Expectation Conformity in Strategic Cognition*

Supplementary Material

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Abstract

This document contains additional applications. All sections, conditions, and results specific to this document have the suffix “S” to avoid confusion with the corresponding parts in the main text. Section S.1 considers situations in which the payoff state is highly dimensional, the players learn about a few dimensions and then act as if the non-explored dimensions did not exist (as in the literature on sparsity). Section S.2 considers games in which players learn about other players' exogenous information (espionage) or take actions that affect other players' ability to learn about their own information (counter-espionage). Section S.3 considers generalized career concerns in which cognition determines the effectiveness of signal-jamming. Section S.4 considers framing games in which cognition affects other players' recollection of favorable (alternatively, unfavorable) information. Section S.5 considers games in which cognition determines the depth of reasoning in the level-k model. Section S.6 collects all proofs for the results in this supplement.

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S.1 Sparsity

Sparsity refers to situations in which a decision maker decides to pay attention only to a few dimensions of a highly-dimensional payoff state and then acts as if the non-explored dimensions did not exist. Sparsity plays an important role in many socio-economic environments (see, e.g., Gabaix, 2014). It is intended to capture a form of bounded rationality. Below we show that its key features are also consistent with a certain representation where all players are fully rational.

We introduce the formal framework in Subsection S.1.1. We then establish a few key properties in Subsection S.1.2. In particular, we show that a player who goes "deeper" into the exploration of the state can perfectly predict the behavior of any player who explores fewer dimensions, whereas a player who explores fewer dimensions than her opponents reasons and acts as if the opponents explored the same dimensions that she explored, despite knowing that this is not the case. In Subsection S.1.3, we then investigate expectation conformity in these games and relate it to whether downstream actions are strategic complements or substitutes. Finally, in Subsection S.1.4 we use expectation conformity to establish equilibrium multiplicity and identify various features of the equilibrium set. Specifically, we show that all pure-strategy equilibria are necessarily symmetric when actions in the downstream game are strategic complements, whereas, when they are substitutes, there exists at most one symmetric pure-strategy equilibrium, typically co-existing with many asymmetric pure-strategy equilibria. Furthermore, while, in the complements case, total welfare is maximal in the symmetric equilibrium featuring the largest cognition, in the substitute case, it is maximal in the asymmetric equilibrium featuring the lowest cognition for the player who is behind in the exploration of the state.

S.1.1 Framework

The players' payoffs are given by

$$u_i(a_i, a_{-i}, \omega) = -(1-\beta)(a_i - g(\omega))^2 - \beta(a_i - \bar{a}_{-i})^2 + \psi(a_{-i}, \omega), \tag{S.1}$$

where $\beta \in (-1, 1)$, $a_i \in A_i = \mathbb{R}$, $a_{-i} \equiv (a_1, ..., a_{i-1}, a_{i+1}, ..., a_n) \in \mathbb{R}^{n-1}$, $\omega \equiv (\omega^k)_{k=1}^K \in \mathbb{R}^K$ for some $K \in \mathbb{N} \cup \{+\infty\}$, $\bar{a}_{-i} \equiv \Sigma_{j \neq i} a_j/(n-1)$, $g : \mathbb{R}^K \to \mathbb{R}$, and $\psi : \mathbb{R}^{K+n-1} \to \mathbb{R}$. The payoff state ω is thus a collection of "fundamental variables." The function g aggregates such variables into a uni-dimensional statistics $g(\omega)$ to be interpreted as a moving target. The variable $\bar{a}_{-i}$ is the average action of player i's opponents. Finally, the scalar β parametrizes the strategic interactions. The case $\beta > 0$ corresponds to a strategic situation in which actions are *strategic complements*, whereas the case $\beta < 0$ corresponds to a situation in which actions are *strategic substitutes.*[1] Finally, the function $\psi(a_{-i}, \omega)$ summarizes various external effects that matter for payoffs (and hence for welfare analysis) but do not play any role for the selection of the individual best responses.

The statistics $g(\omega)$ takes the form $g(\omega) = (1 + \Sigma_{k=1}^K \omega^k)/(1-\beta)$. It is commonly believed

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[1] The formulas below extend to the case in which $\beta = 0$, but this case is not interesting, for it corresponds to a non-strategic situation.

that each dimension ω^k is drawn independently from the other dimensions from a distribution F^k with zero mean and variance σ_k^2. As shown below, the combination of these assumptions with the quadratic payoff specification in (S.1) permits us to capture the essence of sparsity in an otherwise fully-rational model.

There is a natural progression in reasoning, whereby certain dimensions must be explored before others. In such situations, a player's cognition corresponds to the number of dimensions explored. Specifically, each player learns the realization of the various dimensions of the state ω in sequence. That is, learning the realization of ω^k requires having learnt the first $k-1$ dimensions $(\omega^1, \ldots, \omega^{k-1})$.[2] This assumption is not really needed for our analysis of the role that expectation conformity plays in such games. It has the advantage, though, of permitting us to interpret each player's cognition $\rho_i \in \mathbb{N}$ with her "depth of knowledge," that is, with the number of dimensions of the state ω the player learns about.[3]

S.1.2 Preliminary analysis

An implication of the above formalism is that a player who decides to learn ρ_i dimensions of the state reasons "as if" the remaining $K - \rho_i$ dimensions did not exist (that is, as if the state had only ρ_i dimensions). This is one of the key features of sparsity. A second key feature (which the above formalization captures) is that a player who goes deeper in the exploration of the state than her opponents is able to perfectly predict their behavior, whereas a player who explores fewer dimensions than her opponents reasons (and acts) as if the opponents explored the same dimensions that she did, despite knowing that this is not the case.

To see why the above properties hold, but also to appreciate their implications for expectation conformity and equilibrium determinacy in the next subsections, let $s_i \equiv (\omega^1, ..., \omega^{\rho_i})$ be the subset of the state ω explored by player i. We then have that, for any (ω, ρ), $Q(s|\omega, \rho)$ is a Dirac measure assigning probability one to the signal vector $s = (s_1, ..., s_n)$, where n is the number of players.

To facilitate the exposition, from now on assume that $n = 2$ (two players) and, without loss of generality, let player 1 be the player with the lowest depth of knowledge, that is, for whom $\rho_1 \leq \rho_2$.[4]

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[2] As discussed below, this assumption can be micro-founded. For example, when the cognitive costs C_i are separable, one can order the dimensions of the state so that the benefit of exploring each dimension relative to its cost is decreasing in k. When $\beta \geq 0$, such an assumption suffices to guarantee that the equilibria of the game in which the players must explore the dimensions in an increasing sequence are also equilibria in a game in which they can choose the order of the explorations. When, instead, $\beta < 0$, the same conclusion holds, provided that, in addition, one assumes that the benefit of exploring each dimension, relative to its cost, declines sufficiently fast with k to make it suboptimal for the players to explore the dimensions not sequentially as a way of differentiating their actions from those of their opponents.

[3] When cognition is not ordered, so that a player can learn dimension k without having learned all previous dimensions, cognition ρ_i takes the form of a vector of zeros and ones, where each entry indexes whether that dimension has been explored. The analysis below can also be extended to accommodate for "clusters," with dimensions correlated within clusters but independent across clusters (say, with each dimension affected by a cluster-specific shock plus a dimension-specific shock).

[4] The qualitative insights carry over to games with more than two players (i.e., to $n > 2$). The structure of the best responses, however, is more tedious. However, in the special case in which the cognitive profiles of interest ρ and $\hat{\rho}$ are symmetric profiles (i.e., $\rho_i = \rho_j$ and $\hat{\rho}_i = \hat{\rho}_j$ for all $i, j \in I$), the results for the $n > 2$ case coincide with those for the $n = 2$ case.

Given the cognitive profile $\rho = (\rho_1, \rho_2)$, there exists a unique continuation BNE σ^ρ and is such that, for any ω, the two players' equilibrium strategies are Dirac distributions assigning probability one to the actions[5]

$$a_1^\rho(s_1) = \frac{1 + \Sigma_{k=1}^{\rho_1} \omega^k}{1-\beta}$$

and[6]

$$a_2^\rho(s_2) = \frac{1 + \Sigma_{k=1}^{\rho_1} \omega^k}{1-\beta} + \Sigma_{k=\rho_1+1}^{\rho_2} \omega^k.$$

The BNE actions thus reflect the property anticipated above that each player acts "as if" each neglected dimension $k > \rho_i$ did not exist. In particular, the equilibrium action of the player who is behind in the exploration of the state (player 1) is invariant in how far ahead the opponent is in the exploration of the state and coincides with the equilibrium action $a_1^{\rho_1,\rho_1}(s_1)$ that the player would choose if the state was commonly known to have only ρ_1 dimensions, with the dimensions $(\omega^1, ..., \omega^{\rho_1})$ commonly known to both players. Similarly, the equilibrium action for the player who is ahead in the exploration of the state (player 2) coincides with the equilibrium action of the player who is behind, $a_1^\rho(s_1)$, augmented by the extra knowledge $\Sigma_{k=\rho_1+1}^{\rho_2} \omega^k$ that player 2 has about the gross return to her action. The action $a_2^\rho(s_2)$ also coincides with the player's equilibrium action when she expects the state to have only ρ_2 dimensions and her opponent to explore only the first $\rho_1 < \rho_2$ dimensions.

When both players expect cognition $\rho = (\rho_1, \rho_2)$, their ex-ante expected payoffs (gross of the cognitive costs, C_i, and of the expectation of the terms $(1-\beta)g(\omega)^2 + \beta \bar{a}_{-i}^2 + \psi(a_{-i}, \omega)$ that do not interact with the players' best responses) are then equal to (see the proof of Proposition S.1 in the Appendix for details)

$$V_1(\rho_1; \rho, \sigma^\rho) = \frac{1 + \Sigma_{k=1}^{\rho_1} \sigma_k^2}{(1-\beta)^2}$$

and

$$V_2(\rho_2; \rho, \sigma^\rho) = V_1(\rho_1; \rho, \sigma^\rho) + \Sigma_{k=\rho_1+1}^{\rho_2} \sigma_k^2.$$

Note that an implication of this structure is that the various shocks ω^k impact the players' best responses and payoffs separately.

To gather more intuition for the above results (but also for some of the properties derived in the two subsections below) consider for a moment a simplified version of the above problem in which $K = 1$ (that is, the state is unidimensional). Abusing notation, then drop the superscript "1" from the state ω and let σ^2 denote the variance of ω. Furthermore, let $\rho = (\rho_i, \rho_j)$ denote an arbitrary cognitive profile that is commonly expected by both players, and abandon for a moment the convention that player 1 is behind in the exploration of the state, so as to treat the two players

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[5] The computation of the equilibrium actions, as well as the derivation of the interim expected gross payoffs below, is in the Appendix at the end of the supplement (in the proof of Proposition S.1).

[6] One can think of dimension $\omega^0 = 1$ as the component of the state that is exogenously known to the players, $\omega^k/(1-\beta)$ as the contribution of dimension k to the return to the player's "investment" a_i when such a component is commonly known, and ω^k as the contribution of the same dimension to the player's return when individually known.

symmetrically.

In this simplified problem, $\rho_i \in \{0, 1\}$, with $\rho_i = 1$ denoting the decision to learn the state, and $\rho_i = 0$ the decision to remain uninformed. When $\rho_i = 1$, player i receives the signal $s_i = \omega$ with certainty, whereas, when $\rho_i = 0$, she receives the null signal $s_i = \emptyset$ with certainty.

In the downstream game, player i's optimal action is then given by

$$a_i = 1 + \mathbb{E}[\omega|\rho_i, s_i] + \beta\mathbb{E}[a_j|\rho_i, s_i],$$

where $\mathbb{E}[\omega|\rho_i, s_i] = s_i = \omega$ if $\rho_i = 1$ (that is, if player i learned the state) and $\mathbb{E}[\omega|\rho_i, s_i] = 0$ if $\rho_i = 0$ (in which case $s_i = \emptyset$).

Paralleling the analysis also for player j, we then have that, when the two players are expected to engage in cognition $\rho = (\rho_i,\rho_j)$ and, instead, player i chooses cognition ρ_i', in the downstream game, player i then optimally chooses an action equal to

$$a_i \quad = \quad \tfrac{1}{1-\beta} + \rho_i' I_i^\rho \omega,$$

where

$$I_i^\rho \equiv \frac{1 + \beta\rho_j}{(1 - \beta^2\rho_i\rho_j)}$$

denotes the *impulse response* of player i's action to the state ω, when player i chooses to learn the state (i.e., $\rho_i' = 1$) and the two players are expected to engage in cognition $\rho = (\rho_i,\rho_j)$. Such an impulse response naturally reflects the following properties: (a) when player j is not expected to learn the state (i.e., $\rho_j = 0$), the value to player i from learning the state comes entirely from being able to respond to it, with no effect on her ability to respond to her opponent's action (the impulse response is then equal to $I_i^\rho = 1$, irrespectively of whether $\rho_i = 0$, or $\rho_i = 1$); (b) when, instead, player j is the only player expected to learn the state (i.e., $(\rho_i, \rho_j) = (0, 1)$), the value to player i from learning the state also accounts for the improvement in her ability to respond to variations in her opponent's action but without the opponent accommodating for the unanticipated joint learning (the impulse response is then equal to $I_i^\rho = 1 + \beta$ which is higher than 1 when action are complements, and lower than 1 when they are substitutes); (c) finally, when both players are expected to learn the state (i.e., $\rho = (1, 1)$), the impulse response is equal to $I_i^\rho = 1/(1 - \beta)$, which is higher than $1+\beta$ no matter whether actions are complements or substitutes, reflecting the benefit that player i derives from player j responding to the state accounting for joint learning.

We can then use the above properties to compute player i's value of information

$$\mathbb{V}_i^\rho \equiv V_i(1; \rho, \sigma^\rho) - V_i(0; \rho, \sigma^\rho)$$

as a function of the cognitive profile ρ expected from the two players. The results are summarized in Table 1 below which represents the impact of agent i's selection of cognition $\rho_i' = 1$ instead of $\rho_i' = 0$ on the player's behavior and payoff.

Hence, under strategic complements ($\beta > 0$), $\mathbb{V}_i^{(1,1)} > \mathbb{V}_i^{(0,1)} > \mathbb{V}_i^{(1,0)} = \mathbb{V}_i^{(0,0)}$. The value of

Equilibrium cognition: $\rho = (\rho_i, \rho_j)$	Shared knowledge: $(1,1)$	Shared ignorance: $(0,0)$	Asymmetric knowledge	
			i informed: $(1,0)$	i uninformed: $(0,1)$
Impulse response, I_i^ρ	$\frac{1}{1-\beta}$	1	1	$1+\beta$
Value of information, $\mathbb{V}_i^\rho$	$\frac{\sigma^2}{(1-\beta)^2}$	σ^2	σ^2	$(1+\beta)^2\sigma^2$

Table 1: Value of information as a function of $\rho = (\rho_i, \rho_j)$.

learning the state is the highest when both players are expected to learn ($\rho = (1,1)$). Learning the state jointly with the opponent but without the opponent expecting the player to learn (i.e., $\rho = (0,1)$) is less valuable but still superior to being the sole learner ($\rho = (1,0)$ or $\rho = (0,0)$). Under strategic substitutes, instead, $\mathbb{V}_i^{(1,0)} = \mathbb{V}_i^{(0,0)} > \mathbb{V}_i^{(1,1)} > \mathbb{V}_i^{(0,1)}$. The value of learning the state is highest when the other player does not learn($\rho = (1,0)$ or $\rho = (0,0)$). Conditional on the other player learning the state, player i is better off if player j anticipates joint learning ($\rho = (1,1)$) than if she doesn't ($\rho = (0,1)$): this is because, in the former case, player j "makes room" to player i by reducing her response to the state.

S.1.3 Expectation conformity

Returning to the case in which $K > 1$, we can then use the properties established in the previous subsection to investigate expectation conformity (EC) in these games and its decomposition into unilateral expectation conformity (UEC) and increasing differences (ID).[7]

Proposition S.1 (sparsity–EC). *(a) Consider any pair of profiles* $(\hat{\rho}, \sigma^{\hat{\rho}})$ *and* (ρ, σ^ρ) *such that* $\hat{\rho}_2 > \rho_2 \geq \hat{\rho}_1 > \rho_1$*. Irrespective of whether* $\beta > 0$ *(strategic complementarity) or* $\beta < 0$ *(strategic substitutability), UEC holds for such profiles (strictly for player 1, weakly for player 2), whereas ID holds as an equality for both players. As a result, EC holds strictly for player 1, but only weakly for player 2. (b) Next, consider any pair of profiles* $(\hat{\rho}, \sigma^{\hat{\rho}})$ *and* (ρ, σ^ρ) *such that* $\hat{\rho}_2 = \hat{\rho}_1 > \rho_2 = \rho_1$*. UEC holds as an equality for these profiles, whereas ID holds if* $\beta > 0$ *but does not hold if* $\beta < 0$ *(that is,* $\beta\Gamma_i^{ID}(\rho, \hat{\rho}) > 0$*,* $i = 1, 2$*).*

Consider first profiles $(\hat{\rho}, \sigma^{\hat{\rho}})$ and (ρ, σ^ρ) such that player 2 is always ahead in the exploration of the state (i.e., $\hat{\rho}_2 > \rho_2 \geq \hat{\rho}_1 > \rho_1$), possibly reflecting significant differences in the cost of cognition

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[7] When we say that EC holds strictly (alternatively, weakly) for player i we mean that the inequality

$$\Gamma_i^{EC}(\rho, \hat{\rho}) \equiv \Big[V_i(\hat{\rho}_i; \hat{\rho}) - V_i(\rho_i; \hat{\rho})\Big] - \Big[V_i(\hat{\rho}_i; \rho) - V_i(\rho_i; \rho)\Big]$$

is strict (alternatively, weak). Similarly for UEC, i.e., for

$$\Gamma_i^{UEC}(\rho, \hat{\rho}) \equiv \Big[V_i(\hat{\rho}_i; \hat{\rho}_i\,, \rho_{-i}) - V_i(\rho_i; \hat{\rho}_i\,, \rho_{-i})\Big] - \Big[V_i(\hat{\rho}_i; \rho_i\,, \rho_{-i}) - V_i(\rho_i; \rho_i\,, \rho_{-i})\Big]$$

and for ID, i.e., for

$$\Gamma_i^{ID}(\rho, \hat{\rho}) \equiv \Big[V_i(\hat{\rho}_i; \hat{\rho}_i\,, \hat{\rho}_{-i}) - V_i(\rho_i; \hat{\rho}_i\,, \hat{\rho}_{-i})\Big] - \Big[V_i(\hat{\rho}_i; \hat{\rho}_i\,, \rho_{-i}) - V_i(\rho_i; \hat{\rho}_i\,, \rho_{-i})\Big].$$

between the two players. As anticipated above, for the player who is ahead in the exploration of the state (player 2), the value of exploring more dimensions is the same no matter whether the opponent (player 1) expects player 2 to explore $\hat{\rho}_2 \geq \rho_1$ or $\rho_2 \geq \rho_1$ dimensions. This is because player 1's action in the downstream game is the same under both expectations (recall that a key feature of the sparsity model is that the behavior of the player who is behind in the exploration of the state is invariant in how far ahead she expects the opponent to be in the exploration of the state). For player 1, instead, the value of exploring more dimensions is larger when the opponent expects her to explore more dimensions. As explained above, this is because player 2 changes her response to the extra dimensions $(\rho_1 + 1, ..., \hat{\rho}_1)$ explored by player 1, when she expects player 1 to learn such dimensions. In particular, player 2 increases her responses to such dimensions when the actions are complements, whereas she reduces her response to such dimensions when they are substitutes. In either case, player 1 benefits from the adjustment in player 2's response. Hence, no matter the sign of β, UEC holds for these profiles (strictly for player 1, weakly for player 2).

That, for such profiles, ID holds as an equality for both players reflects the separability of the expected payoffs in the explored dimensions. For player 1, the value of expanding her cognition from ρ_1 to $\hat{\rho}_1 \leq \rho_2 < \hat{\rho}_2$ is invariant in how far ahead player 2 is in the exploration of the state, reflecting the fact that player 1 treats all dimensions that she does not explore as if they did not exist. Likewise, the value that player 2 assigns to expanding her cognition from ρ_2 to $\hat{\rho}_2$ is invariant to whether she expects player 1's cognition to be $\hat{\rho}_1$ or ρ_1 for, in either case, player 1 does not respond to the dimensions $(\rho_2 + 1, ..., \hat{\rho}_2)$.

Next, consider profiles $(\hat{\rho}, \sigma^{\hat{\rho}})$ and (ρ, σ^{ρ}) such that $\hat{\rho}_2 = \hat{\rho}_1 > \rho_2 = \rho_1$. The reason for considering such profiles is the interest in understanding whether multiple symmetric (pure-strategy) equilibria are possible in these games. When a player expands her cognition starting from the cognitive level of her opponent, the value she assigns to the expansion is the same no matter whether the opponent anticipates the expansion. This is because the opponent does not respond to any dimension that she does not explore and responds to the explored dimensions as if the unexplored ones did not exist. Hence, UEC holds as an equality for these profiles. Fixing the opponent's expectations about her own cognition, we have that the value that player i assigns to exploring more dimensions is larger when the opponent also explores them and actions are strategic complements, whereas it is lower when the opponent also explores the extra dimensions and actions are strategic substitutes. This is because, as explained above, the value of joint learning is higher than the value of sole learning when actions are complements, whereas it is lower when actions are substitutes.

S.1.4 Equilibrium analysis

When combined with Proposition 1 in the main text, the results in part (a) of Proposition S.1 thus suggests that, when the players face asymmetric cost functions, these games may admit multiple asymmetric (pure-strategy) equilibria, both in the complements and in the substitutes case. The results in part (b), instead, suggest that, when the cost functions are symmetric, these games are likely to feature a unique symmetric (pure-strategy) equilibrium when actions are substitutes, and multiple symmetric (pure-strategy) equilibria when actions are complements. The next two propositions verify these conjectures by considering cost functions that are separable over the explored dimensions. The propositions also provide a more detailed account of the type of multiplicity these games are prone to.

Proposition S.2 (**sparsity–equilibrium-analysis-complements**). *Suppose that the actions in the primitive game are strategic complements (i.e., $\beta \in (0,1)$) and that cognitive costs are symmetric across players and take the form $C_i(\rho_i) = \Sigma_{k=1}^{\rho_i} c_k$, with $c \equiv \left(c^k\right)_{k=1}^{K} \in \mathbb{R}_{++}^{K}$ such that σ_k^2/c_k is strictly decreasing in k.*

- *All (pure-strategy) equilibria are symmetric.*
- *Let*
$$\underline{k} \equiv \min\left\{k \middle| \sigma_k^2 \leq c_k\right\} \qquad \text{and} \qquad \bar{k}(\beta) \equiv \max\left\{k \middle| \frac{\sigma_k^2}{(1-\beta)^2} \geq c_k\right\}.$$
Any level of cognition $k^* \in [\underline{k}, \bar{k}(\beta)] \cap \mathbb{N}$ can be part of a symmetric (pure-strategy) equilibrium, and only these levels can be sustained in a symmetric (pure-strategy) equilibrium.
- *Suppose that there are no external payoff effects, meaning that individual payoffs u_i depend on the state ω and on other agents' actions a_{-i} only through the effects that the latter have on individual best responses.*[8] *Then the (pure-strategy) equilibria are Pareto ranked, with the players' net payoff increasing in the equilibrium depth of knowledge k^*.*
- *All equilibria of the game in which cognition is ordered are also equilibria of the game in which cognition is unordered.*[9] *Furthermore, when, for any $k \in [\underline{k}, \bar{k}(\beta)] \cap \mathbb{N}$,*
$$\frac{\sigma_{k-1}^2}{c_{k-1}} > \frac{\sigma_k^2}{c_k(1-\beta)^2}, \tag{S.2}$$
the converse is also true: any equilibrium of the game in which cognition is unordered is also an equilibrium of the game in which cognition is ordered.

That, when actions are complements and costs are symmetric across players, all equilibria are symmetric follows from the fact that the net benefit of learning the k-th dimension is larger when players jointly learn that dimension than when a player is the sole learner of that dimension. This property in turn implies that, if an asymmetric equilibrium existed in which player 2 learns more dimensions than player 1, then player 1 would have a profitable deviation. Hence there do not exist equilibria with asymmetric levels of cognition. It is also easy to see that, when the two players

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[8] When payoffs are as in (S.1), this occurs when $\psi(a_{-i}, \omega) = (1-\beta)g(\omega)^2 + \beta\bar{a}_{-i}^2$. In this case, the term $-(1-\beta)g(\omega)^2$ from the first addendum in (S.1) is perfectly offset by the corresponding term in ψ and likewise for the externality term $\beta\bar{a}_{-i}^2$ originating in the second addendum in (S.1). As a result, individual payoffs u_i depend on ω and on other agents' actions a_{-i} only through the effect that the latter terms have on individual best responses.

[9] Recall that cognition is ordered when each dimension k can be explored only if all lower dimensions $l = 1, ..., k-1$ are also explored. Cognition is unordered when players can explore each dimension k without having explored all lower dimensions $l < k$.

select the same cognition, they then play the same actions. Hence asymmetric equilibria where the asymmetry originates solely in the actions in the primitive game do not exist either.

That, in any symmetric equilibrium, the cognitive level k^* must exceed $\underline{k}$ follows from the fact that, if this was not the case, then a player would have incentives to deviate and learn the extra dimension $k^* + 1$ even if such dimension is explored solely (the net value of being the sole learner of dimension k is equal to $\sigma_k^2 - c_k$, which is strictly positive for any $k < \underline{k}$). Likewise, that the equilibrium depth of knowledge k^* must not exceed $\bar{k}(\beta)$ follows from the fact the net benefit of learning any dimension $k > \bar{k}(\beta)$ is negative even when learnt jointly with the other player $(\sigma_k^2/(1-\beta)^2 - c_k < 0$ for $k > \bar{k}(\beta))$. That, for any $k \in [\underline{k}, \bar{k}(\beta)] \cap \mathbb{N}$, a symmetric equilibrium with depth of cognition k^* exists in turn follows from the characterization of the payoff functions $V_i(\rho_i'; \rho)$ in the previous subsection, along with the fact that σ_k^2/c_k is decreasing in k.

Finally, that all equilibria in the game in which cognition is ordered are also equilibria in the game in which cognition is unordered follows from the fact that (a) the benefit σ_k^2/c_k of learning dimension k, relative to its cost, is decreasing in k, along with (b) the separability of the players' payoffs in the explored dimensions. That the converse is also true when Condition (S.2) holds for any $k \in [\underline{k}, \bar{k}(\beta)] \cap \mathbb{N}$ follows from the fact that any equilibrium of the game in which cognition is unordered is necessarily symmetric, with the smallest dimension explored greater than $\underline{k}$ and the largest one smaller than $\bar{k}(\beta)$, along with the fact that, by virtue of Condition (S.2), when exploring dimension $k \in [\underline{k}, \bar{k}(\beta)] \cap \mathbb{N}$ jointly with the other player is profitable, the net benefit of exploring any lower dimension $k' < k$ is strictly positive even if such a dimension is explored solely.

The above results thus imply that, when downstream actions are complements and payoffs are symmetric across players and separable over the dimensions explored, the depth of knowledge is not uniquely pinned down in equilibrium. However, in the absence of external payoff effects, the players are better off the larger the depth of knowledge. This is because more knowledge always permits the players to better align their actions with the underlying state.

We now turn to games in which downstream actions are strategic substitutes. Consistently with the convention above, we refer to player 2 as the player going deeper in the exploration of the state.

Proposition S.3 (sparsity–equilibrium-analysis–substitutes). *Suppose that the actions in the downstream game are strategic substitutes (i.e., $\beta \in (-1, 0)$) and that cognitive costs are symmetric across players and take the linear form $C_i(\rho_i) = \Sigma_{k=1}^{\rho_i} c_k$, with $c \equiv \left(c^k\right)_{k=1}^K \in \mathbb{R}_{++}^K$ such that σ_k^2/c_k is strictly decreasing.*

- *A (pure-strategy) symmetric equilibrium exists if and only if there exists a cognitive level k^* satisfying*
$$\frac{\sigma_{k^*+1}^2}{c_{k^*+1}} \leq 1 \leq \frac{\sigma_{k^*}^2}{(1-\beta)^2 c_{k^*}}. \tag{S.3}$$
When a symmetric (pure-strategy) equilibrium exists, it is unique and its cognitive level k^* satisfies (S.3).

- *There may exist asymmetric (pure-strategy) equilibria. In any such equilibrium, the two players' cognitive levels ρ_1 and ρ_2 belong to $[\underline{\underline{k}}(\beta), \bar{\bar{k}}]$ where*

  $$\underline{\underline{k}}(\beta) \equiv \min \left\{ k \middle| \sigma_{k+1}^2 (1+\beta)^2 \leq c_{k+1} \right\} \qquad \text{and} \qquad \bar{\bar{k}} \equiv \max \left\{ k \middle| \sigma_k^2 \geq c_k \right\}$$

  and are such that

  $$\frac{\sigma_{\rho_1+1}^2 (1+\beta)^2}{c_{\rho_1+1}} \leq 1 \leq \frac{\sigma_{\rho_1}^2}{(1-\beta)^2 c_{\rho_1}} \tag{S.4}$$

  and

  $$\frac{\sigma_{\rho_2+1}^2}{c_{\rho_2+1}} \leq 1 \leq \frac{\sigma_{\rho_2}^2}{c_{\rho_2}}. \tag{S.5}$$

  Player 1's equilibrium payoff is increasing in her depth of knowledge ρ_1 and invariant in player 2's depth of knowledge ρ_2. Player 2's equilibrium payoff is decreasing in player 1's depth of knowledge ρ_1 and, in case there are multiple solutions ρ_2 to the double inequality in (S.5), is invariant in ρ_2.[10] *The sum of the two players' equilibrium payoffs is maximal under the (pure-strategy) equilibrium featuring the lowest cognition for player 1 (the one behind in the exploration of the state).*

- *All the equilibria of the game in which cognition is ordered are also equilibria of the game in which cognition is unordered. Furthermore, when, for any $k \in [\underline{\underline{k}}(\beta), \bar{\bar{k}}]$,*

  $$\frac{\sigma_{k-1}^2 (1+\beta)^2}{c_{k-1}} > \frac{\sigma_k^2}{c_k}, \tag{S.6}$$

  the converse is also true: any equilibrium of the game in which cognition is unordered is also an equilibrium of the game in which cognition is ordered.

That, with substitutes, such games admit at most one symmetric equilibrium follows from the fact that the gross benefit of learning any dimension is largest when learnt solely. Furthermore, when a dimension is jointly learnt, the gross benefit of learning it is larger when the opponent expects joint learning than when she expects to be the sole learner (recall the discussion about EC and its components in the previous section).[11] The double inequality in Condition (S.3) then guarantees that, starting from a situation where both players learn the same dimensions, no player has a profitable deviation. Clearly, there is at most one dimension k^* satisfying the double inequality in (S.3). Hence, there exists at most one symmetric (pure-strategy) equilibrium and, when such an equilibrium exists, the equilibrium cognition k^* is given by the unique solution to the double inequality in Condition (S.3).

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[10] This is because multiple solutions to the two inequalities in (S.5) are possible only if either $\frac{\sigma_{\rho_2+1}^2}{c_{\rho_2+1}} = 1$ or $\frac{\sigma_{\rho_2}^2}{c_{\rho_2}} = 1$. In the first case, both ρ_2 and $\rho_2 + 1$ can be sustained in equilibrium. In the second case, both ρ_2 and $\rho_2 - 1$ can be sustained in equilibrium. In either case, under the largest solution, player 2 is indifferent between learning all the equilibrium dimensions and stopping one dimension earlier.

[11] Use also Table 1 to verify such properties.

Next, consider asymmetric (pure-strategy) equilibria. In any such equilibrium, the value to player 2 (the one ahead in the exploration of the state) of increasing (alternatively, decreasing) her cognition by one dimension is invariant in player 1's depth of knowledge. Hence, the number of dimensions explored by player 2 in any such equilibrium must satisfy the double inequality in (S.5). For player 1, instead, the number of dimensions explored in equilibrium depends on what player 2 expects her to do. From the discussion about EC preceding Proposition S.1 in the previous subsection, observe that the term $\sigma_{\rho_1+1}^2(1+\beta)^2 - c_{\rho_1+1}$ in the left-hand side of (S.4) is the net benefit (to player 1) of exploring the (ρ_1+1)-th dimension when player 2 also explores such a dimension but does not expect her to explore it. The term $\sigma_{\rho_1}^2/(1-\beta)^2 - c_{\rho_1}$ in the right-hand side of (S.4), instead, is the net benefit to player 1 of exploring the ρ_1-th dimension when player 2 also explores such a dimension and expects player 1 to do the same. The double inequality, along with the monotonicity of the benefits/cost ratio σ_k^2/c_k in k, then implies that player 1 does not have profitable deviations.

The ranking of the asymmetric equilibria in terms of the players' payoffs naturally reflects the fact that each player's *equilibrium* payoff is increasing in the number of dimensions explored. Hence, these games too are never conducive to situations in which the players are worse off in equilibria in which more knowledge is acquired.

That player 1's equilibrium payoff is invariant in player 2's equilibrium depth of knowledge follows from the fact that both players' response to the dimensions jointly learnt is invariant in the number of dimensions explored solely by player 2, along with the separability of the payoffs. That, instead, player 2's equilibrium payoff is decreasing in player 1's depth of knowledge follows from the fact that, with substitutes, learning a dimension solely is always more valuable than learning it jointly. That the sum of the two players' equilibrium payoffs

$$2\frac{1+\Sigma_{k=1}^{\rho_1}\sigma_k^2}{(1-\beta)^2} + \Sigma_{k=\rho_1+1}^{\rho_2}\sigma_k^2 - 2\Sigma_{k=1}^{\rho_1}c_k - \Sigma_{k=\rho_1+1}^{\rho_2}c_k$$

is maximal under the equilibrium featuring the lowest cognition for player 1 follows from the fact that the net benefit $\sigma_k^2/(1-\beta)^2 - c_k$ that player 1 derives from learning each dimension that player 2 also learns is smaller than the loss $\sigma_k^2 - \sigma_k^2/(1-\beta)^2$ that player 2 incurs when player 1 learns the additional dimension.

Finally, consider the comparison of the equilibria in the game in which cognition is ordered with those in the game in which cognition is unordered. That any equilibrium of the game in which cognition is ordered is also an equilibrium in the game in which cognition is unordered follows from the monotonicity of the relative benefits, σ_k^2/c_k, along with the separability of the payoffs in the dimensions explored. That, under Condition (S.6), any equilibrium in the game in which cognition is unordered coincides with one of the equilibria in the game in which cognition is ordered follows from the following properties. In any equilibrium of the game in which cognition is unordered, the smallest dimension explored is necessarily greater than $\underline{\underline{k}}(\beta)$ whereas the largest one is necessarily smaller than $\bar{\bar{k}}$, exactly as in the game in which cognition is ordered. Condition (S.6) in turn implies that, when the net value to player i of exploring dimension k is positive, so is the value of exploring any lower dimension $k' < k$. This is so irrespective of whether dimensions k and k' are explored solely by player i or jointly with player j, and irrespective of player j's expectations about player i's cognition. These properties follow from the discussion about EC and its drivers in the preceding subsection. Hence, whenever a player explores dimension k, she necessarily explores also any lower dimension $k' < k$.[12]

S.2 Learning about others' beliefs

In this section, we consider a different class of games (inspired by the literature on espionage and noisy communication—see, e.g., Dewatripont and Tirole, 2005, and Calvo-Armengol et al. 2015), in which the players' cognition affects their understanding of other players' information and hence their beliefs. Examples of such situations include instances in which players invest to interpret other players' culture, cognitive traits, and other dimensions of their personality that are responsible for their view of the game, but also instances in which players spy on other players' information to better predict their behavior, as in certain industrial-espionage games. We also consider the possibility that players invest in making themselves understood or, alternatively, in preventing others from understanding them. Examples include situations in which players share their information with others, as those described in the main text, but also situations where firms invest in counter-espionage to guard themselves against rival firms scooping on their research output.

S.2.1 Framework

The primitive game has the same linear-quadratic payoff structure as in the two applications in the main text. Each player is endowed with a "primary" signal equal to

$$s_i^P = \omega + \varepsilon_i,$$

with ε_i drawn from a standard Normal distribution. Such a signal captures the player's exogenous private information (equivalently, his primitive beliefs). In addition to s_i^P, player i receives an endogenous "secondary" signal

$$s_i^S = s_j^P + \gamma_j + \phi_i$$

about the opponent's primary signal, with γ_j drawn from a Normal distribution with mean zero and precision t_j, and with ϕ_i drawn from a Normal distribution with mean zero and precision l_i. The variables $(\omega, \varepsilon_1, \varepsilon_2, \gamma_1, \gamma_2, \phi_1, \phi_2)$ are jointly independent. To ease the exposition, we assume that the variable ω is drawn from an improper uniform prior over the entire real line. As explained in the main text, such an assumption facilitates the derivation of the players' beliefs (which are properly defined despite the improperness of the prior) without significant effect on any of the results.

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[12] Note that Condition (S.6) also implies that there can be at most one dimension explored solely by one of the two players.

A player's cognition affects the precision of her secondary signal (espionage) and/or the precision of her opponent's secondary signal (counter-espionage). Specifically, player i's cognition is given by the vector $\rho_i = (l_i, t_i)$. The term l_i captures player i's investment to learn player j's information (through the precision of the noise term ϕ_i in s_i^S). The term t_i, instead, captures player i's investment to influence player j's learning about player i's primary information (through the precision of the noise term γ_i in s_j^S). Consequently, l_i corresponds to the *self-directed* component of player i's cognition, whereas t_i corresponds to the *manipulative* component of i's cognition. A larger t_i represents a larger investment by player i to make herself understood (i.e., to ease player j's interpretation of i's primitive information, as in the information sharing application in the main text). Alternatively, a smaller t_i can be interpreted as a larger investment in counter-espionage, that is, in making it difficult for player j to "spy" on i's primitive information.

S.2.2 Preliminary analysis

We start by illustrating how to accommodate for interactions between the two forms of cognition (self-directed and manipulative) and then specialize the analysis to the case where cognition is purely self-directed (espionage). The case where cognition is manipulative (counter-espionage) is mathematically equivalent to the analysis of information sharing in the main text.

While it is natural to assume that player i's cognitive cost is increasing in the effort l_i that she puts to interpret her opponent's information (alternatively, in the intensity of her spying activity), the dependence of C_i on t_i is context-specific. If t_i is a proxy for the effort that player i puts in making herself understood, as in Dewatripont and Tirole (2005), then a higher t_i is likely to come with a higher cost (making oneself understood is costly). If, instead, t_i is linked to investments in counter-espionage aimed at preventing the opponent from spying on her primitive information, then a higher t_i, by reflecting a smaller investment in such defensive measures, naturally comes with a lower cost.

Given any pair of cognitive profiles $\rho = (\rho_1, \rho_2)$, with $\rho_i = (l_i, t_i)$, $i = 1, 2$, let

$$r_i^\rho \equiv \frac{l_i t_j}{l_i + t_j} \qquad \text{and} \qquad r_j^\rho \equiv \frac{l_j t_i}{l_j + t_i}$$

denote the endogenous precisions of the total noise $\eta_i \equiv \phi_i + \gamma_j$ and $\eta_j \equiv \phi_j + \gamma_i$ in the two players' secondary signals. Fixing $\rho = (\rho_1, \rho_2)$, one can verify that when, in the primitive game, player i expects player j to select, for each combination of primary and secondary signals $s_j = (s_j^P, s_j^S)$, with probability one an action $a_j(s_j) = m_j s_j^P + (1 - m_j) s_j^S$ that is a convex combination of s_j^P and s_j^S, her best response is to follow a strategy in the downstream game that, for each $s_i = (s_i^P, s_i^S)$, also selects with probability one an action $a_i(s_i) = m_i s_i^P + (1 - m_i) s_i^S$ that is a convex combination of s_i^P and s_i^S, with

$$m_i = \frac{1 + r_i^\rho(1 + \beta) - 2\beta r_i^\rho m_j}{1 + 2 r_i^\rho}. \tag{S.7}$$

When actions in the downstream game are strategic complements ($\beta > 0$), the higher the sensitivity of player j's period-2 action to her primary signal (that is, the larger m_j is), the smaller the weight that player i's best response assigns to player i's primary signal (equivalently, the larger the weight $1 - m_i$ to the secondary signal s_i^S). This property naturally reflects the desire for player i to align her action with player j's action. The opposite is true in case of strategic substitutes ($\beta < 0$). Furthermore, given any pair of cognitive profiles $\rho = (\rho_1, \rho_2)$, there exists a unique linear continuation BNE $\sigma^\rho = (\sigma_1^\rho, \sigma_2^\rho)$ for the primitive game and is such that, for each $s_i = (s_i^P, s_i^S)$, σ_i^ρ selects with certainty the equilibrium action $a_i^\rho(s_i) = m_i^\rho s_i^P + (1 - m_i^\rho)s_j^S$ with

$$m_i^\rho = \frac{1 + 2r_j^\rho + r_i^\rho(1-\beta) + 2r_i^\rho r_j^\rho(1-\beta^2)}{1 + 2(r_i^\rho + r_j^\rho) + 4r_i^\rho r_j^\rho(1-\beta^2)}. \tag{S.8}$$

One can also verify that, when the two players are expected to engage in cognition $\rho = (\rho_i, \rho_j)$ and, instead, player i selects cognition $\rho_i' = (l_i', t_i')$, for any $s_i = (s_i^P, s_i^S)$, player i's best response to player j following the equilibrium strategy σ_j^ρ consists in selecting with certainty the action $a_i^{\rho_i';\rho}(s_i) = m_i^{\rho_i';\rho} s_i^P + (1 - m_i^{\rho_i';\rho})s_j^S$, where

$$m_i^{\rho_i';\rho} = \frac{1 + r_i^{\rho_i',\rho_j}(1+\beta) - 2\beta r_i^{\rho_i',\rho_j} m_j^\rho}{1 + 2r_i^{\rho_i',\rho_j}}$$

is the sensitivity of player i's action to his primary signal and where

$$r_i^{\rho_i',\rho_j} \equiv \frac{t_j l_i'}{t_j + l_i'} \qquad \text{and} \qquad r_j^{\rho_i',\rho_j} \equiv \frac{t_i' l_j}{t_i' + l_j}$$

denote the precisions of the total noises $\eta_i \equiv \phi_i + \gamma_j$ and $\eta_j \equiv \phi_j + \gamma_i$ in the two players' secondary signals that obtain when player j conforms to cognition $\rho_j = (l_j, t_j)$ whereas player i deviates to cognition $\rho_i' = (l_i', t_i')$.

S.2.3 Espionage

We now specialize the noisy-communication model above to the case in which cognition is purely self-directed: players "spy" on each other, that is, take actions to gather (noisy) information about the opponents' beliefs and information (formally captured by their primitive signals). Formally, we assume that γ_j is identically equal to zero in which case the noise $\eta_i \equiv \phi_i + \gamma_j$ in each player i's secondary signal coincides with ϕ_i. Abusing notation, we then let player i's cognition ρ_i coincides with the precision of the noise η_i in her secondary signal. We hen interpret such a secondary signal as the outcome of industrial espionage, as in Kozlovskaya (2018) and Adriani and Sonderegger (2020). Alternatively, ρ_i can be interpreted as the result of various cognitive activities that help the player interpret the opponent's view of the game (see, e.g., Angeletos and La'O, 2013, for a model in which players receive information about the noise in other players' beliefs, but where such a noise is exogenous, and Calvo-Armengol et al., 2015, and Sethi and Yildiz, 2016, 2018, for models in which players choose the precision of the information they receive from other players).

Using the characterization in the previous section, we can establish the following result:

Proposition S.4 (**espionage**). *Let $(\hat{\rho}, \sigma^{\hat{\rho}})$ and (ρ, σ^{ρ}) be two arbitrary profiles. UEC always holds for these profiles, irrespective of whether the actions in the downstream game are strategic complements $(\beta > 0)$ or strategic substitutes $(\beta < 0)$. ID holds if and only if $\beta\,(\hat{\rho}_i - \rho_i)\,(\hat{\rho}_j - \rho_j) \leq 0$.*

Holding player j's cognition fixed, the value to player i of spying on player j (more broadly, of investing in cognitive activities that help her understand player j's primitive view of the game) is higher, the more player j expects player i to invest in cognition (that is, the more player j expects player i to spy on her). This is because, independently of the sign of β, when player i spies more, she then relies more on her secondary signal.[13] When $\beta > 0$, this induces player j to rely more on her primary signal—player j likes being spied.[14] In turn, this makes it even more valuable for player i to rely on her secondary signal and hence to spy more. When, instead, $\beta < 0$, player j responds less to her primary signal when she expects player i to spy more. This is because player j wants to distance herself from player i when actions are strategic substitutes. That player j relies less on her primary signal, however, further boosts player i's incentives to spy: this is because player i can now learn player j's information without ending up aligning her action much with player j's. Thus UEC holds in these games, no matter whether the primitive actions are strategic complements or substitutes.

Next, consider ID. When $\beta > 0$, the value to player i of spying on player j is higher the less player j spies, whereas the opposite is true when $\beta < 0$. Recall that, no matter the sign of β, when player j spies more, she relies more on her secondary signal and less on her primary one. When $\beta > 0$, this reduces player i's incentives to spy, whereas the opposite is true when $\beta < 0$.[15]

Summarizing, UEC always holds in these games, irrespective of whether actions are complements or substitutes. ID, instead, is reversed relative to the case where players learn about a payoff state: a player's incentives to spy on her opponents are stronger when her opponents themselves spy more and the downstream actions are substitutes, whereas they are weaker when the opponents spy more and the downstream actions are complements.

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[13] To see this, note that the formula in (S.8), when specialized to the case of pure espionage under consideration here, is equal to

$$m_i^{\rho} = \frac{1 + 2\rho_j + \rho_i(1-\beta) + 2\rho_i\rho_j(1-\beta^2)}{1 + 2(\rho_i + \rho_j) + 4\rho_i\rho_j(1-\beta^2)}.$$

It is then easy to verify that m_i^{ρ} is decreasing in ρ_i, which means that the sensitivity $1 - m_i^{\rho}$ of player i's action to her secondary signal s_i^S is increasing in ρ_i.

[14] To see this, note that the formula in (S.7), when applied to player j, is equal to

$$m_j = \frac{1 + \rho_j(1+\beta) - 2\beta\rho_j m_i}{1 + 2\rho_j}.$$

It is immediate to see that m_j is decreasing in m_i and hence increasing in $1 - m_i$.

[15] One can also identify conditions under which, when ID does not hold, UEC is nonetheless strong enough to guarantee EC, as well as conditions under which ID prevails over UEC, thus inducing a negative form of expectation conformity. These conditions, however, are not particularly illuminative and hence we do not discuss them here.

S.2.4 Counter-espionage

We do not present formal results here as they are qualitatively similar to those reported in the main text for the case of information sharing. Contrary to the case where cognition is self-directed (the espionage games examined above), UEC holds when downstream actions are complements but not when they are substitutes. When $\beta < 0$, the more player j expects player i to invest in counter-espionage, the more she responds to her secondary signal. This reinforces player i's incentives to invest in counterespionage. The opposite is true when $\beta > 0$. Importantly, these games feature a negative form of ID: a player's incentives to invest in counter-espionage are stronger the less the other players invest in counter-espionage. This is true irrespective of whether actions are complements or substitutes.

S.3 Generalized Career Concerns

In this section, we consider a model of manipulative cognition that encompasses most of the cases considered in the career-concern literature. Career concerns are an instance of "signal jamming," akin to those studied in the Industrial Organization literature, where players engage in manipulative cognition to affect other players' behavior.[16]

We show that when a worker's talent and effort are substitutes, as in Holmström (1999) additive model, EC never obtains. When, instead, they are complements, as in Dewatripont et al (1999)'s multiplicative model, EC typically obtains. Consistently with the results in Proposition 1 in the main text, the equilibrium is thus unique in Holmström (1999), whereas multiple equilibria are possible in Dewatripont et al. (1999).[17] We also show that, when this is the case, the worker is typically better off in the low-effort equilibrium (a form of cognitive trap). Whether or not the above conclusions are robust to the possibility that the labor market also invests in information acquisition then depends on the specific assumptions one makes on the output and information-acquisition technologies, as we discuss below.[18]

We first introduce the framework in Subsection S.3.1 and then characterize equilibrium actions and expectation conformity in Subsection S.3.2.

S.3.1 Framework

As in the career-concerns literature, a worker exerts effort to convince a competitive labor market that her talent is high (see, e.g., Holmström (1999), Dewatripont et al. (1999), and Hörner and

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[16] Signal jamming occurs, for example, when a firm secretly cuts its price so as to reduce its rivals' profits and induce them to believe that demand is low (or that costs are high) and exit the market. See Fudenberg and Tirole (1986).

[17] See also Hörner and Lambert (2019) for a more recent analysis of these games.

[18] Another class of signal-jamming games giving rise to EC is intra-personal memory management. In these games, a player receives information that she may try to remember, or repress, to influence the behavior of her future selves. Both repression and cognitive discipline are typically dictated by the expectations of future selves, with distinct welfare implications. These games were introduced in Bénabou and Tirole (2002). See also Gottlieb (2014, 2022). Dessi (2008) applies similar ideas in the context of cultural transmission with multiple agents. Bénabou (2013) and Bénabou and Tirole (2006) show how memory management and collective decisions interact to produce collective delusions.

Lambert (2019)). Contrary to what examined in that literature, we allow the market to also invest to better interpret the information contained in the worker's performance.

Specifically, let player 1 be the worker and player 2 the competitive labor market, and assume that payoffs are equal to

$$u_1(a_1, a_2, \omega) = a_2$$

for player 1 and

$$u_2(a_1, a_2, \omega) = -(a_2 - \omega)^2$$

for player 2, where $a_2 \in \mathbb{R}$ is player 2's action (e.g., a wage offer) and where $\omega \in \mathbb{R}$ is the underlying state of Nature (e.g., the worker's talent). Denote player 1's effort by ρ_1 and player 2's effort by ρ_2. Given $\rho = (\rho_1, \rho_2)$, player 2 receives a signal

$$s_2 = A(\rho) + M(\rho)\omega + R(\rho)\varepsilon_2$$

about player 1's talent ω, where A is an "additive" term akin to the one in Holmström (1999)'s original model, M is a "multiplicative" term akin to the one in Dewatripont et al. (1999), and R is a term capturing player 2's effort to reduce the noise in the signal. Each of these functions is non-negative, with A and M non-decreasing, and R non-increasing. Holmström (1999)'s original model corresponds to $A(\rho) = \rho_1$ and $M(\rho) = R(\rho) = 1$, all $\rho = (\rho_1, \rho_2)$ (only the worker invests and effort has an additive effect on performance). The multiplicative model of Dewatripont et al. (1999) corresponds to $M(\rho) = \rho_1$, $A(\rho) = 0$, and $R(\rho) = 1$, all $\rho = (\rho_1, \rho_2)$ (again, only the worker invests, but the impact of effort on performance now depends on the state ω).

As in the rest of the literature, suppose that ω is normally distributed with mean $\omega_0 > 0$ and variance $1/h_\omega$, and that ε_2 is normally distributed with mean 0 and variance $1/h_\varepsilon$.

S.3.2 Equilibrium and expectation conformity

Fixing $\rho = (\rho_1, \rho_2)$, for any s_2, we have that player 2's optimal action is equal to

$$a_2^\rho(s_2) = \mathbb{E}[\tilde{\omega}|s_2; \rho] = \left[\frac{M^2(\rho)h_\varepsilon}{M^2(\rho)h_\varepsilon + R^2(\rho)h_\omega}\right]\left[\frac{s_2 - A(\rho)}{M(\rho)}\right] + \left[\frac{R^2(\rho)h_\omega}{M^2(\rho)h_\varepsilon + R^2(\rho)h_\omega}\right]\omega_0.$$

Given $\rho = (\rho_1, \rho_2)$, for any actual choice ρ_1' by player 1, player 1's ex-ante expected payoff (gross of the cognitive cost but net of all terms that do not depend on her actual choice ρ_1') is equal to

$$V_1(\rho_1'; \rho, \sigma^\rho) = \frac{M(\rho)h_\varepsilon}{M^2(\rho)h_\varepsilon + R^2(\rho)h_\omega}\left[M(\rho_1', \rho_2)\omega_0 + A(\rho_1', \rho_2)\right].$$

Likewise, given $\rho = (\rho_1, \rho_2)$, for any actual choice ρ_2' by player 2, player 2's ex-ante expected payoff (gross of the cognitive cost but net of all terms that do not depend on her actual choice ρ_2') is equal to

$$V_2(\rho_2'; \rho, \sigma^\rho) = -\frac{M^2(\rho_1, \rho_2')R^2(\rho_1, \rho_2')h_\varepsilon + R^4(\rho_1, \rho_2')h_\omega}{\left(M^2(\rho_1, \rho_2')h_\varepsilon + R^2(\rho_1, \rho_2')h_\omega\right)^2}.$$

It is then easy to see that, when only player 1 invests (the case considered in the literature), UEC—and hence EC—never obtains in Holmström (1999) additive model, where the optimal level of ρ_1' is implicitly given by

$$C_1'(\rho_1') = \frac{h_\varepsilon}{h_\varepsilon + h_\omega}$$

and is independent of the level ρ_1 expected by player 2. Instead, UEC–and hence EC—can easily obtain in the multiplicative model of Dewatripont et al. (1999) where the optimal level of ρ_1' solves

$$C_1'(\rho_1') = \frac{\rho_1 h_\varepsilon \omega_0}{\rho_1^2 h_\varepsilon + h_\omega}$$

and is increasing in ρ_1 for $\rho_1 \leq \sqrt{h_\omega / h_\varepsilon}$. Consistently with the results in Proposition 1 in the main body, the equilibrium is thus unique in Holmström (1999), whereas multiple equilibria are possible in the multiplicative model of Dewatripont et al. (1999). Furthermore, when this is the case, the worker is better off in the low-effort equilibrium. This multiplicative version of this game is thus prone to expectation traps. Whether or not the above conclusions are robust to the possibility that player 2 (the labor market) also invests to influence the quality of the information received depends on the assumptions one makes about the A, M, and R functions, as indicated in the proposition below. For any $\rho = (\rho_1, \rho_2)$, let

$$L(\rho) \equiv M(\rho)\omega_0 + A(\rho),$$

$$G(\rho) \equiv \frac{M(\rho) h_\varepsilon}{M^2(\rho) h_\varepsilon + R^2(\rho) h_\omega},$$

and

$$Z(\rho) \equiv \frac{M^2(\rho) R^2(\rho) h_\varepsilon + R^4(\rho) h_\omega}{\left(M^2(\rho) h_\varepsilon + R^2(\rho) h_\omega\right)^2}.$$

Proposition S.5 (generalized-career-concerns). *Take any pair of profiles* $(\hat{\rho}, \sigma^{\hat{\rho}})$ *and* (ρ, σ^{ρ}) *with* $\hat{\rho}_i \geq \rho_i$, $i = 1, 2$. *UEC trivially holds as equality for player 2. It holds for player 1 if the function* G *is non-decreasing in* ρ_1. *ID holds for player 1 if the function* G *is increasing in* ρ_2 *and the function* L *is supermodular. ID holds for player 2 if the function* Z *is submodular.*

First, consider player 2. That UEC holds as equality for this player follows from the fact that, in the primitive game, only player 2 has an action. As a consequence, fixing ρ_1, the effect on player 2's payoff of changing ρ_2 is invariant in player 1's expectations of ρ_2. When it comes to player 1, instead, the effect on player 1's payoff of changing ρ_1 naturally depend on player 2's expectations of ρ_1 for the latter determine the sensitivity of a_2 to the signal s_2. Furthermore, when both players invest, the value to each player of changing her own cognition naturally depends on the opponent's cognition, as ρ_1 and ρ_2 jointly shape the statistical properties of the jointly controlled signal. It should thus not surprise that whether ID and hence EC holds for either player depends on the monotonicity and modularity of the functions L, G, and Z that control for the relevant statistical properties of s_2.

S.4 Framing and defensive memory management

In many situations of interest, some players use "frames" (broadly interpreted as the design of contextual purchasing experiences or other manipulative devices aimed at inducing other agents to act favorably to them). In this section, we consider a stylized persuasion game in which a Sender (the persuader) manipulates a Receiver's recollection of information relevant for a decision.[19] The analysis is motivated by the design of contextual purchasing experiences, in the spirit of Salant and Siegel (2018).[20]

S.4.1 Framework

We capture the situations mentioned above as follows. Player 2 (the receiver) has a payoff equal to

$$u_2(a_1, a_2, \omega) = -\left(a_2 - \omega\right)^2$$

where $a_2 \in \mathbb{R}$ is player 2's action and where $\omega \in \mathbb{R}$ is the underlying state of Nature. Player 1 (the persuader) has a payoff

$$u_1(a_1, a_2, \omega) = a_2$$

that is invariant in ω and in her own action, and increasing in player 2's action.[21] In other words, player 2 wants to "do the right thing" (i.e., align her action with the underlying state ω), whereas player 1 wants player 2 to take as high an action as possible (e.g., to increase her purchases of player 1's product, irrespectively of whether or not this is good for player 2). This payoff structure is what in the recent literature has been referred to as "persuasion with transparent motives" (see, e.g., Lipnowski and Ravid (2020)).

Player 2, the receiver, is originally endowed with a primary (exogenous) signal $s_2^P = \omega + \varepsilon$ but recalls such a signal only imperfectly. Such a primary signal may represent the information a buyer received about a seller's product from exogenous sources, or past experiences. In such a context, a "frame" by player 1 is a device influencing player 2's ability to recollect her primary signal. Importantly, such a frame may operate asymmetrically across states, facilitating the recollection of information favorable to player 1 relative to the less favorable one. The choice of a frame may also depend on the information that player 1 herself has about the state. However, because this channel is not essential to the results, we do not consider it here. Instead, we allow the receiver, player 2, to exert effort to increase her recollection of the primary information, thus reducing the effect of player 1's manipulation. We interpret such efforts broadly as "defensive memory management." Allowing for

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[19] See also Schwartzstein and Sunderam (2021) for an alternative model of manipulative persuasion in which the Sender influences the receivers by proposing models that organize past data to make predictions. Related is also Gibbons et al. (2021). That paper studies the effects of frames (interpreted as simplified descriptions of complex strategic situations) on the collective performance of a group of agents.

[20] What distinguishes these situations from those examined in the Bayesian persuasion literature (see, Bergemann and Morris, 2019, and Kamenica, 2019, for overviews) is the Sender's inability to commit to her choice of a frame (i.e., to her information structure).

[21] The above payoff specification is thus the same as in Section S.3.

such efforts also permits us to investigate whether ID holds in this context.

Let $\rho_1, \rho_2 \in \mathbb{R}_+$ and denote by $r(s_2^P; \rho)$ the probability that player 2 recalls her primary signal when the latter takes value s_2^P and the two players engage in cognition $\rho = (\rho_1, \rho_2)$. Let $s_2^R \in \mathbb{R} \cup \{\emptyset\}$ denote player 2's recalled signal, with $s_2^R = \emptyset$ in case player 2 does not recall, and $s_2^R = s_2^P$ in case she does recall. Without loss of generality, then let $s_2^P = \omega$, with ω drawn from $\mathbb{R}$ according to some cdf F.

Interpret ρ_1 as player 1's choice of a frame and assume that ρ_1 increases uniformly the probability that player 2 recalls any positive signal and leaves it unaltered the probability that player 2 recalls any negative signal. Such a stark structure is not essential to the results. What matters is that the likelihood that the receiver recollects information that is more favorable to the persuader relative to the less favorable one is non-decreasing in ρ_1. Formally, there exist non-negative and non-decreasing functions r^+ and r^- such that, when the state is ω and the two players' cognitive choices are $\rho = (\rho_1, \rho_2)$, the probability that player 2 recollects the state is equal to

$$r(\omega; \rho) = \begin{cases} r^-(\rho_2) & \text{if} \quad \omega < 0 \\ r^+(\rho_1, \rho_2) & \text{if} \quad \omega \geq 0. \end{cases}$$

Given $\rho = (\rho_1, \rho_2)$, then let

$$\mathbb{E}\left[\tilde{\omega} | s_2^R; \rho\right] = \begin{cases} \omega & \text{if} \quad s_2^R = \omega \\ \bar{\omega}(\rho) & \text{if} \quad s_2^R = \emptyset \end{cases}$$

denote player 2's posterior expectation of the state (equivalently, of her optimal action) given the recalled information. Let $\omega^- = \mathbb{E}\left[\tilde{\omega} | \tilde{\omega} < 0\right]$ and $\omega^+ = \mathbb{E}\left[\tilde{\omega} | \tilde{\omega} \geq 0\right]$, where both expectations are under the prior distribution F. We then have that, in the absence of any recollection of her primitive information, player 2's expected value of ω is equal to

$$\bar{\omega}(\rho) = \frac{(1 - r^-(\rho_2))F(0)\omega^- + (1 - r^+(\rho_1, \rho_2))(1 - F(0))\omega^+}{(1 - r^-(\rho_2))F(0) + (1 - r^+(\rho_1, \rho_2))(1 - F(0))}.$$

Note that $\bar{\omega}(\rho)$ is weakly decreasing in ρ_1, that is, in the beliefs player 2 has about player 1's use of manipulative frames. It may either be increasing or decreasing in player 2's own cognition, ρ_2. In particular, $\bar{\omega}(\rho)$ is decreasing in ρ_2 if $\frac{dr^-(\rho_2)}{d\rho_2} = \frac{\partial r^+(\rho_1, \rho_2)}{\partial \rho_2}$ and $r^+(\rho_1, \rho_2) \geq r^-(\rho_2)$, that is, if more cognition by player 2 has an equal effect on her ability to recollect positive and negative information, and if the likelihood that she recollects positive information is no smaller than the likelihood that she recollects negative information. On the other hand, $\bar{\omega}(\rho)$ is increasing in ρ_2, when $\frac{dr^-(\rho_2)}{d\rho_2} > \frac{\partial r^+(\rho_1, \rho_2)}{\partial \rho_2}$ and $r^+(\rho_1, \rho_2) \cong r^-(\rho_2)$.

S.4.2 Expectation conformity

Given the quadratic loss function, for any cognitive profile ρ, and any recalled memory s_2^R, player 2's optimal action is equal to

$$a_2^{\rho}(s_2^R) = \mathbb{E}\left[\tilde{\omega}|s_2^R;\rho\right]$$

implying that, for any profile $\rho = (\rho_1, \rho_2)$ and any actual choice of frame ρ_1', player 1's ex-ante expected gross payoff when the two players are expected to engage in cognition $\rho = (\rho_1, \rho_2)$ and, instead, player 1 chooses ρ_1' is equal to

$$V_1(\rho_1';\rho,\sigma^{\rho}) = F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\rho) + r^-(\rho_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\rho_1',\rho_2))\bar{\omega}(\rho) + r^+(\rho_1',\rho_2)\omega^+\right].$$

Similarly, given any cognitive profile $\rho = (\rho_1, \rho_2)$ and any actual choice ρ_2' of memory management, player 2's ex-ante expected gross payoff when the two players are expected to engage in cognition $\rho = (\rho_1, \rho_2)$ and, instead, player 2 invests ρ_2' in memory management is equal to

$$V_2(\rho_2';\rho,\sigma^{\rho}) = -F(0)(1-r^-(\rho_2'))\mathbb{E}\left[\left(\bar{\omega}(\rho_1,\rho_2') - \tilde{\omega}\right)^2|\tilde{\omega} < 0\right]$$

$$-(1-F(0))(1-r^+(\rho_1,\rho_2'))\mathbb{E}\left[\left(\bar{\omega}(\rho_1,\rho_2') - \tilde{\omega}\right)^2|\tilde{\omega} \geq 0\right].$$

The next result illustrates how cognitive expectations shape the players' incentives to engage in manipulative framing and defensive memory management in such environments.

Proposition S.6 (**framing and memory management**). *Consider any pair of profiles $(\hat{\rho}, \sigma^{\hat{\rho}})$ and (ρ, σ^{ρ}). UEC always holds for such profiles (weakly for the receiver, player 2, and strongly for the persuader, player 1). ID holds for player 1 (the persuader) if and only if*

$$[r^+(\hat{\rho}_1,\hat{\rho}_2) - r^+(\rho_1,\hat{\rho}_2)][\omega^+ - \bar{\omega}(\hat{\rho}_1,\hat{\rho}_2)] \geq [r^+(\hat{\rho}_1,\rho_2) - r^+(\rho_1,\rho_2)][\omega^+ - \bar{\omega}(\hat{\rho}_1,\rho_2)] \tag{S.9}$$

which is the case for example when (a) $\hat{\rho}_1 \geq \rho_1$, $\hat{\rho}_2 \geq \rho_2$, (b) r^+ is weakly supermodular, and (c) $\bar{\omega}$ is weakly decreasing in ρ_2.

The persuader's incentives to use more manipulative framing are thus stronger when she is expected to use such frames. This is because, as anticipated above, the more the receiver expects the persuader to manipulate, the more she interprets the lack of recollection of information as a signal of the state being unfavorable to the persuader. Shen then reacts in a less favorable way, boosting the persuader's incentives to use stronger manipulative framing to reduce the risk that the receiver does not recall.

Next, consider the receiver, player 2. Her optimal action depends only on her beliefs about player 1's manipulation and not on her belief about player 1's expectation of her own defensive cognition. The reasons are the same as in the generalized career-concerns models of the previous section. As a result, UEC also holds for player 2 but in the trivial sense of player 2's incentives being invariant in player 1's expectations about her cognition.

The second part of the proposition identifies a condition under which player 1's incentives to engage in manipulative framing are stronger the more player 2 invests in defensive memory management. The condition holds, for example, when a larger ρ_2 increases the marginal effect of player 1's manipulation on player 2's recollection of positive information and, in the absence of any recollection, the lower is player 2's optimal action. ID for player 1 also holds when r^+ is submodular (that is, when the more player 2 invests in defensive memory management, the smaller the marginal effect of player 1's manipulation on player 2's recollection of positive information) provided that, in the absence of any recollection, player 2's optimal action is smaller when player 2 invests more in memory management than when she invests less.

Whether increasing differences holds for player 2 (the receiver) is more convoluted and depends on a complicated condition which we do not discuss here.

S.5 Endogenous depth of reasoning

The analysis in all the situations described above assumes that the players are fully rational, and that cognition takes the form of learning about payoff states and/or other players' beliefs. In this section, we consider an alternative class of cognitive games in which payoffs are common knowledge, but where players are boundedly rational and cognition determines the players' ability to compute iterated best responses. The analysis builds on the level-k model, in which k is the depth of reasoning, that is, the maximal number of steps of iterated best responses performed by the player. The earlier literature (see, e.g., Crawford, Costa-Gomes, and Iriberri, 2013, for a detailed overview) assumes that k is exogenous. Alaoui and Penta (2016, 2017, 2018) are the first to endogenize k. Our analysis differs from theirs in that we allow the value of expanding cognition to depend on a player's beliefs about (a) her opponents' depth of reasoning, and (b) her opponents' expectations about her own depth of reasoning.

A key difference relative to the settings considered above is that, in this model, a player understands that, by increasing her cognition, she may end up with a lower payoff. The reason is that a player who increases her depth of reasoning but not to the point of correctly identifying the opponent's true mixed action may find herself trapped into a cognitive loop that induces her to select a mixed action that is farther away from her best response. The model also shares a few notable features with the sparsity model of Section S.1: (i) a player who goes significantly deeper into the computation of the best responses than her opponent does not gain any advantage in predicting her opponent's behavior vis-a-vis a player who goes slightly deeper; (ii) once at her cognitive capacity, a player is unable to respond to variations in her opponent's behavior due to a deeper understanding of the game.

We show that EC plays a key role also in this model. In particular, we consider a simplified version of the celebrated Arad and Rubinstein (2012)'s 11-20 game. This game, which is intended for experimental work, captures, in a stark and simplified manner, some of the forces that arise in strategic situations where players benefit from matching, or undercutting by little, the rivals' actions (e.g., oligopoly games with imperfect substitutes where firms compete in prices).

We show that this game exhibits a negative form of EC: a player's incentives to expand her depth of reasoning are lower the higher the depth of reasoning expected from her by the opponent (UEC) and the higher the opponent's depth of reasoning. Arguments similar to those establishing Proposition 1 in the main text then imply that, when the cognitive costs are strictly increasing, this model never features multiple (pure-strategy) equilibria.

S.5.1 The environment

Consider the following two-player game in which payoffs are common knowledge. For each $i = 1, 2$, and each $k \in \mathbb{N}$, there is a mixed action $\alpha_i^k \in \Delta(A_i)$ such that α_i^k is a best response for player i to player j playing according to α_j^{k-1}, with α_i^0 specified exogenously, but reflecting a natural "anchor" that depends on the stage-2 game under consideration (up to this point, the formalism is the same as in the original model, where k is exogenous). Each player's cognitive level $\rho_i \in \mathbb{N}$ determines the player's endogenous depth of reasoning, that is, the number of steps of iterated best responses performed by the player. A player with depth of reasoning ρ_i who expects his opponent to have performed $\rho_j \geq \rho_i - 1$ steps of iterated best responses plays $\alpha_i^{\rho_i}$ in the stage-2 game. A player with depth of reasoning ρ_i who, instead, expects his opponent to have performed $\rho_j < \rho_i - 1$ steps of iterated best responses plays $\alpha_i^{\rho_j+1}$ in the stage-2 game. Formally, for any cognitive profile $\rho = (\rho_i, \rho_j)$, and any ρ_i', player i's period-2 mixed action when the two players are expected to engage in cognition ρ and, instead, player i chooses cognition ρ_i' is given by

$$\sigma_i^{\rho_i';\rho} = \begin{cases} \alpha_i^{\rho_i'} & \text{if} \quad \rho_i' \leq \min\{\rho_i + 1, \rho_j\} + 1 \\ \alpha_i^{\min\{\rho_i+1,\rho_j\}+1} & \text{if} \quad \rho_i' > \min\{\rho_i + 1, \rho_j\} + 1. \end{cases}$$

The idea is that player i plays the action corresponding to his cognitive capacity, unless, given the player's beliefs over the two players' cognitive capacities, player i believes his cognitive capacity exceeds the level that is necessary to perfectly predict the opponent's mixed action. This modeling of the stage-2 behavior is the same as in Alaoui and Penta (2016, 2017, 2018). As anticipated above, the key point of departure is in how players choose ρ_i. In Alaoui and Penta (2016, 2017, 2018), the choice of ρ_i is determined by a cost-benefit analysis in which both the costs and the benefits do not depend on a player's beliefs about her opponents' cognitive sophistication and about their expectations of player i's own cognitive capacity. Here, instead, we allow for such dependence and investigate its implications for the selection of the cognitive levels.

Consistently with the notation in the paper, denote by

$$V_i(\rho_i'; \rho, \sigma^\rho) = U_i(\sigma_i^{\rho_i';\rho}, \sigma_j^\rho; \rho_i', \rho_j)$$

the ex-ante expected gross value of choosing cognition ρ_i' when the two players are expected to engage in cognition $\rho = (\rho_i, \rho_j)$ and, instead, player i chooses cognition ρ_i'. Contrary to the case in which cognition coincides with the choice of an information structure, in this model, σ^ρ should be interpreted as the action profile that emerges when the two players engage in cognition ρ (clearly,

σ^ρ need not be a BNE). Likewise, V_i need not coincide with player i's value function. This naturally reflects the limited cognitive ability of the players (recall that this model is meant to be a description of the strategic reasoning of boundedly rational agents). Also note that we dropped ω from the player's payoff function because, as explained above, in this game, payoffs are common knowledge (equivalently, $|\Omega| = 1$).

In the spirit of Alaoui and Penta (2016, 2017, 2018), also assume that, when it comes to choosing their depth of reasoning, the players correctly perceive how V_i depends on the players' cognition, even if they are not able to determine their correct best responses. Importantly, in this cognitive game, a player understands that, by increasing her cognition, she may end up with a lower payoff. This may happen despite the players' cognitive choices being covert. As mentioned above, the reason is that a player who increases her depth of reasoning but not to the point of being able to correctly identify the opponent's true mixed action may find herself trapped into a cognitive loop that induces her to select a stage-2 mixed action that is farther away from her true best response than the one identified by computing a smaller number of iterated best responses.

S.5.2 Discussion

As mentioned above, the players correctly understand how their gross payoffs V_i depend on their own cognition, their opponent's cognition, and their opponent's expectation about their own cognition. This may feel at odds with the assumption that the players are boundedly rational and cannot iterate their best responses to identify the rationalizable actions. It may also look strange that players be able to predict how their stage-2 actions depend on their actual cognition, on their opponent's cognition, and on their opponent's expectations about their own cognition, without however being able to compute the precise best responses. These concerns are normal in models of bounded rationality. The model, however, should not be interpreted literally. It is meant to capture *forces* that shape the choice of the depth of reasoning. It seems plausible that such a choice depends on a player's expectations of her opponent's sophistication, as well as on her beliefs about her opponent's expectation of her own sophistication. That a player's perceived (gross) payoff $V_i(\rho_i'; \rho, \sigma^\rho)$ from choosing cognition ρ_i' when the two players are expected to choose cognition ρ_i and ρ_j correctly reflects the dependence of the stage-2 actual actions on the players' cognitive levels (and hence coincides with the player's true payoff) is not essential. What matters is that a player correctly anticipates the forces that shape her stage-2 action, how they depend on the two players' actual and perceived depth of reasoning, and how such forces in turn affect her payoff. In particular, what matters is that a player understands that (a) more cognition need not translate into higher payoffs when insufficient to identify the opponent's action (it can even backfire by bringing a player's action more farther apart from her true best response), (b) going significantly deeper into the understanding of the game than the opponent need not bring any advantage relative to going slightly deeper, and (c) once at her cognitive capacity, a player is unable to respond to variations in her opponent's behavior due to a deeper understanding of the game. These features seem plausible and extend beyond the specific formalization above.

S.5.3 Arad and Rubinstein "11–20" game

For concreteness, we illustrate the role of expectation conformity in a specific game that has received considerable attention in the level-k literature. The stage-2 game described below was first introduced in Arad and Rubinstein (2012), and then simplified by Alaoui and Penta (2016). The players simultaneously announce an integer between 11 and 20. The players receive a number of tokens equal to the integer they announce. However, if a player announces an integer equal to the one announced by her opponent minus one, she receives extra x tokens, where $x \geq 20$. If the two players announce the same integer, they receive 10 tokens in addition to the integer they announce. Each token corresponds to one payoff unit. Letting $A_i = \{11, 12, ..., 20\}$, $i = 1, 2$, we thus have that the ex-post payoffs are equal to

$$u_i(a_i, a_j) = \begin{cases} a_i + x & \text{if} \quad a_i = a_j - 1 \\ a_i + 10 & \text{if} \quad a_i = a_j \\ a_i & \text{otherwise.} \end{cases}$$

This game, which is intended for experimental work, captures, in a stark and simplified manner, some of the forces that arise in certain strategic situations where players benefit from matching, or undercutting by little, the rivals' actions. For example, the two players could be firms selling imperfectly substitutable goods to different segments of the market. If firm i's price exceeds the rival's, firm i sells only to those consumers who do not value the rival's product (the "loyalists"). Firm i is a monopolist on this segment of the market and its monopolistic price on this segment is $a_i = 20$. Reducing the price below $a_i = 20$ without attracting consumers who see the two goods as substitutes comes with a loss of profits. When, instead, firm i matches its rival's price, in addition to selling to its loyalists, it also sells to 1/2 of those consumers who see the two products as substitutes. If it undercuts its rival, it sells to all consumers who see the two products as substitutes. However, if it undercuts the rival by a lot, the extra profits from conquering the contestable buyers are less than the losses from the loyalists. The Arad and Rubinstein (2012) game is meant to be a (highly simplified) version of the strategic situation that firms face in such circumstances.

Following Arad and Rubinstein (2012) and Alaoui and Penta (2016), let the "anchor" α_i^0 be the degenerate mixed action that selects the largest integer 20 with probability one. This action is often interpreted as the most natural one in the absence of strategic reasoning. The version originally introduced in Arad and Rubinstein (2012) does not feature the bonus of 10 tokens in case the players announce the same integer. The simplification proposed by Alaoui and Penta (2016) has two advantages. It implies that, if the game was played by fully rational players, the unique rationalizable action would have both players select $a_i = 11$ with certainty. It also implies that, for all i, and all $k \geq 9$, α_i^k is the degenerate mixed action that selects the integer 11 with certainty; that is, iterated best responses converge to the unique rationalizable action after 9 iterations. Because of this property, we simplify the analysis by assuming that $\rho_i \in \{0, 1, ..., 9\}$, $i = 1, 2$. We then have the following result (the proof follows directly from the arguments after the proposition):

Proposition S.7. *(a) Consider any pair of profiles* $(\hat{\rho}, \sigma^{\hat{\rho}})$ *and* (ρ, σ^{ρ}) *such that* $\hat{\rho}_1 > \rho_1$*,* $\rho_2 = \rho_1 + 1$*, and* $\hat{\rho}_2 = \hat{\rho}_1 + 1$*. Then*

$$\Gamma_1^{UEC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) = \Gamma_2^{UEC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) = 0,$$

whereas

$$\Gamma_1^{ID}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) < 0 < \Gamma_2^{ID}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big).$$

(b) Next, let $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ *and* $\rho = (\rho_1, \rho_2)$ *be such that* $\hat{\rho}_2 = \hat{\rho}_1 > \rho_2 = \rho_1$*. Then, for* $i = 1, 2$*,* $\Gamma_i^{UEC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) = 0$*, whereas* $\Gamma_i^{ID}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) < 0$*.*

Hence, this game features a negative form of expectation conformity, at least with respect to the cognitive profiles under consideration. Consider first case (a). The idea behind this specific pair of cognitive profiles is the following. Suppose the two players are known to have different cognitive costs, with player 2 being the "leader" (that is, the player with the lowest cognitive cost). Further assume that both players' cognitive costs are strictly increasing in their cognition. Then in any equilibrium in which the follower's cognition is equal to ρ_1, the leader's cognition is equal to $\rho_2 = \rho_1 + 1$. Similarly, in any equilibrium in which the follower's cognition is equal to $\hat{\rho}_1 > \rho_1$, the leader's cognition is equal to $\hat{\rho}_2 = \hat{\rho}_1 + 1$.[22] We are interested in whether *multiple asymmetric equilibria* are possible in such a situation, driven by expectation conformity. The answer is no. To see why this is the case, consider first the situation faced by the follower (player 1). Fixing player 2's cognitive level to $\rho_2 = \rho_1 + 1$, the value to player 1 of expanding her cognition from ρ_1 to $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ is (weakly) smaller when player 2 expects player 1 to choose cognition $\hat{\rho}_1$ than when she expects her to choose cognition $\rho_1 < \hat{\rho}_1$. To see this, note that the gross value to player 1 of expanding cognition from ρ_1 to $\hat{\rho}_1$ when player 2 expects player 1 to choose cognition $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ is equal to

$$\begin{aligned} &V_1(\hat{\rho}_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1, \rho_2}) - V_1(\rho_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1, \rho_2}) \\ &= [20 - \rho_2 + 10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x-1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1)] - (20 - \rho_1) \\ &= 10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x-1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1) - (\rho_2 - \rho_1). \end{aligned}$$

This is because player 2 announces $a_2 = 20 - \rho_2$ when she chooses cognition $\rho_2 = \rho_1 + 1$ and expects player 1 to choose cognition $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$, whereas player 1, when she chooses cognition $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ and expects player 2 to choose cognition $\rho_2 = \rho_1 + 1$, she announces $a_1 = 20 - \rho_2$ if $\hat{\rho}_1 = \rho_1 + 1 = \rho_2$ and $a_1 = 20 - \rho_2 - 1$ if $\hat{\rho}_1 > \rho_1 + 1 = \rho_2$. When, instead, player 1 chooses cognition ρ_1 and expects player 2 to choose cognition $\rho_2 = \rho_1 + 1$, she then announces $a_1 = 20 - \rho_1$.

Likewise, when player 2 expects player 1 to choose cognition $\rho_1 = \rho_2 - 1$, she announces $a_2 = 20 - \rho_1 - 1 = 20 - \rho_2$. It follows that the gross value to player 1 of increasing her cognition from ρ_1 to $\hat{\rho}_1$ when player 2 expects her to choose cognition ρ_1 is the same as when player 2 expects her to

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[22] It is also easy to verify that there exists no equilibrium in which the follower's cognition is strictly higher than the leader's.

choose cognition $\hat{\rho}_1$, implying that

$$\Gamma_1^{UEC}((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})) \equiv \left[V_1(\hat{\rho}_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1, \rho_2}) - V_1(\rho_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1, \rho_2})\right]$$

$$- \left[V_1(\hat{\rho}_1; (\rho_1, \rho_2), \sigma^{\rho_1, \rho_2}) - V_1(\rho_1; (\rho_1, \rho_2), \sigma^{\rho_1, \rho_2})\right] = 0.$$

Next, consider player 2 (the leader) and fix player 1's cognition to be equal to ρ_1. The gross value to player 2 of expanding her cognition from $\rho_2 = \rho_1 + 1$ to $\hat{\rho}_2 = \hat{\rho}_1 + 1$ is the same no matter whether player 1 expects her to choose cognition ρ_2 or $\hat{\rho}_2$. This is because, in either case, player 1's ability to predict player 2's action is bounded by player 1's own cognitive capacity. In fact, player 1 announces $a_1 = 20 - \rho_1$, that is the action identified by ρ_1 steps of iterated best responses, irrespective of how far ahead she thinks player 2 is in the understanding of the game. This last property, which is the same as is Alaoui and Penta (2016), is similar in spirit to the one discussed in the context of sparsity in games. Hence, for player 2 as well, $\Gamma_2^{UEC}(\rho, \hat{\rho}) = 0$.

Next, consider ID, focusing again on the cognitive profiles of part (a) in the proposition. For player 1, the gross value of expanding her cognition from ρ_1 to $\hat{\rho}_1$ when player 2 expects her to choose $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ and chooses cognition $\hat{\rho}_2 = \hat{\rho}_1 + 1$ is equal to

$$V_1(\hat{\rho}_1; (\hat{\rho}_1, \hat{\rho}_2), \sigma^{\hat{\rho}_1, \hat{\rho}_2}) - V_1(\rho_1; (\hat{\rho}_1, \hat{\rho}_2), \sigma^{\hat{\rho}_1, \hat{\rho}_2}) = (20 - \hat{\rho}_1) - (20 - \rho_1) = -(\hat{\rho}_1 - \rho_1).$$

This is because, in this case, player 2 announces $a_2 = 20 - \hat{\rho}_1 - 1$, whereas player 1 announces $a_1 = 20 - \hat{\rho}_1$ when choosing cognition $\hat{\rho}_1$ and $a_1 = 20 - \rho_1$ when choosing cognition ρ_1. The increase in cognition thus induces player 1 to lower her announcement, without, however, matching player 2's announcement, or undercutting it by one.

When, instead, player 2 expects player 1 to choose cognition $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ and chooses cognition ρ_2, she then announces $a_2 = 20 - \rho_2$, in which case the value to player 1 of expanding her cognition from ρ_1 to $\hat{\rho}_1$ is equal to

$$V_1(\hat{\rho}_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1, \rho_2}) - V_1(\rho_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1, \rho_2})$$

$$= \left[20 - \rho_2 + 10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x - 1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1)\right] - (20 - \rho_1).$$

Hence,

$$\Gamma_1^{ID}\left((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\right) \equiv \left[V_1(\hat{\rho}_1; (\hat{\rho}_1, \hat{\rho}_2), \sigma^{\hat{\rho}_1, \hat{\rho}_2}) - V_1(\rho_1; (\hat{\rho}_1, \hat{\rho}_2), \sigma^{\hat{\rho}_1, \hat{\rho}_2})\right]$$

$$- \left[V_1(\hat{\rho}_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1, \rho_2}) - V_1(\rho_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1, \rho_2})\right]$$

$$= -(\hat{\rho}_1 - \rho_1) - \left[10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x - 1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1) - (\rho_2 - \rho_1)\right]$$

$$= -(\hat{\rho}_1 - \rho_2) - \left[10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x - 1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1)\right] < 0.$$

The reason why this game features a negative form of increasing differences for the player with the highest cognitive cost (equivalently, with the lowest expected cognitive level) is that player 2, when expecting player 1's cognition to be equal to $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$, announces $a_2 = 20 - \hat{\rho}_1 - 1$ when choosing the high cognitive level $\hat{\rho}_2 = \hat{\rho}_1 + 1$, whereas she announces $a_2 = 20 - \rho_1 + 1$ when choosing the low cognitive level $\rho_2 = \rho_1 + 1$. Thus player 1 (the one who is expected to be behind in the exploration of the game) suffers from an increase in cognition by her opponent.

Next consider player 2 (the one who is expected to be ahead in the exploration of the game). When player 1 expects her to choose cognition $\hat{\rho}_2$, the gross value of increasing her cognition from $\rho_2 = \rho_1 + 1$ to $\hat{\rho}_2 = \hat{\rho}_1 + 1$ is equal to

$$V_2(\hat{\rho}_2; (\hat{\rho}_1, \hat{\rho}_2), \sigma^{\hat{\rho}_1, \hat{\rho}_2}) - V_2(\rho_2; (\hat{\rho}_1, \hat{\rho}_2), \sigma^{\hat{\rho}_1, \hat{\rho}_2}) = 20 - \hat{\rho}_2 + x - [(20 - \rho_2) + 10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1)] > 0$$

when player 1's cognition is equal to the high level $\hat{\rho}_1$ and is equal to

$$V_2(\hat{\rho}_2; (\rho_1, \hat{\rho}_2), \sigma^{\rho_1, \hat{\rho}_2}) - V_2(\rho_2; (\rho_1, \hat{\rho}_2), \sigma^{\rho_1, \hat{\rho}_2}) = 0$$

when player 1's cognition is equal to the low level ρ_1 (In this latter case, player 2 expects her capacity to be large enough to perfectly predict player 1's announcement, no matter whether she chooses $\rho_2 = \rho_1 + 1$ or $\hat{\rho}_2 = \hat{\rho}_1 + 1 > \rho_2$). It follows that, for the leader, this game features positive increasing differences:

$$\Gamma_2^{ID}\left((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\right) \equiv \left[V_2(\hat{\rho}_2; (\hat{\rho}_1, \hat{\rho}_2), \sigma^{\hat{\rho}_1, \hat{\rho}_2}) - V_2(\rho_2; (\hat{\rho}_1, \hat{\rho}_2), \sigma^{\hat{\rho}_1, \hat{\rho}_2})\right]$$

$$\left[V_2(\hat{\rho}_2; (\rho_1, \hat{\rho}_2), \sigma^{\rho_1, \hat{\rho}_2}) - V_1(\rho_2; (\rho_1, \hat{\rho}_2), \sigma^{\rho_1, \hat{\rho}_2})\right] > 0.$$

Combining the results for unilateral expectation conformity with those for increasing differences, we conclude that, in this game, $\Gamma_1^{EC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) < 0 < \Gamma_2^{EC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big)$. Arguments similar to those establishing Proposition 1 in the main body then imply that, when the cognitive costs are strictly increasing, there cannot exist multiple asymmetric equilibria.

Next, consider the cognitive profiles in part (b) of the proposition. What motivates considering such profiles is their relation to the possibility of *multiple symmetric equilibria*. The result in the proposition implies that such a multiplicity is not possible.

First, consider UEC and, without loss of generality, focus on player 2. When player 1 chooses cognition ρ_1, in the stage-2 game, she then announces $a_1 = 20 - \rho_1$, no matter whether she expects player 2 to choose $\rho_2 = \rho_1$ or $\hat{\rho}_2 > \rho_2 = \rho_1$. This is because, in either case, player 1 is at her cognitive capacity. As a consequence, the value to player 2 of increasing her cognition from ρ_2 to $\hat{\rho}_2 > \rho_2$ is positive but invariant to player 1's expectations about player 2's cognition. From the definition of $\Gamma_2^{UEC}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big)$, we then have that

$$\Gamma_2^{UEC}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) \equiv \big[V_2(\hat{\rho}_2;(\rho_1,\hat{\rho}_2),\sigma^{\rho_1,\hat{\rho}_2}) - V_2(\rho_2;(\rho_1,\hat{\rho}_2),\sigma^{\rho_1,\hat{\rho}_2})\big]$$

$$- \left[V_2(\hat{\rho}_2;(\rho_1,\rho_2),\sigma^{\rho_1,\rho_2}) - V_2(\rho_2;(\rho_1,\rho_2),\sigma^{\rho_1,\rho_2})\right]$$

$$= (20-\rho_1-1+x)-(20-\rho_1)-[(20-\rho_1-1+x)-(20-\rho_1)] = 0.$$

Because the two players face the same situation under the cognitive profiles under consideration, the same conclusion applies to player 1, that is, $\Gamma_1^{UEC}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) = 0$.

Next, consider ID. When player 1 expects player 2 to choose a higher cognitive level $\hat{\rho}_2$, she then announces $a_1 = 20-\rho_1$ when choosing the low cognitive level $\rho_1=\rho_2$ and $a_1 = 20-\hat{\rho}_1$ when choosing the high cognitive level $\hat{\rho}_1=\hat{\rho}_2$ (in both cases, player 1 is constrained by her cognitive capacity). The value to player 2 of increasing her cognition from ρ_2 to $\hat{\rho}_2$ is then equal to

$$V_2(\hat{\rho}_2;(\hat{\rho}_1,\hat{\rho}_2),\sigma^{\hat{\rho}_1,\hat{\rho}_2}) - V_2(\rho_2;(\hat{\rho}_1,\hat{\rho}_2),\sigma^{\hat{\rho}_1,\hat{\rho}_2}) = 20-\hat{\rho}_2+10-(20-\rho_2)$$

when player 1 chooses the high cognitive level $\hat{\rho}_1=\hat{\rho}_2$, whereas it is equal to

$$V_2(\hat{\rho}_2;(\rho_1,\hat{\rho}_2),\sigma^{\rho_1,\hat{\rho}_2}) - V_2(\rho_2;(\rho_1,\hat{\rho}_2),\sigma^{\rho_1,\hat{\rho}_2}) = 20-\rho_2-1+x-(20-\rho_2+10)$$

when player 1 chooses the low cognitive level $\rho_1=\rho_2$. Because $x>20$, we then have that

$$\Gamma_2^{ID}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) \equiv \big[V_2(\hat{\rho}_2;(\hat{\rho}_1,\hat{\rho}_2),\sigma^{\hat{\rho}_1,\hat{\rho}_2}) - V_2(\rho_2;(\hat{\rho}_1,\hat{\rho}_2),\sigma^{\hat{\rho}_1,\hat{\rho}_2})\big]$$

$$- \big[V_2(\hat{\rho}_2;(\rho_1,\hat{\rho}_2),\sigma^{\rho_1,\hat{\rho}_2}) - V_2(\rho_2;(\rho_1,\hat{\rho}_2),\sigma^{\rho_1,\hat{\rho}_2})\big] < 0.$$

The same conclusion applies to player 1. This game thus features a form of negative ID with respect to the profiles under consideration: increasing cognition is more valuable when the opponent chooses a lower cognition. Again, arguments similar to those in Proposition 1 in the main body then imply that this game, despite being played by boundedly rational players, cannot feature multiple symmetric (pure-strategy) equilibria. The reason is that, in this game, the benefit of increasing cognition (starting from a symmetric situation) is lower when the opponent is also expected to increase her cognition, because the opponent calls a lower number after expanding her cognition. Hence, no matter the cognitive costs, this game features a unique symmetric equilibrium.

S.6 Appendix: Proofs

Proof of Proposition S.1. We start by establishing the general properties of the individual best responses and of the ex-ante gross expected payoff functions V_i discussed in the paragraphs preceding the proposition. We then establish the properties in the proposition.

First note that the players' payoffs can be rewritten as

$$u_i(a_i, a_{-i}, \omega) = 2a_i \left[(1-\beta)g(\omega) + \beta \bar{a}_{-i}\right] - a_i^2 + \left\{\psi(a_{-i}, \omega) - (1-\beta)g(\omega)^2 - \beta \bar{a}_{-i}^2\right\}.$$

Hereafter, we disregard the terms in the curly bracket because they have no impact on individual best responses and cancel out when computing the differential payoffs in the definition of UEC and ID. Using the fact that $g(\omega) = (1 + \Sigma_{k=1}^{K} \omega^k)/(1-\beta)$, we then proceed as if the agents' payoffs were given by

$$\hat{u}_i(a_i, a_{-i}, \omega) = 2a_i \left(1 + \Sigma_{k=1}^{K} \omega^k + \beta \bar{a}_{-i}\right) - a_i^2. \tag{S.10}$$

One can then interpret a_i as the agent's investment into a risky activity, 1 as the component of the gross return to such an investment that is commonly known, $(\omega^k)_{k=1}^{K}$ as the components to the gross return that the players learn about, $\beta \bar{a}_{-i}$ as an investment spillover (positive for $\beta > 0$ and negative for $\beta < 0$), and a_i^2 as a quadratic adjustment cost.

As mentioned in the main text, to further simplify the derivations, we assume that there are only two players, so that $n = 2$.

Downstream best responses, equilibrium actions, and equilibrium payoffs.

Fix the cognitive profile $\rho = (\rho_1, \rho_2)$, with $\rho_1 \leq \rho_2$, and note that, given any ω, and $s_i = (\omega^1, ..., \omega^{\rho_i})$, the BNE strategies σ^ρ must select with certainty the actions

$$a_i = 1 + \bar{\omega}^{\rho_i} + \beta \mathbb{E}\left[a_j | \rho_i, s_i\right]$$

for $i, j = 1, 2$, $j \neq i$, where

$$\bar{\omega}^{\rho_i} \equiv \Sigma_{k=1}^{\rho_i} \omega^k.$$

Because both players' payoffs in the primitive game are strictly quasi-concave in their own actions, the BNE strategies must be Dirac measures assigning probability one to actions satisfying the above optimality conditions. Hereafter, we abuse notation and denote the primitive-game equilibrium actions by $a_i^\rho(s_i)$.

That player 2 goes deeper in the exploration of the state than player 1 implies that, for any $s_1 = (\omega^1, ..., \omega^{\rho_1})$ and $s_2 = (\omega^1, ..., \omega^{\rho_2}) = (s_1, \omega^{\rho_1+1}, ..., \omega^{\rho_2})$, $\mathbb{E}\left[a_1^\rho(s_1) | \rho_2, s_2\right] = a_1^\rho(s_1)$. Player's 2 equilibrium action must thus satisfy the optimality condition

$$a_2^\rho(s_2) = 1 + \bar{\omega}^{\rho_2} + \beta a_1^\rho(s_1)$$

for all $s_2 = (\omega^1, ..., \omega^{\rho_2})$. In turn, this implies that, for any $s_1 = (\omega^1, ..., \omega^{\rho_1})$, player 1's equilibrium action in the primitive game must satisfy

$$a_1^\rho(s_1) = 1 + \bar{\omega}^{\rho_1} + \beta \left[1 + \bar{\omega}^{\rho_1} + \beta a_1^\rho(s_1)\right].$$

Combining the above two best responses, we then have that the primitive-game equilibrium actions are given by

$$a_1^{\rho}(s_1) = \frac{1+\bar{\omega}^{\rho_1}}{1-\beta}$$

and

$$a_2^{\rho}(s_2) = a_1^{\rho}(s_1) + \bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1}.$$

In each state ω, the two players' equilibrium interim expected gross payoffs are then equal to

$$\mathbb{E}\left[\hat{u}_1(a_1^{\rho}(s_1), a_2^{\rho}(s_2), \omega)|\rho_1, s_1\right] = 2a_1^{\rho}(s_1)\left(1+\bar{\omega}^{\rho_1} + \beta a_1^{\rho}(s_1)\right) - \left(a_1^{\rho}(s_1)\right)^2 = \left(\frac{1+\bar{\omega}^{\rho_1}}{1-\beta}\right)^2$$

and

$$\mathbb{E}\left[\hat{u}_2(a_1^{\rho}(s_1), a_2^{\rho}(s_2), \omega)|\rho_2, s_2\right] = 2a_2^{\rho}(s_2)\left(1+\bar{\omega}^{\rho_2} + \beta a_1^{\rho}(s_1)\right) - \left(a_2^{\rho}(s_2)\right)^2 = \left(\frac{1+\bar{\omega}^{\rho_2} - \beta(\bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1})}{1-\beta}\right)^2.$$

Integrating over states, we then have that the ex-ante expected gross payoffs are equal to

$$V_1(\rho_1; \rho, \sigma^{\rho}) = \frac{1+\Sigma_{k=1}^{\rho_1}\sigma_k^2}{(1-\beta)^2}$$

and

$$V_2(\rho_2; \rho, \sigma^{\rho}) = V_1(\rho_1; \rho, \sigma^{\rho}) + \Sigma_{k=\rho_1+1}^{\rho_2}\sigma_k^2.$$

Given the derivations above, it is easy to verify that player 1's actions and payoff coincide with those in a fictitious environment in which (a) all dimensions ω_k, $k > \rho_1$, are equal to zero and such an event is commonly known, and (b) all dimensions ω_k, $k \leq \rho_1$, are commonly known by the two players. It is also easy to verify that player 2's actions and payoff coincide with those in a fictitious environment in which (a) all dimensions $k > \rho_2$ are equal to zero and such an event is commonly known, (b) all dimensions $k \leq \rho_1$ are commonly known, (c) it is commonly known that player 1 believes that all dimensions $k > \rho_1$ are identically equal to zero, and (d) player 2 knows all dimensions ω_k with $k \leq \rho_2$. In this sense, each player reasons, and acts, "as if" all dimensions $k > \rho_i$ simply "do not exist", as claimed in the main text.

Player 1's deviations.

Now suppose that the two players are expected to engage in cognition $\rho = (\rho_1, \rho_2)$, with $\rho_1 < \rho_2$, and player 1 deviates and selects cognition ρ_1' with $\rho_1 < \rho_1' \leq \rho_2$. To shorten the formulas, with an abuse of notation, let

$$s_1 = (\omega^1, ..., \omega^{\rho_1})$$

and

$$s_1' = (\omega^1, ..., \omega^{\rho_1}, \omega^{\rho_1+1}, ..., \omega^{\rho_1'}) = (s_1, \omega^{\rho_1+1}, ..., \omega^{\rho_1'}).$$

Player 1's optimal strategy in the primitive game then selects, for any s_1', with probability one the action

$$\begin{aligned} a_1^{\rho_1';\rho}(s_1') = 1+\bar{\omega}^{\rho_1'}+\beta\mathbb{E}\left[a_2^{\rho}(s_2)|\rho_1',s_1'\right] &= 1+\bar{\omega}^{\rho_1'}+\beta\mathbb{E}\left[a_2^{\rho}(s_2)|\rho_1',s_1'\right] \\ &= 1+\bar{\omega}^{\rho_1'}+\beta\left(a_1^{\rho}(s_1)+\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1}\right) \\ &= a_1^{\rho}(s_1)+(1+\beta)(\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1}). \end{aligned}$$

It follows that player 1's interim expected payoff following the deviation is equal to

$$\mathbb{E}\left[\hat{u}_1(a_1^{\rho_1';\rho}(s_1'),a_2^{\rho}(s_2),\omega)|\rho_1',s_1'\right] = 2a_1^{\rho_1';\rho}(s_1')\left[1+\bar{\omega}^{\rho_1'}+\beta\left(a_1^{\rho}(s_1)+\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1}\right)\right]-\left(a_1^{\rho_1';\rho}(s_1')\right)^2$$

which simplifies to

$$\mathbb{E}\left[\hat{u}_1(a_1^{\rho_1';\rho}(s_1'),a_2^{\rho}(s_2),\omega)|\rho_1',s_1'\right] = 2a_1^{\rho}(s_1)\left(1+\bar{\omega}^{\rho_1}+\beta a_1^{\rho}(s_1)\right)-\left(a_1^{\rho}(s_1)\right)^2$$

$$+2a_1^{\rho}(s_1)(1+\beta)(\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1})+2(1+\beta)(\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1})\left(1+\bar{\omega}^{\rho_1}+\beta a_1^{\rho}(s_1)\right)$$

$$+2(1+\beta)^2(\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1})^2-(1+\beta)^2(\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1})^2-2a_1^{\rho}(s_1)(1+\beta)(\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1}).$$

Integrating over states, we then have that player 1's ex-ante expected gross payoff following the deviation is equal to

$$V_1\big(\rho_1';\rho,\sigma^{\rho}\big) = V_1\big(\rho_1;\rho,\sigma^{\rho}\big)+(1+\beta)^2\Sigma_{k=\rho_1+1}^{\rho_1'}\sigma_k^2.$$

Next, consider the case where the two players are expected to choose cognition $\rho=(\rho_1,\rho_2)$, with $\rho_1\leq\rho_2$, and player 1 deviates and selects cognition ρ_1' with $\rho_1'>\rho_2$. Continue to let

$$s_1=(\omega^1,...,\omega^{\rho_1})$$

and

$$s_1'=(\omega^1,...,\omega^{\rho_1},\omega^{\rho_1+1},...,\omega^{\rho_1'})=(s_1,\omega^{\rho_1+1},...,\omega^{\rho_1'}).$$

Then, for any s_1', player 1's optimal behavior in the primitive game consists in choosing with probability one the action

$$\begin{aligned} a_1^{\rho_1';\rho}(s_1') &= 1+\bar{\omega}^{\rho_1'}+\beta\mathbb{E}\left[a_2^{\rho}(s_2)|\rho_1',s_1'\right] \\ &= 1+\bar{\omega}^{\rho_1}+\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_1}+\beta\left(a_1^{\rho}(s_1)+\bar{\omega}^{\rho_2}-\bar{\omega}^{\rho_1}\right) \\ &= a_1^{\rho}(s_1)+(1+\beta)(\bar{\omega}^{\rho_2}-\bar{\omega}^{\rho_1})+\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_2} \\ &= a_2^{\rho}(s_2)+\beta(\bar{\omega}^{\rho_2}-\bar{\omega}^{\rho_1})+\bar{\omega}^{\rho_1'}-\bar{\omega}^{\rho_2}. \end{aligned}$$

It follows that player 1's interim expected payoff following the deviation is equal to

$$\mathbb{E}\left[\hat{u}_1(a_1^{\rho_1';\rho}(s_1'),a_2^{\rho}(s_2),\omega)|\rho_1',s_1'\right] = 2a_1^{\rho_1';\rho}(s_1')\left(1+\bar{\omega}^{\rho_1'}+\beta a_2^{\rho}(s_2)\right)-\left(a_1^{\rho_1';\rho}(s_1')\right)^2$$

which simplifies to

$$\mathbb{E}\left[\hat{u}_1(a_1^{\rho_1';\rho}(s_1'), a_2^{\rho}(s_2), \omega)|\rho_1', s_1'\right] = 2a_2^{\rho}(s_2)\left(1 + \bar{\omega}^{\rho_2} + \beta a_1^{\rho}(s_1)\right) - \left(a_2^{\rho}(s_2)\right)^2$$

$$+2\beta(\bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1})\left(1 + \bar{\omega}^{\rho_1} + \beta a_1^{\rho}(s_1)\right) + 2\beta(1+\beta)(\bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1})^2 + 2\beta(\bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1})(\bar{\omega}^{\rho_1'} - \bar{\omega}^{\rho_2})$$

$$+2(\bar{\omega}^{\rho_1'} - \bar{\omega}^{\rho_2})\left[1 + \bar{\omega}^{\rho_2} + \beta\left(a_1^{\rho}(s_1) + \bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1}\right)\right] + 2(\bar{\omega}^{\rho_1'} - \bar{\omega}^{\rho_2})^2$$

$$-\beta^2(\bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1})^2 - (\bar{\omega}^{\rho_1'} - \bar{\omega}^{\rho_2})^2 - 2\beta(\bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1})(\bar{\omega}^{\rho_1'} - \bar{\omega}^{\rho_2}).$$

Integrating over states we then have that his ex-ante expected gross payoff following the deviation is equal to

$$\begin{array}{rcl} V_1\left(\rho_1'; \rho, \sigma^{\rho}\right) & = & V_2\left(\rho_2; \rho, \sigma^{\rho}\right) + 2\beta\left(1 + \frac{\beta}{2}\right)\Sigma_{k=\rho_1+1}^{\rho_2}\sigma_k^2 + \Sigma_{k=\rho_2+1}^{\rho_1'}\sigma_k^2 \\ & = & V_1\left(\rho_1; \rho, \sigma^{\rho}\right) + (1+\beta)^2\,\Sigma_{k=\rho_1+1}^{\rho_2}\sigma_k^2 + \Sigma_{k=\rho_2+1}^{\rho_1'}\sigma_k^2. \end{array}$$

Finally, suppose that the two players are expected to choose cognition $\rho = (\rho_1, \rho_2)$, with $\rho_1 \leq \rho_2$, and player 1 deviates to some lower cognition $\rho_1' < \rho_1$. Let

$$s_1' = (\omega^1, ..., \omega^{\rho_1'})$$

and

$$s_1 = (\omega^1, ..., \omega^{\rho_1'}, \omega^{\rho_1'+1}, ..., \omega^{\rho_1}) = (s_1', \omega^{\rho_1'+1}, ..., \omega^{\rho_1}).$$

In this case, player 1's optimal action is equal to

$$\begin{array}{rcl} a_1^{\rho_1';\rho}(s_1') & = & 1 + \bar{\omega}^{\rho_1'} + \beta\mathbb{E}\left[a_2^{\rho}(s_2)|\rho_1', s_1'\right] \\ & = & 1 + \bar{\omega}^{\rho_1'} + \beta\mathbb{E}\left[a_1^{\rho}(s_1) + \bar{\omega}^{\rho_2} - \bar{\omega}^{\rho_1}|\rho_1', s_1'\right] \\ & = & 1 + \bar{\omega}^{\rho_1'} + \beta\frac{1+\bar{\omega}^{\rho_1'}}{1-\beta} \\ & = & a_1^{\rho_1', \rho_2}(s_1'). \end{array}$$

It is evident that player 1's ex-ante expected payoff is then equal to

$$V_1\left(\rho_1'; \rho, \sigma^{\rho}\right) = V_1\left(\rho_1'; (\rho_1', \rho_2), \sigma^{\rho_1', \rho_2}\right) = \frac{1 + \Sigma_{k=1}^{\rho_1'}\sigma_k^2}{(1-\beta)^2}.$$

Player 2's deviations.

Next, consider deviations by the player who is expected to be ahead in the exploration of the state. Namely, suppose that the two players are expected to choose cognition $\rho = (\rho_1, \rho_2)$, with $\rho_1 \leq \rho_2$, and player 2 deviates to cognition ρ_2' with $\rho_2' \geq \rho_1$. Let

$$s_1 = (\omega^1, ..., \omega^{\rho_1})$$

and

$$s_2' = (\omega^1, ..., \omega^{\rho_1}, \omega^{\rho_1+1}, ..., \omega^{\rho_2'}) = (s_1, \omega^{\rho_1+1}, ..., \omega^{\rho_2'}).$$

Player 2's optimal action in the primitive game is then given by

$$a_2^{\rho_2';\rho}(s_2') \quad = \quad 1 + \bar{\omega}^{\rho_2'} + \beta\mathbb{E}\left[a_1^{\rho}(s_1)|\rho_2', s_2'\right] = 1 + \bar{\omega}^{\rho_2'} + \beta a_1^{\rho}(s_1) = a_2^{\rho_1,\rho_2'}(s_2').$$

That is, the optimal action for player 2 coincides with the action that she would choose in equilibrium if his deviation was observable. The ex-ante expected gross payoff that player 2 obtains following such a deviation is thus equal to

$$V_2\big(\rho_2'; \rho, \sigma^{\rho}\big) = V_2\big(\rho_2'; (\rho_1, \rho_2'), \sigma^{\rho_1,\rho_2'}\big) = V_1(\rho_1; \rho, \sigma^{\rho}) + \Sigma_{k=\rho_1+1}^{\rho_2'}\sigma_k^2.$$

Finally, consider the payoff that player 2 obtains by deviating to cognition $\rho_2' < \rho_1$. Let

$$s_2' = (\omega^1, ..., \omega^{\rho_2'})$$

and

$$s_1 = (\omega^1, ..., \omega^{\rho_2'}, \omega^{\rho_2'+1}, ..., \omega^{\rho_1}).$$

Player 2's optimal action in the downstream game is then equal to

$$a_2^{\rho_2';\rho}(s_2') \quad = \quad 1 + \bar{\omega}^{\rho_2'} + \beta\mathbb{E}\left[a_1^{\rho}(s_1)|\rho_2', s_2'\right] = 1 + \bar{\omega}^{\rho_2'} + \beta\tfrac{1+\bar{\omega}^{\rho_2'}}{1-\beta} = \tfrac{1+\bar{\omega}^{\rho_2'}}{1-\beta} = a_2^{\rho_1,\rho_2'}(s_2').$$

Again, player 2's optimal action coincides with the action that he would take if his deviation was observable. In turn, this also implies that his ex-ante expected gross payoff following such a deviation is equal to

$$V_2\big(\rho_2'; \rho, \sigma^{\rho}\big) = V_2\big(\rho_2'; (\rho_1, \rho_2'), \sigma^{\rho_1,\rho_2'}\big) = \frac{1 + \Sigma_{k=1}^{\rho_2'}\sigma_k^2}{(1-\beta)^2}.$$

Expectation conformity.

We are now ready to establish the results in the proposition. To see whether UEC, consider first the case where the player who is expected to be ahead is the same under all relevant profiles and, consistently with the notation above, let this player be player 2. That is, consider first the profiles $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ and $\rho = (\rho_1, \rho_2)$ such that $\hat{\rho}_2 > \rho_2 \geq \hat{\rho}_1 > \rho_1$. Then

$$\Gamma_1^{UEC}\big((\rho, \sigma^{\rho}), (\hat{\rho}, \sigma^{\hat{\rho}})\big) \equiv \Big[V_1\big(\hat{\rho}_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1,\rho_2}\big) - V_1\big(\rho_1; (\hat{\rho}_1, \rho_2), \sigma^{\hat{\rho}_1,\rho_2}\big)\Big]$$

$$-\Big[V_1\big(\hat{\rho}_1; (\rho_1, \rho_2), \sigma^{\rho_1,\rho_2}\big) - V_1\big(\rho_1; (\rho_1, \rho_2), \sigma^{\rho_1,\rho_2})\big)\Big]$$

$$= \tfrac{\Sigma_{k=\rho_1}^{\hat{\rho}_1}\sigma_k^2}{(1-\beta)^2} - (1+\beta)^2\Sigma_{k=\rho_1}^{\hat{\rho}_1}\sigma_k^2 = \tfrac{1-(1-\beta^2)^2}{(1-\beta)^2}\Sigma_{k=\rho_1}^{\hat{\rho}_1}\sigma_k^2 > 0$$

and

$$\Gamma_2^{UEC}\big((\rho,\sigma^\rho),(\hat\rho,\sigma^{\hat\rho})\big) = \Big[V_2\big(\hat\rho_2;(\rho_1,\hat\rho_2),\sigma^{\rho_1,\hat\rho_2}\big) - V_2\big(\rho_2;(\rho_1,\hat\rho_2),\sigma^{\rho_1,\hat\rho_2}\big)\Big]$$

$$-\Big[V_2\big(\hat\rho_2;(\rho_1,\rho_2),\sigma^{\rho_1,\rho_2}\big) - V_2\big(\rho_2;(\rho_1,\rho_2),\sigma^{\rho_1,\rho_2}\big)\Big] = 0.$$

Hence UEC holds strictly for player 1 and weakly (i.e., as an equality) for player 2.

Next, consider ID. Using the results above, we have that

$$\Gamma_1^{ID}\big((\rho,\sigma^\rho),(\hat\rho,\sigma^{\hat\rho})\big) = \Big[V_1\big(\hat\rho_1;(\hat\rho_1,\hat\rho_2),\sigma^{\hat\rho_1,\hat\rho_2}\big) - V_1\big(\rho_1;(\hat\rho_1,\hat\rho_2),\sigma^{\hat\rho_1,\hat\rho_2}\big)\Big]$$

$$-\Big[V_1\big(\hat\rho_1;(\hat\rho_1,\rho_2),\sigma^{\hat\rho_1,\rho_2}\big) - V_1\big(\rho_1;(\hat\rho_1,\rho_2),\sigma^{\hat\rho_1,\rho_2}\big)\Big] = 0$$

and

$$\Gamma_2^{ID}\big((\rho,\sigma^\rho),(\hat\rho,\sigma^{\hat\rho})\big) = \Big[V_2\big(\hat\rho_2;(\hat\rho_1,\hat\rho_2),\sigma^{\hat\rho_1,\hat\rho_2}\big) - V_2\big(\rho_2;(\hat\rho_1,\hat\rho_2),\sigma^{\hat\rho_1,\hat\rho_2}\big)\Big]$$

$$-\Big[V_2\big(\hat\rho_2;(\rho_1,\hat\rho_2),\sigma^{\rho_1,\hat\rho_2}\big) - V_2\big(\rho_2;(\rho_1,\hat\rho_2),\sigma^{\rho_1,\hat\rho_2}\big)\Big] = 0.$$

Combining the results for UEC with those for ID, we conclude that EC holds strictly for player 1 but only weakly (as an equality) for player 2.

Next, consider profiles $\hat\rho = (\hat\rho_1,\hat\rho_2)$ and $\rho = (\rho_1,\rho_2)$ such that $\hat\rho_2 = \hat\rho_1 > \rho_2 = \rho_1$. That UEC holds as an equality for these profiles follows directly from the fact that the action of the player who is behind is invariant in the number of additional dimensions $\hat\rho_j - \rho_j$ that the player who is ahead is expected to explore. That $\Gamma_i^{ID}\big((\rho,\sigma^\rho),(\hat\rho,\sigma^{\hat\rho})\big) > 0$ if $\beta > 0$ and $\Gamma_i^{ID}\big((\rho,\sigma^\rho),(\hat\rho,\sigma^{\hat\rho})\big) < 0$ if $\beta < 0$ follows from the fact that the marginal gross value of learning any dimension $k > \rho_i$ is equal to $\sigma_k^2/(1-\beta)^2$ when learnt jointly and σ_k^2 when learnt alone. Q.E.D.

Proof of Proposition S.2. First, we prove that all equilibria are symmetric. To see this, use the derivations in the proof of Proposition S.1 to observe that, in any equilibrium in which cognition is symmetric, so are the equilibrium actions in the primitive game. Hence, if an asymmetric equilibrium exists, it must feature asymmetric cognitive choices. Thus assume an equilibrium exists in which $\rho_1 < \rho_2$. The arguments in the proof of Proposition S.1 imply that, in any such equilibrium, the two players' ex-ante expected payoffs (disregarding again all external effects that have no influence on individual best responses) are given by

$$V_1(\rho_1;\rho,\sigma^\rho) - C(\rho_1) = \frac{1+\Sigma_{k=1}^{\rho_1}\sigma_k^2}{(1-\beta)^2} - \Sigma_{k=1}^{\rho_1}c_k$$

and

$$V_2(\rho_2;\rho,\sigma^\rho) - C(\rho_2) = V_1(\rho_1;\rho,\sigma^\rho) - C(\rho_1) + \Sigma_{k=\rho_1+1}^{\rho_2}\left(\sigma_k^2 - c_k\right).$$

Now suppose that player 1 deviates to cognition $\rho_1 < \rho_1' \leq \rho_2$. The derivations in the proof of

Proposition S.1 imply that her payoff is equal to

$$V_1\left(\rho_1'; \rho, \sigma^\rho\right) - C(\rho_1') = V_1(\rho_1; \rho, \sigma^\rho) - C(\rho_1) + \Sigma_{k=\rho_1+1}^{\rho_1'} \left[(1+\beta)^2\sigma_k^2 - c_k\right].$$

The optimality of player 2's behavior in the putative equilibrium implies that $\Sigma_{k=\rho_1+1}^{\rho_1'}\left(\sigma_k^2 - c_k\right) > 0$. Because $\beta > 0$, we then have that player 1 has a profitable deviation. Hence, all equilibria are symmetric.

Next, observe that, in any symmetric equilibrium with cognitive profile ρ in which the depth of reasoning is $\rho_1 = \rho_2 = k^*$, the players' net equilibrium payoffs are equal to

$$V_i(k^*; (k^*, k^*), \sigma^{k^*, k^*}) - C(k^*) = \frac{1 + \Sigma_{k=1}^{k^*}\sigma_k^2}{(1-\beta)^2} - \Sigma_{k=1}^{k^*} c_k,$$

$i = 1, 2$. For the two players not to have profitable deviations, it must be that neither a unilateral increase nor a unilateral decrease in cognition increases the players' payoff. Suppose that player 2 deviates to cognition $\rho_2 \geq k^* + 1$. The derivations in the proof of Proposition S.1 imply that her net payoff following the deviation is equal to

$$V_i(k^*; (k^*, k^*), \sigma^{k^*, k^*}) - C(k^*) + \Sigma_{k=k^*+1}^{\rho_2}(\sigma_k^2 - c_k),$$

where the last term describes the net gain/loss. Because σ_k^2/c_k is decreasing, such deviations are unprofitable, for all $\rho_2 > k^*$, if and only if $\sigma_{k^*+1}^2 - c_{k^*+1} \leq 0$, i.e., if and only if $k^* \geq \underline{k} \equiv \min\left\{k \middle| \sigma_k^2 \leq c_k\right\}$.

Next, suppose that player 1 deviates to cognition $\rho_1' \leq k^* - 1$. The derivations in the proof of Proposition S.1 imply that her expected net payoff following the deviation is equal to

$$V_i(k^* - 1; (k^* - 1, k^* - 1), \sigma^{k^*-1, k^*-1}) - C(k^* - 1)$$

$$= V_i(k^*; (k^*, k^*), \sigma^{k^*, k^*}) - C(k^*) - \Sigma_{k=\rho_1'+1}^{k^*}\left(\frac{\sigma_k^2}{(1-\beta)^2} - c_k\right),$$

where the last term describes the net gain/loss. Again, because σ_k^2/c_k is decreasing, such deviations are unprofitable, no matter ρ_1', if and only if $\frac{\sigma_{k^*}^2}{(1-\beta)^2} \geq c_{k^*}$, which is equivalent to

$$k^* \leq \bar{k}(\beta) \equiv \max\left\{k \middle| \frac{\sigma_k^2}{(1-\beta)^2} \geq c_k\right\}.$$

Hence, for any $k^* \in [\underline{k}, \bar{k}(\beta)] \cap \mathbb{N}$, a symmetric equilibrium with cognition k^* exists, and there is no symmetric equilibrium with cognition $k^* < \underline{k}$ or $k^* > \bar{k}(\beta)$.

Now assume that there are no external payoff effects (as explained in the main text, this is the case when the function ψ takes the form $\psi(a_{-i}, \omega) = (1-\beta)g(\omega)^2 + \beta\bar{a}_{-i}^2$). That equilibria are Pareto ranked then follows from the fact that, in the symmetric equilibrium with depth of knowledge k^*, the equilibrium payoffs are given by

$$V_i(k^*;(k^*,k^*),\sigma^{k^*,k^*}) - C(k^*) = \frac{1+\Sigma_{k=1}^{k^*}\sigma_k^2}{(1-\beta)^2} - \Sigma_{k=1}^{k^*}c_k$$

which are increasing in k^*.

Finally, that all the equilibria of the game in which cognition is ordered also also equilibria in the game in which cognition is unordered and that, under Condition (S.2), the converse is also true follows from the arguments in Section S.1. Q.E.D.

Proof of Proposition S.3. From the arguments in the proof of Proposition S.2, observe that, if a symmetric equilibrium exists, its cognitive level k^* must satisfy

$$\sigma_{k^*+1}^2 - c_{k^*+1} \leq 0 \leq \frac{\sigma_{k^*}^2}{(1-\beta)^2} - c_{k^*},$$

or, equivalently,

$$\frac{\sigma_{k^*+1}^2}{c_{k^*+1}} \leq 1 \leq \frac{\sigma_{k^*}^2}{(1-\beta)^2 c_{k^*}}. \tag{S.11}$$

The first inequality guarantees that a player does not benefit from learning the k^*+1 dimension, whereas the second inequality that she does not benefit from limiting her knowledge to the k^*-1 dimension, when both players are expected to learn k^* dimensions. Because σ_k^2/c_k is strictly decreasing, there can be at most one k^* satisfying the above two inequalities. Furthermore, when such a k^* exists, existence of a symmetric equilibrium also follows from the fact that σ_k^2/c_k is strictly decreasing, which implies that, if no local deviations are profitable, then nor are any of the global ones (the arguments are the same as in the proof of Proposition S.2).

Next, consider asymmetric equilibria. Suppose the two players are expected to select $\rho = (\rho_1, \rho_2)$, with $\rho_1 < \rho_2$. Then player 2's payoff from choosing any cognitive level $\rho_2' \geq \rho_1$ is equal to

$$V_1(\rho_1;\rho,\sigma^\rho) + \Sigma_{k=\rho_1+1}^{\rho_2'}(\sigma_k^2 - c_k).$$

Hence, for player 2 to choose $\rho_2 > \rho_1$, it must be that

$$\sigma_{\rho_{2+1}}^2 - c_{\rho_{2+1}} \leq 0 \leq \sigma_{\rho_2}^2 - c_{\rho_2},$$

or, equivalently,

$$\frac{\sigma_{\rho_{2+1}}^2}{c_{\rho_{2+1}}} \leq 1 \leq \frac{\sigma_{\rho_2}^2}{c_{\rho_2}}. \tag{S.12}$$

The above double inequality, along with the monotonicity of σ_k^2/c_k in k, implies that, when player 1 chooses cognition ρ_1, player 2 prefers cognition ρ_2 to any cognition $\rho_2' \geq \rho_1$. Below we identify conditions under which, when the two players are expected to select cognition $\rho = (\rho_1, \rho_2)$, with $\rho_1 < \rho_2$, player 1 prefers cognition ρ_1 to cognition $\rho_1' < \rho_1$. As shown in the proof of Proposition S.1, the same conditions imply that player 2 also prefers ρ_1 to any $\rho_2' < \rho_1$, and hence, by transitivity,

ρ_2 to any $\rho_2' < \rho_1$.[23]

Thus, consider player 1 (the one who selects the lowest cognitive level). As shown in the proof of Proposition S.1, when the two players are expected to select cognition $\rho = (\rho_1, \rho_2)$, with $\rho_1 < \rho_2$, if player 1 were to increase her cognition to ρ_1', with $\rho_1 < \rho_1' \leq \rho_2$, her payoff would be equal to

$$V_1(\rho_1; \rho, \sigma^\rho) + (1+\beta)^2 \Sigma_{k=\rho_1+1}^{\rho_1'} \sigma_k^2 - \Sigma_{k=1}^{\rho_1'} c_k.$$

If, instead, she were to select cognition $\rho_1' > \rho_2$, her payoff would be equal to

$$V_1(\rho_1; \rho, \sigma^\rho) + (1+\beta)^2 \Sigma_{k=\rho_1+1}^{\rho_2} \sigma_k^2 + \Sigma_{k=\rho_2+1}^{\rho_1'} \sigma_k^2 - \Sigma_{k=1}^{\rho_1'} c_k.$$

Finally, if she were to reduce her cognition to $\rho_1' < \rho_1$, her payoff would be equal to

$$\frac{1 + \Sigma_{k=1}^{\rho_1'} \sigma_k^2}{(1-\beta)^2} - \Sigma_{k=1}^{\rho_1'} c_k = V_1(\rho_1; \rho, \sigma^\rho) - \Sigma_{k=1}^{\rho_1} c_k - \Sigma_{k=\rho_1'+1}^{\rho_1} \left(\frac{\sigma_k^2}{(1-\beta)^2} - c_k \right).$$

Hence, for player 1 not to have profitable deviations, it must be that

$$(1+\beta)^2 \sigma_{\rho_1+1}^2 - c_{\rho_1+1} \leq 0 \leq \frac{\sigma_{\rho_1}^2}{(1-\beta)^2} - c_{\rho_1}$$

or, equivalently,

$$\frac{\sigma_{\rho_1+1}^2 (1+\beta)^2}{c_{\rho_1+1}} \leq 1 \leq \frac{\sigma_{\rho_1}^2}{(1-\beta)^2 c_{\rho_1}}. \tag{S.13}$$

The two inequalities in (S.13), along with the monotonicity of σ_k^2 / c_k in k, also imply that player 1 prefers ρ_1 to any $\rho_1' \leq \rho_2$. Furthermore, when paired with Condition (S.12), Condition (S.13) also implies that player 1 prefers ρ_1 to any $\rho_1' > \rho_2$.

We thus conclude that Conditions (S.12) and (S.13), which are equivalent to Conditions (S.5) and (S.4) in the main text, are both necessary and sufficient for existence of an asymmetric equilibrium with cognition $\rho = (\rho_1, \rho_2)$, with $\rho_1 < \rho_2$. That, in any asymmetric equilibrium, the cognitive levels ρ_1 and ρ_2 belong to $[\underline{k}(\beta), \bar{k}] \cap \mathbb{N}$ then follows from the above results along with the fact that $\rho_1 < \rho_2$.

Next, consider the equilibrium payoffs. In any asymmetric equilibrium, player 1's equilibrium payoff is equal to

$$\frac{1 + \Sigma_{k=1}^{\rho_1} \sigma_k^2}{(1-\beta)^2} - \Sigma_{k=1}^{\rho_1} c_k,$$

whereas player 2's equilibrium payoff is equal to

$$\frac{1 + \Sigma_{k=1}^{\rho_1} \sigma_k^2}{(1-\beta)^2} - \Sigma_{k=1}^{\rho_1} c_k + \Sigma_{k=\rho_1+1}^{2} \left(\rho_k^2 - c_k \right).$$

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[23] Indeed, from the proof of Proposition S.1, we have that the gross payoff that player 2 obtains from choosing cognition $\rho_2' < \rho_1$ is equal to $\left[1 + \Sigma_{k=1}^{\rho_2'} \sigma_k^2 \right] / (1-\beta)^2$ which is the same payoff that player 1 obtains from choosing cognition $\rho_1' = \rho_2'$ when player 2 is expected to choose any cognitive level $\rho_2 \geq \rho_1$.

It is then easy to see that player 1's equilibrium payoff is increasing in her depth of knowledge ρ_1, whereas player 2's equilibrium payoff is decreasing in player 1's depth of knowledge and, in case there are multiple solutions ρ_2 to the double inequality in (S.12), is invariant in ρ_2. It is also easy to see that the sum of the two players' payoffs is maximal under the equilibrium featuring the lowest cognition for player 1 (the one learning the smallest number of dimensions).

That all the equilibria of the game in which cognition is ordered are also equilibria in the game in which cognition is unordered and, under Condition (S.6), that the converse is also true follows from the arguments in Section S.1. Q.E.D.

Derivation of the conditions at the beginning of Section S.2

When player i expects the opponent to engage in cognition $\rho_j = (l_j, t_j)$ and chooses cognition $\rho_i' = (l_i', t_i')$ then, given $s_i = (s_i^P, s_i^S)$, player i's expectation of the fundamental variable ω is equal to

$$\mathbb{E}\left[\omega | s_i; \rho_i', \rho_j\right] = \frac{1}{1 + h_i^S(\rho_i', \rho_j)} s_i^P + \frac{h_i^S(\rho_i', \rho_j)}{1 + h_i^S(\rho_i', \rho_j)} s_i^S$$

where 1 is the precision of player i's primary signal and where

$$h_i^S(\rho_i', \rho_j) \equiv [var(\varepsilon_j + \gamma_j + \phi_i)|\rho_i', \rho_j]^{-1} = \frac{r_i^{\rho_i', \rho_j}}{1 + r_i^{\rho_i', \rho_j}}$$

is the total precision of player i's secondary signal, with

$$r_i^{\rho_i', \rho_j} \equiv [var(\gamma_j + \phi_i)|\rho_i', \rho_j]^{-1} = \left[\frac{1}{t_j} + \frac{1}{l_i'}\right]^{-1} = \frac{t_j l_i'}{t_j + l_i'}.$$

Similarly,

$$\mathbb{E}\left[s_j^S | s_i; \rho_i', \rho_j\right] = s_i^P$$

and

$$\mathbb{E}\left[s_j^P | s_i; \rho_i', \rho_j\right] = \mathbb{E}\left[\mathbb{E}\left[s_j^P | s_i, \omega; \rho_i', \rho_j\right] | s_i; \rho_i', \rho_j\right] = \mathbb{E}\left[\omega + \mathbb{E}\left[\varepsilon_j | s_i, \omega; \rho_i', \rho_j\right] | s_i; \rho_i', \rho_j\right]$$

$$= \mathbb{E}\left[\omega + \frac{r_i^{\rho_i', \rho_j}}{1 + r_i^{\rho_i', \rho_j}} (s_i^S - \omega) | s_i; \rho_i', \rho_j\right] = \frac{1}{1 + r_i^{\rho_i', \rho_j}} \mathbb{E}\left[\omega | s_i; \rho_i', \rho_j\right] + \frac{r_i^{\rho_i', \rho_j}}{1 + r_i^{\rho_i', \rho_j}} s_i^S$$

$$= \frac{1}{1 + 2r_i^{\rho_i', \rho_j}} s_i^P + \frac{2r_i^{\rho_i', \rho_j}}{1 + 2r_i^{\rho_i', \rho_j}} s_i^S.$$

Now, suppose that, in the primitive game, player i expects player j to follow a strategy that, for any $s_j = (s_j^P, s_j^S)$ selects with probability one the action

$$a_j(s_j) = m_j s_j^P + (1 - m_j) s_j^S$$

for some scalar m_j. Given ρ_i', when player i expects player j to engage in cognition ρ_j, then, for any $s_i = (s_i^P, s_i^S)$, player i's best response consists in choosing with certainty the action

$$\begin{aligned} a_i &= (1-\beta)\mathbb{E}\left[\omega | s_i; \rho_i', \rho_j\right] + \beta\mathbb{E}\left[a_j(s_j) | s_i; \rho_i', \rho_j\right] \\ &= (1-\beta)\tfrac{1+r_i^{\rho_i',\rho_j}}{1+2r_i^{\rho_i',\rho_j}} s_i^P + (1-\beta)\tfrac{r_i^{\rho_i',\rho_j}}{1+2r_i^{\rho_i',\rho_j}} s_i^S \\ &\quad + \beta m_j \left[\tfrac{1}{1+2r_i^{\rho_i',\rho_j}} s_i^P + \tfrac{2r_i^{\rho_i',\rho_j}}{1+2r_i^{\rho_i',\rho_j}} s_i^S\right] + \beta(1-m_j)s_i^P. \end{aligned}$$

Player i's best reply thus consists in choosing with probability one an action $a_i(s_i) = m_i s_i^P + (1 - m_i)s_i^S$ that is also linear in s_i with

$$m_i = \frac{1 + r_i^{\rho_i',\rho_j}(1+\beta) - 2\beta r_i^{\rho_i',\rho_j} m_j}{1 + 2r_i^{\rho_i',\rho_j}}.$$

After some algebra, one can verify that, for any cognitive profile $\rho = (\rho_i, \rho_j)$, the downstream game admits a unique linear continuation equilibrium and the latter is such that, for any $s_i = (s_i^P, s_i^S)$, σ_i^ρ, $i = 1, 2$, selects with certainty the action $a_i^\rho(s_i) = m_i^\rho s_i^P + (1 - m_i^\rho)s_i^S$, with

$$m_i^\rho = \frac{1 + 2r_j^\rho + r_i^\rho(1-\beta) + 2r_i^\rho r_j^\rho(1-\beta^2)}{1 + 2(r_i^\rho + r_j^\rho) + 4r_i^\rho r_j^\rho(1-\beta^2)}. \tag{S.14}$$

The above results also imply that, when the two players are expected to engage in cognition $\rho = (\rho_i, \rho_j)$ and, instead, player i selects cognition ρ_i', in the primitive game, player i's best response to player j following the strategy σ_j^ρ described above is, for any $s_i = (s_i^P, s_i^S)$, to select with certainty the action

$$a_i^{\rho_i';\rho}(s_i) = m_i^{\rho_i';\rho} s_i^P + (1 - m_i^{\rho_i';\rho})s_i^S$$

where

$$m_i^{\rho_i';\rho} = \frac{1 + r_i^{\rho_i',\rho_j}(1+\beta) - 2\beta r_i^{\rho_i',\rho_j} m_j^\rho}{1 + 2r_i^{\rho_i',\rho_j}} \tag{S.15}$$

with m_j^ρ as in (S.14) but applied to player j.

Next, let $\eta_1 \equiv \phi_1 + \gamma_2$ and $\eta_2 \equiv \phi_2 + \gamma_1$ and observe that, for any $(\omega, \varepsilon_1, \varepsilon_2, \eta_1, \eta_2)$, and any (m_1, m_2), when, in the primitive game, for any $s_i = (s_i^P, s_i^S)$, each player $i = 1, 2$ selects with probability one the action $a_i = m_i s_i^P + (1 - m_i)s_i^S$, then

$$a_i - \omega = m_i\varepsilon_i + (1 - m_i)(\varepsilon_j + \eta_i)$$

and

$$a_i - a_j = \left[m_i - 1 - m_j\right]\varepsilon_i + \left[1 - m_i - m_j\right]\varepsilon_j + (1 - m_i)\eta_i - (1 - m_j)\eta_j.$$

In turn, this implies that, when the two players are expected to engage in cognition $\rho = (\rho_i, \rho_j)$ and, instead, player i selects cognition ρ_i', player i's ex-ante expected payoff (gross of the cognitive cost C_i but net of all terms that do not affect individual best responses) is equal to

$$V_i\left(\rho_i'; \rho, \sigma^{\rho}\right) = -(1-\beta)\left(m_i^{\rho_i';\rho}\right)^2 - (1-\beta)\left(1 - m_i^{\rho_i';\rho}\right)^2\left(1 + \tfrac{1}{r_i^{\rho_i',\rho_j}}\right)$$

$$-\beta\left[m_i^{\rho_i';\rho} - \left(1 - m_j^{\rho}\right)\right]^2 - \beta\left[\left(1 - m_i^{\rho_i';\rho}\right) - m_j^{\rho}\right]^2 - \beta\left(1 - m_i^{\rho_i';\rho}\right)^2 \tfrac{1}{r_i^{\rho_i',\rho_j}} - \beta\left(1 - m_j^{\rho}\right)^2 \tfrac{1}{r_j^{\rho_i',\rho_j}}.$$

Finally, take any pair of cognitive levels for player i, $\hat{\rho}_i$ and ρ_i, and let $\rho' = (\rho_i', \rho_j')$ and $\rho'' = (\rho_i'', \rho_j'')$ be two arbitrary cognitive profiles. Then let

$$D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') \equiv \left[V_i\left(\hat{\rho}_i; \rho', \sigma^{\rho'}\right) - V_i\left(\rho_i; \rho', \sigma^{\rho'}\right)\right] - \left[V_i\left(\hat{\rho}_i; \rho'', \sigma^{\rho''}\right) - V_i\left(\rho_i; \rho'', \sigma^{\rho''}\right)\right].$$

Using the characterization of the V_i functions above, after some algebra, we have that

$$D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') = -2\left(m_i^{\rho_i;\rho'} - m_i^{\rho_i;\rho''}\right)\left[m_i^{\rho_i;\rho'} + m_i^{\rho_i;\rho''} - 2 + (1-\beta)\right]$$

$$+2\left(m_i^{\hat{\rho}_i;\rho'} - m_i^{\hat{\rho}_i;\rho''}\right)\left[m_i^{\hat{\rho}_i;\rho'} + m_i^{\hat{\rho}_i;\rho''} - 2 + (1-\beta)\right]$$

$$+4\beta m_j^{\rho''}\left(m_i^{\hat{\rho}_i;\rho''} - m_i^{\rho_i;\rho''}\right) - 4\beta m_j^{\rho'}\left(m_i^{\hat{\rho}_i;\rho'} - m_i^{\rho_i;\rho'}\right)$$

$$+\left(1 - m_i^{\rho_i;\rho'}\right)^2 \tfrac{1}{r_i^{\rho_i,\rho_j'}} + \beta\left(1 - m_j^{\rho'}\right)^2 \tfrac{1}{r_j^{\rho_i,\rho_j'}} - \left(1 - m_i^{\hat{\rho}_i;\rho'}\right)^2 \tfrac{1}{r_i^{\hat{\rho}_i,\rho_j'}} - \beta\left(1 - m_j^{\rho'}\right)^2 \tfrac{1}{r_j^{\hat{\rho}_i,\rho_j'}}$$

$$+\left(1 - m_i^{\hat{\rho}_i;\rho''}\right)^2 \tfrac{1}{r_i^{\hat{\rho}_i,\rho_j''}} + \beta\left(1 - m_j^{\rho''}\right)^2 \tfrac{1}{r_j^{\hat{\rho}_i,\rho_j''}} - \left(1 - m_i^{\rho_i;\rho''}\right)^2 \tfrac{1}{r_i^{\rho_i,\rho_j''}} - \beta\left(1 - m_j^{\rho''}\right)^2 \tfrac{1}{r_j^{\rho_i,\rho_j''}}.$$

Observe that UEC holds for player i if $D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') \geq 0$ for $\rho' = (\hat{\rho}_i, \rho_j)$ and $\rho'' = (\rho_i, \rho_j)$, whereas ID holds for player i if $D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') \geq 0$ for $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\hat{\rho}_i, \rho_j)$. In the proof of Proposition S.4 below we use the $D_i(\rho_i, \hat{\rho}_i; \rho', \rho'')$ function above to establish all the results about UEC and ID when cognition takes the form of pure espionage.

Proof of Proposition S.4. As explained in Section S.2.3, we abuse notation by letting each player's cognition coincides with the precision of the noise in the player's observation of the opponent's primary signal. Given any pair of cognitive levels $\hat{\rho}_i$ and ρ_i for player i, and any pair of cognitive profiles $\rho' = (\rho_i', \rho_j')$ and $\rho'' = (\rho_i'', \rho_j'')$, we then have that $r_i^{\rho_i,\rho_j'} = r_i^{\rho_i,\rho_j''} = \rho_i$, $r_i^{\hat{\rho}_i,\rho_j'} = r_i^{\hat{\rho}_i,\rho_j''} = \hat{\rho}_i$, $r_j^{\rho_i,\rho_j'} = r_j^{\hat{\rho}_i,\rho_j'} = \rho_j'$, $r_j^{\rho_i,\rho_j''} = r_j^{\hat{\rho}_i,\rho_j''} = \rho_j''$, $r_i^{\rho'} = \rho_i'$, $r_i^{\rho''} = \rho_i''$, $r_j^{\rho'} = \rho_j'$, and $r_j^{\rho''} = \rho_j''$.

Replacing the above formulas in the difference $D_i(\rho_i,\hat{\rho}_i;\rho',\rho'')$ in payoff differentials across the two profiles (see the characterization above, just before the beginning of the proof of the proposition), we then have that

$$D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') = 2(1-\beta)\left(m_i^{\rho_i;\rho'} - m_i^{\rho_i;\rho''}\right) - 2(1-\beta)\left(m_i^{\hat{\rho}_i;\rho'} - m_i^{\hat{\rho}_i;\rho''}\right)$$

$$+4\beta m_j^{\rho''}\left(m_i^{\hat{\rho}_i;\rho''} - m_i^{\rho_i;\rho''}\right) + 4\beta m_j^{\rho'}\left(m_i^{\rho_i;\rho'} - m_i^{\hat{\rho}_i;\rho'}\right)$$

$$+\left(m_i^{\rho_i;\rho'} - m_i^{\rho_i;\rho''}\right)\left(m_i^{\rho_i;\rho'} + m_i^{\rho_i;\rho''} - 2\right)\left[\tfrac{1+2\rho_i}{\rho_i}\right]$$

$$-\left(m_i^{\hat{\rho}_i;\rho'} - m_i^{\hat{\rho}_i;\rho''}\right)\left(m_i^{\hat{\rho}_i;\rho'} + m_i^{\hat{\rho}_i;\rho''} - 2\right)\left[\tfrac{1+2\hat{\rho}_i}{\hat{\rho}_i}\right].$$

Furthermore, in this case,

$$m_i^{\rho_i;\rho'} - m_i^{\rho_i;\rho''} = -\frac{2\beta\rho_i}{1+2\rho_i}(m_j^{\rho'} - m_j^{\rho''})$$

and

$$m_i^{\hat{\rho}_i;\rho'} - m_i^{\hat{\rho}_i;\rho''} = -\frac{2\beta\hat{\rho}_i}{1+2\hat{\rho}_i}(m_j^{\rho'} - m_j^{\rho''}).$$

Replacing the above expressions into the formula for D above, after some algebra, we have that

$$D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') = 4\beta\left(\hat{\rho}_i - \rho_i\right)\frac{m_j^{\rho'} - m_j^{\rho''}}{\left(1+2\rho_i\right)\left(1+2\hat{\rho}_i\right)}\left(\beta m_j^{\rho'} + \beta m_j^{\rho''} + 1 - \beta\right).$$

Observe that, independently of the sign of β, $\beta m_j^{\rho'} + \beta m_j^{\rho''} + 1 - \beta > 0$. Hence,

$$D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') \overset{\text{sgn}}{=} \beta\left(\hat{\rho}_i - \rho_i\right)\left(m_j^{\rho'} - m_j^{\rho''}\right).$$

Next observe that

$$m_j^{\rho'} - m_j^{\rho''} = \tfrac{n_i(\rho',\rho'')}{d_i(\rho',\rho'')},$$

where

$$n_i(\rho', \rho'') \equiv \left[1 + 2\rho_i' + \rho_j'(1-\beta) + 2\rho_i'\rho_j'(1-\beta^2)\right]\left[1 + 2(\rho_i'' + \rho_j'')\right]$$

$$+4\rho_i''\rho_j''(1-\beta^2)\left[1 + 2\rho_i' + \rho_j'(1-\beta)\right]$$

$$-\left[1 + 2\rho_i'' + \rho_j''(1-\beta) + 2\rho_i''\rho_j''(1-\beta^2)\right]\left[1 + 2(\rho_i' + \rho_j')\right]$$

$$-4\rho_i'\rho_j'(1-\beta^2)\left[1 + 2\rho_i'' + \rho_j''(1-\beta)\right]$$

and

$$d_i(\rho', \rho'') \equiv \left[1 + 2(\rho_i' + \rho_j') + 4\rho_i'\rho_j'(1-\beta^2)\right]\left[1 + 2(\rho_i'' + \rho_j'') + 4\rho_i''\rho_j''(1-\beta^2)\right].$$

Clearly, $d_i(\rho', \rho'') > 0$, whereas

$$\tfrac{n_i(\rho',\rho'')}{1+\beta} = \rho''_j - \rho'_j + 2\rho'_i\rho''_j - 2\rho''_i\rho'_j + 2\rho''_i\rho''_j(1-\beta)\left[2\rho'_i + 1 - 2\rho'_j\beta\right] - 2\rho'_i\rho'_j(1-\beta)\left[2\rho''_i + 1 - 2\rho''_j\beta\right].$$

Hence,

$$D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') \overset{\text{sgn}}{=} \beta(\hat{\rho}_i - \rho_i)\frac{n_i(\rho', \rho'')}{1+\beta}.$$

To see that this game satisfies UEC for player i, then take $\rho' = (\hat{\rho}_i, \rho_j)$ and $\rho'' = (\rho_i, \rho_j)$. We then have that

$$\tfrac{n_i(\rho',\rho'')}{1+\beta} = 2\rho_j\beta\left[1 + 2\rho_j(1-\beta)\right](\hat{\rho}_i - \rho_i).$$

We thus conclude that, irrespective of the sign of β, $D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') \geq 0$, which implies that the game satisfies UEC.

Next, to see whether this game satisfies ID for player i, take $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\hat{\rho}_i, \rho_j)$. We then have that

$$\tfrac{n_i(\rho',\rho'')}{1+\beta} = -\left(\hat{\rho}_j - \rho_j\right)\left[1 + 2\hat{\rho}_i + 4\hat{\rho}_i^2(1-\beta) + 2\hat{\rho}_i(1-\beta)\right]$$

and hence that $D_i(\rho_i, \hat{\rho}_i; \rho', \rho'') \overset{\text{sgn}}{=} -\beta(\hat{\rho}_i - \rho_i)\left(\hat{\rho}_j - \rho_j\right)$. We conclude that the game satisfies ID for player i if and only if $\beta(\hat{\rho}_i - \rho_i)\left(\hat{\rho}_j - \rho_j\right) \leq 0$. Q.E.D.

Proof of Proposition S.5. First, consider UEC. Using the expressions for $V_1(\rho'_1; \rho, \sigma^\rho)$ and $V_2(\rho'_2; \rho, \sigma^\rho)$ in Section S.3, we have that

$$\Gamma_1^{UEC}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) = \tfrac{M(\hat{\rho}_1,\rho_2)h_\varepsilon}{M^2(\hat{\rho}_1,\rho_2)h_\varepsilon + R^2(\hat{\rho}_1,\rho_2)h_\omega}\left[M(\hat{\rho}_1, \rho_2)\omega_0 + A(\hat{\rho}_1, \rho_2) - M(\rho_1, \rho_2)\omega_0 - A(\rho_1, \rho_2)\right]$$

$$-\tfrac{M(\rho_1,\rho_2)h_\varepsilon}{M^2(\rho_1,\rho_2)h_\varepsilon + R^2(\rho_1,\rho_2)h_\omega}\left[M(\hat{\rho}_1, \rho_2)\omega_0 + A(\hat{\rho}_1, \rho_2) - M(\rho_1, \rho_2)\omega_0 - A(\rho_1, \rho_2)\right]$$

and $\Gamma_2^{UEC}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) = 0$. Hence, for any pair of profiles $\rho = (\rho_1, \rho_2)$ and $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ with $\hat{\rho}_i \geq \rho_i$, $i = 1, 2$, we have that $\Gamma_1^{UEC}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) = 0$ if the function L is constant in ρ_1. When, instead, L is strictly increasing in ρ_1, then $\Gamma_1^{UEC}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) \geq 0$ if the function G is non-decreasing in ρ_1 (with the inequality strict when G is strictly increasing in ρ_1). When, instead, G is strictly decreasing in ρ_1, then $\Gamma_1^{UEC}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) < 0$.

Next, consider ID. Using again the formulas for the functions $V_1(\rho'_1; \rho, \sigma^\rho)$ and $V_2(\rho'_2; \rho, \sigma^\rho)$, we have that

$$\Gamma_1^{ID}\big((\rho, \sigma^\rho), (\hat{\rho}, \sigma^{\hat{\rho}})\big) = \tfrac{M(\hat{\rho}_1,\hat{\rho}_2)h_\varepsilon}{M^2(\hat{\rho}_1,\hat{\rho}_2)h_\varepsilon + R^2(\hat{\rho}_1,\hat{\rho}_2)h_\omega}\left[M(\hat{\rho}_1, \hat{\rho}_2)\omega_0 + A(\hat{\rho}_1, \hat{\rho}_2) - M(\rho_1, \hat{\rho}_2)\omega_0 - A(\rho_1, \hat{\rho}_2)\right]$$

$$-\tfrac{M(\hat{\rho}_1,\rho_2)h_\varepsilon}{M^2(\hat{\rho}_1,\rho_2)h_\varepsilon + R^2(\hat{\rho}_1,\rho_2)h_\omega}\left[M(\hat{\rho}_1, \rho_2)\omega_0 + A(\hat{\rho}_1, \rho_2) - M(\rho_1, \rho_2)\omega_0 - A(\rho_1, \rho_2)\right]$$

and

$$\Gamma_2^{ID}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) = -\tfrac{M^2(\hat{\rho}_1,\hat{\rho}_2)R^2(\hat{\rho}_1,\hat{\rho}_2)h_\varepsilon+R^4(\hat{\rho}_1,\hat{\rho}_2)h_\omega}{(M^2(\hat{\rho}_1,\hat{\rho}_2)h_\varepsilon+R^2(\hat{\rho}_1,\hat{\rho}_2)h_\omega)^2} + \tfrac{M^2(\hat{\rho}_1,\rho_2)R^2(\hat{\rho}_1,\rho_2)h_\varepsilon+R^4(\hat{\rho}_1,\rho_2)h_\omega}{(M^2(\hat{\rho}_1,\rho_2)h_\varepsilon+R^2(\hat{\rho}_1,\rho_2)h_\omega)^2}$$

$$+\tfrac{M^2(\rho_1,\hat{\rho}_2)R^2(\rho_1,\hat{\rho}_2)h_\varepsilon+R^4(\rho_1,\hat{\rho}_2)h_\omega}{(M^2(\rho_1,\hat{\rho}_2)h_\varepsilon+R^2(\rho_1,\hat{\rho}_2)h_\omega)^2} - \tfrac{M^2(\rho_1,\rho_2)R^2(\rho_1,\rho_2)h_\varepsilon+R^4(\rho_1,\rho_2)h_\omega}{(M^2(\rho_1,\rho_2)h_\varepsilon+R^2(\rho_1,\rho_2)h_\omega)^2}.$$

Hence, for any pair of profiles $\rho = (\rho_1, \rho_2)$ and $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ with $\hat{\rho}_i \geq \rho_i$, $i = 1, 2$, we have that $\Gamma_1^{ID}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) \geq 0$ if the function G is increasing in ρ_2 and the function L is supermodular (strictly, if G is strictly increasing and/or if L is strictly supermodular). Similarly, for the same profiles, we have that $\Gamma_2^{ID}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) \geq 0$ if the function Z is submodular (with the inequality strict, if Z is strictly submodular). Q.E.D.

Proof of Proposition S.6. First, consider UEC. Given any pair of profiles $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ and $\rho = (\rho_1, \rho_2)$, the value for player 2 (the receiver) of going from cognition ρ_2 to cognition $\hat{\rho}_2$ when player 1's cognition is ρ_1 is invariant in the level of cognition that player 1 expects player 2 to select. Hence, UEC trivially holds for player 2, albeit never strictly. Next, consider player 1 (the persuader). Using the characterization of the ex-ante expected gross payoffs in the paragraphs preceding the proposition, we have that

$$\Gamma_1^{UEC}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) \equiv \Big[V_1\big(\hat{\rho}_1;(\hat{\rho}_1,\rho_2),\sigma^{\hat{\rho}_1,\rho_2}\big) - V_1\big(\rho_1;(\hat{\rho}_1,\rho_2),\sigma^{\hat{\rho}_1,\rho_2}\big)\Big]$$

$$-\Big[V_1\big(\hat{\rho}_1;(\rho_1,\rho_2),\sigma^{\rho_1,\rho_2}\big) - V_1\big(\rho_1;(\rho_1,\rho_2),\sigma^{\rho_1,\rho_2})\big)\Big]$$

$$= F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^-(\rho_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\hat{\rho}_1,\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^+(\hat{\rho}_1,\rho_2)\omega^+\right]$$

$$-F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^-(\rho_2)\omega^-\right] - (1-F(0))\left[(1-r^+(\rho_1,\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^+(\rho_1,\rho_2)\omega^+\right]$$

$$-F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\rho_1,\rho_2) + r^-(\rho_2)\omega^-\right] - (1-F(0))\left[(1-r^+(\hat{\rho}_1,\rho_2))\bar{\omega}(\rho_1,\rho_2) + r^+(\hat{\rho}_1,\rho_2)\omega^+\right]$$

$$+F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\rho_1,\rho_2) + r^-(\rho_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\rho_1,\rho_2))\bar{\omega}(\rho_1,\rho_2) + r^+(\rho_1,\rho_2)\omega^+\right].$$

After simplifying, we have that

$$\Gamma_1^{UEC}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) = [1-F(0)]\left[r^+(\hat{\rho}_1,\rho_2) - r^+(\rho_1,\rho_2)\right]\left[\bar{\omega}(\rho_1,\rho_2) - \bar{\omega}(\hat{\rho}_1,\rho_2)\right] \geq 0$$

where the inequality follows from the fact that

$$\bar{\omega}(\rho) = \frac{(1-r^-(\rho_2))F(0)\omega^- + (1-r^+(\rho_1,\rho_2))(1-F(0))\omega^+}{(1-r^-(\rho_2))F(0) + (1-r^+(\rho_1,\rho_2))(1-F(0))}$$

is decreasing in ρ_1. Hence, UEC holds strictly for player 1, for any pair of cognitive profiles $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ and $\rho = (\rho_1, \rho_2)$ such that $\hat{\rho}_1 \neq \rho_1$ (irrespective of the sign of $\hat{\rho}_1 - \rho_1$).

Next, consider ID. Note that

$$\Gamma_1^{ID}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big) \equiv \Big[V_1\big(\hat{\rho}_1;(\hat{\rho}_1,\hat{\rho}_2),\sigma^{\hat{\rho}_1,\hat{\rho}_2}\big) - V_1\big(\rho_1;(\hat{\rho}_1,\hat{\rho}_2),\sigma^{\hat{\rho}_1,\hat{\rho}_2}\big)\Big]$$

$$-\Big[V_1\big(\hat{\rho}_1;(\hat{\rho}_1,\rho_2),\sigma^{\hat{\rho}_1,\rho_2}\big) - V_1\big(\rho_1;(\hat{\rho}_1,\rho_2),\sigma^{\hat{\rho}_1,\rho_2}\big)\Big]$$

$$= F(0)\left[(1-r^-(\hat{\rho}_2))\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2)+r^-(\hat{\rho}_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\hat{\rho}_1,\hat{\rho}_2))\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2)+r^+(\hat{\rho}_1,\hat{\rho}_2)\omega^+\right]$$

$$-F(0)\left[(1-r^-(\hat{\rho}_2))\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2)+r^-(\hat{\rho}_2)\omega^-\right] - (1-F(0))\left[(1-r^+(\rho_1,\hat{\rho}_2))\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2)+r^+(\rho_1,\hat{\rho}_2)\omega^+\right]$$

$$-F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2)+r^-(\rho_2)\omega^-\right] - (1-F(0))\left[(1-r^+(\hat{\rho}_1,\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2)+r^+(\hat{\rho}_1,\rho_2)\omega^+\right]$$

$$+F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2)+r^-(\rho_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\rho_1,\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2)+r^+(\rho_1,\rho_2)\omega^+\right].$$

After simplifying, we have that

$$\frac{\Gamma_1^{ID}\big((\rho,\sigma^{\rho}),(\hat{\rho},\sigma^{\hat{\rho}})\big)}{(1-F(0))} = \left[r^+(\hat{\rho}_1,\hat{\rho}_2)-r^+(\rho_1,\hat{\rho}_2)\right]\left[\omega^+-\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2)\right]$$

$$-\left[r^+(\hat{\rho}_1,\rho_2)-r^+(\rho_1,\rho_2)\right]\left[\omega^+-\bar{\omega}(\hat{\rho}_1,\rho_2)\right].$$

Hence, ID holds for player 1 with respect to the cognitive profiles $\hat{\rho}=(\hat{\rho}_1,\hat{\rho}_2)$ and $\rho=(\rho_1,\rho_2)$ if and only if Condition (S.9) in the proposition holds. It is easy to see that Condition (S.9) is satisfied when properties (a)-(c) in the proposition hold.

Finally, note that

$$\frac{\partial\bar{\omega}(\rho)}{\partial\rho_2} \overset{sgn}{=} \left[-\frac{dr^-(\rho_2)}{d\rho_2}F(0)\omega^- - \frac{\partial r^+(\rho_1,\rho_2)}{\partial\rho_2}(1-F(0))\omega^+\right]\left[(1-r^-(\rho_2))F(0)+(1-r^+(\rho_1,\rho_2))(1-F(0))\right]$$

$$-\left[(1-r^-(\rho_2))F(0)\omega^- + (1-r^+(\rho_1,\rho_2))(1-F(0))\omega^+\right]\left[-\frac{dr^-(\rho_2)}{d\rho_2}F(0) - \frac{\partial r^+(\rho_1,\rho_2)}{\partial\rho_2}(1-F(0))\right].$$

After some algebra, we have that

$$\frac{\partial\bar{\omega}(\rho)}{\partial\rho_2} \overset{sgn}{=} \frac{dr^-(\rho_2)}{d\rho_2} - \frac{\partial r^+(\rho_1,\rho_2)}{\partial\rho_2} - r^+(\rho_1,\rho_2)\frac{dr^-(\rho_2)}{d\rho_2} + \frac{\partial r^+(\rho_1,\rho_2)}{\partial\rho_2}r^-(\rho_2)\ .$$

Therefore, $\bar{\omega}(\rho)$ is decreasing in ρ_2 for example when $\frac{dr^-(\rho_2)}{d\rho_2}=\frac{\partial r^+(\rho_1,\rho_2)}{\partial\rho_2}$ and $r^+(\rho_1,\rho_2)\geq r^-(\rho_2)$, whereas $\bar{\omega}(\rho)$ is increasing in ρ_2 when $\frac{dr^-(\rho_2)}{d\rho_2}>\frac{\partial r^+(\rho_1,\rho_2)}{\partial\rho_2}$ and $r^+(\rho_1,\rho_2)\cong r^-(\rho_2)$. Q.E.D.

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