Exposure to the Unexpected, Duty of Disclosure, and Contract Design[*]

Alessandro Pavan[†] and Jean Tirole[‡]

January 18, 2022

Abstract

We introduce the concept of "relative exposure to the unexpected" and show that, when parties to a contract covertly engage in information acquisition prior to designing the contract, the concept underlies a variety of phenomena such as expectation conformity (the parties' tendency to conform to the level of cognition that is expected of them), over-cognition, and the desirability of mandatory disclosure laws. The paper develops a simple framework that captures the above phenomena in a unified manner and sheds light on a few policy implications.

Keywords: cognition, expectation conformity, contracts, adverse selection, caveat emptor, mandatory disclosures.

JEL numbers: C72; C78; D82; D83; D86.

1 Introduction

A key activity of public and private decision-makers is to design, negotiate, and enter contractual agreements, such as private contracts, laws or international treaties. To this purpose, they set aside other activities in order to think and brainstorm with colleagues about the implications of alternative designs, they allocate scarce cognitive resources to remembering central features, and they hire engineering, financial or legal experts. Cognitive activities simultaneously determine the degree of

contractual incompleteness and influence transactional frictions that arise from asymmetric information. They are therefore central to the functioning of markets, regulation and political decisions.

This paper studies a simple contracting environment in which the parties covertly engage in information acquisition prior to designing the contract, and cognition changes the nature of the contract between the two parties. Its contribution is three-fold. First, it brings together within a unified framework and generalizes a number of otherwise disconnected contributions. Second, it demonstrates that a sufficient statistic, the "relative exposure to the unexpected," underlies a variety of concepts and phenomena that are relevant for this type of situations. Third, it analyzes the welfare implications of mandatory disclosure laws.

In the model, described in Section 2, parties can opt for a standard contract. This contract may or may not be adequate. If it is not adequate, the parties are exposed to a loss of efficiency, whose relative incidence, which we label "relative exposure to the unexpected," plays a key role both in contract design and in the subsequent contract renegotiation. For example, the physical design may turn out to be inappropriate for the buyer's needs or unexpectedly costly for the seller to produce; alternatively the parties may omit to index the contract to some key contingencies, with serious impact on risk-sharing or behavior. Cognition prior to signing the contract can allow the parties to prevent such inefficiencies and write a better contract. However, in addition to its effect on efficiency, it also involves rent-seeking aspects. As a consequence, the resulting contract may be too complete or too incomplete, depending on the situation under consideration.

Section 3 analyses the case in which only one party can engage in cognition, with the latter taking the form of information acquisition. Both the incentive to acquire information and that to disclose it increase with the information acquirer's relative exposure to the unexpected. Expectation conformity (namely, a player's incentives to conform to what expected out of them), and its corollary (the possibility of multiple equilibrium cognitive patterns) obtain if and only if the information acquirer is relatively more exposed to the unexpected; in this case, the party is always better off in a low-cognition equilibrium.

Section 4 studies mandatory disclosure. The notion of mandatory disclosure is complex, and has been the object of tensions in contract law for a long time.[1] For example, in Macquarie International Health Clinic Pty Ltd vs Sydney South West Area Health Service (2010, NSWCA 268), the Court held that the obligation of "good faith" does not require parties to compromise their own commercial interests, but that parties must cooperate, including disclosing information, in a reasonable way to achieve the contract's objectives. The Macquarie decision allows for the possibility of delay in disclosure; in some cases, indeed, a party may refrain from disclosing at the contractual stage, but then disclose and renegotiate before news about contractual inefficiencies publicly accrue. Our results identify precise conditions under which a party has incentives to delay disclosure and show that such conditions are related to a party's relative exposure to the unexpected. The possibility of delay in disclosure raises the issue of the welfare benefits of making disclosure a legal requirement, which we

---

[1]See Kronman (1978) seminal paper on the topic.

also investigate in Section 4.

Section 4.1 shows that, in the absence of disclosure regulation, strategic dissimulation of information at the contracting stage occurs when the other party to the contract is relatively more exposed to the unexpected and post-contract renegotiation does much to limit the damage associated with a wrong design. In this case, mandatory disclosure, studied in Section 4.2, has both positive and negative effects. On the positive side, it prevents efficiency losses associated with delayed voluntary disclosure. On the negative side, it involves two costs. First, the information acquirer, having fewer options on what to do with the information, is less keen on gathering socially useful information in the first place (we show that late disclosures occur only when information collection is socially insufficient). Second, the information acquirer is deterred from disclosing information that is acquired after the contract is signed whenever, as is likely, it cannot be proven whether this information was acquired prior to or after the contracting date. So, while mandatory disclosure is always optimal for exogenously acquired information, its optimality when information is endogenously acquired requires strong conditions. In that, the optimal treatment of contract-relevant information bears resemblance with intellectual property law.[2]

Finally, Section 4.3 investigates the optimal mechanism design when the (balanced-budget) court (a) either observes whether a party discloses information post contract, (b) or observes that the party concealed information prior to contracting (observing only that the contract is renegotiated does not alter the equilibrium outcomes, as the parties can undo any penalty that one pays to the other because of renegotiation). We show that, in general, optimally-designed penalties do better than simple mandatory-disclosure rules, but the key trade-offs in their design are similar to those associated with the choice of whether or not to make disclosure mandated.

Section 5 extends the analysis to two-sided cognition and analyses contract completeness. It finds that the cognitive pattern is skewed (only one party acquires information) if and only if the information acquirer socially over-invests in information acquisition. We use these results to discuss the broader impacts of disclosure laws. Section 6 concludes.

Relationship to the literature. The paper is related to several literatures. The informal literature on transaction costs economics (e.g., Williamson (1975, 1985)) and the more formal work on ownership (e.g., Grossman and Hart (1986), Hart and Moore (1990)) identified the difficulty of foreseeing actions and contingencies as a foundation for otherwise unsatisfactory contracts. More recently and more closely related to this work, Gabaix and Laibson (2006) emphasized the strategic lack of disclosure in an environment with "shrouded attributes," but did not analyze information acquisition. The cognitive approach to incomplete contracting is taken up in Bolton and Faure-Grimaud (2010), Tirole (2009), Von Thadden and Zhao (2012) and Zhao (2015). Relative to these papers, we bring a unifying framework and obtain new insights regarding two-sided cognition, mandatory

---

[2]The latter specifies that an inventor is entitled to benefiting from their innovation if the latter is novel, non-obvious, and useful. The goal of intellectual property law is to reward inventors without generating socially costly "underserved" rents.

disclosure, and regulation.

The idea of modeling contractual incompleteness as an investment determining the probability that players learn the need to modify a default contract traces back to Bajari and Tadelis (2001). The key contribution is in relating such investment to the subsequent decision of whether or not to disclose information proving the need to change the contract, and in using the analysis do discuss the welfare merits of mandatory disclosure and other related policy interventions.

The paper builds on the framework developed in Tirole (2009). It shares with that paper the idea that pre-contractual cognition can be insufficient or excessive (as it reflects rent-seeking) and the focus on how cognition impacts the degree of contractual incompleteness.[3] That earlier paper, set in a context akin to that of Example 1 below, goes in some directions that are not (but could be) explored in this paper: the role of competition, and the effect of vertical integration and relationship contracting. The current paper's contribution is different. First, it identifies a summary statistic, the relative exposure to the unexpected, that encapsulates the various results on the extent of disclosure, expectation conformity, excess- or under-cognition, or the impact of disclosure regulation. Second, it analyses the welfare implications of disclosure laws.

Disclosure games, in which a sender holds hard (verifiable) information and decides whether to reveal it to a receiver who then takes an action affecting both parties, have received signifiant attention.[4] The corresponding literature has investigated factors that prevent unraveling (uncertainty as to whether the sender has received information or costly disclosure; see Dekel et al (2018) for recent developments in this literature) or lead to the disclosure of bad news (various forms of reputation; see Bourjade and Jullien (2011), Dziuda (2011), Grubb (2011), and Ispano (2018)). Most papers on disclosure games (e.g., Okuno-Fujiwara et al (1990)) take the information structure as given; exceptions include Matthews and Postlewaite (1985), Shavell (1994) and Dang (2008).

In parallel, the information transmission literature pioneered by Crawford-Sobel (1982) posits that the sender holds soft (unverifiable) information and transmits a message to the receiver, who then takes an action affecting both.[5] Reviewing this equally sizeable literature lies outside the scope

---

[3]Related angles are examined, among others, in Spier (1992), Gabaix and Laibson (2006), and Bolton and Faure--Grimaud (2010).

[4]See Sobel (2013) for a survey. Okuno-Fujiwara et al (1990) assume that cognition is exogenous and contains a general analysis of when voluntary disclosure leads to full disclosure. Shavell (1994) provides the basic analysis of costly information acquisition prior to disclosure (earlier work on the topic includes Farrell (1985) and Sobel (1989)). The paper, however, does not emphasize strategic complementarities between anticipated and optimal cognition. Rather, it provides a careful analysis of an informal idea of Kronman (1978), according to which mandated information disclosure is likely to reduce incentives for information acquisition; the paper further shows that the impact of required disclosure is rather asymmetric: It discourages buyers but not sellers from acquiring socially valuable information. Kartik et al (2017) consider a multi-sender model of disclosure of hard information; among other things, the paper shows that disclosure strategies are strategic substitutes under a disclosure cost and strategic complements under a concealment cost. Another recent contribution to the literature is Hoffmann et al (2020) in which the sender secretly collects information about the receiver's preferences and selectively discloses, subject to absorption capacity constraints limiting the number of dimensions over which information can be disclosed (which is another impediment to unraveling).

[5]Much of the attention has focused on the polar cases of hard and soft information. In general, how hard (per se

of this paper. Most papers again take the information structure as given.[6]

A rich literature has discussed the implications of the possibility of acquiring information during bargaining (after a contract offer is on the table) rather than before bargaining. In Cremer and Khalil (1992), a contract is designed so as to alter the incentives to the other party's information acquisition (see also Lewis and Sappington (1997), and Szalay (2009) for a more general treatment). A principal/buyer wants to buy an amount of input from an agent/seller. The agent learns his marginal cost for free when starting the production process, i.e. after signing the contract; the agent can alternatively pay a fixed cost to learn his marginal cost prior to accepting the principal's contract. So, information acquisition is purely wasteful, and the optimal contract is designed so as to deter any information acquisition. More generally, a contract offer may be used to induce or deter information acquisition, but in practice much information is acquired prior to bargaining, vindicating the current study. Cremer and Khalil (1994) and Cremer et al. (1998a,b) study information acquisition prior to contracting. For example, in Cremer et al. (1998a), the agent can learn his marginal cost of production, or choose not to learn it, prior to contracting (he learns it costlessly after contracting). The principal, who does not observe the agent's information acquisition strategy and can only guess it, then offers a contract. The principal, who moves second, cannot prevent socially wasteful information acquisition. The menu offered by the principal embodies three options if the are two cost levels, as the agent in equilibrium plays a mixed strategy and may not acquire information. Our work complements this line of research by providing a more flexible framework that we use to investigate the welfare effects of mandatory disclosure laws.

The present paper also connects to a large legal literature on *caveat emptor* ("Let the buyer beware"). Caveat emptor, which has a long tradition dating back to the Roman times, provides in common law a safe harbor for a seller not to disclose information to the buyer.[7] Several arguments have been made in its favor. The first is that, under symmetric information, the seller's liability may alter consumer choices when the buyer's relative tastes for price and defects are heterogenous (Buchanan (1970)). The main justification is to avoid frivolous lawsuits by having buyers bear the risk of loss and thereby imposing upon them a duty to inspect. In particular, caveat emptor holds it that the seller has no duty to disclose patent or obvious defects to the buyer.

The possibility of asymmetric information has over time led courts and legislative enactments

---

informative) communicated information is depends on the – endogenous – communication efforts exerted by the sender and the receiver (see Dewatripont and Tirole (2005)).

[6]An exception is Pei (2015), who allows the sender to choose his information structure from a "rich set" (if a partition belongs to the sender's choice set, then any coarser partition also belongs to this set). The paper shows that, in contrast with Crawford-Sobel (1982), all equilibria are such that the sender communicates all he knows to the receiver, and characterizes the equilibrium outcomes. Gentzkow and Kamenica (2017) obtain a related result in a game with hard information. They show that, if information acquisition is costly and observable by the decision maker (who observes the information structure, but not the realized signal), then disclosure requirements have no effect on the set of pure-strategy equilibria.

[7]French law, by contrast, tends to view dissimulation as contrary to good faith bargaining.

to what Johnson (2008) calls *"caveat emptor light"*, more in line with Kronman (1978)'s influential theory of information as a property right. Kronman (1978) makes a distinction between casually and deliberately acquired information. Information acquired by the homeowner as a by-product of living in the house (the presence of termites, the occasional flooding of the basement) should be disclosed, while that deliberately acquired by the seller is akin to a property and should not be subject to a disclosure obligation. If the presence of termites or the occasional flooding are just redistributive (they affect the sale price, but not who should own the house for what purpose), this precept runs counter Cooter and Ulen (2016), who distinguish between "productive" and "redistributive" facts, and argue that the redistributive facts should not be subject to a disclosure duty.

The present paper's contribution relative to this legal literature is to offer a formal framework in which this legal debate can be analyzed. It considers a situation that is more general than a sale (in the model, both parties have a post-contractual stake, and indeed may want to renegotiate the contract to their mutual advantage) and it shows the role played by a sufficient statistic (a player's relative exposure to the unexpected) in determining bargaining behavior and outcomes, and in assessing the merits of alternative disclosure laws.

2   Model

2.1   Description

Players and contingencies. Two risk-neutral players engage in a contractual relationship with unknown payoffs. The state space is binary, with $\Omega = \{\omega, \widehat{\omega}\}$, and it is common knowledge that each party assigns prior probabilities q and $\widehat{q}$ to ω and $\hat{\omega}$, respectively. In state ω, the players' initial (pre-search) and final (post-search) information is $\varnothing$ (no signal), regardless of any cognitive effort (there is no snag/flaw to be discovered, say). In state of nature $\widehat{\omega}$, instead, player i learns $\widehat{\omega}$ with probability ρ_i and nothing (keeps the information set $\varnothing$) with probability $1 - \rho_i$. The draws are conditionally independent between the two players (this assumption plays no role until Section 5.2). Information $\widehat{\omega}$ is *hard* and therefore can be disclosed to player $j \neq i$ if player i decides so. The cognition cost is $C_i(\rho_i)$, with $C_i'(\rho_i) > 0$ for $\rho_i > 0$, $C_i''(\rho_i) > 0$ for all ρ_i, $C_i'(0) = C_i(0) = 0$, and $C_i'(1) = \infty$.

A prominent interpretation of this information structure goes as follows: Knowing the state of nature allows a trade, a technological choice, or a contract design to match the state. In state ω, a known ("business as usual" or "boiler plate") choice, a, is optimal. This choice is also available in state $\widehat{w}$. However, state $\widehat{\omega}$ optimally requires an original response, $\widehat{a}$, which becomes known to the players only after learning the state. The very act of conceptualizing state $\widehat{\omega}$ reveals the nature of $\widehat{a}$.[8] The model is fully Bayesian: Players are uninformed, but rational, in that they know that they

---

[8]There is little loss of generality in assuming that designing a solution comes for free with the discovery that the standard design is not the appropriate one. Suppose that the first search only shows the party that there is a snag with the standard design (i.e., reveals that the state is not ω). One could then envision a second search in which the party

don't know. Accordingly, beliefs are a martingale.[9]

An alternative interpretation of the model is in terms of contract incompleteness. Contracts often fail to describe some states of nature or actions to be undertaken in certain states of nature. Incompleteness is often motivated by the presence of "unforeseen contingencies." The question as to whether omitted contingencies are truly unforeseeable or just too costly to conceive and include in the contract can be sidestepped under an approach in terms of "I did not think about/I did not have this in mind when making the decision".

Payoffs. The players can contract on an action and transfer money between themselves. As already mentioned, action a (alternatively, $\widehat{a}$) is jointly optimal, i.e., maximizes the sum of the two players' gross surplus, in state ω (alternatively, $\widehat{\omega}$). Initially, only the state space and action a are known to the players. Searching for information leads each player to either learn nothing, or to learn both $\widehat{\omega}$ and $\widehat{a}$: As we noted, learning that the state is $\widehat{\omega}$ also reveals to a player what's to be done in that state of nature (that is, action $\widehat{a}$) and conversely.

We let U_i (alternatively, $\widehat{U}_i$) denote player i's *gross* surplus in state ω (alternatively, $\widehat{\omega}$) when the optimal action for that state is taken (a when the state is ω and $\hat{a}$ when the state is $\hat{\omega}$). We also denote by $U = \Sigma_i U_i$ and $\widehat{U} = \Sigma_i \widehat{U}_i$. We then denote by $\delta \geq 0$ the deadweight loss associated with choosing action a in state $\widehat{\omega}$ (recall that, in state ω, only action a is available; see footnote 9). Such a loss can then be decomposed into the respective losses to the two players, δ_i, with $\Sigma_i \delta_i = \delta$. These payoffs should be interpreted as post-renegotiation payoffs in case the initial contract is eventually renegotiated, in which case δ can be interpreted as the "adjustment cost," which is incurred in case the parties fail to specify the right design $\hat{a}$ upfront at the initial contractual stage. These payoffs are gross of monetary transfers between the two players and of cognitive costs.

The assumption that payoffs are renegotiation-inclusive deserves some comment. An improper design, by definition, leads to a higher cost or to a lower customer satisfaction (or both). When the state of nature is realized, it may be too late to change the design, and the deadweight loss is fully incurred (the "no-renegotiation case"). Alternatively, the loss may be reduced, or even eliminated, by altering the design and incurring some associated adjustment cost (the "renegotiation case"). We would expect that the earlier the state of nature is realized, the lower the adjustment cost (for example, fewer investments will have been sunk by the seller and the buyer).

Figure 1 represents the players' joint payoffs as a function of the state and the action specified at the contractual stage.

---

looks for the appropriate solution. The resulting search cost could be subsumed in the overall cognition cost function of learning the design for the state $\hat{\omega}$. So the assumption should not be taken too literally.

[9]The binary-choice structure can alternatively be derived by assuming that the action set is composed of the status-quo action a, and of a large number of known, but ex-ante indistinguishable alternative actions. With probability q, all these other actions give very negative gross payoffs to the players. With probability $1 - q$, instead, one of the alternative actions gives a higher total surplus than action a, whereas the remaining alternatives to the status quo still give very negative gross payoffs. The "relevant alternative" can be labeled $\widehat{a}$ and the state in which there is such a relevant alternative is labelled $\widehat{\omega}$.

state of nature

Figure 1: **Joint surplus**

This formalism exhibits flexibility also along another dimension, which we will later make use of. In a first interpretation of the model, the information held by the parties can be disclosed voluntarily prior to contracting or is not disclosed at all until the state of nature is publicly realized. As we clarify in due time, the incentives the players face in such a situation are the same as under a *mandatory disclosure* regime in which any post-contractual disclosure is deemed by courts a violation of good-faith bargaining (a party disclosing information after a contract was signed but before the state is publicly revealed may be interpreted as concealing information that was already available at the contractual stage, even when this is not the case). A different situation arises when a party can conceal information at the contractual stage and reveal it after the contract is signed but before the state of nature is fully revealed, without courts ruling against such behavior (we will refer to such a possibility as *delayed disclosure*). Formally, such a situation is similar to the first one that arises when players can disclose only prior to contracting one, but with a lower realized deadweight loss in case of delayed disclosure.

We assume that each player's outside option (that is, her payoff in case of no trade with the other party) is equal to zero. Finally, we assume that $U, \hat{U} \geq 0$, which means that, under the appropriate design, there are gains from trade (at least weakly) in either state.[10]

Bargaining outcome. Let w_i denote player i's bargaining power, or weight, in any negotiation between the two players, with $\Sigma_i w_i = 1$. We assume that transfers negotiated between the two players are such that player i (alternatively, j) reaps a fraction w_i (alternatively, w_j) of the total gains from trade expected by any third-party (e.g., an arbitrator), given the information disclosed by the two players, and given the parties' (equilibrium) cognitive strategies. For example, suppose that, in equilibrium, player i is expected to engage in cognition ρ_i and disclose with probability d_i

---

[10]The assumption that $\hat{U} \geq 0$ is without loss of generality; once the parties learn that the state is $\hat{\omega}$, because they face no residual uncertainty anymore, they can always decide to leave the relationship and enjoy their outside options, if this is the efficient course of action. The assumption that $U \geq 0$, instead, is with loss of generality; it rules out the possibility that the parties stay in the relationship when, under the default design a, their total surplus is below their outside options, because they hope that the state is $\hat{\omega}$ and that this event will eventually become known to them. This possibility, however, does not seem particularly relevant for the applications the analysis is meant for.

in state $\hat{\omega}$ (as explained above, disclosure is possible only in this state) and likewise that player j is expected to engage in cognition ρ_j and disclose with probability d_j. Let q' (alternatively, $\hat{q}'$) denote the posterior probability of the state being ω (alternatively, $\hat{\omega}$) in the absence of any disclosure, given the players' equilibrium strategies.[11] Using Bayes' rule

$$q' = \frac{q}{q + \hat{q}(1 - \rho_i d_i)(1 - \rho_j d_j)} = 1 - \hat{q}'.$$

In case of no disclosure, the transfer t_i to player i (which is equal to $-t_j$ where t_j is the transfer to player j) is thus given by the unique solution to

$$q'U_i + \hat{q}'\left(\widehat{U}_i - \delta_i\right) + t_i = w_i\left[q'U + \hat{q}'\left(\widehat{U} - \delta\right)\right] \tag{1}$$

and yields player i a fraction w_i of the total surplus $q'U + \hat{q}'\left(\widehat{U} - \delta\right)$ expected under the equilibrium strategies. Note that the term in square brackets in the right-hand side is the total surplus expected by a third party in case of no disclosure. The transfer t_i is thus determined so as to to give to player i a fraction w_i of the expected total surplus.

Likewise, when one of the two players discloses, thus revealing that the state is $\hat{\omega}$ (recall that the disclosed information is hard), then the transfer to player i is set to satisfy

$$\widehat{U}_i + t_i = w_i\widehat{U}. \tag{2}$$

The same division of surplus is used to determine transfers ex-post, in case parties renegotiate the initial contract. That is, ex-post transfers are determined so that each player i receives a fraction w_i of the total surplus from renegotiating the initial contract. We also assume that the gross payoffs in each of the two states are such that, when the transfers between the two players are determined by the rule described above, each player prefers contracting with the other player to her outside option (accounting for the fact that, off path, beliefs may differ from equilibrium beliefs q' and $\hat{q}'$). In the Appendix, we identify the precise conditions that guarantee that this is the case and show that these conditions are always met when, given δ and $U_i - \widehat{U}_i$, the gross surplus U_i is large enough for all i.

The advantage of the protocol above is that it permits us to capture the incentives the two players face when it comes to the collection of hard information and the choice of whether or not to disclose it in the simplest possible way. In particular, it eliminates both signaling through the choice of t_i (with the associated multiplicity of equilibrium offers driven by the players' out-of-equilibrium beliefs) and the possibility that a player collects information not to disclose it, but to reject offers by the opponent that deliver a payoff below their outside option. While both possibilities are certainly of interest, they are not novel and distract from what the paper delivers. Importantly, the outcomes of the above protocol coincide with those under the familiar Rubinstein-Stahl sequential bargaining game when (a) offers are frequent (i.e., when the delay between offers vanishes), (b) the players' participation

---

[11]Hereafter, we use the "prime" sign to denote a third party's posterior beliefs, in the absence of disclosure, computed using the players' equilibrium strategies.

constraints under the above protocol are satisfied, and (c) players hold passive beliefs.[12] The last assumption, which amounts to the refinement that players do not change their beliefs over the information held by the opponent when they receive an off-path offer, seems justified in our setting, given that all types of each player have the same preferences over the negotiated price, which makes signaling implausible.

The following concept will play an important role in our analysis:

Definition 1. Player i's **_relative exposure to the unexpected_** is given by

$$\sigma_i \equiv \left[U_i - \left(\widehat{U}_i - \delta_i \right) \right] - w_i \left[U - \left(\widehat{U} - \delta \right) \right].$$

Player i is *relatively more exposed to the unexpected* if $\sigma_i > 0$.

Intuitively, player i loses (or gains) gross surplus $U_i - \left(\widehat{U}_i - \delta_i \right)$ when the default design a is specified in the contract and the realized state is $\widehat{\omega}$ rather than ω. From an ex-ante viewpoint, the player may be able to appropriate a fraction of the total loss $U - \left(\widehat{U} - \delta \right)$, the magnitude of which depends on the player's bargaining weight w_i. Note that the measure σ_i is indeed a relative exposure measure as $\Sigma_i \sigma_i = 0$. Accordingly, we say that player i is relatively more exposed if $\sigma_i > 0$.

The parameter σ_i will play an important role as a sufficient statistic underlying our various results. It allows us to give a formal meaning to Cooter and Ulen (2016) distinction between "productive and redistributive facts". The authors acknowledge that information acquisition usually unveils both productive and redistributive elements. In our model, σ_i has a clear redistributive/zero-sum nature, whereas $\widehat{q}\delta$ captures the productive stake in information acquisition (equivalently, the expected losses that the two players can avoid by discovering that the state is $\hat{\omega}$ and writing a contract specifying the appropriate action $\hat{a}$ for that state).

Timing. In Section 3, we will assume that disclosure occurs either prior to contracting or never until the state of nature is publicly realized (as anticipated above, this situation will be shown to be equivalent to one in which disclosure is mandatory by law). This assumption does not rule out renegotiation *after* the state of nature is publicly disclosed, as shown by the two illustrations discussed in Subsection 2.2. By contrast, the case of delayed disclosure will be studied in Section 4. The timing, in the case of pre-contractual disclosure, is summarized in Figure 2 and goes as follows:

(1) The parties secretly and non-cooperatively choose how much information to acquire (formally captured by the probability ρ_i of learning the state $\widehat{\omega}$, when relevant);

(2) Parties either learn that the state is $\widehat{\omega}$ or do not receive any information (i.e., receive the null signal $\varnothing$);

(3) Parties simultaneously choose whether to disclose $\widehat{\omega}$, if relevant, to the other party;

---

[12]The proof follows from familiar arguments and is thus omitted; it is, however, available upon request.

10

(4) In the absence of any disclosure, the transfer t_i between the two parties is determined according to (1) in conjunction with action a. If one of the parties discloses, in which case the state $\hat{w}$ is revealed, the action $\hat{a}$ is chosen and the transfer t_i is determined according to (2).

(5) The state of nature is publicly realized.[13]

Note that one can without loss of generality assume that if exerted, the outside option is exerted at stage (4) as a party can always choose to not invest in cognition before.
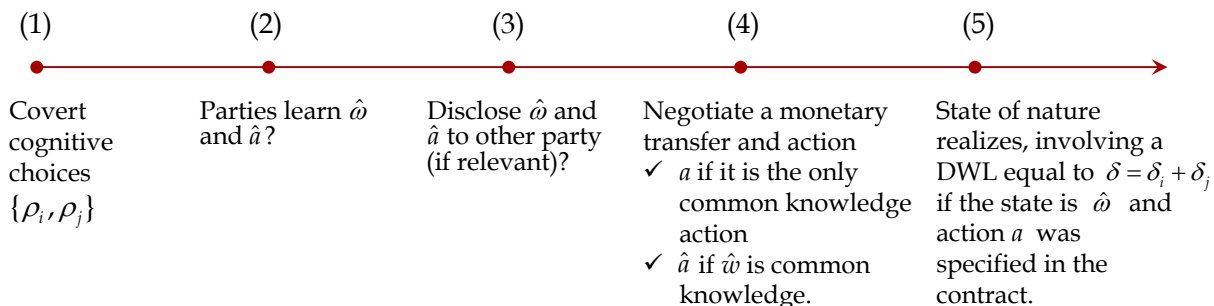
(1)	(2)	(3)	(4)	(5)
Covert cognitive choices $\{\rho_i, \rho_j\}$	Parties learn $\hat{\omega}$ and $\hat{a}$?	Disclose $\hat{\omega}$ and $\hat{a}$ to other party (if relevant)?	Negotiate a monetary transfer and action ✓ a if it is the only common knowledge action ✓ $\hat{a}$ if $\hat{w}$ is common knowledge.	State of nature realizes, involving a DWL equal to $\delta = \delta_i + \delta_j$ if the state is $\hat{\omega}$ and action a was specified in the contract.

Figure 2: **Timing**

2.2 Illustrations

Example 1: Buyer-seller game.
The seller's cost of supplying the buyer is known and equal to c. The buyer's gross surplus is B if the design matches the state of nature, but only $b < B$ if design a is chosen in state $\hat{\omega}$. If design a is chosen and state $\hat{\omega}$ is revealed, the buyer can still enjoy the full surplus B but only if the seller incurs some adjustment cost $\alpha \geq 0$ (such a cost may reflect the changes to the product necessary to deliver the value B to the buyer in state $\hat{\omega}$). Figures 3 and 4 summarize the gross payoffs of the two parties without and with renegotiation, respectively.

1. *No renegotiation (the ex-post adjustment cost is large: $\alpha \geq B - b$).*

	ω	$\widehat{\omega}$
a	$B, -c$	$b, -c$
$\hat{a}$		$B, -c$

Figure 3: **Buyer-seller game in the absence of ex-post renegotiation**

---

[13]As noted above, once the state is publicly revealed, the parties can still renegotiate to their mutual advantage. This is the case if action a is specified in the original contract, the state of nature is $\hat{\omega}$, and the adjustment cost to replace action a with action $\hat{a}$ is not too large: see the examples in Section 2.2.

11

Hence, the gross payoffs satisfy the following conditions:

$$U = \widehat{U} = B - c, \quad \delta_B = B - b, \quad \delta_S = 0, \quad \delta = B - b.$$

As a result,

$$\sigma_B = \delta_B - w_B\delta = w_S\delta = w_S(B - b) > 0,$$

implying that the buyer is relatively more exposed to the unexpected.

2. *Ex-post renegotiation* (adjustment cost $\alpha < B - b$).

	ω	$\widehat{\omega}$
a	$B, -c$	$b + w_B(B - b - \alpha),$ $-c + w_S(B - b - \alpha)$
$\hat{a}$		$B, -c$

Figure 4: **Buyer-seller game with ex-post renegotiation**

Compared to the case of a large adjustment cost, the deadweight loss is smaller, but the relative exposure is the same. To see this, note that

$$\delta_B = (B - b) - w_B[(B - b) - \alpha] = w_S(B - b) + w_B\alpha$$

and

$$\delta_S = -w_S(B - b - \alpha) = \alpha - \delta_B$$

and hence

$$\delta = \alpha \quad \text{and} \quad \sigma_B = w_S(B - b).$$

Note that, for $\alpha = B - b$, the expressions are the same as for the no-renegotiation case.

This model allows one to get at the notion of contract incompleteness in an otherwise familiar modeling environment. Contract incompleteness is then related to the amount of pre-contractual cognition and is measured by the probability that the wrong design is adopted in state of nature $\hat{\omega}$; this is a rather natural definition, especially when some adjustment/renegotiation occurs in that configuration.

Figures 3 and 4 describe *symmetric* versions of the buyer-seller game: The players' payoffs are state independent provided that the optimal (state-dependent) action is specified in the original contract. More generally, one can allow these payoffs to depend on the state; a case in point is the shrouded attributes model, to which we turn next.

Example 2: Generalized Gabaix and Laibson (2006) shrouded attributes model

A seller may disclose to a buyer that the satisfactory consumption of a basic good may require the supply of an add-on also provided by the seller. The cost to the seller of the basic good is equal to c. The add-on may or may not be needed for the buyer to be able to enjoy the basic good. Let ω correspond to the state in which the add-on is not needed and $\hat{\omega}$ the state in which it is. The cost of the add-on to the seller is $\hat{c}$. Gabaix and Laibson (2006) assume an exogenous information structure in which the seller is perfectly informed whereas the buyer is unable to invest in cognition. Although their model is phrased in terms of boundedly rational agents, the key insights can be illustrated in the context of our framework. The situation examined in Gabaix and Laibson (2006) corresponds to a special version of our model in which stages (1) and (2) are irrelevant (and hence omitted in the timing described in Figure 5), $\rho_S = 1$ and $\rho_B = 0$.
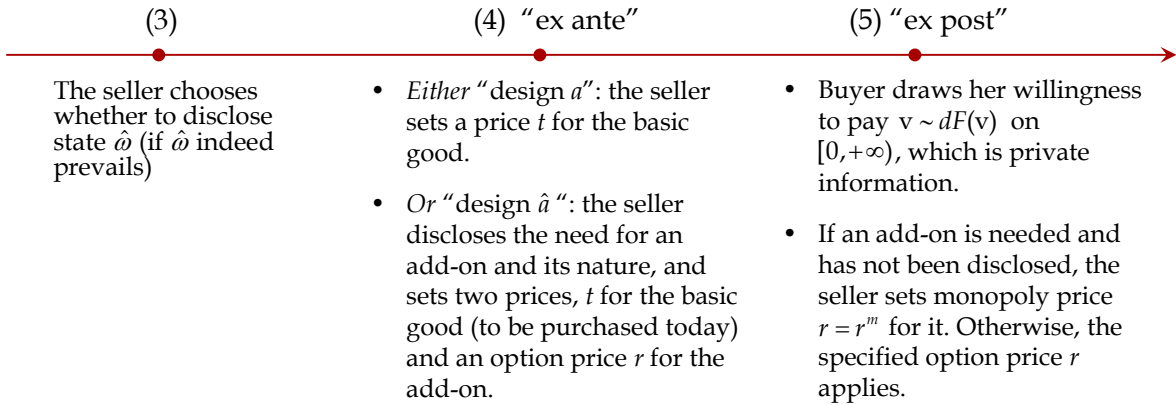
(3)	(4) "ex ante"	(5) "ex post"
The seller chooses whether to disclose state $\hat{\omega}$ (if $\hat{\omega}$ indeed prevails)	• *Either* "design a": the seller sets a price t for the basic good. • *Or* "design $\hat{a}$": the seller discloses the need for an add-on and its nature, and sets two prices, t for the basic good (to be purchased today) and an option price r for the add-on.	• Buyer draws her willingness to pay v $\sim dF($v$)$ on $[0,+\infty)$, which is private information. • If an add-on is needed and has not been disclosed, the seller sets monopoly price $r = r^m$ for it. Otherwise, the specified option price r applies.

Figure 5: **Timing in the exogenous-information shrouded-attributes model**

For simplicity, assume that, in state $\widehat{w}$ in which the add-on is needed, the basic good brings no value to the buyer, unless combined with the add-on. The value the buyer assigns to the good (in its basic configuration in state ω or together with the add-on in state $\widehat{\omega}$) is v, with the latter drawn from $\mathbb{R}_+$ according to the distribution F and privately observed by the buyer ex-post. i.e., at stage (5). The motivation for a random willingness to pay is that it generates a downward-sloping demand (and hence a monopoly distortion) in case of ex-post contracting (i.e., in case the need for the add-on is disclosed only ex-post, after the initial contract is signed) without introducing adverse selection on the buyer side at the ex-ante contracting stage. Let r^m denote the monopoly price for the add-on in state $\hat{\omega}$ when the state is revealed only ex-post, after the initial contract is signed. Because, in this state, the value for the basic good alone is zero, the value the buyer assigns to the add-on coincides with the value v the buyer assigns to the basic good together with the add-on (equivalently, one can think of v as the buyer's value for the "satisfactory" version of the good, which coincides with the basic specification in state ω and to the combination of the basic configuration with the add-on in state $\widehat{\omega}$). Hence, the ex-post monopoly price for the add-on in state $\widehat{\omega}$ is the value of r for which $(r - \hat{c})[1 - F(r)]$ is the highest. The lack of ex-ante adverse selection implies that if the need for the add-on is disclosed at the ex-ante stage, then it is optimal for the seller to price it at marginal

13

cost, in which case there is no ex-post distortion. Let $\mathcal{S}(r) = \int_r^\infty (\mathrm{v} - r)dF(\mathrm{v})$ denote the buyer's expected surplus when the price for the add-on is r, gross of the price of the basic good. Gabaix and Laibson (2006)'s model thus corresponds to a buyer-seller game in which the payoffs are asymmetric across the two states when $\hat{c} > 0$. The two players' payoffs (gross of the price paid by the buyer for the basic good) are represented in Figure 6.

	ω	$\widehat{\omega}$
a	$\int_0^\infty \mathrm{v}dF(\mathrm{v})\,,\ \ -c$	$\int_{r^m}^\infty (\mathrm{v} - r^m)dF(\mathrm{v})\,,$ $-c + (r^m - \hat{c})[1 - F(r^m)]$
$\hat{a}$		$\int_{\hat{c}}^\infty (\mathrm{v} - \hat{c})dF(\mathrm{v}),\ \ -c$

Figure 6: **Payoffs in the Gabaix and Laibson (2006) model (gross of the ex-ante transfer)**

Hence, modulo the exogeneity of the information structure, this model is also a special case of our general model in which the following conditions hold:

- $w_S = 1$ (the seller is a price setter);

- $\delta = \big[\mathcal{S}(\hat{c}) - \mathcal{S}(r^m)\big] - (r^m - \hat{c})\big[1 - F(r^m)\big]$ is the deadweight loss associated with non-disclosure and ex-post monopoly pricing (so the "wrong design" de facto corresponds to a contractual failure in which the seller, by not disclosing, fails to commit to the cost-based add-on price);

- $\sigma_B = \int_0^{r^m} \mathrm{v}\,dF(\mathrm{v}) + \big[1 - F(r^m)\big]r^m$ (the buyer's relative exposure to the unexpected is equal to the buyer's loss of utility under ex-post monopoly pricing and combines the loss from foregone consumption (in case $v \in [0, r^m]$) and the extra payment (in case $\mathrm{v} > r^m$).

2.3 Special cases of cognition

The analysis in the next sections allows for fairly flexible payoff specifications and for different forms of cognition. The following special cases, however, may be particularly salient in applications.

1. *Inner-looking cognition*: The uncertainty that player i faces is about her own payoff only: $U_j = \hat{U}_j = \hat{U}_j - \delta_j$. For example, in a natural variant of the buyer-seller game described above, the unknown state of nature might raise the production cost from c to $C > c$ (or possibly lower it), without affecting the buyer's valuation (i.e., $b = B$).

14

2. *Outer-looking cognition*: The uncertainty that player i faces is about player j's payoff only: $U_i = \hat{U}_i = \hat{U}_i - \delta_i$. This situation is precisely the one described in the buyer-seller game of Example 1 above, where the seller may learn about whether the design of the good fits the buyer's needs, and where the seller's cost is fixed at c.

3. *Mixed case*: Player i's cognition uncovers information that is payoff-relevant for both players. For example, in the Gabaix and Laibson (2006) model considered above, ex-post renegotiation makes the state of nature relevant for both parties.

3 One-sided cognition under voluntary disclosure

Suppose, first, that only player i can search for the state of nature $\widehat{\omega}$ (player j will then be referred to as "the other player"). For example, only the seller is able to find out whether a design flaw may prevent the buyer from fully enjoying the consumption experience (outer-looking cognition), or only the buyer may invest in learning whether the design matches her own needs (inner-looking cognition).

3.1 Disclosure decision

In this subsection, we take the cognitive effort $\rho_i = \rho^*$ by player i as given and investigate player i's incentive to disclose.[14]

When player i is expected to exert cognition ρ^* and disclose with probability $d^* \in (0,1)$, the posterior probability that player j assign to the state being ω in the absence of any disclosure (which coincides with the probability assigned to the same event by the third party, e.g., an arbitrator negotiating the transfer on behalf of the two players) is equal to

$$q'(\rho^*, d^*) = \frac{q}{1 - (1-q)\rho^* d^*}. \tag{3}$$

Suppose first that player i is expected to disclose with certainty when he finds that the state is $\widehat{\omega}$ (i.e., $d^* = 1$) (Note that this is necessarily the case when the cost of information is strictly positive, as there is no point in searching if it is weakly suboptimal to disclose).[15]

Suppose player i learns that the state is $\widehat{\omega}$. By disclosing this information, he obtains a payoff equal to $w_i\widehat{U}$, that is, his share w_i of the (complete-information) total surplus $\widehat{U}$. By not disclosing,

---

[14]Because player i is the only player to search and disclose information, we drop the subscript i from ρ_i and d_i to ease the notation.

[15]There are in general two possible motivations for player i to acquire information. The first (on which we focus here) is to disclose it so as to avoid the deadweight loss of a wrong contract design. The second is to decide whether or not to trade at all with player j (this alternative motivation is ruled out by the assumption that, given the transfer t_i proposed by the third party, player i's expected surplus is higher than his outside option, no matter the player's posterior beliefs). See also the discussion in the Appendix.

instead, he obtains

$$\widehat{U}_i - \delta_i + t_i,$$

where t_i is the transfer associated with the default design a, as given in Condition (1). Simple algebra reveals that player i prefers disclosing to not disclosing if and only if

$$w_i\delta \geq q'(\rho^*, 1)\sigma_j \tag{4}$$

where σ_j is player j's relative exposure to the unexpected.

Condition (4) says that player i prefers to disclose whenever his share of the deadweight loss, $w_i\delta$, exceeds the cross-subsidy embodied in the transfer t_i, namely $q'(\rho^*, 1)\sigma_j$. This cross-subsidy is proportional to the posterior probability $q'(\rho^*, 1)$ that player j (as well as the third party negotiating the transfer t_i on behalf of the two players) assigns to the erroneous state, ω, in the absence of disclosure, when player i is expected to search with intensity ρ^* and disclose with certainty. The right-hand-side of (4) is thus a rent that player i obtains thanks to the mis-pricing induced by the third party's incorrect beliefs.

If Condition (4) is violated, disclosing information when expected to do so is not sequentially rational for player i. In particular, when $w_i\delta \leq q\sigma_j$, because, for any probability $d^* > 0$ by which player i discloses, $q'(\rho^*, d^*) > q = q'(\rho^*, 0)$, there is no equilibrium in which player i discloses information with positive probability after learning that the state is $\widehat{\omega}$. If, instead,

$$q\sigma_j < w_i\delta < q'(\rho^*, 1)\sigma_j,$$

then, holding fixed player i's cognition at ρ^*, we have that, while disclosing with certainty cannot be part of an equilibrium, disclosing with probability $d^* \in (0, 1)$, with the latter appropriately defined, can be consistent with player i's rationality. In fact, using again Condition (1), we have that, when player j and the third party expect player i do disclose with probability d^*, player i is indifferent between disclosing and not disclosing if and only if $q'(\rho^*, d^*)\sigma_j = w_i\delta$, which implies that d^* must be equal to the unique solution to[16]

$$\frac{q}{1 - (1 - q)\rho^* d^*} = \frac{w_i\delta}{\sigma_j}. \tag{5}$$

The following proposition summarizes the above results:

Proposition 2. [*incentive to disclose prior to contracting*] *Suppose that player i's cognition is fixed at $\rho_i = \rho^*$. In the voluntary disclosure game, player i's disclosure behavior is unique.*

   1. *Player i discloses with certainty if $w_i\delta \geq q'(\rho^*, 1)\sigma_j$;*

---

[16]In this case, the ex-ante expected deadweight loss is equal to

$$DWL \equiv \widehat{q}(1 - \rho^* d^*)\delta = q\left[\frac{\sigma_j}{w_i\delta} - 1\right]\delta.$$

2. *Player i does not disclose if $w_i \delta \leq q\sigma_j$;*

3. *Player i randomizes between disclosing and not disclosing if $q\sigma_j < w_i\delta < q'(\rho^*, 1)\sigma_j$. In this case, the probability d^* with which player i discloses is given by the unique solution to Equation (5).*

3.1.1 Applications

- *Full bargaining power.*

In some applications, the informed player has full bargaining power so that $w_i = 1$. Condition (4) then simplifies to

$$\delta \geq q'(\rho^*, 1)\big[U_j - (\widehat{U}_j - \delta_j)\big].$$

The left-hand side is the total deadweight loss from not changing the design when it is optimal to do so, whereas the right-hand side is the surplus that i can extract from j because of the latter's misperception of the state. In turn this is equal to the extra utility $U_j - (\widehat{U}_j - \delta_j)$ that j expects to derive from the state being ω instead of $\widehat{\omega}$, scaled by the probability $q'(\rho^*, 1)$ that j assigns to the state being ω when i is expected to exert cognition ρ^* and disclose with certainty. In other words, the left-hand side is the efficiency gain from taking the correct action in state $\widehat{\omega}$ whereas the right-hand side is the speculative gain from taking advantage of the opponent's misperception of the state.

- *Symmetric buyer-seller game.*

In Example 1, $\sigma_B = w_S(B - b)$. So, in the absence of renegotiation (for example because the adjustment costs are high), Condition (4) boils down to $w_B \geq -q'(\rho^*, 1)w_S$ if $i = B$, and $w_S \geq q'(\rho^*, 1)w_S$ if $i = S$. In other words, when renegotiation is too costly, player i always discloses, no matter whether he is a buyer or a seller.

In the presence of renegotiation, instead, the buyer always discloses, whereas the seller discloses only if the deadweight loss (then equal to the adjustment cost α) is large enough: $\alpha \geq q'(\rho^*, 1)(B-b)$.

- *Gabaix-Laibson (2006) model.*

The result in Proposition 2 implies that the seller opts for shrouded attributes (that is, does not disclose the need for the add-on when the latter is needed) if $w_S\delta \leq q\sigma_B$, or, applying the specific expressions for these variables and remembering that $w_S = 1$ in the Gabaix and Laibson (2006) model,

$$\int_{\widehat{c}}^{r^m} (\mathrm{v} - \widehat{c})dF(\mathrm{v}) \leq q\Big[\int_0^{r^m} \mathrm{v}dF(\mathrm{v}) + \big[1 - F(r^m)\big]r^m\Big]. \tag{6}$$

The left-hand side of (6) is the deadweight loss associated with non-disclosure and the concomitant monopoly pricing; this loss is entirely borne by the seller who has full bargaining power as a price setter. The right-hand side is the product of the posterior probability of the good state (which is

equal to the prior probability in a no-disclosure equilibrium) and the buyer's loss of utility when the state is $\widehat{\omega}$ instead of ω. This loss is decomposed into foregone consumption ($v \leq r^m$) and extra payment ($v > r^m$).

3.2 Cognitive choice

We now turn to player i's choice of cognition. Let $t_i(\rho^*, d^*)$ be the transfer that player i obtains in the absence of any disclosure when he is expected to exert cognition ρ^* and disclose with probability d^*. Using Condition (1), we have that

$$t_i(\rho^*, d^*) = w_i U - U_i + \hat{q}'(\rho^*, d^*)\sigma_i, \tag{7}$$

where $\hat{q}'(\rho^*, d^*) \equiv 1 - q'(\rho^*, d^*)$.

Let us first look for a cognition-free equilibrium ($\rho^* = 0$, $d^* \in [0, 1]$). Because $C_i'(0) = 0$, such an equilibrium exists if and only if there is no net gain from disclosure: $w_i \delta \leq q\sigma_j$ (from Proposition 2).

We therefore focus on the more interesting case in which in equilibrium player i chooses cognition $\rho^* > 0$ and discloses with certainty upon learning that the state is $\widehat{\omega}$. Note that the mixed-strategy region in part 3 of Proposition 2 can exist only if information is exogenous. As explained above, player i puts zero effort in acquiring information if the latter is costly and if it is weakly optimal ex-post not to disclose.

Let $V_i(\rho, \rho^*)$ denote player i's gross payoff when he chooses cognition ρ and is expected to choose cognition ρ^* and disclose with certainty in case he finds that the state is $\hat{\omega}$. Using the result in Proposition 2, for i to strictly prefer to disclose after learning that the state is $\hat{\omega}$, it must be that $w_i \delta > q'(\rho^*, 1)\sigma_j$. Clearly, the same condition also implies that player i strictly prefers to disclose when his actual cognition is ρ (recall that cognition is covert). Hence, when $w_i \delta > q'(\rho^*, 1)\sigma_j$,

$$V_i(\rho, \rho^*) = \hat{q}\Big[\rho w_i \widehat{U} + (1 - \rho)\big(\hat{U}_i - \delta_i + t_i(\rho^*, 1)\big)\Big] + q\big[U_i + t_i(\rho^*, 1)\big]. \tag{8}$$

For ρ^* to be an equilibrium cognitive level, it must be that

$$\rho^* \in \arg\max_{\rho} \{V_i(\rho, \rho^*) - C_i(\rho)\}$$

and hence ρ^* must satisfy the first-order-condition

$$C_i'(\rho^*) = \hat{q}\Big[w_i \widehat{U} - \big(\widehat{U}_i - \delta_i + t_i(\rho^*, 1)\big)\Big].$$

Equivalently, plugging the formula for $t_i(\rho^*, 1)$ from (7) and using $\sigma_i = -\sigma_j$, we have that ρ^* must satisfy

$$C_i'(\rho^*) = \hat{q}\Big[w_i \delta - q'(\rho^*, 1)\sigma_j\Big]. \tag{9}$$

As explained above, the right-hand side of Condition (9) is the benefit of disclosing information, net of its opportunity cost. Consistently with the result in Proposition 2, Condition (9) admits a

solution $\rho^* > 0$ if and only if $w_i\delta > q\sigma_j$, as $q'(\rho^*, 1)$ is increasing in ρ^* and is such that $q'(0,1) = q$. Furthermore, in this case, ρ^* is unique if $\sigma_j \geq 0$, as $q'(\rho^*, 1)$ is strictly increasing in ρ^*.

Comparing the equilibrium cognitive level to its efficient counterpart, we have that player i over-invests in cognition if the private benefit of investing in information acquisition (the right-hand side of (9)) exceeds the social benefit ($\hat{q}\delta$). This happens if and only if party i is heavily exposed, in the sense that

$$q'(\rho^*, 1)\sigma_i > w_j\delta. \tag{10}$$

Proposition 3. *[over-cognition] Player i over-invests in cognition (relative to what is socially efficient) only if player i is relatively more exposed to the unexpected than player j, that is, only if $\sigma_i > 0$.*

Next, we turn to expectation conformity, which we define as follows. Consider full disclosure and two levels of cognition ρ and ρ'. We say that expectation conformity holds if player i has more incentives to choose ρ' than ρ when expected to do so.[17] That is, expectation conformity holds if and only if

$$\Gamma_i^{EC}(\rho, \rho') \equiv \left[V_i(\rho', \rho') - V_i(\rho, \rho')\right] - \left[V_i(\rho', \rho) - V_i(\rho, \rho)\right] > 0.$$

Now suppose that player i finds it optimal to disclose information both when he is expected to engage in cognition ρ and when he is expected to engage in cognition ρ' (recall that there is no point for i to acquire any information if he then conceals it). Simple computations then show that

$$\Gamma_i^{EC}(\rho, \rho') = q\hat{q}(\rho' - \rho)\left[\frac{1}{1 - \hat{q}\rho'} - \frac{1}{1 - \hat{q}\rho}\right]\sigma_i.$$

Thus expectation conformity holds if and only if player i is relatively more exposed to the unexpected, i.e. $\sigma_i \geq 0$. In this case, the higher the cognition effort expected from him, the lower the probability of the bad state in the absence of disclosure and so the less favorable the agreement to player i whenever he does not disclose any information.[18] This raises player i's incentives to acquire information.

Finally, note that the exposure to the unexpected is also the key to the existence of multiple equilibria and to "cognitive traps," that is the possibility that player i is better off in a low-cognition equilibrium. To see this, first note that, when $\Gamma_i^{EC} < 0$, i.e., when $\sigma_i < 0$, the equilibrium level of cognition is unique.[19] Next, observe that, when $\Gamma_i^{EC} > 0$, i.e., when $\sigma_i > 0$, for any pair (ρ, ρ'), one can construct cost functions such that both ρ and ρ' are equilibrium cognitive levels. Finally, observe that

$$sgn\left(\frac{dV_i(\rho^*, \rho^*)}{d\rho^*}\right) = sgn\left(\frac{\partial t_i(\rho^*)}{\partial \rho^*}\right) = -sgn(\sigma_i),$$

---

[17]See Pavan and Tirole (2021) for a more general analysis of expectation conformity in games.

[18]Use (7) to verify that, when $\sigma_i > 0$, the transfer $t_i(\rho^*, 1)$ to player i is decreasing in the cognition ρ^* that player j expects from i.

[19]As anticipated above, when $\sigma_i < 0$ (equivalently, when $\sigma_j > 0$), Condition (9) has a unique solution.

which implies that, when multiple equilibria are possible (which is the case only if $\sigma_i > 0$) player i is better off in the low-cognition equilibrium. Summarizing the analysis above, we have the following result:

Proposition 4. [equilibrium cognition]
(i) Cognition $\rho^* > 0$ can be sustained in equilibrium if and only if it satisfies Condition (9).[20]
(ii) In any equilibrium in which $\rho^* > 0$, disclosure occurs with certainty.
(iii) If and only if player i is relatively more exposed to the unexpected ($\sigma_i > 0$), then player j's anticipation of a higher cognition by player i increases the latter player's stake in cognition (expectation conformity), thus raising the possibility of multiple equilibria.
(iv) In case of multiple equilibria, player i is better off in a low-cognition equilibrium.

4 Strategic delay and mandatory disclosure

4.1 Delayed disclosure

So far, we assumed that a party acquiring information discloses it before contracting if he discloses it at all; in the absence of intent of pre-contractual disclosure, the party would not acquire any information. This property no longer holds if the information can be withheld and disclosed at some stage between the contracting date and the date at which the state of nature is publicly realized. In this subsection, there is no penalty for withholding information other than the deadweight loss of embarking on the wrong design.

To illustrate, we modify the course of events in Figure 2 by allowing for interim renegotiation at stage (5).[21] If the state of nature is $\widehat{\omega}$ and action a specified by the contract is renegotiated into $\widehat{a}$ at stage (5), the total deadweight loss is only $\beta\delta$ where $\beta \in [0,1]$: Some useless investments can be avoided, while others have already been sunk prior to this early renegotiation stage. We will not need to specify whose investments have been sunk by stage (5), as we will see that only the total deadweight loss $\beta\delta = \beta_i\delta_i + \beta_j\delta_j$ matters.

The possibility of interim renegotiation raises the informed party's incentive to disguise her information at the contracting stage because, when concealing that the state is $\widehat{\omega}$, she is then able to limit the damage done by concealment while still acquiring a good bargaining position (if party j is relatively more exposed, that is, if $\sigma_j > 0$).

We also allow party i to exogenously receive information at stage (5): She learns the state $\widehat{\omega}$ (if it occurs) with probability μ.[22] Namely, we assume that (a) party i's pre-contracting probability of

---

[20]Note that, given ρ^*, the function $V_i(\rho, \rho^*) - C_i(\rho)$ is globally concave in ρ.

[21]Such interim renegotiation is not to be confused with the ex-post renegotiation that may occur after the state of Nature is publicly revealed (such as those considered in the buyer-seller or in the shrouded-attributes games of Subsection 2.2).

[22]In a multi-period extension, one would expect the probability of receiving information, μ, and the remaining deadweight loss, β, to be positively correlated: As time elapses, more information is acquired and more investments

being informed in state $\widehat{\omega}$, ρ_i, belongs to $[0, 1-\mu]$ and the cost function satisfies the same properties as earlier, with $\lim_{\rho \to 1-\mu} C_i'(\rho) = +\infty$, and (b) the total probability of learning state $\widehat{\omega}$ when it prevails is $\rho + \mu$.[23] We represent these changes in the environment in Figure 7, which describes only the contractual/post-contractual events (the pre-contractual ones are the same as in the previous sections).
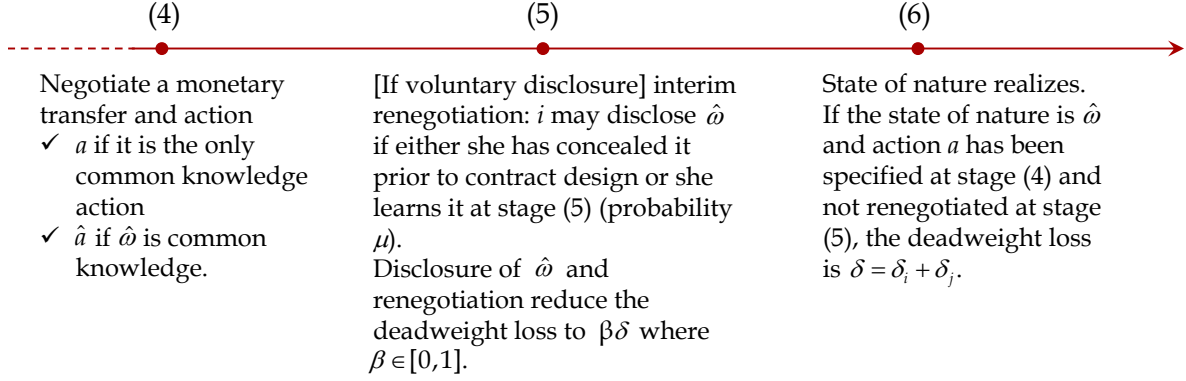

Figure 7: **Interim renegotiation**

Suppose that party i is expected to engage in cognition ρ^* and disclose with probability d^* at stage (3). When, at stage (2), party i learns that the state is $\widehat{\omega}$ but does not disclose this information till stage (5), she receives a payoff equal to

$$\left(\widehat{U}_i - \delta_i\right) + t_i + w_i(1 - \beta)\delta,$$

where $t_i = t_i(\rho^*, d^*)$ now solves

$$t_i + q'(\rho^*, d^*)U_i + \widehat{q}'(\rho^*, d^*)\left[\widehat{U}_i - \delta_i + \frac{\mu + \rho^*(1 - d^*)}{1 - \rho^* d^*}w_i(1 - \beta)\delta\right]$$

$$= w_i \left\{q'(\rho^*, d^*)U + \widehat{q}'(\rho^*, d^*)\left[(\widehat{U} - \delta) + \frac{\mu + \rho^*(1 - d^*)}{1 - \rho^* d^*}(1 - \beta)\delta\right]\right\} \qquad (11)$$

with $q'(\rho^*, d^*) = 1 - \widehat{q}'(\rho^*, d^*)$ as defined in the previous section. One can verify that the transfer t_i that solves (11) is the same as in the previous section (as given in (1) or, equivalently, by (7)).

The condition for player i to disclose for sure at stage (3) ($d^* = 1$) then becomes

$$w_i \widehat{U} \geq \widehat{U}_i - \delta_i + t_i + w_i(1 - \beta)\delta$$

which, after substituting for $t_i = t_i(\rho^*, 1)$, can be rewritten as

$$w_i \beta \delta \geq q'(\rho^*, 1)\sigma_j, \qquad (12)$$

[23]One could alternatively assume independence between the two events (the total probability of receiving information being equal to $\rho + (1 - \rho)\mu$) or various forms of positive or negative correlation between the two events, without any significant change in the qualitative results. One could also let stage-(5) information acquisition, which here is assumed to be exogenous, be subject to moral hazard, again without any significant change to the insights.

where, as earlier, $\sigma_j \equiv U_j - (\widehat{U}_j - \delta_j) - w_j[U - (\widehat{U} - \delta)]$ denotes player j's relative exposure to the unexpected.

Player i's equilibrium cognitive level ρ^*, when the latter is positive and player i discloses with certainty at stage (3), then continues to be given by the solution to[24]

$$C_i'(\rho^*) = \widehat{q}[w_i \delta - q'(\rho^*, 1)\sigma_j]. \tag{13}$$

In the special case in which renegotiation can take place at no cost before any investment is sunk (i.e., $\beta = 0$), it is optimal for player i to disclose prior to contracting if and only if he is relatively more exposed to the unexpected (i.e., if and only if $\sigma_i \geq 0$). In this case, the specification of the contractual terms is just a zero-sum game. More generally, we have the following result:

Proposition 5. *[**delayed disclosure**] Let $\rho^{ND}(\beta)$ and ρ^D solve $C_i'(\rho^{ND}) = \widehat{q}w_i(1 - \beta)\delta$ and $C_i'(\rho^D) = \widehat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j]$, respectively.[25] There exist thresholds $\beta^*, \beta^{**} \in [0, 1]$, with $\beta^* \leq \beta^{**}$, such that the following are true:*

1. *when $\beta \leq \beta^*$, the cognitive level is given by $\rho^{ND}(\beta)$ and disclosure at stage (3) occurs with probability zero;*

2. *when $\beta^* < \beta < \beta^{**}$, the cognitive level is equal to $\rho^{ND}(\beta)$ and disclosure occurs with probability $d^*(\beta) \in (0, 1)$ given by the unique solution to $q\sigma_j/[1 - (1 - q)\rho^{ND}(\beta)d^*] = w_i\beta\delta$;*

3. *when $\beta \geq \beta^{**}$, the cognitive level is equal to ρ^D and disclosure occurs with certainty.[26]*

Proof of Proposition 5. From the discussion preceding the proposition, we have that, when $w_i\beta\delta \leq q\sigma_j$, player i never discloses at stage (3). That, in this case, the equilibrium level of cognition is given by $\rho^{ND}(\beta)$ follows from the fact that the benefit of cognition comes from the possibility to disclose at stage (5), thus reducing the deadweight loss from δ to $\beta\delta$ (the marginal benefit of cognition is thus equal to $\widehat{q}w_i(1 - \beta)\delta$).

When, instead, $w_i\beta\delta > q\sigma_j$, then disclosure at stage (3) occurs with positive probability under any equilibrium. In particular, when $q\sigma_j < w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, which requires that $\sigma_j > 0$, in the unique equilibrium, player i engages in cognition $\rho^{ND}(\beta)$ and then discloses with probability $d^*(\beta) \in (0, 1)$ given by the unique solution to $q\sigma_j/[1 - (1 - q)\rho^{ND}(\beta)d] = w_i\beta\delta$. To see why this is the case, note that, given cognition $\rho^{ND}(\beta)$, if player i were expected to disclose with certainty, the net

---

[24]The reason why the condition for the equilibrium value of ρ^* is unaffected by the possibility of a late disclosure is that the increase in the probability of finding information at stage (3) comes with a reduction in the probability of not finding any information at all (the probability μ of receiving information at stage (5) is fixed).

Note also that for a given d^*,
$$V(\rho, \rho^*) = \quad q[U_i + t_i(\rho^*, d^*)] + \widehat{q}\big[(1 - \mu - \rho)[\widehat{U}_i - \delta_i + t_i(\rho^*, d^*)]$$
$$+ \rho d^*(w_i\widehat{U}) + [\mu + \rho(1 - d^*)][\widehat{U}_i - \delta_i + t_i(\rho^*, d^*) + w_i(1 - \beta)\delta]\big].$$

[25]The assumption that $\lim_{\rho \to 1 - \mu} C_i'(\rho) = +\infty$ implies that $\rho^{ND}(\beta), \rho^D < 1 - \mu$ for all $\beta \in [0, 1]$.

[26]As discussed above, equation $C_i'(\rho^D) = \widehat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j]$ can have multiple solutions only when $\sigma_j < 0$. In this case, $\beta^* = \beta^{**} = 0$ as shown in the proof of the proposition.

benefit of disclosure would be equal to $w_i\beta\delta - q'(\rho^{ND}(\beta), 1)\sigma_j < 0$, whereas, if he were expected to disclose with probability zero, the net benefit of disclosing at stage (3) would be equal to $w_i\beta\delta - q\sigma_j > 0$, implying that, in either case, player i would have a profitable deviation. Hence, when $q\sigma_j < w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, in any equilibrium in which cognition is equal to $\rho^{ND}(\beta)$, the equilibrium probability of disclosure, $d^*(\beta)$, is given by the unique solution to $w_i\beta\delta - q'(\rho^{ND}(\beta), d^*)\sigma_j = 0$. Note that the last condition implies that, when player j and the third party expect player i to engage in cognition $\rho^{ND}(\beta)$ and discloses with probability $d^*(\beta)$, player i is indifferent between disclosing and not disclosing at stage (3). Also note that, because $\rho^{ND}(\beta)$ is decreasing in β, $d^*(\beta)$ is increasing in β.

To see that, when $q\sigma_j < w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, there exists no equilibrium with cognition other than $\rho^{ND}(\beta)$, first recall that, because $q\sigma_j < w_i\beta\delta$, in any equilibrium, disclosure must occur with positive probability. Next, observe that, in any equilibrium in which player i discloses with probability less than 1, player i must be indifferent between disclosing and concealing at stage (3), implying that the marginal benefit from information acquisition is equal to $\hat{q}w_i(1-\beta)\delta$.[27] Because player i's net payoff is strictly concave in ρ_i, player i's cognitive level must necessarily be equal to $\rho^{ND}(\beta)$. Finally, observe that there exists no equilibrium in which disclosure occurs with certainty. To see this, note that, if such an equilibrium existed, then player i's equilibrium cognitive level ρ^D would have to solve $C_i'(\rho^D) = \hat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j]$, with $q'(\rho^D, 1)\sigma_j \leq \beta w_i\delta$. Because $q'(\rho, 1)$ is increasing in ρ, and because $w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, it must be that $\rho^D < \rho^{ND}(\beta)$. However, the definitions of ρ^D and $\rho^{ND}(\beta)$ imply that

$$C_i'(\rho^D) = \hat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j] > \hat{q}[w_i\delta - \beta w_i\delta] = C_i'(\rho^{ND}(\beta))$$

which is inconsistent with $\rho^D < \rho^{ND}(\beta)$. Hence, when $q\sigma_j < w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, in equilibrium, disclosure occurs with probability less than 1.

Lastly, suppose that $w_i\beta\delta \geq q'(\rho^{ND}(\beta), 1)\sigma_j$. Then the arguments above imply that the unique equilibrium is such that $\rho = \rho^D$ and disclosure occurs with certainty at stage (3).

The result in the proposition then follows from the arguments above by observing that, when $\sigma_j \leq 0$, $\beta^* = \beta^{**} = 0$. When, instead, $\sigma_j > 0$, β^* is given by the unique solution to

$$w_i\beta^*\delta = q\sigma_j$$

whereas $\beta^{**} \geq \beta^*$ is given by the unique solution to

$$w_i\beta^{**}\delta = q'\left(\rho^{ND}(\beta^{**}), 1\right)\sigma_j.$$

Note that, for $\beta = \beta^{**}$, $\rho^{ND}(\beta^{**}) = \rho^D$. That $\rho^{ND}(\beta)$ is strictly decreasing in β then implies that, for $0 \leq \beta < \beta^{**}$, $\rho^{ND}(\beta) > \rho^D$. Q.E.D.

---

[27]Note that this is the marginal benefit in case player i conceals at stage (3).

4.2 Mandatory disclosure

We now investigate the effects of mandatory disclosure, by which we mean policies that severely punish any late disclosure. As in the previous subsection, we assume that the court is unable to determine whether information disclosed at stage (5) was received at stage (5) or it was received at stage (2) and disclosed only at stage (5) for strategic/opportunistic reasons. Disclosures at stage (5) are severely punished, no matter when the information was received. Hence, when disclosure is mandatory, party i either discloses at stage (3) or never.

Note that mandatory disclosure is irrelevant in the absence of interim renegotiation (that is, when delayed disclosure at stage (5) is either not feasible or not useful to reduce the welfare losses). This is because, as stated in Proposition 4, in that case, in equilibrium, party i either acquires no information or discloses it with certainty prior to contracting. Things are different when interim renegotiation is possible. As shown above, in this case, party i may find it optimal to acquire information and then conceals it till stage (5). Mandatory disclosure then has a bite, as it affects both the incentives for i to disclose information at stage (3) and the incentives to acquire information in the first place. Our next result summarizes the implications of mandatory disclosure. Let ρ^M and d^M denote the equilibrium cognition and probability of disclosure under a mandatory disclosure law.

Proposition 6. *[**mandatory disclosure**] Under mandatory disclosure, $\rho^M = d^M = 0$ if $w_i\delta \leq q\sigma_j$, and $\rho^M = \rho^D$ and $d^M = 1$ if $w_i\delta > q\sigma_j$, with ρ^D satisfying $C_i'(\rho^D) = \hat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j]$. If the equilibrium under voluntary disclosure involves disclosure with probability less than one (i.e., if $\beta < \beta^{**}$) and disclosing becomes mandatory, then cognition is reduced.[28] There exists μ^* such that, for all $\mu \geq \mu^*$, a mandatory disclosure law reduces welfare. Similarly, for β either small or close to (but smaller than) β^{**}, a mandatory disclosure law reduces welfare.*

Proof of Proposition 6. When disclosure is mandatory, the equilibrium cognition is the same as in the game of Section 3 in which interim disclosure (i.e., after the contract is signed but before the state is publicly revealed) is not feasible. This means that $\rho^M = 0$ if $w_i\delta \leq q\sigma_j$ and $\rho^M = \rho^D$, with ρ^D satisfying $C_i'(\rho^D) = \hat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j]$ if $w_i\delta > q\sigma_j$. Because $\rho^M < \rho^{ND}(\beta)$, we thus have that, when $\beta < \beta^{**}$, mandatory disclosure reduces cognition. The net benefit of a mandatory-disclosure law (when it has an effect, i.e., when $\beta < \beta^{**}$) is the difference ΔW between the welfare deadweight loss under voluntary disclosure and the welfare deadweight loss under mandatory disclosure:

$$\Delta W(\beta, \mu) = \hat{q}\delta\left[\left(1 - \rho^{ND}(\beta) - \mu\right) + \mu\beta + \rho^{ND}(\beta)\left(1 - d^*(\beta)\right)\beta - \left(1 - \rho^M\right)\right]$$

$$= \hat{q}\delta[\rho^{ND}(\beta)\left(1 - d^*(\beta)\right)\beta - \left(\rho^{ND}(\beta) - \rho^M\right) - (1 - \beta)\mu]. \tag{14}$$

The benefit of the law is to avoid the deadweight loss $\beta\delta$ from delay when, under voluntary disclosure, early disclosure happens with probability $d^*(\beta) < 1$, where $d^*(\beta)$ is the value in part 2 of Proposition 5.

---

[28]Note that $\beta^{**} > 0$ only when $\sigma_j > 0$ in which case ρ^D is unique.

The law, however, has two costs. The first one is that it reduces valuable cognition as $\rho^M < \rho^{ND}(\beta)$. The second one is that it deters player i from disclosing information that is received exogenously with probability μ post contract, i.e., at stage (5).[29]

Because $\Delta W(\beta, \mu)$ is decreasing in μ, there exists μ^* such that, for all $\mu \geq \mu^*$, $\Delta W(\beta, \mu) < 0$.[30] It is also easy to see to see that, when $\beta = 0$, $\Delta W(\beta, \mu) = -\widehat{q}\delta[\rho^{ND}(0) - \rho^M + \mu] < 0$. In this case, interim renegotiation entails no deadweight loss and, perhaps unexpectedly, the mandatory-disclosure requirement reduces welfare by disincentivizing information collection (it also makes player i reluctant to disclose information acquired after the contract, but, as we argued earlier, μ is likely to be close to 0 if β is). By continuity of $\Delta W(\beta, \mu)$ in β, mandatory disclosure reduces welfare for β strictly positive but small. Lastly, observe that, when $\beta = \beta^{**}$, $d^*(\beta) = 1$, in which case $\Delta W(\beta, \mu) < 0$. By continuity, $\Delta W(\beta, \mu) < 0$ also for β strictly below β^{**} but close to β^{**}. Q.E.D.

Figure 8 summarizes the analysis in the last two propositions for the case in which $0 < \sigma_j < w_i\delta/q$. Note that this case is the most interesting one. When $\sigma_j \leq 0$, there is always disclosure, whether mandated or not (i.e. $\beta^* = \beta^{**} = 0$) and hence mandatory disclosure has no bite. When, instead, $\sigma_j \geq w_i\delta/q$, then a fortiori $\sigma_j \geq w_i\beta\delta/q$ and so no-disclosure prevails whenever disclosure is voluntary ($\beta^* = \beta^{**} = 1$). Furthermore, there is no information acquisition under mandatory disclosure ($\rho^M = 0$) in which case mandatory disclosure is clearly welfare-reducing: $\Delta W < 0$.

The corollary below summarizes the implications of the above results for the structure of optimal disclosure laws. To relate the results to the policy debate, we find it useful to amend our maintained assumption about the cost of cognition as follows: there exists $\rho^{ex} \in [0, 1)$ such that $C_i(\rho_i) = 0$ for all $\rho_i \leq \rho^{ex}$, $C_i'(\rho_i), C_i''(\rho_i) > 0$ for all $\rho_i > \rho^{ex}$, $C_i'(\rho^{ex}) = 0$, and $C_i'(1 - \mu) = \infty$ (note that the baseline model is nested with $\rho^{ex} = 0$). The idea is that player i may be endowed with information acquired exogenously for free prior to contracting. We then have the following result (the proof follows from inspection of the formula for ΔW in (14)):

Corollary 7. [**optimal disclosure law**] *The optimal disclosure law is:*

- *mandatory disclosure when the arrival of exogenous pre-contractual information is frequent (ρ^{ex} large), the arrival of post-contractual information is rare (μ small), and β is close to β^*;*

- *voluntary disclosure when the arrival of exogenous pre-contractual information is rare (ρ^{ex} small), the arrival of post-contractual information is frequent (μ large) and β is either close to zero or to β^{**}.*

The corollary formalizes Kronman (1978)'s and Eisenberg (2003)'s informal argument that mandatory-disclosure laws must distinguish between the cases of information casually acquired prior to contracting and information that results from deliberate search (which, according to Kronman, must benefit

---

[29]Recall that the court is unable to distinguish between information disclosed at stage (5) that is received at stage (5) and information disclosed at stage (5) that is received at stage (3) and concealed for strategic reasons. As a result, any late disclosure is punished by the court, no matter when the information was received.

[30]Recall that, because $\lim_{\rho \to 1-\mu} C_i'(\rho) = +\infty$, $\rho^M, \rho^{ND}(\beta) \leq 1 - \mu$.
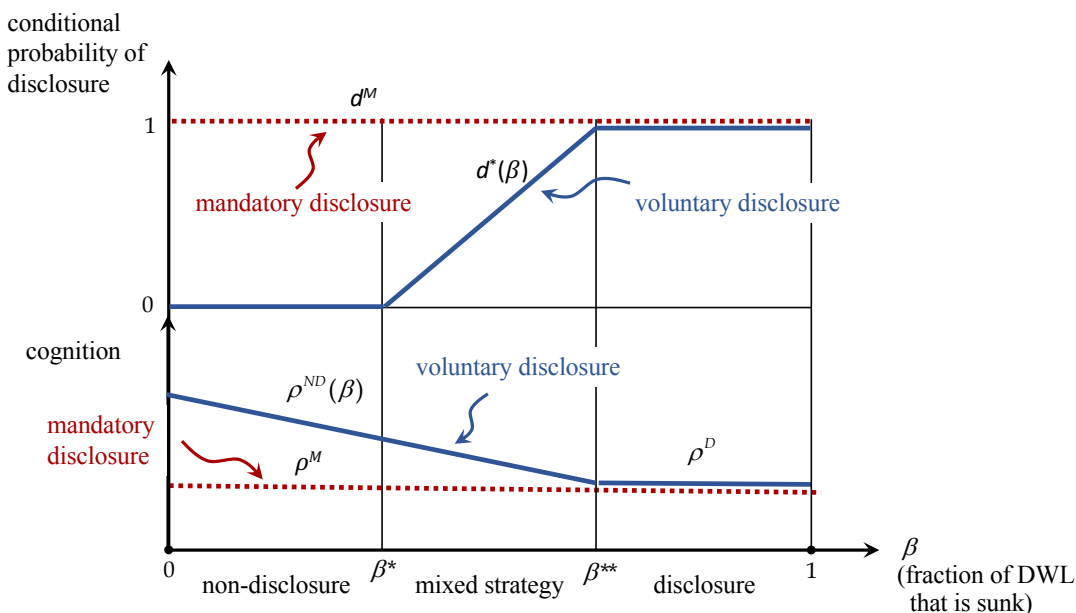
Figure 8: **Voluntary vs mandatory disclosure (for $0 < \sigma_j < w_i \delta / q$)**

from a legal no-disclosure privilege, in effect a property right). When $\sigma_j > 0$, i.e., when mandatory disclosure matters, mandatory disclosure reduces the incentive of party i to acquire information. It may also make party i reluctant to disclose information acquired post contracting.

It would seem reasonable to protect particularly exposed parties (those for whom $\sigma_j > 0$) against concealment of contract-relevant information; that insight is indeed correct when information is exogenous (a person skilled in the art has casually acquired information prior to contracting through the unfolding of past professional relationships), but not if information is endogenous: When σ_j is large (party j is very exposed to the unexpected), the information-acquiring party does not acquire any information under mandatory disclosure, as she has to reveal the bad news prior to contracting. A mandatory disclosure law then de facto mandates disclosure of non-existing information. In contrast, it deters disclosure of information casually acquired after the contract is signed and it prevents the pre-contracting acquisition of useful information that would have occurred in the absence of a disclosure requirement.

4.3 Broader class of mechanisms

Can other mechanisms be employed, that make contracts more efficient? Let us discuss in more detail the objective function of, and the information and instruments available to, the court or arbitrator. We assume that the court has no redistributive purposes and only aims at raising the efficiency of the relationship; so the court is preoccupied with the level of cognition and the avoidance of deadweight

losses, not with the monetary transfers paid or received by the two parties. In terms of instruments, the court can impose a penalty $p \geq 0$ on party i; the mechanism is balanced in that the penalty is paid to party j.

One can envision alternative information structures for the court, ranked by their fineness:

(a) At the very least, the court observes that the initial contract is renegotiated at stage (5) (implying that party i has disclosed information revealing that the state is $\hat{\omega}$ at that stage), but nothing more. In particular, it does not know whether the information that gave rise to the renegotiation was received at stage (2) or at stage (5).

(b) The court can directly observe that party i disclosed relevant information at stage (5). However, it cannot determine whether the information disclosed at stage (5) was received at stage (2) or at stage (5). This is the information structure considered when studying mandatory disclosure laws.[31]

(c) When party i conceals information at stage (3), the court receives evidence of this behavior with some strictly positive probability; in this case, p stands for the expected penalty paid by party i (nominal penalty times probability of detection) for non-disclosure at stage (3).

Below we consider the role of penalties under each of these information structures.

(a) Suppose that the court can only verify that renegotiation took place (the new contract differs from the initial one). In case renegotiation occurs, party i is charged a penalty p that is paid to party j. Such a penalty is neutral (as in Tirole 1986) because its payment is passed through in the contract renegotiation. To see this, let t'_i denote the transfer received by party i as part of the renegotiation; it is given by $(1 - \beta_i)\delta_i + t'_i - p = w_i(1 - \beta)\delta$. Because the penalty is paid only when the two parties agree to renegotiate the initial contract, the negotiated ex-post transfer t'_i offsets one-for-one the penalty, making the latter irrelevant.

(b) Next, suppose that the court can directly observe that party i disclosed relevant information at stage (5) that proves that the state is $\hat{\omega}$. It can then force party i to pay a penalty $p \geq 0$ to party j irrespective of whether renegotiation occurs. As a result, the penalty does not impact the renegotiation process. This implies that party i's net benefit from disclosing information that is still private at stage (5) is, regardless of the stage of accrual, equal to $w_i(1 - \beta)\delta - p$. So if $p > w_i(1 - \beta)\delta$ there is never any disclosure at stage (5). The outcome is then the one under mandatory disclosure discussed in Section 4.2. When, instead, $p = 0$, the outcome is the one under the voluntary-disclosure regime discussed in Section 5.

---

[31] In richer settings in which disclosure can occur over an interval of time, the court can also set a deadline between the contracting date and the time at which information becomes public such that the party disclosing information is charged a penalty when disclosing after the contracting date but before the deadline. The optimal choice of such a deadline is then determined by the speed by which the deadweight loss from specifying a wrong design at the contracting date increases with time, relative to the speed by which exogenous information is expected to arrive after the contracting date.

Clearly, any $p, p' > w_i(1-\beta)\delta$ are equivalent (both in terms of payoffs and welfare) because they discourage party i from disclosing information at stage (5) no matter when it was received. Next, observe that, for any $p \leq w_i(1-\beta)\delta$, when party i is expected to engage in cognition ρ^*, disclose at stage (3) with probability d^*, and disclose with certainty at stage (5) (both the information received and withheld at stage (3) and the one received exogenously at stage (5)) the transfer $t_i(\rho^*, d^*; p)$ that party i receives at stage (4) in the absence of any disclosure now solves

$$t_i + q'(\rho^*, d^*)U_i + \widehat{q}'(\rho^*, d^*)\Big[\widehat{U}_i - \delta_i + \tfrac{\mu+\rho^*(1-d^*)}{1-\rho^*d^*}[w_i(1-\beta)\delta - p]\Big]$$
$$= w_i\Big\{q'(\rho^*, d^*)U + \widehat{q}'(\rho^*, d^*)\Big[(\widehat{U} - \delta) + \tfrac{\mu+\rho^*(1-d^*)}{1-\rho^*d^*}(1-\beta)\delta\Big]\Big\}.$$

That is, relative to the absence of the penalty (i.e., to $p = 0$), the stage-(4) transfer $t_i(\rho^*, d^*; p)$ to party i increases by $\widehat{q}'(\rho^*, d^*)\tfrac{\mu+\rho^*(1-d^*)}{1-\rho^*d^*}p$:

$$t_i(\rho^*, d^*; p) = t_i(\rho^*, d^*; 0) + \widehat{q}'(\rho^*, d^*)\frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*}p.$$

Party i then finds it optimal to disclose at stage (3) only if

$$w_i\widehat{U} \geq \widehat{U}_i - \delta_i + t_i(\rho^*, d^*; p) + w_i(1-\beta)\delta - p$$

which, using the expression for $t_i(\rho^*, d^*; p)$ derived above, can be rewritten as

$$w_i\beta\delta \geq q'(\rho^*, d^*)\sigma_j - \Big[1 - \widehat{q}'(\rho^*, d^*)\frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*}\Big]p. \tag{15}$$

Holding ρ^* and d^* fixed, we thus have that the net effect of the penalty on player i's payoff in case she withholds information at stage (3) is equal to

$$\Big[1 - \widehat{q}'(\rho^*, d^*)\frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*}\Big]p > 0.$$

Thus, for any $p \leq w_i(1-\beta)\delta$, an equilibrium in which party i engages in cognition ρ^*, discloses with probability $d^* > 0$ at stage (3) and discloses with certainty at stage (5) (no matter when the information available at stage (5) was received) exists if and only if Condition (15) is satisfied (with the condition holding as an equality when $d^* \in (0, 1)$) and

$$C'_i(\rho^*) = \hat{q}\Big[w_i\delta - q'(\rho^*, d^*)\sigma_j - \widehat{q}'(\rho^*, d^*)\frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*}p\Big]. \tag{16}$$

Note that, when $d^* \in (0, 1)$, because party i is indifferent between disclosing and not disclosing at stage (3), the marginal benefit of cognition (the right-hand-side of (16)) is also equal to

$$\hat{q}\Big[w_i(1-\beta)\delta - p\Big]$$

which is the benefit in case player i does not plan to discloses at stage (3).

The following result then holds:

Proposition 8. *Suppose that the court can verify whether or not party i discloses relevant information at stage (5) but cannot determine whether the information was received at stage (2) or at stage (5).*

1. Small penalties for late disclosures (formally, $p \leq w_i(1-\beta)\delta$) come with the same trade-offs as mandatory-disclosure laws (which are formally equivalent to large penalties $p > w_i(1-\beta)\delta$): They increase the incentives for disclosure at stage (3) but reduce the marginal benefits to cognition.

2. Under the welfare-maximizing equilibrium, welfare is always higher under a penalty equal to $p = w_i(1-\beta)\delta$ than under a mandatory-disclosure law, with the comparison strict for example when cognition under a mandatory-disclosure law is zero.

3. When, in the absence of any policy intervention, the equilibrium features disclosure with certainty at stage (3), the unique optimal penalty is $p^* = 0$.

Proof of Proposition 8. The result in part 1 follows from the discussion preceding the proposition. The result in part 2 follows from the fact that any equilibrium when $p > w_i(1-\beta)\delta$ is also an equilibrium when $p = w_i(1-\beta)\delta$. When, under a policy of mandatory disclosure, the unique equilibrium features no cognition, then it must be that $w_i\delta \leq q'\sigma_j$. In this case, when $p = w_i(1-\beta)\delta$, an equilibrium exists in which cognition is equal to zero but party i discloses with certainty the information received exogenously at stage (5). The equilibrium when $p = w_i(1-\beta)\delta$ then Pareto-dominates the one under a mandatory-disclosure law. The result in part 3 is an immediate implication of part 1: Any $p > 0$ reduces cognition without augmenting the probability of disclosure. Q.E.D.

(c) Finally, suppose that the court can detect with some strictly positive probability whether party i withheld information at stage (2) and, in case it receives evidence of such a behavior, charges party i a penalty. Then, interpret p as the penalty that party i expects to pay in case information is withheld at stage (3) (thus p is the product of the probability of detection and the fine). The probability the court finds out that party i withheld information at stage (3) is independent of whether or not party i discloses at stage (5). As a result, party i always discloses the information she possess at stage (5), irrespective of when it was received and of the magnitude of p. This is the key difference with respect to the scenario (b) considered above.

Because of this difference, when party i is expected to exert cognition ρ^* and disclose at stage (3) with probability d^*, the transfer that she receives at stage (4) in case of no disclosure at stage (3) now solves

$$t_i + q'(\rho^*, d^*)U_i + \widehat{q}'(\rho^*, d^*)\left[\widehat{U}_i - \delta_i + \frac{\mu + \rho^*(1-d^*)}{1-\rho^* d^*}w_i(1-\beta)\delta - \frac{\rho^*(1-d^*)}{1-\rho^* d^*}p\right]$$
$$= w_i\left\{q'(\rho^*, d^*)U + \widehat{q}'(\rho^*, d^*)\left[(\widehat{U} - \delta) + \frac{\mu + \rho^*(1-d^*)}{1-\rho^* d^*}(1-\beta)\delta\right]\right\}.$$

Equivalently,

$$t_i(\rho^*, d^*; p) = w_i U - U_i - \widehat{q}'(\rho^*, d^*)\sigma_j + \widehat{q}'(\rho^*, d^*)\frac{\rho^*(1-d^*)}{1-\rho^* d^*}p.$$

For party i to find it optimal to disclose at stage (3) with strictly positive probability, it must be that

$$w_i \widehat{U} \geq \widehat{U}_i - \delta_i + t_i(\rho^*, d^*; p) + w_i(1 - \beta)\delta - p.$$

Using the above expression for $t_i(\rho^*, d^*; p)$, we can rewrite the above condition for disclosure as follows:

$$w_i \beta \delta \geq q'(\rho^*, d^*)\sigma_j - \left[1 - \widehat{q}'(\rho^*, d^*)\frac{\rho^*(1 - d^*)}{1 - \rho^* d^*}\right] p. \tag{17}$$

An equilibrium in which party i engages in cognition ρ^* and discloses with probability $d^* > 0$ at stage (3) then exists if and only if Condition (17) is satisfied (with the condition holding as an equality when $d^* \in (0, 1)$) and

$$C_i'(\rho^*) = \hat{q}\left[w_i \delta - q'(\rho^*, d^*)\sigma_j - \widehat{q}'(\rho^*, d^*)\frac{\rho^*(1 - d^*)}{1 - \rho^* d^*}p\right]. \tag{18}$$

As in scenario (b) above, when $d^* \in (0, 1)$, because party i must be indifferent between disclosing and not disclosing at stage (3), the return to cognition in case party i discloses (the right-hand-side of (18)) must coincide with the return to cognition

$$\hat{q}\left[w_i(1 - \beta)\delta - p\right]$$

in case party i conceals at stage (3). We then have the following result:

Proposition 9. *Suppose that, in case party i conceals information at stage (3), the court finds evidence of such a behavior with strictly positive probability and then charges party i a fine.*

1. Penalties for concealing information involve the same trade-offs as mandatory-disclosure laws: They increase the incentives for early disclosure but reduce the benefits to higher cognition.

2. A policy of mandatory-disclosure is strictly dominated by a large penalty that is paid by party i only when the court finds evidence of information concealment.

3. When, in the absence of any regulation, the equilibrium features disclosure with certainty at stage (3), any penalty $p \geq 0$ is optimal.

Proof of Proposition 9. The result in part 1 follows from the arguments preceding the proposition. The result of part 2 follows from the fact that cognition and stage-(3) disclosure are the same under the two policies, but information received exogenously at stage (5) is disclosed with probability 1 under a penalty for concealment and with probability zero under a mandatory-disclosure law. Finally, Part 3 is an immediate implication of the fact that, by disclosing with probability one at stage (3), party i does not pay any penalty. Q.E.D.

5 Two-sided cognition

5.1 Over-cognition implies one-sided cognition

Under unilateral information acquisition, no matter whether disclosure is mandated or voluntary, player i over-invests in cognition if and only if Condition (10) holds.[32] The same condition implies that, under player i's cognition, player j would not want to disclose that the state is $\widehat{\omega}$ if he knew it, and therefore would not want to acquire information, however small the cost of doing so: see Condition (4)). The intuition for this result is that excess cognition by player i occurs when the private benefit of information exceeds the social benefit. Put it differently, information, at the margin, reduces j's welfare; and so player j has no incentive to acquire this information, however cheap.

A simple implication of the above results is that there can be over-cognition only by the player who is relatively more exposed to the unexpected (i.e., $\sigma_i > 0$), for example the buyer in the buyer-seller game.[33]

Proposition 10. *[over-cognition and two-sided cognition]*
Irrespective of whether disclosure is mandatory or voluntary:
(i)   Under one-sided cognition, player i engages in excess cognition if and only if Condition (10) is satisfied;
(ii)   Under two-sided cognition, player j does not want to engage in any cognition, even when $C_j'(0) = 0$, if and only if (10) is satisfied.

5.2 Actual cognition by both players

Next, we look for an equilibrium in which both parties invest in cognition (and disclose information), which, as we saw, requires that $w_i\delta + q'\sigma_i > 0$ for all i, where q' is the posterior belief that the state is ω conditional on none of the parties having disclosed that the state is $\widehat{\omega}$. Assume that the search outcomes are independent.[34] Such an equilibrium must satisfy the following properties.

---

[32]To see this, recall that, under voluntary disclosure, if player i discloses with certainty at stage (4), then his cognitive level is given by $\rho^D = \rho^*$ with the latter satisfying $C_i'(\rho^*) = \widehat{q}[w_i\delta - q'(\rho^*, 1)\sigma_j]$, whereas, if he discloses with probability less than 1 then his cognitive level is given by $\rho^{ND}(\beta)$ with the latter satisfying $C_i'(\rho^{ND}) = \widehat{q}w_i(1 - \beta)\delta$. The social marginal benefit of cognition is equal to $\widehat{q}\delta$. Because $\rho^{ND}(\beta) \geq \rho^D$, we then have that, when Condition (10) holds, player i necessarily over-invests. When, instead, Condition (10) is violated, because $\widehat{q}w_i(1 - \beta)\delta < \widehat{q}\delta$, then necessarily player i underinvests.

[33]Condition (10) applied to the buyer-seller game under buyer cognition (i.e., $i = B$) takes the following form:
• in the absence of ex-post renegotiation: $q'(\rho^*, 1) > 1$, which is impossible, irrespective of the equilibrium level of ρ^*, so that there is never over-cognition;
• under ex-post renegotiation: $q'(\rho^*, 1)(B - b) > \alpha$, which is satisfied if, for example, the adjustment cost α is close to zero — to see this, note that, in this application, ρ^* solves $C_B'(\rho^*) = \widehat{q}\left[w_B\alpha + \frac{q(1-w_B)(B-b)}{1-(1-q)\rho^*}\right]$ along with the fact that $q'(\rho^*, 1) = q/\left[1 - (1 - q)\rho^*\right]$.
[34]In the symmetric case, for example, the socially optimal level of cognition per party, ρ^{FB}, is then given by $C'(\rho^{FB}) = \widehat{q}(1 - \rho^{FB})\delta$.

For all i, under mandatory disclosure, or if $\sigma_i \geq 0$, either $\rho_i = 0$ if $w_i \delta \leq q' \sigma_j$, or ρ_i is given by the solution to

$$C_i'(\rho_i) = \hat{q}(1 - \rho_j)(w_i \delta - q' \sigma_j), \tag{19}$$

where the probability that the state is ω in case neither player discloses is given by

$$q' = \frac{q}{q + \hat{q}(1 - \rho_i)(1 - \rho_j)}. \tag{20}$$

In the absence of mandatory disclosure and if $\sigma_i < 0$, ρ_i is given by the solution to to[35]

$$C_i'(\rho_i) = \hat{q}(1 - \rho_j)w_i \delta. \tag{21}$$

Note that the first-order condition ((19)) can be rewritten as

$$C_i'(\rho_i) = \hat{q}\left[(1 - \rho_j)w_i \delta + (-\sigma_j)\frac{q}{\frac{q}{1 - \rho_j} + \hat{q}(1 - \rho_i)}\right]. \tag{22}$$

If $\sigma_j \leq 0$, that is, if player i is relatively more exposed to the unexpected, player i's reaction curve (ρ_i as a function of ρ_j) is downward-sloping. But strategic substitutability holds also if $\sigma_j > 0$. In fact, the derivative of the right-hand-side of Condition (22) with respect to ρ_j is $\hat{q}\left[-w_i \delta + \sigma_j (q')^2\right]$, which is negative from the active cognition condition.

Assuming that the functions C_i are sufficiently convex so that the reaction curves cross only once, thus defining a stable equilibrium, we then have the following result:

Proposition 11. [actual two-sided cognition] *Assume independent searches and a stable equilibrium. Then, in the two-sided cognition region, cognitive efforts are locally strategic substitutes, reflecting the public good nature of information. Suppose that player i is relatively more exposed to the unexpected ($\sigma_i > 0$); then lifting the mandatory disclosure requirement increases j's cognition and reduces i's cognition.*

6   Concluding remarks

Cognition, broadly defined to include financial and human investments, attention, rehearsal and retrieval strategies, is at the core of informational asymmetries, and therefore frictions in contracting. This paper introduces the concept of relative exposure to the unexpected and shows that the latter is a unifying factor underlying players' disclosure decisions, their incentives to conform to other players' expectations, and the welfare implications of pre-contractual disclosure obligations. In this respect, the paper is only a first pass at a much broader and richer set of questions.

Disclosure is a central topic in law and economics. We have formalized some of the relevant trade-offs and welfare recommendations. Clearly, applications abound beyond the framework developed

---

[35]Note that in this case player j always discloses.

here. A case in point is patenting, as the search for prior art embodies both efficiency and rent-seeking aspects. As in this paper, the inventor may casually or deliberately acquire information about relevant prior art and choose whether or not to disclose this information to the patent office when applying for a patent. By concealing prior art, the intellectual property (IP) owner can exploit the monopoly power conferred by the patent beyond what motivated by the necessity to incentivize R&D toward novel content. In this context, the patent office does not negotiate a monetary transfer, but chooses the patent's coverage and scope, both of which impact the inventor's welfare. Disclosure rules play an important role also for the diligence of search, the opposition process, and the choice of what to include in a standard. Identifying the exact role of various policy interventions requires developing a proper framework.

Another case in point is disclosure in standard-setting processes. Many standard setting organizations (SSOs) require that any IP owner participating in working group discussions disclose any potentially relevant patent rights they own that they know of, or reasonably should know of. Such requirements allow the SSO members to assess the nature of IP ownership covering the proposed standard. On the other hand, search and disclosure are costly for IP owners; their patent portfolios may have thousands of items, and they may need to search for "the needle in the haystack". Furthermore, early disclosure of plans may limit their ability to get future patent awards and may convey information to rivals about what the IP owner intends to do with its existing patents. Again, the terms of the disclosure requirement are complex (type of disclosures, timing of disclosure) and the duty of disclosure is worth theoretical and empirical investigations.

Finally, many contracts (such as laws, international agreements) involve more than two protagonists and their negotiation obeys certain protocols which are likely to condition information acquisition. We leave these topics and the many other issues related to cognition-intensive contracting for future research.

7 Appendix: Participation Constraints

In this Appendix, we show that the assumption that U_i is large (for given $U_i - \hat{U}_i$), $i = 1, 2$, guarantees that, under the assumed bargaining protocol, all the equilibria in the baseline model in which participation is compulsory are also equilibria when players can reject the third party's offers. The analysis below considers the case in which $\beta = 1$ (the model of Sections 2 and 3). Because each player's payoff is weakly higher when disclosure can be delayed to stage (5) than when it must be done at stage (3) or never thereafter, the conditions below guarantee participation also in the model of Section 4 in which $\beta < 1$. Furthermore, because $\hat{U} \geq 0$ and because, under the assumed protocol, each player gets a payoff equal to $w_i \hat{U}$, $i = 1, 2$, in case of disclosure, to establish the result, it suffices to consider the payoff that each player expects in the absence of disclosure.

7.1 One-sided cognition

Consider first the case where only player i engages in cognition.

Player j. In the absence of any disclosure by player i, when the proposed transfer is the one in (1), player j's expected payoff when accepting the offer is equal to $w_j[q'U + \hat{q}'(\hat{U} - \delta)]$, where q' and $\hat{q}'$ are shortcuts for $q'(\rho^*, d^*)$ and $\hat{q}'(\rho^*, d^*) = 1 - q'(\rho^*, d^*)$, respectively, with $q'(\rho^*, d^*)$ as defined in (3). Because U and $\hat{U}$ are non-negative and because $q' \geq q$, a sufficient condition for player j's participation constraint to be satisfied is that the expected aggregate surplus under the common prior $(q, \hat{q})$ is non-negative:

$$qU + \hat{q}(\hat{U} - \delta) \geq 0. \tag{23}$$

Clearly, because in any equilibrium in which player i does not engage in cognition, $q' = q$, Condition (23) is also necessary to guarantee participation in equilibria involving no cognition.

Player i. The analysis is more complex for the player engaging in cognition, because the decision of whether or not to accept the third party's offer depends on whether the player's actual cognition is equal to the equilibrium one and on the information received by the player. After choosing $\rho_i = \rho^*$, where ρ^* is the equilibrium cognitive level, player i clearly prefers accepting the mediator's contract to her outside option. To see this, consider first the case in which player i discovers that the state is $\hat{\omega}$. By disclosing, player i obtains $w_i \hat{U}$ which is strictly higher than his outside option. If not disclosing (followed by accepting the mediator's offer) gives player i a payoff even higher than the one under disclosure, such payoff must be higher than the one under the outside option. Hence, no matter whether on path player i discloses or not, upon learning that the state is $\hat{\omega}$, player i's equilibrium payoff is always higher than his outside option.[36]

Next, consider the case in which player i does not find evidence that the state is $\hat{\omega}$. His equilibrium payoff is then equal to $w_i[q'U + \hat{q}'(\hat{U} - \delta)]$, where again q' and $\hat{q}'$ are shortcuts for $q'(\rho^*, d^*)$ and $\hat{q}'(\rho^*, d^*) = 1 - q'(\rho^*, d^*)$, respectively. The latter's payoff is thus weakly greater than 0 under the same assumptions that guarantee that player j's equilibrium payoff is non-negative (e.g., when (23) is satisfied).

Now suppose that player i deviates and chooses cognition $\rho_i \neq \rho^*$. Suppose first that $w_i \delta \leq q \sigma_j$. The analysis in Section 3 implies that, when participation is mandatory, in the (unique) equilibrium, player i exerts no cognition, so that $\rho^* = 0$. Thus consider the situation that player i faces when he deviates to $\rho_i > 0 = \rho^*$. If, after deviating, player i learns that the state is $\hat{\omega}$, condition $w_i \delta \leq q \sigma_j$ implies that, by concealing and then accepting the mediator's offer, player i obtains more than by disclosing and then accepting the mediator's offer. Because the latter's strategy gives him a payoff equal to $w_i \widehat{U} \geq 0$, concealing and then accepting the mediator's offer is at lest as profitable than dropping out and taking the outside option.

---

[36]When, on path, player i discloses, it is possible that, by not disclosing, player i obtains a payoff $\hat{U}_i - \hat{\delta}_i + t_i < 0$ when learning that the state is $\hat{\omega}$. This, however, has no implications for the results.

When, instead, player i does not receive evidence that the state is $\hat{\omega}$, because

$$t_i = w_i[qU + \hat{q}(\hat{U} - \delta)] - qU_i - \hat{q}(\hat{U}_i - \delta_i),$$

his payoff from accepting the mediator's offer is equal to

$$q'U_i + \hat{q}'(\hat{U}_i - \delta_i) + t_i = w_i[qU + \hat{q}(\hat{U} - \delta)] + (q' - q)[U_i - (\hat{U}_i - \delta_i)], \qquad (24)$$

where here we abuse notation and let $q' \equiv q'(\rho_i)$ denote player i's posterior that the state is ω when exerting cognition ρ_i and not finding any evidence that the state is $\hat{\omega}$; similarly, we let $\hat{q}' \equiv 1 - q'(\rho_i)$ denote player i's posterior that the state is $\hat{\omega}$ when player i's cognition is ρ_i and he finds no evidence that the state is $\hat{\omega}$.

Under Condition (23), the above payoff is positive for any q' (equivalently, for all ρ_i) if and only if

$$w_i[qU + \hat{q}(\hat{U} - \delta)] + (1 - q)[U_i - (\hat{U}_i - \delta_i)] \geq 0. \qquad (25)$$

Hence, jointly, Conditions (23) and (25) imply that, when, in the game with compulsory participation, player i prefers exerting no cognition and then accepting the third party's offer to any other strategy, the same is true in the game in which participation is voluntary.

Next, suppose that $w_i\delta > q\sigma_j$. Recall that, when participation is compulsory, player i always exerts some cognition in any equilibrium. Now take any equilibrium in the game with compulsory participation in which player i engages in cognition $\rho^* > 0$ and discloses with certainty and let $q'(\rho^*, 1)$ and $\hat{q}'(\rho^*, 1) = 1 - q'(\rho^*, 1)$ denote the posterior probability that player j and the third party assign to the state being ω and $\hat{\omega}$, respectively, in the absence of any disclosure, when player i is expected to choose cognition ρ^* and disclose with certainty (clearly, the same probabilities also coincide with player i's beliefs when player i chooses cognition ρ^* and does not receive any evidence that the state is $\hat{\omega}$). In any such equilibrium, player i's ex-ante expected payoff, net of the cognitive cost, is equal to

$$(1 - \hat{q}\rho^*)w_i[q'(\rho^*, 1)U + (1 - q'(\rho^*, 1))(\hat{U} - \delta)] + \hat{q}\rho^* w_i\hat{U} - C_i(\rho^*)$$

In the game with voluntary participation, the following is thus a necessary condition for player i to find it optimal to follow the same strategy as in the game in which participation is compulsory:

$$(1 - \hat{q}\rho^*)w_i[q'(\rho^*, 1)U + (1 - q'(\rho^*, 1))(\hat{U} - \delta)] + \hat{q}\rho^* w_i\hat{U} - C_i(\rho^*) \geq 0. \qquad (26)$$

Note that, by the law of iterated expectation, the expression in the left-hand-side of (26) is bounded from below by

$$w_i[qU + (1 - q)(\hat{U} - \delta)] - C_i(\rho^*).$$

Next, recall that, in the game in which participation is compulsory, in any equilibrium with cognition ρ^*, necessarily $w_i\delta > q'(\rho^*, 1)\sigma_j$. Now suppose that, in the game with voluntary participation,

player i deviates to some cognition $\rho_i \neq \rho^*$. When player i learns that the state is $\hat{\omega}$, disclosing and then accepting the third party's offer yields him a gross payoff $w_i \hat{U} \geq 0$ which is higher than the payoff that player i can obtain by either concealing and then accepting the mediator's offer or dropping out and enjoying the outside option. When, instead, player i does not receive evidence that the state is $\hat{\omega}$, accepting the third party's offer yields a gross payoff equal to

$$q'(\rho_i, 1)U_i + \hat{q}'(\rho_i, 1)(\hat{U}_i - \delta_i) + t_i = w_i[q'(\rho^*, 1)U + \hat{q}'(\rho^*, 1)(\hat{U} - \delta)] + (\hat{q}'(\rho_i, 1) - \hat{q}'(\rho^*, 1))[U_i - (\hat{U}_i - \delta_i)],$$

where $\hat{q}'(\rho_i, 1) - \hat{q}'(\rho^*, 1) \in [-(1-q), 1-q]$. Player i then prefers accepting the third party's offer to his outside option if and only if

$$w_i[q'(\rho^*, 1)U + \hat{q}'(\rho^*, 1)(\hat{U} - \delta)] + (\hat{q}'(\rho_i, 1) - \hat{q}'(\rho^*, 1))[U_i - (\hat{U}_i - \delta_i)] \geq 0. \tag{27}$$

Note that a sufficient condition for (27) to hold is that

$$w_i[q'U + \hat{q}'(\hat{U} - \delta)] \geq (1-q) \left| U_i - (\hat{U}_i - \delta_i) \right|. \tag{28}$$

We conclude that Conditions (23), (25), (26), and (27), with the latter holding for all ρ_i, are sufficient to guarantee that all the equilibria of the game in which participation is compulsory are also equilibria in the game in which participation is voluntary. These conditions are always satisfied when, given $U_i - \hat{U}_i$, U_i is large, $i = 1, 2$. Also note that, Conditions (25) and (27) are implied by Condition (23) in the following special cases:

- Outer-looking cognition: $U_i = \hat{U}_i = \hat{U}_i - \delta_i$.

- Inner-looking cognition with unknown opportunity: $U_i = \hat{U}_i - \delta_i$. The idea is that the known design has also known consequences so that $U_i = \hat{U}_i - \delta_i$. But a better design in state $\hat{\omega}$ might improve player i's utility (say, a cost reduction equal to δ_i).

7.2 Two-sided cognition

Next, consider equilibria in which both players engage in cognition. As the analysis in Section 5 reveals, these equilibria involve strategic effects similar to those in the case of one-sided cognition. Conditions (23), (25) and (27) above thus guarantee that the equilibria in the game with voluntary participation coincide with those in the game in which participation is mandatory also under two-sided cognition.

References

Bajari, P., and S. Tadelis (2001) "Incentives versus Transaction Costs: a Theory of Procurement Contracts," *Rand Journal of Economics*, 32(3): 387–407.

Bolton, P., and A. Faure-Grimaud (2010) "Satisficing Contracts," *Review of Economic Studies*, 77(3): 937–971.

Bourjade, S., and B. Jullien (2011) "The Roles of Reputation and Transparency on the Behavior of Biased Experts," *Rand Journal of Economics*, 42: 575–594.

Buchanan, J. (1970) "In Defense of Caveat Emptor," *The University of Chicago Law Review*, 38: 64–73.

Cooter, R, and T. Ulen (2016) *Law and Economics*, Berkeley Law Books (6th edition).

Crawford, V., and J. Sobel (1982) "Strategic Information Transmission," *Econometrica*, 50: 1431–51.

Crémer, J., and F. Khalil (1992) "Gathering information before signing a contract," *American Economic Review*, 82(3): 566–578.

Crémer, J., and F. Khalil (1994) "Gathering information before the contract is offered: the case with two states of nature," *European Economic Review*, 38(3-4): 675–682.

Crémer, J., Khalil, F., and J.C. Rochet (1998a) "Contracts and Productive Information Gathering." *Games and Economic Behavior*, 25(2): 174–193.

Crémer, J., Khalil, F., and J.C. Rochet (1998b) "Strategic Information Gathering before a Contract is Offered." *Journal of Economic Theory*, 81(1): 163–200.

Dang, T.V. (2008) "Bargaining with Endogenous Information," *Journal of Economic Theory*, 140: 339–354.

Dekel, Eddie, Ben-Porath, Elchanan, and Lipman, Barton, (2018), "Disclosure and Choice," *The Review of Economic Studies*, 85(3), 1471-1501.

Dewatripont, M., and J. Tirole (2005) "Modes of Communication," *Journal of Political Economy*, 113: 1217–1238.

Dziuda, W. (2011) "Strategic Argumentation," *Journal of Economic Theory*, 146(4): 1362–1397.

Eisenberg, M. (2003) "Disclosure in Contract Law," *California Law Review*, 91: 1645–1691.

Farrell, J. (1985) "Owner-Consumers and Efficiency," *Economics Letters* 19: 303-306.

Gabaix, X., and D. Laibson (2006) "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets," *Quarterly Journal of Economics*, 121(2): 505–40.

Gentzkow, M., and E. Kamenica (2017) "Disclosure of Endogenous Information," *Economic Theory Bulletin*, 5(1): 47–56.

Grossman, S., and O. Hart (1986) "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94(4): 691-719.

Grubb, M. (2011) "Developing a Reputation for Reticence," *Journal of Economics, Management and Strategy*, 20: 225–268.

Hart, O., and J. Moore (1990) "Property Rights and the Nature of the Firm," *Journal of Political Economy* 98(6): 1119–1158.

Hoffmann, F., Inderst, R., and M. Ottaviani (2020) "Persuasion through Selective Disclosure: Implications for Marketing, Campaigning, and Privacy Regulation," *Management Science*, https://doi.org/10.1287/mnsc.2019.3455.

Ispano, A. (2018) "Information Acquisition and the Value of Bad News," *Games and Economic Behavior*, 110: 165–173.

Johnson, A. (2008) "An Economic Analysis of the Duty to Disclose Information: Lessons Learned from the Caveat Emptor Doctrine," *San Diego Law Review*, 45 (1): 79–132.

Kartik, N., Xu F., and Suen W, (2017), "Investment in Concealable Information by Biased Experts," *Rand Journal of Economics* 48 (1): 24–43.

Kronman, A. (1978) "Mistake, Disclosure, Information and the Law of Contracts," *Journal of Legal Studies*, 7: 1–34.

Lewis. T., and D. Sappington (1997) "Information Management in Incentive Problems," *Journal of Political Economy*, 105(4): 796–821.

Matthews, S., and A. Postlewaite (1985) "Quality Testing and Disclosure," *Rand Journal of Economics*, 16: 328–340.

Okuno-Fujiwara, M., Postlewaite, A., and K. Suzumura (1990) "Strategic Information Revelation," *Review of Economic Studies*, 57(1): 25–47.

Pavan, A., and J. Tirole (2021) "Expectation Conformity in Strategic Cognition," mimeo Northwestern University and Toulouse School of Economics.

Pei, D. (2015) "Communication with endogenous information acquisition," *Journal of Economic Theory*, 160: 132–149.

Ravid, D., Roesler, A.K., and B. Szentes (2019) "Learning before Trading: On the Inefficiency of Ignoring Free Information," mimeo, University of Chicago.

Segal, I. (1999) "Complexity and Renegotiation: A Foundation for Incomplete Contracts," *Review of Economic Studies*, 66(1): 57–82.

Shavell, S. (1994) "Acquisition and Disclosure of Information Prior to Sale," *Rand Journal of Economics*, 25(1): 20–36.

Sobel, J. (1989) "An Analysis of Discovery Rules," *Law and Contemporary Problems*, 52(1): 133-159.

Sobel, J. (2013) "Giving and Receiving Advice," in *Advances in Economics and Econometrics*, D. Acemoglu, M. Arellano, and E. Dekel (eds.), Cambridge University Press, 305–341.

Spier, K. (1992) "Incomplete Contracts and Signaling," *Rand Journal of Economics*, 23(3): 432–443.

Szalay, D. (2009) "Contracts with Endogenous Information," *Games and Economic Behavior*, 65(2): 586–625.

Tirole, J. (1986) "Procurement and Renegotiation," *Journal of Political Economy*, 94: 235–259.

Tirole, J. (2009) "Cognition and Incomplete Contracts," *American Economic Review*, 99(1): 265–294.

Von Thadden, E.-L., and X. Zhao (2012) "Incentives for Unaware Agents," *Review of Economic Studies*, 79 (3): 1151–1174.

Williamson, O. (1975) *Markets and Hierarchies: Analysis and Antitrust Implications*, The Free Press, New York.

Williamson, O. (1985) *The Economic Institutions of Capitalism: Firms, Markets, Relational Contracting*, The Free Press, New York.

Zhao, X. (2015) "Strategic Mis-Selling and Pre-Purchase Deliberation," Working Paper, available at SSRN: https://ssrn.com/abstract=2131315 or http://dx.doi.org/10.2139/ssrn.2131315.