Multilateral vertical contracting*

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Abstract

We develop a flexible and tractable framework of secret contracting in multilateral vertical relationships, which places no restriction on tariffs and accounts for their impact on downstream competition. We show that equilibrium tariffs are cost-based and replicate the outcome of a multi-brand oligopoly, a finding in line with the analysis of a recent merger.

We provide a micro-foundation for this framework. We then use it to analyze the effect of RPM and price parity provisions, and of resale vs. agency business models. Next, we extend the framework to endogenize the distribution network; we also consider mergers and show that their impact on the distribution network can dominate price effect. Finally, we adapt this framework to the case of public contracting.

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1 Introduction

We propose a flexible, tractable model of multilateral contracting for the analysis of competition and network formation in vertical chains.

Upstream markets often involve intricate networks of interlocking relationships. For instance, competing supermarkets carry the same rival brands, health insurers deal with the same care providers, and pay-TV operators offer the same channels. Likewise, PC OEMs develop computers based on Intel and AMD chips, and Airbus and Boeing offer a choice of engines from General Electric, Rolls Royce and Pratt & Whitney. Yet, the vertical contracting literature mostly focuses on simpler market structures. Indeed, much of the early literature focuses on an upstream or downstream monopolist,[1] or on competing vertical structures (e.g., franchise networks).[2]

Several papers address multilateral relations but impose various restrictions. For instance, upstream competition comes from fringe suppliers[3] or perfect substitutes,[4] or contracts are restricted to linear or two-part tariffs.[5] Other papers, prompted by merger waves and policy debates in pay-TV[6] and healthcare[7] markets, either assume away the interplay between wholesale agreements and downstream outcomes (by restricting attention to lump-sum transfers), or account for it only partially (by assuming that upstream and downstream prices are set simultaneously).[8]

In the first part of this paper, we develop a model of multilateral relationships with upstream and downstream price competition,[9] allowing for any distribution of bargaining power. We do not restrict the tariffs that can be negotiated, and take

[1]See, e.g., Mathewson and Winter (1984) and Rey and Tirole (1986) on vertical coordination, Hart and Tirole (1990), O'Brien and Shaffer (1992) and McAfee and Schwartz (1994) on supplier's opportunism, and Bernheim and Whinston (1985, 1986, 1998) on exclusive dealing.

[2]See, e.g., Bonanno and Vickers (1988) and Rey and Stiglitz (1988, 1995) on strategic delegation.

[3]This is a frequent assumption in the literature on private labels (see, e.g., Mills, 1995, and Gabrielsen and Sørgard, 2007). See also Hart and Tirole (1990) and Innes and Hamilton (2009).

[4]See, e.g., Salinger (1988), Ordover *et al.* (1990), de Fontenay and Gans (2005, 2014), and Nocke and White (2007, 2010).

[5]See, e.g., Dobson and Waterson (2007), Rey and Vergé (2010) and Allain and Chambolle (2011).

[6]See, e.g., Chipty and Snyder (1999) on the impact of horizontal mergers, Crawford and Yurukoglu (2012) on bundling, and Crawford *et al.* (2018) on vertical integration.

[7]See, e.g., Gowrisankaran *et al.* (2015) on hospital mergers, and Ho and Lee (2017) on competition among health insurance providers.

[8]Among the most recent papers, Gowrisankaran *et al.* (2015) and Ho and Lee (2017) follow the former approach, whereas Crawford *et al.* (2018) adopt the latter.

[9]Nocke and Rey (2018) study multilateral relations with downstream competition in quantities.

into account their impact on downstream competition. As supply contracts are often private, we mostly focus on secret contracting,[10] which raises a modelling issue: when receiving an out-of-equilibrium offer, a firm must conjecture about the contracts signed by its rivals; as Bayesian updating does not restrict off-equilibrium beliefs, there are typically many (perfect Bayesian) equilibria. This has led the literature to rely on reasonable beliefs, such as passive or wary beliefs. Unfortunately, with downstream competition in prices, these beliefs create existence or tractability issues.[11] We define instead a bargaining equilibrium as follows. First, upstream negotiations are modelled using a *Nash-in-Nash* approach, which relies on the contract equilibrium concept developed by Crémer and Riordan (1987) and Horn and Wolinsky (1988): each contract is bilaterally efficient given the other equilibrium contracts, and the gains from trade are shared according to firms' bilateral bargaining power.[12] Second, given their negotiated contracts, downstream firms compete in prices.

We first show that equilibrium tariffs are *cost-based* (as long as the downstream behavior is smooth, in a sense made precise later): marginal input prices reflect marginal costs of production – an insight in line with Nilsen *et al.* (2016), who find that an upstream merger between Norwegian egg producers only affected inframarginal input prices. This is because, in any bilateral negotiation, the two firms internalize the full margin generated by their channel, which induces the supplier to price aggressively and undercut the margins it charges to others. It follows that the downstream outcome is the same as if each downstream firm could produce all inputs at cost. The tariffs however affects the division of profit, with more convex (resp., concave) tariffs giving a larger share to upstream (resp., downstream) firms.

We then provide a micro-foundation for these bargaining equilibria, which relies on a (non-cooperative, sequential) game of delegated negotiations: each firm relies on different agents to negotiate with its various partners and, for each channel, one side is randomly selected to make a take-it-or-leave-it offer. Any bargaining equilibrium

[10]At the end of the paper, we show how to apply our framework to the case of public contracts.

[11]Both issues arise even in the absence of upstream competition; see Rey and Vergé (2004).

[12]O'Brien and Shaffer (1992) apply this approach in an upstream monopoly setting. Since then, it has been used in both the theoretical literature (e.g., Gans, 2007; Milliou and Petrakis, 2007; Allain and Chambolle, 2011) and the empirical literature (e.g., Crawford and Yurukoglu, 2012; Grennan, 2013; Gowrisankaran *et al.*, 2015). Because it combines the cooperative Nash-bargaining solution (for each vertical channel) with a non-cooperative Nash-equilibrium concept (across channels), Collard-Wrexler *et al.* (2019) have coined the terminology *Nash-in-Nash bargaining*.

outcome can be sustained by a sequential equilibrium of the delegated negotiations game; conversely, any *regular* equilibrium outcome of this game (in a sense made precise) corresponds to a bargaining equilibrium. Compared with a game of direct negotiations, the candidate equilibria that survive single-channel deviations are the same in the two games; assuming delegated negotiations however ensures existence, by ruling out deviations involving multiple channels. A preliminary stage in which one side gets to make an offer, with the above lottery as a back-up in case of rejection, can be used to generate deterministic outcomes; a similar approach provides a micro-foundation for linear tariffs and/or publicly observable contracts.

To illustrate the flexibility of our framework, we first study the impact of classic vertical restraints such as resale price maintenance (RPM hereafter) and price parity agreements (PPAs hereafter). Allowing RPM generates many equilibria: as retail prices are separately negotiated, firms can agree on any arbitrary marginal transfer prices (and share the profits as desired through, e.g., lump-sum fees), which however affect their negotiations with other partners. Furthermore, if price floors can sustain supra-competitive prices when brands are more substitutable than stores, price ceilings can do the same in the opposite case. This finding challenges the current antitrust approach towards RPM, which views inter-brand competition as likely to prevent anti-competitive effects, and treats maximum RPM more favorably than minimum RPM. By contrast PPAs, which require retailers to charge the same price for all brands, have no substantial impact on retail prices. They may limit the joint profit that a retailer can generate with a given supplier, but pricing at marginal cost still makes the retailer the residual claimant on this joint profit; as a result, equilibrium contracts are again cost-based. This contrasts with the view, common in policy circles, that retail PPAs are akin to RPM and should therefore be banned; it also suggests that the anti-competitive effects highlighted by the literature depends on the nature of the contracts that are considered (e.g., linear vs. non-linear tariffs).

We also use our framework to compare business models. Switching from the traditional resale model to the agency model often used by online marketplaces amounts to turning the model *upside-down*: platforms charge transaction-based commissions to suppliers, who control the final prices. Equilibrium tariffs are again cost-based and the final outcome is the same as if suppliers were directly competing against each other at all retail locations. Which business model delivers lower prices thus depends whether competition is fiercer among suppliers or retailers.

In the second part of the paper, we turn to network formation. As the Nash-in-Nash approach implies that every channel is active in equilibrium,[13] we add a preliminary stage where firms choose which channels to activate. To deal with coordination problems, we focus on coalition-proof Nash equilibria (CPNE).[14]

We apply this approach to the case of a successive duopoly. Adding a distribution channel increases demand, but it also dissipates profit through intrabrand competition. As a result, suppliers activate both channels only when downstream firms are sufficiently differentiated. Furthermore, for the case of linear demands, there is always a unique CPNE, with the complete network if downstream firms are sufficiently differentiated, and exclusive dealing otherwise.

Finally, we study the impact of mergers on prices and channel decisions. For any given network, in the absence of efficiency gains a downstream merger raises prices (by eliminating downstream competition), whereas an upstream merger has no impact on final prices (as marginal input prices remain cost-based). However, accounting for the impact on the distribution network can give rise to very different insights. Pre-merger, firms may limit the number of channels to avoid profit dissipation through intrabrand competition. A downstream merger eliminates this concern and tends to expand the network, to the point that it may benefit consumers and increase social welfare. By contrast, an upstream merger enables suppliers to coordinate their channel decisions and can trigger vertical foreclosure, which harms consumers and social welfare. Finally, a vertical merger induces the integrated supplier to charge higher (marginal) prices to rivals, which tends to raise retail prices and reduce consumer surplus and social welfare, but it may also expand or restrict the distribution network.

The paper is organized as follows. We first outline our setting (Section 2), characterize the bargaining equilibria (Section 3), and provide a micro-foundation (Section 4), before studying vertical restraints and alternative business models (Section 5). We then endogenize the channel network (Section 6), and examine the impact of mergers in this extended setting (Section 7). Finally, we apply our approach to publicly

[13]See, e.g., de Fontenay and Gans (2014) and Collard-Wexler *et al.* (2019).

[14]See Bernheim *et al.* (1987).

observable contracts (Section 8), before providing concluding remarks (Section 9).

2 The model

2.1 Secret contracting in non-linear tariffs

We consider a vertical chain with $n \geq 2$ competing manufacturers, $M_1, ..., M_n$, and $m \geq 2$ competing retailers, $R_1, ..., R_m$.[15] We assume constant returns to scale[16] and denote M_i's unit cost by c_i, for $i \in \mathcal{I} \equiv \{1, ..., n\}$, and R_j's unit cost by γ_j, for $j \in \mathcal{J} \equiv \{1, ..., m\}$.[17] The demand for brand i at store j, $D_{ij}(\mathbf{p})$, is continuously differentiable in the price vector $\mathbf{p} = (p_{ij})_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ whenever positive.[18]

We assume that wholesale agreements are purely vertical: the contract between M_i and R_j specifies a transfer, t_{ij}, based solely on M_i's sales through R_j, q_{ij}. This excludes *horizontal* clauses such as exclusive dealing or market-share discounts,[19] but allows for any non-linear tariffs $t_{ij}(q_{ij})$. We moreover focus on secret contracting: the terms negotiated between M_i and R_j (including whether they reached an agreement) are private information to the two parties. Finally, we assume that wholesale negotiations can influence retail pricing decisions. We thus consider the following timing:

Stage 1: Each M_i negotiates with each R_j a non-linear tariff $t_{ij}(q_{ij})$; these bilateral negotiations are simultaneous and secret.

Stage 2: Retailers simultaneously set retail prices for all the brands that they carry.

2.2 Bargaining equilibrium

For tractability we follow the contract equilibrium approach pioneered by Crémer and Riordan (1987) and Horn and Wolinsky (1988), which requires contracts to be bilaterally efficient. We moreover allow for balanced bargaining, and denote by $\alpha_{ij} \in [0, 1]$ the bargaining power of M_i in its negotiation with R_j. Specifically, in stage 2, each R_j chooses its prices, given the contracts it negotiated, and assuming that its

[15] The analysis can be transposed to other vertically related industries.

[16] Allowing for non-linear cost functions is straightforward but notationally cumbersome.

[17] For ease of exposition, we use subscripts i and h for manufacturers, and j and k for retailers.

[18] This allows for kinks where demand becomes zero (e.g., when demand is linear).

[19] We consider price parity and other provisions in Section 5.

rivals set the equilibrium retail prices. In stage 1, each M_i and each R_j negotiates a tariff that: (i) maximizes their joint profit, given the other equilibrium contracts and R_j's induced retail pricing behavior; and (ii) gives a share α_{ij} of the bilateral gains from trade to M_i.[20]

To state this formally, we express the price vector as $\mathbf{p} = (\mathbf{p}_j, \mathbf{p}_{-j})$, where $\mathbf{p}_j = (p_{hj})_{h\in\mathcal{I}} = (p_{ij}, \mathbf{p}_{-i,j})$[21] is the vector of R_j's prices and $\mathbf{p}_{-j}$ the vector of all other retailers' prices. A *bargaining equilibrium* is then defined as follows:

Definition 1 (bargaining equilibrium)**.** *Fix $\boldsymbol{\alpha} \in [0,1]^{mn}$. A bargaining equilibrium is a vector of price responses $(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}$, together with a vector of equilibrium tariffs $\mathbf{t}^\mathbf{e} = (\mathbf{t}_j^\mathbf{e})_{j\in\mathcal{J}}$ and a vector of equilibrium prices $\mathbf{p}^\mathbf{e} = (\mathbf{p}_j^\mathbf{e})_{j\in\mathcal{J}}$, such that:*

- *In stage 2, for every $j \in \mathcal{J}$, the price response $\mathbf{p}_j^R(\cdot)$:*
  - *maximizes R_j's profit for any $\mathbf{t}_j = (t_{ij})_{i\in\mathcal{I}}$ negotiated by R_j in stage 1,* [22] *given rivals' equilibrium prices, $\mathbf{p}_{-j}^\mathbf{e}$:*

$$\mathbf{p}_j^R(\mathbf{t}_j) \in \arg\max_{\mathbf{p}_j} \pi_j(\mathbf{p}_j, \mathbf{p}_{-j}^\mathbf{e}; \mathbf{t}_j) \equiv \sum_{i\in\mathcal{I}} [(p_{ij}-\gamma_j) D_{ij}(\mathbf{p}_j, \mathbf{p}_{-j}^\mathbf{e}) - t_{ij}(D_{ij}(\mathbf{p}_j, \mathbf{p}_{-j}^\mathbf{e}))];$$

  - *satisfies $\mathbf{p}_j^R(\mathbf{t}_j^\mathbf{e}) = \mathbf{p}_j^\mathbf{e}$.*
- *In stage 1, for every $(i,j) \in \mathcal{I}\times\mathcal{J}$, the equilibrium tariff t_{ij}^e:*
  - *maximizes the joint profit of M_i and R_j, given R_j's other equilibrium tariffs, $\mathbf{t}_{-i,j}^\mathbf{e}$, its rivals' equilibrium prices, $\mathbf{p}_{-j}^\mathbf{e}$, and R_j's price response, $\mathbf{p}_j^R(\mathbf{t}_j)$; that is, using $q_{hk}^R(t_{ij}) \equiv D_{hk}(\mathbf{p}_j^R(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}), \mathbf{p}_{-j}^\mathbf{e})$:*

$$t_{ij}^e \in \arg\max_{t_{ij}} \left\{ \begin{array}{c} [p_{ij}^R(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}) - c_i - \gamma_j] q_{ij}^R(t_{ij}) \\ + \sum_{k\in\mathcal{J}\setminus\{j\}} \left[t_{ik}^e(q_{ik}^R(t_{ij})) - c_i q_{ik}^R(t_{ij})\right] \\ + \sum_{h\in\mathcal{I}\setminus\{i\}} \left[[p_{hj}^R(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}) - \gamma_j] q_{hj}^R(t_{ij}) - t_{hj}^e(q_{hj}^R(t_{ij}))\right] \end{array} \right\},$$

  - *gives M_i a share α_{ij} of the additional profit generated by their relationship.*

[20] We focus on efficient bilateral negotiations, which the firms can achieve as they can share their joint profit (e.g., via a fixed fee) without affecting it.

[21] With the convention that $p_{ij} = \infty$ when R_j does not carry M_i's brand.

[22] We assume that a tariff can be adopted only if it induces a well-behaved retail pricing problem. Alternatively, we could restrict attention to continuous tariffs and bounded demands.

3 Equilibrium analysis

For the case of an upstream monopoly, O'Brien and Shaffer (1992) show that secret contracting leads to *cost-based* tariffs, that is, marginal wholesale prices reflect marginal costs. We show below that this insight carries over to the case of upstream competition, as long as the retail behavior is smooth, in a sense made precise. We then turn to existence and to the impact of tariffs on the division of profits.

3.1 Cost-based tariffs

To introduce the notion of smooth retail behavior, fix a candidate bargaining equilibrium with tariffs $\mathbf{t^e}$ and retail prices $\mathbf{p^e}$, and consider a bilateral deviation adding a wholesale price w_{ij} to $t^e_{ij}(q_{ij})$, which thus becomes $\hat{t}_{ij}(q_{ij}; w_{ij}) \equiv t^e_{ij}(q_{ij}) + w_{ij}q_{ij}$. Let

$$\hat{\mathbf{p}}^i_j(w_{ij}) \equiv \mathbf{p}^R_j(\hat{t}_{ij}(\cdot; w_{ij}), \mathbf{t}^e_{-i,j}) \quad \text{and} \quad \hat{q}^j_{ik}(w_{ij}) \equiv D_{ik}(\hat{\mathbf{p}}^i_j(w_{ij}), \mathbf{p^e}_{-j})$$

denote R_j's price response and the resulting sales of brand i at store k, for $k \in \mathcal{J}$, and let $\boldsymbol{\delta}^i$ denote the $m \times m$ matrix of *diversion ratios* for brand i across stores:

$$\boldsymbol{\delta}^i_{j,k} \equiv -\frac{d\hat{q}^j_{ik}}{dw_{ij}}(0) / \frac{d\hat{q}^j_{ij}}{dw_{ij}}(0),$$

which measures the impact of a marginal reduction in M_i's sales by R_j on its sales by R_k. We say that the equilibrium retail behavior is *smooth* if the tariffs and price responses are differentiable, and the diversion ratio matrices are nonsingular:

Definition 2 (smooth retail behavior). *The equilibrium retail behavior is smooth if:*

(i) for every $j \in \mathcal{J}$, the tariffs $\mathbf{t}^{\mathbf{e}}_j$ and the price response $\hat{\mathbf{p}}^i_j(w_{ij})$ are differentiable;

(ii) for every $i \in \mathcal{I}$, the diversion ratio matrix $\boldsymbol{\delta}^i$ is nonsingular.

We have:

Proposition 1 (cost-based tariffs). *In any equilibrium in which the retail behavior is smooth, tariffs are cost-based: $(t^e_{ij})'(q^e_{ij}) = c_i$ for every $(i, j) \in \mathcal{I} \times \mathcal{J}$.*

Proof. See Appendix A. ■

When negotiating with one retailer, a manufacturer has an incentive to "undercut" the margins it charges to other retailers; thus, in equilibrium, its margins must all be zero. To see this, let consider a candidate equilibrium in which M_i charges (smooth) margins $u_{ik}^e \equiv (t_{ik}^e)'(q_{ik}^e) - c_i$ to each R_k. In its negotiation with R_j, M_i can add a wholesale price w_{ij} and adjust it so as to induce the quantity q_{ij} that maximizes their joint profit, taking into account that R_j will adjust its prices so as to maximize its own profit. It follows that M_i's upstream margin must neutralize the marginal impact of q_{ij} on its own profit. As increasing q_{ij} by one unit would alter M_i's sales through every other R_k by $\boldsymbol{\delta}_{j,k}^i$ unit, the negotiated margins must satisfy:

$$u_{ij}^e = \sum_{k \in \mathcal{J} \setminus \{j\}} \boldsymbol{\delta}_{j,k}^i u_{ik}^e.$$

In practice, we would expect an increase in M_i's sales by R_j to reduce its sales by the other retailers but expand its total sales, in which case the negotiated margin indeed undercuts those charged to R_j's rivals.[23] More generally, as long as the diversion ratio matrix $\boldsymbol{\delta}^i$ is nonsingular, M_i's equilibrium margins must all be zero.

Remark 1 (multi-product firms and/or selective distribution)**.** The proof provided in Appendix A covers a more general setting allowing for multi-brand manufacturers and multi-store (or multi-format) retailers, as well as for selective distribution networks.

Remark 2 (smooth retail behavior)**.** For the case of an upstream monopoly, O'Brien and Shaffer (1992) show that the equilibrium retail behavior is always smooth. Unfortunately, the reasoning does not carry over to the case of upstream competition, as R_j's response to M_i's tariff, say, now depends on M_i's rivals' tariffs; hence, it may not be smooth if these other tariffs are discontinuous. Yet, we suspect that, in equilibrium, tariffs and price responses are likely to be smooth.

3.2 Existence

To ensure existence, we assume that a Nash equilibrium would indeed exists in the *multi-brand oligopoly* where each retailer could produce all brands at cost:[24]

[23]That is, if $\boldsymbol{\delta}_{j,k}^i \geq 0$ for $k \neq j$ and $\sum_{k \neq j} \boldsymbol{\delta}_{j,k}^i < 1$, then u_{ij}^e is a contraction of $(u_{ik}^e)_{k \neq j}$.

[24]See Vives (1999) for a discussion of the appropriate underlying assumptions on demand.

Assumption A (multi-brand oligopoly). There exists a price vector $\mathbf{p}^*$ satisfying:

$$\forall j \in \mathcal{J}, \quad \mathbf{p}_j^* \in \arg\max_{\mathbf{p}_j} \pi_j(\mathbf{p}_j, \mathbf{p}_{-j}^*), \text{ where } \pi_j(\mathbf{p}) \equiv \sum_{i\in\mathcal{I}} (p_{ij} - c_i - \gamma_j) D_{ij}(\mathbf{p}).$$

Let $\Delta_j^i \equiv \pi_j(\mathbf{p}^*) - \max_{\mathbf{p}_{-i,j}} \pi_j((\infty, \mathbf{p}_{-i,j}), \mathbf{p}_{-j}^*) \in [0, \pi_j(\mathbf{p}^*)]$ denote the contribution of brand i to R_j's profit in this multi-brand oligopoly equilibrium.[25] The next Proposition shows that any multi-brand oligopoly outcome can be sustained by cost-based two-part tariffs, and characterizes the associated profits:

Proposition 2 (existence)**.** *There exists a bargaining equilibrium in which:*

(*i*) $\mathbf{p}^{\mathbf{e}} = \mathbf{p}^*$ *and, for every* $(i,j) \in \mathcal{I} \times \mathcal{J}$, $t_{ij}^e(q_{ij}) \equiv \alpha_{ij}\Delta_j^i + c_i q_{ij}$;

(*ii*) *for every* $(i,j) \in \mathcal{I} \times \mathcal{J}$, M_i*'s and* R_j*'s equilibrium profits are given by:*

$$\Pi_{M_i}^e \equiv \sum_{j\in\mathcal{J}} \alpha_{ij}\Delta_j^i \geq 0 \quad \textit{and} \quad \Pi_{R_j}^e \equiv \pi_j(\mathbf{p}^*) - \sum_{i\in\mathcal{I}} \alpha_{ij}\Delta_j^i \geq 0.$$

Proof. See Appendix B. ■

If the other channels adopt such tariffs, then the joint variable profit of M_i and R_j accounts for the full margins on R_j's sales of all brands, and only for those. To maximize this profit, it suffices to make R_j the residual claimant, which a cost-based two-part tariff precisely achieves. All retailers then behave as if supplied at cost.

Tariffs being cost-based, if R_j were to delist M_i, it would benefit from increased sales of rival brands, whereas M_i would not benefit from possibly increased sales of its brand by other retailers. Hence, M_i obtains a positive profit only if it contributes to the profit generated by R_j (i.e., $\Delta_j^i > 0$) *and* it has some bargaining power (i.e., $\alpha_{ij} > 0$). By contrast, as long as there is some brand substitution (i.e., $\Delta_j^i < \pi_j(\mathbf{p}^*)$), R_j obtains a positive profit, regardless of its bilateral bargaining power.

3.3 Division of profits

Proposition 1 shows that, with appropriate cost-based tariffs, any multi-brand oligopoly equilibrium $\mathbf{p}^*$ can be sustained as a bargaining equilibrium. It is straightforward to

[25] $\Delta_j^i \geq 0$ follows from the fact that R_j can choose not to sell brand i (e.g., by setting $p_{ij} = +\infty$); $\Delta_j^i \leq \pi_j(\mathbf{p}^*)$ follows from (imperfect) brand substitutability.

check that, conversely, the profits identified by Proposition 1 constitute the equilibrium profits whenever $\mathbf{p}^*$ is sustained with two-part tariffs. However, we show in Online appendix A that, under mild regularity assumptions, other tariffs can sustain different divisions of the industry profit. Consider for instance a convex tariff of the form $t_{ij}^{\sigma}(q_{ij}) = F_{ij} + c_i q_{ij} + \sigma[q_{ij} - D_{ij}(\mathbf{p}^*)]^2$, where $\sigma > 0$. Introducing the quadratic term does not affect the amount paid if R_j sticks to $\mathbf{p}_j^*$, but increases it if R_j were to modify its prices and/or stop carrying another brand, thereby weakening R_j's bargaining position in its negotiations with the other suppliers. Conversely, manufacturers obtain a smaller share of the profit when tariffs are concave (i.e., $\sigma < 0$).

4 Micro-foundation

We now show that a bargaining equilibrium is an equilibrium outcome of a non-cooperative game in which each side gets to make an offer with a probability reflecting its bargaining power.[26] As already mentioned, with secret contracting these games have many equilibria, which has led the literature to focus on specific beliefs. The above contract equilibrium approach is in line with passive beliefs: when negotiating its own contract, a channel assumes that the others stick to the equilibrium tariffs when negotiating its own contract.[27] Unfortunately, with downstream price competition, deviating on multiple channels may destroy any candidate equilibria with passive beliefs.[28] To avoid this, we assume that firms delegate the negotiations to partner-specific agents.[29] Specifically, each M_i has m agents, $M_i^1, ..., M_i^m$, each R_j has n agents, $R_j^1, ..., R_j^n$, and the negotiation between M_i and R_j is handled by M_i^j

[26]See Collard-Wexler *et al.* (2019) for a micro-foundation of the Nash-in-Nash approach when the gains from trade are determined by the network of active channels. In our setting, however, the tariffs affect these gains through their impact on downstream competition.

[27]See McAfee and Schwartz (1994); Hart and Tirole (1990) call this *market-by-market bargaining*.

[28]For the case of an upstream monopoly, single-channel deviations characterize a single candidate PBEPB (O'Brien and Shaffer, 1992) but, when downstream firms are insufficiently differentiated, the supplier can benefit from deviating simultaneously on multiple channels (Rey and Vergé, 2004).

For downstream Cournot competition, existence of a PBEPB has been established by Hart and Tirole (1990) for an upstream monopoly, and extended by Nocke and Rey (2018) for an upstream duopoly. McAfee and Schwartz (1995) note however that existence problems arise again when negotiated tariffs become publicly observable before downstream decisions are made.

[29]For the micro-foundation of their bargaining model with threat of replacement, Ho and Lee (2019) adopt a similar approach to avoid the simultaneous replacement of multiple partners.

and R_j^i, each agent seeking to maximize the profit of its firm. The firms and their agents play the following *delegated negotiations* game Γ:

Stage 1. Two-step bilateral negotiations; for each channel $M_i - R_j$:

First step. Nature randomly picks one side to make a take-it-or-leave-it offer: it selects M_i^j with probability α_{ij} and R_j^i with probability $1-\alpha_{ij}$; the selection is only observed by M_i^j and R_j^i, and selections are independent across channels.

Second step. The selected agent, M_i^j or R_j^i, offers a tariff $t_{ij}(q_{ij})$ to its counterpart, who accepts or rejects it; offers are simultaneous and secret, and acceptance decisions are also simultaneous and secret.

Stage 2. Each R_j observes the tariffs negotiated by its agents, or the lack thereof; retailers then simultaneously set retail prices for the brand(s) that they carry.

We look for the sequential equilibria of this game Γ, which requires beliefs to be *consistent*.[30] This implies that a deviation by one player conveys no information on other players' simultaneous moves.[31] Hence, in stage 1, the receiver of a deviant offer does not revise its beliefs about the tariffs negotiated by the other agents; and in stage 2, a retailer that faces a deviant contract believes that the other retailers still face the equilibrium tariffs and therefore stick to their equilibrium prices.

Let $\theta_{ij} \in \Theta_{ij} \equiv \{M_i^j, R_j^i\}$ denote the selected proposer in the negotiation between M_i and R_j, $\boldsymbol{\theta}_j \equiv (\theta_{ij})_{i\in\mathcal{I}} \in \Theta_j \equiv \Pi_{i\in\mathcal{I}}\Theta_{ij}$, and $\boldsymbol{\theta} \equiv (\boldsymbol{\theta}_j)_{j\in\mathcal{J}} \in \Theta \equiv \Pi_{j\in\mathcal{J}}\Theta_j$. A sequential equilibrium of game Γ consists of price responses, $(\hat{\mathbf{p}}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}$, together with equilibrium tariffs, $(\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}$ (where $\hat{\mathbf{t}}^{\boldsymbol{\theta}} = (\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j})_{j\in\mathcal{J}}$), a equilibrium prices, $(\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}$ (where $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = (\hat{\mathbf{p}}_j^{\boldsymbol{\theta}_j})_{j\in\mathcal{J}}$), and beliefs $\mathbf{b} \equiv \{(b_{M_i^j}, b_{R_j^i})_{(i,j)\in\mathcal{I}\times\mathcal{J}}, (b_{R_j})_{j\in\mathcal{J}}\}$, such that:[32]

(i) In stage 2, for every $j \in \mathcal{J}$:

- for any tariffs $\mathbf{t}_j$, the price response $\hat{\mathbf{p}}_j^R(\mathbf{t}_j)$ maximizes R_j's expected profit, given the other retailers' equilibrium prices, $\hat{\mathbf{p}}_{-j} = (\hat{\mathbf{p}}_k^{\boldsymbol{\theta}_k})_{k\in\mathcal{J}\setminus\{j\},\theta\in\Theta_k}$;
- for every $\boldsymbol{\theta}_j \in \Theta_j$, $\hat{\mathbf{p}}_j^{\boldsymbol{\theta}_j} = \hat{\mathbf{p}}_j^R(\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j})$.

(ii) In stage 1, for every $(i,j) \in \mathcal{I}\times\mathcal{J}$ and every selected agent $\theta_{ij} \in \Theta_{ij}$:

[30] See Kreps and Wilson (1982).

[31] Fudenberg and Tirole (1991) refer to this principle as *no-signaling-what-you-don't-know*.

[32] Sequential equilibria have been defined for finite action spaces. We adapt here the definition by focusing on equilibrium tariffs and unilateral deviations from these tariffs.

- the proposer and the receiver (regardless of the offered tariff) believe that all other agents stick to their equilibrium behavior; they thus expect R_j to charge $\hat{\mathbf{p}}_j^R(t_{ij}, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}})$ and its rivals to charge $\hat{\mathbf{p}}_{-j}^{\boldsymbol{\theta}_{-j}} = (\hat{\mathbf{p}}_k^{\boldsymbol{\theta}_k})_{k \in \mathcal{J} \setminus \{j\}}$;
- the proposer offers a tariff that maximizes its firm's expected profit, among those that do not decrease the expected profit of the receiver's firm.[33]

As firms can share profit (e.g., through lump-sum transfers), their agents always seek to maximize their joint profit, and the proposing side appropriates the bilateral gains from trade. The tariff negotiated by M_i^j and R_j^i thus induces R_j to maximize M_i and R_j's joint profit, as in a bargaining equilibrium, and the probability α_{ij} plays the same role as M_i's bargaining power in its bilateral relationship with R_j. Building on this, we show below that any bargaining equilibrium can be replicated as an equilibrium of game Γ; the converse moreover holds for any equilibrium of game Γ with *regular* tariffs and price responses, defined as follows:

Definition 3 (regular price responses and tariffs)**.** *In game* Γ*:*

(i) the price responses $(\hat{\mathbf{p}}_j^R)_{j \in \mathcal{J}}$ *are said to be regular if they are invariant to lump-sum changes in tariffs: for any* $j \in \mathcal{J}$*, any tariffs* $\mathbf{t}_j$ *and any vector of fixed fees* $\mathbf{f} = (f_{ij})_{i \in \mathcal{I}} \in \mathbb{R}^n$*,* $\hat{\mathbf{p}}_j^R(\mathbf{t}_j + \mathbf{f}_j) = \hat{\mathbf{p}}_j^R(\mathbf{t}_j)$*; and*

(ii) the tariffs $\mathbf{t}^{\boldsymbol{\theta}}$ *are said to be regular if they depend on which side makes the offer only through a lump-sum transfer: for any* $(i, j) \in \mathcal{I} \times \mathcal{J}$*,* $t_{ij}^{M_i^j}(q_{ij}) - t_{ij}^{R_j^i}(q_{ij})$ *does not depend on* q_{ij}*.*

A retailer's price response is trivially regular unless the retailer is indifferent between several optimal prices; regularity then requires the price response to be independent of lump-sum changes in the retailer's expected profit function. Tariffs are regular when which side makes the offer does not affect firms' bargaining positions with other partners.[34] Together, these two requirements imply that bilateral bargaining power has no impact either on retail prices: $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}}$ for any $\boldsymbol{\theta} \in \Theta$.

The next Proposition establishes an equivalence between the bargaining equilibria and "regular" equilibria of game Γ:

[33] Without loss of generality, attention can be restricted to acceptable tariffs, as the "null" tariff $t_{\emptyset}$, equal to 0 for $q_{ij} = 0$ and to $+\infty$ for $q_{ij} > 0$, is acceptable and mimics rejection.

[34] As discussed in Section 3.3, the shape of the tariff affects the outcome of the other negotiations.

Proposition 3 (micro-foundation)**.**

(i) *For any bargaining equilibrium $\mathcal{B}$, there exists a sequential equilibrium of game Γ, with regular price responses and tariffs, that yields the same retail outcome and gives all firms the same expected profits as $\mathcal{B}$.*

(ii) *Conversely, for any sequential equilibrium $\mathcal{E}$ of game Γ with regular price responses and tariffs, there exists a bargaining equilibrium yielding the same retail outcome and giving all firms the same expected profits as $\mathcal{E}$.*

Proof. See Appendix C. ■

Any bargaining equilibrium $\{(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}, \mathbf{t}^{\mathbf{e}}, \mathbf{p}^{\mathbf{e}}\}$ can thus be replicated as a sequential equilibrium of game Γ with regular price responses and tariffs. The proof relies on contingent tariffs, $\hat{t}_{ij}^{M_i^j} = t_{ij}^e + F_{ij}^{R_i}$ and $\hat{t}_{ij}^{R_j^i} = t_{ij}^e - F_{ij}^{M_i}$, where the fees $F_{ij}^{M_i}$ and $F_{ij}^{R_j}$ leave the receiving agent indifferent between accepting or rejecting the offer. The construction also relies on price responses that coincide with $(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}$ when profits are single-peaked, and may otherwise slightly differ to ensure their regularity.

Conversely, any sequential equilibrium of game Γ with regular price responses $(\hat{\mathbf{p}}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}$ and regular tariffs $(\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}$ (implying $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}}$ for any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$) can be replicated as a bargaining equilibrium. There, the proof relies on the same price responses and on the expected tariffs $t_{ij}^e = E_{\theta_{ij}}[\hat{t}_{ij}^{\boldsymbol{\theta}_{ij}}]$.

Remark 3 (on the role of delegated negotiations)**.** To assess the role of delegation, consider the *direct negotiations* game Γ^{Direct}, similar to Γ except that each firm assigns the same agent to negotiate with all of its partners. The receiver of an unexpected offer may then wonder about the offers made to its rivals – the consistency requirement imposed on sequential equilibria has little bite in game Γ^{Direct}. As already noted, the literature often focuses on passive beliefs, which is in line with the bargaining equilibrium approach and the spirit of delegated negotiations. And indeed, any Perfect Bayesian Equilibrium with passive beliefs (PBEPB hereafter) of game Γ^{Direct} is a sequential equilibrium of game Γ. Unfortunately, the converse does not hold: the sequential equilibria of game Γ constitute the only candidate PBEPBs of game Γ^{Direct}, as characterized by single-channel deviations, but they may not survive a firm deviating on its offers to multiple partners (see footnote 28). In other words, delegating negotiations to distinct agents does not affect the set of candidate PBEPB outcomes but ensures existence, by preventing multi-channel deviations.

Remark 4 (deterministic outcomes)**.** In game Γ, the retail outcome is deterministic but the tariffs depend on which side makes the offer. To ensure that the equilibrium tariffs, too, are deterministic, it suffices to add the following preliminary stage:

Stage 0**.** For each $M_i - R_j$ pair, M_i^j offers a tariff $t_{ij}(q_{ij})$ to R_j^i, who then accepts or rejects it;[35] all offers are simultaneous and secret, and all acceptance decisions are also simultaneous and secret. If the offer is accepted, the game directly proceeds to stage 2, otherwise it proceeds to stage 1.

Let $\hat{\Gamma}$ denote this extended game, and consider a bargaining equilibrium $\mathcal{B} = \{(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}, \mathbf{t}^\mathbf{e}, \mathbf{p}^\mathbf{e}\}$ together with the associated equilibrium of game Γ identified by Proposition 3, $\mathcal{E} = \{(\hat{\mathbf{p}}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}}, (\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\theta\in\Theta}, \mathbf{b}\}$; by construction, they satisfy $t_{ij}^e = E_{\theta_{ij}}[\hat{t}_{ij}^{\theta_{ij}}]$ (for any $(i,j) \in \mathcal{I} \times \mathcal{J}$) and yield the same retail prices ($\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \mathbf{p}^\mathbf{e}$ for any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$) and the same expected profits, $\{\Pi_{M_i}^e\}_{i\in\mathcal{I}}$ and $\{\Pi_{R_j}^e\}_{j\in\mathcal{J}}$. Suppose now that, in the modified game $\hat{\Gamma}$, all players adopt the same strategies and beliefs as in $\mathcal{E}$ for stages 1 and 2, and consider the negotiation for channel $M_i - R_j$ in stage 0. As subsequent negotiations (in case of rejection at stage 0) are bilaterally efficient, and R_j^i can secure $\Pi_{R_j}^e$ by proceeding to stage 1, it is optimal for M_i^j to offer the expected tariff t_{ij}^e and for R_j^i to accept it. It follows that there is an equivalence between the bargaining equilibria and the deterministic equilibria of the extended game $\hat{\Gamma}$.

Remark 5 (linear tariffs)**.** When tariffs are restricted to be linear, bilateral negotiations are no longer efficient and Nash bargaining amounts to maximizing $\left(\Delta_{M_i}^{ij}\right)^{\alpha_{ij}} (1-\Delta_{R_j}^{ij})^{1-\alpha_{ij}}$, where $\Delta_{M_i}^{ij}$ and $\Delta_{R_j}^{ij}$ denotes the gains from trade for the two firms. We show in Online appendix B that a bargaining equilibrium can still be replicated as an equilibrium of game $\hat{\Gamma}$ for appropriate probabilities $\boldsymbol{\beta} = (\beta_{ij})_{(i,j)\in\mathcal{I}\times\mathcal{J}}$; the probabilities $\boldsymbol{\beta}$ however depend on more variables than the weights $\boldsymbol{\alpha}$.

5 Vertical restraints and agency model

To illustrate the flexibility of our approach, we first consider the impact of vertical restraints, namely resale price maintenance (RPM) and price parity agreements (PPAs)

[35] The analysis would be unaffected if R_j^i was selected to make the offer in this stage.

– provisions that have triggered heated policy debates, particularly with the development of online platforms. We then discuss the impact of a switch to the agency business model often adopted by these online retail platforms.

5.1 Resale price maintenance

We suppose here that each $M_i - R_j$ pair can contract not only on a (non-linear) tariff $t_{ij}(q_{ij})$, but also on the retail price p_{ij}. The timing remains unchanged, with the caveat that in case of RPM, R_j sets the price p_{ij} that has been agreed upon.

Allowing for RPM does not destabilize the above cost-based tariff equilibria. Indeed, if the other channels sign cost-based tariffs, then a cost-based tariff t_{ij} induces R_j to maximize its joint profit with M_i, and there is no need for contracting on p_{ij}.[36] However, RPM can sustain many other outcomes, even with simple two-part tariffs.

In addition, either minimum RPM (i.e., price floors) or maximum RPM (i.e., price caps) can achieve that result, depending on whether manufacturers or retailers are closer substitutes. To illustrate this, we consider the case of symmetric firms,[37] assume that $\mathbf{p}^* = (p^*, ..., p^*)$, and denote the demand for symmetric prices by $D(p) \equiv D_{ij}(p, ..., p)$, and the interbrand and intrabrand demand price sensitivities by:

$$\lambda_M(p) \equiv \sum_{h \in \mathcal{I} \setminus \{i\}} \frac{\partial D_{hj}}{\partial p_{ij}}(p, \ldots, p) \text{ and } \lambda_R(p) \equiv \sum_{k \in \mathcal{J} \setminus \{j\}} \frac{\partial D_{ik}}{\partial p_{ij}}(p, \ldots, p).$$

We have:

Proposition 4 (RPM). *Under mild regularity conditions:*[38]

(*i*) *any price vector* $\mathbf{p}$ *can generically be sustained with RPM; and*

(*ii*) *when firms are symmetric, any price* $p > p^*$ *for which* $\lambda_M(p) > \lambda_R(p)$ *(resp.,* $\lambda_M(p) < \lambda_R(p)$*) can be sustained with minimum RPM (resp., maximum RPM).*

[36] Using RPM however reduces R_j's profit, as R_j can no longer adjust p_{ij} if another negotiation breaks down: this reduces R_j's disagreement payoff and, therefore, its equilibrium payoff.

[37] Symmetry among manufacturers means that $c_i = c$ and $D_{ij}(\mathbf{p}) = D_{hj}(\sigma_{ih}^M(\mathbf{p}))$ for any $i \neq h \in I$ and any $j \in J$, where $\sigma_{ih}^M(\mathbf{p})$ swaps the prices of brands i and h in each retailer's stores; symmetry among retailers means $\gamma_j = \gamma$ and $D_{ij}(\mathbf{p}) = D_{ik}(\sigma_{jk}^R(\mathbf{p}))$ for any $j \neq k \in J$ and any $i \in I$, where $\sigma_{jk}^R(\mathbf{p})$ swaps R_j's and R_k's prices for each brand.

[38] Mostly quasi-concavity conditions on firms' profits; see Online appendix C for details.

Proof. See Online Appendix C. ■

Under RPM, the joint profit of M_i and R_j no longer depends on their own tariff, as R_j's prices are directly negotiated. They can thus agree on any wholesale price w_{ij} (using a fixed fee to share their profit as desired), which however affects their other negotiations over $(p_{ik})_{k \neq j}$ and $(p_{hj})_{h \neq i}$. As there are as many wholesale prices as retail price targets, it follows that, generically, any retail prices can be sustained.

By construction, the equilibrium upstream margins induce M_i and R_j to stick to $p > p^*$ when they maximize their joint profit. Replacing the margins on M_i's sales to rival retailers with those on R_j's sales on rival brands would transform that joint profit into the one that R_j would face if tariffs were cost-based tariffs, a situation in which R_j would undercut p. It follows that, when there is more substitution upstream, the equilibrium margins must be negative; price floors are then needed to counter retailers' excessive incentives to lower prices. When instead there is more substitution downstream, positive upstream margins are required, and price caps are then needed to counter retailers' excessive incentives to raise prices.[39]

5.2 Price parity agreements

We now turn to PPAs, which require the retailer to price the manufacturer's brand at the same level as (or no less/more than) competing brands. These provisions have triggered debates about their potential anti-competitive effects, some agencies considering that they have the same adverse effects as RPM.[40]

To shed some light on this debate, we now consider a variant of our setting in which, in the second stage, retailers must charge the same price on all brands. We find that PPAs have little impact on the equilibrium outcome:[41]

Proposition 5 (price parity agreements)**.** *Under PPAs, in the class of bargaining equilibria based on differentiable tariffs and positive quantities:*

[39]Price floors thus have no effect in this case; by contrast, O'Brien and Shaffer (1992) and Allain and Chambolle (2011) show that industry-wide price floors are always anticompetitive.

[40]See, for instance, the UK Office of Fair Trading 2010 decision in the tobacco case (Decision CA98/01/2010, Case CE/2596-03).

[41]The proof of Proposition 5 does not raise any particular difficulty and is thus omitted. We refer the interested reader to Rey and Vergé (2019).

(i) equilibrium tariffs are all cost-based; and,

(ii) if $\mathbf{p}^*$ *is symmetric across brands, then it remains an equilibrium outcome.*

The insight of Proposition 1 thus carries over when retailers must set uniform prices across brands. PPAs thus have no impact on equilibrium tariffs, which remain cost-based. If in addition the equilibrium prices are already symmetric absent PPAs, then PPAs have no impact on retail prices either.

5.3 Agency model

We have focused so far on the *resale* business model usually adopted by brick-and-mortar retailers: distributors buy goods from suppliers and resell them to consumers. Online platforms often adopt instead an *agency* business model: suppliers sell directly to consumers, and platforms obtain commissions based on sales.

Modelling the agency model requires turning the framework *upside-down*: manufacturers are downstream and control retail prices; retailers/platforms are upstream and charge commissions to their partners. The timing thus becomes:

Stage 1: Each pair negotiates a (possibly non-linear) commission schedule based on the volume of sales achieved by the manufacturer on the retailer's platform.

Stage 2: Manufacturers simultaneously set the retail prices for their products, for each platform that carries them.

It follows that, as long as the manufacturers' behavior is smooth, marginal commissions must reflect marginal distribution costs; the equilibrium outcome is therefore that of competition between *multi-store* firms. Whether this is more competitive than the previous multi-brand oligopoly depends on whether manufacturers or retailers are closer substitutes. In particular, for symmetric firms, we have:

Proposition 6 (agency versus resale)**.** *If* $\lambda_R(\cdot) > \lambda_M(\cdot)$ *(resp.,* $\lambda_M(\cdot) > \lambda_R(\cdot)$*), then switching from the resale to the agency model increases (resp., decreases) the equilibrium retail prices.*

Proof. See Online appendix D. ■

Price parity agreements (now requiring manufacturers to set the same prices on all platforms) have again no impact on the equilibrium outcome beyond imposing symmetry. That is, equilibrium tariffs remain cost-based and, when firms are symmetric at both stages of the vertical chains (and the equilibrium prices are symmetric in the absence of PPAs), price parity agreements do not affect the equilibrium retail prices either. These insights are in sharp contrast with the recent literature on price parity agreements. However, so far this literature has focused on either linear commissions[42] or constant revenue-sharing rules,[43] which generate contractual inefficiencies; instead, we allow here for general non-linear commissions and thus for efficient bilateral contracting.[44]

6 Endogenous network

Tariffs being cost-based, intrabrand competition dissipates profits when retailers are close substitutes; firms would then benefit from limiting the number of distribution channels. Yet, the above Nash-in-Nash approach predicts that all channels are always active: as long as firms are differentiated, every bilateral negotiation generates positive gains from trade; hence, every vertical pair wants to activate its channel.

To endogenize the channel network, the framework must therefore allow manufacturers and/or retailers to select their negotiating partners.[45] Prompted by the observation that many insurers limit the set of hospitals to which they offer access, several papers have introduced a preliminary stage along these lines:[46] Ho and Lee (2019) let the insurers unilaterally select their hospital networks, whereas Liebman

[42]See Boik and Corts (2016) and Johansen and Vergé (2017).

[43]See Johnson (2017) and Foros *et al.* (2017).

[44]Allowing for direct sales by suppliers would amount to adding a platform (the direct sales channel) offering intermediation services at cost, and would not affect the above insights.

[45]For an earlier analysis of buyer-seller network formation without downstream competition, see, e.g., Kranton and Minehart (2001).

[46]To capture insurers' incentive to limit their hospital networks, these papers propose variants of the Nash-in-Nash approach that allow insurers, in case of disagreement with a selected hospital, to replace it with another one from outside the network. Opting for selective networks then induces hospitals to compete for inclusion and enables insurers to obtain more favorable terms. In our setting, intrabrand competition already provides an incentive to limit the size of distribution networks.

(2018) lets them choose the size of these networks; Ghili (2020) relies instead on the notion of pairwise stability proposed by Jackson and Wolinsky (1996).[47] We explore here an alternative approach, in which the distribution network is determined through a simultaneous *veto-game*. This approach turns out to remain reasonably tractable and predicts the emergence of selective distribution networks when retailers are close substitutes, as intuition suggests.

Formally we assume that manufacturers and retailers first choose which channels to activate, each firm having veto power. That is, each firm announces which partner(s) it wishes to deal with (if any); these announcements are simultaneous and publicly observable, and a channel becomes active if and only if both partners wish so. This preliminary stage determines the channel network, which then gives rise a bargaining equilibrium defined along the same lines as before. It is well-known that veto games are subject to coordination problems – in particular, there always exists a trivial equilibrium in which no channel becomes active. To avoid this, we focus on Coalition-Proof Nash Equilibria (CPNE hereafter) – see Bernheim *et al.* (1987).

As the number of potential networks grows geometrically with the number of firms, in this section we focus on the simplest relevant case with two symmetric manufacturers, M_A and M_B, two symmetric retailers, R_1 and R_2, and symmetric bargaining sharing rules, i.e., $\alpha_{ij} = \alpha$ for $i \in \{A, B\}$ and $j \in \{1, 2\}$. To ensure that continuation payoffs are properly defined, throughout this section, we focus on bargaining equilibria based on two-part tariffs,[48] and assume that the retail price response is smooth and unique; Proposition 1bis then ensures that tariffs are cost-based.

6.1 Bargaining equilibria

We provide in Online appendix E.1 a complete characterization of the bargaining equilibria for each distribution network, and summarize here their main features.

• **Bilateral monopoly:** a single channel is active, say $i - j$. M_i and R_j obtain $\Pi_M^m \equiv \alpha\pi^m$ and $\Pi_R^m \equiv (1 - \alpha)\,\pi^m$, respectively, where π^m denotes the monopoly

[47] Lee and Fong (2013) adopt instead an infinite horizon framework in which, at the beginning of every period, firms can, at some cost, add new links or cut existing ones.

[48] The analysis is thus valid when only two-part tariffs are feasible, or when firms favor two-part tariffs when they are indifferent between those and other non-linear tariffs.

profit obtained generated by the channel.

• **Exclusive dealing**: two unconnected channels are active, say $i-j$ and $h-k$. Manufacturers' and retailers' profits are $\Pi_M^{ED} \equiv \alpha\pi^{ED}$ and $\Pi_R^{ED} \equiv (1-\alpha)\,\pi^{ED}$, where π^{ED} denotes the per-channel profit in a duopoly where the two products are differentiated both upstream and downstream.

• **Upstream foreclosure:** a single manufacturer deals with both retailers. Manufacturer's and retailers' profits are respectively $\Pi_M^{UF} \equiv 2\alpha\pi^{UF}$ and $\Pi_R^{UF} \equiv (1-\alpha)\,\pi^{UF}$, where π^{UF} denotes the per-channel profit in a duopoly where the two products are differentiated only downstream.

• **Downstream foreclosure:** a single retailer deals with both manufacturers. The manufacturers' and the retailer's profits are $\Pi_M^{DF} \equiv \alpha\left(2\pi^{DF}-\pi^m\right)$ and $\Pi_R^{DF} \equiv 2\,(1-\alpha)\,\pi^{DF} + 2\alpha\left(\pi^m - \pi^{DF}\right)$, where π^{DF} denotes the per-channel profit when a downstream monopolist sells both brands.

• **Single exclusion:** a single channel, say $h-k$, is excluded. M_i and R_j thus have two partners, whereas M_h and R_k have one partner.

• **Interlocking relationships:** all channels are active; firms' profits are then:

$$\Pi_M^{IR} = 2\alpha(2\pi^* - \hat{\pi}^*) \text{ and } \Pi_R^{IR} = 2(1-\alpha)\pi^* + 2\alpha(\hat{\pi}^* - \pi^*),$$

where π^* denotes the equilibrium per-channel profit, whereas $\hat{\pi}^*$ denotes the profit that a retailer could achieve by dropping one brand.[49]

Two observations readily follow from manufacturers being imperfect substitutes:

• *Bilateral monopoly versus downstream foreclosure:* in both networks there is a single retailer, carrying only one brand in the first case, and both brands in the second case; brand differentiation yields: $2\pi^{DF} > \pi^m > \pi^{DF} > 0$.

• *Upstream foreclosure versus exclusive dealing:* in both networks there are two mono-brand retailers, carrying the same brand in the first case, and different brands in the second case; brand differentiation yields: $\pi^{ED} > \pi^{UF} > 0$.

[49]Using symmetry, π^* and $\hat{\pi}^*$ correspond to the profits π_j^* and π_j^{ij} defined in Proposition 2.

6.2 Equilibrium network

We now study the CPNE of the network formation game. For expositional purposes, we restrict attention to cases where all firms have bargaining power (i.e., $\alpha \in (0,1)$).[50]

We first note that at least two channels are active – otherwise, any excluded vertical pair would profitably activate its channel. Furthermore, upstream foreclosure cannot arise: the excluded supplier (say, M_h) and either retailer (say, R_j) would gain from activating their channel (possibly in addition to the channel $i-j$): M_h would benefit from avoiding exclusion and R_j would benefit from dealing with a different supplier than R_k (as $\Pi_R^{ED} > \Pi_R^{UF}$).

Intuitively, dual distribution is profitable only if retailers are sufficiently differentiated; otherwise, intrabrand competition dissipates profits without adding much demand. The following Proposition confirms this intuition by considering the polar cases where retailers are either perfect substitutes or local monopolies:[51]

Proposition 7 (endogenous network - polar cases)**.**

(*i*) *When retailers are local monopolies, the unique CPNE yields interlocking relationships.*

(*ii*) *When instead retailers are perfect substitutes:*

- *if* $\pi^{ED} + \pi^m > 2\pi^{DF}$, *the unique CPNE yields exclusive dealing;*
- *if* $\pi^{ED} + \pi^m < 2\pi^{DF}$, *the unique CPNE yields downstream foreclosure.*

Proof. See Online appendix E.2. ■

Interestingly, firms' relative bargaining power has no impact on the equilibrium network. When retailers are local monopolies, opening an additional channel always benefits both partners. When instead retailers are perfect substitutes, the network choice is driven by manufacturers, who want to deal with a single retailer; the relevant comparison is therefore between downstream foreclosure and exclusive dealing. As

[50] When $\alpha = 0$, coalition-proofness has little bite, as manufacturers obtain no profit anyway; when $\alpha = 1$, retailers obtain no profit unless they carry both brands, which limits the scope for profitable deviations. However, as α tends to 0 or 1, there is a (generically) unique equilibrium.

[51] While we have so far ruled out these extreme cases for expositional purposes, it is straightforward to extend the previous analysis, as long as manufacturers remain imperfect substitutes.

manufacturers obtain a share α of their contributions to their retailer's profit, the outcome follows from a comparison between these contributions – i.e., the channel profit π^{ED} under exclusive dealing, and the additional profit from expanding the brand portfolio, $2\pi^{DF} - \pi^m$, under downstream foreclosure.

To provide further insights, we study below the following linear demand specification, in which costs are normalized to zero and, for $i \neq h \in \{A, B\}$ and $j \neq k \in \{1, 2\}$, the (inverse) demand for brand i at store j is given by, for some $\mu, \rho \in (0, 1)$:

$$P(q_{ij}, q_{hj}, q_{ik}, q_{hk}) = 1 - q_{ij} - \mu q_{hj} - \rho q_{ik} - \mu\rho q_{hk}.$$

The parameters μ and ρ reflect the degree of substitution between manufacturers and between retailers.[52] The next proposition confirms the previous insights:

Proposition 8 (endogenous network - linear demand)**.** *For the above linear demand, there exists $\rho^*(\mu) \in (0, 1)$, which is a decreasing function of μ, such that:*

- *if $\rho < \rho^*(\mu)$, then the unique CPNE yields interlocking relationships;*
- *if instead $\rho \geq \rho^*(\mu)$, then the unique CPNE yields exclusive dealing.*

Proof. See Online appendix E.3. ■

There is again a unique CPNE, which does not depend on firms' relative bargaining powers (α): interlocking relationships arise when retailers are sufficiently differentiated, otherwise firms prefer avoiding intrabrand competition. Furthermore, exclusive always arises in the latter case, as $\pi^{ED} + \pi^m > 2\pi^{DF}$ for the linear demand.[53]

[52]To limit the number of parameters, the price sensitivity across both manufacturers and retailers is supposed to be the product of those across manufacturers (μ) and across retailers (ρ). Similar insights obtain when making this assumption for the demand D rather than the inverse demand P, or when normalizing demand so as to ensure that $P(q, q, q, q)$ remains constant as μ and ρ evolve.

[53]This analysis provides a micro-foundation for networks of exclusive relations, which have been the focus of many studies – see, e.g., Bonanno and Vickers (1988), Horn and Wolinsky (1988), and Milliou and Petrakis (2007).

7 Mergers

We now consider the effect of horizontal and vertical mergers. Whereas the literature on mergers often focuses on price effects,[54] our approach provides a natural framework for studying the impact on distribution networks as well. As we will see, taking this dimension into consideration can yield very different conclusions.

For the sake of exposition, we stick to the above successive duopoly setting and maintain the focus on equilibria based on two-part tariffs.

7.1 Downstream merger

A merger between R_1 and R_2 creates a multi-location retail monopolist, R. Equilibrium tariffs remain cost-based[55] but eliminating downstream competition raises prices to the monopoly level.

Beyond this classic horizontal effect, a downstream merger may also affect the distribution network: pre-merger, exclusivity can arise to avoid downstream competition; by creating a retail monopoly, the merger eliminates this motivation and makes interlocking relationships more likely. Indeed, for the linear demand specification, the unique CPNE always involves interlocking relationships. The merger may therefore benefit consumers by expanding product variety. This is for instance the case when retailers are good enough substitutes, so that exclusive dealing arises pre-merger, and brand differentiation is so large that prices are then close to the monopoly level. The following proposition confirms this intuition for the linear demand specification:

Proposition 9 (downstream merger)**.**

(i) *A downstream merger yields monopolistic retail prices for any given distribution network but makes interlocking relationships more likely. Hence, it reduces*

[54] Regarding horizontal mergers in vertically related markets see, e.g., von Ungern-Stenberg (1996) and Dobson and Waterson (1997) for downstream mergers and Horn and Wolinsky (1988) and Ziss (1995) for upstream mergers. More recently, Milliou and Sandonis (2018) consider the impact on product portfolio.

There is also a substantial literature on vertical integration and foreclosure; see, e.g., Salinger (1988), Ordover, Saloner and Salop (1990), Hart and Tirole (1990) and, more recently, Nocke and Rey (2018). We extend the insights of the last two papers to multiple upstream firms and price competition downstream.

[55] See Remark 1. This is also in line with Bernheim and Whinston (1985, 1986, 1998).

consumer surplus and total welfare when interlocking relationships already arise pre-merger, but otherwise expands the distribution network and can then increase consumer surplus and total welfare – all the more so if, pre-merger, the two channels are substantially differentiated.

(*ii*) *For the linear demand specification considered above, the merger reduces consumer surplus and total welfare whenever $\rho < \rho^* (\mu)$; when instead $\rho \geq \rho^* (\mu)$, there exist $\hat{\mu}_S (\rho)$ and $\hat{\mu}_W (\rho)$, which are decreasing in ρ and satisfy $\hat{\mu}_S(1) = \hat{\mu}_W(1) = 0$, such that the merger increases consumer surplus (resp., total welfare) whenever $\mu < \hat{\mu}_S(\rho)$ (resp. $\mu < \hat{\mu}_W(\rho)$).*

Proof. See Online appendix F.1. ■

Pre-merger, the condition $\rho \geq \rho^* (\mu)$ ensures that exclusive dealing arises and the conditions $\mu < \hat{\mu}_s (\rho)$ (for $s \in \{S, W\}$) ensure that brand differentiation induces high prices; as a result, the network-expansion effect of the merger more than compensates the price increase to the monopoly level; taking into consideration this network effect thus reverses the standard conclusion based on prices.

7.2 Upstream merger

A merger between M_A and M_B creates a multi-brand upstream monopolist, M. Equilibrium tariffs remain cost-based;[56] hence, for any given distribution network, the merger affects neither wholesale nor retail prices, but only the division of profit.

The merger may however affect consumers by altering the equilibrium network. For example, when retailers are close substitutes, M may decide to sell both brands through a unique retailer, so as to avoid downstream competition; competing manufacturers may instead distribute their products through different retailers, so as to improve their bargaining position. Likewise, where competing manufacturers would opt for interlocking relationships, M may instead limit the distribution of one brand to improve the profitability of its other brand. We have:

Proposition 10 (upstream merger)**.**

[56]See Remark 1. This is also in line with O'Brien and Shaffer (1992).

(*i*) *An upstream merger does not affect retail prices for in any given distribution network but may generate (complete or partial) vertical foreclosure, in which case it reduces consumer surplus and total welfare.*

(*ii*) *For the linear demand specification, there exist* $\tilde{\rho}(\mu)$ *and* $\overline{\rho}(\mu)$*, which are decreasing in* μ *and satisfy* $0 < \tilde{\rho}(\mu) < \rho^*(\mu) < \overline{\rho}(\mu) < 1$ *for* $\mu > 0$*, such that:*

- *if* $\rho \geq \overline{\rho}(\mu)$*, then the merger alters the network from exclusive dealing to downstream foreclosure;*
- *if instead* $\tilde{\rho}(\mu) < \rho < \rho^*(\mu)$*, then the merger alters the network from interlocking relationships to exclusive dealing;*
- *otherwise, the merger has no impact on the network.*

Proof. See Online appendix F.2. ■

Hence, despite the absence of direct price effects, taking into consideration the effect of an upstream merger on the distribution network can give rise to competition concerns: an *horizontal merger* between suppliers may trigger *vertical foreclosure*, as the merged entity may stop supplying one of the retailers.

7.3 Vertical merger

We conclude this section with an analysis of vertical integration, which, as is well-known, gives rise to (partial) foreclosure.[57] We report here the main findings and refer the interested reader to Rey and Vergé (2019) for a detailed analysis.

A merger between M_i and R_j creates a vertically integrated firm, I, that interacts with the independent M_h and R_k. M_h's tariffs remain cost-based but, as R_k now competes with I's downstream subsidiary, either I stop supplying R_k, or it increases its wholesale price ($w_{ik} > c$). In addition, in the latter case, I takes into account its the upstream margin earned on R_k's sales and thus competes less aggressively on the downstream market. As a result, whenever R_k initially carries M_i's brand, the merger raises retail prices, which reduces consumer surplus and total welfare.

[57] See the literature on vertical integration and foreclosure mentioned in footnote 54.

The merger may also alter the distribution network. Where an independent M_i can limit intrabrand competition only through exclusivity, I can now achieve this by raising w_{ik}; hence, the merger may induce R_k to carry both brands rather than M_h's brand only. However, I also internalizes the impact of carrying M_h's brand on the profitability of its own brand, which makes interlocking relationships less likely. Indeed, we have:

Proposition 11 (vertical merger)**.** *For the above linear demand specification:*

(i) *When $\rho < \rho^* (\mu)$ (interlocking relationships pre-merger), a vertical merger raises the wholesale price charged to the independent retailer, and either does not affect the network or induces the integrated firm to drop the rival brand; it thus increases retail prices and reduces both consumer surplus and total welfare.*

(ii) *When instead $\rho \geq \rho^* (\mu)$ (exclusive dealing pre-merger), there exists $\rho^{IR} (\mu, \alpha)$ and $\rho^{ED} (\mu, \alpha) > \max \left\{\rho^* (\mu), \rho^{IR} (\mu, \alpha)\right\}$ such that:*

- *If $\rho > \rho^{ED} (\mu, \alpha)$, then the merger has no network or price effect; it thus has no impact on consumer surplus and total welfare.*
- *If instead $\rho \leq \rho^{IR} (\mu, \alpha)$, then the merger fully expands the distribution network, which increases consumer surplus and total welfare.*
- *Otherwise, the merged firm supplies the rival retailer, but charges a positive margin; as a result, the merger reduces consumer surplus (and also reduces total welfare if retailers are close enough substitutes).*

The analysis of the case of a linear demand shows that the parameter regions in which a vertical merger is either neutral or pro-competitive are rather small, suggesting that the upward price pressure that it creates is likely to dominate any network expansion benefit.

8 Public contracting

While our assumption of secret contracting is natural for many industries, it is worth noting that the same framework, as well as its micro-foundation, can also be used when wholesale tariffs are publicly observed by all firms before retail prices are set. To see this, we modify the retail pricing stage presented in Section 2 as follows:[58]

Stage 2 (public contracting): Retailers, having observed all wholesale tariffs, simultaneously set retail prices for the brand(s) that they carry.

A bargaining equilibrium in this modified setting is defined as follows. In stage 2, retail prices constitute a Nash equilibrium given the negotiated tariffs. In stage 1, each $M_i - R_j$ pair negotiates a tariff $t_{ij}(q_{ij})$ that: (i) maximizes the joint profit of M_i and R_j, given the other equilibrium contracts and the resulting retail price equilibrium; and (ii) gives a share α_{ij} of the bilateral gains from trade to M_i. Formally:

Definition 4 (public contracting)**.** *A bargaining equilibrium with public contracting is a retail price response $\mathbf{p}^R(\mathbf{t}) = (\mathbf{p}_j^R(\mathbf{t}))_{j\in\mathcal{J}}$, together with a vector of equilibrium tariffs $\mathbf{t}^\mathbf{e} = (\mathbf{t}_j^\mathbf{e})_{j\in\mathcal{J}}$ and a vector of equilibrium prices $\mathbf{p}^\mathbf{e} = (\mathbf{p}_j^\mathbf{e})_{j\in\mathcal{J}}$, such that:*

- *In stage 2, the retail price response $\mathbf{p}^R(\cdot)$ satisfies the following conditions:*
  - *for any $\mathbf{t}$ negotiated in stage 1, $\mathbf{p}^R(\mathbf{t})$ constitutes a Nash equilibrium:*
  $$\forall j \in \mathcal{J}, \mathbf{p}_j^R(\mathbf{t}) \in \arg\max_{\mathbf{p}_j} \sum_{i\in\mathcal{I}} \left[\begin{array}{c} (p_{ij} - \gamma_j) D_{ij}(\mathbf{p}_j, \mathbf{p}_{-j}^R(\mathbf{t})) \\ -t_{ij}(D_{ij}(\mathbf{p}_j, \mathbf{p}_{-j}^R(\mathbf{t}))) \end{array} \right];$$
  - $\mathbf{p}^R(\mathbf{t}^\mathbf{e}) = \mathbf{p}^\mathbf{e}$.
- *In stage 1, for every $(i,j) \in \mathcal{I} \times \mathcal{J}$, the equilibrium tariff t_{ij}^e:*
  - *maximizes the joint profit of M_i and R_j, given R_j's other equilibrium tariffs, $\mathbf{t}_{-i,j}^\mathbf{e}$, and the retail price response, $\mathbf{p}^R(\mathbf{t})$; that is, $t_{ij} = t_{ij}^e$ maximizes:*
  $$\begin{aligned} &\left(p_{ij}^R\left(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}, \mathbf{t}_{-j}^\mathbf{e}\right) - c_i - \gamma_j\right) D_{ij}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}, \mathbf{t}_{-j}^\mathbf{e}\right)\right) \\ &+ \sum_{k\in\mathcal{J}\backslash\{j\}} \left[t_{ik}^e\left(D_{ik}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}, \mathbf{t}_{-j}^\mathbf{e}\right)\right)\right) - c_i D_{ik}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}, \mathbf{t}_{-j}^\mathbf{e}\right)\right)\right] \\ &+ \sum_{h\in\mathcal{I}\backslash\{i\}} \left[\begin{array}{c} \left(p_{hj}^R\left(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}, \mathbf{t}_{-j}^\mathbf{e}\right) - \gamma_j\right) D_{hj}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}, \mathbf{t}_{-j}^\mathbf{e}\right)\right) \\ -t_{hj}^e\left(D_{hj}\left(\mathbf{p}^R\left(t_{ij}, \mathbf{t}_{-i,j}^\mathbf{e}, \mathbf{t}_{-j}^\mathbf{e}\right)\right)\right) \end{array} \right]; \end{aligned}$$
  - *gives M_i and R_j shares α_{ij} and $1 - \alpha_{ij}$ of the bilateral gains from trade.*

[58] We still assume here that wholesale negotiations are simultaneous and secret; hence, a bilateral deviation (or break-down) does not affect the outcome of the other negotiations.

These bargaining equilibria can be generated by the game of delegated negotiations Γ^P, derived from game Γ by replacing its stage 2 with the above "Stage 2 (public contracting)". As tariffs are publicly observed by all firms at the beginning of stage 2, each continuation game constitutes a proper subgame, and it is thus natural to look for the subgame perfect equilibria (SPEs hereafter) of game Γ^P. Ensuring the existence of Nash equilibria for any set of wholesale tariffs is however problematic – for example, sufficiently concave tariffs would generate convex profit functions and discontinuous price responses. A solution consists in focusing on two-part tariffs, which allow for bilateral efficiency without raising convexity issues: the existence of continuation equilibria is then guaranteed if, in the downstream market where m multi-product firms compete against each other, there exists a Nash equilibrium for any profile of constant unit costs.

In what follows, we therefore focus on two-part tariffs of the form $t_{ij}(q_{ij}) = F_{ij} + w_{ij}q_{ij}$, which we denote by $t_{ij} = \{w_{ij}, F_{ij}\}$. For the sake of exposition, we further assume that, in case of multiple equilibria, the selection of the continuation equilibrium depends on unit costs (and thus on wholesale prices), and not on fixed costs (franchise fees); that is, the price response can be expressed as $\mathbf{p}^R(\mathbf{w})$, where $\mathbf{w} = (\mathbf{w}_j)_{j\in\mathcal{J}}$ denotes the vector of wholesale prices. With this restriction, the next proposition establishes a perfect correspondence between the bargaining equilibria and the SPEs of game Γ^P:

Proposition 12 (micro-foundation: public two-part tariffs)**.**

(*i*) *For any bargaining equilibrium of the form* $\mathcal{B} = \{\mathbf{p}^R(\mathbf{w}), \mathbf{t}^\mathbf{e} = \{\mathbf{w}^\mathbf{e}, \mathbf{F}^\mathbf{e}\}, \mathbf{p}^\mathbf{e}\}$, *there exist* $(\mathbf{F}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}$ *such that* $\mathcal{E} = \{\mathbf{p}^R(\mathbf{w}), (\hat{\mathbf{t}}^{\boldsymbol{\theta}} = \{\mathbf{w}^\mathbf{e}, \mathbf{F}^{\boldsymbol{\theta}}\})_{\boldsymbol{\theta}\in\Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \mathbf{p}^\mathbf{e})_{\boldsymbol{\theta}\in\Theta}\}$ *constitutes a SPE of game* Γ^P*, giving all firms the same expected profits as* $\mathcal{B}$*.*

(*ii*) *Conversely, for any SPE of game* Γ^P *of the form* $\mathcal{E} = \{\hat{\mathbf{p}}^R(\mathbf{w}), (\hat{\mathbf{t}}^{\boldsymbol{\theta}} = \{\hat{\mathbf{w}}, \hat{\mathbf{F}}^{\boldsymbol{\theta}}\})_{\boldsymbol{\theta}\in\Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}})_{\boldsymbol{\theta}\in\Theta}\}$, $\mathcal{B} = \{\hat{\mathbf{p}}^R(\mathbf{w}), \mathbf{t}^\mathbf{e} = \{\hat{\mathbf{w}}, \mathbf{F}^\mathbf{e} = E_{\boldsymbol{\theta}}[\hat{\mathbf{F}}^{\boldsymbol{\theta}}]\}, \hat{\mathbf{p}}\}$ *constitutes a bargaining equilibrium, giving all firms the same expected profits as* $\mathcal{E}$*.*

Proof. See Online appendix G.1. ■

The intuition is the same as for secret contracts. Consider the bilateral negotiation between M_i and R_j, say, in game Γ^P. Given the retail price response

$\mathbf{p}^R(w_{ij}, \mathbf{w}^{\mathbf{e}}_{-i,j}, \mathbf{w}^{\mathbf{e}}_{-j})$, the selected agent chooses the wholesale price w_{ij} that maximizes the joint profit of the two firms. Hence, which side makes the offer has no impact on the wholesale prices, which coincide with bargaining equilibrium ones. Which side makes the offer however affects the fixed fees, as the selected agent appropriates the bilateral gains from trade; as a result, expected fixed fees coincide with those negotiated in a bargaining equilibrium.

Intuitively, the equilibrium wholesale prices are now above cost. Indeed, starting from cost-based tariffs, a marginal increase in w_{ij}, say, generates only a second-order loss of efficiency in the bilateral relationship between M_i and R_j (as $w_{ij} = c_i$ would then maximize the joint profit of M_i and R_j), but generates a first-order strategic benefit, by inducing the other retailers to raise their prices (assuming, as is often the case, that retail prices are strategic complements). This, in turn, implies that retail prices and industry profit are higher under public contracting than under secret contracting. Yet, we would expect the outcome to be somewhat competitive.

To explore this further, consider a market structure with symmetric costs ($c_i = c$ and $\gamma_j = \gamma$) and demands, and such that (i) a uniform increase in a retailer's prices decreases its demand; and (ii) total industry profit is concave in prices and maximal for symmetric monopoly prices: $p^M_{ij} = p^M$. Suppose further that:

Assumption $\mathbf{A}^P$(public contracting) The price response $\mathbf{p}^R(\mathbf{w})$ is unique and differentiable in $\mathbf{w}$. Furthermore, starting from a symmetric outcome where publicly observable wholesale prices are all equal to w, increasing w_{ij}, for some $(i,j) \in \mathcal{I} \times \mathcal{J}$:

- (i) increases all retail prices;
- (ii) decreases the total quantity sold by M_i;
- (iii) increases the total quantity sold by any other M_h, for $h \neq i$, as well as the total quantity sold by any other R_k, for $k \neq j$.

The uniqueness of the price response ensures that symmetric wholesale prices yield symmetric retail prices; hence, under secret contracting (in which $w^*_{ij} = c$), the equilibrium is also symmetric: $p^*_{ij} = p^*$. The next proposition confirms the intuition that public contracting generates in that case higher prices and profits:

Proposition 13 (public contracting raises prices and profits)**.** *Under Assumption A^P, any bargaining equilibrium with symmetric public two-part tariffs $t_{ij}^P = \{w^P, F^P\}$ generates positive upstream margins (i.e., $w^P > c$) and symmetric retail prices $p_{ij} = p^P$ that lie between the competitive and monopoly levels: $p^* < p^P < p^M$.*

Proof. See Online appendix G.2. ■

Remark: on the role of delegated negotiations. Proposition 12 shows that bargaining equilibria can again be interpreted as (here, subgame perfect) equilibria of a delegated negotiations game. Assuming that firms delegate the bilateral negotiations with their partners to distinct agents again ensures existence. This is achieved not only by limiting the scope for multilateral deviations, as for secret contracting, but also by limiting the scope for multilateral responses to a deviation, which would otherwise arise with public contracting. For example, in the direct negotiation game considered by Rey and Vergé (2010), in which manufacturers have all the bargaining power in the bilateral negotiations, a small reduction in the wholesale price charged by M_i to R_j, say, may induce *any* of R_j's rivals to reject the offer made by *any* of M_i's rivals. As a result, even for a simple successive linear duopoly model such as the one considered in Proposition 13, multi-channel deviations and responses prevent the existence of a SPE in most of the parameter range. Assuming delegated negotiations also affect pricing incentives, however. For example, when negotiating with R_j, M_i^j takes as given the fixed fee that M_i^k is negotiating with R_k. By contrast, in the case of direct negotiations, M_i would take into account the fact that a reduction in w_{ij}, which is likely to induce R_j to price more aggressively and reduces R_k's profit, would induce a reduction in the fixed fee that could be charged to R_k. Ignoring this effect is thus likely to induce manufacturers to price more aggressively.

Remark: deterministic outcomes and linear tariffs. As for game Γ, the retail equilibrium outcome of game Γ^P is deterministic but the negotiated tariffs depend on which side makes the offer. It is however straightforward to extend again the game so as to ensure that the equilibrium tariffs, too, are deterministic. Consider the extended game $\hat{\Gamma}^P$, in which in a preliminary stage 0, one side can make an offer; the game proceeds as in Γ^P if the offer is rejected, otherwise it proceeds directly to the retail pricing stage. The same reasoning as for secret tariffs (with the caveat that any change in the tariffs accepted at stage 0 or 1 is now observed by all retailers before stage 2) applies; as a result, there is an equivalence between the bargaining equilibria, the equilibria of game Γ^P, and the deterministic equilibria of the modified game $\hat{\Gamma}^P$.

The same reasoning carries over to the case of public linear tariffs: as before, any bargaining equilibrium with public linear tariffs can be replicated as an equilibrium of the extended game $\hat{\Gamma}^P$ for appropriate bargaining parameters $\boldsymbol{\beta}$.

9 Conclusion

In the first part of this paper, we develop a general yet tractable framework for the analysis of multilateral vertical relations. The key features are (secret, bilateral) upstream negotiations, followed by downstream price competition. The setting allows for arbitrary number of firms, degree of product differentiation, cost or demand asymmetry, and bargaining power, and places no restriction on the tariffs that can be negotiated.

An appealing feature of this framework lies in its tractability. We show that equilibrium tariffs are cost-based, whenever they induce a smooth downstream behavior; the equilibrium outcome thus replicates that of a multi-brand oligopoly. The shape of tariffs, however, affects the division of the profits, downstream firms getting a higher (resp., lower) share of the industry profit when tariffs are convex (resp., concave).

We provide a micro-foundation that relies on a non-cooperative game of delegated negotiations. The bargaining equilibria correspond to the sequential equilibria of this game, and correspond to the candidate perfect Bayesian equilibria with passive beliefs of a game of direct negotiations, as characterized by single-channel deviations; focusing on delegated negotiations ensures existence by discarding the possibility of multi-channel deviations.

To illustrate the versatility of this framework, we consider several extensions. We first consider resale price maintenance (RPM) provisions, where the retail price of a product is contractually set by its manufacturer. We show that any prices can (generically) be sustained in equilibrium and that both maximum and minimum RPM can be used to raise prices above their competitive levels, an insight at odds with the current legal treatment of RPM, which treats minimum RPM substantially more harshly than maximum RPM.

We then turn to price parity agreements that restrict a retailer's pricing policy across competing brands. While antitrust agencies have sometimes viewed these price parity agreements as a restriction of competition, similar to minimum RPM, in our setting these contractual clauses are instead rather ineffective – they do not substantially affect the equilibrium outcome, beyond imposing symmetry.

We also use our framework to study the agency business model widely adopted by online retailers and intermediation platforms. This amounts to turning the initial resale setting upside-down: manufacturers are now downstream and set final price, whereas retailers (or intermediation platforms) are upstream. The above insight carries over: as long as firms can negotiate non-linear commissions, these must be cost-based. The equilibrium outcome then replicates that of direct competition between multi-platform firms. Likewise, price parity agreements (linking prices across distribution platforms) do not substantially affect the equilibrium outcome, beyond imposing symmetry.

In the second part of this paper, we endogenize the channel network by introducing a preliminary stage in which firms choose which channels to activate. To obtain a complete characterization, we restrict attention to successive symmetric duopolies. In the polar case where downstream firms are local monopolies, the unique (coalition-proof) equilibrium has all channels being active. When downstream firms are instead perfect substitutes, the unique equilibrium involves either exclusive dealing (each upstream firm dealing with a different downstream firm) or downstream foreclosure (both upstream firms dealing with a common downstream firm). When demand is linear, there is always a unique (coalition-proof) equilibrium, with all channels being active if retailers are sufficiently differentiated, and exclusive dealing otherwise.

Finally, we use our extended framework to study the impact of mergers on the network as well as on prices. Interestingly, this may lead to rather different conclusions than when focusing on price effects. In particular, a downstream merger may expand the distribution network and benefits consumers and society, despite the elimination of downstream competition. Conversely, an upstream merger can trigger vertical foreclosure and be anti-competitive despite the absence of direct price effects.

That upstream contract terms are private and not observable by rival suppliers or customers appears plausible in many markets. Yet, other markets may be more transparent. We show how to adapt the above framework to the case of publicly observable two-part tariffs. The case of (either secret or publicly observable) linear contracts is considered as well. Which assumption about the informational context or the relevant type of tariffs provides the best fit could be empirically tested.

It would also be interesting to compare the predictions of our (static) network formation framework with those of alternative approaches, such as the dynamic approach developed by Lee and Fong (2013) (using Markov-perfection as an equilibrium selection device). Finally, the flexibility and tractability of the approach studied in this paper makes it a good instrument to study firms' decisions over other dimensions, such as product portfolio or investment in production capacity or innovation.

References

Allain, M.-L., and C. Chambolle (2011): "Anti-competitive effects of resale-below-cost laws," *International Journal of Industrial Organization*, 29(4), 373-385.

Bernheim, B.D., B. Peleg and M.D. Whinston (1987): "Coalition-Proof Nash Equilibria I. Concepts," *Journal of Economic Theory*, 42(1), 1-12.

Bernheim, B.D., and M.D. Whinston (1985): "Common Marketing Agency as a Device for Facilitating Collusion," *Rand Journal of Economics*, 16(2), 269-281.

Bernheim, B.D., and M.D. Whinston (1986): "Common Agency," *Econometrica*, 54(4), 923-942.

Bernheim, B.D., and M.D. Whinston (1998): "Exclusive Dealing," *Journal of Political Economy*, 106, 64-103.

Boik, A., and K.S. Corts (2016): "The Effects of Platform Most-Favored-Nation Clauses on Competition and Entry," *Journal of Law and Economics*, 59(1), 105-134.

Bonanno, G., and J. Vickers (1988): "Vertical Separation," *Journal of Industrial Economics*, 36, 257-265.

Chipty, T., and C.M. Snyder (1999): "The role of firm size in bilateral bargaining: A study of the cable television industry," *Review of Economics and Statistics*, 81(2), 326-340.

Collard-Wexler, A., R.S. Lee and G. Gowrisankaran (2019): "'Nash-in-Nash' Bargaining: A Microfoundation for Applied Work," *Journal of Political Economy*, 127(1), 163-195.

Crawford, G.S., and A. Yurukoglu (2012): "The Welfare Effects of Bundling in Multichannel Television Markets," *American Economic Review*, 102(2), 643-685.

Crawford, G.S., R.S. Lee, M.D. Whinston and A. Yurukoglu (2018): "The Welfare Effects of Vertical Integration in Multichannel Television Markets," *Econometrica*, 86(3), 891–954.

Crémer, J., and M. Riordan (1987): "On Governing Multilateral Transactions with Bilateral Contracts," *Rand Journal of Economics*, 18(3), 436-451.

de Fontenay, C., and J. Gans (2005): "Vertical Integration in the Presence of Upstream Competition," *Rand Journal of Economics*, 35(3), 544-572.

de Fontenay, C., and J. Gans (2014): "Bilateral Bargaining with Externalities," *Journal of Industrial Economics*, 62(4), 756-788.

Dobson, P., and M. Waterson (2007): "The Competition Effects of Industry-wide Vertical Price Fixing in Bilateral Oligopoly," *International Journal of Industrial Organization*, 25(5), 935-962.

Dobson, P. and M. Waterson (1997): "Countervailing Power and Consumer Prices", *Economic Journal*, 107(441), 418-430.

Foros, Ø., H.J. Kind and G. Shaffer (2017): "Apple's Agency Model and the Role of Most-Favored Nation Clauses," *Rand Journal of Economics*, 48(3), 673-703.

Fudenberg, D., and J. Tirole (1991): "Perfect Bayesian Equilibrium and Sequential Equilibrium," *Journal of Economic Theory*, 53(2), 236-260.

Gabrielsen, T., and L. Sørgard (2007), "Private labels, price rivalry, and public policy," *European Economic Review*, 51(2), 403-424.

Gans, J. (2007): "Vertical Contracting When Competition For Orders Precedes Procurement," *Journal of Industrial Economics*, 55(2), 325-346.

Ghili, S. (2020): "Network Formation and Bargaining in Vertical Markets: The Case of Narrow Networks in Health Insurance", mimeo.

Gowrisankaran, G., A. Nevo and R. Town (2015): "Mergers When Prices Are Negotiated: Evidence from the Hospital Industry," *American Economic Review*, 175(1), 172-203.

Grennan, M. (2013): "Price Discrimination and Bargaining: Empirical Evidence from Medical Devices," *American Economic Review*, 103(1), 145-177.

Hart, O., and J. Tirole (1990): "Vertical Integration and Market Foreclosure," *Brookings Papers on Economic Activity: Microeconomics*, 205-286.

Ho, K., and R.S. Lee (2017): "Insurer Competition in Health Care Markets," *Econometrica*, 85(2), 379-417.

Ho, K., and R.S. Lee (2019): "Equilibrium Provider Networks: Bargaining and Exclusion in Health Care Markets", *American Economic Review*, 109(2), 473-522.

Horn, H., and A. Wolinsky (1988): "Bilateral monopolies and incentives for merger," *Rand Journal of Economics*, 19(3), 408-419.

Innes, R., and S. Hamilton (2009): "Vertical Restraints and Horizontal Control," *Rand Journal of Economics*, 40(1), 120-143.

Jackson, M., and A. Wolinsky (1996): "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory*, 71(1), 44-74.

Johansen, B.O., and T. Vergé (2017): "Platform Price Parity Clauses and Direct Sales," Working Papers in Economics 01/17, University of Bergen.

Johnson, J. (2017): "The Agency Model and MFN Clauses," *Review of Economic Studies*, 84(3), 1151–1185.

Kranton, R.E., and D.F. Minehart (2001): "A Theory of Buyer-Seller Networks", *American Economic Review*, 91(3), 485-508.

Kreps, D.M., and R. Wilson (1982): "Sequential Equilibria," *Econometrica*, 50(4), 863-894.

Lee, R.S., and K. Fong (2013): "Markov-Perfect Network Formation: An Applied Framework for Bilateral Oligopoly and Bargaining in Buyer-Seller Networks," mimeo.

Liebman, E. (2018): "Bargaining in Markets with Exclusion: An Analysis of Health Insurance Networks", mimeo.

Mathewson, F., and R. Winter (1984): "An Economic Theory of Vertical Restraints," *Rand Journal of Economics*, 15(1), 27-38.

McAfee, R. P., and M. Schwartz (1994): "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity and Uniformity," *American Economic Review*, 84(1), 210-230.

McAfee, R. P., and M. Schwartz (1995): "The non-existence of pairwise-proof equilibrium," *Economic Letters*, 49(3), 251-259.

Milliou, C., and E. Petrakis (2007): "Upstream Horizontal Mergers, Vertical Contracts, and Bargaining," *International Journal of Industrial Organization*, 25(5), 963-987.

Milliou, C., and J. Sandonis (2018): "Manufacturer Mergers and Product Variety in Vertically Related Markets", *Journal of Industry, Competition and Trade*, 18(1), 1-

24.

Mills, D. (1995): "Why retailers sell private labels," *Journal of Economics and Management Strategy*, 4(3), 509-528.

Nilsen, Ø.A., L. Sørgard and S.A. Ulsaker (2016): "Upstream merger in a successive oligopoly: Who pays the price,"*International Journal of Industrial Organization*, 48, 143–172.

Nocke, V., and P. Rey (2018): "Exclusive Dealing and Vertical Integration in Interlocking Relationships," *Journal of Economic Theory*, 177, 183-221.

Nocke, V., and L. White (2007): "Do Vertical Mergers Facilitate Upstream Collusion?," *American Economic Review*, 97(4), 1321-1339.

Nocke, V., and L. White (2010): "Vertical merger, collusion, and disruptive buyers," *International Journal of Industrial Organization*, 28(4), 350-354.

O'Brien, D., and G. Shaffer (1992): "Vertical Control with Bilateral Contracts," *Rand Journal of Economics*, 23(3), 299-308.

Ordover, J., G. Saloner and S.C. Salop (1990): "Equilibrium Vertical Foreclosure," *American Economic Review*, 80(1), 127-142.

Rey, P., and J.E. Stiglitz (1988): "Vertical Restraints and Producers Competition," *European Economic Review*, 32(2-3), 561-568.

Rey, P., and J.E. Stiglitz (1995): "The Role of Exclusive Territories in Producer's Competition," *Rand Journal of Economics*, 26(3), 431-451.

Rey, P., and J. Tirole (1986): "The Logic of Vertical Restraints," *American Economic Review,* 76(5), 921-939.

Rey, P., and T. Vergé (2004): "Bilateral Control with Vertical Contracts," *Rand Journal of Economics*, 35(4), 728-746.

Rey, P., and T. Vergé (2010): "Resale Price Maintenance And Interlocking Relationships," *Journal of Industrial Economics*, 58(4), 928-961.

Rey, P., and T. Vergé (2019): "Secret contracting in multilateral relations," TSE Working Paper 16-744.

Salinger, M.A. (1988): "Vertical Mergers and Market Foreclosure," *Quarterly Journal of Economics*, 103(2),345–356.

Vives, X. (1999): *Oligopoly Pricing: Old Ideas and New Tools,* MIT Press.

Von Ungern-Sternberg, T. (1996), "Countervailing Power Revisited", *International Journal of Industrial Organization*, 14(4), 507-519.

Ziss, S. (1995): "Vertical Separation and Horizontal Mergers", *Journal of Industrial Economics*, 43(1), 63-75.

Appendix

A Tariffs are cost-based: Proof of Proposition 1

We provide a proof showing that Proposition 1 holds more generally when allowing for multi-product firms and partial networks.

Setting. Each M_i sells n_i brands and each R_j operates m_j stores (or store formats, e.g., online versus offline); let $B_i \equiv \{1, ..., n_i\}$ denote M_i's brand portfolio, $S_j \equiv \{1, ..., m_j\}$ denote R_j's set of stores, and $q_{ij,bs}$ denote the quantity of brand $b \in B_i$ sold at store $s \in S_j$. We maintain the assumption of constant returns to scale and denote by $c_{i,b}$ the unit cost of producing brand $b \in B_i$ and by $\gamma_{j,s}$ the unit cost of operating store $s \in S_j$.

We allow the distribution network to be incomplete and, for every $i \in \mathcal{I}$, denote by $\mathcal{J}_i \subseteq \mathcal{J}$ the set of retailers carrying (some of) M_i's brands; also, for any $j \in \mathcal{J}_i$, we denote by $B_{ij} \subseteq B_i$ the set of M_i's brands carried by R_j and by $S_{ij}(b) \subseteq S_j$ the set of R_j's stores carrying brand $b \in B_{ij}$.

The demand for brand $b \in B_{ij}$ at store $s \in S_{ij}(b)$, denoted by $D_{ij,bs}(\mathbf{p})$, is supposed to be differentiable (when positive) in the price vector $\mathbf{p} = (\mathbf{p}_j)_{j \in \mathcal{J}}$, where $\mathbf{p}_j = ((p_{ij,bs})_{b \in B_{ij}, s \in S_{ij}(b)})_{i \in \mathcal{I}}$ represents R_j's vector of prices; let $D_{ij,b}(\mathbf{p}) \equiv \sum_{s \in S_{ij}(b)} D_{ij,bs}(\mathbf{p})$ denote the aggregate demand for brand $b \in B_{ij}$ at R_j's stores, and $\mathbf{D}_{ij}(\mathbf{p}) \equiv (\mathbf{D}_{ij,b}(\mathbf{p}))_{b \in B_{ij}}$ denotes the vector of these aggregate demands.

To maximize the scope for *internal* coordination, we assume that each M_i negotiates with each R_j a single tariff based on the aggregate quantities of M_i's brands sold at R_j's stores, $t_{ij}(\mathbf{Q}_{ij})$, where $\mathbf{Q}_{ij} \equiv (Q_{ij,b})_{b \in B_{ij}}$ and $Q_{ij,b} \equiv \sum_{s \in S_{ij}(b)} q_{ij,bs}$; that is, the tariff covers all brands and does not discriminate among R_j's stores.[59] Let $\mathbf{t} \equiv (\mathbf{t}_j)_{j \in \mathcal{J}}$, where $\mathbf{t}_j \equiv ((t_{ij,b})_{b \in B_{ij}})_{i \in \mathcal{I}}$ is the vector of tariffs that R_j faces.

[59] This setting can readily be adapted to situations in which the parties negotiate a separate tariff for each brand (or brand category) and/or each store (or store format).

Smooth retail behavior. Fix a candidate bargaining equilibrium with retail prices $\mathbf{p^e}$ and tariffs $\mathbf{t^e}$, and for any given $i \in \mathcal{I}$ and $j \in \mathcal{J}_i$, suppose that the tariff $t^e_{ij}(\mathbf{Q}_{ij})$ is increased by constant wholesale prices $\mathbf{w}_{ij} = (w_{ij,b})_{b \in B_{ij}}$; that is, it becomes $\hat{t}_{ij}(\mathbf{Q}_{ij}; \mathbf{w}_{ij}) \equiv t^e_{ij}(\mathbf{Q}_{ij}) + \sum_{b \in B_{ij}} w_{ij,b} Q_{ij,b}$. Let

$$\hat{\mathbf{p}}^i_j(\mathbf{w}_{ij}) \in \arg\max_{\mathbf{p}_j} \Bigg\{ \sum_{h \in \mathcal{I}} \sum_{b \in B_{hj}} \{ \sum_{s \in S_{hj}(b)} [(p_{hj,bs} - \gamma_{j,s}) D_{hj,bs}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j})] - t^e_{hj}(\mathbf{D}_{hj}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}))\} \\ - \sum_{b \in B_{ij}} w_{ij,b} D_{ij,b}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}) \Bigg\}$$

denote the resulting price response, $\hat{Q}^j_{ik,b}(\mathbf{w}_{ij}) \equiv D_{ik,b}(\mathbf{p}^i_j(\mathbf{w}_{ij}), \mathbf{p}^{\mathbf{e}}_{-j})$ denote the aggregate quantity of brand $b \in B_{ij}$ sold at R_k's stores, given R_j's price response $\hat{\mathbf{p}}^i_j(\mathbf{w}_{ij})$, and $\hat{\mathbf{Q}}^j_{ik}(\mathbf{w}_{ij}) \equiv (\hat{Q}^j_{ik,b}(\mathbf{w}_{ij}))_{b \in B_{ij}}$ denote the vector of these aggregate quantities. Finally, for any $\hat{b} \in B_{ij}$, $k \in \mathcal{J}$ and $b \in B_{ik}$, let:

$$\delta^i(j, \hat{b}; k, b) \equiv -\frac{\partial \hat{Q}^j_{ik,b}}{\partial w_{ij,\hat{b}}}(\mathbf{0}) / \frac{\partial \hat{Q}^j_{ij,\hat{b}}}{\partial w_{ij,\hat{b}}}(\mathbf{0})$$

denote the diversion ratio from the sales of brand $\hat{b}$ by R_j to those of brand b by R_k, and $\boldsymbol{\delta}^i$ denote the matrix of these diversion ratios.[60]

We say that the equilibrium retail behavior is *smooth* if, for every $i \in \mathcal{I}$: (i) for every $j \in \mathcal{J}_i$, the tariff $\mathbf{t}^e_{ij}$ and the price responses $\hat{\mathbf{p}}^i_j(\mathbf{w}_{ij})$ are differentiable; and (ii) the diversion ratio matrix $\boldsymbol{\delta}^i$ is nonsingular.

Proposition 1bis (cost-based tariffs - multi-product firms and arbitrary networks)**.** *In any equilibrium in which the retail behavior is smooth, tariffs are cost-based:*

$$\forall i \in \mathcal{I},\ \forall j \in \mathcal{J}_i,\ \forall b \in B_{ij}, \quad \frac{\partial t^e_{ij}}{\partial Q_{ij,b}}(\mathbf{Q}^e_{ij}) = c_{i,b}.$$

Proof. In their bilateral negotiation, M_i and R_j could adjust $\mathbf{w}_{ij}$ so as to maximize

[60] Specifically, $\boldsymbol{\delta}^i$ is the $M^i \times M^i$ matrix with elements $\boldsymbol{\delta}^i_{\ell(j,\hat{b}),\ell(k,b)} = \delta^i(j, \hat{b}; k, b)$ where, for any $j \in \mathcal{J}_i$, $M^i_j \equiv \sum_{k \in \mathcal{J}_i, k<j} |B_{ik}|$, $M^i \equiv \sum_{j \in \mathcal{J}_i} |B_{ij}|$ and, for any $b \in B_{ij}$, $\ell(j, b) \equiv M^i_j + r(b)$, where $r(b)$ is the rank of b in B_{ij}.

their joint profit, given by:

$$\hat{\pi}_j^i(\mathbf{w}_{ij}) + \sum_{b \in B_i} w_{ij,b} \hat{Q}_{ij,b}^j(\mathbf{w}_{ij}) + \sum_{k \in \mathcal{J}_i} [t_{ik}^e(\hat{\mathbf{Q}}_{ik}^j(\mathbf{w}_{ij})) - \sum_{b \in B_{ik}} c_{i,b} \hat{Q}_{ik,b}^j(\mathbf{w}_{ij})],$$

where $\hat{\pi}_j^i(\mathbf{w}_{ij})$ denotes the profit obtained by R_j thanks to its best-response $\hat{\mathbf{p}}_j^i(\mathbf{w}_{ij})$, which, from the envelope theorem, satisfies, for any $b \in B_{ij}$:

$$\frac{\partial \hat{\pi}_j^i}{\partial w_{ij,b}}(\mathbf{w}_{ij}) = -\hat{Q}_{ij,b}^j(\mathbf{w}_{ij}).$$

It follows that, for any $j \in \mathcal{J}_i$ and any $\hat{b} \in B_{ij}$, the first-order condition with respect to $w_{ij,\hat{b}}$, evaluated at $\mathbf{w}_{ij} = 0$, yields:

$$\sum_{k \in \mathcal{J}_i} \sum_{b \in B_{ik}} u_{ik,b}^e \frac{\partial \hat{Q}_{ik,b}^j}{\partial w_{ij,\hat{b}}}(\mathbf{0}) = 0, \tag{1}$$

where

$$u_{ik,b}^e \equiv \frac{\partial t_{ik}^e}{\partial Q_{ik,b}}(\mathbf{Q}_{ik}^e) - c_{i,b}$$

denotes the equilibrium margin charged by M_i to R_k on brand $b \in B_i$. Dividing (1) by $\frac{\partial \hat{Q}_{ij,\hat{b}}^j}{\partial w_{ij,\hat{b}}}(\mathbf{0})$, the first-order conditions can be expressed as $\boldsymbol{\delta}^i \cdot (\mathbf{u}_i^e)^{\mathbf{T}} = \mathbf{0}$, where $\mathbf{u}_i^e \equiv (u_{ij,b}^e)_{b \in B_i, j \in \mathcal{J}_i}$; the diversion ratio matrix $\boldsymbol{\delta}^i$ being nonsingular, M_i's equilibrium margins must all be zero. ■

B Existence: Proof of Proposition 2

Fix a candidate equilibrium $(\mathbf{t}^e, \mathbf{p}^e)$ in which $\mathbf{p}^e = \mathbf{p}^*$ and, for every $(i,j) \in \mathcal{I} \times \mathcal{J}$, $t_{ij}^e(q_{ij}) = F_{ij}^e + c_i q_{ij}$, where F_{ij}^e remains to be determined. Consider the negotiation between some M_i and R_j. Given their other equilibrium tariffs, $(t_{ik}^e)_{k \in \mathcal{J} \setminus \{j\}}$ and $(t_{hj}^e)_{h \in \mathcal{I} \setminus \{i\}}$, and the other retailers' equilibrium prices, $\mathbf{p}_{-j}^*$, they seek to maximize their joint profit, equal to:

$$(p_j - c_i - \gamma_j) D_{ij}(\mathbf{p}_j, \mathbf{p}_{-j}^*) + \sum_{k \in \mathcal{J} \setminus \{j\}} F_{ik}^e + \sum_{h \in \mathcal{I} \setminus \{i\}} [(p_{hj} - c_h - \gamma_j) D_{hj}(\mathbf{p}_j, \mathbf{p}_{-j}^*) - F_{hj}^e].$$

Assumption A ensures that this joint profit is maximal for $\mathbf{p}_j^*$. Furthermore, given R_j's other equilibrium tariffs, $\mathbf{t}_{-i,j}^*$, a tariff t_{ij} leads R_j to maximize its own profit, equal to:

$$\left(p_{ij}-\gamma_j\right) D_{ij}\left(\mathbf{p}_j,\mathbf{p}_{-j}^*\right)-t_{ij}\left(D_{ij}\left(\mathbf{p}_j,\mathbf{p}_{-j}^*\right)\right)+\sum_{h\in\mathcal{I}\setminus\{i\}}\left[\left(p_{hj}-c_i-\gamma_j\right) D_{hj}\left(\mathbf{p}_j,\mathbf{p}_{-j}^*\right)-F_{hj}^*\right].$$

A cost-based two-part tariff in the form $t_{ij}\left(q_{ij}\right)=F_{ij}+c_i q_{ij}$ is then optimal, as it makes R_j's variable profit equal to the joint variable profit of M_i and R_j.

We now determine the fixed fees. Let Δ_{M_i} and Δ_{R_j} denote the gains from trade for M_i and R_j. Under Nash bargaining, M_i obtains a fraction α_{ij} of the total gains:

$$\Delta_{M_i}=\alpha_{ij}\left(\Delta_{M_i}+\Delta_{R_j}\right). \tag{2}$$

In the candidate equilibrium, M_i and R_j respectively obtain:

$$\Pi_{M_i}^*=\sum_{k\in\mathcal{J}} F_{ik}^e \quad \text{and} \quad \Pi_{R_j}^*=\pi_j(\mathbf{p}^*)-\sum_{h\in\mathcal{I}} F_{hj}^e.$$

If the negotiation between M_i and R_j were to break down, M_i would collect the fees from the other retailers and R_j would maximize its profit from the other brands. Hence, $\Delta_{M_i}=F_{ij}^e$ and $\Delta_{R_j}=\Delta_j^i-F_{ij}^e$; the Nash bargaining rule (2) thus yields: $F_{ij}^e=\alpha_{ij}\Delta_j^i$.

C Micro-foundation: Proof of Proposition 3

Part (i)**.** Fix a bargaining equilibrium $\mathcal{B}=\{(\mathbf{p}_j^R(\mathbf{t}_j))_{j\in\mathcal{J}},\mathbf{t}^{\mathbf{e}},\mathbf{p}^{\mathbf{e}}\}$ and, for any $(i,j)\in\mathcal{I}\times\mathcal{J}$, denote the equilibrium profits of M_i and R_j by (using $q_{hk}^e=D_{hk}\left(\mathbf{p}^{\mathbf{e}}\right)$)

$$\Pi_{M_i}^e\equiv\sum_{k\in\mathcal{J}}\left[t_{ik}^e(q_{ik}^e)-c_i q_{ik}^e\right] \text{ and } \Pi_{R_j}^e\equiv\sum_{h\in\mathcal{I}}\left[(p_{hj}^e-\gamma_j)q_{hj}^e-t_{hj}^e(q_{hj}^e)\right].$$

Likewise, in case of a negotiation break-down between M_i and R_j, we adopt the convention $t_{ij}=\emptyset$ and $p_{ij}=+\infty$, and denote the resulting prices by $\mathbf{p}_j^{ij}=(+\infty,\mathbf{p}_{-i,j}^{ij})\equiv\mathbf{p}_j^R(\emptyset,\mathbf{t}_{-i,j}^{\mathbf{e}})$, where $\mathbf{t}_{-i,j}^{\mathbf{e}}=(t_{hj}^e)_{h\in\mathcal{I}\setminus\{i\}}$, and the resulting profits by (using $q_{hk}^{ij}=$

$D_{hk}(\mathbf{p}_j^{ij}, \mathbf{p}_{-j}^e))$

$$\Pi_{M_i}^{ij} \equiv \sum_{k \in \mathcal{J} \setminus \{j\}} \left[t_{ik}^e(q_{ik}^{ij}) - c_i q_{ik}^{ij} \right] \text{ and } \Pi_{R_j}^{ij} \equiv \sum_{h \in \mathcal{I} \setminus \{i\}} \left[(p_{hj}^{ij} - \gamma_j) q_{hj}^{ij} - t_{hj}^e(q_{hj}^{ij}) \right].$$

To construct a corresponding equilibrium for game Γ, we first define, for any $(i, j) \in \mathcal{I} \times \mathcal{J}$ and $\theta_{ij} \in \Theta_{ij}$, the tariff $\hat{t}_{ij}^{\theta_{ij}} = t_{ij}^e + \hat{F}_{ij}^{\theta_{ij}}$, where

$$\hat{F}_{ij}^{\theta_{ij}} \equiv \begin{cases} \Pi_{R_j}^e - \Pi_{R_j}^{ij} & \text{if } \theta_{ij} = M_i^j, \\ -(\Pi_{M_i}^e - \Pi_{M_i}^{ij}) & \text{if } \theta_{ij} = R_j^i. \end{cases}$$

By construction, the tariff $\hat{t}_{ij}^{\theta_{ij}}$ coincides with t_{ij}^e in expectation:

Lemma 1 (bargaining fees)**.** *For every $(i, j) \in \mathcal{I} \times \mathcal{J}$, $E_{\theta_{ij}}[\hat{t}_{ij}^{\theta_{ij}}] = t_{ij}^e$.*

Proof. Together, the definition of $(\hat{F}_{ij}^{\theta_{ij}})_{\theta_{ij} \in \Theta_{ij}}$ and the Nash bargaining rule yield: $E_{\theta_{ij}}[\hat{F}_{ij}^{\theta_{ij}}] = \alpha_{ij} \hat{F}_{ij}^{M_i^j} + (1 - \alpha_{ij}) \hat{F}_{ij}^{R_j^i} = \alpha_{ij}(\Pi_{R_j}^e - \Pi_{R_j}^{ij}) - (1 - \alpha_{ij})(\Pi_{M_i}^e - \Pi_{M_i}^{ij}) = 0.$ ■

Next, for every $j \in \mathcal{J}$, we define the price response $\hat{\mathbf{p}}_j^R(\cdot)$ as follows:[61]

- for every $\boldsymbol{\theta}_j \in \Theta_j$, $\hat{\mathbf{p}}_j^R(\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j}) = \mathbf{p}_j^R(\mathbf{t}_j^{\mathbf{e}}) = \mathbf{p}^{\mathbf{e}}$ (where $\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j} = (\hat{t}_{ij}^{\theta_{ij}})_{i \in \mathcal{I}}$);
- for every $i \in \mathcal{I}$, every $\boldsymbol{\theta}_{-i,j} = (\theta_{hj})_{h \in \mathcal{I} \setminus \{i\}}$ and any $t_{ij} \notin \{\hat{t}_{ij}^{M_i^j}, \hat{t}_{ij}^{R_j^i}\}$, $\hat{\mathbf{p}}_j^R(t_{ij}, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}}) = \mathbf{p}_j^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}})$ (where $\hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}} = (\hat{t}_{hj}^{\theta_{hj}})_{h \in \mathcal{I} \setminus \{i\}}$) – this applies as well in case of negotiation break-down with M_i, namely, $\hat{\mathbf{p}}_j^R(\emptyset, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}}) = \mathbf{p}_j^R(\emptyset, \mathbf{t}_{-i,j}^{\mathbf{e}}) = \mathbf{p}_j^{ij}$;
- in all other cases, $\hat{\mathbf{p}}_j^R(\mathbf{t}_j) = \mathbf{p}_j^R(\mathbf{t}_j)$.

Let $\hat{\mathbf{p}}^{\boldsymbol{\theta}} \equiv (\hat{\mathbf{p}}_j^R(\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j}))_{j \in \mathcal{J}}$. We now show that $\{(\hat{\mathbf{p}}_j^R(\mathbf{t}_j))_{j \in \mathcal{J}}, (\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta} \in \Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta} \in \Theta}\}$, together with consistent beliefs, constitutes a sequential equilibrium $\mathcal{E}$ of game Γ, yielding essentially the same outcome as $\mathcal{B}$. By construction, $\mathcal{E}$ yields the same prices as $\mathcal{B}$, regardless of which side gets to make the offers, even in case of a negotiation break-down. Furthermore, from Lemma 1 the expected equilibrium tariffs coincide

[61]In essence, $\hat{\mathbf{p}}_j^R(\cdot)$ coincides with $\mathbf{p}_j^R(\cdot)$; it may only depart from $\mathbf{p}_j^R(\cdot)$ if R_j's profit has multiple maxima and $\mathbf{p}_j^R(\cdot)$ picks different optimal prices when tariffs are altered by some constants. However, even in that case, both $\hat{\mathbf{p}}_j^R(\cdot)$ and $\mathbf{p}_j^R(\cdot)$ pick optimal prices.

with $\mathbf{t}^{\mathbf{e}}$; hence, $\mathcal{E}$ also yields the same expected profits and disagreement payoffs as $\mathcal{B}$ (where the expectation refers to which side makes the offers).

It is straightforward to check that, for every $j \in \mathcal{J}$, $\hat{\mathbf{p}}_j^R(\cdot)$ constitutes an appropriate price response for R_j in stage 2. Belief consistency implies that, whatever the tariffs $\mathbf{t}_j$ negotiated by its own agents (including if some negotiations failed), R_j expects its rivals' agents to have negotiated the equilibrium tariffs; it thus expects its rivals to charge the equilibrium prices, $\mathbf{p}_{-j}^{\mathbf{e}}$. It follows that setting $\mathbf{p}_j = \hat{\mathbf{p}}_j^R(\mathbf{t}_j)$ is indeed optimal, as altering the tariffs by some constants does not affect the set of optimal prices.

We now consider stage 1 and study the bilateral negotiation between M_i and R_j, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$. In $\mathcal{B}$, the tariff t_{ij}^e maximizes their joint profit, given the other retailers' equilibrium prices, $\mathbf{p}_{-j}^{\mathbf{e}}$, the equilibrium tariffs $\mathbf{t}_{i,-j}^{\mathbf{e}} = (t_{ik}^e)_{k\in\mathcal{J}\setminus\{j\}}$ and $\mathbf{t}_{-i,j}^{\mathbf{e}} = (t_{hj}^e)_{h\in\mathcal{I}\setminus\{i\}}$ negotiated with the other partners, and R_j's price response $\mathbf{p}_j^R(\cdot)$; that is, $t_{ij} = t_{ij}^e$ maximizes

$$\Pi_{i-j}^R(t_{ij}) \equiv \Pi_{i-j}(\mathbf{p}_j^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}}))$$

where:

$$\begin{aligned}\Pi_{i-j}(\mathbf{p}_j) \equiv\ & (p_{ij} - c_i - \gamma_j) D_{ij}(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}) \\ & + \sum_{k\in\mathcal{J}\setminus\{j\}} \left[t_{ik}^e(D_{ik}(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}})) - c_i D_{ik}(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}) \right] \\ & + \sum_{h\in\mathcal{I}\setminus\{i\}} \left[(p_{hj} - \gamma_j) D_{hj}(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}}) - t_{hj}^e(D_{hj}(\mathbf{p}_j, \mathbf{p}_{-j}^{\mathbf{e}})) \right].\end{aligned}$$

In the above candidate equilibrium of game Γ, the agents M_i^j and R_j^i expect M_i's and R_j's other agents to negotiate the equilibrium tariffs and the other retailers to charge the equilibrium prices. Hence, when signing a tariff t_{ij} they expect R_j to charge $\hat{\mathbf{p}}_j^R(t_{ij}, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}})$ for any realization $\boldsymbol{\theta}_{-i,j} \equiv (\theta_{hj})_{h\in\mathcal{I}\setminus\{i\}} \in \boldsymbol{\Theta}_{-i,j} \equiv \Pi_{h\in\mathcal{I}\setminus\{i\}}\Theta_{hj}$. Therefore, they expect the joint profit of their two firms to be given by:

$$\hat{\Pi}_{i-j}^R(t_{ij}) \equiv E_{\boldsymbol{\theta}_{i-j}^{ij}} \left[\hat{\Pi}_{i-j}(\hat{\mathbf{p}}_j^R(t_{ij}, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}}); \boldsymbol{\theta}_{i-j}^{ij}) \right],$$

where $\boldsymbol{\theta}^{ij}_{i-j} \equiv ((\theta_{ik})_{k\in\mathcal{J}\setminus\{j\}}, (\theta_{hj})_{h\in\mathcal{I}\setminus\{i\}}) \in \boldsymbol{\Theta}^{ij}_{i-j} \equiv \{\Theta_i \times \Theta_j\} \setminus \Theta_{ij}$ and:

$$\begin{aligned}\hat{\Pi}_{i-j}(\mathbf{p}_j; \boldsymbol{\theta}^{ij}_{i-j}) \;\; \equiv \;\; & (p_{ij} - c_i - \gamma_j) D_{ij}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}) \\ & + \sum_{k\in\mathcal{J}\setminus\{j\}} [\hat{t}^{\theta_{ik}}_{ik}(D_{ik}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j})) - c_i D_{ik}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j})] \\ & + \sum_{h\in\mathcal{I}\setminus\{i\}} [(p_{hj} - \gamma_j) D_{hj}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}) - \hat{t}^{\theta_{hj}}_{hj}(D_{hj}(\mathbf{p}_j, \mathbf{p}^{\mathbf{e}}_{-j}))]\end{aligned}$$

For any $\boldsymbol{\theta}_{-i,j} \in \boldsymbol{\Theta}_{-i,j}$, $\hat{\mathbf{p}}^R_j(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}) = \mathbf{p}^R_j(t^e_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j})$ if $t_{ij} \in \{\hat{t}^{M^j_i}_{ij}, \hat{t}^{R^i_j}_{ij}\}$, otherwise $\hat{\mathbf{p}}^R_j(t_{ij}, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}) = \mathbf{p}^R_j(t_{ij}, \mathbf{t}^{\mathbf{e}}_{-i,j})$. From Lemma 1, $E_{\boldsymbol{\theta}^{ij}_{i-j}}[\hat{\Pi}_{i-j}(\mathbf{p}_j; \boldsymbol{\theta}^{ij}_{i-j})] = \Pi_{i-j}(\mathbf{p}_j)$. Hence, $\hat{\Pi}^R_{i-j}(t_{ij}) = \Pi^R_{i-j}(t^e_{ij})$ if $t_{ij} \in \{\hat{t}^{M^j_i}_{ij}, \hat{t}^{R^i_j}_{ij}\}$, otherwise $\hat{\Pi}^R_{i-j}(t_{ij}) = \Pi^R_{i-j}(t_{ij})$, which is maximal for $t_{ij} = t^e_{ij}$. Therefore, $\Pi^R_{i-j}(t_{ij})$ is maximal for any $t_{ij} \in \{\hat{t}^{M^j_i}_{ij}, \hat{t}^{R^i_j}_{ij}\}$.

As already noted, M_i and R_j have the same disagreement payoffs in $\mathcal{B}$ and in $\mathcal{E}$. $\hat{t}^{M^j_i}_{ij}$ moreover gives R_j its disagreement payoff, and $\hat{t}^{R^i_j}_{ij}$ gives M_i its disagreement payoff: with $\hat{t}^{M^j_i}_{ij} = t^e_{ij} + \hat{F}^{M^j_i}_{ij}$, R_j obtains an expected profit (where the expectation is taken over $\boldsymbol{\theta}_{-i,j}$) equal to $\Pi^e_{R_j} - \hat{F}^{M^j_i}_{ij} = \Pi^{ij}_{R_j}$, and with $\hat{t}^{R^i_j}_{ij} = t^e_{ij} + \hat{F}^{R^i_j}_{ij}$, M_i obtains an expected profit (where the expectation is taken over $\boldsymbol{\theta}_{i,-j}$) equal to $\Pi^e_{M_i} - \hat{F}^{R^i_j}_{ij} = \Pi^{ij}_{M_i}$. As $\hat{t}^{M^j_i}_{ij}$ maximizes the joint bilateral profit and gives R_j its disagreement payoff, constitutes an optimal tariff for M^j_i; likewise, $\hat{t}^{R^i_j}_{ij}$ constitutes an optimal tariff for R^i_j.

Part (ii)**.** Fix a sequential equilibrium of game Γ, $\mathcal{E} = \{(\hat{\mathbf{p}}^R_j(\mathbf{t}_j))_{j\in\mathcal{J}}, (\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}, \mathbf{b}\}$, with regular price responses and tariffs, implying $\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}}$ for any $\boldsymbol{\theta} \in \Theta$. Consider now the tariffs $\mathbf{t}^{\mathbf{e}} = (t^e_{ij})_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ where, for every $i \in \mathcal{I}$ and every $j \in \mathcal{J}$:

$$t^e_{ij}(q_{ij}) \equiv E_{\theta_{ij}}\left[\hat{t}^{\theta_{ij}}_{ij}(q_{ij})\right] = \alpha_{ij}\hat{t}^{M^j_i}_{ij}(q_{ij}) + (1 - \alpha_{ij})\,\hat{t}^{R^i_j}_{ij}(q_{ij}).$$

We now show that $\mathcal{B} \equiv \{(\hat{\mathbf{p}}^R_j(\mathbf{t}_j))_{j\in\mathcal{J}}, \mathbf{t}^{\mathbf{e}}, \hat{\mathbf{p}}\}$ constitutes a bargaining equilibrium yielding essentially the same outcome as $\mathcal{E}$. The regularity of the tariffs implies that, for any $j \in \mathcal{J}$ and $h \in \mathcal{I}$, $\hat{t}^{M^j_h}_{hj}(\cdot)$ and $\hat{t}^{R^h_j}_{hj}(\cdot)$ differ only by a constant; hence, t^e_{hj} also differs from $\hat{t}^{M^j_h}_{hj}(\cdot)$ or $\hat{t}^{R^h_j}_{hj}(\cdot)$ only by a constant. The regularity of price responses then yields:

$$\hat{\mathbf{p}}^R_j(\mathbf{t}^{\mathbf{e}}_j) = \hat{\mathbf{p}}^R_j(\hat{\mathbf{t}}^{\boldsymbol{\theta}_j}_j) = \hat{\mathbf{p}}_j \text{ and } \hat{\mathbf{p}}^R_j(\emptyset, \mathbf{t}^{\mathbf{e}}_{-i,j}) = \hat{\mathbf{p}}^R_j(\emptyset, \hat{\mathbf{t}}^{\boldsymbol{\theta}_{-i,j}}_{-i,j}).$$

Prices and expected tariffs are thus the same in $\mathcal{B}$ and in $\mathcal{E}$, even if a negotiation fails; it follows that expected profits and disagreement payoffs are also the same.

In stage 2, each R_j expects its rivals to charge the same prices as in $\mathcal{E}$ (where they do not depend on the realization of $\boldsymbol{\theta}_{i-j}^{ij}$). Hence, $\hat{\mathbf{p}}_j^R(\mathbf{t}_j)$ still constitutes an appropriate price response to any tariffs $\mathbf{t}_j$ negotiated in stage 1.

We now consider stage 1 and study the bilateral negotiation between M_i and R_j, for some $i \in \mathcal{I}$ and $j \in \mathcal{J}$. In $\mathcal{E}$, the tariff offered by the selected agent, θ_{ij}, maximizes the expected profit of its firm, among those that are acceptable by the other agent. As profits can be shared through lump-sum fees (which do not affect retailers' pricing decisions, as price responses are regular), it follows that the tariff $\hat{t}_{ij}^{\theta_{ij}}$ maximizes the expected joint profit of M_i and R_j, given the other retailers' prices and the tariffs negotiated with the other partners; that is, $t_{ij} = \hat{t}_{ij}^{\theta_{ij}}$ maximizes:

$$\hat{\Pi}_{i-j}^R(t_{ij}) = E_{\boldsymbol{\theta}_{i-j}^{ij}}\left[\hat{\Pi}_{i-j}(\hat{\mathbf{p}}_j^R(t_{ij}, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}}); \boldsymbol{\theta}_{i-j}^{ij})\right].$$

From the regularity of the price responses and of the tariffs, $\hat{p}_{ij}^R(t_{ij}, \hat{\mathbf{t}}_{-i,j}^{\boldsymbol{\theta}_{-i,j}}) = \hat{p}_{ij}^R(t_{ij}, \mathbf{t}_{-i,j}^{\mathbf{e}})$; and from Lemma 1, $E_{\boldsymbol{\theta}_{i-j}^{ij}}[\hat{\Pi}_{i-j}(\mathbf{p}_j; \boldsymbol{\theta}_{i-j}^{ij})] = \Pi_{i-j}(\mathbf{p}_j)$ for any $\mathbf{p}_j$. Therefore, $\hat{\Pi}_{i-j}^R(t_{ij}) = \Pi_{i-j}^R(t_{ij})$, the joint profit of M_i and R_j in their bilateral negotiation of the bargaining game, taking as given R_j's other equilibrium tariffs, $\mathbf{t}_{-i,j}^{\mathbf{e}}$, as well as rivals' equilibrium prices, $\mathbf{p}_{-j}^{\mathbf{e}}$, and R_j's price response, $\mathbf{p}_j^R(\mathbf{t}_j)$. As $t_{ij} = \hat{t}_{ij}^{\theta_{ij}}$ maximizes $\hat{\Pi}_{i-j}^R(t_{ij})$, it also maximizes $\Pi_{i-j}^R(t_{ij})$; and as R_j's price response is regular and t_{hk}^e differs from $\hat{t}_{hk}^{\theta_{hk}}$ only by some constant, $t_{ij} = t_{ij}^e$, too, maximizes $\Pi_{i-j}^R(t_{ij}) = \hat{\Pi}_{i-j}^R(t_{ij})$. Finally, it is straightforward to check that t_{ij}^e gives M_i and R_j shares α_{ij} and $1 - \alpha_{ij}$, respectively, of the bilateral gains from trade.

Online appendix for
Secret contracting with multilateral relations

A Division of profits

Fix a multi-brand oligopoly equilibrium $\mathbf{p}^*$, a given R_j, and consider the following quadratic tariffs, for every $i \in \mathcal{I}$:

$$t_{ij}^{\sigma}(q_{ij}) \equiv F_{ij}(\sigma) + c_i q_{ij} + \sigma(q_{ij} - q_{ij}^*)^2,$$

where $q_{ij}^* = D_{ij}(\mathbf{p}^*)$ and $F_{ij}(\sigma)$ remains to be determined. It is straightforward to check that, for any $\sigma > 0$, $\mathbf{p}_j^*$ remains a best-response to $\mathbf{p}_{-j}^*$ when R_j faces the tariffs $\mathbf{t}_j^{\sigma} = (t_{ij}^{\sigma})_{i \in \mathcal{I}}$. To see this, let

$$\pi_j^{\sigma}(\mathbf{p}_j) \equiv \pi_j(\mathbf{p}_j, \mathbf{p}_{-j}^*) - \sum_{i \in \mathcal{I}} \sigma \left[D_{ij}\left(\mathbf{p}_j, \mathbf{p}_{-j}^*\right) - q_{ij}^*\right]^2$$

denote R_j's variable profit, given the tariffs $\mathbf{t}_j^{\sigma}$ and rivals' equilibrium prices $\mathbf{p}_{-j}^*$; for $\sigma > 0$ and any $\mathbf{p}_j$, we have:

$$\pi_j^{\sigma}(\mathbf{p}_j) \leq \pi_j(\mathbf{p}_j, \mathbf{p}_{-j}^*) \leq \pi_j(\mathbf{p}_j^*, \mathbf{p}_{-j}^*) = \pi_j^{\sigma}(\mathbf{p}_j^*),$$

where the first inequality stems from $\sigma > 0$, the second one from the fact that $\mathbf{p}^* = (\mathbf{p}_j^*, \mathbf{p}_{-j}^*)$ is a Nash equilibrium, and the equality from $D_{ij}(\mathbf{p}^*) = q_{ij}^*$. For the sake of exposition, we assume that $\mathbf{p}_j^*$ constitutes R_j's unique best-response to $\mathbf{p}_{-j}^*$ for $\sigma = 0$, and remains a best-response as long as σ is not too negative:

Assumption A'. $\pi_j^0(\mathbf{p}_j) = \pi_j(\mathbf{p}_j, \mathbf{p}_{-j}^*)$ is uniquely maximal for $\mathbf{p}_j = \mathbf{p}_j^*$ and there exists $\sigma_R > 0$ such that $\mathbf{p}_j^* \in \arg\max_{\mathbf{p}_j} \pi_j^{\sigma}(\mathbf{p}_j)$ as long as $\sigma \geq -\sigma_R$.

However, R_j's outside option in its bilateral negotiation with a given M_i, $\pi_j^{ij}(\sigma) \equiv \max_{\mathbf{p}_{-i,j}} \pi_j^{\sigma}(\infty, \mathbf{p}_{-i,j})$, decreases with σ; indeed, the envelope theorem yields:

$$\frac{d\pi_j^{ij}}{d\sigma}(\sigma) = -\sum_{h \in \mathcal{I} \setminus \{i\}} \left[q_j^{ij}(\sigma) - q_{ij}^*\right]^2 < 0.$$

where $q_{hj}^{ij}(\sigma)$ denotes the quantity of brand h sold by R_j when dropping brand i and facing the tariffs $\mathbf{t}_j^\sigma$.[1] Building on this insight leads to:

Proposition A.1 (division of profit)**.** *There exists $\overline{\sigma} > 0$ such that, for any σ satisfying $|\sigma| < \overline{\sigma}$, there exists an equilibrium in which the tariffs are $(\mathbf{t}_j^\sigma, \mathbf{t}_{-j}^*)$, for some $\mathbf{F}_j^\sigma = (F_{ij}^\sigma)_{i \in \mathcal{I}}$, the retail prices are $\mathbf{p}^*$, and each M_i obtains a profit, $\Pi_{M_i}^\sigma$, which strictly increases with σ.*

Proof. We have already seen that, for any $\sigma \geq -\sigma_R$, $\mathbf{p}^*$ constitutes a retail price equilibrium when these contracts are in place. In the negotiation between M_i and R_j, given their other equilibrium tariffs, $(t_{hj}^\sigma)_{h \in \mathcal{I} \setminus \{i\}}$, and the other retailers' equilibrium prices, $\mathbf{p}_{-j}^*$, the two firms seek to maximize their joint variable profit, which coincides with $\pi_j^\sigma(\mathbf{p}_j)$; it is thus maximal for $\mathbf{p}_j = \mathbf{p}_j^*$, which agreeing on t_{ij}^σ precisely achieves.

The gains from trade are shared according to the Nash bargaining rule. Each M_i derives all of its profit through the fixed fees:

$$\Pi_{M_i}^\sigma = F_{ij}^\sigma + \sum_{k \in \mathcal{J} \setminus \{j\}} F_{ik}^*,$$

whereas R_j obtains $\Pi_{R_j}^\sigma = \pi_j^* - \sum_{h \in \mathcal{I}} F_{hj}^\sigma$. If the negotiation were to break down, M_i would collect the other retailers' fees whereas R_j would adjust its prices $\mathbf{p}_{-i,j}$ so as to maximize $\pi_j^\sigma(\infty, \mathbf{p}_{-i,j})$. Hence, their bilateral gains from trade are given by:

$$\Delta_{M_i} = F_{ij}^\sigma \quad \text{and} \quad \Delta_{R_j} = \pi_j^* - F_{ij}^\sigma - \pi_j^{ij}(\sigma),$$

and the Nash bargaining rule (2) yields: $F_{ij}^\sigma = \alpha_{ij}\left[\pi_j^* - \pi_j^{ij}(\sigma)\right]$. Therefore:

$$\Pi_{M_i}^\sigma = \alpha_{ij}\left[\pi_j^* - \pi_j^{ij}(\sigma)\right] + \sum_{k \in \mathcal{J} \setminus \{j\}} F_{ik}^*,$$

which is strictly increasing in σ. ■

B Micro-foundation for linear tariffs

Consider a bargaining equilibrium $\mathcal{B}(\boldsymbol{\alpha})$ for given bargaining weights $\boldsymbol{\alpha} = (\alpha_{ij})_{(i,j) \in \mathcal{I} \times \mathcal{J}}$. Let $\mathbf{w}^\mathbf{e}(\boldsymbol{\alpha})$ and $\mathbf{p}^\mathbf{e}(\boldsymbol{\alpha})$ denote the equilibrium wholesale and retail prices and, for any $(i,j) \in \mathcal{I} \times \mathcal{J}$, let $\Pi_{M_i}^e(\boldsymbol{\alpha})$ and $\Pi_{R_j}^e(\boldsymbol{\alpha})$ denote M_i's and R_j's profits, and $\Pi_{M_i}^{ij}(\boldsymbol{\alpha})$ and

[1] For the sake of exposition, we assume that dropping brand i indeed induces R_j to adjust the sales of at least one other brand (i.e., $q_{hj}^{ij}(\sigma) \neq q_{ij}^*$ for some $h \neq i$).

$\Pi^{ij}_{R_j}(\boldsymbol{\alpha})$ denote their disagreements payoffs – i.e., the profits that they would obtain if the other equilibrium tariffs remained unchanged but they decided not to deal with each other. From the Nash bargaining rule, we have:

$$w^e_{ij}(\boldsymbol{\alpha}) = \arg\max_{w_{ij}} \left\{ \left[\Pi^R_{M_i}(w_{ij};\boldsymbol{\alpha}) - \Pi^{ij}_{M_i}(\boldsymbol{\alpha})\right]^{\alpha_{ij}} \left[\Pi^R_{R_j}(w_{ij};\boldsymbol{\alpha}) - \Pi^{ij}_{R_j}(\boldsymbol{\alpha})\right]^{1-\alpha_{ij}} \right\}$$

where $\Pi^R_{M_i}(w_{ij};\boldsymbol{\alpha})$ and $\Pi^R_{R_j}(w_{ij};\boldsymbol{\alpha})$ denote M_i's and R_j's profits in the continuation equilibrium where R_j faces wholesale prices $\mathbf{w}_j = (w_{ij}, \mathbf{w}^e_{-i,j}(\boldsymbol{\alpha}))$ and expects all other retailers to face wholesale prices $\mathbf{w}^e_{-j}(\boldsymbol{\alpha})$ (and thus to charge retail prices $\mathbf{p}^{\mathbf{e}}_{-j}(\boldsymbol{\alpha})$). The resulting equilibrium profits, $\Pi^e_{M_i}(\boldsymbol{\alpha})$ and $\Pi^e_{R_j}(\boldsymbol{\alpha})$, lie on the Pareto frontier generated by varying w_{ij}. By construction, these Pareto frontiers are downward sloping; for the sake of exposition, we assume that they are also continuous.

We now show that $\mathcal{B}(\boldsymbol{\alpha})$ can be replicated as an equilibrium of the game $\hat{\Gamma}(\boldsymbol{\beta})$ for appropriate bargaining probabilities $\boldsymbol{\beta} = \left(\beta_{ij}\right)_{(i,j)\in\mathcal{I}\times\mathcal{J}}$. For the sake of exposition, we assume here that, in stage 0, the upstream agent is always selected to make an offer to its counterpart. The analysis would be qualitatively the same if the downstream agent were instead selected, although the choice of the probabilities $\boldsymbol{\beta}$ would be affected.

Specifically, we construct an equilibrium in which, at stage 0, the wholesale tariffs $\mathbf{w}^{\mathbf{e}}(\boldsymbol{\alpha})$ are offered and accepted. Firms therefore obtain the same profits as in $\mathcal{B}(\boldsymbol{\alpha})$. Fix $i \in \mathcal{I}$ and $j \in \mathcal{J}$, and consider what happens if R^i_j rejects M^j_i's offer but all other tariffs are accepted at stage 0. With probability β_{ij}, M^j_i gets to make a (final) counter-offer and thus leaves R^i_j a profit just equal to the disagreement payoff $\Pi^{ij}_{R_j}(\boldsymbol{\alpha})$. With probability $1-\beta_{ij}$, R^i_j gets to make the final offer and thus achieves at least $\Pi^e_{R_j}(\boldsymbol{\alpha})$: it can secure this profit by offering a wholesale price $w_{ij} = w^e_{ij}(\boldsymbol{\alpha})$, which M^j_i is willing to accept (as it is the outcome of the Nash bargaining rule, which never harms the negotiating parties). Therefore, R^i_j's outside option in game $\hat{\Gamma}(\boldsymbol{\beta})$ is of the form $\hat{\Pi}_j\left(\beta_{ij}\right)$, which is non-decreasing in β_{ij} and such that $\hat{\Pi}_j(0) = \Pi^{ij}_{R_j}(\boldsymbol{\alpha}) \le \Pi^e_{R_j}(\boldsymbol{\alpha}) \le \hat{\Pi}_j(1)$. There thus exists β_{ij} such that $\hat{\Pi}_j(\beta_{ij}) = \Pi^e_{R_j}(\boldsymbol{\alpha})$. This value of β_{ij} induces M^j_i, in stage 0, to choose a wholesale price w_{ij} that lies on the Pareto frontier and gives R_j at least $\Pi^e_{R_j}(\boldsymbol{\alpha})$; this leads M^j_i to choose $w_{ij} = w^e_{ij}(\boldsymbol{\alpha})$. As a consequence, any bargaining equilibrium outcome can be replicated as the equilibrium outcome of some game $\hat{\Gamma}(\boldsymbol{\beta})$.

The reverse statement also holds for any equilibrium $\hat{\mathcal{E}}(\boldsymbol{\beta})$ of the game $\hat{\Gamma}(\boldsymbol{\beta})$ that is Pareto efficient and such that, in stage 0, each downstream agent (i) is indifferent between accepting the initial offer or waiting for the stochastic game Γ, and (ii) accepts the offer. Let $\hat{\mathbf{w}}^{\mathbf{e}}(\boldsymbol{\beta})$ and $\hat{\mathbf{p}}^{\mathbf{e}}(\boldsymbol{\beta})$ denote the equilibrium wholesale and retail prices, $\hat{\Pi}^{e}_{M_i}(\boldsymbol{\beta})$ and $\hat{\Pi}^{e}_{R_j}(\boldsymbol{\beta})$ denote M_i's and R_j's profits, and $\hat{\Pi}^{ij}_{M_i}(\boldsymbol{\beta})$ and $\hat{\Pi}^{ij}_{R_j}(\boldsymbol{\beta})$ denote their disagreements payoffs – i.e., the profits that they would obtain if the other equilibrium tariffs remained unchanged but they decided not to deal with each other. These profits also constitute the profits that M_i^j and R_j^i can respectively secure in game Γ when not selected to make the final offer at stage 1. Finally, denote by $\bar{\Pi}^{i}_{R_j}(\boldsymbol{\beta})$ the maximal profit that R_j can achieve if it has to give M_i a profit at least equal to $\hat{\Pi}^{ij}_{M_i}(\boldsymbol{\beta})$. Recall that, in equilibrium, R_j is indifferent between accepting and rejecting the initial offer made by M_i^j at stage 0, implying that its equilibrium profit $\hat{\Pi}^{e}_{R_j}(\boldsymbol{\beta})$ satisfies: $\hat{\Pi}^{e}_{R_j}(\boldsymbol{\beta}) = \beta_{ij}\hat{\Pi}^{ij}_{R_j}(\boldsymbol{\beta}) + \left(1-\beta_{ij}\right)\bar{\Pi}^{i}_{R_j}(\boldsymbol{\beta})$. Suppose now that a bargaining equilibrium yields the wholesale prices $w_{hk} = \hat{w}^{e}_{hk}(\boldsymbol{\beta})$ for every $hk \neq ij$, and study the bargaining between M_i and R_j for different values of the bargaining weight α_{ij}. For $\alpha_{ij} = 0$, the outcome is the same as if R_j^i were to make a take-it-or-leave-it offer; R_j's profit is therefore equal to $\bar{\Pi}^{i}_{R_j}(\boldsymbol{\beta})$. When $\alpha_{ij} = 1$, the bargaining solution is the same as if M_i^j were to make a take-it-or-leave-it offer and R_j's profit is therefore equal to $\hat{\Pi}^{ij}_{R_j}(\boldsymbol{\beta})$. As α_{ij} continuously increases from 0 to 1, R_j's profit continuously decreases from $\bar{\Pi}^{i}_{R_j}(\boldsymbol{\beta})$ to $\hat{\Pi}^{ij}_{R_j}(\boldsymbol{\beta})$; hence, there exists a value for α_{ij} for which the bargaining solution gives R_j a profit equal to $\hat{\Pi}^{e}_{R_j}(\boldsymbol{\beta})$. As the outcome moreover lies on the Pareto frontier, it must be the case that M_j obtains $\hat{\Pi}^{e}_{M_i}(\boldsymbol{\beta})$, and $w_{ij} = \hat{w}^{e}_{ij}(\boldsymbol{\beta})$. Therefore, there exists a vector of bargaining weights $\boldsymbol{\alpha}$ and an associated bargaining equilibrium $\hat{\mathcal{B}}(\boldsymbol{\alpha})$ yielding the same outcome as $\hat{\mathcal{E}}(\boldsymbol{\beta})$.

C RPM: Proof of Proposition 4

Part (i). For the sake of exposition, we suppose that bilateral profits are well-behaved when firms rely on two-part tariffs:

Assumption B. For any $(i,j) \in \mathcal{I} \times \mathcal{J}$, any wholesale prices $(w_{hk})_{(h,k)\neq(i,j)\in\mathcal{I}\times\mathcal{J}}$ and any prices $(p_{hk})_{(h,k)\neq(i,j)\in\mathcal{I}\times\mathcal{J}}$, the gross joint profit of M_i and R_j, given by:

$$\Pi_{M_i-R_j}(p_{ij}) = \left(p_{ij}-c_i-\gamma_j\right) D_{ij}(\mathbf{p}) + \sum_{k\in\mathcal{J}\setminus\{j\}} (w_{ik}-c_i)\, D_{ik}(\mathbf{p}) + \sum_{h\in\mathcal{I}\setminus\{i\}} \left(p_{hj}-w_{hj}-\gamma_j\right) D_{hj}(\mathbf{p}),$$

is strictly quasi-concave[2] in p_{ij} and maximal for a finite price level.

We now show that, under Assumption B, any price vector can generically be sustained with RPM. For simplicity, we focus here on outcomes in which all quantities are positive. Let $\mathbf{\Lambda}(\mathbf{p})$ denote the $nm \times nm$ matrix such that the term in row $l(i,j) \equiv (i-1)\,m + j$ and column $l(h,k)$, for $i,h \in \mathcal{I}$ and $j,k \in \mathcal{J}$, is given by:

$$\Lambda_{l(i,j),l(h,k)}(\mathbf{p}) = \begin{cases} \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) & \text{if } h \neq i \text{ and } k = j, \\ -\frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) & \text{if } h = i \text{ and } k \neq j, \\ 0 & \text{otherwise.} \end{cases}$$

Fix a price vector $\mathbf{p}$ satisfying $|\mathbf{\Lambda}(\mathbf{p})| \neq 0$. We now construct a vector of wholesale prices, $\mathbf{w}$, and a vector of fixed fees, $\mathbf{F}$, such that each pair $M_i - R_j$ agreeing the retail price p_{ij} and on the two-part tariff $\{w_{ij}, F_{ij}\}$ constitutes a bargaining equilibrium. To achieve this, we first select the wholesale prices that induce the desired retail prices, and then adjust the fixed fees so as to share the gains from trade as desired.

The joint profit $\Pi_{M_i-R_j}$ does not depend on t_{ij} and, from Assumption B, is strictly quasi-concave in the channel's own price. Therefore, M_i and R_j have no incentive to deviate from the equilibrium retail price p_{ij} if the first-order condition holds, namely:

$$D_{ij}(\mathbf{p}) + \left(p_{ij}-c_i-\gamma_j\right)\frac{\partial D_{ij}}{\partial p_{ij}}(\mathbf{p}) + \sum_{k\in\mathcal{J}\setminus\{j\}} (w_{ik}-c_i)\frac{\partial D_{ik}}{\partial p_{ij}}(\mathbf{p}) + \sum_{h\in\mathcal{I}\setminus\{i\}} \left(p_{hj}-w_{hj}-\gamma_j\right)\frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}) = 0.$$

Combining the nm first-order conditions for every $i \in \mathcal{I}$ and $j \in \mathcal{J}$ thus yields:

$$\mathbf{\Lambda}(\mathbf{p}) \cdot (\mathbf{w}-\mathbf{c})^{\mathbf{T}} = \boldsymbol{\mu}(\mathbf{p})^{\mathbf{T}}, \tag{3}$$

where $\mathbf{w}_{l(i,j)} = w_{ij}$, $\mathbf{c}_{l(i,j)} = c_i$, and

[2] If $D_{ij}(\cdot)$ drops to zero when the price p_{ij} is high enough, then the strict quasi-concavity should hold in the price range where $D_{ij}(\cdot) > 0$. A similar comment applies to Assumptions C and D.

$$\boldsymbol{\mu}_{l(i,j)}(\mathbf{p}) \equiv D_{ij}(\mathbf{p}) + \sum_{h\in\mathcal{I}} \left(p_{hj} - c_h - \gamma_j\right) \frac{\partial D_{hj}}{\partial p_{ij}}(\mathbf{p}).$$

Hence, if $|\boldsymbol{\Lambda}(\mathbf{p})| \neq 0$, there exists a unique vector of wholesale prices, $\mathbf{w}(\mathbf{p})$ satisfying the above first-order conditions for every $(i,j) \in \mathcal{I} \times \mathcal{J}$. The fixed fee $F_{ij}(\mathbf{p})$, for every $(i,j) \in \mathcal{I} \times \mathcal{J}$, is then uniquely identified using the Nash bargaining rule.

Part (ii). Suppose now that firms are symmetric and $\mathbf{p}^* = (p^*, ..., p^*)$. The equilibrium condition $\mathbf{p}_j^* \in \arg\max_{\mathbf{p}_j} \pi_j(\mathbf{p}_j, \mathbf{p}_{-j}^*)$ implies $\mu(p^*) = 0$, where, letting $\lambda(p) \equiv -\frac{\partial D_{ij}}{\partial p_{ij}}(p, \ldots, p)$ denote the own-price demand sensitivity:

$$\mu(p) \equiv D(p) - (p - c - \gamma)[\lambda(p) - \lambda_M(p)].$$

We will assume that, at higher prices, retailers would like to undercut each other:

Assumption C. For any $p > p^*$, starting from $\mathbf{p} = (p, ..., p)$, a retailer facing cost-based tariffs would wish to undercut p: $\mu(p) < 0$.

For any vector of symmetric prices $(p, \ldots, p)$, the condition $|\boldsymbol{\Lambda}(\mathbf{p})| \neq 0$ amounts to $\lambda_M(p) \neq \lambda_R(p)$ and the equilibrium condition (3) boils down to:

$$w(p) \equiv c + \frac{\mu(p)}{\lambda_M(p) - \lambda_R(p)}, \tag{4}$$

where $\lambda(p) \equiv -\frac{\partial D_{ij}}{\partial p_{ij}}(p, \ldots, p)$ and $\mu(p) = D(p) - (p - c - \gamma)[\lambda(p) - \lambda_M(p)]$.

Suppose now that all retail prices are set to p and all wholesale prices are set to $w(p)$, characterized by (4). By adjusting the price p_{ij}, R_j could obtain a profit, gross of fixed fees, equal to:

$$[p_{ij} - w(p) - \gamma]\, D_{ij}(p_{ij}, p, ..., p) + \sum_{h\in\mathcal{I}\setminus\{i\}} [p - w(p) - \gamma]\, D_{hj}(p_{ij}, p, ..., p).$$

The impact of a marginal increase in p_{ij} on that profit is given by:

$$\begin{aligned} D(p, ..., p) - [p - w(p) - \gamma][\lambda(p) - \lambda_M(p)] &= \mu(p) + [w(p) - c][\lambda(p) - \lambda_M(p)] \\ &= \mu(p)\frac{\lambda(p) - \lambda_R(p)}{\lambda_M(p) - \lambda_R(p)}. \end{aligned}$$

where the second equality follows from (4). As retailers are imperfect substitutes, $\lambda(p) > \lambda_R(p)$: when the price of a particular brand increases in one store, inducing consumers to buy less of that brand in that store, they only partially report the lost demand for the brand to different stores. As $\mu(p) < 0$ from Assumption C, it follows that retailers have an incentive to lower their prices if $\lambda_M(p) > \lambda_R(p)$, in which case price floors are needed to sustain p; price caps are instead needed if $\lambda_M(p) < \lambda_R(p)$.

To conclude the proof, we now introduce our last technical assumption, ensuring that the joint profit of M_i and R_j remains well-behaved when R_j faces price caps or price floors for the other brands (whether or not these constraints are binding). For any w and $p > p^*$, and any $(i,j) \in \mathcal{I} \times \mathcal{J}$, let $\hat{\mathbf{p}}_j^{ij}(p_{ij}; w, p) = (\hat{p}_{hj}^{ij}(p_{ij}; w, p))_{h \in \mathcal{I} \setminus \{i\}}$ denote the prices that R_j would like to charge on the other brands, conditional on charging p_{ij} for brand i and on facing price floors (if $\lambda_M(p) > \lambda_R(p)$) or price caps (if $\lambda_M(p) < \lambda_R(p)$) set to p on the other brands. We assume that:

Assumption D. For any $(i,j) \in \mathcal{I} \times \mathcal{J}$, any $p > p^*$, and any $w < c$ (if $\lambda_M(p) > \lambda_R(p)$) or any $w > c$ (if $\lambda_M(p) < \lambda_R(p)$), the gross joint profit of M_i and R_j, given by:

$$
\begin{aligned}
& (p_{ij} - c - \gamma) \, D_{ij} \left(\left(p_{ij}, \hat{\mathbf{p}}_j^{ij} \left(p_{ij}; w, p \right) \right), p, ..., p \right) \\
& + \sum_{k \in \mathcal{J} \setminus \{j\}} (w - c) \, D_{ik} \left(\left(p_{ij}, \hat{\mathbf{p}}_j^{ij} \left(p_{ij}; w, p \right) \right), p, ..., p \right) \\
& + \sum_{h \in \mathcal{I} \setminus \{i\}} \left(\hat{p}_{hj}^{ij} \left(p_{ij}; w, p \right) - w - \gamma \right) D_{hj} \left(\left(p_{ij}, \hat{\mathbf{p}}_j^{ij} \left(p_{ij}; w, p \right) \right), p, ..., p \right),
\end{aligned}
$$

is strictly quasi-concave in p_{ij} and maximal for a finite price level.

From (4), the required wholesale price, $w(p)$, lies below cost if if $\lambda_M(p) > \lambda_R(p)$) and above cost if instead $\lambda_M(p) > \lambda_R(p)$. The relevant price constraints (price caps or price floors) are therefore binding for $w = w(p)$ and $p_{ij} = p$. By continuity, this remains true when p_{ij} slightly departs from p. It follows that (4) still constitutes the relevant first-order condition when fixed RPM is replaced with minimum RPM (when $\lambda_M(p) > \lambda_R(p)$) or maximum RPM (when $\lambda_M(p) < \lambda_R(p)$). Assumption D then ensures that global deviations in p_{ij} are not profitable either.

D Agency versus resale: Proof of Proposition 6

In the resale model, the outcome is that of a multi-brand oligopoly in which each R_j, for $j \in J$, can produce every brand $i \in \mathcal{I}$ at cost and seeks to maximize its profit, given by $\pi_j^{Resale}(\mathbf{p}) \equiv \sum_{i\in\mathcal{I}}(p_{ij} - c_i - \gamma_j)D_{ij}(\mathbf{p})$. By assumption, the equilibrium outcome is unique (and thus symmetric), and characterized by first-order conditions. The equilibrium price is thus the unique solution to $\phi^{Resale}(p) = 0$, where, using $\lambda(p) \equiv -\frac{\partial D_{ij}}{\partial p_{ij}}(p, ..., p)$:

$$\phi^{Resale}(p) \equiv D(p) - (p - c - \gamma)[\lambda(p) - \lambda_M(p)].$$

Similarly, in the agency model, the outcome is that of a multi-store oligopoly in which each M_i, for $i \in I$, can distribute at cost in each retail location $j \in \mathcal{J}$ and seeks to maximize its profit, given by $\pi_i^{Agency}(\mathbf{p}) \equiv \sum_{j\in\mathcal{J}}(p_{ij} - c_i - \gamma_j)D_{ij}(\mathbf{p})$**.** The equilibrium price is thus the unique solution to $\phi^{Agency}(p) = 0$, where:

$$\phi^{Agency}(p) \equiv D(p) - (p - c - \gamma)[\lambda(p) - \lambda_R(p)].$$

By construction, both equilibrium prices exceed $c + \gamma$ (otherwise, firms would make a loss). The conclusion then follows from the observation that $\phi^{Agency}(c + \gamma) = \phi^{Resale}(c + \gamma) > 0$ and, in the range $p > c + \gamma$, the sign of $\phi^{Agency}(p) - \phi^{Resale}(p)$ is the same as that of $\lambda_R(p) - \lambda_M(p)$.

E Endogenous network

We focus in what follows on a symmetric case, with two manufacturers, labelled M_A and M_B for convenience, and two retailers, R_1 and R_2. Manufacturers' and retailers' unit costs are respectively denoted by $c_A = c_B = c$ and $\gamma_1 = \gamma_2 = \gamma$, and, for any price vector $\mathbf{p} \equiv (p_{A1}, p_{B1}, p_{A2}, p_{B2})$, any $i \neq h \in \{A, B\}$ and any $j \neq k \in \{1, 2\}$, the demand for brand i at store j is given by:

$$D_{ij}(\mathbf{p}) \equiv D(p_{ij}, p_{hj}, p_{ik}, p_{hk}),$$

where the function $D(.)$ is continuously differentiable. Bargaining sharing rules, too, are symmetric: $\alpha_{ij} = \alpha$ for every $(i, j) \in \mathcal{I} \times \mathcal{J}$.

E.1 Bargaining equilibria

With interlocking relationships, the manufacturers' and retailers' equilibrium profits are $\Pi_M^{IR} \equiv 2\alpha(2\pi^* - \hat{\pi}^*)$ and $\Pi_R^{IR} \equiv 2(1-\alpha)\pi^* + 2\alpha(\hat{\pi}^* - \pi^*)$, where, using symmetry, π^* and $\hat{\pi}^*$ denote the profits π_j^* and π_j^{ij} defined in Proposition 2. We now characterize, for every other channel network, the unique bargaining equilibrium based on two-part tariffs. For the sake of exposition, we assume that demand satisfies standard regularity assumption ensuring that two-part tariffs induce a smooth retail behavior. Equilibrium wholesale prices are therefore at cost (see Proposition 1bis in Appendix A): $w_{ij} = c$ for any active channel $i - j$.

E.1.1 Bilateral monopoly

When a single channel is active, say $i - j$, it generates a profit

$$\pi^m \equiv \max_p (p - c - \gamma) D(p, \infty, \infty, \infty).$$

As M_i and R_j would both obtain zero profit in case of disagreement, their equilibrium profits are respectively equal to $\Pi_M^m \equiv \alpha\pi^m$ and $\Pi_R^m \equiv (1-\alpha)\pi^m$.

E.1.2 Exclusive dealing

Suppose now that two unconnected channels are active, say $i - j$ and $h - k$. In equilibrium, each channel generates a profit $\pi^{ED} \equiv (p^{ED} - c - \gamma) D(p^{ED}, \infty, \infty, p^{ED})$, where $p^{ED} \equiv \arg\max_p (p - c - \gamma) D(p, \infty, \infty, p^{ED})$. As M_i and R_j would again obtain zero profit in case of disagreement, their equilibrium profits are respectively equal to $\Pi_M^{ED} \equiv \alpha\pi^{ED}$ and $\Pi_R^{ED} \equiv (1-\alpha)\pi^{ED}$.

E.1.3 Upstream foreclosure

In the case where a single manufacturer, say M_i, deals with both retailers, in equilibrium each channel generates a profit $\pi^{UF} \equiv (p^{UF} - c - \gamma) D(p^{UF}, \infty, p^{UF}, \infty)$, where $p^{UF} = \arg\max_p (p - c - \gamma) D(p, \infty, p^{UF}, \infty)$. If the negotiation with R_j were to break down, R_j would be excluded and obtain zero profit, whereas M_i would still obtain the fixed fee charged to the other retailer. It follows that the bilateral gains from trade are π^{UF}, and the manufacturer and each retailer's equilibrium profits are respectively equal to $\Pi_M^{UF} \equiv 2\alpha\pi^{UF}$ and $\Pi_R^{UF} \equiv (1-\alpha)\pi^{UF}$.

E.1.4 Downstream foreclosure

In the case where a single retailer, say R_j, deals with both manufacturers, in equilibrium, each channel generates a profit $\pi^{DF} \equiv \max_p (p - c - \gamma) D(p, p, \infty, \infty)$. If the negotiation with M_i were to break down, M_i would be excluded and obtain zero profit. In equilibrium, it thus obtains a share α of the bilateral gains from trade, which are here equal to $2\pi^{DF} - \pi^m$, as M_i and R_j jointly earn $2\pi^{DF} - F_{hj}$ when reaching an agreement, and $\pi^m - F_{hj}$ otherwise. In equilibrium, each manufacturer's profit is equal to $\Pi_M^{DF} \equiv \alpha(2\pi^{DF} - \pi^m)$, whereas R_j obtains $\Pi_R^{DF} \equiv 2(1-\alpha)\pi^{DF} + 2\alpha(\pi^m - \pi^{DF})$.

E.1.5 Single exclusion

Suppose finally that a single channel, say $h - k$, remains inactive. All firms are thus directly or indirectly connected, as M_i deals with both retailers, and R_j deals with both manufacturers. For the sake of exposition, we use the subscripts J, M and R to refer respectively to the joint channel of the two multi-channel firms (here, $i - j$), the other channel of the multi-channel manufacturer (here, $i - k$), and the other channel of the multi-channel retailer (here, $h - j$). The equilibrium retail prices satisfy:

$$\left(p_J^{SE}, p_R^{SE}\right) = \underset{(p_J, p_R)}{\arg\max} \left\{ (p_J - c - \gamma) D\left(p_J, p_R, p_M^{SE}, \infty\right) + (p_R - c - \gamma) D\left(p_R, p_J, \infty, p_M^{SE}\right) \right\},$$
$$p_M^{SE} = \underset{p_M}{\arg\max}\, (p_M - c - \gamma) D\left(p_M, \infty, p_J^{SE}, p_R^{SE}\right).$$

In what follows, we assume that these prices are unique. We denote by

$$\pi_m^{SE} \equiv \left(p_J^{SE} - c - \gamma\right) D\left(p_J^{SE}, p_R^{SE}, p_M^{SE}, \infty\right) + \left(p_R^{SE} - c - \gamma\right) D\left(p_R^{SE}, p_J^{SE}, \infty, p_M^{SE}\right)$$

the profit generated by the multi-channel retailer (R_j), and by

$$\pi_s^{SE} \equiv \left(p_M^{SE} - c - \gamma\right) D\left(p_M^{SE}, \infty, p_J^{SE}, p_R^{SE}\right)$$

the profit generated by the single-channel retailer (R_k). Finally, let

$$\hat{\pi}_J \equiv \max_p\, (p - c - \gamma) D\left(p, \infty, p_M^{SE}, \infty\right) \text{ and } \hat{\pi}_R = \max_p\, (p - c - \gamma) D\left(p, \infty, \infty, p_M^{SE}\right)$$

denote the profit that the multi-channel retailer (R_j) could generate by focusing instead, respectively, on the joint channel ($M_i - R_j$), and on the other channel ($M_h - R_j$). By construction, $\hat{\pi}_J < \pi_m^{SE}$ and $\hat{\pi}_R < \pi_m^{SE}$, as R_j relies here on a single channel.

From Nash bargaining (equation (2)), the fixed fees are of the form $F = \alpha\Delta$, where Δ is the extra joint profit generated by a successful negotiation. We thus have:

$$F_{ij} = \alpha\left(\pi_m^{SE} - \hat{\pi}_R\right) > 0,\ F_{hj} = \alpha\left(\pi_m^{SE} - \hat{\pi}_J\right) > 0 \text{ and } F_{ik} = \alpha\pi_s^{SE} > 0.$$

Manufacturers' profits are therefore given by $\Pi_{Mm}^{SE} \equiv \alpha(\pi_m^{SE} + \pi_s^{SE} - \hat{\pi}_R)$ and $\Pi_{Ms}^{SE} \equiv \alpha(\pi_m^{SE} - \hat{\pi}_J)$, where the subscripts Mm and Ms respectively refer to the multi-channel and single-channel manufacturers; using a similar convention, retailers' profits are given by $\Pi_{Rm}^{SE} \equiv (1-\alpha)\pi_m^{SE} + \alpha(\hat{\pi}_J + \hat{\pi}_R - \pi_m^{SE})$ and $\Pi_{Rs}^{SE} \equiv (1-\alpha)\pi_s^{SE}$.

E.2 Proof of Proposition 7

In what follows, we assume that manufacturers and retailers hold some bargaining power, that is, $\alpha \in (0, 1)$.

E.2.1 No retail competition

When retailers are local monopolies, each downstream market can be analyzed separately. Furthermore, in each market, all firms prefer to be active, and the retailer prefers carrying both brands rather than one, as:

$$\Pi_R^{DF} = 2(1-\alpha)\pi^M + 2\alpha(\pi^m - \pi^M) > (1-\alpha)2\pi^M \geq (1-\alpha)\pi^m = \Pi_R^m,$$

where the inequalities stem from imperfect brand substitutability. It follows that interlocking relationships constitutes the only CPNE network.

E.2.2 Perfect retail substitutes

When retailers are perfect substitutes, each manufacturer must deal with a single retailer, otherwise intrabrand competition would eliminate any profit, and the manufacturer could profitably deviate to exclusivity. Hence, the only candidate CPNE networks are exclusive dealing and downstream foreclosure.

As active firms obtain a positive profit, exiting the market is not a profitable deviation, and coalitions cannot deviate either by activating more than two channels, as at least one manufacturer (who has to be part of the coalition) would be dealing with both retailers and thus obtain zero profit. Furthermore, an active retailer prefers downstream foreclosure to either bilateral monopoly or exclusive dealing:

$$\Pi_R^{DF} = 2(1-\alpha)\pi^{DF} + 2\alpha(\pi^m - \pi^{DF}) > (1-\alpha)2\pi^{DF} > (1-\alpha)\max\{\pi^m, \pi^{ED}\} = \max\{\Pi_R^m, \Pi_R^{ED}\},$$

where the first inequality stems from the fact that a channel profit is maximal when all other channels are inactive, and the second one from the fact that the industry profit is greater when a single retailer carries both brands than when it carries a single brand ($2\pi^{DF} > \pi^m$) or when the two brands are carried by different retailers ($2\pi^{DF} > 2\pi^{ED}$).

We now study the other potential deviations for each candidate CPNE.

Exclusive dealing. Consider a candidate CPNE in which, say, M_i deals with R_j whereas M_h deals with R_k. In the light of the above remarks, deviations leading to fewer, or to more active channels are irrelevant, as at least one firm would exit. Likewise, deviations leading to more active channels or to upstream foreclosure are also irrelevant, as intrabrand competition would then dissipate all profits. Therefore, the only relevant deviation is for a coalition to move to downstream foreclosure. Suppose, for instance, that M_i and R_k agree to open their channel (in addition to the $h-k$ channel) and foreclose R_j (that is, M_i and R_k now deal with each other, whereas M_i stops dealing with R_j but R_k keeps dealing with M_h). Based on the above remarks, this deviation is always profitable for R_k. By contrast, it is profitable for M_i if and only if:

$$\Pi_M^{DF} = \alpha(2\pi^{DF} - \pi^m) > \alpha\pi^{ED} = \Pi_M^{ED}.$$

It follows that exclusive dealing is a CPNE network if and only if $\pi^{ED} \geq 2\pi^{DF} - \pi^m$.

Downstream foreclosure. Consider now a candidate CPNE in which the two manufacturers deal with a single common retailer, say, R_j. Using the same reasoning as above, the only relevant deviation is now for a coalition involving R_k and one manufacturer, say M_h, to move to exclusive dealing. This deviation is always profitable for R_k and is profitable for M_h if and only if $\Pi_M^{ED} > \Pi_M^{DF}$. It follows that downstream foreclosure is a CPNE network if and only if $\pi^{ED} \leq 2\pi^{DF} - \pi^m$.

E.3 Proof of Proposition 8

In what follows, we exclude the extreme cases where brands or retailers are either perfect substitutes or "local monopolies", i.e., $\mu, \rho \in (0,1)$. We also continue to assume that all firms hold some bargaining power, i.e., $\alpha \in (0,1)$.

From Section E.1, active firms always obtains a positive profit, $\Pi_R^{ED} > \Pi_R^{UF}$ and $\Pi_R^{DF} > \Pi_R^m$. In addition, for the linear demand, $\Pi_M^{ED} > \Pi_M^{DF}$, $\Pi_R^{IR} > \Pi_{Rs}^{SE}$ and:[3]

(P1) $\Pi_M^{ED} \gtrless \Pi_{Mm}^{SE} \Longleftrightarrow \rho \gtrless \rho^*(\mu)$, where $\rho^*(\mu) \in (0,1)$ and is strictly decreasing in μ;[4]

(P2) $\rho \leq \rho^*(\mu) \Longrightarrow \{\Pi_{Rm}^{SE} > \Pi_R^{ED}$ and $\Pi_M^{IR} > \Pi_{Ms}^{SE}\}$,[5]

(P3) $\rho \geq \rho^*(\mu) \Longrightarrow \Pi_M^{ED} > \Pi_M^{IR}$.

For the sake of exposition, we characterize a deviation by the network configuration that it implements. Larger coalitions can achieve a broader range of deviations but are more fragile, as they also face a broader set of sub-coalitions. Furthermore, exit is never profitable. Hence, without loss of generality we can restrict attention to deviations that cannot be achieved by smaller coalitions, and in which all deviating firms remain active. We can further focus on equilibrium strategies in which firms only want to deal with their equilibrium partners, as this minimizes the number of possible deviations.

At least two channels must be active as any excluded vertical pair could profitably activate its channel. Furthermore, as $\Pi_R^{ED} > \Pi_R^{UF}$ and $\Pi_M^{ED} > \Pi_M^{DF}$, foreclosure is also ruled out (both upstream and downstream), as the excluded firm, together with either potential partner, could form a mutually profitable coalition that switches to exclusive dealing. Hence, the only candidate networks for a CPNE are: exclusive dealing, single exclusion and interlocking relationships. We consider them in turn.

[3]Specifically, for the linear demand: (i) $\pi_m^{SE} \geq \pi^{UF}$, implying $\Pi_{Rm}^{SE} > \Pi_R^{UF}$; (ii) $\pi^{ED} > 2\pi^{DF} - \pi^m$, implying $\Pi_M^{ED} > \Pi_M^{DF}$ for any $\alpha > 0$; and (iii) $2\pi^* > \pi_s^{SE}$, which, together with $\hat{\pi}^* > \pi^*$ (from brand substitutability), implies $\Pi_R^{IR} > \Pi_{Rs}^{SE}$.

[4]The threshold $\rho^*(\mu)$ is the unique solution in $[0,1]$ to $\pi^{ED} = \pi_m^{SE} + \pi_s^{SE} - \hat{\pi}_R$, which amounts to: $(4+\mu^4\rho^4)(4-8\rho+3\rho^2-\rho^3)+4\mu^2\rho^2(1-\rho+2\rho^2)+\mu^4\rho^4(1+\rho)-4\mu\rho(2-\mu\rho+\mu^2\rho^2)(6-11\rho+6\rho^2-\rho^3) = 0$.

[5]Specifically: (i) $\rho \leq \rho^*(\mu)$ implies $\pi_m^{SE} > \pi^{ED}$, which, together with $\hat{\pi}_J + \hat{\pi}_R > \pi_m^{SE}$ (from brand substitutability), implies $\Pi_{Rm}^{SE} > \Pi_R^{ED}$; and (ii) $\Pi_M^{IR} > \Pi_{Ms}^{SE}$ amounts to $2(2\pi^* - \hat{\pi}^*) > \pi_m^{SE} - \hat{\pi}_J$, which holds if and only if $\rho < \rho^*(0)$, and thus holds for any $\rho \leq \rho^*(\mu)$.

E.3.1 Exclusive dealing

Consider a candidate CPNE in which M_i deals exclusively with R_j, say, and likewise for M_h and R_k. This constitutes a Nash equilibrium, as unilateral deviations can only lead to exit. Furthermore: (i) any deviation collectively achievable by the manufacturers, by the retailers, or by a pair of existing partners, can also be achieved unilaterally; and (ii) any deviation by a three-firm coalition can also be achieved by a two-firm coalition. Therefore, we only need to consider deviations by the coalition $\{M_i, R_k\}$ (or by symmetry, $\{M_h, R_j\}$) or by the *grand coalition* (all four players).

Deviations by the coalition $\{M_i, R_k\}$. We can focus on deviations in which M_i and R_k activate their channel, as all others can be achieved unilaterally. Furthermore:

(i) deviating to downstream foreclosure (channels $i - k$ and $h - k$) is not profitable for M_i, as $\Pi_M^{ED} > \Pi_M^{DF}$; likewise, deviating to upstream foreclosure (channels $i - j$ and $i - k$) is not profitable for R_k, as $\Pi_R^{ED} > \Pi_R^{UF}$;

(ii) a deviation to bilateral monopoly (channel $i - k$ only) is not self-enforcing, as R_k prefers downstream foreclosure ($\Pi_R^{DF} > \Pi_R^m$), which it could achieve by maintaining channel $h - k$ as well.

The only remaining deviation is to single exclusion, in which only channel $h-j$ remains inactive. Starting from this configuration, the most relevant unilateral deviation is for both members of the coalition to switch back to exclusive dealing, which is preferred to downstream foreclosure by M_i and to upstream foreclosure by R_k (see (i) above). Hence, the coalition's deviation to single exclusion is self-enforcing whenever it is profitable for both of its members; from (P1) and (P2), this is the case if and only if $\rho < \rho^*(\mu)$.

Deviations by the grand coalition. The only deviation uniquely achievable by the grand coalition is to interlocking relationships. From (P3), this deviation is not profitable for the manufacturers when $\rho \geq \rho^*(\mu)$.

Summing-up, exclusive dealing is a CPNE network if and only if $\rho \geq \rho^*(\mu)$.

E.3.2 Interlocking relationships

Consider now a candidate CPNE with interlocking relationships. All channels being active, every firm is willing to deal with every partner. Hence, any deviation by a coalition of three or four firms can be achieved by a smaller coalition. We therefore restrict attention to unilateral and pairwise deviations.

Together, the manufacturers could deviate to exclusive dealing. From (P3), this deviation is profitable if $\rho \geq \rho^*(\mu)$, in which case it is also self-enforcing, as a manufacturer cannot unilaterally increase its profit: switching to the other retailer leads to downstream foreclosure and yields $\Pi_M^{DF} < \Pi_M^{ED}$, whereas adding a second channel leads to single exclusion and yields $\Pi_{Mm}^{SE} \leq \Pi_M^{ED}$ (from (P1)).

Conversely, unilateral deviations are not profitable when $\rho < \rho^*(\mu)$: as dropping a channel induces single exclusion, a deviating retailer would obtain $\Pi_{Rs}^{SE} < \Pi_R^{IR}$, whereas a deviating manufacturer would obtain $\Pi_{Ms}^{SE} < \Pi_M^{IR}$ (from (P2)).

Furthermore, pairwise deviations are not self-enforcing when $\rho < \rho^*(\mu)$. As non-members' channel remains active, the relevant deviations (for which both coalition members also remain active) are exclusive dealing and foreclosure; however:

- exclusive dealing can be achieved by any pairwise coalition but would be destabilized by a switch to single exclusion (which any member of the coalition could achieve by maintaining both of its channels), as $\Pi_{Mm}^{SE} > \Pi_M^{ED}$ (from (P1)) and $\Pi_{Rm}^{SE} > \Pi_R^{ED}$ (from (P2));
- downstream foreclosure can only be achieved by the coalition of manufacturers and would be destabilized by a switch to exclusive dealing (which any manufacturer could achieve by switching to the other retailer), as $\Pi_M^{ED} > \Pi_M^{DF}$;
- upstream foreclosure can only be achieved by the coalition of retailers and would also be destabilized by a switch to exclusive dealing (which any retailer could achieve by switching to the other manufacturer), as $\Pi_R^{ED} > \Pi_R^{UF}$.

Interlocking relationships is thus a CPNE network if and only if $\rho < \rho^*(\mu)$.

E.3.3 Single exclusion

Finally, consider a candidate CPNE with only one inactive channel, say $h - j$. When $\rho > \rho^*(\mu)$, M_i can profitably drop channel $i - k$, thus inducing exclusive dealing, as $\Pi_M^{ED} > \Pi_{Mm}^{SE}$. When instead $\rho \leq \rho^*(\mu)$, the coalition $\{M_h, R_j\}$ can profitably activate the missing channel, thus inducing interlocking relationships, as $\Pi_R^{IR} > \Pi_{Rs}^{SE}$ and $\Pi_M^{IR} > \Pi_{Ms}^{SE}$ (from (P2)). Being mutually profitable, this deviation is also self-enforcing, as the most relevant deviation would be to switch back to single exclusion.

It follows that single exclusion never constitutes a CPNE network.

The insights of Proposition 8 are illustrated in Figure 1.

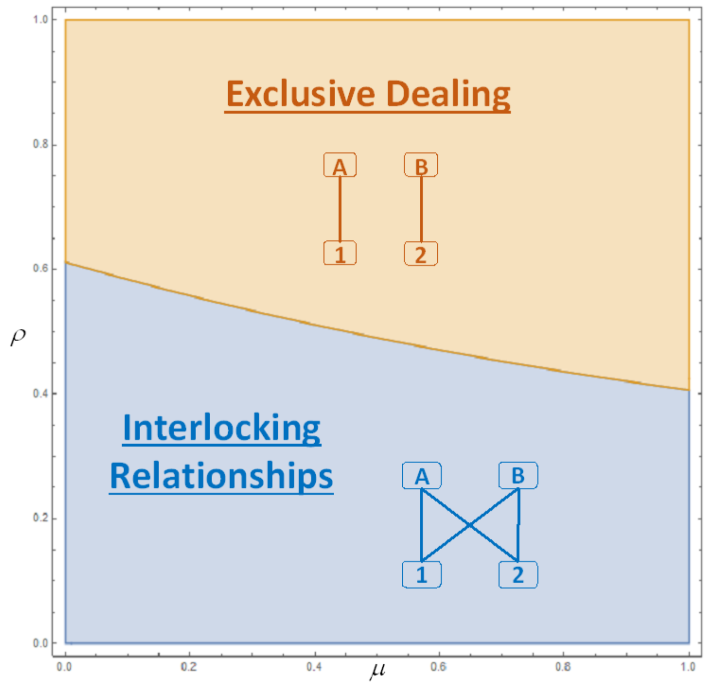

Figure 1: Equilibrium distribution network

F Mergers

F.1 Downstream merger: Proof of Proposition 9

Equilibrium distribution network. As before, we restrict attention to smooth retail responses. This amount here to assuming that R's monopolistic price response,

$$\mathbf{p}^{\mathbf{m}}(\mathbf{w}) \equiv \arg\max_{\mathbf{p}} \sum_{i\in\{A,B\}} \sum_{j\in\{1,2\}} (p_{ij} - w_i - \gamma_j) D_{ij}(\mathbf{p})$$

is unique and differentiable. From Proposition 1bis, the equilibrium tariffs are therefore cost-based: $w_{ij} = c$ for $i \in \{A, B\}$ and $j \in \{1, 2\}$.

In the linear demand example, it can then be easily shown that R prefers to carry both brands at both locations and, conversely, a manufacturer strictly prefers to distribute its brand at both locations, regardless of whether the rival brand is also present. It follows that the unique coalition-proof distribution network involves interlocking relationships.[6]

Impact on prices and consumer surplus. The retail merger creates a downstream monopolist that sells all brands at all stores but offers them at industry-wide monopoly prices. Hence, the merger harms consumers whenever all brands are already sold at all stores pre-merger. The merger may however benefit consumers by expanding the distribution network. This is for instance the case when retailers are close substitutes, so that exclusive dealing arises pre-merger, and brands do not compete (maximal differentiation between brands), as the two brands are then already sold at monopoly prices pre-merger; hence, in that case consumers are not affected by any price increase, but benefit post-merger from increased variety as they can find both brands at both locations.

For the linear demand specification, interlocking relationships arise pre-merger whenever $\rho < \rho^* (\mu)$, in which case the merger harms consumers and society, as it raises prices without any off-setting benefit in variety. When instead $\rho \geq \rho^* (\mu)$, the merger expands the distribution network, from exclusive dealing to interlocking relationships. In that case, there exist two thresholds $\hat{\mu}_S (\rho)$ and $\hat{\mu}_W (\rho)$ (where $\hat{\mu}_S (\rho)$ and $\hat{\mu}_W (\rho)$ are decreasing function of ρ such that $\hat{\mu}_S (1) = \hat{\mu}_W (1) = 0$ and $\hat{\mu}_S (\rho) < \hat{\mu}_W (\rho)$ for $\rho < 1$)[7] such that, despite price increases, by expanding the distribution network the downstream merger increases consumer surplus if and only if $\mu < \hat{\mu}_S (\rho)$, and increases total welfare if and only if $\mu < \hat{\mu}_W (\rho)$. Hence, the downstream merger benefits consumer if and only if: (i) retailers are close enough substitutes, namely, $\rho > \rho^* (\mu)$, so that the pre-merger distribution network involves exclusive dealing; and yet the combination of brand and retail differentiation yields prices that are so high that increasing them further to the monopoly level does not offset the benefit from expanding the network. These insights are illustrated by Figure 2.

[6] For a detailed analysis, see Rey and Vergé (2019).

[7] The threshold $\hat{\mu}_S (\rho)$ is the unique solution in $[0, 1]$ to:

$$2 (1 - \rho) - 2 (1 + \rho) \mu - 3\rho^2\mu^2 + \rho^3\mu^3 = 0;$$

whereas the threshold $\hat{\mu}_W (\rho)$ is the unique solution in $[0, 1]$ to:

$$6 (1 - \rho) - 2 \left(3 + \rho - 2\rho^2\right) \mu + (4 - 5\rho) \mu^2 + 3\rho^3\mu^3 = 0.$$

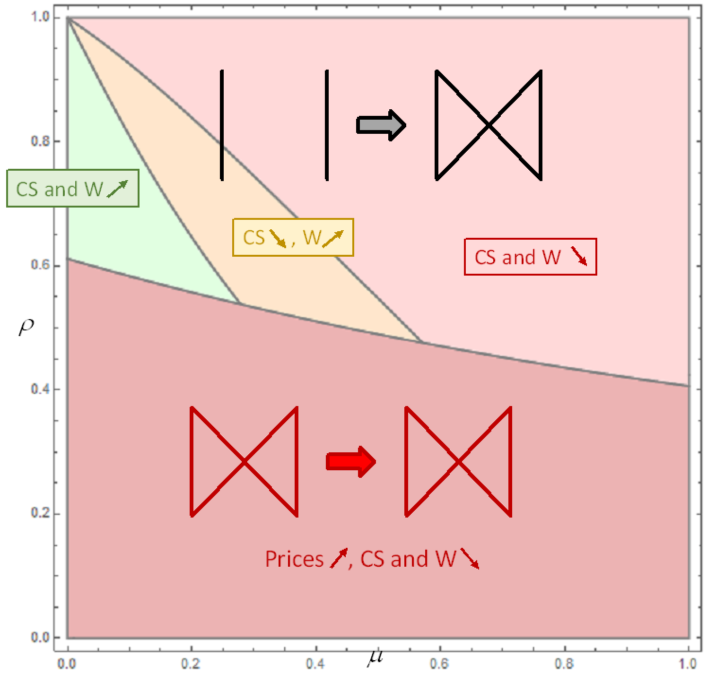

Figure 2: Impact of a downstream merger

F.2 Upstream Merger: Proof of Proposition 10

Equilibrium distribution network. A merger between M_A and M_B creates a multi-brand monopolist M. To fix ideas, we assume that it offers each R_j a unique tariff $t_j(q_{Aj}, q_{Bj})$, and restrict again attention to smooth retail behavior, which requires retail prices responses to be differentiable and the diversion matrix to be nonsingular. Proposition 1bis then implies that equilibrium tariffs are cost-based.

If the negotiation between M and R_j were to break-down, R_j would be excluded whereas M would still obtain the fixed fee F_k. It follows that, in any given network, M obtains a share α of the profit generated by each active channel. Hence, decisions about activating or not a channel are independent of α. In addition, the per-channel profits are the same as in the pre-merger case.

For the linear demand, regardless of its rival's strategy, a retailer always prefers to activate as many channels as possible. Therefore, the CPNE network must be M's preferred one. It is then straightforward to show that there exist two thresholds, $\tilde{\rho}(\mu)$

and $\bar{\rho}(\mu)$, which are decreasing in μ and satisfy $\rho^*(\mu) < \tilde{\rho}(\mu) < \bar{\rho}(\mu)$ for $\mu \in (0,1]$, $\tilde{\rho}(0) = \rho^*(0)$ and $\bar{\rho}(0) = 1$, such that the CPNE network involves:[8]

- interlocking relationships if and only if $\rho \leq \tilde{\rho}(\mu)$;
- exclusive dealing if and only if $\tilde{\rho}(\mu) \leq \rho < \bar{\rho}(\mu)$;
- downstream foreclosure if and only if $\rho \geq \bar{\rho}(\mu)$.

Impact on prices and consumer surplus. For any given distribution network, the merger affects neither wholesale nor retail prices, but only the division of profit. The merger may however alter the equilibrium distribution network and therefore have an impact on variety and prices, and thus on consumer surplus and welfare.

- When $\rho \geq \bar{\rho}(\mu)$ (i.e., retailers are close substitutes), M prefers selling both brands to a single common retailer (downstream foreclosure) so as to avoid downstream competition and increase industry profit rather than selling each one of them to a different retailer (exclusive dealing). In that case, the merger has little impact on variety (two channels are available pre- as well as post-merger, which involve distinct brands and either the same retailer or two closely substitutable retailers), but raises prices by avoiding downstream competition. Hence, it reduces consumer surplus and total welfare.
- When $\rho^*(\mu) \leq \rho < \tilde{\rho}(\mu)$, M extends the distribution network and opts for interlocking relationships instead of exclusive dealing, which increases consumer surplus and total welfare by both increasing variety and decreasing prices.

Hence, for the linear demand specification, the merger may either have no impact on consumer surplus and welfare (when the network is unaffected), a positive impact (for a small set of parameter with intermediate degree of substitution between retailers), or a negative impact (when retailers are close substitutes). These insights are illustrated by Figure 3.

[8]The threshold $\tilde{\rho}(\mu)$ is the unique solution in $[0,1]$ to
$4(1-\mu) - 4(2-\mu-\mu^2)\rho + 3(1+\mu-2\mu^2)\rho^2 - (1+4\mu-3\mu^2-2\mu^3)\rho^3 + \mu(1+\mu-2\mu^2)\rho^4 = 0$,
whereas $\bar{\rho}(\mu)$ is the unique solution in $[0,1]$ to: $4 - 4(1+\mu)\rho + 3\mu\rho^2 - \mu^2\rho^3 = 0$.

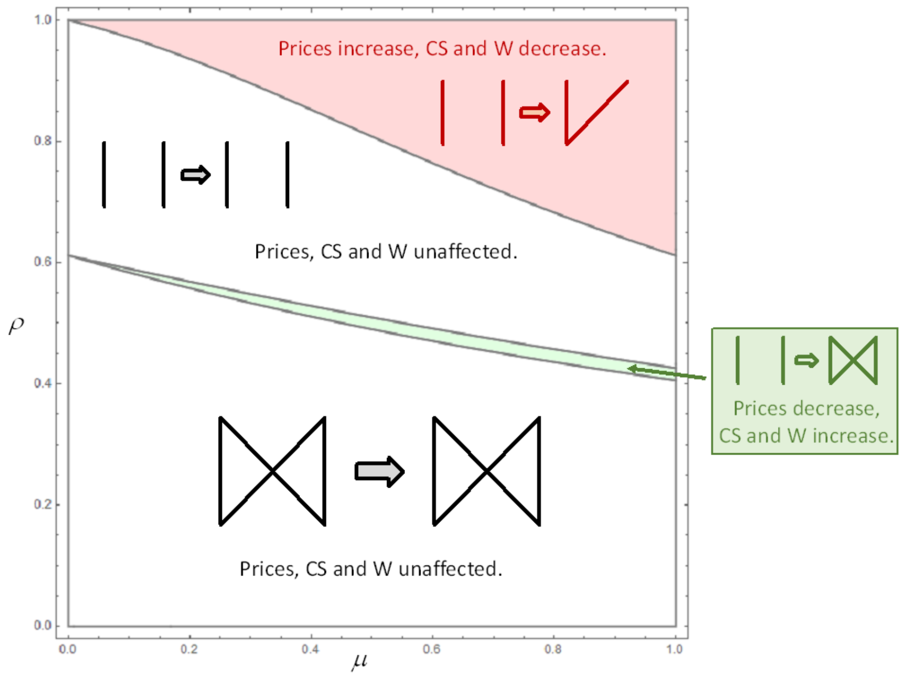

Figure 3: Impact of an upstream merger

G Public contracting

G.1 Proof of Proposition 12

Part (i). Fix a bargaining equilibrium with public two-part tariffs in which the retail price response depends only on wholesale prices, $\mathcal{B} = \left\{\mathbf{p}^R(\mathbf{w}), \mathbf{t}^{\mathbf{e}}, \mathbf{p}^{\mathbf{e}}\right\}$, and, for any $(i,j) \in \mathcal{I} \times \mathcal{J}$, denote the equilibrium profits of M_i and R_j by (using $q^e_{hk} = D^e_{hk}(\mathbf{p}^{\mathbf{e}})$):

$$\Pi^e_{M_i} \equiv \sum_k \left[(w_{ik} - c_i)q^e_{ik} + F^e_{ik}\right] \text{ and } \Pi^e_{R_j} \equiv \sum_h \left[(p^e_{hj} - w^e_{hj} - \gamma_j)q^e_{hj} - F^e_{hj}\right].$$

Likewise, in case of a negotiation break-down between M_i and R_j, we adopt the convention $w_{ij} = p^R_{ij} = \infty$, and denote the resulting prices by $\mathbf{p}^{ij} = \mathbf{p}^R\left(\infty, \mathbf{w}^{\mathbf{e}}_{-i,j}, \mathbf{w}^{\mathbf{e}}_{-j}\right)$, and the resulting profits by (using $q^{ij}_{hk} = D_{hk}(\mathbf{p}^{ij})$)

$$\Pi^{ij}_{M_i} \equiv \sum_{k \in \mathcal{J}\setminus\{j\}} \left[(w_{ik} - c_i)q^{ij}_{ik} + F^e_{ik}\right] \text{ and } \Pi^{ij}_{R_j} \equiv \sum_{h \in \mathcal{I}\setminus\{i\}} \left[(p^{ij}_{hj} - w^e_{hj} - \gamma_j)q^{ij}_{hj} - F^e_{hj}\right].$$

To construct a corresponding equilibrium for game Γ, we first define, for any $(i,j) \in \mathcal{I} \times \mathcal{J}$ and $\theta_{ij} \in \Theta_{ij}$, the tariff $\hat{t}^{\theta_{ij}}_{ij} \equiv t^e_{ij} + \hat{F}^{\theta_{ij}}_{ij}$, where

$$\hat{F}_{ij}^{\theta_{ij}} \equiv \begin{cases} \Pi_{R_j}^e - \Pi_{R_j}^{ij} & \text{if } \theta_{ij} = M_i^j, \\ -\left(\Pi_{M_i}^e - \Pi_{M_i}^{ij}\right) & \text{if } \theta_{ij} = R_j^i. \end{cases}$$

By construction, the tariffs $(\hat{t}_{ij}^{\theta_{ij}})_{\theta_{ij}\in\Theta_{ij}}$ coincide with t_{ij}^e in expectation:

Lemma G.1 (bargaining fees: public two-part tariffs)**.** *For every* $(i,j) \in \mathcal{I} \times \mathcal{J}$, $E_{\theta_{ij}}[\hat{t}_{ij}^{\theta_{ij}}] = t_{ij}^e$.

Proof. Together, the definition of $(\hat{F}_{ij}^{\theta_{ij}})_{\theta_{ij}\in\Theta_{ij}}$ and the Nash bargaining rule yield: $E_{\theta_{ij}}[\hat{F}_{ij}^{\theta_{ij}}] = \alpha_{ij}\hat{F}_{ij}^{M_i^j} + (1-\alpha_{ij})\hat{F}_{ij}^{R_j^i} = \alpha_{ij}(\Pi_{R_j}^e - \Pi_{R_j}^{ij}) - (1-\alpha_{ij})(\Pi_{M_i}^e - \Pi_{M_i}^{ij}) = 0.$ ∎

Let $\hat{\mathbf{p}}^{\boldsymbol{\theta}} \equiv \mathbf{p}^{\mathbf{e}}$ for every $\boldsymbol{\theta} \in \Theta$ and, for every $j \in \mathcal{J}$, $\hat{\mathbf{t}}_j^{\boldsymbol{\theta}_j} \equiv (\hat{t}_{ij}^{\boldsymbol{\theta}_{ij}})_{i\in\mathcal{I}}$. We now show that $\{\mathbf{p}^R(\mathbf{w}), (\hat{\mathbf{t}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}})_{\boldsymbol{\theta}\in\Theta}\}$ constitutes a subgame perfect equilibrium $\mathcal{E}$ of game Γ^P yielding the same expected profits as $\mathcal{B}$. By construction, the wholesale and retail prices are the same as in $\mathcal{B}$, regardless of which side makes the offers, even in case of a negotiation break-down. Furthermore, from Lemma G.1, the expected tariffs coincide with $\mathbf{t}^e$; hence, $\mathcal{E}$ also yields the same expected profits and disagreement payoffs as $\mathcal{B}$ (where the expectation refers to which side gets to make the offers).

In stage 2, for any publicly observed wholesale prices $\mathbf{w}$, the retail price response $\mathbf{p}^R(\mathbf{w})$ is by definition a Nash equilibrium of the continuation game. Turning to stage 1, consider the bilateral negotiation between M_i and R_j, for some $(i,j) \in \mathcal{I} \times \mathcal{J}$. Their agents, M_i^j and R_j^i, expect all other agents to negotiate the equilibrium tariffs, which satisfy $E_{\theta_{hk}}[\hat{t}_{hk}^{\theta_{hk}}] = t_{hk}^e$ for any $(h,k) \in \mathcal{I} \times \mathcal{J}$. Hence, when signing a tariff $t_{ij} = \{w_{ij}, F_{ij}\}$ they anticipate for their firms an expected joint profit equal to:

$$\begin{aligned} \Pi_{M_i-R_j}^R(w_{ij}) \;\equiv\;& [p_{ij}^R(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}}) - c_i - \gamma_j] D_{ij}(\mathbf{p}^R(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}})) \\ &+ \sum_{k\in\mathcal{J}\setminus\{j\}} [(w_{ik}^e - c_i) D_{ik}(\mathbf{p}^R(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}})) + F_{ik}^e] \\ &+ \sum_{h\in\mathcal{I}\setminus\{i\}} \left\{ \begin{array}{l} [p_{hj}^R(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}}) - w_{hj}^e - \gamma_j] \\ \times D_{hj}(\mathbf{p}^R(w_{ij}, \mathbf{w}_{-i,j}^{\mathbf{e}}, \mathbf{w}_{-j}^{\mathbf{e}})) - F_{hj}^e \end{array} \right\}, \end{aligned} \tag{5}$$

which coincides with the bilateral joint profit that M_i and R_j seek to maximize in $\mathcal{B}$. Hence, the selected agent, θ_{ij}, is willing to choose $w_{ij} = w_{ij}^e$ and set the fixed fee so as to leave the other agent indifferent between accepting or rejecting the offer, which is achieved by charging $F_{ij}^e + \hat{F}_{ij}^{\theta_{ij}}$.

Part (ii). Fix a subgame perfect equilibrium of game Γ^P in two-part tariffs of the form $\mathcal{E} = \{\hat{\mathbf{p}}^R(\mathbf{w}), (\hat{\mathbf{t}}^{\boldsymbol{\theta}} = \{\hat{\mathbf{w}}, \hat{\mathbf{F}}^{\boldsymbol{\theta}}\})_{\boldsymbol{\theta}\in\Theta}, (\hat{\mathbf{p}}^{\boldsymbol{\theta}} = \hat{\mathbf{p}})_{\boldsymbol{\theta}\in\Theta}\}$, and consider the fixed fees $\mathbf{F^e} = (F^e_{ij})_{(i,j)\in\mathcal{I}\times\mathcal{J}}$ where, for every $(i,j) \in \mathcal{I}\times\mathcal{J}$:

$$F^e_{ij} = E_{\theta_{ij}}\left[\hat{F}^{\theta_{ij}}_{ij}\right] = \alpha_{ij}\hat{F}^{M^j_i}_{ij} + (1-\alpha_{ij})\,\hat{F}^{R^i_j}_{ij}. \tag{6}$$

We now show that $\mathcal{B} \equiv \{\hat{\mathbf{p}}^R(\mathbf{w}), \mathbf{t^e} = \{\hat{\mathbf{w}}, \mathbf{F^e}\}, \mathbf{p^e} = \hat{\mathbf{p}}\}$ constitutes a bargaining equilibrium – by construction, it gives all firms the same expected profits as $\mathcal{E}$.

By definition, in stage 2, for any publicly observed wholesale prices $\mathbf{w}$, the retail price response $\hat{\mathbf{p}}^R(\mathbf{w})$ is a Nash equilibrium of the continuation game. We now turn to stage 1, and study the bilateral negotiation between M_i and R_j, for some $(i,j) \in \mathcal{I}\times\mathcal{J}$. In $\mathcal{E}$, the tariff offered by the selected agent, θ_{ij}, maximizes the expected profit of its firm, among those that are acceptable by the other agent. As profits can be shared through fixed fees (which do not affect retailers' pricing decisions), it follows that the wholesale price $\hat{w}_{ij}$ maximizes the expected joint profit of M_i and R_j, given all the other equilibrium tariffs and the retail price response. As the equilibrium wholesale prices negotiated with the other firms, $(\hat{w}_{ik})_{k\neq j}$ and $(\hat{w}_{hj})_{h\neq i}$, do not depend on which side gets to make the offers, and the equilibrium expected fixed fees are equal to $(F^e_{ik})_{k\neq j}$ and $(F^e_{hj})_{h\neq i}$, it follows that $w_{ij} = \hat{w}_{ij}$ maximizes $\Pi^R_{M_i-R_j}(w_{ij})$, given by (5). To conclude the proof, it suffices to note that, by construction, the fixed fees given by (6) share the gains from trade according to the Nash bargaining rule.

G.2 Proof of Proposition 13

Fix a symmetric equilibrium with wholesale prices $\mathbf{w}^P = (w^P, ..., w^P)$. The uniqueness of the price response $\mathbf{p}^R(\cdot)$ ensures that the retail price equilibrium is also symmetric: $\mathbf{p}^P \equiv \mathbf{p}^R(\mathbf{w}^P) = (p^P, ..., p^P)$, where $p_{ij} = p^P$ maximizes:

$$\left(p_{ij} - w^P - \gamma\right) D_{ij}\left(p_{ij}, \mathbf{p}^P_{-i,j}, \mathbf{p}^P_{-j}\right) + \sum_{h\in\mathcal{I}\setminus\{i\}} \left(p^P - w^P - \gamma\right) D_{hj}\left(p_{ij}, \mathbf{p}^P_{-i,j}, \mathbf{p}^P_{-j}\right).$$

Letting $q^P \equiv D_{ij}(\mathbf{p}^P)$ and $d^P \equiv p^P - w^P - \gamma$ denote the equilibrium quantity and downstream margin, we thus have:

$$0 = q^P + d^P \sum_{h\in\mathcal{I}} \frac{\partial D_{hj}}{\partial p_{ij}}\left(\mathbf{p}^P\right) = q^P + d^P \sum_{h\in\mathcal{I}} \frac{\partial D_{ij}}{\partial p_{hj}}\left(\mathbf{p}^P\right),$$

where the first equality stems from the optimality of p_{ij} and the second one follows from symmetry. As $q^P > 0$ and increasing all of R_j's prices reduces the demand for channel $i - j$, it follows that the equilibrium downstream margin is positive:

$$d^P > 0. \tag{7}$$

Consider now the bilateral negotiation between M_i and R_j, for some $(i, j) \in \mathcal{I} \times \mathcal{J}$. In their bilateral negotiation, M_i and R_j choose w_{ij} so as to maximize their (variable) joint profit, which, using $u^P \equiv w^P - c$, $\mathbf{p}^r(w_{ij}) \equiv \mathbf{p}^R(w_{ij}, \mathbf{w}^P_{-i,j}, \mathbf{w}^P_{-j})$, and:

$$\pi^r_j(w_{ij}) \equiv \max_{\mathbf{p}_j}\{(p_{ij} - w_{ij} - \gamma)D_{ij}(\mathbf{p}_j, \mathbf{p}^r_{-j}(w_{ij})) + \sum_{h \in \mathcal{I}\setminus\{i\}} (p_{hj} - w^P - \gamma)D_{hj}(\mathbf{p}_j, \mathbf{p}^r_{-j}(w_{ij}))\},$$

can be expressed as:

$$\pi^r_j(w_{ij}) + (w_{ij} - c)D_{ij}(\mathbf{p}^r(w_{ij})) + u^P \sum_{k \in \mathcal{J}\setminus\{j\}} D_{ik}(\mathbf{p}^r(w_{ij})).$$

The equilibrium wholesale price $w_{ij} = w^P$ thus satisfies:

$$\frac{d\pi^r_j}{dw_{ij}}(w^P) + q^P + u^P \frac{dQ^r_{M_i}}{dw_{ij}}(w^P) = 0,$$

where $Q^r_{M_i}(w_{ij}) \equiv \sum_{k \in \mathcal{J}} D_{ik}(\mathbf{p}^r(w_{ij}))$; using the envelope theorem then yields:

$$u^P = d^P \frac{\sum_{h \in \mathcal{I}} \sum_{k \in \mathcal{J}\setminus\{j\}} \sum_{g \in \mathcal{I}} \frac{\partial D_{hj}}{\partial p_{gk}}(\mathbf{p}^P) \frac{dp^r_{gk}}{dw_{ij}}(w^P)}{-\frac{dQ^r_{M_i}}{dw_{ij}}(w^P)} > 0, \tag{8}$$

where the inequality stems from (7), together with product substitutability and the fact that, from Assumption A^P, raising w_{ij} increases all retail prices and reduces M_i's total quantity, $Q^r_{M_i}(w_{ij})$. As raising wholesale prices increases retail ones, this in turn implies that the equilibrium retail prices are above the competitive level: $p^P > p^*$.

Alternatively, the joint variable profit of M_i and R_j can be expressed as:

$$\Pi^r(w_{ij}) - u^P \sum_{h \in \mathcal{I}\setminus\{i\}} Q^r_{M_h}(w_{ij}) - d^P \sum_{k \in \mathcal{J}\setminus\{j\}} Q^r_{R_k}(w_{ij}),$$

where $Q^r_{R_k}(w_{ij}) \equiv \sum_{h \in \mathcal{I}} D_{hk}(\mathbf{p}^r(w_{ij}))$, and $\Pi^r(w_{ij}) \equiv (u^P + d^P) \sum_{k \in \mathcal{J}} Q^r_{R_k}(w_{ij})$ denotes the industry profit in the continuation Nash equilibrium $\mathbf{p}^r(w_{ij})$. It follows that the equilibrium wholesale price $w_{ij} = w^P$ satisfies:

$$\frac{d\Pi^r}{dw_{ij}}(w^P) = u^P \sum_{h \in \mathcal{I} \setminus \{i\}} \frac{dQ^r_{M_h}}{dw_{ij}}(w^P) + d^P \sum_{k \in \mathcal{J} \setminus \{j\}} \frac{dQ^r_{R_k}}{dw_{ij}}(w^P) > 0,$$

where the inequality stems from (7) and (8), together with the property that, from Assumption A^P, raising w_{ij} increases the total quantities sold by every other manufacturer M_h, for $h \neq i$, and by every other retailer R_k, for $k \neq j$.

To conclude the proof, let $\Pi(p)$ denote the industry profit for retail prices all equal to p, $\Pi^R(\mathbf{w})$ denote the industry profit in the Nash equilibrium $\mathbf{p}^R(\mathbf{w}^P)$, and $p^S(w)$ denote the symmetric Nash equilibrium retail price for wholesale prices all equal to w. By construction, we have

$$\Pi'(p^P)\frac{dp^S}{dw}(w^P) = \left.\frac{d}{dw}\Pi(p^S(w))\right|_{w=w^P} = \left.\frac{d}{dw}\Pi^R(w, ..., w)\right|_{w=w^P} = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \frac{d\Pi^r}{dw_{ij}}(w^P) > 0.$$

As Nash equilibrium prices are increasing in all wholesale prices, $p^S(w)$ also increases with w; hence, $\Pi'(p^P) > 0$. As the assumed concavity of the industry profit function implies that of $\Pi(p)$, it follows that p^P lies below the monopoly level: $p^P < p^M$.