INDUSTRIAL MONETARY POLICY*

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Abstract

"Industrial monetary policies" refer to liquidity support programs through which authorities shape the location and continuation of financial activity on their soil. Banks' locus of activity depends on their prospect of receiving liquidity support and therefore on factors such as countries' relative resilience. We predict that global banks are global in life and national in death; and that they become less global when banking is competitive, times are turbulent, and international risk sharing (say, through swap lines) weak. We analyse both the competitive benefits of industrial monetary policies as well as their limitations, such as currency appreciation.

Keywords: Hegemon, cross-border banking, liquidity support, too big to fail, international monetary system, home bias, regulatory competition, exhorbitant duty.

JEL numbers: D43, E58, F33, G15, G21.

1 Introduction

Countries are eager to attract and maintain economic activity on their soil. A large literature correspondingly studies how this desire shapes trade agreements and protectionist policies (from export and plant-creation subsidies to import tariffs). It focuses on industry and by and large ignores the banking system. While some insights gleaned in this literature also apply to finance, there are also cross-border banking specificities, such as the key role of public liquidity provision and the presence of supervisors. This paper aims at extending the trade literature to the banking system and thereby at shedding light on some ongoing debates.

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A case in point is the European banking market. Commentators bemoan the strong home bias of the banks, even of the biggest ones. Clearly a European banking passport does not guarantee a diversified European presence. More generally, the decomposition of the assets of large cross-border banks into home country/rest of region/rest of the world reveals a limited diversification.[1] Of course inertia and the dominance of home banks in their turf prior to banking liberalization partly account for this strong country specialization. This paper focuses on another source of specialization: the greater ability to access liquidity when specializing.[2]

Section 2 first describes the bare-bones model. There are two identical countries, one bank, and three dates: A date 0 at which the bank chooses how much and where to invest, a date 1 at which it may require liquidity support, and a date 2 at which long-term (private and social) benefits of these investments are reaped (date 2 stands for the "future"). The bank is initially not credit constrained when it "acquires" (say, SME or retail) customers at date 0. Acquiring "projects" or customers is a costly activity. This investment need not be one-shot, as at date 1 the bank may have to reinvest in the funded projects so as to enable them to continue.[3].

To capture public liquidity provision in a stark and simple way, we assume that there is no store of value;[4] that is, we abstract from the standard liquidity-underinvestment, or investment-in-the-wrong-kind-of-liquidity market failure that has received much attention in the economics literature; the latter has also stressed the need for public provision of liquidity in tail events even in the presence of abundant stores of values. Because there are no stores of value at date 0 in the model, only the state can supply the required date-1 liquidity. Countries supply liquidity because they care about the continuation of activity at home. In Section 2, we assume that the states do not commit to liquidity provision; rather they supply the liquidity in a time-consistent manner.

Our first key assumption is that money is (at least in part) fungible: The countries cannot ensure that all the money they supply at date 1 will be used domestically. For perfect fungibility for instance, banks allocate their date-1 funds internationally as is optimal for them. This creates a leakage (some of the funds are channeled abroad) and an associated free-riding problem (at date 1, a country benefits from the liquidity provided by the other country). Section 2 shows that, despite the symmetry of the model, the bank specializes in a country: The bank wants to be the national champion of one country so as to be able to count on its support in case of

[1] Schoenmaker (2013, Chapter 13).

[2] While public liquidity provision has been the object of much scrutiny, its translation to the international context has received much less attention. Two policy developments suggest that such attention is warranted. The swap lines extended by the Fed in 2008 and the more formal arrangements offered by China to boost the renminbi in international transactions (see Bahaj-Reis 2022 for a study of their consequences) suggest that international liquidity provision can be used strategically. Simultaneously, concern has been expressed over the ability of multinational banks to secure sufficient support, and plans are drawn at the European level to facilitate banks' recovery and resolution. It is also often argued that European banks are not diversified enough and that both competition and financial stability might be promoted through pan-European banking. The model sheds light on these developments.

[3] Interpreting projects as SME borrowers, one may have in mind the standard learning-by-lending argument. The incumbent lender over time knows more about the borrower than alternative lenders and will underbid the new lender whenever profitable, leaving the latter with an adversely selected sample of negative-value customers, with the outcome being an ex-post monopoly position for the historical lender (see Section 9.4 in Tirole 2006 for an exposition of the standard argument).

[4] More generally, there is an insufficient amount of stores of value to cover at least some states of nature.

distress. And that country bears the brunt of the rescue. The degree of specialization may vary from partial to complete.

Because countries are ex-ante identical, the bare-bones model does not make a prediction as to which country the bank favors. Section 2 therefore relaxes the symmetry assumption in order to derive predictions as to which country will attract banking activity. We investigate three determinants. A larger room for fiscal or monetary manoeuver offers a comparative advantage to the country. So does a higher eagerness to rescue banks. Differences in the distributions of domestic shocks are relevant as well. While it is still the case that a single country ends up rescuing the bank, two differences with the symmetric-shock-distributions case emerge. First, the marginal leakage attached to liquidity support by one country depends not only on the banks' relative presence in the country relative to abroad, but also on who benefits most from a bit more liquidity; in particular the support may now come from the low-presence country. Second, both the marginal and inframarginal leakages matter for the determination of which country brings the liquidity. We identify the determinants of the bank's country choice.

Our second key assumption is that a world of countries does not behave as a borderless one. In our context, international date-1 coordination of liquidity provision would lead to better support for the bank. In practice though, and despite numerous attempts at policy reforms (e.g., resolution planning, MoUs requiring information sharing), international coordination has mostly failed in 2008 and during other episodes of cross-border bank distress. Cross-border bailouts end up in disaster. In Mervyn King's oft-cited quip: *"Global banks are global in life and national in death"*. Indeed, "Coasian agreements" require not only (a) coordination (often difficult to achieve given the urgency of banking rescues), but also (b) informational commonality (say, about the countries' willingness to rescue the bank or about the bank's exact exposures) as well as (c) the availability of public funds in both countries.[5] If any of these conditions is violated, Coasian bargaining breaks down and the insights of the uncoordinated case carry over qualitatively.

Section 3 looks at date-0 public policies. At that date, a country may want to attract banking capital - we call this industrial monetary policy (IMP) - in two ways. The first approach results directly from the analysis in Section 2. Because there is higher trust in a country's liquidity support when the country is resilient, the country can keep room for manoeuver on the fiscal side (keeping spare borrowing capacity) or the monetary side (being able to supply liquidity without risking inflation). The other important determinant is the country's eagerness, rather than ability, to bring liquidity support; in this respect, turning a blind eye on a bank being systemic is a commitment to bring it liquidity support. Section 3 studies another form of IMP: pledging to act as a lender of last resort (LOLR). LOLR differs from the provision of liquidity considered hitherto, in that it requires a commitment, perhaps sustained by reputation, while the provision of liquidity studied in Section 2 is ex-post voluntary; put differently, LOLR involves going beyond what the country would naturally do at date 1. We show how a conditional form of LOLR enables countries to attract investments onto their own soil, while an unconditional one tends to produce the opposite of what was intended and is often self-defeating. We

[5] Pre-arranged lines of credit between countries would alleviate one obstacle to efficient negotiations, namely the possibility that one country does not have the funds needed to compensate the other; but they raise the prospect of moral hazard and furthermore, coordination failures have other sources as we just saw.

also show that exchange rate appreciations are an unavoidable byproduct of successful industrial monetary policies. Exchange rate appreciations, in turn, are a limiting factor for industrial monetary policies by endogenously making them costlier to operate.

Section 4 studies competition in banking with ex-ante identical countries. At date 0, identical banks compete à la Cournot and pick their footprint in each country; so, they choose their overall scale and their degree of specialization. At date 1, each bank can receive liquidity support from the countries. We show that, under reasonable conditions, the only equilibria in this otherwise symmetric environment are asymmetric equilibria in which the banks specialize in different countries. We then demonstrate that Coasian bargains would lead not only to a better allocation of support to economies at date 1, but also, by limiting the incentive to become a national champion, to more banking competition at date 0. We also show that the more competitive the banking system (as captured by the number of banks divided by the number of countries), the more banks specialize. They always operate in a single country for a large enough bank/country ratio.

While the core analysis assumes fungibility, we are agnostic as to regulators' ability to ringfence, which depends on institutional details (allocation of supervisory duties, legal regimes for resolution…). Even when a bank is supervised, it can use its private information to reshuffle liquidity across countries ahead of shocks; it can also redistribute net worth through complex contracts with international suppliers or investors, or by engaging in country-specific risktaking. To be certain, supervision reduces these margins for manoeuver, but only to a limited extent. There is agreement among scholars that the home country can achieve a higher level of ringfencing if the bank has branches rather than subsidiaries abroad. Subsidiaries on the other hand require much more capital and liquidity than branches as the bank holding cannot pool resources and use excess cash in one country to meet obligations in another (Cerutti et al 2010).

Section 5 relaxes the perfect fungibility assumption. We assume that the country that regulates a bank can at least partly ringfence the use of cash at date 1, with a ringfencing parameter going from 0 (perfect fungibility) to 1 (perfect ringfencing). We show that banks want to incorporate in (and thereby be supervised by) the country in which they have their strongest presence. Ringfencing has a benefit and a cost: In the absence of Coasian bargains, it makes the home country more eager to exert its responsibility; but it also creates a misallocation of liquidity within the bank.

Section 6 reviews the relevant literature and Section 7 concludes.

We conclude this overview of the paper with two related questions: *What makes banks "banks" in this model? And why did we title the paper "Industrial Monetary Policy"?* Concerning the former question, our theory hinges on the entities' ability to count on state support, were they in dire straits. History has taught us that this is certainly the case for the financial industry. To be certain, this willingness of authorities to accommodate entities whose activity is highly valued by the state extends beyond banking (for example one can think of national champions such as Boeing and Airbus); and the theory developed below would apply as well to such sensitive industries. But state support in dire straits is particularly prominent in the banking sector, and so there is little abuse of terminology.

This discussion also points at when location matters within the financial industry. The state is particularly eager to preserve the integrity of the financial system with regard to the "core functions": lending to SMEs and serving retail depositors (note that these core functions show up on both the asset and liability sides of their balance sheets). Rescues are also likely when there are externalities on the payment system or more generally onto other regulated banks and insurance companies; the externality may be generated by counterparty exposures or by the threat of fire sales. Such concerns may also induce authorities to bail out non-bank actors (like in 2008 AIG, whose bankruptcy might have generated domino effects onto regulated banks and insurance companies). A contrario, the location of a hedge fund in Bermuda is likely to be driven by other considerations than the prospect of state support, which is much lower than for the above-mentioned actors.

Regarding the title, we note that liquidity support to banks often operates through the central bank. This support can be targeted to a specific bank, which can use the discount window and emergency liquidity assistance (ELA).[6] The Fed's or ECB's supervisory arm can also exercise supervisory leniency. Alternatively, in the presence of macroeconomic shocks, the support may aim at the banking sector as a whole: asset purchase programs (corporate bonds, public debt, ABS, covered bonds), monetary bailouts (consisting in lowering the interest rate to facilitate the banks' refinancing),[7] and swaps with other central banks. While liquidity support is prominent in the banking world, there are alternative strategies for attracting banking activity such as issuing stores of value and granting fiscal subsidies to investment. These strategies are more outside the Central Bank's remit, but also relevant. We later comment on their limits and stress the benefits of liquidity support.

2 A bank's international diversification strategy

We start with a single bank in a given strategic environment (monopoly, oligopoly or competitive). This will allow us to study its diversification trade off (Section 2) as well as the country's optimal industrial monetary policy (Section 3). We will later analyze how banking rivalry impacts behaviors (Section 4).

2.1 Model Description

There are two identical countries, A and B, and one cross-border bank which acquires "clients" or "projects" in the two countries. There are three dates $\{0,1,2\}$, and players do not discount the future.

[6] The Bagehot doctrine aims at restricting such support to pure liquidity crises, with enough collateral to make the central bank bear no risk. But the line between monetary and fiscal policy has always been a thin one, except if the authorities know for sure that the run is a pure one, unrelated to fundamentals (and if so a coordination failure is required to explain why banks do not borrow on the market given that their liabilities are risk-free). The line has been particularly blurred since 2008 (and recently, in reaction to Covid, the list of assets that banks can use as collateral has been extended and the measure applied to determine these assets' value ("haircut") has been made less strict).

[7] See Farhi-Tirole (2012).

Date 0

The bank picks a mass q^k of clients (a continuum of them) in each country $k \in \{A, B\}$. The unit cost of customer acquisition in country k is $c(q^k)$. This unit cost function satisfies $c(0) = 0$, $c' > 0$, and $qc'' + c' > 0$, where the latter condition is sufficient to guarantee the convexity of the acquisition cost $c(q)q$ and (when we introduce banking competition) strategic substituability.[8]

This assumption also generates a cost benefit of diversification, i.e. $c(q^A)q^A + c(q^B)q^B \geq 2c(\frac{q^A+q^B}{2})(\frac{q^A+q^B}{2})$ for all (q^A, q^B). Let

$$\hat{c}(q^k) \equiv c(q^k) + q^k c'(q^k)$$

denote the marginal acquisition cost in country k. From our previous assumption, $\hat{c}$ is increasing.

Let $q \equiv \sum_k q^k$ stand for total bank size, and $\sigma^k \equiv \frac{q^k}{q}$ denote the bank's relative presence in country k. The parameter $\sigma \equiv \max_{k \in \{A,B\}} \sigma^k$ is a measure of the bank's country specialization. The bank is (fully) diversified if $\sigma = 1/2$. For conciseness, country k such that $\sigma^k = \sigma > 1/2$ will be called the "home country" or "high-presence country", while the other country is the "foreign country" or "low-presence country".

The bank has initial resources at date 0. Unless otherwise specified, we will assume that the bank is not credit constrained. That is, for its optimal size q^* and specialization σ^*, the bank's date-0 resources exceed $C(q^*, \sigma^*)$, where C is the bank's investment cost function:

$$C(q, \sigma) \equiv c(\sigma q)\sigma q + c((1 - \sigma)q)(1 - \sigma)q.$$

Our assumptions imply that $C_q > 0$ and $C_{qq} > 0$, and (in the relevant range $\sigma \geq 1/2$) $C_\sigma \geq 0$, (with strict inequality unless $\sigma = 1/2$), $C_{\sigma\sigma} > 0$, and $C_{\sigma q} > 0$, where $C_q = \partial C/\partial q$, etc. For example, for a linear unit cost $c(q) = q$ (see footnote 8 for foundations), the acquisition cost is quadratic:

$$C(q, \sigma) = [\sigma^2 + (1 - \sigma)^2]q^2.$$

The bank's date-0 establishment choices are commonly observable.

Date 1

In each country, projects are hit by liquidity shocks at date 1. The distribution of the bank's projects' liquidity shocks on its various projects, $F(\rho) \sim [0, +\infty)$, is the same in both countries and is common knowledge. So these are no macroeconomic shocks, only project-specific shocks (we later develop an alternative interpretation involving a macroeconomic shock). These shocks are to be understood as net of any date-1 revenue; we do not allow net date-1 revenues to be strictly positive only for notational simplicity.[9]

[8] As an illustration, suppose that there is a mass 1 of borrowers and that each borrower at date 0 requires an investment of 1. A borrower has endowment y that is distributed according to $G(y)$ on $[0, 1]$. For simplicity assume that this endowment is not observable by the bank. The supply of loans q^k in country k determines a cutoff $y^k = G^{-1}(1 - q^k)$ and a cost $c(q^k) \equiv 1 - y^k$. For instance, for a uniform distribution of borrower endowment on $[0, 1]$, $c(q^k) \equiv q^k$.

[9] Strictly positive net revenues for some projects can easily be accommodated as they are used to cover other projects' liquidity shocks. See Subsection 2.4.

The cumulative distribution F is twice continuously differentiable. For conciseness its density f satisfies $f' \leq 0$ (as it is for uniform, exponentiel and Pareto distributions); this condition guarantees the concavity of the objective functions. The weak concavity of F is also sufficient for the hazard rate $f(\rho)/F(\rho)$ to be monotonic, and indeed for the stronger property that the elasticity of the cumulative distribution, $f(\rho)\rho/F(\rho)$, be decreasing in ρ and always less than or equal to 1.[10] Finally, the truncated mean $E[\rho|\rho \leq \rho^*]$ is smaller than $\rho^*/2$, and the distribution is thin-tailed (in that $\lim_{\rho\to\infty} f(\rho)\rho = 0$) provided it has a mean.[11]

Countries, which have observed the bank's investments $\{q^k\}$ in the two countries, face costs of public funds, $\{\lambda^k\}$ for country k, and simultaneously select liquidity support[12] $\{T^k \geq 0\}$. So the bank receives $T = T^A + T^B$. We will let $\kappa^k \in [0, q^k]$ denote the continuation scale in country k. That is, a fraction κ^k/q^k of initial investments is safeguarded thanks to the bank's date-1 reinvestment in country k. As discussed in the introduction, λ^k stands for country k's shadow cost of raising resources- whether fiscal or monetary- to support the bank.

Date 2

The bank enjoys its private benefit, $[\sum_m \kappa^m]b$. Country k receives social benefit $\kappa^k\beta$.[13]

Objective functions

Whether transfers are ringfenced (as in Section 5) or not (as will be the case until then), the bank allocates the available money within a given country to those projects that require the smallest reinvestment. Letting ρ^k denote the bank's cutoff in country k,[14] $\kappa^k = F(\rho^k)q^k$. The bank's utility is then equal to the expected continuation benefit minus the cost of acquisition:

$$U = [\sum_k F(\rho^k)q^k]b - [\sum_k c(q^k)q^k].$$

Countries do not necessarily care about the bank itself, but they internalize the continuation of economic activity on their soil. This feature is, as we discussed, what makes banks "banks" in our model. Country k's welfare comprises two terms: The benefit of the economic activity net of the transfers needed to maintain it; and the rent S of date-0 project owners. The rent S satisfies $dS/dq = qc'(q)$ (the product of the number of inframarginal units times the necessary price increase brought about by the marginal one). It is increasing and convex and is given

[10] $f(\rho)\rho = \int_0^\rho f(\rho)dx \leq \int_0^\rho f(x)dx \leq F(\rho)$.

[11] Suppose not. Then $\exists \epsilon > 0$ $s.t.$ $\forall \underline{\rho} : \exists \hat{\rho} \geq \underline{\rho} : f(\hat{\rho})\hat{\rho} \geq \epsilon$. Consider a sequence $\{\hat{\rho}_n\}_{n\in\mathbb{N}}$ satisfying $f(\hat{\rho}_n)\hat{\rho}_n \geq \epsilon$ such that $\hat{\rho}_n \geq 2\hat{\rho}_{n-1}$. Denote $\hat{\rho}_0 = 0$. Therefore,

$$\int_0^\infty f(\rho)d\rho = \sum_{n=1}^\infty \int_{\hat{\rho}_{n-1}}^{\hat{\rho}_n} f(\rho)d\rho \geq \sum_{n=1}^\infty (\hat{\rho}_n - \hat{\rho}_{n-1}) f(\hat{\rho}_n) \geq \sum_{n=1}^\infty (\hat{\rho}_n - \hat{\rho}_{n-1}) \frac{\epsilon}{\hat{\rho}_n} \epsilon \sum_{n=1}^\infty (1 - \frac{\hat{\rho}_{n-1}}{\hat{\rho}_n}) \geq \epsilon \sum_{n=1}^\infty \frac{1}{2} = \infty,$$

a contradiction.

[12] In the model these are pure bailouts as we have assumed that there is no pledgeable income that the bank can return at date 2. With (say, a random) date-2 income, this liquidity support takes the more familiar form of a risky collateralized loan to the banks. More on this later.

[13] For micro-foundations of b and β, see e.g., Farhi-Tirole (2021).

[14] That is, projects that require reinvestment $\rho \leq \rho^k$ are continued.

by:[15]

$$S(q^k) = \int_0^{q^k} xc'(x)dx.$$

The intertemporal welfare of country k is the sum of the date-0, date-1 and date-2 welfares:

$$W^k = S(q^k) + [F(\rho^k)q^k\beta - \lambda^k T^k].$$

Absence of ringfencing

We assume that money is fungible: Countries cannot target their liquidity assistance to reinvestment within the country. This implies that the bank allocates funds as it sees optimal, i.e. to the projects that have the lowest cost of reinvestment and so the cutoff in the two countries coincide:

$$\rho^A = \rho^B = \rho^*.$$

2.2 Date-1 liquidity provision and date-0 bank size and diversification

Let us assume for the moment that the two countries face identical costs of public funds: For all k, $\lambda^k = \lambda$.

Countries' incentives under full fungibility

At date 1, the bank receives transfers from both countries, in total amount $T \equiv T^A + T^B$. It then efficiently allocates this liquidity to the least-continuation-cost projects, so as to maximize the date-2 private (and social) value. Let ρ^* denote the bank's cutoff for continuation ($\rho^A = \rho^B = \rho^*$). The date-1 budget constraint given aggregate size q writes:

$$\left[\int_0^{\rho^*} \rho dF(\rho)\right] q = T. \tag{1}$$

Furthermore

$$\kappa^k = F(\rho^*)q^k.$$

Country k chooses its support level at date 1 so as to solve:

$$\max_{\{T^k\}}\{\beta\kappa^k - \lambda T^k\} = \max\{\beta F(\rho^*)q^k - \lambda T^k\}$$

[15]In the example given in footnote 8,

$$S(q^k) = \int_{G^{-1}(1-q^k)}^{\infty} (y - y^k)dG(y);$$

so for example with a quadratic acquisition cost,

$$S(q^k) = (q^k)^2/2.$$

with $\rho^*(T^A + T^B)$ given by (1), and so:[16]

$$\frac{\partial(\beta\kappa^k - \lambda T^k)}{\partial T^k} = \frac{\beta\sigma^k}{\rho^*} - \lambda.$$

This implies that, unless $\sigma = 1/2$, only the home country (country k such that $\sigma^k \equiv \sigma > 1/2$) brings liquidity to the bank. The cutoff is given by [17]

$$\rho^* = \hat{\beta}\sigma, \tag{2}$$

where $\hat{\beta}$ is the countries' cost-adjusted willingness to pay for continuation:

$$\hat{\beta} \equiv \frac{\beta}{\lambda}.$$

If $\sigma = 1/2$, then $\rho^* = \hat{\beta}/2$; we can assume for example that each country contributes for half of the liquidity provision.[18]

The bank's continuation scale is therefore $F(\hat{\beta}\sigma)$ where, recall, $\sigma \in [1/2, 1]$ denotes the bank's degree of specialization. The continuation scale is higher, the lower the probability of leakage, where leakage can be measured by the fraction, $1 - \sigma$, that benefits the other country. Because the efficient level of support for the two countries is $\rho^* = \hat{\beta}$, there is free riding between the two countries unless the bank is fully specialized, i.e. not cross-border ($\sigma = 1$). The leakage associated with the inability to ringfence induces the home country to bring too little liquidity to the bank, from the point of view of the other country (and of course from the point of view of the bank).

Finally, let us note that the high-presence country indeed wants to bring the liquidity level corresponding to the first-order condition. Its net utility is given by $\lambda q\Big[F(\hat{\beta}\sigma)\hat{\beta}\sigma - \int_0^{\hat{\beta}\sigma} \rho dF(\rho)\Big]$. The term in brackets is equal to 0 for $\hat{\beta} = 0$ and is increasing in $\hat{\beta}$ and therefore always positive. Put differently, if the country is willing to bring liquidity to withstand a marginal shock, it is a fortiori willing to bring liquidity that will serve to withstand (lower) inframarginal shocks. As we will see, this property need not hold when the shock distributions are asymmetric: There may be much more leakage of liquidity for the inframarginal shocks than for the marginal one.

Optimal size and diversification

At date 0, the bank chooses an overall presence, q, and a specialization $\sigma \in [\frac{1}{2}, 1]$, so as to maximize:

$$U(q, \sigma) \equiv F(\hat{\beta}\sigma)bq - C(q, \sigma)$$

[16] Note that $\frac{\partial^2(\beta F(\rho^*(T^A+T^B))q^k - \lambda T^k)}{\partial(T^k)^2} < 0$.

[17] We assumed that the distribution F has support on $[0, +\infty)$ so there is no issue with an upper bound. When we use the uniform distribution for illustrative purposes, though, we must assume that $\hat{\beta}\sigma$ does not exceed the upper bound.

[18] There are actually a continuum of equilibria, all yielding the same date-1 continuation scale, and therefore the same bank behavior at date 0: Any $\{T^A, T^B\}$ such that $T^A + T^B = [\int_0^{\hat{\beta}/2} \rho dF(\rho)]q$ is an equilibrium of the liquidity provision stage. Because the bank is not affected by who is bringing the overall support T, this indeterminacy has no impact on the date-1 equilibrium outcome beyond the redistributive aspect.

or

$$\max_{\{q,\sigma\}}[F(\hat{\beta}\sigma)q]b - c(\sigma q)\sigma q - c((1-\sigma)q)(1-\sigma)q.$$

For simplicity, we assume that this program is concave.[19] Recalling that $\hat{c}(q) \equiv c(q) + c'(q)q$, the first-order conditions with respect to q and σ, respectively, are for an interior solution[20]

$$F(\hat{\beta}\sigma)b = \sigma\hat{c}(\sigma q) + (1-\sigma)\hat{c}((1-\sigma)q) \tag{3}$$

and

$$f(\hat{\beta}\sigma)\hat{\beta}b = \hat{c}(\sigma q) - \hat{c}((1-\sigma)q). \tag{4}$$

Let (q^{nc}, σ^{nc}) denote the bank's optimal choices under no-commitment (i.e. when the countries act in a time-consistent manner). The first observation comes from looking at condition (4): Necessarily $\sigma^{nc} > 1/2$. Starting with equal presence in the two countries, the marginal cost $\hat{c}$ for the bank of investing is the same in both and so a slight increase in σ has no effect on the investment cost (the RHS of (4) is 0): The cost in terms of diversification is second order. But the specialization brings in more liquidity support and increases bank payoff to the first order (this is the term of $f(\hat{\beta}\sigma)\hat{\beta}b$ on the LHS of (4)). Condition (3) says the marginal benefit of overall size expansion, $F(\hat{\beta}\sigma)b$, is equal to its average marginal cost.

Proposition 1 *(country specialization). Under fungibility, symmetric countries, and time-consistent support policies:*

(i) Date-1 liquidity support is brought only by the home country, increasingly so as the bank specializes more, i.e. as σ increases.

(ii) At date 0, the bank chooses to specialize partly or fully ($\sigma^{nc} \in (\frac{1}{2}, 1]$). The bank diversifies maximally when support is expected to be generous ($\lim_{\hat{\beta}\to+\infty} \sigma = 1/2$). Conversely, when expected support is meager, the bank specializes ($\lim_{\hat{\beta}\to 0} \sigma = 1$). Finally, for any $\hat{\beta}$, the bank fully specializes ($\sigma = 1$) for a uniform distribution ($f' = 0$) and diversifies ($\sigma < 1$) otherwise ($f' < 0$).

(iii) More turbulent times (in the sense of a uniform shift θ toward higher shocks: $F(\rho - \theta)$) lead to more specialization, as the bank is keen on securing liquidity from the home country: Provided that $\hat{c}(0) = 0$, full specialization ($\sigma^{nc} = 1$) obtains for a wider set of parameters; also, with linear cost, σ^{nc} weakly increases with θ.

Proof. Part (i) was proved earlier.

(ii) Consider now the bank's choice of specialization. The condition for full specialization is simply obtained by substituting $\sigma = 1$ into equation (3), using the fact that $\hat{c}(0) = c(0) = 0$, and checking that the LHS of (4) exceeds its RHS. Next, when $\hat{\beta} \to +\infty$, (nearly) all shocks are

[19] Under our assumptions, $C_{qq} > 0$ and $C_{\sigma\sigma} > 0$. However $C_{qq}C_{\sigma\sigma} - (C_{q\sigma})^2$ is not always positive. On the other hand, we only need $F(\hat{\beta}\sigma)qb - C(q,\sigma)$ to be concave, with $\partial^2(F(\hat{\beta}\sigma)qb)/\partial q\partial\sigma = f(\hat{\beta}\sigma)\hat{\beta}b$, so that for $\hat{\beta}b$ large enough concavity in garanteed.

[20] If $\sigma = 1$, condition (4) becomes $f(\hat{\beta})\hat{\beta}b \geq \hat{c}(q)$.

covered by the home country and so cost minimization is the bank's overriding consideration: because the distribution of shocks has thin tails ($lim_{\rho\to+\infty}f(\rho)\rho = 0$) , condition (4) shows that the bank must diversify maximally when $\hat{\beta}$ is large. Conversely, $\lim_{\hat{\beta}\to 0} F(\hat{\beta})/f(\hat{\beta})\hat{\beta} = 1$, and so (almost) full specialization is expected for low support.

(iii) Consider a uniform shift θ in the distribution of shocks, which becomes $F(\rho-\theta)$. A larger θ means larger shocks. Simple computations show that (a) full specialization ($\sigma^{nc} = 1$) obtains for a larger set of parameter as θ increases, and (b) in the quadratic acquisition cost case ($c(q) = q$, and so $\hat{c}(q) = 2q$), σ^{nc} is a weakly increasing function of θ. This is easily understood: The bank faces a trade-off between specializing more to secure more liquidity support from its home country and conquering the marginally more lucrative foreign market (i.e., benefiting from diversification). The first concern looms larger when shocks become more likely. ∎

Quadratic acquisition cost example. Suppose $c(q) = q$ (and so $\hat{c}(q) = 2q$). Combining (3) and (4) yields

$$\frac{f(\hat{\beta}\sigma)(\hat{\beta}\sigma)}{F(\hat{\beta}\sigma)} = \frac{2\sigma^2 - \sigma}{2\sigma^2 - 2\sigma + 1}. \tag{5}$$

The RHS of (5) is increasing in σ and is equal to 1 for $\sigma = 1$. The LHS of (5) represents the elasticity of the distribution function $F(\rho)$. Because this elasticity is decreasing in ρ, then the equilibrium specialization σ^{nc} is a decreasing function of the eagerness $\hat{\beta}$ to support banking activity. Intuitively, when the countries are very eager to provide liquidity support, a high specialization is no longer needed to obtain this support.

We can complete this analysis by looking at the case of two specific distributions.

(a) *Exponential distribution.* Suppose that $F(\rho) = 1 - e^{-\rho}$ on $[0, +\infty)$. Then

$$\frac{f(\rho)\rho}{F(\rho)} = \frac{\rho}{e^{\rho} - 1}$$

is smaller than 1 and decreasing in ρ. So there is never full specialization ($1/2 < \sigma^{nc} < 1$) from Proposition 1 (ii). And σ^{nc} is decreasing in β and increasing in λ. Furthermore, we verify the more general properties:

$$\lim_{\hat{\beta}\to+\infty} \sigma^{nc} = \frac{1}{2} \quad \text{and} \quad \lim_{\hat{\beta}\to 0} \sigma^{nc} = 1.$$

(b) *Uniform distribution.* For $F(\rho) = \rho$ on $[0, 1]$, $f(\rho)\rho/F(\rho) \equiv 1$. So Proposition 1 (ii) implies that $\sigma^{nc} = 1$ regardless of $\hat{\beta}$.

2.3 Does the home country benefit from being the home country?

The answer is obviously yes if $\sigma^{nc} = 1$. But more generally the home country bears the brunt of the liquidity support and it is not obvious that it is better off than the foreign country. Let V^h and V^f denote their payoffs. Then

$$V^h - V^f = \left[S(\sigma q) + \left[F(\hat{\beta}\sigma)\beta\sigma - \lambda \int_0^{\hat{\beta}\sigma} \rho dF(\rho)\right]q\right] - [S((1-\sigma)q) + F(\hat{\beta}\sigma)\beta(1-\sigma)q].$$

or

$$V^h - V^\ell = \left[F(\hat{\beta}\sigma)\beta(2\sigma - 1) - \lambda \int_0^{\hat{\beta\sigma}} \rho dF(\rho)\right] q + [S(\sigma q) - S((1-\sigma)q)]$$

Proposition 2 *(benefit from being the home country)*

(i) *For* $\sigma = 1/2$, $V^h < V^f$.

(ii) *For* $\sigma \geq 2/3$, $V^h > V^f$.

(iii) *There exists a* $\underline{\sigma} \in (1/2, 2/3)$ *such that* $V^h > V^f$ *iff* $\sigma > \underline{\sigma}$.

A corollary of Proposition 2 (combined with part (iii) of Proposition 1) is that as times become more turbulent, it is more likely that $V^h > V^f$.[21]

Capital controls. Suppose now that countries can limit the investment on their soil, perhaps through a date-0 capital control or through prudential requirements. Country k can set a cap $\bar{q}^k$ on the bank's domestic investment: $q^k \leq \bar{q}^k$. Consider the game in which at date-1, countries simultaneously set ceilings $\bar{q}^k$ on the bank's investment. In order to avoid a multiplicity of equilibria in the absence of capital controls, suppose that when indifferent, the bank picks country B as the home country.[22] The no-capital-control equilibrium is then unique and has the bank invest $q^A = (1-\sigma^{nc})q^{nc}$ in country A and $q^B = \sigma^{nc} q^{nc}$ in country B. Countries A and B obtain no-capital-control welfares V^f and V^h, respectively.

Proposition 3 *(capital controls) The following is an equilibrium behavior of the capital control game:*

(i) *When* $V^h \geq V^f$ *at the unconstrained date-0 investment policy* $\{q^{nc}, \sigma^{nc}\}$*, countries do not impose capital controls and the outcome is the same as in the absence of capital controls.*

(ii) *When* $V^{ff} = V^f - \left[F(\hat{\beta}\sigma^{nc}) - F\left(\frac{\hat{\beta}}{2}\right)\right]\beta(1-\sigma^{nc})q^{nc} \leq V^h < V^f$ *at the unconstrained date-0 investment policy* $\{q^{nc}, \sigma^{nc}\}$*, country A imposes capital control* $\bar{q}^A = (1-\sigma^{nc})q^{nc}$ *and country B imposes no capital control. The outcome is again the same as in the absence of capital control. [When* $V^h < V^{ff}$*, the equilibrium of the capital control game is in mixed strategies.]*

When being the home country is beneficial despite the associated duty, country A would like to challenge country B's dominance; however, country A cannot make itself more attractive by imposing a capital control on the bank's investment there (part (i) of the proposition). When being the home country is penalizing (part (ii) of the proposition), country B would like to switch roles with country A, and can do so in the absence of capital control by country A: it suffices that country B make itself less attractive through a capital control a bit below q^B. But country A can protect itself against such passing the buck by imposing a capital control

[21] Of course turbulent times also reduce the overall scale, so here we discuss only relative payoffs.

[22] One may justify this lexicographic choice by imagining that the bank has a (vanishingly small) preference for country B. For example, the marginal investment cost in country B might be slightly smaller than in country A, with the difference going to 0. Or country B be might be slightly more eager to rescue the bank than country A.

at level q^A, which in equilibrium has no effect, but deters country B from switching roles: To make country A the home country, country B must then choose a lower capital control; but that implies that the liquidity support will be meager- an effect captured by the difference between V^f and V^{ff}.

2.4 Discussion of modeling choices and simple extensions

Correlation of shocks. Due to risk neutrality, the model admits two equivalent interpretations. The one developed above has a continuum of independent projects whose realized shock distribution is, by the law of large numbers, identical to the distribution of shocks $F(\rho)$ for an arbitrary project. Alternatively, the projects sponsored by the bank all face the same shock ρ drawn from $F(\rho)$. In this correlated-shocks interpretation, $F(\rho^*)$ is to be interpreted as the probability that all projects continue, rather than as the fraction of surviving projects. In this case, the bank may need no cash with an arbitrary probability (then $F(0) > 0$). The level of λ^k thus stands more generally for the availability of public funds in times of global stress, that is in those states of nature in which the banks need liquidity support.

Capital requirements. Because we want to focus on liquidity shortages in the simplest manner, we assumed that the bank faces no date-0 solvency constraint. We did relax this assumption, along the lines of Farhi-Tirole (2012). The insights remain the same, unsurprisingly given that our focus is on liquidity provision, not on ex-ante solvency.

More than two countries. The analysis is unchanged when the cross-border bank is potentially present in n countries. Specialization in a single country can still arise, actually under the exact same condition as when there are only two countries. In particular, for $\hat{c}(0) = 0$, specialization occurs iff $f(\hat{\beta})\hat{\beta}/F(\hat{\beta}) \geq 1$ (actually equal to 1 –the case of the uniform distribution of shocks– given our simplifying assumption that F is weakly concave). If this condition is violated, then $\sigma^{nc} < 1$, and there is more diversification, the larger the number of countries; intuitively, there are more opportunities and so the cost of not diversifying is higher while the gain from not diversifying (the increased access to financial support) is independent of the number of countries.[23]

Date-1 revenue and date-2 pledgeable income, and prudential supervision. Nothing would be altered if the bank produced income at date 1 and/or pledgeable income at date 2 in case of project continuation. To the extent that the date-2 pledgeable income is bailinable,[24] it plays the same role as a date-1 revenue, except that the revenue is contingent on continuation. The analysis directly carries through, replacing "liquidity shocks" by "net liquidity shocks". We now show how to incorporate date-1 income into the analysis.

Let the bank receive date-1 revenue rq where $r \geq 0$ is the per-unit revenue. In the absence of public liquidity provision, the bank faces cutoff ρ_0^* such that $[\int_0^{\rho_0^*} \rho dF(\rho)]q = rq$ provided that

[23] Let $C^n(q,\sigma) \equiv c(\sigma q)\sigma q + (n-1)c\left(\frac{1-\sigma}{n-1}q\right)\left(\frac{1-\sigma}{n-1}\right)q$, so that $C^2(q,\sigma) = C(q,\sigma)$. It is optimal for the bank to select a specialization σ in a country to solve $\max_{\{q,\sigma\}} F(\hat{\beta}\sigma)bq - C^n(q,\sigma)$.

[24] That is, the corresponding claims can be transferred or sold, letting the firms' contribute to the liquidity support and thereby reducing the public funds commitment. See Dewatripont-Tirole (2019) and Clayton-Schaab (2021) for analyses of optimal bailinability.

the bank is eager enough to continue ($\rho_0^* \leq b$), which we will assume (the bank does not produce excess liquidity by itself). If the bank cannot divert any of the liquidity and under potential public liquidity provision, the home country brings the liquidity shortfall whenever $\rho_0^* < \hat{\beta}\sigma$ and the bank does not need liquidity support if $\rho_0^* \geq \hat{\beta}\sigma$.

Suppose now that, when left unsupervised/monitored, the bank can abscond with the liquidity[25]; say, it can grab ξrq where $\xi \in (0,1]$, where $1-\xi$ is the deadweight loss associated with diverting money. Introduce the possibility that countries can, at a cost and in a non-contractible fashion, take steps to prevent liquidity diversion by the bank (see Appendix C for more details). The equilibrium allocation of supervisory activities is described in part (ii) of the following proposition.

Proposition 4 *(inside liquidity and supervision by the home country).*

(i) There exists a level of inside liquidity r_0 such that for $r < r_0$, then $\sigma = \sigma^{nc}$, where σ^{nc} is the level of specialization in the absence of inside liquidity. For $r > r_0$, then the bank's continuation is self-financed.

(ii) Suppose that the bank, if left unmonitored, can divert cash to its own benefit and that the countries can engage in costly supervision to prevent such diversion. If $\rho_0^* \leq \hat{\beta}\sigma$, then only the home country, which has to cover any increase in the liquidity shortfall, has an incentive to engage in costly supervision. If $\rho_0^* > \hat{\beta}\sigma$, the home country has more incentives to supervise the bank (both because of the larger scale of its operations in the country and because it will supply the liquidity); in equilibrium, either the home country is the only supervisor or the foreign country may join in.

2.5 Asymmetric countries and the bank's choice of home country

With symmetric countries, the bank is indifferent regarding its choice of home country. This is no longer so if we introduce asymmetries.

2.5.1 Asymmetric eagerness to bring liquidity support

Suppose that countries are symmetrical except that $\hat{\beta}^A > \hat{\beta}^B$. That is, the stability of the banking sector is more important to country A ($\beta^A > \beta^B$) or/and country A has more financial muscle than country B ($\lambda^A < \lambda^B$). Then for any degree of specialization $\sigma > \frac{1}{2}$, there is more liquidity support if the bank makes country A its home country:

$$\hat{\beta}^A\sigma > \max\left\{\hat{\beta}^B\sigma, \hat{\beta}^A(1-\sigma)\right\}.$$

This simple result captures the idea that dominance is facilitated by healthy public finances, as the latter allow the country to bring liquidity support in dire straits. Similarly, a country which is eager to maintain activity on its soil is more attractive to the bank.

[25]It is then optimal for the bank to divert the entire liquidity, as any shortfall to cover shocks below $\hat{\beta}\sigma$ will be covered by the home country. Note also that diversion is optimal for the bank whenever ρ_0^* is smaller or even slightly bigger than $\hat{\beta}\sigma$.

Proposition 5 *(asymmetric eagerness to bring liquidity support). Suppose that countries differ only in their resilience ($\lambda^A \neq \lambda^B$) or/and their willingness to keep activity on their soil ($\beta^A \neq \beta^B$). Then the country with the highest index $\hat{\beta}^k\sigma^k$ provides the liquidity. The bank specializes in the highest-$\hat{\beta}^k$ country. The bank is therefore more likely to specialize in country A if (i) country A is more resilient ($\lambda^A < \lambda^B$) and/or (ii) country A is more eager to keep activity on its soil ($\beta^A > \beta^B$).*

2.5.2 Asymmetric shock distributions

Suppose that the bank's distribution of shocks is country contingent (but countries are otherwise symmetrical). Let $F^k(\rho)$ denote the distribution in country k, with density $f^k(\rho)$. Generalizing the previous analysis, we have

$$\frac{1}{\lambda}\frac{\partial(\beta\kappa^k - \lambda T^k)}{\partial T^k} = \frac{\hat{\beta}}{\rho^*}\frac{f^k(\rho^*)q^k}{\sum_m f^m(\rho^*)q^m} - 1,$$

where the cutoff is given by:

$$\sum_m \int_0^{\rho^*} \rho q^m dF^m(\rho) = T.$$

A country's incentive to rescue the bank now depends not only on the relative presence of the bank in the country, but also on the relative densities around the cutoff shock (on the likelihood ratio $f^k(\rho^*)/f^l(\rho^*)$): It may now be the case that liquidity support be brought (solely) by country l even though $\sigma^l < 1/2$. All depends on where in the shock distribution the cutoff lies.

Intuitively, the bank wants to specialize in the country in which the rescue makes a big difference for the country at the margin, i.e. with a comparatively high density of shocks at the margin (i.e., country B if the likelihood ratio satisfies $f^B(\rho^*)/f^A(\rho^*) > 1$ at the cutoff ρ^*). However, inframarginal leakage may invalidate this conclusion, as illustrated in Figure 1: While country B (the high-shocks country) benefits more from 1 unit of extra liquidity at $\rho = \rho^*$, country A benefits more from liquidity provision overall and may actually be the rescuing country. Proposition 6 shows however that it is still the case that a single country supplies liquidity. For tractability, we here specialize to exponential distributions of shocks, with country A being the low-shocks country: For some $\delta > 0$,

$$F^A(\rho) = 1 - e^{-\rho} \text{ on } [0, +\infty) \text{ and } F^B(\rho) = 1 - e^{-(\rho-\delta)} \text{ on } [\delta, +\infty).$$

Letting $\tilde{\sigma}^A \equiv \frac{\sigma^A}{\sigma^A + e^\delta \sigma^B}$ and $\tilde{\sigma}^B \equiv 1 - \tilde{\sigma}^A$ denote the "modified presences", country A benefits more than country B from marginal liquidity when $\sigma^* > \delta$ if and only if $\tilde{\sigma}^A > \tilde{\sigma}^B$ or $\sigma^A > e^\delta \sigma^B$.

Proposition 6 *(differential liquidity shocks). Suppose exponential shock distributions with country A the low-shocks country. Country k's willingness to bring liquidity support at the margin hinges on the product of two leakage-related coefficients: the bank's relative presence in the country, σ^k, and the likelihood ratio $f^k(\rho^*)/f^l(\rho^*)$, which measures the relative fraction of projects that will be supported at the margin in country k. There may be a region where the high-shocks country benefits more from liquidity support at the margin, and yet the low-shocks country supplies the liquidity. More specifically, there exists a unique equilibrium, satisfying:*

- *The low-shocks country, country A, supplies the entire liquidity if either $\hat{\beta}\sigma^B \leq \delta$ (country B would not benefit from providing liquidity alone) or $\tilde{\sigma}^A \geq \tilde{\sigma}^B$ (country A has a higher stake at the margin when $\rho^* > \delta$).*

- *The high-shocks country, country B, supplies the entire liquidity if $\tilde{\sigma}^A < \tilde{\sigma}^B$ and $\hat{\beta} \geq \hat{\beta}^\dagger$ for some $\hat{\beta}^\dagger$.*

- *Otherwise, country A supplies some deterministic liquidity T^A and country B randomizes between not supplying any liquidity and topping up with liquidity T^B to bring the cutoff ρ^* to $\hat{\beta}\tilde{\sigma}^B$.*

The choice of specialization with asymmetric shock distributions is complex in general. One can however obtain a few partial results: a) for low willingnesses to support (technically: for $\hat{\beta} \leq \delta$), the bank specializes fully in country A (that is $\sigma^A = 1$), as shocks in country B will never be covered; b) for high willingnesses to support ($\hat{\beta} \to +\infty$), the bank gets its liquidity from country B and $\tilde{\sigma}^B > 1/2$.

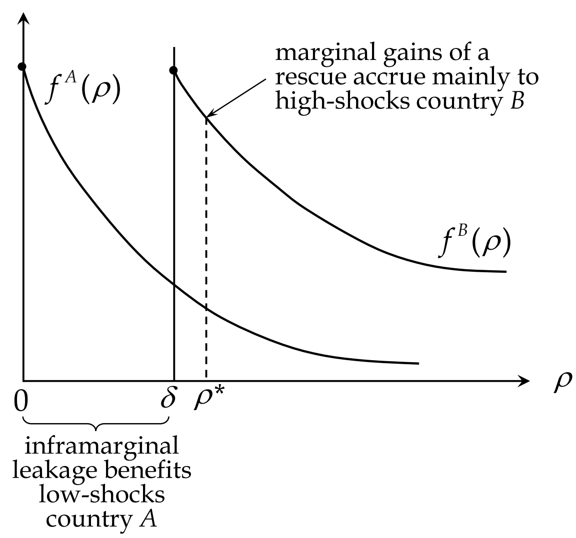

Figure 1: Discrepancy between marginal and inframarginal leakage when shocks are asymmetric (exponential distribution).

2.6 Coasian bargains and their breakdowns

Suppose that the two countries face the same shadow cost λ and bargain efficiently at date 1 (say, they share the gains from trade). The countries jointly provide a socially efficient level of support to the bank, that satisfies

$$\rho^* = \hat{\beta}.$$

Anticipating a Coasian bargain, the bank chooses $\{q, \sigma\}$ so as to solve:

$$\max_{\{q,\sigma\}} F(\hat{\beta})bq - C(q, \sigma).$$

For any size q, the continuation scale is independent of specialization, and so the bank just minimizes cost:

$$\sigma = \frac{1}{2}. \tag{6}$$

Scale q is given by

$$F(\hat{\beta})b = \hat{c}\left(\frac{q}{2}\right). \tag{7}$$

For $c(q^k) = q^k$ for example, $q = F(\hat{\beta})b$. In contrast, in the absence of Coasian bargain, $q = \frac{F(\hat{\beta}\sigma)b}{2[\sigma^2+(1-\sigma)^2]}$. So there is more investment under a Coasian bargain for two reasons: The prospect of a stronger liquidity support ($F(\hat{\beta}) \geq F(\hat{\beta}\sigma)$) and a cost benefit from diversification ($\frac{1}{2} < \sigma^2+(1-\sigma)^2$). More generally, the cost benefit from country diversification implies a higher investment under Coasian bargaining.

Proposition 7 *(higher investment under Coasian bargaining). The bank invests more if it expects a Coasian bargain: It receives a stronger liquidity support; and because this liquidity support does not require country specialization, it enjoys the cost benefit of country diversification.*

Uncertain fund availability at country level. In Appendix E, we study then bank's strategy when ex-ante symmetrical countries face perfectly negatively correlated shocks on the availability of their public funds. Such asymmetries ($\lambda^A \neq \lambda^B$) make Coasian bargains harder to reach as the distressed (high λ) country cannot compensate the richer (low λ) country. Ideally, the two countries should at date 0 sign a swap agreement to equalize their availability of public funds and facilitate the date-1 advent of a Coasian bargain. In the absence of date-0 (and date-1) agreement, we provide conditions for a bank to specialize or diversify. A sufficient condition for diversification is our sustained assumption $f' \leq 0$. In contrast, with sufficiently fat tails, specialization emerges.

Observation: If the failure of Coasian bargaining results from an asymmetry in the countries' capabilities to bring support at date 1 and it is ex ante unknown which country will be unable to bring liquidity support, mutual credit lines give rise to a Pareto improvement.

3 Industrial monetary policy

Liquidity support interventions studied so far have been ex-post (i.e. time-consistent) interventions. We now study whether countries would want to attract banks at date 0 by promising domestic LOLR services. We add a prior stage ("stage −1") at which countries may commit to a level of liquidity support. They do this conditionally (contingent on the absolute or relative presence in the country) or unconditionally. Suppose again a single bank and two countries. In a first step, only country A has cash to bring liquidity: $\lambda^A = \lambda$ and $\lambda^B = +\infty$ (in that sense, country A is an "hegemon"); in the following, $\hat{\beta}$ will therefore stand for $\beta/\lambda^A = \beta/\lambda$. Regardless of any ex-ante commitment, the bank will necessarily specialize in country A, as it can count on no support from country B.

3.1 Unconditional commitment

Let us first assume that the hegemon commits to a level of support $T \geq 0$ that is unconditional on the bank's footprints in the two countries or on the use of the liquidity. If $T < [\int_0^{\hat{\beta}\sigma} \rho dF(\rho)]q$, not topping up the committed liquidity T to the level that would prevail in the absence of a committed line, is not time-consistent; so we assume that the mutually beneficial renegotiation to the higher liquidity support enabling $\rho^* = \hat{\beta}\sigma$ takes place.[26] When facing a promised liquidity support T, the bank therefore solves at date 0:

$$\max_{\{q,\sigma,\rho^*\}} F(\rho^*)bq - C(q,\sigma)$$

s.t.

$$\left[\int_0^{\rho^*} \rho dF(\rho)\right] q \leq \max\left\{T, \left[\int_0^{\hat{\beta}\sigma} \rho dF(\rho)\right] q\right\}$$

Given that the bank's liquidity constraint is always binding, the optimal choice $\{q,\sigma\}$ may exhibit one of two configurations:

Proposition 8 *(unconditional liquidity support). Suppose that the hegemon commits to a liquidity support $T \geq 0$ at date 0. There exists $T_1 \in (0, T^{nc})$ such that*

- *for $T \leq T_1$, the commitment is irrelevant and the bank chooses the time-consistent outcome $\{q^{nc}, \sigma^{nc}\}$,*
- *for $T > T_1$, the bank fully diversifies* ($\sigma = 1/2$).

Proof. Low commitment: Suppose that, in equilibrium, the bank chooses $\{q,\sigma\}$ such that $T \leq \left[\int_0^{\hat{\beta}\sigma} \rho dF(\rho)\right] q$. Then $\rho^* = \hat{\beta}\sigma$ and the corresponding allocation, which therefore maximizes $F(\hat{\beta}\sigma)bq - C(q,\sigma)$, could be equally achieved in the absence of commitment ($T = 0$).

High commitment: Assume, next, that, in equilibrium, $T > \left[\int_0^{\hat{\beta}\sigma} \rho dF(\rho)\right] q$. Let $R(T/q) > \hat{\beta}\sigma$ be defined by

$$\int_0^{R(T/q)} \rho dF(\rho) \equiv \frac{T}{q}.$$

The bank's optimum then solves

$$\max_{\{q,\sigma\}} F(R(T/q))bq - C(q,\sigma)$$

and so

$$\sigma = \frac{1}{2}.$$

[26] In contrast, if T is larger than what sustains $\rho^* = \hat{\beta}\sigma$, renegotiation cannot occur as long as there are no gains from trade (i.e. unless T is extremely high), which never occurs for the ex-ante optimal T.

In words, the country's commitment is either irrelevant (low commitment) or induces a diversification that is unwanted by country A (high commitment). Because the concepts of "low commitment" and "high commitment" are endogenous, let us look at which prevails. The difference of the bank's utility between diversification and specialization is

$$\Delta(T) \equiv \max_{\{q\}} \left\{ F\left(R\left(\frac{T}{q}\right)\right)q - C\left(q, \frac{1}{2}\right)\right\} - \left[F(\hat{\beta}\sigma^{nc})bq^{nc} - C(q^{nc}, \sigma^{nc})\right].$$

Let T^{nc} denote the level of liquidity support in the absence of commitment ($R(\frac{T^{nc}}{q^{nc}}) = \hat{\beta}\sigma^{nc}$). Because $q = q^{nc}$ is an option for the bank when it diversifies

$$\Delta(T^{nc}) \geq C(q^{nc}, \sigma^{nc}) - C\left(q^{nc}, \frac{1}{2}\right) > 0.$$

Because Δ is increasing in T and $\Delta(0) < 0$, there exists $T_1 < T^{nc}$ such that the bank diversifies iff $T \geq T^*$. ■

Proposition 8 shows that an unconditional support either is irrelevant or induces the bank to diversify at $\sigma = 1/2$ rather than specialize at $\sigma^{nc} > 1/2$. For a given size q, the commitment necessarily reduces country A's welfare. So, if for instance the bank has an overall managerial capacity constraint $\bar{q}$ that lies below or exceeds slightly q^{nc}, the unconditional support can only reduce welfare. In the absence of such a capacity constraint (our paradigm so far), a higher commitment T leads to a larger bank size (as shown by a revealed preference argument), and so country A might benefit from the expanded scale even though it captures only half of the global benefit.

Overall, a commitment to an unconditional liquidity support is hindered by the non-appropriation by country A of two externalities, on country B and on the bank: Country B benefits from the diversification and from an increased scale. And, unless country A is able to capture the increase in its rent, the bank gains from the liquidity support policy.[27]

3.2 Conditional commitment

Suppose now that country A can condition the liquidity support T on (a) a presence $\{q^k\}_{k\in\{A,B\}}$ in the two countries and (b) possibly a date-0 transfer $T_0 (\gtrless 0)$. The contract is thus as complete as it can be given the fungibility constraint.

A key question is whether the hegemon can charge the bank at date 0 for the enhanced liquidity support. This in turn hinges on whether the bank has free cash flow to pay for the additional liquidity support (TU: utility is then transferable) or not (NTU: utility is non transferable). At one extreme, the bank had just enough net worth to finance its date-0 investment cost $C(q, \sigma)$ in the absence of industrial monetary policy (IMP); this NTU paradigm is also a good characterization for the situation of credit rationing familiar in corporate finance. At the other extreme, the bank's date-0 endowment is very large, so charging for the additional liquidity is not an issue. Accordingly, we will consider both the cases of "no date-0 transfer" and "date-0 transfer".

[27] Whether an unconditional support can benefit country A hinges on whether the bank is credit constrained (unless its endowment much exceeds the investment cost, country A will not be able to capture much of the bank's extra rent). We will study date-0 transfers in the next subsection.

NTU. The worst-case scenario for IMP corresponds to limited instruments (no date-0 transfer). Appendix G shows that even in this case, the hegemon deviates from the time-consistent solution. Intuitively, increasing the liquidity support T slightly above T^{nc} imposes only a second-order loss on the hegemon and leads to a first-order gain for the bank. However, the hegemon must (a) secure itself some quid-pro-quo; and (b) protect itself from the incentive for diversification induced by the commitment to some level of liquidity support (see Section 3.1). Objective (b) can be achieved through the specification of presences $\{q^k\}_{k\in\{A,B\}}$, while objective (a) results from the bank's willingness to increase q in reaction to enhanced liquidity support (or else from a slight contractual increase in q^A).

TU. Let us consider now (the simpler case of) transferable utility. The hegemon can offer a transfer ($T_0 \gtrless 0$) at date 0. The maximization of hegemon surplus subject to the bank's utility being no less than U^{nc} writes[28]

$$\max_{\{q,\sigma,\rho^*\}} S(\sigma q) + \left[F(\rho^*)[\beta\sigma + \lambda b] - \lambda \int_0^{\rho^*} \rho dF(\rho)\right] q - \lambda C(q,\sigma)$$

yielding, letting $\hat{S} \equiv S/\lambda$,

$$\begin{aligned} \rho^* &= \hat{\beta}\sigma + b && (8)\\ C_\sigma(q,\sigma) &\leq F(\rho^*)\hat{\beta} q + \hat{S}'(\sigma q) q \quad \text{(with equality if } \sigma < 1) && (9)\\ C_q(q,\sigma) &= F(\rho^*)[\hat{\beta}\sigma + b] - \int_0^{\rho^*} \rho dF(\rho) + \hat{S}'(\sigma q)\sigma && (10) \end{aligned}$$

For a given specialization σ, liquidity is supplied in greater amount than in the time consistent case ($\rho^{nc} = \hat{\beta}\sigma^{nc}$) because of the internalization of the bank's benefit from continuation under TU.

Note that the choice of specialization still satisfies $\sigma > 1/2$, and obeys two opposite considerations. First, a higher specialization directly benefits the hegemon. Second, the investment cost is raised by specialization directly at the time-consistent outcome; this second consideration calls for less specialization, combined with a promise of sufficient liquidity support (which itself calls for a commitment not to diversify too much).

Proposition 9 *(Hegemon's exhorbitant duty). The bank specializes in the hegemon because of its resilience, and this regardless of the hegemon's ability to commit to a support policy. The hegemon always provides for more continuation[29] under IMP (but may accept less specialization) than under non commitment:*

$$\rho^* = \hat{\beta}\sigma + \frac{\mu}{\lambda} b > \rho^{nc} = \hat{\beta}\sigma$$

where $\mu = \lambda$ under transferable utility, and $\mu \leq \lambda$ under non-transferable utility.

The optimal policy comes with an "exorbitant duty" (Gourinchas-Rey 2022): The hegemon must provide more liquidity than it would wish ex post in exchange of attracting more investment into the country.

[28]The shadow price of the constraint $U \geq U^{nc}$ is then equal to λ.

[29]So we here compare the cutoffs ρ rather than the liquidity support T, as the scale q need not be the same in both environments.

3.3 Exchange rate appreciation

We now introduce a date-1 exchange rate. We show that exchange rate appreciations are an unavoidable byproduct of successful industrial monetary policies. Exchange rate appreciations, in turn, are a limiting factor for industrial monetary policies by endogenously making them costlier to operate.

Assume that consumers in each country k have identical preferences $E[c_0^{k,A} + c_0^{k,B} + u(c_1^{k,A}, c_1^{k,B}) + c_2^{k,A} + c_2^{k,B}]$, where $c_t^{k,m}$ is the consumption of country k of good produced in country m at date t. Assume that investment in country k requires reinvestment in the goods produced in this country. Denote by ω the date-1 endowment of home goods in country k, which we assume are owned by consumers of country k.[30] Then the exchange rate e of country A at date 1 is the relative price of good B vs. good A.

With Cobb-Douglas preferences $u(c_1^{k,A}, c_1^{k,B}) = 2(c_1^{k,A} c_1^{k,B})^{\frac{1}{2}}$, the date-1 exchange rate is given by equating the demand for good A with the supply of good A net of (lump-sum) taxes levied to finance the bank's continuation in both countries. A specialization in country A increases the demand for date-1 A-goods and appreciates the exchange rate. This appreciation in turn increases country A's opportunity cost of providing date-1 liquidity. Appendix H analyzes the two-way interaction between liquidity support and exchange rate appreciation, both for a time-consistent support policy and for an IMP.

Proposition 10 *(currency appreciation). The country that attracts the most investment (the hegemon) sees its date-1 exchange rate appreciate when it follows its time-consistent support policy, and a fortiori when it engages in IMP. This appreciation makes it more costly to provide liquidity and limits the liquidity support; in particular it dampens the benefit of an industrial monetary policy.*

The introduction of an exchange rate, combined with the exhorbitant duty studied previously, allows us to develop another interpretation of what was called "projects" or "clients" in the model. Prior to 2008, foreigners purchased a substantial amount of American assets and took on dollar liabilities; with the global crisis, they needed dollars to honor their liabilities. The swap lines granted by the Fed to other Central Banks allowed them to overcome the initial dollar shortage. In the framework of the model, one can replace "projects" by "US assets", which foreigners invested in at "date 0" (i.e. prior to 2008), creating a rent (S) for their issuers. At "date 1" (the financial crisis), US financial institutions would have been hurt by a fire sale of these assets. That gave the US a stake (β) in the continuation of activities on their soil (limited sales by foreigners). Furthermore, the US may well have gone beyond what it would have done in a time-consistent manner; its liquidity provision may have reflected the desire to keep its reputation[31] for providing liquidity to the world in times of stress and therefore comfort its position as a hegemon. This obedience to the "exorbitant duty" may be illustrated by the rescue of AIG;[32] this rescue as well as the other bailouts underline the special capability of the

[30] This endowment is non-storable. Also it cannot be pledged at date 0 to create date-0 stores of value (think of ω as date-1 labor income).

[31] In the case of the US, the IMP strategy seems to rely more on reputation than on formal contracts.

[32] The Congressional Oversight Panel (2010) estimated that about $62 billion of TARP money targeted to AIG went to European banks and other foreign institutions.

US to operate large-scale liquidity provision.

3.4 Multiple hegemon wannabes

Suppose now that $\lambda^A = \lambda^B = \lambda$, and so each country has the ability to attract investment on their soil through a commitment to future support. Each country competes à la Nash to attract the bank onto its territory. Assuming transferable utility (TU), country k's IMP offer is a vector $(T_0^k, T^k, q^k, \sigma^k)$, where T_0^k is a (positive or negative) date-0 transfer to the bank, and T^k, the date-1 liquidity support, without loss of generality satisfies the credibility constraint $R(T^k/q^k) \geq \hat{\beta}\sigma^k$. More formally, the timing goes as follows: (1) The two countries select their offers. (2) The bank either picks one of these offers or proceeds without commitment (and then receives utility U^{nc}). Letting the bank's, the home country's and the foreign country's utilities be denoted by U, V^h and V^f:

$$\begin{aligned} U(T_0, T, q, \sigma) &\equiv T_0 + F(R(T/q))bq - C(q, \sigma), \\ V^h(T_0, T, q, \sigma) &\equiv S(\sigma q) + \left[F(R(T/q))\beta\sigma - \lambda \int_0^{R(T/q)} \rho dF(\rho) \right] q - \lambda T_0, \end{aligned}$$

and for the foreign country (whose payoff is determined by the home country's policy (T^*, q^*, σ^*)).

$$V^f(T^*, q^*, \sigma^*) \equiv F(R(T^*/q^*))\beta(1 - \sigma^*)q^*,$$

then a symmetric equilibrium $(T_0^*, T^*, q^*, \sigma^*)$ solves

$$\max_{\{T_0, T, q, \sigma\}} U(T_0, T, q, \sigma)$$

s.t.

$$V^h(T_0, T, q, \sigma) \geq V^f(T^*, q^*, \sigma^*)$$

and

$$U(T_0, T, q, \sigma) \geq U^{nc}.$$

The latter constraint, the bank's participation constraint, is non-binding. Furthermore, the contract between a country and the bank must be bilaterally efficient, and so must satisfy conditions (8) through (10). In the Bertrand-like outcome, each country is indifferent between engaging in IMP with the bank and so being the home country, and passing on this opportunity and being the foreign country.

Proposition 11 *(IMP competition to attract the bank). Suppose that countries are symmetrical, and in particular have the same cost of funds λ . Their competition to attract the bank under TU delivers the same $\{T, q, \sigma\}$ as under an hegemon (where country A had cost of funds λ , and country B was drained of funds). The only difference relates to the date-0 transfer T_0 , which is more favorable to the bank than under an hegemon.*

4 Banking competition

So far, we have looked at a single bank in isolation. Returning to the model of Section 2 ($\lambda^k = \lambda$ for all k, no commitment, and no Coasian bargain), we now investigate the consequences of our analysis for the intensity of banking competition. To illustrate the more general model, consider two cross-border banks, 1 and 2 (indexed by subscripts i, j). At date 0, they compete à la Cournot. Let q_i^k denote the size of bank i in country k. Letting $q^k \equiv \Sigma_i q_i^k$, the unit cost of customer acquisition is $c(q^k)$ in country k, with again $c'(q^k) > 0$ and $q^k c''(q^k) + c'(q^k) > 0$ (which will guarantee that investments are strategic substitutes). The countries face the same shadow cost λ of public funds at date 1.

Let $\sigma_i^k \equiv q_i^k / q_i$ denote the relative presence of bank i in country k, and $\sigma_i \equiv \max_{k\in\{A,B\}} \sigma_i^k$ denote its degree of specialization. As in Section 2 liquidity support is determined by σ_i: The cutoff for bank i is

$$\rho_i^* = \hat{\beta}\sigma_i.$$

4.1 Date-0 competition

Bank i solves:

$$\max_{\{q_i^A, q_i^B\}} \{F(\hat{\beta}(\max_{k\in\{A,B\}}(\frac{q_i^k}{q_i^A + q_i^B}))(q_i^A + q_i^B)b - \sum_{k\in\{A,B\}} c(q_i^k + q_j^k)q_i^k\}.$$

This yields the following first-order condition with respect to q_i^k:

If $0 < q_i^k < q_i^l$ (the LHS of (11) must be weakly negative if $q_i^k = 0$ and weakly positive if $q_i^k = q_i^l$).

$$F(\hat{\beta}(\frac{q_i^l}{q_i^A + q_i^B}))b - c(q^k) - c'(q^k)q_i^k - f(\hat{\beta}(\frac{q_i^l}{q_i^A + q_i^B}))\frac{q_i^l}{q_i^A + q_i^B}\hat{\beta}b = 0. \tag{11}$$

If $q_i^k > q_i^l \geq 0$ (the LHS of (12) is weakly negative if $q_i^k = q_i^l$)

$$F(\hat{\beta}(\frac{q_i^k}{q_i^A + q_i^B}))b - c(q^k) - c'(q^k)q_i^k + f(\hat{\beta}(\frac{q_i^k}{q_i^A + q_i^B}))\frac{q_i^l}{q_i^A + q_i^B}\hat{\beta}b = 0. \tag{12}$$

Conditions (11) and (12) are the standard Cournot conditions except for the last terms on the RHSs (those proportional to the density f), which reflect the concern about receiving liquidity support. An increased scale in the low-presence country dilutes the benefit for the high-presence country to supply liquidity, increasing the cost for the bank of diversifying across countries. And conversely for the choice of scale in the high-presence country.

Thus a symmetric, balanced outcome, in which banks invest equally in each country cannot exist: The LHS of (11) would be non-negative and the LHS of (12) would be non-positive at $q_i^k = q_i^l$, which is impossible.

It can also be shown that, given that countries are symmetrical, banks do not pick the same home country. The reason for this is that entry into a relatively unserved market is more profitable than entry in a competitive market. We will therefore look for an equilibrium in which the banks both specialize, but in different countries.

4.2 Equilibrium analysis

We first look for an *partial-specialization equilibrium* in which the two banks invest in both countries and choose to specialize in different countries, with the same degree of specialization $\sigma \in (\frac{1}{2}, 1)$.

It is instructive to consider a bank's choice of specialization in say country k, given a fixed size q_i:

$$\max_{\{\sigma_i\}} [F(\hat{\beta}\sigma_i)b - c(\sigma_i q_i + q_j^k)\sigma_i - c((1-\sigma_i)q_i + q_j^l)(1-\sigma_i)]q_i.$$

In such a "symmetric"[33] equilibrium, for which we omit bank and country indices, the first-order condition with respect to σ_i yields:

$$f(\hat{\beta}\sigma)b\hat{\beta} = c'(q)q(2\sigma - 1).$$

Let q denote the total volume in a given country. Conditions (11) and (12) can be rewritten as

$$\begin{aligned} F(\hat{\beta}\sigma)b - c(q) - c'(q)(1-\sigma)q - f(\hat{\beta}\sigma)\sigma\hat{\beta}b &= 0 \\ F(\hat{\beta}\sigma)b - c(q) - c'(q)\sigma q + f(\hat{\beta}\sigma)(1-\sigma)\hat{\beta}b &= 0. \end{aligned}$$

Adding and subtracting these two conditions yields the condition obtained above together with a new condition:

$$f(\hat{\beta}\sigma)b\hat{\beta} = c'(q)q(2\sigma - 1) \tag{13}$$

$$F(\hat{\beta}\sigma)b = c(q) + c'(q)q[\sigma^2 + (1-\sigma)^2]. \tag{14}$$

For $\sigma > \frac{1}{2}$, $\sigma^2 + (1-\sigma)^2 > \frac{1}{2}$: As earlier, the marginal cost of investment is higher under specialization than when banks diversify their portfolio, because an increase in the bank's size has a larger inframarginal effect on the cost of acquisition in the market in which the bank is more present.[34]

This in turn implies that

$$F(\hat{\beta}\sigma)b = \left(\sigma - \frac{1}{2}\right)f(\hat{\beta}\sigma)\hat{\beta}b + H\left(\frac{f(\hat{\beta}\sigma)\hat{\beta}b}{2\left(\sigma - \frac{1}{2}\right)}\right) \tag{15}$$

where H is an increasing function defined implicitly by $H(X) = c(q) + c'(q)\frac{q}{2}$ when $X = c'(q)q$.[35]

[33] It is symmetric in magnitudes, but, as we have seen, banks specialize in different countries. A sufficient condition for the second-order condition with respect to σ_i to be satisfied is $f' \leq 0$, which we already assumed. The objective function is also concave in q.

[34] Note also that a small diversification has only a second-order effect on the marginal cost: $\left.\frac{d(\sigma^2+(1-\sigma)^2)}{d\sigma}\right|_{\sigma=1/2} = 0$. So the liquidity effect necessarily dominates starting from full diversification.

[35] For instance for $c(q) = q$ (see footnote 8), $H(X) = \frac{3}{2}X$; and more generally $H' \geq 1$ if and only if $c' \geq c''q$.

A *full-specialization equilibrium* in which banks both stay in their home country and do not attempt to invade the other's territory has banks operate at scale q^M (where "M" stands for (local) monopoly) and requires that the following condition be satisfied:

$$\frac{f(\hat{\beta})\hat{\beta}}{F(\hat{\beta})} \geq \frac{c'(q^M)q^M}{c'(q^M)q^M + c(q^M)}.$$

The RHS of this inequality decreases with the ratio of the average cost $c(q)/q$ over marginal cost $c'(q)$. Note that the condition for full specialization with multiple banks is weaker than that $(f(\hat{\beta})\hat{\beta}/F(\hat{\beta}) \geq 1)$ for a single bank: There is less incentive for bank i to diversify if bank j is strong in bank i's low-presence country.[36]

It is again interesting to investigate the consequences of a shift θ in the distribution (the distribution writes $F(\rho - \theta)$). A higher θ can be interpreted as more turbulent times. With this interpretation, more turbulent times lead to banking renationalization (a higher specialization).

To show that competition is stronger under a Coasian bargain, note that bank's size q in the absence of Coasian bargain is given by

$$c(q) + [\sigma^2 + (1-\sigma)^2]qc'(q) = F(\hat{\beta}\sigma)b.$$

This is to be compared with a similar equation in the Coasian bargain case:

$$c(q) + \frac{1}{2}qc'(q) = F(\hat{\beta})b.$$

So overall, there is more banking activity under a Coasian bargain for two reasons: A stronger liquidity support and a smaller market power.

We summarize the conclusions in the following proposition (the extension to a more general oligopoly can be found in Appendix I):

Proposition 12 *(banking competition). Consider an ex-ante symmetric oligopoly of I banks choosing their footprints in K symmetric countries. In the absence of commitment,*

(i) Banks specialize and they do so in different countries.

(ii) Full specialization obtains for a wider range of parameters than in the single-bank case. More generally, an increase in banking competition (as captured by the number of banks divided by the number of countries) makes it more likely that the banks fully specialize.

[36]Consider a power function acquisition cost $c(q) = q^\eta$ with $\eta > 0$ (which includes the linear case $c(q) = q$). Then $H(X) = (\frac{1}{2} + \frac{1}{\eta})X$ and (15) can be rewritten as

$$\frac{f(\beta\sigma)\beta\sigma}{F(\beta\sigma)} = \frac{\sigma^2 - (\sigma/2)}{\sigma^2 - \sigma + \frac{1}{2}(1 + \frac{1}{\eta})}. \tag{16}$$

The LHS of (16) is non-decreasing in σ, while the RHS is (always) decreasing in σ on $[1/2, 1]$. Thus condition (16) has a unique solution. And this solution is a corner solution of full specialization if and only if $\frac{f(\hat{\beta})\hat{\beta}}{F(\hat{\beta})} \geq \frac{\eta}{1+\eta}$.

(iii) More turbulent times lead to banking renationalization.

(iv) A Coasian bargain increases competition by reducing specialization.

5 Ringfencing

Our interest here lies with the implications of supervision on the allocation of liquidity within the cross-border bank. Assume that $\lambda^k = \lambda$ for both countries. We return to the focus on a single bank (for expositional simplicity only). We caricature reality by assuming that the home country can retain more liquidity in the country than under fungibility:

Assumption (regulation and ringfencing). When the home country k regulates the bank (case on which we focus for expositional convenience), the fraction of its total liquidity T that is used by the bank in country k is equal to $s^k T$, where $s^k \equiv \sigma^k + \alpha(1-\sigma^k)$. Similarly, we define $s^l \equiv (1-\alpha)\sigma^l$ as the fraction of the liquidity that is employed in the bank's foreign country.

The parameter α in $[0,1]$ is the ringfencing parameter; the case $\alpha = 0$ corresponds to perfect fungibility, the case $\alpha = 1$ describes perfect ringfencing.[37] Presumably α is larger in case of a branch than for a subsidiary.[38]

Suppose that the home country k (and only country k) brings liquidity to the bank; its ex-post utility is then $\beta F(\rho^k)q^k - \lambda T^k$, where $[\int_0^{\rho^k} \rho dF(\rho)]q^k = s^k T$. The threshold ρ^k in country k is then given by:

$$\rho^k = \hat{\beta} s^k.$$

The cutoff in the host country is then $\mu^l \leq \rho^k$ where

$$\frac{s^k}{s^l} = \frac{\int_0^{\rho^k} \rho dF(\rho)}{\int_0^{\mu^l} \rho dF(\rho)} \frac{\sigma^k}{\sigma^l} \Longleftrightarrow 1 + \frac{\alpha}{(1-\alpha)\sigma^k} = \frac{\int_0^{\rho^k} \rho dF(\rho)}{\int_0^{\mu^l} \rho dF(\rho)}.$$

Because projects in the branch's host country l are sacrificed, that would cost little to rescue, we must check that country l does not want to supply liquidity as well. Only country k brings liquidity if and only if $\mu^l > \hat{\beta} s^l = \hat{\beta}(1-\alpha)\sigma^l$, or

$$1 + \frac{\alpha}{(1-\alpha)\sigma^k} \leq \frac{\int_0^{\hat{\beta} s^k} \rho dF(\rho)}{\int_0^{\hat{\beta} s^l} \rho dF(\rho)}. \tag{17}$$

As ringfencing becomes more potent (α increases), the LHS of this inequality (which is equal $\left[s^k/\sigma^k\right]/\left[s^l/\sigma^l\right]$) increases, suggesting that the increased leakage makes country l less eager to

[37] This formulation of ringfencing is only one of several assumptions with similar consequences that we could entertain. For example, the ringfencing parameter could apply only to the transfer made by country k and to revenues earned in country k.

[38] Suppose that the bank specializes in country A and is supervised by that country (as we argued, it is efficient to have supervision by the country that will bring liquidity). Then, the cash flow located in country B can more easily be repatriated if the bank's presence there takes the form of a branch.

bring liquidity; however the RHS also increases with α, as few projects continue in country l and so the marginal cost of rescuing projects in that country is small.

Example (power law distribution). Let $F(\rho) = \rho^{\upsilon}$ on $[0,1]$ with $\upsilon > 0$. Inequality (17) then writes (assuming interior solutions for the cutoffs):

$$\left(\frac{(1-\alpha)\sigma^l}{\sigma^k + \alpha\sigma^l}\right)^{\upsilon+1}\left[1 + \frac{\alpha}{(1-\alpha)\sigma^k}\right] \leq 1.$$

This inequality is satisfied for any $\alpha \in [0,1]$ provided that $\sigma^k \geq 1/2$.

Assume that only country k (the home country, with regulates the bank) brings liquidity -as is indeed consistent with equilibrium for the power law distribution.

Observation. Ringfencing has two opposite consequences: First, it implies a *misallocation of the bank's resources* among the projects in the two countries; valuable projects are stopped in the host-country/cash-poor establishment (where "poor" is relative to presence), that would have been pursued had they been located in the home-country/cash-rich one. Second, ringfencing *alters liquidity support*; it unambiguously increases liquidity support if regulation takes place in the country with the highest bank presence.[39]

6 Related literature

This paper connects to numerous literatures.

Dominant currency paradigms. There is a large literature on dominant currencies, motivated inter alia by the observation that the dollar is still dominant today in international bond issuance,[40] in the currency denomination of invoices (dollar invoicing is much larger than the share of the US in foreign trade[41]), and in the choice of foreign exchange reserves. A strong, large and resilient economy are oft-invoked factors that might be conducive to such dominance (as is the case for our theory, in which the country's ability to supply liquidity in difficult times is crucial for becoming an hegemon). Complementarities are also often emphasized: between currency used for working capital and currency used for invoicing sales in Bahaj-Reis (2022), strategic complementarities in pricing in Gopinath et al (2020). This paper abstracts from the important nominal issues (the section on exchange rates indeed focused on real exchange rates) and looks at policies that are directly targeted at making the country attractive for banking investment.[42]

Farhi et al (2011) focus on the role of stores of value and argue that the United States may be facing a modern version of Triffin's dilemma, with a growing demand for U.S. issued safe assets

[39]By contrast, it would tend to reduce liquidity support if regulation takes place in the low-presence country, unless the establishments in the two countries have similar sizes.

[40]Except for green bonds, for which the Eurozone fares well.

[41]E.g. Amiti et al (2022).

[42]There is a debate as to whether network externalities are sufficiently strong (and the cost of obtaining protection from future exchange rate changes sufficiently high) so as to predict that international currency status is a natural monopoly: see Eichengreen et al (2018) and the literature reviewed next.

but a limited fiscal capacity; they discuss ways to increase the global supply of safe assets by pooling resources, using swap lines more systematically or encouraging the emergence of alternative safe assets from large and fiscally sound economies. Farhi and Maggiori (2018) analyze the international demand for reserve assets and argue that a multipolar world may be more unstable than one with a Hegemon. In their model, a reserve country issues a "safe" bond, for which there is a demand by risk-averse investors in the rest of the world. The country may later on default at a cost, possibly destroying the reserve value of the bond. A higher level of borrowing triggers a higher risk of (endogenous) default: this is Triffin's dilemma. A monopoly issuer may produce too much of the reserve asset: it tends to produce too little because of the standard monopoly distortion, but also may produce too much of the reserve asset in the absence of commitment. More generally, Gourinchas and Rey (2022) argue that the exorbitant privilege of a country that lies at the center of the international monetary system comes with an exorbitant duty, and document that the US provides insurance to the rest of the world, especially in times of global stress.[43]

Regulatory competition. The 1988 Basel Accord aimed at avoiding a regulatory race to the bottom and establish a level-playing field in terms of capital requirements. Morrison and White (2009) revisit the costs and benefits of a level-playing field. They assume that regulators in different countries are differentially apt at curbing bank risk taking. An inexperienced regulator must offset his lack of competency by asking for higher capital charges, even for healthy banks which would be safe even with lower capital requirements. When banks are mobile, talented-regulator countries exert negative externalities on untalented-regulator ones: Healthy banks want to migrate to the former (non-insured depositors still trust the banks located there despite low capital requirements), leaving an adversely selected sample to the latter. While a level-playing field eliminates this cherry-picking externality, it also penalizes countries with a talented regulator, as capital requirements must adjust to the lowest common denominator (i.e. high capital requirements, not the lowest common denominator envisioned in the Basel debate). Overall, the level-playing field has ambiguous welfare effects.[44]

Clayton and Schaab (2022) analyse regulatory externalities in a world of multinational banks, each with a domestic base and regulator. Multinational banks exert fire sale externalities in their various countries of operation. Clayton and Schaab show that countries' setting of quantity regulations, such as solvency and liquidity requirements, do not achieve the coordinated optimum, but that price-based regulation does as individual regulators adopt a Pigovian approach that directly addresses the externality at stake.

Countries' lack of internalization of externalities of cross-border banks led Schoenmaker (2013) to suggest the existence of a "financial trilemma"; any two of three following three objectives

[43]An analogy can be drawn between a commitment to supply liquidity and the traditional commitment by suppliers to supply an add-on at a low price in the future (e.g. Shepard 1987, Farrell-Gallini 1988).

[44]While Morrison and White study regulatory competition among countries, in Carletti et al (2016)'s principal-agent problem, a centralized supervisor (e.g., the ECB) has full control rights over banks, but relies on local supervisors to collect the information necessary to act. Local supervisors are assumed to be less inclined to intervene relative to a centralized supervisor. Because the local supervisors do not control the use of information under a centralized structure, they collect less information than in a decentralized structure in which they would have the control rights. The centralization of supervision then has an ambiguous welfare impact despite the improvement in the objective function of the decision-maker.

can be combined, but one has to give: (1) a stable financial system; (2) international banking; (3) national financial policies for supervision and resolution.

Our paper also connects to the broader literature on the impediments of the free flow of capital. While Feldstein-Horioka (1980) and the contributions that followed it stressed impediments originating with the government or a mere cost of doing business abroad (taxes, FX and regulatory risks, or transaction costs), we emphasize a novel channel that originates with the banks' desire to count on support in times of stress; relatedly, the government can create impediments through new routes (IMP, ring fencing).

Competition in banking. In a closed economy framework, the literature has highlighted other industrial organization trade-offs than those emphasized here. Most prominent here is the debate (reviewed in Vives 2011) concerning a possible trade-off between competition and financial stability. The standard "charter value" argument states that a bank has less incentive to take the risk of failure if it expects to be profitable in the future; a reduction in competition therefore reduces risk. This charter value argument suggests either adjusting regulation (more supervision, higher capital and liquidity requirements) when the bank is in a more competitive environment or loosening the competition policy rules. There are other effects too: First, high banking concentration may make the banks too-big-to-fail, inducing them to take on more risk. Second, for wholesale deposits, the higher interest rates generated by competition may exacerbate bank runs. We refer to Vives' paper for the policy implications of these theories.

Strategic choice of balance sheet. The paper belongs to the line of contributions emphasizing how public funds can be brought into play by banking choices.[45] These choices may be a shortage of liquid assets or an excess of short-term debt, creating the prospect of bankruptcy or fire sales in case of a subsequent macroeconomic shock. It may also involve an excessive investment in the domestic sovereign bond, generating a doom loop. Our paper differs from the literature in that it shows how a home bias emerges endogenously from the banks' desire to garner liquidity support in distress and looks at the consequences of this choice for industrial monetary policy. The paper shows that another and so far neglected negative side effect of the too-big-to-fail conundrum is that it also perpetuates home bias.

Branches vs. subsidiaries. Section 5 relates to the large law and economics literature on the difference of regulatory and governance environments for subsidiaries and branches.[46] On the institutional front, a branch in the host country exposes the parent bank to liabilities in case of branch distress and therefore to higher[47] risk than a subsidiary, which has its own capital, board of directors and for which limited liability protects the parent bank. Modes of supervision also differ (the following is only the broad picture, as practice in the matter is somewhat heterogeneous). In all cases, the home supervisor supervises the entire banking group, including the affiliate (whether a subsidiary or a bank); the supervisor can limit the bank's range of

[45] Examples of papers in this literature are Clayton-Schaab (2021, 2022), Farhi-Tirole (2012, 2018, 2021). Early papers in the literature on fire sales externalities include Shleifer-Vishny (1992), Allen-Gale (1994), Lorenzoni (2008), Stein (2012) and Kurlat (2016).

[46] Useful reviews are provided in Danisewicz et al (2017), Fiechter et al (2011) and Schoenmaker (2013).

[47] Of course the parent bank has "its name on the door" and so may feel compelled to rescue its subsidiary despite the absence of legal obligation to do so. This reputation risk makes the distinction not as strong as it might appear.

activities and locations.[48] While the branch must obey the host country's regulations, it is most often not supervised by that country.[49] A subsidiary is by contrast supervised by both countries (indeed the host country is the lead supervisor for the locally incorporated subsidiary). There is a consensus that, whether for governance or regulatory reasons, the bank can more easily relocate liquidity across borders in the case of a branch. And subsidiaries require more capital and liquidity than a branch.[50]

The branch vs. subsidiary choice is guided by various factors, some unrelated to the analysis of this paper. Differentials between corporate tax rates making a branch particularly appealing in a high-tax country is an obvious one. Another is the affiliate's source of funding –retail vs. wholesale-; retail deposit funding makes the subsidiary regime more likely, if only because the host country- which then will be in charge of deposit insurance- will often demand to supervise the affiliate.

The theoretical literature on the choice between the two is scant. Dell'Ariccia and Marquez (2010) show that this choice should be influenced by the type of host country risk; in case of (uninsurable) economic shock, the limited parent-affiliate liability afforded by the subsidiary structure pleads for the latter as it limits the propagation of shocks; by contrast the branch structure allows faster repatriation and protects the bank against expropriation by the host country.[51] Dell'Ariccia and Marquez predict that subsidiaries will take on more risk and will be larger. Cerutti et al (2007) find empirical evidence in support of this theory.

Transmission of shocks through a cross-border banking system. Beyond the general literature on the cross-country propagation of shocks through the financial system (e.g. Giannetti-Laeven 2012), the banking literature points at a stronger propagation of shocks in the home country to the host country when parent banks operate through branches in the host country and they are short of tier-1 capital (Danisewicz et al 2017).

Contribution games. Liquidity support granted to a cross-border bank is in part a public good. As such, the paper is related to the sizeable literature on the non-cooperative provision of public goods. Within that literature, the most closely related paper is Compte-Jehiel (2003), which shows that an exogenous asymmetry in payoffs can substantially alleviate the free-riding

[48]See Fiechter et al (2011).

[49]Note, though, that the host country may have responsibility for supervising the branch's liquidity position, as is the case within the European Union or under the Basel Concordat.

[50]They also facilitate the writing of living wills. Cross-border bank resolutions are known to be particularly tricky. Bolton and Oehmke (2019) look at the role of bailinable securities (loss-absorbing capital) to prevent bankruptcy in a multinational banking context (our model by contrast focuses on public liquidity support once these securities have been bailed in; adding bailinable securities- e.g. along the lines of Clayton-Schaab (2021) and Dewatripont-Tirole (2019)- to our model would be doable provided we added date-2 pledgeable income). A single point of entry for bank resolution is efficient when countries can commit to cooperate at the crisis stage (the equivalent of a date-1 Coasian bargain in our model). Bolton and Oehmke interpret this a supranational regulation. Multiple points of entry by contrast may be advantageous when no such commitment is feasible and resolution will involve large asymmetries in transfers. Calzolari and Loranth (2011) study the non-cooperative decisions of multiple regulators of a multinational bank. A bank at date 0 invests in projects in both countries using local deposits. The regulator (regulators in the case of a subsidiary) decide at date 1 whether to liquidate the bank's country operations/project or let it continue, on the basis of a signal on the likely date-2 return. Regulators stand for the interests of depositors. The paper stresses the externalities in liquidation decisions.

[51]Through forced holdings of government bonds, forced lending to government-connected borrowers, capital controls, etc.

problem by "designating" a natural provider of the public good. While the applications differ, a similar effect is at work here. Compte and Jehiel do not investigate the consequences for the endogenous emergence of asymmetries.

7 Summary and alleys for future research

This paper is a first foray into the two-way interaction between banks' investment and location strategies and countries' willingness and ability to support their financial sector in dire straits. It predicts that, as the conventional wisdom has it, global banks are global in life and national in death. This gives them an incentive to have a strong presence in one country when international diversification would be cheaper. Such country specialization is more extensive (a) when banking competition is stronger, (b) when times are turbulent, and (c) when international risk sharing (say, through swap lines) is weaker.

We investigated three determinants of a high presence of financial activity in a country: (a) room for fiscal and monetary manoeuver, (b) high eagerness to rescue banks, and (c) the distribution of shocks. Although these determinants were studied in the context of time-consistent policies, they also have a flavor of IMP to the extent that they are themselves in part endogenous; for instance, a country may become welcoming of financial institutions by building resilient public finances or by making its polity less hostile to finance. IMP can alternatively build on an "exorbitant duty", namely an explicit promise for support beyond the time-consistent level. When unconditional, such commitments often backfire as they encourage a lower relative presence in the country. When conditional however, they always benefit the home country through either a higher presence or- when feasible- an ex-ante payment by the bank for the liquidity commitment. Except perhaps for the equilibrium payment, the IMP allocation is the same whether there is a hegemon or competition among hegemon wannabes. Finally, an IMP comes with an exchange rate appreciation, a limiting factor for the policy.

A number of extensions seem desirable. In particular, while liquidity provision will always be part of the broader picture due to the limitations on alternative instruments, its interaction with these instruments is worth studying. One, directly in line with the model, would be a relaxation of prudential standards, which would increase the risk of resort to public funds. Conversely, the regulatory trend towards more bailinability should indirectly benefit cross-border diversification. Another is the issuance of government bonds that can serve as cheap stores of value as long as they remain safe.[52] In particular, American sovereign bonds are a handy instruments to hedge risk in a world in which many investments and transactions are in dollars. But the bond-issuing country then is faced with Triffin's dilemma. Finally, countries can try to attract activity onto their soil by subsidizing investment. This third alternative also faces limits in its effectiveness. As the literature has shown, capital subsidies do not obliterate the need for liquidity support.

[52] As in He et al (2019).

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Appendix

A Proof of Proposition 2 (benefit from being the home country)

(i) Substituting $\sigma = 1/2$, one can see that $V^h - V^f = -\lambda\left[\int_0^{\hat{\beta}/2} \rho dF(\rho)\right]q < 0$.

(ii) Ignoring the positive rent differential term $S(\sigma q) - S((1-\sigma)q)$, a sufficient condition for the home country to be better off is:

$$A \equiv F(\hat{\beta}\sigma)\beta(2\sigma - 1) - \lambda \int_0^{\hat{\beta}\sigma} \rho dF(\rho) > 0 \;\Rightarrow\; V^h > V^f.$$

From the concavity of $F(\cdot)$:

$$\frac{\int_0^{\hat{\beta}\sigma} \rho dF(\rho)}{F(\hat{\beta}\sigma)} = \mathbb{E}[\rho \mid \rho \leq \hat{\beta}\sigma] \leq \frac{\hat{\beta}\sigma}{2} \Longrightarrow A \geq F(\hat{\beta}\sigma)\beta(\frac{3}{2}\sigma - 1).$$

(iii) Since $F(\cdot)$ is concave, the elasticity is less than or equal to 1. Thus, $\partial A/\partial\sigma \geq 0$. Therefore,

$$\frac{\partial}{\partial\sigma}(V^h - V^f) = q\left[\frac{\partial A}{\partial\sigma} + S'(\sigma q) + S'((1-\sigma)q)\right] > 0.$$

■

B Proof of Proposition 3 (capital controls)

(i) Suppose that $V^h \geq V^f$. First, note that if country B imposes a capital control in $[q^A, q^B)$, the roles are reversed and so country A becomes the home country with investment q^B and country B becomes the foreign country with investment q^A; because $V^h \geq V^f$, country B does not gain from doing this. Country B choosing a capital control in $[0, q^A)$ inflicts an even worse loss on country B, using the forthcoming characterization showing that the foreign country does not benefit from imposing a capital control (except to prevent role switching).

So, let the binding constraint be $q^A \leq \bar{q}^A$. When it is binding ($q^A = \bar{q}^A$), the bank solves over q^B or equivalently $\sigma = q^B/(q^B + q^A)$:

$$\max_{\sigma}\left[F(\hat{\beta}\sigma)\frac{q^A}{1-\sigma}\right]b - c\left(\frac{\sigma q^A}{1-\sigma}\right)\frac{\sigma q^A}{1-\sigma} + c(q^A)q^A,$$

which results in:

$$\left[(1-\sigma)\hat{\beta}f(\hat{\beta}\sigma) + F(\hat{\beta}\sigma)\right]b = \hat{c}\left(\frac{\sigma q^A}{1-\sigma}\right). \tag{A1}$$

From here one can see that the left-hand side is weakly decreasing in σ (as $f' \leq 0$), while $\frac{1}{1-\sigma}$ is increasing in σ. Therefore, σq^A decreases as σ increases, which means that the elasticity of σ with respect to q^A, is greater than -1:

$$\frac{d}{d\sigma}(\sigma q^A) = q^A + \sigma\frac{dq^A}{d\sigma} \leq 0 \Longrightarrow \frac{q^A}{\sigma}\frac{d\sigma}{dq^A} \geq -1.$$

On the other hand, the utility of country A is:

$$U^A(\sigma, q^A) = F(\hat{\beta}\sigma)\beta q^A + S(q^A),$$

and so

$$\begin{aligned}\frac{d}{dq^A}U^A &= \frac{d\sigma}{dq^A}\hat{\beta}f(\hat{\beta}\sigma)\beta q^A + F(\hat{\beta}\sigma)\beta + S'(q^A)\\ &= F(\hat{\beta}\sigma)\beta\left[1 + \frac{q^A}{\sigma}\frac{d\sigma}{dq^A}\cdot\frac{\hat{\beta}\sigma f(\hat{\beta}\sigma)}{F(\hat{\beta}\sigma)}\right] + S'(q^A) \geq 0.\end{aligned}$$

Increasing q^A benefits directly country A, but it might reduce the size of the bailout granted by country B. This result nonetheless show that the direct effect dominates.

(ii) Suppose now that $V^h < V^f$. Provided that country B imposes no capital control, country A incurs no cost in setting $\bar{q}^A = q^A = (1-\sigma^{nc})q^{nc}$. Can country B make more than V^h? If it sets control $\bar{q}^B \in [q^A, q^B)$, then it remains the home country, and, from the envelope theorem, reduces its welfare. From the characterization in (i), the best deviation of B in $[0, q^A)$ is q^A (actually $q^A - \varepsilon$, to become the foreign country). This yields B utility

$$V^{ff} \equiv F(\hat{\beta}/2)\beta q^A + S(q^A) = V^f - \Big[F(\hat{\beta}\sigma^{nc}) - F(\hat{\beta}/2)\Big]\beta q^A.$$

So, if $V^{ff} \leq V^h$, country B does not gain from becoming the foreign country either. ■

C Proof of Proposition 4 (supervision by the home country)

(i) First, fix the degree of specialization σ. Two cases are then possible:

1. *Low revenue.* If $\rho_0^* \leq \hat{\beta}\sigma$, the continuation scale is unchanged (at $F(\hat{\beta}\sigma)q^k$ in country k) relative to the $r = 0$ benchmark of Proposition 1: The home country provides additional liquidity $[\int_0^{\hat{\beta}\sigma} \rho dF(\rho) - r]q$, conditional on the bank's revenue being reinvested. Note that the date-1 revenue does not benefit the bank.[53] Rather, it serves to reduce the public outlay.

2. *High revenue.* If $\rho_0^* > \hat{\beta}\sigma$ (which requires $\rho_0^* > \hat{\beta}/2$), there is no public liquidity provision, and also no specialization as specialization is costly for the bank and motivated only by the desire to garner liquidity support.

Next, consider the choice of specialization. If $\rho_0^* \leq \beta/2$, then $\sigma = \sigma^{nc}$, while if $\rho_0^* \geq \beta$, then $\sigma = 1/2$. More generally, either the bank sets σ high enough that is resorts to public liquidity ($\rho_0^* < \hat{\beta}\sigma$), and then the optimal solution does not depend on the unit revenue r; as we noted, this revenue does not benefit the bank. The specialization is then the same, σ^{nc}, as when $r = 0$. Or $\rho_0^* \geq \hat{\beta}\sigma$, so continuation is entirely self-financed. Necessarily $\sigma = 1/2$ as diversification minimizes the cost of acquisition. Because the utility $\max_q F(\rho_0^*(r))bq - C(q, \frac{1}{2})$ is increasing in r, there exists a cut-off r such that the bank does without public liquidity iff $r > r_0$.

(ii) This analysis suggests that (a) were the date-1 revenue subject to moral hazard, careful monitoring of activities that determine this revenue should be undertaken; and (b) the natural supervisor is the country with the largest presence, as this country will end up footing the bill. Indeed, when $\rho_0^* \leq \hat{\beta}\sigma$, only the home country has an incentive to supervise as the continuation scale does not depend on whether the bank absconds with the liquidity or not. When $\rho_0^* > \hat{\beta}\sigma$ however, the bank might reduce its continuation scale by diverting some of the liquidity, and so the two countries might monitor simultaneously. Suppose however the following monitoring technology: In the absence of monitoring, the bank can consume the date-1 cash (as a private benefit) before the country can harness it as a contribution toward the desired liquidity provision. At cost $\psi(z)q$, (where $\psi' > 0$, $\psi'' \geq 0$), a prudential supervisor can prevent this diversion of money with probability z. To rule out supervision by the two countries, let us

[53] This would not be the case if either the bank had some bargaining power or there were states of nature without liquidity needs (or with minor ones).

further assume that if both countries monitor, then the probability of preventing cash diversion is $z = \max\{z^A, z^B\}$; the idea is that monitoring involves going first for the easy sources of diversion. [We could of course assume that a fresh pair of eyes per se improves monitoring, but in practice having multiple supervisors need not be the most efficient approach.] Then only the high-presence country monitors.

■

D Proof of Proposition 6 (asymmetric distributions of shocks)

Let $\tilde{\sigma}^A \equiv \frac{\sigma^A}{\sigma^A + e^\delta \sigma^B}$ and $\tilde{\sigma}^B \equiv 1 - \tilde{\sigma}^A$. The variables $\tilde{\sigma}_A$ and $\tilde{\sigma}_B$ are the density-adjusted shares in countries A and B. For exponential distributions, the FOC are:

$$\frac{1}{\lambda}\frac{\partial}{\partial T^A}(\beta \kappa^A - \lambda T^A) = \frac{\hat{\beta}\tilde{\sigma}^A}{\rho^*} - 1$$

and

$$\frac{1}{\lambda}\frac{\partial}{\partial T^B}(\beta \kappa^B - \lambda T^B) = \frac{\hat{\beta}\tilde{\sigma}^B}{\rho^*} - 1 \quad \text{(for } \rho^* \geq \delta\text{; } = -1 \text{ for } \rho^* < \delta\text{)}.$$

As anticipated and depicted in Figure 1, these first-order conditions imply that at the margin the "bang-for-the-buck" is higher in the high-shocks country, country B, whenever $\rho^* \geq \delta$.[54] But this ignores the individual rationality constraint for country B; suppose country B (and not country A) supplies the liquidity. Then country B must prefer doing so rather than not supplying any liquidity:

$$\beta F^B(\rho^*)\sigma^B q \geq \lambda\left[\sum_k \sigma^k \int_0^{\rho^*} \rho dF^k(\rho)\right] q \tag{A2}$$

where $\rho^* \equiv \hat{\beta}\tilde{\sigma}^B$. That (A2) need not be satisfied is also illustrated in Figure 1. There, $\hat{\beta}$ is chosen such that ρ^* barely exceeds δ. While marginal liquidity goes mainly to country B, almost no inframarginal liquidity does. So the LHS of (A2) is close to 0, which is not the case for the RHS.

One must thus distinguish between the marginal leakage for country k (raising the cutoff from ρ^* to $\rho^* + d\rho^*$ involves the same marginal cost for both countries, and a marginal benefit $\beta(\sigma^k q f^k(\rho^*) d\rho^*)$ and the inframarginal leakage (the total benefit is $\beta(\sigma^k q F^k(\rho^*)))$.

(a) *Only country A provides liquidity*

If country A provides the liquidity,

$$\rho^* = \begin{cases} \hat{\beta} \text{ if } \hat{\beta} \leq \delta \\ \delta \text{ if } \delta \leq \hat{\beta} \leq \delta/\tilde{\sigma}^A \\ \hat{\beta}\tilde{\sigma}^A \text{ if } \hat{\beta} \geq \delta/\tilde{\sigma}^A \end{cases}$$

[54] More formally, fix some $\{q, \sigma \geq 1/2\}$. Then $\hat{\beta}\sigma/[\sigma + e^\delta(1-\sigma)] < \hat{\beta}e^\delta\sigma/[e^\delta\sigma + (1-\sigma)]$, and so there more continuation if the bank specializes in country B than if it specializes in country A.

To see this, note that country A's incentive per unit of output (normalized by the shadow cost of funds) is at most $\hat{\beta}$, the incentive that prevails when all the liquidity serves to sustain projects in country A. This is indeed the case if $\hat{\beta} = \rho^* \leq \delta$. For slightly higher $\hat{\beta}$, the liquidity is shared, actually in the favor of country B at the margin. And so ρ^* remains at δ until $\hat{\beta} \geq \delta/\tilde{\sigma}^A$, at which point country A is willing to provide more liquidity even though it mostly benefits country B at the margin.

A necessary condition for the absence of top-up by country B in the region in which $\rho^* \geq \delta$ is given by the FOCs:

$$\tilde{\sigma}^A \geq \tilde{\sigma}^B.$$

Conversely, if $\tilde{\sigma}^A \geq \tilde{\sigma}^B$, it is easy to check that the IR constraint for country A,

$$\hat{\beta} F^A(\rho^*)\sigma^A \geq \Sigma_k \sigma^k \int_0^{\rho^*} dF^k(\rho),$$

is satisfied: the country's objective function is concave and the cutoff ρ^* is optimal for country A.

So country A supplies the entire liquidity if and only if either $\hat{\beta}\sigma^B \leq \delta$ or $\{\hat{\beta}\sigma^B > \delta$ and $\tilde{\sigma}^A \geq \tilde{\sigma}^B\}$

(b) *Only country B provides liquidity*

For country B to exclusively supply the liquidity it must be the case that

$$\tilde{\sigma}^B > \tilde{\sigma}^A,$$

Using the envelope theorem, (A1) is satisfied iff

$$\hat{\beta} \geq \hat{\beta}^\dagger$$

for some $\hat{\beta}^\dagger < +\infty$ (since the difference between the LHS and the RHS goes to $+\infty$ as $\hat{\beta}$ goes to $+\infty$).

(c) *Both may provide liquidity*

Let us show that for $\tilde{\sigma}^B > \tilde{\sigma}^A$ but $\hat{\beta} < \hat{\beta}^\dagger$, there is a unique equilibrium . In this equilibrium, country A supplies T^A for sure and country B mixes between $T^B = 0$ and a top up of T^A, T^B, that increases ρ^* from some $\tilde{\rho} < \delta$ to $\hat{\beta}\tilde{\sigma}^B$.

Given the (mixed) strategy of country B, the country A, solves:

$$\max_{T^K} \mathbb{E}_{T^B}\Big[\hat{\beta} q \sigma^A F^A(\rho^*) - T^A\Big], \text{ where}$$

$$q \int_0^{\rho^*} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho = T^A + T^B.$$

The FOC is:

$$\mathbb{E}_{T^B}\left[\frac{1}{h(\rho^*)}\right] = \frac{1}{\hat{\beta}},$$

where

$$h(\rho^*) = \begin{cases} \rho^* & \text{if } \rho^* < \delta \\ \frac{\rho^*}{\tilde{\sigma}^A} & \text{if } \rho^* > \delta \end{cases}$$

Note that $h(\rho^*)$ is strictly increasing in ρ^* and ρ^* is strictly increasing in T^A. Thus, regardless of the distribution of T^B, the left-hand side of the FOC is decreasing in T^A and has a unique solution. As a result, in any equilibrium, country A plays a pure strategy. This similarly holds for country B, with the difference that it may be indifferent between $T^B = 0$ or:

$$T^B = -T^A + \int_0^{\hat{\beta}\tilde{\sigma}^B} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho.$$

Therefore, the only non-pure equilibrium is when country B mixes between the above T^B with probability p and $T^B = 0$ with probability $1 - p$.

Denote by $\tilde{\rho}$ as the cutoff ρ^* induced by T^A alone:

$$T^A = \int_0^{\tilde{\rho}} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho.$$

Country B is indifferent iff:

$$\begin{aligned} \hat{\beta}\sigma^B F^B(\tilde{\rho}) &= \hat{\beta}\sigma^B F^B(\hat{\beta}\tilde{\sigma}^B) - T^B \\ &= \hat{\beta}\sigma^B F^B(\hat{\beta}\tilde{\sigma}^B) + \int_0^{\tilde{\rho}} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho - \int_0^{\hat{\beta}\tilde{\sigma}^B} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho. \end{aligned}$$

If $\tilde{\rho} > \delta$, the only solution for that is $\tilde{\rho} = \hat{\beta}\tilde{\sigma}^B$, a contradiction. Therefore, country B could be indifferent only when

$$\tilde{\rho} \leq \delta \leq \hat{\beta}\tilde{\sigma}^B \leq \hat{\beta}.$$

Thus, the indifference condition is:

$$0 = \hat{\beta}\sigma^B F^B(\hat{\beta}\tilde{\sigma}^B) + \int_0^{\tilde{\rho}} \rho\sigma^A f^A(\rho) d\rho - \int_0^{\hat{\beta}\tilde{\sigma}^B} \rho\Big(\sigma^A f^A(\rho) + \sigma^B f^B(\rho)\Big) d\rho.$$

Simplifying this:

$$\hat{\beta}\tilde{\sigma}^B e^{-\delta} + e^{-\hat{\beta}\tilde{\sigma}^B} = \tilde{\sigma}^A(1 + \tilde{\rho})e^{-\tilde{\rho}} + \tilde{\sigma}^B(1 + \delta)e^{-\delta}.$$

The right-hand side is decreasing in $\tilde{\rho}$ and $(1 + \tilde{\rho})e^{-\tilde{\rho}}$ goes from 1 to 0 as $\tilde{\rho}$ goes from 0 to ∞. Therefore, $\tilde{\rho}$ exists iff

$$\tilde{\sigma}^B(1 + \delta)e^{-\delta} \leq e^{-\hat{\beta}\tilde{\sigma}^B} + \hat{\beta}\tilde{\sigma}^B e^{-\delta} \leq \tilde{\sigma}^A + \tilde{\sigma}^B(1 + \delta)e^{-\delta},$$

where the second inequality is equivalent to $\hat{\beta} \leq \hat{\beta}^\dagger$. Moreover, the left-hand side is convex in $\hat{\beta}$ and minimized at $\hat{\beta}\tilde{\sigma}^B = \delta$. So, the first inequality always holds. At this point $\tilde{\rho}$ is equal to δ. Thus, $\tilde{\rho}$ is less than or equal to δ when it exists: There is no leakage with probability $1 - p$.

Furthermore, country A chooses T^A such that:

$$\frac{p}{\hat{\beta}\tilde{\sigma}^B/\tilde{\sigma}^A} + \frac{1-p}{\tilde{\rho}} = \frac{1}{\hat{\beta}},$$

which means:

$$p = \frac{\hat{\beta} - \tilde{\rho}}{\hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho}}.$$

Since $\tilde{\rho} \leq \hat{\beta}$, for the probability p to be well-defined, it is necessary to have: $\tilde{\sigma}^A \leq \tilde{\sigma}^B$. Because:

$$0 \leq \frac{\hat{\beta} - \tilde{\rho}}{\hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho}} \Rightarrow 0 \leq \hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho},$$

and

$$\frac{\hat{\beta} - \tilde{\rho}}{\hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho}} \leq 1 \Rightarrow \hat{\beta} - \tilde{\rho} \leq \hat{\beta} - \frac{\tilde{\sigma}^A}{\tilde{\sigma}^B}\tilde{\rho} \Rightarrow \tilde{\sigma}^A \leq \tilde{\sigma}^B.$$

In summary, the mentioned mixed equilibrium exists iff

$$\tilde{\sigma}^A \leq \tilde{\sigma}^B \quad \text{and} \quad \frac{\delta}{\tilde{\sigma}^B} \leq \hat{\beta} \leq \hat{\beta}^{\dagger}.$$

■

E Breakdowns of the Coasian bargain and their implications

As noted in the introduction, Coasian bargains require coordination, informational commonality (say, about the willingness of each country, β^k, to rescue the banks) as well as the availability of public funds in both countries. If either condition is violated, Coasian bargaining breaks down. The two causes of Coasian breakdown offer similar insights and we will here content ourselves with the case of different shadow costs of public funds. We will do so in a stark manner by considering negatively correlated shocks and by assuming that at date 1, one country has cash ($\lambda^k = \lambda$) while the other is broke or has reached its indebtedness limit imposed by a treaty or by financial markets ($\lambda^l = \infty$). Which country will have money at date 1 is not known at date 0, and each country is equally likely to be that country. The implicit assumption of negative correlation of course makes ex-post Coasian bargains impossible. The allocation of activity between the two countries solves:

$$\max_{\{\sigma,q\}}\left[\frac{F(\hat{\beta}\sigma) + F(\hat{\beta}(1-\sigma))}{2}b - c(\sigma q)\sigma - c((1-\sigma)q)(1-\sigma)\right]q$$

yielding first-order conditions

$$\frac{F(\hat{\beta}\sigma) + F(\hat{\beta}(1-\sigma))}{2}b = \sigma\hat{c}(\sigma q) + (1-\sigma)\hat{c}((1-\sigma)q)$$

$$\frac{f(\hat{\beta}\sigma) - f(\hat{\beta}(1-\sigma))}{2}b\beta = \hat{c}(\sigma q) - \hat{c}((1-\sigma)q).$$

Whether or not the bank specializes depends finely on the shock distribution. In particular, if $f' \leq 0$ as we have assumed, then $\sigma = 1/2$. The next example exhibits specialization over a range of parameters.

Binary shock distribution: Suppose that at date 1, within each country a fraction x of (distressed) projects face shock ρ while the remaining fraction of (healthy) projects face shock 0. Then the bank will be able to continue full scale under full diversification if and only if $\hat{\beta}/2 \geq \rho$; the equilibrium is then the same as under a Coasian bargain. By contrast, in the interesting case in which $\hat{\beta}/2 < \rho < \hat{\beta}$,

- Either the bank specializes and picks $\sigma = \rho/\hat{\beta}$ (is insured against liquidity shocks) and total scale $q \equiv q^I(x)$ given by $\sigma\hat{c}(\sigma q) + (1-\sigma)\hat{c}((1-\sigma)q) = (1-\frac{x}{2})b$.
- Or the bank diversifies maximally ($\sigma = 1/2$ for all i) and total scale is given by $q \equiv q^D(x)$ given by $\hat{c}(\frac{q}{2}) = (1-x)b$.

Let

$$C(\sigma, q) \equiv \sigma\hat{c}(\sigma q) + (1-\sigma)\hat{c}((1-\sigma)q),$$

and U^I and U^D denote the bank's utilities when $\sigma = \rho/\hat{\beta}$ (insurance) and when $\sigma = 1/2$ (diversification). We have

$$\frac{\partial U^I}{\partial x} = \frac{\partial}{\partial x}\left[\max_q \left\{\left(1-\frac{x}{2}\right)bq - C\left(\frac{\rho}{\hat{\beta}}, q\right)\right\}\right] = -b\frac{q^I(x)}{2}$$

and

$$\frac{\partial U^D}{\partial x} = \frac{\partial}{\partial x}\left[\max_q \left\{(1-x)bq - C\left(\frac{1}{2}, q\right)\right\}\right] = -bq^D(x).$$

Assume that $\partial(q^D(x) - (q^I(x)/2))/\partial x \geq 0$, as is the case for a linear cost ($c(q) = q \Rightarrow \hat{c}(q) = 2q$, and so $q^I(x) = \frac{(1-\frac{x}{2})b}{2[\sigma^2+(1-\sigma)^2]}$ and $q^D(x) = (1-x)b$) or more generally for power functions ($\hat{c}(q) = \gamma q^\eta$ with $\gamma > 0$ and $\eta \geq 1$). Because $U^I(0) < U^D(0)$ and $U^I(1) > U^D(1)$, there exists $x^* \in (0,1)$ such that for $x < x^*$, the bank diversifies ($\sigma = 1/2$) and for $x > x^*$, it specializes ($\sigma = \rho/\hat{\beta} > 1/2$).

F Industrial monetary policy: The binary-shock case

The shock structure is binary. A fraction x of projects face shock ρ while the remaining fraction face shock 0. We assume that $\hat{\beta}/2 < \rho < \hat{\beta}$, so that the bank will have to downsize by a factor x at date 1 if it fully diversifies. Let $\sigma \in (\frac{1}{2}, 1)$ be defined by $\sigma\hat{\beta} = \rho$.

Finally, let us assume a linear cost structure ($c(q^k) = q^k$). Let U^I (for "insured") and U^D (for "diversified") denote the bank's utility when it chooses to specialize at $\sigma = \frac{\rho}{\hat{\beta}}$ in country A and to diversify fully at $\sigma = \frac{1}{2}$:

$$U^I = \max_q\{bq - c(\sigma q)\sigma q - c((1-\sigma)q)(1-\sigma)q\}$$

and

$$U^D = \max_q\{2[(1-x)b\frac{q}{2} - c(\frac{q}{2})\frac{q}{2}]\}.$$

In the linear cost case ($c(q) = q$),

$$q^I = \frac{b}{2[(\frac{\rho}{\beta})^2 + (1 - \frac{\rho}{\hat{\beta}})^2]}$$

and

$$q^D = (1 - x)b.$$

Suppose that the bank would want to diversify in the absence of LOLR ($U^D > U^I$):

$$(1 - x)^2 > \frac{1}{2[(\frac{\rho}{\hat{\beta}})^2 + (1 - \frac{\rho}{\hat{\beta}})^2]}.$$

Country A's welfare is then equal to $S(\frac{q^D}{2}) + \beta(1-x)\frac{q^D}{2}$. Will the country with cash want to grant LOLR to the bank?

Unconditional and free-of-charge liquidity support. Suppose first that country A commits to liquidity support T . The bank then keeps diversifying. Let $\xi = min\{1 - x + \frac{T}{\rho q}, 1\}$ denote the fraction of activity that continues at date 1, where as earlier q denotes the bank's overall size.[55]

(i) For $T < x\rho[(1 - x)b]$, the bank solves

$$\max_q\{(1 - x + \frac{T}{\rho q})bq - c(\frac{q}{2})q\} = \max_q\{(1 - x)bq - c(\frac{q}{2})q + \frac{Tb}{\rho}\}$$

yielding

$$q = q^D = (1 - x)b.$$

Country A's welfare is now $S(\frac{q^D}{2}) + \beta[(1 - x) + \frac{T}{\rho q^D}]\frac{q^D}{2} - \lambda T$. Country A's welfare is reduced:

$$\Delta W^A = T(\frac{\beta}{2\rho} - \lambda) < 0.$$

Such interventions only benefit the bank and reduce welfare: LOLR interventions that reduce date-1 downsizing without changing the date-0 scale cannot improve welfare since, if desirable for country A, they could be performed at date 1 anyway.

(ii) For $T \in [x\rho(1 - x)b, x\rho b]$, the bank optimally chooses a size that leads to continuation with probability 1 and diversifies so as to reduce investment cost:

$$q = \frac{T}{\rho x}.$$

And country A welfare,

$$W = S\left(\frac{T}{2\rho x}\right) + \left[\frac{\beta}{2\rho x} - \lambda\right]T,$$

is convex in T. So, the optimum does not lie in the interior of this intermediate region.

[55] $\xi = \kappa/q$.

(iii) For $T \geq x\rho b$, then

$$q = b.$$

The welfare gain (or loss) at $T = x\rho b$[56] relative to the absence of transfer is:

$$\Delta W^A = S(\frac{b}{2}) - S(\frac{(1-x)b}{2}) + \rho bx[\frac{\beta}{\rho}(1-\frac{x}{2}) - \lambda].$$

Therefore, there can be a welfare gain from large unconditional liquidity support if $\frac{\hat{\beta}}{\rho}(1-\frac{x}{2}) > 1$, i.e. if x is not too large. The source of the gains is two-fold: First, banks acquire more clients, which generates more inframarginal surplus; second, banks continue more often, which, given their larger scale, can also potentially generate more social benefits (note a potential complementarity here).

Optimal intervention. Now consider a contract between Country A and the bank specifying in the two countries a presence $\{q^k\}_{k\in\{A,B\}}$, a continuation scale $\{\xi^k\}_{k\in\{A,B\}} \in [1-x, 1]$, a liquidity allocation $\{T^k\}_{k\in\{A,B\}}$, and a date-0 transfer between the bank and country A, so as to maximize the joint surplus of the two players:

$$U + W^A = S(q^A) + \xi^A \beta q^A + \sum_k [(\xi^k b - c(q^k) - (\xi^k - (1-x))\rho]q^k.$$

We can consider two cases

- fungibility: $\xi^A = \xi^B$;[57]
- ring-fencing: ξ^A and ξ^B can be chosen separately.

Optimal quantities satisfy:

$$(b+\hat{\beta})\xi^A = c(q^A) + [\xi^A - (1-x)]\rho$$
$$b\xi^B = c(q^B) + c'(q^B)q^B + [\xi^B - (1-x)]\rho.$$

As for continuation scales, under ringfencing, $\xi^B = 1$ if $b \geq \rho$ and $\xi^B = 1-x$ otherwise. Similarly, $\xi^A = 1$ if $b+\beta \geq \rho$ and $\xi^A = 1-x$ otherwise. Under fungibility, $\xi = \xi^A = \xi^B = 1$ if $(\beta + b - \rho)q^A + (b-\rho)q^B > 0$.

G Proof of Proposition 9 (IMP in the NTU and TU cases)

Let us investigate the possibility of a counterfavor by the bank to the hegemon's increased liquidity provision. Under NTU, this amounts to finding $\{q, \sigma, \rho^*\}$ such that

$$V^h = S(\sigma q) + \left[F(\rho^*)\beta\sigma - \lambda \int_0^{\rho^*} \rho dF(\rho)\right]q > V^{nc} = S(\sigma^{nc}q^{nc}) + \left[F(\hat{\beta}\sigma^{nc})\beta\sigma^{nc} - \lambda \int_0^{\hat{\beta}\sigma^{nc}} \rho dF(\rho)\right]q^{nc}$$

[56] Any transfer beyond this level is wasteful.

[57] Formally, the bank is indifferent between the two countries when it comes to covering shocks of magnitudes ρ. However, if the binary distribution approximates a smoother one, then the cutoff rule will be the same for both countries, as in Section 2.

and

$$U = F(\rho^*)bq - C(q,\sigma) \geq U^{nc} = F(\hat{\beta}\sigma^{nc})bq^{nc} - C(q^{nc}, \hat{\beta}\sigma^{nc}).$$

Considering the maximization of V^k and letting μ denote the shadow cost of the constraint, the FOC with respect to ρ^* is $\rho^* = \hat{\beta}\sigma + \frac{\mu}{\lambda}b$. Thus more liquidity is supplied than in the absence of commitment.[58] Finally, $\mu \leq \lambda$ results from the fact that the hegemon is constrained by the bank's inability to transfer money at date 0 (NTU).

Assume now TU (so that $\mu = \lambda$), that the acquisition cost is quadratic ($c(q^k) = q^k$), that $F(\rho)$ is uniform on $[0,1]$ and that $\bar{q} = 1$ for notational simplicity.

We know that for uniform distribution $\sigma^{nc} = 1$. So we only need to prove that there is not necessarily full specialization under IMP. Using equations (8) and (9), we obtain

$$C_\sigma(1,\sigma) = F(\hat{\beta}\sigma + b)\hat{\beta} + \hat{S}'(\sigma).$$

Under quadratic acquisition cost, $C(1,\sigma) = [\sigma^2 + (1-\sigma)^2]$ and $\hat{S}(\sigma) = \sigma^2/2\lambda$. Finally, for a uniform distribution $F(\rho) = \rho$. And so

$$2(2\sigma - 1) = (\hat{\beta}\sigma + b)\hat{\beta} + \frac{\sigma}{\lambda}.$$

So if $2 > (\hat{\beta} + b)\hat{\beta} + (1/\lambda)$, that is if $\hat{\beta}$ is small enough, $\sigma^{\text{IMP}} < 1$.

H Exchange rate appreciation in the binary case

For conciseness, we obtain the proposition in the binary-shock case (see Appendix F). A fraction x of projects face shock ρ while the remaining fraction face shock 0. We assume that $\hat{\beta}/2 < \rho < \hat{\beta}$, so that the bank will have to downsize by a factor x at date 1 if it fully diversifies. Let $\sigma \in (\frac{1}{2}, 1)$ be defined by $\sigma\hat{\beta} = \rho$.

Exchange rate without commitment. We first examine the equilibrium in the time-consistent case when ringfencing is impossible. We then extend the analysis to accommodate IMP and ringfencing. How does the bank split liquidity support at date 1? For simplicity, we assume that the liquidity is given to the bank in a way that neutralizes changes in the exchange rate brought about by changes in the split (this can be done by making the liquidity support contingent on the exchange rate, for example via an appropriate currency composition). This ensures that the bank does not seek to manipulate the exchange rate when it decides its split, and chooses the same cutoff for all countries.[59] There are two possible equilibria: I (insured) and D

[58]To obtain some intuition, let us contemplate a small deviation (dq, $d\sigma$, $d\rho^*$) from the no-commitment outcome along the bank's indifference curve. Provided that $d\rho^* = \hat{\beta}d\sigma$, the bank's optimization with respect to q and σ in the no-commitment case implies that the loss dU is second-order in dq and $d\sigma$. In contrast, the hegemon's gain is first-order if $V_q^h dq + V_\sigma^h d\sigma > 0$ (using the envelope theorem). So the hegemon can compensate for the bank's second-order loss (by increasing T slightly above $\hat{\beta}d\sigma$), and thus a Pareto-improvement can be achieved. If the cost C must meet a budget constraint, then one must pick $\{dq, d\sigma\}$ such that $V_q^h dq + V_\sigma^h d\sigma > 0$ and $C_q dq + C_\sigma d\sigma = 0$ (where V_q, V_σ, C_q, C_σ are all positive). The vectors of partial derivatives are generically non-colinear and so such a vector $\{dq, d\sigma\}$ can be found.

[59]Alternatively, we could assume a large number of (non-competing) banks without market power on the exchange rate.

(diversified). The equilibrium depends on which of these equilibria yields higher utility to the bank.

In the first possible equilibrium, the bank is specialized in country A, country A's exchange rate is appreciated $e < 1$, and country A provides liquidity support. Country A provides liquidity (and then $\xi^A = \xi^B = \xi$ because the bank does not manipulate the exchange rate) up to the point where

$$\beta q^A - \frac{\rho}{e^{\frac{1}{2}}} q^A - \rho e^{\frac{1}{2}} q^B - \frac{1}{2e} \frac{\omega - [\xi - (1-x)]\rho q^A - e[\xi - (1-x)]\rho q^B}{e^{\frac{1}{2}}} \frac{\partial e}{\partial \xi} = 0$$

or at a corner ($\xi = 1$ or $\xi = 1 - x$) if the condition holds as an inequality ($\geq$ or $\leq$), where

$$e = \frac{\omega - [\xi - (1-x)]\rho q^A}{\omega - [\xi - (1-x)]\rho q^B} < 1$$

and so

$$\frac{\partial e}{\partial \xi} = -\frac{\rho(q^A - q^B)}{[\omega - [\xi - (1-x)]\rho q^B]^2} < 0.$$

Let $\xi(q^A, q^B)$ be the solution. At date 0, the bank solves[60]

$$U^I = \max_{\{q^A, q^B\}} \{b\xi(q^A, q^B)(q^A + q^B) - c(q^A)q^A - c(q^B)q^B\},$$

where "I" stands for "insured". Specializing increases liquidity support, but appreciates the exchange rate, which triggers the two offsetting forces described above. The solution $\{q^A, q^B, e, \xi\}$ is then given by the solution to this system of four equations.

In the second possible equilibrium, the bank diversifies, and $e = 1$. Its utility under diversification ("D") is

$$U^D = \max_q \{2[(1-x)b\frac{q}{2} - c(\frac{q}{2})\frac{q}{2}]\}$$

and

$$q^D = (1-x)b$$

exactly as above.

Industrial policy and ringfencing. With Cobb-Douglas preferences and a binary shock, the exchange rate is given by:

$$\frac{\omega - [\xi^A - (1-x)]\rho q^A + e[\omega - [\xi^B - (1-x)]\rho q^B]}{2} = \omega - [\xi^A - (1-x)]\rho q^A$$

[60]For example, suppose that the equilibrium is such that $\xi = 1$. This means that

$$U^I = \max_{\{q^A, q^B\}} \{b(q^A + q^B) - c(q^A)q^A - c(q^B)q^B\}$$

s.t.

$$\beta \frac{q^A}{q^A + q^B} - \frac{\rho}{e^{\frac{1}{2}}} \frac{q^A}{q^A + q^B} - \rho e^{\frac{1}{2}} \frac{q^B}{q^A + q^B} + \frac{1}{2e} \frac{\omega - x\rho q^A - ex\rho q^B}{e^{\frac{1}{2}}} \frac{\rho(q^A - q^B)l}{(\omega - x\rho q^B)^2} = 0$$

where

$$e = \frac{\omega - x\rho q^A}{\omega - x\rho q^B}$$

which can be rewritten as:

$$e = \frac{\omega - [\xi^A - (1-x)]\rho q^A}{\omega - [\xi^B - (1-x)]\rho q^B}.$$

The exchange rate of country A is more appreciated (e lower), the higher is q^A, the lower is q^B, the higher is ξ^A, and the lower is ξ^B:

$$\frac{\partial e}{\partial \xi^A} = e\frac{-\rho q^A}{\omega - [\xi^B - (1-x)]\rho q^B} < 0 \quad \frac{\partial e}{\partial \xi^B} = e\frac{\rho q^B}{\omega - [\xi^B - (1-x)]\rho q^B} > 0$$

$$\frac{\partial e}{\partial q^A} = \frac{-[\xi^A - (1-x)]\rho}{\omega - [\xi^B - (1-x)]\rho q^B} < 0 \quad \frac{\partial e}{\partial q^B} = e\frac{[\xi^B - (1-x)]\rho}{\omega - [\xi^B - (1-x)]\rho q^B} > 0.$$

The joint surplus of the bank and country A is now given by:

$$U + W^A = S(q^A) + \xi^A \beta q^A + \sum_k [\xi^k b - c(q^k)]q^k + \frac{\omega - [\xi^A - (1-x)]\rho q^A - e[\xi^B - (1-x)]\rho q^B}{e^{\frac{1}{2}}}.$$

This expression is decreasing in e:

$$\frac{\partial(U + W^A)}{\partial e} = -\frac{1}{2e}\frac{\omega - [\xi^A - (1-x)]\rho q^A + e[\xi^B - (1-x)]\rho q^B}{e^{\frac{1}{2}}}.$$

Assuming ringfencing (ξ^A and ξ^B can be selected independently), the first-order conditions are

$$\hat{\beta} + b = \frac{\rho}{e^{\frac{1}{2}}} - \frac{1}{q^A}\frac{\partial(U + W^A)}{\partial e}\frac{\partial e}{\partial \xi^A}$$

$$b = \rho e^{\frac{1}{2}} - \frac{1}{q^B}\frac{\partial(U + W^A)}{\partial e}\frac{\partial e}{\partial \xi^B}$$

$$(b + \hat{\beta})\xi^A = c(q^A) + [\xi^A - (1-x)]\frac{\rho}{e^{\frac{1}{2}}} - \frac{\partial(U + W^A)}{\partial e}\frac{\partial e}{\partial q^A}$$

$$b\xi^B = c(q^B) + c'(q^B)q^B + [\xi^B - (1-x)]\rho e^{\frac{1}{2}} - \frac{\partial(U + W^A)}{\partial e}\frac{\partial e}{\partial q^B}.$$

The desire of country A to expand bank activities more in country A than in country B leads to more liquidity injections in country A at date 1, which in turn increases the demand for country A's currency and appreciates country A's exchange rate. As for banking activities, there are new forces: Liquidity injections in A (B) are more (less) costly because the exchange rate of country A is appreciated at date 1 (this pushes towards lower q^A, higher q^B, lower ξ^A, higher ξ^B); liquidity injections in A (B) appreciate (depreciate) the country A exchange rate and help (hurt) country A's terms of trade manipulation (this pushes towards higher q^A, lower q^B, higher ξ^A, lower ξ^B). The first force captures exchange rate appreciations as a limiting factor for industrial monetary policies.

I Proof of Proposition 12 (with more than two countries and two banks)

There are K countries (indexed by $k \in \{1,...,K\}$) and I banks (indexed by $i \in \{1,\dots,I\}$). For analytical simplicity, suppose that $I = nK$ (in Section 4, $I = K = 2$ and $n = 1$). An increase in n, the banks-to-countries ratio, i.e. the intensity of competition, amounts to an increase in competition in the banking sector, while an increase in K leads, ceteris paribus, to an increase in banks' international diversification. We look for an equilibrium in which:

- All banks have the same scale q, which they allocate in proportion σ to one country, the home country, and $\frac{1-\sigma}{K-1}$ to each of the other countries.
- For any country k, exactly n banks select this country as their home country. Let $Q \equiv nq$ denote the size of the banking sector in any given country.

Following the steps of Section 4, bank i solves:

$$\max_{\{\sigma_i,q_i\}}\left\{F(\sigma_i\beta)q_i b - c(Q+\sigma_i q_i - \sigma q)\sigma_i q_i - (K-1)c(Q+\frac{(1-\sigma_i)q_i}{K-1} - \frac{(1-\sigma)q}{K-1})\frac{(1-\sigma_i)q_i}{K-1}\right\}$$

The first-order condition with respect to σ_i is:

Either $\sigma \le 1$ and

$$f(\sigma\beta)b\beta = \frac{c'(Q)Q}{n}\left[\sigma - \frac{1-\sigma}{K-1}\right]$$

Or $\sigma = 1$ and

$$f(\sigma\beta)b\beta \ge \frac{c'(Q)Q}{n}.$$

As for the first-order condition with respect to q_i, we obtain:

$$F(\sigma\beta)b = c(Q) + \frac{c'(Q)Q}{n}\left[\sigma^2 + \frac{(1-\sigma)^2}{K-1}\right]$$

In particular, banks fully specialize (they are present in a single country: $\sigma = 1$) if and only if:

$$\frac{f(\beta)\beta}{F(\beta)} \ge \frac{c'(Q)Q}{c'(Q)Q + nc(Q)}.$$

For the power cost function ($c(Q) = Q^\eta$), this inequality writes:

$$\frac{f(\beta)\beta}{F(\beta)} \ge \frac{\eta}{\eta + n}$$

So the stronger the competition, the more likely are banks to not be cross-border. More generally, whenever there exists $\varepsilon > 0$ such that $\frac{c(Q)}{c'(Q)Q} \ge \varepsilon$ for all Q, then as $n \to \infty$, banks specialize fully.

■