

Vertical Foreclosure and Multi-Segment Competition*

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Abstract

This paper analyzes a supplier's incentives to foreclose downstream entry when entrants have stronger positions in different market segments, thus bringing added value as well as competition. We first consider the case where wholesale contracts take the form of linear tariffs, and characterize the conditions under which the competition-intensifying effect dominates, thereby leading to foreclosure. We then show that foreclosure can still occur with non-linear tariffs, even coupled with additional provisions such as resale price maintenance.

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1 Introduction

Many industries have seen the emergence of new distribution channels, such as mobile virtual network operators (MVNOs) in the telecommunication industry, or platforms such as Amazon.com or Alibaba.com in online retailing. These new channels often appeal to different types of customers. For example, MVNOs typically offer cheaper “no-frills” services, targeting price-sensitive or younger customers. Similarly, online platforms attract a broad audience whereas established brick-and-mortar stores appeal more to consumers with high brand loyalty. A challenge for these entrants, however, is to secure access to established suppliers. For example, MVNOs do not possess their own networks and therefore need access to existing networks. Similarly, online platforms must convince manufacturers to distribute their products through their channels.¹

When deciding on whether to grant access to their products, the incumbents face a trade-off. Entrants bring value by attracting different types of consumers. In theory, the incumbents may benefit from this through appropriate wholesale arrangements. In practice, however, it may be difficult to limit entry to specific segments and, more generally, to control entrants’ marketing strategies; they may then compete with the incumbents, thereby dissipating profit, and may even end-up challenging incumbents’ core business.

To study this trade-off, we develop a simple framework with one incumbent at both upstream and downstream levels, and two market segments. The downstream incumbent has a strong position in the high-end segment, and faces an entrant bringing value in the low-end segment. We first characterize the drivers of the incumbents’ decision to accommodate entry or foreclose the market when contractual arrangements are limited to linear wholesale tariffs. We then show that general non-linear tariffs – even coupled with additional vertical restraints such as resale price maintenance (RPM) – may not suffice to maximize industry profit or ensure entry accommodation, as the entrant will target the high-end segment whenever the margins are larger there. As a result, foreclosure may occur.

The literature on vertical foreclosure often focuses on linear tariffs,² thus leaving open the question of whether foreclosure may still occur when more elaborate contracts are feasible. The few papers allowing non-linear wholesale tariffs (e.g., Hart and Tirole, 1990, O’Brien and Shaffer, 1992) indeed emphasize that full exclusion is never optimal when the entrant offers a differentiated good, as non-linear tariffs allow the supplier to extract the higher industry profits.³ In contrast, we find that when firms can target specific market

¹In the telecommunication industry, MNOs foreclosed MVNOs in several countries (e.g., Spain or Poland) without regulation of the market (see European Commission, 2006). Also, many established apparel producers offer no or only a small selection of their products on Internet retailers.

²See, e.g., Ordober et al. (1990), Chen (2001), Sappington (2005), Ordober and Shaffer (2007), Hoeffler and Schmidt (2008), and Bourreau et al. (2011).

³See Rey and Tirole (2007) for a summary of foreclosure incentives with non-linear tariffs.

segments, non-linear tariffs, even augmented with RPM, do not guarantee entry.⁴

2 The Model

There are two incumbent firms, U and D . U costlessly supplies an input to D , who transforms it into a final good using a one-to-one technology. There is a unit mass of consumers, with a proportion x having valuations V_D and unit costs C , and the others having valuations v_D and unit cost c ; the first group of consumers constitutes the “high-end” market segment and are more valuable:

$$V_D - C > v_D - c.$$

We will consider two scenarios, in which either the incumbents are vertically integrated, or they can engage in efficient contracting (e.g., two-part tariffs); all results are valid in both scenarios. Throughout the paper, “the incumbents” will refer to the integrated firm in the former scenario, and to the upstream supplier in the latter scenario.

A new firm E can enter the downstream market, with a comparative advantage in the low-end segment: for the sake of exposition, we suppose that it faces the same costs as D in each segment, C and c , but offer different values to consumers, V_E and v_E , satisfying:

$$v_E > v_D \text{ and } V_D > V_E.$$

Downstream firms can discriminate consumers across the two segments: each firm $i = D, E$ sets two prices, P_i in the high-end and p_i in the low-end segment.⁵ By contrast, we assume that wholesale arrangements cannot be made contingent on targeted segments; that is, the tariff is only based on the quantity bought by E , not on which consumers E sells to.⁶

Absent entry, the industry maximizes its profit by setting $P_D = V_D$ and $p_D = v_D$, yielding a profit of

$$\Pi^m \equiv x(V_D - C) + (1 - x)(v_D - c).$$

We will assume that foreclosure is more profitable than removing D from the market,

⁴Gabrielsen and Johansen (2017) demonstrate that non-linear tariffs and RPM are not sufficient to maximize the industry profit if downstream firms must also make investment decisions. However, they are not concerned with market foreclosure.

⁵Our insights extend to second-degree price discrimination (i.e., when firms cannot distinguish between consumers but can offer several types of contracts), provided that high-end consumers generate larger volumes. See Online Appendix D for a formal analysis.

⁶In practice, firms may not be able or allowed to assign specific customer segments to their distributors – in Europe, for example, manufacturers are prevented from restricting brick-and-mortar retailers to open online stores as well.

that is:

$$\Pi^m > x(V_E - C) + (1 - x)(v_E - c). \quad (1)$$

Throughout the paper, we assume that the incumbents accommodate entry whenever they are indifferent between foreclosing it or not.

3 Linear tariffs

We first consider the case of linear tariffs. The game then consists of the following two stages: in the first stage, U offers a wholesale price w_E to E (and a two-part tariff (F_D, w_D) to D , if separated), which E (and D) can either accept or reject. In the second stage, D and E compete in prices.

We then have:

Proposition 1 *The incumbents foreclose entry if and only if*

$$x[(V_E - C) - (v_E - c)] > (1 - x)(v_E - v_D). \quad (2)$$

Proof. The incumbents can secure Π^m by charging $w_E = +\infty$ (and $(F_D = \Pi^m, w_D = 0)$, under separation). Furthermore, in order for the incumbents to obtain more profit than Π^m , E must be serving the low-end segment, which requires $w_E \leq v_E - c$.

Thus, consider $w_E \leq v_E - c$ and a candidate equilibrium in which E serves the low-end segment. If E supplies only the low-end segment, then the price at which D serves the high-end segment must satisfy $V_D - P_D \geq V_E - (C + w_E)$, as E is willing to serve high-end consumers at any price above $C + w_E$; hence, the incumbents cannot obtain more than

$$\Pi_I = x(P_D - C) + (1 - x)w_E \leq x(V_D - V_E) + w_E.$$

If instead E supplies both segments, then the incumbents' profit is equal to w_E . It follows that the maximal profit that the incumbents can obtain does not exceed:

$$x(V_D - V_E) + v_E - c.$$

Comparing this expression with Π^m shows that foreclosure occurs whenever (2) holds.

Conversely, entry occurs whenever (2) is not met. Suppose that U sets $w_E = v_E - c$, inducing E to offer $p_E = v_E$ in the low-end segment and $P_E = v_E - c + C$ in the high-end segment. If $v_E - c > V_E - C$, then E cannot actively compete in the high-end segment (as $P_E > V_E$) and D can thus charge $P_D = V_D$ in that segment. The incumbents (with

$w_D = V_D - C$, under separation)⁷ can then obtain the entire monopoly profit, equal to

$$\Pi^M = x(V_D - C) + (1 - x)(v_E - c) > \Pi^m.$$

If instead $v_E - c \leq V_E - C$, then D can serve the high-end segment at price $P_D = C + V_D - V_E + v_E - c$. Under vertical integration, the incumbents thus obtain:

$$\Pi = x(V_D - V_E) + v_E - c,$$

which exceeds Π^m if (2) does not hold. Under separation, charging $w_D = v_D - c$ ensures that D is not willing to serve the low-end segment but is willing to serve the high-end segment at this price P_D , as the margin $P_D - C = V_D - V_E + v_E - c$ exceeds w_D (using $V_D > V_E$ and $v_E > v_D$). ■

The intuition is as follows. Accommodating entry is profitable only if E serves the low-end segment, in which case U optimally charges $w_E = v_E - c$ to extract E 's profit in that segment. If $v_E - c \geq V_E - C$, then E cannot compete in the high-end segment (as $w_E + C$ exceeds V_E), and the incumbents obtain the maximal industry profit; hence, foreclosure does not occur in equilibrium. By contrast, if $v_E - c < V_E - C$, then E can compete in the high-end segment as well, which reduces the profit that the incumbents can achieve in that segment. Foreclosure is then optimal if this profit-dissipation effect (measured by the left-hand side of (2)) off-sets the added-value brought by E in the low-end segment (measured by the right-hand side). As can be seen from (2), this is more likely when the high-end segment is large (x high), E is relatively less competitive in the low-end segment ($(V_E - C) - (v_E - c)$ large), or adds little value to D in that segment ($v_E - v_D$ small).

When foreclosure occurs, mandating entry (e.g., by imposing a cap on the wholesale price w_E not exceeding $v_E - c$) can only benefit consumers,⁸ and increase social welfare.

4 Additional contractual provisions

From the previous discussion, if $v_E - c \geq V_E - C$, then the incumbents obtain the maximal industry profit. Otherwise, competition in the high-end segment prevents full industry profit maximization, which in turn can lead to foreclosure. We now analyze whether these insights carry over with more elaborate wholesale contracts.

We first note that non-linear tariffs alone do not restore industry profit maximization:

Proposition 2 *If $v_E - c < V_E - C$, non-linear tariffs do not allow the incumbents to achieve industry profit maximization.*

⁷This in particular ensures that D is not willing to contest E in the low-end segment.

⁸Consumers benefit from a lower price in the high-end market, and may obtain a greater value in the low-end market if $w_E < v_E - c$.

Proof: See Online Appendix A.

The intuition is that non-linear tariffs cannot prevent E from diverting part of its sales to the high-end segment, and E has indeed an incentive to do so whenever its margin there is larger than the margin in the low-end segment. A common way of controlling further the behavior of downstream firms is to restrict their prices, a practice known as Resale Price Maintenance (RPM). In the spirit of the above analysis, we consider here “industry-wide” RPM, where the price restrictions cannot be made contingent on consumer segments.⁹ If both types of consumers wish to buy at the maintained price, then the effectiveness of such provisions depends on the extent to which firms have to satisfy demand. We will consider two polar scenarios: in the first one (no rationing), firms cannot ration demand in any way; in the second scenario, firms can not only ration demand, but moreover select which consumers to serve. The next proposition shows that combining RPM with non-linear tariffs enables industry profit maximization in a larger range of situations, but not always.

Proposition 3 *If $v_E - c < V_E - C$ and non-linear contracts are allowed, then:*

- (i) *If $V_E \leq v_E$, then RPM restores industry profit maximization, regardless of whether firms can ration demand.¹⁰*
- (ii) *If instead $V_E > v_E$ and $C \geq c$, then RPM restores industry profit maximization when firms can select which buyers to serve.*
- (iii) *Finally, if $V_E > v_E$ but $C < c$, then RPM cannot restore industry profit maximization, regardless of whether firms can ration demand.*

Proof: See Online Appendix B.

If $V_E > v_E$ and $C \geq c$, then, in order to achieve profit maximization with industry-wide RPM, E must be able to ration demand selectively (namely, to refuse selling to high-end consumers). If instead E could simply ration demand but not choose its buyers, then under usual rationing schemes some high-end consumers would end-up buying from E .

The above propositions characterize the conditions under which the incumbents can obtain the industry monopoly profit. When this is not the case, they may be tempted to foreclose entry. The following Proposition confirms this possibility; for the sake of exposition, we focus on the case where firms can selectively ration consumers:

⁹If the incumbents could monitor sales and set separate RPM prices on each segment, then the contract would impose a price floor on the high-end market segment and foreclosure would not occur.

¹⁰In that case, simple linear tariffs actually suffice to achieve industry profit maximization.

Proposition 4 *Suppose that firms can use RPM and select which buyers to serve, $C < c$ and $V_E > v_E$. Regardless of whether non-linear contracts are allowed, foreclosure occurs if and only if*

$$x(V_E - v_E) > (1 - x)(v_E - v_D).$$

Proof: See Online Appendix C.

Hence, foreclosure can still occur with RPM. Comparing the condition of Proposition 4 with that of Proposition 1 shows that it occurs for a smaller parameter range with RPM than without (because $C < c$).

We conclude with a discussion of the impact of non-linear tariffs and RPM on consumers and society. Obviously, enlarging the set of instruments available has no impact when the incumbents achieve the industry monopoly outcome with linear tariffs, which is the case if $v_E - c \geq V_E - C$. When instead $v_E - c < V_E - C$, allowing non-linear tariffs and RPM can only increase the profit derived from accommodating entry, which enhances total welfare. The impact on consumers is more ambivalent, and depends on what happens in the benchmark situation (i.e., with linear tariffs and no RPM):

- When foreclosure occurs, consumers obtain no surplus. In that situation, enlarging the set of instruments benefits consumers if: (i) it induces entry; and (ii) it does not allow the incumbents to achieve the industry monopoly outcome. This is indeed the case when $V_E > v_E$, $C < c$, and:

$$x[(V_E - C) - (v_E - c)] > (1 - x)(v_E - v_D) \geq x(V_E - v_E).$$

In that case, allowing (non-linear tariffs and) RPM induces the incumbent to accommodate entry, and high-end consumers benefit from some competition (D serves them at price $\hat{P}_D < V_D$).

- When instead entry occurs, consumers obtain some surplus; in that situation, which arises when

$$x[(V_E - C) - (v_E - c)] \leq (1 - x)(v_E - v_D),$$

allowing non-linear tariffs and RPM harms consumers, by allowing the incumbents to increase the price charged in the high-end segment (namely, from $P_D = V_D - [(V_E - C) - (v_E - c)]$ to either $\hat{P}_D = P_D + c - C$ if $c > C$ and $V_E > v_E$, and up to V_D otherwise.

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Online Appendix (Not for Publication)

A Proof of Proposition A

To maximize industry profit, D must serve high-end consumers at price V_D and E must serve the low-end segment at price v_E . However, E can divert one unit from low-end consumers and sell it in to the high-end segment at price (slightly below) V_E , and has indeed the incentive to do so if $V_E - C > v_E - c$.

B Proof of Proposition B

(i) If $V_E \leq v_E$, then consumers from the high-end segment are not interested in buying from E at price $p_E = v_E$. The incumbents can thus obtain the industry monopoly profit by charging $w_E = v_E - c$, together with an RPM provision requiring E to charge $p_E = v_E$. E is willing to accept this contract and can then only sell to the low-end segment at that price. Furthermore, under integration, D has no incentive to compete in the low-end market, as U 's upstream margin, $w_E = v_E - c$, exceeds D 's downstream margin, $v_D - c$; similarly, under separation, charging $w_D = V_D - C$ ensures that D does not want to sell to the low-end segment (as $w_D = V_D - C > v_D - c$, which is the largest margin D could obtain in the low-end segment) and appropriates all of D 's profit.

(ii) If instead $V_E > v_E$ and $C \geq c$, charging $w_E = v_E - c$ achieves industry profit maximization if E can select its buyers: E is then willing to accept this contract and to serve the low-end segment at price $p_E = v_E$, and is not willing to serve the high-end segment at that price;¹ furthermore, as in case (i), D has no incentive to compete with E in the low-end segment under integration, and under separation when $w_D = V_D - C$.

(iii) Finally, suppose that $V_E > v_E$ but $C < c$. From the proof of Proposition 2, as $v_E - c < V_E - C$, the incumbents cannot achieve industry profit maximization if they do not sign an RPM contract with E . With an industry-wide RPM provision (for E , or for both downstream firms), D must charge $P_D = V_D$, and E must charge $p_E = v_E$ in both segments. If $V_E > v_E$, consumers from the high-end segments are then willing to buy from E , and E finds it profitable to divert sales from the low-end to the high-end segment whenever $C < c$.

C Proof of Proposition C

To characterize the profit that the incumbents can achieve by accommodating entry, we first provide a lower bound, and show that it cannot be improved.

¹In the limit case $C = c$, E is indifferent between serving that segment or not, and is thus willing to serve only the low-end segment.

Suppose that U offers to supply E at wholesale price $w_E = v_E - c$, together with a RPM provision requiring E to set $p_E = v_E$. Under integration, E then wins the competition for the low-end segment (as the integrated firm's upstream margin, $w_E = v_E - c$, exceeds its own downstream margin, $v_D - c$); it follows that D wins the competition for the high-end segment, at price $\hat{P}_D \equiv V_D - V_E + v_E$: indeed, D slightly undercuts E in that segment, as the associated margin, $\hat{P}_D - C = V_D - V_E + v_E - C$, exceeds the integrated firm's upstream margin, $w_E = v_E - c$.² Under separation, U can achieve the same outcome (and appropriate all the profits) by supplying D at wholesale price $w_D = \hat{P}_D - C$: D (resp., E) then sells the high-end (resp., low-end) market at price \hat{P}_D (resp., v_E) and cannot profitably serve the other segment. Hence, the resulting profit for the incumbents is given by:

$$\hat{\Pi} = x(V_D - V_E + v_E - C) + (1 - x)(v_E - c).$$

Suppose now that there exists an alternative equilibrium in which the incumbents accommodate entry and obtain a profit $\Pi > \hat{\Pi}$. This profit must be at least equal to Π^m , otherwise the incumbents would not accommodate entry; using (1), it must therefore satisfy:

$$\begin{aligned} \Pi &> \max \left\{ \hat{\Pi}, x(V_E - C) + (1 - x)(v_E - c) \right\} \\ &= x \left(\max \left\{ \hat{P}_D, V_E \right\} - C \right) + (1 - x)(v_E - c). \end{aligned} \quad (3)$$

Furthermore, as the entrant cannot lose money in equilibrium, the incumbents' profit, Π , cannot exceed the maximal industry profit, and thus:

$$x(\max \{P_D, V_E\} - C) + (1 - x)(v_E - c) \geq \Pi, \quad (4)$$

where P_D denotes D 's equilibrium price. To see this, note that $v_E - c$ is an upper bound on the margin obtained in the low-end segment, and $\max \{P_D, V_E\} - C$ an upper bound on the margin obtained in the high-end segment: either D sells, or E sells at a price that cannot exceed V_E . It follows from (3) and (4) that $P_D > \hat{P}_D$.

Next, we note that E must be serving (part of) the low-end segment: otherwise, foreclosure would be more profitable. This, in turn, requires $p_E \leq v_E$. High-end consumers would therefore be willing to buy from E at price p_E (as $p_E \leq v_E$ and $P_D > \hat{P}_D$). Moreover, as $C < c$, E prefers serving the high-end segment at p_E than serving low-end consumers at the same price. As it can select which buyers to serve, it follows that E must be serving the entire high-end segment, at some price $P_E \leq V_E$, as well as (part of) the low-end segment. As the margin on the low-end segment cannot exceed $v_E - c$, the

²This amounts to $V_D - V_E + v_E - C > v_E - c$, which follows from $V_D > V_E$ and $C < c$.

resulting industry profit is thus bounded by:

$$x(V_E - C) + (1 - x)(v_E - c).$$

From (1), this profit is lower than Π^m , a contradiction.

It follows that the incumbents' profit must be equal to $\hat{\Pi}$. Comparing $\hat{\Pi}$ with Π^m yields the condition in the Proposition.

D Second-Degree Price Discrimination

In this Appendix we show how our analysis can apply to the case of second-degree price discrimination, where firms cannot explicitly discriminate consumers (e.g., because they do not identify consumers' types, or because of a legal ban on such discrimination) but can offer multiple contracts, designed for different types.

To this end, we modify the baseline model by assuming that high-end consumers must get at least $N > 1$ units in order to obtain the utility V_D or V_E ; otherwise, their utility is zero. The other assumptions remain unchanged: low-end consumers only need one unit to obtain the utility v_D or v_E ,³ and the costs of serving high-end and low-end consumers are still C and c , respectively (note that C now denotes the total cost of supplying N units to a high-end consumer).

To discriminate consumers, each firm $i = D, E$ can offer two options: a single unit at price p_i and a bundle of N units at price P_i . For the sake of exposition, we will assume that low-end consumers cannot “unbundle” multi-unit options (high-end consumers may however satisfy their needs by combining several single-unit options). As a result, if N is large, namely, if

$$Nv_D \geq V_D,$$

then absent entry the incumbents can still extract all the surplus from consumers (by charging V_D and v_D to high-end and low-end consumers, respectively) and obtain:

$$\Pi^f = x(V_D - C) + (1 - x)(v_D - c).$$

For the sake of exposition, we focus here on the case where the wholesale contract between U and E simply consists of a linear tariff, and denote the wholesale price by w_E . Following the same reasoning as in the proof of Proposition 1, it can be checked that, to maximize their profit, the incumbents must charge $w_E = v_E - c$ and induce E to supply low-end consumers (and only those) at price $p_E = v_E$. High-end consumers are then not tempted to buy from E (at unit price $p_E = v_E$), as the above condition $Nv_D \geq V_D$

³For example, the low-end segment may correspond to households and the high-end segment to businesses, requiring larger volumes.

(combined with $v_E > v_D$ and $V_D > V_E$) implies $Nv_E > V_E$. However, instead of making zero profit in the low-end segment, E could target high-end consumers by offering them N units at a price above the associated cost, equal to $C + (v_E - c)N$; if this cost is lower than V_E and larger than v_E ,⁴ then E indeed exerts competitive pressure on D , preventing it to charge more than

$$\hat{P}_D = V_D - V_E + C + (v_E - c)N.$$

Summing-up, if $V_E - C > (v_E - c)N \geq v_E - C$, the maximal profit from accommodating entry is

$$x[V_D - C - (V_E - C) + (v_E - c)N] + (1 - x)(v_E - c).$$

Comparing this profit with Π^f , foreclosure is optimal if and only if

$$x((V_E - C) - (v_E - c)N) > (1 - x)(v_E - v_D).$$

We therefore obtain a very similar conclusion as in the baseline model: E still has an incentive to target the most profitable segment, which limits the incumbents' ability to exploit high-end consumers; this may induce the incumbents to foreclose entry.

⁴The case $C + (v_E - c)N < v_E$ does not look plausible and would induce E to sell the N -unit package at a minimal price $\hat{P} = x[C + (v_E - c)N] + (1 - x)v_E$.