Online Appendix for "Meritocracy and Homophily in Collegial Organizations"

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A Proof of Lemma 1

A.1 Value functions for majority and minority members with the canonical strategies

Let us consider the canonical strategies:

(i) Members of the majority (all) vote for the majority candidate if the latter is at least as talented as the minority candidate.

(ii) When the minority candidate is more talented, members of a type-X majority (all) vote for the majority candidate with probabilities $\{\sigma_X(M)\}_{M\in\{k,\dots,N-1\}}$ with $\sigma_X(k)\in\{0,1\}$ and $\sigma_X(M)=0$ if $M>k$.

Value function for a majority member. In this Subsection only, Let $V^-_{i,X}$ denote the expected value function conditional on the minority candidate being more talented, and $V^+_{i,X}$ denote the expected value function conditional on the complementary event.[1] The value function for a majority member writes for any $k\le M\le N-1$,[2],

$$V_{M,X}=xV^-_{M,X}+(1-x)V^+_{M,X} \tag{3}$$

$$\text{where}\quad\begin{cases} V^-_{M,X}=\sigma_X(M)\Big[b_X+\delta\Big(\dfrac{M}{N-1}V_{M,X}+\big(1-\dfrac{M}{N-1}\big)V_{M+1,X}\Big)\Big] \\ \qquad\qquad +(1-\sigma_X(M))\Big[s+\delta\Big(\dfrac{M-1}{N-1}V_{M-1,X}+\big(1-\dfrac{M-1}{N-1}\big)V_{M,X}\Big)\Big] \\ V^+_{M,X}=b_X+\dfrac{\overline{x}}{1-x}s+\delta\Big(\dfrac{M}{N-1}V_{M,X}+\Big(1-\dfrac{M}{N-1}\Big)V_{M+1,X}\Big) \end{cases}$$

[1] To alleviate the notation with respect to the text, we omit the superscript r,r' referring to the two groups' strategies, and use the superscript instead to decompose the value function depending on the current-period candidates draw (and thus recruit).

[2] Equation (3) applies even when $M=N-1$ as the majority size $M+1$ becomes irrelevant (its probability being nil).

With probability x, the (type-X) majority faces a trade-off between selecting a talented minority member (yielding payoff s) and picking the less talented majority candidate (yielding payoff b_X). With probability $1-x$, the majority candidate is at least as talented as the minority one, and the majority candidate brings average payoff $b_X + \overline{x}s/(1-x)$, where $\overline{x}/(1-x)$ is the conditional probability of that candidate's being talented. Recruiting a majority candidate when the majority has size M in period t yields an end-of-period majority size of $M+1$. From the perspective of a majority member, three events might then happen at the beginning of period $t+1$ before the vote takes place: *(i)* with probability $1/N$ (which is already embedded in the discount factor $\delta \equiv \delta_0(1-1/N)$), the majority member quits the organization, which gives him zero payoff; *(ii)* with probability M/N, *another* majority member quits, and thus the majority size decreases to M; *(iii)* with probability $(N-M-1)/N$, a minority member quits, and thus the majority size remains equal to $M+1$.

Value function for a minority member. If the majority recruits the majority candidate in period t, then at the beginning of period $t+1$: *(i)* with probability $1/N$, the minority member quits the organization, which gives her zero payoff; *(ii)* with probability $(M+1)/N$, a majority member quits, and thus the majority size decreases to M; *(iii)* with probability $(N-M-2)/N$, *another* minority member quits, and thus the majority size remains equal to $M+1$. The value function for a (type-X) minority member writes for any $k \leq M \leq N-2$:

$$V_{N-M-1,X} = xV^{-}_{N-M-1,X} + (1-x)V^{+}_{N-M-1,X} \tag{4}$$

$$\text{where } \begin{cases} V^{-}_{N-M-1,X} = \sigma_Y(M)\delta\left(\dfrac{M+1}{N-1}V_{N-M-1,X} + \left(1-\dfrac{M+1}{N-1}\right)V_{N-M-2,X}\right) \\ \qquad\qquad +(1-\sigma_Y(M))\left[s + b_X + \delta\left(\dfrac{M}{N-1}V_{N-M,X} + \left(1-\dfrac{M}{N-1}\right)V_{N-M-1,X}\right)\right] \\ V^{+}_{N-M-1,X} = \dfrac{\overline{x}}{1-x}s + \delta\left(\dfrac{M+1}{N-1}V_{N-M-1,X} + \left(1-\dfrac{M+1}{N-1}\right)V_{N-M-2,X}\right) \end{cases}$$

A.2 Continuation values with the canonical strategies

Let us begin with a useful result, which we will use repeatedly throughout our analysis. We index the canonical strategies by $r \in \{m, e\}$, where m stands for the meritocratic strategy and e for the basic-entrenchment one. To alleviate the notation, we omit the subscript $X \in \{A, B\}$ as we restrict our attention to a single group.

Lemma A.1. ***(Majority continuation values)*** *Fix $V_{k-1} \in \mathbb{R}$ (continuation value upon losing control) and consider the value function $(V_M^r(V_{k-1}))_{M \geq k}$ associated with the canonical strategy $r \in \{m, e\}$ given V_{k-1}. Then,*

(i) For $r = e$, the continuation value $V_M^e(V_{k-1})$ increases with $M \geq k$ and has decreasing differences (i.e., $u_M^e(V_{k-1}) \equiv V_{M+1}^e(V_{k-1}) - V_M^e(V_{k-1})$ decreases with $M \geq k$), strictly so if $x > 0$.

(ii) For $r = m$, if $V_k^m(V_{k-1}) \geq V_{k-1}$, the continuation value $V_M^m(V_{k-1})$ increases with $M \geq k$ and has decreasing differences (i.e., $u_M^m(V_{k-1}) \equiv V_{M+1}^m(V_{k-1}) - V_M^m(V_{k-1})$ decreases with $M \geq k$), strictly so if $V_k^m(V_{k-1}) > V_{k-1}$.

Proof. Let $r \in \{m, e\}$. By definition of the canonical strategies, for any $M \in \{k+1, ..., N-1\}$,

$$V_M^r(V_{k-1}) = (\overline{x} + x)s + (1-x)b + (1-x)\delta\left[\frac{M}{N-1}V_M^r(V_{k-1}) + \left(1 - \frac{M}{N-1}\right)V_{M+1}^r(V_{k-1})\right] + x\delta\left[\frac{M-1}{N-1}V_{M-1}^r(V_{k-1}) + \left(1 - \frac{M-1}{N-1}\right)V_M^r(V_{k-1})\right].$$

For $M = k$, the same recursive equation holds for the meritocratic strategy ($r = m$), while for the basic-entrenchment strategy ($r = e$),

$$V_k^e(V_{k-1}) = \overline{x}s + b + \delta\left[\frac{k}{N-1}V_k^e(V_{k-1}) + \left(1 - \frac{k}{N-1}\right)V_{k+1}^e(V_{k-1})\right].$$

Consequently, for any $M \in \{k+1, ..., N-1\}$, letting $u_M^r(V_{k-1}) \equiv V_{M+1}^r(V_{k-1}) - V_M^r(V_{k-1})$,

$$\left[1 - \delta x\left(1 - \frac{M}{N-1}\right) - \delta(1-x)\frac{M}{N-1}\right]u_M^r(V_{k-1}) = \delta x\frac{M-1}{N-1}u_{M-1}^r(V_{k-1}) + \delta(1-x)\left(1 - \frac{M+1}{N-1}\right)u_{M+1(V_{k-1})}^r. \tag{5}$$

Moreover, for $M = k$, the meritocratic strategy (still) yields (5), i.e.

$$\left[1 - \delta x\left(1 - \frac{k}{N-1}\right) - \delta(1-x)\frac{k}{N-1}\right]u_k^m(V_{k-1}) = \delta x\frac{k-1}{N-1}u_{k-1}^m(V_{k-1}) + \delta(1-x)\left(1 - \frac{k+1}{N-1}\right)u_{k+1}^m(V_{k-1}).$$

whereas the basic-entrenchment strategy yields

$$u_k^e(V_{k-1}) = x(s-b) + \delta(1-x)\left[\left(1 - \frac{k+1}{N-1}\right)u_{k+1}^e(V_{k-1}) + \frac{k}{N-1}u_k^e(V_{k-1})\right]. \quad (6)$$

We show the result by contradiction and by induction. Suppose $u_{N-2}^r(V_{k-1}) \leq 0$. Then, Equation (5) for $M = N-2$ implies

$$\left[1 - \delta\frac{x}{N-1} - \delta(1-x)\frac{N-2}{N-1}\right]u_{N-2}^r(V_{k-1}) = \delta x\frac{N-3}{N-1}u_{N-3}^r(V_{k-1})$$

Therefore, $u_{N-3}^r(V_{k-1}) \leq 0$ and $u_{N-3}^r(V_{k-1}) \leq u_{N-2}^r(V_{k-1})$. We then proceed by induction to show that for any $M \in \{k+1, ..., N-2\}$, $u_{M-1}^r(V_{k-1}) \leq u_M^r(V_{k-1}) \leq 0$. Suppose the result holds for all indices in $\{M+1, ..., N-2\}$. Then, (5) implies

$$\left[1 - \delta x\left(1 - \frac{M}{N-1}\right) - \delta(1-x)\frac{M}{N-1}\right]u_M^r(V_{k-1})$$
$$\geq \delta x\frac{M-1}{N-1}u_{M-1}^r(V_{k-1}) + \delta(1-x)\left(1 - \frac{M+1}{N-1}\right)u_M^r(V_{k-1})$$

i.e.,

$$\left[1 - \delta x\left(1 - \frac{M}{N-1}\right) - \delta(1-x)\frac{N-2}{N-1}\right]u_M^r(V_{k-1}) \geq \delta x\frac{M-1}{N-1}u_{M-1}^r(V_{k-1})$$

Consequently, $u_{M-1}^r(V_{k-1}) \leq u_M^r(V_{k-1}) \leq 0$. The result follows by induction. In particular, one has $u_k^r(V_{k-1}) \leq u_{k+1}^r(V_{k-1}) \leq 0$.

However, consider the basic-entrenchment strategy and suppose $x > 0$ (the case $x = 0$ is analogous). (6) then implies that

$$0 \geq \left[1 - \delta(1-x)\frac{N-2}{N-1}\right]u_k^e(V_{k-1}) \geq x(s-b) > 0,$$

which is a contradiction. Similarly, consider the meritocratic strategy and suppose that $V_k^m(V_{k-1}) > V_{k-1}$ (the weak inequality case is analogous). Using (5) in $M = k$ allows to extend the induction argument to show that $u_{N-2}^m \leq 0$ implies $u_{k-1}^m(V_{k-1}) \leq 0$, i.e. $V_k^m(V_{k-1}) \leq V_{k-1}$, which is a contradiction.

Therefore, for any $r \in \{m, e\}$, $u_{N-2}^r(V_{k-1}) > 0$. Using (5), one then has by induction that

$$u_k^r(V_{k-1}) > ... > u_{N-2}^r(V_{k-1}) > 0.$$

as was to be shown. □

A.3 Proof of Lemma 1

Let v (resp. w) denote the incremental value brought to a member of the majority by the minority (resp. majority) candidate. So $v \in \{0, s\}$, $w \in \{b, b+s\}$, and $v > w$ if and only if $(v, w) = (s, b)$ (otherwise $v < w$). Throughout the Online Appendix, we refer to the incremental value brought by current-period hires as a "flow payoff" (slighlty abusing vocabulary as this incremental value captures the discounted sum of present and future quality and homophily benefits, if any).

Let $\mathcal{C} \equiv \Big[0, ((\overline{x}+x)s + (1-x)b)/(1-\delta)\Big]$. All vectors of value functions $(V_k, .., V_{N-1})$ necessarily belong to $\mathcal{C}^k$ as for any $s \geq b$, $\mathbb{E}_{v,w}[\max(v, w)] = (\overline{x}+x)s + (1-x)b$. By construction, given any $V_{k-1} \in \mathcal{C}$, the majority faces an optimal control problem, and there exists a unique sequence of majority value functions $(V_k(V_{k-1}), ..., V_{N-1}(V_{k-1}))$ solving the Bellman equations:

$$\forall i \geq k, \qquad V_i = \mathbb{E}_{v,w}\Bigg[\max\Bigg\{v + \delta\bigg(\frac{i-1}{N-1}V_{i-1} + \bigg(1 - \frac{i-1}{N-1}\bigg)V_i\bigg), \\ w + \delta\bigg(\frac{i}{N-1}V_i + \bigg(1 - \frac{i}{N-1}\bigg)V_{i+1}\bigg)\Bigg\}\Bigg]$$

Hence, rewriting (1), the majority's choice at size M between two candidates with profiles (v, w) is determined by the following comparison:

$$v - w - \delta\left[\frac{M-1}{N-1}(V_M - V_{M-1}) + \left(1 - \frac{M}{N-1}\right)(V_{M+1} - V_M)\right] \lesseqgtr 0. \tag{7}$$

Given any $V_{k-1} \in \mathcal{C}$, the majority can always guarantee a sequence of value functions such that $V_M > V_{k-1}$ for any $M \geq k$, for instance by following the meritocratic strategy (making meritocratic recruitments at all majority sizes) as such a strategy yields a flow payoff equal to $\mathbb{E}_{v,w}[\max(v, w)] = (\overline{x}+x)s + (1-x)b \geq (1-\delta)V_{k-1}$ at all majority sizes.

Hence in particular, the solution to the Bellman equations given V_{k-1} satisfies $V_{k+1}(V_{k-1}) > V_{k-1}$, and thus for $M = k$, (7) writes as

$$v - w - \delta\frac{k-1}{N-1}(V_{k+1}(V_{k-1}) - V_{k-1}) \leq v - w.$$

Hence, it is never optimal for a majority with size k to recruit the minority candidate whenever $v < w$ (i.e. whenever the majority candidate is at least as talented as the minority one).

Fix $V_{k-1} \in \mathcal{C}$. Let us show that for any $V_{k-1} \in \mathcal{C}$, the majority's best response among pure Markov Perfect strategy is either meritocracy or basic entrenchment.

Consider the sequence of value functions $(V_M^{\mathrm{e}}(V_{k-1}))_{M\geq k}$ generated by the basic-entrenchment strategy given V_{k-1}: the sequence $(V_M^{\mathrm{e}}(V_{k-1}))_{M\geq k}$ is defined recursively by (5)-(6), i.e. satisfies

$$V_k^e(V_{k-1}) = \mathbb{E}[w] + \delta\left[\frac{k}{N-1}V_k^e(V_{k-1}) + \frac{k-1}{N-1}V_{k+1}^e(V_{k-1})\right]$$

and for any $M \geq k+1$,

$$\begin{aligned}V_M^{\mathrm{e}}(V_{k-1}) = \mathbb{E}_{v,w}[\max(v,w)] + \delta x\left[\frac{M-1}{N-1}V_{M-1}^{\mathrm{e}}(V_{k-1}) + \frac{N-M}{N-1}V_M^{\mathrm{e}}(V_{k-1})\right] \\ + \delta(1-x)\left[\frac{M}{N-1}V_M^{\mathrm{e}}(V_{k-1}) + \frac{N-M-1}{N-1}V_{M+1}^{\mathrm{e}}(V_{k-1})\right]\end{aligned}$$

Let us distinguish three cases, depending on whether $s - b - \delta\frac{k-1}{N-1}\Big(V_{k+1}^e(V_{k-1}) - V_{k-1}\Big)$ is strictly negative, strictly positive, or nil.

Case 1. Suppose the following inequality holds:

$$s - b - \delta\frac{k-1}{N-1}\Big(V_{k+1}^e(V_{k-1}) - V_{k-1}\Big) < 0. \tag{8}$$

Let us show that the sequence of majority value functions $(V_M^{\mathrm{e}}(V_{k-1}))_{M\geq k}$ solves the Bellman equations given V_{k-1}. By Lemma A.1, $u_M^{\mathrm{e}} \equiv V_{M+1}^{\mathrm{e}} - V_M^{\mathrm{e}} > 0$ and $u_{M+1}^e \leq u_M^e$ for all $M \geq k$. Hence, (7) implies that given the continuation values induced by the basic-entrenchment strategy, it is strictly optimal for the majority at any majority size $M \geq k$ to recruit its in-group candidate whenever he is at least as talented as the minority one (as then $w \geq v$).

Moreover, (8) and (7) imply that given the continuation values induced by the basic-entrenchment strategy, it is optimal for the majority at size $M = k$ to recruit the majority candidate even when he is less talented than the minority candidate. In addition, (6) together with the inequality $u_{k+1}^{\mathrm{e}} \leq u_k^{\mathrm{e}}$ imply that

$$\left[1 - \delta(1-x)\frac{N-2}{N-1}\right]u_k^{\mathrm{e}} \leq x(s-b)$$

and therefore, using again (6),

$$\delta\left(\frac{k-2}{N-1}u^{\mathrm{e}}_{k+1}+\frac{k}{N-1}u^{\mathrm{e}}_{k}\right)\le\frac{\delta x\frac{N-2}{N-1}}{1-\delta(1-x)\frac{N-2}{N-1}}(s-b)<s-b,$$

where the second inequality follows from $\delta<(N-1)/N$. Hence, by monotonicity of the sequence $(u^{\mathrm{e}}_M)_{M\ge k}$,[3] for any majority size $M\ge k+1$, (7) implies that, given the continuation values induced by the basic-entrenchment strategy, it is strictly optimal for the majority to recruit the minority candidate whenever she is more talented than the majority candidate.

Therefore, the sequence of majority value functions $(V^{\mathrm{e}}_M(V_{k-1}))_{M\ge k}$ solves the Bellman equations given V_{k-1}, and as the latter have a unique solution, $V_M(V_{k-1})=V^{\mathrm{e}}_M(V_{k-1})$ for any $M\ge k$. Identifying the strategies from the value functions (using (7)), if (8) holds, the majority's best response to V_{k-1} among pure Markov Perfect strategies is thus the basic-entrenchment strategy.

Case 2. Suppose the following inequality holds:

$$s-b-\delta\frac{k-1}{N-1}\Big(V^{e}_{k+1}(V_{k-1})-V_{k-1}\Big)>0, \tag{9}$$

To alleviate the notation, let

$$\Delta\equiv s-b-\delta\frac{k-1}{N-1}\Big(V^{e}_{k+1}(V_{k-1})-V_{k-1}\Big).$$

Consider the sequence of value functions $(V^{\mathrm{m}}_M(V_{k-1}))_{M\ge k}$ generated by the meritocratic strategy given V_{k-1}: the sequence $(V^{\mathrm{m}}_M(V_{k-1}))_{M\ge k}$ is defined recursively by (5) for any $M\ge k$, i.e. satisfies for all $M\ge k$

$$\begin{aligned}V^{\mathrm{m}}_M(V_{k-1})=\mathbb{E}_{v,w}[\max(v,w)]&+\delta x\left[\frac{M-1}{N-1}V^{\mathrm{m}}_{M-1}(V_{k-1})+\frac{N-M}{N-1}V^{\mathrm{m}}_{M}(V_{k-1})\right]\\&+\delta(1-x)\left[\frac{M}{N-1}V^{\mathrm{m}}_{M}(V_{k-1})+\frac{N-M-1}{N-1}V^{\mathrm{m}}_{M+1}(V_{k-1})\right]\end{aligned}$$

with $V^{\mathrm{m}}_{k-1}(V_{k-1})\equiv V_{k-1}$.

[3] As $u^{\mathrm{e}}_{M+1}\le u^{\mathrm{e}}_M$ for all $M\ge k+1$,

$$\delta\left(\frac{N-M-1}{N-1}u^{\mathrm{e}}_M+\frac{M-1}{N-1}u^{\mathrm{e}}_{M-1}\right)\le\delta\left(\frac{k-2}{N-1}u^{\mathrm{e}}_{k+1}+\frac{k}{N-1}u^{\mathrm{e}}_k\right)<s-b.$$

Then, using the recursive expressions of the continuation values induced by the meritocratic and basic-entrenchment strategies, for any $M \geq k+1$,

$$\begin{aligned}
&V^{\mathrm{m}}_M(V_{k-1}) - V^{\mathrm{e}}_M(V_{k-1}) \\
&\quad = \delta x \left[\frac{M-1}{N-1} \Big(V^{\mathrm{m}}_{M-1}(V_{k-1}) - V^{\mathrm{e}}_{M-1}(V_{k-1}) \Big) + \frac{N-M}{N-1} \Big(V^{\mathrm{m}}_M(V_{k-1}) - V^{\mathrm{e}}_M(V_{k-1}) \Big) \right] \\
&\qquad + \delta(1-x) \left[\frac{M}{N-1} \Big(V^{\mathrm{m}}_M(V_{k-1}) - V^{\mathrm{e}}_M(V_{k-1}) \Big) + \frac{N-M-1}{N-1} \Big(V^{\mathrm{m}}_{M+1}(V_{k-1}) - V^{\mathrm{e}}_{M+1}(V_{k-1}) \Big) \right]
\end{aligned}$$

and for $M = k$,

$$\begin{aligned}
&V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1}) \\
&\quad = x\Delta + \delta x \frac{k}{N-1} \Big(V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1}) \Big) \\
&\qquad + \delta(1-x) \left[\frac{k}{N-1} \Big(V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1}) \Big) + \frac{k-1}{N-1} \Big(V^{\mathrm{m}}_{k+1}(V_{k-1}) - V^{\mathrm{e}}_{k+1}(V_{k-1}) \Big) \right].
\end{aligned}$$

Using the recursive expressions of the $(V^m_M - V^e_M)_M$ yields that[4]

$$V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1}) > \dots > V^{\mathrm{m}}_{N-1}(V_{k-1}) - V^{\mathrm{e}}_{N-1}(V_{k-1}) > 0.$$

As a consequence, by the recursive expression of $V^{\mathrm{m}}_{k+1}(V_{k-1}) - V^{\mathrm{e}}_{k+1}(V_{k-1})$,

$$\left[1 - \delta(1-x) - \delta x \frac{k-1}{N-1} \right] \Big(V^{\mathrm{m}}_{k+1}(V_{k-1}) - V^{\mathrm{e}}_{k+1}(V_{k-1}) \Big) < \delta x \frac{k}{N-1} \Big(V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1}) \Big),$$

and thus by the recursive expression of $V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1})$,[5]

$$\left[1 - \delta(1-x) - \delta x \frac{k-1}{N-1} \right] \Big(V^{\mathrm{m}}_{k+1}(V_{k-1}) - V^{\mathrm{e}}_{k+1}(V_{k-1}) \Big) < \frac{\delta x \frac{k}{N-1}}{1 - \delta(1-x) - \delta x \frac{k}{N-1}} x\Delta.$$

Therefore,

$$\delta \frac{k-1}{N-1} \Big(V^{\mathrm{m}}_{k+1}(V_{k-1}) - V^{\mathrm{e}}_{k+1}(V_{k-1}) \Big) < \Delta,$$

[4] One may for instance proceed as in the proof of Lemma A.1 and suppose by contradiction that $V^{\mathrm{m}}_{N-1}(V_{k-1}) - V^{\mathrm{e}}_{N-1}(V_{k-1}) \leq 0$.

[5] Indeed, as $V^{\mathrm{m}}_{k+1}(V_{k-1}) - V^{\mathrm{e}}_{k+1}(V_{k-1}) < V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1})$,

$$V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1}) < x\Delta + \left(\delta x \frac{k}{N-1} + \delta(1-x) \right) \big(V^{\mathrm{m}}_k(V_{k-1}) - V^{\mathrm{e}}_k(V_{k-1}) \big).$$

and hence, by definition of Δ,

$$s - b - \delta \frac{k-1}{N-1}\Big(V^m_{k+1}(V_{k-1}) - V_{k-1}\Big) > 0. \tag{10}$$

Consequently, (10) and (7) imply that given the continuation values induced by the meritocratic strategy, it is optimal for the majority at size $M = k$ to recruit the minority candidate whenever she is more talented than the majority candidate.

By construction, $V_{k-1} \in \mathcal{C}$, and thus $V_{k-1} \leq \mathbb{E}_{v,w}[\max(v,w)]/(1-\delta)$. Therefore, $V^m_k(V_{k-1}) \geq V_{k-1}$. Consequently, by Lemma A.1, $u^m_{k-1} \geq u^m_k \geq ... \geq u^m_{N-2} \geq 0$. Hence, (7) implies that given the continuation values induced by the meritocratic strategy, it is indeed strictly optimal at any majority size $M \geq k$ for the majority to recruit the majority candidate whenever he is at least a talented as the minority candidate (as then $v > w$). Moreover, (10), the monotonicity of the sequence $(u^m_M)_{M \geq k-1}$, and (7) imply that given the continuation values induced by the meritocratic strategy, it is strictly optimal at any majority size $M \geq k$ for the majority to recruit the minority candidate whenever she is more talented than the minority candidate.

Therefore, the sequence of value functions $(V^{\mathrm{m}}_M(V_{k-1}))_{M \geq k}$ generated by the meritocratic strategy given V_{k-1} solves the Bellman equations, and as the latter have a unique solution, $V_i(V_{k-1}) = V^{\mathrm{m}}_i(V_{k-1})$ for any $i \geq k$. Identifying the strategies from the value functions (using (7)), if (9) holds, the majority's best response to V_{k-1} among pure Markov Perfect strategies is thus the meritocratic strategy.

Case 3: Suppose that the following equality holds:

$$s - b - \delta \frac{k-1}{N-1}\Big(V^e_{k+1}(V_{k-1}) - V_{k-1}\Big) = 0,$$

i.e. the majority is indifferent between $\sigma(k) = 0$ and $\sigma(k) = 1$. Then, the above arguments imply that the sequences of value functions induced by the basic-entrenchment and meritocratic strategies both solve the Bellman equations. Identifying the strategies from the value functions (using (7)), the majority has then two best responses (yielding the same continuation values): meritocracy and basic entrenchment.

B Proof of Lemma 2

The result for $N = 4$ derives from straightforward computations.[6] We assume in the following that $N \geq 6$.

Proof of (i). Consider first the basic-entrenchment strategies. For any $M \in \{k-1, ..., N-2\}$, let V_M^e denote the continuation value function with the basic-entrenchment strategy for both group, and let $u_M^e \equiv V_{M+1}^e - V_M^e$. As argued in the proof of Lemma A.1 (see Online Appendix A.2), the recursive expressions of the continuation value function yield for any $M \in \{k+1, ..., N-2\}$,

$$\left[1 - \delta x \left(1 - \frac{M}{N-1}\right) - \delta(1-x)\frac{M}{N-1}\right] u_M^e = \delta x \frac{M-1}{N-1} u_{M-1}^e + \delta(1-x)\left(1 - \frac{M+1}{N-1}\right) u_{M+1}^e, \tag{5}$$

and for $M = k$,

$$u_k^e = x(s-b) + \delta(1-x)\left[\left(1 - \frac{k+1}{N-1}\right) u_{k+1}^e + \frac{k}{N-1} u_k^e\right] \tag{6}$$

Therefore, the result follows straightforwardly from claim (i) in Lemma A.1.

Consider now the meritocratic strategies. Let let V_M^m denote the continuation value function with the basic-entrenchment strategy for both group, and let $u_i^m \equiv V_{i+1}^m - V_i^m$ for any $i \in \{1, ..., N-2\}$. By construction, Equation (5) holds for any $M \in \{k, ..., N-2\}$. We use the same argument as in the proof of Lemma A.1 (by contradiction and by induction).

Hence, assume by contradiction that $u_{N-2}^m \leq 0$. Then, by induction, this implies that for any $M \in \{k, ..., N-2\}$, $u_{M-1}^m \leq u_M^m \leq 0$, and thus in particular $u_{k-1}^m \leq u_k^m \leq 0$.

Consider now u_1^m. Writing the recursive expression of the value function in $M \in$

[6] Using (3) and (4), the basic-entrenchment strategies yield

$$\left[1 - \frac{2\delta}{3}(1-x)\right](V_3^{\mathrm{e}} - V_2^{\mathrm{e}}) = x(s-b)$$

and thus $V_1^{\mathrm{e}} = \overline{x}s/(1-\delta) < (b + \overline{x}s)/(1-\delta) < V_2^{\mathrm{e}} < V_3^{\mathrm{e}}$. Similarly for the meritocratic equilibrium:

$$\begin{cases} \left[1 - \frac{x\delta}{3} - \frac{2\delta}{3}(1-x)\right](V_3^{\mathrm{m}} - V_2^{\mathrm{m}}) = \frac{x\delta}{3}(V_2^{\mathrm{m}} - V_1^{\mathrm{m}}) \\ \left[1 - \delta(1-x)\right](V_2^{\mathrm{m}} - V_1^{\mathrm{m}}) = (1-2x)b + \delta\frac{(1-x)}{3}(V_3^{\mathrm{m}} - V_2^{\mathrm{m}}) \end{cases}$$

and thus $V_1^{\mathrm{m}} < V_2^{\mathrm{m}} < V_3^{\mathrm{m}}$, and $V_2^{\mathrm{m}} - V_1^{\mathrm{m}} > V_3^{\mathrm{m}} - V_1^{\mathrm{m}}$.

$\{k+1,...,N-1\}$ (thus writing V^m_{N-M-1} as a function of V^m_{N-M-2}, V^m_{N-M-1} and V^m_{N-M}), and then substracting the expression in $N-M-1$ from the expression in $N-M$ (and rearranging) yields for any $M \in \{k+1,...,N-2\}$:

$$\left[1-\delta(1-x)\frac{M}{N-1}-\delta x\left(1-\frac{M}{N-1}\right)\right]u^m_{N-M-1} = \delta(1-x)\left(1-\frac{M+1}{N-1}\right)u^m_{N-M-2}+\delta x\frac{M-1}{N-1}u^m_{N-M} \quad (11)$$

and in particular,

$$\left[1-\delta\frac{x}{N-1}-\delta(1-x)\frac{N-2}{N-1}\right]u^m_1 = \delta x\frac{N-3}{N-1}u^m_2$$

By the usual induction argument using (11), $u^m_1 > 0$ implies $0 < u^m_1 < u^m_2 < ... < u^m_{k-2} < u^m_{k-1}$, which contradicts $u^m_{k-1} \leq 0$. Hence $u^m_1 \leq 0$ and the same induction argument now implies $0 \geq u^m_1 \geq u^m_2 \geq ... \geq u^m_{k-2} \geq u^m_{k-1}$.

However, substracting Equation (3) in k and Equation (4) in $k-1$ yields after rearranging:

$$\left[1-\delta(1-x)\right]u^m_{k-1} = (1-2x)b+\delta(1-x)\left[\left(1-\frac{k}{N-1}\right)u^m_k+\left(1-\frac{k+1}{N-1}\right)u^m_{k-2}\right] \quad (12)$$

The contradiction then obtains by summing the above equation together with Equations (5) and (11) over all indices $i \in \{1,...,N-2\}$ (and rearranging), which gives:

$$\left(1-\delta\frac{x}{N-1}-\delta(1-x)\right)(u^m_1+u^m_{N-2})+\left(1-\delta\right)\sum_{i=2}^{N-3}u^m_i = (1-2x)b > 0$$

If $x < 1/2$, this contradicts the fact that $u^m_i \leq 0$ for all $i \in \{1,...,N-2\}$. Therefore, $u^m_{N-2} > 0$. By induction, Equation (5) then implies that $0 < u^m_{N-2} < ... < u^m_{k-1}$.[7]

The proof of claim *(ii)* relies on the same induction arguments as the proof of *(i)* and is thus omitted for the sake of brevity.

Claim *(iii)* again derives from arguments analogous to the ones used in the proofs of *(i)* and *(ii)*. The result is obvious with *(i)* for the meritocratic equilibrium. The result for the basic-entrenchment equilibrium obtains by considering the sequence $V^e_i - V^e_{N-1-i}$

[7] If $x = 1/2$, the same argument yields that $V^m_i = V^m_{i+1}$ for all i.

for $i \in \{k, ..., N-2\}$ and using (3)-(4).[8]

Suppose by contradiction that $V_k^e - V_{k-1}^e \leq 0$. This implies with (5)-(11) that $V_{k+1}^e - V_{k-2}^e < V_k^e - V_{k-1}^e \leq 0$, and thus by induction that $V_{N-1}^e - V_1^e < V_{N-2}^e - V_1^e < ... < V_k^e - V_{k-1}^e \leq 0$, which contradicts $V_{N-1}^e \geq V_{N-2}^e$ as shown above.[9] Hence, $V_k^e - V_{k-1}^e > 0$. If $V_{k+1}^e - V_{k-2}^e \leq 0$, the same contradiction is reached again as then $V_{N-1}^e - V_1^e < V_{N-2}^e - V_1^e < ... < V_{k+1}^e - V_{k-2}^e \leq 0$ (Again, one could sum over $i \in \{k+1, ..., N-2\}$ the analogues of (5)-(11) and note that the RHS is positive whenever $x \leq 1/2$.) The result obtains by induction: for any $i \in \{k, ..., N-2\}$, $V_i^e - V_{N-1-i}^e > 0$.

C Proof of Proposition 2

Lemma 1 implies that in any pure-strategy Markov Perfect equilibrium (if any), each group plays either the meritocratic, or the basic-entrenchment strategy. Proposition 1 then implies that in the symmetric case ($b_A = b_B = b$), both groups must play the same trategy, i.e. any pure-strategy Markov Perfect equilibrium (if any) is symmetric, and either meritocratic or basically entrenched. This establishes claim (i).

Let us now characterize the existence regions of these two equilibria.

C.1 A necessary and sufficient condition for existence

Lemma C.1. *There exists no profitable one-shot deviation from the meritocratic strategy (resp. the basic-entrenchment strategy) at any majority size and for any realization of the candidates' vertical types if and only if there exists no profitable deviation when $M = k$ and the minority candidate is strictly more talented.*

[8] Namely, using that, for $M \in \{k+1, ..., N-3\}$,

$$\left[1 - \delta(1-x)\frac{M}{N-1} - \delta x\left(1 - \frac{M-1}{N-1}\right)\right](V_M^e - V_{N-M-1}^e) - (1-2x)b + \frac{\delta}{N-1}\left[(1-x)u_{N-M-2}^e + xu_{N-M-1}^e\right]$$
$$= \delta(1-x)\left(1 - \frac{M}{N-1}\right)(V_{M+1}^e - V_{N-M-2}^e) + \delta x\frac{M-1}{N-1}(V_{M-1}^e - V_{N-M}^e)$$

while for $M = k$ and $M = N-2$,

$$\left[1 - \delta\frac{k}{N-1}\right](V_k^e - V_{k-1}^e) = b - \frac{\delta}{N-1}u_{k-2}^e + \delta\left(1 - \frac{k}{N-1}\right)(V_{k+1}^e - V_{k-2}^e),$$
$$\left[1 - \delta(1-x)\frac{N-2}{N-1} - \delta x\frac{2}{N-1}\right](V_{N-2}^e - V_1^e) = (1-2x)b - \frac{\delta x}{N-1}u_1 + \delta\frac{(1-x)}{N-1}(V_{N-1}^e - V_1^e) + \delta x\frac{N-3}{N-1}(V_{N-3}^e - V_2^e)$$

The result follows, as we know from above that in the basic-entrenchment equilibrium, $u_i^e \leq 0$ for any $i \leq k-2$.

[9] Another contradiction would be reached by summing as above the analogues of (5)-(11) and noting that the RHS is positive whenever $x \leq 1/2$.

Proof. We know by Lemma 1 that a group's best response when it has the majority (group size $i \geq k$) is either meritocracy or basic entrenchment. The two strategies coincide at all majority sizes and all profiles of current-period candidates, except at majority size $M = k$ when the minority candidate is strictly more talented than the majority candidate. The result follows. □

C.2 Existence regions

Let us introduce the notation for transition probabilities for group sizes *from the perspective of (in- or out-) group members*: for any horizontal group within the organization, we refer to the transition probability from group sizes i to j *from an (in- or out-) group member's perspective* as the probability that the group's size goes from i to j *conditional on the given member being still a member of the organization then.*[10]

Namely, for regime $\mathrm{r} \in \{\mathrm{e}, \mathrm{m}\}$, let $p^{\mathrm{r}}_{i,j}$ be the one-period transition probability from an in-group member's perspective, i.e., the probability that a group size moves from $i \geq 1$ to $j \geq 1$ from one period to another conditional on the given group member still being in the organization in the following period (which has probability $(N-1)/N$). As an illustration, for any $M > k$ and in the basic-entrenchment equilibrium ($\mathrm{r} = \mathrm{e}$), $p^{\mathrm{r}}_{i,j}$ is the probability from a majority member's perspective that the majority size moves from $i \geq k$ to $j \geq k$ from one period to another conditional on the majority member still being in the organization in the following period. Consequently,

$$\begin{cases} p^{\mathrm{e}}_{M,M+1} = (1-x)\left(1 - \dfrac{M+1}{N}\right)\dfrac{N}{N-1} = (1-x)\left(1 - \dfrac{M}{N-1}\right) \\ p^{\mathrm{e}}_{M,M} = \left[(1-x)\dfrac{M}{N} + x\left(1 - \dfrac{M}{N}\right)\right]\dfrac{N}{N-1} = (1-x)\dfrac{M}{N-1} + x\left(1 - \dfrac{M-1}{N-1}\right) \\ p^{\mathrm{e}}_{M,M-1} = x\dfrac{M-1}{N}\dfrac{N}{N-1} = x\dfrac{M-1}{N-1} \\ p^{\mathrm{e}}_{M,j} = 0 \qquad \text{if } |M-j| > 1. \end{cases} \tag{13}$$

and

$$\begin{cases} p^{\mathrm{e}}_{k,k+1} = \left(1 - \dfrac{k+1}{N}\right)\dfrac{N}{N-1} = 1 - \dfrac{k}{N-1} \\ p^{\mathrm{e}}_{k,k} = \dfrac{k}{N}\dfrac{N}{N-1} = \dfrac{k}{N-1} \\ p^{\mathrm{e}}_{k,k-1} = 0 \end{cases} \tag{14}$$

[10] We use the same convention as for the value functions, denoting by i a post-departure, pre-vote size.

For any $i, j \in \{1, ..., N-1\}$ and $t \geq 0$, let $\pi^{\mathrm{r}}_{i,j}(t)$ be the t-period transition probability from i to j in regime r from an in-group member's perspective, i.e. the probability that starting from i, the group's size is equal to j after t periods conditional on the group member still being in the organization. Hence, for any $i, j \in \{1, ..., N-1\}$ and $t \geq 1$,

$$\pi^{\mathrm{r}}_{i,j}(t+1) = p^{\mathrm{r}}_{j-1,j}\pi^{\mathrm{r}}_{i,j-1}(t) + p^{\mathrm{r}}_{j,j}\pi^{\mathrm{r}}_{i,j}(t) + p^{\mathrm{r}}_{j+1,j}\pi^{\mathrm{r}}_{i,j+1}(t),$$

and $\pi^{\mathrm{r}}_{i,j}(1) = p^{\mathrm{r}}_{i,j}$.

Similarly, let $\hat{p}^{\mathrm{r}}_{i,j}$ be the transition probability from an *out-group* member's perspective, i.e. the probability that a group's size moves from $i \geq k$ to j from one period to another conditional on the other group member still being in the organization in the following period (which has probability $(N-1)/N$). As an illustration, for any $M > k$ and in the basic-entrenchment equilibrium, $\hat{p}^{\mathrm{e}}_{i,j}$ is the transition probability from a minority member's perspective, and thus

$$\begin{cases} \hat{p}^{\mathrm{e}}_{M,M+1} = (1-x)\left(1 - \dfrac{M+2}{N}\right)\dfrac{N}{N-1} = (1-x)\left(1 - \dfrac{M+1}{N-1}\right) \\ \hat{p}^{\mathrm{e}}_{M,M} = \left[(1-x)\dfrac{M+1}{N} + x\left(1 - \dfrac{M+1}{N}\right)\right]\dfrac{N}{N-1} = (1-x)\dfrac{M+1}{N-1} + x\left(1 - \dfrac{M}{N-1}\right) \\ \hat{p}^{\mathrm{e}}_{M,M-1} = x\dfrac{M}{N}\dfrac{N}{N-1} = x\dfrac{M}{N-1} \\ \hat{p}^{\mathrm{e}}_{M,j} = 0 \qquad \text{if } |M-j| > 1. \end{cases} \tag{15}$$

and

$$\begin{cases} \hat{p}^{\mathrm{e}}_{k,k+1} = \left(1 - \dfrac{k+2}{N}\right)\dfrac{N}{N-1} = 1 - \dfrac{k+1}{N-1} \\ \hat{p}^{\mathrm{e}}_{k,k} = \dfrac{k+1}{N}\dfrac{N}{N-1} = \dfrac{k+1}{N-1} \\ \hat{p}^{\mathrm{e}}_{k,k-1} = 0 \end{cases} \tag{16}$$

For any $i, j \in \{1, ..., N-1\}$, and $t \geq 0$, let $\hat{\pi}^{\mathrm{r}}_{i,j}(t)$ be the t-period transition probability from i to j in regime r from an out-group member's perspective, i.e. the probability that starting from i, the group's size is equal to j after t periods conditional on the out-group member still being in the organization. Hence, for any $i, j \in \{1, ..., N-1\}$ and $t \geq 1$,

$$\hat{\pi}^{\mathrm{r}}_{i,j}(t+1) = \hat{p}^{\mathrm{r}}_{j-1,j}\hat{\pi}^{\mathrm{r}}_{i,j-1}(t) + \hat{p}^{\mathrm{r}}_{j,j}\hat{\pi}^{\mathrm{r}}_{i,j}(t) + \hat{p}^{\mathrm{r}}_{j+1,j}\hat{\pi}^{\mathrm{r}}_{i,j+1}(t)$$

and $\pi^{\mathrm{r}}_{i,j}(1) = p^{\mathrm{r}}_{i,j}$.

For the meritocratic equilibrium, transition probabilities are given by (13) for in-group members, and by (15) for out-group members at all group sizes ($i \in \{1, ..., N-1\}$).

Note that because probabilities sum to 1,

$$\begin{cases} \left(\sum_{i=k+1}^{N-1} \hat{\pi}^{\mathrm{e}}_{k,i}(t)\right) - \left(\sum_{i=k+1}^{N-1} \pi^{\mathrm{e}}_{k+1,i}(t)\right) = -\left(\hat{\pi}^{\mathrm{e}}_{k,k}(t) - \pi^{\mathrm{e}}_{k+1,k}(t)\right) \\ \left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k+1,i}(t)\right) - \left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k-1,i}(t)\right) = -\left[\left(\sum_{i=1}^{k-1} \pi^{\mathrm{m}}_{k+1,i}(t)\right) - \left(\sum_{i=1}^{k-1} \pi^{\mathrm{m}}_{k-1,i}(t)\right)\right] \end{cases} \tag{17}$$

C.2.1 Proof of claims (ii) and (iii)

We now turn to the statement of the existence result. Building on Lemma C.1, let us examine the case in which the majority is tight ($M = k$) and the minority candidate is more talented.

Necessary and sufficient condition for existence of the meritocratic equilibrium. Leaving control considerations aside, choosing the less-deserving majority candidate when the majority is tight involves a cost $s - b$. To evaluate the impact of a potential switch of control, which occurs with conditional probability $(k-1)/(N-1)$, note that in a meritocratic equilibrium, the present discounted expected quality of future appointees does not depend on the allocation of control. The only impact of the change in control is thus linked to homophily benefits when the two candidates equally talented (which has probability $1-2x$), as control allows one to select the in-group candidate. So, a necessary condition of existence of a meritocratic equilibrium is:

$$s - b \geq \delta \frac{k-1}{N-1}(1-2x) b \sum_{t=0}^{+\infty} \delta^t \left[\left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k+1,i}(t)\right) - \left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k-1,i}(t)\right)\right],$$

and the meritocratic equilibrium exists only if

$$\frac{s}{b} \geq \rho^{\mathrm{m}} \equiv 1 + \delta \frac{k-1}{N-1}(1-2x) \sum_{t=0}^{+\infty} \delta^t \left[\left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k+1,i}(t)\right) - \left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k-1,i}(t)\right)\right]$$

Lemma C.1 implies that this condition is in fact also sufficient: as intuitive, deviations from meritocracy are less appealing further away from a tight majority size, i.e. from immediate control considerations.

Necessary and sufficient condition for existence of the basic-entrenchment equilibrium. Choosing the less talented majority candidate yields a direct payoff loss $s - b$. If the majority has size k, then with probability $(k-1)/(N-1)$, the surrendering of control translates into a permanent loss of homophily benefits whenever the two candidates are equally talented, which has probability $1 - 2x$. This cost is equal to

$$\frac{\delta}{1-\delta}(1-2x)b$$

Moreover, because the new majority will itself be basically entrenched, i.e. always voting for its own candidate whenever the majority is tight, the surrendering of control entails an additional loss of homophily benefit proportional to $2xb$ whenever the majority is tight, along with the difference in homophily benefits associated with meritocratic decisions, i.e. choosing a talented minority candidate instead of an untalented majority candidate, at any majority size $M \geq k+1$. The latter would seem unwarranted as the two groups then agree on the decision to pick the more talented candidate; its existence comes from the fact that transition probabilities depend on one's perspective. Put together, these two terms add up to

$$\delta\frac{k-1}{N-1}2xb\sum_{t=0}^{+\infty}\delta^t\pi^{\mathrm{e}}_{k+1,k}(t) - \delta\frac{k-1}{N-1}xb\sum_{t=0}^{+\infty}\delta^t\left[\left(\sum_{i=k+1}^{N-1}\hat{\pi}^{\mathrm{e}}_{k,i}(t)\right) - \left(\sum_{i=k+1}^{N-1}\pi^{\mathrm{e}}_{k+1,i}(t)\right)\right]$$

Another way to interpret the homophily payoff terms consists in noticing that the expected per-period payoff of a majority (resp. minority) member is equal to $(1-x)b$ (resp. xb) whenever the majority is not tight ($M \geq k+1$), while it is equal to b (resp. 0) when majority is tight ($M = k$).

Finally, because the new majority is itself basically entrenched, and since the shift in control implies that perspectives change, the surrendering of control yields a differential quality payoff equal to

$$\begin{aligned}\delta\frac{k-1}{N-1}(\overline{x}+x)s\sum_{t=0}^{+\infty}\delta^t\left[\left(\sum_{i=k+1}^{N-1}\hat{\pi}^{\mathrm{e}}_{k,i}(t)\right) - \left(\sum_{i=k+1}^{N-1}\pi^{\mathrm{e}}_{k+1,i}(t)\right)\right] \\ + \delta\frac{k-1}{N-1}\overline{x}s\sum_{t=0}^{+\infty}\delta^t\left(\hat{\pi}^{\mathrm{e}}_{k,k}(t) - \pi^{\mathrm{e}}_{k+1,k}(t)\right)\end{aligned}$$

So overall a necessary condition for the existence of the basic-entrenchment equilibrium is

$$
\begin{aligned}
b - s \geq \delta \frac{k-1}{N-1}(\overline{x}+x)s \sum_{t=0}^{+\infty} \delta^t \left[\left(\sum_{i=k+1}^{N-1} \hat{\pi}^{\mathrm{e}}_{k,i}(t) \right) - \left(\sum_{i=k+1}^{N-1} \pi^{\mathrm{e}}_{k+1,i}(t) \right) \right] \\
+ \delta \frac{k-1}{N-1} \overline{x} s \sum_{t=0}^{+\infty} \delta^t \left(\hat{\pi}^{\mathrm{e}}_{k,k}(t) - \pi^{\mathrm{e}}_{k+1,k}(t) \right) - \frac{k-1}{N-1} \frac{\delta}{1-\delta}(1-2x)b \\
- \delta \frac{k-1}{N-1} 2xb \sum_{t=0}^{+\infty} \delta^t \pi^{\mathrm{e}}_{k+1,k}(t) + \delta \frac{k-1}{N-1} xb \sum_{t=0}^{+\infty} \delta^t \left[\left(\sum_{i=k+1}^{N-1} \hat{\pi}^{\mathrm{e}}_{k,i}(t) \right) - \left(\sum_{i=k+1}^{N-1} \pi^{\mathrm{e}}_{k+1,i}(t) \right) \right]
\end{aligned}
$$

Let Inequality (18) be the inequality:

$$
1 + \delta \frac{k-1}{N-1} x \sum_{t=0}^{+\infty} \delta^t \left(\pi^{\mathrm{e}}_{k+1,k}(t) - \hat{\pi}^{\mathrm{e}}_{k,k}(t) \right) > 0. \tag{18}
$$

Define ρ^{e} as

$$
\rho^{\mathrm{e}} \equiv \begin{cases} \dfrac{1 + \frac{k-1}{N-1}\frac{\delta}{1-\delta}(1-2x) + \delta \frac{k-1}{N-1} x \sum_{t=0}^{+\infty} \delta^t \left(\pi^{\mathrm{e}}_{k+1,k}(t) + \hat{\pi}^{\mathrm{e}}_{k,k}(t) \right)}{1 + \delta \frac{k-1}{N-1} x \sum_{t=0}^{+\infty} \delta^t \left(\pi^{\mathrm{e}}_{k+1,k}(t) - \hat{\pi}^{\mathrm{e}}_{k,k}(t) \right)} & \text{if (18) holds,} \\ +\infty \qquad \text{otherwise.} \end{cases}
$$

Hence, the basic-entrenchment equilibrium exists only if $s/b \leq \rho^{\mathrm{e}}$. As the series term in (18) is negative for all t (see Lemma C.2 below), the basic-entrenchment equilibrium might exist for all values of s and b for δ sufficiently close to 1, and thus we set $\rho^{\mathrm{e}} = +\infty$ if (18) fails. Nonetheless, we show that for a positive rate of time preference (which we assumed) – i.e. $\delta < (N-1)/N$ –, the basic-entrenchment equilibrium exists only on a finite interval: $\rho^{\mathrm{e}} < +\infty$ (see Section C.2.3 for the proof of this result).

Lemma C.1 yields that this necessary condition is also sufficient. Hence, the basic-entrenchment (resp. meritocratic) equilibrium exists if and only if $s/b \leq \rho^{\mathrm{e}}$ (resp. $s/b \geq \rho^{\mathrm{m}}$).

Lastly, we show that the cutoffs ρ^{e} and ρ^{m} satisfy the following inequalities:[11]

$$
1 \leq 1 + \delta \frac{k-1}{N-1}(1-2x) \leq \rho^{\mathrm{m}} \leq 1 + \frac{\delta}{1-\delta}\frac{k-1}{N-1}(1-2x) < \rho^{\mathrm{e}} < +\infty \tag{19}
$$

The upper and lower bounds on ρ^{m} may be decomposed as follows: $(1-2x)$ is the probability of a homophily benefit from control, $(k-1)/(N-1)$ the (conditional) probability

[11] The proof that $\rho^{\mathrm{e}} < +\infty$ is delayed to Section C.2.3.

of losing the majority when its end-of-period size is k, while δ (resp. $\delta/(1-\delta)$) are the time-discounted weights corresponding to a transient (resp. permanent) loss of control.[12]

The bounds on ρ^{e} and ρ^{m} in Inequality (19) derive from the following lemma.

Lemma C.2. *For all $t \geq 0$,*

(i) $\pi^{e}_{k+1,k}(t) \leq \hat{\pi}^{e}_{k,k}(t)$

(ii) $\sum_{i \geq k} \pi^{m}_{k+1,i}(t) \geq \sum_{i \geq k} \pi^{m}_{k-1,i}(t)$

Proof. We use a result relying on the properties of monotone Markov chains.[13]
(i) Define the process $M(t)$ (resp. $\hat{M}(t)$) as the probability distribution over majority sizes $\{k, ..., N-1\}$ from a majority (resp. minority) member's perspective. Hence, the i-th component of $M(t)$ is the probability (from the perspective of a majority member) that the majority be of size $k-1+i$ at period t. In particular, if at time 0 the majority is known to have size $k+1$, then $M(0) = (0, 1, 0, ..., 0)$, and at any later time t, $M(t) = \left(\pi^{\mathrm{e}}_{k+1,k}(t), ..., \pi^{\mathrm{e}}_{k+1,N-1}(t)\right)$. Similarly, if at time 0 the majority is known to have size k, then $\hat{M}(0) = (1, 0, , ..., 0)$, and at any later time t, $\hat{M}(t) = \left(\hat{\pi}^{\mathrm{e}}_{k,k}(t), ..., \hat{\pi}^{\mathrm{e}}_{k,N-1}(t)\right)$.

Let P (resp. $\hat{P}$) be the stochastic matrix associated with the process $M(t)$ (resp. $\hat{M}(t)$). As a consequence, for any $i, j \in \{1, ..., k\}$,

$$P_{ij} = p^{\mathrm{e}}_{k+i-1,k+j-1}, \qquad \text{and} \qquad \hat{P}_{ij} = \hat{p}^{\mathrm{e}}_{k+i-1,k+j-1}$$

We first note that for any $i > i'$ and any $j^* \in \{1, ..., k\}$, $\sum_{j \geq j^*} P_{ij} \geq \sum_{j \geq j^*} P_{i'j}$, i.e. $P_{i\cdot}$ *stochastically dominates* $P_{i'\cdot}$ whenever $i > i'$. Hence, P is *stochastically monotone*, and by the same argument, so is $\hat{P}$.

We then note that P and $\hat{P}$ are stochastically comparable, with $P_{i\cdot}$ stochastically dominating $\hat{P}_{i\cdot}$ for any $i \in \{1, ..., k\}$. Furthermore, the process $M(t)$ starts from the initial state $M(0) = (0, 1, 0, ...)$ which stochastically dominates the initial state of the process $\hat{M}(t)$, that is $\hat{M}(0) = (1, 0, , 0, ...)$.

Hence, a standard argument implies that for any $t > 0$, the distribution $M(t)$ stochastically dominates the distribution $\hat{M}(t)$ (see for instance Theorem 3.31 in Kijima 1997).[14]

[12] Note that ρ^{m} reaches its upper bound as x goes to 0. In the limit, it is equal to $1 + \frac{\delta}{1-\delta}\frac{k-1}{N-1}$, which is intuitive: the majority weights the current-period payoff $s-b$ against the constant homophily loss in future periods due to the permanent loss of control (times its probability of occurrence $(k-1)/(N-1)$).

[13] See, e.g., Kijima, M. (1997). "Monotone Markov Chains". In: *Markov Processes for Stochastic Modeling.* Springer, Boston, MA. https://doi.org/10.1007/978-1-4899-3132-0_3.

[14] A sketch of the proof is as follows. Proceed by induction on t. The result for $t = 0$ holds as the

In particular, we have that for any $t > 0$,

$$\sum_{i=k+1}^{N-1} \pi^{\mathrm{e}}_{k+1,i}(t) \geq \sum_{i=k+1}^{N-1} \hat{\pi}^{\mathrm{e}}_{k,i}(t),$$

which is equivalent to: $\pi^{\mathrm{e}}_{k+1,k}(t) \leq \hat{\pi}^{\mathrm{e}}_{k,k}(t)$.

(ii) The second inequality can be shown with the same technique as the one used in the proof of claim (i), by considering the process of one's successive in-group sizes in the meritocratic equilibrium, either starting from the initial state $k+1$ or $k-1$. Indeed, the same conditions are satisfied, as (a) both processes (of probability distribution over one's successive in-group sizes) share the same transition matrix[15] which is stochastically monotone, and (b) the initial state with mass 1 in $k+1$ stochastically dominates the initial state with mass 1 in $k-1$. Hence, the stochastic-comparison argument applies, yielding that the process of one's in-group size starting from $k+1$ stochastically dominates at any time $t \geq 0$ the process starting from $k-1$, and thus in particular,

$$\sum_{i \geq k} \pi^{\mathrm{m}}_{k+1,i}(t) \geq \sum_{i \geq k} \pi^{\mathrm{m}}_{k-1,i}(t).$$

□

C.2.2 Proof of claim (iv)

The result derives from the explicit expressions of the existence thresholds together with Lemma C.2. Indeed, by Lemma C.2, for all $t \geq 0$,

$$\pi^{\mathrm{e}}_{k+1,k}(t) - \hat{\pi}^{\mathrm{e}}_{k,k}(t) \leq 0, \qquad \text{and} \qquad \left(\sum_{i \geq k} \pi^{\mathrm{m}}_{k+1,i}(t)\right) - \left(\sum_{i \geq k} \pi^{\mathrm{m}}_{k-1,i}(t)\right) \geq 0$$

Using term-by-term differentiation of the series yields the result: $\partial\rho^{\mathrm{m}}/\partial\delta_0 \geq 0, \partial\rho^{\mathrm{e}}/\partial\delta_0 \geq 0$ for all $\delta_0 \in [0,1)$. Moreover, using term-by-term differentiation of the series for ρ^{m} and

initial state $M(0) = (0,1,0,...)$ stochastically dominates the initial state $\hat{M}(0) = (1,0,,0,...)$. Suppose that $M(t)$ stochastically dominates $\hat{M}(t)$. Then, since P stochastically dominates $\hat{P}$, we have that $\hat{M}(t)P$ stochastically dominates $\hat{M}(t)\hat{P}$. Since P is stochastically monotone, $M(t)P$ stochastically dominates $\hat{M}(t)P$. Thus, by transitivity, $M(t)P$ stochastically dominates $\hat{M}(t)\hat{P}$. In other words, $M(t+1)$ stochastically dominates $\hat{M}(t+1)$, which concludes the proof.

[15]Namely, the matrix P^{m} with components $P_{ij} = p^{\mathrm{m}}_{i,j}$ for any $i,j \in \{1,...,N-1\}$.

explicit computations for ρ^{e} yields

$$\left.\frac{\partial \rho^{\mathrm{m}}}{\partial \delta_0}\right|_{\delta_0=0} = \frac{k-1}{N}(1-2x) \qquad \text{and} \qquad \left.\frac{\partial \rho^{\mathrm{e}}}{\partial \delta_0}\right|_{\delta_0=0} = \frac{k-1}{N}.$$

C.2.3 Basic entrenchment exists only on a finite interval ($\rho^{\mathrm{e}} < \infty$)

We show in this section that $\rho^{\mathrm{e}} < \infty$.[16] The result immediately follows from the explicit expression of ρ^{e} for $k = 2$. Hence, let $k \geq 3$. Let us stress that this result (for general k) is not obvious as strategic complementarity could a priori induce the existence of the basic-entrenchment equilibrium even for arbitrarily large s/b. Checking that $\rho^{\mathrm{e}} < \infty$ thus requires some computations, in particular as the majority size has different transition probabilities from the perspective of a majority member and from the one of a minority member (due to a member's conditioning on still being a member in the next periods).

Let V_i^{e} denote the value function in the basic-entrenchment equilibrium, and define as before $u_i^{\mathrm{e}} \equiv V_{i+1}^{\mathrm{e}} - V_i^{\mathrm{e}}$. Fix $s > 0$. For any $i \in \{1, ..., N-2\}$, u_i^{e} is continuous with respect to $b \in [0, +\infty)$.

The (one-shot) deviation differential payoff from basic entrenchment to meritocracy in $M = k$ is equal to

$$s - b + \delta\frac{k-1}{N-1}(V_{k-1}^{\mathrm{e}} - V_{k+1}^{\mathrm{e}}) = s - b - \delta\frac{k-1}{N-1}(u_{k-1}^{\mathrm{e}} + u_k^{\mathrm{e}})$$

Fix $b = 0$. If the above payoff is strictly positive for $b = 0$, then by continuity, it must be so on a neighbourhood of 0. Hence, there would exist $\overline{\rho} > 0$ such that for any $s/b > \overline{\rho}$, there exists a strictly profitable deviation from basic entrenchment to meritocracy, which would yield the result: $\rho^{\mathrm{e}} < \infty$. We thus show that for $b = 0$:

$$s - \delta\frac{k-1}{N-1}(u_{k-1}^{\mathrm{e}} + u_k^{\mathrm{e}}) > 0 \tag{20}$$

[16] The proof also yields that $\rho^{\mathrm{e}\dagger}|_{s^\dagger > b} < \infty$ (thus in particular for $x^\dagger \geq 1/2$), where $\rho^{\mathrm{e}\dagger}$ defined in Proposition 11 (see Section 4.4.1).

Using (3)-(4) and (5)-(11), the above inequality can be written as

$$\frac{\delta x \dfrac{k-1}{N-1}}{1-\delta x-\delta(1-x)\left[\dfrac{k+1}{N-1}+\dfrac{k-2}{N-1}a_{k+1}\right]} \tag{21}$$
$$\times\left(1-\frac{\delta x\dfrac{k}{N-1}}{1-\delta(1-x)\left[\dfrac{k}{N-1}+\dfrac{k-2}{N-1}b_{k+1}\right]}-\frac{\delta x\dfrac{k-2}{N-1}}{1-\delta(1-x)\left[\dfrac{k+1}{N-1}+\dfrac{k-3}{N-1}b_{k+2}\right]}\right)<1$$

where the vectors $(a_{k+l})_{l=1}^{k-2}$, $(b_{k+l})_{l=1}^{k-2}$ are defined recursively by

$$\begin{cases} a_{k+l}=\dfrac{\delta x\dfrac{k+l}{N-1}}{1-\delta(1-x)\left[\dfrac{k+l+1}{N-1}+\dfrac{k-l-2}{N-1}a_{k+l+1}\right]-\delta x\dfrac{k-l-1}{N-1}} \\ a_{N-2}=\dfrac{\delta x\dfrac{N-2}{N-1}}{1-\delta(1-x)-\delta\dfrac{x}{N-1}} \end{cases}$$

and

$$\begin{cases} b_{k+l}=\dfrac{\delta x\dfrac{k+l-1}{N-1}}{1-\delta(1-x)\left[\dfrac{k+l}{N-1}+\dfrac{k-l-2}{N-1}b_{k+l+1}\right]-\delta x\dfrac{k-l-1}{N-1}} \\ b_{N-2}=\dfrac{\delta x\dfrac{N-3}{N-1}}{1-\delta(1-x)\dfrac{N-2}{N-1}-\delta\dfrac{x}{N-1}} \end{cases}$$

Indeed, computations using (3)-(4) and (5)-(11) for the basic entrenchment equilibrium, give that:

$$\begin{cases} \left[1-\delta(1-x)\dfrac{k+1}{N-1}-\delta x\right]\left(V^{\mathrm{e}}_{k+1}-V^{\mathrm{e}}_{k-1}\right) \\ \qquad =xs+\delta(1-x)\dfrac{k-2}{N-1}\left(V^{\mathrm{e}}_{k+2}-V^{\mathrm{e}}_{k-2}\right)-\delta x\dfrac{k}{N-1}u^{\mathrm{e}}_{k}+\delta x\dfrac{k-2}{N-1}u^{\mathrm{e}}_{k-2} \\ V^{\mathrm{e}}_{k+2}-V^{\mathrm{e}}_{k-2}=a_{k+1}\left(V^{\mathrm{e}}_{k+1}-V^{\mathrm{e}}_{k-1}\right) \\ u^{\mathrm{e}}_{k+1}=b_{k+1}u^{\mathrm{e}}_{k} \\ u^{\mathrm{e}}_{k-3}=b_{k+2}u^{\mathrm{e}}_{k-2} \end{cases}$$

and thus, by rearranging,[17] (20) is equivalent to (21).

We thus show that for any $x \in (0,1)$ and $\delta \in [0,(N-1)/N)$, inequality (21) is satisfied.[18]. By construction, $(a_{k+l})_{l=1}^{k-2}$ and $(b_{k+l})_{l=1}^{k-2}$ are increasing with l, and for any l, $b_{k+l} < a_{k+l} < 1$. Moreover, for any l, a_{k+l} and b_{k+l} are increasing with respect to x and δ.[19] Therefore, the term on the first line (resp. second line) in (21) is strictly increasing (resp. decreasing) with respect to x and δ.

Using the inequality $b_{k+1} < b_{k+2} < 1$, a sufficient condition for (21) to be satisfied is

$$\begin{aligned} &\delta x \frac{k-1}{N-1}\left(1-\delta x\frac{N-2}{N-1}-\delta(1-x)\left[\frac{k}{N-1}+\frac{k-2}{N-1}b_{k+1}\right]\right) \\ &\quad / \left[\left(1-\delta(1-x)\left[\frac{k}{N-1}+\frac{k-2}{N-1}b_{k+1}\right]\right)\left(1-\delta x-\delta(1-x)\left[\frac{k+1}{N-1}+\frac{k-2}{N-1}a_{k+1}\right]\right)\right] < 1 \end{aligned} \tag{22}$$

or equivalently,

$$\begin{aligned} &\delta x \frac{k-1}{N-1}\left(1-\delta x\frac{N-2}{N-1}-\delta(1-x)\left[\frac{k}{N-1}+\frac{k-2}{N-1}b_{k+1}\right]\right) \\ &\quad - \left(1-\delta(1-x)\left[\frac{k}{N-1}+\frac{k-2}{N-1}b_{k+1}\right]\right)\left(1-\delta x-\delta(1-x)\left[\frac{k+1}{N-1}+\frac{k-2}{N-1}a_{k+1}\right]\right) < 0 \end{aligned} \tag{23}$$

The above inequalities are strictly stronger than (21) for any $x \in (0,1)$, and coincide with (21) in $x = 1$.

We now show that for any $x \in [0,1]$, (i) the LHS in (23) increases with δ over

[17]Using in particular that (3)-(4) imply:

$$\begin{cases} u^{\mathrm{e}}_{k} = xs + \delta(1-x)\left[\left(1-\frac{k+1}{N-1}\right)u^{\mathrm{e}}_{k+1} + \frac{k}{N-1}u^{\mathrm{e}}_{k}\right] \\ u^{\mathrm{e}}_{k-2} = -xs + \delta(1-x)\left[\frac{k+1}{N-1}u^{\mathrm{e}}_{k-2} + \left(1-\frac{k+2}{N-1}\right)u^{\mathrm{e}}_{k-3}\right] \end{cases}$$

[18]The case $x \geq 1/2$ is equivalent to the homogamic-evaluation-capability setting with $x^{\dagger} \geq 1/2$. Indeed, the homogamic-evaluation-capability equivalent of (20) is:

$$\frac{\delta x \frac{k-1}{N-1}}{1-\delta x^{\dagger}-\delta(1-x^{\dagger})\left[\frac{k+1}{N-1}+\frac{k-2}{N-1}a^{\dagger}_{k+1}\right]} \times \left(1 - \frac{\delta x^{\dagger}\frac{k}{N-1}}{1-\delta(1-x^{\dagger})\left[\frac{k}{N-1}+\frac{k-2}{N-1}b^{\dagger}_{k+1}\right]} - \frac{\delta x^{\dagger}\frac{k-2}{N-1}}{1-\delta(1-x^{\dagger})\left[\frac{k+1}{N-1}+\frac{k-3}{N-1}b^{\dagger}_{k+2}\right]}\right) < \frac{x}{x^{\dagger}}$$

with the corresponding families $(a^{\dagger}_{k+l})_{l=1}^{k-2}$, $(b^{\dagger}_{k+l})_{l=1}^{k-2}$ defined as before by replacing x with $x^{\dagger}$.

[19]These results can be shown by downward induction starting from $l = N-2$.

$[0, (N-1)/N]$, and (ii) its maximum (thus with $\delta = (N-1)/N$) is strictly negative.

(i) In order to alleviate the notation, let C_a and C_b be defined as

$$C_a \equiv \frac{k+1}{N-1} + \frac{k-2}{N-1}a_{k+1}, \qquad \text{and} \qquad C_b \equiv \frac{k}{N-1} + \frac{k-2}{N-1}b_{k+1}$$

Since $b_{k+1} < a_{k+1} < 1$, we have that $C_b < C_a < 1$. Using a downward induction argument on the sequences $(a_{k+l})_l$ and $(b_{k+l})_l$ yields that $\partial a_{k+1}/\partial\delta > \partial b_{k+1}/\partial\delta$.[20] As a consequence,

$$\begin{aligned}\phi(\delta) &\equiv \frac{\partial a_{k+1}}{\partial\delta}\Big[1-\delta(1-x)C_b\Big] + \frac{\partial b_{k+1}}{\partial\delta}\left[1-\delta(1-x)C_a - \delta x\left(1+\frac{k-1}{N-1}\right)\right]\\ &\geq \frac{\partial b_{k+1}}{\partial\delta}\left[2-\delta(1-x)(C_a+C_b) - \delta x\left(1+\frac{k-1}{N-1}\right)\right] > 0\end{aligned}$$

Denoting by $\varphi(\delta)$ the partial derivative of the LHS in (23) with respect to δ, we have after rearranging:

$$\begin{aligned}\varphi(\delta) = {} & x\left(1+\frac{k-1}{N-1}\right) + (1-x)(C_a+C_b)\\ & - 2\delta\left[x(1-x)\left(1+\frac{k-1}{N-1}\right)C_b + (1-x)^2C_aC_b + x^2\frac{k-1}{N-1}\frac{N-2}{N-1}\right]\\ & + \delta(1-x)\frac{k-2}{N-1}\left(\frac{\partial a_{k+1}}{\partial\delta}\Big[1-\delta(1-x)C_b\Big] + \frac{\partial b_{k+1}}{\partial\delta}\left[1-\delta(1-x)C_a-\delta x\left(1+\frac{k-1}{N-1}\right)\right]\right)\end{aligned}$$

---

[20]The result follows from the observation that

$$\frac{\partial a_{N-2}}{\partial\delta} = \frac{x\dfrac{N-2}{N-1}}{\left(1-\delta(1-x)-\delta\dfrac{x}{N-1}\right)^2} > \frac{x\dfrac{N-3}{N-1}}{\left(1-\delta(1-x)\dfrac{N-2}{N-1}-\delta\dfrac{x}{N-1}\right)^2} = \frac{\partial b_{N-2}}{\partial\delta}$$

and for any $l \in \{1, ..., k-3\}$,

$$\begin{aligned}\frac{\partial a_{k+l}}{\partial\delta} &= \frac{x\dfrac{k+l}{N-1} + \delta^2x(1-x)\dfrac{k+l}{N-1}\dfrac{k-l-2}{N-1}\dfrac{\partial a_{k+l+1}}{\partial\delta}}{\left(1-\delta(1-x)\left[\dfrac{k+l+1}{N-1}+\dfrac{k-l-2}{N-1}a_{k+l+1}\right]-\delta x\dfrac{k-l-1}{N-1}\right)^2}\\ &> \frac{x\dfrac{k+l-1}{N-1} + \delta^2x(1-x)\dfrac{k+l-1}{N-1}\dfrac{k-l-2}{N-1}\dfrac{\partial b_{k+l+1}}{\partial\delta}}{\left(1-\delta(1-x)\left[\dfrac{k+l}{N-1}+\dfrac{k-l-2}{N-1}b_{k+l+1}\right]-\delta x\dfrac{k-l-1}{N-1}\right)^2} = \frac{\partial b_{k+l}}{\partial\delta}\end{aligned}$$

Let $\psi(\delta) \equiv \varphi(\delta) - \delta(1-x)\dfrac{k-2}{N-1}\phi(\delta)$. We then note that $\psi(\delta) \geq 0$ [21], and therefore, $\varphi(\delta) > 0$ for any $x \in [0,1]$. Consequently, the LHS in (23) is strictly increasing with respect to δ, and thus reaches its maximum over $[0,(N-1)/N]$ in $\delta = (N-1)/N$.

(ii) We now let $\delta = (N-1)/N$ and show that the LHS in (23) with $\delta = (N-1)/N$ is strictly negative. Indeed, the latter then writes as

$$\begin{aligned}
LHS \equiv\ & x\frac{k-1}{N}\left(1 - x\frac{N-2}{N} - (1-x)\left[\frac{k}{N} + \frac{k-2}{N}b_{k+1}\right]\right) \\
& - \left(1 - (1-x)\left[\frac{k}{N} + \frac{k-2}{N}b_{k+1}\right]\right)\left(1 - x\frac{N-1}{N} - (1-x)\left[\frac{k+1}{N} + \frac{k-2}{N}a_{k+1}\right]\right) \\
=\ & x\frac{k-1}{N}\left(\frac{1}{N} + (1-x)\frac{k-2}{N}(a_{k+1} - b_{k+1})\right) \\
& - \left(\frac{k+1}{N} - \frac{1-x}{N} - (1-x)\frac{k-2}{N}b_{k+1}\right)\left(1 - x\frac{N-1}{N} - (1-x)\left[\frac{k+1}{N} + \frac{k-2}{N}a_{k+1}\right]\right)
\end{aligned}$$

where b_{k+1} and a_{k+1} are evaluated in $\delta = (N-1)/N$. Using that $b_{k+1} < 1$, we get after rearranging that

$$\begin{aligned}
LHS \leq\ & x\frac{k-1}{N}\left(\frac{1}{N} + (1-x)\frac{k-2}{N}(a_{k+1} - b_{k+1})\right) \\
& - \left(\frac{2}{N} + x\frac{k-1}{N}\right)\left(1 - x\frac{N-1}{N} - (1-x)\left[\frac{k+1}{N} + \frac{k-2}{N}a_{k+1}\right]\right) \\
=\ & -\frac{2}{N^2} - (1-x)\frac{2}{N}\frac{k-2}{N}\Big[1 - a_{k+1}\Big] - x(1-x)\frac{k-1}{N}\frac{k-2}{N}\Big[1 - 2a_{k+1} + b_{k+1}\Big]
\end{aligned}$$

Hence, a sufficient condition for the LHS in (23) to be strictly negative is that $1 - 2a_{k+1} + b_{k+1} > 0$. This actually holds,[22] which concludes the proof.

---

[21]Indeed, the expressions of ϕ and φ yield after rearranging:

$$\begin{aligned}
\psi(\delta) =\ & x\left(1 + \frac{k-1}{N-1}\right) + (1-x)(C_a + C_b) - 2\delta\left[x(1-x)\left(1 + \frac{k-1}{N-1}\right)C_b + (1-x)^2 C_a C_b + x^2\frac{k-1}{N-1}\frac{N-2}{N-1}\right] \\
=\ & x\left[1 + \frac{k-1}{N-1} - \delta(1-x)\left(1 + \frac{k-1}{N-1}\right)C_b - \delta x\left(\frac{N-2}{N-1}\right)^2\right] \\
& + (1-x)\left[\Big(C_a - \delta x C_b - \delta(1-x)C_a C_b\Big) + \left(C_b - \delta x\frac{k-1}{N-1}C_b - \delta(1-x)C_a C_b\right)\right] \geq 0
\end{aligned}$$

where the last inequality stems from the fact that $k/(N-1) < C_b < C_a < 1$.

[22]The argument is as follows. One first notes that since for any $l \in \{1,...,k-2\}$, $\partial a_{k+l}/\partial\delta \geq \partial b_{k+l}/\partial\delta > 0$, the term $[1 - 2a_{k+l} + b_{k+l}]$ is strictly bounded below by its value for $\delta = (N-1)/N$. The rest of the argument derives from downward induction showing the result for any l with $\delta = (N-1)/N$.

D Proof of Proposition 3

We first show the result for majority members. For any $i \in \{k, ..., N-1\}$, let $v_i \equiv V_i^{\text{m}} - V_i^{\text{e}}$. By construction, for any $i \geq k+1$, the recursive expressions of V_i^{m} and V_i^{e} yield:

$$\left[1 - \delta(1-x)\frac{i}{N-1} - \delta x\left(1 - \frac{i-1}{N-1}\right)\right]v_i = \delta(1-x)\left(1 - \frac{i}{N-1}\right)v_{i+1} + \delta x\frac{i-1}{N-1}v_{i-1}, \tag{24}$$

while for $i = k$,

$$v_k = \Delta + \delta\left[\frac{k}{N-1}v_k + \left(1 - \frac{k}{N-1}\right)v_{k+1}\right]$$

where $\Delta \equiv x(s-b) + \delta x\frac{k-1}{N-1}\left(V_{k-1}^{\text{m}} - V_{k+1}^{\text{m}}\right) > 0$, this last inequality stemming from $s/b > \rho^{\text{m}}$. Hence,

$$\left[1 - \delta\frac{k}{N-1}\right]v_k = \Delta + \delta\left(1 - \frac{k}{N-1}\right)v_{k+1} \tag{25}$$

Assume by contradiction that $v_{N-1} \leq 0$. Then, Equation (24) for $i = N-1$ implies that $v_{N-2} \leq v_{N-1} \leq 0$, and thus by induction that $v_k \leq v_{k+1} \leq ... \leq v_{N-1} \leq 0$. However,

Explicit computations yield that for $\delta = (N-1)/N$,

$$\left[1 - 2a_{N-2} + b_{N-2}\right] = \frac{(1-x)\frac{2}{N^2}}{\left(1 - (1-x)\frac{N-1}{N} - \frac{x}{N}\right)\left(1 - (1-x)\frac{N-2}{N} - \frac{x}{N}\right)} \geq 0$$

Then, for any $l \in \{1, ..., k-3\}$, the term $[1 - 2a_{k+l} + b_{k+l}]$ with $\delta = (N-1)/N$ has the same sign as

$$\begin{aligned}
&\left(1 - (1-x)\left[\frac{k+l+1}{N} + \frac{k-l-2}{N}a_{k+l+1}\right] - x\frac{k-l-1}{N}\right)\left(1 - (1-x)\left[\frac{k+l}{N} + \frac{k-l-2}{N}b_{k+l+1}\right] - x\frac{k-l-1}{N}\right) \\
&\quad - 2x\frac{k+l}{N}\left(1 - (1-x)\left[\frac{k+l}{N} + \frac{k-l-2}{N}b_{k+l+1}\right] - x\frac{k-l-1}{N}\right) \\
&\quad + x\frac{k+l-1}{N}\left(1 - (1-x)\left[\frac{k+l+1}{N} + \frac{k-l-2}{N}a_{k+l+1}\right] - x\frac{k-l-1}{N}\right) \\
&= (1-x)\left[\frac{k-l-1}{N} - \frac{k-l-2}{N}a_{k+l+1}\right]\left[\frac{k-l}{N} - x\frac{k-l-2}{N} - (1-x)\frac{k-l-2}{N}b_{k+l+1}\right] + x(1-x)\frac{k+l-1}{N}\frac{2}{N} \\
&\quad + x(1-x)\frac{k+l-1}{N}\frac{k-l-2}{N}\left[1 - 2a_{k+l+1} + b_{k+l+1}\right] \\
&\geq x(1-x)\frac{k+l-1}{N}\frac{k-l-2}{N}\left[1 - 2a_{k+l+1} + b_{k+l+1}\right].
\end{aligned}$$

Equation (25) then yields $0 \geq (1-\delta)v_k \geq \Delta > 0$, which is a contradiction. Hence, $v_{N-1} > 0$, and by induction using Equation (24), $v_k > v_{k+1} > ... > v_{N-1} > 0$, which concludes the proof.

The result for minority members follows from analogous computations, noting that for $M \geq k+1$, meritocracy and basic entrenchment yield the same flow payoffs and transition probabilities, while in $M = k$ (minority size $N - M - 1 = k - 1$),

$$V^{\mathrm{m}}_{k-1} - V^{\mathrm{e}}_{k-1} = x(s+b) + x\delta\left[\frac{k-2}{N-1}(V^{\mathrm{m}}_{k-1} - V^{\mathrm{m}}_{k-2}) + \frac{k}{N-1}(V^{\mathrm{m}}_{k} - V^{\mathrm{m}}_{k-1})\right] + \delta\left[\frac{k-2}{N-1}(V^{\mathrm{m}}_{k-2} - V^{\mathrm{e}}_{k-2}) + \frac{k+1}{N-1}(V^{\mathrm{m}}_{k-1} - V^{\mathrm{e}}_{k-1})\right],$$

where $V^{\mathrm{m}}_{k-2} \leq V^{\mathrm{m}}_{k-1} \leq V^{\mathrm{m}}_{k}$ by Lemma 2. Hence, $V^{\mathrm{m}}_{i} \geq V^{\mathrm{e}}_{i}$ for any $i \leq k-1$.

Lastly, as a by-product of the proof, we have that the gap between the value functions in the two equilibria, $V^{\mathrm{m}}_{i} - V^{\mathrm{e}}_{i}$, decreases as the majority size moves further away from $M = k$.[23]

E Complements on Section 2.2.2

We first describe the ergodic distributions of majority sizes. Since, by convention, payoffs in a given period accrue after the current-period vote and before the next-period departure, we are interested in the *end-of-period* distribution of majority sizes. Let us index the end-of-period majority size by $i \in \{k, ..., N\}$. Let ν^{r}_i denote the ergodic probability of majority size i at the end of a period in regime $\mathrm{r} \in \{\mathrm{e}, \mathrm{m}\}$ (see Online Appendix E for their expressions). The next Lemma shows that basic entrenchment leads to larger majorities, as intuitive:

Lemma E.1. ***(End-of-period ergodic distributions)*** *The probability distribution $\{\nu^e_i\}$ strictly first-order stochastically dominates $\{\nu^m_i\}$.*

Ergodic quality. By taking the fixed point of the dynamic equation for (expected) aggregate quality in the ergodic state,[24] aggregate per-period expected quality S^{r}, $\mathrm{r} \in$

[23] The result for $i \leq k-1$ can be established using analogous computations to the case $i \geq k$, relying on the recursive expressions of the minority value functions.

[24] The aggregate quality at the end of period $t+1$ is the aggregate quality at the end of period t minus the (expected) loss due to a member's departure, plus the (expected) contribution of the recruited candidate. For the meritocratic equilibrium,

$$S^{\mathrm{m}}_{t+1} = \frac{N-1}{N} S^{\mathrm{m}}_t + (N-1)[\overline{x} + x]\tilde{s},$$

$\{\mathrm{e}, \mathrm{m}\}$ is

$$\begin{cases} S^{\mathrm{m}} \equiv N(N-1)\Big(\overline{x}+x\Big)\tilde{s} \\ S^{\mathrm{e}} \equiv N(N-1)\Big[\nu^{\mathrm{e}}_{k+1}\dfrac{k+1}{N}\overline{x}+\Big(1-\nu^{\mathrm{e}}_{k+1}\dfrac{k+1}{N}\Big)\Big(\overline{x}+x\Big)\Big]\tilde{s} \end{cases}$$

Unsurprisingly, the ergodic quality of a meritocratic organization exceeds that of a basically-entrenched one:

$$S^{\mathrm{m}} - S^{\mathrm{e}} = N(N-1)\nu^{\mathrm{e}}_{k+1}\frac{k+1}{N}x\tilde{s} > 0.$$

Ergodic homophily benefit. A basically-entrenched organization always dominates a meritocratic one in terms of ergodic aggregate homophily benefit ($B^{\mathrm{m}} < B^{\mathrm{e}}$): *(a)* the function $\Big(i \mapsto i(i-1)+(N-i)(N-i-1)\Big)$ is strictly increasing for $i \in \{k, ..., 2k\}$, and *(b)* the probability distribution $\{\nu^{\mathrm{e}}_i\}$ strictly first-order stochastically dominates $\{\nu^{\mathrm{m}}_i\}$ from Lemma E.1.

E.1 Proof of Lemma E.1

We show successively that:

(i) $\nu^{\mathrm{e}}_k = 0$

(ii) for any $i \geq k+1$, we have that: $\dfrac{\nu^{\mathrm{e}}_{i+1}}{\nu^{\mathrm{e}}_i} = \dfrac{\nu^{\mathrm{m}}_{i+1}}{\nu^{\mathrm{m}}_i} = \dfrac{1-x}{x}\dfrac{N-i}{i+1}$,

(iii) $\nu^{\mathrm{e}}_k + \nu^{\mathrm{e}}_{k+1} < \nu^{\mathrm{m}}_k + \nu^{\mathrm{m}}_{k+1}$

and so, that the probability distribution $\{\nu^{\mathrm{e}}_i\}$ strictly first-order stochastically dominates $\{\nu^{\mathrm{m}}_i\}$.

Claim *(i)* derives from the fact that i refers to the size of the majority at the end of

where $\overline{x} = x + (1-2x)\alpha$ is the probability of an in-group (or out-group) candidate being of type s. Similarly for the basic-entrenchment equilibrium,

$$S^{\mathrm{e}}_{t+1} = \frac{N-1}{N}S^{\mathrm{e}}_t + (N-1)\Big[\nu^{\mathrm{e}}_{k+1}\frac{k+1}{N}\overline{x} + \Big(1-\nu^{\mathrm{e}}_{k+1}\frac{k+1}{N}\Big)[\overline{x}+x]\Big]\tilde{s}$$

the period $i \in \{k, ..., 2k\}$. Note that in regime $\mathrm{r} \in \{\mathrm{e}, \mathrm{m}\}$,

$$\nu_N^{\mathrm{r}} = (1-x)\nu_N^{\mathrm{r}} + \frac{1-x}{N}\nu_{N-1}^{\mathrm{r}}$$

and for $k+2 \leq i < N$, $$\nu_i^{\mathrm{r}} = (1-x)\frac{N-(i-1)}{N}\nu_{i-1}^{\mathrm{r}} + \left[(1-x)\frac{i}{N} + x\frac{N-i}{N}\right]\nu_i^{\mathrm{r}} + x\frac{i+1}{N}\nu_{i+1}^{\mathrm{r}}$$

Claim *(ii)* follows by backward induction starting from $i = N$ and going down until $k+2$ included. Note that the explicit expression of the ergodic distribution in the basic-entrenchment equilibrium obtains with claims *(i)* and *(ii)* by writing $\Sigma_{i=k+1}^{N}\nu_i^{\mathrm{e}} = 1$. The explicit expression of the ergodic distribution in the meritocratic equilibrium obtains similarly noting that $(1-x)N\nu_k^{\mathrm{m}} = x(k+1)\nu_{k+1}^{\mathrm{m}}$. One has in particular that

$$\begin{cases} \nu_{k+1}^{\mathrm{m}}\left[\frac{x}{1-x}\frac{k+1}{N} + 1 + \sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i \prod_{j=1}^{i}\frac{k-j}{k+1+j}\right] = 1 \\ \nu_{k+1}^{\mathrm{e}}\left[1 + \sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i \prod_{j=1}^{i}\frac{k-j}{k+1+j}\right] = 1 \end{cases}$$

Lastly, claims *(i)* and *(ii)* together imply claim *(iii)*.

Remark. The ergodic probability for the majority size to be equal to k at the beginning of a period in the basic-entrenchment equilibrium writes as $\nu_{k+1}^{\mathrm{e}}(k+1)/N$, and thus by the above expression, decreases with k.

F Proof of Proposition 4

Let ρ^W be uniquely defined by

$$qN(N-1)\left[1 + \frac{x}{1-x}\frac{k+1}{N} + \sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i \prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]\rho^W$$
$$= \frac{2}{1-x}\left[1 + \sum_{i=1}^{k-1}(i+1)^2\left(\frac{1-x}{x}\right)^i \prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]$$

We show that $W^{\mathrm{m}} \geq W^{\mathrm{e}}$ if and only if $s/b \geq \rho^W$. The result then obtains by showing that $\rho^W < 1$ for all parameter values.

Let us first establish the explicit expression of ρ^W. By construction, we have that

$$B^{\mathrm{m}} - B^{\mathrm{e}} = \sum_{i=k}^{N}\left(\nu_i^{\mathrm{m}} - \nu_i^{\mathrm{e}}\right)\left[i(i-1) + (N-i)(N-i-1)\right]\tilde{b}$$

Hence, computations using the explicit expressions of the ergodic distributions (see Section E above) yield after rearranging:

$$\left[\frac{x}{1-x}\frac{k+1}{N}+1+\sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]\left[1+\sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]\left(B^{\mathrm{m}}-B^{\mathrm{e}}\right)$$
$$=-\frac{2x}{1-x}\frac{k+1}{N}\left[1+\sum_{i=1}^{k-1}(i+1)^2\left(\frac{1-x}{x}\right)^i\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]\tilde{b}$$

Similar computations for $(S^{\mathrm{m}}-S^{\mathrm{e}})$ yield:

$$\left[\frac{x}{1-x}\frac{k+1}{N}+1+\sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]\left[1+\sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]\left(S^{\mathrm{m}}-S^{\mathrm{e}}\right)$$
$$=N(N-1)x\frac{k+1}{N}\left[\frac{x}{1-x}\frac{k+1}{N}+1+\sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]\tilde{s}$$

The expression of ρ^W follows. Lastly, the inequality $\rho^W<1$ derives from the observations that for any $x\in[0,1/2]$, $N(N-1)>2(l+1)^2/(1-x)$ for any $l\leq k-2$, and that[25]

$$N(N-1)\left[1+\left(\frac{1-x}{x}\right)^{k-1}\prod_{j=1}^{k-1}\frac{k-j}{k+1+j}\right]>\frac{2}{1-x}\left[1+k^2\left(\frac{1-x}{x}\right)^{k-1}\prod_{j=1}^{k-1}\frac{k-j}{k+1+j}\right].$$

G Proof of Proposition 5

Let us formalize equal-treatment rules. Suppose that the majority controls in each period not only recruitment, but also the allocation of a nonrival, excludable common good (e.g., allocating a given facility, or resource to specific groups, with the two groups favoring distinct uses for the facility or resource). That is, suppose that, in each period, the majority can adapt a facility to its members' exclusive preferences: namely, choose its location on a Hotelling segment ($l\in[0,1]$), with the bliss points of the two

[25] Indeed, as the inequality $N(N-1)<2k^2/(1-x)$ holds if and only if $x>(k-1)/(N-1)$, we have that for any $x\in[0,1/2]$, the difference between the LHS minus the RHS is bounded below by

$$N(N-1)\left[1+\left(\frac{1-x}{x}\right)^{k-1}\prod_{j=1}^{k-1}\frac{k-j}{k+1+j}\right]-4\left[1+k^2\left(\frac{1-x}{x}\right)^{k-1}\prod_{j=1}^{k-1}\frac{k-j}{k+1+j}\right]$$
$$>N(N-1)\left[1+\left(\frac{k}{k-1}\right)^{k-1}\prod_{j=1}^{k-1}\frac{k-j}{k+1+j}\right]-4\left[1+k^2\left(\frac{k}{k-1}\right)^{k-1}\prod_{j=1}^{k-1}\frac{k-j}{k+1+j}\right]$$
$$>N(N-1)-4-N>0,$$

where the second inequality derives from $\left(\frac{k}{k-1}\right)^{k-1}\prod_{j=1}^{k-1}\frac{k-j}{k+1+j}<1$, while the third holds for any $N\geq 4$.

horizontal groups located at the extremes (resp. at 0 and 1).[26] An equal-treatment rule thus forces the facility to be located at an equal distance of both extremes ($l = 1/2$). Access to the facility yields a per-period payoff $\tilde{\varsigma} - \tilde{c}(l)$ to a member when the facility is located at distance l from the member's bliss point, where the transportation cost $\tilde{c}$ is strictly increasing and convex. Therefore, locating the facility at a group's bliss point instead of at the other group's bliss point delivers an extra per-period payoff equal to the cost differential, $\tilde{c}(1) - \tilde{c}(0)$, to all majority members, which constitutes an extra benefit from being in the majority.[27] Thus, unequal treatment magnifies the incentive for entrenchment. Conversely, mandating equal treatment of the organization's members – here, equal access/distance to the resource – reduces the value of the majority's decision rights, lowers the majority's payoff and raises the minority's one for a given recruitment policy, and thus fosters meritocracy.

Lastly, the sum of members' payoffs from accessing the facility (leaving aside quality and homophily payoffs) when it is located at a distance l from the majority's bliss point is equal to $N\tilde{\varsigma} - M\tilde{c}(l) - (N-M)\tilde{c}(1-l)$. Consequently, allocative efficiency may involve locating the facility at the majority's bliss point – e.g., for sufficiently large M when $\tilde{c}$ is strictly convex.

The proof of Proposition 2 goes through (see Online Appendix C), adding the discounted payoff $\varsigma - c(l)$, where $\varsigma \equiv \tilde{\varsigma}/\big[1-\delta_0(1-2/N)\big]$, and $c(l) \equiv \tilde{c}(l)/\big[1-\delta_0(1-2/N)\big]$ with $c(l) = c(0)$ for a majority member, and $c(l) = c(1)$ for a minority member unless equal treatment is enforced, in which case $c(l) = c(1/2)$ for both majority and minority members. Hence, the existence regions of basic entrenchment and meritocracy now depend on the transportation cost. In particular, when equal treatment is not enforced, meritocracy exists if and only if

$$s \geq b + \delta \frac{k-1}{N-1}\Big[(1-2x)b + c(1) - c(0)\Big] \sum_{t=0}^{+\infty} \delta^t \left[\left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k+1,i}(t)\right) - \left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k-1,i}(t)\right)\right].$$

where the transition probabilities π^{m} induced by the meritocratic equilibrium do not depend on $\varsigma, c(1), c(0)$. By contrast, when equal treatment is enforced, meritocracy exists

[26] The allocation is decided at the start of the period, after the departure has occurred and prior to the recruitment (i.e., among the $N-1$ members left, so that the majority is clearly defined), and holds for the period.

[27] It can in fact be interpreted as a special case of nonlinear homophily benefits (see Section 4.3).

if and only if

$$s \geq b + \delta \frac{k-1}{N-1}(1-2x)b \sum_{t=0}^{+\infty} \delta^t \left[\left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k+1,i}(t) \right) - \left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k-1,i}(t) \right) \right],$$

which is the same condition as in our baseline model.

As noted in the proof of Proposition 2 (see Lemma C.2), for all $t \geq 0$,

$$\left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k+1,i}(t) \right) - \left(\sum_{i=k}^{N-1} \pi^{\mathrm{m}}_{k-1,i}(t) \right) \geq 0,$$

with a strict inequality for $t = 0$. Consequently, when equal treatment is not enforced, the existence region of meritocracy increases as the cost differential $c(1) - c(0) > 0$ decreases, and it is always strictly smaller than when equal treatment is enforced.

The results for ergodic payoffs follows from the same computations as in the proof of Proposition 4 (see Online Appendix F).

H Proof of Proposition 6

H.1 Proof of claim (i)

Whenever a minority quota is implemented, we refer to the "existence region of (constrained) meritocracy" as the set of values of s/b for which there exists an equilibrium in which recruitments are meritocratic (i.e., a talented candidate is always recruited against a strictly less talented candidate) whenever the quota R is not binding. The result is (almost) immediate for a quota of 1.[28] We thus focus on $R \geq 2$.

Consider a quota $R = k - l$ with $l \in \{1, ..., k-2\}$, and denote by $\tilde{V}$ the value function from recruiting the most talented candidate (and breaking ties in favor of the majority candidate) at all majority sizes at which the quota R is not binding (omitting the superscript m), and let $\tilde{u}_i \equiv \tilde{V}_{i+1} - \tilde{V}_i$. We will first show that the sequence $(\tilde{u}_i)_{i \geq k-1}$ is such that $\tilde{u}_{k+l-1} < 0$, and such that it satisfies at least one of the following assertions: (A_1) it decreases with i, or (A_2) it is always strictly negative.[29] As in the baseline case, the monotonicity property (A_1) would imply that the most tempting deviation from meritocracy to basic entrenchment is when the majority has size k and the minority candidate is strictly

[28] As will be clear shortly, the argument is significantly shorter in this case than with $R \geq 2$ since the minority's value function in the basic-entrenchment equilibrium writes as in the baseline model with no affirmative action (due to the conditioning on still being a member next period).

[29] By contrast, in the baseline setting without affirmative action, the sequence $(u_i)_{i \geq k-1}$ is positive for any i and decreases with i.

more talented than the majority candidate, while (A_2) would imply that deviations from constrained meritocracy to basic entrenchment at any size $i \geq k$ are non-profitable as they yield a deviation payoff bounded above by

$$-(s-b)+\delta\left[\left(1-\frac{i}{N-1}\right)\tilde{u}_i+\frac{i-1}{N-1}\tilde{u}_{i-1}\right]<0$$

Lastly, that $\tilde{u}_{k+l-1}$ is negative suggests that there may be profitable deviations from meritocracy with ties broken in favor of the majority candidate to meritocracy with ties broken in favor of the minority candidate when s/b is high enough (more on this below).

We first suppose by contradiction that $\tilde{u}_{k+l-1} \geq 0$. The usual induction argument relying on (5) then yields that $\tilde{u}_{k-1} > \tilde{u}_k > ... > \tilde{u}_{k+l-1} \geq 0$. Yet, summing as in the proof of Lemma 2, the above recursive expression for $\tilde{u}_{k+l-1}$ with (12) and (5) over indices k to $k+l-2$, and rearranging, yields on the LHS a weighted sum of $\tilde{u}_{k-1}, ..., \tilde{u}_{k+l-1}$ which is strictly positive, while on the RHS:

$$-xs-(1-x)b+(1-2x)b+\delta(1-x)\frac{k-2}{N-1}\tilde{u}_{k-2}=-x(s+b)+\delta(1-x)\frac{k-2}{N-1}\tilde{u}_{k-2},$$

and so $\tilde{u}_{k-2} > 0$. Summing (11) at $k-2$ to the above sum, and rearranging, yields on the LHS a weighted sum of $\tilde{u}_{k-1}, ..., \tilde{u}_{k+l-1}$ which is strictly positive, and on the RHS:

$$-x(s+b)+\delta(1-x)\frac{k-3}{N-1}\tilde{u}_{k-3},$$

Hence, $\tilde{u}_{k-3} > 0$, and by repeating this argument, $\tilde{u}_i > 0$ for any $i \in \{k-l-1, ..., k+l-1\}$. Yet summing the above recursive expressions of $\tilde{u}_{k-l-1}$ and $\tilde{u}_{k+l-1}$ together with (5)-(11)-(12) for $i \in \{k-l, ..., k+l-2\}$, yields after rearranging, on the LHS a weighted sum of all $\tilde{u}_i$ which is strictly positive, while on the RHS: $-x(s+b)+xs-(1-x)b=-b<0$, which is a contradiction. Consequently, $\tilde{u}_{k+l-1} < 0$.

To show that the sequence $(\tilde{u}_i)_{i\geq k-1}$ satisfies either (A_1) or (A_2) (or both), we proceed by induction considering the lowest index i^- such that $\tilde{u}_i < 0$ for any $i \geq i^-$. We first note that if $i^- \geq k$, then (5) brings by induction that[30]

$$\tilde{u}_{k+l-1} < \tilde{u}_{k+l-2} < ... < \tilde{u}_{i^-+1} < \tilde{u}_{i^-} < 0 < \tilde{u}_{i^--1} < \tilde{u}_{i^--2} < ... < \tilde{u}_{k-1},$$

[30] The inequalities $\tilde{u}_{k+l-1} < \tilde{u}_{k+l-2} < ... < \tilde{u}_{i^-+1} < \tilde{u}_{i^-} < 0$ can be established by induction using the recursive expressions of the $\tilde{u}_i$ from $i = i^-$ up to $i = k+l-2$.

which yields that (A_1) holds. If $i^- \leq k-1$, then (A_2) holds.

Consequently, to show that with affirmative action, the existence region of (constrained) meritocracy expands towards lower values of s/b, it is sufficient to consider deviations from meritocracy to basic entrenchment when the majority is tight and faces an untalented majority candidate and a talented minority candidate, and to show that the condition for non-profitability is looser for any s/b with affirmative action than in the baseline setting (without affirmative action).

Explicit computations yield[31]

$$\begin{cases} \tilde{u}_{k+l-1} = -xs - (1-x)b + \delta x \left[\dfrac{k-l}{N-1} \tilde{u}_{k+l-1} + \dfrac{k+l-2}{N-1} \tilde{u}_{k+l-2} \right] \\ \tilde{u}_{k-l-1} = xs - (1-x)b + \delta x \left[\dfrac{k-l-1}{N-1} \tilde{u}_{k-l-1} + \dfrac{k+l-1}{N-1} \tilde{u}_{k-l} \right] \end{cases} \tag{26}$$

Thus using (5) at $k+l-1$ and (11) at $k-l-1$, together with the fact that $u_i \geq 0$ for all i in the baseline setting, one gets[32]

$$\begin{cases} \left[1 - \delta x \dfrac{k-l}{N-1}\right] \left(\tilde{u}_{k+l-1} - u_{k+l-1}\right) < -xs - (1-x)b + \delta x \dfrac{k+l-2}{N-1} \left(\tilde{u}_{k+l-2} - u_{k+l-2}\right) \\ \left[1 - \delta x \dfrac{k-l-1}{N-1}\right] \left(\tilde{u}_{k-l-1} - u_{k-l-1}\right) < xs - (1-x)b + \delta x \dfrac{k+l-2}{N-1} \left(\tilde{u}_{k-l} - u_{k-l}\right) \end{cases}$$

[31] By definition of affirmative action with quota R, in any equilibrium

$$\begin{cases} \tilde{V}_{k+l} = \overline{x}s + \delta \left[\dfrac{k+l-1}{N-1} \tilde{V}_{k+l-1} + \dfrac{k-l}{N-1} \tilde{V}_{k+l} \right] \\ \tilde{V}_{k-l-1} = \overline{x}s + \delta \left[\dfrac{k-l-1}{N-1} \tilde{V}_{k-l-1} + \dfrac{k+l}{N-1} \tilde{V}_{k-l} \right] \end{cases}$$

Hence, in the meritocratic equilibrium,

$$\begin{cases} \tilde{u}_{k+l-1} = -xs - (1-x)b + \delta x \left[\dfrac{k-l}{N-1} \tilde{u}_{k+l-1} + \dfrac{k+l-2}{N-1} \tilde{u}_{k+l-2} \right] \\ \tilde{u}_{k-l-1} = xs - (1-x)b + \delta x \left[\dfrac{k-l-1}{N-1} \tilde{u}_{k-l-1} + \dfrac{k+l-1}{N-1} \tilde{u}_{k-l} \right] \end{cases}$$

[32] Note that the omitted terms write for the first equation as

$$-\delta(1-x) \left[\frac{k+l-1}{N-1} u_{k+l-1} + \frac{k-l-1}{N-1} u_{k+l} \right],$$

which is thus proportional to $(-b)$ (see proof of Lemma 2 for details). Similarly for the second equation.

Therefore, using (5) at $k+l-2$ and (11) at $k-l$, one gets

$$\begin{cases} \left[1-\delta x\frac{k-l+1}{N-1}-\delta(1-x)\frac{k+l-2}{N-1}-\delta(1-x)\frac{k-l}{N-1}\frac{\delta x\frac{k+l-2}{N-1}}{1-\delta x\frac{k-l}{N-1}}\right]\left(\tilde{u}_{k+l-2}-u_{k+l-2}\right) \\ \qquad <\frac{\delta(1-x)\frac{k-l}{N-1}}{1-\delta x\frac{k-l}{N-1}}\Big[-xs-(1-x)b\Big]+\delta x\frac{k+l-3}{N-1}\left(\tilde{u}_{k+l-3}-u_{k+l-3}\right) \\ \left[1-\delta x\frac{k-l}{N-1}-\delta(1-x)\frac{k+l-1}{N-1}-\delta(1-x)\frac{k-l-1}{N-1}\frac{\delta x\frac{k+l-1}{N-1}}{1-\delta x\frac{k-l-1}{N-1}}\right]\left(\tilde{u}_{k-l}-u_{k-l}\right) \\ \qquad <\frac{\delta(1-x)\frac{k-l-1}{N-1}}{1-\delta x\frac{k-l-1}{N-1}}\Big[xs-(1-x)b\Big]+\delta x\frac{k+l-2}{N-1}\left(\tilde{u}_{k-l+1}-u_{k-l+1}\right) \end{cases}$$

We begin by noting that

$$\frac{k-l}{N-1}\left[1-\delta x\frac{k-l-1}{N-1}\right]>\frac{k-l-1}{N-1}\left[1-\delta x\frac{k-l}{N-1}\right],$$

and[33]

$$\begin{aligned}\delta x\delta(1-x)\left(\frac{k-l}{N-1}\right)^2\frac{k+l-2}{N-1}\left[1-\delta x\frac{k-l-1}{N-1}\right]\\ >\delta x\delta(1-x)\frac{k-l+1}{N-1}\frac{k-l-1}{N-1}\frac{k+l-1}{N-1}\left[1-\delta x\frac{k-l}{N-1}\right]-\frac{\delta x}{N-1},\end{aligned}$$

[33]To see this, we observe that: $(k-l)(k+l-2)=(k-l+1)(k+l-1)-(2k-1)$, and as a consequence, using the above inequality,

$$\begin{aligned}\left(\frac{k-l}{N-1}\right)^2\frac{k+l-2}{N-1}\left[1-\delta x\frac{k-l-1}{N-1}\right]\\ >\frac{k-l+1}{N-1}\frac{k-l-1}{N-1}\frac{k+l-1}{N-1}\left[1-\delta x\frac{k-l}{N-1}\right]-\frac{k-l}{N-1}\left[1-\delta x\frac{k-l-1}{N-1}\right]\frac{1}{N-1},\end{aligned}$$

The inequality thus obtains using that $\delta(1-x)\frac{k-l}{N-1}<1-\delta x\frac{k-l}{N-1}$.

Hence, we have that[34]

$$\frac{k-l+1}{N-1}\left[1-\delta x\frac{k-l}{N-1}-\delta(1-x)\frac{k+l-1}{N-1}-\delta(1-x)\frac{k-l-1}{N-1}\frac{\delta x\frac{k+l-1}{N-1}}{1-\delta x\frac{k-l-1}{N-1}}\right]$$
$$>\frac{k-l}{N-1}\left[1-\delta x\frac{k-l+1}{N-1}-\delta(1-x)\frac{k+l-2}{N-1}-\delta(1-x)\frac{k-l}{N-1}\frac{\delta x\frac{k+l-2}{N-1}}{1-\delta x\frac{k-l}{N-1}}\right]$$

By downward (resp. upward) induction on $(\tilde{u}_i - u_i)$ for $i \geq k$ (resp. for $i \leq k-2$), we get that

$$C_1\Big(\tilde{u}_{k-1}-u_{k-1}\Big) < -C_2xs - C_3(1-x)b < 0 \tag{27}$$

where C_1, C_2 and C_3 are strictly positive constants that depend on the parameters k, l and x. Let us detail the induction argument. Using (5)-(11), we obtain two sequences $(a_j)_{0\leq j\leq l-2}$ and $(b_j)_{0\leq j\leq l-2}$ such that for any $j \leq l-2$,

$$\begin{cases} a_j\Big(\tilde{u}_{k+j}-u_{k+j}\Big) \\ \quad < -\Big[xs+(1-x)b\Big]\dfrac{\delta(1-x)\frac{k-l}{N-1}}{1-\delta x\frac{k-l}{N-1}}\displaystyle\prod_{n=j+1}^{l-2}\left(\frac{\delta(1-x)}{a_n}\frac{k-n-1}{N-1}\right)+\delta x\frac{k+j-1}{N-1}\Big(\tilde{u}_{k+j-1}-u_{k+j-1}\Big) \\ b_j\Big(\tilde{u}_{k-j-2}-u_{k-j-2}\Big) \\ \quad < \Big[xs-(1-x)b\Big]\dfrac{\delta(1-x)\frac{k-l-1}{N-1}}{1-\delta x\frac{k-l-1}{N-1}}\displaystyle\prod_{n=j+1}^{l-2}\left(\frac{\delta(1-x)}{b_n}\frac{k-n-2}{N-1}\right)+\delta x\frac{k+j}{N-1}\Big(\tilde{u}_{k-j-1}-u_{k-j-1}\Big) \end{cases}$$

where

$$\begin{cases} a_{j-1}=1-\delta x\dfrac{k-j}{N-1}-\delta(1-x)\dfrac{k+j-1}{N-1}-\delta(1-x)\dfrac{k-j-1}{N-1}\dfrac{\delta x\frac{k+j-1}{N-1}}{a_j} \\ b_{j-1}=1-\delta x\dfrac{k-j-1}{N-1}-\delta(1-x)\dfrac{k+j}{N-1}-\delta(1-x)\dfrac{k-j-2}{N-1}\dfrac{\delta x\frac{k+j}{N-1}}{b_j} \end{cases}$$

[34]Note that

$$\frac{k-l+1}{N-1}\left[1-\delta(1-x)\frac{k+l-1}{N-1}\right]=\frac{k-l}{N-1}\left[1-\delta(1-x)\frac{k+l-2}{N-1}\right]+\frac{1-\delta(1-x)}{N-1}.$$

We first note that by induction[35]

$$\forall j \leq l-1, \qquad \frac{\delta(1-x)}{a_j}\frac{k-j-1}{N-1} < 1, \qquad \text{and} \qquad \frac{\delta(1-x)}{b_j}\frac{k-j-2}{N-1} < 1 \tag{28}$$

Hence, using (12), the coefficient C_1 in (27) is given by

$$1-\delta(1-x) - \frac{\delta(1-x)}{a_0}\frac{k-1}{N-1}\delta x\frac{k-1}{N-1} - \frac{\delta(1-x)}{b_0}\frac{k-2}{N-1}\delta x\frac{k}{N-1} > 1-\delta > 0$$

Using (12) further implies that the coefficient C_3 in (27) is strictly positive. We then show by downward induction on j that for any $j \leq l-1$,

$$\frac{1}{a_j}\frac{k-j-1}{N-1} > \frac{1}{b_j}\frac{k-j-2}{N-1},$$

which will yield that $C_2 > 0$. The initialization ($j = l-1$) derives from the observation in footnote 35 (the case $j = l-2$ has also been established above). As for the induction, i.e. to show that $a_{j-1}(k-j-1) < b_{j-1}(k-j)$, we note that for any $j \geq 0$, the induction hypothesis implies that[36]

$$\frac{k-j-1}{N-1}\frac{k+j-1}{N-1}\frac{1}{a_j}\frac{k-j-1}{N-1} > \frac{k-j}{N-1}\frac{k+j}{N-1}\frac{1}{b_j}\frac{k-j-2}{N-1} - \frac{1}{a_j}\frac{k-j-1}{N-1}\frac{1}{N-1}$$

and thus, using (28),

$$\delta x\delta(1-x)\frac{k-j-1}{N-1}\frac{k+j-1}{N-1}\frac{1}{a_j}\frac{k-j-1}{N-1} > \delta x\delta(1-x)\frac{k-j}{N-1}\frac{k+j}{N-1}\frac{1}{b_j}\frac{k-j-2}{N-1} - \frac{\delta x}{N-1}$$

Therefore, using the recursive expression of a_{j-1} and b_{j-1}, we have that

$$a_{j-1}(k-j-1) < b_{j-1}(k-j) - \frac{1-\delta}{N-1} < b_{j-1}(k-j),$$

[35] The initialization with $j = l-1$ stems from the observation that

$$\delta(1-x)\frac{k-l}{N-1} < 1 - \delta x\frac{k-l}{N-1}, \qquad \text{and} \qquad \delta(1-x)\frac{k-l-1}{N-1} < 1-\delta x\frac{k-l-1}{N-1}$$

Moreover,

$$\delta(1-x)\frac{k-l}{N-1}\left[1-\delta x\frac{k-l-1}{N-1}\right] > \delta(1-x)\frac{k-l-1}{N-1}\left[1-\delta x\frac{k-l}{N-1}\right], \qquad \text{i.e.} \frac{1}{a_{l-1}}\frac{k-l}{N-1} > \frac{1}{b_{l-1}}\frac{k-l-1}{N-1}$$

[36] Indeed, we have that $(k-j-1)(k+j-1) = (k-j)(k+j) - (2k-1)$, and

$$\frac{k-j}{N-1}\left[1-\delta(1-x)\frac{k+j}{N-1}\right] = \frac{k-j-1}{N-1}\left[1-\delta(1-x)\frac{k+j-1}{N-1}\right] + \frac{1-\delta(1-x)}{N-1}$$

as was to be shown.

This in turn implies that $\left(\tilde{u}_k - u_k\right) < 0$. Therefore,

$$s - b - \delta\frac{k-1}{N-1}(u_{k-1} + u_k) > s - b - \delta\frac{k-1}{N-1}(\tilde{u}_{k-1} - u_k),$$

i.e., the non-profitability condition for a deviation from meritocracy to basic entrenchment is (strictly) looser with a quota R than without.

Remark: For s/b sufficiently high, meritocracy with reverse favoritism is an equilibrium: the majority always picks the most talented candidate and breaks ties in favor of the minority candidate. Let $b = 0 < s$. We first note that in the unconstrained, meritocratic equilibrium, this implies that $u_i = 0$ for any $i \in \{1, ..., N-2\}$. The above computations then apply, switching the weights $1-x$ and x (except for the flow payoffs of $\tilde{u}_{k+l-1}$ and $\tilde{u}_{k-l-1}$ which remain respectively given by $-xs$ and xs). Hence, $\tilde{u}_i < 0$ for any $i \geq k-1$. Consequently, the deviation differential payoff from reverse-favoritism meritocracy to standard-favoritism meritocracy at majority size M is given by

$$\delta\left(\frac{M-1}{N-1}\tilde{u}^{\mathrm{m}}_{M-1} + \frac{N-1-M}{N-1}\tilde{u}^{\mathrm{m}}_{M}\right) < 0,$$

which yields the result. By contrast, the same argument implies that meritocracy with standard favoritism is no longer an equilibrium for s/b sufficiently high.[37]

Remark: Comparing the continuation value functions. The same computations as in the proof of Proposition 3 apply (see Online Appendix D). Therefore, whenever (constrained) meritocracy and (constrained) basic entrenchment coexist, at any majority size the (constrained) meritocratic equilibrium is preferred to the (constrained) basic-entrenchment equilibrium by (current) majority members.[38]

[37]Considering $b = 0 < s$, and observing that in the meritocratic equilibrium, $u_i = 0$ for any $i \in \{1, ..., N-2\}$ and using the above computations in order to get that $\tilde{u}_i < 0$.

[38]Building on analogous computations, it can be shown that the same preference also holds in several cases for all (current) minority members. By mimicking the argument in Online Appendix D, we have that $\tilde{V}^{\mathrm{m}}_i \geq \tilde{V}^{\mathrm{e}}_i$ for any $i \leq k-1$ if

$$s + b + \delta\left(\frac{k}{N-1}\tilde{u}^{\mathrm{m}}_{k-1} + \frac{k-2}{N-1}\tilde{u}^{\mathrm{m}}_{k-2}\right) > 0,$$

This inequality holds in particular whenever δ is small.

H.2 Proof of claim (ii)

Let $N \geq 4$ and $1 \leq l \leq k-1$. The ergodic aggregate quality of a basically-entrenched organization under laissez-faire and a meritocratic one under affirmative action with quota l write respectively:

$$\begin{cases} S^{\mathrm{e}} = N(N-1)\left[\dfrac{k+1}{N}\nu^{\mathrm{e}}_{k+1}\overline{x} + \left(1 - \dfrac{k+1}{N}\nu^{\mathrm{e}}_{k+1}\right)\left(\overline{x}+x\right)\right]\tilde{s} \\ S^{\mathrm{m,AA}} = N(N-1)\left[\dfrac{l}{N}\nu^{\mathrm{m,AA}}_{N-l}\overline{x} + \left(1 - \dfrac{l}{N}\nu^{\mathrm{m,AA}}_{N-l}\right)\left(\overline{x}+x\right)\right]\tilde{s} \end{cases}$$

and thus:

$$S^{\mathrm{m,AA}} - S^{\mathrm{e}} = N(N-1)\left[\frac{k+1}{N}\nu^{\mathrm{e}}_{k+1} - \frac{l}{N}\nu^{\mathrm{m,AA}}_{N-l}\right]x\tilde{s}$$

Explicit computations (see Lemma E.1 and its proof in Section E) yield:

$$\begin{cases} \nu^{\mathrm{e}}_{k+1}\left[1 + \displaystyle\sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^{i}\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right] = 1 \\ \nu^{\mathrm{m,AA}}_{N-l}\left[1 + \displaystyle\sum_{i=1}^{k-l-1}\left(\frac{x}{1-x}\right)^{i}\prod_{j=1}^{i}\frac{N-l+1-j}{l+j} + \left(\frac{x}{1-x}\right)^{k-l}\frac{k+1}{N}\prod_{j=1}^{k-l-1}\frac{N-l+1-j}{l+j}\right] = 1 \end{cases}$$

Consequently, $S^{\mathrm{m,AA}} - S^{\mathrm{e}}$ has same sign as

$$(k+1)\left[1 + \sum_{i=1}^{k-l-1}\left(\frac{x}{1-x}\right)^{i}\prod_{j=1}^{i}\frac{N-l+1-j}{l+j} + \left(\frac{x}{1-x}\right)^{k-l}\frac{k+1}{N}\prod_{j=1}^{k-l-1}\frac{N-l+1-j}{l+j}\right] \\ - l\left[1 + \sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^{i}\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]$$

We then note that the above expression is strictly negative for x in a neighbourhood of 0, and strictly positive for x in a neighbourhood of 1. Moreover, since $x/(1-x)$ (resp. $(1-x)/x$) strictly increases (resp. decreases) with $x \in (0, 1/2)$, there exists a unique $x_{\mathrm{AA}}(l) \in (0, 1/2]$ such that for any $x < x_{\mathrm{AA}}(l)$ (resp. $x > x_{\mathrm{AA}}(l)$), the above expression is strictly negative (resp. positive).

Lastly, we note that by construction, $x_{\mathrm{AA}}(l)$ is such that

$$(k+1)\left[1+\sum_{i=1}^{k-l-1}\left(\frac{x_{\mathrm{AA}}(l)}{1-x_{\mathrm{AA}}(l)}\right)^i\prod_{j=1}^{i}\frac{N-l+1-j}{l+j}+\left(\frac{x_{\mathrm{AA}}(l)}{1-x_{\mathrm{AA}}(l)}\right)^{k-l}\frac{k+1}{N}\prod_{j=1}^{k-l-1}\frac{N-l+1-j}{l+j}\right]$$
$$=l\left[1+\sum_{i=1}^{k-1}\left(\frac{1-x_{\mathrm{AA}}(l)}{x_{\mathrm{AA}}(l)}\right)^i\prod_{j=1}^{i}\frac{k-j}{k+1+j}\right]$$

The LHS strictly decreases with l for any given x fixed, and strictly increases with x for any fixed l. By contrast, the RHS strictly increases with l for any fixed x, and strictly decreases with x for any fixed l. Hence, $x_{\mathrm{AA}}(l)$ strictly increases with l.

I Proof of Proposition 7

Proof of claim (i). With blind hiring, recruiting the candidate with the highest talent signal z is a dominant strategy, strictly so when there is a unique such candidate. The welfare analysis depends on the precision of the talent signal (z).

If the signal z perfectly reveals a candidate's talent, blind hiring thus leads to the (or "a") most talented candidate being recruited in each period, breaking ties randomly between equally talented candidates. Blind hiring then achieves the same ergodic quality (S) as meritocracy (strictly higher than basic entrenchment), but a lower ergodic aggregate homophily (B) than both meritocracy and basic entrenchment.

If the signal z does not perfectly reveal a candidate's talent, blind hiring leads to another candidate than the most talented one(s) being recruited with strictly positive probability. The less precise the signal, the higher this probability, and thus the larger the quality loss. When the signal is pure noise and thus uninformative about the candidates' talent, blind hiring leads to the (or "a") most talented candidate being recruited with probability at most $(1-x)$.[39] By contrast, under basic entrenchment, the most talented candidate is selected with probability 1 at any majority size $M \geq k+1$, and with probability $(1-x)$ at $M = k$.

Proof of claim (ii). Suppose $s = b$. We show that for any V_{k-1} in the feasible range, the full-entrenchment strategy is a strictly dominant strategy for the majority, within the

[39] This probability is strictly lower than $(1-x)$ if and only if the probability that the two candidates of each horizontal group are equally talented is strictly lower than 1. Indeed, the majority then randomly recruits one candidate among the four available (with uniform distribution), and the probability that a given candidate among the four is a most talented candidate is bounded above by $(1-x)$, strictly so unless the two candidates of each type always have the same talent.

set of level-l super-entrenchment strategies, where $l \geq 0$. Specifically, in the case of semi-blind hiring, we define level-l super-entrenchment strategies as in Section 2.1.1 whenever the majority faces exactly one candidate from each horizontal group, and extend these strategies to the events in which the majority faces either two majority candidates, or two minority candidates, by assuming that in any such event, the majority picks a most talented candidate among the two.[40] Let us first observe that because the probability that at the first round, the two minority candidates have strictly higher signals z than the two majority candidates is strictly positive, the transition probability from any two majority size $M \geq k$ to $M-1$ is strictly positive regardless of the majority's strategy.

When $s = b$, the full-entrenchment strategy yields the highest feasible flow payoff at any majority size $M \geq k$. Hence, for any strategy by the other group, $V_{k-1} \leq V_k^{\text{fe}}(V_{k-1})$, where $V_M^{\text{fe}}(V_{k-1})$ is the majority's continuation value at size M given the full-entrenchment strategy and V_{k-1}.

Moreover, suppose by contradiction that there exists V_{k-1} in the feasible range such that $V_{k-1} = V_k^{\text{fe}}(V_{k-1})$. Then, starting from group size $k-1$, a group must receive the maximum flow payoff at the current and at all future periods. However, as $x < 1/2$, the probability that the majority faces in the second round a majority candidate and a minority candidate who are equally talented is strictly positive, and the majority then chooses the majority candidate, which yields a strictly lower flow payoff to the (current-period) minority members than to the (current-period) majority members. Therefore, for any V_{k-1} in the feasible range, $V_{k-1} < V_k^{\text{fe}}(V_{k-1})$.

Lastly, for any majority size $M \geq k$, the full-entrenchment strategy is the unique strategy to minimize among all level-l entrenchment strategies the probability that starting from M, group size $k-1$ is reached.

Therefore, for $s/b = 1$, full entrenchment is a strictly dominant strategy for the majority. By continuity, it remains a strictly dominant strategy for any $s/b > 1$ in a neighborhood of 1, which yields the result.

J Proof of Proposition 8

We first show that, as claimed in the text, it is an equilibrium for the principal not to intervene when it is uninformed.

Let us first argue that given the members' basic-entrenchment strategy, there is no

[40] In any such event, choosing a most talented candidate is a dominant strategy, strictly so when one candidate is strictly more talented than the other.

current-period benefit for the principal to intervene at any majority size. Indeed, there is no current-period benefit for the principal to intervene whenever the majority is not tight ($M \geq k+1$) – or whenever it is tight and meritocratic – as then the majority's choice maximizes the organization's quality and, by resolving ties in favor of the majority candidate, it also maximizes the homophily payoff conditional on maximizing the organization's quality. Hence, for $s > b$ and $q \geq 1$, the majority's choice is optimal from the principal's point of view.[41]

Similarly, there is no current-period benefit for the principal to intervene when the majority is tight ($M = k$). Indeed, since a tight entrenched majority always votes for its own candidate, its vote carries no information on the candidates' respective talents. Hence, the principal picks the (or "a" if there is a tie) most talented candidate with probability $1-2x+(1/2)(2x) = 1-x$, which is the same probability of the basically-entrenched majority choosing the most talented candidate. However, when the majority is tight, it takes the homophily-maximizing decision with probability 1, while the principal can only do so with probability 1/2 as it does not observe horizontal types.

Let us now consider the distribution of future majority sizes, to show that the principal has no future-periods benefits from an intervention in the current period. At any majority size $M \geq k$, by picking the minority candidate instead of the majority one, the principal sets the organization on a path on which the distribution of future majority sizes is stochastically dominated at any future time by the one on the no-intervention/original path (using the same argument as in the proof of Proposition 2, see the proof of Lemma C.2 in Online Appendix C.2.1). Hence, at any future time, the organization is more likely to be in the tight-majority state ($M = k$) following the principal's appointment of the minority candidate. Yet, the (expected) current-period welfare-increment (for incumbent members from the current-period recruit) is minimal at state $M = k$, equal to $\overline{x}s(N-1) + bk$, while it is equal to $(\overline{x}+x)s(N-1) + (1-x)bM + xb(N-1-M) > \overline{x}s(N-1) + bk$ at any majority size $M \geq k+1$.[42]

Hence, an uninformed principal cannot outperform the majority's decision.[43]

[41]Fix $s > b$. Since the quality payoff accrues to all members of the organization, while the homophily benefit only accrues to the in-group members, this optimality persists for q in a lower neighbourhood of 1.

[42]At any majority size $M \geq k+1$, the principal "mistakenly" picking an untalented majority candidate instead of a talented minority candidate yields a lower aggregate welfare as in equilibrium, the majority itself prefers recruiting the talented minority candidate instead of an untalented majority candidate.

[43]Even if the principal observed horizontal types (still without observing the vertical types), a non-intervention equilibrium would still exist as the principal could not strictly improve on the basically-entrenched majority's choices. (The above argument would go through as in particular, when the

J.1 Proof of claim (i)

Let $\lambda > 0$ be the probability that the principal learns the quality of the candidates. We look for equilibria in which the principal intervenes whenever informed that meritocracy is violated (and only then). We consider the organization members' strategy and we show that, given such an intervention policy for the principal:

(a) for s/b sufficiently close to 1, there exists a profitable deviation from basic entrenchment in $k+1$ (the unique equilibrium when s/b is close to 1 and $\lambda = 0$) toward super-entrenchment at level 1. The argument then extends to any level of super-entrenchment.

(b) for s/b sufficiently close to 1, full entrenchment is an equilibrium.

(c) for any s/b sufficiently close to 1, the full-entrenchment equilibrium is the unique symmetric MPE in pure strategies.

In the next Section, to prove claim (ii), we will show that for any s/b sufficiently close to 1 and for any λ in an intermediate range, if the majority is (basically, super- or fully) entrenched, it is optimal for the principal to intervene whenever it is informed that the current-period recruitment violates meritocracy.

(a). For $i \geq k$, let V_i be the majority value function in the basic-entrenchment equilibrium when the principal is informed with probability λ and intervenes whenever informed that meritocracy is violated. Consider a deviation at $M = k+1$ from basic-entrenchment to level-1 super-entrenchment. The (one-shot) differential payoff from the deviation at $M = k+1$ writes

$$\begin{aligned}\Delta &\equiv (1-\lambda)\left[b - s + \delta\left(\frac{k+1}{N-1}V_{k+1} + \frac{k-2}{N-1}V_{k+2}\right) - \delta\left(\frac{k}{N-1}V_k + \frac{k-1}{N-1}V_{k+1}\right)\right] \\ &= (1-\lambda)\left[b - s + \delta\left(\frac{k-2}{N-1}u_{k+1} + \frac{k}{N-1}u_k\right)\right]\end{aligned}$$

where $u_i \equiv V_{i+1} - V_i$. The sequence $(u_i)_{1\leq i \leq N-2}$ satisfies Equation (5) for any $i \geq k+1$,

basically-entrenched majority is tight, its recruitment choice does not reveal any information about the quality of candidates.)

and Equation (11) for any $i \leq k-3$, while

$$\begin{cases} \left[1-\delta(1-x)\frac{k}{N-1}-\delta x\lambda\frac{k-1}{N-1}\right]u_k = x(1-\lambda)(s-b)+\delta(1-x)\frac{k-2}{N-1}u_{k+1}+\delta x\lambda\frac{k-1}{N-1}u_{k-1} \\ \left[1-\delta(1-x\lambda)\right]u_{k-1} = (1-2x\lambda)b+\delta(1-x\lambda)\left[\frac{k-2}{N-1}u_{k-2}+\frac{k-1}{N-1}u_k\right] \\ \left[1-\delta(1-x)\frac{k+1}{N-1}-\delta x\lambda\frac{k-2}{N-1}\right]u_{k-2} = -x(1-\lambda)(s+b)+\delta(1-x)\frac{k-3}{N-1}u_{k-3}+\delta x\lambda\frac{k}{N-1}u_{k-1} \end{cases} \tag{29}$$

Summing up on all indices yields[44]

$$\left[1-\delta\frac{x}{N-1}-\delta(1-x)\right](u_1+u_{N-2})+\Big(1-\delta\Big)\sum_{i=2}^{N-3}u_i = (1-2x)b > 0 \tag{30}$$

Fix $b>0$. For any $s \geq b$, the same argument as the one used in the proof of Lemma 2 yields $u_k > u_{k+1} > ... > u_{N-2} > 0$.[45] The differential deviation payoff is thus strictly positive if and only if

$$\delta\left(\frac{k-2}{N-1}u_{k+1}+\frac{k}{N-1}u_k\right) > s-b \tag{31}$$

Consequently, for $s=b$, (31) is satisfied as it writes

$$\delta\left(\frac{k-2}{N-1}u_{k+1}+\frac{k}{N-1}u_k\right) > 0$$

Lastly, since for fixed b, $(u_i)_i$ is continuous with respect to s, this implies that for any s/b sufficiently close to 1, there exists a strictly profitable (one-shot) deviation from basic-entrenchment to level-1 super-entrenchment.

The same argument can be adapted to show that, for s/b sufficiently close to 1, there exist profitable deviations from any level $l \in \{1,...,k-2\}$ of super-entrenchment toward super-entrenchment at a higher level $l' > l$, and thus in particular toward full-entrenchment.

(b). We now show the existence of the full-entrenchment equilibrium for s/b suffi-

[44] Assuming $k \geq 4$. The expression for $k \in \{2,3\}$ writes differently on the LHS but has the same implication.

[45] Put succinctly, one supposes by contradiction that $u_{N-2} \leq 0$ and reaches a contradiction showing by induction, using (5) together with the above system, that this implies $u_{k-1} \leq 0$. Then, if $u_1 \leq 0$, (11) implies $u_i \leq 0$ for all i, which contradicts (30); whereas if $u_1 > 0$, (11) implies $u_{k-1} > 0$ and we reach again a contradiction. Hence, $u_{N-2} > 0$ and the same induction argument using (5) thus brings the result.

ciently close to 1. Let now V_i, u_i correspond to the full-entrenchment strategies. The deviation differential payoff from full-entrenchment to super-entrenchment at a lower level ($l \leq k-2$) in $M = N-1$ whenever the minority candidate is more talented writes

$$\Delta \equiv (1-\lambda)\left[s - b - \delta \frac{N-2}{N-1} u_{N-2}\right]$$

Explicit computation with (3)-(4) yield:

$$u_{N-2} = \delta(1-x\lambda)\frac{N-2}{N-1}u_{N-2} + \delta x\lambda \left[\frac{N-3}{N-1}u_{N-3} + \frac{1}{N-1}u_{N-2}\right]$$

and more generally for any $M \geq k$,

$$u_M = \delta(1-x\lambda)\left[\frac{M}{N-1}u_M + \left(1 - \frac{M+1}{N-1}\right)u_{M+1}\right] + \delta x\lambda\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right]$$

while for any $i \leq k-2$,

$$u_i = \delta(1-x\lambda)\left[\frac{i-1}{N-1}u_{i-1} + \left(1 - \frac{i}{N-1}\right)u_i\right] + \delta x\lambda\left[\frac{i-1}{N-1}u_i + \left(1 - \frac{i+1}{N-1}\right)u_{i+1}\right]$$

with

$$\left[1 - \delta\big(1 - x\lambda\big)\right]u_{k-1} = (1-2x\lambda)b + \delta(1-x\lambda)\left[\frac{k-1}{N-1}u_k + \frac{k-2}{N-1}u_{k-2}\right]$$

Summing up over all indices yields

$$\left[1 - \delta\left(1 - x\lambda\frac{N-2}{N-1}\right)\right]u_{N-2} + \left[1 - \delta\left(1 - \frac{x\lambda}{N-1}\right)\right]u_1 + (1-\delta)\sum_{i=2}^{N-3} u_i = (1-2x\lambda)b > 0 \tag{32}$$

Fix $b > 0$ and let $s = b$. The usual argument implies that $u_{N-2} > 0$.[46] Hence, the differential deviation payoff when the majority has size $N-2$ writes for $s = b$ as

$$\Delta = -(1-\lambda)\delta\frac{N-2}{N-1}u_{N-2} < 0.$$

By continuity, the inequality holds for s/b in a neighbourhood of 1.

Since $u_k > u_{k+1} > ... > u_{N-2} > 0$, the most profitable (one-shot) deviation from

[46] Indeed, if not, then the above equations imply by induction that $u_k \leq u_{k+1} \leq ... \leq u_{N-2} \leq 0$ and thus $0 \geq u_1 \geq u_2 \geq ... \geq u_{k-1}$, which yields to a contradiction with (32). Therefore, $u_{N-2} > 0$, and by induction again $u_k > u_{k+1} > ... > u_{N-2} > 0$.

full-entrenchment is when the majority has size $N-1$ and a talented minority candidate faces an untalented majority candidate. As a consequence, the above necessary condition is also sufficient.

Hence, full-entrenchment is an equilibrium for s/b in a neighbourhood of 1.

(c). Lastly, we show that for s/b in a neighbourhood of 1, full-entrenchment equilibrium is the unique (pure-strategy) symmetric MPE. To this end, we show that, for s/b in a neighbourhood of 1, any (pure-strategy) symmetric MPE is monotonic, in the sense that a stronger majority makes more meritocratic recruitments. Together with (a), this establishes the uniqueness of full entrenchment.

Let $s = b > 0$. We show that in any symmetric MPE, the differential value function $(u_M)_{M \geq k-1}$ is strictly positive and strictly decreases with M. Since the difference between the payoffs from a meritocratic, resp. an entrenched recruitment at majority size M whenever the minority candidate is strictly more talented than the majority one writes as

$$s - b - \delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right],$$

the monotonicity of $(u_M)_M$ implies the monotonicity of the equilibrium. Moreover, if the strict monotonicity of $(u_M)_M$ obtains for $s = b$, then by continuity, it persists for s/b in a neighbourhood of 1, which implies that, for s/b in such a neighbourhood, any symmetric MPE is monotonic.

For $s = b > 0$, we have that

$$\begin{cases} u_{k-1} = (1-2x\lambda)b + \delta(1-x\lambda)\left[\frac{k-2}{N-1}u_{k-2} + u_{k-1} + \frac{k-1}{N-1}u_k\right] & \text{in an equilibrium in which the majority is entrenched in } k, \\ u_{k-1} = (1-2x)b + \delta(1-x)\left[\frac{k-2}{N-1}u_{k-2} + u_{k-1} + \frac{k-1}{N-1}u_k\right] & \text{in an equilibrium in which it is meritocratic in } k. \end{cases}$$

and for any majority size $M \leq N-2$,

$$\begin{cases} u_M = \delta(1-x\lambda)\left[\frac{M}{N-1}u_M + \left(1-\frac{M+1}{N-1}\right)u_{M+1}\right] + \delta x\lambda\left[\frac{M-1}{N-1}u_{M-1} + \left(1-\frac{M}{N-1}\right)u_M\right] \\ \qquad\qquad \text{in an equilibrium in which the majority is entrenched in } M, M+1, \\ u_M = \delta(1-x)\left[\frac{M}{N-1}u_M + \left(1-\frac{M+1}{N-1}\right)u_{M+1}\right] + \delta x\left[\frac{M-1}{N-1}u_{M-1} + \left(1-\frac{M}{N-1}\right)u_M\right] \\ \qquad\qquad \text{in an equilibrium in which the majority is meritocratic in } M, M+1, \\ u_M = \delta(1-x)\left[\frac{M}{N-1}u_M + \left(1-\frac{M+1}{N-1}\right)u_{M+1}\right] + \delta x\lambda\left[\frac{M-1}{N-1}u_{M-1} + \left(1-\frac{M}{N-1}\right)u_M\right] \\ \quad \text{in an equilibrium in which the majority is entrenched (resp. meritocratic) in } M (\text{resp. } M+1), \\ u_M = \delta(1-x\lambda)\left[\frac{M}{N-1}u_M + \left(1-\frac{M+1}{N-1}\right)u_{M+1}\right] + \delta x\left[\frac{M-1}{N-1}u_{M-1} + \left(1-\frac{M}{N-1}\right)u_{M+1}\right] \\ \quad \text{in an equilibrium in which the majority is meritocratic (resp. entrenched) in } M (\text{resp. } M+1), \end{cases}$$

together with similar expressions for u_i when $i \leq k-2$.

Let us first show that $u_i > 0$ for all $i \in \{k-1, ..., N-2\}$. We proceed by induction. Suppose by contradiction that $u_{N-2} < 0$. Then, the above recursive expressions imply that $u_{k-1} < u_k < ... < u_{N-2} < 0$.[47] Therefore, the majority is meritocratic at all majority sizes $M \geq k$.[48] But then, Lemma 2 implies that $u_{k-1} > u_k > ... > u_{N-2}$, a contradiction.

Suppose now (again by contradiction) that $u_{N-2} = 0$. The above recursive expressions then imply that $u_{k-2} < u_{k-1} = u_k = ... = u_{N-2} = 0$, and thus that $Vk-2 > V_{k-1} = V_k = ... = V_{N-1}$. However, this implies that at all majority sizes $M \geq k$, the majority recruits the majority candidate whenever he is at least as talented as the minority candidate (and the majority is indifferent when he is strictly less talented than the minority candidate): since the majority recruits its own candidate at least a fraction $1-x$ of the time, and the minority candidate at most a fraction $x < 1-x$ of

[47] Indeed, the above recursive expressions imply that there exists $(a,b) \in \{(1-x\lambda, x\lambda),\ (1-x,x),\ (1-x, x\lambda),\ (1-x\lambda, x)\}$ such that

$$u_{N-3} = \frac{1-\delta\frac{N-2}{N-1}a - \frac{\delta}{N-1}b}{\delta\frac{N-3}{N-1}b}u_{N-2}.$$

Hence, $u_{N-2} < 0$ implies $u_{N-3} < u_{N-2} < 0$. The result obtains by induction on the majority size.

[48] Indeed, as $s = b$, the differential payoff between recruiting a talented minority candidate instead of an untalented majority candidate is equal to

$$-\delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1-\frac{M}{N-1}\right)u_M\right] > 0.$$

the time at all majority sizes, V_M must be strictly higher than V_{k-1} for all $M \geq k$, a contradiction.

Therefore, $u_{N-2} > 0$, and the above system then implies that $u_i > 0$ for all $i \in \{k-1, ..., N-2\}$ as was to be shown.

Let us now show that $uk - 1 > u_k > ... > u_{N-2}$. Using that $u_{N_2} > 0$ and $u_{N-3} > 0$, the above system evaluated at $M = N - 2$ implies that, for any (pure) strategies, $u_{N-2} < u_{N-3}$. Proceeding recursively for $M \geq k$, $0 < u_{M+1} < u_M$ and $u_{M-1} > 0$ implies by the same argument that $u_M < u_{M-1}$. Therefore, the sequence $(u_M)_{M \geq k}$ strictly decreases with M.

Remark: Non-ergodic welfare comparison. Proposition 3 yields that, whenever meritocracy co-exists with basic entrenchment, the former is preferred by all members of the organization at any majority size. The result goes through in this setting.

Namely, we show that for any $l \geq 2$, whenever super-entrenchment at level $l-1$ and super-entrenchment at level l co-exist in equilibrium, the former is preferred by all (current) members of the organization at any majority size. The result for majority members relies on the same computations as in the proof of Proposition 3 (see Online Appendix D), using that since super-entrenchment at level $l-1$ is an equilibrium[49],

$$s - b - \delta\left(\frac{k+l-1}{N-1} u_{k+l-1}^{\mathrm{e},l-1} + \frac{k-l-1}{N-1} u_{k+l}^{\mathrm{e},l-1}\right) \geq 0$$

where $u_i^{\mathrm{e},l-1} = V_{i+1}^{\mathrm{e},l-1} - V_i^{\mathrm{e},l-1}$ with $V_i^{\mathrm{e},l-1}$ the value function of being in a group of size i in the super-entrenchment at level $l-1$ equilibrium. The result for minority members also relies on analogous computations to the ones in the proof of Proposition 3 (see Online Appendix D): using the recursive expressions of the value function for minority members in a similar fashion, we have that $V_i^{\mathrm{e},l-1} \geq V_i^{\mathrm{e},l}$ for any $i \leq k-1$ if

$$s + b + \delta\left(\frac{k-l-1}{N-1} u_{k-l-1}^{\mathrm{e},l-1} + \frac{k+l-1}{N-1} u_{k-l}^{\mathrm{e},l-1}\right) \geq 0 \tag{33}$$

We thus show that this inequality holds, using the recursive expressions of $(u_i^{\mathrm{e},l-1})_i$. We distinguish two cases.

(1) if $u_{k-l}^{\mathrm{e},l-1} \geq 0$, then $0 \geq u_1^{\mathrm{e},l-1} \geq u_2^{\mathrm{e},l-1} \geq ... \geq u_{k-l}^{\mathrm{e},l-1}$.[50] Hence, inequality (33) holds.

[49]Indeed, this implies that in equilibrium, meritocratic recruitments are the majority's best response whenever it has size $k+l$, hence the inequality.

[50]This can be shown by the usual argument, supposing by contradiction that $u_1^{\mathrm{e},l-1} < 0$, which implies by the recursive expressions of $(u_i^{\mathrm{e},l-1})_i$, that $0 > u_1^{\mathrm{e},l-1} > ... > u_{k-l}^{\mathrm{e},l-1}$, hence a contradiction. Therefore,

(2) if $u^{e,l-1}_{k-l} \leq 0$, then $u^{e,l-1}_{k-l} \leq u^{e,l-1}_{k-l-1}$ and $u^{e,l-1}_{k-l} \leq u^{e,l-1}_{k-l+1}$. Indeed,

- consider the first inequality and suppose by contradiction that $u^{e,l-1}_{k-l-1} < u^{e,l-1}_{k-l}$. By the usual (contradiction and induction) argument, this implies that $u_1 < ... < u^{e,l-1}_{k-l} \leq 0$. However, by summing the recursive expressions of $u^{e,l-1}_i$ for $i = 1, ...k - l - 1$, and rearranging, we get

$$\left[1 - \delta\frac{x}{N-1} - \delta(1-x)\right]u^{e,l-1}_1 + (1-\delta)\sum_{i=2}^{k-l-2} u^{e,l-1}_i + \left[1 - \delta\left(1 - (1-x)\frac{k-l-1}{N-1}\right)\right]u^{e,l-1}_{k-l-1}$$
$$= \delta x\frac{k+l-1}{N-1}u^{e,l-1}_{k-l} > \delta x\frac{k+l-1}{N-1}u^{e,l-1}_{k-l-1}$$

  Therefore, as $u^{e,l-1}_1 < u^{e,l-1}_{k-l-1}$, rearranging implies that

$$\left[2 - \delta\left(1 + \frac{k+l}{N-1}\right)\right]u^{e,l-1}_{k-l-1} + (1-\delta)\sum_{i=2}^{k-l-2} u^{e,l-1}_i > 0,$$

  which is a contradiction, as $u^{e,l-1}_1 < ... < u^{e,l-1}_{k-l} \leq 0$. Consequently, $u^{e,l-1}_{k-l} \leq u^{e,l-1}_{k-l-1}$.

- consider the second inequality and suppose by contradiction that $u^{e,l-1}_{k-l} > u^{e,l-1}_{k-l+1}$. Using the recursive expression of $u^{e,l-1}_{k-l+1}$, this implies that $u^{e,l-1}_{k-l+2} < u^{e,l-1}_{k-l+1} < 0$, and by induction that $0 > u^{e,l-1}_{k-1}$. However, we know from the above computations that $u^{e,l-1}_i > 0$ for any $i \geq k-1$, and thus in particular, $u^{e,l-1}_{k-1} > 0$, which contradicts the above implication. Hence, $u^{e,l-1}_{k-l} \leq u^{e,l-1}_{k-l+1}$.

Therefore, if $u^{e,l-1}_{k-l} \leq 0$, then $u^{e,l-1}_{k-l} \leq u^{e,l-1}_{k-l-1}$ and $u^{e,l-1}_{k-l} \leq u^{e,l-1}_{k-l+1}$, and thus

$$s + b + \delta\left(\frac{k-l-1}{N-1}u^{e,l-1}_{k-l-1} + \frac{k+l-1}{N-1}u^{e,l-1}_{k-l}\right) \geq s + b + \delta\frac{N-2}{N-1}u^{e,l-1}_{k-l},$$

and using the recursive expression of $u^{e,l-1}_{k-l}$,[51]

$$\left[1 - \delta\left[1 - x(1-\lambda)\right]\frac{N-2}{N-1}\right]u^{e,l-1}_{k-l} \geq -(1-\lambda)x(s-b).$$

$u^{e,l-1}_1 \geq 0$, and the recursive expressions of $(u^{e,l-1}_i)_i$ now imply that $0 \geq u^{e,l-1}_1 \geq u^{e,l-1}_2 \geq ... \geq u^{e,l-1}_{k-l}$.

[51] Namely,

$$u^{e,l-1}_{k-l} = -(1-\lambda)x(s-b) + \delta(1-x)\left[\frac{k-l-1}{N-1}u^{e,l-1}_{k-l-1} + \frac{k+l+1}{N-1}u^{e,l-1}_{k-l}\right] + \delta x\lambda\left[\frac{k-l}{N-1}u^{e,l-1}_{k-l} + \frac{k+l-2}{N-1}u^{e,l-1}_{k-l+1}\right]$$

As a consequence,

$$s+b+\delta\left(\frac{k-l-1}{N-1}u^{\mathrm{e},l-1}_{k-l-1}+\frac{k+l-1}{N-1}u^{\mathrm{e},l-1}_{k-l}\right)\geq s+b-\frac{\delta x(1-\lambda)(N-2)}{N-1-\delta\big[1-x(1-\lambda)\big](N-2)}(s-b)$$
$$\geq s+b-\frac{k-1}{k+1}(s-b)>0$$

Hence, inequality (33) holds in both cases ($u^{\mathrm{e},l-1}_{k-l}\lesseqgtr 0$), as was to be shown.

J.2 Proof of claim (ii)

Suppose the principal maximizes quality. With the above arguments, for any $\lambda>0$, the principal's strategy "overruling whenever informed that meritocracy is violated" and the majority's full entrenchment is an equilibrium for s/b close to 1. But it may not be unique – e.g., if λ is close to 0, no overruling and basic entrenchment is an equilibrium for s/b close to 1. We argue that for λ sufficiently close to 1, "overruling whenever informed that meritocracy is violated" and full entrenchment is the unique equilibrium (with our equilibrium concept).

Specifically, let us show that for any λ sufficiently close to 1, for any s/b sufficiently close to 1, if the majority is (basically, super or fully) entrenched, it is optimal for the principal to intervene whenever it is informed that the current-period recruitment violates meritocracy.

Note first that the principal cannot expand the existence region of meritocracy by its interventions as the prospect of its overruling a majority's decision only scales down (by a strictly positive factor) the one-shot deviation differential payoff from meritocracy to entrenchment. Hence, under our assumption that the meritocratic equilibrium is selected whenever it exists, the principal fails to expand the region where meritocracy prevails.

As noted in the text, for $\lambda=1$ (perfectly informed principal), the principal can reproduce the equilibrium path of the meritocratic regime, which strictly dominates in terms of quality the equilibrium path of the basic-entrenchment regime. Hence, by continuity, keeping members' strategies fixed, for λ sufficiently close to 1, it is optimal for the principal to intervene whenever informed. Moreover, by the same argument as in our initial remark about a blind principal, whenever the principal is not informed, it cannot outperform an entrenched majority's choice in terms of aggregate welfare. Indeed, it selects the (or "a" in case of a tie) most talented candidate with the same probability as the majority in the current period, while making a choice that is suboptimal in terms of homophily payoffs in the current period, and its intervention induces a distribution over future majority sizes that yields the same future quality payoffs as the non-intervention distribution,[52] but that is dominated by the latter in terms of future homophily payoffs. Therefore, for λ close to 1, given the members' strategy (basic, super- or full entrenchment), it is optimal for the principal to intervene if and only if it is informed that the current-period recruitment violates meritocracy.

Consequently, by claim (i), for s/b close to 1 (such that in particular, basic entrenchment is the unique equilibrium under *laissez-faire*) and λ close to 1, the unique equilibrium is for the principal to intervene if and only if it is informed that meritocracy is violated, and for the majority to be fully entrenched.

Let us now show that, for s/b close to 1 and λ in an intermediate range, the principal achieves a higher ergodic quality when it commits not to intervene. To provide an intuition, consider s/b close to 1 and λ close to 1 so that in the unique equilibrium, the organization is fully-entrenched. Since the principal is only informed with probability strictly below 1, it cannot compensate all the "un-meritocratic" recruitments made by the fully-entrenched majority. Hence, at any majority size $M \geq k+1$, i.e. at which the majority would have made meritocratic recruitments under *laissez-faire*, the principal would be better off in terms of flow welfare, if it could commit not to intervene. By contrast, whenever the majority is tight ($M = k$), basic entrenchment would have prevailed under *laissez-faire*, and so the principal's intervention improves the flow welfare.

To make things precise, let us consider s/b sufficiently close to 1 and λ sufficiently close to 1 such that the unique equilibrium is for the majority to fully entrench and for the principal to intervene if and only if informed that the current-period recruitment violates meritocracy. Ergodic aggregate quality is then strictly higher when the principal commits not to intervene if and only if

$$N(N-1)(1-\lambda)xs > N(N-1)\nu^{\mathrm{e}}_{k+1}\frac{k+1}{N}xs, \qquad \text{i.e.} \qquad \lambda < 1 - \nu^{\mathrm{e}}_{k+1}\frac{k+1}{N},$$

which yields the result. The range of values of λ for which the result holds is non-empty in particular whenever x is sufficiently small, as ν^{e}_{k+1} goes to 0 when x goes to 0. It is also non-empty whenever δ is sufficiently small, as it is then a strictly dominating strategy

[52] With full entrenchment, the expected talent of the recruit does not depend on majority size.

for the principal to intervene whenever informed that the current recruitment violates meritocracy (as $s > b$ and quality benefits accrue to all organization members, while homophily ones only to in-groups).

K Proof of Proposition 9

We successively consider the three incentive schemes mentioned in the text.

K.1 Material rewards for talent in the organization

K.1.1 Non-budget-balanced incentives

Letting T denote the ergodic expected per-period transfer and ξ its shadow cost, the ergodic welfare function in the presence of transfers becomes: $W = qS + B - \xi T$. (The budget-neutral case described in the text thus corresponds to $\xi = 0$.)

Proposition K.1. ***(Rewarding quality: Costly incentives)*** *Costly incentives raise welfare W only if the organization is neither naturally meritocratic nor too recalcitrant to meritocracy: For any cost of public funds $\xi \geq 0$ ($\xi = 0$ for budget-neutral incentives), there exists $\rho_\xi \in [1, \rho^m)$, strictly increasing with ξ and satisfying $\rho_0 = 1$, such that quality assessment exercises raise welfare W if and only if $s/b \in [\rho_\xi, \rho^m)$.*

The intuition behind Proposition K.1 is that for high s/b, the organization embraces meritocracy by itself and so spending public funds is wasteful. When instead the organization has little appetence for meritocracy (s/b small), the principal must pour large amounts of money on the organization to be effective, and this may prove too costly. It is thus only in the intermediate range that a boost promotes meritocracy and quality at a reasonable cost.[53]

K.1.2 Proofs of Propositions 9 and K.1

Consider a basically-entrenched organization, i.e. by the equilibrium selection (by Proposition 3, meritocracy thus prevails whenever it exists as an equilibrium), suppose $s/b < \rho^{\mathrm{m}}$. Let $T/N \equiv \eta y$ denote equal the minimal expected bonus per member needed

[53] The optimal transfer is equal to 0 for s/b below a certain threshold (which increases with the cost of public funds ξ), jumps discontinuously strictly above zero at this threshold, and then decreases with s/b above the threshold, down to zero when $s/b = \rho^{\mathrm{m}}$.

for the organization to move from basic entrenchment to meritocracy.[54] For the sake of exposition, we first assume that the principal does not value members' homophily benefits, and thus letting ξ be the cost of public funds,[55] the principal's objective function writes as the ergodic welfare with per-period welfare given by $W = qS - \xi T$.[56] Note that such an objective constitutes an *upper* bound on the admissible cost of a policy as (ergodic aggregate) homophily payoffs decrease when the organization goes from basic entrenchment to meritocracy (see Section 2.2.2). From previous computations on ergodic welfare, the (ergodic) efficiency gain from moving from basic entrenchment to meritocracy writes as $S^{\mathrm{m}} - S^{\mathrm{e}} = N(N-1)\nu^{\mathrm{e}}_{k+1}\frac{k+1}{N}x\frac{\tilde{s}}{1-\delta} > 0$. Rewarding quality is thus optimal for the principal if and only if

$$\xi\eta y N^2(\overline{x}+x) \leq N(N-1)\nu^{\mathrm{e}}_{k+1}\frac{k+1}{N}x\tilde{s}$$

where $N[\overline{x}+x]$ is the average number of talented members in a meritocratic organization, and ν^{e}_{k+1} the objective ergodic probability of majority size $k+1$ in the basic-entrenchment equilibrium (see Section 2.2.2). The above inequality rewrites as a condition on the administrative cost of public funds:[57]

$$\xi \leq \frac{(k+1)(N-1)}{N^2}\cdot\frac{x\nu^{\mathrm{e}}_{k+1}}{\overline{x}+x}\cdot\frac{\tilde{s}}{\eta y} = \frac{(k+1)(N-1)}{N^2}\cdot\frac{x\nu^{\mathrm{e}}_{k+1}}{\overline{x}+x}\cdot\frac{\frac{s}{b}}{\rho^{\mathrm{m}}-\frac{s}{b}}$$

[54]Namely,

$$\frac{s^{+}(\eta,y)}{b} = \rho^{\mathrm{m}}, \qquad \text{i.e.} \qquad \eta y = \left(\frac{b}{s}\rho^{\mathrm{m}}-1\right)\tilde{s} > 0$$

[55]The interpretation of ξ depends on the principal's welfare objective. If it is solely concerned with maximizing the (ergodic aggregate) quality of the organization, then ξ is the total cost of intervention, i.e. the sum of the payment and its shadow cost. By contrast, if the principal internalizes the "material" welfare of members, i.e. the sum of their quality payoffs and (possibly) rewards for quality (as opposed to their non-material welfare which consists of homophily benefits), then ξ is only the shadow cost of public funds.

[56]This objective may be interpreted as the limit of the main objective as qs/b and ξ/b go to $+\infty$, with qs/ξ constant.

[57]By Inequality (19), a lower bound on the RHS of the above equation is given by

$$\frac{(k+1)(N-1)}{N^2}\cdot\frac{x\nu^{\mathrm{e}}_{k+1}}{\overline{x}+x}\cdot\frac{\tilde{s}}{\eta y} \geq \frac{(k+1)(N-1)^2}{(k-1)N^2}\cdot\frac{x(1-2x)\nu^{\mathrm{e}}_{k+1}}{\overline{x}+x}\cdot\frac{(1-\delta)}{\delta}$$

Note that the RHS strictly increases with s/b and goes to $+\infty$ as s/b goes to ρ^{m}.[58] The result follows. The same argument applies if the principal's objective writes as $W = qS + B - \xi T$, yielding a higher threshold ρ_ξ (as $B^{\mathrm{m}} < B^{\mathrm{e}}$).

K.2 Fines for unequal treatment & reputational (symbolic) incentives

The two incentive schemes can be modelled in the same way. For conciseness, we focus on reputational (symbolic) incentives. Proposition 9 obtains from the same arguments as in the proof of Proposition 2 (see Online Appendix C). Adding a reputational cost $\frac{\vartheta}{M}$ for each majority member (present at the moment of the vote) for recruiting an untalented majority candidate against a talented minority one, and a corresponding reputational gain $\frac{\vartheta}{N-1-M}$ for minority members.[59] Our analysis goes through if the reputation gain accrues not (only) to minority members, but (also) to individuals external to the organization (e.g., members of a meritocratic majority in another organization). [Similarly, our analysis of fines goes through whether the proceeds redistributed to all members, to minority members alone, or to members of other organizations.]

The proof of Proposition 2 goes through for ϑ below a threshold. In particular, meritocracy then exists if and only if for all $M \geq k$,[60]

$$s \geq b - \frac{\vartheta}{M} + \delta\left(\frac{M-1}{N-1}(V^m_M - V^m_{M-1}) + \frac{N-1-M}{N-1}(V^m_{M+1} - V^m_M)\right),$$

where V^m_M is the continuation value in the meritocratic equilibrium for a majority member at majority size M, and does not depend on ϑ.

[58] The monotonicity of the RHS with respect to N is non-trivial. Namely, although the first two terms decrease with $N \geq 4$, so that $(k+1)(N-1)\nu^{\mathrm{e}}_{k+1}/N^2$ decreases with N, the comparative statics of ρ^{m} with respect to N are non-trivial. Nonetheless, for N large, the first two terms $(k+1)(N-1)\nu^{\mathrm{e}}_{k+1}/N^2$ are in $O(1/N)$, while for $\delta_0 < 1$, ρ^{m} is in $O(1)$. Therefore, the RHS is in $O(1/N)$ for N large, which is intuitive: the upper bound on the admissible cost of public funds is inversely proportional to the size of the organization, i.e. to the number of individuals to whom the bonus must be distributed.

[59] The normalization captures a fixed loss of reputation, ϑ, for the majority following its decision to discriminate, shared equally among the M majority members who took part in the vote. Similarly, to ensure that reputations are zero-sum, the minority enjoys a (relative) reputation boost, shared equally within the minority.

[60] When ϑ is sufficiently small, the necessary and sufficient condition is the inequality with $M = k$ (as in our baseline specification):

$$s \geq b - \frac{\vartheta}{k} + \delta\frac{k-1}{N-1}(1-2x)b\sum_{t=0}^{+\infty}\delta^t\left[\left(\sum_{i=k}^{N-1}\pi^{\mathrm{m}}_{k+1,i}(t)\right) - \left(\sum_{i=k}^{N-1}\pi^{\mathrm{m}}_{k-1,i}(t)\right)\right].$$

By contrast, for larger ϑ, as the individual reputation loss, $\frac{\vartheta}{M}$, decreases with the majority size, larger majorities may be more tempted to discriminate than thinner majorities.

Consequently, the existence region of meritocracy increases as ϑ increases, i.e., as discrimination becomes more salient.

L "Weak link" principle: Proof of Proposition 2'

We know from Lemma 1 that in any pure-strategy MPE, any majority plays either the meritocratic strategy or the basic-entrenchment strategy.

Suppose $b_B < b_A < s$. The existence regions of basic entrenchment and meritocracy are a corollary of Proposition 2. So is the uniqueness of basic entrenchment (resp. meritocracy) among MPEs in pure strategies whenever $s < \min\left(\rho^{\mathrm{m}} b_A, \rho^{\mathrm{e}} b_B\right)$ (resp. $s > \min\left(\rho^{\mathrm{m}} b_A, \rho^{\mathrm{e}} b_B\right)$).

Let us consider the case $\rho^{\mathrm{e}} b_B < s < \rho^{\mathrm{m}} b_A$.[61] The basic-entrenchment strategy for type-A agents and the meritocratic strategy for type-B agents constitute an MPE in pure strategies as $s < \rho^{\mathrm{m}} b_A$ (implying that type-A members best-reply to type-B members' meritocratic strategy with the basic-entrenchment strategy), and $s > \rho^{\mathrm{e}} b_B$ (implying that type-B members best-reply to type-A members' basic-entrenchment strategy with the meritocratic strategy). Uniqueness follows from Lemma 1, and the inequality $\rho^{\mathrm{m}} < \rho^{\mathrm{e}}$ (from Proposition 2).

Lastly, if $\rho^{\mathrm{m}} b_A < s < \rho^{\mathrm{e}} b_B$, then by Proposition 2, the meritocratic and basic-entrenchment equilibria coexist as $\rho^{\mathrm{e}} > s/b_B > s/b_A > \rho^{\mathrm{m}}$. The same argument as in the proof of Proposition 3 yields the Pareto-comparison.

Remark: Asymmetric patience. The proof for the result with asymmetric patience (mentioned in Section 4.1) follows from analogous arguments, noting that Proposition 2.(iv) yields that $\rho^{\mathrm{e}}(\delta_0)$ and $\rho^{\mathrm{m}}(\delta_0)$ increase with δ_0.

M Proof of Proposition 10

We use a fixed-point argument to prove the existence of a class of equilibria characterized by a weakly decreasing decision rule $(\Delta_M)_M$ [62]. Let $\overline{u}$ be given by

$$\overline{u} \equiv \frac{1}{1-\delta}\left(\mathbb{E}\Big[(s+b)\mathbf{1}\{\hat{s}-s \leq b\}\Big] + \mathbb{E}\Big[\hat{s}\mathbf{1}\{\hat{s}-s > b\}\Big]\right)$$

[61] The thresholds are those computed in the proof of Proposition 2 (see Online Appendix C) and do not depend on s, nor b_A, b_B.

[62] We thus focus on equilibria such that the decision rule only depends on the majority size.

Note that $\Big(\mathbb{E}\Big[(s+b)\mathbf{1}\{\hat{s}-s\le b\}\Big]+\mathbb{E}\Big[\hat{s}\mathbf{1}\{\hat{s}-s>b\}\Big]\Big)$ is the highest flow payoff a majority member can guarantee, and consequently, $\overline{u}$ represents an upper bound on the majority's expected utility from a recruitment (i.e. its expected utility in the absence of control consideration). We define K as the set of sequences $(u_M)_{M\in\{k-1,...,N-2\}}$ such that (i) for any M, $u_M\in\Big[0,\overline{u}\Big]$ and (ii) the sequence $(u_M)_M$ is weakly decreasing. By construction, the set K is non-empty, compact and convex.

As earlier, let $\{V_i\}$ denote the value functions and $V\equiv(V_1,...,V_{N-1})$. For $i\in\{k-1,...,N-2\}$, let $u_i\equiv V_{i+1}-V_i$. In the equilibria we look for, whenever the majority has size $M\in\{k,...,N-1\}$, it favors a majority candidate with (discounted) talent s against a minority candidate with (discounted) talent $\hat{s}$ if and only if[63]

$$\hat{s}+\delta\left[\frac{M-1}{N-1}V_{M-1}+\left(1-\frac{M-1}{N-1}\right)V_M\right]\le s+b+\delta\left[\frac{M}{N-1}V_M+\left(1-\frac{M}{N-1}\right)V_{M+1}\right],$$

i.e. if and only if

$$\hat{s}-s\le b+\delta\left[\frac{M-1}{N-1}u_{M-1}+\left(1-\frac{M}{N-1}\right)u_M\right]$$

We denote by $\overline{s}\in[b,+\infty)$ the lowest real number such that $\mathbb{P}(\hat{s}-s\le\overline{s})=1$ if it exists, and let $\overline{s}=+\infty$ otherwise. We first consider the "decision-rule" (cutoff) mapping $D:K\longrightarrow\Big[0,\min(b+\delta\overline{u},\overline{s})\Big]^k, u\longmapsto(D_M)_{M\in\{k,...,N-1\}}$, where

$$D_M(u)\equiv\begin{cases}b+\delta\left[\frac{M-1}{N-1}u_{M-1}+\left(1-\frac{M}{N-1}\right)u_M\right] & \text{if } b+\delta\left[\frac{M-1}{N-1}u_{M-1}+\left(1-\frac{M}{N-1}\right)u_M\right]<\overline{s}\\ \overline{s} & \text{otherwise}\end{cases}$$

Taking $V_{k-1}\ge 0$ as fixed, we consider the "value-function" mapping T defined as $T:\Big[0,+\infty\Big]^k\times\Big[b,\overline{s}\Big]^k\longrightarrow\Big[0,+\infty\Big]^k,\Big((V_M)_M,(\Delta_M)_M\Big)\longmapsto(T_M)_M$, where

$$\begin{aligned}T_M(V,\Delta)\equiv{}&\mathbb{E}\Big[(s+b)\mathbf{1}\{\hat{s}-s\le\Delta_M\}\Big]+\delta\mathbb{P}(\hat{s}-s\le\Delta_M)\left[\frac{M}{N-1}V_M+\left(1-\frac{M}{N-1}\right)V_{M+1}\right]\\&+\mathbb{E}\Big[\hat{s}\mathbf{1}\{\hat{s}-s>\Delta_M\}\Big]+\delta\mathbb{P}(\hat{s}-s>\Delta_M)\left[\frac{M-1}{N-1}V_{M-1}+\left(1-\frac{M-1}{N-1}\right)V_M\right]\end{aligned}$$

[63]The assumption that ties are broken in favor of the majority candidate comes withouth loss of generality when vertical types are continuously distributed within each group.

In order to alleviate the notation, we define the functions h and h_1 as

$$\begin{cases} h(X) \equiv \mathbb{E}\Big[(s+X)\mathbf{1}\{\hat{s}-s \leq X\}\Big] + \mathbb{E}\Big[\hat{s}\mathbf{1}\{\hat{s}-s > X\}\Big] \\ h_1(X) \equiv X - h(X) \end{cases}$$

Fix $V_{k-1} \geq 0$. Given a sequence $u \equiv (u_M)_{M\in\{k-1,...,N-2\}} \in K$, we define the sequence $V(u) \equiv (V_M)_{M\in\{k,...,N-1\}}$ by upward induction by letting $V_M \equiv u_{M-1} + V_{M-1}$. Lastly, we define the mapping $\Upsilon : u \longmapsto \Upsilon(u)$ from K into itself by

$$\Upsilon_M(u) \equiv \min\left\{T_{M+1}\Big(V(u), D(u)\Big) - T_M\Big(V(u), D(u)\Big), h(b)/(1-\delta)\right\}$$

for any $M \in \{k-1, ..., N-2\}$ (with the convention that $T_{k-1}\Big(V(u), D(u)\Big) \equiv V_{k-1}$). While bounding above $\Upsilon(u)$ is necessary to the argument, it does not threaten the existence of an equilibrium: indeed, $h(b)$ is the highest flow payoff (quality and homophily) that a majority member can guarantee.[64] Hence, we have by construction that for any $u \in K$ and any $i \in \{k-1, ..., N-2\}$, $\Upsilon_i(u) \leq \overline{u}$. With an abuse of notation, we omit in the following the min operator.

We now check that the mapping Υ is well-defined, i.e. that $\Upsilon(u) \in K$ for any $u \in K$. Rearranging the above expression for $T_M\Big(V(u), D(u)\Big)$ yields:

$$\begin{aligned} T_M\Big(V(u), D(u)\Big) = {} & \mathbb{E}\Big[s\mathbf{1}\{\hat{s}-s \leq D_M(u)\}\Big] + \mathbb{E}\Big[\hat{s}\mathbf{1}\{\hat{s}-s > D_M(u)\}\Big] \\ & + \mathbb{P}(\hat{s}-s \leq D_M(u))\left[b + \delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right]\right] \\ & + \delta\left[\frac{M-1}{N-1}V_{M-1} + \left(1 - \frac{M-1}{N-1}\right)V_M\right] \end{aligned}$$

[64] Indeed, for any joint distribution of types, the quantity

$$\mathbb{E}[(s+b)\mathbf{1}\{\hat{s}-s \leq X\}] + \mathbb{E}[\hat{s}\mathbf{1}\{\hat{s}-s > X\}]$$

decreases with $X \geq b$.

We thus distinguish two cases.

(A) If $D_M(u) < \overline{s}$ for all $M \geq k$, then[65]

$$\begin{aligned}
T_M\Big(V(u), D(u)\Big) &= \mathbb{E}\Big[s\mathbf{1}\{\hat{s} - s \leq D_M\}\Big] + \mathbb{E}\Big[\hat{s}\mathbf{1}\{\hat{s} - s > D_M\}\Big] + \mathbb{P}(\hat{s} - s \leq D_M)D_M \\
&\qquad + \delta\left[\frac{M-1}{N-1}V_{M-1} + \left(1 - \frac{M-1}{N-1}\right)V_M\right] \\
&= h(D_M) + \delta\left[\frac{M-1}{N-1}V_{M-1} + \left(1 - \frac{M-1}{N-1}\right)V_M\right] \qquad (34)
\end{aligned}$$

Consequently, if $D_M(u) < \overline{s}$ [66], plugging the above expressions in the equality $\Upsilon_M(u) = T_{M+1}(V, D) - T_M(V, D)$, and using the expression of D_M as a function of u, yields

$$\begin{aligned}
\Upsilon_M(u) &= h(D_{M+1}) - h(D_M) + \delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right] \\
&= h(D_{M+1}) + h_1(D_M) - b \qquad (35)
\end{aligned}$$

Since $u \in K$, we have that (i) $u_M \geq 0$ for any M and thus by construction $D_M \geq b$, and (ii) the sequence $(u_M)_M$ is decreasing, and thus so is the sequence $(D_M)_M$. As a consequence, $D_M \geq D_{M+1} \geq b$.

Henceforth, we restrict our attention to the set $\mathcal{G}$ of joint distributions such that the functions h_1 and $(h - h_1)$ are strictly increasing over $[b, +\infty) \cap \text{Supp}(\hat{s} - s)$ [67]. This set notably includes the set of continuous joint symmetric distributions[68], as well as the case where the majority candidate has a fixed type $s \geq 0$ and the minority candidate a type $s + D$ where D is a (full support) random variable with a continuously differentiable

[65] Note that in this case the mapping T can be defined as $T : \big[0, V_{k-1} + k\overline{u}\big]^k \times \big[b, b + \overline{u}\big]^k \longrightarrow \big[0, V_{k-1} + k\overline{u}\big]^k$.

[66] By monotonicity (as $u \in K$), $D_M(u) < \overline{s}$ implies that $D_{M'} < \overline{s}$ for any $M' > M$.

[67] Note that $(h - h_1)$ being strictly increasing implies that h is strictly increasing, as $h(X) - h_1(X) = 2[h(X) - X/2]$.

[68] Indeed, letting F be the marginal c.d.f. of s and $\hat{s}$, then

$$\forall \Delta > 0, \qquad h(\Delta) = \int_0^{\overline{s}} (s + \Delta)F(s + \Delta)dF(s) + \int_\Delta^{\overline{s}} \hat{s}F(\hat{s} - \Delta)dF(\hat{s}),$$

and thus, for any $\Delta \in (0, \overline{s})$,

$$h'(\Delta) = \int_0^{\overline{s}} F(s + \Delta)dF(s) + \int_0^{\overline{s}-\Delta} (s + \Delta)f(s + \Delta)dF(s) - \int_\Delta^{\overline{s}} \hat{s}f(\hat{s} - \Delta)dF(\hat{s}) = \int_0^{\overline{s}} F(s + \Delta)dF(s),$$

and thus $h'(\Delta) \in (1/2, 1)$ since $\int_0^{\overline{s}} F(s)dF(s) = 1/2$.

distribution over $(-s,s)$ symmetric around 0.[69]

As a consequence, for any $u \in K$, $\Upsilon_M(u) \geq 0$ and the sequence $(\Upsilon_M(u))_{M\geq k}$ is decreasing as it inherits the monotonicity of the sequence $(D_M)_M$. Moreover, for any $M \geq k$,

$$\Upsilon_M(u) \leq h(D_M) + h_1(D_M) - b = \delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right] < \delta\frac{N-2}{N-1}u_{k-1} \leq \overline{u}$$

It thus remains to check that $\Upsilon_{k-1}(u) \geq \Upsilon_k(u)$. By monotonicity of h and $(h - h_1)$ and using the above computations, a sufficient condition for this inequality to hold writes as:

$$(1-\delta)V_{k-1} \leq h(b).$$

This condition imposes an upper bound on V_{k-1}. Recall that $h(b)$ is the highest flow payoff (quality and homophily) that a majority member can guarantee. Therefore, for any symmetric joint distribution of types, any (increasing and concave) equilibrium value function must satisfy $V_{k-1} < h(b)/(1-\delta)$. Hence assuming this inequality hold does not threaten the existence of an equilibrium. We thus fix in the following V_{k-1} such that the above inequality holds. Hence, under the above conditions, $\Upsilon(u) \in K$.

(B) We now consider the case where $\overline{s} < +\infty$ and $D_M(u) = \overline{s}$ for some M. (Note that as $u_M \leq \overline{u} < \infty$, the case $D_M(u) = \overline{s}$ can only arise when $\overline{s} < \infty$.)

We first note that, within the class of equilibria with $u \in K$ (and thus a decreasing sequence $(\Delta_M)_M$), $\Delta_k = \overline{s}$ implies that $\Delta_{k+1} < \overline{s}$. Hence, whenever the majority is not tight, it recruits a minority candidate with a strictly positive probability: $\Delta_M < \overline{s}$ for any $M \geq k+1$.[70]

[69]Indeed, denoting by F the c.d.f. of D, we have for any $\Delta \in (0, \overline{s})$,

$$h(\Delta) = \int_{-s}^{\Delta}(s+\Delta)dF(D) + \int_{\Delta}^{s}(s+D)dF(D), \qquad \text{and thus} \qquad h'(\Delta) = F(\Delta) \in (1/2, 1)$$

[70]Indeed, suppose by contradiction that $\Delta_k = \Delta_{k+1} = \overline{s}$. Then, by construction,

$$u_k = \delta\left[\frac{k}{N-1}u_k + \frac{k-2}{N-1}u_{k+1}\right]$$

Since $u \in K$, this yields that $u_k = u_{k+1} = 0$, which contradicts the initial assumption as $b < \overline{s}$.

Consequently, we only need to consider the case where $D_{k+1}(u) < D_k(u) = \overline{s} < \infty$ [71]. We first show that $\Upsilon_k(u) \in [\Upsilon_{k+1}(u), \overline{u}]$. By construction,

$$T_k\Big(V(u), D(u)\Big) = \mathbb{E}[s] + b + \delta\left[\frac{k}{N-1}V_k + \left(1 - \frac{k}{N-1}\right)V_{k+1}\right],$$

and thus, since $D_{k+1} < \overline{s}$ implies that $T_{k+1}(V, D)$ is given by (34),

$$\Upsilon_k(u) = h(D_{k+1}) - \mathbb{E}[s] - b$$

By monotonicity of the sequence $(D_M)_M$ and since the functions h and h_1 are increasing, we have that $\Upsilon_k(u) \geq \Upsilon_{k+1}(u)$. It thus remains to check that $\Upsilon_{k-1}(u) \geq \Upsilon_k(u)$. A sufficient condition for this inequality to hold writes as[72]

$$(1-\delta)V_{k-1} \leq \mathbb{E}[s] + b + \frac{k}{N-2}(\overline{s} - b)$$

This second inequality is looser than the condition[73] in case (A) and is thus satisfied for

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[71]Indeed, note that if $D_{k+1}(u) < \overline{s}$, then $D_{k+1}(\Upsilon(u)) < \overline{s}$ as

$$\begin{aligned} D_{k+1}(\Upsilon(u)) &< b + \delta\left[\frac{k}{N-1}\Big(h(D_{k+1}(u)) - \mathbb{E}[s] - b\Big) + \frac{k-2}{N-1}\Big(h(D_{k+2}(u) + h_1(D_{k+1}(u)) - b\Big)\right] \\ &< \left(1 - \delta\frac{N-2}{N-1}\right)b + \delta\left[\frac{k}{N-1}\big(h(D_{k+1}(u)) - \mathbb{E}[s]\big) + \frac{k-2}{N-1}D_{k+1}(u)\right] \\ &< \left(1 - \delta\frac{N-2}{N-1}\right)b + \delta\frac{N-2}{N-1}\overline{s} < \overline{s} \end{aligned}$$

[72]Indeed, a sufficient condition for $\Upsilon_{k-1}(u) \geq \Upsilon_k(u)$ is

$$2(\mathbb{E}[s] + b) - (1-\delta)V_{k-1} + \delta u_{k-1} \geq h\left(b + \delta\frac{N-2}{N-1}u_k\right) - \delta\frac{k-1}{N-1}u_k,$$

which by monotonicity of h and $h - h_1$ holds in particular if

$$\begin{aligned} & 2(\mathbb{E}[s] + b) - (1-\delta)V_{k-1} + \delta u_{k-1} \geq h\left(b + \delta\frac{N-2}{N-1}u_{k-1}\right) - \delta\frac{k-1}{N-1}u_{k-1}, \\ \text{i.e.} \quad & (1-\delta)V_{k-1} \leq 2(\mathbb{E}[s] + b) - h\left(b + \delta\frac{N-2}{N-1}u_{k-1}\right) + \delta\left(1 + \frac{k-1}{N-1}\right)u_{k-1} \end{aligned}$$

Hence, by monotonicity of $X \mapsto X - h(X)$ and since u_{k-1} must satisfy $\delta(N-2)/(N-1)u_{k-1} \geq (\overline{s} - b)$, a sufficient condition for this inequality to hold is

$$(1-\delta)V_{k-1} \leq 2(\mathbb{E}[s] + b) - h(\overline{s}) + (\overline{s} - b) + \frac{k}{N-2}(\overline{s} - b),$$

which yields the result as $h(\overline{s}) = \mathbb{E}[s] + \overline{s}$.

[73]Indeed, for any joint distribution such that $(\hat{s} - s)$ is symmetrically distributed around 0,

$$h(b) \leq \mathbb{E}[s] + b + \frac{k}{N-2}(\overline{s} - b)$$

$V_{k-1} \leq h(b)/(1-\delta)$ (which must be the case in any equilibrium as discussed above).

Therefore, fixing $V_{k-1} \in [0, h(b)/(1-\delta)]$, Υ is a well-defined continuous mapping from K into itself. By Brouwer's fixed point theorem, it admits a fixed point.

Moreover, as the functions h and h_1 are strictly increasing over $(b, \bar{s})$, Υ is order-preserving on the complete lattice K. Hence, fixing $V_{k-1} \in [0, h(b)/(1-\delta)]$, by Tarski's fixed point theorem, there exists a unique least fixed point of Υ, $u^*(V_{k-1})$. We select this least fixed point for the equilibrium condition.

By construction, the mapping $(V_{k-1}, u) \mapsto \Upsilon_{V_{k-1}}(u)$ is continuous on $[0, h(b)/(1-\delta)] \times K$. Hence, since Υ is order-preserving, the mapping $V_{k-1} \mapsto u^*(V_{k-1})$ is continuous.

Given u^* and the corresponding sequence of cutoffs $D(u^*)$, define for $M \geq k+1$,

$$V^*_M(V_{k-1}) = V^*_{M-1}(V_{k-1}) + u^*_{M-1}(V_{k-1}),$$

and $V^*_k(V_{k-1}) = V_{k-1} + u^*_{k-1}(V_{k-1})$. Then, using the recursive expression of the value function for group size $i \leq k-1$, define $(V^*_i(V_{k-1}))_{i \leq k-1}$ such that for all $i \leq k-1$,

$$\begin{aligned} V^*_i = {} & \mathbb{E}[s\mathbf{1}\{\hat{s} - s \leq D_{N-1-i}(u^*)\}] + \mathbb{E}[(\hat{s}+b)\mathbf{1}\{\hat{s} - s > D_{N-1-i}(u^*)\}] \\ & + \delta\mathbb{P}(\hat{s} - s \leq D_{N-1-i}(u^*))\left[\frac{i-1}{N-1}V^*_{i-1} + \left(1 - \frac{i-1}{N-1}\right)V^*_i\right] \\ & + \delta\Big(1 - \mathbb{P}(\hat{s} - s \leq D_{N-1-i}(u^*))\Big)\left[\frac{i}{N-1}V^*_i + \left(1 - \frac{i}{N-1}\right)V^*_{i+1}\right]. \end{aligned}$$

By construction, the mapping defined from $[0, h(b)/(1-\delta)]$ into itself that assigns to $V_{k-1} \in [0, h(b)/(1-\delta)]$ the value $V^*_{k-1}(V_{k-1})$ is continuous. By Brouwer's fixed-point theorem, it admits a fixed point. This establishes existence.

We now show that any equilibrium characterized by a sequence of cut-offs $(\Delta_M)_{M \geq k}$ is such that (a) $\Delta_M > b$ for any $M \geq k$, and (b) the sequence $(\Delta_M)_M$ is strictly decreasing.

(a) We first argue that in any equilibrium, $\Delta_M > b$ for any $M \geq k$. We show this by downward induction. Suppose that $\Delta_{N-1} \leq b$. Then[74], this implies that $u_{N-2} \leq 0$, i.e. $V_{N-2} \geq V_{N-1}$. Hence the continuation payoff for a majority of size $N-1$ is bounded below by δV_{N-1}. By deviating from Δ_{N-1} to the value that maximizes the flow payoff, a

[74]Using that by construction,

$$\Delta_{N-1} = b + \delta\frac{N-2}{N-1}u_{N-2}$$

majority with size $N-1$ gets a utility greater than

$$\max_X \left\{ \mathbb{E}[(s+b)\mathbf{1}\{\hat{s}-s \leq X\}] + \mathbb{E}[\hat{s}\mathbf{1}\{\hat{s}-s > X\}] \right\} + \delta V_{N-1}$$

Hence, this would imply that

$$(1-\delta)V_{N-1} \geq \max_X \left\{ \mathbb{E}[(s+b)\mathbf{1}\{\hat{s}-s \leq X\}] + \mathbb{E}[\hat{s}\mathbf{1}\{\hat{s}-s > X\}] \right\}$$

which is a contradiction as the RHS is the highest attainable flow payoff (and $\delta > 0$).[75] Therefore $V_{N-1} > V_{N-2}$, and thus $\Delta_{N-1} > b$. Suppose now that $V_{M'+1} > V_{M'}$ for any $M' \geq M$, and that $V_M \leq V_{M-1}$. Therefore, the continuation payoff for a majority of size M is bounded below by δV_M. Hence, by deviating from Δ_M to the value that maximizes the flow payoff, a majority with size M gets a utility greater than

$$\max_X \left\{ \mathbb{E}[(s+b)\mathbf{1}\{\hat{s}-s \leq X\}] + \mathbb{E}[\hat{s}\mathbf{1}\{\hat{s}-s > X\}] \right\} + \delta V_M,$$

which again leads to a contradiction. Consequently, $u_{M-1} > 0$, while $u_M > 0$ by the induction hypothesis. Hence, since by construction we have that either $\Delta_M = \overline{s} > b$, or

$$\Delta_M = b + \delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right], \tag{36}$$

this implies that $\Delta_M > b$. By induction, the inequality holds for any majority size $M \geq k$.

(b) We thus show that the sequence $(\Delta_M)_{M\geq k}$ is strictly decreasing. We first consider the case where for any $M \geq k$, $\Delta_M < \overline{s}$, and therefore (36) holds, and

$$u_M = h(\Delta_{M+1}) + \Delta_M - h(\Delta_M) - b \tag{37}$$

Suppose by contradiction that $\Delta_{N-1} \geq \Delta_{N-2}$. By the above equations,

$$\begin{aligned}
\Delta_{N-1} &= b + \delta\frac{N-2}{N-1}u_{N-2} \\
&= \left(1 - \delta\frac{N-2}{N-1}\right)b + \delta\frac{N-2}{N-1}\Big[h(\Delta_{N-1}) + \Delta_{N-2} - h(\Delta_{N-2})\Big] \\
&\leq \left(1 - \delta\frac{N-2}{N-1}\right)b + \delta\frac{N-2}{N-1}\Delta_{N-1}
\end{aligned}$$

[75] Indeed, the above inequality holds only if $\Delta_M = \arg\max_X \left\{\mathbb{E}[(s+b)\mathbf{1}\{\hat{s}-s \leq X\}] + \mathbb{E}[\hat{s}\mathbf{1}\{\hat{s}-s > X\}]\right\}$ for all $M \geq k$, and $V_i = V_{N-1}$ for all $i \in \{1, ..., N-2\}$, which is impossible.

where the inequality derives from the strict monotonicity of h_1. Hence $\Delta_{N-1} \leq b$, which contradicts the above result. Therefore $\Delta_{N-1} < \Delta_{N-2}$. We henceforth proceed by induction. Suppose $\Delta_{M'+1} < \Delta_{M'}$ for any $M' \geq M$, and suppose by contradiction that $\Delta_M \geq \Delta_{M-1}$. By (37), using the monotonicity of h_1, we have that

$$u_M < \Delta_M - b, \qquad \text{and} \qquad u_{M-1} \leq \Delta_M - b,$$

and therefore,

$$\Delta_M < \left(1 - \delta\frac{N-2}{N-1}\right)b + \delta\frac{N-2}{N-1}\Delta_M,$$

i.e. $\Delta_M < b$, which is a contradiction. Hence for any $M \geq k$, $\Delta_{M+1} < \Delta_M$, as was to be shown.

We now consider the case where there exists $M \geq k$ such that $\Delta_M = \bar{s}$. This implies that $\Delta_{M+1} < \bar{s}$ as otherwise the explicit expressions of V_M and V_{M+1} would give that

$$\delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right] = 0, \qquad \text{and thus} \qquad \Delta_M = b < \bar{s},$$

which is a contradiction. Hence suppose by contradiction that $\Delta_{N-1} = \bar{s}$, then $\Delta_{N-2} < \bar{s} = \Delta_{N-1}$. Yet the above computations[76] thus yield that $\Delta_{N-1} \leq b < \bar{s}$, which is a contradiction. Therefore, $\Delta_{N-1} < \bar{s}$, and as a consequence, the above computations yield that $\Delta_{N-2} > \Delta_{N-1}$. We again proceed by induction. Suppose $\Delta_{M'+1} < \Delta_{M'}$ for any $M' \geq M$. If $\Delta_M < \bar{s}$, the above computations apply, yielding that $\Delta_M < \Delta_{M-1}$. Hence, suppose by contradiction that $\Delta_M = \bar{s} \geq \Delta_{M-1}$. As noted above, this implies that $\Delta_{M-1} < \bar{s}$ and (37) holds in $M-1$, and thus $u_{M-1} \leq \Delta_M - b$. Moreover, using the explicit expressions of V_{M+1} and V_M,

$$\begin{aligned} u_M &= h(\Delta_{M+1}) - h(\Delta_M) + \delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right] \\ &< \delta\left[\frac{M-1}{N-1}u_{M-1} + \left(1 - \frac{M}{N-1}\right)u_M\right] \end{aligned}$$

where the inequality follows from the monotonicity of h. Therefore, $u_M < u_{M-1}$. As a

[76]Using that as $\Delta_{N-1} = \bar{s}$,

$$\Delta_{N-1} \leq b + \delta\frac{N-2}{N-1}u_{N-2} \leq \left(1 - \delta\frac{N-2}{N-1}\right)b + \delta\frac{N-2}{N-1}\Delta_{N-1}$$

consequence,

$$\begin{aligned}\Delta_M = \overline{s} \leq b + \delta\bigg[\frac{M-1}{N-1}u_{M-1} + \bigg(1 - \frac{M}{N-1}\bigg)u_M\bigg] \\ < b + \delta\frac{N-2}{N-1}u_{M-1} < \bigg(1 - \delta\frac{N-2}{N-1}\bigg)b + \delta\frac{N-2}{N-1}\Delta_M,\end{aligned}$$

i.e. $\Delta_M < b < \overline{s}$, which is a contradiction. Hence, for any $M \geq k$, $\Delta_{M+1} < \Delta_M$, as was to be shown.

We then turn to showing that equilibria can be ranked from more to less meritocratic. Consider the class of equilibria characterized by a decreasing decision rule $(\Delta_M)_{M\in\{k,\dots,N-1\}}$. We refer in the following to an equilibrium by its decision rule $\Delta \equiv (\Delta_M)_{M\in\{k,\dots,N-1\}}$. Let Δ and Δ' be two equilibria within this class. We now show that

(i) $\Delta_k < \Delta'_k$ implies that $\Delta_M < \Delta'_M$ for any $M \geq k+1$,

(ii) $\Delta_k = \Delta'_k \in [b, \overline{s}]$ implies that $\Delta_M = \Delta'_M < \overline{s}$ for any $M \geq k+1$,

(i) Assume that $\Delta_k < \Delta'_k < \overline{s}$ (computations are analogous in the case $\Delta_k < \Delta'_k = \overline{s}$). By monotonicity, $\Delta_M < \overline{s}$ and $\Delta'_M < \overline{s}$ for any $M \geq k+1$, and thus, with the above notation,

$$\begin{aligned}\Delta_M &= b + \delta\bigg[\frac{M-1}{N-1}u_{M-1} + \bigg(1 - \frac{M}{N-1}\bigg)u_M\bigg] \\ &= \bigg(1 - \delta\frac{N-2}{N-1}\bigg)b + \delta\bigg[\frac{M-1}{N-1}\Big[h(\Delta_M) + h_1(\Delta_{M-1})\Big] + \bigg(1 - \frac{M}{N-1}\bigg)\Big[h(\Delta_{M+1}) + h_1(\Delta_M)\Big]\bigg]\end{aligned}$$

Consequently, for any $M \geq k+1$,

$$\begin{aligned}&h_{2,M}(\Delta_M) - h_{2,M}(\Delta'_M) \\ &\quad= \delta\frac{M-1}{N-1}\Big[h_1(\Delta_{M-1}) - h_1(\Delta'_{M-1})\Big] + \delta\bigg(1 - \frac{M}{N-1}\bigg)\Big[h(\Delta_{M+1}) - h(\Delta'_{M+1})\Big] \quad (38)\end{aligned}$$

where the function $h_{2,M}$ is given by

$$h_{2,M}(X) \equiv X - \delta\frac{M-1}{N-1}h(X) - \delta\bigg(1 - \frac{M}{N-1}\bigg)h_1(X),$$

We note that $h_{2,M}$ is strictly increasing over $[b, \overline{s}]$ [77]. By monotonicity of h_1, we get for $M = k + 1$ that

$$h_{2,k+1}(\Delta_{k+1}) - h_{2,k+1}(\Delta'_{k+1}) < \delta\left(1 - \frac{k+1}{N-1}\right)\left[h(\Delta_{k+2}) - h(\Delta'_{k+2})\right]$$

Suppose by contradiction that $\Delta_{k+1} \geq \Delta'_{k+1}$. Then by monotonicity, $\Delta_{k+2} \geq \Delta'_{k+2}$. By summing Equation (38) in $k + 1$ and $k + 2$ and rearranging, we get that

$$\begin{aligned}
&\left[h_{2,k+1}(\Delta_{k+1}) - \delta\frac{k+1}{N-1}h_1(\Delta_{k+1})\right] - \left[h_{2,k+1}(\Delta'_{k+1}) - \delta\frac{k+1}{N-1}h_1(\Delta'_{k+1})\right] \\
&\qquad + \left[h_{2,k+2}(\Delta_{k+2}) - \delta\frac{k-2}{N-1}h(\Delta_{k+2})\right] - \left[h_{2,k+2}(\Delta'_{k+2}) - \delta\frac{k-2}{N-1}h(\Delta'_{k+2})\right] \\
&= \delta\frac{k}{N-1}\left[h_1(\Delta_k) - h_1(\Delta'_k)\right] + \delta\left(1 - \frac{k+2}{N-1}\right)\left[h(\Delta_{k+3}) - h(\Delta'_{k+3})\right]
\end{aligned}$$

Since for any $M \geq k+1$, the functions $h_{2,M} - \delta\frac{M}{N-1}h_1$ and $h_{2,M} - \delta\frac{N-M}{N-1}h$ are strictly increasing over $[b, \overline{s}]$, the above equality implies that $\Delta_{k+3} \geq \Delta'_{k+3}$. We now proceed by induction: suppose that $\Delta_j \geq \Delta'_j$ for any $j \in \{k+1, ..., M\}$. Then by summing Equation (38) over the indices $k + 1, ..., M$ and rearranging,

$$\begin{aligned}
&\left[h_{2,k+1}(\Delta_{k+1}) - \delta\frac{k+1}{N-1}h_1(\Delta_{k+1})\right] - \left[h_{2,k+1}(\Delta'_{k+1}) - \delta\frac{k+1}{N-1}h_1(\Delta'_{k+1})\right] \\
&\qquad + \left[h_{2,M}(\Delta_M) - \delta\frac{N-M}{N-1}h(\Delta_M)\right] - \left[h_{2,M}(\Delta'_M) - \delta\frac{N-M}{N-1}h(\Delta'_M)\right] \\
&\qquad + \sum_{j=k+2}^{M-1}\left(\left[h_{2,j}(\Delta_j) - \delta\frac{j}{N-1}h_1(\Delta_j) - \delta\frac{N-j}{N-1}h(\Delta_j)\right]\right. \\
&\qquad\qquad \left. - \left[h_{2,j}(\Delta'_j) - \delta\frac{j}{N-1}h_1(\Delta'_j) - \delta\frac{N-j}{N-1}h(\Delta'_j)\right]\right) \\
&= \delta\frac{k}{N-1}\left[h_1(\Delta_k) - h_1(\Delta'_k)\right] + \delta\left(1 - \frac{M}{N-1}\right)\left[h(\Delta_{M+1}) - h(\Delta'_{M+1})\right]
\end{aligned}$$

Since for any $j \geq k + 1$, the functions $h_{2,j} - \delta\frac{j}{N-1}h_1 - \delta\frac{N-j}{N-1}h$ are strictly increasing over $[b, \overline{s}]$, we get that $\Delta_{M+1} \geq \Delta'_{M+1}$. Hence by induction, we have that $\Delta_M \geq \Delta'_M$ for

[77] Indeed, we may rewrite the function $h_{2,M}$ as: $h_{2,M}(X) = \left[1 - \delta\left(1 - \frac{M}{N-1}\right)\right]h_1(X) + \left[1 - \delta\frac{M-1}{N-1}\right]h(X)$.

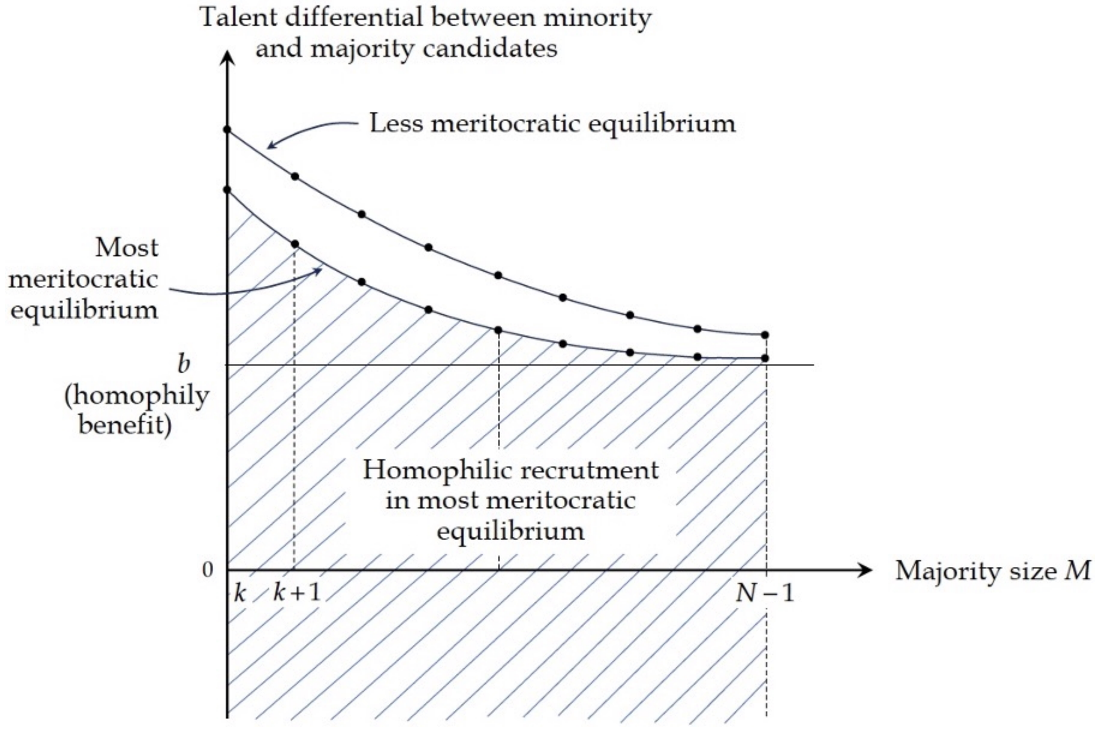

Figure 5: Decision rules for two distinct equilibria.

any $M \geq k+1$. But by summing (38) over all these indices and rearranging yields

$$\begin{aligned} 0 \leq & \left[h_{2,k+1}(\Delta_{k+1}) - \delta\frac{k+1}{N-1}h_1(\Delta_{k+1})\right] - \left[h_{2,k+1}(\Delta'_{k+1}) - \delta\frac{k+1}{N-1}h_1(\Delta'_{k+1})\right] \\ & + \left[h_{2,N-1}(\Delta_{N-1}) - \delta\frac{1}{N-1}h(\Delta_{N-1})\right] - \left[h_{2,N-1}(\Delta'_{N-1}) - \delta\frac{1}{N-1}h(\Delta'_{N-1})\right] \\ & + \sum_{j=k+2}^{N-2}\Bigg(\left[h_{2,M}(\Delta_M) - \delta\frac{M}{N-1}h_1(\Delta_M) - \delta\frac{N-M}{N-1}h(\Delta_M)\right] \\ & \qquad - \left[h_{2,M}(\Delta'_M) - \delta\frac{M}{N-1}h_1(\Delta'_M) - \delta\frac{N-M}{N-1}h(\Delta'_M)\right]\Bigg) \\ = & \, \delta\frac{k}{N-1}\left[h_1(\Delta_k) - h_1(\Delta'_k)\right] < 0 \end{aligned}$$

which is a contradiction. Therefore, $\Delta_{k+1} < \Delta'_{k+1}$. The result then obtains by induction, supposing by contradiction that $\Delta_j < \Delta'_j$ for any $j \in \{k, ..., M-1\}$ and that $\Delta_M \geq \Delta'_M$, and considering the sums of (38) over appropriate indices so as to reach a contradiction. (ii) We note that the above argument yields that if $\Delta_k = \Delta'_k \in [b, \bar{s}]$, then $\Delta_M = \Delta'_M$ for any $M \geq k+1$. As a consequence, any two distinct equilibria with a decreasing decision rule satisfy either "$\Delta_M < \Delta'_M$ for all $M \geq k$", or "$\Delta_M > \Delta'_M$ for all $M \geq k$". Figure 5 provides an illustration.

Non-ergodic welfare. Lastly, we turn to comparing the equilibria in terms of non-ergodic welfare. Consider two equilibria described by a decreasing decision rule denoted respectively by Δ and Δ' such that $\Delta \prec \Delta'$, and let $(V_i)_{i\in\{1,...,N-1\}}$ and $(V_i')_{i\in\{1,...,N-1\}}$ be the corresponding equilibrium value functions. For any $M \geq k$, we have by construction that

$$\begin{aligned} V_M = &\mathbb{E}[(s+b)\mathbf{1}\{\hat{s}-s \leq \Delta_M\}] + \mathbb{E}[\hat{s}\mathbf{1}\{\hat{s}-s > \Delta_M\}] \\ &+ \delta\mathbb{P}(\hat{s}-s\leq\Delta_M)\left[\frac{M}{N-1}V_M + \left(1-\frac{M}{N-1}\right)V_{M+1}\right] \\ &+ \delta\Big(1-\mathbb{P}(\hat{s}-s\leq\Delta_M)\Big)\left[\frac{M-1}{N-1}V_{M-1} + \left(1-\frac{M-1}{N-1}\right)V_M\right] \end{aligned}$$

We first note that $\Delta_k < \Delta_k'$ implies that $\Delta_k < \overline{s}$. Hence, for any $M \geq k$,

$$\Delta_M = b + \delta\left[\frac{M-1}{N-1}u_{M-1} + \frac{N-M-1}{N-1}u_M\right],$$

and therefore, for any $M \geq k$,

$$\begin{aligned} &\left[1-\delta\left(1-\frac{M-1}{N-1}\right)\Big[1-\mathbb{P}(\hat{s}-s\leq\Delta_M')\Big] - \delta\frac{M}{N-1}\mathbb{P}(\hat{s}-s\leq\Delta_M')\Big)\right](V_M - V_M') \\ &= \mathbb{E}\Big[(\hat{s}-s-\Delta_M)\mathbf{1}\{\Delta_M < \hat{s}-s\leq\Delta_M'\}\Big] + \delta\mathbb{P}(\hat{s}-s\leq\Delta_M')\left(1-\frac{M}{N-1}\right)(V_{M+1}-V_{M+1}') \\ &\quad + \delta\Big(1-\mathbb{P}(\hat{s}-s\leq\Delta_M')\Big)\frac{M-1}{N-1}(V_{M-1}-V_{M-1}') \end{aligned} \tag{39}$$

Two cases arise depending on whether $\Delta_k' = \overline{s}$. If so, then the result for majority members follows by the usual argument (by contradiction and by induction). Hence, for any $\delta \in [0,(N-1)/N)$, any "meritocratic" equilibrium (i.e. with $\Delta_k < \overline{s}$) is preferred at any majority size by all majority members to the basic-entrenchment equilibrium ($\Delta_k' = \overline{s}$).

If $\Delta_k' < \overline{s}$, we need to adapt the arguments in the proof of Lemma 2 and Proposition 3. Suppose by contradiction that $V_{N-1} \leq V_{N-1}'$. Then equation (39) implies that $V_{N-2} - V_{N-2}' \leq V_{N-1} - V_{N-1}' \leq 0$, and thus by induction that $V_{k-1} - V_{k-1}' \leq V_k - V_k' \leq V_{k+1} - V_{k+1}' \leq ... \leq V_{N-1} - V_{N-1}' \leq 0$. However, since $\Delta_k < \Delta_k' < \overline{s}$, we have that

$$b + \delta\frac{k-1}{N-1}\Big(V_{k+1}-V_{k-1}\Big) < b + \delta\frac{k-1}{N-1}\Big(V_{k+1}'-V_{k-1}'\Big),$$

and thus, $V_{k-1} - V_{k-1}' > V_{k+1} - V_{k+1}'$, which contradicts the above inequality. Hence,

$V_{N-1} \geq V'_{N-1}$, and (39) implies by induction that $V_{k-1} - V'_{k-1} \geq V_k - V'_k \geq ... \geq V_{N-1} - V'_{N-1} \geq 0$. Therefore, a more meritocratic equilibrium is preferred at any majority size by all majority members to a less meritocratic equilibrium.

Similarly, for any $i \leq k-1$, we have by construction that

$$\begin{aligned} V_i = & \mathbb{E}[s\mathbf{1}\{\hat{s} - s \leq \Delta_{N-1-i}\}] + \mathbb{E}[(\hat{s} + b)\mathbf{1}\{\hat{s} - s > \Delta_{N-1-i}\}] \\ & + \delta\mathbb{P}(\hat{s} - s \leq \Delta_{N-1-i})\left[\frac{i-1}{N-1}V_{i-1} + \left(1 - \frac{i-1}{N-1}\right)V_i\right] \\ & + \delta\Big(1 - \mathbb{P}(\hat{s} - s \leq \Delta_{N-1-i})\Big)\left[\frac{i}{N-1}V_i + \left(1 - \frac{i}{N-1}\right)V_{i+1}\right] \end{aligned}$$

Hence, for any $i \leq k-1$,

$$\begin{aligned} & \left[1 - \delta\left(1 - \frac{i-1}{N-1}\right)\mathbb{P}(\hat{s} - s \leq \Delta'_{N-1-i}) - \delta\frac{i}{N-1}\Big[1 - \mathbb{P}(\hat{s} - s \leq \Delta'_{N-1-i})\Big]\right](V_i - V'_i) \\ & = \mathbb{E}\left[\left[\hat{s} - s + b + \delta\left(\frac{i-1}{N-1}u_{i-1} + \frac{N-1-i}{N-1}u_i\right)\right]\mathbf{1}\{\Delta_{N-1-i} < \hat{s} - s \leq \Delta'_{N-1-i}\}\right] \\ & \quad + \delta\mathbb{P}(\hat{s} - s \leq \Delta'_{N-1-i})\frac{i-1}{N-1}(V_{i-1} - V'_{i-1}) \\ & \quad + \delta\Big(1 - \mathbb{P}(\hat{s} - s \leq \Delta'_{N-1-i})\Big)\left(1 - \frac{i}{N-1}\right)(V_{i+1} - V'_{i+1}) \end{aligned} \tag{40}$$

Hence, for δ close to 0, the expectation term on the RHS of (40) is strictly positive, and thus $V_i > V'_i$ for all $i \in \{1, ..., k-1\}$.

N Complements on non-linear homophily benefits

A non-linear homophily benefit does not require enlarging the state space, as the size of the majority is still a sufficient statistics looking forward. (To alleviate the notation, as we consider nonlinear yet symmetric benefits, we omit the horizontal-group subscript $X \in \{A, B\}$.)

Let $\tilde{\mathcal{B}}(i)$ denote the per-period homophily benefit enjoyed by a member whose in-group has size i (thus, in the linear case, $\tilde{\mathcal{B}}(i) \equiv (i-1)\tilde{b}$). In this Section – and only in this Section –, we change the definition of the value function: let now V_i be the forward-looking discounted sum of future homophily and quality payoff for a member with in-group size i *net of the quality stock alone* (and not of the homophily stock). Indeed, the current quality stock (sum of members' talent) is still irrelevant looking forward, and we thus take it out of the value function to alleviate the expressions. By contrast, the current homophily stock (majority size) affects the incremental lifetime homophily contribution of a new in-group member.

With this new definition, the (forward-looking net-of-quality-stock) continuation value function of a majority member at majority size M is given by

$$\tilde{\mathcal{B}}(M+1) + s_M + \delta\left[\frac{M}{N-1}V_M + \left(1 - \frac{M}{N-1}\right)V_{M+1}\right]$$

if the majority recruits the majority candidate with talent $s_M \in \{0, s\}$, and by

$$\tilde{\mathcal{B}}(M) + s_m + \delta\left[\frac{M-1}{N-1}V_{M-1} + \left(1 - \frac{M-1}{N-1}\right)V_M\right]$$

if the majority recruits the minority candidate with talent $s_m \in \{0, s\}$.

Proposition N.1. ***(Non-linear homophily benefits)***

(i) With strictly concave homophily benefits $\tilde{\mathcal{B}}$, symmetric MPEs are still either meritocratic or basically-entrenched if $\tilde{\mathcal{B}}(k+1) - \tilde{\mathcal{B}}(k) \leq \tilde{s}$, and are super-entrenched if $\tilde{\mathcal{B}}(k+2) - \tilde{\mathcal{B}}(k+1)$ is sufficiently large.

(ii) With strictly convex homophily benefits, when there exists a threshold size such that $\tilde{\mathcal{B}}(M+1) - \tilde{\mathcal{B}}(M) < \tilde{s}$ for any M below the threshold and $\tilde{\mathcal{B}}(M+1) - \tilde{\mathcal{B}}(M) > \tilde{s}$ for any M above the threshold,[78] then in any symmetric MPE, there exists a threshold size such that recruitments are entrenched for majority sizes above the threshold. In any equilibrium, on path, a group eventually forms a fully-entrenched majority.

With convex homophily benefits such as considered in claim (ii) of Proposition N.1, and a low discount factor, thin majorities are meritocratic, while large ones are entrenched. In other words, larger majorities then discriminate more than thinner ones.

N.0.1 Proof of Proposition N.1: Concave homophily benefits

Let $\tilde{\mathcal{B}}(i)$ be strictly concave in the number of in-group members i, i.e. $\tilde{\mathcal{B}}(i+1) - \tilde{\mathcal{B}}(i)$ be strictly decreasing in i. Suppose $\tilde{\mathcal{B}}(k+1) - \tilde{\mathcal{B}}(k) \leq \tilde{s}$. Hence,

$$0 \leq \tilde{\mathcal{B}}(N) - \tilde{\mathcal{B}}(N-1) < \tilde{\mathcal{B}}(N-1) - \tilde{\mathcal{B}}(N-2) < ... < \tilde{\mathcal{B}}(k+1) - \tilde{\mathcal{B}}(k) \leq \tilde{s}. \quad (41)$$

[78] The threshold can be equal to $N-1$ for δ sufficiently low, and must be weakly below $N-2$ otherwise.

The proof follows from arguments similar to the ones in the proof of Lemma 1 (see Online Appendix A). Consider first a given value of V_{k-1} (in a well-chosen set), and the majority's optimal control problem given V_{k-1}. Let $(V_M(V_{k-1}))_{M\geq k}$ be the (unique) solution to this problem, i.e. such that for all $M \geq k$, the Bellman equation holds:

$$V_M = \mathbb{E}_{\tilde{v}_M,\tilde{w}_M}\Bigg[\max\Bigg\{\tilde{w}_M + \delta\bigg(\frac{M}{N-1}V_M + \bigg(1-\frac{M}{N-1}\bigg)V_{M+1}\bigg)\bigg),$$
$$\tilde{v}_M + \delta\bigg(\frac{M-1}{N-1}V_{M-1} + \bigg(1-\frac{M-1}{N-1}\bigg)V_M\bigg)\Bigg\}\Bigg],$$

where $\tilde{v}_M \in \{\tilde{\mathcal{B}}(M), \tilde{\mathcal{B}}(M)+s\}$ (resp. $\tilde{w}_M \in \{\tilde{\mathcal{B}}(M+1), \tilde{\mathcal{B}}(M+1)+s\}$) is the flow-homophily and flow-and-future-quality value to a majority member when a minority (resp. majority) candidate is recruited at majority size M.

Consider then $M = k$, and suppose that:

$$\delta\frac{k-1}{N-1}\Big(V_{k+1}(V_{k-1}) - V_{k-1}\Big) > s - \Big(\tilde{\mathcal{B}}(k+1) - \tilde{\mathcal{B}}(k)\Big),$$

i.e. that, by the Bellman equation at $M = k$,

$$V_k(V_{k-1}) = \tilde{\mathcal{B}}(k+1) + \overline{x}s + \delta\bigg[\frac{k}{N-1}V_k(V_{k-1}) + \frac{k-1}{N-1}V_{k+1}(V_{k-1})\bigg].$$

Computations analogous to the ones in the proof of Lemma A.1 (see Online Appendix A) then yield that the value function from the basic-entrenchment strategy satisfies the above recursive equation for V_k and the Bellman equations for $M \geq k+1$. Indeed, for any $M \geq k+1$, the value function with the basic-entrenchment strategies satisfies:

$$\begin{aligned}V^{\mathrm{e}}_{M+1} - V^{\mathrm{e}}_M =&(1-x)(\tilde{\mathcal{B}}(M+2) - \tilde{\mathcal{B}}(M+1)) + x(\tilde{\mathcal{B}}(M+1) - \tilde{\mathcal{B}}(M))\\ &+ \delta(1-x)\bigg[\frac{M}{N-1}(V^{\mathrm{e}}_{M+1} - V^{\mathrm{e}}_M) + \bigg(1-\frac{M+1}{N-1}\bigg)(V^{\mathrm{e}}_{M+2} - V^{\mathrm{e}}_{M+1})\bigg]\\ &+ \delta x\bigg[\frac{M-1}{N-1}(V^{\mathrm{e}}_M - V^{\mathrm{e}}_{M-1}) + \bigg(1-\frac{M}{N-1}\bigg)(V^{\mathrm{e}}_{M+1} - V^{\mathrm{e}}_M)\bigg]\end{aligned}$$

while for $M = k$,

$$\begin{aligned}V^{\mathrm{e}}_{k+1} - V^{\mathrm{e}}_k =&(1-x)\Big[\tilde{\mathcal{B}}(k+2) - \tilde{\mathcal{B}}(k+1)\Big] + xs\\ &+ \delta(1-x)\bigg[\frac{k}{N-1}(V^{\mathrm{e}}_{k+1} - V^{\mathrm{e}}_k) + \frac{k-2}{N-1}(V^{\mathrm{e}}_{k+2} - V^{\mathrm{e}}_{k+1})\bigg].\end{aligned}$$

Hence, using the same recursive technique as in Online Appendix A), $V^{\mathrm{e}}_{M+1} \geq V^{\mathrm{e}}_M$ for all $M \geq k$, which implies that the majority's recruitment of the in-group candidate whenever he is at least as talented as the out-group candidate is optimal at any $M \geq k$. Moreover, by the same type of argument, the sequence $(V^{\mathrm{e}}_{M+1} - V^{\mathrm{e}}_M)_{M\geq k}$ is decreasing.[79] Hence, by the same argument as in the proof of Lemma 1 (Online Appendix A.3),

$$\delta\left[\frac{k}{N-1}(V^{\mathrm{e}}_{k+1} - V^{\mathrm{e}}_k) + \frac{k-2}{N-1}(V^{\mathrm{e}}_{k+2} - V^{\mathrm{e}}_{k+1})\right] \leq \frac{\delta\frac{N-2}{N-1}}{1-\delta(1-x)\frac{N-2}{N-1}}\Big((1-x)\Big[\tilde{\mathcal{B}}(k+2) - \tilde{\mathcal{B}}(k+1)\Big] + xs\Big).$$

As a consequence, since by construction, $s = \tilde{s}/\big(1-\delta(N-2)/(N-1)\big)$ and by (41), $\tilde{s} > \tilde{\mathcal{B}}(k+2) - \tilde{\mathcal{B}}(k+1)$, the above inequality yields that:

$$\delta\left[\frac{k}{N-1}(V^{\mathrm{e}}_{k+1} - V^{\mathrm{e}}_k) + \frac{k-2}{N-1}(V^{\mathrm{e}}_{k+2} - V^{\mathrm{e}}_{k+1})\right] \leq s - \Big[\tilde{\mathcal{B}}(k+2) - \tilde{\mathcal{B}}(k+1)\Big].$$

Since the sequences $(\tilde{\mathcal{B}}(M+1) - \tilde{\mathcal{B}}(M))_{M\geq k}$ and $(V^{\mathrm{e}}_{M+1} - V^{\mathrm{e}}_M)_{M\geq k}$ are decreasing, we have for all $M \geq k+1$,

$$s - \Big(\tilde{\mathcal{B}}(M+1) - \tilde{\mathcal{B}}(M)\Big) \geq \delta\left[\frac{M-1}{N-1}(V^{\mathrm{e}}_M - V^{\mathrm{e}}_{M-1}) + \left(1 - \frac{M}{N-1}\right)(V^{\mathrm{e}}_{M+1} - V^{\mathrm{e}}_M)\right],$$

and thus the sequence (V^{e}_M) satisfies the Bellman equations for $M \geq k+1$.

Therefore, $V_M(V_{k-1}) = V^{\mathrm{e}}_M$ for all $M \geq k$. It can then be checked that the only strategy consistent with this value function is the one of basic-entrenchment.

Similarly, if on the opposite, the solution to the Bellman equations satisfies:

$$\delta\frac{k-1}{N-1}\Big(V_{k+1}(V_{k-1}) - V_{k-1}\Big) < s - \Big(\tilde{\mathcal{B}}(k+1) - \tilde{\mathcal{B}}(k)\Big), \tag{42}$$

the same arguments as the ones used above and in the proofs of Lemma 1 show that letting V^{m} denote the value function corresponding to the meritocratic strategies, $V_M(V_{k-1}) = V^{\mathrm{m}}_M(V_{k-1})$ for all $M \geq k$. And again, the meritocratic strategy is the only one consistent with this value function.

Lastly, in the meritocratic and in the basic-entrenchment equilibria, $V_{N-1} \geq ... \geq$

[79]The result can be shown by contradiction and by induction, proceeding as in the proof of Lemma 2 and using (41).

$V_{k+1} \geq V_k \geq V_{k-1}$. Hence in particular, a necessary condition for the equilibrium to be meritocratic, i.e., for inequality (42) to hold is:

$$\tilde{\mathcal{B}}(k+1) - \tilde{\mathcal{B}}(k) \leq s.$$

If instead, $\tilde{\mathcal{B}}(k+1) - \tilde{\mathcal{B}}(k) > s$, then meritocracy cannot be an equilibrium. Moreover, by the same logic, if $\tilde{\mathcal{B}}(k+2) - \tilde{\mathcal{B}}(k+1) > s$, basic entrenchment cannot be an equilibrium (as recruiting an untalented majority candidate against a talented minority candidate at majority size $k+1$ then yields a strictly profitable deviation), and only super-entrenchment can be an equilibrium.[80]

N.0.2 Proof of Proposition N.1: Convex homophily benefits

Let us note that, by considering flow incremental payoffs, the result clearly holds for any δ sufficiently low. In particular, for any δ sufficiently low, the unique equilibrium features meritocratic recruitments below the threshold, and entrenched ones above.

Let us now consider the general case ($\delta \in (0,1)$). Let $\tilde{\mathcal{B}}(\cdot)$ be convex, and let $\underline{M} \geq k$ be such that $\tilde{\mathcal{B}}(M+1) - \tilde{\mathcal{B}}(M) < \tilde{s}$ (resp. $> \tilde{s}$) for any $M < \underline{M}$ (resp. $M \geq \underline{M}$). Let us further assume that $\underline{M} \leq N-2$.

Consider a given value of V_{k-1} (in a well-chosen set), and let $(V_M(V_{k-1}))_{M \geq k}$ be the (unique) solution to the majority's optimal control problem given V_{k-1}, i.e. such that for all $M \geq k$, the Bellman equation holds:

$$\begin{aligned} V_M = \mathbb{E}_{v_M, w_M}\Bigg[\max\Bigg\{&\tilde{v}_M + \delta\Bigg(\frac{M}{N-1}V_M + \Bigg(1 - \frac{M}{N-1}\Bigg)V_{M+1}\Bigg)\Bigg), \\ &\tilde{w}_M + \delta\Bigg(\frac{M-1}{N-1}V_{M-1} + \Bigg(1 - \frac{M-1}{N-1}\Bigg)V_M\Bigg)\Bigg\}\Bigg], \end{aligned}$$

where $\tilde{v}_M \in \{\tilde{\mathcal{B}}(M), \tilde{\mathcal{B}}(M)+s\}$ (resp. $\tilde{w}_M \in \{\tilde{\mathcal{B}}(M+1), \tilde{\mathcal{B}}(M+1)+s\}$).

Consider the strategy consisting in always recruiting the majority candidate at any majority size $M \geq \underline{M}$ (entrenched recruitments), denoting by $(V^*_M)_{M \geq \underline{M}}$ its induced value function. For any $M \geq \underline{M}$,

$$V^*_{M+1} - V^*_M = \tilde{\mathcal{B}}(M+2) - \tilde{\mathcal{B}}(M+1) + \delta\left[\frac{M}{N-1}(V^*_{M+1} - V^*_M) + \frac{N-M-2}{N-1}(V^*_{M+2} - V^*_{M+1})\right]$$

[80] Using the same recursive method as in the proof of Lemma 2 (see Online Appendix B), it can be shown that the value function generated by the level-l super-entrenchment strategies increases with majority size for any $M \geq k+l$: $V_{k+l} \leq V_{k+l+1} \leq ... \leq V_{N-1}$.

By convexity,

$$\tilde{\mathcal{B}}(N) - \tilde{\mathcal{B}}(N-1) \geq ... \geq \tilde{\mathcal{B}}(\underline{M}+1) - \tilde{\mathcal{B}}(\underline{M}) > \tilde{s}, \tag{43}$$

and thus the sequence $(V^*_{M+1} - V^*_M)_{M \geq \underline{M}}$ is positive and increasing. Moreover,

$$V^*_{M+1} - V^*_M \geq \frac{\tilde{\mathcal{B}}(M+2) - \tilde{\mathcal{B}}(M+1)}{1 - \delta\frac{N-2}{N-1}} \geq \frac{\tilde{\mathcal{B}}(\underline{M}+1) - \tilde{\mathcal{B}}(\underline{M})}{1 - \delta\frac{N-2}{N-1}} > s,$$

for any $M \geq \underline{M}$. As a consequence, for any $M \geq \underline{M}+1$,

$$s - (\tilde{\mathcal{B}}(M+1) - \tilde{\mathcal{B}}(M)) < \delta\left[\frac{M-1}{N-1}(V^*_M - V^*_{M-1}) + \left(1 - \frac{M}{N-1}\right)(V^*_{M+1} - V^*_M)\right].$$

Hence, if the solution to the Bellman equations satisfies

$$V_M = \tilde{B}(M+1) + \overline{x}s + \delta\left[\frac{M}{N-1}V_M + \frac{N-M-1}{N-1}V_{M+1}\right] \tag{44}$$

for some $M \geq \underline{M} - 1$, then the strategy of entrenched recruitments at majority sizes $M' \geq M$ solves the Bellman equations for $M' \geq M$, and thus the unique solution to the Bellman equations is such that the majority candidate is always recruited, regardless of his talent, at all majority sizes $M' \geq M$.[81]

Suppose by contradiction that there exists no such majority size, i.e. the solution to the Bellman equations corresponds to meritocratic recruitments at all majority sizes $M \geq \underline{M} - 1$. Then, the same arguments as in the linear case apply to majority sizes $M < \underline{M} - 1$, yielding that the solution of the Bellman equations is given either by the meritocratic or basic-entrenchment strategies. Yet, we now argue that with these strategies, the majority has a profitable deviation whenever it has majority size $N-1$ and its in-group candidate is less talented.

Indeed, consider the value function $(V_M)_M$ generated by one such strategy. Building on previous arguments, $V_{M+1} \geq V_M$ for all $M \geq k$. In addition, using their recursive

[81] In other words, a sufficient condition for entrenched recruitments to obtain at all majority sizes above a threshold is that there exists $M \geq \underline{M} - 1$ such that the solution to the Bellman equations satisfies (44) at majority size M.

expressions yields for any $M \geq \max(k+1, \underline{M})$,

$$\begin{aligned}
V_M &= \overline{x}s + (1-x)\tilde{\mathcal{B}}(M+1) + x\tilde{\mathcal{B}}(M) + xs \\
&\quad + \delta(1-x)\left(\frac{M}{N-1}V_M^* + \frac{N-M-1}{N-1}V_{M+1}^*\right) + \delta x\left(\frac{M-1}{N-1}V_{M-1}^* + \frac{N-M}{N-1}V_M^*\right) \\
&< \overline{x}s + (1-x)\tilde{\mathcal{B}}(N) + x\tilde{\mathcal{B}}(N-1) - (N-M-1)\tilde{s} + xs \\
&\quad + \delta(1-x)\left(\frac{M}{N-1}V_M^* + \frac{N-M-1}{N-1}V_{M+1}^*\right) + \delta x\left(\frac{M-1}{N-1}V_{M-1}^* + \frac{N-M}{N-1}V_M^*\right)
\end{aligned}$$

where the inequality follows from (43). Then, using the recursive expression of V_M and that $V_{N-1} \geq V_{N-2} \geq ... \geq V_k$ yields that

$$\begin{aligned}
&\left[1 - \delta(1-x)\frac{N-2}{N-1} - \delta x\right]V_{N-2} \\
&\qquad < \overline{x}s + (1-x)\tilde{\mathcal{B}}(N) + x\tilde{\mathcal{B}}(N-1) - \tilde{s} + xs + \delta\frac{(1-x)}{N-1}V_{N-1} \\
&\qquad < \overline{x}s + \tilde{\mathcal{B}}(N) - \tilde{s} + x\frac{\delta\frac{N-2}{N-1}}{1-\delta\frac{N-2}{N-1}}\tilde{s} + \delta\frac{(1-x)}{N-1}V_{N-1}.
\end{aligned}$$

In addition, as the majority can secure at least the entrenchment payoff at $M = N-1$,

$$(1-\delta)V_{N-1} \geq \overline{x}s + \tilde{\mathcal{B}}(N).$$

As a consequence,

$$\begin{aligned}
\tilde{\mathcal{B}}(N) - \tilde{\mathcal{B}}(N-1) + \delta\frac{N-2}{N-1}\Big[V_{N-1} - V_{N-2}\Big] &> \tilde{s} + \delta\frac{N-2}{N-1}\left[\frac{\frac{1-\delta(1-x)\frac{N-2}{N-1}}{1-\delta\frac{N-2}{N-1}}}{1-\delta(1-x)\frac{N-2}{N-1} - \delta x}\right]\tilde{s} \\
&> \tilde{s} + \delta\frac{N-2}{N-1}\left[\frac{1}{1-\delta\frac{N-2}{N-1}}\right]\tilde{s} \\
&= s,
\end{aligned}$$

i.e. recruiting the in-group candidate against a more talented out-group candidate is a profitable deviation for the majority when it has size $N-1$.

O Homogamic evaluation capability: Proof of Proposition 11 and complements

Before stating the general result (for $s^\dagger \lesseqgtr b$), let us build the intuition for the case: $s^\dagger > b$. For this case to arise, majority members need to be sufficiently optimistic about the average quality of minority candidates. That is, the draws in talent must be sufficiently uncorrelated (i.e. x large) and the average ability of a candidate high enough (i.e. $\overline{x}$ large). [Had we assumed non-Bayesian beliefs, a further condition would have been the absence of prejudice about the minority.]

Intuitively, when $s^\dagger > b$, the model becomes similar to our baseline setup, yet with two key differences:

(i) The probability that the minority candidate is assessed by majority members as strictly more talented (in expectation) than the majority one increases from x to $x^\dagger \equiv x + (1-2x)(1-\alpha) > x$. In other words, minority candidates may get the benefit of the doubt.

(ii) The stand-alone cost of an entrenched vote is smaller as $s^\dagger - b < s - b$.

When $s^\dagger > b$, we show that, except perhaps when the majority is tight ($M = k$), whenever the majority candidate lacks talent, the majority gives the benefit of the doubt to, and picks the minority candidate. Consequently, the minority candidate may be selected even though the two candidates are equally talented.

Proposition 11 in the text and its implications for welfare follow from the general results in the next Proposition and its Corollary.

Proposition O.1. ***(Meritocratic and basic-entrenchment equilibria with homogamic evaluation capability)***

(i) If $s^\dagger \leq b$, the majority coopts only candidates of the in-group and therefore becomes homogeneous.

(ii) If $s^\dagger > b$, there exist finite thresholds $\rho^{e\dagger}$ and $\rho^{m\dagger}$ such that[82]

- *The basic-entrenchment equilibrium – in which the majority always chooses the majority's candidate for $M = k$, while for all $M \geq k+1$, the majority chooses*

[82] If $b < s^\dagger$ and $x^\dagger \leq 1/2$, then $\rho^{\mathrm{m}\dagger} < \rho^{\mathrm{e}\dagger}$. If $b < s^\dagger$ and $x^\dagger \geq 1/2$, then $\rho^{\mathrm{m}\dagger} \leq x^\dagger/x$, and thus the meritocratic equilibrium exists for all $s/b \geq x^\dagger/x$.

the majority's candidate if talented, and chooses the minority's candidate (of unknown talent) otherwise – exists if and only if $s/b \leq \rho^{e\dagger}$.

- *The meritocratic equilibrium – in which the minority candidate is elected against an untalented majority candidate for all $M \geq k$ – exists if and only if $s/b \geq \rho^{m\dagger}$.*

Corollary O.2. ***(Welfare)*** *(i) Whenever $s^\dagger \leq b$, by leading to full entrenchment, homogamic evaluation capability lowers ergodic aggregate welfare relative to perfect information. (ii) As with perfect information, the meritocratic equilibrium dominates the basic-entrenchment equilibrium in terms of ergodic aggregate welfare for any $s/b > 1$ whenever $x^\dagger$ is below or close to $1/2$, or close to 1.*[83] *Furthermore, the meritocratic and basic-entrenchment equilibria with homogamic evaluation capability yield a lower ergodic aggregate welfare than their perfect-information counterparts.*

O.1 Proof of Proposition O.1

The same arguments as with perfect information apply, with the appropriate changes in payoffs and with $x^\dagger$ replacing x in the transition probabilities.

The properties of the value functions of the meritocratic and basic-entrenchment equilibria with homogamic evaluation capability depend on whether $x^\dagger \leq 1/2$. If $x^\dagger \leq 1/2$, they exhibit the same features – monotonicity and concavity/convexity – as their perfect-information counterparts (indeed, the proof of Lemma 2 goes through replacing x by $x^\dagger$). By contrast, if $x^\dagger > 1/2$, the value function in the meritocratic equilibrium (if it exists) now decreases with group size $i \in \{1, ..., N-1\}$ [This observation immediately gives that for $x^\dagger > 1/2$, the meritocratic equilibrium exists for any $s^\dagger > b$.], and is concave for the minority ($i \leq k-1$) and convex for the majority ($i \geq k$). Similarly, in the basic-entrenchment equilibrium (if it exists), the value function increases less over $\{k, ..., N-1\}$ than it decreases over $\{1, ..., k-1\}$, whereas with $x^\dagger \leq 1/2$, the opposite holds: the distinction stems from the fact that the (weighted) sum of differences $V^{\mathrm{e}}_{i+1} - V^{\mathrm{e}}_i$ is equal to $(1-2x^\dagger)b$. As a consequence, with $x^\dagger \geq 1/2$, in the basic-entrenchment equilibrium, it is not the case in general that $V^{\mathrm{e}}_i \geq V^{\mathrm{e}}_{N-i-1}$ for any $i \geq k$, while in the meritocratic equilibrium, $V^{\mathrm{m}}_i \leq V^{\mathrm{m}}_{N-i-1}$ for any $i \geq k$ (the curse of control in action).

[83] Whenever they coexist, the meritocratic equilibrium is (still) preferred to the basic-entrenchment equilibrium by all members at any majority size.

Let the quantities $Y^\dagger$ and $Z^\dagger$ be given by

$$\begin{cases} Y^\dagger \equiv 1 + \delta\dfrac{k-1}{N-1}x^\dagger \displaystyle\sum_{t=0}^{+\infty}\delta^t\Big(\pi^{\mathrm{e}\dagger}_{k+1,k}(t) - \hat{\pi}^{\mathrm{e}\dagger}_{k,k}(t)\Big) \\ Z^\dagger \equiv 1 + \dfrac{k-1}{N-1}\dfrac{\delta}{1-\delta}(1-2x^\dagger) + \delta\dfrac{k-1}{N-1}x^\dagger \displaystyle\sum_{t=0}^{+\infty}\delta^t\Big(\pi^{\mathrm{e}\dagger}_{k+1,k}(t) + \hat{\pi}^{\mathrm{e}\dagger}_{k,k}(t)\Big) \end{cases}$$

where the probabilities $\pi^{\mathrm{e}\dagger}_{i,j}(t)$ (resp. $\hat{\pi}^{\mathrm{e}\dagger}_{i,j}(t)$) are taken *(a)* following the basic-entrenchment equilibrium strategies described in Proposition 11, and *(b)* from a majority member's perspective (resp. minority member's perspective) with transition parameter $x^\dagger$ instead of x. Define then $\rho^{\mathrm{e}\dagger}$ as

$$\rho^{\mathrm{e}\dagger} \equiv \begin{cases} \dfrac{x^\dagger}{x}\dfrac{Z^\dagger}{Y^\dagger} & \text{if } Y^\dagger > 0 \\ +\infty & \text{otherwise.} \end{cases}$$

The same argument as the one used in the proof of $\rho^{\mathrm{e}} < +\infty$ [84] yields that for any $\delta \in [0, (N-1)/N)$ and $x^\dagger \in [0,1)$, $\rho^{\mathrm{e}\dagger} < \infty$.

Similarly, let $\rho^{\mathrm{m}\dagger}$ be defined as

$$\rho^{\mathrm{m}\dagger} \equiv \frac{x^\dagger}{x}\left[1 + \frac{k-1}{N-1}(1-2x^\dagger)\delta\sum_{t=0}^{+\infty}\delta^t\left[\left(\sum_{i=k}^{N-1}\pi^{\mathrm{m}\dagger}_{k+1,i}(t)\right) - \left(\sum_{i=k}^{N-1}\pi^{\mathrm{m}\dagger}_{k-1,i}(t)\right)\right]\right]$$

where the probabilities $\pi^{\mathrm{m}\dagger}_{i,j}(t)$ are taken *(a)* following the meritocratic equilibrium strategies described in Proposition 11, and *(b)* from the perspective of a member of the group with initial size i, with transition parameter $x^\dagger$ instead of x. We show that the thresholds $\rho^{\mathrm{m}\dagger}$ and $\rho^{\mathrm{e}\dagger}$ are the homogamic-evaluation-capability counterparts of ρ^{m} and ρ^{e} in the baseline setting.

The proof of Proposition 11 is analogous to that of Proposition 2. As mentioned, when $x^\dagger \leq 1/2$, the value functions in the basic entrenchment and meritocratic equilibria with homogamic evaluation capability exhibit features similar to the ones of their perfect-information counterparts. Namely, the sequence $(V^{\mathrm{e}\dagger}_M)_{M\geq k}$ remains increasing and concave. By contrast, the monotonicity of the sequence $(V^{\mathrm{m}\dagger}_M)_{M\geq k}$ may differ: it is increasing (and concave) if $x^\dagger \leq 1/2$, whereas it is decreasing (and convex) if $x^\dagger > 1/2$. Moreover, in this latter case it may then be that $V^{\mathrm{e}\dagger}_k < V^{\mathrm{e}\dagger}_{k-1}$. Nonetheless, for $x^\dagger > 1/2$, the sequence $(V^{\mathrm{m}\dagger}_M)_{M\geq k}$ being decreasing implies that its differences $(V^{\mathrm{m}\dagger}_{M+1} - V^{\mathrm{m}\dagger}_M)$ are

[84] Cf. Section C.2.3.

negative and thus recruiting the minority candidate against an untalented majority candidate is optimal (as $s^\dagger > b$): hence, for $x^\dagger > 1/2$, the meritocratic equilibrium exists whenever $s^\dagger > b$. Lastly, in both cases, because of discounting, a talented majority candidate is still preferred to the minority candidate (with unknown talent) at any majority size.

We thus consider $x^\dagger \in [0,1]$ henceforth. As noted above, the argument used in step 1 of the proof of Proposition 2 applies to both equilibria[85], thus yielding that (except in the meritocratic equilibrium for $x^\dagger > 1/2$), the most profitable deviation from these candidate equilibria is when the majority is tight and faces an untalented majority candidate together with an unknown-quality minority one. We thus focus on step 2 and consider one-shot deviations in majority size $M = k$ when the majority candidate is untalented.

A (one-shot) deviation in majority size k from the basic-entrenchment strategy (defined in Proposition 11), i.e. picking the minority candidate (of unknown talent) instead of the untalented majority candidate, yields a payoff equal to:[86]

$$\Delta^{\mathrm{e},\dagger} \equiv s^\dagger - b + \delta\frac{k-1}{N-1}xs\sum_{t=0}^{+\infty}\delta^t\left(\pi^{\mathrm{e}\dagger}_{k+1,k}(t) - \hat{\pi}^{\mathrm{e}\dagger}_{k,k}(t)\right) + \delta\frac{k-1}{N-1}x^\dagger b\sum_{t=0}^{+\infty}\delta^t\left(\sum_{i\geq k+1}\hat{\pi}^{\mathrm{e}\dagger}_{k,i}(t)\right)$$
$$- \delta\frac{k-1}{N-1}x^\dagger b\sum_{t=0}^{+\infty}\delta^t\pi^{\mathrm{e}\dagger}_{k+1,k}(t) - \frac{k-1}{N-1}\frac{\delta}{1-\delta}(1-x^\dagger)b$$

where the probabilities $\pi^{\mathrm{e}\dagger}_{i,j}(t)$ (resp. $\hat{\pi}^{\mathrm{e}\dagger}_{i,j}(t)$) are taken *(a)* following the basic-entrenchment equilibrium strategies described in Proposition 11, and *(b)* from a majority member's perspective (resp. minority member's perspective) with transition parameter $x^\dagger$ instead of x. By construction, $s^\dagger/s = x/x^\dagger$. Rearranging yields

$$\Delta^{\mathrm{e},\dagger} = \frac{x}{x^\dagger}s\left[1 + \delta\frac{k-1}{N-1}x^\dagger\sum_{t=0}^{+\infty}\delta^t\left(\pi^{\mathrm{e}\dagger}_{k+1,k}(t) - \hat{\pi}^{\mathrm{e}\dagger}_{k,k}(t)\right)\right]$$
$$- b\left[1 + \frac{k-1}{N-1}\frac{\delta}{1-\delta}(1-2x^\dagger) + \delta\frac{k-1}{N-1}x^\dagger\sum_{t=0}^{+\infty}\delta^t\left(\pi^{\mathrm{e}\dagger}_{k+1,k}(t) + \hat{\pi}^{\mathrm{e}\dagger}_{k,k}(t)\right)\right]$$

[85] For both equilibria when $x^\dagger \leq 1/2$ and for the basic-entrenchment equilibrium when $x^\dagger \geq 1/2$, the argument goes through replacing x by $x^\dagger$ and s by $s^\dagger$ when appropriate. In particular, in the basic-entrenchment equilibrium, for $x^\dagger \in [0,1]$, analogous computations yield that at majority size $M = k+1$,

$$\delta\left(\frac{k-2}{N-1}u^{\mathrm{e}\dagger}_{k+1} + \frac{k}{N-1}u^{\mathrm{e}\dagger}_{k}\right) \leq \frac{\delta\dfrac{k}{N-1}}{1-\delta\dfrac{k}{N-1}}\frac{1}{1-\overline{x}}\big(xs - (1-\overline{x})b\big) < s^\dagger - b.$$

[86] Indeed, the difference between the expected maximum of both candidates' talents and the expected quality of the majority candidate writes as before $(\overline{x} + (1-\overline{x})x/x^\dagger)s - \overline{x}s = xs$.

which yields the result for the existence region of the basic-entrenchment equilibrium.

Similarly for the meritocratic equilibrium, consider the (one-shot) deviation of a majority member voting in k the untalented majority candidate instead of the minority one. Such a deviation yields a payoff equal to:

$$\Delta^{\mathrm{m},\dagger} = b - s^\dagger + \delta\frac{(k-1)}{N-1}(1-x^\dagger)b\sum_{t=0}^{+\infty}\delta^t\left[\left(\sum_{i\geq k}\pi^{\mathrm{m}\dagger}_{k+1,i}(t)\right)-\left(\sum_{i\geq k}\pi^{\mathrm{m}\dagger}_{k-1,i}(t)\right)\right]$$
$$+\delta\frac{(k-1)}{N-1}x^\dagger b\sum_{t=0}^{+\infty}\delta^t\left[\left(\sum_{i\leq k-1}\pi^{\mathrm{m}\dagger}_{k+1,i}(t)\right)-\left(\sum_{i\leq k-1}\pi^{\mathrm{m}\dagger}_{k-1,i}(t)\right)\right]$$

i.e. by rearranging,

$$\Delta^{\mathrm{m},\dagger} = -\frac{x}{x^\dagger}s + b\left[1+\delta(1-2x^\dagger)\frac{(k-1)}{N-1}\sum_{t=0}^{+\infty}\delta^t\left[\left(\sum_{i\geq k}\pi^{\mathrm{m}\dagger}_{k+1,i}(t)\right)-\left(\sum_{i\geq k}\pi^{\mathrm{m}\dagger}_{k-1,i}(t)\right)\right]\right]$$

The result for the existence region of the meritocratic equilibrium follows. Lastly, the proof for $\rho^{\mathrm{e},\dagger} < +\infty$ is in Section C.2.3.

Note moreover that Lemma C.2 holds with the transition probabilities $\pi^{\mathrm{e}\dagger}$ and $\pi^{\mathrm{m}\dagger}$ [87], and this establishes the inequality $\rho^{\mathrm{m}\dagger} < \rho^{\mathrm{e}\dagger}$ for $x^\dagger \leq 1/2$, as well as the inequality $\rho^{\mathrm{m}\dagger} \leq x^\dagger/x$ for $x^\dagger \geq 1/2$ (noted in the text).[88]

O.2 Proof of Corollary O.2

The same argument as the one used in the proof of Proposition (3) yields that, whenever they co-exist, the meritocratic equilibrium is preferred to the basic-entrenchment equilibrium by all members at any majority size.

We now consider ergodic per-period aggregate welfare. We first show that with homogamic evaluation capability, meritocracy dominates basic entrenchment. To this end, we show that the result of Proposition 4, proved in Online Appendix F, holds replacing x with $x^\dagger \in [0,1]$. Analogous computations to the ones in Online Appendix F show that

[87] Indeed, the proof holds for any $x \in [0,1]$ as the stochastic matrices P and $\hat{P}$ (introduced in the proof of Lemma C.2) remain stochastically monotone and stochastically comparable (with P stochastically dominating $\hat{P}$) for any $x \in [0,1]$.

[88] If $b < s^\dagger$ and $x^\dagger \geq 1/2$, then $\rho^{\mathrm{m}\dagger} \leq x^\dagger/x$, and thus the meritocratic equilibrium exists for all $s/b \geq x^\dagger/x$. Lastly, $s^\dagger$ and $x^\dagger$ both depend on x, and thus the value of $x^\dagger$ constrains the possible values of $s^\dagger$: in particular, for $x^\dagger \geq 1/2$ (and thus $\alpha \leq 1/2$), $s^\dagger$ decreases with $x^\dagger$, and $s^\dagger = 0$ when $x^\dagger = 1$. As a consequence, for any $b > 0$, the inequality $s^\dagger > b$ can only hold for $x^\dagger$ sufficiently below 1.

meritocracy dominates basic entrenchment if and only if

$$N(N-1)x\left[\frac{x^\dagger}{1-x^\dagger}\frac{k+1}{N}+1+\sum_{i=1}^{k-1}\left(\frac{1-x^\dagger}{x^\dagger}\right)^i\prod_{l=1}^{i}\frac{k-l}{k+1+l}\right]q\tilde{s}$$
$$>\frac{2x^\dagger}{1-x^\dagger}\left[1+\sum_{i=1}^{k-1}(i+1)^2\left(\frac{1-x^\dagger}{x^\dagger}\right)^i\prod_{l=1}^{i}\frac{k-l}{k+1+l}\right]\tilde{b} \tag{45}$$

where $q \geq 1$. By Proposition 11, a necessary condition for meritocracy and basic entrenchment to exist is $b < s^\dagger$, i.e. $xs > x^\dagger b$. Therefore, a sufficient condition for (45) to be satisfied is

$$N(N-1)\left[\frac{x^\dagger}{1-x^\dagger}\frac{k+1}{N}+1+\sum_{i=1}^{k-1}\left(\frac{1-x^\dagger}{x^\dagger}\right)^i\prod_{l=1}^{i}\frac{k-l}{k+1+l}\right]$$
$$>\frac{2}{1-x^\dagger}\left[1+\sum_{i=1}^{k-1}(i+1)^2\left(\frac{1-x^\dagger}{x^\dagger}\right)^i\prod_{l=1}^{i}\frac{k-l}{k+1+l}\right]$$

By Online Appendix F, the above inequality holds for any $x^\dagger \in [0, 1/2]$, as well as for $x^\dagger$ greater than but close to 1/2. Moreover, it clearly holds for $x^\dagger$ close to 1. [Numerical simulations suggest it holds for any $x^\dagger[0, 1]$.]

We then turn to the ergodic aggregate welfare comparison of homogamic evaluation capability with respect to perfect information: we show that meritocracy and basic entrenchment with homogamic evaluation capability are dominated by their perfect-information counterparts. We proceed as in Section 2.2.2.

We first note that in both equilibria, the ergodic distribution of majority sizes with perfect information first-order stochastically dominates the one with homogamic evaluation capability. Using the notation introduced in Section 2.2.2, we denote by $\nu_i^{\mathrm{r}\dagger}$ the ergodic probability of state i at the end of a period in regime $\mathrm{r} \in \{\mathrm{e}, \mathrm{m}\}$, and show that for $\mathrm{r} \in \{\mathrm{e}, \mathrm{m}\}$, the probability distribution $\{\nu_i^{\mathrm{r}}\}$ first-order stochastically dominates $\{\nu_i^{\mathrm{r}\dagger}\}$. Indeed, for $\mathrm{r} \in \{\mathrm{e}, \mathrm{m}\}$, consider the stochastic matrices P^{r} and $P^{\mathrm{r}\dagger}$ associated with the probability distribution over (end-of-period) majority sizes in equilibrium r respectively with perfect information and homogamic evaluation capability, from an outsider's perspective[89]. By construction, both P^{r} and $P^{\mathrm{r}\dagger}$ are stochastically monotone, and the two are stochastically comparable, with $P^{\mathrm{r}}_{i.}$ stochastically dominating $P^{\mathrm{r}\dagger}_{i.}$ for any row index i as $x^\dagger \geq x$. Therefore, the ergodic distribution of majority sizes in equilibrium r

[89]Namely, for any $i, j \in \{1, ..., k\}$, the matrix component P^{r}_{ij} (resp. $P^{\mathrm{r}\dagger}_{ij}$) is the probability (from an outsider's perspective) that the (end-of-period) majority size moves from $k+i-1$ to $k+j-1$ from one period to another in equilibrium $\mathrm{r} \in \{\mathrm{e}, \mathrm{m}\}$ with perfect information (resp. with homogamic evaluation capability).

with perfect information (first-order) stochastically dominates the one with homogamic evaluation capability.

As a consequence, since the aggregate homophily payoff at a given majority size strictly increases with the majority size, perfect information yields a higher ergodic aggregate homophily payoff than homogamic evaluation capability in equilibrium $\mathrm{r} \in \{\mathrm{e}, \mathrm{m}\}$. Moreover, by Section 2.2.2, the difference in aggregate per-period expected quality between perfect information and homogamic evaluation capability writes as

$$S^{\mathrm{r}} - S^{\mathrm{r}\dagger} = \begin{cases} 0 & \text{if } \mathrm{r} = \mathrm{m}, \\ N(N-1)\Big[\nu_{k+1}^{\mathrm{e}\dagger} - \nu_{k+1}^{\mathrm{e}}\Big]\dfrac{k+1}{N} x\tilde{s} & \text{if } \mathrm{r} = \mathrm{e}. \end{cases}$$

Hence, since the probability distribution $\{\nu_i^{\mathrm{e}}\}$ first-order stochastically dominates $\{\nu_i^{\mathrm{e}\dagger}\}$, $S^{\mathrm{r}} - S^{\mathrm{r}\dagger} \geq 0$. Therefore, meritocracy and basic entrenchment with homogamic evaluation capability are dominated by their perfect-information counterparts in terms of ergodic per-period aggregate welfare.

In order to establish the welfare claim in (i), we show that (perfect-information) basic entrenchment dominates full entrenchment. The aggregate ergodic quality in the full-entrenchment equilibrium writes as $S^{\mathrm{f}} = N(N-1)\overline{x}\tilde{s}$, and thus using the computations of Section 2.2.2, the difference between the ergodic efficiency of a basically-entrenched and fully-entrenched organization is given by

$$S^{\mathrm{e}} - S^{\mathrm{f}} = N(N-1)\left[1 - \nu_{k+1}^{\mathrm{e}}\frac{k+1}{N}x\right]\tilde{s}$$

Similarly, the difference ergodic homophily benefits is given by

$$B^{\mathrm{e}} - B^{\mathrm{f}} = \sum_{i=k+1}^{N} \nu_i^{\mathrm{e}}\Big[i(i-1) + (N-i)(N-i-1) - N(N-1)\Big]$$

Building on Online Appendix E, explicit computations[90] then yield that $q(S^{\mathrm{e}} - S^{\mathrm{f}}) + B^{\mathrm{e}} - B^{\mathrm{f}} > 0$ for any $s > b$, hence the result.

[90]With the explicit expressions for the ergodic probabilities ν_i^{e} derived in Online Appendix E, $q(S^{\mathrm{e}} - S^{\mathrm{f}}) + B^{\mathrm{e}} - B^{\mathrm{f}}$ has the same sign as

$$\left[N(N-1)\left(1 - \frac{k+1}{N}x\right)q\tilde{s} + 2(k+1)(1-k)\tilde{b}\right] + \sum_{i=1}^{k-1}\left(\frac{1-x}{x}\right)^i \prod_{l=1}^{i}\frac{k-l}{k+1+l}\left[N(N-1)q\tilde{s} + 2(i+k+1)(i-k+1)\tilde{b}\right]$$

The second part of the welfare claim in (ii) stems from the explicit expressions of ρ^{m} and $\rho^{\mathrm{m}\dagger}$ which imply that for δ close to 0,

$$\rho^{\mathrm{m}} = 1 + (1-2x)\frac{k-1}{N-1}\delta + O(\delta^2), \qquad \text{and} \qquad \rho^{\mathrm{m}\dagger} = \frac{x^\dagger}{x}\left[1 + (1-2x^\dagger)\frac{k-1}{N-1}\delta\right] + O(\delta^2),$$

and thus $\rho^{\mathrm{m}} < \rho^{\mathrm{m}\dagger}$. The first part derives from the above results, namely that meritocracy and basic entrenchment with homogamic evaluation capability are dominated by their perfect-information counterparts, and that meritocracy dominates basic entrenchment with homogamic evaluation capability as well as with perfect information.

P Complements on uncertain voting participation or identification of group allegiance

Imperfect identification of group allegiance. Our modelling of uncertain voting participation also applies to imperfect identification of group allegiance. As an illustration, let us introduce the possibility that a candidate be able to mascarade as belonging to the other group and thereby be elected. Namely, let us assume there is a probability $\vartheta \in (0, 1/2)$ that the best candidate of the majority group[91] is incorrectly identified (tagged as belonging to majority group, when actually belonging to the minority group). To avoid having to consider complicated coming-out strategies of misidentified members, we further assume that the real identity of the newly elected member is revealed after the vote and before curent-period payoffs accrue.

The probability of a fully-entrenched majority with size $M = N-1$ losing control, is strictly positive and proportional to ϑ^k, as it takes k consecutive occurrences of "bad luck" to topple its grip on the organization. By the above argument on uncertain voting participation (replacing the probability of the majority losing the vote with the probability of recruiting a minority candidate incorrectly identified), there exists a non-empty neighbourhood of 1 such that for s/b in this neighbourhood, the only monotone equilibrium is the full-entrenchment equilibrium.

The result obtains by noting that for any $x \leq 1/2$,

$$N(N-1)\left(1 - \frac{k+1}{N}x\right) > 2(k+1)(1-k),$$

and that for any $i \in \{1, ..., k-1\}$, $2(i+k+1)(i-k+1) > 2(k+2)(2-k) > -N(N-1)$.

[91] We further assume that all candidates of the majority group are equally "unreliable" (incorrectly identified with the same probability). Otherwise, an untalented yet fully "reliable" candidate (i.e. identified as perfectly belonging to the majority) might then be preferred to a talented yet "unreliable" candidate.

This analysis of turncoats presumes that candidates identified as sympathetic to the majority may actually favor the minority. A milder version of the same idea is that candidates identified as belonging to a given horizontal group still prefer in-group colleagues all else being equal, but that the intensity of candidates' homophily-vs-quality preferences varies and is not observable. So a majority recruit may for instance put a much higher weight on talent relative to homophily than the average majority member[92] and therefore resist an entrenched strategy. Anticipating this possibility, the majority may again want to be super-entrenched, so as to minimize the probability of a switch in control.

Remark on our model of absenteeism. Absenteeism raises the question of what happens when the numbers of majority and minority members who show up are equal (or if no-one shows up). The key assumption behind the statement of the Λ function is that a process is in place, which will guarantee a decision in case of such draws. One can envision a variety of such processes. For example, the majority leader might take the decision. Or the assembly of members might reconvene as many times as is needed to break the tie (technically, an infinite number of times if one wants to reach a decision with probability 1. Otherwise, the results are just limit results). Similarly, one could add a quorum rule given such reconvening; this quorum, for a given absenteeism process, would generate a different Λ function, but still one satisfying our assumptions. The Λ function captures all kinds of processes and all forms of correlation among members' absences, as long as the process delivers an outcome.

P.1 Proof of Proposition 12

We look for monotonic (in the sense that a stronger majority makes more meritocratic recruitments), pure-strategy symmetric MPEs. When looking for level-l super-entrenchment equilibria, we now look for equilibria in which (a) the majority is super-entrenched to level l and (b) the minority always votes for its in-group candidate whenever it is pivotal with a strictly positive probability, i.e. whenever $M \leq k + l - 1$.

Let us thus define the strategy corresponding to "super-entrenchment to level l" for any group with size i such that $i \geq k$ or $\Lambda(N-1-i) > 0$, as the strategy that coincides with the previous level-l super-entrenchment strategy for the majority (group size $i \geq k$),

[92] For example, a small fraction of majority candidates might have homophily benefit zb, where $z < 1$, and thus favor the meritocratic strategy over an entrenched one, despite their colleagues in the majority favoring the latter over the former.

and that consists in always voting for the in-group candidate for the minority whenever $\Lambda(M) > 0$, i.e. whenever the minority is pivotal with a strictly positive probability. Formally, generalizing $\sigma(i)$ to be the probability that a group with size $i \geq 1$ votes for the out-group candidate when the latter is more talented than the out-group candidate, super-entrenchment strategies are defined by:

(i) $\sigma(i) = 0$ for all $i \in \{N-k-l, ..., k+l\}$ and $\sigma(i) = 1$ for $i \geq k+l+1$,

(ii) at any group size $i \geq N-k-l$, each group votes for its in-group candidate whenever she is equally or more talented than the out-group candidate.

We denote by V_i the corresponding value function and u_i its first-difference.

Proof for existence. Let $s = b > 0$. The usual computations[93] (see proof of Lemma 2) yield that for any $i \geq k+l$ and for any $i \leq k-2-l$, $u_i = 0$. The usual argument then applies: using that for group sizes $i \in \{k, ..., k+l-1\}$,

$$\left[1 - \delta\Lambda(i)\left(1 - \frac{i}{N-1}\right) - \delta(1-\Lambda(i+1))\frac{i}{N-1}\right]u_i$$
$$= [\Lambda(i) - \Lambda(i+1)]b + \delta\Lambda(i)\frac{i-1}{N-1}u_{i-1} + \delta(1-\Lambda(i+1))\left(1 - \frac{i+1}{N-1}\right)u_{i+1},$$

while for group sizes $i \in \{k-2-l, ..., k-2\}$,

$$\left[1 - \delta\Lambda(N-i-2)\frac{i}{N-1} - \delta(1-\Lambda(N-i-1))\left(1 - \frac{i}{N-1}\right)\right]u_i$$
$$= [\Lambda(N-i-2) - \Lambda(N-i-1)]b + \delta\Lambda(N-i-2)\left(1 - \frac{i+1}{N-1}\right)u_{i+1}$$
$$+ \delta(1-\Lambda(N-i-1))\frac{i-1}{N-1}u_{i-1},$$

and lastly for group size $k-1$:

$$[1 - \delta(1-\Lambda(k))]u_{k-1} = (1-2\Lambda(k))b + \delta(1-\Lambda(k))\frac{k-1}{N-1}u_k + \delta(1-\Lambda(k))\frac{k-2}{N-1}u_{k-2},$$

one first shows that $u_i > 0$ for any $i \in \{k-1-l, ..., k+l-1\}$. The only non-trivial case for profitable deviations is thus when the out-group candidate is more talented than the in-group one. Therefore, since in such a case, for $s = b$, the one-shot deviation differential

[93] This could be seen by using the recursive expressions for the sequence $(u_i)_i$ and supposing by contradiction that $u_i \neq 0$ for some $i \geq k+l$ or $\leq k-2-l$.

payoff is given by

$$-\delta(1-\Lambda(i))\left[\left(1-\frac{i}{N-1}\right)u_i+\frac{i-1}{N-1}u_{i-1}\right]<0$$

at group size $i \in \{k, ..., N-1\}$, and by

$$-\delta\Lambda(N-1-i)\left[\left(1-\frac{i}{N-1}\right)u_i+\frac{i-1}{N-1}u_{i-1}\right]<0$$

at group size $i \in \{N-k-l, ..., k-1\}$, super-entrenchment to level l is an equilibrium.

The result obtains by continuity for s/b in a neighbourhood of 1.

Proof for uniqueness. We now show that, for s/b close to 1, super-entrenchment at level l is the unique symmetric MPE such that a stronger majority makes more meritocratic recruitments. Hence, we consider the class of equilibria such that a stronger majority makes more meritocratic recruitments, and show that, for any candidate equilibrium within this class, for s/b close to 1, the majority is super-entrenched in $k+l$. By monotonicity, this implies that all candidate equilibria within this class must feature an entrenched majority at majority sizes $M \in \{k, ..., k+l\}$. We will then show that the minority best-replies to this strategy by voting for the in-group candidate whenever it is pivotal with a strictly positive probability, i.e. at any majority size $M \leq k+l-1$.

We henceforth consider a candidate equilibrium within the class of symmetric MPEs such that a stronger majority makes more meritocratic recruitments. We begin by noting that when $s=b$, a group's flow payoff whenever it is pivotal does not depend on its making a meritocratic or entrenched recruitment (as the difference between the two is equal to $x(s-b)=0$) and is strictly positive (proportional to $\overline{x}s+b$). Moreover, for $s=b$, the flow differential payoff in the expression of u_i writes as $[\Lambda(i)-\Lambda(i+1)]b$ (resp. $[\Lambda(i)-\Lambda(i+1)](1-2x)b$) if the minority follows entrenchment (resp. meritocracy) at majority sizes i and $i+1$, as $[\Lambda(i)-\Lambda(i+1)]b-2x\Lambda(i)b$ if the minority follows meritocracy at majority size i and entrenchment at majority size $i+1$, and as $[\Lambda(i)-\Lambda(i+1)]b+2x\Lambda(i+1)b$ if the minority follows entrenchment at majority size i and meritocracy at majority size $i+1$. In particular, the flow-payoff term in u_{k+l-1} writes as $\Lambda(k+l-1)b$ if the minority is entrenched at majority size $k+l-1$ (resp. $\Lambda(k+l-1)(1-2x)b$ if it votes meritocratically). By contrast, for any $i \geq k+l$, the flow-payoff term in u_i is equal to 0.

We now show that, for $s=b$, in any symmetric MPE such that a stronger majority makes more meritocratic recruitments

$$\frac{k+l-1}{N-1}u_{k+l-1} + \left(1 - \frac{k+l}{N-1}\right)u_{k+l} > 0$$

Suppose by contradiction that the above LHS is weakly lower than 0, and thus that the majority votes meritocratically at size $k+l$. Suppose first that $u_{k+l} \leq 0$. By monotonicity within the equilibrium class, the majority votes meritocratically at any size $i \geq k+l$, and thus the recursive expression of u_i for $i \geq k+l$ is given by (5) and yields[94] that $u_{k+l-1} \leq u_{k+l} \leq ... \leq u_{N-2} \leq 0$. Then, summing up (and rearranging) the recursive expression of u_{k+l-1} and u_i for $i \geq k+l$ (and rearranging) yields on the LHS a (positively) weighted sum of u_i, $i \geq k+l-1$, which is thus (weakly) negative, and on the RHS the sum of the flow-payoff term in u_{k+l-1}, which is strictly positive (as noted above, since $\Lambda(k+l-1) > 0 = \Lambda(k+l)$), and of a term proportional to u_{k+l-2}. Therefore, $u_{k+l-2} < 0$. We proceed by induction in order to show that $u_i < 0$ for any $i \in \{k-1, ..., k+l-2\}$. Let $M \in \{k, ..., k+l-2\}$, and suppose $u_i \leq 0$ for any $i \geq M$. Summing and rearranging as above the recursive expressions of the differential value function u_i over indices $i \in \{M, ..., N-2\}$ gives on the LHS a weighted sum of u_i for $i \in \{M, ..., N-2\}$, which is weakly negative with the induction hypothesis, while on the RHS a first term proportional to u_{M-1} and a second term which is the sum of the flow-differential payoffs, equal either to $\Lambda(M)(1-2x)b$, $\Lambda(M)b$ or $[\Lambda(M)+\Lambda(M+1)2x]b$, and is thus strictly positive. Therefore, $u_{M-1} < 0$.

Hence, by induction, $u_i < 0$ for any $i \in \{k-1, ..., k+l-2\}$. Therefore, the majority is meritocratic at any majority size $i \geq k$. As a consequence, the flow differential payoffs in the expression of u_i for $i \leq k-1$ write as $[\Lambda(N-i-2) - \Lambda(N-i-1)](1-2x)b > 0$ for any $i \in \{k-l-1, ..., k-2\}$, and 0 for any $i \leq k-l-2$.

Let us consider the minority's incentives. Suppose by contradiction that $u_{k-l-1} \leq 0$. Then, the recursive expression of u_i for $i \leq k-l-2$ is given by (11) and yields that $u_{k-l-1} \leq ... \leq u_1 \leq 0$. Furthermore, since the flow differential payoffs are positive for $i \in \{k-l-1, ..., k-2\}$, we have that $u_i \leq 0$ for $i \in \{1, ..., k-1\}$. Therefore, the minority votes meritocratically whenever it is pivotal with a strictly positive probability. Hence, the sum of the flow differential payoffs over all indices $i \in \{1, ..., N-2\}$ writes as

$$2\Lambda(k)(1-2x)b + [1-2\Lambda(k)](1-2x)b = (1-2x)b > 0$$

[94]This can be seen by supposing by contradiction that $u_{N-2} > 0$, and reaching a contradiction using (5). The result then obtains by downward induction, using again (5).

where the second term is the flow differential payoff in u_{k-1}. Yet this contradicts $u_i \leq 0$ for all $i \in \{1, ..., N-2\}$.

Hence, $u_{k-l-1} > 0$. The recursive expressions of the differential value function (11) now yield that $0 < u_1 < ... < u_{k-l-1}$. Supposing by contradiction that $u_{k-l} \leq 0$ yields again that $u_i \leq 0$ for $i \in \{k-l, ..., k-1\}$. Hence, by summing the recursive expressions of u_i for $i \in \{k-l, ..., N-2\}$ and rearranging yields on the LHS a weighted sum of the differential value function u_i for $i \in \{k-l, ..., N-2\}$, which is weakly negative, while on the RHS, a term proportional to u_{k-l-1} (and thus strictly positive) and the sum of the flow differential payoffs, which is strictly positive. This is a contradiction, and thus $u_{k-l} > 0$. Using repeatedly the same argument, we have by induction that $u_i > 0$ for any $i \leq k-2$, and as a consequence, the minority is entrenched whenever it has size $i \in \{k-l, ..., k-2\}$, i.e. whenever the majority has size $i \in \{k+1, ..., k+l-1\}$.

Back to the majority, summing again the recursive expression of the differential value function u_i over indices $i \geq k-1$ yields after rearranging, on the LHS a weighted sum of the differential value function u_i for $i \in \{k-1, ..., N-2\}$, which is weakly negative, while on the RHS, a term proportional to u_{k-2} (and thus strictly positive) and the sum of the flow differential payoffs, which is equal to $[1 - \Lambda(k)](1-2x) > 0$. Hence, the RHS is strictly positive, which is a contradiction. Therefore, $u_{k+l} > 0$, and thus using the recursive expression of u_i for $i \geq k+l$ (namely (5) as we suppose that the majority votes meritocratically at size $k+l$), we have that $u_{k+l-1} > u_{k+l} > u_{k+l+1} > ... > u_{N-2} > 0$.

Consequently, for $s = b$,

$$s - b + \delta \left[\frac{k+l-1}{N-1} u_{k+l-1} + \left(1 - \frac{k+l}{N-1}\right) u_{k+l} \right] > 0$$

and thus the majority is entrenched when it has size $k+l$.[95] By continuity with respect to s/b, this inequality holds for any s/b sufficiently close to 1, yielding the majority's entrenchment at size $k+l$.

Hence, for $s/b > 1$ sufficiently close to 1, any candidate equilibrium such that a larger majority makes more meritocratic recruitments is such that the majority makes entrenched recruitments at majority sizes $M \leq k+l$, and using the same arguments as in the proof of Proposition 2, meritocratic recruitments at majority sizes $M \geq k+l+1$.[96]

[95] Note that, as $s = b$, entrenchment at size $k+l$ implies that $u_{k+l} = u_{k+l+1} = ... = u_{N-2} = 0$ (as all flow-payoff terms in the recursive expressions of u_i for $i \geq k+l$ are thus nil).

[96] In fact, the argument implies that for s/b sufficiently close to 1, in any symmetric, possibly non-monotonic MPE, the majority makes entrenched recruitments when it has size $k+l$, and thus by the same arguments as in the proof of Proposition 2, and makes meritocratic recruitments when it has size $M \geq k+l+1$. The requirement that a stronger majority makes more meritocratic recruitments

The usual recursive arguments (considering first $s = b$ then using the value functions' continuity with respect to s/b) then yield that for s/b sufficiently close to 1, the minority uniquely best-replies to such strategies by being entrenched whenever it is pivotal with a strictly positive probability.

This establishes, for s/b sufficiently close to 1, the uniqueness of the level-l super-entrenchment equilibrium within the class of equilibria such that a stronger majority makes more meritocratic recruitments.

Q Proof of Proposition 13

We first show that when candidates reapply, meritocratic strategies do not sustain an equilibrium for s/b in some interval $[1, \rho^{\mathrm{m}} + \epsilon)$ with $\epsilon > 0$. We then show that the meritocratic *equilibrium path* starting from an initial state with empty storage is no longer an equilibrium path for s/b in some interval $[1, \rho^{\mathrm{m}} + \epsilon)$ with $\epsilon > 0$: an equilibrium may be observationally equivalent to a meritocratic equilibrium by exhibiting the same recruitment path, without necessarily being meritocratic off the equilibrium path (more on this below).

Let us define the meritocratic equilibrium as the equilibrium in which the majority always recruits the best candidate available[97] for any stocks of candidates, and look for necessary conditions for the meritocratic equilibrium to exist. We show the latter are more often binding when candidates reapply than when they cannot. Namely, when candidates reapply, we exhibit one deviation that is profitable for s/b a bit above ρ^{m} (and for all $s/b \in [1, \rho^{\mathrm{m}}]$). Note that we do not derive a sufficient condition for existence.

Two effects (which we will successively illustrate) are at play, shrinking the existence region of meritocracy: (i) the ability to recall a talented minority candidate increases the value of basic entrenchment; and (ii) the preferential treatment given by the majority to its in-group talented candidate(s) in store makes an incumbent majority with a large number of talented minority candidates in store less willing to relinquish control.

To illustrate both forces at play, consider first $x = 1/2$ (so that $\rho^{\mathrm{m}} = 1$), and $s/b = 1$. Suppose the majority has size k, and no talented majority candidate available[98] but an infinite number of talented minority ones in store. Recruiting a talented minority

then yields that for s/b sufficiently close to 1, the majority must make entrenched recruitments at sizes $M \leq k + l - 1$ too.

[97]Namely the best candidate among current-period and stored candidates, breaking ties in favor of in-group candidates as before.

[98]Namely, it has no such candidate in store, and the current-period majority candidate is untalented.

candidate instead of an untalented majority one gives a differential payoff equal to

$$s - b + \delta \frac{k-1}{N-1}\left(\frac{s}{1-\delta} - V_{k+1,0,\infty}\right) = \delta \frac{k-1}{N-1}\left(\frac{s}{1-\delta} - V_{k+1,0,\infty}\right)$$

where $V_{k+1,0,\infty}$ is the majority value function when it has size $k+1$, no talented majority candidate in store and an infinite number of talented minority ones in store. Since for $x = 1/2$, a majority with size $k+1$ can secure in each period an (expected) flow quality payoff equal to $\tilde{s}$, and for at least the first two periods, an (expected) flow homophily payoff equal to $\tilde{b}/2$ [99], we have that $V_{k+1,0,\infty} > s/(1-\delta)$. Furthermore, as the majority cannot do better than $\tilde{s}$ in terms of flow quality payoff, the term $[s/(1-\delta) - V_{k+1,0,\infty}]$ does not decrease with s, but strictly decreases with b. Therefore, the above differential payoff is strictly negative for any s/b in an upper neighbourhood of 1. Because of time discounting ($\delta_0 < 1$), the result holds when the majority has in store a sufficiently large finite number of talented minority candidates. Hence, for $x = 1/2$, there exists a strictly profitable deviation away from meritocracy for $s/b \in [\rho^{\mathrm{m}}, \rho^{\mathrm{m}} + \epsilon)$.

Consider now $x < 1/2$ (so that $\rho^{\mathrm{m}} > 1$), and $s/b = \rho^{\mathrm{m}}$. A necessary condition for the meritocratic equilibrium to exist is that a repeated deviation towards basic entrenchment whenever the majority is tight ($M = k$) and has no talented majority candidate available and exactly one talented minority candidate available, be non profitable. Upon permanently deviating to basic entrenchment, the majority has one talented minority candidate in store, and either size k or $k+1$. Yet, for $x < 1/2$, a basically-entrenched majority's value function strictly increases with the number of talented minority candidates in store[100]. Hence, when candidates reapply, a permanent deviation away from meritocracy becomes more profitable. Furthermore, an inspection of the additional payoff due to storability shows that the latter increases with s and decreases with b. Intuitively, this derives from the fact that having a talented minority

[99] In particular, reverting to the meritocratic strategy yields to the current majority group an (expected) flow payoff equal to $\tilde{s} + \tilde{b}/2$ as long as it retains control over the organization, and equal to $\tilde{s}$ after it has relinquished it to the other group.

[100] Indeed, a basically-entrenched majority solves an optimal control problem. Moreover, as $x < 1/2$, the majority faces two untalented current-period candidates with a strictly positive probability $(1 - 2x > 0)$, in which case, whenever it is not tight ($M > k$) and whenever it has a talented minority candidate in store, it recruits the latter, thus receiving a strictly positive differential payoff with respect to the empty-storage state. Indeed, the differential payoff from recruiting a stored talented minority candidate instead of an untalented majority candidate whenever the majority is not tight, is bounded below by:

$$s - b - x(s-b)\frac{\delta k/(N-1)}{1 - \delta k/(N-1)} > (1-x)(s-b) > 0$$

candidate in store leads to the latter being recruited (at some point, with strictly positive probability) instead of a (talented or untalented) in-group candidate or an untalented out-group candidate, thus yielding a positive quality gain and a positive homophily loss with respect to the payoff when candidates cannot reapply. Therefore, since in the absence of storability, we have the equivalence between the profitability of one-shot and permanent deviations[101], there exists a profitable deviation away from meritocracy for $s/b > \rho^{\mathrm{m}}$ (and for all $s/b \in [1, \rho^{\mathrm{m}}]$) , i.e. the existence region of meritocracy shrinks.

Finally we show that the meritocratic equilibrium *path* starting from an initial state with empty storage is no longer an equilibrium path for s/b in some interval $[\rho^{\mathrm{m}}, \rho^{\mathrm{m}} + \epsilon)$ with $\epsilon > 0$. We first note that, on the meritocratic equilibrium path starting from an initial state with empty storage, storage is never used[102]. Considering the repeated deviation to basic-entrenchment described above yields that, for $x < 1/2$, there exists a strictly profitable deviation away from this equilibrium path for s/b slightly above ρ^{m}. Hence, when $x < 1/2$, then for s/b in some interval $[\rho^{\mathrm{m}}, \rho^{\mathrm{m}}+\epsilon)$ with $\epsilon > 0$, the meritocratic equilibrium path starting from an initial state with empty storage is no longer so.

R Hierarchies and the glass ceiling

For simplicity, we look at the continuous-time version of our model. Consider a large two-tier organization with a mass 1 of senior positions and a mass $J > 1$ of junior positions. A higher J corresponds to a "more pyramidal" organization. Between times t and $t + dt$, a fraction $\chi^S dt$ of seniors departs and is replaced by juniors promoted to seniority; a fraction $\chi^J Jdt$ of juniors departs as well. To offset these two flows out of the junior pool, a fraction $\hat{\chi} Jdt$ of new juniors is recruited (where $J\hat{\chi} = \chi^S + J\chi^J$). The flow of talented majority (minority) candidates is Xdt. We will assume that $X \leq J\hat{\chi}$ (otherwise the organization would be homogenous, and the absence of minority juniors would deprive us of an analysis of the glass ceiling). Seniors have control over hiring and promotion decisions.

As noted in the text, a glass ceiling in such hierarchical organizations results from control being located at the senior level. This operates through two channels:

[101] Hence, when candidates cannot reapply, the above repeated deviation yields a zero differential payoff for $s/b = \rho^{\mathrm{m}}$.

[102] Indeed, as we assume $\alpha = 0$, the organization faces at most one new talented candidate each period, and on the meritocratic equilibrium path, recruits her/him.

- *Concern for control:* as earlier in the paper, control allows groups to engage in favoritism. Because control is located at the senior level, this in turn implies some discrimination in promotions, which in general exceeds that at the hiring level (if any). A concern for control and the concomitant discrimination may arise even in large organizations, either because of shocks, or because the talent pool is larger in the minority.

- *Differential mingling effect:* for organizational reasons, senior members tend to hang around more with senior members than with junior ones. Their homophily concerns are therefore higher for promotions than for hiring decisions.

Because the second effect is at this stage of the paper newer, we illustrate it through a simple example, which can be much enriched in ways that we later discuss. Assume that senior members enjoy (expected lifetime) homophily benefits from in-group senior and junior members, which we denote respectively by b^S and b^J. The differential mingling effect is captured by $b^S > b^J$. A fraction $x \leq 1/2$ of new hires are in-group talented juniors, and similarly for the out-group ones: $xJ\hat{\chi}dt = Xdt$. Talent is observed prior to hiring. A talented member brings quality benefits to seniors equal to s^J when junior, and $s^S > s^J$ when senior. Assume that $s^l > b^l$ at both levels $l \in \{J, S\}$, and that $s^S - s^J > b^S - b^J$ (these two conditions generalize the previous assumption that quality matters to the majority).

In this framework, majority members are never worried about losing control, as the promotion of those who wil bring them the highest net benefits will always be tilted in favor of in-group juniors. This leads us to focus on the *majority's pecking order*: A promotion yields discounted net benefit to a majority senior member equal to 1) $s^S - s^J + b^S - b^J$ in the case of an in-group talented member; 2) $s^S - s^J$ for an out-group talented member; 3) $b^S - b^J$ for an in-group untalented member; 4) 0 for an out-group untalented member. This pecking order implies that promotion decisions will be tilted in favor of in-group members (except in the non-generic case in which all talented juniors are promoted and no untalented one is). In contrast, the junior population is balanced in composition; indeed, there is no rationale for the majority to discriminate at the hiring state as long as $s^J > b^J$.

When $X < \chi^S < 2X$, i.e. equivalently $x < 1 \Big/ \Big[1 + J\chi^J/\chi^S\Big] < 2x$, in steady state the organization promotes all talented in-group juniors, a fraction z of talented out-group juniors, and no untalented juniors. The flows in and out of the junior and senior pools must balance, yielding respectively: $J\hat{\chi} = \chi^S + J\chi^J$, and $J\hat{\chi}x(1+z) = \chi^S$.

We define the glass ceiling index as the relative probability of promotion of talented majority and minority members, minus 1:[103]

$$\gamma \equiv \frac{1}{z} - 1 = \frac{2X - \chi^S}{\chi^S - X} \in (0, \infty).$$

In this region, the glass ceiling index is invariant with how pyramidal the organization is (J)[104], decreases with the frequency of senior-level vacancies (χ^S) and increases with the flow of talented candidates (X). Covering all parameter regions, the glass ceiling index is monotonic with χ^S/X.[105]

Proposition R.1. ***(Glass ceiling)*** *In the hierarchical organization's steady state, hiring at the junior level is meritocratic. By contrast, there exists a glass ceiling for minority juniors.*

This environment can be enriched in interesting ways. First, one may distinguish between talent and "senior potential"; only a fraction of talented members have the potential to make a more important contribution at the senior level; furthermore it may take time for the organization to discover who has such senior potential (there is a time of reckoning). Second, talented members may have outside opportunities. Talented women may then quit the organization due to a discouragement effect: either they have been identified as lacking senior potential (their male counterparts by contrast staying in the organization), or the delay in being promoted is not worth the wait. Finally, the possibility of outside recruitment at the senior level would impact the glass ceiling effect.

S Negative homophily

As claimed in the text (see footnote 6), the case $\tilde{b} < 0$, corresponding to *negative homophily*, can be accommodated in our model. Indeed, the set of possible flow payoffs in any period still writes as $\{\tilde{s}, 0, \tilde{s}+\tilde{b}, \tilde{b}\}$. Hence, for $\tilde{b} < 0$, two cases must be distinguished:

[103] This definition of the glass ceiling index only looks at flows and is a conservative estimate of the glass ceiling; indeed, were we to look at stock, the glass ceiling effect would be stronger because the share of talented minority juniors promoted to seniority (over the whole stock of such juniors) would be below z (whenever $z < x$, the steady state of the junior pool features a mixture of talented minority and untalented majority juniors).

[104] An increase in J has two opposite effects: it makes it more difficult for a junior to be promoted, and talented minority members are the first to be left out; but it also makes talented juniors scarcer in the junior pool, increasing the minority members' probability of promotion.

[105] Indeed, for $\chi^S > 2X$, the senior majority hires all talented juniors and (some) untalented in-group juniors, and thus $\gamma = 0$, whereas for $\chi^S < X$, it promotes no out-group talented juniors, only talented in-group ones, and thus we set $\gamma = +\infty$.

- $\tilde{s}+\tilde{b} < 0$ (i.e. $-1 < \tilde{s}/\tilde{b} < 0$): the majority always votes for the minority candidate. The (end-of-period) majority size converges to k, which is an absorbing state. The majority then switches and control alternates between the two groups.

- $\tilde{s}+\tilde{b} > 0$ (i.e. $\tilde{s}/\tilde{b} < -1$): there always exists an equilibrium in which the majority votes for the most talented candidate with a tie-breaking rule in favor of the minority candidate.

Let us provide a few more details on the second case ($\tilde{s}/\tilde{b} < -1$). Indeed, the same computations as in the proof of Lemma 2 (see Online Appendix B) yield that, letting $u_i \equiv V_{i+1} - V_i$, with V_i the value function with such strategies, $0 < u_1 < ... < u_{k-1}$ and $u_{k-1} > ... > u_{N-2} > 0$, with

$$u_{k-1} = -(1-2x)b + \delta x \left[\frac{k-1}{N-1} u_k + u_{k-1} + \frac{k-2}{N-1} u_{k-2} \right],$$

and thus in particular,

$$\left[1 - 2\delta x \frac{N-2}{N-1} \right] u_{k-1} < -(1-2x)b.$$

As a consequence, deviations that yield a lower current-period flow payoff, together with a lower (in a first-order stochastic sense) distribution of next-period in-group sizes are strictly unprofitable. Moreover, as $0 < u_{N-2} < ... < u_{k-1}$, the deviation differential payoff for the majority from picking its in-group candidate instead of an at-least-as-talented out-group candidate (hence opting for a higher distribution of next-period in-group sizes at the expense of a lower current-period flow payoff) is maximal when both candidates have the same talent and the majority has size k. It then writes as

$$b + \delta \frac{k-1}{N-1}(u_{k-1} + u_k) < b + \delta \frac{N-2}{N-1} u_{k-1} < 0$$

using the above upper bound on u_{k-1}. Therefore, such a deviation is never profitable for the majority, and thus these strategies form an equilibrium.

T More than two horizontal groups

Suppose three groups $X \in \{A, B, C\}$ are located along the Hotelling line at -1, 0 and 1 respectively. The homophily benefit for a group X when the hired candidate is X' is $b[1 - d(X, X')]$ where $d(X, X')$ is the distance between the group and the candidate's locations. Each period, one member quits and is replaced by one of the candidates through majority voting. As preferences are single-peaked, we assume that the Condorcet winner is elected. But, as with two groups, group preferences embody not only the static preference among current candidates, but also possible losses of control in the future.

There is either no talented candidate (probability $1-3x$) or a single talented candidate (probability $x \leq 1/3$ that it belongs to any of the three groups).

One can show that for $s/b < 1$, $s/b > 1$ and close to 1, and s/b large, the unique equilibria are the analogs of the equilibria in the two-group environment.

Indeed, for $s/b < 1$, the unique equilibrium is full entrenchment, i.e., each group fully entrenches itself when majoritarian on its own, and if neither group A nor C has a majority on its own, the smallest flank sides with group B against the largest flank, and group B eventually reaches a majority on its own (and subsequently fully entrenches itself).

For s/b close to 1, the unique equilibrium is basic entrenchment with two flanks and a center:

- Groups A and C entrench themselves when majoritarian on their own: if they obtain the majority alone (they have at least $N/2$ members), they keep it. When their majority is not at stake, they vote for group B's candidate when (s)he is strictly more talented than their own candidate, and they vote for their own candidate otherwise.

- If neither group A nor group C has a majority on its own, group B is pivotal, i.e., has a de facto although not necessarily a de jure majority. It then behaves meritocratically as long as its majority is not tight.

- Group B has a "tight de facto majority" if the largest flank is in a position of acquiring control next period, were its candidate selected today. When group B has a tight de facto majority, the smallest flank sides with it to block the potential formation of an entrenched majority of members of the largest flank.

In this equilibrium, the smallest flank, which has an even higher stake than the center in preventing entrenchment by the opposite flank, always sides with the moderate group against the largest flank, and allows the moderate group to retain control even if it does not by itself enjoy majority control. Just like the basically-entrenched majority in the two-group paradigm, the center with respect to both flanks, and each flank with respect to the center behave meritocratically as long as their majority (whether on its own for a flank, or with another group's support for the center) is not tight.

Lastly, for s/b sufficiently large, the unique equilibrium is meritocracy:

- All groups always vote for the (single) talented candidate whenever there is one.
- In the absence of a talented candidate, group B's in-group candidate is elected.