LE MULTIPLICATEUR BUDGÉTAIRE

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L3–TSE: Macroeconomics

[redacted]

Part I

Exercice I: Habit Persistence and The Keynesian Multiplier

We consider a simple keynesian model with habit persistence in consumption.
The consumption function takes the following form:

$$C_t = \lambda C_{t-1} + (1-\lambda)C_t^\star$$

where

$$C_t^\star = c(Y_t - T_t)$$

$c \in (0,1)$ is the Marginal Propensity to Consume. The parameter $\lambda \in [0,1)$ measures the degree of habit persistence in consumption. C_t is the real consumption, Y_t the real aggregate income and T_t the level of taxation on this income.

The aggregate resources constraint is given by:

$$Y_t = C_t + G_t$$

For simplicity, we omit private investment. The level of government spending G_t is exogenous.

Let L the lag operator, such that:

$$LC_t = C_{t-1}$$

1. Assume first that $T_t = 0$. Using the lag operator L, determine the equilibrium output as a function of G_t. More precisely, determine the function $B(L)$, such that equilibrium output is expressed as:

   $$Y_t = B(L)G_t$$

2. Compute the short run multiplier. This multiplier is obtained by imposing $L = 0$ in the representation $Y_t = B(L)G_t$.

3. Compute the long run multiplier. This multiplier is obtained by imposing $L = 1$ in the representation $Y_t = B(L)G_t$.

4. Discuss the two obtained government spending multipliers.

5. Assume now that $G_t = T_t$. Using the lag operator L, determine the equilibrium output as a function of G_t. More precisely, determine the function $B(L)$, such that equilibrium output is expressed as:

   $$Y_t = B(L)G_t$$

   Compute the short run and long–run multipliers. Discuss.

Exercice II: A Benchmark Model

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household's utility function is given by

$$\log(c_t) - \frac{\eta}{1+\nu} n_t^{1+\nu} \tag{1}$$

where real consumption is denoted c_t and labor supply n_t. $\eta > 0$ is a scale parameter and $\nu \geq 0$ is the inverse of the Frishean elasticity of labor supply. The time t budget constraint of the representative household is

$$c_t \leq w_t n_t - T_t + \Pi_t \tag{2}$$

where w_t is the real wage, T_t is a lump-sum tax and Π_t are the profits received from the firm. The representative household thus maximizes (1) subject to (2).

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the following technology

$$y_t = a n_t^{\alpha},$$

where $a > 0$ is the level of the technology and $\alpha \in (0, 1]$. The profit function is given by:

$$\Pi_t = y_t - w_t n_t$$

Government spending is entirely financed by taxes,

$$T_t = g_t.$$

The homogenous good can be used for private and public consumption. Accordingly, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)
2. Determine the optimality condition of the firm.
3. Determine the equilibrium output.
4. Compute the log–linearization of equilibrium output around the determinist steady-state.
5. Compute the output multiplier and discuss the value of this multiplier with respect to ν and α.
6. Compute the consumption multiplier and discuss the value of this multiplier with respect to ν and α.

Exercice III: Consumption, Labor Supply and the Multiplier

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household's utility function is given by

$$\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\nu} n_t^{1+\nu}$$

where real consumption is denoted c_t and labor supply n_t. $\eta > 0$ is a scale parameter and $\nu \geq 0$ is the inverse of the Frishean elasticity of labor supply. $\sigma > 0$ is a parameter that governs the sensitivity of consumption. Note that when $\sigma = 1$, we retrieve the log utility function. The time t budget constraint of the representative household is

$$c_t \leq w_t n_t - T_t + \Pi_t$$

where w_t is the real wage, T_t is a lump-sum tax and Π_t are the profits received from the firm. The representative household thus maximizes (1) subject to (2).

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the following constant return–to–scale technology

$$y_t = a n_t,$$

where $a > 0$ is the level of the technology. The profit function is given by:

$$\Pi_t = y_t - w_t n_t$$

Government spending is entirely financed by taxes,

$$T_t = g_t.$$

The homogenous good can be used for private and public consumption. Accordingly, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)
2. Determine the optimality condition of the firm.
3. Determine the equilibrium output.
4. Compute the log–linearization of equilibrium output around the determinist steady-state.
5. Compute the output multiplier and discuss the value of this multiplier with respect to ν and σ.
6. Compute the consumption multiplier and discuss the value of this multiplier with respect to ν and σ.

Part II

Exercice I: Taxes on the Labor Input and the Multiplier

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household's utility function is given by

$$\log(c_t) - \eta n_t$$

where real consumption is denoted c_t and labor supply n_t. $\eta > 0$ is a scale parameter. The time t budget constraint of the representative household is

$$c_t \leq w_t n_t - T_t + \Pi_t$$

where w_t is the real wage, T_t is a lump-sum tax and Π_t are the profits received from the firm.

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the following constant return–to–scale technology

$$y_t = a n_t,$$

where $a > 0$ is the level of the technology. We assume that the firm must pay a proportional tax on the labor input. So the profit function is given by:

$$\Pi_t = y_t - (1 + \tau_{w,t}) w_t n_t$$

Government spending is entirely financed by the taxes on the labor input,

$$g_t = \tau_{w,t} w_t n_t$$

Government spending is exogenously fixed and the tax rate $\tau_{w,t}$ will endogenously adjust to satisfy the government budget constraint.

The homogenous good can be used for private and public consumption. Accordingly, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)

2. Determine the optimality condition of the firm.

3. Determine the equilibrium output.

4. Determine the value of the output multiplier.

5. Determine the value of the consumption multiplier.

Exercice II: Public Spending in Utility Function and the Multiplier

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household's utility function is given by

$$\log(c_t^\star) - \eta n_t$$

where

$$c_t^\star = c_t + \alpha_g g_t$$

The parameter α_g accounts for the complementarity/substitutability between private consumption c_t and public spending g_t. When $\alpha_g = 0$, we recover the standard business cycle model in which government spending operates through income effects on the labor supply. When the parameter $\alpha_g > 0$, government spending is a substitute for private consumption. When the parameter $\alpha_g < 0$, the equilibrium private consumption and output can react positively to an increase in government spending. The real consumption is denoted c_t and n_t is the labor supply. $\eta > 0$ is a scale parameter. The time t budget constraint of the representative household is

$$c_t \leq w_t n_t - T_t + \Pi_t$$

where w_t is the real wage, T_t is a lump-sum tax and Π_t are the profits received from the firm.

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the following constant return–to–scale technology

$$y_t = a n_t,$$

where $a > 0$ is the level of the technology. The profit function is given by:

$$\Pi_t = y_t - w_t n_t$$

Government spending is entirely financed by taxes,

$$T_t = g_t.$$

The homogenous good can be used for private and public consumption. Accordingly, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)
2. Determine the optimality condition of the firm.
3. Determine the equilibrium output.
4. Compute the output multiplier and discuss the value of this multiplier with respect to α_g.
5. Compute the consumption multiplier and discuss the value of this multiplier with respect to α_g.

Exercice III: Labor Supply, Public Spending in Utility and the Multiplier

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household's utility function is given by

$$\log\left(c_t^\star - \eta\frac{\eta}{1+\nu}n_t^{1+\nu}\right)$$

where

$$c_t^\star = c_t + \alpha_g g_t$$

The parameter α_g accounts for the complementarity/substitutability between private consumption c_t and public spending g_t. The real consumption is denoted c_t and n_t is the labor supply. $\eta > 0$ is a scale parameter and $\nu \geq 0$ is the inverse of the elasticity of labor supply. The time t budget constraint of the representative household is

$$c_t \leq w_t n_t - T_t + \Pi_t$$

where w_t is the real wage, T_t is a lump-sum tax and Π_t are the profits received from the firm.

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the following constant return–to–scale technology

$$y_t = a n_t,$$

where $a > 0$ is the level of the technology. The profit function is given by:

$$\Pi_t = y_t - w_t n_t$$

Government spending is entirely financed by taxes,

$$T_t = g_t.$$

The homogenous good can be used for private and public consumption. Accordingly, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)
2. Determine the optimality condition of the firm.
3. Determine the equilibrium output.
4. Compute the output and consumption multiplier and discuss the result.

Part III

Exercice I: Endogenous Public Spending

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household's utility function is given by

$$\log(c_t) - \frac{\eta}{1+\nu} n_t^{1+\nu}$$

where real consumption is denoted c_t and labor supply n_t. $\eta > 0$ is a scale parameter and $\nu \geq 0$ is the inverse of the Frishean elasticity of labor supply. The time t budget constraint of the representative household is

$$c_t \leq w_t n_t - T_t + \Pi_t$$

where w_t is the real wage, T_t is a lump-sum tax and Π_t are the profits received from the firm.

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the following technology

$$y_t = a n_t^{\alpha},$$

where $a > 0$ is the level of the technology and $\alpha \in (0, 1]$. The profit function is given by:

$$\Pi_t = y_t - w_t n_t$$

Government spending is entirely financed by taxes,

$$T_t = g_t.$$

The literature has emphasized the relevance of stabilizing government spending rule. Here, we specify a feedback rule of the following form

$$g_t = \left(\frac{y_t}{y_{t-1}}\right)^{-\varphi_g} \exp(u_t)$$

where $\varphi_g \geq 0$, *i.e.* government spending stabilizes aggregate activity. The random term u_t represents the discretionary part of the policy.

The homogenous good can be used for private and public consumption. Accordingly, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)
2. Determine the optimality condition of the firm.
3. Determine the equilibrium output.
4. Compute the log–linearization of equilibrium output around the determinist steady-state.
5. Discuss the dynamic effects on output and consumption of an increase in the discretionary part u_t of the government policy.

Exercice II: Externality in Production and the Multiplier

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household's utility function is given by

$$\log(c_t) - \frac{\eta}{1+\nu} n_t^{1+\nu}$$

where real consumption is denoted c_t and labor supply n_t. $\eta > 0$ is a scale parameter and $\nu \geq 0$ is the inverse of the Frishean elasticity of labor supply. The time t budget constraint of the representative household is

$$c_t \leq w_t n_t - T_t + \Pi_t$$

where w_t is the real wage, T_t is a lump-sum tax and Π_t are the profits received from the firm.

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the following technology

$$y_t = a n_t s_t,$$

where $a > 0$ is the level of the technology. Here s_t is an externality on production specified as

$$s_t = \bar{n}_t^{\varphi}$$

where $\varphi \geq 0$ governs the degree of productive externality. $\bar{n}_t$ represents the average level of labor. Notice that the technology displays constant returns-to-scale at the private level, but increasing returns at the social (aggregate) level when $\varphi > 0$. The profit function is given by:

$$\Pi_t = y_t - w_t n_t$$

Government spending is entirely financed by taxes,

$$T_t = g_t.$$

The homogenous good can be used for private and public consumption. Accordingly, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)
2. Determine the optimality condition of the firm.
3. Determine the equilibrium output.
4. Compute the log–linearization of equilibrium output around the determinist steady-state (we assume that steady–state exists and is unique).
5. Compute the output multiplier and discuss the value of this multiplier with respect to φ.
6. Compute the consumption multiplier and discuss the value of this multiplier with respect to φ.

Exercice III: Externality in Labor Supply and the Multiplier

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household's utility function is given by

$$\log(c_t) - \frac{\eta}{1+\nu}\left(\frac{n_t}{\bar{n}_t^{\vartheta}}\right)^{1+\nu}$$

where $\bar{n}_t$ represents the average labor supply in the economy. The parameter ϑ measures the external effect of other households'labor supply on individual utility. For example, when $\vartheta > 0$, individual and aggregate labor supplies are complement. The real consumption is denoted c_t and n_t is labor supply. $\eta > 0$ is a scale parameter and $\nu \geq 0$ is the inverse of the Frishean elasticity of labor supply. The time t budget constraint of the representative household is

$$c_t \leq w_t n_t - T_t + \Pi_t$$

where w_t is the real wage, T_t is a lump-sum tax and Π_t are the profits received from the firm.

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the following technology

$$y_t = a n_t,$$

where $a > 0$ is the level of the technology. The profit function is given by:

$$\Pi_t = y_t - w_t n_t$$

Government spending is entirely financed by taxes,

$$T_t = g_t.$$

The homogenous good can be used for private and public consumption. Accordingly, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

1. Determine the optimality condition of the households and then deduce the Marginal Rate of Substitution (MRS)
2. Determine the optimality condition of the firm.
3. Determine the equilibrium output.
4. Compute the log–linearization of equilibrium output around the determinist steady-state (we assume that steady–state exists and is unique).
5. Compute the output multiplier and discuss the value of this multiplier with respect to ϑ.
6. Compute the consumption multiplier and discuss the value of this multiplier with respect to ϑ.