Lockdowns and PCR tests: A cost-benefit analysis of exit strategies

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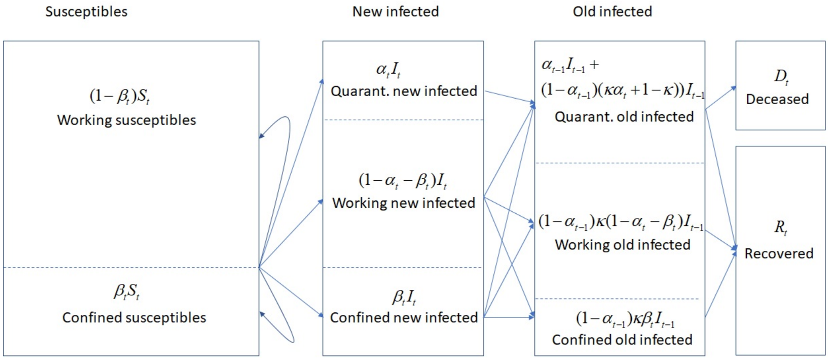
This a a very preliminary work. My model relies on very uncertain epidemiological parameters.

A dynamic of susceptible, infected and recovered persons



A SIR model

- One period = one week.
- {Susceptible,Infectious,Recovered,Dead} = $\{S, I, R, D\}$.
- Infected people remain infectious for 2 weeks. I_t is the number of people becoming infected at the beginning of week t.
- $I_t^{tot} = I_t + I_{t-1}$. I_t are asymptomatic in week t, but
  - a fraction $1-\kappa$ of I_{t-1} exhibits acute symptoms in week t, a fraction π_d of them will die at the end of that week. The others survive are become immune.
  - a fraction κ of I_{t-1} remains asymptomatic in week t, and then will become immune at the end of that week.
- In total,

$$\Delta R_{t+1} = ((1-\kappa)(1-\pi_d) + \kappa)I_{t-1}$$

$$\Delta D_{t+1} = (1-\kappa)\pi_d I_{t-1}$$

- I assume that immunity is observable.

Policy instruments

- Immunized people are sent back to work.
- The government can impose partial quarantine and partial PCR testing.
  - Weekly frequency.
- Group of people perceived as Susceptible at the beginning of week t: $\hat{S}_t = S_{t-1} + (1 - \alpha_{t-1}) I_{t-1}$.
- In this group, there exist old and new infected persons.
  - a fraction α_t of $\hat{S}_t$ is tested for the presence of the virus (100% efficient);
    - The positives are quarantined for 2 weeks (with the symptomatic cases);
    - The negatives are sent back to work;
  - a fraction β_t of $\hat{S}_t$ is confined;
  - a fraction $(1 - \alpha_t - \beta_t)$ of $\hat{S}_t$ is sent to work.

Transmission process

- Each infected person infects π persons per week, with
  - $\pi = \pi_0$ if the person is quarantined;
  - $\pi = \pi_1 > \pi_0$ if the person is confined;
  - $\pi = \pi_2 > \pi_1$ if the person is sent back to work.
- Mean number of transmission by the newly infected persons in week t:

$$\overline{\pi}_t/(1-\xi\beta_t) = \pi_0\alpha_t + \pi_1\xi\beta_t + \pi_2(1-\alpha_t-\xi\beta_t)$$

- Mean number of transmission by the old infected persons in week t:

$$\begin{aligned}\tilde{\pi}_t/(1-\xi\beta_t) &= \pi_0(\alpha_{t-1}+(1-\alpha_{t-1})(\kappa\alpha_t+1-\kappa)) \\ &\quad +\pi_1(1-\alpha_{t-1})\kappa\xi\beta_t \\ &\quad +\pi_2(1-\alpha_{t-1})\kappa(1-\alpha_t-\xi\beta_t)\end{aligned}$$

- Efficiency rate of confinement: ξ. If $\xi = 1$, 100% confinement kills the pandemic in 2 weeks (unrealistic).

Other assumptions

- Total transmission rate per infected person: $R_t = \overline{\pi}_t + \tilde{\pi}_t$.
- Why do we want to "flatten the curve"?
  - Mortality rate among symptomatic cases depends upon the capacity C of ICUs:

$$\pi_{dt} = \begin{cases} \pi_{dmin}, & \text{if } (1-\kappa)I_{t-1} < C \\ \pi_{dmax} > \pi_{dmin}, & \text{if } (1-\kappa)I_{t-1} > C \end{cases}$$

- After 52 weeks, a vaccine is found, and the pandemic is stopped.
- The pandemic is also stopped in week $t+1$ if I_t is below I_{min}, thanks to an intensive search of the remaining clusters. [This is critically important.]

SIR dynamics

$$S_t + I_{t-1} + I_t + R_t + D_t = 1 \quad \forall t$$

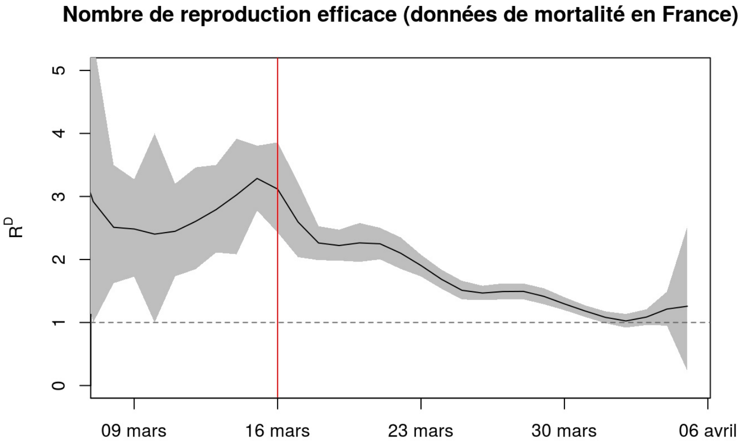
$$S_0 = 1 - \epsilon; \quad I_0 = \epsilon; \quad I_{-1} = R_0 = D_0 = 0$$

$$\begin{aligned}
I_{t+1} &= (\overline{\pi}_t I_t + \tilde{\pi}_t I_{t-1}) S_t \\
S_{t+1} &= S_t - I_{t+1} \\
R_{t+1} &= R_t + (\kappa + (1-\kappa)(1-\pi_{dt})) I_{t-1} \\
D_{t+1} &= D_t + (1-\kappa)\pi_{dt} I_{t-1}
\end{aligned}$$

SIR dynamics

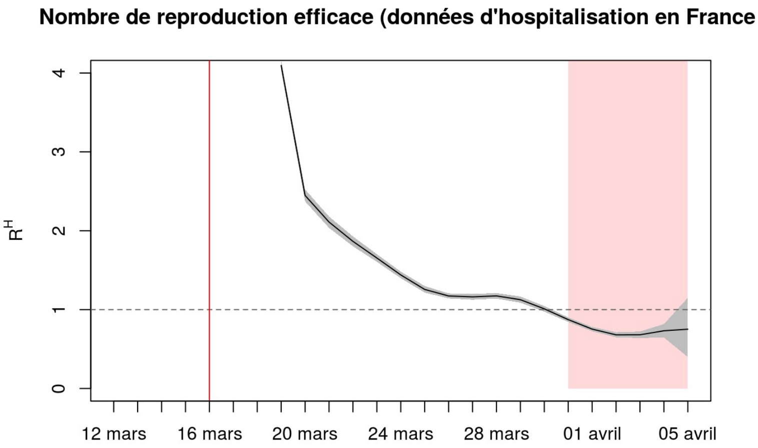
	Value	Description
	1	Size of the population
	2	Weeks of infection
π_0	0.1	Weekly reprod. rate of quarantined positives
π_1	0.5	Weekly reprod. rate of confined positives
π_2	1.2	Weekly reprod. rate of unconstrained positives
ξ	0.5	Efficiency rate of the confinement
κ	0.5	Prop. of asymptomatic positives in 2d week of infection
π_{dmin}	0.02	Prob. of dying if symptomatic positive (under capacity)
π_{dmax}	0.04	Prob. of dying if symptomatic positive (over capacity)
C	0.002	Health care capacity for covid
I_{min}	10^{-5}	Extinction threshold of the pandemic
ϵ	5×10^{-4}	Initial fraction of infection

Reproduction rate in France: $\pi_2 = 1.2$



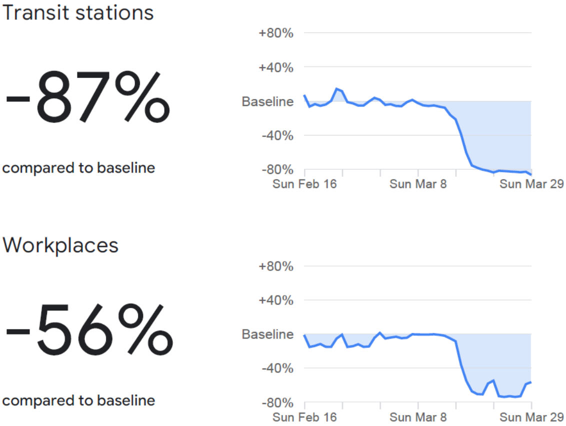
Source: "Limites et délais dans l estimation du nombre de reproduction", Laboratoire MIVEGEC, CNRS, IRD, Université de Montpellier, `http://alizon.ouvaton.org/Rapport5_R.html`

Reproduction rate in France: $\pi_1 = 0.5$



Source: "Limites et délais dans l estimation du nombre de reproduction", Laboratoire MIVEGEC, CNRS, IRD, Université de Montpellier, `http://alizon.ouvaton.org/Rapport5_R.html`

Efficiency of confinement $\xi = 0.5$



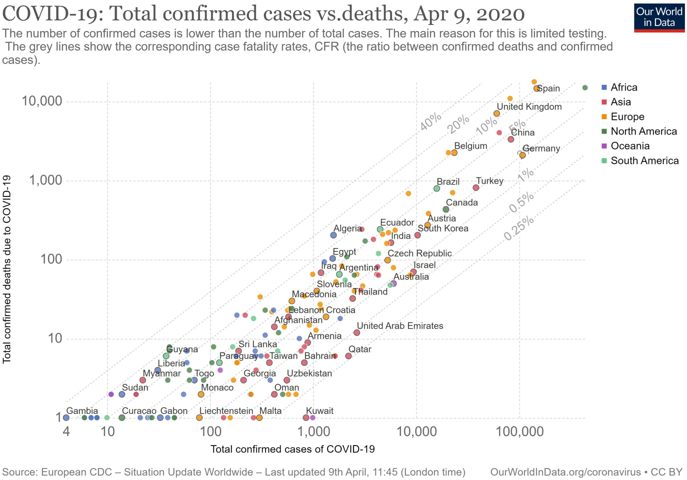
Source: Google mobility index for France : `https://www.google.com/covid19/mobility/`

Proportion of asymptomatic positives $\kappa = 0.5$

Case	% asymptomatic
Diamond Princess cruise	18%
Vo'Eugenia (Northern Italy)	50-75%
Japanese nationals evacuated from Wuhan	31%
LTC nursing King county Washington	57%
Iceland	50%
WHO Q&A	80%
CDC	25%

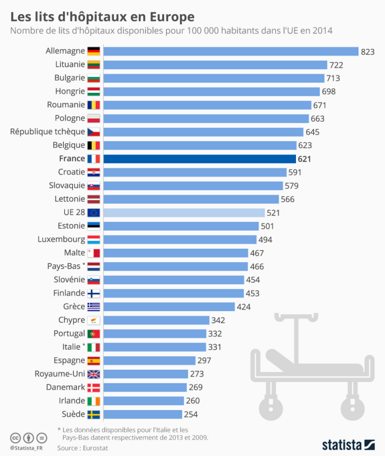
Source (excluding Chinese data):
`https://www.cebm.net/covid-19/covid-19-what-proportion-are-asymptomatic/`

Mortality rate among symptomatic cases: $\pi_{dmin} = 2\%$ and $\pi_{dmax} = 4\%$



Source: `https://ourworldindata.org/covid-mortality-risk`

Bed capacity for covid in hospitals: $C = 2/1000$

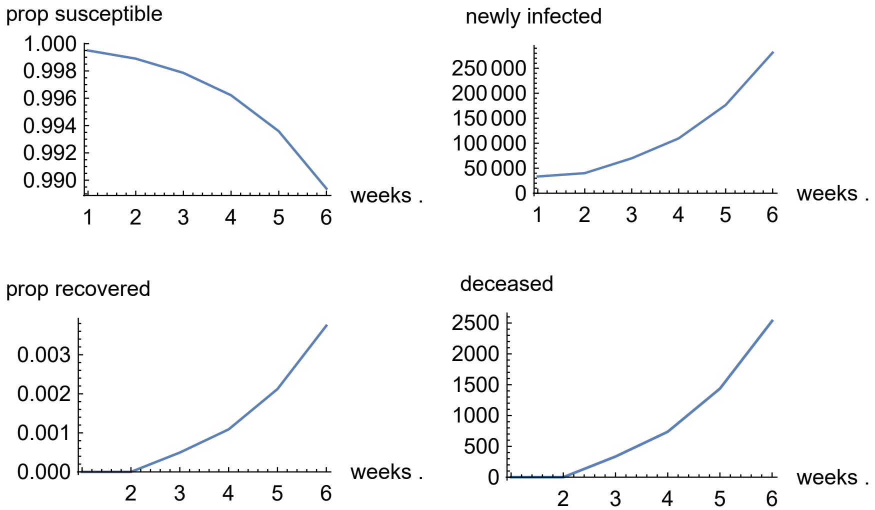


Source: `https://fr.statista.com/infographie/7564/les-lits-dhopitaux-en-europe/`

Initial Laisser-faire phase

- Case study: France. Population 67 millions. Bed capacity: 134,000.
- We start in mid-February with a first wave of $\epsilon = 5 \times 10^{-4}$ infections, i.e., 33,500 persons.
- No specific policy implemented: No confinement, no test.
- The transmission rate is $R_0 = 1.85$.
- After 5 weeks (France: confinement on March 17),
  - New infections goes up to 281,348 persons/week (50% need hosp.);
  - Total deceased: 2,534 persons;
  - Proportion susceptible: 98.9%;
  - Proportion immune: 0.4%.

Five weeks of laisser-faire

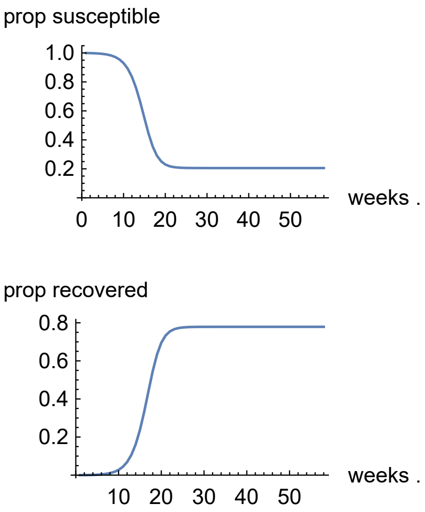


Permanent Laisser-faire

$$\beta_t = 0$$

- Simulation of the "herd immunity" strategy.
- No specific policy implemented: No confinement, no test.
- The transmission rate is $R_0 = 1.85$.
- Global impacts:
  - Total deceased: 1,053,590 persons;
  - Asymptotic proportion immune: 77.88%;
  - Peak infection wave: 7.4 million persons.

Permanent Laisser-faire

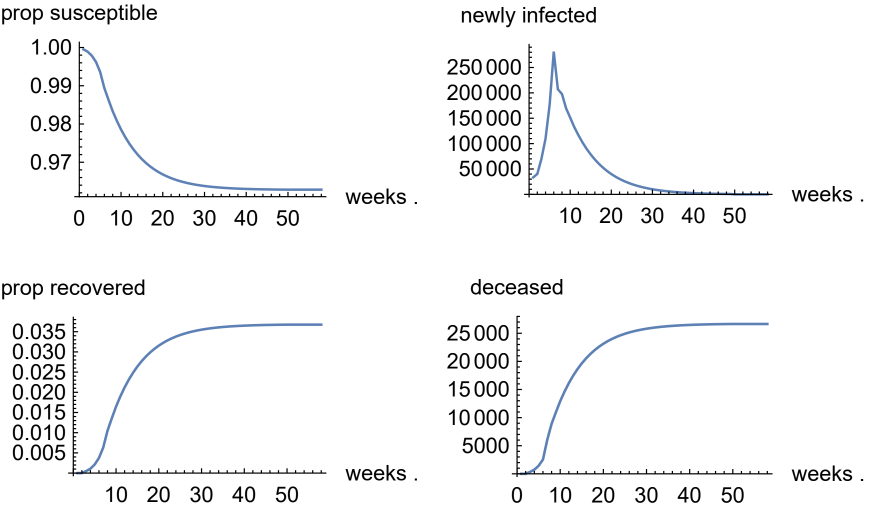


Suppression through long confinement

$$\beta_t = 0.8$$

- The pandemic is suppressed through a confinement until $I_t < I_{min}$.
- Because of essential services, we assume that $\beta_t = 0.8$ until suppression.
- The transmission rate is $R_0 = 0.86$.
- Global impacts:
  - Full confinement equivalent: 34.07 weeks (exit week 50);
  - Total deceased: 26,654 persons;
  - Proportion immune at exit: 3.67%;
  - Peak infection wave: 281,000 persons.
- Raising β from 0.8 to 1 would reduce confinement to 19.64 full weeks, and the number of deaths to 15,330 persons.

Suppression through long confinement



Cost-benefit analysis of the suppression strategy

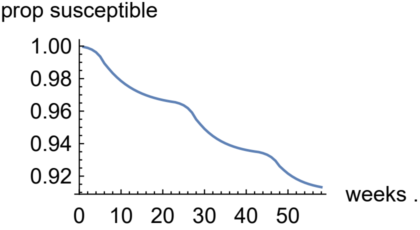
- Value unit: Billion of euros (BEUR)
- We assume that 50% of confined people can continue to work. Thus, one week of confinement yields of 1/104 of annual GDP. French GDP $\simeq 2,400$ BEUR.
- We assume a value of one life lost equaling 0.001 BEUR.

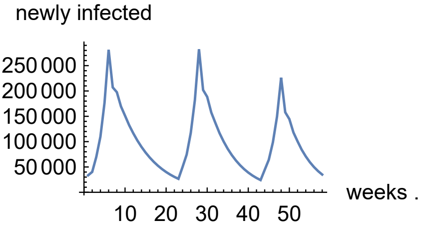
	Laisser-faire	Suppression
Lives lost	1,053,590	26,654
Value lives lost	1,054	27
Weeks lost	0	34.07
Value weeks lost	0	786
Net loss (BEUR)	1,054	813

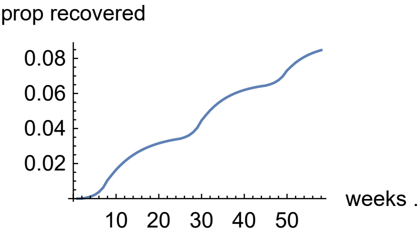
Stop-and-Go strategy

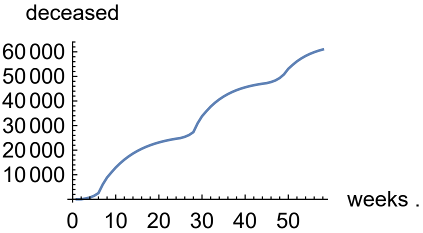
- The confinement ($\beta = 0.8$) is stopped if $I_t < I_a = 0.1C/(1-\kappa)$, and it is restarted if $I_t > I_b = 0.8C/(1-\kappa)$.
- Three sequences of confinement/deconfinement.
- Global impacts:
  - Full confinement equivalent: 31.93 weeks;
  - Total deceased: 60,956 persons;
  - Proportion immune at exit: 8.47%;
  - Peak infection wave: 282,000 persons.
- Compared to the suppression strategy: More deaths but smaller GDP losses. Net loss: 798 BEUR. Marginally better.

Stop-and-Go strategy









Linear strategy

$$\beta_t = 0.8\frac{0.5(I_t + I_{t-1}) - I_b}{I_a - I_b}$$

- The confinement rate β_t is linearly increasing with the rate of infection.
- The confinement rate decreases slowly from 60% to 50% during the year.
- Global impacts:
    - Full confinement equivalent: 29.19 weeks;
    - Total deceased: 92,076 persons;
    - Proportion immune at exit: 13.34%;
    - Peak infection wave: 281,000 persons.
- Compared to Stop-and-Go: More deaths and smaller GDP loss. Net loss: 765 BEUR. Marginally better.

Linear strategy