Liquidity Supply across Multiple Trading Venues [1]

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Abstract

Financial markets are increasingly fragmented. How to supply liquidity in this environment? Using an inventory model, we analyze how two strategic intermediaries compete across two venues that can be hit simultaneously by liquidity shocks of equal or opposite signs. Although order flow is fragmented ex-ante, we show that intermediaries might strategically consolidate it ex-post, improving global liquidity. We also find that local spreads co-move together across venues as a result of global inventory management. Using Euronext proprietary data, we uncover new evidence of inventory control across venues and find that local spreads vary in a way uniquely predicted by the model.

Keywords: Market fragmentation, multi-venue market-making, bid-ask spreads

JEL Classification code: G10, G12, G20

A "market making strategy" is defined as a *"strategy involving posting firm, simultaneous two-way quotes [...] on a single trading venue or across different trading venues, with the result of providing liquidity on a regular and frequent basis to the overall market"*. Directive 2014/65/UE, MiFID II, May 15, 2014

1 Introduction

In the last decade, advances in technology and changes in regulation both in the U.S. (RegNMS) and in Europe (MiFID) have fostered the proliferation of alternative trading venues. As a result, it has become much easier for intermediaries to engage in market-making simultaneously across more than one trading venue. For instance, KCG Holdings Inc., one of the largest U.S. trading firms, trades NYSE-listed securities in a broad set of trading platforms which include ARCA, GETMATCHED, BATS-Z, NYSE, EDGA, NASDAQ, BATS-Y, BX, or LIGHTPOOL. Recent empirical evidence (e.g., Brogaard et al, 2014; Jovanovic and Menkveld, 2015; Menkveld, 2013; van Kervel, 2014) shows that high frequency traders, namely financial institutions which have invested in high speed trading capacity, informally undertake this multi-venue market-making role.

In this paper, we develop a market-making model to analyze how risk-averse intermediaries strategically supply liquidity across multiple trading venues. We test the predictions of our model using a proprietary dataset from Euronext on multi-traded stocks, in which we can uniquely identify financial institutions involved in multi-venue market-making strategies and compute their inventory across venues.

Intuitively, when there exists a single market and a market-maker is in a long position, she revises quotes downward to increase the chances to shed some of her risky position. In a fragmented environment, the size of this downward price revision should however take into account what may happen in other venues. Her quoting aggressiveness within one venue should reflect her willingness to absorb liquidity shocks at other venues, as well as the degree of competition in those venues.

We develop this intuition using an inventory model based on Ho and Stoll (1983), in which two risk averse market-makers compete to simultaneously post prices in two venues that are exogenously hit by buy or sell liquidity shocks. We introduce an asymmetry by assuming that the venue termed as the dominant market receives a larger shock than the alternative venue termed as the satellite market. Liquidity shocks might be of the same signs or of opposite signs. We call the sum of liquidity shocks across venues the global order flow.

When liquidity shocks have the same sign across venues, an intermediary faces a "dual liability risk": in case her quotes are simultaneously hit, the market-maker executes, say, a cumulated buy transaction. One might expect that the premium due to this additional risk leads to larger spreads, but this is not always the case. The market-maker is willing to consolidate the global order flow when her inventory is very large since it allows her to better lay off her risky inventory. She faces, however, the competition of the opponent that might choose to compete in a single venue and not in both of them. This forces the market-maker to set very aggressive quotes across venues to be sure to execute the global order flow. In this case, ex ante fragmentation (the existence of two venues) increases within-venue competition and leads to ex post consolidation of the global order flow. In contrast, when the market-maker's inventory is lower, she chooses to execute only the shock that best mean-revert her risky inventory. She thus refrains from competing in the other venue. The global order flow remains ex post fragmented, since shocks do not interact and are absorbed by different intermediaries.

When liquidity shocks across venues have opposite signs, the impact of a cumulated transaction across venues is smaller due to an "offsetting" effect. This might not be desirable for risk-averse intermediaries. For instance, when a market-maker's inventory position is very long, she is reluctant to absorb a sell shock that would exacerbate her risky inventory exposure. She will thus post attractive prices only in the venue hit by a buy shock to reduce her inventory risk. In this case, the global order flow remains ex post fragmented and competition is very weak in the venue hit by the sell shock. When her inventory is low, the market-maker is willing to execute shocks that offset each other to keep her inventory low. She thus posts competitive prices to attract the entire order flow, again leading to ex post consolidation.

The model shows that market-makers post prices within one venue that depend on the sign and the magnitude of the shock in the other venue. The interdependence of quoting aggressiveness across venues in turn impacts the tightness of spreads in each venue. Interestingly, ex ante, this impact may be positive or negative. Local liquidity, measured here by expected spreads, only worsens in some cases: namely when the probability to observe shocks with the same sign is high. Global liquidity, measured by ex ante total transaction costs, may also be better or worse, depending on the magnitude of the shock hitting the dominant venue and on the probability that shocks have the same sign across venues. The existence of strategic multi-venue intermediaries makes liquidity, measured by spreads, interconnected across venues. Our paper thus proposes a new explanation - multi-venue inventory management - for the commonality of liquidity across venues.

Our results still hold if we relax some of the model's assumptions. First, we analyze the case of a global liquidity demander that optimally splits his liquidity demand across venues. We show that, even if the liquidity demand is endogenized, the market remains ex ante fragmented. Second, we investigate whether intermediaries would prefer trading together to share risks in a pre-trading stage. We find that, in some cases, multi-venue intermediaries prefer not to trade in the inter-dealer market but trade directly in the customer-dealer marker.

To test the model, we adopt a two-step empirical approach. In the first step, we investigate whether inventory effects across venues are present in our data, a test that, to the best of our knowledge, has never been performed. This step is meant to empirically validate our assumption that intermediaries manage risk by controlling inventory across venues. In the second step, we test the main prediction of our model, i.e., that bid-ask spreads within one venue vary with the way the global order flow fragments across venues and with the divergence in intermediaries' inventories. In particular, spreads should decrease when shocks have the same signs across venues and when divergence is high. This result is uniquely predicted by our model and it is the opposite of what an adverse-selection-based model would predict.

Our analysis uses a proprietary dataset on multi-venue traded stocks from Euronext on a four-month period in 2007. When Euronext was created in 2000 as a result of the merger of several European Stock Exchanges, the stocks which used to be multi-listed in different Exchanges fell into the Euronext jurisdiction. Within Euronext, trading rules in all markets were harmonized and structured based on the Paris Bourse limit order book model. Order books remained separate with price-time priority enforced within each market, but not across markets. Moreover, during that period (that is, before the implementation of MiFID in November 2007), Euronext collected the overwhelming majority of the trades. This environment therefore provides an excellent laboratory to test our predictions.

In our dataset, orders and trades sent to or executed in any limit order book are flagged with a unique identifier and the account used by the financial institution. This enables us to identify 46 multi-venue intermediaries, that is, members acting either as proprietary traders or as exchange-regulated market-makers, who post order messages (submission, revision, or cancellation) and trade at least once in each of the two exchanges on which the stock is traded. Due to the supremacy of Euronext during the sample period, our reconstitution of intermediaries' net positions is a good proxy for their aggregate inventory. Figure 1 illustrates our data. The top graph shows the multi-venue quoting activity of a Euronext intermediary trading the French gaz utility Suez both on Euronext Paris and Euronext Brussels on January 19, 2007. The bottom graph shows the aggregate inventory. Interestingly this inventory tends to mean-revert over the day. Her quoting aggressiveness also varies across hours and across venues (quotes of the satellite venue are more distant from the midpoint of the consolidated book).

[INSERT FIGURE 1]

In accordance with Figure 1, our empirical analysis finds evidence of inventory effects. Using a logit model, we find that multi-venue intermediaries, in particular formally registered market-makers, are more likely to submit messages aiming at mean-reverting inventory in a venue when their preexisting orders have been passively hit in the other venue. This result validates our hypothesis that aggregate inventory is a driver of multi-venue market-making strategies. It also makes this paper one of the first to uncover evidence on cross-venue inventory effects. More importantly, our empirical analysis shows that bid-ask spreads within one venue are significantly lower when our measure of divergence in inventories is high and when liquidity shocks across venues have the same sign, in line with our main prediction.

Our paper contributes to the theoretical literature on multi-market trading. Traditional models including Pagano (1989), Chowdhry and Nanda (1991), Bernhardt and Hughson (1997), Easley et al (1996), and Foucault and Menkveld (2008) assume that quotes are competitively set by independent pools of market makers in multiple markets to satisfy the zero-profit condition. They focus on the routing or order splitting decisions of strategic liquidity demanders, who can either be informed or not. We instead exogenously fix order flows routed towards each market to focus on the inter-dependent quoting strategies of multi-venue market-makers. As Seppi (1997) and Parlour and Seppi (2003), we model competition for order flow based on liquidity provision when market-makers are not perfectly competitive.

We also contribute to the empirical literature on how traders operate in multi-market environments. Menkveld (2008) and Halling, Moulton, and Panayidès (2013) focus on how liquidity demanders adjust their trading strategies to multi-trading. In contrast, we investigate how liquidity suppliers strategically trade in a multi-venue environment. Our empirical analysis is most closely related to van Kervel (2014) and Jovanovic and Menkveld (2015). van Kervel (2014) finds that trades on the most active venues for 10 FTSE100 stocks are often followed by immediate cancellations of limit orders on competing venues, which would be expected in the presence of multi-venue market-makers that strategically balance their aggregate inventory. Jovanovic and Menkveld (2015) statistically identify a multi-venue intermediary actively trading across Euronext and Chi-X, and find that the participation of this intermediary has an impact on spreads and volumes. Our model of strategic competition across venues also corroborates their findings. Since each institution in our sample is identified by a unique identifier across the multiple limit order books we are able to precisely compute the aggregate inventory and analyze the related quoting strategies of intermediaries who exploit the multi-market environment. Our results thus extend and complement the existing empirical findings.

The paper is organized as follows. Section 2 describes the model and investigates the price formation in a two-venue market-making environment. Section 3 describes the data, provides summary statistics and tests the main implications of the model. Section 4 concludes the paper. All proofs are available in the Appendix.

2 The Model

2.1 The basic setting

We consider the market for a risky asset with a random final cash flow $\tilde{v}$ which is normally distributed with expected value μ and variance σ^2. There are two types of market participants: investors who demand liquidity and intermediaries who supply liquidity.

A fragmented market. We suppose that the risky security trades in two trading venues, denoted D and S, that we assume to be visible. The venues can be exogenously hit by buy or sell shocks. By convention, a buy (resp. sell) shock generates a buy (resp. sell) liquidity demand denoted $Q > 0$ (resp. $Q < 0$). We call the sum of the liquidity demands the "global" order flow.[1] We assume that the liquidity demand sent to venue D, denoted Q_D, is larger than that routed to venue S, i.e., $|Q_D| > |Q_S|$. We thus term venue D as the dominant market, and venue S as the satellite market. Note that the sign of the global order flow is equal to the sign of the liquidity demand routed to venue D.

Intermediaries' reservation price. Liquidity is supplied by two intermediaries $i = 1, 2$. Each intermediary i is endowed with a nonzero inventory position of the risky asset I_i, where I_i is the realization of the random variable $\tilde{I}_i$ uniformly distributed on $[I_d, I_u]$. Intermediaries are *risk-averse* and have the following common CARA utility function:

$$u(\tilde{w}_i) = -\exp(-\rho\tilde{w}_i), \tag{1}$$

where ρ is the coefficient of absolute risk aversion, and $\tilde{w}_i$ the terminal wealth of market-maker i. All random variables are independent and their distributions are common knowledge.

As Ho and Stoll (1983) demonstrate, market-maker i's reservation price r_i to execute the incoming liquidity demand Q is such that:

$$r_i(Q) = \mu - \rho\sigma^2 I_i + \frac{\rho\sigma^2}{2}Q. \tag{2}$$

Note that the marginal valuation of intermediary i, $(\mu - \rho\sigma^2 I_i)$, depends on the risk of holding an inventory position. An intermediary in a long position is reluctant to increase her exposure to inventory risk and therefore posts relatively low ask and bid prices to attract sell orders. The second component of reservation prices ($\frac{\rho\sigma^2 Q}{2}$) represents the price impact of a trade and is thus increasing in the trade size Q. For ease of exposition, in what follows we consider that market-maker 1 is endowed with a longer inventory position, i.e., $I_1 \geq I_2$.

[1] In the base model, the global order flow exogenously fragments across venues D and S. We address the case of endogenizing order flow by, say, a global liquidity demander that would optimally split orders across venues through a smart order routing engine in section 2.4.1.

Quoting strategies of multi-venue intermediaries. We assume that intermediaries have access to all trading venues at the same time. Conditional on observing Q_D and Q_S, multi-venue market-makers thus post *simultaneously* their quotes in venues D and S. The market-maker who posts the lowest ask price (resp. highest bid price) in venue m executes $Q_m > 0$ (resp. $Q_m < 0$), for $m = D, S$.

A multi-venue quoting strategy for market-maker i is a couple of quoted prices (p_i^D, p_i^S) where p_i^D is the price posted by market-maker i in market D and p_i^S is the price posted by i in market S (which is an ask price if $Q_m > 0$ or a bid price if $Q_m < 0$). In the next section, we analyze the Nash equilibria of the quoting game.

Note that in our set-up, market-makers must manage their inventory by keeping track of orders across all trading venues. Because making the market "globally" (i.e., across various venues) affects intermediary's total exposure to inventory risk, only aggregate inventory matters as opposed to ordinary inventory that guides an intermediary taking risks just in one venue.[2]

Figure 2 shows the extensive form of the trading game. We denote ζ_m the probability that a liquidity shock hits venue m ($m = D, S$) and assume that $\zeta_D > \zeta_S$ (consistently with venue D being the dominant market). The probability that shocks simultaneously hit both venues is denoted λ ($\equiv \zeta_D \times \zeta_S$). The cases in which there is only one shock (either in venue D or S) occurring with probability $(1 - \lambda)$ are not explicitly analyzed (because they correspond to the case of a single venue already analyzed in the literature). The probability that shocks have the same sign is γ. The analysis of price formation across venues provided below is restricted to the case in which the global order flow is net-buying $Q_D + Q_S > 0$ (with probability $\lambda/2$) and thus such that $Q_D > 0$, while Q_S might be a buy or sell liquidity demand: $Q_S > 0$ (with probability γ) or $Q_S < 0$ (with probability $(1 - \gamma)$). Symmetric results are obtained for a net-selling global order flow.

[INSERT FIGURE 2]

[2]Our definition of aggregate inventory is close to the definition of *equivalent* or total inventory emphasized by Ho and Stoll (1983) and discussed in Naik and Yadav (2003). However, while equivalent inventory is the overall position of an intermediary across all stocks, aggregate inventory is the cumulated net inventory position of an intermediary in a single stock but across all available trading venues.

2.2 Equilibrium quotes in fragmented markets

Consider the benchmark case in which liquidity demands are batched and sent to a single venue, which is the case analyzed by Ho and Stoll (1983). The market-maker with the longer inventory position (market-maker 1 by assumption) posts the most competitive ask price, by quoting the reservation price of her shorter opponent ($(a^{batch})^* = r_2(Q_D+Q_S)-\varepsilon$, where ε corresponds to one tick). This section analyzes how market fragmentation alters this result.

2.2.1 Preliminary results

The outcome of whether or not the fragmented order flow might be consolidated ex post (through the execution by a single intermediary) depends on the conditions described by Lemma 1.

Lemma 1 *Assume that* $I_1 > I_2$ *and that* $Q_D + Q_S > 0$.

1. If $(I_1 - I_2 - Q_D)Q_S \leq 0$*, and if an equilibrium exists, then it is such that the global order flow remains fragmented: orders submitted to the different venues are executed by different intermediaries. Conversely, if* $(I_1 - I_2 - Q_D)Q_S > 0$*, and if an equilibrium exists, then it is characterized by the ex post consolidation of the global order flow, through a multi-venue execution by a single intermediary.*

2. If there exists an equilibrium such that the global order flow remains fragmented ex post, then the longer intermediary absorbs the shock of the dominant venue, while the shorter intermediary absorbs the shock of the satellite venue. If there exists an equilibrium characterized by the ex post consolidation of the order flow, then the longer intermediary executes the global order flow.

Lemma 1 states the conditions that determine whether the global order flow is ex post consolidated or remains fragmented, viz.: (i) the price impact of a single or multiple trades, (ii) intermediaries' aggregate inventory and (iii) the divergence in intermediaries' inventories.

First, when shocks have the same sign, the price impact of trading in the two venues is *cumulative*. When shocks have opposite signs, the converse *offsetting* effect is observed: trading in both venues enables intermediaries to reduce the price impact of a single trade.

Second, in our model, intermediaries' inventory is affected by all trades, either in venue S or in venue D. Intermediaries' willingness to trade thus depends on their aggregate inventory position across all venues. By assumption, market-maker 1 is endowed with a larger aggregate inventory ($I_1 > I_2$). She faces more needs to sell than market-maker 2. She is thus willing to post more aggressive selling prices across venues on average. Price aggressiveness however depends on the competition induced by market-maker 2, which in turn depends on his aggregate inventory.

Third, intermediaries' willingness to absorb a single or multiple shocks depends on the divergence in their aggregate inventories. When the divergence is high ($I_1 - I_2 > Q_D$), market-maker 1's inventory is very large, she is willing to execute all possible buy orders to lay off her inventory, i.e., to execute $Q_D + Q_S$ when Q_D and Q_S have the same sign, or only execute Q_D when Q_D and Q_S have opposite signs. In the latter case, absorbing $Q_S < 0$ would instead exacerbate her inventory exposure. When the divergence in inventories is low ($I_1 - I_2 \leq Q_D$), it is more profitable for her to absorb less buy orders, i.e., either only Q_D when shocks have same signs, or $Q_D + Q_S$ when shocks have opposite signs.

Note that liquidity demands across venues can be interpreted as substitutes (resp. complements) when Q_D and Q_S have the same signs, since the marginal gain of trading $Q_D > 0$ when a market-maker also trades $Q_S > 0$ is lower (resp. higher) than when she does not trade Q_S. Substitutability is a key determinant of our results, in line with the outcome of the Vickrey-Clarke-Groves (VCG) mechanism for combinatorial auctions.[3]

2.2.2 Equilibrium quotes

In our model, best prices might differ across venues for two reasons.[4] First, intermediaries are not constrained to post competitive prices for the entire order flow, and may choose to compete just in a single venue. Second, intermediaries post quotes reflecting the price impact of trades of different size (second-degree price differentiation). Besides,

[3]In combinatorial auctions, multiple items, which are related but not necessarily identical (like the multiple shocks in our model), are sold simultaneously. Bidders may submit bids on packages of items. A single bidder wins the bundle of items in the VCG mechanism under a condition similar to the one under which an intermediary absorbs all shocks (ex post consolidation), as stated by Lemma 1. See Vickrey (1961), Clarke (1971) and Groves (1973) or Ausubel and Milgrom (2006) for a discussion of the VCG mechanism.

[4]In Europe, a consolidated tape in which all trades and quotes of all exchanges and multi-trading facilities would be recorded does not exist, orders sometimes execute at prices which may differ from the best existing quoted prices in the market (trade-throughs are allowed).

it is worth noticing that, similar to inventory models in a single venue like Ho and Stoll (1983) or Biais (1993), the divergence in intermediaries' inventories is a determinant of the competitiveness of their quotes across venues. When the divergence is low (resp. high), market-maker 1's inventory position is close to (resp. away from) that of market-maker 2, and market-makers are less (resp. more) able to post aggressive prices. The combination of these characteristics leads to unexpected intra-venues competition effects resulting in equilibrium prices described by Proposition 1 below.

Proposition 1 *Assume that* $I_1 > I_2$ *and* $Q_D + Q_S > 0$.

1. If $(I_1 - I_2 - Q_D)Q_S > 0$, *there exists a Nash equilibrium, in which market-maker 1, with the larger inventory, consolidates the global order flow by posting the best prices across venues, while market-maker 2, with the smaller inventory, quotes his own reservation prices, that is:*

1.1. If $Q_S > 0$, *market-maker 1 posts the best ask prices in venue D and venue S:*

$$\left((a_1^D)^*, \quad (a_1^S)^*\right) = \left(r_2(Q_D) - \varepsilon, \quad r_2(Q_S) - \varepsilon\right);$$

1.2. If $Q_S < 0$, *market-maker 1 posts the best ask price in venue D and the best bid price in venue S:*

$$\left(\left(a_1^D\right)^*, \quad \left(b_1^S\right)^*\right) = \left(r_2(Q_D) - \rho\sigma^2\left(-Q_S\right) - \varepsilon, \quad r_2\left(Q_S\right) + \varepsilon\right);$$

2. If $(I_1 - I_2 - Q_D)Q_S \leq 0$, *there exists a unique Nash equilibrium, in which market-maker 1, holding the larger inventory, posts the best price in the dominant market while market-maker 2 posts the best price in the satellite market, that is:*

2.1. If $Q_S > 0$, *market-makers post the following ask prices in venues D and S:*

$$\begin{aligned}
\left((a_1^D)^*, \quad a_1^S\right) &= \left(r_2(Q_D) + \rho\sigma^2 Q_S\left(\frac{Q_D - (I_1 - I_2)}{Q_D}\right) - \varepsilon, \quad r_1(Q_S) + \rho\sigma^2 Q_D\right), \\
\left(a_2^D, \quad (a_2^S)^*\right) &= \left(r_2(Q_D) + \rho\sigma^2 Q_S\left(\frac{Q_D - (I_1 - I_2)}{Q_D}\right), \quad r_1(Q_S) + \rho\sigma^2 Q_D - \varepsilon\right);
\end{aligned}$$

2.2. If $Q_S < 0$, *market-makers post the following ask prices in venue D and bid prices* *in venue S:*

$$\left((a_1^D)^*, \quad b_1^S\right) = \left(r_2(Q_D) - \rho\sigma^2(-Q_S) - \varepsilon, \quad r_1(Q_S) + \rho\sigma^2 Q_D\right),$$
$$\left(a_2^D, \quad (b_2^S)^*\right) = \left(r_2(Q_D) - \rho\sigma^2(-Q_S), \quad r_1(Q_S) + \rho\sigma^2 Q_D + \varepsilon\right);$$

where ε corresponds to one tick.

To help to understand Proposition 1, we use Figure 3, which shows the best prices as a function of the divergence in inventories. Panel A illustrates the case in which two buy shocks simultaneously hit venue D and venue S. Panel B illustrates the case of shocks of opposite directions. In both cases, the vertical line Q_D separates the region in which the divergence in intermediaries' inventories is high (the right-hand side of the graph, corresponding to $I_1 - I_2 > Q_D$) from the region in which divergence is low (the left-hand side, corresponding to $I_1 - I_2 \leq Q_D$).

[INSERT FIGURE 3]

The case of simultaneous buy shocks. Panel A shows that when the divergence in inventories is high, equilibrium selling prices ($(a^D)^*$ and $(a^S)^*$) are more competitive than the benchmark price. Market-maker 1's inventory is so large (compared to her opponent's) that she is willing to absorb both shocks. Market-maker 2 might however choose to compete in a single venue, forcing market-maker 1 to post more aggressive prices across venues to be sure to consolidate the entire order flow. In this case, ex ante fragmentation (the existence of multiple venues) increases intra-venue competition leading to consolidation ex post. Accordingly, relative to the benchmark, ex post transaction costs are lower: $TC - TC^{batch} = (a^D)^* Q_D + (a^S)^* Q_S - (a^{batch})^*(Q_D + Q_S) = -\rho\sigma^2 Q_D Q_S < 0$.

Consider the case of a low divergence in inventories, illustrated by the region to the left of the vertical line Q_D. As the divergence in inventories decreases, equilibrium ask prices are less and less competitive compared to the benchmark. Interestingly, the equilibrium selling price in the satellite venue might even be higher than the one of the dominant venue despite a smaller quantity to execute. The intuition for this result is as follows. First, in this region, market-maker 1 is not ready to execute the entire order flow since her inventory is large but not very divergent.[5] She is keen to absorb the single larger buy shock, which

[5]This situation might be interpreted as a capacity constraint in our two-sided two-market Bertrand competition model with asymmetric costs among liquidity suppliers.

provides the best way to reduce her inventory imbalance, and reluctant to absorb the other buy shock in the satellite venue. She therefore posts less competitive prices in the satellite venue letting her opponent absorb the smaller shock in that venue. This results in ex post fragmentation of the global order flow. Second, when $I_1 - I_2 \to Q_D$, market-maker 1 is indifferent between executing the entire order flow or the single liquidity demand Q_D because her trading profits are identical.[6] At the other extreme, when $I_1 - I_2 \to 0$, market-makers' profits are equal (where there is no divergence, market-makers' private values are symmetric). At the limit, the longer market-maker executes the larger demand, while the shorter market-maker executes the smaller demand. A higher equilibrium price in the satellite market must therefore compensate the smaller quantity executed by market-maker 2 for the equal profits condition to hold. In between these extreme cases, as we move leftwards from the vertical line Q_D to the y-axis, the equilibrium ask price in the satellite market varies from smaller to higher than that in the dominant market. Therefore there exists an intersection point p at which selling prices are equal across venues leading to an outcome identical to the benchmark.[7]

In this region of low divergence in inventories, ex ante fragmentation has an ambiguous effect on price competition. To the left of p, intermediaries cannot post very different prices because their inventories are closer, and competition is weaker ($TC - TC^{batch} \geq 0$). To the right of p, inventories are more divergent, prices are more competitive, and ex post transaction costs are smaller than those paid in the benchmark: $TC - TC^{batch} < 0$.

The case of opposite shocks. Panel B illustrates the case of shocks of opposite sign. Subfigure (a) depicts the best selling price in the dominant venue (which is hit by a buy shock) as a function of divergence in inventories. Subfigure (b) draws the best buying price in the satellite venue (which is hit by a sell shock). Panel B shows that price competition between intermediaries is weaker compared to the one existing when shocks have the same signs.

The equilibrium selling price in venue D $(a^D)^*$ is more competitive than the benchmark price. The opposite holds in the satellite venue. The best buying price is less and less competitive as the divergence in inventories increases. When market-maker 1's inventory

[6]Note that, when $I_1 - I_2 \to Q_D$ market-maker 2 is indifferent between executing nothing or Q_S because he has zero profit in both cases.

[7]At p, we can show that the binding constraint is the following: market-maker 1 must be indifferent to execute only Q_D or the entire order flow $Q_D + Q_S$.

is large (the region to the right of the vertical line Q_D), she is very keen to execute the buy demand Q_D to mean-revert her inventory. Simultaneously, she is reluctant to add more inventory by executing the sell demand Q_S. This is anticipated by market-maker 2 who therefore posts a non aggressive price in venue S, less and less aggressive that market-maker 1 inventory is larger. Ex post transaction costs thus worsen: $TC - TC^{batch} = \rho\sigma^2(I_1 - I_2 - Q_D)(-Q_S) \geq 0$ in this region. In contrast, when market-maker 1' s inventory is close to her competitor's (the region to the left of the vertical line Q_D), she is willing to execute the entire order flow $Q_D + Q_S$ to benefit from the offsetting effect. Executing the entire order flow has a smaller impact on inventory compared to a single trade in venue D or S. The ability of market-maker 2 to compete in just one venue forces market-maker 1 to post attractive, but not too aggressive prices due to the low divergence in inventories. In this case of ex post consolidation of the entire order flow, there is a multiplicity of equilibria. We select the equilibrium in which there is price continuity at $I_1 - I_2 = Q_D$ in venue D. At any equilibrium though, the weighted averaged price paid by liquidity demanders is equal to $r_2(Q_D + Q_S)$, that is, the price formed at equilibrium in the batch auction. Ex post transaction costs are thus equal to those paid in the benchmark in this region.

It is worth noticing that in case the global order flow remains fragmented ("Ex post fragmentation"), intermediaries obtain a better allocation of risk compared to the batch auction, as shown in the following corollary.

Corollary 1 *The fragmented market generates a more efficient outcome in risk sharing among intermediaries than the batch market in the sense that intermediaries bear lower aggregate security risk in the fragmented market.*

The better allocation of risk does not however necessarily lead to more competitive prices as detailed above since intermediaries have less incentives to undercut each other. This result is in the spirit of the one obtained in Biais et al (1998).

2.3 Assessing ex ante execution quality

In our model, because intermediaries manage their inventory globally, they place quotes in one venue taking into account the impact of a potential trade in the other venue. The interdependent quoting aggressiveness across venues in turn impacts local liquidity, measured here by bid-ask spreads, and global liquidity.

2.3.1 Interconnected liquidity

Using Proposition 1, we compute the expected (half-) spreads in the dominant and the satellite venues for any set of inventory positions and any sign for liquidity demands Q_D and Q_S. For ease of exposition, we denote by ϕ_m the magnitude of the shock routed to venue m scaled by the distribution support $(I_u - I_d)$: $-\phi_m = \frac{Q_m}{I_u - I_d}$ for a sell shock and $\phi_m = \frac{Q_m}{I_u - I_d}$ for a buy shock $(m = D, S)$. Proposition 2 follows.

Proposition 2 *Under the assumption that $\phi_D < 1$, the expected (half-) spreads in the dominant and the satellite venues respectively write:*

$$E\left(\underline{s}^D\right) = \rho\sigma^2(I_u - I_d)\left[\frac{1}{2}(\phi_D - \frac{2I_d + I_u}{3}) + \zeta_S\phi_S\left[\gamma(\phi_D - \frac{(\phi_D)^2}{3}) - (1-\gamma)\right]\right], \quad (3)$$

$$E\left(\underline{s}^S\right) = \rho\sigma^2(I_u - I_d)\left[\frac{1}{2}(\phi_S - \frac{2I_d + I_u}{3}) + \zeta_D\phi_D\left[\phi_D - \frac{(\phi_D)^2}{3} - (1-\gamma)\right]\right], \quad (4)$$

where γ is the probability that order flows routed to D and to S have the same sign and ζ_m is the probability that a liquidity shock hits venue m $(m = D, S)$.

In line with the intuition explained above, the first component of the expected best offer in the two venues (Eq. (3) and (4)) is the *direct* price impact of the order flow routed to that venue. It corresponds to the expected best offer that would prevail if ϕ_{-m} is zero (with probability $1 - \zeta_{-m}$). The second component consists of the *indirect* price impact of trading in another venue (ϕ_{-m}) resulting from the interdependent quoting strategies across venues. This impact may be positive or negative depending on the value of the parameters γ and ϕ_D. In particular, expected spreads in the two venues are increasing with γ the probability that shocks have the same signs across venues. When γ is high ($\gamma \to 1$), local expected spreads are negatively impacted by ex ante fragmentation. Interestingly if γ is sufficiently low, the opposite occurs. This result suggests that the empirical findings of Degryse et al (2014) uncovering a negative impact of fragmentation on local liquidity (that is, on $E(s^m)$) might be explained by a high probability to observe order flows with the same signs across venues. It is also worth noticing that the shock that hits the dominant market has a bigger impact on spreads in the satellite market than the reverse (given that $\zeta_D > \zeta_S$, $\phi_D > \phi_S$, and $(1-\gamma)(\phi_D - \frac{(\phi_D)^2}{3}) \geq 0$).

Proposition 2 shows that local expected spreads are indirectly influenced by orders sent to other venues due to the presence of strategic multi-venue intermediaries. They make the liquidity (measured by quoted spreads) of different venues interrelated in our model, as stated by the following Proposition:

Proposition 3 *Spreads co-vary jointly:*

$$cov(s^D, s^S) = \lambda\big(\rho\sigma^2(I_u - I_d)\big)^2 \Bigg[\frac{3\gamma-1}{36} - (\phi_D)^2\left(\frac{1}{6} - \frac{2}{9}\phi_D + \gamma\frac{(\phi_D)^2}{12}\right) - \gamma\phi_D\phi_S\left(\frac{(\phi_D)^4}{9} - \frac{2(\phi_D)^3}{3} + \frac{5(\phi_D)^2}{4} - \frac{8\phi_D}{9} + \frac{1}{6}\right)\Bigg] \quad (5)$$

where $\lambda = \zeta_D\zeta_S$ is the probability to observe two simultaneous shocks in venues D and S.

Our model therefore proposes a new explanation for the interconnectedness of trading venues, namely the inventory management strategies of multi-venues market-makers. This explanation is distinct from those found in the literature which have focused on arbitrage strategies (Foucault et al, 2014; Rahi and Zigrand, 2013), duplicate strategies (van Kervel, 2014) or directional trading strategies (Chowdhry and Nanda, 1991).[8]

2.3.2 Market quality

While the previous section analyzes local liquidity, this section investigates global liquidity (measured by ex ante total transaction costs) to determine whether market performance improves or worsens when liquidity is strategically supplied across multiple venues.

From Proposition 2, we compute expected transaction costs in a fragmented market. The next corollary compares them to expected transaction costs that would prevail in a batch market (our benchmark).

Corollary 2 *Expected transaction costs are lower in a fragmented market than in a batch market if and only if $\gamma > \frac{1}{3}$ and ϕ_D is neither too large, nor too small ($\Phi^1_\gamma < \phi_D < \Phi^2_\gamma$).*

The intuition of the corollary is as follows. First, recall that ex post transaction costs are strictly lower in a fragmented market when shocks have the same signs. In particular, competition heats up when there is a high divergence in inventories. Therefore, if the

[8] See Cespa and Foucault (2014) for interconnectedness across different assets.

probability to observe shocks of the same signs is high ($\gamma \to 1$), the probability to observe more aggressive quoting strategies than in the benchmark increases with the probability of a high divergence in inventories, i.e., ϕ_D should not be too large ($\phi_D < \Phi_\gamma^2$). Conversely, when shocks are more likely to have opposite sign ($\gamma \to 1/3$), quoting aggressiveness increases with the probability of low divergence in inventories, i.e., ϕ_D should not be too small ($\Phi_\gamma^1 < \phi_D$). Note that when the probability to get shocks of opposite sign is too high ($\gamma < 1/3$), transaction costs are always larger in a fragmented market.

From Proposition 2 and Proposition 3 we deduce that when the probability of having shocks of the same sign is high, even if local spreads are in average larger, global liquidity improves due to a stronger competition across venues. The opposite effect is found when the probability of having shocks of opposite sign is high.

The ambiguous result of multi-venue market-making on market performance is consistent with the mixed empirical evidence investigating the impact of market fragmentation (see, e.g., the literature review in O'Hara and Ye, 2011). In our model, there exist cases in which the longer market-maker competes fiercely to consolidate the fragmented order flow, which has a positive impact on transaction costs. Opposite effects are found when she refrains to compete for the entire order flow and restricts competition to a single venue. Few theoretical models find positive impacts of fragmentation of trading. Foucault and Menkveld (2008) show that even if time priority is not enforced across limit order books, the consolidated depth may be larger due to the presence of liquidity suppliers who consolidate the market through their queue jumping strategy across limit order books.

2.4 Discussion

The aim of this section is to assess the impact of relaxing some of the model's assumptions. We analyze two extensions. First, we relax the hypothesis that the market is exogenously fragmented. Second, we investigate whether intermediaries would prefer trading and sharing risks together in a pre-trading stage.

2.4.1 Endogenous fragmentation of the total order flow

Consider the case of a global liquidity demander that must trade a given quantity denoted Q. He optimizes execution costs and thus splits optimally orders across venues.[9] Note that the strategic decision to spatially split up orders extends the case in which shocks have exogenously the same sign in our previous set-up. As in section 2.1, we suppose that market-maker 1 is longer than market-maker 2 and that Q is a buy order flow, i.e., $Q > 0$ (results for the case of a sell order flow, or when market-maker 2 is longer than market-maker 1, are deduced by symmetry).

We consider that the global liquidity demander enjoys some private benefits denoted δ_m to trade in venue m. We assume that $\delta_D > \delta_S$, consistently with the dominant market defined above, and that $\delta_D - \delta_S < \rho\sigma^2 Q$.[10,11] The liquidity demander chooses the proportion α of the order flow routed to market D (and $(1-\alpha)$ to market S) so as to minimize transaction costs.[12] We show that there exists an equilibrium such that $\alpha \in [\frac{1}{2}; 1)$, that is, such that the liquidity demander optimally splits orders across venues and sends a larger demand to the dominant market.[13] In this interval, transaction costs write:

$$TC(\alpha) = [((a^D)^*(\alpha Q) - \delta_D - \mu)\alpha + ((a^S)^*((1-\alpha)Q) - \delta_S - \mu)(1-\alpha)] \times Q.$$

In the Appendix, we show the existence and the characterization of an equilibrium α^*. This yields the following proposition.

Proposition 4 *If $r_2(Q) - r_1(Q) > (\delta_D - \delta_S)$, there exists an interior equilibrium α^*, such that it is optimal for the global liquidity demander to split orders across venues.*

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[9] See Degryse et al (2013) for an analysis of "order splitting" by liquidity demanders over time rather than over venues.

[10] Numerous studies (Froot and Dabora, 1999; Gagnon and Karolyi, 2010; Foerster and Karolyi, 1999; Shleifer and Vishny, 1997; or Stulz, 2005) document the existence of a domestic bias, due to investment barriers, e.g., regulatory barriers, taxes, or information constraints. In Europe, brokerage fees charged in 2013 to trade in a foreign country or trading venue are 15 to 40% higher than those charged to trade in a national exchange, but the situation was even worse back in 2007 (see documents on Fees and Commissions of various brokers from 2007 to 2013). Differences in private benefits might also capture differences in terms of maker/taker spreads.

[11] When $\delta_D - \delta_S \geq \rho\sigma^2 Q$, the private benefits of trading in venue D are so large that it is never optimal for investors to split the quantity to be traded across trading platforms.

[12] Because markets are transparent in our set up, we assume that liquidity demanders perfectly anticipate what the best bid and ask prices are.

[13] A complete proof of the existence and characterization of all the equilibria is available on request.

The liquidity demander trades off the benefits of price competition through fragmentation (related to the divergence of inventories, $I_1 - I_2$) to the private benefits of sending the entire demand to the dominant market ($\delta_D - \delta_S$), that is, when $r_2(Q) - r_1(Q) > (\delta_D - \delta_S)$. We conclude that, even when the demand splitting is endogenized, it is still the case that the market remains ex ante fragmented.

2.4.2 Introduction of an inter-dealer market

In this section, we analyze if our results are sensitive to the introduction of an inter-dealer market in which intermediaries are able to optimally share inventory risks (stage 1) before setting quotes in the customer-dealer market (stage 2). It could be the case that they prefer sharing risks in an inter-dealer market to avoid multi-venue competition in the customer-dealer market.

In a conservative approach, we assume that intermediaries independently and unstrategically maximize their expected profit in the inter-dealer market, then their expected profit in the customer-dealer market (the model is solved sequentially).[14]

In the first stage, we find that at the symmetric equilibrium, intermediaries perfectly share inventory risk in the inter-dealer market, that is, they trade a quantity $q^* = \frac{I_1 - I_2}{2}$ at price $p^* = \mu - \rho\sigma^2 \frac{I_1 + I_2}{2}$ such that their new inventory positions (I'_1, I'_2) write $I'_1 = I'_2 = \frac{I_1 + I_2}{2}$. In the second stage, we simply use the equilibrium in the customer-dealer market derived in section 2.2 for the limit case where $I'_1 \to I'_2$. Finally, we compute and compare the intermediaries' expected profits whether they trade or not in the inter-dealer market. This yields the following corollary.

Corollary 3 *The set of parameters for which intermediaries choose not to trade in the inter-dealer market is non-empty.*

As illustrated by Figure 4, there exist cases (white squared surface) in which intermediaries find more profitable ex ante not to trade in the inter-dealer market (for different values of γ and q_S) and trade directly in the customer-dealer market.

[14] In the case in which intermediaries strategically trade in the inter-dealer market *after* observing the realization of the order flows in venue D and S, we find that they may find optimal to reinforce the divergence in inventories in order to maximize their trading profit in the customer-dealer market. The inter-dealer market is not a way to optimize risk-sharing, but to enhance divergence in inventories. Multi-venue competition in the customer-dealer is thus emphasized in this case.

2.5 Testable implications

To establish the external validity of our modeling approach, we adopt a two-step empirical strategy. In the first step, we investigate whether inventory effects across venues are present in the Euronext limit-order book environment. This step is meant to empirically validate our assumption that aggregate inventory is a driver of intermediaries multi-venue market-making strategies.[15] In the second step, we proceed to test the main prediction of our model, derived from Proposition 1.

2.5.1 Testing the validity of a cross-venue inventory model

Our model assumes that intermediary i' multi-venue market-making strategy is governed by her aggregate inventory, defined at time t as the cumulated net volume of transactions across all trading venues : $I_{i,t} = I_{i,0} + \sum_{\tau=0}^{\tau=t} Q_{D,\tau} + \sum_{\tau=0}^{\tau=t} Q_{S,\tau}$ where $I_{i,0}$ is the initial inventory. Our model implies that intermediaries should react to a change in their aggregate inventory by adjusting quotes in *all* venues. In particular, after a trade, say in venue S, that increases the inventory exposure, a multi-venue intermediary should update quotes in venue S, but also in venue D to elicit inventory-reducing orders. We specifically focus on cross-venue inventory effects that, to the best of our knowledge, have never been investigated. Formulating our hypothesis in the context of the limit-order-book environment of Euronext, we test whether, for instance, after executing a sell order in the satellite venue that increases the total inventory exposure, a multi-venue market-maker is more likely to cancel an existing buy order in the dominant market, or modify it for a less aggressive price (negative revision), or post a new sell limit order in the dominant market or modify an existing sell order for a more aggressive price (positive revision). We thus posit the following hypothesis:

Hypothesis 1 *Multi-venue market-makers should update existing limit orders or submit new orders in one venue after a trade in another venue, in a direction that is associated with their inventory changes.*

[15] The literature has so far focused mostly on within-venue inventory effects in the context of dealer markets and the specialist-based model of the New York Stock Exchange (NYSE). See, among others, Hansh et al (1998) and Reiss and Werner (1998) for London equity dealers, Bjønnes and Rime (2005) for foreign exchange dealers, Panayidès (2007) for NYSE specialists. Raman and Yadav (2014) uncover some within-venue inventory effects for limit order traders on the National Stock Exchange, India.

We acknowledge that other trading strategies, such as cross-venue arbitrage, could lead to order placement patterns that resemble those due to inventory considerations. In case, say, the bid price in venue S jumps above the best ask in venue D, an arbitrageur might step in and sell one share in venue S, and buy one in venue D to reduce the existing price discrepancy. The buy and sell orders submissions from the arbitrageur are empirically similar to inventory-driven strategies. A way to distinguish these strategies is to take into account the aggressiveness of the initial transaction. In case there is an arbitrage opportunity, we expect arbitrageurs to post aggressive orders in a venue simultaneously/after an active transaction in another venue.[16] In contrast, after a passive transaction (existing limit orders passively hit), we expect more messages related to inventory management. We thus control for arbitrage opportunities and for the transaction aggressiveness in our empirical analysis.

2.5.2 Testing the main prediction of the model

Proposition 1 describes equilibrium prices in the dominant and satellite venues. Within venue, for the same liquidity demand, price competitiveness varies with the sign of the shock to absorb in the other venue and with the divergence in intermediaries' inventories. As illustrated by Figure 3, when shocks across venues have the same signs, competition gets more intense as the divergence in inventories increases. We thus expect tighter bid-ask spreads when both conditions hold simultaneously. We thus deduce the following hypothesis :

Hypothesis 2 *Variations in spreads in one venue depend on both the directions of order flows across venues (identical or opposite), and the divergence in intermediaries' inventories.*

Note that spreads vary more with the divergence in inventories in the satellite venue than in the dominant venue. In particular, when shocks have the same sign and divergence in inventories is low, competition is weaker than in the dominant venue. Recall that, despite a shock of a smaller magnitude, the best ask price in the satellite venue is higher than the one in the dominant venue (region to the left of the point p on Figure 3). In contrast,

[16] We call a transaction "active" when intermediaries trade through a liquidity demanding order like a market or marketable order.

when divergence is high (region to the right of the vertical line Q_D), competition heats up and the best ask price in the satellite venue is smaller than in the dominant venue (reflecting a smaller quantity to absorb).

This prediction is interesting because it allows us to distinguish our theory from a competing adverse-selection hypothesis: in case an informed trader would split his orders across venues, the adverse selection component of multi-venue market-makers should increase. Transaction costs, measured by quoted bid-ask spreads, should thus increase in all venues if order flows across venues have same direction. Our model predicts however that if we introduce an interaction term between the order flow direction and a measure of divergence in inventories, it should have a negative impact on spreads.[17]

3 Empirical analysis

3.1 Forming the sample

Our analysis uses a proprietary dataset from Euronext on multi-listed stocks. Euronext was created in 2000 as a result of the merger of three European exchanges, namely Amsterdam, Brussels and Paris. The Lisbon exchange joined in 2002. Before the introduction of the Universal Trading Platform (UTP) in 2009, each of the four exchanges maintained their domestic market. As a result, firms could be multi-listed on several Euronext exchanges; for example, Suez was traded in Paris and Brussels.

Our sample consists of all multi-traded stocks within Euronext, spanning four months (79 trading days) from January 1, 2007 to April 30, 2007.[18] The data on orders and quotes are provided by Euronext. Euronext files also provide us with the identification of the member participating in each quote or transaction, and whether the member is acting as an agent or as a principal (that is, either as a proprietary trader or an exchange-regulated market maker). The data assigns a unique identifier to each member, enabling us to trace members' inventory changes and quoting behavior across time, across stocks, and across exchanges. During the sample period, Euronext exchanges followed the same market

[17] Comerton-Forde et al (2010) relate variations in spreads and specialists' inventories. They focus however on the level of inventories aggregated across all specialists to show that this measure and the tightness of the funding market significantly impact variations in spreads on the NYSE.

[18] Four trading days are dropped in January due to missing data about best limits.

model (same trading hours, and same trading rules), and the payment of membership fees granted access to all Euronext markets. Note also that, during this period (pre-MiFID environment), trading was concentrated in Euronext.[19] For all these reasons, Euronext is an excellent environment to test the predictions of our model. Other stock-level information comes from Compustat Global.

We keep firms that trade in euros using a continuous trading session in at least two exchanges on which they are traded. We also restrict our analysis to members acting in their capacity as a principal who post order messages (submission, revision, cancellation) and trade at least once in each of the two exchanges on which the stock is traded. Overall, we follow 46 members, denominated as "multi-venue intermediaries". Because these members do not necessarily follow the same stocks, our sample finally consists of 178 pairs (stock, member), among which 20% involve an exchange-regulated market-maker, called thereafter Designated Market-Maker (DMM) (see Panel C of Table 1).[20]

The final sample contains 20 firms with at least one multi-venue intermediary, trading continuously in two Euronext exchanges. Among them, 11 are traded on Euronext Amsterdam, 12 are traded on Euronext Brussels and 17 on Euronext Paris. To determine which is the dominant market (market D in the model) and which is the satellite market (market S in the model), we use the primary market as the (exogenous) dominant platform.

3.1.1 Measuring liquidity

We measure the spread in the market m as the equally-weighted average bid-ask spread for stock j, during a twenty-minutes interval t.[21] We focus on the relative bid-ask spread $RBAS\_m$, and the variation of the relative spread between two consecutive intervals, $\Delta RBAS\_m$, where $m = DOM, SAT$.

[19]Some French stocks were traded on the London Stock Exchange or the Deutsche Böerse, while some Dutch stocks were traded in Xetra. Gresse (2012) finds a market share of 96.45% for CAC40 stocks and even 99.99% for other SBF120 stocks in October 2007. Degryse et al (2014) show that Euronext concentrates the trading volume of the 52 Amsterdam Exchange Index Large and Mid cap constituents on our sample period.

[20]Our paper does not compare the liquidity provision of exchange-regulated market-makers versus endogenous market-makers, as Anand and Venkatamaran (2014) do using Toronto Stock Exchange data. We however keep trace of their difference in trading behaviors as suggested by the literature.

[21]We compute both equally-weighted and time-weighted averages of the quoted spreads. As the results for the two weighting schemes are virtually identical, we restrict the presentation to the equally-weighted spread measures.

3.1.2 Measuring aggregate inventory

In our dataset, the initial inventory position (I_0) of members is not observable. Moreover, members differ in the amount of capital at risk they commit to their trading activities and/or in their risk aversion, which makes inventories not comparable to each other. We thus follow Hansh et al's (1998) methodology by building standardized inventory positions to deal with these unobservable characteristics. Let $IP^s_{i,t}$ denote the inventory position of multi-venue intermediary i in stock s at the end of day t. We use the record of all trades executed by i across venues, plus the direction of these trades to obtain her inventory. We thus construct a time series for each intermediary's inventory position in each stock across all Euronext venues from the start to the end of our sample period. Since at the time more than 95% of the volumes were traded in Euronext, our inventory variable is a good proxy for intermediaries' aggregate inventories. We compute the mean ($\overline{IP}^s_i$) and the standard deviation (σ^s_i) for each of these inventory series. The standardized inventory is defined as

$$I^s_{i,t} = \frac{IP^s_{i,t} - \overline{IP}^s_i}{\sigma^s_i}.$$

We then build a measure of divergence in inventories. Let $I^s_{M,t}$ denote the median inventory at time t in stock s, and let $ID_{i,t} = |I^s_{i,t} - I^s_{M,t}|$ denote the member i's inventory position relative to the median inventory. The larger ID_i, the more divergent the inventory position of member i relative to the median is, and the more aggressively she will quote, in order to reduce her inventory exposure (Hansh et al, 1998). We take the mean of inventory divergence across intermediaries at time t in each stock s, $\overline{RI}^s_t$, to get a proxy of divergence in intermediaries' inventories ($I_1 - I_2$ in our model).

3.1.3 Determining the direction of order flows across venues

The model's predictions depend on whether liquidity demands sent across venues have the same or the opposite direction. We proxy liquidity demand by the net order flow in market m (i.e., trade imbalance) in stock s during a twenty-minutes interval t, $TrIMB\_m$, as the number of buyer-initiated trades minus the number of seller-initiated trades.[22] The dummy variable d_POS takes the value of one if order flows have the same direction across venues ($TrIMB\_DOM \times TrIMB\_SAT > 0$) on a given twenty-minutes interval,

[22]Note that our data specify the sign of trades.

and zero otherwise. Note that we exclude the first and last five minutes of trading in order to avoid contamination by specific trading behaviors during the open or close of the markets.[23]

3.1.4 Control variables

In our regression specifications, we control for the existence of arbitrage opportunities. This is necessary because, by buying the asset in one venue and reselling it in the other venue, arbitrageurs behave as inventory-driven market-makers. The dummy d_AO takes the value of one if the best bid in one venue exceeds the best ask in the other venue, i.e., $max(Bid\_SAT, Bid\_DOM) > min(Ask\_SAT, Ask\_DOM)$. We also expect arbitrageurs to use more often active transactions (marketable orders) than passive transactions (non-aggressive limit orders) to take fast arbitrage opportunities. We thus use the dummy d_AT which takes the value of one if the origin of transaction executed by the member is a market/marketable order, and zero if it is a limit order hit. In some regressions, we also control for the pending time to the next market close ($TimeClos$), the (log) transaction size in number of shares ($TrSize$), and the number of trades $NbTr$.

3.2 Summary statistics

Table 1 presents summary statistics for our sample. Panel A presents statistics across stocks. The average (median) firm has a stock price of 53.3 (50.09) Euros, a market cap of 30.6 (20.4) billion Euros, and 9 (5) multi-venue intermediaries trading on the stock. There is an average number of 3 realized arbitrage opportunities per day, and 59% of order flows across venues have the same direction. Panel B presents statistics computed within each market. Relative (quoted) spreads of the satellite market are five to ten times larger than those of the dominant market, depending if one takes means or medians. The daily number of trades is much smaller (twenty five times less in average) in the satellite market, reflecting lack of trade activity, and transaction size is also much smaller. Surprisingly, the daily number of best limit updates is only three times less in average in the satellite venue. This suggests that the satellite market is not a very active trading place, but it is closely monitored. T-tests of the difference in means between the two markets (not

[23]On February 19, 2007, the closing fixing moved from 5:25 pm to 5:30 pm. We therefore drop all observations before 9:05 am and after 5:20 pm.

shown) confirm the statistical significance of these differences. Panel C presents statistics computed for each multi-venue intermediary. There is considerable heterogeneity in terms of member trading activity, resulting from our conservative selection. The average multi-venue member makes 70 trades per day in the dominant market and 9 trades in the satellite market, but the median member only does 8 and 1 respectively. Panel C also shows the mean reversion parameter in members' aggregate inventory, obtained by estimating the following regression model of inventory time series for each pair (stock, member),

$$\Delta I_{it} = \alpha + \beta I_{it-1} + \varepsilon_t,$$

where ΔI_{it} is the change in aggregate inventory from the previous trade. Mean reversion predicts that $\beta < 0$ (if $\beta = 0$, it has a unit root and it is non-stationary). Across the 178 pairs, Panel C shows that the average mean-reversion parameter (β) is -0.073, which means that multi-venue members reduce, in average, inventory by 7.3% during the next trade.

3.3 Multivariate analysis

3.3.1 Inventory management across venues

The first step of our empirical analysis is to validate that inventory management matters for multi-venue members trading across several limit order books. Panel C already shows that aggregate inventories of some members are mean-reverting, which is consistent with the model. We now investigate whether a multi-venue member sends inventory-driven messages in one venue in response to a transaction in another venue (that is, a transaction that causes a change in her aggregate inventory). We focus on messages routed to the dominant market after a transaction in the satellite market, because effects in the more liquid market should be more easily detected. For example, after a buy in the satellite market, a multi-venue member should cancel or negatively revise existing buy orders – or submit new sell orders or positively revise sell orders in the dominant market. The opposite should occur after a sell. We implement the following Logit regression:

$$\begin{aligned} Pr(d\_i) &= \alpha + \beta_1 d\_DMM + \beta_2 |I_{i,\tau-1}| + \beta_3 d\_DMM \times |I_{i,\tau-1}| \\ &\quad + \beta_4 d\_AO_{s,\tau} + \beta_5 log(TrSize_{s,\tau}) + \beta_6 TimeClos_{s,\tau} + \varepsilon_{s,\tau}, \end{aligned} \quad (6)$$

where d_i is the dummy variable that takes 1 if member i sends a message in the dominant market in direction of inventory following a trade at time τ in the satellite market.[24] The explanatory variables are the lagged absolute inventory position of member i, the dummy variable for designated market-makers, and the interaction between both. We control for the existence of an arbitrage opportunity at the time of the trade, the size of the trade, and the pending time to the close. Our specification also includes firm fixed-effects to control for time-invariant firm heterogeneity. We run the regression both after an active and a passive transaction.

The results of the Logit analysis are presented in Table 2. Panel A reports the results for order submissions after a passive transaction, while Panel B reports the results for order submissions after an active transaction. First, in both cases, the likelihood that multi-venue intermediaries use "inventory-driven" strategies is larger when they are dedicated market-makers. Second, these trading strategies seem different according to whether the change in aggregate inventory has been caused by a passive transaction or an active transaction, consistently with the discussion of Hypothesis 1. The probability to post cross-venue inventory-driven messages is negatively related to the existence of an arbitrage opportunity when the transaction is passive, while it is significantly positively related when it is active.

In particular, Panel A shows that, when the transaction is passive, dedicated market-makers are more likely to use cross-venue inventory-driven messages, even more likely when their aggregate inventory is large. This finding validates the assumption of the model that intermediaries manage inventory risk across multiple venues. When the transaction is active, Panel B shows that the coefficients of the dummy Arbitrage Opportunity and the dummy for designated market-maker are positive and significant. This suggests that multi-venue designated market-makers take arbitrage opportunities by posting aggressive orders in the two venues. This is in line with the role that Euronext assigns to designated market-makers in cross-listed stocks. Note that, in this case, the aggregate inventory of dedicated market-makers has no significant impact, supporting the notion that the observed sequence of messages is driven by an arbitrage trading strategy.

In summary, these results are consistent with Hypothesis 1 of intermediaries using cross-venue strategies to manage inventory.

[24] Messages are tracked through their first 10 seconds after a trade.

3.3.2 Spreads

To test the main prediction of our model, we estimate the relation between the variation in twenty-minute bid-ask spreads in the satellite market and the price competition among multi-venue members which is related to the divergence in their inventories ($\overline{RI}^s$) and to the direction of order flows across venues (i.e., whether the dummy d_POS is equal to one). We run the following panel regression model:

$$\Delta RBAS\_SAT_t^s = \alpha + \beta_1 \overline{RI}_{t-1}^s + \beta_2 d\_POS_t^s + \beta_3 d\_POS_t \times \overline{RI}_{t-1} + \beta_4 NbTr\_SAT_t^s + \varepsilon_t^s. \quad (7)$$

Proposition 1 predicts that the sign of the order flows routed across venues impacts the spreads. More specifically, we expect the following sign: $\beta_2 > 0$. We also expect that in case of a large inventory divergence and same direction of shocks across venues, members compete more fiercely to execute all orders across venues, implying $\beta_3 < 0$. This interaction term allows us to distinguish our predictions from those of a adverse selection model, since the latter would predict $\beta_3 \geq 0$. Finally, the number of trades in the satellite market, $NbTr\_SAT$, controls for the impact of trades.

All specifications include day dummies and use clustered standard errors by stock to accommodate the possibility that relative spreads are strongly correlated within firms.

Table 3 presents estimation results. We report two specifications: the first with time fixed effects (Column 1) and the second with day and firm fixed-effects. The main conclusions from the analysis are as follows. First, spreads in the satellite market vary with the direction of order flows across venues (coeff. 0.108, t-stat. 2.14 in column 1), consistently with our predictions. Second, the variable of interest which is the interaction term between the direction of order flows and inventory divergence has a negative and statistical significant impact on spreads (coeff. -0.12, t-stat. -2.00). Spreads in the satellite market are thus significantly lower when there exists intermediaries holding large aggregate inventory and when order flows across venues have the same sign, supporting Hypothesis 2. This result is consistent with the case of intense competition among intermediaries illustrated by the case of "Ex post consolidation" in Panel A of Figure 3, and uniquely predicted by our model. Results for other control variables are not statistically significant. Overall, the results in Table 3 corroborate the main implication of the model.

4 Conclusion

We develop a multi-venue inventory model in which two risk averse intermediaries quote a single asset in two venues that may be hit simultaneously by shocks of equal or opposite signs. Intra-venue competition is driven by the divergence in intermediaries' inventories and the sign and magnitude of the shock in the other venue. Counter-intuitively, we find that cases in which market-makers are willing to absorb both shocks, leading to an ex post consolidation of liquidity. We show that local expected spreads may be positively or negatively impacted by interdependent market-making strategies across the two venues. Our model has interesting policy implications as we show that ex ante fragmentation may decrease total transaction costs (a measure of global liquidity). The intuition for this result is that intra-venue competition and inter-venue competition are interrelated: the possibility to compete in a single venue forces in some cases competitors to post aggressive quotes across all venues.

Our model also yields unique empirical predictions. In particular, we show that local spread depends: (i) on the way order flow fragments between venues; (ii) on the divergence of intermediaries' inventories; and (iii) on the interaction between the two. We exploit the co-existence of multiple order books for the same security within Euronext to test our model. First, we uncover new evidence of cross venue inventory effects validating the hypothesis that aggregate inventory management drives order placement across venues. Second, our panel regression analysis reveals that local bid-ask spreads vary with the sign of the order flow in the alternative venue and with the interaction between order flow and the dispersion in intermediaries' inventories (measuring divergence in inventories). These findings are in line with the predictions of the model and cannot be explained by alternative theories, e.g., adverse selection. Our results complement the existing literature on liquidity commonality. They suggest that multi-venue inventory management is an alternative mechanism to the information channel that explains common factors in liquidity. Effects could be emphasized if we now consider an intermediary trading a portfolio of assets whose returns are more or less correlated together. The intermediary' quotes placement across venues should take into account her aggregate inventories in the other assets and how they fluctuate together. The impact of multi-venue multi-asset market-making raises challenging questions related to liquidity spillover across assets and across venues. While this is an issue outside the scope of this paper, we believe it is an interesting topic for future research.

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Figure 1: One day of two-venue quotes placement and aggregate inventory of a Euronext multi-venue intermediary trading Suez

Figure 1 plots the aggregate inventory of a Euronext intermediary trading Suez and the prices that she posts in Euronext Paris and Euronext Brussels, compared to the midpoint during that trading day, January 19, 2007. The intermediary is a formally registered market-maker in Suez. The top graph plots three series of prices. The pink dash-dotted line plots the midpoint computed as the average between the consolidated best ask and best bid, i.e., the lowest ask (resp. the highest bid) across the dominant and the satellite market. The hollow circles depict the prices that the market-maker posts in the satellite market while the dark-blue triangles depict her quotes in the dominant market. Euronext Paris and Euronext Brussels are limit order books: the figure only depicts the liquidity supply activity of the market-maker (limit order placement). The bottom graph plots the aggregate euro inventory of the market-maker for the day, which is computed using the record of all signed market-makers' trades multiplied by the price of transaction across all trading venues.

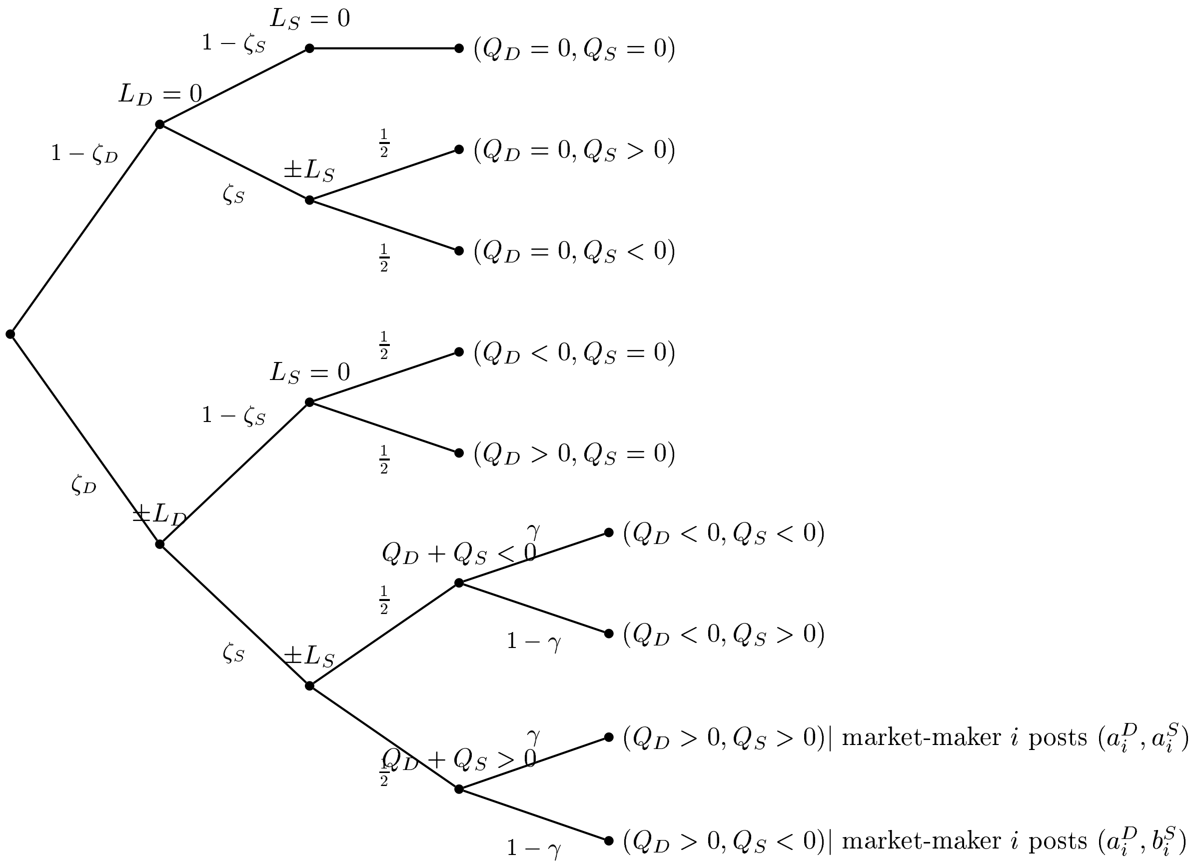
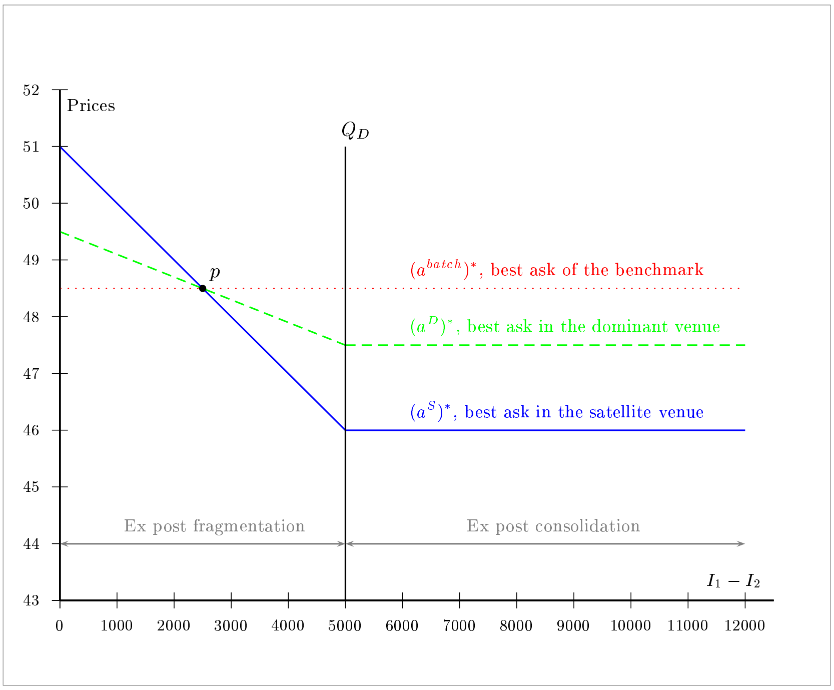
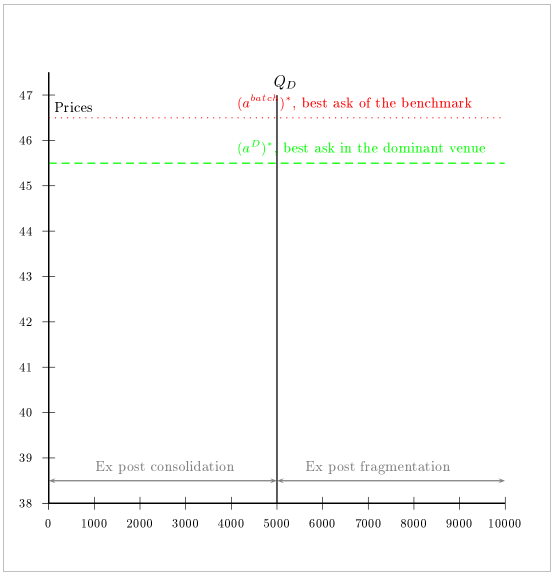


Figure 2: Tree of the quoting game across trading venues

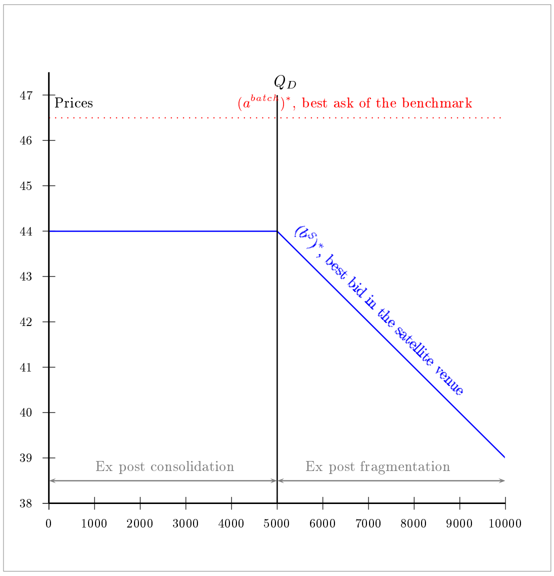
Figure 2 represents the tree of the trading game. At date 1 (not represented on the Figure), market-maker i is endowed with an inventory position denoted I_i. At date 2, venue m is hit by a liquidity shock with probability ζ_m, $m = D, S$. The probability that shocks simultaneously hit both venues is denoted λ ($= \zeta_D \times \zeta_S$). The probability that shocks have the same sign is denoted γ. The paper analyzes price formation across venues when the global order flow is net-buying ($Q_D + Q_S > 0$, which occurs with probability $\lambda/2$). Symmetric results are obtained for a net-selling global order flow. At date 3, market-maker i posts simultaneously a price in venue D and a price in venue S. We denote a_i^m (resp. b_i^m) the ask price (resp. bid price) that i posts in venue m if $Q_m > 0$ (resp. $Q_m < 0$).



(Panel A)



(a)



(b)

(Panel B)

Figure 3: Illustration of Proposition 1

Figure 3 illustrates Proposition 1. Panel A shows equilibrium selling prices in a fragmented market when buy shocks hit simultaneously venues D and S. Panel B depicts equilibrium prices when a buy shock hits venue D (Panel B (a)) and a sell shock hits venue S (Panel B (b)). The dotted red line depicts the benchmark selling price of the batch market, the green dashed line plots the best selling price in the dominant venue, and the plain blue line plots the best ask (Panel A) or best bid (Panel B) price in the satellite venue. We call p the intersection point of the 3 equilibrium prices in Panel A. The vertical line Q_D separates the region in which there is a low divergence in intermediaries' inventories ($I_1 - I_2 \leq Q_D$) from the region in which there is a high divergence in inventories ($I_1 - I_2 > Q_D$). Parameters are $Q_D = 5,000$, $|Q_S| = 2,000$, $I_u = 15,000$, $I_d = 0$, $\mu = 50$, $\sigma^2 = 0.001$, $\rho = 1$, $I_2 = 5,000$, I_1 is varying.

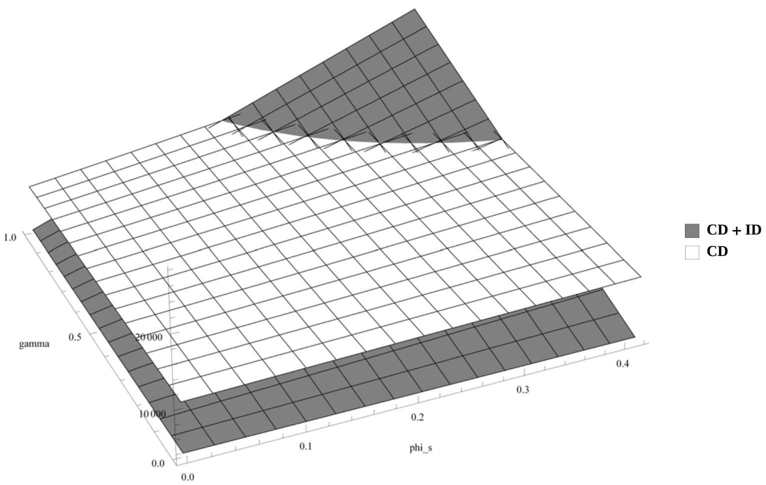


Figure 4: Impact of the inter-dealer market on dealers' expected profits.

Figure 4 represents intermediaries' expected profits with or without an initial trading round in an inter-dealer market, as a function of γ (the probability that shocks have the same sign) and ϕ_S, for $\phi_S \leq \phi_D$ and $\phi_D \leq I_u - I_d$. The white squared surface plots the expected trading profit in the customer-dealer market (CD) only, the grey squared surface plots the total expected trading profit if intermediaries engage in an inter-dealer round before trading in the customer-dealer market (CD+ID).

Table 1

Summary Statistics

This table reports summary statistics for the data used in this study. The sample consists of 20 multi-listed, continuously-traded stocks on Euronext exchanges, from January 1, 2007 through April 30, 2007 (79 trading days). The quotes and trades data comes from Euronext, and other stock-level information comes from Compustat Global.
Panel A reports the daily mean across the 20 stocks for the variables used in this study. Market capitalization is price times shares outstanding, in millions of Euros. Number of Trades is the number of transactions per day across the total number of trading venues. Number of Messages is the daily total number of orders (submissions, revisions, cancellations) across the total number of trading venues. Trade Size is the daily average size of transactions across trading venues. Number of Realized Arbitrage Opportunities is the daily number of times the best bid in the dominant (resp. satellite) market is greater than the best ask in the satellite (resp. dominant) market and buy and sell trades by the same intermediary are observed during the window of the arbitrage opportunity. Number of multi-venue intermediaries is the total number of market-makers as defined in Section 3.1. Average Inventory Divergence (RI_m) is the average divergence in market-makers' inventories, where inventories are measured each 20 minutes interval. d_POS is a dummy variable that takes the value of one if order flows across venues have the same direction.
Panel B reports summary statistics by market type. It contains news variables. Bid-Ask Spread is the equally-weighted average difference between the best bid and the best ask during the day. Relative Spread is equal to the equally-weighted average of ratio between the spread and the midpoint. Number of Best Limits Updates is the total number of times there is a change in the best limits. Percentage of Active Trades is the ratio of the number of transactions caused by a market or a marketable order over the total number of transactions in the trading venue. Percentage of Passive Trades is the ratio of the number of limit order hit over the total number of transactions in the trading venue. Percentage of Cancellations (resp. New Submissions) is the ratio of the number of cancellations (resp. new submissions) over the total number of messages in the trading venue. Percentage of Revisions is the ratio of the number of revised orders (messages other than new submissions and cancellations) over the total number of messages in the trading venue.
Panel C reports summary statistics by multi-venue intermediaries. d_DMM is the dummy that take one if the multi-venue intermediary is an exchange-regulated market-maker, also called Dedicated Market-Maker (DMM) in the stock. Number of Trades in D is the average daily number of transactions executed in the dominant venue. Number of Trades in S is the average daily number of transactions executed in the satellite venue. Percentage of Passive Transactions in S is the ratio of the number of limit order posted by the intermediary i which are hit in the satellite market over the total number of transactions. Percentage of Messages in Direction of Inventory is the ratio of the number of messages submitted within 10 seconds in the dominant market after a transaction in the satellite market which are in direction of inventory management over the total number of messages submitted within 10 seconds in the dominant market after a transaction in the satellite market. Delay to submit a message in direction of inventory is the number of second between a transaction in S and an inventory-driven message in D.

Table 1

Summary statistics (cont.)

Panel A. Summary statistics by stock

	N	Mean	Std. Dev.	Q1	Median	Q3
Market Capitalization (in billion)	1197	30589	33500	2396	20438	50089
Price	1577	53.30	36.40	25	50	70
Number of Trades	1577	2645	3142	73	1635	4213
Number of Messages	1577	9850	9924	1524	7079	15355
Trade Size	1553	491	576	33	304	1617
Number of Arbitrage Opportunities	1577	3	9	0	0	3
Number of multi-venue intermediaries	1577	9	9	3	5	10
Average inventory divergence, RI_m	1577	.62	.36	.38	.59	.82
d_POS	1224	.59	.29	.45	.60	.76

Panel B. Summary statistics by venue

B.1 Dominant venue

	N	Mean	Std. Dev.	Q1	Median	Q3
Bid-Ask Spread	1577	.11	.13	.022	.06	.16
Relative Bid-Ask Spread	1577	.28	.37	.07	.12	.27
Number of Best limits Updates	1577	6059	5095	776	4847	9655
Number of Trades	1577	2577	3108	73	1449	4055
Percentage of Active Trades	1577	45	26	28	39	54
Percentage of Passive Trades	1577	55	26	45	60	72
Percentage of Cancelations	1407	12	13	0	9	21
Percentage of Revisions	1407	33	36	4	17	60
Percentage of New Submissions	1407	22	17	5	25	34
Transaction Size	1577	620	684	192	360	779

B.2 Satellite venue

	N	Mean	Std. Dev.	Q1	Median	Q3
Bid-Ask Spread	1564	1.24	2.38	.066	.33	1.55
Relative Bid-Ask Spread	1564	1.87	3.28	.24	1,00	1.98
Number of Best limits Updates	1551	2614	3797	81	794	4040
Number of Trades	1109	95	385	0	3	20
Percentage of Active Trades	1109	31	28	0	30	45
Percentage of Passive Trades	1109	69	28	55	70	100
Percentage of Cancelations	1395	8	11	0	4	10
Percentage of Revisions	1395	79	26	70	90	98
Percentage of New Submissions	1395	8	12	0	4	11
Transaction Size	1109	348	369	100	250	485

Panel C. Summary statistics by multi-venue intermediary

	N	Mean	Std. Dev.	Q1	Median	Q3
Dummy for Dedicated Market-Maker	178	0,19	0,39	0,00	0,00	1,00
Average Mean Reversion of Inventory	178	-0,073	0,150	-0,314	-0,013	0,001
Number of Trades in D	178	70	131	0	8	377
Number of Trades in S	178	9	28	0	1	69
Percentage of Messages in Direction of Inventory	110	66	30	0	66	100
Percentage of Passive Transactions in S	178	53	30	0	52	98
Delay to submit a message in Direction of Inv.	110	3	2	0	3	8

Table 2

Likelihood of Expected Inventory-driven Message following a Transaction in the Satellite Market

This table presents estimates of the relation between the likelihood of an inventory-driven message posted by the intermediary i in the dominant market after a trade in the satellite market. The left-hand side variable is Indicator of Expected Message, a dummy variable that takes the value 1 if the message has the expected value. Left-hand side variables are described in caption of Table 1. $DMM\times$Standardized Inventory is an interaction term equal to the product of DMM and Standardized Inventory. Panel A shows regression specifications in the subsample of passive transactions. Panel B shows regression specifications in the subsample of active transactions. All specifications include firm fixed effects and t-statistics are calculated using standard errors clustered by liquidity supplier. The symbols ***, **, * denote significance levels of 1%, 5% and 10%, respectively for the two-tailed hypothesis test that the coefficient equals zero.

Panel A. Passive Transactions

Dependent variable:	Indicator of Expected Message			
	(1)		(2)	
Log Trade Size	0,032		0,032	
	(1,05)		(1,05)	
Standardized Inventory	0,018		-0,02	
	(0,56)		(-0.55)	
DMM	1,522	***	1,377	***
	(3,70)		(3,42)	
Arbitrage Opportunity	-0,310	***	-0,309	***
	(-3.31)		(-3.33)	
Time to close	0,025		0,025	
	(1,38)		(1,36)	
DMM × Standardized Inventory			0,187	**
			(2,33)	
Intercept	0,217		0,243	
	(0,66)		(0,74)	
Firm FEs	Yes		Yes	
N	18 022		18 022	
Pseudo R^2	0,06		0,06	

Panel B. Active Transactions				
Dependent variable:	Indicator of Expected Message			
	(1)		(2)	
Log Trade Size	-0,015		-0,014	
	(-0.45)		(-0.45)	
Standardized Inventory	-0,005		0,043	
	(-0.08)		(0,59)	
DMM	0,646	**	0,733	***
	(2,44)		(3,76)	
Arbitrage Opportunity	0,597	***	0,603	***
	(4,46)		(4,58)	
Time to close	0,013		0,014	
	(0,80)		(0,81)	
DMM × Standardized Inventory			-0,125	
			(-0.67)	
Intercept	1,402	**	1,348	**
	(2,30)		(2,10)	
Firm FEs	Yes		Yes	
N	9 100		9 100	
Pseudo R^2	0,06		0,06	

Table 3

Determinants of Relative Spreads in the Satellite Market

This table presents estimates of the relation between changes in relative bid-ask spreads in the satellite market and the divergence in intermediaries' inventories and the direction of order flows across venues. The left-hand side variable is the Change in Relative Spread of the Satellite market in the 20-minutes interval. The right-hand-side variables are defined in caption of Table 1. Same Direction×Lag AbsoluteRI is an interaction term equal to the product of Same Direction and Lag AbsoluteRI. t-statistics are calculated using standard errors clustered by firm. The symbols ***, **, * denote significance levels of 1%, 5% and 10%, respectively for the two-tailed hypothesis test that the coefficient equals zero.

Dependent variable:	Change in Relative Spread of Market S			
	(1)		(2)	
Same Direction	0,108	**	0,105	**
	(2,14)		(2,13)	
Lag Absolute RI	0,087		0,076	
	(1,14)		(1,34)	
Same Direction × Lag Absolute RI	-0,12	**	-0,119	**
	(-2.00)		(-2.01)	
Number of Trades in Market S	-0,050		0,004	
	(-1.30)		(0,12)	
Intercept	-0,078		-0,065	
	(-0.93)		(-1.03)	
Time FEs	Yes		Yes	
Firm FEs	No		Yes	
N	11 172		11 172	
Adjusted R^2	0,01		0,03	

5 Appendix – Proofs

Preliminary element used in the proofs: intermediaries' trading profits

Market-maker i's trading profit is given by:

$$V_i\left(p_1^D, p_2^D, p_1^S, p_2^S\right) = \begin{cases} \underbrace{p_i^D Q_D + p_i^S Q_S - r_i\left(Q_D+Q_S\right)\left(Q_D+Q_S\right)}_{\equiv v_i(Q_D+Q_S)} & \text{if } p_i^D Q_D < p_{-i}^D Q_D \text{ and } p_i^S Q_S < p_{-i}^S Q_S, \\ \underbrace{\left(p_i^D - r_i(Q_D)\right) Q_D}_{\equiv v_i(Q_D)} & \text{if } p_i^D Q_D < p_{-i}^D Q_D \text{ and } p_i^S Q_S > p_{-i}^S Q_S, \\ \underbrace{\left(p_i^S - r_i\left(Q_S\right)\right) Q_S}_{\equiv v_i(Q_S)} & \text{if } p_i^D Q_D > p_{-i}^D Q_D \text{ and } p_i^S Q_S < p_{-i}^S Q_S, \\ 0 & \text{if } p_i^D Q_D > p_{-i}^D Q_D \text{ and } p_i^S Q_S > p_{-i}^S Q_S. \end{cases}$$

where p_i^D denotes the price set by market-maker i in venue D, and p_i^S denotes the price posted by i in venue S, $i = 1, 2$. The price p_i^m is an ask price if $Q_m > 0$ and a bid price if $Q_m < 0$, $m = D, S$.[25]

Proof of Lemma 1

Case 1. We first look for the necessary conditions that must be simultaneously fulfilled to guarantee the existence of an equilibrium in which a single market-maker executes the entire order flow ("ex post consolidation").

Market-maker $i \in \{1, 2\}$ executes the entire order flow in equilibrium if and only if the ask price a_D prevailing in venue D (in which $Q_D > 0$), and the ask price p_S (resp. the bid price) prevailing in venue S are the maximum (resp. minimum in venue S if $Q_S < 0$) prices such that: (i) trading $Q_D + Q_S$ is profitable for market-maker i (i.e., $v_i(Q_D + Q_S) \geq 0$), and (i') not for market-maker $-i$ (i.e., $v_{-i}(Q_D + Q_S) < 0$); (ii) trading the total order flow is more profitable for market-maker i than trading only Q_D (i.e., $v_i(Q_D + Q_S) \geq v_i(Q_D)$) or (ii') only Q_S (i.e., $v_i(Q_D + Q_S) \geq v_i(Q_S)$); (iii) it is not profitable for market-maker $-i$ to undercut market-maker i neither in venue D (i.e., $v_{-i}(Q_D) < 0$) nor (iii') in venue S (i.e., $v_{-i}(Q_S) < 0$). Using the definition of market-makers' reservation prices and trading profits, these conditions rewrite as

[25] As in Biais (1993), the utility function of intermediaries given in Eq. (1) is linearized, under the assumption $Q_D < I_u - I_d$. Note that, in our transparent setting, the criticism on the linear approximation used by Biais (1993) for opaque markets raised by de Frutos and Manzano (2002) does not apply. The assumption $Q_D < I_u - I_d$ also guarantees that market-maker i has a probability to post the best price in venue m which is strictly greater than 0 and strictly lower than 1, for $i = 1, 2$ and $m = D, S$.

follows:

$$
\begin{aligned}
\text{i} &: a_D Q_D + p_S Q_S \geq r_i(Q_D + Q_S)(Q_D + Q_S),\\
\text{i'} &: a_D Q_D + p_S Q_S < r_{-i}(Q_D + Q_S)(Q_D + Q_S);\\
\text{ii} &: a_D \geq r_i(Q_D) + \rho\sigma^2 Q_S,\\
\text{ii'} &: p_S Q_S \geq \left(r_i(Q_S) + \rho\sigma^2 Q_D\right) Q_S;\\
\text{iii} &: a_D < r_{-i}(Q_D),\\
\text{iii'} &: p_S Q_S < r_{-i}(Q_S) Q_S.
\end{aligned}
$$

• *Suppose that market-maker 1 trades* Q_D+Q_S. If $(I_1-I_2-Q_D)Q_S \leq 0$, then $\left(r_1(Q_S) + \rho\sigma^2 Q_D\right) Q_S \geq r_2(Q_S)Q_S$. Thus conditions (ii') and (iii') cannot hold simultaneously. A necessary condition for such an equilibrium to exist is thus $(I_1 - I_2 - Q_D)Q_S > 0$, i.e., either $I_1 - I_2 > Q_D$ if $Q_S > 0$ or $I_1 - I_2 < Q_D$ if $Q_S < 0$.

• *Suppose that market-maker 2 trades* $Q_D + Q_S$. Recall that by assumption $I_1 > I_2$ (implying that $r_1(Q_D + Q_S) < r_2(Q_D + Q_S)$) and $Q_D + Q_S > 0$. Thus conditions (i) and (i') cannot simultaneously hold for market-maker 2. Therefore, there exists no equilibrium such that market-maker 2 trades the total order flow.

Case 2. We now look for the necessary conditions that must be simultaneously fulfilled to guarantee the existence of an equilibrium in which the different parts of the order flow are executed by different market-makers ("ex post fragmentation").

There exists an equilibrium such that market-maker $i \in \{1, 2\}$ trades Q_D and market-maker $-i$ trades Q_S if and only if the ask price a_D prevailing in venue D (in which $Q_D > 0$), and the ask or bid price p_S prevailing in venue S (in which $Q_S > 0$ or $Q_S < 0$) are the maximum (resp. minimum in market S if $Q_S < 0$) prices such that: (I) market-maker i is better off trading Q_D rather than Q_S (i.e., $v_i(Q_D) > v_i(Q_S)$) and (I') market-maker $-i$ is better off trading Q_S rather than Q_D (i.e., $v_{-i}(Q_S) > v_{-i}(Q_D)$); (II) market-maker $-i$ is better off trading Q_S only rather than $Q_D + Q_S$ (i.e., $v_{-i}(Q_D + Q_S) < v_{-i}(Q_S)$) and (II') market-maker i is better off trading Q_D only rather than $Q_D + Q_S$ (i.e., $v_i(Q_D + Q_S) < v_i(Q_D)$) ; (III) trading Q_D is profitable for market-maker i (i.e., $v_i(Q_D) \geq 0$) and (III') trading Q_S is profitable for market-maker $-i$ (i.e.,

$v_i(Q_S) \geq 0$). These conditions may be rewritten as follows:

$$\text{I}: a_D > r_i(Q_D) + (p_S - r_i(Q_S))\,\frac{Q_S}{Q_D},$$
$$\text{I '}: p_S Q_S > r_{-i}(Q_S)Q_S + (a_D - r_{-i}(Q_D))\,Q_D;$$
$$\text{II}: a_D < r_{-i}(Q_D) + \rho\sigma^2 Q_S,$$
$$\text{II'}: p_S Q_S < \left(r_i(Q_S) + \rho\sigma^2 Q_D\right) Q_S;$$
$$\text{III}: a_D \geq r_i(Q_D),$$
$$\text{III'}: p_S Q_S \geq r_{-i}(Q_S)Q_S.$$

• *Suppose that market-maker 1 trades Q_D and market-maker 2 trades Q_S.* If $(I_1 - I_2 - Q_D)Q_S \geq 0$, then conditions II' and III' cannot hold simultaneously. A necessary condition for such an equilibrium to exist is thus $(I_1 - I_2 - Q_D)Q_S < 0$, that is, either $I_1 - I_2 < Q_D$ if $Q_S > 0$ or $I_1 - I_2 > Q_D$ if $Q_S < 0$.

• *Suppose that market-maker 1 trades Q_S and market-maker 2 trades Q_D.* If $Q_S < 0$, then conditions II and III cannot hold simultaneously, since $r_1(Q_D) + \rho\sigma^2 Q_S < r_2(Q_D)$. If $Q_S > 0$, a necessary condition for conditions I and I' to hold simultaneously is

$$r_1(Q_S) + (a_D - r_1(Q_D))\,\frac{Q_D}{Q_S} < r_2(Q_S) + (a_D - r_2(Q_D))\,\frac{Q_D}{Q_S},$$

which is never true since $I_1 > I_2$ and $|Q_D| > |Q_S|$. Consequently, there exists no equilibrium in which the longer market-maker (here, market-maker 1) would be the first buyer in venue S while the shorter market-maker (market-maker 2) would be the first seller in venue D.

Finally, the limit case in which $(I_1 - I_2 - Q_D)Q_S = 0$ is analyzed in the proof of Proposition 1 below. ■

Proof of Proposition 1

From Lemma 1, there are various cases to consider, depending on the signs of $I_1 - I_2 - Q_D$ and Q_S.

Case 1.1. $Q_S > 0$ and $I_1 - I_2 > Q_D$ (i.e., $(I_1 - I_2 - Q_D)Q_S > 0$). From Lemma 1 (Case 1), we know that market-maker 1 consolidates the entire order flow by posting the best ask price in both venues D and S. The ask prices a_D and a_S are the maximum prices that satisfy the set of conditions i to iii':

$$\begin{aligned}
\text{ii and iii} &: r_1(Q_D) + \rho\sigma^2 Q_S \le a_D < r_2(Q_D),\\
\text{ii' and iii'} &: r_1(Q_S) + \rho\sigma^2 Q_D \le a_S < r_2(Q_S),\\
\text{i} &: r_1(Q_D+Q_S)(Q_D+Q_S) \le a_D Q_D + a_S Q_S,\\
\text{i'} &: a_D Q_D + a_S Q_S < r_2(Q_D+Q_S)(Q_D+Q_S).
\end{aligned}$$

From the two first inequalities, $(a_D)^* = r_2(Q_D) - \varepsilon$ and $(a_S)^* = r_2(Q_S) - \varepsilon$ are natural candidates for the equilibrium, as they are the maximum prices that satisfy conditions ii and iii, ii' and iii'. Straightforward computations show that they also satisfy conditions i and i' (details are omitted for brevity).

Case 1.2. $Q_S < 0$ and $I_1 - I_2 < Q_D$ (i.e., $(I_1 - I_2 - Q_D)Q_S > 0$). From Lemma 1 (Case 1), we know that market-maker 1 consolidates the entire order flow by posting the best ask price in venue D and the best bid price in venue S. The ask price a_D in venue D and the bid price b_S in venue S are respectively the maximum and the minimum prices that satisfy the set of conditions i to iii':

$$\begin{aligned}
\text{ii and iii} &: r_1(Q_D) - \rho\sigma^2(-Q_S) \le a_D < r_2(Q_D),\\
\text{ii' and iii'} &: r_2(Q_S) < b_S \le r_1(Q_S) + \rho\sigma^2 Q_D,\\
\text{i and i'} &: r_1(Q_D+Q_S)(Q_D+Q_S) \le a_D Q_D + b_S Q_S < r_2(Q_D+Q_S)(Q_D+Q_S).
\end{aligned}$$

The natural candidates for the equilibrium $a_D = r_2(Q_D) - \varepsilon$ and $b_S = r_2(Q_S) + \varepsilon$ do not satisfy condition i'. Consequently, the constraint i' is binding at equilibrium, and equilibrium prices must be such that:

$$(a_D)^* = r_2(Q_D+Q_S) + ((b_S)^* - r_2(Q_D+Q_S))\frac{(-Q_S)}{Q_D} - \varepsilon. \tag{8}$$

First, we input the expression of $(a_D)^*$ defined in Eq. (8) above in market-maker 1's trading profit (conditional on the fact that she executes Q_D and Q_S): $v_1(Q_D+Q_S) = \rho\sigma^2(I_1 - I_2)(Q_D + Q_S)$. This trading profit does not depend on equilibrium prices. Consequently, there exists a continuum of prices that may sustain the equilibrium. Second, inputting $(a_D)^*$ defined in Eq.

(8) into conditions ii to iii', the equilibrium price in venue S must satisfy:

$$\text{ii and iii}: \rho\sigma^2(I_2 - I_1)\frac{Q_D}{-Q_S} + r_2(Q_S) \le (b_S)^* < \rho\sigma^2(I_1 - I_2)\frac{Q_D}{-Q_S} + r_2(Q_S) - \rho\sigma^2 Q_D,$$
$$\text{ii' and iii'}: r_2(Q_S) < (b_S)^* \le r_1(Q_S) + \rho\sigma^2 Q_D.$$

Since $I_1 > I_2$, we have $\rho\sigma^2(I_2 - I_1)\frac{Q_D}{-Q_S} < r_2(Q_S)$ and $r_1(Q_S) + \rho\sigma^2 Q_D < \left(\rho\sigma^2(I_2 - I_1)\right)\frac{Q_D}{-Q_S} + r_2(Q_S) - \rho\sigma^2 Q_D$. Thus the second inequality is constraining both the minimum and the maximum possible price in market S. Within all equilibria defined by:

$$(a_D)^* = r_2(Q_D + Q_S)\frac{Q_D + Q_S}{Q_D} + (b_S)^*\frac{(-Q_S)}{Q_D} - \varepsilon,$$
$$(b_S)^* \in (r_2(Q_S) + \epsilon, r_1(Q_S) + \rho\sigma^2 Q_D + \epsilon],$$

we select the only equilibrium that is continuous at $I_1 - I_2 = Q_D$, that is, $(a_D)^* = r_2(Q_D) - \rho\sigma^2(-Q_S) - \epsilon$, from which we deduce that $(b_S)^* = r_2(Q_S) + \epsilon$.

Case 2.1. $Q_S > 0$ and $I_1 - I_2 < Q_D$ (i.e., $(I_1 - I_2 - Q_D)Q_S < 0$). From Lemma 1 (Case 2), we know that dealer 1 executes Q_D while dealer 2 executes Q_S. The ask prices a_D and a_S are the maximum prices that satisfy the set of conditions I to III':

$$\text{II and III}: r_1(Q_D) \le a_D < r_2(Q_D) + \rho\sigma^2 Q_S,$$
$$\text{II' and III'}: r_2(Q_S) \le a_S < r_1(Q_S) + \rho\sigma^2 Q_D,$$
$$\text{I}: a_D > r_1(Q_D) + (a_S - r_1(Q_S))\frac{Q_S}{Q_D},$$
$$\text{I'}: a_S > r_2(Q_S) + (a_D - r_2(Q_D))\frac{Q_D}{Q_S}.$$

The candidates for the equilibrium $a_D = r_2(Q_D) + \rho\sigma^2 Q_S - \varepsilon$ and $a_S = r_1(Q_S) + \rho\sigma^2 Q_D - \varepsilon$ from the two first inequalities do not satisfy condition I'. Consequently, the constraint I' is binding at equilibrium, and equilibrium prices must be such that:

$$(a_D)^* = r_2(Q_D) + ((a_S)^* - r_2(Q_S))\frac{Q_S}{Q_D} - \varepsilon. \tag{9}$$

First, notice that under the condition in Eq. (9), condition I always holds (given that $(I_1 - I_2)(Q_D - Q_S) > 0$). Second, inputting $(a_D)^*$ defined in Eq. (9) into conditions (II and III) and

(II' and III') yields the following restrictions on $(a_S)^*$:

$$\text{II and III} : r_2(Q_S) + (r_1(Q_D) - r_2(Q_D))\,\frac{Q_D}{Q_S} \leq (a_S)^* < r_2(Q_S) + \rho\sigma^2 Q_D,$$
$$\text{II' and III'} : r_2(Q_S) \leq (a_S)^* < r_1(Q_S) + \rho\sigma^2 Q_D.$$

Third, we compute market-makers' equilibrium profits. The trading profit of market-maker 2 (conditional on the fact that he executes Q_S) writes: $v_2(Q_S) = ((a_S)^* - r_2(Q_S))Q_S$. We use the expression of $(a_D)^*$ defined in Eq. (9) to compute the trading profit of market-maker 1 (conditional on the fact that she executes Q_D) as a function of $(a_S)^*$: $v_1(Q_D) = \left(r_2(Q_D) + ((a_S)^* - r_2(Q_S))\frac{Q_S}{Q_D} - r_1(Q_D)\right) Q_D$.

We observe that market-makers' profits are both strictly increasing in $(a_S)^*$. Consequently, market-makers' reaction functions are such that the best ask price in venue S is the highest possible one. From conditions (II and III) and (II' and III'), and under the hypothesis that $I_1 - I_2 < Q_D$, we deduce that condition (II' and III') is binding and that $(a_S)^*$ is such that:

$$(a_S)^* = r_1(Q_S) + \rho\sigma^2 Q_D - \varepsilon, \tag{10}$$

from which we deduce that:

$$(a_D)^* = r_2(Q_D) + \rho\sigma^2 Q_S - \rho\sigma^2(I_1 - I_2)\frac{Q_S}{Q_D} - \varepsilon. \tag{11}$$

Consequently, there exists a unique equilibrium such that market-maker 1 posts $(a_D)^*$ (defined in Eq. (11)) and trades Q_D while market-maker 2 posts the best ask price equal to $(a_S)^*$ (defined in Eq. (10)) and trades Q_S.

Case 2.2. $Q_S < 0$ and $I_1 - I_2 > Q_D$ (i.e., $(I_1 - I_2 - Q_D)Q_S < 0$). From Lemma 1 (Case 2), we know that market-maker 1 executes Q_D while market-maker 2 executes Q_S. The ask price a_D in venue D and the bid price b_S in venue S are respectively the maximum and the minimum prices that satisfy the set of conditions I to III':

$$\text{II and III} : r_1(Q_D) \leq a_D < r_2(Q_D) + \rho\sigma^2 Q_S,$$
$$\text{II' and III'} : r_1(Q_S) + \rho\sigma^2 Q_D < b_S \leq r_2(Q_S),$$
$$\text{I} \;: a_D > r_1(Q_D) + (b_S - r_1(Q_S))\,\frac{Q_S}{Q_D},$$
$$\text{I '} : b_S < r_2(Q_S) + (r_2(Q_D) - a_D)\,\frac{Q_D}{-Q_S}.$$

From the two first inequalities, $a_D = r_2(Q_D) - \rho\sigma^2(-Q_S) - \varepsilon$ and $b_S = r_1(Q_S) + \rho\sigma^2 Q_D + \varepsilon$ are natural candidates for the equilibrium. Straightforward computations show that they also satisfy conditions I and I'.

Case 3: $I_1 - I_2 = Q_D$, i.e., $(I_1 - I_2 - Q_D)Q_S = 0$. Notice that if $I_1 - I_2 = Q_D$, then the equilibrium described in 1.1. cannot be sustained because conditions ii' and iii' (i.e., $r_1(Q_S) + \rho\sigma^2 Q_D \leq a_S = r_2(Q_S)$) cannot hold simultaneously due to the strict inequality, which contradicts the equality $r_1(Q_S) + \rho\sigma^2 Q_D = r_2(Q_S)$. If $(a_D)^* = r_2(Q_D) - \varepsilon$ and $(a_S)^* = r_2(Q_S) = r_1(Q_S) + \rho\sigma^2 Q_D$ however, conditions i, i', ii, ii' and iii hold. Thus at these prices, market-maker 2 becomes indifferent between trading Q_S or not, and market-maker 1 is indifferent between executing $Q_D + Q_S$ or Q_D. ■

Proof of Corollary 1

In our set up (identical risk aversion and identical pre-trade inventory distribution), we can measure intermediaries' aggregate posttrade risk by the sum of the variance of their posttrade wealths (Yin, 2005). In the batch auction, the longer intermediary executes the entire order flow, thus the aggregate posttrade risk, denoted by $(\sigma^2_{agg})^{batch}$, is equal to:

$$(\sigma^2_{agg})^{batch} = Var((I_1 - Q_D - Q_S)\tilde{v}) + Var((I_2)\tilde{v}). \tag{12}$$

In a fragmented market, posttrade allocations depend on the sign of $(I_1 - I_2 - Q_D)\, Q_S$.

• If $(I_1 - I_2 - Q_D)\, Q_S > 0$, the aggregate posttrade risk is similar to that in the batch auction, because the longer intermediary consolidates the entire order flow:

$$(\sigma^2_{agg})^{cons} = Var((I_1 - Q_D - Q_S)\tilde{v}) + Var((I_2)\tilde{v}) = (\sigma^2_{agg})^{batch}.$$

• If $(I_1 - I_2 - Q_D)\, Q_S \leq 0$, shocks are absorbed by different intermediaries (ex post fragmentation) and the aggregate posttrade risk is equal to:

$$(\sigma^2_{agg})^{frag} = Var((I_1 - Q_D)\tilde{v}) + Var((I_2 - Q_S)\tilde{v}). \tag{13}$$

Then, subtracting Eq. (13) from Eq. (12) is equal to $(\sigma^2_{agg})^{frag} - (\sigma^2_{agg})^{batch} = 2Q_S(I_1 - I_2 - Q_D) < 0$, which is negative in the case considered here.■

Proof of Proposition 2

We decompose the proof into two results, depending on the sign of Q_S.

Notations. For ease of computation in the proof, we use the following notations $q_m = Q_m$ for a net-buying order flow and $q_m = -Q_m$ for a net-selling order flow ($m = S, D$). Let us also define $v_d = \mu - \rho\sigma^2 I_d$, $v_u = \mu - \rho\sigma^2 I_u$, $x = \mu - \rho\sigma^2 I_1$ and $y = \mu - \rho\sigma^2 I_2$. The support of the uniform distribution function of x and y simplifies to $[v_u, v_d]$. We also note $d = \rho\sigma^2 q_D$ and $s = \rho\sigma^2 q_S$. Finally, let $a^{m,+}$ (resp. $a^{m,-}$) be the best ask price of venue m when liquidity demands have the same sign (resp. opposite sign) across venues.

Result 1 *Suppose that shocks have the same sign (with probability γ). Then, the expected ask prices quoted in the venues D and S are equal to:*

$$E\left(\underline{a}^{m,+}\right) = \mu - \rho\sigma^2 \frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_m}{2} + \rho\sigma^2 q_{-m}\left(\frac{q_D}{I_u - I_d} - \frac{1}{3}\left(\frac{q_D}{I_u - I_d}\right)^2\right), m = S, D. \quad (14)$$

Proof. We first compute the expected ask that prevails in venue D. By definition,

$$E\left(\underline{a}^{D,+}\right) = E\left(\min\left(a_1^D, a_2^D\right) \mathbb{1}_{Q_D>0} \mathbb{1}_{Q_S>0}\right).$$

Given Proposition 1, the notations defined above, and the symmetry of our hypotheses, the latter equation writes:

$$\begin{aligned} E\left(\underline{a}^{D,+}\right) &= \frac{2}{(v_d - v_u)^2}\Bigg[\int_{v_u}^{v_d - d}\int_{x+d}^{v_d}(y + \frac{d}{2})dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}\left(y + \frac{d}{2} + s\left(\frac{d - (y - x)}{d}\right)\right)dydx \\ &- \int_{v_u}^{v_d - d}\int_{x+d}^{v_d}\left(y + \frac{d}{2} + s\left(\frac{d - (y - x)}{d}\right)\right)dydx\Bigg]. \end{aligned} \quad (15)$$

We now turn to the expected ask prevailing in venue S using a similar reasoning. The expression writes:

$$\begin{aligned} E\left(\underline{a}^{S,+}\right) &= E\left(\min\left(a_1^S, a_2^S\right) \mathbb{1}_{Q_D>0} \mathbb{1}_{Q_S>0}\right) \\ &= \frac{2}{(v_d - v_u)^2}\Bigg[\int_{v_u}^{v_d - d}\int_{x+d}^{v_d}(y + \frac{s}{2})dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}\left(x + \frac{s}{2} + d\right)dydx \\ &- \int_{v_u}^{v_d - d}\int_{x+d}^{v_d}\left(x + \frac{s}{2} + d\right)dydx\Bigg]. \end{aligned} \quad (16)$$

Computations based on Eq. (15) and on Eq. (16) yield the expressions given in Eq. (14) for $m = D$ and $m = S$ respectively. Q.E.D.

Result 2 *Suppose that shocks have opposite signs (with probability $1 - \gamma$), then the expected ask* *prices in venues D and S respectively write:*

$$E\left(\underline{a}^{D,-}\right) = \mu - \rho\sigma^2 \frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_D}{2} - \rho\sigma^2 q_S, \quad (17)$$

$$E\left(\underline{a}^{S,-}\right) = \mu - \rho\sigma^2 \frac{2I_d + I_u}{3} + \frac{\rho\sigma^2 q_S}{2} - \rho\sigma^2 q_D + \frac{(q_D)^2}{(I_u - I_d)} - \frac{(q_D)^3}{3\,(I_u - I_d)^2}. \quad (18)$$

Proof. We first compute the expected best ask prevailing in venue D (considering a sell shock in venue S):

$$E\left(\underline{a}^{D,-}\right) = E\left(\min\left(a_1^D, a_2^D\right) \mathbb{1}_{Q_D>0} \mathbb{1}_{Q_S<0}\right),$$

which rewrites:

$$E\left(\underline{a}^{D,-}\right) = \frac{2}{(v_d - v_u)^2}\Big(\int_{v_u}^{v_d-d}\int_{v_u}^{x+d}(y + \frac{d}{2} - s)dydx + \int_{v_u}^{v_d}\int_{x}^{v_d}(y + \frac{d}{2} - s)dydx - \int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y + \frac{d}{2} - s)dydx\Big). \quad (19)$$

Eq. (17) immediately follows.

Symmetrically, the expected best ask prevailing in market S (considering now a sell shock in venue D) writes:

$$E(\underline{a}^{S,-}) = \frac{2}{(v_d - v_u)^2}\Big(\int_{v_u-d}^{v_d}\int_{v_u}^{x+d}(x + \frac{s}{2} + d)dydx + \int_{v_u}^{v_d}\int_{v_u}^{x}(y + \frac{s}{2})dydx - \int_{v_u-d}^{v_d}\int_{v_u}^{x+d}(y + \frac{s}{2})dydx\Big). \quad (20)$$

Computations yield Eq. (18). Q.E.D.

Let us define the half-spread as $s^m = a^m - \mu$ and $\phi_m = \frac{q_m}{I_u - I_d}$. Proposition 2 is then obtained from Results 1 and 2 considering the extensive form of the game represented in Figure 2. ■

Proof of Proposition 3

By definition, $Cov(s^D, s^S) = \gamma Cov(\underline{a}^{D,+} - \mu, \underline{a}^{S,+} - \mu) + (1-\gamma)Cov(\underline{a}^{D,-} - \mu, \mu - \overline{b}^{S,-}) = \gamma Cov(\underline{a}^{D,+}, \underline{a}^{S,+}) - (1-\gamma)Cov(\underline{a}^{D,-}, \overline{b}^{S,-})$. We thus decompose the proof into two results, depending on the sign of shocks across venues (similar or opposite).

Result 3 *Suppose that shocks have the same sign (with probability γ). The covariance between* *the ask price in venue D and the one in venue S is equal to:*

$$\frac{Cov(\underline{a}^{D,+},\underline{a}^{S,+})}{(\rho\sigma^2)^2(I_u-I_d)^2}=\frac{1}{18}-\phi_D\left(-\frac{\phi_D-\phi_S}{6}+\frac{2(\phi_S-\phi_D)}{9}\phi_D+\frac{15\phi_S-\phi_D}{12}\phi_D^2+\frac{2\phi_S}{3}\phi_D^3+\frac{\phi_S}{9}\phi_D^4\right), \tag{21}$$

where $\phi_m=\frac{q_m}{(I_u-I_d)}$, $m=D,S$.

Proof. By definition, $E\left(\underline{a}^{D,+}\underline{a}^{S,+}\right)=E\left(\min\left(a_1^D,a_2^D\right)\times\min\left(a_1^S,a_2^S\right)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S>0}\right)$. Using Proposition 1, and notations defined above, we get:

$$\begin{aligned}E\left(\underline{a}^{D,+}\underline{a}^{S,+}\right)=\frac{2}{(v_d-v_u)^2}\Bigg[&\int_{v_u}^{v_d-d}\int_{v_u}^{v_d-d}(y+\frac{s}{2})(y+\frac{d}{2})dydx\\&+\int_{v_u}^{v_d}\int_{x}^{v_d}(x+\frac{s}{2}+d)(y+\frac{d}{2}+s\left(\frac{d-(y-x)}{d}\right))dydx\\&-\int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(x+\frac{s}{2}+d)(y+\frac{d}{2}+s\left(\frac{d-(y-x)}{d}\right))\Bigg].\end{aligned} \tag{22}$$

To compute $Cov(\underline{a}^{D,+},\underline{a}^{S,+})=E(\underline{a}^{D,+}\underline{a}^{S,+})-E(\underline{a}^{D,+})E(\underline{a}^{S,+})$, we use the expression above and Result 1 for expressions of $E\left(\underline{a}^{D,+}\right)$ and $E\left(\underline{a}^{S,+}\right)$. Computations yield Eq. (21). Q.E.D.

Result 4 *Suppose that shocks have opposite signs (*$1-\gamma$*). The covariance between the best price in venue D and the one in venue S writes:*

$$\frac{Cov\left(\underline{a}^{D,-},\overline{b}^{S,-}\right)}{(\rho\sigma^2)^2(I_u-I_d)^2}=\frac{1}{36}+\frac{(\phi_D)^2}{36}\left(3\left(\phi_D\right)^2-8\phi_D+6\right). \tag{23}$$

Proof. If a sell shock hits venue S, the expected best bid in venue S is such that $E(\overline{b}^{S,-})=E(max(b_1^S,b_2^S)\mathbb{1}_{Q_D>0}\mathbb{1}_{Q_S<0})$, or:

$$\begin{aligned}E(\overline{b}^{S,-})=\frac{2}{(v_d-v_u)^2}(&\int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(x+\frac{s}{2}+d)dydx+\int_{v_u}^{v_d}\int_{x}^{v_d}(y+\frac{s}{2})dydx\\&-\int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y+\frac{s}{2})dydx)\end{aligned} \tag{24}$$

When a buy shock hits venue D, the expected best ask price of venue D is thus described by Eq. (17). Then $E(\underline{a}^{D,-}\overline{b}^{S,-})$ writes:

$$\begin{aligned}E(\underline{a}^{D,-}\overline{b}^{S,-})=\frac{2}{(v_d-v_u)^2}[&\int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y+\frac{d}{2}-s)(x+\frac{s}{2}+d)dydx\\&+\int_{v_u}^{v_d}\int_{x}^{v_d}(y+\frac{d}{2}-s)(y+\frac{s}{2})dydx-\int_{v_u}^{v_d-d}\int_{x+d}^{v_d}(y+\frac{d}{2}-s)(y+\frac{s}{2})dydx].\end{aligned} \tag{25}$$

Using Equations (17), (24) and (25), we can deduce the expression of $Cov(\underline{a}^{D,-}, \overline{b}^{S,-})$ described in Eq. (23). Q.E.D.

Proposition 3 is obtained from Results 3 and 4 considering the extensive form of the game represented in Figure 2. ■

Proof of Corollary 2

Remind that $\underline{a}^{batch}$ denotes the lowest ask price in the benchmark model in which the global order flow is batched and executed by the intermediary with the larger inventory. From Ho and Stoll (1983), we know that:

$$E\left(\underline{a}^{batch}\right) = \mu - \rho\sigma^2 \frac{2I_d + I_u}{3} + \frac{\rho\sigma^2(q_m + q_{-m})}{2}. \tag{26}$$

Using Eq. (14), (17) and (24) and the symmetry of the game, we deduce that the difference in transactions costs between a fragmented and a batch market is:

$$\begin{aligned} \Delta TC &= \gamma\left(E\left(\underline{a}^{D,+}\right) q_D + E\left(\underline{a}^{S,+}\right) q_S - E\left(\underline{a}^{batch}\right)(q_D + q_S)\right) \\ &+ (1-\gamma)\left(E\left(\underline{a}^{D,-}\right) q_D - E\left(\overline{b}^{S,-}\right) q_S - E\left(\underline{a}^{batch}\right)(q_D - q_S)\right). \end{aligned}$$

After straightforward computations the latter expression is equal to:

$$\Delta TC = \rho\sigma^2 q_S\left(I_u - I_d\right)\left(-\frac{(\gamma+1)}{3}\right) P_\gamma(\phi_D), \tag{27}$$

where $P_\gamma(x) = x^3 - 3x^2 + \frac{3}{(\gamma+1)}x + \frac{(\gamma-1)}{(\gamma+1)}$ for $x \in [0,1]$, and $\phi_D = \frac{q_D}{I_u - I_d}$.

To investigate whether transaction costs are larger or smaller in the batch auction, let us analyze the sign of the cubic polynomial P_γ. First, note that:

$$P'_\gamma(x) = 3x^2 - 6x + \frac{3}{(1+\gamma)} = 3\left(x - \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)\right)\left(x - \left(1 + \sqrt{\frac{\gamma}{1+\gamma}}\right)\right).$$

Given that $x \in [0,1]$, then $x - \left(1 + \sqrt{\frac{\gamma}{1+\gamma}}\right) < 0$, and the sign of $P'_\gamma(x)$ only depends on the sign of $\left(x - \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)\right)$. P_γ is increasing if $x < \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)$ and is decreasing if $x > \left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)$. Thus, the local maximum is $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) = \frac{\gamma\left(-1+2\sqrt{\frac{\gamma}{1+\gamma}}\right)}{1+\gamma}$.

- Consider the case where $\gamma \leq \frac{1}{3}$. Straightforward computations show that $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) \leq 0$ (with $P_\gamma(1 - \sqrt{\frac{\gamma}{1+\gamma}}) = 0$ if $\gamma = \frac{1}{3}$). We therefore deduce that $P_\gamma \leq 0$, i.e., $\Delta TC \geqslant 0$ if $\gamma \leq \frac{1}{3}$.
- Consider now the case where $\gamma > \frac{1}{3}$. We can show that $P_\gamma > 0$, or, equivalently, $\Delta TC < 0$

iff $x \in [\Phi^1_\gamma, \Phi^2_\gamma]$ where $P_\gamma(\Phi^1_\gamma) = 0 = P_\gamma(\Phi^2_\gamma)$. Note that if $\gamma = 1$, then it is direct to show that $P_1 > 0$ if $x \in [0, \frac{(3-\sqrt{3})}{2}]$, or equivalently, $\Delta TC < 0$ iff $\phi_D < \frac{(3-\sqrt{3})}{2}$.■

Proof of Proposition 4

Given the equilibrium prices $\left((a^D)^*, (a^S)^*\right)$ derived in Proposition 1, transaction costs write:

$$TC(\alpha) = [((a^D)^*(\alpha Q) - \delta_D - \mu)\alpha + ((a^S)^*((1-\alpha)Q) - \delta_S - \mu)(1-\alpha)] \times Q. \qquad (28)$$

We want to show that there exists an interior equilibrium, that is, an $\alpha^* \in [\frac{1}{2}, 1)$ that minimizes transaction costs $TC(.)$.

• We first conjecture that there exists an equilibrium characterized by a high divergence in intermediaries' inventories, i.e., $\frac{1}{2} \leq \alpha < \frac{I_1 - I_2}{Q}$. The first order condition (FOC) yields:

$$\alpha^H = \frac{1}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2 Q}.$$

The two conditions for an interior equilibrium $\alpha \in [\frac{1}{2}, 1)$ to exist are thus: i. a condition ensuring that our conjecture holds, i.e., $\alpha^H < \frac{I_1 - I_2}{Q}$, and ii. a condition ensuring that the equilibrium is interior, i.e., $\alpha^H < 1$. The latter always holds under our assumption $\delta_D - \delta_S < \rho\sigma^2 Q$. Condition i. rewrites as follows:

$$I_1 - I_2 > \frac{1}{2}\left(Q + \frac{\delta_D - \delta_S}{\rho\sigma^2}\right). \qquad (29)$$

• We now conjecture that there exists an equilibrium characterized by a low divergence in intermediaries' inventories, i.e., $\alpha \geq \frac{I_1 - I_2}{Q}$. The FOC yields:

$$\alpha^L = \frac{1}{2} - \frac{\delta_D - \delta_S}{2\rho\sigma^2 Q} + \frac{(I_1 - I_2)}{Q}.$$

The three conditions for an interior equilibrium to exist are such that: (i) our conjecture must hold, i.e., $\alpha^L \geq \frac{I_1 - I_2}{Q}$; (ii) there exists an interior equilibrium, i.e., $\alpha^L < 1$; and (iii) $\alpha^L \geq \frac{1}{2}$. Condition (i) always holds under our assumption $\delta_D - \delta_S < \rho\sigma^2 Q$. Condition (ii) translates into $I_1 - I_2 < \frac{Q}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2}$, which is the complement of the condition (29) above. Notice that if $I_1 - I_2 = \frac{Q}{2} + \frac{\delta_D - \delta_S}{2\rho\sigma^2}$, then there exists an equilibrium such that $\alpha^* = 1$. Condition (iii) imposes $I_1 - I_2 \geq \frac{\delta_D - \delta_S}{2\rho\sigma^2}$ (or $r_2(Q) - r_1(Q) > \delta_D - \delta_S$).[26]■

Proof of Corollary 3

First stage: the inter-dealer market (ID). If market-maker 1 sells a quantity q at price p

[26]If $I_1 - I_2 < \frac{\delta_D - \delta_S}{2\rho\sigma^2}$, there is no solution to the FOC in$[\frac{1}{2}, 1)$. There is a corner equilibrium: $\alpha^* = 1$.

to market-maker 2 in the inter-dealer market, the profits of market-maker 1 and 2 respectively write:

$$\left(v_1^{ID} = \left[p - \mu - \frac{\rho\sigma^2}{2}(q - 2I_1)\right] q; v_2^{ID} = \left[\mu - \frac{\rho\sigma^2}{2}(q + 2I_2) - p\right] q\right).$$

We maximize market-makers' profits with respect to q to find market-maker 1's supply function, and market-maker 2's demand function. The crossing of the supply and demand curves yields the following symmetric equilibrium in the inter-dealer market:

$$\left(q_{ID}^* = \frac{I_1 - I_2}{2}; p_{ID}^* = \mu - \rho\sigma^2 \frac{I_1 + I_2}{2}\right).$$

At equilibrium in the inter-dealer market, market-makers' profits write $\left(v_1^{ID}\right)^* = \left(v_2^{ID}\right)^* = \frac{\rho\sigma^2}{8}(I_1 - I_2)^2$. Notice that market-makers find it optimal to perfectly share risk: after trading in the inter-dealer market, market-makers 1 and 2 end up with the same inventory position, $I_1' = I_2'$.

Second stage: the customer-dealer market (CD). Given market-makers' inventory positions after their trades in the inter-dealer market, their equilibrium profits in the customer-dealer market can be computed at the limit when $I_1' \to I_2'$ using the formula derived in the proof of Proposition 1. We find: $\left(v_1^{CD|ID}\right)^* = \left(v_2^{CD|ID}\right)^* = \rho\sigma^2 q_D q_S$.

Comparison. We finally compute market-makers' expected profits in the presence of an inter-dealer market, namely $V^{CD+ID} = E\left(\left(v_i^{CD|ID}\right)^* + \left(v_i^{ID}\right)^*\right)$, and compare them with the expected profits they obtain in the absence of an inter-dealer market, namely $\left(V^{CD}\right)^* = E\left(\left(v_i^{CD}\right)^*\right)$. Computations yield:

$$V^{CD+ID} = \frac{\rho\sigma^2}{48}(I_u - I_d)^2 + \gamma\rho\sigma^2 q_D q_S, \tag{30}$$

and

$$\begin{aligned} V^{CD} &= \frac{\rho\sigma^2}{6}(I_u - I_d)\left(q_D + (2\gamma - 1)q_S\right) \\ &+ \frac{\rho\sigma^2 q_S}{(I_u - I_d)^2} \times \left[\begin{array}{l} (1-\gamma)(I_u - I_d)^3 - \left(3(1-\gamma)q_D + \frac{1}{2}\gamma q_S\right)(I_u - I_d)^2 \\ + \left\{(1-\gamma)q_D + \frac{1}{2}\gamma q_S\right\} q_D \left(3(I_u - I_d) - q_D\right) \end{array}\right]. \end{aligned} \tag{31}$$

To assess the impact of the existence of an inter-dealer market on intermediaries' expected profits, one needs to compare the expressions given in Eq. (30) and (31). Closed-form solutions are difficult to interpret. However there exist parameters' values such that intermediaries would prefer not to share risk in an inter-dealer market, that is, $V^{CD} > V^{CD+ID}$. Figure 4 shows that intermediaries are better off trading ex ante in an inter-dealer market only when (i) the probability that shocks have the same sign, γ, is high, and (ii) the size of the liquidity demand sent to the satellite venue, q_S, is small. ■