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Country Solidarity in Sovereign Crises*

Jean Tirole†

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Abstract

When will solidarity, which emerges spontaneously from the fear of spillovers, be reinforced through contracting? The optimal pact between countries that differ substantially in their probability of distress is a simple debt contract with market financing, a borrowing cap, but no joint liability. While joint liability augments total surplus, the borrowing country cannot compensate the deep-pocket guarantor.

By contrast, the optimal pact between two countries symmetrically exposed to shocks with an arbitrary correlation is a simple debt contract with joint liability, provided that shocks are sufficiently independent, spillovers sufficiently large, liquidity needs moderate and available sanctions sufficiently tough.

Keywords: Sovereign debt, solidarity, joint liability, bailouts.

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†Toulouse School of Economics (TSE) and Institute for Advanced Study in Toulouse (IAST); jean.tirole@tse-fr.eu.

1 Introduction

The ongoing Eurozone crisis has sparked a vivid controversy on country solidarity: Should Eurozone countries continue to informally stand by to, if they so wish, secure their peers' access to borrowing? Or should Europeans reinforce their solidarity by issuing Eurobonds, with full joint-and-several liability, or through other formal risk sharing mechanisms such as a common deposit or unemployment insurance scheme?

The crisis also raises the question of the perimeter of the solidarity area. The policy debate, negotiations and actual bailout policies all take it for granted that, just as it fell to the US to rescue Mexico in 1995, Eurozone countries are the natural providers of insurance to each other; even non-Eurozone European countries are exempted from contributing to bailouts.[1] This assumption is at first sight puzzling. After all, insurance economics points at the desirability of spreading risk broadly, rather than allocating it to a small group of countries, which moreover may face correlated risk. Indeed, alternative cross-insurance mechanisms, such as the IMF's Flexible Credit Line, the Chiang Mai Initiative, or credit lines offered to countries by consortia of banks, already exist, that do not involve insurance among countries within a monetary zone.

In analyzing the determinants of international solidarity and their impact on institutions and sovereign borrowing, this paper distinguishes between two forms of solidarity: ex post (spontaneous) and ex ante (contractual). Ex post, the impacted countries may stand by the troubled country because they want to avoid the externality or collateral damage inflicted by the latter's default. Ex ante, they may commit to support levels beyond what they would spontaneously offer ex post, through joint-and-several liability or alternative risk-sharing mechanisms. Spontaneous and contractual bailouts, which respectively correspond roughly to the European approach to date and to the various Eurobonds proposals, are not equivalent.

First, and as will be much emphasized in the first part of the paper, joint-and-several liability redistributes resources from healthy countries to fragile ones as the latter have no means to compensate the former for the resulting insurance (they would have to borrow even more to do that). Second, even in the absence of initial asymmetries, joint liability affects the countries' borrowing capability and probability of default. A failure to stand by a failing country implies a cost of own default on top of the collateral damage incurred when the failing country defaults. Joint liability therefore may create domino effects and increase default costs if the guarantor does not have deep pockets. Conversely, it reduces the occurrence of default if debt levels are moderate enough so as to allow the guarantor to stand by its promise to cover the other country's debt if

[1] While the IMF has large programs in the Eurozone, the brunt of the risk is borne by Eurozone countries and the ECB.

needed.

The paper investigates when countries are willing to extend solidarity by entering ex-ante risk sharing arrangements. It looks at optimal contracts in two environments. In the *asymmetric case*, a risky country (the "agent") under laissez-faire borrows from the market; when defaulting, it exerts an externality on some other country (the "principal"), which has deep pockets. The latter, anticipating this externality or the cost of rescuing the borrower, can enter an optimal ex-ante agreement so to limit the amount borrowed from the market by the risky country and to bring extra support in case of borrower distress. In the *symmetric case*, both countries borrow and are risky. They therefore can exert negative default externalities on each other. Again we look at bargaining for the optimal contract between the two countries, with the status-quo point given by individually-optimal borrowing from the market. Let us now describe these two cases in more detail.

Section 2 sets up the asymmetric model. The borrowing country's income realization (or equivalently its willingness to accept sacrifices) is unobserved by third parties, and there are states of nature in which the country cannot (or does not want to) repay. The country's default imposes a negative externality or collateral damage on the other country. The latter, who has a priori no comparative advantage relative to the market in lending to the borrowing country, may thus be willing to assume some of the borrowing country's debt to prevent collateral damages.

The narrowness of the tax base (that is, who stands for the "principal" in this model) is then rationalized by the heterogeneity in countries' willingness to stand by the failing country: Countries that have a larger stake in avoiding a country's default are more likely to bail out that country. Consequently, a borrowing country's collateral is provided by the collateral damage its default creates onto peer countries, in short by its nuisance power. The collateral damage cost admits both economic and political considerations. Economic spillovers include reduced trade, banking exposures and the fear of a run on other countries. The end of the European construction would involve a sizeable political cost; non-Eurozone political costs are evidenced by various countries' access to cash through their nuisance power (collapse of USSR and fear of nuclear weapons proliferation, assistance to North Korea, US support to Saudi Arabia, Pakistan or Israel) or conversely bailouts motivated by the desire to gain geopolitical influence.[2] Yet another non-economic motivation for bailing out another country is

[2]As Roubini (2004) notes, "*Even before the September 11 events, but more so afterwards, the U.S. tendency to support financial aid to countries that are considered as friends, allies or otherwise strategically or systemically important (Turkey, Pakistan, Indonesia, and possibly Brazil) has clearly emerged, more strongly than during the previous administration. Even in the case of Argentina, where IMF support was eventually cut off leading to the sovereign default of this country, political considerations have been dominant: the August 2001 augmented package was pushed for political rather than economic reasons.*"

empathy, be it driven by ethnic, religious, vicinity or other considerations.

Under laissez-faire, the borrowing country borrows from foreign private creditors without ex ante contracting with the principal. Unregulated borrowing in general generates overborrowing from the point of view of the principal. A Pareto improvement may then be obtained through a contract between the agent and the principal. Whether gains from trade can be realized however is not a foregone conclusion; for, borrowing environments are by essence ones of non-transferable utility.

The ex-ante optimal contract, studied in Section 3, differs from laissez-faire when the borrowing country's liquidity needs (the benefit from borrowing) are intermediate: for low needs, the country does not borrow while for very high liquidity needs, the country borrowing to the hilt maximizes joint surplus subject to the agent's incentive constraint. By contrast, for intermediate needs, ex-ante contracting is desirable and, in exchange of a transfer from the principal, specifies a cap on private sector borrowing.[3]

In general, the optimal contract strictly requires borrowing from the private sector. This surplus-creating role of the private market may sound surprising in view of the assumption that the market and the principal have no relative advantage in lending, as they are equally patient and do not observe the income realization/willingness to reimburse debt. However, any sanction inflicted upon the agent negatively impacts the welfare of the principal, but not that of the investors who have lent to the country. The spillover effect means that, unlike the market, the principal lacks credibility in imposing sanctions on the agent.

Furthermore, and a central result of our analysis, the optimal contract in the asymmetric case is a simple debt contract and involves no joint-and-several liability. The intuition goes as follows: Joint liability allows the debtor country to borrow more by making it more credible that it will be bailed out in case of hardship. But, because the absence of cash is the essence of borrowing, it has no ability to compensate the guarantor for the extra involvement. Thus, asymmetric situations in which the potential guarantor is unlikely to enter distress lead to an implicit form of solidarity (ex-post bailouts), but no explicit solidarity.

By contrast, in the symmetric environment studied in Section 4, in which the two countries borrow, are risky and inflict symmetric collateral damages upon each other when defaulting, borrowing countries have a "currency" with which to pay for the formal insurance they receive through joint-and-several liability: they can reciprocate by offering their guarantor some insurance in a situation in which the fortunes are reversed. We show that joint-and-several liability (cum joint monitoring of countries'

[3]This conclusion is in line with standard models of sovereign borrowing, which predict that countries will spontaneously cap their borrowing so as to make their repayment credible; but the borrowing cap is here conditioned by the externality imposed on the official sector by high debt.

indebtedness) then may emerge as part of the optimal arrangement. More precisely, joint liability (in contrast with currency areas) is optimal provided that country shocks are sufficiently independent, liquidity needs moderate, available sanctions sufficiently tough, and spillovers sufficiently large.

The redistributive consideration is absent in the symmetric case. Only the impacts of joint liability on borrowing capability and probability of default are relevant. Intuitively, under joint liability, countries will want to keep their borrowing moderate as they have to factor in their extra obligation to come to the other country's rescue if needed (domino effects play an important role in constraining borrowing, but are avoided in an efficient pact); so joint liability is in fact associated with a reduced borrowing capability. On the other hand, joint liability creates insurance opportunities between two countries: default then occurs only when both countries are in distress; by contrast, insurance opportunities are limited under individual liability because each country is then too indebted to be willing or able to rescue the other country. Contractual solidarity must thus trade off the cost from reduced borrowing capability against the reduction in the probability of default. High liquidity needs favor high borrowing and therefore the absence of joint liability. By contrast, high collateral damages and a lack of correlation of shocks both make the reduction in default probability and therefore joint liability attractive.

Section 5 summarizes the main findings and concludes with some alleys for future research.

Relationship to the literature: The literature on sovereign defaultable debt[4] has two (complementary) strands. One line, starting with Eaton and Gersovitz (1981) (e.g., Sachs 1984; Krugman 1985; Eaton et al 1986, Bulow and Rogoff 1989 b, Fernandez and Rosenthal 1990), stresses the deterring effect of sanctions, such as trade embargoes, seizure of assets or military interventions. An increase in the cost of default makes the country more prone to repay, but raises the cost of default when the latter occurs due to particularly low resources. Dellas and Niepelt (2012) assume that the cost of default is higher when defaulting on the official sector, as the latter can avail itself of a different set of sanctions. They thereby obtain an optimal mix of private and official sector borrowing, that delivers the optimal sanction. On the empirical front, Rose (2005) shows that debt renegotiations imply a substantial and long-lasting decline in trade.[5]

[4]See e.g., chapter 6 of Obstfeld and Rogoff (1996) and Sturzenegger and Zettelmeyer (2007) for reviews of this literature. The following obviously does not do justice to this very rich literature. For example, it leaves aside the large literature on self-fulfilling liquidity crises initiated by Calvo (1988).

[5]In Fernandez and Rosenthal (1990), the debtor, when repaying in full, receives a "bonus", not paid by the creditor, and interpreted as an improved access to international markets. They show that creditors forgive enough of the debt so as to incentivize the debtor to eventually repay in full. Mitchener and Weidenmier (2010) study "supersanctions" (gunboat diplomacy, seizure of railway assets, foreign administration to collect customs and taxes...) during the gold exchange standard period (1870-1913)

Another line emphasizes that default tarnishes the country's reputation and limits its future access to international financial markets. On the theory side, Eaton and Gersovitz (1981) also developed a model in which sovereign borrowing serves to smooth country consumption and reimbursement is enforced by the threat of being excluded from international capital markets. Kletzer (1984) is the first paper to consider asymmetric information in sovereign debt; it uses the punishment threat of a credit embargo to enforce payments. Bulow and Rogoff (1989 a) argued that reputational concerns per se may not create access to international finance: a country cannot borrow if it can still save at going rates of interest after default. Some of the subsequent literature revisited Bulow and Rogoff's provocative analysis. Hellwig and Lorenzoni (2009) showed that borrowing is feasible under maintained access to savings if the Bulow-Rogoff assumption that the rate of interest exceeds the rate of growth is relaxed. Cole and Kehoe (1995), Eaton (1996) and Kletzer and Wright (2000) stress that commitment is two-sided, as lenders may not comply with the punishment required to maintain discipline. Wright (2002) formalizes banks' tacit collusion to punish a country in default. Cole and Kehoe (1998) argue that opportunistic behavior in the financial market may tarnish the sovereign's overall reputation and create a collateral loss in the relationship with third parties (e.g. domestic constituencies). Cole and Kehoe (2000) study a country's dynamic debt management in a DSGE reputation model.

On the empirical front, Aguiar and Gopinath (2006) show how the presence of trend shocks improves the ability of Eaton-Gersovitz style models to account for actual rate of defaults and other empirical facts for emerging markets. Second, while a number of scholars have documented that defaulting countries recoup unexpectedly quickly access to international capital markets, Cruces and Trebesch (2013) show that large haircuts are associated with high subsequent bond yield spreads and long periods of capital market exclusion.

These papers focus on the allocation of risk between the country and foreign creditors. So does the work of Gennaioli et al (2014) and Mengus (2013 a,b), which stresses the role of domestic banking exposures in the sovereign's decision to default.[6] Arteta and Hale (2008), Borensztein and Panizza (2009) and Gennaioli et al (2014) provide empirical evidence on the internal cost of default. Jeske (2006) and Wright (2006) analyze the impact of the allocation of country liabilities between private and public *borrowing*. The innovation in these papers is the introduction of resident default on international borrowing (associated with a lack of enforcement of foreign claims on domestic resi-

and find that such sanctions were very effective in resuscitating access to capital markets after default.

[6]This holds even if the sovereign can engage in bailouts of domestic banks, provided that it has incomplete information on the quality of balance sheets: see Mengus (2013 a,b). Models of moral hazard (e.g., Tirole 2003) often stress the benefits of a home bias in savings on the government's incentive to behave.

dents by domestic enforcement institutions), on top of standard default on public debt.

By contrast, this paper takes a shot at analyzing the equilibrium allocation of claims on the sovereign between the private and official sectors as well as the split within the official sector; to this purpose it introduces two features that are traditionally absent in the literature: collateral damage costs and the possibility of cross-insurance among countries.[7]

Corsetti et al (2006) develop a model of mixed private-public financing, in which international institutions serve as a lender of last resort and prevent self-fulfilling liquidity runs. They emphasize the role of the precision of the international institution's information, and show that official lending may not increase moral hazard. Persson and Tabellini (1996) study cross-country fiscal externalities when political institutions are not integrated but (a varying degree of) fiscal integration is in place. Bolton and Jeanne (2011) show how monetary integration may create a premium on a healthy country's debt through the collateral demand by banks in weaker ones, and that joint liability destroys this premium.

Bulow and Rogoff (1988) build an infinite-horizon framework of a recurrent debt renegotiation among three players: the debtor country, creditor banks, and consumers in creditor countries, who benefit from the debtor country's exports and therefore are willing to contribute in order to avoid the debtor country's default and concomitant trade sanctions. The anticipation of future side-payments by consumers implies that bank lenders (the "market" in my model) are willing to lend more, which benefits the borrowing country.

Niepmann and Eisenlohr (2013) consider private defaults rather than defaults on sovereign debt. Spillovers are associated with cross exposures between banks of different countries. Contagion thus arises from international balance sheets. The paper looks at the optimal bailout of banks (which requires using distortionary taxation), and show that efficient risk sharing requires that the healthy country should finance a larger fraction of the bailout of the distressed country's banks than the distressed country does. This risk sharing arrangement however does not emerge from uncoordinated behaviors.

This paper is complementary to these papers in its emphasis on optimal design, debt limits, the emergence of joint liability, the role of contagion and the benefits from market financing.

Finally, the paper offers some similarities with the literature on the "cross-pledging" of the revenues in several activities by a single agent (Diamond, 1984) and among

[7]In the banking context, Rochet and Tirole (1996) derives optimal cross-exposures as the outcome of a trade-off between the incentive to monitor and the risk of contagion.

agents (literature on group lending and microfinance).[8] It has been shown in the latter literature that group lending can increase entrepreneurs' access to capital either by mobilizing social capital or by inducing mutual monitoring. Relative to this literature, the paper adds bailouts (the group lending literature assumes that joint liability is the only vector of solidarity) and the requirement that the exercise of even contractual solidarity must respect the guarantor's willingness or ability to pay constraint.

2 Asymmetric environment

(a) *Description*

There are two periods ($t = 1, 2$) and no discounting between the two periods. There are three risk neutral economic agents: the borrowing country (A, the agent), which is cashless at date 1, the international financial market/private investors (M), and another country (P, the principal). The principal is affected by a default of the borrowing country and has deep pockets (so, this is the asymmetric version of the model). The private financial market is competitive.

A borrowing contract between the country and international investors (M or/and P) specifies a reimbursement-contingent sanction $c \in [0, C]$ on the country. C denotes the maximal direct punishment that can be imposed upon the country. The sanction c generates an "externality" or "collateral damage" indirect cost rc on the other country where $r < 1$ denotes the spillover-own default cost ratio. We conveniently take spillovers costs to be proportional to own default costs, but the key property is that tougher sanctions also create larger spillover costs.

We will also allow direct sanctions on the principal in case a pact involving the principal is signed; for instance, joint liability may have to be enforced through direct sanctions on the principal if the latter does not abide by its commitment: See Section 3. Were the principal to be sanctioned, we will then assume symmetrical spillover (the same r coefficient) costs for the agent for notational simplicity, but none of our qualitative results hinges on this assumption.

Each country's objective is to maximize the expectation of total (date 1 + date 2) consumption net of sanction costs. The timing, described as in Figure 1, goes as follows.

Date 1: Borrowing.

The only difference between "laissez-faire" and an "optimal pact" is that in the latter case the agent contracts with the principal before borrowing from the market. In

[8] See, e.g., Tirole (2006, section 4.6) for a review of that literature's main themes, as well as Tirole (2010) for a recent contribution to the economics of extended liability.

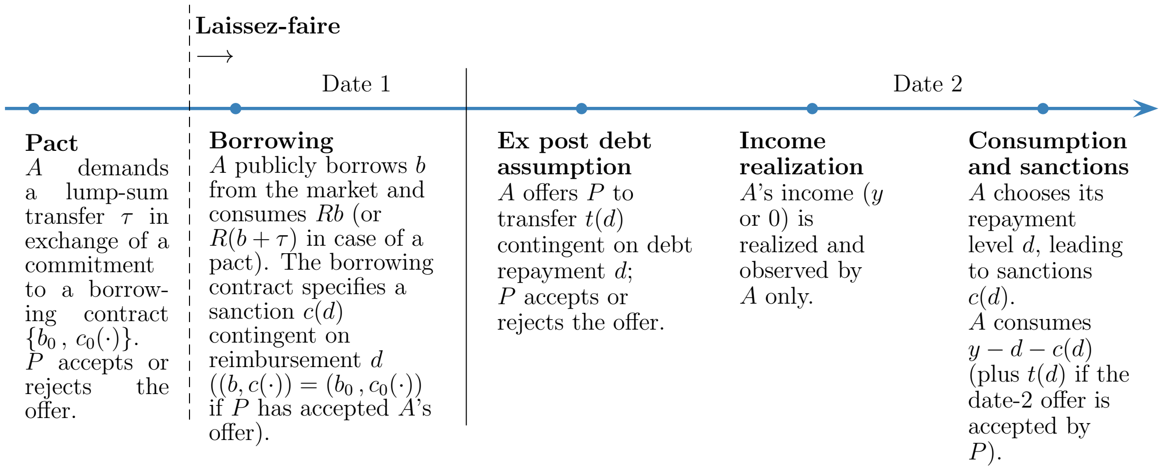

Figure 1: Timing

the case of an ex-ante pact, we will see that there is no loss of generality involved in assuming that the agent borrows only from the market and demands a payment τ in exchange of a commitment to a given borrowing contract.

At date 1, the agent has no money, borrows b from the market and values this borrowing at Rb. The parameter R measures the intensity of the agent's liquidity needs: current consumption needs or, in an extension of the model, quality of his investment opportunities.[9] A *borrowing contract* specifies a sanction $c(d) \in [0, C]$ for each level of debt repayment d. It is publicly observable.

A special case of a borrowing contract is a simple debt contract; in a *simple debt contract* (which will turn out to be optimal), the borrowing contract specifies debt d to be repaid at date 2 to private investors and a sanction c if and only if the debt d is not fully repaid (for simplicity, we will abuse notation and denote by c and d a non-contingent sanction and debt).

Date 2

[9] Rb is most simply interpreted as the agent's date-1 consumption. When R stands for the value of investment opportunities, one must be careful to distinguish investments in non-tradables (which are private benefits and therefore akin to consumption) and investments in tradables (that are likely to raise date-2 income available for repayment). The situation in which borrowing can serve to invest in tradables rather than in non tradables or to consume can be formalized by assuming that the probability of a high income is contingent on the amount of borrowing ($\alpha(b)$, with α increasing and concave), the principal's preferences with respect to agent borrowing can be shown to be ambiguous, even excluding any bailout. On the one hand, the principal benefits from a higher level of borrowing because this increases the probability α that the agent will be able to repay its debt; on the other hand, more borrowing is associated with a higher debt reimbursement and, in an optimal contract, higher sanctions on the country and therefore higher spillovers on the principal (this can be seen more formally by looking at the optimal program for the agent: max $\{\alpha(b)(y - d) - [1 - \alpha(b)]c\}$ subject to $c \geq d$ and $b \leq \alpha(b)d$. The expected externality on the principal is then $\{-[1 - \alpha(b)]rc\}$).

In practice, countries are mostly worried by over-borrowing by the countries that might inflict collateral damage rather than by their under-borrowing; relatedly, the widespread concern is that indebted countries borrow to consume or to invest in the non-tradable sector (e.g. real estate). Thus, our formulation captures actual peer concerns about over-borrowing.

Income realization.

At date 2, the agent receives a random income, equal to y with probability α (good state of nature, G) and 0 with probability $1 - \alpha$ (bad state of nature, B). Only the agent observes the realization. Income "0" is to be interpreted as some incompressible, minimum level of consumption below which the agent is not disposed to go. Equivalently, the market and the principal are uncertain as to whether A is willing to make sacrifices to reimburse the debt (i.e., as to the level of the incompressible level of consumption).[10]

We assume for expositional convenience[11] that $C \geq y$, which means that one can design sanctions that are strong enough to rule out strategic default.

Debt assumption/bailout.

A offers P a contract specifying a transfer $t(d)$ conditional on A repaying d to investors (this stage actually matters only if no pact has been signed at date 1). P then accepts or rejects this offer; $t(d) = 0$ for all d in case of rejection. (For example, in the case of a simple debt contract with investors, one can restrict attention to offers in which P brings conditional support $t(d) = d - \widehat{d}$, with $\widehat{d} \leq d$, provided that the agent reimburses the private investors. The remaining debt burden on A is then $\widehat{d}$.)

Repayment decision.

Finally, the agent chooses repayment d, leading to sanction $c(d)$. For the moment, we assume commitment to the sanction. Later on (see Proposition 2), we will revisit this commitment assumption and show that it is fine in the case of market financing, but questionable in the case of official financing; we will then conclude that the country must borrow from private investors to achieve commitment. Thus, we can proceed by assuming commitment, as long as we keep this point in mind for the implementation of the optimal pact.

(b) *Discussion of modeling choices*

Key ingredients. The key ingredients of the theory presented here are: a) *borrowing country sovereignty*: the latter can borrow in the marketplace if it wants to; and b) *externality*: default imposes costs not only on the defaulting country, but also on the other country. These ingredients ensure that rescues may occur and that the country's borrowing conditions depend on the possibility of solidarity by the deeper pocket country.

[10]Were the state of nature verifiable, then contingent debt contracts could be written, that deliver a higher utility to the agent. The latter would then be tempted to renege in the *good* state of nature, as optimal insurance would call for debt forgiveness in the bad state and a high repayment in the good state. See Grossman and van Huyck (1988) for the view that if states of nature are verifiable, the sovereign's ability to default partially or fully can, under some conditions, mimic an optimal state-contingent debt contract. In practice, some of the state of nature may be observable and therefore some state-contingency is to be expected. Furthermore, the government's overall attitude may partly reveal the underlying state. In this spirit, Trebesch (2009) finds that domestic firms suffer more in their access to credit when the government has employed coercive actions instead of good faith debt renegotiations.

[11]The case $C < y$ delivers similar results, but is a bit more cumbersome.

Non-essential modeling choices. In contrast with these essential modeling choices, we could make a variety of alternative assumptions concerning less essential ones, leading to quantitatively different, but qualitatively similar results. First, we could posit different information structures for the principal and the market; indeed, in the first version of the paper (Tirole 2012), the principal was assumed to observe the agent's shock realization and to collude with the agent regarding the announcement of this realization; the principal is interestingly in a weaker position under symmetric information as it knows exactly how much is required to prevent default; furthermore, the principal then has an effective role as a lender beyond that of a bailout entity. On the other hand, the version presented here is simpler because the principal has no comparative advantage in lending (actually, we will show that it has a comparative disadvantage, due to its lack of credibility as an enforcer of sanctions). Second, if debt repayment were a more protracted event (the country could delay default by incurring sacrifices), a war of attrition between the principal (delaying debt assumption) and the agent (delaying default) could occur.[12]

Recouping through sanctions. We can think of sanction c as endowing the lenders with the ability to ask a court to recoup country's assets or seize its exports abroad, or more generally to enforce sanctions on the country. We are studying optimal contracts between the debtor country and its lenders. Note also that our model ignores the amount collected by market investors when the country defaults. This is only for notational simplicity; it is straightforward to add an investor payoff εc to default penalty enforcement as long as it is smaller than the defaulting country's cost due to this enforcement ($0 < \varepsilon < 1$); we here take the limit as ε tends to 0 to avoid carrying around payment recovery terms.

Partial vs. full repayment. In the optimal borrowing contract of our two-outcome framework (y or 0), the country will either honor the full liability or repay nothing. In practice, countries rarely default fully and the cost of default seems to comprise a fixed cost of not repaying in full, and a variable cost that increases with the actual size of default. Such partial defaults and punishments do arise in the optimal contract of our model with a continuum of outcomes, but the treatment is then more complex than with two outcomes.

(c) *No-externality benchmark: optimal borrowing contract with private investors*

Suppose that there is no principal. Equivalently, as will be shown later, the principal incurs no spillover cost ($r = 0$).

[12] Another informational case which could lead to a war of attrition occurs when the principal does not observe the shock but the agent asks for the date-2 transfer after observing the shock.

An incentive-compatible borrowing contract is a 4-uple $\{d^G, d^B, c^G, c^B\}$ such that $c^\omega \in [0, C]$ for $\omega \in \{G, B\}$, $d^G \le y$, $d^B = 0$, and $d^G + c^G \le d^B + c^B = c^B$. Using the competitive capital market assumption ($b = \alpha d^G$), the country's welfare is

$$U_A = R[\alpha d^G] + \alpha[y - d^G - c^G] + (1-\alpha)(-c^B).$$

At the optimum, $c^G = 0$ (no sanction if the country reveals the high state) and $d^G = c^B$. Thus *the optimal contract is a simple debt contract*, with a pre-specified debt repayment demand $d (= d^G) \le y$ and sanction $c = d$ ($= c^B$) if d is not repaid in full. The agent can borrow up to $b = \alpha c$ from the market, and reimburse $d = c$, the highest credible reimbursement, in the good state, at the cost of default in the bad state. The agent then receives utility

$$U_A = R(\alpha c) + \alpha(y - c) - (1-\alpha)c.$$

Thus the agent solves

$$\max_{\{c \in [0,y]\}} \{(\alpha R - 1)c + \alpha y\}.$$

The agent can either refrain from borrowing ($b = 0$) and receive utility αy, or borrow maximally ($b = \alpha y$) and receive utility $(\alpha R - 1)y + \alpha y$ (the linearity of the objective function implies that we can focus on these two alternatives). Thus the agent borrows from the market if[13]

$$\alpha R > 1.$$

Proposition 1 (optimal borrowing contract in the absence of externality). *Suppose that $r = 0$. At the optimum, there is no borrowing if $R < 1/\alpha$. If $R > 1/\alpha$, the optimal borrowing contract is a simple debt contract with nominal debt $d = y$ and sanction $c = y$ if repayment is lower than y.*

3 Solidarity in the asymmetric environment

We now introduce an externality rc on the principal for sanction c on the agent. Let

$$e \equiv ry.$$

[13]For expositional convenience, we ignore non-generic cases (such as $\alpha R = 1$), for which there are multiple optima.

denote the externality when the sanction is equal to y. This is the externality incurred in the absence of debt assumption by the principal in state B when the agent's repayment is at its maximal level in state G, enforced by the threat of sanction $c = y$.

3.1 Optimal pact

We suppose that prior to borrowing at date 1 the agent makes a take-it-or-leave-it contractual offer to the principal. If the principal turns down the offer, the outcome is the laissez-faire one. We let $U_P^{LF} \leq 0$ denote the principal's reduced-form utility under laissez-faire, i.e., if there is no agreement on a pact at date 1 (we will study its determination in Section 3.2; note that the possibility of debt assumption by P may alter A's borrowing relative to the level given in Proposition 1). Thus the agent puts the principal at its reservation utility U_P^{LF}. While the theory is easily generalized to more even distributions of bargaining power at date 1, giving no bargaining power to the principal is particularly interesting because it gives the best chance to joint-liability demands by the agent.

We adopt a mechanism design approach. The principal can make a date-1 contribution τ in exchange of having a say on the borrowing contract. The equilibrium allocation is described by

- ✓ the date-1 disbursements b by the market and τ by the principal, such that $b + \tau \geq 0$ (since the agent has no money at date 1);
- ✓ the equilibrium effective debt repayment d_A^ω by the agent in state of nature $\omega \in \{G, B\}$;
- ✓ the net date-2 payment by the principal t^ω in state of nature $\omega \in \{G, B\}$;[14]
- ✓ the total punishments $\hat{c}_A^\omega$ and $\hat{c}_P^\omega$ for the agent and principal in state of nature $\omega \in \{G, B\}$.

"Total punishments" refer to the cost borne by countries from own default and the other country's default. Recall that the sanction c_i on country i inflicts an externality rc_i on the other country, where the direct cost exceeds the indirect one ($r < 1$). We allow for the possibility of sanctions on the two countries and so the total sanction inflicted upon i is:

$$\hat{c}_i^\omega \equiv c_i^\omega + rc_j^\omega$$

[14]The accounting convention is that t^ω goes to private investors (equivalently, it can go to the agent, who can use it to pay investors back). $t^\omega > 0$ in case of a bailout, and < 0 if the principal receives money. Note also that this notation refers to the *actual* repayments and does not imply that state-contingent debt can be issued.

Let $\widehat{C} \equiv \left\{(\hat{c}_A, \hat{c}_P) | \exists (c_A, c_P) \in [0, C]^2 \text{ such that } \hat{c}_i = c_i + rc_j\right\}$ denote the set of feasible punishments. Let $\hat{c} \equiv (1+r)C$ denote the maximal overall cost that can be inflicted upon a country.[15]

Because there is revelation only by the agent and the initial contract can be designed so as to be bilaterally efficient, the recontracting possibility between A and P at date 2 as pictured in Figure 1 is irrelevant.[16] We will see in Section 4 that this is no longer the case when the two countries are exposed to shocks that they must disclose.

Let

$$R^* \equiv \begin{cases} \dfrac{1}{\alpha - (1-\alpha)r} & \text{if } \alpha > (1-\alpha)r \\ +\infty & \text{otherwise.} \end{cases}$$

Thus $R^* > 1/\alpha$ for $r > 0$.

Proposition 2 (optimal pact in the asymmetric case). *Except for the level of date-1 transfer from P to A, the optimal pact between the agent and the principal is independent of the principal's utility U_P^{LF} under laissez-faire:*
(i) For $R < R^*$, the agent commits not to borrow ($b^* = 0$) and so there is no sanction on the equilibrium path ($\hat{c}_i^\omega = 0$ for all ω and i). The optimal pact can be implemented through a fixed date-1 transfer $\tau = -U_P^{LF}$ in exchange of a commitment by the agent not to borrow at all.
(ii) For $R > R^*$, the agent pays back in the good state of nature the maximum(y) it can pay in that state, resulting in borrowing ability αy; and the principal chips in $e = ry$ in that state. Consequently, direct and indirect sanctions are y and e in the bad (default) state. Formally: $b + \tau = \alpha(y+e)$, $d_A^G = \hat{c}_A^B = y$, $\hat{c}_P^B = e$ and $\hat{c}_i^G = 0$ for $i \in \{A, P\}$.
(iii) There is no need for joint liability in the optimal pact, regardless of R. For $R > R^*$, the optimal pact can be implemented without any ex-ante pact through a simple debt contract in which the agent borrows from the market $d = y + e$ and incurs sanction $c = y$ in the absence of full repayment, the principal then offering a bailout e contingent on d being repaid. [Alternatively, the optimal allocation can be implemented through a pact. For instance, the principal can transfer $\tau = \alpha e$ against the agent's commitment of borrowing no more than y from the market.]
(iv) If furthermore sanctions are enforced only if it is in the interest of the lender to enforce them, market financing is required whenever borrowing is optimal.

To obtain intuition for parts (i) and (ii) of the Proposition (which are proved in the Appendix), suppose that the agent in equilibrium repays an amount d in the good

[15]The assumption of cost additivity is made only to avoid adding new notation and is not required for the theory to carry through. For instance, a country incurring trade sanctions may be less affected by trade sanctions on neighbouring countries than it would be if it did not face such sanctions itself (sub-additivity); conversely, political or military weakness due to economic difficulties may generate super-additive effects. I am agnostic as to which prevails.

[16]This will result from the contract described in Proposition 2 being collusion-proof.

state of nature. The minimum sanction that makes this incentive compatible is $c = d$, implying total cost $\alpha d + (1-\alpha)d = d$ on the agent and $(1-\alpha)rd$ on the principal. The net benefit for the agent is thus $(R\alpha - 1)d$ and the net cost for the principal $(1-\alpha)rd$. If $R\alpha - 1 < (1-\alpha)rR$, i.e. $R < R^*$, then $d = 0$ is optimal, as the principal is willing to transfer $(1-\alpha)rd$ at date 1 to the agent, raising the latter's utility by $R(1-\alpha)rd$. By contrast, if $R \geq R^*$, the agent, who cannot commit to repay more than y in the good state of nature, already obtains its maximum utility by borrowing to the hilt and so no pact can raise this utility without hurting the principal.

The implementation of the optimal contract (part (iii) of the Proposition) is straightforward. Note that when $R > R^*$ and in the laissez-faire implementation, the principal contributes at level e conditional on repayment of total debt $d = y + e$, implying total cost for the principal associated with agent borrowing equal to $\alpha e + (1-\alpha)r\hat{c}_A^B = e$. And so joint liability is not required. Joint liability is not used either in the second implementation.

To understand why the joint-liability option serves no purpose, note that the potential benefit of joint liability is that the agent, who values date-1 resources at R, can borrow more if investors can turn to the principal if the debt is not paid by the agent. However the principal must be compensated for that sacrifice; it de facto becomes a second lender. Because the principal has the same rate of time preference and the same information as the market and thus has no comparative advantage in the lending activity, there are no gains from trade in that direction.

Finally, let us show that market financing is required for $R > R^*$, i.e., whenever there is borrowing and sanctions are to be time-consistent (part (iv) of the Proposition). We have noted so far that the principal has the same discount factor and the same information as the market. The spillover effects however put the principal at a comparative *disadvantage* in lending relative to the market. To see this, suppose that there is no borrowing from the private sector ($b = 0$) and that the principal transfers $\tau = \alpha(y + e)$ at date 1 and threatens the agent with sanction $\hat{c}_A^B = y$ in case of non-repayment of debt $d_A^G = y$. Then it cannot be an equilibrium for the agent to reimburse y in state G and nothing in state B; for, the principal then would not enforce the sanction in the absence of reimbursement since P would then sanction itself and lose e.

By contrast, the market credibly sanctions the agent in case of non-repayment provided that there is an arbitrarily small amount of money to be recouped in the process (recall that we took the limit as ε, and therefore the amount of money εc_A^B to be recouped (for sanction c_A^B that is inflicted in the absence of repayment), tends to 0). More generally, mixed financing with sanctions imposed by market investors and the official sector in proportion of their relative stakes in the country is not desirable. The optimum requires delegating all sanctions to the market and then is equivalent to pure market financing.

Discussion

(a) A number of recent policy proposals by economists, think-tanks and politicians[17] have proposed introducing contractual solidarity through a two-tier borrowing structure: blue bonds, for which the Eurozone would be jointly liable, and red bonds, for which no such solidarity would operate.[18] Blue bond issues would be capped at a fraction of GDP (say 60 %). These proposals all insist on a number of features: budgetary supervision (a policy that in our model would be akin to controlling moral hazard on the choice of α), joint liability on the blue bonds, no bail-out clause on the red bonds, and seniority of blue bonds over red bonds. Our analysis shows that joint liability is unlikely to emerge under asymmetric conditions.

(b) The optimality of full market borrowing is due to the fact that the model takes away all comparative advantage from the principal: it has no informational advantage, and on top of that it does not like to sanction a default because it will shoot itself in the foot. If the principal could observe the borrower's state more easily than the market, then there may be a lending role especially in a repeated context, where the principal can be incentivized to punish even if it is costly. Part (iv) of the Proposition therefore should not be interpreted as a statement that the market always wins over the principal (anticipating on our other applications of our model, think of informal lending by the extended family versus lending by micro finance organizations). But the overall thrust is that one needs the "outside market" for loans.

(c) A spread on the agent's sovereign debt appears when the agent opts for a risky strategy. Proposition 2 therefore implies that high spreads correlate with high liquidity needs (high R). By contrast, because this model has no shortage of international stores of value, the agent's borrowing pattern has no impact on the principal's borrowing conditions: there is just no spread there. By contrast, if there were a shortage of safe financial instruments in the principal's economy, safe instruments' premium would

[17]Variants of Eurobonds have been advocated by most leading European politicians, multi-lateral organizations (e.g., the IMF), the media (e.g., *The Economist*), and in several economists' proposals that have attracted wide attention in policy circles. See in particular Delpla and von Weizsacker (2010), Euro-nomics group (2011), and Hellwig and Philippon (2011). Related proposals include the European Commission's green paper on "stability bonds" (2011), the Tremonti-Juncker proposal (2010), and the German Council of Economic Experts' "European Redemption Pact" (2011). See Claessens et al (2012) for an extensive overview and discussion of the various proposals.

Most of these proposals advocate coupling Eurobonds with borrowing limits. For example, Olivier Blanchard, IMF's chief economist, argues in the *Financial Times Deutschland* (April 23, 2012) that: "*When there was no fiscal treaty nor budgetary discipline instruments, the Germans had good reason to reject bearing the brunt of irresponsible policies by other states. But now we have a fiscal treaty. The Germans should accept that the Eurozone is going by way of Eurobonds.*" The European Financial Stability Facility created in 2010 can issue bonds backed by guarantees given by the Euro area member states.

[18]The particular terminology is due to Delpla and von Weizsacker (2010). See also the closely related Eurobill proposal of Hellwig and Philippon (2011).

increase due to a flight to quality, as in Bolton and Jeanne (2011). These properties will also hold under laissez-faire.

3.2 Laissez-faire

While Proposition 2 contained the main insights for the asymmetric case and qualitatively does not depend on the exact value of the principal's laissez-faire payoff U_P^{LF}, it is interesting to investigate the agent's date-1 borrowing behavior when there are externalities ($r \geq 0$) but no pact has been signed at date 1. We restrict attention, *in this section only*, to simple debt contracts (although we will prove in the Appendix that such contracts can be optimal in the full class of feasible contracts).

Proposition 3 (optimal borrowing under laissez faire). *In the absence of contract with the principal, the agent always borrows from the market at date 1. The agent's optimal simple debt contract is*

✓ *either a high-debt policy (borrowing $\alpha(y + rc)$ against debt claim $y + rc$ and defaulting in the bad state), where $c = y$ if $R\alpha r < 1 - \alpha$, and $c = C$ if $R\alpha r > 1 - \alpha$.*

✓ *or a low-debt one (borrowing d_L against debt claim d_L and never defaulting, thanks to the principal's ex-post debt assumption). The safe debt level is $d_L \equiv (1 - \alpha)rC$ if $rC < y$, and $d_L = rC$ if $rC > y$.*

(i) The agent picks the high-debt policy if $R \geq R^{LF}$ for some threshold $R^{LF} \leq R^*$.
(ii) The high-debt policy is more likely, the greater the probability of a good state and the more pressing the agent's liquidity needs.
(iii) The principal' s welfare U_P^{LF} is lowest when the agent's liquidity needs are high.

This result sheds light on why P and A may gain from contracting before A receives financing from the market. When the agent's liquidity needs are not pressing ($R < R^{LF}$), the principal knows that borrowing will be limited under laissez-faire; as there is then no default, there are no gains from contracting between the principal and the agent at date 1, and laissez-faire prevails. As liquidity needs increase ($R^{LF} < R < R^*$), the agent, who would over-borrow under laissez-faire, is offered a "bribe" by the principal to limit its borrowing.[19] When R starts exceeding R^*, though, risky borrowing becomes jointly efficient. The parties cannot improve on laissez-faire then. It is only when the agent's high borrowing is inefficient, which occurs for intermediate values of R, that the principal and the agent gain from a pact.

[19] A control over private borrowing is in general required. Otherwise, the agent might well overborrow, preventing the optimum from being reached. This argument is a variant of the classic dilution problem (e.g., Bizer-de Marzo, 1992; Segal 1999), but with a twist: Overborrowing is here motivated by the desire to trigger an uncontracted-for bailout.

*Intuition.*The proof of Proposition 3 can be found in the Appendix. To grasp the intuition for it, consider a simple debt contract when the maximal sanction, C, is exactly equal to y. Either the agent borrows $d_H = y + e$ (where, recall, $e = ry$) and then receives conditional support e and pays back y in the good state. The agent then defaults in the bad state. This yields principal welfare $U_P(d_H) = -e$ and gross agent welfare:

$$U_A(d_H) = R\alpha(y + e) - (1 - \alpha)y.$$

Or the agent borrows $d_L = (1 - \alpha)e$ which the principal covers rather than risking an agent default and concomitant spillover cost e with probability $1 - \alpha$, and so no default occurs. Then principal welfare is $U_P(d_H) = -(1 - \alpha)e$ and gross agent welfare is

$$U_A(d_L) = R(1 - \alpha)e + \alpha y.$$

Note that:

$$U_A(d_H) \geq U_A(d_L) \iff R \geq R^{LF} = 1/[\alpha + (2\alpha - 1)r]$$

where $R^{LF} < R^*$ (whenever R^{LF} is finite; if $(1 - 2\alpha)r \geq \alpha$, then $R^{LF} = R^* = +\infty$).

Finally, note that for $r = 0$ (the no-externality or no-principal case), $R^{LF} = R^* = 1/\alpha$.

4 Contractual solidarity behind the veil of ignorance

Consider now the *symmetric* version of the two-country model. Both countries borrow at date 1. Country $i \in \{1, 2\}$ values cash b_i available at date 1 at Rb_i. At date 2, each country either has income y (is "intact" or "healthy") or has no income (is "distressed"). Only the country knows its income realization. The state of nature is now $\omega = (\omega_1, \omega_2)$ where $\omega_i \in \{G, B\}$. The probability that k countries have income y is p_k (with $\Sigma_{k=0}^{2} p_k = 1$). By keeping these probabilities general, we allow arbitrary patterns of correlation between income shocks. Let

$$\alpha \equiv p_2 + (p_1/2)$$

denote the unconditional probability of being intact, and

$$\beta \equiv p_2/[p_2 + (p_1/2)]$$

the probability of the other country being healthy when the country itself is healthy. Positive (resp. negative) correlation corresponds to $\beta > \alpha$ or $4p_2p_0 > p_1^2$ (resp. $\beta < \alpha$ or $4p_2p_0 < p_1^2$).

For comparative statics purposes, we will occasionally define an index ρ of correlation. Suppose that the incomes are perfectly positively correlated with probability ρ (they are both equal to y or to 0), and perfectly negatively correlated with probability $1-\rho$ (one is equal to y and the other to 0, with equal probabilities), the marginal probability of y remaining equal to α. Then

$$p_2 = \alpha\left[1-(1-\rho)(1-\alpha)\right],\ p_1 = 2\alpha(1-\alpha)(1-\rho) \text{ and } p_0 = (1-\alpha)\left[1-(1-\rho)\alpha\right].$$

Perfect positive (negative) correlation corresponds to $\rho = 1$ ($\rho = 0$). More generally, we will say that the countries are more correlated if ρ increases.

The (symmetric) laissez-faire outcome does not influence even quantitatively the optimal (symmetric) contract, and so need not be derived. Let us investigate the conditions under which joint liability, which creates a risk of domino effect and thereby increases default costs if the amount of borrowing is high, emerges from an optimal pact. We again distinguish between country i's own sanction cost, c_i, and the (smaller) collateral damage this sanction imposes on the other country, rc_i (where $r < 1$). Without loss of generality, we focus on incentive compatible (truthful) mechanisms.

Let $\hat{c}_0$ (respectively $\hat{c}_2$) denote the total sanction cost per country when both are in distress (respectively, healthy). When $k=1$, we will distinguish between the pain, $\hat{c}_1^G$, inflicted upon the country that is in a good state (has income y), and that, $\hat{c}_1^B$, inflicted upon the country in a bad state (with zero income). Let $\hat{c}_1 \equiv \left(\hat{c}_1^B + \hat{c}_1^G\right)/2$ denote the per-country average pain when $k=1$. And let d_k denote the expected, per-country reimbursement to private creditors in state of nature k. Obviously, $d_0 = 0$. Similarly, when $k=1$, the healthy country pays $2d_1$.

As often in mechanism design, the strategy for finding the optimal arrangement will consist in considering a subconstrained program and checking that its solution can indeed be implemented. Consider the following program:

$$\max\left\{R\left[\Sigma_{k=0}^{2}p_k d_k\right] - \Sigma_{k=0}^{2}p_k\left(d_k+\hat{c}_k\right)\right\} \tag{II}$$

$$= \max\left\{(R-1)(p_2 d_2 + p_1 d_1) - \left[p_2\hat{c}_2 + p_1\frac{\hat{c}_1^G+\hat{c}_1^B}{2} + p_0\hat{c}_0\right]\right\}$$

subject to the truthtelling constraint in the good state

$$\beta\left(d_2+\hat{c}_2\right) + (1-\beta)\left(2d_1+\hat{c}_1^G\right) \le \beta\hat{c}_1^B + (1-\beta)\hat{c}_0$$

and to feasibility constraints

$$d_2 \le y$$

and

$$2d_1 \leq y$$

The objective function is the difference between a country's date-1 benefit derived from borrowing $b = \Sigma_{k=0}^{2} p_k d_k$, and the date-2 cost, which includes monetary reimbursement and the pain associated with sanctions and their spillovers. The per-country expected payoff is equal to αy (a constant) plus the maximand in (II), which represents the net per-country utility.

In the absence of further constraints, the full information allocation would be feasible when the countries are perfectly positively correlated ($p_1 = 0 \iff \beta = 1$).[20] Having each country reimburse y when healthy and 0 when distressed, and no default on the equilibrium path, can be obtained by setting $\hat{c}_2 = \hat{c}_0 = 0, \;\; d_2 = 2d_1 = y$, and $\hat{c}_1^B = y$. This discussion provides a first rationale for the following condition:

Collusion-proofness. The overall contract is collusion-proof if the countries cannot profitably increase their welfares through a side contract written before the state of nature is revealed. Formally, a side contract is a revelation game between the two parties in which both countries announce their own state of nature $\omega_i \in \{G, B\}$ in an incentive-compatible way to a mediator prior to their public announcements. The side contract specifies public announcements $\hat{\omega}(\omega)$ as well as side transfers from i to j, $t_{ij}(\omega)$, such that $t_{ij}(\omega) + t_{ji}(\omega) = 0$. For more on the modeling of side contracting used here, see, e.g., Laffont and Martimort (1997, 2000).

The second rationale for the collusion-proofness requirement is that its introduction makes the framework consistent with that of Sections 2 and 3 and its date-2 side contract between A and P. Note further that any collusion that would occur after the agent learns the income realization can be achieved before he learns it (technically, there are fewer individual rationality constraints in the collusion subform).

We consider a subset of the constraints imposed by collusion proofness, leaving it to the proposed implementation to check overall collusion proofness. Consider state (G, G). If $d_2 + \hat{c}_2 > \hat{c}_0$, the two countries could declare themselves income-free and be better off. We thus require that

$$d_2 + \hat{c}_2 \leq \hat{c}_0.$$

Similarly it must not be the case that in state (G, G), the two parties gain from randomizing between declaring (G, B) and declaring (B, G):

$$d_2 + \hat{c}_2 \leq d_1 + \frac{\hat{c}_1^G + \hat{c}_1^B}{2}$$

[20]This point is closely related to that in Crémer and Mclean (1988), although there are a number of differences, including the limited liability enjoyed by the agents.

To see that such collusion does not interfere with truth-telling to the mediator of the side contract, consider a side contract in which the countries misreport when the state of nature is (G, G). This change only facilitates the fulfillment of the truth-telling constraint.[21] These two constraints do not exhaust the set of collusion-proofness constraints, but in the Appendix we verify that the derived contracts satisfy the missing constraints.

A rapid inspection of this program shows that at the optimum $\hat{c}_2 = 0$ and $\hat{c}_1^G \equiv \phi(\hat{c}_1^B)$: to reward truth telling, one punishes countries as little as possible when they declare a high ability/willingness to pay.

Rewriting the program then yields:

$$\max_{\left\{d_2 \geq 0, d_1 \geq 0, \left(\hat{c}_0, \hat{c}_1^B\right) \in \left[0, \hat{c}\right]^2\right\}} \left\{(R-1)\left(p_2 d_2 + p_1 d_1\right) - \left[p_1 \frac{\phi(\hat{c}_1^B) + \hat{c}_1^B}{2} + p_0 \hat{c}_0\right]\right\} \tag{II}$$

s.t.

$$p_2 d_2 + p_1 d_1 + \frac{p_1}{2} \phi(\hat{c}_1^B) \leq p_2 \hat{c}_1^B + \frac{p_1}{2} \hat{c}_0 \tag{1}$$

$$d_2 \leq y \tag{2}$$

$$2 d_1 \leq y \tag{3}$$

$$d_2 \leq \hat{c}_0 \tag{4}$$

$$d_2 \leq d_1 + \frac{\phi(\hat{c}_1^B) + \hat{c}_1^B}{2} \tag{5}$$

Feasible contacts, i.e. contracts satisfying (1) through (5), include two prominent classes (as we will later show, optimal allocations can often be implemented by a contract in these classes):

Individual liability contracts (IL). An individual debt contract is characterized by

$$d_2 = y \quad , \quad d_1 = y/2 \quad , \quad d_0 = 0$$

and

$$\hat{c}_2 = 0 \quad , \quad \hat{c}_1^B = y \quad , \quad \hat{c}_1^G = ry \qquad , \qquad \hat{c}_0 = (1+r)y,$$

[21] Letting $U_{\omega_i \omega_j}$ denote the expected utility of party i in state ω_i when the other party is in state ω_j, the truth telling constraint can be rewritten as:

$$\beta U_{GG} + (1-\beta) U_{GB} \geq \beta U_{BG} + (1-\beta) U_{BB} + y.$$

This constraint is still satisfied when U_{GG} is replaced by some $\widehat{U}_{GG} \geq U_{GG}$. Furthermore, the truthtelling constraint when $\omega_i = B$ is not affected because a distressed country cannot mimic a healthy one if there is any borrowing $(p_2 d_2 + p_1 d_1 > 0)$.

delivering utility

$$U^{\mathrm{IL}} \equiv \Big[(R-1)\alpha - (1-\alpha)(1+r)\Big] y.$$

Note that constraints (4) and (5) are slack in the *IL* contract, which is also shown in the Appendix to satisfy the ignored collusion-proofness; so individual contracts do not invite collusion. The borrowing constraint level is

$$b^{IL} \equiv (p_2 + \frac{p_1}{2}) y = \alpha y.$$

Joint liability contracts (JL). A contract with joint liability is characterized by

$$d_2 = d_1 = y/2 \quad , \quad d_0 = 0$$

and

$$\hat{c}_1^B = \hat{c}_1^G = 0 \quad , \quad \hat{c}_0 = \Big(\frac{p_2 + p_1}{p_1}\Big) y,$$

where the value of $\hat{c}_0$ is the minimal sanction when $k = 0$ that guarantees individual truthtelling.

Note that, unlike individual debt contracts, joint liability contracts are not always feasible as they require sufficient sanctions:

$$\hat{c} \equiv (1+r)C \geq \Big(\frac{p_2 + p_1}{p_1}\Big) y.$$

JL contracts then deliver utility

$$U^{\mathrm{JL}} \equiv \Big[(R-1)\frac{(1-p_0)}{2} - p_0\Big(\frac{p_2 + p_1}{p_1}\Big)\Big] y.$$

Note that constraints (2) and (4) are slack in the *JL* contract. The Appendix checks that the *JL* contract satisfies the missing collusion-proofness constraints and not only (4) and (5); it is therefore collusion-proof. The borrowing level is

$$b^{\mathrm{JL}} \equiv (p_2 + p_1) y/2 < b^{\mathrm{IL}},$$

provided that $p_2 > 0$.

We first compare these two classes of contracts:

$$U^{\mathrm{IL}} \geq U^{\mathrm{JL}} \iff \frac{R-1}{2} \geq (1+r)\Big(\frac{p_1 + 2p_0}{2p_2}\Big) - \frac{p_0\big(p_2 + p_1\big)}{p_2 p_1}.$$

Note that either can dominate: For instance, individual liability dominates for R

large, while for $p_2 = p_0$ (i.e. $\alpha = 1/2$), the RHS of this inequality, equal to $(1+r)[1+(p_1/2p_2)] - [1 + p_2/p_1]$, goes to $+\infty$ as p_2 goes to 0 ($\rho \to 1$) and so joint liability dominates.

Proposition 4 (individual vs. joint liability). *There is more borrowing under individual liability: $b^{IL} > b^{JL}$ (unless $p_2 = 0$). Individual liability contracts are more attractive relative to joint liability contracts ($U^{IL} - U^{JL}$ increases),*
(i) the higher the liquidity needs (R),

(ii) the higher the correlation (ρ) ,

(iii) the more limited the feasible sanctions (the lower C is),

(iv) the smaller the spillovers (the lower r is).

The proof of Proposition 4 is straightforward, so we will focus on the intuition. There are costs and benefits to joint liability. The requirement of debt assumption combined with limited resources imply lower debt levels, which is costly if liquidity needs are high. On the other hand, joint liability reduces sanction costs in the state in which only one of the countries is healthy; so joint liability is more attractive if this event is likely.[22] Third, we have seen that joint liability requires sufficient sanctions ($C \geq (p_2 + p_1)y/p_1(1+r)$) while individual liability does not. Finally, joint liability becomes more attractive under high spillovers for two reasons; first, spillovers increase the scope for sanctions ($\hat{c} = (1+r)C$); second, the country's welfare, U^{IL}, under independent liability decreases with spillovers while that, U^{JL}, under joint liability is independent of spillovers.

We now study whether IL or JL contracts are optimal contracts. To this purpose, let us define the following notion:

A contract is a *quasi-IL contract* if
(i) $d_2 = 2d_1 = y$ (maximum reimbursement)
(ii) $\hat{c}_1^G = r\hat{c}_1^B$ and constraints (1), (4) and (5) are satisfied.

So a quasi-IL contract involves maximum borrowing/reimbursement; it is a bit more flexible in the allocation of sanctions $\left(\hat{c}_0\,,\, \hat{c}_1^B\right)$ to achieve truthtelling (constraint (1)). The flexibility however is rather limited by the requirements that $y \leq \hat{c}_0$ and $y \leq \hat{c}_1^B(1+r)$ (constraints (4) and (5)).

Proposition 5 below provides a partial characterization of the optimum, comforting the findings in Proposition 4.

[22] For example, for $r = 0$, individual liability strictly dominates in case of independence or positive correlation.

Proposition 5 (optimal contract). *Let*

$$A \equiv (R-1)\frac{p_1}{2} - p_0 \quad and \quad B \equiv (R-1)\Big(p_2 - \frac{p_1 r}{2}\Big) - p_1\Big(\frac{1+r}{2}\Big).$$

(i) *If $A < 0$ and $B < 0$, borrowing is suboptimal ($b^* = 0$).*

(ii) *If $A > 0$ and $B > 0$, the optimal contract is a quasi-IL contract.*

If $p_0 p_2 - \frac{p_1^2}{4} = \frac{p_1}{2} r\Big(p_0 + \frac{p_1}{2}\Big)$ (as is the case for independence and no externality), the IL contract is optimal.

(iii) *If $A > 0 > B$, the optimal contract is the JL contract (provided that it is feasible, i.e., that $\hat{c} \geq \big[(p_2 + p_1)/p_1\big]y$).*

Joint liability is therefore optimal when externalities (r) are large and correlation (ρ) low.

We conclude that for a wide range of parameters,[23] either the joint-liability contract or (a contract very similar to) the individual liability contract is optimal. The comparative statics on the factors favouring one or the other furthermore confirm those of Proposition 4.

Interestingly, at the optimal contract, joint liability does not generate domino effects; this does not imply that the threat of contagion under joint liability plays no role; indeed, this very threat of contagion is what leads the countries to moderate their borrowing relative to what they borrow in the absence of joint liability.

5 Conclusion

Summary. Solidarity is driven by the fear that spillovers from the distressed country's default negatively affect the rescuer. This paper's first contribution was to provide formal content to the intuitive notion that collateral damages of a country's default are de facto collateral for the country.

The paper's second contribution was to unveil the conditions under which joint-and-several liability may emerge. Standard liquidity provision or risk sharing models presume that accord is reached behind the veil of ignorance. Once the veil of ignorance is lifted (as is currently the case in the Eurozone), healthy countries have no incentive to accept obligations beyond the implicit ones that arise from spillover externalities. Put differently, it is not in the self-interest of healthy countries to accept joint-and-several liability, even though they realize that they will be hurt by a default and thus will

[23]When $A < 0 < B$, the solution to this program violates the missing collusion-proofness constraints, and the analysis is then more complex.

ex post show some solidarity in order to prevent spillovers. In this "non-transferable utility" environment, gains from trade exist as total surplus can be increased, but they cannot be realized. An ex-ante transfer from distressed countries to healthy ones to compensate them for, and make them accept the future liability is ruled out as it would just add to the distressed countries' indebtedness.

Third, the paper showed that by contrast, in a more symmetrical, mutual-insurance context, contractual solidarity in the form of joint liability is optimal provided that country shocks are sufficiently independent, spillovers costs sufficiently large, liquidity needs moderate and feasible sanctions sufficient. While domino effects do not arise in equilibrium, the contagion risk leads to a reduction in borrowing relative to its maximal level under individual borrowing.

While joint liability has the potential to increase borrowing relative to individual liability, the overall picture that emerges from the analysis is that the option of declaring joint liability actually does not lead to higher borrowing levels: Either the potential guarantor has deep pockets and then it has no incentive to enter joint liability because it cannot be compensated for the service it provides to the other country. Or the two parties have shallow pockets and then to avoid domino effects they keep their borrowing limited when opting for joint liability.[24] Finally, we have seen that spillovers confer an advantage on market borrowing as they make sanctions by the official sector less credible than market-imposed sanctions.

Returning to the puzzle stated in the introduction, both the bailout contributions and the policy debate about Eurobonds and the banking union mostly concern a very limited insurance pool, namely the Eurozone, while basic principles of insurance economics would call for a much broader solidarity area. Although the following suggestions are no substitute for a careful analysis, the model arguably sheds light on the puzzle. First, the monetary union has drastically increased the degree of financial integration among Eurozone countries.[25] Financial integration implies increased spillovers from default. Second, the establishment of the monetary union in large part was driven by a political project. Abandoning the Euro, or letting some Eurozone countries default would have a substantial symbolic impact. These two factors are likely explanations for the otherwise peculiar risk-sharing arrangement.

Research alleys. On the theoretical front, the paper is only a first attempt at under-

[24]As one referee pointed out, it would be interesting to study dynamic environments in which countries would be exposed to liquidity shocks rather than to solvency/pledgeable income shocks. For example, in a three-period model, one of the countries might have a high R at the intermediate date. Its ability to borrow under joint liability/ harness the other country's pledgeable income would allow it to borrow more in order to meet this liquidity shock.

[25]Until the recent "re-nationalization". Kalemli-Ozcan et al (2010) shows that financial integration was driven more by the elimination of currency risk than by trade in goods.

standing the fundamentals of country solidarity, whether reluctantly provided or more pro-actively contracted for. There are many interesting alleys for future research in this area alone. For instance, one might extend the analysis of Section 4 to consider extended solidarity; first losses could be covered by an inner circle of countries within a solidarity area and macro shocks within this area might be partly insured by an outer solidarity area (rest of the world, IMF).

Another fascinating topic for future analysis would result from asymmetries of information about collateral damages and the concomitant posturing behaviors in the international community. Yet another class of extensions consists in studying repeated bailouts.[26]

Similarly, one may build on this paper to investigate the impact of fiscal unions. A fiscal union creates an automatic risk sharing mechanism and thus correlates income realizations; it further generates some joint liability through the issuance of federal debt. And, as is well-known, the increase in correlation facilitates the conduct of monetary policy as well. Nonetheless, states still enjoy some degree of subsidiarity; the implications of fiscal federalism for solidarity are definitely worth investigating.

The paper has assumed that troubled countries can resort only to hard default to escape the burden of liabilities in adverse times. Either they are highly inflation averse or their commitment to a currency union precludes any debt monetization. Broadening the analysis to allow for debt monetization would be worthwhile.[27]

Another extension of this paper's framework consists in endogenizing spillovers. While empathy suffering and trade and political disruptions are in part exogenous, counterparty risk is determined by domestic prudential supervision as well as other mechanisms (such as the ECB's recent LTRO facility that led to some "running for home"). This paper's previous version accordingly endogenized spillovers. Under one-way insurance, the principal generally, although not always, chooses to minimize its exposure to the risky country. By contrast, mutual insurance often leads countries to contractually maximize their cross-exposures.

Finally, the paper's modeling and implications focused on its international finance motivation. Its potential scope of applications however is broader. A corporation may guarantee a key supplier's debts by integrating it as its division, or by keeping it independent and promising to cover its liabilities. Banks may enter various kinds of contractual agreements, including credit lines, which imply varying degrees of soli-

[26]de Soyres (2013) derives the choice of the maturity structure of a sovereign that may be repeatedly bailed out. She finds that long-term debt arises for moderate liquidity needs. By contrast, short-term debt is more likely to be raised in situations of high liquidity needs. She then shows that the rescuer may assume short-term debt in order to avoid an impending default but will rather target long-term debt if the country does not want to repay because its total debt is too large.

[27]Recent work on debt monetization includes Aguiar et al (2013) and Corsetti and Dedola (2012).

darity. Individuals choose between giving a helping hand to members of their family (children) or friends facing financial straits and more formally standing surety for them, thereby facilitating their access to credit or housing. Integrating the specificities of these other contexts would be of much interest.

These and the many related topics on solidarity are left to future research.

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Appendix

Proof of Proposition 2

Consider the following program, consisting in maximizing the agent's utility subject to incentive and participation constraints:

$$\max \left\{ U_A = R(b+\tau) + \alpha(y - d_A^G - \hat{c}_A^G) + (1-\alpha)(-\hat{c}_A^B) \right\}, \tag{I}$$

subject to $\hat{c}^\omega \in \hat{C}$ for all ω, to the principal's and the market's participation constraints:

$$\begin{aligned} &-\tau - \alpha\left(t^G + \hat{c}_P^G\right) - (1-\alpha)\left(t^B + \hat{c}_P^B\right) \geq U_P^{LF} \\ &-b + \alpha\left(d_A^G + t^G\right) + (1-\alpha)t^B \geq 0, \end{aligned}$$

and to feasibility and incentive compatibility:

$$d_A^G \leq y \quad \text{and} \quad d_A^G + \hat{c}_A^G \leq \hat{c}_A^B.$$

Adding up the participation constraints and replacing in U_A yields

$$U_A \leq R[\alpha\left(d_A^G - \hat{c}_P^G\right) + (1-\alpha)\left(-\hat{c}_P^B\right) - U_P^{LF}] + \alpha\left(y - d_A^G - \hat{c}_A^G\right) + (1-\alpha)\left(-\hat{c}_A^B\right).$$

So let us maximize the RHS of this inequality subject to $(\hat{c}_A^\omega, c_A^\omega) \in \widehat{C}$ for all ω and to the agent's feasibility and incentive constraints. A quick inspection of the program shows that they should be no punishment in the good state of nature ($\hat{c}_A^G = \hat{c}_P^G = 0$, which is feasible), that punishment should not exceed what is necessary for incentive compatibility: $d_A^G = \hat{c}_A^B$, and that the principal should be minimally punished in the bad state: $\hat{c}_P^B = \phi(\hat{c}_A^B)$ where

$$\phi(\hat{c}_A) \equiv \min_{\{(\tilde{c}_A, \tilde{c}_P) \in \widehat{C} | \tilde{c}_A = \hat{c}_A\}} \{\hat{c}_P\}.$$

Note that $\phi' = r$ for $\hat{c}_A < C$ (and $\phi' = 1/r$ for $C < \hat{c}_A < (1+r)C$, but this range is irrelevant as $\hat{c}_A^B = d_A^G \leq y \leq C$).

Substituting, the upper bound $\widehat{U}_A$ is reached by solving the new program:

$$\widehat{U}_A = \max_{\{\hat{c}_A^B \leq y\}} \left\{ R\left[\alpha \hat{c}_A^B - (1-\alpha)\phi(\hat{c}_A^B) - U_P^{LF}\right] + \alpha y - \hat{c}_A^B \right\}.$$

Note that

$$\frac{\partial \widehat{U}_A}{\partial \hat{c}_A^B} = R[\alpha - (1-\alpha)\phi'] - 1$$

In the relevant range ($\hat{c}_A^B \le y \le C$), then

$$\frac{\partial \widehat{U}_A}{\partial \hat{c}_A^B} = R[\alpha - (1-\alpha)r] - 1,$$

which yields parts (i) and (ii) of the Proposition. ■

Proof of Proposition 3

Consider a simple debt contract, with reimbursement $d > 0$ and sanction $c > 0$ if d is not fully reimbursed. Suppose first that, at date 2, A and P agree on contingent transfers (t^B, t^G) such that there is no default in either state: $t^B \ge d$ and so $t^B - d > -c$. Furthermore, $t^G \ge t^B$ (otherwise $t^B - d > \max\{t^G - d, -c\}$). Thus, if the outcome is no default at all, the expected cost to P is at least d. On the other hand, P's utility is bounded below by $-rC$. We are thus led to consider two cases:

(i) if $C > y/r$, the highest debt that the principal may assume in both states is $d = rC > y$, enforced by sanction $c = C$. Indeed, for $d = rC$, the spillover cost is $d = rC$ in each state in which P does not bring support. A can thus offer P to transfer $t^G = t^B = d$; the principal exposes itself to a loss rC in both states of nature if he refuses the offer. Then $U_A = \alpha y + RrC$.

(ii) if $C < y/r$, then the highest such debt is $(1-\alpha)rC$ since d will be repaid by the agent in state G even in the absence of support by P.

The agent's maximal utility in the absence of default is:

$$U_A(d_L) = Rd_L + \alpha y \quad \text{where} \quad d_L \equiv \begin{cases} (1-\alpha)rC \text{ if } rC < y \\ rC \text{ if } rC > y \end{cases}$$

Suppose now that there is default only in state B. Then $d_H \equiv y + rc$ is the maximum debt that P is willing to help assume in state G, enforced by sanction $c \ge y$. For a sanction $c \in [y, C]$, the agent's utility is then $R[\alpha(y + rc)] - (1-\alpha)c$ and so the optimal c is equal to C if $R\alpha r > 1 - \alpha$. On the other hand if $R\alpha r < 1 - \alpha$, the agent is better off setting $c = y$ and obtaining $R\alpha(y + e) - (1-\alpha)y$.

Let $\kappa \equiv y/C$. When $R\alpha r > 1 - \alpha$, then

✓ if $r < \kappa$, $\quad R^{LF} \equiv \begin{cases} \dfrac{\alpha\kappa + (1-\alpha)}{\alpha\kappa + (2\alpha - 1)r} & \text{if} \quad \alpha\kappa + (2\alpha-1)r > 0 \\ +\infty & \text{otherwise} \end{cases}$

✓ if $r > \kappa$, $\quad R^{LF} \equiv \begin{cases} \dfrac{\alpha\kappa + (1-\alpha)}{\alpha\kappa - (1-\alpha)r} & \text{if} \quad \alpha\kappa + (\alpha - 1)r > 0 \\ +\infty & \text{otherwise} \end{cases}$

When $R\alpha r < 1 - \alpha$, then

✓ if $r < \kappa$, $\quad R^{LF} \equiv \begin{cases} \dfrac{1}{\alpha(1+r) - (1-\alpha)\dfrac{r}{\kappa}} & \text{if} \quad \alpha(1+r) - (1-\alpha)\dfrac{r}{\kappa} > 0 \\ +\infty & \text{otherwise} \end{cases}$

✓ if $r > \kappa$, $\quad R^{LF} \equiv \begin{cases} \dfrac{1}{\alpha(1+r) - \dfrac{r}{\kappa}} & \text{if} \quad \alpha(1+r) - \dfrac{r}{\kappa} > 0 \\ +\infty & \text{otherwise} \end{cases}$ ■

Finally, let us show that in some region of the parameter space, the simple debt contract is optimal. To show this, let us derive an upper bound on agent welfare,

$$U_A = (R-1)\left[\alpha d_A^G + (1-\alpha)d_A^B\right] + \alpha y + \left[\alpha t^G + (1-\alpha)t^B\right] - \left[\alpha c_A^G + (1-\alpha)c_A^B\right],$$

where d_A^ω is the agent's reimbursement to the market in state ω, t^ω the transfer from the principal and c_A^ω the sanction. Feasibility in state B requires that $d_A^B \leq t^B$. Incentive compatibility in state G requires that $d_A^G - t^G \leq d_A^G - t^G + c_A^G \leq d_A^B - t^B + c_A^B \leq c_A^B$. Finally, let $H \leq rC$ denote the maximum expected hardship that can be imposed upon the principal. Necessarily,

$$\alpha t^G + (1-\alpha)(t^B + rc_A^B) \leq H.$$

Adding up the feasibility and incentive constraints (with weights $1-\alpha$ and α) implies that

$$\alpha d_A^G + (1-\alpha)d_A^B \leq \alpha t^G + (1-\alpha)t^B + \alpha c_A^B.$$

And so

$$\begin{aligned} U_A &\leq (R-1)\alpha c_A^B + R[\alpha t^G + (1-\alpha)t^B] + \alpha y - (1-\alpha)c_A^B \\ &\leq (R-1)\alpha c_A^B + R[H - (1-\alpha)rc_A^B] + \alpha y - (1-\alpha)c_A^B \\ &\leq [R[\alpha - (1-\alpha)r] - 1]\, c_A^B + \alpha y + RH \\ &\leq \alpha y + RH \leq \alpha y + RrC \end{aligned}$$

for $R \leq R^*$. Thus, in case (i) above and for $R \leq R^*$, the simple debt contract is optimal in the class of all contracts.

Proof of Proposition 5

Constraints (4) and (5) in the text ensure that the two parties do not want to collude when the state is (G, G). Let us first discuss remaining collusion-proofness constraints, those in state (B, B) and in state (G, B) (or equivalently (B, G). In state (B, B), mimicking state (G, G) or state (G, B) is unfeasible if reimbursements are positive, which will be the case except in the trivial case in which no borrowing is optimal.

In state (G, B), two further constraints must be satisfied. First, $d_1 + [(\hat{c}_1^G + \hat{c}_1^B)/2] \leq \hat{c}_0$ is a sufficient condition for the absence of gains from trade from mimicking (B, B) (the necessary and sufficient condition is more complex as the collusive arrangement must respect the truthtelling requirement). Second, if $2d_2 \leq y$ and $d_1 + [(\hat{c}_1^G + \hat{c}_1^B)/2] > d_2 + \hat{c}_2$, there are potential gains from masquerading as state (G, G). A sufficient condition for collusion-proofness is therefore that either $2d_2 > y$ or that $d_1 + [(\hat{c}_1^G + \hat{c}_1^B)/2] \leq d_2 + \hat{c}_2$.

One can check that IL and JL contracts are indeed collusion proof. Because both contracts satisfy constraints (4) and (5), the countries' welfares cannot be improved in state (G, G). So consider state (G, B), say. State (G, G) cannot be mimicked under IL (because total income y is lower than $2d_2 = 2y$) and does not bring any increase in total surplus under JL (total reimbursement is y and there is no punishment under (G, B) and under (G, G)). Similarly, declaring (B, B) brings about a reduction in total surplus ($2\hat{c}_0 > 2d_1 + \hat{c}_1^G + \hat{c}_1^B$) in either case and so there is no possible gain from trade. Finally, state (B, B), with no available income, cannot be misrepresented if there is any reimbursement in the other states, which is the case.

Let us assume that borrowing $(p_1 d_1 + p_2 d_2)$ is strictly positive.

First, we show that the truthtelling constraint (1) must be binding. If it is not, then $\hat{c}_0 = d_2$. Suppose that (5) is not binding either; then $d_1 = y/2$ and $\hat{c}_1^G = \hat{c}_1^B = 0$. Either $(R-1)p_2 < p_0$ and then $d_2 = \hat{c}_0 = 0$ and then (1) is violated. Or (omitting non-generic cases) $(R-1)p_2 > p_0$ and then $d_2 = \hat{c}_0 = y$, violating (5).

So (5) must be binding if (1) is not. One can then rewrite the program as max $\{(R-1)(p_2 d_2 + p_1 d_1) - p_1(d_2 - d_1) - p_0 d_2\}$ subject to $d_2 \leq y$ and $2d_1 \leq y$. So $d_1 = y/2$. Either $(R-1)p_2 - p_1 - p_0 < 0$ and then $d_2 = 0$, which contradicts the assumption that (5) is binding; or $(R-1)p_2 - p_1 - p_0 > 0$ and then $d_2 = y$ and constraint (1) is violated as $\hat{c}_1^B(1+r)/2 = y/2$. We thus conclude that (1) must be binding.

Substituting (1) into the objective function and letting

$$A \equiv (R-1)\frac{p_1}{2} - p_0$$

and

$$B \equiv (R-1)\Big(p_2 - \frac{p_1 r}{2}\Big) - p_1\Big(\frac{1+r}{2}\Big),$$

the per-country utility becomes

$$U = A\hat{c}_0 + B\hat{c}_1^B$$

(i) $d_1 = d_2 = 0$ is optimal if $A < 0$ and $B < 0$.
(ii) Suppose that $A > 0$ and $B > 0$ (again we ignore non-generic cases for conciseness). Then maximizing sanctions (so as to maximize borrowing) is optimal and $d_2 = y = 2d_1$. Embodying (2) and (3) into (1), one must verify:

$$C\hat{c}_0 + D\hat{c}_1^B \leq p_2 y + \frac{p_1}{2} y \tag{1'}$$

where $C \equiv p_1/2$ and $D = p_2 - (p_1 r/2)$.

So either $A/C > B/D$ and then minimum weight must be put on $\hat{c}_1^B$ (relative to $\hat{c}_0$). Condition (5) is then binding, implying $\hat{c}_1^B = y/(1+r)$ and $\hat{c}_0$ then determined by (1′) satisfied with equality; the missing collusion-proofness constraints are then satisfied. Or $A/C < B/D$ and then a priori (4) is binding ($\hat{c}_0 = y$) and $\hat{c}_1^B$ is given by (1′) satisfied with equality ($\hat{c}_1^B = p_2 y/[p_2 - (p_1 r/2)]$). However, one of the missing collusion-proofness constraints is then violated (that specifying $d_1 + (1+r)\hat{c}_L^B/2 \leq \hat{c}_0$) and must be reintroduced. But the optimal contract is still a quasi-IL contract.[28] In either case, the optimum is a quasi-IL contract. Note that

$$\frac{A}{C} \geq \frac{B}{D} \iff p_0 p_2 - \frac{p_1^2}{4} \leq \frac{p_1}{2} r\left(p_0 + \frac{p_1}{2}\right)$$

$$\iff \left(\frac{1+r}{\gamma(1-\gamma)}\right)\left(\frac{1-\rho}{\rho}\right)^2 + \left(\frac{2r}{\gamma}\right)\left(\frac{1-\rho}{\rho}\right) \geq 4.$$

(iii) Suppose next that $A > 0 > B$, which can be rewritten as:

$$\frac{1+r}{\frac{2p_2}{p_1} - r} > R - 1 > \frac{2p_0}{p_1}$$

(which corresponds to r large and a low correlation index ρ). The fact that $0 > B$ calls for $\hat{c}_1^B = 0$. Constraint (5) must then be binding and so $d_2 = d_1 = y/2$.
Constraint (1) yields

$$\hat{c}_0 = \frac{p_2 + p_1}{p_1} y.$$

Provided that $\hat{c}_0 \leq \hat{c}$, then the optimal contract is the joint liability contract. ■

[28] In either case provided that the resulting values are smaller than $\hat{c}$. Otherwise one must add the constraint that $\hat{c}_0 \leq \hat{c}$ (respectively $\hat{c}_1^B \leq \hat{c}$).