partial m^*(r^*(s^*),s^*)}{\partial s} + \lambda G(m^*(r^*(s^*),s^*);\rho^*(s^*))$$

$$+(1+\lambda)s^*\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*)).$$

Hence, when $s^* > 0$, $dr^*(s^*)/ds > 0$, and

$$\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*)) > 0,$$

necessarily $\partial \hat{W}(r^*(s^*),s^*)/\partial r > 0$. That is, under the welfare-maximizing policy s^*, welfare always increases with the friendliness of the follower's response when cognition reduces the probability of engagement (i.e., when $G_\rho(m^*(r(s^*),s^*);\rho^*(s^*)) < 0$) and the comparative statics of the equilibrium r and ρ have the same monotonicity as those of the best responses (i.e., r^* increases and ρ^* decreases with the subsidy).

To gauge some intuition about whether a higher subsidy triggers a larger response by the follower when information is endogenous than when it is exogenous, note that

$$\frac{dr^*(s^*)}{ds} - \frac{d\hat{r}(s^*)}{ds} \stackrel{sgn}{=} \frac{\partial \delta_F(r^*(s^*),m)}{\partial m}\frac{\partial}{\partial \rho}M^-(m^*(r^*(s^*),s^*);\rho^*(s^*))\frac{d\rho^*(s^*)}{ds},$$

where $\partial \delta_F(r^*(s^*),m)/\partial m$ is the sensitivity of the follower's payoff to the state (which is invariant in m under the maintained assumption that δ_F is affine in m). Hence, in the lemons case (i.e., when Assumption 2 holds, in which case $\partial \delta_F(r^*(s^*),m)/\partial m > 0$), an increase in the subsidy leads to a larger response by the follower under endogenous information when

$$\frac{\partial}{\partial \rho}M^-(m^*(r^*(s^*),s^*);\rho^*(s^*))\frac{d\rho^*(s^*)}{ds} > 0 \tag{14}$$

and a smaller response when the inequality is reversed. The opposite conclusions holds in the anti-lemon case (i.e., under Assumption 2', in which case $\partial \delta_F(r^*(s^*),m)/\partial m < 0$). This is also intuitive. Consider the case in which Assumption 2 holds (i.e., the lemons case). The follower responds more to an increase in the subsidy when information is endogenous than when it is exogenous if the increase in the subsidy leads to a reduction in cognition and, as a result of it, an alleviation of the adverse selection problem.

When applied to the Akerlof's model, the above insights lead to the following:

Proposition 7 (*double dividend of the subsidy in Akerlof's model*). *Consider the Akerlof's model of Subsection 2.2 and let s^* denote the optimal subsidy when information is endogenous. Assume that $G(m;\rho^*(s^*))/g(m;\rho^*(s^*))$ is increasing in m, information structures are consistent with the MPS order, and $G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*)) < 0$ (meaning that cognition aggravates adverse selection). Then, when information is exogenous and equal to $\rho^*(s^*)$, the optimal subsidy, s^{**}, satisfies $s^{**} < s^*$.*

Recall that the property that $G(m; \rho^*(s^*))/g(m; \rho^*(s^*))$ is increasing in m guarantees that Assumption 2 holds in the Akerlof's model (that is, the friendliness of the follower's reaction increases with the leader's cognition if and only if a higher cognition alleviates adverse selection, i.e., it increases $M^-(m^*(r^*(s^*), s^*; \rho^*(s^*)))$). That information structures are consistent with the MPS order and $G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*)) < 0$ in turn implies that, under the optimal subsidy s^*, cognition aggravates adverse selection (see Proposition 1). Therefore, under the assumptions in the proposition, starting from s^*, if the government were to cut the subsidy, it would trigger a larger reduction in the price offered by the buyer when information is endogenous than when it is exogenous. The optimal subsidy is thus larger under endogenous information.

The same conclusions apply to the application of Example (a) in the Supplement where the government directly controls the price at which the sellers can trade in their assets. Proposition 7 above then implies that, when information is endogenous, it is optimal for the government to design an asset-purchase program with a higher price.

The results above point to a general insight. When increasing trade is socially beneficial, cognition aggravates adverse selection, and a friendlier reaction by player F reduces the marginal value of information for player L (which is the case under the conditions for EC in Proposition 1 and for cognitive traps in Proposition 3), the social value of subsidizing trade is higher when information is endogenous than when it is exogenous. This is because subsidizing trade comes with a *double dividend*: in addition to inducing player L to engage more often, it induces L to acquire less information which in turn alleviates adverse selection and further boosts welfare.

Table 1 in the Appendix summarizes some of the key results from this section and the previous ones.

7 Flexible Information Acquisition

The analysis in the previous sections assumes that the experiments player L has access to can be ranked according to the Blackwell order (equivalently, that the distributions $G(\cdot; \rho)$ of the posterior mean induced by such experiments are consistent with the mean-preserving-spread order — Assumption 3). The key forces responsible for EC identified in Proposition 1, however, extend to more general information structures. To see this, consider an arbitrary experiment $q : \Omega \to \Delta(Z)$ mapping states into probability distributions over a rich (Polish) space of signal realizations Z. Note that any such experiment, when combined with the prior G over Ω, leads to a distribution G^q of the posterior mean, m. Furthermore, when combined with the optimal engagement strategy (that is, with the strategy that, for any reaction r by player F, specifies to engage if and only $m \le m^*(r)$), the experiment q leads to a stochastic choice rule $\sigma^q : \Omega \to [0, 1]$ specifying the probability that player L engages in each state.

Following the rational inattention literature, one can think of player L as choosing directly the rule $\sigma : \Omega \to [0, 1]$ subject to an appropriate specification of the cost functional $\mathcal{C}(\sigma)$, with the interpretation that, for any σ, $\mathcal{C}(\sigma)$ is the cost of the cheapest experiment $q : \Omega \to \Delta(Z)$ that permits L to implement the stochastic choice rule σ. A couple of cost functionals that have been considered in the literature are those linked to "mutual information" and "maximal slope". Below we discuss both specifications and explain how our results are broadly consistent with these specifications.

For any experiment q, let

$$I^q = \int_\omega \int_z \ln(q(z|\omega))q(\mathrm{d}z|\omega)dG(\omega) - \int_z \ln\left(\int_\omega q(z|\omega)dG(\omega)\right)\int_\omega q(z|\omega)dG(\omega)$$

denote the mutual information between the random variables ω and z, where z is the random variable obtained by combining the prior G with the signal q. Now suppose that there exists a function $c : \mathbb{R}_+ \to \mathbb{R}_+$ such that, for any q, the cost of experiment q is given by $\mathbf{C}(q) = c(I^q)$. To facilitate the comparison with the analysis in the previous sections in which cognition is indexed by a uni-dimensional parameter ρ, with higher values indexing superior information structures, let player L's cognition determine the easiness by which L can absorb information. Specifically, assume that, for any $\rho \in \mathbb{R}_+$, player L's marginal cost of entropy reduction is $1/\rho$. To be able to process information at marginal cost $1/\rho$, player L must make a cognitive investment whose cost is $C(\rho)$, with the function C satisfying the same assumptions as in the baseline model. The difference is that, once ρ is chosen, player L can now pick *any* experiment q of her choice, with each experiment costing her $c(I^q)/\rho$. For simplicity, one can then assume that c is the identity function (i.e., $c(I^q) = I^q$ for any q) so that the cost of each experiment q is given by the mutual information between its realizations z and the state ω (equivalently, by the reduction in entropy brought by the experiment), scaled by the (inverse) of L's cognition, ρ.

Alternatively, one can let $\rho \in \mathbb{R}_+$ denote player L's "information capacity." Under this interpretation, L first purchases capacity ρ at cost $C(\rho)$ and then chooses the experiment that maximizes her expected payoff among those whose mutual information between ω and the realization z of the selected experiment is no greater than ρ.

It is well known that, for any cognitive investment ρ and any anticipated reaction r by player F, the experiment $q^{\rho,r}$ that maximizes player L's expected payoff net of the above cognitive cost is binary, i.e., for any ω, it assigns positive probability only to two signal realizations. Without loss of generality, label these signal realizations by $z = 1$ and $z = 0$, and interpret $z = 1$ as a "recommendation to engage" and $z = 0$ as a "recommendation to not engage." Letting $q^{\rho,r}(1|\omega)$ denote the probability that signal $q^{\rho,r}$ recommends $z = 1$ when the state is ω and $q^{\rho,r}(1) \equiv \int_\omega q^{\rho,r}(1|\omega)dG(\omega)$ the total probability of $z = 1$ under the experiment $q^{\rho,r}$, we have that the optimal signal is given by the solution to the following functional equation (see, e.g., Woodford (2009) and Yang (2015)):[25]

$$\delta_L(r,\omega) = \frac{1}{\rho}\left[\ln\left(\frac{q^{\rho,r}(1|\omega)}{1 - q^{\rho,r}(1|\omega)}\right) - \ln\left(\frac{q^{\rho,r}(1)}{1 - q^{\rho,r}(1)}\right)\right].$$

Next, consider the case in which the cost of inducing a stochastic choice rule $\sigma : \Omega \to [0,1]$ is given by $\mathcal{C}(\sigma) = c\left(\sup\{|\sigma'(\omega)|\}\right)$, where the function $c : \mathbb{R}_+ \cup \{+\infty\} \to \mathbb{R}_+ \cup \{+\infty\}$ is non-decreasing and satisfies $c(0) = 0$ and $c(k) < \infty$ for all $k \in \mathbb{R}_+$. Here $\sigma'(\omega)$ is the derivative of σ at ω. At any point of discontinuity of σ, $\sigma'(\omega) = +\infty$, whereas at any point ω at which σ is continuous but non-differentiable, $\sigma'(\omega)$ is the maximum between the left and the right derivative. Examples of this cost functional can be found in Robson (2001), Rayo and Becker (2007), Netzer (2009), and more recently Morris and Yang

---

[25]The formula below is for when $1/\rho$ measures the marginal cost of entropy reduction. Conclusions similar to those reported below hold for the case where ρ determines the information capacity, i.e., the maximal level of entropy reduction, as in Sims (2003)'s original work on rational inattention (see also Mackowiak and Wiederholt (2009)).

(2021).

Again, to see the connection with the analysis in the previous sections, one can think of player L's cognition as determining the maximal slope of her stochastic choice rule, selected by the player at cost $C(\rho)$ with C satisfying the same properties as in the baseline model. Given ρ, player L then selects the experiment that maximizes her expected payoff, among those inducing a stochastic choice rule σ whose maximal slope is no greater than ρ. For any ρ and r, the optimal experiment can be taken to be binary and, for any ω, it recommends $z = 1$ (i.e., engagement) with probability $q^{\rho,r}(1|\omega)$ given by

$$
q^{\rho,r}(1|\omega) = \begin{cases} 1 & \text{if} & \omega \leq m^*(r) - \frac{1}{2\rho} \\ \frac{1}{2} - \rho(\omega - m^*(r)) & \text{if} & m^*(r) - \frac{1}{2\rho} < \omega \leq m^*(r) + \frac{1}{2\rho} \\ 0 & \text{if} & \omega > m^*(r) + \frac{1}{2\rho} \end{cases}
$$

where $m^*(r)$ is the same engagement cutoff as in the previous sections.

What distinguishes the two examples of flexible information acquisition above from the analysis in the previous sections is that, for any cognitive choice ρ, there are multiple experiments that share the same cost (parametrized by ρ). After choosing ρ, player L then chooses the experiment that maximizes her expected payoff among those whose cost is $C(\rho)$, with the optimal choice depending on the anticipated reaction r by player F.

It is evident that, in each of the two cases of flexible information acquisition described above, under the optimal experiment $q^{\rho,r}$, when player L receives the signal $z = 1$ (equivalently, when she engages), her posterior mean, which is given by

$$
\mathbb{E}[\omega|z = 1; q^{\rho,r}] = \int \omega \frac{q^{\rho,r}(1|\omega)}{q^{\rho,r}(1)} dG(\omega),
$$

is less than $m^*(r)$, and likewise, after receiving signal $z = 0$,

$$
\mathbb{E}[\omega|z = 0; q^{\rho,r}] = \int \omega \frac{1 - q^{\rho,r}(1|\omega)}{1 - q^{\rho,r}(1)} dG(\omega)
$$

is greater than $m^*(r)$.

For any anticipated cognition $\rho^\dagger$, any reaction r by player F, and any cutoff m^*, then let $M^-(m^*; \rho^\dagger, r)$ denote the expected value of m conditional on $m \leq m^*$, when player L chooses cognition $\rho^\dagger$ and then selects the optimal experiment $q^{\rho^\dagger,r}$ anticipating a reaction r by player F. Then note that, for any $\rho^\dagger$ and r, when the cutoff is equal to $m^*(r)$,

$$
M^-(m^*(r); \rho^\dagger, r) = \mathbb{E}[\omega|z = 1; q^{\rho^\dagger,r}]
$$

and $\partial M^-(m^*(r); \rho^\dagger, r)/\partial \rho^\dagger \overset{\text{sgn}}{=} A(m^*(r); \rho^\dagger, r)$, with

$$
A(m^*(r); \rho^\dagger, r) \equiv \left[m^*(r) - M^-(m^*(r); \rho^\dagger, r)\right] G_\rho(m^*(r); \rho^\dagger, r) - \int_{-\infty}^{m^*(r)} G_\rho(m; \rho^\dagger, r) dm
$$

where, for any m, $G(m; \rho^\dagger, r)$ denotes the probability that L's posterior mean is less than m under the

experiment $q^{\rho^\dagger,r}$ and where $G_\rho(m;\rho^\dagger,r)$ denotes the partial derivative of such a probability with respect to ρ, evaluated at $\rho = \rho^\dagger$; such a derivative is computed accounting for the fact that, when ρ changes, the optimal experiment $q^{\rho,r}$ (which also depends on the expected reaction r) changes.

As in the baseline model, the sign of A determines whether an increase in anticipated cognition aggravates or alleviates the adverse selection problem. Consistently with the baseline model, we then continue to interpret $\mathrm{A}(\rho^\dagger) \equiv A(m^*(r(\rho^\dagger));\rho^\dagger,r(\rho^\dagger))$ as the "adverse selection effect." As in the baseline model, $r(\rho^\dagger)$ denotes the equilibrium reaction by player F in a fictitious setting in which player L's cognition is exogenously fixed at $\rho^\dagger$. However, differently from the baseline model, in this fictitious setting, player L chooses the distribution $G(\cdot;\rho^\dagger,q)$ over her posterior mean m by selecting an experiment $q : \Omega \to \Delta(Z)$.[26] The equilibrium reaction $r(\rho^\dagger)$ is thus computed jointly with the equilibrium choice of experiment q and the equilibrium engagement strategy $a(\cdot)$.

Next, let

$$\Pi(\rho;r) \equiv U_L(0) + \int_{-\infty}^{m^*(r)} \delta_L(r,m)dG(m;\rho,r) = U_L(0) + \int_{-\infty}^{+\infty} \delta_L(r,\omega)q^{\rho,r}(1|\omega)dG(\omega)$$

denote the payoff, gross of the cost, that player L obtains by choosing cognition ρ when expecting a reaction r by player F (with the expectation computed under the optimal experiment $q^{\rho,r}$) and then let

$$\begin{aligned}
B(\rho;\rho^\dagger) &\equiv& -\frac{\partial^2 \Pi(\rho;r(\rho^\dagger))}{\partial \rho \partial r} = -\int_{-\infty}^{m^*(r(\rho^\dagger))} \frac{\partial \delta_L(r(\rho^\dagger),m)}{\partial r}dG_\rho(m;\rho,r(\rho^\dagger)) \\
&=& -\frac{\partial \delta_L(r(\rho^\dagger),m^*(r(\rho^\dagger)))}{\partial r}G_\rho\Big(m^*(r(\rho^\dagger));\rho,r(\rho^\dagger)\Big) \\
&& + \int_{-\infty}^{m^*(r(\rho^\dagger))} \frac{\partial^2 \delta_L(r(\rho^\dagger),m)}{\partial r \partial m}G_\rho(m;\rho,r(\rho^\dagger))dm.
\end{aligned}$$

As in the baseline model, the function $B(\rho;\rho^\dagger)$ measures how a reduction in the friendliness of F's reaction around $r(\rho^\dagger)$ affects L's marginal benefit of cognition when the latter is equal to ρ. Consistently with the baseline model, we will continue to refer to $B(\rho;\rho^\dagger)$ as the "benefit of friendlier reactions" effect.

The following proposition establishes the precise sense in which results analogous to those in Proposition 1 extend to this setting when the cost of cognition is determined by either entropy reduction or the maximum slope of the induced stochastic choice rule.

Proposition 8 (*EC under flexible information acquisition*). *Suppose that Assumptions 1 and 2 hold and that cognition ρ determines either the marginal cost of entropy reduction or the maximum-slope of the induced stochastic choice rule.*
(i) EC holds at $(\rho,\rho^\dagger)$ if and only if the adverse selection and the benefit of friendlier reactions effects are of opposite sign: $\mathrm{A}(\rho^\dagger)\mathrm{B}(\rho;\rho^\dagger) < 0$.
(ii) A sufficient condition for cognition to aggravate adverse selection at $\rho^\dagger$ is that, $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ is increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$ at $\rho = \rho^\dagger$.

---

[26]Recall that $\rho^\dagger$ only pins down the marginal cost of entropy reduction (alternatively, the maximal level of entropy reduction) or the maximal slope of the induced stochastic choice rule, leaving player L with flexibility over her choice of experiment $q : \Omega \to \Delta(Z)$.

(iii)   A sufficient condition for a reduction in the friendliness of F's reaction at $r(\rho^\dagger)$ to raise L's value of cognition at ρ is that, in addition to $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ be increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$, the total probability $q^{\rho,r(\rho^\dagger)}(1) \equiv \int q^{\rho,r(\rho^\dagger)}(1|\omega)dG(\omega)$ player L engages is non-increasing in ρ.

(iv)   Therefore, a sufficient condition for EC to hold at $(\rho, \rho^\dagger)$ is that the conditions in parts (ii) and (iii) jointly hold.

(v)   Suppose that $M^-(m^*(r(\rho^\dagger)); \rho)$ is decreasing in ρ at $\rho = \rho^\dagger$, implying that $A(\rho^\dagger) < 0$, and that $\partial^2 \delta_L(r, m)/\partial r \partial m = 0$. Then $q^{\rho,r(\rho^\dagger)}(1)$ non-increasing in ρ at $\rho = \rho^\dagger$ is necessary and sufficient for EC at $(\rho, \rho^\dagger)$.

As in the baseline model, EC obtains when, and only when, the adverse-selection effect is of opposite sign than the benefit of friendlier reactions effect, i.e., when $A(\rho^\dagger)B(\rho; \rho^\dagger) < 0$. The intuition is the same as in the baseline model.

When information is flexible, an increase in cognition (starting from $\rho^\dagger$) aggravates the severity of the adverse selection problem when it induces L to select an experiment that makes her engage with a higher probability at low states (namely for $\omega < m^*(r(\rho^\dagger))$) and with a lower probability at high states (namely for $\omega > m^*(r(\rho^\dagger))$), relative to the total probability $q^{\rho,r(\rho^\dagger)}(1)$ of engaging. This is because, in the eyes of player F, such changes make the engagement decision by player L a more informative signal of the state being less favorable to player F. When, in addition to the last property described, cognition also reduces the overall probability $q^{\rho,r(\rho^\dagger)}(1)$ that player L engages, starting from the actual level ρ selected by player L, a reduction in the friendliness of player F's reaction (starting from $r(\rho^\dagger)$) increases L's marginal value of cognition at ρ. The property that $q^{\rho,r(\rho^\dagger)}(1)$ decreases with ρ is equivalent to the property in the baseline model that, in the eyers of player L, cognition reduces the probability of trade (i.e., $G_\rho\left(m^*(r(\rho^\dagger)); \rho\right) < 0$). This condition is both necessary and sufficient for EC when L's payoff is separable in r and ω (as in the Akerlof's model) and cognition always aggravates the adverse selection problem in the eyes of player F. The results above are the analogs of those established in Proposition 1 for the case where ρ is a mean-preserving-spread index, thus establishing the robustness of the key insights to the more flexible information structures considered in this section.

8   Conclusions

We investigate how incentives to acquire information in generalized lemons problems depend on other players' expectations about the acquired information. We show how expectation conformity, i.e., the value to conform to other players' expectations, is affected by (a) the impact of information on the severity of the adverse selection problem, (b) the sensitivity of the marginal value of information to the friendliness of other players' reactions, and (c) the overall value of engagement, as captured by the size of the gains from trade.

We then use the characterization to shed light on the connection between expectation conformity and cognitive traps, and on the role of disclosure of hard information in such games, whereby players engage in activities that prove how well or poorly informed they are.

Finally, we show how the results change in the anti-lemons case, that is, in settings in which the players' payoffs are aligned, and how a benevolent government can improve upon the laissez-faire equilibrium

by subsidizing (alternatively, taxing) trade.

There are many venues for future research. First, in more applied work geared at understanding the role of endogenous information in financial trading, it would be interesting to investigate how the type of security issued to finance a project affects the incentives for information acquisition and the resulting severity of the adverse selection problem. Second, it would be interesting to study how public disclosures by benevolent governments impact the incentives for private information acquisition. For example, in the context of stress testing, the announcement that a bank failed a test may induce a conservative response by potential asset buyers which may induce asset owners to collect more information, which in turn aggravates the severity of the adverse selection problem. This is an angle that does not seem to have been accounted for in the design of the optimal stress tests. Lastly, it would be interesting to extend the analysis by allowing both sides of the market to acquire information and investigate how strategic complementarity/substitutability in information acquisition is affected by the adverse selection problem.

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9 Appendix

Proof of Proposition 6. Using the envelope theorem, we have that[27]

$$W'(s) = \int_{-\infty}^{m^*(r^*(s),s)} \left[\frac{\partial \delta_L(r^*(s),m)}{\partial r} \frac{dr^*(s)}{ds} + 1 \right] dG(m; \rho^*(s))$$

$$-\frac{d}{ds} \left[(1+\lambda)sG(m^*(r^*(s),s); \rho^*(s)) \right].$$

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[27]Here we are using the fact that, given s and $r^*(s)$, $m^*(r^*(s),s)$ and $\rho^*(s)$ maximize the leader's payoff $\int_{-\infty}^{\hat{m}} (\delta_L(r^*(s),m) + s) \, dG(m; \rho) - C(\rho)$ over $(\hat{m}, \rho)$.

The first line is simply the effect of a change in the subsidy on the leader's expected payoff (holding $\rho^*(s)$ and $m^*(r^*(s), s)$ fixed by usual envelope-theorem arguments). The second line is the (total) effect of a change in the subsidy on the cost of the program to the government.

Note that $W'(s)$ can be expressed as

$$W'(s) = \frac{dr^*(s)}{ds} \int_{-\infty}^{m^*(r^*(s),s)} \frac{\partial \delta_L(r^*(s),m)}{\partial r} dG(m; \rho^*(s)) - s(1+\lambda) \frac{d}{ds} \left[G(m^*(r^*(s), s); \rho^*(s)) \right]$$

$$-\lambda G(m^*(r^*(s), s); \rho^*(s)).$$

When $W(s)$ is quasi-concave in s, the optimal s is thus strictly positive when

$$W'(0) = \frac{dr^*(0)}{ds} \int_{-\infty}^{m^*(r^*(0),0)} \frac{\partial \delta_L(r^*(0),m)}{\partial r} dG(m; \rho^*(0)) - \lambda G(m^*(r^*(0),0); \rho^*(0)) > 0$$

and strictly negative when the above inequality is reversed.

Because δ_L is affine in m, it can be expressed as $\delta_L(r, m) = a_L(r)m + b_L(r)$, for some functions $a_L(r)$ and $b_L(r)$. Assumption 1 then implies that, for any r and m, $a_L(r) < 0$ and $a'_L(r)m + b'_L(r) > 0$. This means that $W'(0) > 0$ when $dr^*(0)/ds > r^\#$, whereas $W'(0) < 0$ when $dr^*(0)/ds < r^\#$, where

$$r^\# \equiv \frac{\lambda}{\frac{\partial}{\partial r}\delta_L(r^*(0), M^-(m^*(r^*(0),0); \rho^*(0))}$$

is a strictly positive constant that depends on the primitives of the problem. For example, in the Akerlof's model, $\delta_L(r, m) = -m + r$, in which case $r^\# = \lambda$.

Next, observe that, for any s, $\rho^*(s)$ and $r^*(s)$ jointly solve the following two conditions:

$$\int_{-\infty}^{m^*(r^*,s)} \delta_F(r^*, m)dG(m; \rho^*) = 0, \tag{15}$$

and

$$\rho^* = \arg\max_\rho \left\{ \int_{-\infty}^{m^*(r^*,s)} \left(\delta_L(r^*, m) + s \right) dG(m; \rho) - C(\rho) \right\}. \tag{16}$$

Because δ_F is affine in m, it can be expressed as $\delta_F(r, m) = a_F(r)m + b_F(r)$, for some functions $a_F(r)$ and $b_F(r)$, with $a_F(r) > 0$ when Assumption 2 holds (lemons), and $a_F(r) < 0$ when Assumption 2' holds (anti-lemons).[28]

Hence, for any s, $r^*(s)$ solves $\delta_F(r^*, M^-(m^*(r^*, s); \rho^*(s))) = 0$. Using the implicit-function theorem, we have that

$$\frac{dr^*(s)}{ds} = -\frac{\frac{d}{ds}\delta_F(r, M^-(m^*(r,s); \rho^*(s)))\big|_{r=r^*(s)}}{\frac{d}{dr}\delta_F(r, M^-(m^*(r,s); \rho^*(s)))\big|_{r=r^*(s)}}$$

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[28]To see this, note that, for any m^*, ρ, and r, $\int_{-\infty}^{m^*} \delta_F(r, m)dG(m; \rho) = G(m^*; \rho)\delta_F(r, M^-(m^*; \rho))$. Now fix s and drop it. The equilibrium r thus solves $a_F(r)M^-(m^*(r); \rho) + b_F(r) = 0$. Hence,

$$\frac{dr}{d\rho} = -\frac{a_F(r)\frac{\partial}{\partial\rho}M^-(m^*(r); \rho)}{\frac{\partial}{\partial r}\delta_F(r, M^-(m^*(r); \rho))}.$$

The denominator in the above expression is negative, by assumption. It follows that $dr/d\rho \overset{sgn}{=} a_F(r)\frac{\partial}{\partial\rho}M^-(m^*(r); \rho)$. Hence, $a_F(r) > 0$ when Assumption 2 holds, whereas $a_F(r) < 0$ when Assumption 2' holds.

35

where the denominator is negative, by assumption. We thus have that $dr^*(0)/ds > r^\#$ if

$$\frac{d}{ds}\delta_F(r^*(0), M^-(m^*(r^*(0), s); \rho^*(s)))\bigg|_{s=0} > \Lambda \equiv \lambda \left| \frac{\frac{d}{dr}\delta_F(r, M^-(m^*(r,0); \rho^*(0)))|_{r=r^*(0)}}{\frac{d}{dr}\delta_L(r, M^-(m^*(r,0); \rho^*(0)))|_{r=r^*(0)}} \right|$$

whereas $dr^*(0)/ds < r^\#$ if the above inequality is reversed. The condition says that, at the laissez-faire equilibrium, holding the follower's reaction fixed at $r^*(0)$, a small subsidy to engagement has a strong enough positive effect on the follower's payoff, accounting for the effect that the subsidy has on both the leader's engagement threshold and cognition. Note that, in the Akerlof's model of Example 1, $\Lambda = \lambda$.

Clearly,

$$\frac{d}{ds}\delta_F(r^*(0), M^-(m^*(r^*(0), s); \rho^*(s)))\bigg|_{s=0} = a_F(r^*(0)) \frac{d}{ds}M^-(m^*(r^*(0), s); \rho^*(s))\bigg|_{s=0}.$$

The result in the proposition then follows by letting $K \equiv \Lambda/a_F(r^*(0))$ and noting that $K > 0$ when $a_F(r^*(0)) > 0$, i.e., when Assumption 2 (lemons) holds, whereas $K < 0$ when $a_F(r^*(0)) < 0$, i.e., when Assumption 2' (anti-lemons) holds. ∎

Proof of Lemma 1. The optimal value of s^* solves $dW(s^*)/ds = 0$. That is, s^* solves

$$\frac{dr^*(s^*)}{ds} \int_{-\infty}^{m^*(r^*(s^*), s^*)} \frac{\partial\delta_L(r^*(s^*), m)}{\partial r} dG(m; \rho^*(s^*))$$

$$-(1+\lambda)s^*g(m^*(r^*(s^*), s^*); \rho^*(s^*)) \left[\frac{\partial m^*(r^*(s^*), s^*)}{\partial r}\frac{dr^*(s^*)}{ds} + \frac{\partial m^*(r^*(s^*), s^*)}{\partial s} \right] \qquad (17)$$

$$-(1+\lambda)s^*\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*)) - \lambda G(m^*(r^*(s^*), s^*); \rho^*(s^*)) = 0.$$

Next, observe that, when information is exogenous and equal to $\rho = \rho^*(s^*)$, using the envelope theorem and the fact that $\hat{r}(s^*) = r^*(s^*)$, we have that

$$\frac{dW^\#(s^*)}{ds} = \frac{\partial\hat{W}(r^*(s^*), s^*)}{\partial r}\frac{d\hat{r}(s^*)}{ds} + \frac{\partial\hat{W}(r^*(s^*), s^*)}{\partial s}$$

where

$$\frac{\partial\hat{W}(r^*(s^*), s^*)}{\partial r} = \int_{-\infty}^{m^*(r^*(s^*), s^*)} \frac{\partial\delta_L(r^*(s^*), m)}{\partial r} dG(m; \rho^*(s^*))$$

$$-(1+\lambda)s^*g(m^*(r^*(s^*), s^*); \rho^*(s^*))\frac{\partial m^*(r^*(s^*), s^*)}{\partial r}$$

and

$$\frac{\partial\hat{W}(r^*(s^*), s^*)}{\partial s} = -(1+\lambda)s^*g(m^*(r^*(s^*), s^*); \rho^*(s^*))\frac{\partial m^*(r^*(s^*), s^*)}{\partial s} - \lambda G(m^*(r^*(s^*), s^*); \rho^*(s^*)).$$

Using (17), we thus have that

$$\frac{dW^\#(s^*)}{ds} = \left(\frac{d\hat{r}(s^*)}{ds} - \frac{dr^*(s^*)}{ds} \right)\frac{\partial\hat{W}(r^*(s^*), s^*)}{\partial r} + (1+\lambda)s^*\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*)).$$

When $W^\#(s)$ is strictly quasi-concave in s, we then have that $s^{**} < s^*$ if $dW^\#(s^*)/ds < 0$ and $s^{**} > s^*$ if the above inequality is reversed, which establishes to the result in the lemma. ∎

Proof of Proposition 7. The result follows from Lemma 1. In fact, in this case,

$$\frac{d\hat{r}(s^*)}{ds} - \frac{dr(s^*)}{ds} = \frac{\frac{\partial}{\partial\rho}M^-(m^*(r^*(s^*),s^*);\rho^*(s^*))\frac{d\rho^*(s^*)}{ds}}{\frac{\partial}{\partial m^*}M^-(m^*(r^*(s),s^*);\rho^*(s^*)) - 1}.$$

Using the fact that

$$\frac{\partial}{\partial\rho}M^-(m^*;\rho) = \frac{G_\rho(m^*;\rho)[m^* - M^-(m^*;\rho)] - \int_{-\infty}^{m^*}G_\rho(m;\rho)dm}{G(m^*;\rho)}$$

and

$$\frac{\partial}{\partial m^*}M^-(m^*;\rho) = \frac{g(m^*;\rho)[m^* - M^-(m^*;\rho)]}{G(m^*;\rho)},$$

along with the fact that $m^*(r^*(s^*),s^*) = r^*(s^*) + s^*$ and $r^*(s^*) = M^-(m^*(r^*(s^*),s^*);\rho^*(s^*)) + \Delta$, we have that

$$\frac{d\hat{r}(s^*)}{ds} - \frac{dr(s^*)}{ds} = \left((s^* + \Delta)G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*)) - \int_{-\infty}^{m^*(r^*(s^*),s^*)}G_\rho(m;\rho^*(s^*))dm\right)\frac{\frac{d\rho^*(s^*)}{ds}}{D},$$

where
$$D \equiv (s^* + \Delta)g(m^*(r^*(s^*),s^*);\rho^*(s^*)) - G(m^*(r^*(s^*),s^*);\rho^*(s^*)) < 0$$

when $G(m;\rho^*(s^*))/g(m;\rho^*(s^*))$ is increasing in m. Hence, $d\hat{r}(s^*)/ds - dr(s^*)/ds < 0$ when

$$G(m;\rho^*(s^*))/g(m;\rho^*(s^*))$$

is increasing in m, information structures are consistent with the MPS order, $G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*)) < 0$, and $d\rho^*(s^*)/ds < 0$.

Furthermore, in this case,

$$\frac{\partial\hat{W}(r^*(s^*),s^*)}{\partial r} = G(m^*(r^*(s^*),s^*);\rho^*(s^*)) - (1+\lambda)s^*g(m^*(r^*(s^*),s^*);\rho^*(s^*)).$$

Using the fact that

$$\frac{dW^\#(s^*)}{ds} = \left(\frac{d\hat{r}(s^*)}{ds} - \frac{dr^*(s^*)}{ds}\right)\frac{\partial\hat{W}(r^*(s^*),s^*)}{\partial r} + (1+\lambda)s^*\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*))$$

as established in the proof of Lemma 1, we then have that $dW^\#(s^*)/ds = \frac{d\rho(s^*)}{ds}\frac{J}{D}$, where

$$J \equiv (\Delta - \lambda s^*)\,G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*))G(m^*(r^*(s^*),s^*);\rho^*(s^*))$$

$$+ \left(\int_{-\infty}^{m^*(r^*(s^*),s^*)}G_\rho(m;\rho^*(s^*))dm\right)[(1+\lambda)s^*g(m^*(r^*(s^*),s^*);\rho^*(s^*)) - G(m^*(r^*(s^*),s^*);\rho^*(s^*))].$$

Note that $J < 0$ when information structures are consistent with the MPS order, $G(m;\rho^*(s^*))/g(m;\rho^*(s^*))$ is increasing in m, and $G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*)) < 0$.[29]

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[29]Note that, under the optimal subsidy s^*, welfare is equal to $G(m^*(r^*(s^*),s^*);\rho^*(s^*))(\Delta - \lambda s^*) - C(\rho^*(s^*))$. Because welfare is non-negative under the laissez-faire equilibrium (i.e., when $s = 0$), it must be that $\Delta > \lambda s^*$. Also note that, when $\Delta > \lambda s^*$, $(1+\lambda)s^*g(m^*(r^*(s^*),s^*);\rho^*(s^*)) - G(m^*(r^*(s^*),s^*);\rho^*(s^*)) < D$ and hence the second line in J is negative

We conclude that, under the assumptions in the proposition, $dW^{\#}(s^*)/ds \overset{sgn}{=} d\rho(s^*)/ds$. To see that, under the assumptions in the proposition, $d\rho(s^*)/ds < 0$, note that

$$\frac{dr^*(s)}{ds} = -\frac{\frac{\partial M^-(m^*(r^*(s),s);\rho^*(s))}{dm^*} + \frac{\partial M^-(m^*(r^*(s),s);\rho^*(s))}{d\rho}\frac{d\rho^*(s)}{ds}}{\frac{\partial M^-(m^*(r^*(s),s);\rho^*(s))}{dm^*} - 1}.$$

Under the assumptions in the proposition,

$$\frac{\partial M^-(m^*(r^*(s),s);\rho^*(s))}{dm^*} - 1 = D \cdot G(m^*(r^*(s^*),s^*);\rho^*(s^*)) < 0$$

and $\partial M^-(m^*(r^*(s),s);\rho^*(s))/d\rho < 0$. Hence, $dr^*(s)/ds < 0$ if $d\rho(s^*)/ds > 0$. This cannot be consistent with the optimality of s^*. In fact, by cutting the subsidy, the planner would then induce a friendlier reaction by the follower, permit the leader to economize on cognition, and save on the costs of public funds. The optimality of s^* thus implies that $d\rho(s^*)/ds < 0$. We conclude that $dW^{\#}(s^*)/ds < 0$. The strict quasi-concavity of $W^{\#}$ then implies that $s^{**} < s^*$. ∎

Proof of Proposition 8.

(i) The proof follows from the same arguments that establish part (i) of Proposition 1.

(ii) Recall that

$$\frac{\partial}{\partial\rho}\left(M^-(m^*\left(r(\rho^\dagger)\right);\rho,r(\rho^\dagger))\right) = \frac{\partial}{\partial\rho}\left(\int \omega\frac{q^{\rho,r(\rho^\dagger)}(1|\omega)}{q^{\rho,r(\rho^\dagger)}(1)}dG(\omega)\right).$$

Both when the cost of information is given by entropy reduction and when it is given by maximum slope, $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ is a decreasing function of ω. Hence, when $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ is increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$, the collection of distributions $\left(F^{\rho,r(\rho^\dagger)}\right)_\rho$, indexed by ρ, with each cdf $F^{\rho,r(\rho^\dagger)}$ defined by the density

$$f^{\rho,r(\rho^\dagger)}(\omega) \equiv \frac{q^{\rho,r(\rho^\dagger)}(1|\omega)}{q^{\rho,r(\rho^\dagger)}(1)}g(\omega)$$

can be ranked according to FOSD, with $F^{\rho,r(\rho^\dagger)} \succ F^{\rho',r(\rho^\dagger)}$ for any $\rho < \rho'$. This means that $M^-(m^*(r(\rho^\dagger));\rho,r(\rho^\dagger))$ is decreasing in ρ at $\rho = \rho^\dagger$ which implies that cognition aggravates adverse selection.

(iii) Note that

$$-\frac{\partial\Pi(\rho;r(\rho^\dagger))}{\partial r} = -q^{\rho,r(\rho^\dagger)}(1)\int \frac{\partial\delta_L(r(\rho^\dagger),\omega)}{\partial r}\frac{q^{\rho,r(\rho^\dagger)}(1|\omega)}{q^{\rho,r(\rho^\dagger)}(1)}dG(\omega). \tag{18}$$

Under Assumption 1, $\partial\delta_L(r(\rho^\dagger),\omega)/\partial r$ is increasing in ω. Hence, when

$$q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$$

is increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$, the integral term in (18) is decreasing in ρ (the arguments are the same as in part (ii)). Hence, given $r = r(\rho^\dagger)$, a sufficient condition for a reduction in the friendliness of player F's reaction to raise the incentive for cognition at ρ is that, in addition to $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ be increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in

when information structures are consistent with the MPS order and $G(m;\rho^*(s^*))/g(m;\rho^*(s^*))$ is increasing in m.

38

ρ for $\omega > m^*(r(\rho^\dagger))$, $q^{\rho, r(\rho^\dagger)}(1)$ is non-increasing in ρ.

(iv) The proof is an immediate implication of parts (ii) and (iii).

(v) The proof follows from the fact that, in this case,

$$B(\rho; \rho^\dagger) = -\frac{\partial \delta_L\left(r(\rho^\dagger), m^*(r(\rho^\dagger))\right)}{\partial r} G_\rho\left(m^*\left(r(\rho^\dagger)\right); \rho, r(\rho^\dagger)\right).$$

Because $A(\rho^\dagger) < 0$, the result in part (i) implies that a necessary and sufficient condition for expectation conformity to hold at $(\rho, \rho^\dagger)$ is that $B(\rho; \rho^\dagger) > 0$ which is the case if and only if $G_\rho\left(m^*\left(r(\rho^\dagger)\right); \rho, r(\rho^\dagger)\right) < 0$. Because, for any ρ,

$$G\left(m^*\left(r(\rho^\dagger)\right); \rho, r(\rho^\dagger)\right) = \int q^{\rho, r(\rho^\dagger)}(1|\omega) dG(\omega) \equiv q^{\rho, r(\rho^\dagger)}(1)$$

the latter property is equivalent to $q^{\rho, r(\rho^\dagger)}(1)$ being non-increasing in ρ.

	Lemons $\frac{dr}{d\rho^{\dagger}} \overset{sgn}{=} \frac{\partial}{\partial\rho^{\dagger}} M^{-}(m^{*};\rho^{\dagger})$	Anti-lemons $\frac{\mathrm{d}r}{\mathrm{d}\rho^{\dagger}} \overset{sgn}{=} -\frac{\partial}{\partial\rho^{\dagger}} \mathrm{M}^{-}(\mathrm{m}^{*};\rho^{\dagger})$
Does an increase in cognition lead to unfriendlier response? $(dr/d\rho^{\dagger} < 0)$	Yes if • MPS and $G_{\rho} < 0$ • or cognition always aggravates adverse selection (e.g., uniform, Pareto, exp.)	No if • MPS and $G_{\rho} < 0$ • or cognition always aggravates adverse selection (e.g., uniform, Pareto, exp.)
Does an unfriendlier response increase L's demand for cognition? $\left(-\frac{\partial^2\Pi(\rho;r)}{\partial r\partial\rho} > 0\right)$	• Yes if MPS and $G_{\rho} < 0$ • No if $G_{\rho} \geq 0$ and $\frac{\partial^2\delta_L}{\partial m\partial r} = 0$ (Akerlof's model +ex. a, c, d in Supplement)	• Yes if MPS and $G_{\rho} < 0$ • No if $G_{\rho} \geq 0$ and $\frac{\partial^2\delta_L}{\partial m\partial r} = 0$ (Spence's model + ex. f, h in Supplement)
(Local) EC / cognitive traps	• Yes if MPS and $G_{\rho} < 0$ • $G_{\rho} < 0$ is NSC if cognition always aggravates adverse selection and $\frac{\partial^2\delta_L}{\partial m\partial r} = 0$	• Yes if cognition always aggravates adverse selection, $G_{\rho} > 0$, and $\frac{\partial^2\delta_L}{\partial m\partial r} = 0$
Engagement channel of subsidy $\frac{\partial}{\partial m^{*}} M^{-}(m^{*};\rho^{\dagger})\frac{\partial m^{*}}{\partial s}$	• Benefits player L	• Hurts player L
Cognition channel of subsidy $\frac{\partial}{\partial\rho^{\dagger}} M^{-}(m^{*};\rho^{\dagger})\frac{d\rho^{\dagger}}{ds}$	• Benefits player L if MPS and $G_{\rho} < 0$	• Hurts player L if MPS and $G_{\rho} < 0$
Total effect of subsidy on welfare	• positive if engagement + cognition channels $> K > 0$	• negative if engagement + cognition channels $> K < 0$

Table 1: summary of a few results