Competitive Cross-Subsidization*

Zhijun Chen[†] Patrick Rey[‡]

15 December 2016

Abstract

Cross-subsidization arises naturally when firms with different comparative advantages compete for consumers with heterogeneous shopping patterns. Firms then face a form of co-opetition, as they offer substitutes for one-stop shoppers and complements for multi-stop shoppers. Competition for one-stop shoppers drives total prices down to cost, but firms subsidize weak products with the profit made on strong products. Firms and consumers would benefit from cooperation limiting cross-subsidization (e.g., through price caps). Banning below-cost pricing instead increases firms' profits at the expense of one-stop shoppers, which calls for a cautious use of below-cost pricing regulations in competitive markets.

JEL Classification: L11, L41.

Keywords: cross-subsidization, shopping patterns, multiproduct competition, co-opetition.

*We are grateful to Stephen Hamilton and Paul Klemperer, as well as to participants at the EARIE 2011 conference, the IIOC 2013 conference, the 2013 conference of the German Economic Association, and the seminars at the University of Melbourne and Monash University, for helpful comments. Financial support from the European Research Council (ERC) under the European Community's Seventh Framework Programme (FP7/2007-2013) Grant Agreement N 340903 is also gratefully acknowledged.

[†]Monash University; e-mail: chenzj1219@gmail.com.

[‡]Toulouse School of Economics, University of Toulouse Capitole, Toulouse, France; e-mail: patrick.rey@tse-fr.eu.

1 Introduction

Multi-product firms compete through a variety of pricing strategies. They may, for instance, engage in bundling,[1] a feature already extensively studied in the literature.[2] They may also engage in cross-subsidization, by pricing some products below cost and compensating the loss with profits from other products. Competition between Apple and Amazon offers a recent illustration. In 2010, in conjunction with the iPad, Apple launched "iBooks", an e-book application with more than 30,000 books pre-loaded free of charge on the iBooks store.[3] The following year, Amazon responded by launching the "Kindle Fire", a tablet computer version of its e-book reader, and offering it at a below-cost price.[4] It is commonly recognized that the iPad offers more functions than the Kindle Fire, while Amazon, which offers more than two million e-books, provides more variety and thus a higher match value than the iBooks store.[5] Hence, each firm has a comparatively stronger product in relation to its rival. Furthermore, both firms sell their comparatively weaker products below cost, and derive profits from their strong products. Moreover, consumers can combine the two firms' strong products, but not the weak ones: iPad users can download a free Kindle Application to access Amazon's e-books, whereas Kindle Fire users have no access to the iBooks store.

[1] Namely, offering a discount on some products, conditional on the purchase of others (mixed bundling), or offering selected products only as a bundle (pure bundling).

[2] For papers accounting for intra-product competition and heterogeneous shopping patterns, see, for example, Matutes and Regibeau (1992), Armstrong and Vickers (2010), and Zhou (2016).

[3] See Appleinsider's report, available at http://appleinsider.com/articles/10/03/25/apple_loads_up_new_ibooks_store_with_free_public_domain_ipad_titles.

[4] The Kindle Fire, which offers access to the Amazon Appstore, streaming movies and TV shows, is sold in the US at a retail price of $199. According to IHS, an information company, "Amazon makes its money not on Kindle hardware, but on the paid content and other products it plans to sell the consumer through the Kindle". IHS estimates Amazon's hardware cost for a Kindle Fire at $201.70, not including "additional expenses such as software, licensing, royalties or other expenditures." See https://technology.ihs.com/389433/amazon-kindle-fire-costs-20170-to-manufacture.

Amazon still sells its Kindle Fire series below cost, and launched a $50 Kindle Fire 7 tablet in 2015. See http://www.ibtimes.com/fire-tablet-vs-ipad-why-amazons-50-tablet-betting-content-king-2102347.

[5] For a comparison between iBooks and Amazon e-books, see, for example: http://appadvice.com/appnn/2015/04/apples-ibooks-versus-amazons-kindle-leading-e-reading-apps-go-head-to-head.

Apple and Amazon's strategies in competitive markets such as tablets and e-books,[6] in which the relevant information is readily available,[7] are somewhat at odds with the existing theory. According to this theory, cross-subsidization arises in the context of regulated or monopolistic markets,[8] or in markets characterized by frictions such as consumers' limited information or bounded rationality.[9] We develop here a new approach, based on the diversity of purchasing patterns.

The literature on competitive multiproduct pricing often assumes that customers engage in "one-stop shopping" and purchase all products from the same supplier. Yet, in practice, many customers engage in multi-stop shopping and rely on several suppliers to fulfill their needs.[10] The choice between these purchasing patterns is driven not only by the diversity and the relative merits of suppliers' offerings, but also by the transaction costs that buyers must bear in order to enjoy the products. As mentioned by Klemperer (1992), these transaction costs include physical costs such as transportation costs, and non-physical costs, such as the opportunity cost of time and the adoption cost of using a new electronic device. Following the terminology of the literature, we will refer to these costs as "shopping costs".[11] Obviously, these costs vary across customers. For example, some consumers may face tighter time constraints and/or dislike shopping, whereas others may be less time-constrained and/or enjoy shopping. Indeed, some users, already familiar with the Kindle system, may be reluctant to switch to the iPad because of the associated learning costs,[12] whereas others may enjoy the adoption of a new device. All other things

[6] Competition is particularly intense in the tablet computer market. Many other firms are also present in the e-book market.

[7] The iPad and the Kindle Fire are each sold at a uniform price, and these prices are readily available on Apple or Amazon's websites. The same applies to iBooks and Amazon's e-books.

[8] For instance, Faulhaber (2005, pp.442) asserts that "under competitive conditions, the issue of cross-subsidy simply does not arise."

[9] See the literature review below.

[10] According to the marketing literature, patronizing multiple stores becomes an important pattern in the grocery retail business; see, for example, Gijsbrechts et al. (2008), who conclude that in the US roughly 75% of grocery shoppers regularly shop at more than one store every week.

[11] This terminology is widely adopted in the literature on multiproduct competition – see, for example, Klemperer and Padilla (1997) and Armstrong and Vickers (2010).

[12] Before the launch of the iPad and the Kindle Fire, readers of Amazon's e-books were mainly using

being equal, customers with high transaction costs tend to favor "one-stop shopping", whereas others are more prone to "multi-stop shopping".

We first note that the diversity of purchasing patterns gives rise to a form of "co-opetition": On the one hand, firms offer substitutes for one-stop shoppers, who look for the best basket of products; while on the other hand, firms offer complements for multi-stop shoppers, who seek to combine suppliers' best products. We show that this duality drastically affects firms' pricing strategies and can lead to cross-subsidization, even in competitive markets.

Specifically, we consider a setting in which two firms offer the same product line (which consists of two products, for the sake of simplicity). Consumers are perfectly informed about prices, as is indeed the case for e-books and tablets. To discard price-discrimination motives, we further assume that consumers have inelastic demands. Altogether, these assumptions allow us to abstract away from the motivations already highlighted in the literature on cross-subsidization (see the literature review below). Our key ingredients are instead that: (i) consumers have heterogeneous shopping costs; and (ii) through lower costs and/or higher consumer value, each firm enjoys a comparative advantage over one product.[13] For the sake of exposition, we initially assume that firms have similar comparative advantages; that is, each firm has a stronger product than its rival, but overall their baskets generate the same surplus. In equilibrium, consumers with high shopping costs engage in one-stop shopping, and competition for these consumers drives firms' aggregate prices down to cost. By contrast, consumers with low shopping costs engage in multi-stop shopping and buy each firm's strong product, by which means the firms make a profit. Cross-subsidization therefore arises naturally, where each firm prices its weak product below cost and subsidizes the loss with the profit from its strong product.

This provides some insights on the outcome of co-opetition. On the one hand, aggregate price levels are "competitive": firms supply one-stop shoppers at cost. If firms could coordinate their pricing strategies, they would raise total prices in order to exploit one-stop shoppers. At the same time, however, a lack of coordination over the prices charged to multi-stop shoppers leads to "double marginalization", as each firm charges a margin

the original Kindle device.

[13] For instance, the iPad for Apple and the e-book catalogue for Amazon.

on its strong product. This causes excessive cross-subsidization and results in not enough multi-stop shopping: limiting cross-subsidization would benefit both firms and consumers.

These insights are quite robust and remain valid in more general settings. We show, in particular, that the analysis applies when the dispersion of shopping costs is limited (as long as both shopping patterns arise in equilibrium), or when one firm offers a better basket than the other, thus enjoying market power over one-stop shoppers. We also extend our framework to account for the development of online sales, which we capture as reducing the shopping costs for "internet-savvy" consumers. We find that this leads to higher prices for multi-stop shoppers.

The prevalence of cross-subsidization in retailing markets has led many countries to adopt specific regulations prohibiting or restricting certain forms of below-cost pricing.[14] These regulations are however quite controversial and have triggered an intense policy debate.[15] To shed some light on this debate, we consider a variant where below-cost pricing is banned. The equilibrium then involves mixed strategies: firms sell weak products at cost but randomize prices for their strong products. Banning below-cost pricing thus results in higher prices for one-stop shoppers (who can no longer purchase the products at cost), and greater profitability for firms (in fact, their expected profits more than double). The impact on multi-stop shoppers is less obvious. However, when weak products offer relatively low value, there is not a lot of one-stop shopping; firms are therefore not overly concerned about losing sales to one-stop shoppers and charge higher prices to multi-stop shoppers as well. Depending on the distribution of shopping costs, this reduction in consumer surplus may exceed the increase in firms' profits and thus result in lower total welfare. This suggests that regulations on below-cost pricing in competitive markets should be carefully evaluated.[16]

---

[14]In the US, half of the states have adopted laws against below-cost resale, and some of the other states have adopted similar rules for gasoline markets; see Calvani (2001). In the EU, below-cost resale is banned in Belgium, France, Ireland, Luxembourg, Portugal, and Spain, and is restricted in Austria, Denmark, Germany, Greece, Italy, and Sweden.

[15]For instance, the OECD (2007) argues that these laws are more likely to harm consumers than benefit them. See Section 5 for a more detailed discussion.

[16]By contrast, Chen and Rey (2012) show that banning below-cost pricing in concentrated markets can discipline the pricing behavior of a dominant firm competing with smaller firms. Such a ban then

Related literature. Cross-subsidization has been extensively studied in the context of regulated markets such as telecommunications, energy, and postal markets, in which historical incumbents fight entry by pricing below cost in liberalized segments,[17] subsidizing their losses with the profits earned in protected segments. There is a small literature of cross-subsidization in unregulated, competitive markets; however, it typically assumes that consumers engage in one-stop shopping, and relies either on consumers' limited information or on bounded rationality.

In a setting where consumers are initially unaware of prices, Lal and Matutes (1994) show that firms advertise a loss-leader product in order to attract consumers.[18] Rhodes (2015) develops a multi-product search model where competing firms randomly advertise one product at a low price, and may even set its advertised price below cost. By contrast, when consumers are aware of prices, Ambrus and Weinstein (2008) show that below-cost pricing does not arise when consumers have inelastic demands or when consumers have sufficiently diverse preferences.[19]

Ellison (2005) and Gabaix and Laibson (2006) study add-on pricing and product shrouding. Firms may price a leading product below cost (such as a hotel room fee) to lure consumers and subsidize the loss with the profit from shrouded add-on prices (such as telephone call charges and internet access fees). Grubb (2009) considers consumers with behavioral biases (such as over-confidence about the usage management) in the mobile-phone-service market, and shows that such bias can lead firms to price below cost on some units within a mobile-service plan. Recently, Johnson (2016) considers a setting in which one-stop shoppers may underestimate their needs, and shows that below-cost pricing may arise when consumers have different biases across products.[20]

---

benefits both consumers and smaller rivals, and enhances social welfare.

[17] Such an exclusionary motive does not appear relevant for the tablet and e-book markets. Amazon can hardly hope to drive the iPad out of the market, and conversely, Apple is probably not primarily aiming to exclude Amazon's e-books.

[18] In equilibrium, consumers stop searching after the first visit, and thus all consumers are one-stop shoppers in their setting.

[19] They find that below-cost pricing arises only when consumers have elastic demands exhibiting a very specific form of complementarity.

[20] There is also a marketing literature on loss leading that focuses on impulsive purchases. For instance,

In the case of tablets and e-books, as already noted, information about Apple and Amazon's prices is readily available to consumers. Furthermore, bounded rationality may be less relevant for simple goods such as e-books than for more complex products such as mobile telephony services. Yet, accounting for the diversity of purchasing patterns enables us to offer a rationale for the observed cross-subsidization, even in the absence of any limitation on consumers' information and rationality.

The paper is organized as follows. Section 2 illustrates the main intuition by way of a simple example. Section 3 develops our baseline framework, with symmetric comparative advantages and a wide range of transaction costs. Section 4 presents our main insights – in equilibrium, both shopping patterns coexist, and firms engage in cross-subsidization despite selling their baskets at cost. Section 5 studies the impact of a ban on below-cost pricing. Section 6 shows that the insights remain valid when transaction costs are bounded (as long as both shopping patterns arise) and when firms have asymmetric comparative advantages. It also explores the effect of a change in the distribution of shopping costs, triggered by the development of online retailing. Finally, Section 7 concludes.

2 A simple example

A numerical example illustrates the main intuition. Consumers wish to buy two goods, A and B, which can both be supplied by two firms, 1 and 2. Firm 1 enjoys a lower unit cost for good A whereas firm 2 enjoys the same cost advantage for good B: $c_1^A = c_2^B = \$10 < c_2^A = c_1^B = \$30$. Finally, consumers face a shopping cost s, reflecting the opportunity cost of the time spent in traffic, parking, selecting products, checking out, and so on. It may also account for consumers' enjoyment or dislike of shopping.[21]

Suppose first that all consumers face a "high" shopping cost, larger than the efficiency gain: $s \geq \Delta c = \$20$. In equilibrium, consumers then behave as one-stop shoppers, that is, they buy both products from the same firm, and thus only the total basket prices, P_1

Hess and Gerstner (1987) show that firms can use loss leader products to lure consumers, who will purchase some other products impulsively. Such impulsive purchases are similar to the "unplanned purchases" analyzed by Johnson (2016).

[21] Consumers' values for A and B are assumed to be larger than production and shopping costs.

and P_2, matter. As the firms face the same total cost of \$40, Bertrand-like competition drives the basket price down to this cost: $P_1 = P_2 = \$40$.

Suppose instead that the shopping cost s is sufficiently low such that, in equilibrium, consumers behave as multi-stop shoppers and purchase each product at the lowest available price. Asymmetric Bertrand competition then leads firms to sell weak products at cost, that is, $p_2^A = p_1^B = \$30$, and strong products at a price equal to (or just below) the rival's cost minus consumers' shopping costs: $p_1^A = p_2^B = \$30 - s$.

Hence, in these two situations, where all consumers adopt the same shopping pattern, firms have no incentive to engage in cross-subsidization.[22] Next, suppose that a fraction of consumers face a high shopping cost, $s_H = \$20$, whereas the others have a low shopping cost, $s_L = \$2$. As before, fierce price competition dissipates profits from one-stop shoppers, and drives basket prices down to total cost:

$$p_1^A + p_1^B = p_2^A + p_2^B = \$40.$$

Yet, as each firm has a cost advantage in one market, it can sell its strong product at a lower price than its rival. Keeping the total price constant for one-stop shoppers, it suffices to undercut the rival's weak product by the amount of $s_L = \$2$ to attract multi-stop shoppers. It follows that the equilibrium prices are given by:

$$p_1^A = p_2^B = \$19,\ p_1^B = p_2^A = \$21.$$

That is, each firm sells its weak product below cost ($\$21 < \$30$) and compensates the loss with the profit from the strong product ($\$19 > \$10$). This pricing strategy does not affect the shopping behavior of high-cost consumers (who still face a total price of \$40), but generates a positive profit from multi-stop shoppers, who buy A from 1 and B from 2 as $p_1^A + p_2^B = \$38 < \$40$, giving each firm a positive margin of $p_1^A - c_1^A = p_2^B - c_2^B = \$9$. Note that this cross-subsidization arises whatever the proportion of high and low shopping costs are. In particular, starting from a situation where all consumers have the same shopping costs, introducing an arbitrarily small number of consumers with a different shopping cost suffices to drastically alter the equilibrium prices and to ensure the adoption of cross-subsidization strategies.

[22] In the first case, where only total basket prices matter, firms may as well offer each product at cost.

3 Baseline model

We now consider more general supply and demand conditions. Consumers are willing to buy one unit of A and one unit of B. Each firm $i \in \{1, 2\}$ can produce a variety of each good, A_i and B_i, at constant unit costs c_i^A and c_i^B. Consumers have homogeneous preferences, and derive utility u_i^j from firm i's variety of good $j = A, B$.[23]

Throughout the analysis, we assume that firm 1 enjoys a comparative advantage in the supply of good A, whereas firm 2 enjoys a comparative advantage for good B. This may reflect a specialization in different product lines, and be driven by better product quality (i.e., $u_1^A > u_2^A$), a lower cost (i.e., $c_1^A < c_2^A$), or a combination of both.[24] For the sake of exposition, we initially focus on the case where firms enjoy the same comparative advantage for their strong products:

$$u_1^A - c_1^A - \left(u_2^A - c_2^A\right) = u_2^B - c_2^B - \left(u_1^B - c_1^B\right) \equiv \delta > 0, \tag{1}$$

implying that their baskets offer the same total value:[25]

$$u_1^A - c_1^A + u_1^B - c_1^B = u_2^A - c_2^A + u_2^B - c_2^B \equiv w > \delta. \tag{2}$$

Our key modelling feature is that consumers incur a shopping cost, s, to visit a firm, and that this cost varies across consumers, reflecting the fact they may be more or less time-constrained, or that they value the shopping experience in different ways. Intuitively, consumers with high shopping costs favor one-stop shopping, whereas those with lower

[23] While we focus here on independent demands for A and B, the analysis carries over when there is partial substitution or complementarity, that is, when the utility derived from enjoying both A_i and B_h is either lower or higher than $u_i^A + u_h^B$.

[24] For instance, Amazon has specialized in the e-books market since the 1990s and has exclusive deals with leading publishers. It provides a much larger variety of e-books and thus a much higher match value for consumers than Apple's iBooks. In contrast, Amazon's Kindle Fire has very limited applications and its value is much lower than Apple's powerful iPad.

[25] That $w > \delta$ reflects the assumption that A_2 and B_2, despite offering less value than A_1 and B_2, nevertheless generate a surplus:

$$u_1^A - c_1^A = u_2^B - c_2^B = \frac{w + \delta}{2} > u_2^A - c_2^A = u_1^B - c_1^B = \frac{w - \delta}{2} > 0.$$

shopping costs can take advantage of multi-stop shopping. Shopping patterns are, however, endogenous and depend on firms' prices. To ensure that both types of shopping patterns arise, we will assume that the shopping cost s is sufficiently dispersed, namely:

Assumption A: The shopping cost s is distributed according to a cumulative distribution function $F(\cdot)$ with positive density function $f(\cdot)$ over $\mathbb{R}_+$.

Finally, we assume that firms compete in prices; that is, the firms simultaneously set their prices, (p_1^A, p_1^B) and (p_2^A, p_2^B),[26] and, having observed all prices, consumers then make their shopping decisions. We will look for the subgame-perfect Nash equilibria of this game.

4 Competitive cross-subsidization

We first show that, in equilibrium, multi-stop and one-stop shopping patterns coexist, with multi-stop shoppers buying strong products and competition for one-stop shoppers driving firms' basket prices down to cost:

Lemma 1 *Under Assumption A, in equilibrium:*

- *(i) there are both multi-stop shoppers and one-stop shoppers;*
- *(ii) multi-stop shoppers buy firms' strong products, A_1 and B_2; and*
- *(iii) firms sell their baskets at cost.*

Proof. See Appendix A. ■

The first two insights are intuitive. Consumers with very low shopping costs (s close to 0) are willing to visit both firms so as to combine products with better value. Conversely, consumers with high shopping costs (s close to w, and thus such that $s > \delta$) are willing to visit one firm at most. The last insight follows directly from the firms' symmetric position, vis-à-vis one-stop shoppers: as their baskets generate the same value w, Bertrand-like competition drives their prices down to cost.

Building on Lemma 1 leads to our main insight:

[26] Allowing for pure or mixed bundling does not affect the analysis; see the remark at the end of Section 4.

Proposition 1 *Under Assumption A, in equilibrium firms sell their weak products below cost.*

Proof. See Appendix B ■

The intuition is fairly simple. As firms sell their baskets at cost to one-stop shoppers, they can only earn a profit from the sales of their strong products to multi-stop shoppers. Hence, it cannot be optimal for the firms to price their strong products below cost. Suppose now that firm i, for example, sells its strong product at cost (and thus sells its weak product at cost as well), and consider the following "cross-subsidization" deviation: maintaining a total margin equal to zero, firm i slightly raises the price on its strong product, reducing the price on the weak product by the same amount. This deviation does not affect the basket price offered to multi-stop shoppers, but generates a profit from multi-stop shoppers, who now pay a higher price for the strong product. As the deviation decreases the value of multi-stop shopping, it may also induce some consumers to switch to one-stop shopping. But this does not affect firm i: initially, it was earning zero profit from multi-stop shoppers, which it also earns from one-stop shoppers, regardless of which firm they go to. Hence, cross-subsidization is profitable.

To go further, we introduce the following regularity condition:

Assumption B: The density function $f(\cdot)$ is continuous and the inverse hazard rate $h(\cdot) \equiv F(\cdot)/f(\cdot)$ is strictly increasing.

The following proposition then establishes the existence of a unique equilibrium:

Proposition 2 *Under Assumptions A and B, there exists a unique equilibrium, in which both firms sell their weak products below cost and cross-subsidize them with their strong products. More precisely, defining:*

$$j(x) \equiv x + 2h(x), \tag{3}$$

we have:

(i) consumers with a shopping cost $s < \tau^*$, where:

$$0 < \tau^* \equiv j^{-1}(\delta) < \delta \, (< w),$$

engage in multi-stop shopping (they visit both firms and buy their strong products), whereas consumers with a shopping cost $\tau^* < s < w$ engage in one-stop shopping and buy both products from the same firm (either one); and

(ii) both firms offer their baskets at cost, but charge the same margin $\rho^* = h(\tau^*) > 0$ on their strong products and the same margin $-\rho^* < 0$ on their weak products.

Proof. See Appendix C. ■

The characterization of this equilibrium builds on Lemma 1. Firms only derive a profit from selling their strong products to multi-stop shoppers, that is, those consumers with a sufficiently low shopping cost, namely:

$$s < \tau \equiv \delta - \rho_1 - \rho_2,$$

where $\rho_1 \equiv p_1^A - c_1^A$ and $\rho_2 \equiv p_2^B - c_2^B$ respectively denote firm 1 and 2's margins on their strong products. Hence, firm i's profit can be expressed as:

$$\pi_i(\rho_1, \rho_2) = \rho_i F(\tau) = \rho_i F(\delta - \rho_1 - \rho_2). \tag{4}$$

The monotonicity of the inverse hazard rate $h(\cdot)$ ensures that the profit function given by (4) is strictly quasi-concave, and the "aggregative game" nature of this profit function then ensures that the equilibrium is unique and symmetric.[27] Specifically, both firms charge the same positive margin ρ^* on their strong products, and the same negative margin $-\rho^*$ on their weak products, where the equilibrium margin ρ^* is characterized by the first-order condition:

$$\rho^* = h(\delta - 2\rho^*).$$

The equilibrium threshold for multi-stop shopping, τ^*, thus satisfies:

$$\tau^* = \delta - 2\rho^* = \delta - 2h(\tau^*),$$

and is therefore given by $\tau^* = j^{-1}(\delta)$, where $j^{-1}(\cdot)$ is strictly increasing. Finally, in equilibrium, each firm earns a positive profit, equal to:

$$\pi^* = \rho^* F(\tau^*) = h(\tau^*) F(\tau^*).$$

[27]See Selten (1970). Here, firm i's profit is a function of its own margin ρ_i and of the threshold τ, which only depends on the sum of the two firms' margins, $\rho_1 + \rho_2$.

The equilibrium characterized by Proposition 2 is consistent with the cross-subsidization strategies adopted by Amazon and Apple in the tablet and e-book markets, where weak products (Kindle Fire and iBooks) are sold below cost whereas strong products (the iPad and Amazon's e-books) are sold with positive margins.

As mentioned in the Introduction, here firms face a form of co-opetition: they offer substitute products to one-stop shoppers, but at the same time they offer complementary products to multi-stop shoppers. Indeed, the firms' baskets are perfect substitutes for one-stop shoppers; as is standard in such a case, fierce competition for these consumers drives the total basket price down to cost. Yet, firms make a profit on multi-stop shoppers, who visit both firms in order to buy their strong products. Furthermore, a *reduction* in the price of one firm's strong product encourages additional consumers to switch from one-stop to multi-stop shopping, thereby *increasing* the other firm's profit. As is usual with complements, the prices of strong products are subject to double marginalization problems. When contemplating an increase in the price of its strong product, firm i balances between the positive impact on its margin ρ_i and the adverse impact on multi-stop shopping, but ignores the negative effect of this reduction in multi-stop shopping activity on the other firm's profit. Firms would therefore benefit from a mutual moderation of the prices charged on these products. Interestingly, while double marginalization is usually associated with excessively high price *levels*, here it yields excessively distorted price *structures*: firms' total prices remain at cost, but they engage in excessive cross-subsidization, compared with what would maximize their joint profit. Keeping total margins equal to zero, firms' joint profit when charging a margin ρ on strong products is given by:

$$2\rho F(\tau) = 2\rho F(\delta - 2\rho),$$

and is maximal for some $\hat{\rho} < \rho^*$.[28]

Letting firms negotiate price cap agreements would enable them to alleviate double marginalization problems by limiting cross-subsidization.[29] If, for instance, firms intro-

[28] A standard revealed preference argument yields $\hat{\rho}F(\delta - 2\hat{\rho}) > \rho^* F(\delta - 2\rho^*) > \hat{\rho}F(\delta - \rho^* - \hat{\rho})$, implying $\hat{\rho} < \rho^*$.

[29] For a discussion of such commercial cooperation and price cap agreements, see Rey and Tirole (2013); Lerner and Tirole (2015) also provide a discussion of price commitments in the context of standard setting.

duce a cap $\hat{\rho}$ on the margins charged on strong products, then in the resulting equilibrium: (i) competition for one-stop shoppers still drives total prices down to cost; but (ii) price caps limit cross-subsidization: firms charge the same margins, $\rho_1 = \rho_2 = \hat{\rho}\,(< \rho^*)$, on strong products and the same margin, $-\hat{\rho}$, on weak ones. Despite an increase in the prices of weak products, the adoption of price caps would nevertheless benefit both consumers and firms. Consumers opting for one-stop shopping would remain supplied at cost, and reducing the prices on strong products would not only benefit multi-stop shoppers but, by limiting double marginalization, would also increase the profit made on these multi-stop shoppers.

Remark: Shopping costs and complements. At first glance, that shopping costs generate complementarities in firms' products might not come as a surprise. Indeed, although consumers have independent demands for goods A and B, as one might expect, one-stop shopping introduces a complementarity between the products offered *within a firm*: cutting the price of A_i, say, is likely to steer one-stop shoppers towards firm i, which in turn boosts the sales of the firm's other product, B_i. This form of complementarity is not specific to our setting and has already been documented, not only in marketing and retailing,[30] but in many other areas as well.[31] More interestingly, however, in our setting, multi-stop shopping introduces a complementarity *across firms*, namely, between their strong products: cutting the price of one firm's strong product induces marginal consumers to switch from one-stop to multi-stop shopping, which boosts the sales of the other firm's strong product.[32]

Remark: Bundling. As consumers have homogeneous valuations, there is no scope here for tying and (pure or mixed) bundling. For instance, if one firm ties both products together physically, consumers are forced to engage in one-stop shopping, and price

[30] See, for example, Messinger and Narasimhan (1997).

[31] These include public services (see, e.g., Dykman (1995) for a study of one-stop career centers set-up by the US Department of Labor to provide employment and related social services), health care (see, e.g., Glick (2007) for multispecialty dental offices or Snow (1996) for long-term-care and managed-care organizations) and legal services (see, e.g., Bahls (1990)).

[32] A similar complementarity for multi-stop shoppers arises when shopping patterns are driven by heterogeneous preferences rather than transaction cost differences; see Armstrong and Vickers (2010).

competition for one-stop shoppers leads to zero profit. Similar reasoning applies to pure bundling when products are costly, to such an extent that it does not pay to add one's favorite variety to a bundle. In principle, a firm may also engage in mixed bundling, and offer three prices: one for its strong product, one for the weak product, and one (involving a discount) for the bundle. However, as one-stop shoppers only purchase the bundle, and multi-stop shoppers only buy the strong product, no consumer will ever pick the weak product on a stand-alone basis. Hence, only two prices matter here: the total price for the bundle, and the stand-alone price for the strong product. As these prices can be implemented using the stand-alone prices for the two products, offering a bundled discount (in addition to these stand-alone prices) cannot generate any additional profit.

5 Resale-below-cost laws

In regulated industries, cross-subsidization has been a well-recognized issue in both theory and practice,[33] and has prompted regulators to impose structural or behavioral remedies.[34] In contrast, in competitive markets, the policy debate is more divided. Although below-cost pricing might be treated as predatory,[35] in many cases (including the Apple vs. Amazon example) there is no such thing as a "predatory phase" followed by a "recoupment phase" (e.g., once rivals have been driven out of the market), which constitute key features of predation scenarios.[36] As mentioned in the Introduction, this has led many countries to adopt specific rules prohibiting or limiting below-cost pricing in retail

---

[33] The seminal paper of Faulhaber (1975) rigorously defines the concept of cross-subsidy and introduces two tests for subsidy-free pricing, which have been widely applied in both regulation and antitrust enforcement. See Faulhaber (2005) for a recent survey.

[34] Such concerns led, for instance, to the break-up of AT&T and the imposition of lines of business restrictions on local telephone companies (*U.S. v. AT&T* 1982). More recently, the European Commission required the German postal operator to stop cross-subsidizing its parcel services with the profit derived from its legal monopoly on letter services (*Deutsche Post* 2001).

[35] See, for example, Bolton, Brodley and Riordan (2000) and Eckert and West (2003) for detailed discussions of how predatory pricing tests should be designed. Rao and Klein (1992) and Berg and Weisman (1992) examine the treatment of cross-subsidization under US antitrust laws.

[36] For instance, the feasibility of recoupment is a necessary condition for a case of predation in the US, since the Supreme Court decision in *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*

markets. These rules, known as Resale-Below-Cost (RBC hereafter) laws, have been the subject of heated policy debates. In Ireland, for example, based on evidence that consumers pay more when grocery goods are subject to the prohibition of below-cost sales, in 2005 the Irish Competition Authority recommended terminating the RBC law.[37] However, the Irish Joint Committee on Enterprise and Small Business recommended keeping the RBC law due to concerns about an increased concentration in grocery retailing and predatory pricing. The Irish example highlights the dilemma of antitrust authorities: RBC laws may prevent dominant retailers from engaging in predatory pricing against smaller or more fragile rivals, but in competitive markets they may also lead to higher prices and thus harm consumers.

We now examine the impact of a ban on below-cost pricing in our setting. We first note that such a ban raises equilibrium basket prices, which benefits firms at the expense of one-stop shoppers:

Proposition 3 *When below-cost pricing is prohibited, in equilibrium each firm obtains a profit at least equal to:*

$$\bar{\pi} \equiv \max_{\rho} \rho F\left(\delta - \rho\right) > 2\pi^{*}.$$

It follows that, compared to the equilibrium that arises in the absence of a ban under Assumptions A and B:

(i) firms more than double their profits; and

(ii) one-stop shoppers face higher prices for the firms' baskets.

Proof. See Appendix D. ∎

The intuition is simple. If the rival offers both of its products at cost, a firm cannot make a profit on one-stop shoppers, but can still make a profit by selling its strong product to multi-stop shoppers. Indeed, charging a margin $\rho < \delta$ induces consumers with shopping cost $s < \tau = \delta - \rho$ to buy both strong products, thus generating a profit $\rho F\left(\delta - \rho\right)$. By

[37] The Irish Competition Authority examined pricing trends under the Groceries Order (the RBC law introduced in Ireland in 1987). The authority found that prices for grocery items covered by the Order had been increasing, while prices for grocery items not covered by the Order had been decreasing; it concluded that, on average, Irish families were paying 500 euros more per year because of the Order. See OECD (2007).

choosing the optimal margin:

$$\bar{\rho} \equiv \arg\max_{\rho} \rho F(\delta - \rho), \tag{5}$$

the firm can thus secure $\bar{\pi}$. To conclude the argument, it suffices to note that, in response to any prices set by the rival, the firm can still obtain at least $\bar{\pi}$ by charging a prohibitive price on its weak product (so as to induce multi-stop shoppers to buy firms' strong products) and a margin of $\bar{\rho}$ on its strong product.

Hence, in any equilibrium, each firm earns a profit at least equal to $\bar{\pi}$. Furthermore, as the rival can no longer subsidize its weak product, each firm now more than doubles its profit:[38]

$$\bar{\pi} = \max_{\rho} \rho F(\delta - \rho) > 2\rho^* F(\delta - 2\rho^*) = 2\pi^*.$$

Finally, equilibrium total margins are positive, as weak products cannot be sold below cost, and strong products are sold with a positive margin. One-stop shoppers thus face higher prices than in the absence of the ban.

Intuitively, banning below-cost pricing should lead the firms to offer their weak products at cost (i.e., $\mu_1 = \mu_2 = 0$). Furthermore, as a firm can obtain at least $\bar{\pi}$ by charging $\bar{\rho}$ to multi-stop shoppers, it will never charge so low a margin that it would obtain less than $\bar{\pi}$, even if it were to attract all shoppers. That is, no firm will ever charge $\rho < \underline{\rho}$, where $\underline{\rho}$ is the lower solution to:

$$\underline{\rho} F\left(w - \underline{\rho}\right) = \bar{\pi}. \tag{6}$$

The next proposition shows that, while there is no pure-strategy equilibrium when below-cost pricing is banned, there exists an equilibrium in which firms indeed sell their weak products at cost, and obtain an expected profit equal to $\bar{\pi}$ by randomizing the margins on their strong products between $\underline{\rho}$ and $\bar{\rho}$:

Proposition 4 *Under Assumption A, when below-cost pricing is prohibited:*

[38]The strict inequality follows from $2\rho^* > \bar{\rho}$, or $\bar{\tau} = \delta - \bar{\rho} > \tau^*$ (note that $\tau^* + 2\rho^* = \delta = \bar{\tau} + \bar{\rho}$). To see this, note that $\bar{\tau} = \delta - \bar{\rho} = \delta - h(\bar{\tau})$, which amounts to $\delta = l(\bar{\tau})$, where $l(\tau) \equiv \tau + h(\tau)$ $< j(\tau) = \tau + 2h(\tau)$, and this implies $\bar{\tau} = l^{-1}(\delta) > j^{-1}(\delta) = \tau^*$.

(i) there exists no equilibrium in pure strategies; and

(ii) there exists a symmetric mixed-strategy equilibrium in which firms obtain an expected profit equal to $\bar{\pi}$ by selling weak products at cost and randomizing the margins on strong products over $[\underline{\rho}, \bar{\rho}]$.

Proof. See Online Appendix A.1. ■

As in the sales model of Varian (1980), firms face a dilemma: they are tempted to exploit "captive" customers (the uninformed consumers in Varian's model, and multi-stop shoppers here) but, at the same time, they want to compete for "price-sensitive" customers (the informed consumers in Varian's model, and one-stop shoppers here). To see why there is no pure-strategy equilibrium, note that competition for one-stop shoppers would again drive *total* basket prices down to cost. But as below-cost pricing is banned, this would require selling *both* products at cost. Obviously, this cannot be an equilibrium, as a firm can make a profit on multi-stop shoppers by charging a small positive margin on its strong product.

The characterization of the mixed-strategy equilibrium is similar to that proposed by Varian (1980) and Baye, Kovenock, and de Vries (1992).[39] In this equilibrium, *ex post*, consumers with a shopping cost below:

$$\tau^b(\rho_1, \rho_2) \equiv \delta - \max\{\rho_1, \rho_2\},$$

favor multi-stop shopping and buy both firms' strong products, whereas consumers with a shopping cost in the range:

$$\tau^b(\rho_1, \rho_2) < s < v^b(\rho_1, \rho_2) \equiv w - \min\{\rho_1, \rho_2\},$$

are one-stop shoppers and buy from the firm that charges the lowest price for its basket.

Let us now examine the impact of a ban on consumers. We first note that marginal consumers are one-stop shoppers, as $v^b > \tau^b$.[40] As banning below-cost pricing raises

[39] Using the analysis of the latter paper, it can moreover be shown that, conditional on pricing weak products at cost, the (mixed-strategy) equilibrium (for the price of strong products) is unique.

[40] This follows directly from $w > \delta$ and $\max\{\rho_1, \rho_2\} \geq \min\{\rho_1, \rho_2\}$.

prices for one-stop shoppers, it follows that this reduces not only the number of one-stop shoppers, but also the total number of consumers – from $F(w)$ to $F\left(v^b(\rho_1,\rho_2)\right)$. Furthermore, the multi-stop shopping cost threshold τ^b satisfies:

$$\tau^b(\rho_1,\rho_2) \geq \bar{\tau} \equiv \tau^b(\bar{\rho},\bar{\rho}) = \delta - \bar{\rho} > \tau^*.$$

Hence, banning below-cost pricing *fosters* multi-stop shopping.

This does not mean that multi-stop shoppers face lower prices, however. In particular, the upper bound $\bar{\rho}$ exceeds the margin ρ^* that arises in the absence of the ban,[41] implying that multi-stop shoppers face higher prices with at least some probability. The next proposition shows that banning below-cost pricing also in fact harms multi-stop shoppers when weak products offer relatively little value, that is, when w is close to δ:[42]

Proposition 5 *Suppose Assumption A holds. Keeping δ constant, for w close enough to δ:*

- *every consumer's expected surplus is lower in the equilibrium characterized by Proposition 4 than in the equilibrium that arises in the absence of a ban; and*
- *total welfare can however be lower or higher, depending on the distribution of shopping costs. For instance, if $F(s) = s^k/k$, then there exists $\hat{k}(w,\delta) > 0$ such that total welfare is lower (resp., higher) when below-cost pricing is banned if $k < \hat{k}(w,\delta)$ (resp., $k > \hat{k}(w,\delta)$).*

Proof. See Online Appendix A.2. ■

The intuition is that, when weak products are "very" weak, there are relatively few one-stop shoppers. Firms can then raise the prices of their strong products, so as to exploit multi-stop shoppers, without being too concerned about losing one-stop shoppers. Indeed, in the limit case where $w = \delta$, the lower bound $\underline{\rho}$ of the equilibrium margin distribution converges to the upper bound $\bar{\rho}\,(> \rho^*)$, and thus multi-stop shoppers certainly face higher prices. By continuity, multi-stop shoppers face higher *expected* prices, as long as weak

[41] To see this, it suffices to note that, from the first-order conditions, ρ^* and $\bar{\rho}$ satisfy respectively, $\rho = h(\delta - \rho^* - \rho)$ and $\rho = h(\delta - \rho)$, where $h(.)$ is an increasing function.

[42] From (1) and (2), the surplus generated by weak products is equal to $u_1^B - c_1^B = u_2^A - c_2^A = (w-\delta)/2$.

products are not too valuable.[43] However, as a ban on below-cost pricing increases firms' profits, the impact on total welfare remains ambiguous, and depends, in particular, on the distribution of shopping costs.

Thus, in competitive markets, RBC laws increase firms' profits but hurt one-stop shoppers. When weak products offer relatively low value, multi-stop shoppers face higher prices as well, in which case banning below-cost pricing increases firms' profits at the expense of consumers. This finding gives support to the conclusion of the OECD (2007) report, which argues that RBC laws are likely to lead to higher prices and thus harm consumers. The reduction in consumer surplus may, moreover, exceed the increase in firms' profits and thus result in lower total welfare. However, when, instead, weak products offer high value, RBC laws may have a positive impact on multi-stop shoppers.[44]

In a setting where consumers are one-stop shoppers who underestimate (some of) their needs, Johnson (2016) finds that banning below-cost pricing has an unambiguously negative impact: it increases the price for potential loss leaders (those products for which consumers do not underestimate their needs) and harms consumers, despite decreasing the prices for the other products. In our setting, a ban on below-cost pricing also raises the price of potential loss leaders (namely, the weak products), but can either increase or decrease the (expected) price of the other products (the strong ones). Also, while one-stop shoppers are worse-off under RBC laws, as in Johnson's paper, we allow for multi-stop shoppers as well, and they can either be worse- or better-off. In spite of these discrepancies, Johnson's paper and this paper both call for the cautious use of below-cost pricing regulations in competitive markets; and where they are implemented, their impact should be carefully evaluated.

[43] The upper bound $\bar{\rho}$ depends only on δ, whereas the lower bound $\underline{\rho}$ depends on both w and δ and, keeping δ constant, converges to $\bar{\rho} > \rho^*$ when w tends to δ. The lower bound $\underline{\rho}$ remains higher than ρ^* as long as w remains below some threshold $\hat{w} > \delta$; and for slightly larger values of w, $\underline{\rho}$ becomes lower than ρ^*, but the expected equilibrium value of ρ remains higher than ρ^*.

[44] However, RBC laws reduce total expected consumer surplus when, for instance, the density of the distribution of shopping costs does not increase between τ^* and δ; see Online Appendix A.3.

6 Extensions

In our baseline model, below-cost pricing emerges as the result of two forces: head-to-head competition for one-stop shoppers drives total basket prices down to aggregate costs, whereas market power over multi-stop shoppers yields positive margins on strong products. We now show that these insights carry over as long as both shopping patterns arise in equilibrium.

6.1 Bounded shopping costs

The baseline model assumes a widespread dispersion of consumers' shopping costs, spanning the entire range from "pure multi-stop shoppers" (consumers with $s = 0$ will always choose the best value offered for each product) to "pure one-stop shoppers" (consumers with $s \geq \delta$ will never visit a second firm). To make a robustness check, we consider here less dispersed distributions of the shopping cost. It is straightforward to check that, as long as one-stop and multi-stop shopping patterns both arise in equilibrium: (i) competition for one-stop shoppers drives total prices down to cost ($m_1^* = m_2^* = 0$); and (ii) multi-stop shoppers buy the strong products. Hence, firms still derive their profits from multi-stop shoppers only, and firm i's profit remains given by (4). Ruling out local deviations then leads to the same characterization as before: $\rho_1^* = \rho_2^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$.

The following propositions confirm that this equilibrium exists whenever consumers' shopping costs are sufficiently diverse. By contrast, when shopping costs are all low enough, active consumers systematically visit both stores and only buy strong products, which firms price above cost. Conversely, when shopping costs are all high enough, consumers visit at most one firm, and symmetric Bertrand competition leads both firms to offer the basket at cost.

We first consider the effect of an upper bound on consumers' shopping costs:

Proposition 6 *Suppose that shopping costs are distributed over $[0, \overline{s}]$, where $\overline{s} > 0$. Then:*

- *if $\overline{s} > j^{-1}(\delta)$, there exists a unique equilibrium, with both types of shopping patterns and the same prices as in the baseline model; and*

- *if instead* $\overline{s} \leq j^{-1}(\delta)$, *there exist multiple equilibria. In each equilibrium: (i) only multi-stop shopping arises; and (ii) weak products are offered at below-cost prices, but firms only sell their strong products, with a positive margin ranging from* $h(\overline{s})$ *to* $\delta - \overline{s} - h(\overline{s})$.

Proof. See Online Appendix B.1. ■

Hence, while firms always price their weak products below cost, it is only when some consumers have high enough shopping costs, namely, when $\overline{s} > j^{-1}(\delta)$, that cross-subsidization actually occurs. Otherwise, all consumers patronize both firms and only buy strong products. Indeed, in the limit case $\bar{s} = 0$, where consumers incur no shopping costs, each firm earns a margin of up to δ on its strong product, reflecting its comparative advantage, as standard asymmetric Bertrand competition suggests.

We now turn to the impact of a lower bound on shopping costs:

Proposition 7 *Suppose that shopping costs are distributed over* $[\underline{s}, +\infty)$, *where* $\underline{s} < w$.[45] *Then:*

- *if* $\underline{s} < \delta/3$, *there exists a unique equilibrium, with both types of shopping patterns and the same prices as in the baseline model;*
- *if instead* $\underline{s} > \delta$, *there exist multiple equilibria in which: (i) only one-stop shopping arises, and (ii) firms make zero profit; and*
- *Finally, if* $\delta/3 \leq \underline{s} \leq \delta$, *both types of equilibria coexist.*[46]

Proof. See Online Appendix B.2. ■

Thus, cross-subsidization arises in equilibrium as long as some consumers have a shopping cost lower than the extra value δ offered by combining both strong products, and it does arise for certain when some consumers have a low enough shopping cost (namely, lower than $\delta/3$).

[45] This assumption is needed for the viability of the markets, as consumers with shopping costs exceeding w never visit any firm.

[46] In the limit case $\underline{s} = \delta$, however, only those consumers with a shopping cost equal to δ may opt for multi-stop shopping.

6.2 Market power

Another feature of our analysis is that firms want to charge higher prices to multi-stop shoppers than to one-stop shoppers. We note here that this is likely to remain the case when firms have little market power over multi-stop shoppers and/or have market power over one-stop shoppers as well.

To see the first point, it suffices to note that our main results (Propositions 1 and 2) still hold when δ becomes arbitrarily small (but remains positive). Likewise, in the simple discrete example considered in Section 2, cross-subsidization keeps emerging in equilibrium, even when very few consumers face a low shopping cost: as long as some consumers face a low cost, firms price their weak products at \$21, which is substantially lower than their cost (\$30). Interestingly, in both instances (that is, as δ goes to 0, or when the proportion of consumers facing a low shopping cost tends to vanish), cross-subsidization keeps arising (and, in the latter case, remains substantial) even though firms' profits tend to 0.

To see the second point, note that, as stressed above, firms offer substitute baskets to one-stop shoppers, and complements to multi-stop shoppers. Hence, even if limited, competition for one-stop shoppers still tends to curb total prices on firms' baskets, whereas double-marginalization tends, instead, to raise prices on strong products.

To study this more formally, consider the baseline model of Section 3, except that firms now have asymmetric comparative advantages. Namely:

$$u_1^A - c_1^A - \left(u_1^B - c_1^B\right) \equiv \bar{\delta} > \underline{\delta} \equiv u_2^B - c_2^B - \left(u_2^A - c_2^A\right),$$

which implies that firm 1 is more efficient in supplying one-stop shoppers:

$$w_1 - w_2 = \bar{\delta} - \underline{\delta} > 0,$$

where:

$$w_1 \equiv u_1^A - c_1^A + u_1^B - c_1^B \text{ and } w_2 \equiv u_2^A - c_2^A + u_2^B - c_2^B,$$

denote the surpluses generated by the firms' baskets. Firm 1 therefore enjoys some market power over one-stop shoppers: in equilibrium, firm 2 still offers its basket at cost ($m_2 = 0$) but firm 1 now attracts all one-stop shoppers and charges them a total margin reflecting its competitive advantage, $m_1 = \bar{\delta} - \underline{\delta}$. Hence, one-stop shoppers obtain a consumer value $v_1 = w_2$, and the multi-stop shopping threshold becomes:[47]

$$\tau = v_{12} - v_1 = \left(w_2 + \bar{\delta} - \rho_1 - \rho_2\right) - w_2 = \underline{\delta} + \mu_1 - \rho_2,$$

where $\mu_1 = m_1 - \rho_1 = \bar{\delta} - \underline{\delta} - \rho_1$ denotes firm 1's margin on its weak product.

As firm 1 sells both products to one-stop shoppers, and sells its strong product to multi-stop shoppers as well, its profit can be expressed as:

$$\begin{aligned} \pi_1 &= \rho_1 F(\tau) + m_1 \left[F(v_1) - F(\tau)\right] \\ &= \left(\bar{\delta} - \underline{\delta}\right) F(w_2) - \mu_1 F\left(\underline{\delta} + \mu_1 - \rho_2\right), \end{aligned}$$

which obviously leads firm 1 to subsidize its weak product: $\mu_1 < 0$.[48] To understand why cross-subsidization still arises when firm 1 enjoys market power over one-stop shoppers as well, consider again the following thought experiment. Increase ρ_1 by a small amount and decrease μ_1 by the same amount, so as to keep the total margin m_1 equal to $\bar{\delta} - \underline{\delta} > 0$. This alteration of the price structure does not affect the profit made on one-stop shoppers (who pay the same price for the basket) but increases the profit made on multi-stop shoppers (who pay a higher price for the strong product). In addition, this induces some multi-stop shoppers (those with s slightly below τ) to switch to one-stop shopping and buy firm 1's weak product as well (instead of buying only its strong product). It is therefore profitable for firm 1 to keep altering the price structure as long as it earns a non-negative margin on its weak product, which leads the firm to sell its weak product below cost.

Remark: Collusion. The scope for below-cost pricing would, however, disappear if firms could coordinate their pricing decisions, for example, through tacit or explicit collusion. Consider, for instance, our baseline setup, but suppose now that firms interact repeatedly

[47] Combining A_1 and B_2 yields a surplus equal to:

$$u_1^A - c_1^A + u_2^B - c_2^B = w_1 + \underline{\delta} = w_2 + \bar{\delta}.$$

[48] Firm 2 only sells its strong product (to multi-stop shoppers), and thus charges a positive margin on it; as $m_2 = 0$, firm 2 thus still prices its weak product below-cost, but consumers do not buy it in equilibrium.

over time and are indeed "sufficiently patient" (that is, their discount factors are close to 1) that they can perfectly collude and maximize their joint profits. Using the total margin, m, and the margin differential between strong and weak products, $t = \rho - \mu$, as decision variables, total industry profit can be expressed as:

$$\Pi = mF(v) + tF(\tau) = mF(w - m) + tF(\delta - t).$$

It is thus separable in m and t and, as $w > \delta$, a revealed preference argument shows that the industry-profit maximizing margins satisfy $m = \rho + \mu > t = \rho - \mu$, and thus $\mu > 0$: there is no below-cost pricing. Hence, cross-subsidization arises here precisely when firms are strongly competing against each other for one-stop shoppers.

6.3 Online retailing

The last decade has seen established retailers developing their online activities. This offers consumers an alternative way of fulfilling their needs, but also has an impact on retail competition and on retailers' pricing strategies. To explore these implications, consider the following variant of the baseline model, where a fraction λ of "internet-savvy" consumers see their shopping costs drop to zero. That is, the distribution of shopping costs can be characterized by a cumulative distribution function $F_\lambda(s)$ and a density $f_\lambda(s)$, where $F_\lambda(0) = \lambda$ and, for $s > 0$:

$$f_\lambda(s) = (1 - \lambda) f(s) \text{ and } F_\lambda(s) = \lambda + (1 - \lambda) F(s).$$

The inverse hazard rate becomes:

$$h_\lambda(s) = h(s) + \frac{\lambda}{1 - \lambda} \frac{1}{f(s)}.$$

Hence:

(i) this hazard rate still increases with s if $f(s)$ does not increase with s, or if λ is not too large;[49] and

[49] If $f'(s) > 0$, then $h_\lambda(s)$ still increases with s in the relevant range $s \in [0, \delta]$ if:

$$\frac{\lambda}{1 - \lambda} < \max_{s \in [0,\delta]} f^2(s) \frac{h'(s)}{f'(s)}.$$

(ii) the hazard rate moreover increases with the proportion λ of "internet-savvy" consumers.

Condition (i) ensures that the equilibrium characterization of Proposition 2 remains valid; condition (ii) then implies that the equilibrium prices charged on strong products increase with λ.

More generally, the following proposition shows that the development of online retailing leads to an increase in the prices of strong products whenever it inflates the inverse hazard rate:

Proposition 8 *Suppose that the development of online retailing affects the distribution of shopping costs in such a way that: (i) the distribution still satisfies Assumptions A and B; and (ii) the inverse hazard rate is inflated. Then there exists a unique equilibrium, in which firms sell their baskets at cost but charge a positive margin on their strong products (and thus a negative margin on their weak products); furthermore, the equilibrium prices of strong products increase with the development of online retailing.*

Proof. See Appendix E. ■

Proposition 8 points out that the development of online sales is not only profitable, but also consistent with an *increase* in the prices of strong products: while one-stop shoppers can still buy firms' baskets at cost, multi-stop shoppers (including those buying online) face higher prices as the proportion of online customers increases. The intuition is straightforward: an increase in the development of online activity, as measured, for instance, by the proportion λ of "internet-savvy" consumers, boosts multi-stop shopping, which benefits the firms but also encourages them to take advantage of this shift in demand by raising the prices of their strong products – at the expense of the less internet-savvy multi-stop shoppers.

6.4 Multiple firms

For the sake of exposition, we have so far focused on a simple setting where two firms enjoy a comparative advantage on two different products. It would be straightforward to extend the analysis to situations where the firms offer more products, and enjoy a comparative advantage on different sets of products. Indeed, as long as consumers have homogeneous valuations for the various combinations of products, the above analysis carries over, interpreting ρ_i as firm i's overall margin on its strong products.

It may be potentially more interesting to extend the analysis to settings where several firms enjoy a comparative advantage on a given product. Intuitively, our basic insights carry over as long as these firms enjoy some market power (e.g., due to product differentiation and heterogenous preferences) over the product in question. To see this, we sketch here a simple variant in which: (i) four firms, 1, 2, 3 and 4, each produce goods A and B with the same unit costs, c_A and c_B; and (ii) consumers derive the same value u_i from good $i = A$, B, except that:

- half of the consumers derive an additional value δ from the variety of good A offered by firm 1, and from the variety of good B offered by firm 2; and
- the other half of the consumers derive an additional value δ from the variety of good A offered by firm 3, and from the variety of good B offered by firm 4.

We further assume that the distribution of valuations is independent from the distribution of the shopping cost. For every one-stop shopper, two firms offer the most attractive basket, with total valuation $u_A + u_B + \delta$: firms 1 and 2 for the first half of consumers, and firms 3 and 4 for the second half. Bertrand competition therefore drives the price of every basket down to the total unit cost, $c_A + c_B$.

Consider now the multi-stop shoppers, and assume that firm i, for example, charges a margin ρ_i on its strong product, and thus a margin $-\rho_i$ on its weak product, whereas other firms all charge a margin ρ^* on their strong products, and a margin $-\rho^*$ on their weak products. As long as the total margin lies below δ, firm i is able to attract half of the consumers whose shopping cost lies below $\tau = \delta - \rho^* - \rho_i$. Hence, its profit is equal to:

$$\pi_i = \frac{1}{2}\rho_i F\left(\delta - \rho^* - \rho_i\right).$$

The same analysis as before then shows that the equilibrium margin remains as described in Proposition 2.

7 Conclusion

We consider the competition between multi-product firms in a setting where: (i) firms enjoy comparative advantages over different goods or services; and (ii) customers have heterogeneous transaction costs. As a result, those with low costs tend to patronize multiple suppliers, whereas those with higher shopping costs are more prone to one-stop shopping. This gives rise to a form of co-opetition, as firms' baskets are substitutes for one-stop shoppers, but their strong products are complements for multi-stop shoppers. As a result, competition for one-stop shoppers drives total basket prices down to total cost but, in order to exploit their market power over multi-stop shoppers, firms price strong products above cost and weak products below cost. Furthermore, the complementarity of firms' strong products generates double marginalization problems, which here take the form of excessive cross-subsidization: indeed, firms would benefit from mutual moderation, for example, by agreeing to put a cap on the prices of strong products. Such bilateral price cap agreements would benefit consumers (one-stop shoppers would remain supplied at cost, and multi-stop shoppers would benefit from lower prices), and would also increase profits by boosting multi-stop shopping.

The legal treatment of cross-subsidization in competitive markets has triggered much debate. We find that banning below-cost pricing substantially benefits firms – their profits more than double – at the expense of one-stop shoppers, and it can also reduce total consumer surplus and social welfare, depending on the value offered by weak products and the distribution of shopping patterns. Our analysis thus calls for a cautious use of resale-below-cost laws in competitive markets.

We have developed these insights using a simple setup, with individual unit demands and homogeneous consumer valuations for the goods. It would be interesting to extend the analysis to other environments, for example, by allowing for more general demand or for correlation (e.g., due to underlying characteristics such as wealth) between customers' preferences and their transaction costs. We leave this task to future research.

Our framework can also be used as a building block to revisit classic issues such as product differentiation strategies or investment in quality. In particular, it suggests that firms have an incentive to target different products as their strong products. The framework can also be used to explore newer issues, such as the development of online sales. While our first exploration suggests that firms indeed have an incentive to reduce the transaction costs of their customers, it would be interesting to model explicitly the investment in online activities, and to study the implications of such investment on transaction costs and purchasing patterns.

References

Ambrus, Attila, and Jonathan Weinstein (2008), "Price Dispersion and Loss Leaders," *Theoretical Economics,* 3:525-537.

Armstrong, Mark, and John Vickers (2010), "Competitive Non-linear Pricing and Bundling," *Review of Economic Studies,* 76:1-31.

Bahls, Jane Easter (1990), "One-Stop Shopping: Are Law Firms Selling Too Many Services?", *Student Lawyer*, 19:20-26.

Baye, Michael, Dan Kovenock and Casper de Vries (1992), "It Takes Two to Tango: Equilibria in a Model of Sales," *Games and Economic Behavior*, 4: 493-510.

Berg, Sanford V., and Dennis L. Weisman (1992), "A Guide to Cross-Subsidization and Price Predation," *Telecommunication Policy*, August:447-459.

Bolton, Patrick, Joseph Brodley and Michael Riordan (2000), "Predatory Pricing: Strategic Theory and Legal Policy," *Georgetown Law Journal,* 88:2239-2330.

Calvani, Terry (2001), "Predatory Pricing and State Below-Cost Sales Statutes in the United States: An Analysis," *Report for the Competition Bureau, Canada*, available at http://www.bureaudelaconcurrence.gc.ca/eic/site/cb-bc.nsf/eng/01292.html.

Chen, Zhijun, and Patrick Rey (2012), "Loss Leading as an Exploitative Practice," *American Economic Review*,102(7):3462-82.

Dykman, Ann (1995), "One-Stop Shopping," *Vocational Education Journal*, 70(8):34-38.

Eckert, Andrew, and Douglas S. West (2003), "Testing for Predation by a Multiproduct Retailer," in *The Pros and Cons of Low Prices*, Stockholm: Swedish Competition Authority.

Ellison, G. (2005), "A Model of Add-On Pricing," *Quarterly Journal of Economics*, 120(2): 585–637.

Faulhaber, Gerald R. (1975), "Cross-Subsidization: Pricing in Public Enterprises," *American Economic Review*, 65(5):966-977.

Faulhaber, Gerald R. (2005), "Cross-Subsidy Analysis with More Than Two Services," *Journal of Competition Law and Economics,* 1(3):441-448.

Gabaix, X., and D. Laibson (2006), "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets," *Quarterly Journal of Economics*, 121(2): 505–540.

Grubb, M. D. (2009), “Selling to Overconfident Consumers,” *American Economic Review*, 99(5): 1770–1807.

Gijsbrechts, Els, Katia Campo and Patricia Nisol. (2008), “Beyond Promotion-Based Store Switching: Antecedents and Patterns of Systematic Multiple-Store Shopping,” *International Journal of Research in Marketing*, 25(1): 5-21.

Glick, Michael (2007), “One-Stop Shopping,” *Journal of the American Dental Association*, 138(3):282–283.

Hess, James and Eitan Gerstner (1987), “Loss Leader Pricing and Rain Check Policy,” *Marketing Science*, 6:358-374.

Johnson, Justin P. (2016), “Unplanned Purchases and Retail Competition,” *American Economic Review*, forthcoming.

Klemperer, Paul (1992), “Equilibrium Product Lines: Competing Head-to-Head may be Less Competitive,” *American Economic Review*, 82:740-755.

Klemperer, Paul, and Jorge Padilla (1997) “Do Firms’ Product Lines Include Too Many Varieties?”, *RAND Journal of Economics*, 28: 472-488.

Lal, Rajiv, and Carmen Matutes (1994), “Retail Pricing and Advertising Strategies,” *Journal of Business,* 67:345-370.

Lerner, Josh, and Jean Tirole (2015), “Standard-Essential Patents,” *Journal of Political Economy*, 123(3):547–586.

Messinger, Paul R., and Chakravarthi Narasimhan (1997), “A Model of Retail Formats Based on Consumers’ Economizing on Shopping Time,” *Marketing Science*, 16:(1):1-23.

OECD (2007), “Resale Below Cost Laws and Regulations,” *OECD Journal of Competition Law and Policy*, 9:169-255.

Rao, P.M., and Joseph A. Klein (1992), “Antitrust Issues Concerning R&D Cross-Subsidization: A Case Study of US Telecommunications,” *Telecommunication Policy,* July:415-424.

Rey, Patrick and Jean Tirole (2013), “Cooperation vs. Collusion: How Essentiality Shapes Co-opetition,” TSE Working Paper n° 13-439.

Rhodes, Andrew (2015), “Multiproduct Retailing,” *Review of Economic Studies*, 82 (1):360 - 390.

Selten, Reinhard (1970), *Preispolitik der Mehrproduktenunternehmung in der Statis-*

chen Theorie, Springer Verlag, Berlin.

Snow, Charlotte (1996), “One-Stop Shopping,” *Modern Healthcare*, 26(48):32-3.

Varian, Hal (1980), “A Model of Sales,” *American Economic Review*, 70: 651-659.

Zhou, Jidong (2016), “Competitive Bundling”, *Econometrica*, forthcoming.

Appendix

Notation. Throughout the exposition:

- we will refer to the two firms as firms i and j, with the convention that $i \neq j \in \{1,2\}$; and
- for each firm $i \in \{1,2\}$, we will denote the social value generated by its strong (resp., weak) product by $\bar{w}_i$ (resp., by $\underline{w}_i$), and we will denote the margin charged on its strong (resp., weak) product by ρ_i (resp., by μ_i). By assumption, we have $\bar{w}_i, \underline{w}_i > 0$ and:

$$\bar{w}_i + \underline{w}_i = w,$$

  for $i \in \{1,2\}$ and, for $j \neq i \in \{1,2\}$:

$$\bar{w}_i - \underline{w}_j = \delta;$$

- the value offered by firm $i \in \{1,2\}$ is thus equal to:

$$v_i \equiv \max\{\underline{w}_i - \mu_i, 0\} + \max\{\bar{w}_i - \rho_i, 0\},$$

  whereas multi-stop shoppers obtain a value:

$$v_{12} = \max\{\bar{w}_1 - \rho_1, 0\} + \max\{\bar{w}_2 - \rho_2, 0\},$$

  if they buy both strong products, and obtain instead a value:

$$\underline{v}_{12} = \max\{\underline{w}_1 - \mu_1, 0\} + \max\{\underline{w}_2 - \mu_2, 0\},$$

  if they buy both weak products; and
- using the "adjusted" margins, defined as:

$$\hat{\mu}_i \equiv \min\{\mu_i, \underline{w}_i\} \text{ and } \hat{\rho}_i \equiv \min\{\rho_i, \bar{w}_i\},$$

  these values can be respectively expressed as:

$$\begin{aligned} v_i &= \underline{w}_i - \hat{\mu}_i + \bar{w}_i - \hat{\rho}_i = w - \hat{\mu}_i - \hat{\rho}_i, \\ v_{12} &= \bar{w}_1 - \hat{\rho}_1 + \bar{w}_2 - \hat{\rho}_2 = w + \delta - \hat{\rho}_1 - \hat{\rho}_2, \\ \underline{v}_{12} &= \underline{w}_1 - \hat{\mu}_1 + \underline{w}_2 - \hat{\mu}_2 = w - \delta - \hat{\mu}_1 - \hat{\mu}_2. \end{aligned}$$

Note that a multi-stop shopper would buy both strong products only if $\rho_i < \bar{w}_i$ (that is, $\hat{\rho}_i = \rho_i$) for both firms, otherwise, the value from such multi-stop shopping, net of shopping costs, would be lower than the net value from one-stop shopping. This implies that:

$$v_{12} = w + \delta - \rho_1 - \rho_2.$$

Likewise, a multi-stop shopper will actually buy both weak products only if $\mu_i < \underline{w}_i$ (that is, $\hat{\mu}_i = \mu_i$) for both firms, implying that:

$$\underline{v}_{12} = w - \delta - \mu_1 - \mu_2.$$

Moreover, for a firm that attracts one-stop shoppers, it is never optimal to charge a margin that exceeds the social value of the product, that is, $\rho_i > \bar{w}_i$ and $\mu_i > \underline{w}_i$ cannot arise in equilibrium where firm i serves some one-stop shoppers. Suppose firm i charges $\mu_i > \underline{w}_i$ and $\rho_i \leq \bar{w}_i$, and one-stop shoppers only buy its strong product. Reducing μ_i such that $\tilde{\mu}_i = \underline{w}_i - \varepsilon > 0$ increases firm i's profit by also selling its weak product to one-stop shoppers and by attracting more one-stop shoppers as $\tilde{v}_i > v_i$. Doing so may also transform some multi-stop shoppers (if indeed there are any multi-stop shoppers buying strong products) into one-stop shoppers, as now $\tilde{\tau} = \delta + \tilde{\mu}_i - \rho_j < \tau$, on which firm i earns a higher profit. Similarly, charging $\rho_i > \bar{w}_i$ is never optimal if firm i attracts some one-stop shoppers. Therefore, without loss of generality we focus on $\mu_i \leq \underline{w}_i$ and $\rho_i \leq \bar{w}_i$ if one-stop shoppers patronize firm i.

The shopping cost thresholds, below which consumers favor picking both strong products rather than patronizing only firm 1 or firm 2, are respectively $\tau_1 = v_{12} - v_1 = \delta + \mu_1 - \rho_2$ and $\tau_2 = v_{12} - v_2 = \delta - \rho_1 + \mu_2$, and $\tau = \min\{\tau_1, \tau_2\}$. Likewise, the thresholds for picking weak products are $\underline{\tau}_1 \equiv \underline{v}_{12} - v_1 = \rho_1 - \mu_2 - \delta$, $\underline{\tau}_2 \equiv \underline{v}_{12} - v_2 = \rho_2 - \mu_1 - \delta$, and $\underline{\tau} = \min\{\underline{\tau}_1, \underline{\tau}_2\}$. Note that $\underline{\tau}_1 = -\tau_2$, $\underline{\tau}_2 = -\tau_1$, and thus $\underline{\tau} = -\tau$. Therefore, in equilibrium, it cannot be the case that some multi-stop shoppers buy strong products, and other buy weak products.

A Proof of Lemma 1

To prove the lemma, we first establish the following claims.

Claim 1 *Some consumers are active in equilibrium.*

Proof. Suppose there is no active consumer. It must be the case that $\max\{v_1, v_2, v_{12}, \underline{v}_{12}\} \leq 0$, and firms make no profit. Consider the following deviation for firm 1: charge $\tilde{\mu}_1 > 0$ and $\tilde{\rho}_1 > 0$ such that $\tilde{m}_1 = \tilde{\rho}_1 + \tilde{\mu}_1 = w - \varepsilon$, for some $\varepsilon \in (0, w)$. Firm 1 then attracts consumers with shopping cost $s \leq \tilde{v}_1 = \varepsilon$ and earns a positive profit, a contradiction. Thus, some consumers must be active in equilibrium. ■

Claim 2 *If there are active one-stop shoppers in equilibrium, then $m_1 = m_2 = 0$.*

Proof. Consider a candidate equilibrium in which some one-stop shoppers are active, which requires $\max\{v_1, v_2\} > 0$. We first show that no firm charges a negative total margin. To see this, suppose firm 1 sets $m_1 < 0$ (and thus $v_1 > w > 0$), say, then:

- if $m_1 < m_2$, firm 1 incurs a loss by attracting one-stop shoppers; then consider the following deviations:
  - if there is no multi-stop shopper, or if firm 1 does not make a profit on multi-stop shoppers, then firm 1 could avoid all losses by increasing both of its prices;
  - if some multi-stop shoppers buy the strong products, and firm 1 makes a profit on them (that is, $\rho_1 > 0$), then firm 1 would benefit from raising the margin on its weak product. Keeping ρ_1 constant, raising the margin on the weak product to $\tilde{\mu}_1 = -\rho_1 = \mu_1 - m_1 > \mu_1$: (i) yields $\tilde{m}_1 = 0$, thus avoiding the loss from one-stop shoppers; and (ii) moreover increases the demand from multi-stop shoppers (on which firm 1 makes a positive margin), as it reduces the value of one-stop shopping without affecting that of multi-stop shopping; and
  - if some multi-stop shoppers buy the weak products, and firm 1 makes a profit on them (that is, $\mu_1 > 0$), then firm 1 could avoid the loss from one-stop shoppers by raising the margin on its strong product to $\tilde{\rho}_1 = -\mu_1$ (yielding $\tilde{m}_1 = 0$). This would also increase the demand of multi-stop shoppers by reducing the value from one-stop shopping without affecting that of multi-stop shopping; and

- if instead $m_1 \geq m_2$ (and thus $m_2 < 0$), then the same argument applies to any firm that attracts one-stop shoppers (firm 2 if $m_1 > m_2$, and at least one of the firms if $m_1 = m_2$).

Next, we show that both firms charging a positive total margin cannot be an equilibrium. Suppose firms set $m_1, m_2 > 0$. Then:

- if any firm, say firm 1, charges a higher margin than its rival ($m_1 > m_2 > 0$ and $v_2 > \max\{v_1, 0\}$), it faces no demand from one-stop shoppers. Consider then the following deviations:
  - if there is no multi-stop shopper, which requires $\max\{v_{12}, \underline{v}_{12}\} \leq v_2$, firm 1 can make a positive profit by undercutting both of its rival's "quality-adjusted" prices by $\varepsilon/2$: for ε positive but small enough, charging $\tilde{\rho}_1 = \mu_2 + \delta - \varepsilon/2$ and $\tilde{\mu}_1 = \rho_2 - \delta - \varepsilon/2$ profitably attracts one-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > v_2$ and $\tilde{m}_1 = m_2 - \varepsilon > 0$), without transforming them into multi-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > \max\{\tilde{v}_{12}, \underline{\tilde{v}}_{12}\} = v_2 + \varepsilon/2$); and
  - if some multi-stop shoppers are active, which requires $\max\{v_{12}, \underline{v}_{12}\} > v_2$ $(> v_1)$, firm 1 can make a profit by keeping its margin constant on the product purchased by multi-stop shoppers, and reducing its other margin so as to yield $\tilde{m}_1 = m_2 - \varepsilon$, with $\varepsilon > 0$. Doing so attracts all one-stop shoppers (as $\tilde{v}_1 = v_2 + \varepsilon > v_2$ and $\tilde{m}_1 = m_2 - \varepsilon > 0$), at the cost of slightly reducing the demand of multi-stop shoppers (as it does not affect the value from multi-stop shopping, $\max\{v_{12}, \underline{v}_{12}\}$, and only increases the value of one-stop shopping by ε, from v_2 to $\tilde{v}_1 = v_2 + \varepsilon$), and is obviously profitable for ε small enough; and

- if both firms charge the same total margin ($m_1 = m_2 > 0$), then $v_2 = v_1$ and $\tau_1 = \tau_2$. At least one firm, say firm 1, does not obtain more than half of the demand from one-stop shoppers. Nonetheless, this firm can attract all one-stop shoppers using the deviations described above for the case $m_1 > m_2$, and the gain from doing so offsets the loss from the slight reduction in demand, if any, from multi-stop shoppers.

Finally, we show that no firm charges a positive total margin in equilibrium. Suppose, for instance, that $m_1 > m_2 = 0$; firm 2 then makes zero profit from one-stop shoppers. If it makes a loss on multi-stop shoppers, then it could profitably deviate by raising all of its prices so as to avoid the loss. If, instead, it supplies multi-stop shoppers at or above cost, then it could profitably deviate by increasing by some $\varepsilon > 0$ the margin of the product *not picked* by multi-stop shoppers, keeping constant its other margin. For ε small enough, firm 2 still supplies all one-stop shoppers, but now makes a profit on them, and doing so, moreover, *increases* the demand from multi-stop shoppers, and thus the profit on them.

We conclude that firms must charge $m_1 = m_2 = 0$ in any equilibrium with active one-stop shoppers. ■

Claim 3 *In equilibrium, active multi-stop shoppers buy the strong products.*

Proof. Suppose that some multi-stop shoppers buy the weak products. Each firm must then offer better value on its weak product than the rival's strong product; that is, each firm must sell its strong product with a margin that exceeds its rival's "quality-adjusted" margin: $\rho_2 \geq \mu_1 + \delta$ and $\rho_1 \geq \mu_2 + \delta$. We show that such a configuration cannot be an equilibrium. We consider two cases:

- suppose first that there are only multi-stop shoppers (buying the weak products). To make a profit, firms must charge non-negative margins on their weak products, that is, μ_1, $\mu_2 \geq 0$. From the above, this implies that each firm sells its strong product with a margin that exceeds its comparative advantage δ: $\rho_2 \geq \delta$ and $\rho_1 \geq \delta$. But then, any firm could profitably undercut its rival. For instance, keeping μ_1 unchanged, by charging $\tilde{\rho}_1 = \mu_2 + \delta - \varepsilon > 0$, firm 1 would also sell its strong product to all previously active consumers, as it now offers better value on A: $\tilde{v}_1^A = v_2^A + \varepsilon$. The deviation may also attract additional one-stop shoppers from which the firm makes a profit as $\tilde{\rho}_1 > 0$ and $\mu_1 \geq 0$; and

- suppose, instead, that there are both one-stop shoppers and multi-stop shoppers. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$. As firms make no profit from one-stop shoppers, they must charge non-negative margins on their weak products, that is, μ_1, $\mu_2 \geq 0$. This implies, however, that margins on strong products are non-positive, say, $\rho_1 = m_1 - \mu_1 \leq 0$, which contradicts the condition $\rho_1 \geq \mu_2 + \delta \geq \delta$.

Therefore multi-stop shoppers must buy strong products in equilibrium. ■

Claim 4 *Some multi-stop shoppers are active in equilibrium.*

Proof. Suppose all active consumers are one-stop shoppers, which requires $\max\{v_1, v_2\} > 0$ and $\max\{v_1, v_2\} \geq \max\{v_{12}, \underline{v}_{12}\}$. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$, and thus firms make zero profit. We show that this configuration cannot be an equilibrium.

By construction, $v_1 + v_2 = v_{12} + \underline{v}_{12}$, as it corresponds to the total value of buying one unit of both products from both firms. Here, we have $v_1 = v_2 \geq \max\{v_{12}, \underline{v}_{12}\}$, and it follows that $v_1 = v_2 = v_{12} = \underline{v}_{12}$; that is, firms must offer the same value on both products, by charging a margin $\delta/2$ on strong products, and subsidizing weak products by the same amount. It follows that it is profitable for any firm to encourage some consumers to only buy its strong product. For instance, increasing μ_1 by $\varepsilon > 0$ and decreasing ρ_1 by the same amount raises both τ_1 and τ_2 by ε, which triggers some multi-stop shopping as $\tau = \varepsilon > 0$; and as $\tilde{\rho}_1 = \delta/2 - \varepsilon > 0$ for ε small enough, firm 1 now makes a positive profit on these multi-stop shoppers. ■

Claim 5 *Some one-stop shoppers are active in equilibrium.*

Proof. Suppose there are only multi-stop shoppers who, from Claim 3, buy the strong products. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have:

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\},$$

which implies $\max\{v_1, v_2\} \leq v_{12}/2$, and the demand from multi-stop shoppers is $F(v_{12}/2)$. As consumers only buy strong products, firms must charge non-negative margins on these products. Without loss of generality, suppose $\rho_2 \geq \rho_1\,(\geq 0)$, and consider the following deviation for firm 1: keeping ρ_1 constant, change μ_1 to:

$$\tilde{\mu}_1 = \frac{w - \delta + \rho_2 - \rho_1}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0,$$

so as to *increase* the value offered to one-stop shoppers to:

$$\tilde{v}_1 = w - \rho_1 - \tilde{\mu}_1 = \frac{w + \delta - \rho_1 - \rho_2}{2} + \varepsilon = \frac{v_{12}}{2} + \varepsilon.$$

This deviation does not affect v_{12} nor τ_2 (which only depends on ρ_1, ρ_2 and μ_2), but it decreases τ_1 to $\tilde{\tau}_1 = \delta + \tilde{\mu}_1 - \rho_2 = v_{12}/2 - \varepsilon$; as initially $\tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1 \, (< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some of the initial multi-stop shoppers to buy both products from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), from which firm 1 earns an extra profit by selling its weak product (as $\tilde{\mu}_1 > 0$), and it, moreover, attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), which generates additional profit (as $\rho_1 \geq 0$ and $\tilde{\mu}_1 > 0$). ■

Claims 4 and 5 establish part (i) of the Lemma. Part (ii) then follows from Claim 3, while part (iii) follows from Claim 2.

B Proof of Proposition 1

Thanks to Lemma 1, the equilibrium characterization is fairly simple. As firms sell their baskets at cost, one-stop shopping yields a consumer value w. Consumers may, however, prefer buying both strong products, that is, purchasing A_1 from firm 1 and B_2 from firm 2, rather than patronizing a single firm. Such multi-stop shopping involves an extra shopping cost s and yields a total value:

$$v_{12} \equiv u_1^A - p_1^A + u_2^B - p_2^B = w + \delta - \rho_1 - \rho_2,$$

where $\rho_1 \equiv p_1^A - c_1^A$ and $\rho_2 \equiv p_2^B - c_2^B$ respectively denote firm 1 and 2's margins on strong products. Consumers favor multi-stop shopping over one-stop shopping if the additional value from mixing-and-matching exceeds the extra shopping cost, that is, if

$$s \leq \tau \equiv v_{12} - w = \delta - \rho_1 - \rho_2.$$

Hence, consumers with a shopping cost $s < \tau$ engage in multi-stop shopping, whereas those with a shopping cost such that $\tau < s < w$ opt for one-stop shopping (and those with a shopping cost $s > w$ do not shop at all). As firms only derive a profit from selling their strong products to multi-stop shoppers, firm i's profit can be expressed as:

$$\pi_i(\rho_1, \rho_2) = \rho_i F(\tau) = \rho_i F(\delta - \rho_1 - \rho_2).$$

It is clearly optimal for firm i to charge a positive margin on its strong product, that is, $\rho_i > 0$. As the basket is offered at cost, this implies that firm i sells its weak product *below cost*.

C Proof of Proposition 2

Thanks to Lemma 1, the equilibrium is interior, and consumers whose shopping cost lies below $\tau^* > 0$ patronize both firms, whereas those whose shopping cost lies between τ^* and w patronize a single firm. The monotonicity of the inverse hazard rate $h(\cdot)$ furthermore ensures that the first-order conditions characterize a unique candidate equilibrium, satisfying $m_1^* = m_2^* = 0$ and $\rho_1^* = \rho_2^* = \rho^*$, such that:

$$\rho^* = h(\tau^*),$$

where:

$$\tau^* = j^{-1}(\delta).$$

We now show that firms cannot benefit from any deviation. Suppose firm 1 charges ρ_1 and μ_1 instead of $\rho_1^* = \rho^*$ and $\mu_1^* = -\rho^*$. Then:

- it cannot make a profit from one-stop shoppers, as it would have to charge $m_1 \leq m_2^* = 0$ to attract them;
- it cannot make a profit either by offering the weak product to multi-stop shoppers, as it would have to charge $\mu_1 \leq \rho_2^* - \delta = \rho^* - \delta < 0$ (as $\rho^* < \delta$) to attract them; and
- thus, it can only make a profit from multi-stop shoppers, and this profit is equal to $\rho_1 F(\tau)$, where $\tau = \min\{\delta + \mu_2^* - \rho_1, \delta + \mu_1 - \rho_2^*\}$; but then:

$$\rho_1 F(\tau) \leq \rho_1 F(\delta + \mu_2^* - \rho_1) = \rho_1 F(\delta - \rho^* - \rho_1) \leq \pi^*,$$

where the inequality comes from the fact that the profit function $\rho_1 F(\delta - \rho^* - \rho_1)$ is quasi-concave, from the monotonicity of $h(\cdot)$, and by construction maximal for $\rho_1 = \rho^* = \rho_1^*$.

D Proof of Proposition 3

We now derive the minmax profit that each firm can earn when below-cost pricing is not allowed. Consider first firm i's response when firm j sets both of its margins to zero, that is, $\mu_j = \rho_j = 0$. Firm i cannot make a profit from one-stop shoppers who can obtain both products at cost from firm j, and thus it can only make a profit by selling its strong product to multi-stop shoppers. The threshold for multi-stop shopping is $\tau = \delta - \rho_i$, and thus the profit from multi-stop shoppers is given by $\rho_i F(\delta - \rho_i)$. Choosing ρ_i so as to maximize this profit gives firm i:

$$\bar{\pi} \equiv \max_{\rho} \rho F(\delta - \rho) > 0,$$

where the inequality stems from $\delta > 0$. The associated margin is given by:

$$\bar{\rho} \in \arg\max_{\rho} \rho F(\delta - \rho).$$

Note that this margin satisfies $\bar{\rho} < (\delta \leq) \bar{w}_i$ for $i \in \{1, 2\}$.[50]

To conclude the argument, it suffices to note that, in response to any rival's margins $\mu_j \geq 0$ and $\rho_j \geq 0$, firm i can always secure at least $\bar{\pi}$ by charging $\mu_i \geq \underline{w}_i$ and $\rho_i = \bar{\rho}$. Choosing $\mu_i \geq \underline{w}_i$ ensures that any multi-stop shoppers will buy both firms' strong products.

Additionally, if $v_j \geq v_i$, then the threshold for multi-stop shopping is given by:

$$\tau = v_{12} - v_j$$

and thus satisfies:

$$\begin{aligned} \tau &= w + \delta - \bar{\rho} - \hat{\rho}_j - \left(w - \hat{\mu}_j - \hat{\rho}_j\right) \\ &= \delta + \hat{\mu}_j - \bar{\rho} \\ &\geq \delta - \bar{\rho}, \end{aligned}$$

[50] In case there are multiple solutions, then any solution satisfies the properties in the proof below.

where the inequality stems from $\hat{\mu}_j = \min\left\{\mu_j, \underline{w}_j\right\} \geq 0$. It follows that firm i obtains at least $\bar{\pi}$:

$$\pi_i = \rho_i F(\tau) = \bar{\rho} F(\tau) \geq \bar{\rho} F(\delta - \bar{\rho}) = \bar{\pi}.$$

If instead $v_j < v_i$, then firm i sells its strong product to both one-stop and multi-stop shoppers, and thus again obtains at least $\bar{\pi}$:

$$\pi_i = \rho_i F\left(\max\left\{v_i, \frac{v_{12}}{2}\right\}\right) \geq \rho_i F(v_i) = \bar{\rho} F(\bar{w}_i - \bar{\rho}) \geq \bar{\rho} F(\delta - \bar{\rho}) = \bar{\pi},$$

where the second inequality stems from $\bar{w}_i > \delta$.

It follows that, in any candidate equilibrium, firms must obtain a positive profit $\pi_i \geq \bar{\pi}$, and thus charge a positive total margin $m_i > 0$ (as $m_i = 0$ would imply $\mu_i = \rho_i = 0$, and thus $\pi_i = 0$).

E Proof of Proposition 8

Let us index the development of online retailing by a parameter λ and suppose that the associated distribution of shopping costs, characterized by a cumulative distribution function $F(s;\lambda)$ with density $f(s;\lambda)$, satisfies Assumptions A and B, and is, moreover, such that:

$$h(s;\lambda) \equiv \frac{F(s;\lambda)}{f(s;\lambda)},$$

increases with λ.

The analysis developed for the baseline model carries over: the equilibrium margin, ρ^*_λ, and the associated multi-stop shopping threshold, $\tau^*_\lambda = \delta - 2\rho^*_\lambda$, are now such that $\rho^*_\lambda = h(\tau^*_\lambda;\lambda)$. Hence, the margin ρ^*_λ satisfies:

$$\rho^*_\lambda = h(\delta - 2\rho^*_\lambda;\lambda).$$

As $h(s;\lambda)$ increases with both s and λ, it follows that ρ^*_λ increases with λ. Conversely, the threshold τ^*_λ is such that:

$$\frac{\delta - \tau^*_\lambda}{2} = h(\tau^*_\lambda;\lambda).$$

Hence, as $h(s;\lambda)$ increases with both s and λ, τ^*_λ decreases as λ increases.

Online Appendix
(Not for Publication)

This Online Appendix first analyzes the case of bounded shopping costs considered in Section 6.1, before studying the mixed-strategy equilibrium generated by RBC laws (5).

A RBC laws

We now turn to the mixed-strategy equilibrium that arise under RBC laws.

A.1 Proof of Proposition 4

We first show that there is no pure strategy Nash equilibrium under RBC laws. We note that in any pure strategy Nash equilibrium each firm $i = 1, 2$ would have to charge $\rho_i, \mu_i \geq 0$, so as to satisfy the RBC laws, and from Proposition 3 we have:

Corollary 1 *Under RBC laws, in any equilibrium, each firm must obtain a positive profit; therefore, each firm should attract some consumers and sell them at least one product with a positive margin.*

Proof. This follows directly from Proposition 3, which implies that under RBC laws, in any equilibrium, each firm i must obtain a profit at least equal to $\pi_i \geq \bar{\pi} > 0$. ■

It follows that, in any equilibrium with pure strategies, some consumers must be active. We successively consider the cases in which one-stop shoppers would be supplied by both firms, one firm, or none (that is, only multi-stop shoppers would be active).

Case (1): Both firms supply one-stop shoppers. This case can only arise when the two firms offer one-stop shoppers the same positive value, $v_1 = v_2 > 0$, implying $\hat{m}_1 = \hat{m}_2$. By construction, at least one firm, say firm i, attracts only a fraction of these one-stop shoppers; and from Corollary 1, firm i must sell at least one good with a positive margin. Suppose firm i deviates by reducing that margin by ε:

- this deviation enables firm i to attract all active one-stop shoppers; and

- in addition, the relevant thresholds for multi-stop shopping, which can initially be expressed as:

$$\begin{aligned} \tau &= v_{12} - \max\{v_1, v_2\} = v_{12} - v_i = \delta - \hat{\rho}_j + \hat{\mu}_i, \\ \underline{\tau} &= \underline{v}_{12} - \max\{v_1, v_2\} = \underline{v}_{12} - v_i = -\delta - \hat{\mu}_j + \hat{\rho}_i, \end{aligned}$$

  can only be lowered by the reduction of firm i's margin.[51] Therefore:

  - if initially there are only one-stop shoppers, then the deviation does not transform any of them into multi-stop shoppers; and
  - if instead there are initially multi-stop shoppers as well, then the deviation can only transform marginal multi-stop shoppers into one-stop shoppers, from which firm i makes a higher profit.

It follows that, for ε small enough, the deviation is profitable.

Case (2): One firm supplies one-stop shoppers. This case arises when, for instance, $v_i > v_j \, (> 0)$, implying $\hat{m}_j > \hat{m}_i$, in which case firm i attracts all one-stop shoppers. From Corollary 1, firm j must also obtain a profit, implying that some multi-stop shoppers must also be active. For this to be the case, firm i must offer a positive value, $v_i^{ms} > 0$, on the product they target. It is moreover straightforward to see that firm i must offer a positive value, $v_i^{os} \equiv v_i - v_i^{ms} > 0$, on its other product as well. Starting from a situation where it would offer no value on this other product, reducing its margin so as to offer a slightly positive value on that product (e.g., $\tilde{v}_i^{os} = \varepsilon > 0$) would not only enable firm i to sell both of its products to one-stop shoppers (with an almost "full" margin on the other product) but, by slightly increasing its overall value, from v_i to $\tilde{v}_i = v_i + \varepsilon$, it would also transform marginal multi-stop shoppers into (more profitable) one-stop shoppers, buying both products from firm i. Therefore, we can restrict attention to firm i's margins such that $\rho_i < \bar{w}_i$ and $\mu_i < \underline{w}_i$. As from Corollary 1, firm i must sell at least one good with a positive margin. We thus have $(m_j \geq) \, \hat{m}_j > \hat{m}_i = m_i > 0$ and

[51] More precisely, as $\tilde{v}_i > v_j$, these thresholds either become $\tilde{\tau} = \tilde{v}_{12} - \tilde{v}_i = \tau - \varepsilon$ and $\underline{\tilde{\tau}} = \underline{\tilde{v}}_{12} - \tilde{v}_i = \underline{\tau}$ (if $\tilde{\mu}_i = \mu_i - \varepsilon$ and $\tilde{\rho}_i = \rho_i$), or $\tilde{\tau} = \tau$ and $\underline{\tilde{\tau}} = \underline{\tilde{\tau}} - \varepsilon$ (if $\tilde{\rho}_i = \rho_i - \varepsilon$ and $\tilde{\mu}_i = \mu_i$).

firm i's profit can be expressed as:

$$\begin{aligned}\pi_i &= m_i\left[F\left(v_i\right)-F\left(\tau\right)\right]+m_i^{ms}F\left(\tau^{ms}\right)\\ &= m_iF\left(v_i\right)-m_i^{os}F\left(\tau^{ms}\right),\end{aligned}$$

where τ^{ms} denotes the threshold for multi-stop shopping, whereas m_i^{ms} and $m_i^{os}=m_i-m_i^{ms}$ respectively denote firm i's margins on the product bought by multi-stop shoppers (as well as by one-stop shoppers), and on the other product (bought only by one-stop shoppers).[52] Note that charging a zero margin on the product bought by multi-stop shoppers is never optimal: starting from $m_i^{ms}=0$, deviating to $\tilde{m}_i^{ms}=\varepsilon$ (where ε is positive but "small") and $\tilde{m}_i^{os}=m_i-\varepsilon$ allows firm i to earn the same profit from one-stop shoppers, but, in addition, it now derives a positive profit from multi-stop shoppers. Moreover, the deviation keeps v_i unchanged but reduces the multi-stop shopping threshold τ^{ms} to $\tilde{\tau}^{ms}=\tau^{ms}-\varepsilon$, and thus transforms marginal multi-stop shoppers into one-stop shoppers, from which firm i makes more profit. Thus, in what follows, we focus on $m_i^{ms}>0$ and distinguish two cases, depending on whether or not firm i charges the monopoly profit-maximizing margin $m^*\equiv\arg\max_m mF\left(w-m\right)$ (which, given the monotonicity of the inverse hazard rate $h\left(\cdot\right)$, is uniquely defined by $h\left(w-m^*\right)=m^*$):

- if $m_i\neq m^*$, then suppose that firm i adjusts its margin on the product bought by multi-stop shoppers to $\tilde{m}_i^{ms}=m_i^{ms}+\varepsilon\left(m^*-m_i\right)$, where $\varepsilon>0$ is small enough to ensure that $\tilde{m}_i<\hat{m}_j$ and $\tilde{m}_i^{ms}>0$. Such a deviation does not change the threshold τ^{ms} (which depends on firm i's prices only through m_i^{os}), and firm i's profit becomes:

$$\tilde{\pi}_i=\left(m_i+\varepsilon\left(m^*-m_i\right)\right)F\left(v_i-\varepsilon\left(m^*-m_i\right)\right)-m_i^{os}F\left(\tau^{ms}\right).$$

  The monotonicity of the inverse hazard rate $h\left(\cdot\right)$ ensures that the first term increases with ε as long as $\tilde{m}_i<m^*$, implying that such a deviation is profitable;

- if $m_i=m^*$, then firm j can benefit from undercutting its rival. Firm j's profit is given by:

$$\pi_j=m_j^{ms}F\left(\tau^{ms}\right),$$

[52] If multi-stop shoppers buy strong products, we thus have $\tau^{ms}=\tau=v_{12}-v_i$, $m^{ms}=\rho_i$ and $m^{os}=\mu_i$; if instead multi-stop shoppers buy weak products, we have $\tau^{ms}=\underline{\tau}=\underline{v}_{12}-v_i$, $m^{ms}=\mu_i$ and $m^{os}=\rho_i$.

where m_j^{ms} denotes firm j's margin on the product bought by multi-stop shoppers. Using:

$$\tau^{ms} = v_i^{ms} + v_j^{ms} - v_i \leq v_j^{ms} < w - m_j^{ms},$$

where the first inequality stems from $v_i = v_i^{os} + v_i^{ms} \geq v_i^{ms}$,[53] and the second follows from the fact that the surplus generated by any single product cannot exceed w, we have:

$$\pi_j = m_j^{ms} F\left(\tau^{ms}\right) < m_j^{ms} F\left(w - m_j^{ms}\right) \leq \pi^* \equiv m^* F\left(w - m^*\right). \tag{7}$$

That is, the maximum profit that firm j can earn from multi-stop shoppers is strictly lower than the monopoly profit derived from one-stop shoppers. Consider now firm j's deviation to $\tilde{\mu}_j = \max\{\rho_i - \delta - \varepsilon/2, 0\}$ and $\tilde{\rho}_j = m^* - \varepsilon - \tilde{\mu}_j$, for some $\varepsilon > 0$:

- if $\rho_i > \delta$, then for ε small enough, $\tilde{\mu}_j = \rho_i - \delta - \varepsilon/2 \left(< \bar{w}_i - \delta = \underline{w}_j\right)$ and $\tilde{\rho}_j = \mu_i + \delta - \varepsilon/2 \left(< \underline{w}_i + \delta = \bar{w}_j\right)$, implying $\tilde{v}_{12} = \underline{\tilde{v}}_{12} = v_i + \varepsilon/2 < \tilde{v}_j = v_i + \varepsilon$, and thus $\tilde{\tau} = \underline{\tilde{\tau}} = -\varepsilon/2 < 0$. Therefore, firm j transforms all multi-stop shoppers into one-stop shoppers, and attracts all one-stop shoppers to whom it charges a total margin of $\tilde{m}_j = m^* - \varepsilon$; and
- if, instead, $\rho_i \leq \delta$, which implies that multi-stop shoppers buy strong products,[54] then $\tilde{\mu}_j = 0$ and $\tilde{\rho}_j = m^* - \varepsilon$ (note that $\rho_i \leq \delta$ then implies $\tilde{\rho}_j < m^* = \rho_i + \mu_i \leq \delta + \underline{w}_i = \bar{w}_j$); firm j then attracts all one-stop shoppers and also serves any remaining multi-stop shoppers (who still buy strong products, as $\tilde{\tau} = \tilde{v}_{12} - \tilde{v}_j = \delta - \rho_i \geq 0$), but makes the same margin $\tilde{m}_j = \tilde{\rho}_j = m^* - \varepsilon$ on both types of shoppers; and

- in both cases, the deviation yields a profit:

$$\tilde{\pi}_j = \tilde{m}_j F\left(\tilde{v}_j\right) = \left(m^* - \varepsilon\right) F\left(w - m^* + \varepsilon\right),$$

which, from (7), makes the deviation profitable for ε small enough.

[53] From the remarks above, this inequality is actually strict, as $v_i^{os} > 0$.

[54] Multi-stop shoppers would buy weak products only if $\underline{v}_{12} > v_i$, or $\underline{\tau} = \underline{v}_{12} - v_i = -\delta - \hat{\mu}_j + \hat{\rho}_i > 0$, which (using $\hat{\rho}_i = \rho_i$, as noted above) implies $\rho_i > \delta + \hat{\mu}_j \geq \delta$.

Case (3): There only exist multi-stop shoppers. This case arises when $v^{ms} \equiv v_1^{ms} + v_2^{ms} \geq 2\max\{v_1, v_2\}$,[55] where, as before, v_i^{ms} denotes the value offered by firm i on the product targeted at multi-stop shoppers. By construction, however, $v_i = v_i^{ms} + v_i^{os}$, where, as before, v_i^{os} denotes the value offered by firm i on its other product. The first condition therefore implies:[56]

$$v_1^{ms} = v_2^{ms} = \frac{v^{ms}}{2} > v_1^{os} = v_2^{os} = 0.$$

But then any firm i can profitably deviate by charging a positive but non-prohibitive margin on its other product, leaving a positive value $\tilde{v}_i^{os} > 0$. This deviation does not affect the value offered to multi-stop shoppers, v^{ms}, but it *increases* the value offered to one-stop shoppers to:

$$\tilde{v}_i = v_i^{ms} + \tilde{v}_i^{os} = \frac{v^{ms}}{2} + \tilde{v}_i^{os} > \frac{v^{ms}}{2}.$$

This deviation thus induces some of the initial multi-stop shoppers (namely, those whose shopping costs lie between $\tilde{\tau}^{ms} = v^{ms} - \tilde{v}_i$ and $v^{ms}/2$) to buy both products from firm i, enabling firm i to earn an additional profit from selling its other product, and it, moreover, attracts more one-stop shoppers (namely, those whose shopping cost lies between $v^{ms}/2$ and $\tilde{v}_i$), generating yet another profit.

To summarize, no pure-strategy satisfying $\rho_i \geq 0$ and $\mu_i \geq 0$ for $i \in \{1, 2\}$ can form a Nash equilibrium in any of the above configurations; hence, there is no pure-strategy Nash equilibrium when below-cost pricing is prohibited.

[55] We must have:

$$v^{ms} - 2s \geq 0 \Longrightarrow v^{ms} - 2s \geq \{v_1, v_2\} - s,$$

which amounts to:

$$s \leq v^{ms}/2 \Longrightarrow s \leq v^{ms} - \max\{v_1, v_2\},$$

or $\max\{v_1, v_2\} \leq v^{ms} - v^{ms}/2 = v^{ms}/2$.

[56] To see this, note that the condition $v^{ms} \geq 2v_j$ amounts to:

$$\begin{aligned} v_1^{ms} + v_2^{ms} &\geq 2\left(v_j^{ms} + v_j^{os}\right) \\ &\Longleftrightarrow v_i^{ms} - v_j^{ms} \geq 2v_j^{os}. \end{aligned}$$

As v_j^{os} cannot be negative (consumers can always opt out), and the condition $v^{ms} \geq 2v_j$ must hold for $j \in \{1, 2\}$, it follows that $0 \geq v_1^{ms} - v_2^{ms} \geq 0$, or $v_1^{ms} = v_2^{ms}$; this, in turn, implies $0 \leq v_j^{os} \leq 0$, or $v_j^{os} = 0$, for $j \in \{1, 2\}$.

We now characterize the mixed-strategy equilibrium. Firm i's profit, as a function of the two firms' margins on their strong products, ρ_1 and ρ_2, is given by:

$$\pi_i^b(\rho_i, \rho_j) \equiv \begin{cases} \rho_i F(w - \rho_i) & \text{if} \quad \rho_i < \rho_j, \\ \rho_i F(\delta - \rho_i) & \text{if} \quad \rho_i > \rho_j. \end{cases}$$

In the first case ($\rho_i < \rho_j$), firm i sells its strong product to both one-stop and multi-stop shoppers, whereas in the second case ($\rho_i > \rho_j$), it sells its strong product only to multi-stop shoppers.

Consider a candidate equilibrium in which each firm i: (i) sells its weak product at cost; (ii) randomizes the margin ρ_i on its strong product according to a distribution $G(\rho)$ over some interval with continuous density $g(\rho)$; and (iii) obtains an expected profit equal to the minmax, $\bar{\pi}$. By construction, the bounds of the support of the distribution must be given by (5) and (6).

Consider consumers' responses to given margins ρ_i and ρ_j:

- consumers buy both goods from firm i if:
  - firm i undercuts its rival:
  $$\rho_j \geq \rho_i;$$
  - one-stop shopping is valuable:
  $$s \leq v_i = w - \rho_i;$$
  - and is more valuable than multi-stop shopping:
  $$s \geq v_{12} - v_i = \delta - \rho_j; \text{ and}$$
- consumers instead engage in multi-stop shopping if:
  $$s \leq v_{12} - \max\{v_1, v_2\},$$
  which amounts to:
  $$s \leq \delta - \rho_i \text{ and } s \leq \delta - \rho_j.$$

Figure 1 depicts the consumers' response.

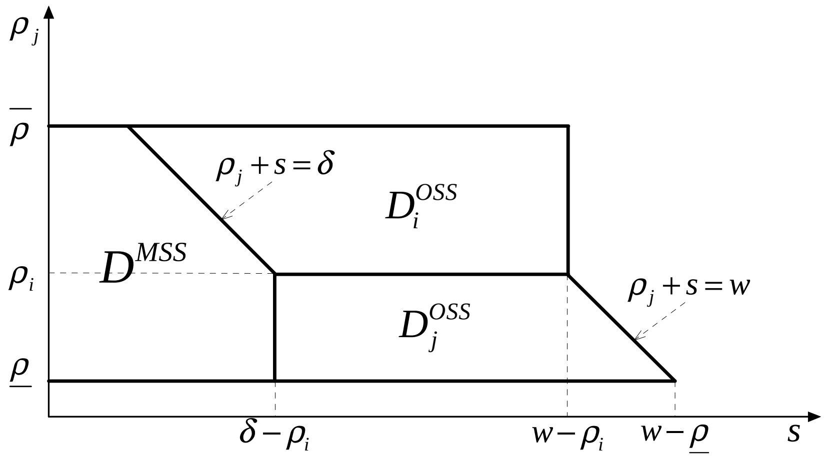


Figure 1

Firm i's expected profit can then be expressed as:

$$\rho_i E\left(D_i^{OSS} + D^{MSS}\right),$$

where D_i^{OSS} represents the demand from one-stop shoppers going to firm i, and D^{MSS} is the demand from multi-stop shoppers. As firm j's margin is distributed according to the distribution function $G\left(\rho_j\right)$, firm i's expected profit can be written as:

$$\begin{aligned} \pi\left(\rho_i\right) &= \rho_i\left[\left(1 - G\left(\rho_i\right)\right) F(w - \rho_i) + G\left(\rho_i\right) F\left(\delta - \rho_i\right)\right] \\ &= \rho_i\left\{F(w - \rho_i) - G\left(\rho_i\right)\left[F(w - \rho_i) - F\left(\delta - \rho_i\right)\right]\right\}. \end{aligned}$$

Hence, for a firm to obtain its minmax profit $\bar{\pi}$, we must have, for all ρ:

$$\rho\left\{F(w - \rho) - G\left(\rho\right)\left[F(w - \rho) - F\left(\delta - \rho\right)\right]\right\} = \bar{\pi},$$

or:

$$G\left(\rho\right) \equiv \frac{\rho F\left(w - \rho\right) - \bar{\pi}}{\rho F\left(w - \rho\right) - \rho F\left(\delta - \rho\right)}. \tag{8}$$

By construction, the function $G\left(\cdot\right)$ defined by (8) is such that $G\left(\underline{\rho}\right) = 0$ and $G\left(\bar{\rho}\right) = 1$; it remains to confirm that it is increasing in ρ in the range $\left[\underline{\rho}, \bar{\rho}\right]$. Differentiating (8) with respect to ρ, we have:

$$G'\left(\rho\right) = \frac{\left[\bar{\pi} - \rho F\left(\delta - \rho\right)\right]\left[F(w - \rho) - \rho f(w - \rho)\right] + \left[\rho F(w - \rho) - \bar{\pi}\right]\left[F\left(\delta - \rho\right) - \rho f\left(\delta - \rho\right)\right]}{\left[\rho F(w - \rho) - \rho F\left(\delta - \rho\right)\right]^2}.$$

As $w > \delta$, and given (5) and (6), the functions $\rho F(w - \rho)$ and $\rho F(\delta - \rho)$ are both increasing in the range $\left[\underline{\rho}, \bar{\rho}\right]$, and moreover satisfy $\underline{\rho} F(w - \underline{\rho}) = \bar{\rho} F(\delta - \bar{\rho}) = \bar{\pi}$ and $\rho F(w - \rho) > \bar{\pi} > \rho F(\delta - \rho)$ for $\underline{\rho} < \rho < \bar{\rho}$. It follows that $G'(\bar{\rho}) = 0$ and $G'(\rho) > 0$ for $\underline{\rho} \leq \rho < \bar{\rho}$.

We now show that the function $G(\cdot)$ supports a symmetric mixed strategy equilibrium. To see this, consider firm i's best response when its rival, firm j, adopts the above strategy. If firm i were to charge a total margin $m_i > \bar{\rho}$, one-stop shoppers would go to the rival and multi-stop shoppers become those consumers whose shopping cost is lower than $v_{12} - v_j = \delta - \rho_i$; hence, firm i would earn a profit equal to $\rho_i F(\delta - \rho_i) \leq \bar{\pi}$. Thus, without loss of generality, we can restrict attention to deviations that are such that $m_i \leq \bar{\rho}$.

Suppose first that firm i prices its weak product above cost (i.e., its total margin satisfies $m_i > \rho_i$), and consider the impact of an increase in the margin on the strong product, ρ_i, keeping constant the total margin m_i. We distinguish between two cases, depending on which firm offers the best prices.

- When the realization of the rival's margin is such that $m_j \left(= \rho_j\right) > m_i$, one-stop shoppers (if any) favor firm i, and thus the multi-stop shopping threshold is $\tau = v_{12} - v_i = \delta + m_i - \rho_i - \rho_j$. Two cases may then arise:

  - if $\tau = v_{12} - v_i \leq v_i$, which amounts to $v_i \geq v_{12}/2$, consumers whose shopping costs lie below τ engage in multi-stop shopping and buy strong products, whereas those with s between τ and v_i buy both products from firm i. Hence, increasing ρ_i:

    - increases the profit earned by selling the strong product to all active consumers (that is, those with $s \leq v_i = w - m_i$); and

    - also induces some multi-stop shoppers to buy firm i's weak product as well, which further enhances firm i's profit.

  - if instead $v_i < v_{12}/2$, consumers whose shopping costs lie below $v_{12}/2$ engage in multi-stop shopping and buy strong products, and all other consumers are inactive. Hence, firm i's profit is equal to:

$$\pi_i(\rho_i) = \rho_i F\left(\frac{v_{12}}{2}\right) = \rho_i F\left(\frac{w + \delta - \rho_1 - \rho_2}{2}\right),$$

which increases with ρ_i: the derivative is equal to:

$$\pi_i'(\rho_i) = F\left(\frac{v_{12}}{2}\right) - \frac{\rho_i f\left(\frac{v_{12}}{2}\right)}{2} = \left[2h\left(\frac{v_{12}}{2}\right) - \rho_i\right] \frac{f\left(\frac{v_{12}}{2}\right)}{2},$$

where the term in brackets is positive, as $v_i < v_{12}/2$ implies $2h(v_{12}/2) > h(v_{12}/2) > h(v_i) = h(w - m_i) > m_i > \rho_i$ (where the penultimate inequality stems from $m_i \leq \bar{\rho}$, the function $m_i F(w - m_i)$ being increasing in m_i in that range).

• When, instead, the realization of the rival's margin is such that $m_j \left(= \rho_j\right) < m_i$, one-stop shoppers (if any) favor firm j; hence, firm i only sells (its strong product) to multi-stop shoppers, and the multi-stop shopping threshold is $\tau = v_{12} - v_j = \delta - \rho_i$. Two cases may again arise:

- if $\tau = v_{12} - v_j \leq v_i$, which amounts to $v_j \geq v_{12}/2$, all consumers whose shopping costs lie below τ engage in multi-stop shopping, and so firm i's profit is equal to:

$$\pi_i(\rho_i) = \rho_i F(\tau) = \rho_i F(\delta - \rho_i),$$

  which increases with ρ_i on the relevant range $\rho_i \leq \bar{\rho}$; and

- if instead $v_j < v_{12}/2$, only those consumers with s below $v_{12}/2$ engage in multi-stop shopping, and so firm i's profit is equal to $\pi_i(\rho_i) = \rho_i F\left(\frac{v_{12}}{2}\right)$. The same reasoning as above then shows that this profit again increases with ρ_i.

Therefore, it is never optimal for a firm to price its weak product above cost: starting from $\rho_i < m_i$, raising ρ_i would always increase firm i's *ex post* profit, and would thus increase its expected profit as well.

Suppose now that firm i sells its weak product at cost: $m_i = \rho_i$. By construction, choosing any ρ_i in the range $\left[\underline{\rho}, \bar{\rho}\right]$ yields the same expected profit, $\bar{\pi}$. It remains to confirm that it is not profitable to pick a margin ρ_i outside the support of G:

- choosing $\rho_i < \underline{\rho}$ attracts all one-stop shoppers and thus yields an expected profit equal to $\pi_i(\rho_i) = \rho_i F(w - \rho_i)$, which increases in ρ_i for $\rho_i \leq \bar{\rho}$, and is thus lower than $\pi_i\left(\underline{\rho}\right) = \bar{\pi}$; and

- choosing $\rho_i > \bar{\rho}$ attracts no one-stop shoppers, and thus the expected profit must be lower than $\rho_i F(\delta - \rho_i) \leq \max_\rho \rho F(\delta - \rho) = \bar{\pi}$.

This establishes the first part of the proposition; the rest has been established in the main text.

A.2 Proof of Proposition 5

We now analyze the impact of banning below-cost pricing on consumer surplus. When below-cost pricing is not prohibited, the equilibrium consumer surplus can be expressed as:

$$\begin{aligned} S^* &= \int_0^w (w-s)\, f(s)ds + \int_0^{\tau^*} (\tau^* - s)\, f(s)ds \\ &= \int_0^w F(s)ds + \int_0^{\tau^*} F(s)ds, \end{aligned}$$

where the second expression relies on integration by parts. The first term in that expression represents the surplus that would be generated if all consumers were one-stop shoppers (and thus bought the bundle at cost), and the second term represents the extra surplus from multi-stop shopping. When, instead, below-cost pricing is banned, *ex post* (i.e., for a given realization of the margins ρ_1 and ρ_2) consumer surplus can be written as:

$$\begin{aligned} S^b(\rho_1,\rho_2) &= \int_0^{v^b(\rho_1,\rho_2)} \left[v^b(\rho_1,\rho_2) - s\right] f(s)ds + \int_0^{\tau^b(\rho_1,\rho_2)} \left[\tau^b(\rho_1,\rho_2) - s\right] f(s)ds \\ &= \int_0^{v^b(\rho_1,\rho_2)} F(s)ds + \int_0^{\tau^b(\rho_1,\rho_2)} F(s)ds. \end{aligned}$$

Thus, the resulting change in *ex post* consumer surplus is given by:

$$\Delta S(\rho_1,\rho_2) = S^b(\rho_1,\rho_2) - S^* = \int_{\tau^*}^{\tau^b(\rho_1,\rho_2)} F(s)ds - \int_{v^b(\rho_1,\rho_2)}^{w} F(s)ds.$$

Banning below-cost pricing generates two opposite effects on consumer surplus. On the one hand, the increase in multi-stop shopping (recall that $\tau^b > \tau^*$) has a positive effect, represented by the first term in the above expression; on the other hand, one-stop shoppers face higher prices than before, causing a loss of consumer surplus represented by the second term. The net effect depends on the value of w, δ, and the distribution of shopping costs, which contribute to determining equilibrium prices.

To explore this further, we fix the parameter δ and examine the sign of ΔS as a function of the social value w. Note that τ^* and $\bar{\rho}$ do not depend on w, whereas $\underline{\rho}(w)$ is the lower solution to $\underline{\rho}F\left(w-\underline{\rho}\right) = \bar{\pi} = \bar{\rho}F(\delta-\bar{\rho})$, and thus decreases in w.

In the limit case where $w = \delta$, the lower bound $\underline{\rho}(w)$ coincides with $\bar{\rho}$; that is, both firms charge $\rho = \bar{\rho}$ with probability one. As $\bar{\rho} > \rho^*$ (and weak products are priced at cost, instead of being subsidized), all prices are higher than before, and thus every consumer's (expected) surplus goes down. By continuity, this remains the case as long as weak products offer sufficiently low value (i.e., as long as w is close enough to δ).

We now consider the impact of a ban on total welfare, that is, on the sum of consumer surplus and firms' profits. When w is close to δ, the equilibrium margin distribution tends to assign a probability mass of 1 on $\bar{\rho}$, and the impact of a ban on *expected* welfare then becomes:

$$\begin{aligned}
\Delta W &= \Delta S(\bar{\rho}, \bar{\rho}) + 2(\bar{\pi} - \pi^*) \\
&= \int_{\tau^*}^{\tau^b(\bar{\rho},\bar{\rho})} F(s)ds - \int_{v^b(\bar{\rho},\bar{\rho})}^{w} F(s)ds + 2(\bar{\pi} - \pi^*) \\
&= 2\Phi(\delta - \bar{\rho}) - \Phi(\delta - 2\rho^*) - \Phi(\delta) + 2(\bar{\pi} - \pi^*),
\end{aligned}$$

where:

$$\Phi(x) \equiv \int_0^x F(s)ds.$$

The sign of ΔW can be either positive or negative, depending on the distribution of shopping costs. To see this, we consider the case where shopping costs are distributed according to $F(s) = s^k/k$. The hazard rate assumption is satisfied for any $k > 0$, and:

$$f(s) = s^{k-1},\ \Phi(s) = \frac{s^{k+1}}{k(k+1)},\ h(s) = \frac{F(s)}{f(s)} = \frac{s}{k}.$$

When below-cost pricing is not prohibited, the equilibrium is characterized by:

$$\begin{aligned}
\rho^* &= h(\delta - 2\rho^*) = \frac{\delta - 2\rho^*}{k} \Leftrightarrow \rho^* = \frac{\delta}{2+k}, \\
\tau^* &= \delta - 2\rho^* = \frac{k\delta}{2+k}, \\
\pi^* &= 2\rho^* F(\delta - 2\rho^*) = 2\rho^* \frac{(\delta - 2\rho^*)^k}{k} = 2k^{k-1}\left(\frac{\delta}{2+k}\right)^{k+1}, \\
v^* &= w = \delta.
\end{aligned}$$

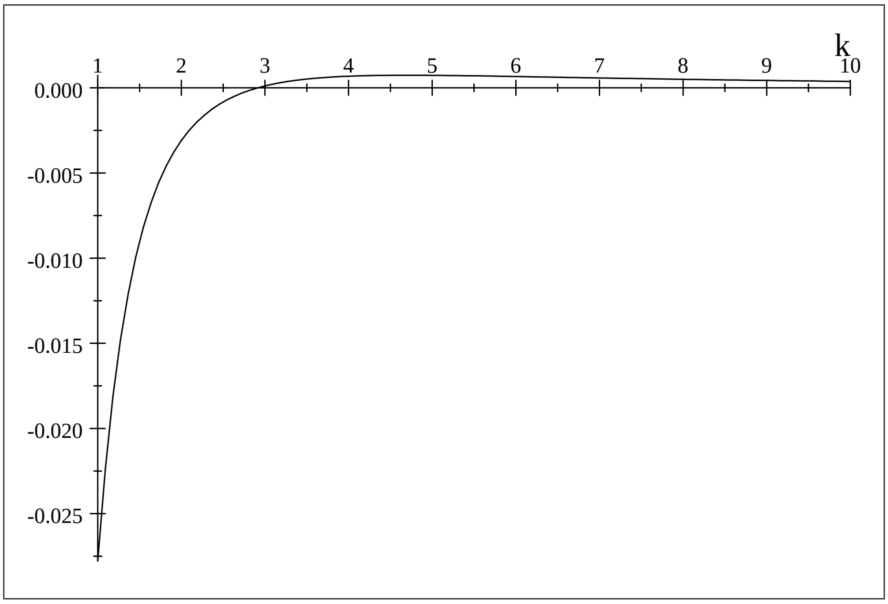
Instead, when below-cost pricing is banned, the equilibrium is characterized as follows:

$$\begin{aligned}
\bar{\rho} &= h(\delta - \bar{\rho}) = \frac{\delta - \bar{\rho}}{k} \Leftrightarrow \bar{\rho} = \frac{\delta}{1+k}, \\
\bar{\tau} &= v^b(\bar{\rho}, \bar{\rho}) = \delta - \bar{\rho} = \frac{k\delta}{1+k}, \\
\bar{\pi} &= 2\bar{\rho}F(\delta - \bar{\rho}) = 2\bar{\rho}\frac{(\delta - \bar{\rho})^k}{k} = 2k^{k-1}\left(\frac{\delta}{1+k}\right)^{k+1}.
\end{aligned}$$

Thus, banning below-cost pricing results in a change of total welfare:

$$\Delta W(k) = 2k^{k-1}\left(\left(\frac{\delta}{1+k}\right)^{k+1} - \left(\frac{\delta}{2+k}\right)^{k+1}\right) + \frac{1}{k(1+k)}\left(2\left(\frac{k\delta}{1+k}\right)^{k+1} - \delta^{k+1} - \left(\frac{k\delta}{2+k}\right)^{k+1}\right).$$

This expression is continuous in k and tends to $-\infty$ when k goes to 0; hence, banning below-cost pricing reduces total welfare when the distribution is not too convex. The following graph represents $\Delta W(k)/\delta^{k+1}$ and shows that banning below-cost pricing instead increases total welfare when the distribution of shopping cost is sufficiently convex (namely, for $k > \hat{k} \simeq 2.9$):



By continuity, for w close enough to δ, there exists $\hat{k}(w, \delta)$ such that banning below-cost pricing reduces total welfare when $k < \hat{k}(\delta)$.

A.3 The impact of RBC laws on consumer surplus

We conclude by noting that RBC laws necessarily decrease (expected) consumer surplus when the density of the distribution of shopping costs does not increase between $\tau^*$and δ.

The impact of RBC laws on total expected consumer surplus can be expressed as the impact on expected social welfare, minus the impact on expected industry profit:

$$E\left[\Delta S\left(\rho_1,\rho_2\right)\right]=E\left[\Delta W\left(\rho_1,\rho_2\right)\right]-E\left[\Delta \Pi\left(\rho_1,\rho_2\right)\right],$$

where:

$$E\left[\Delta \Pi\left(\rho_1\rho_2\right)\right]=2\left(\bar{\pi}-\pi^*\right),$$

and $\Delta W\left(\rho_1\rho_2\right)$ can be obtained by comparing the two regimes:

- when firms are allowed to price below-cost, social welfare is equal to:

  $$W^*=\int_0^w\left(w-s\right)dF\left(s\right)+\int_0^{\tau^*}\left(\delta-s\right)dF\left(s\right),$$

  where the first term is the social welfare that would be generated if all consumers were one-stop shoppers, and the second term represents the additional welfare from multi-stop shopping; and

- under RBC laws, *ex post* social welfare is equal to:

  $$W^b\left(\rho_1,\rho_2\right)=\int_0^{v^b\left(\rho_1,\rho_2\right)}\left(w-s\right)dF\left(s\right)+\int_0^{\tau^b\left(\rho_1,\rho_2\right)}\left(\delta-s\right)dF\left(s\right),$$

  where:

  $$v^b\left(\rho_1,\rho_2\right)=w-\min\left\{\rho_1,\rho_2\right\}\text{ and }\tau^b\left(\rho_1,\rho_2\right)=\delta-\max\left\{\rho_1,\rho_2\right\}.$$

Hence, the impact of a ban on *ex post* social welfare is given by:

$$\Delta W\left(\rho_1,\rho_2\right)=\int_{\tau^*}^{\tau^b\left(\rho_1,\rho_2\right)}\left(\delta-s\right)dF\left(s\right)-\int_{v^b\left(\rho_1,\rho_2\right)}^{w}\left(w-s\right)dF\left(s\right),\tag{9}$$

and the impact of RBC laws on total expected consumer surplus can thus be expressed as:

$$
\begin{aligned}
E\left[\Delta S\left(\rho_1,\rho_2\right)\right] &= E\left[\Delta W\left(\rho_1,\rho_2\right)\right]-2\left(\bar{\pi}-\pi^*\right)\\
&= E\left[\Delta W\left(\rho_1,\rho_2\right)-2\left(\bar{\pi}-\pi^*\right)\right]\\
&= E\left[\int_{\tau^*}^{\tau^b(\rho_1,\rho_2)}\left(\delta-s\right)dF\left(s\right)-\int_{v^b(\rho_1,\rho_2)}^{w}\left(w-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right]\\
&\leq E\left[\int_{\tau^*}^{\tau^b(\rho_1,\rho_2)}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right]\\
&= E\left[\int_{\delta-2\rho^*}^{\delta-\max\{\rho_1,\rho_2\}}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right)\right]\\
&= E\left[\phi\left(\max\left\{\rho_1,\rho_2\right\}\right)\right],
\end{aligned}
$$

where:

$$
\phi\left(\rho\right)\equiv\int_{\delta-2\rho^*}^{\delta-\rho}\left(\delta-s\right)dF\left(s\right)-2\left(\bar{\pi}-\pi^*\right).
$$

It follows that RBC laws reduce expected consumer surplus whenever $E\left[\phi\left(\rho\right)\right]<0$, where the function $\phi\left(\rho\right)$ decreases as ρ increases:

$$
\phi'\left(\rho\right)=-\rho f\left(\delta-\rho\right)<0.
$$

We have:

Proposition 9 *If $f\left(s\right)$ is non-increasing for $s\in\left[\tau^*,\delta\right]$, then RBC laws reduce total expected consumer surplus.*

Proof. It suffices to show that $\phi\left(0\right)\leq 0$. Using $\tau^*=\delta-2\rho^*$, we have:

$$
\begin{aligned}
\phi\left(0\right) &= \int_{\tau^*}^{\delta}\left(\delta-s\right)f\left(s\right)ds-2\left(\bar{\pi}-\pi^*\right)\\
&\leq \int_{\tau^*}^{\delta}\left(\delta-s\right)f\left(\tau^*\right)ds-2\pi^*\\
&= \left[-\frac{\left(\delta-s\right)^2}{2}\right]_{\tau^*}^{\delta}\times\frac{F\left(\tau^*\right)}{\rho^*}-2\rho^*F\left(\tau^*\right)\\
&= \left[\frac{\rho^2}{2}\right]_0^{2\rho^*}\times\frac{F\left(\tau^*\right)}{\rho^*}-2\rho^*F\left(\tau^*\right)\\
&= 0,
\end{aligned}
$$

where the first inequality stems from the assumed monotonicity of $f(\cdot)$ on the range $[\tau^*, \delta]$ and from the fact that:

$$\bar{\pi} = \max_{\rho} \rho F(\delta - \rho) \geq 2\rho^* F(\delta - 2\rho^*) = 2\pi^*,$$

and the equality that follows uses the first-order condition characterizing ρ^*, namely:

$$\rho^* f(\tau^*) = F(\tau^*).$$

It follows that $\phi(\rho) < 0$ for any $\rho > 0$, and thus:

$$E[\Delta S(\rho_1, \rho_2)] \leq E[\phi(\max\{\rho_1, \rho_2\})] < 0.$$

■

B Bounded shopping costs

We now consider the case where shopping costs are bounded either above ($s \leq \overline{s}$) or below ($s > \underline{s}$).

B.1 Proof of Proposition 6

Suppose that consumers' shopping costs are distributed over $[0, \overline{s}]$, where $\overline{s} > 0$. It is straightforward to check that the first four claims in the proof of Lemma 1 still hold; that is, in any equilibrium, there exist active multi-stop shoppers who buy the strong products, and in addition, if there are active one-stop shoppers, then $m_1 = m_2 = 0$.

We first note that the equilibrium identified in the baseline model still exists when $\overline{s}$ is large enough:

Claim 6 *When $\overline{s} > j^{-1}(\delta)$, then there exists an equilibrium with both types of shoppers: consumers with a shopping cost lower than $\tau^* = j^{-1}(\delta)$ engage in multi-stop shopping, and face a margin $\rho^* = h(\tau^*)$ on each strong product; whereas those with a higher cost favor one-stop shopping.*

Proof. As shown in the text, there is a unique candidate equilibrium where both types of shopping patterns arise, and is as described in the Claim. The existence of one-stop shopping, however, requires $\overline{s} > \tau^* = j^{-1}(\delta)$. Conversely, when this condition holds, the margins $m_1^* = m_2^* = 0$ and $\rho_1^* = \rho_2^* = h(\tau^*)$ do support an equilibrium: indeed the reasoning of the proof of Proposition 2 ensures that no deviation is profitable. ■

Next, we show that one-stop shopping cannot arise if $\overline{s}$ is too low:

Claim 7 *When $\overline{s} \leq j^{-1}(\delta)$, then one-stop shopping does not arise in equilibrium.*

Proof. Suppose there exist some one-stop shoppers, which requires $\tau < \min\{\max\{v_1, v_2\}, \overline{s}\}$. Competition for these one-stop shoppers leads to $m_1 = m_2 = 0$, and thus $\tau_1 = \tau_2 = \delta - \rho_1 - \rho_2 < \overline{s}$, which implies $\rho_1 + \rho_2 > \delta - \overline{s} > 2h(\overline{s})$. Therefore, at least one of the margins on strong products must exceed $h(\overline{s})$. Suppose $\rho_1 > h(\overline{s})$; then $\rho_1 > h(\overline{s}) > h(\tau)$, as $\overline{s} > \tau$ and $h(\cdot)$ is strictly increasing. Consider now the following deviation: decrease ρ_1 to $\tilde{\rho}_1$ and increase μ_1 by the same amount, so as to maintain the total margin. This does not affect the profit from one-stop shoppers (which remains equal to zero), but yields a profit from multi-stop shoppers, equal to $\tilde{\pi}_1 = \tilde{\rho}_1 F(\tilde{\tau})$, where $\tilde{\tau} = \delta - \tilde{\rho}_1 - \rho_2$. As $d\tilde{\pi}_1/d\tilde{\rho}_1|_{\tilde{\rho}_1=\rho_1} = -f(\tau)(\rho_1 - h(\tau))$, which is strictly negative as $\rho_1 > h(\tau)$, such a deviation is profitable. Hence, one-stop shopping does not arise in equilibrium. ■

Claims 6 and 7 together establish the first part of the Proposition. We now characterize the equilibria where all consumers are multi-stop shoppers.

Claim 8 *When $\overline{s} \leq j^{-1}(\delta)$, any margin profile such that $\rho_1 \in [h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$, $\mu_2 = \rho_1 - \delta + \overline{s}$ and $\mu_1 = \rho_2 - \delta + \overline{s}$, constitutes an equilibrium in which all active consumers are multi-stop shoppers.*

Proof. Suppose there are only multi-stop shoppers who, from Claim 3, buy the strong products. Consumers are willing to visit both firms if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$), but would prefer one-stop shopping if $s > \tau = v_{12} - \max\{v_1, v_2\}$; hence, we must have $\tau \geq \min\{v_{12}/2, \overline{s}\}$, and the demand from multi-stop shoppers is $F(\min\{v_{12}/2, \overline{s}\})$. As consumers only buy strong products, firms must charge non-negative margins on these products: $\rho_1, \rho_2 \geq 0$.

If $\overline{s} < \min\{v_{12}/2, \tau\}$, each firm can profitably deviate by slightly raising the price for its strong product: this increases the margin without affecting the demand, equal to $F(\overline{s})$. Hence, without loss of generality, we can assume $\overline{s} \geq \min\{v_{12}/2, \tau\}$. The condition $\tau \geq \min\{v_{12}/2, \overline{s}\}$ then implies that either $v_{12}/2 \leq \min\{\tau, \overline{s}\}$, or $v_{12}/2 \geq \tau = \overline{s}$. We consider these two cases in turn.

Consider the first case, and note that the condition:

$$\frac{v_{12}}{2} \leq \tau = v_{12} - \max\{v_1, v_2\}$$

then implies $\max\{v_1, v_2\} \leq v_{12}/2$. Without loss of generality, suppose $\rho_2 \geq \rho_1 (\geq 0)$, and consider the following deviation for firm 1: keeping ρ_1 constant, reduce μ_1 so as to offer $\tilde{v}_1 = v_{12}/2 + \varepsilon$, which amounts to charging:

$$\tilde{\mu}_1 = \frac{w - \delta + \rho_2 - \rho_1}{2} - \varepsilon \geq \frac{w - \delta}{2} - \varepsilon > 0.$$

This deviation does not affect v_{12} or $\tau_2 = v_{12} - v_2$, but it decreases τ_1 to $\tilde{\tau}_1 = v_{12} - \tilde{v}_1 = v_{12}/2 - \varepsilon$; as initially $\tau_2 \geq \tau \geq v_{12}/2$, it follows that the multi-stop shopping threshold becomes $\tilde{\tau} = \tilde{\tau}_1 (< v_{12}/2) < \tilde{v}_1$. This adjustment thus induces some multi-stop shoppers to buy everything from firm 1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $v_{12}/2$), on which firm 1 earns an extra profit from selling its weak product (as $\tilde{\mu}_1 > 0$), and it, moreover, attracts some additional one-stop shoppers (those whose shopping cost lies between $v_{12}/2$ and $\tilde{v}_1$), generating additional profit (as $\rho_1 \geq 0$ and $\tilde{\mu}_1 > 0$).

Hence, we cannot have an equilibrium of the type $v_{12}/2 \leq \min\{\tau, \overline{s}\}$.

Consider now the second case: $\overline{s} = \tau \leq v_{12}/2$. Note first that if $\tau = \tau_i = v_{12} - v_i < \tau_j = v_{12} - v_j$, then firm i could again profitably deviate by increasing the margin on its strong product without affecting the demand (as τ_i does not depend on ρ_i). Hence, we must have $\overline{s} = \tau = \tau_1 = \tau_2$, and thus $v_1 = v_2$, or $m_1 = m_2 = m$.

We now show that firms' margins on weak products must satisfy $\mu_1, \mu_2 \leq -h(\overline{s})$, and margins on strong products must satisfy $\rho_1, \rho_2 \geq h(\overline{s})$. To see this, note that firm 1, say, could induce some multi-stop shoppers to buy its weak product B as well, by reducing the margin on its weak product, so that $\tilde{\tau}_1 = \delta + \tilde{\mu}_1 - \rho_2 < \tau_1 (= \delta + \mu_1 - \rho_2) = \overline{s}$, keeping the total margin constant: $\tilde{\rho}_1 + \tilde{\mu}_1 = m_1$. By so doing, firm 1 would earn a profit equal to:

$$\begin{aligned}\pi_1 &= \tilde{\rho}_1 F(\tilde{\tau}_1) + m_1(F(\overline{s}) - F(\tilde{\tau}_1)) \\ &= m_1 F(\overline{s}) - \tilde{\mu}_1 F(\delta + \tilde{\mu}_1 - \rho_2).\end{aligned}$$

To rule out such a deviation, μ_1 must satisfy:

$$\mu_1 \in \arg\max_{\tilde{\mu}_1 \le \mu_1} -\tilde{\mu}_1 F(\delta + \tilde{\mu}_1 - \rho_2),$$

which, given the monotonicity of $h(\cdot)$, amounts to:

$$\mu_1 \le -h(\overline{s}).$$

Alternatively, firm 1 could discourage some multi-stop shoppers by increasing $\tilde{\rho}_1$, so that $\tilde{\tau}_2 = \delta + \mu_2 - \tilde{\rho}_1 < \tau_2 (= \delta + \mu_2 - \rho_1) = \overline{s}$, keeping $\tilde{\mu}_1$ unchanged. Doing so yields a profit equal to:

$$\pi_1 = \tilde{\rho}_1 F(\tilde{\tau}_2).$$

Ruling out this deviation thus requires:

$$\rho_1 \in \arg\max_{\tilde{\rho}_1 \ge \rho_1} \tilde{\rho}_1 F(\delta + \mu_2 - \tilde{\rho}_1),$$

or:

$$\rho_1 \ge h(\overline{s}).$$

The conditions $\mu_2 \le -h(\overline{s})$ and $\rho_2 \ge h(\overline{s})$ can be derived using the same logic.

Therefore, the margins for any candidate equilibria must satisfy (using $\tau = \delta + \mu_1 - \rho_2 = \overline{s}$): $-h(\overline{s}) \ge \mu_1 = \rho_2 - \delta + \overline{s} \ge h(\overline{s}) - \delta + \overline{s}$, implying $\overline{s} + 2h(\overline{s}) \le \delta$. Hence, an equilibrium with only multi-stop shopping exists only when $\overline{s} \le j^{-1}(\delta)$. Conversely, when this condition holds, any margins satisfying $\rho_1, \rho_2 \in [h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$, $\mu_2 = \rho_1 - \delta + \overline{s}$ and $\mu_1 = \rho_2 - \delta + \overline{s}$ constitute an equilibrium in which all consumers are multi-stop shoppers. ■

Claims 7 and 8 together establish the second part of the Proposition.

B.2 Proof of Proposition 7

Suppose that consumers' shopping costs are distributed over $[\underline{s}, +\infty)$, where $\underline{s} < w$. We first show that part of Lemma 1 still applies:

Lemma 2 *Suppose that consumer shopping costs are distributed over* $[\underline{s}, +\infty)$*, where* $\underline{s} < w$*. Then, in equilibrium:*

- *some one-stop shoppers are active;*
- $m_1 = m_2 = 0$*; and*
- *active multi-stop shoppers buy strong products.*

Proof. It is straightforward to check that the first three claims of the proof of Lemma 1 remain valid: in equilibrium, some consumers are active (Claim 1); $m_1 = m_2 = 0$ whenever there are active one-stop shoppers (Claim 2), and active multi-stop shoppers buy the strong products (Claim 3). Furthermore, Claim 3 establishes part (iii) of the Lemma, whereas Claim 2 implies that part (ii) follows from part (i). Finally, to complete the proof, it suffices to note that the proof of Claim 5 remains valid, which yields part (i). ∎

We now proceed to establish the proposition. We first note that multi-stop shopping must arise when some consumers have low enough shopping costs:

Lemma 3 *If* $\underline{s} < \delta/3$*, some multi-stop shoppers are active in equilibrium.*

Proof. Suppose all active consumers are one-stop shoppers. From Claim 2, price competition for one-stop shoppers then leads to $m_1 = m_2 = 0$. Ruling out multi-stop shopping requires $v = w \geq \underline{v}_{12} - \underline{s} = w - \delta - \mu_1 - \mu_2 - \underline{s}$, or (using $m_1 = m_2 = 0$) $\rho_1 + \rho_2 \leq \delta + \underline{s}$. If firm 2, say, is the one that charges less on its strong product (i.e., $\rho_2 \leq \rho_1$), then we must have $\rho_2 \leq (\delta + \underline{s})/2$. Consider the following deviation for firm 1: charge $\tilde{\rho}_1 = \varepsilon > 0$ and $\tilde{\mu}_1 = -\varepsilon$ such that the total margin remains zero. The multi-stop shopping threshold becomes:

$$\tilde{\tau} = \delta - \tilde{\rho}_1 - \rho_2 \geq \delta - \varepsilon - \frac{\delta + \underline{s}}{2} = \frac{\delta - \underline{s}}{2} - \varepsilon.$$

As $\delta > 3\underline{s}$ (implying $(\delta - \underline{s})/2 > \underline{s}$), it follows that $\tilde{\tau} > \underline{s}$ for ε sufficiently small. Hence, firm 1 can induce some consumers to engage in multi-stop shopping and make a profit on them. ■

Next, we show that there indeed exists an equilibrium with multi-stop shopping as long as some consumers' shopping costs are not too large:

Lemma 4 *If $\underline{s} < \delta$, there exists an equilibrium exhibiting both types of shopping patterns, in which firms' total margins are zero ($m_i^* = 0$) and the margins on their strong products are equal to $\rho_i^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$.*

Proof. Suppose $\underline{s} < \delta$. As discussed in the text, the unique candidate equilibrium exhibiting both types of shopping patterns is such that: (i) both firms charge zero total margins ($m_i^* = 0$) and a positive margin on their strong products equal to $\rho_i^* = \rho^* = h(\tau^*)$, where $\tau^* = j^{-1}(\delta)$; and (ii) consumers with a shopping cost lying between $\underline{s}$ and τ^* engage in multi-stop shopping, whereas those with a shopping cost lying between τ^* and w are one-stop shoppers. Therefore, this type of equilibrium exists when $\underline{s} < \tau^* = j^{-1}(\delta)$. As the function $j(\cdot)$ is strictly increasing and satisfies $j(\underline{s}) = \underline{s} + 2h(\underline{s}) = \underline{s}$, the condition $\underline{s} < \tau^*$ amounts to $\underline{s} < \delta$.

Conversely, these margins indeed constitute an equilibrium. By construction, given the equilibrium prices charged by the other firm, a firm cannot make a profit on one-stop shoppers, and charging ρ^* on the strong product maximizes the profit that a firm earns from multi-stop shoppers. ■

It follows that the analysis of the baseline model still applies when the lower bound is small enough, namely, when $\underline{s} < \delta/3$. From Lemmas 2 and 3, both types of shopping patterns must arise in equilibrium; Lemma 4 then ensures that the unique candidate identified in the text is indeed an equilibrium. This establishes the first part of the Proposition.

We now turn to the second part of the Proposition, and first note that multi-stop shopping cannot arise when all consumers have high shopping costs:

Lemma 5 *If $\underline{s} > \delta$, there are no multi-stop shoppers in equilibrium.*

Proof. Suppose, to the contrary, there are some active multi-stop shoppers. From Lemma 2, $m_1 = m_2 = 0$ and multi-stop shoppers must buy strong products; hence,

$\tau = \delta - \rho_1 - \rho_2 > \underline{s}$. As $\underline{s} > \delta$, it follows that $\rho_1 + \rho_2 < 0$; hence, at least one firm must charge a negative margin on its strong product and incur a loss from serving multi-stop shoppers. But this cannot be an equilibrium, as that firm could avoid the loss by increasing its prices. ■

Finally, we show that when all consumers have large enough shopping costs, there exists equilibria with no multi-stop shoppers.

Lemma 6 *There exist equilibria with one-stop shopping if and only if $\underline{s} \geq \delta/3$. In these equilibria, margins satisfy: (i) $\rho_1 + \mu_1 = \mu_2 + \rho_2 = 0$; (ii) $\delta - \underline{s} \leq \rho_1, \rho_2, \rho_1 + \rho_2 \leq \delta + \underline{s}$; and (iii) $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$.*

Proof. Consider a candidate equilibrium with only one-stop shopping. From Lemma 2, $m_1 = m_2 = 0$ and thus $\tau = \delta - \rho_1 - \rho_2$. For firm 1, say, it cannot be profitable to deviate by attracting one-stop shoppers, as this would require a negative total margin $\tilde{m}_1 < 0$. Firm 1 could, however, deviate so as to induce some consumers to engage in multi-stop shopping; more specifically:

- it could induce some consumers to buy both strong products by charging $\tilde{\rho}_1$ such that $\tilde{\tau}_2 = \delta - \tilde{\rho}_1 + \mu_2 = \delta - \tilde{\rho}_1 - \rho_2 > \underline{s}$, or $\tilde{\rho}_1 < \delta - \underline{s} - \rho_2$; and
- alternatively, it could induce some consumers to buy both weak products by charging $\tilde{\mu}_1$ such that $\underline{\tilde{\tau}}_2 = -\delta + \rho_2 - \tilde{\mu}_1 > \underline{s}$, or $\tilde{\mu}_1 < \rho_2 - \delta - \underline{s}$.

Ruling out the first type of deviation requires $\rho_2 \geq \delta - \underline{s}$, while preventing the second type of deviation requires $\rho_2 \leq \delta + \underline{s}$. Therefore, the equilibrium margin ρ_2 must lie between $\delta - \underline{s}$ and $\delta + \underline{s}$. Applying the same logic to rule out firm 2's deviations requires the equilibrium margin ρ_1 to lie between $\delta - \underline{s}$ and $\delta + \underline{s}$ as well. Moreover, the margins cannot exceed the social values, which requires $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$.

Conversely, any margins that satisfy: (i) $\rho_1 + \mu_1 = \mu_2 + \rho_2 = 0$; (ii) $\delta - \underline{s} \leq \rho_1, \rho_2, \rho_1 + \rho_2 \leq \delta + \underline{s}$; and (iii) $-\underline{w}_1 \leq \rho_1 \leq \bar{w}_1$ and $-\underline{w}_2 \leq \rho_2 \leq \bar{w}_2$ constitute an equilibrium in which all active consumers are one-stop shoppers and both firms earn zero profit.

The above analysis shows that equilibrium margins must satisfy: (i) $\delta - \underline{s} \leq \rho_1, \rho_2$, implying $\rho_1 + \rho_2 \geq 2\delta - 2\underline{s}$; and (ii) $\rho_1 + \rho_2 \leq \delta + \underline{s}$. These two conditions then lead to

$2\delta - 2\underline{s} \leq \delta + \underline{s}$, which amounts to $\delta/3 \leq \underline{s}$. It thus follows that such an equilibrium exists if and only if $\delta/3 \leq \underline{s}$. ■

Combining Lemmas 5, 6 and 2 yields the second part of the Proposition, whereas Lemmas 4 and 6 together yield the last part. **Q.E.D.**