Expectation Conformity in Strategic Cognition*

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Abstract

We study situations in which players shape their own understanding of the game, as well as others', through information acquisition, communication, and other forms of strategic cognition. We introduce the concept of expectation conformity: incentives to change cognition are stronger when others expect such changes and/or make similar changes themselves. Expectation conformity provides a revealed-preference characterization of equilibrium determinacy in cognitive games. When it fails, equilibrium cognition is unique, independently of cognitive costs. When it holds, suitable costs generate multiple equilibria. We illustrate the implications in two-player-constant-sum and linear-quadratic-Gaussian games. The online supplement considers other forms of cognition.

Keywords: cognition, expectation conformity, equilibrium multiplicity, information acquisition, information manipulation.

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1 Introduction

In many strategic situations, players can shape both their own understanding of the game and the understanding of others. Firms acquire information (about demand and technology) before competing in markets, and managers frame the context in which they operate by disclosing selective information about firm-specific and sectorial data to affect the beliefs and decisions of investors, employees, and other relevant actors. These cognitive activities are costly, and their value depends not only on the environment itself, but also on the cognitive choices others expect a player to make and the cognitive choices others themselves make.

This paper studies such "cognitive games," that is, strategic situations in which players' understanding of the game is endogenous. Cognition may take various forms: information acquisition about payoff-relevant states, communication and information sharing, signal jamming, industrial espionage and counter-espionage, framing, memory management, or iterative strategic reasoning. Across these environments, a common force emerges: players' incentives to engage in cognition reflect the interplay between expectations and the form of strategic interaction.

Our objective is to understand when cognition generates complementarities strong enough to sustain multiple equilibria. More specifically, we ask: when does a player become more willing to change cognition when others expect her to do so, and when others themselves adjust their cognition accordingly? We refer to this force as *expectation conformity* (EC).

Expectation conformity combines two distinct effects. The first one, which we call *unilateral expectation conformity* (UEC), captures the impact of others' expectations about a player's cognition on that player's incentives to invest in cognition. For example, when cognition is *self-directed* (that is, it influences a player's own understanding of the game without affecting others') and takes the form of information acquisition, a player may value information more when others expect her to acquire more information. When cognition is *manipulative* (that is, it influences other players' understanding of the game, as when a player shares her information with others, or engages in signal jamming), a player may have stronger incentives to shape other players' information when others expect such behavior. The second effect is an across-player complementarity that we label *increasing differences* (ID). This force captures how the value of cognition depends on the actual cognitive choices of others. For example, when cognition is self-directed information acquisition, the value of becoming more informed may increase when others themselves become more informed.

We are also interested in understanding how the equilibrium cognitive choices depend on the type of strategic interaction, in particular on whether actions in the primitive game are strategic complements or substitutes.

The paper's main contribution is to show that expectation conformity provides a general revealed-preference characterization of the drivers of equilibrium multiplicity in cognitive games. We establish three central results. First, if expectation conformity fails across all pairs of strategy profiles, then irrespective of the cognitive costs, the game admits at most one equilibrium. Second, if expectation conformity holds for a pair of profiles, suitable cognitive costs can sustain both profiles as equilibria. Third, the decomposition of EC into UEC and ID clarifies the economic forces responsible for multiplicity and identifies whether cognition itself behaves as a strategic complement or substitute. Importantly, the analysis does not require solving explicitly for the equilibrium cognitive choices. Instead, it relates the potential for equilibrium indeterminacy to properties of the primitive game and to the form of cognition under investigation. This makes the characterization applicable also to environments in which solving for equilibrium cognition may be difficult.

We first study a benchmark class of environments: two-player-constant-sum games. In such games, EC either fails or holds only weakly. As a result, if multiple equilibria exist, players must be indifferent across their equilibrium cognitive choices. Thus, two-player-constant-sum games rule out strict forms of cognitive complementarity.

In contrast, EC naturally emerges in many non-constant-sum games. To illustrate, we consider linear-quadratic-Gaussian environments in which players seek to match their actions with an underlying state while also coordinating with, or differentiating from, others. We consider two forms of cognition. First, we assume cognition is self-directed and takes the form of the choice of a Blackwell experiment about an exogenous Gaussian state. Second, we assume cognition is manipulative and takes the form of noisy information sharing; players receive exogenous private information and choose what to communicate to others. In both cases, we characterize when strategic complements and substitutes in the primitive game generate EC, and use the characterization to determine whether the cognitive game can admit multiple equilibria. Among other things, we show that strategic complementarity in actions can lead to multiple symmetric equilibria in which the information the players acquire (alternatively, share with others) takes the form of a Gaussian experiment (alternatively, a Gaussian garbling), whereas this is not possible under strategic substitutes, no matter the cost functions.

In the Online Supplement, we consider other forms of cognition. (a) First, we consider situations in which the payoff state is highly dimensional, the players learn about a few dimensions and then act as if the non-explored dimensions did not exist (as in the literature on sparsity). (b) Second, we consider games in which players learn about other players' exogenous information (espionage) or take actions that affect other players' ability to learn about their own information (counter-espionage). (c) Third, we consider generalized career-concerns in which cognition determines the effectiveness of signal-jamming. (d) Forth, we consider framing games in which cognition affects other players' recollection of favorable (alternatively, unfavorable) information. (e) In all these settings, cognition takes the form of an information structure (for oneself, when self-directed, or for other players, when manipulative). The last section of the Online Supplement considers situations in which payoffs are common knowledge and cognition affects a player's ability to compute iterated best responses (specifically, it endogenizes the depth of reasoning in the level-k model). The exposition in the main text is self-contained and does not require the reading of the Online Supplement. The purpose of the latter is to illustrate the role of EC in situations different from those discussed in the main text, which can be useful for scholars working in related areas.

Summarizing, the paper suggests that EC can be a unifying lens for understanding the possibility of equilibrium multiplicity in cognitive games. The concept isolates the forces that make cognition itself complementary or substitutable across players without committing to specific cognitive costs and explicitly characterizing the equilibrium cognitive choices. We believe this perspective can be useful for understanding a wide class of environments involving information acquisition, communication, framing, memory, and strategic reasoning.

Organization. We wrap up the Introduction with a brief discussion of the most pertinent literature. Section 2 describes cognitive games. Section 3 introduces EC and its decomposition into UEC and ID. Section 4 relates EC to the determinacy of equilibria. Section 5 considers two-player-constant-sum games. Sections 6 and 7 consider linear-quadratic-Gaussian games in which cognition is, respectively, self-directed and takes the form of information acquisition (Section 6) and manipulative and takes the form of noisy information sharing (Section 7). Section 8 concludes. Omitted proofs are in the Appendix at the end of the document.

Related Literature: The paper is mostly related to two strands of the literature. The first one investigates information acquisition in linear-quadratic games (e.g., Hellwig and Veldkamp (2009), Myatt and Wallace (2012), Bergemann and Morris (2013), Colombo et al (2014), and Pavan, (2025)).* In these papers, information acquisition is self-directed and is about an exogenous payoff state. Denti (2023) and Hebert and La'O (2023), instead, allow players to also learn about others' actions. Dewatripont and Tirole (2005), Che and Kartik (2009), Calvo'-Armengol et al. (2015), Sethi and Yildiz (2016), Kozlovskaya (2018), Adriani and Sonderegger (2020), and Denti (2023) in turn allow players to learn about other players' beliefs, and/or, to communicate to others their information.

The second strand is the literature on psychological games (e.g., Geanakoplos et al. (1988), and Battigalli and Dufwenberg (2009)). This literature studies games where players' (first and

* Related is also the literature on rational inattention. See Maćkowiak et al. (2023) for an overview.

higher-order) beliefs over other players' intentions enter directly into payoffs, for example in the form of sequential reciprocity and regret. In contrast, we consider situations in which players, at a cost, learn about, and/or manipulate, other players' intentions, but where beliefs are instrumental to the actions in the primitive game.

While the analysis touches upon themes in these literatures, to the best of our knowledge, this is the first paper to investigate the relationship between EC, the determinacy of equilibria, and the complementarity/substitutability of the cognitive choices across various specifications of the cognitive game.

2 Cognitive games

Primitive game. There are $n \in \mathbb{N}$ players, indexed by $i \in I \equiv \{1, \ldots, n\}$, with $n \geq 2$. Each player i has an action set A_i, with a_i denoting a representative element. The player receives a gross payoff $u_i(\alpha_i, \alpha_{-i}, \omega)$ when, in state of nature $\omega \in \Omega$, she chooses the mixed action $\alpha_i \in \Delta(A_i)$ and the other players choose the mixed actions $\alpha_{-i} \equiv (\alpha_1, ..., \alpha_{i-1}, \alpha_{i+1}, ..., \alpha_n) \in \prod_{j \neq i} \Delta(A_j)$. The players share a common prior F on Ω.[†] When α_i is a Dirac measure assigning probability one to action $a_i \in A_i$ and likewise α_{-i} is a collection of Dirac measures with each α_j assigning probability one to some action $a_j \in A_j$, $j \neq i$, we abuse notation and denote by $u_i(a_i, a_{-i}, \omega)$ player i's payoff under the pure-action profile (a_i, a_{-i}), where $a_{-i} \equiv (a_1, ..., a_{i-1}, a_{i+1}, ..., a_n) \in \prod_{j \neq i} A_j$.

Cognition. Prior to playing the primitive game, the players simultaneously engage in cognition, resulting in a collection of information structures. We index player i's cognition by ρ_i and denote by $C_i(\rho_i)$ her cost of cognition ρ_i. At this stage, we do not specify the space in which ρ_i lives, nor impose restrictions on the cost functions C_i, as both vary across contexts.

Player i's action in the primitive game is measurable with respect to the sigma algebra of information sets induced by the cognitive choices $\rho \equiv (\rho_i, \rho_{-i})$, with $\rho_{-i} \equiv (\rho_1, ..., \rho_{i-1}, \rho_{i+1}, ..., \rho_n)$. We capture the above measurability constraints as follows. Given ρ, each player i observes a signal realization $s_i \in S_i$, where S_i is a Polish space, with the vector $s \equiv (s_1, \ldots, s_n)$ drawn from $S \equiv \prod_{i=1}^n S_i$ according to a distribution $Q(s|\omega, \rho)$. We assume that cognition is *covert*; for any i and any (ω, ρ_i), the support of the marginal distribution of Q over S_i is invariant in ρ_{-i}.[‡]

[†] Che and Kartik (2009) by contrast look at information acquisition under heterogeneous priors. Our definition of expectation conformity and most of our results do not hinge on the common prior. assumption However, the exposition is facilitated by such an assumption.

[‡] The role of this assumption is to facilitate the analysis by abstracting from the discussion of sequential rationality at information sets where beliefs are unrestricted.

We say that cognition is *"self-directed"* when the distribution from which each player's information is drawn is not affected by other players' cognitive choices. That is, there exists a collection of distributions $(Q_i)_{i=1}^n$ such that, for any (s, ω, ρ), $Q(s|\omega, \rho) = \oplus_{i \in I} Q_i(s_i|\omega, \rho_i)$. We say that cognition is *"manipulative"* if, irrespectively the signals s are drawn independently across players, the distribution from which a player's signal s_i is drawn depends on other players' cognitive choices ρ_{-i}.

A special case of self-directed cognition is when each ρ_i is a Blackwell experiment about the exogenous payoff state, that is, a mapping $\rho_i : \Omega \to \Delta(S_i)$. In this case, we use $\rho_i' \succ \rho_i$ to denote that ρ_i' is Blackwell more informative than ρ_i. When, for any pair (ρ_i', ρ_i), with $\rho_i' \succ \rho_i$, $C_i(\rho_i') \geq C_i(\rho_i)$, we say that C_i is Blackwell monotone.

Continuation equilibria. A (behavioral) strategy $\sigma_i \in \Delta(A_i)^{S_i}$ for player i in the primitive game is a measurable mapping $\sigma_i : S_i \to \Delta(A_i)$ that specifies, for each $s_i \in S_i$, a mixed action $\sigma_i(s_i) \in \Delta(A_i)$.

When players engage in cognition ρ and play in the downstream game according to σ, player i's expected payoff (gross of the cognitive cost C_i) is equal to

$$U_i(\sigma; \rho) \equiv \int_\omega \Big[\int_s u_i(\sigma_i(s_i), \sigma_{-i}(s_{-i}), \omega) Q(\mathrm{d}s|\omega, \rho) \Big] F(\mathrm{d}\omega).$$

Player i's *net* payoff is the above gross payoff, minus the cognitive cost $C_i(\rho_i)$.

For any vector ρ of cognitive choices, let σ^ρ denote a *Bayes-Nash Equilibrium* (BNE) in the Bayesian game induced by ρ, when ρ is common knowledge. Throughout, we will focus on ρ for which a BNE exists and to games that admit at least one such ρ.

For any $\rho = (\rho_i, \rho_{-i})$, any BNE σ^ρ, and any ρ_i', we denote by

$$V_i(\rho_i'; \rho, \sigma^\rho) \equiv \sup_{\sigma_i \in \Delta(A_i)^{S_i}} U_i(\sigma_i, \sigma_{-i}^\rho; \rho_i', \rho_{-i})$$

the maximal gross payoff that player i can obtain by selecting cognition ρ_i' and then optimizing over his mixed actions, when the other players choose ρ_{-i} and play according to the BNE strategies $\sigma_{-i}^\rho \equiv (\sigma_1^\rho, ..., \sigma_{i-1}^\rho, \sigma_{i+1}^\rho, ..., \sigma_n^\rho)$. By the definition of the BNE σ^ρ, when $\rho_i' = \rho_i$,

$$V_i(\rho_i; \rho, \sigma^\rho) = U_i(\sigma^\rho; \rho).$$

We assume there exist scalars $(k_i)_{i=1}^n \in \mathbb{R}_{++}^n$ such that $|V_i(\rho_i'; \rho, \sigma^\rho)| \leq k_i$, for all $(\rho_i', \rho, \sigma^\rho)$, all i.

Full cognitive game: timing and equilibrium. The game unfolds as follows:

1. the players simultaneously choose cognition $(\rho_i)_{i=1}^n$;

2. they then receive private signals $(s_i)_{i=1}^n$;
3. they choose mixed actions $(\alpha_i)_{i=1}^n$;
4. finally, for any ω, they collect payoffs $u_i(\alpha_i, \alpha_{-i}, \omega) - C_i(\rho_i)$.

Definition 1. The strategy profile (ρ, σ^ρ) is an e*quilibrium* of the cognitive game if, for any i and any ρ_i',

$$V_i(\rho_i; \rho, \sigma^\rho) - C_i(\rho_i) \geq V_i(\rho_i'; \rho, \sigma^\rho) - C_i(\rho_i').$$

3 Expectation conformity

Let (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ denote two distinct strategy profiles.

Definition 2 (expectation conformity). Expectation conformity (EC) holds for player i with respect to (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ if

$$V_i\big(\hat{\rho}_i; (\hat{\rho}, \sigma^{\hat{\rho}})\big) - V_i\big(\rho_i; (\hat{\rho}, \sigma^{\hat{\rho}})\big) \geq V_i\big(\hat{\rho}_i; (\rho, \sigma^\rho)\big) - V_i\big(\rho_i; (\rho, \sigma^\rho)\big). \qquad \Big(EC_i\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big)\Big)$$

When we say that EC holds for (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ we mean *for all players*.
Let us decompose the difference

$$\Gamma_i^{EC}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) \equiv \Big[V_i\big(\hat{\rho}_i; (\hat{\rho}, \sigma^{\hat{\rho}})\big) - V_i\big(\rho_i; (\hat{\rho}, \sigma^{\hat{\rho}})\big)\Big] - \Big[V_i\big(\hat{\rho}_i; (\rho, \sigma^\rho)\big) - V_i\big(\rho_i; (\rho, \sigma^\rho)\big)\Big]$$

into a within-player ***unilateral expectation conformity* (UEC)** term

$$\Gamma_i^{UEC}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) \equiv \Big[V_i\big(\hat{\rho}_i; (\hat{\rho}_i, \rho_{-i}), \sigma^{\hat{\rho}_i, \rho_{-i}}\big) - V_i\big(\rho_i; (\hat{\rho}_i, \rho_{-i}), \sigma^{\hat{\rho}_i, \rho_{-i}}\big)\Big]$$

$$-\Big[V_i\big(\hat{\rho}_i; (\rho_i, \rho_{-i}), \sigma^{\rho_i, \rho_{-i}}\big) - V_i\big(\rho_i; (\rho_i, \rho_{-i}), \sigma^{\rho_i, \rho_{-i}}\big)\Big]$$

which captures the impact of the other players' expectation of player i's cognition on player i's value of cognition, fixing the other players' cognitive choices at ρ_{-i}, and an across-players ***increasing differences* (ID)** term,

$$\Gamma_i^{ID}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) \equiv \Big[V_i\big(\hat{\rho}_i; (\hat{\rho}_i, \hat{\rho}_{-i}), \sigma^{\hat{\rho}_i, \hat{\rho}_{-i}}\big) - V_i\big(\rho_i; (\hat{\rho}_i, \hat{\rho}_{-i}), \sigma^{\hat{\rho}_i, \hat{\rho}_{-i}}\big)\Big]$$

$$-\Big[V_i\big(\hat{\rho}_i; (\hat{\rho}_i, \rho_{-i}), \sigma^{\hat{\rho}_i, \rho_{-i}}\big) - V_i\big(\rho_i; (\hat{\rho}_i, \rho_{-i}), \sigma^{\hat{\rho}_i, \rho_{-i}}\big)\Big]$$

which captures the impact of other players' cognition on player i's value of cognition, holding fixed the other players' expectations of player i's cognition. Clearly, in settings in which only player i engages in cognition, ID is muted and EC and UEC coincide. When, instead, multiple players engage in cognition, it is useful to investigate whether EC comes from UEC, ID, or a combination of the two. The decomposition offers a deeper understanding of what drives equilibrium multiplicity in the cognitive game.

Remark. The notion of EC, and its relation to equilibrium determinacy, extend to games in which cognition takes the form of covert investments other than the choice of an information structure (e.g., the depth of reasoning in the level-k model in the Online Supplement or the distribution of the payoff state ω , as in the hold-up model).

4 Equilibrium Determinacy

A necessary condition for (ρ, σ^ρ) and $(\hat\rho, \sigma^{\hat\rho})$ to be equilibrium profiles (for some cost functions $(C_i)_{i\in I}$) is that $\Gamma_i^{EC}\big((\rho,\sigma^\rho),(\hat\rho,\sigma^{\hat\rho})\big) \geq 0$ for all i. The following proposition says that this condition is also sufficient, for some cost functionals:

Proposition 1 (equilibrium determinacy). *(a) If EC holds for (ρ, σ^ρ) and $(\hat\rho, \sigma^{\hat\rho})$, there exist $(C_i)_{i\in I}$ such that (ρ, σ^ρ) and $(\hat\rho, \sigma^{\hat\rho})$ are both equilibrium profiles. Furthermore, if cognition is self-directed information acquisition, C_i can be taken to be Blackwell monotone. (b) If there exists i such that $EC_i\big((\rho,\sigma^\rho),(\hat\rho,\sigma^{\hat\rho})\big)$ is not satisfied, there are no $(C_i)_{i\in I}$ for which both (ρ, σ^ρ) and $(\hat\rho, \sigma^{\hat\rho})$ are equilibrium profiles. (c) If there is no pair (ρ, σ^ρ) and $(\hat\rho, \sigma^{\hat\rho})$ of distinct profiles such that EC holds for (ρ, σ^ρ) and $(\hat\rho, \sigma^{\hat\rho})$, the cognitive game has at most one equilibrium, for any $(C_i)_{i\in I}$.*

Proof. Part (a). For (ρ, σ^ρ) and $(\hat\rho, \sigma^{\hat\rho})$ to be equilibrium profiles for the costs $(C_i)_{i\in I}$, it is both necessary and sufficient that, for all i, and all $\tilde\rho_i$,

$$V_i(\rho_i;\rho,\sigma^\rho) - C_i(\rho_i) \geq V_i(\tilde\rho_i;\rho,\sigma^\rho) - C_i(\tilde\rho_i) \tag{1}$$

and

$$V_i(\hat\rho_i;\hat\rho,\sigma^{\hat\rho}) - C_i(\hat\rho_i) \geq V_i(\tilde\rho_i;\hat\rho,\sigma^{\hat\rho}) - C_i(\tilde\rho_i). \tag{2}$$

Let $(C_i)_{i\in I}$ be any profile of cost such that, for all i and all $\tilde\rho_i$,

$$C_i\big(\tilde\rho_i\big) = \max\big\{0, K_i + V_i\big(\tilde\rho_i;\rho,\sigma^\rho\big) - V_i\big(\rho_i;\rho,\sigma^\rho\big), K_i + d_i + V_i\big(\tilde\rho_i;\hat\rho,\sigma^{\hat\rho}\big) - V_i\big(\hat\rho_i;\hat\rho,\sigma^{\hat\rho}\big)\big\} \tag{3}$$

with

$$d_i \in [V_i\big(\hat\rho_i;\rho,\sigma^\rho\big) - V_i\big(\rho_i;\rho,\sigma^\rho\big), V_i\big(\hat\rho_i;\hat\rho,\sigma^{\hat\rho}\big) - V_i\big(\rho_i;\hat\rho,\sigma^{\hat\rho}\big)]$$

and $K_i \geq \max\{0, -d_i\}$. It is easy to see that, when EC holds for (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$, under the above costs, Conditions (1) and (2) are satisfied, implying that (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ are equilibrium profiles for these cost functions.

Next, suppose that cognition is self-directed information acquisition. Because more information cannot hurt a player when acquired covertly, for any $(\rho^\#, \sigma^{\rho^\#})$ and any (ρ'_i, ρ''_i), with $\rho''_i \succ \rho'_i$, $V_i\big(\rho''_i; (\rho^\#, \sigma^{\rho^\#})\big) \geq V_i\big(\rho'_i; (\rho^\#, \sigma^{\rho^\#})\big)$. The formula in (3) then guarantees that for any (ρ'_i, ρ''_i), with $\rho''_i \succ \rho'_i$, $C_i\big(\rho''_i\big) \geq C_i\big(\rho'_i\big)$, implying that the above cost functions are Blackwell monotone.

Next, consider parts (b) and (c). If (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ are both equilibrium profiles, it must be that, for all i,

$$V_i\big(\hat{\rho}_i; \hat{\rho}, \sigma^{\hat{\rho}}\big) - V_i\big(\rho_i; \hat{\rho}, \sigma^{\hat{\rho}}\big) \geq C_i\big(\hat{\rho}_i\big) - C_i\big(\rho_i\big) \geq V_i\big(\hat{\rho}_i; \rho, \sigma^\rho\big) - V_i\big(\rho_i; \rho, \sigma^\rho\big). \tag{4}$$

Hence if $EC_i\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big)$ does not hold for some i, there are no $(C_i)_{i \in I}$ for which (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ are both equilibrium profiles. This also means that, when, for all (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$, there exists an i such that $EC_i\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big)$ is violated, irrespective of $(C_i)_{i \in I}$, there is at most one equilibrium. □

5 Two-Player-Constant-Sum Games

Suppose that $n = 2$ and that, for all $(\alpha_i, \alpha_j, \omega)$,

$$u_i(\alpha_i, \alpha_j, \omega) + u_j(\alpha_i, \alpha_j, \omega) = k(\omega),$$

where k is an arbitrary function of the state of Nature. The overall game obviously is not zero-sum: Any cognition, when costly, reduces total surplus and amounts to pure rent-seeking. Constant-sum games have remarkable properties. For example, a player can only benefit from having (and being known to have) more information (e.g., Lehrer and Rosenberg (2006)). Another interesting property is given by the following result:[§]

Proposition 2 (constant-sum games). *For all (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$, $\Sigma_i \Gamma_i^{EC}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) \leq 0$. As a consequence, if there are multiple equilibria, in none of them does a player have a strict preference for her equilibrium cognition over her cognition in any other equilibrium.*

[§] We are grateful to Gabriel Carroll for conjecturing that zero-sum games fail to satisfy expectation conformity.

Proof. The constant-sum property implies that

$$\Sigma_i \left\{ \left[U_i\big(\sigma_i^{\hat{\rho}}, \sigma_j^{\hat{\rho}}; \hat{\rho}_i, \hat{\rho}_j\big) - U_i\big(\sigma_i^{\rho}, \sigma_j^{\hat{\rho}}; \rho_i, \hat{\rho}_j\big) \right] - \left[U_i\big(\sigma_i^{\hat{\rho}}, \sigma_j^{\rho}; \hat{\rho}_i, \rho_j\big) - U_i\big(\sigma_i^{\rho}, \sigma_j^{\rho}; \rho_i, \rho_j\big) \right] \right\} = 0. \quad (5)$$

Next observe that, for any $l, m \in \{1, 2\}$, $m \neq l$,

$$U_l\big(\sigma_l^{\rho}, \sigma_m^{\hat{\rho}}; \rho_l, \hat{\rho}_m\big) \leq \sup_{\sigma_l \in \Delta(A_l)^{S_l}} U_l(\sigma_l, \sigma_m^{\hat{\rho}}; \rho_l, \hat{\rho}_m)$$

and

$$U_l\big(\sigma_l^{\hat{\rho}}, \sigma_m^{\rho}; \hat{\rho}_l, \rho_m\big) \leq \sup_{\sigma_l \in \Delta(A_l)^{S_l}} U_l(\sigma_l, \sigma_m^{\rho}; \hat{\rho}_l, \rho_m).$$

Hence, Condition (5) implies that

$$\Sigma_i \, \Gamma_i^{EC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) \leq 0 \quad (6)$$

Because equilibrium multiplicity requires that

$$\Gamma_i^{EC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) \geq 0$$

for all i (see Proposition 1), the inequality in (6) implies that, if (ρ, σ^{ρ}) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ are both equilibrium profiles, then $\Gamma_i^{EC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) = 0$ for all i. Using (4), we then have that

$$V_i\big(\hat{\rho}_i; \hat{\rho}, \sigma^{\hat{\rho}}\big) - V_i\big(\rho_i; \hat{\rho}, \sigma^{\hat{\rho}}\big) = C_i\big(\hat{\rho}_i\big) - C_i\big(\rho_i\big) = V_i\big(\hat{\rho}_i; \rho, \sigma^{\rho}\big) - V_i\big(\rho_i; \rho, \sigma^{\rho}\big). \quad (7)$$

In each equilibrium, each player must thus be indifferent between her cognitive level in that equilibrium and her cognitive level in any other equilibrium. □

The result does not mean that the equilibrium payoffs are the same in all equilibria. To illustrate this point in the simplest possible terms, suppose i's payoff is $(a_i - a_j)\omega$, for $i, j = 1, 2$, $j \neq i$. Further assume that $A_i = \{1, -1\}$, for $i = 1, 2$, and that ω is drawn from $\Omega = \{-1, 1\}$ with equal probability. Lastly, suppose that cognition is self-directed, with $\rho_i \in \{1, \emptyset\}$, $i = 1, 2$. When $\rho_i = 1$, player i perfectly learns ω. When, instead, $\rho_i = \emptyset$, player i receives no information about ω. Let $C_i(1) = 1$ and $C_i(\emptyset) = 0$. The game admits multiple pure-strategy equilibria. In one, both players learn the state and then match the state with their action. In another one, neither player learns the state and then selects action $a_i = 1$ with probability one, $i = 1, 2$. The equilibrium payoffs under the first equilibrium are equal to -1 for both players, whereas the equilibrium payoffs under the second equilibrium are equal to 0. The first equilibrium can be interpreted as a "cognitive trap". Importantly, note that, in contrast to two-player constant-sum games, n-player constant-sum games in which $n > 2$ may admit multiple strict equilibria. To see this, take a non-constant-sum two-player game admitting multiple strict equilibria (such as those in the next section). Have this game played twice, by players 1 and 2 and by players 3 and 4, respectively. Use players 3 and 4 (alternatively, players 1 and 2) as passive "budget balancers" in the game played by 1 and 2 (alternatively, 3 and 4). The transformed game is a constant-sum game that admits multiple strict equilibria.

6 Self-Directed Cognition in Linear-Quadratic Games

A class of games that has received prominent attention are linear-quadratic-Gaussian games in which players independently learn about a payoff-relevant state. A central question in this literature is whether multiple equilibria are possible. The question has been answered only for specific costs. For example, when actions in the primitive game are strategic complements and each player chooses whether or not to pay attention to a discrete number of information sources (with each choice binary), Hellwig and Veldkamp (2009) showed that these games typically admit multiple symmetric equilibria. In contrast, when attention is a continuous choice and its marginal cost vanishes as the attention to each source vanishes, Myatt and Wallace (2012) showed that the same games typically admit a unique symmetric equilibrium. The results for strategic substitutes remain unexplored.

EC contributes a different angle. By focusing on the benefits of conforming to other players' expectations, it helps determine whether these games are prone to multiplicity for some cost functions, without requiring that one selves for the equilibria themselves, which may be difficult for general cost functions.

To illustrate, suppose that $n = 2$, $A_i = A_j = \mathbb{R}$ and

$$u_i(a_i, a_{-i}, \omega) = -(1-\beta)(a_i - \omega)^2 - \beta(a_i - a_j)^2, \tag{8}$$

where $\beta \in (-1, +1)$, and with ω drawn from $\Omega = \mathbb{R}$ according to a Normal distribution with mean y and precision h. Actions in the primitive game are *strategic complements* when $\beta \in (0, 1)$ and *strategic substitutes* when $\beta \in (-1, 0)$. Cognition is self-directed and corresponds to the selection of a Blackwell experiment $\rho_i : \Omega \to \Delta(S_i)$, with $S_i = \mathbb{R}$. The restriction of β to $(-1, +1)$ guarantees that, for any $\rho = (\rho_1, \rho_2)$, the BNE in the downstream game is unique when strategies are square-integrable, an assumption we maintain throughout.

Motivated by the literature on rational inattention (e.g., Maćkowiak et al, 2023) and the literature on robust predictions in games of incomplete information (e.g., Bergemann and

Morris (2013)), we are interested in equilibria in which the players' actions are themselves Gaussian. Under natural restrictions on the cost functions (namely, monotonicity with respect to garblings—see, e.g., Denti (2023)), this amounts to the players choosing Blackwell experiments that, for any ω, draw a signal s_i from a Normal distribution with mean ω and some precision h_i. For any profile of Gaussian experiments, the unique BNE is in linear strategies. For any pair of profiles $\rho = (\rho_1, \rho_2)$ and $\hat{\rho} = (\hat{\rho}, \hat{\rho}_2)$ of Gaussian experiments, we denote by (h_1, h_2) and $(\hat{h}_1, \hat{h}_2)$ the corresponding profiles of signal precisions.

Proposition 3 (information acquisition). *Let (ρ, σ^ρ) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ be strategy profiles in which the experiments are Gaussian. UEC holds for these profiles, irrespective of whether the actions in the primitive game are strategic complements or substitutes. ID holds for player i if and only if $\beta(\hat{h}_i - h_i)(\hat{h}_j - h_j) \geq 0$. Finally, EC holds for player i if and only if*

$$\beta(\hat{h}_i - h_i)(\hat{h}_j - h_j)(\hat{h}_i + h)\left[h_i(1+\beta) + h\right] + \beta^2(\hat{h}_i - h_i)^2\hat{h}_j\left[h_j(1+\beta) + h\right] \geq 0. \tag{9}$$

Proof. See Appendix. □

Holding fixed the precision of the opponent's Gaussian experiment, the value to player i of increasing the precision of her own Gaussian experiment is higher when player j expects player i to select a more informative experiment. To understand why, suppose actions in the primitive game are strategic complements. When player j expects player i to choose a more informative experiment, she also expects i's action a_i to respond more to the experiment's realization s_i. Player j then optimally adjusts her action a_j by responding more to her own signal s_j. This in turn increases player i's incentives to acquire more information to better align her action with j's. Next, suppose actions are strategic substitutes. When player j expects player i to acquire more information, she also expects i's action to respond more to s_i. Player j then responds less to s_j to differentiate her action from i's. This in turn increases i's incentives to acquire more information to differentiate her action from j's.

To understand the second part of the proposition, suppose that $\hat{h}_i > h_i$. Holding player j's expectation about i's experiment fixed, the value to player i of acquiring more information is higher when either (a) player j also acquires more information and actions are strategic complements, or (b) player j acquires less information and actions are strategic substitutes. This is because, in either case, when a player acquires more information she responds more to it.

The last part of the proposition combines the above properties to identify necessary and sufficient conditions for the game to admit multiple equilibria in which the players acquire Gaussian experiments, for some cost functions. Without loss, let $\hat{\rho}_i$ be Blackwell more informative than ρ_i (i.e., $\hat{h}_i > h_i$). When actions are complements, there exist Blackwell monotone cost functions for which the game admits multiple (possibly asymmetric) equilibria, with both players acquiring more information in one equilibrium than in the other (i.e., such that $\hat{h}_i > h_i$, $i = 1, 2$). This is not possible when actions are substitutes. On the other hand, in this latter case, there exist Blackwell monotone cost functions for which the game admits multiple equilibria in which one player acquires more information in one equilibrium and the other player in the other equilibrium (i.e., such that $\hat{h}_i > h_i$ and $\hat{h}_j < h_j$). This is not guaranteed under strategic complements.

When applied to symmetric profiles, Proposition 3 leads to the following:

Corollary 1. *When $\beta > 0$, there exist Blackwell monotone cost functions for which the game admits multiple symmetric equilibria in which the selected experiments are Gaussian. When, instead, $\beta < 0$, no matter the cost functions, the game admits at most one symmetric equilibrium in which the selected experiments are Gaussian.*

Proof. See Appendix. □

The result for complements is consistent with what noticed by Hellwig and Veldkamp (2009) for specific cost functions. The one for substitutes is, to the best of our knowledge, new.

7 Manipulative Cognition in Linear-Quadratic Games

An example of manipulative cognition is noisy information sharing. Consider the game of the previous section but now assume that each player is endowed with an exogenous "primary" signal

$$s_i^P = \omega + \varepsilon_i,$$

with ε_i drawn from a standard Normal distribution. We capture noisy information sharing by assuming that each player i chooses a Blackwell garbling of her primary signal s_i^P to pass on to the rival. We refer to such a garbled endogenous signal as "secondary" and denote it by s_j^S. Again, following the pertinent literature (e.g., Vives (2006) and Calvo'-Armengol et al. (2015)), we are interested in whether multiple (possibly asymmetric) equilibria exist in which the garbling are Gaussian. Specifically, we are interested in whether, for appropriate costs, the players' secondary signals take the form

$$s_j^S = s_i^P + \gamma_i$$

with γ_i drawn from a Normal distribution with mean zero and precision h_i.

To ease the exposition, we consider the limit in which the precision h of the prior goes to 0. This assumption facilitates the description of the posterior beliefs but is not essential to the results. For each pair $\rho = (\rho_1, \rho_2)$ and $\hat{\rho} = (\hat{\rho}, \hat{\rho}_2)$ of Gaussian garblings, let (h_1, h_2) and $(\hat{h}_1, \hat{h}_2)$ be the corresponding profiles of signal precisions. Again, we restrict attention to square-integrable strategies which guarantees uniqueness of the BNE for any profile of garblings.

Proposition 4 (information sharing). *Let (ρ, σ^{ρ}) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ be strategy profiles in which the information each player passes on to the rival takes the form of a Gaussian garbling of her exogenous information. UEC holds for such profiles if $\beta > 0$, but does not hold if $\beta < 0$. Irrespectively of the sign of β, ID holds for player i if and only if $(\hat{h}_i - h_i)(\hat{h}_j - h_j) \leq 0$. Finally, EC holds for player i if and only if*

$$\beta(\hat{h}_i - h_i)\left[(\hat{h}_i - h_i)(1 + 2h_j)(1 + 2(1-\beta)\hat{h}_j) - 2\beta h_i(\hat{h}_j - h_j)(1 + 2(1-\beta)\hat{h}_i)\right] \geq 0. \quad (10)$$

Proof. See Appendix. □

Contrary to the case in Section 6, whether or not UEC holds in these games thus depends on whether the actions in the primitive game are strategic complements or substitutes. To gather intuition, fix the precision h_j of the noise in player i's secondary signal (which is controlled by player j). When player j expects player i to share more information (that is, to pass on a more precise secondary signal, i.e., to choose $\hat{h}_i \geq h_i$), irrespective of the sign of β, player j's action responds more to her secondary signal and less to her primary signal. When actions are complements, this increases the incentives for player i to share more information so as to better align j's action with hers, whereas the opposite is true when actions are substitutes.

Next, consider ID. Fix the precision h_i of the secondary signal that player j expects to receive from player i. When player j sends a more precise signal to player i (that is, when $\hat{h}_j \geq h_j$), she then relies more on her primary signal when actions are complements, and more on her secondary signal when they are substitutes. As a result, no matter the sign of β, the incentives for player i to reciprocate by sending player j a more precise secondary signal are smaller, implying that this game satisfies a negative form of increasing differences: players share less information with rivals when they expect rivals to share more information with them.

Finally, consider EC. The result in the proposition implies that, when actions are strategic complements, there exist Blackwell monotone cost functions under which the game admits multiple asymmetric equilibria in which both players select Gaussian garblings of their exogenous information, with one player investing more in one equilibrium, the other player in the other equilibrium (i.e., $\hat{h}_i > h_i$ and $\hat{h}_j < h_j$). When, instead, actions are substitutes, no matter the costs, there cannot exist multiple equilibria in which both players garble less in one equilibrium than the other (i.e., such that $\hat{h}_i > h_i$ for $i = 1, 2$). When applied to symmetric profiles, the proposition implies the following:

Corollary 2. *When $\beta > 0$, there exist Blackwell monotone cost functions for which the game admits multiple symmetric equilibria in which each player sends to the rival a Gaussian garbling of her exogenous information. When, instead, $\beta < 0$, for any collection of cost functions, the game admits at most one symmetric equilibrium in which the shared information is Gaussian.*

Proof. See Appendix. □

8 Concluding Remarks

We study "cognitive games," that is, strategic situations where players engage in activities shaping their own understanding of the game, as well as others'. We focus on a specific aspect of such games: expectation conformity. The latter is driven by the interaction between two synergies, (a) the value to conform to other players' expectations about one's own cognition (unilateral expectation conformity), and (b) the value to adapt to other players' cognitive choices (increasing differences).

Expectation conformity (or the lack thereof) plays an important role when it comes to the determinacy of equilibria, both when cognition is self-directed (i.e., it affects a player's own understanding of the game without affecting others') as well as when it is manipulative (i.e., it affects other players' understanding of the game).

In future work, it would be interesting to extend the analysis to dynamic situations in which cognition is partially observable (as when players can disclose the type of experiments they conducted) and agents adjust their cognition as the game progresses. This last possibility introduces new effects, such as the possibility that players sacrifice current flow payoffs to enhance their understanding of the game and exploit the acquired knowledge in subsequent periods (as in the multi-armed bandit literature). It also introduces the possibility for the players to signal their cognition to other players, a dimension that appears relevant in certain problems of interest.

It would also be interesting to investigate whether players over- or under-invest in cognition (relative to what is socially efficient) and identify interventions that can mitigate the inefficiencies.

Appendix

Proof of Proposition 3. Let $\rho = (\rho_i, \rho_j)$ be a profile of Gaussian experiments, with precisions (h_i, h_j). For any i and any s_i, optimality requires that player i's action is equal to

$$a_i = \beta \mathbb{E}[\tilde{a}_j | s_i, \rho] + (1-\beta)\mathbb{E}[\tilde{\omega} | s_i, \rho].$$

Because $|\beta| < 1$, the best-response operator is a contraction in L^2 implying that there exists a unique BNE in square-integrable strategies. Joint Gaussianity in turn implies that the unique BNE is linear. Specifically, for any i, any s_i, the unique BNE strategy σ_i^ρ selects with probability one the action $a_i = m_i^\rho s_i + (1-m_i^\rho) y$, where y is the mean of the common Gaussian prior (with precision h) and[¶]

$$m_i^\rho = (1-\beta) h_i \frac{h_j(1+\beta)+h}{(h_i+h)(h_j+h)-\beta^2 h_i h_j}.$$

Now suppose that the two players are expected to choose Gaussian experiments $\rho = (\rho_i, \rho_j)$, with precisions (h_i, h_j), and then follow the BNE strategies σ^ρ. When player i deviates and selects a Gaussian experiment ρ_i', with precision h_i', and then optimally adjusts the rule she follows to map her signal realizations s_i into her actions, her ex-ante expected payoff, gross of the cost $C_i(\rho_i')$, is equal to

$$V_i\left(\rho_i'; \rho, \sigma^\rho\right) = -\left(m_i^{\rho_i';\rho}\right)^2 \tfrac{1}{h_i'} - \beta \left(m_j^\rho\right)^2 \tfrac{1}{h_j}$$

$$-\left[1-\beta+\left(m_i^{\rho_i';\rho}\right)^2 - 2(1-\beta)m_i^{\rho_i';\rho} + \beta\left(m_j^\rho\right)^2 - 2\beta m_i^{\rho_i';\rho} m_j^\rho\right] \tfrac{1}{h},$$

where

$$m_i^{\rho_i';\rho} \equiv \frac{(1-\beta)h_i'}{h_i'+h} + \frac{\beta h_i'}{h_i'+h} m_j^\rho.$$

In other words, $m_i^{\rho_i';\rho}$ is the sensitivity of player i's action to her signal s_i when the two players are expected to choose the Gaussian experiments $\rho = (\rho_i, \rho_j)$ and play according to the BNE strategies σ^ρ and, instead, player i chooses the Gaussian experiment ρ_i' and then optimally adjusts the rule she follows to map each signal realization s_i into her action a_i.

To see whether this game satisfies UEC, ID, and EC, take any two Gaussian experiments $\hat{\rho}_i$ and ρ_i with precisions respectively equal to $\hat{h}_i$ and h_i, and let $(\rho', \sigma^{\rho'})$ and $(\rho'', \sigma^{\rho''})$ be any pair of profiles in which the players' experiments are Gaussian (with precisions (h_i', h_j') and (h_i'', h_j'')).

Then let

[¶] The formulas for the equilibrium sensitivities are obtained after a few simplifications which are omitted for brevity.

$$D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') \equiv \left[V_i\big(\hat{\rho}_i; \rho', \sigma^{\rho'}\big) - V_i\big(\rho_i; \rho', \sigma^{\rho'}\big)\right] - \left[V_i\big(\hat{\rho}_i; \rho'', \sigma^{\rho''}\big) - V_i\big(\rho_i; \rho'', \sigma^{\rho''}\big)\right]$$

with $\sigma^{\rho'}$ and $\sigma^{\rho''}$ denoting the BNEs associated with ρ' and ρ'', respectively.

Observe that, for each player i, UEC holds if $D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') \geq 0$ for $\rho' = (\hat{\rho}_i, \rho_j)$ and $\rho'' = (\rho_i, \rho_j)$. ID holds if $D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') \geq 0$ for $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\hat{\rho}_i, \rho_j)$. EC holds if $D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') \geq 0$ for $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\rho_i, \rho_j)$, respectively.

Using the characterization of the V_i functions above, we have that[‖]

$$D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') = \frac{\hat{h}_i - h_i}{(h_i + h)\left(\hat{h}_i + h\right)} \beta \left(m_j^{\rho'} - m_j^{\rho''}\right) \left[2(1-\beta) + \beta\left(m_j^{\rho'} + m_j^{\rho''}\right)\right]$$

from which we obtain that

$$D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') \overset{\text{sgn}}{=} \beta(\hat{h}_i - h_i)\left(m_j^{\rho'} - m_j^{\rho''}\right).$$

To see whether this game satisfies UEC for player i, then take $\rho' = (\hat{\rho}_i, \rho_j)$ and $\rho'' = (\rho_i, \rho_j)$. We have that $\frac{\partial m_j^{\rho}}{\partial h_i} \overset{\text{sgn}}{=} \beta$. We thus conclude that, independently of the sign of β, $D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') \geq 0$, which implies that UEC holds.

To see whether the game satisfies ID for player i, take $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\hat{\rho}_i, \rho_j)$. Using the fact that, for any $\rho = (\rho_i, \rho_j)$, m_j^{ρ} is non-decreasing in h_j, we have that $D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') \overset{\text{sgn}}{=} \beta(\hat{h}_i - h_i)(\hat{h}_j - h_j)$. We thus have that ID holds for player i if and only if $\beta(\hat{h}_i - h_i)(\hat{h}_j - h_j) \geq 0$.

Finally, to see whether this game satisfies EC for player i, take $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\rho_i, \rho_j)$. Using the formulas above we have that the sign of $D_i(\hat{\rho}_i, \rho_i; \rho', \rho'')$ is equal to the sign of

$$\beta(\hat{h}_j - h_j)\left(\hat{h}_i - h_i\right)\left(\hat{h}_i + h\right)\left[h_i(1+\beta) + h\right] + \beta^2\left(\hat{h}_i - h_i\right)^2 \hat{h}_j\left[h_j(1+\beta) + h\right],$$

which yields the result in the proposition. Q.E.D.

Proof of Corollary 1. Consider two symmetric profiles (ρ, σ^{ρ}) and $(\hat{\rho}, \sigma^{\hat{\rho}})$, where ρ and $\hat{\rho}$ are Gaussian experiments, with precisions $h_i = h_j = h^L$ and $\hat{h}_i = \hat{h}_j = h^H$, with $h^H > h^L$, and where σ^{ρ} and $\sigma^{\hat{\rho}}$ are the BNEs associated with these profiles. The result then follows from Proposition 3 by noting that Condition (9) holds when $\beta \in [0, 1)$ but not when $\beta \in (-1, 0)$. Q.E.D.

Proof of Proposition 4. For any proper Gaussian prior with mean $y = 0$ and precision h, and any profile of Gaussian garblings $\rho = (\rho_1, \rho_2)$, with precisions (h_i, h_j), the continuation game admits a unique BNE in square-integrable strategies, and this BNE is linear in (s_i^P, s_i^S).

[‖] Again, the formula for D_i is obtained after a few algebraic simplifications that are omitted for brevity.

The arguments are the same as in the proof of Proposition 3. The formulas below are for the limit in which $h \to 0$.

When player i expects the opponent to share information according to a Gaussian garbling ρ_j with precision h_j and shares her own information according to a Gaussian garbling ρ_i' with precision h_i', then, given $s_i = (s_i^P, s_i^S)$, player i's expectation of the fundamental variable ω is equal to

$$\mathbb{E}\left[\omega|s_i;\rho_i',\rho_j\right] = \frac{1}{1+h_i^S(\rho_i',\rho_j)}s_i^P + \frac{h_i^S(\rho_i',\rho_j)}{1+h_i^S(\rho_i',\rho_j)}s_i^S$$

where

$$h_i^S(\rho_i',\rho_j) \equiv \left[var(\varepsilon_j+\gamma_j)|\rho_i',\rho_j\right]^{-1} = \frac{h_j}{1+h_j}$$

is the total precision of player i's secondary signal. Similarly,

$$\mathbb{E}\left[s_j^S|s_i;\rho_i',\rho_j\right] = s_i^P,$$

and

$$\mathbb{E}\left[s_j^P|s_i;\rho_i',\rho_j\right] = \tfrac{1}{1+2h_j}s_i^P + \tfrac{2h_j}{1+2h_j}s_i^S.$$

For any profile $\rho = (\rho_i, \rho_j)$ of Gaussian garblings, with precisions (h_i, h_j), the unique (square-integrable) BNE is such that, for any i, any (s_i^P, s_i^S), σ_i^ρ selects with certainty the action $a_i = m_i^\rho s_i^P + (1-m_i^\rho)s_i^S$, with

$$m_i^\rho = \frac{1+2h_i+h_j(1-\beta)+2h_ih_j(1-\beta^2)}{1+2(h_i+h_j)+4h_ih_j(1-\beta^2)}. \tag{11}$$

Now suppose the two players are expected to choose the Gaussian garblings $\rho = (\rho_i, \rho_j)$, with precisions (h_i, h_j), and then follow the associated BNE strategies σ^ρ. When player i deviates and selects the Gaussian garbling ρ_i', with precision h_i', for any $s_i = (s_i^P, s_i^S)$, she then optimally selects the action

$$a_i = m_i^{\rho_i';\rho}s_i^P + (1-m_i^{\rho_i';\rho})s_i^S$$

where

$$m_i^{\rho_i';\rho} \equiv \frac{1+h_j(1+\beta)-2\beta h_j m_j^\rho}{1+2h_j}, \tag{12}$$

with m_j^ρ as in (11) but applied to player j.

In turn, this implies that, when the two players are expected to choose the Gaussian garblings $\rho = (\rho_i, \rho_j)$, with precisions (h_i, h_j), and, instead, player i chooses the Gaussian garbling ρ_i', with precision h_i', her ex-ante expected payoff (gross of the cognitive cost C_i but net of all terms that do not affect individual best responses) is equal to

$$V_i\big(\rho_i';\rho,\sigma^\rho\big) = -(1-\beta)\left(m_i^{\rho_i';\rho}\right)^2 - (1-\beta)\left(1-m_i^{\rho_i';\rho}\right)^2\left(1+\tfrac{1}{h_j}\right)$$

$$-\beta\left[m_i^{\rho_i';\rho} - \left(1-m_j^\rho\right)\right]^2 - \beta\left[\left(1-m_i^{\rho_i';\rho}\right) - m_j^\rho\right]^2 - \beta\left(1-m_i^{\rho_i';\rho}\right)^2\tfrac{1}{h_j} - \beta\left(1-m_j^\rho\right)^2\tfrac{1}{h_i'}.$$

Finally, take any pair of Gaussian garblings for player i, $\hat{\rho}_i$ and ρ_i (with precisions $\hat{h}_i$ and h_i, respectively) and let $\rho' = (\rho_i', \rho_j')$ and $\rho'' = (\rho_i'', \rho_j'')$ be two arbitrary profiles of Gaussian garblings, with precisions (h_i', h_j') and (h_i'', h_j''), respectively. As in the proof of the previous proposition, let

$$D_i(\hat{\rho}_i,\rho_i;\rho',\rho'') \equiv \left[V_i\big(\hat{\rho}_i;\rho',\sigma^{\rho'}\big) - V_i\big(\rho_i;\rho',\sigma^{\rho'}\big)\right] - \left[V_i\big(\hat{\rho}_i;\rho'',\sigma^{\rho''}\big) - V_i\big(\rho_i;\rho'',\sigma^{\rho''}\big)\right],$$

where $\sigma^{\rho'}$ and $\sigma^{\rho''}$are the BNEs associated with the profiles ρ' and ρ''.

Using the characterization of the V_i functions above, after some algebra, we have that

$$D_i(\hat{\rho}_i,\rho_i;\rho',\rho'') = \beta\left[1-m_j^{\rho'} - \left(1-m_j^{\rho''}\right)\right]\left(2-m_j^{\rho'}-m_j^{\rho''}\right)\left(\tfrac{1}{h_i}-\tfrac{1}{\hat{h}_i}\right),$$

from which we obtain that $D_i(\hat{\rho}_i,\rho_i;\rho',\rho'') \overset{\text{sgn}}{=} \beta\left(\hat{h}_i - h_i\right)H_i(\rho',\rho'')$, where

$$H_i(\rho',\rho'') \equiv h_i' - h_i'' + 2h_i'h_j'(1-\beta) + 2h_i'h_j'' - 2h_i''h_j''(1-\beta) - 2h_i''h_j'$$

$$+4\beta h_i'h_i''\left(h_j''-h_j'\right)(1-\beta) + 4\left(h_i'-h_i''\right)h_j'h_j''(1-\beta).$$

To see whether this game satisfies UEC for player i, take $\rho' = (\hat{\rho}_i, \rho_j)$ and $\rho'' = (\rho_i, \rho_j)$. For these profiles, we have that

$$H_i(\rho',\rho'') = \left(\hat{h}_i - h_i\right)\left(1+4h_j^2(1-\beta)+2h_j(1-\beta)+2h_j\right).$$

Hence, $D_i(\hat{\rho}_i,\rho_i;\rho',\rho'') \overset{\text{sgn}}{=} \beta$. This game thus satisfies UEC for complements but not for substitutes.

Next, to see whether this game satisfies ID for player i, take $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\hat{\rho}_i, \rho_j)$. For these profiles, we have that

$$H_i(\rho',\rho'') = -2\beta\hat{h}_i\left(\hat{h}_j - h_j\right)\left(1+2\hat{h}_i(1-\beta)\right).$$

Hence, $D_i(\hat{\rho}_i,\rho_i;\rho',\rho'') \overset{\text{sgn}}{=} -\left(\hat{h}_i - h_i\right)\left(\hat{h}_j - h_j\right)$. Independently of the sign of β, this game thus satisfies the form of negative ID in the proposition.

Finally, to see whether this game satisfies EC for player i, consider the profiles $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$

and $\rho'' = (\rho_i, \rho_j)$. For these profiles

$$H_i(\rho', \rho'') = (\hat{h}_i - h_i)(1 + 2h_j)(1 + 2(1-\beta)\hat{h}_j) - 2\beta h_i(\hat{h}_j - h_j)(1 + 2(1-\beta)\hat{h}_i).$$

and hence

$$D_i(\hat{\rho}_i, \rho_i; \rho', \rho'') \overset{\text{sgn}}{=}$$
$$\beta\left(\hat{h}_i - h_i\right)\left[(\hat{h}_i - h_i)(1 + 2h_j)(1 + 2(1-\beta)\hat{h}_j) - 2\beta h_i(\hat{h}_j - h_j)(1 + 2(1-\beta)\hat{h}_i)\right]$$

which yield the result in the proposition. Q.E.D.

Proof of Corollary 2. Consider two symmetric profiles (ρ, σ^{ρ}) and $(\hat{\rho}, \sigma^{\hat{\rho}})$ where ρ and $\hat{\rho}$ are Gaussian garblings, with precisions $h_i = h_j = h^L$ and $\hat{h}_i = \hat{h}_j = h^H$, with $h^H > h^L$, and where σ^{ρ} and $\sigma^{\hat{\rho}}$ are the (unique square-integrable) BNEs associated with these profiles. The result then follows from Proposition 4 along with the fact that the Condition for EC in (10) is satisfied for $\beta \in [0, 1)$ but not for $\beta \in (-1, 0)$. Q.E.D.

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