LA CRITIQUE DE LUCAS

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[redacted]

Part I

Exercice I: Public Spending and Private Consumption

Burda Michael et Wyplosz Charles, (2003), Macroéconomie, Une perspective européenne, Bruxelles, De Boeck, coll. ”Ouvertures, économiques prémisses, 3ème édition”, p. 406-407.

“Il est généralement accepté que l’impact des politiques actuelles sur le comportement contemporain du secteur privé dépend des anticipations des agents. Ainsi, la fonction de consommation relie la consommation contemporaine à la richesse et au revenu disponible courant. Cependant, la richesse, mesurée par la valeur actualisée du flux net de revenus futurs, n’est pas observable. Outre que les agents n’aiment pas en faire état, beaucoup de ses composantes, essentiellement liées au capital humain, n’ont pas de valeur marchande. Ce que l’on peut observer, c’est la liaison entre la consommation et le revenu disponible. Cette liaison considérant la richesse comme donnée, elle invalide la relation établie plus haut entre revenu disponible et consommation courante. En effet, la richesse dépend des anticipations des agents et peut donc se modifier rapidement. Ainsi, une réduction temporaire de la fiscalité affecte le revenu disponible courant, non la richesse. On n’en attend donc pas de grand impact sur la consommation. Par contre, la même réduction fiscale, si elle est perçue comme permanente, peut accroître considérablement la richesse et stimuler la consommation, rompant la relation observée antérieurement entre les deux variables.

La critique de Lucas établit que l’impact des politiques économiques s’apprécie mal au travers de l’expérience du passé. Les anticipations des agents privés, et donc leurs réactions, ne sont pas seulement, voire principalement, basées sur les mesures de politique économique. Elles dépendent de quelque chose de plus profond, de la perception qu’ont les agents des règles de conduite du gouvernement, ce que l’on appelle le régime de politique économique. Lorsque les agents privés et les marchés financiers s’attendent à un changement de régime, ils sont susceptibles d’adopter des comportements très différents de ceux que suggère le passé.

L’une des implications de cette critique est que les pouvoirs publics prennent moins de risques s’ils procèdent par petites étapes. Plus généralement, le risque de déclencher des modifications complexes du système économique constitue un argument supplémentaire à l’encontre de l’intervention active des pouvoirs publics dans la vie économique. Il contribue également à expliquer la prudence accrue dont font désormais preuve les gouvernements. En tout état de cause, les politiques économiques doivent joindre aux mesures contemporaines des engagements contraignants pour l’avenir. Les sections qui suivent illustrent concrètement la critique de Lucas. [...]

On sait que le multiplicateur transforme une variation exogène de la demande en une variation de la production. Les opposants des politiques budgétaires actives affirment qu’une action budgétaire sur la demande peut rendre ce multiplicateur négatif. Ce pourrait être le cas, notamment, lorsqu’un accroissement permanent des dépenses publiques est perçu comme une réduction de la richesse privée. En effet, la contrainte budgétaire de l’Etat exige que ces dépenses accrues trouvent leur contrepartie, en valeur actualisée, dans des recettes fiscales plus élevées. Même si l’équivalence ricardienne ne se vérifie pas totalement, un accroissement des dépenses publiques constitue inéluctablement une ponction sur la richesse privée. Si, en conséquence, la dépense privée diminue davantage que ne s’est accrue la demande publique, le risque de récession est réel. A l’inverse, une politique budgétaire qui se veut restrictive peut s’avérer expansionniste si les citoyens, convaincus qu’un jour ou l’autre l’Etat ne sera plus en mesure d’honorer ses engagements, avaient d’avance réduit leurs dépenses.

Pour paradoxal qu’il paraisse, ce raisonnement s’appuie sur des expériences de stabilisation menées au cours des années 1980. En Allemagne, au Danemark et en Irlande, des resserrements budgétaires ont donné lieu à de fortes croissances économiques."

Propose a simple model of consumption in which public spending can reduce private consumption. You can use and combine the two examples of the course on the intertemporal budget constraint of the government and the consumption function in the permanent income model.

Exercice II: Monetary Policy

Suppose that the economy is described by the following Aggregate Supply (AS)–Aggregate Demand (AD) equations

$$y_t = \alpha(p_t - E_{t-1}p_t) \quad \text{(AS)}$$

$$y_t = m_t - p_t \quad \text{(AD)}$$

where y_t is aggregate output, p_t the level of price and m_t the money demand. All the variables are expressed in logs. E_{t-1} is the rational expectation operator and it represent the expectation conditional on the information set in period $t-1$. The parameter α is positive.

Assume that the ECB's monetary policy is initially

$$\bar{m}_t = \mu_1 + \bar{m}_{t-1} + \varepsilon_t$$

where $\bar{m}_t$ represents the supply of money and ε_t is an unexpected shock to monetary policy ($E_{t-1}\varepsilon_t = 0$). The parameter μ_1 represents a constant growth rate of money supply ($\bar{m}_t - \bar{m}_{t-1}$).

1. Solve the model, *i.e.* express the level of output y_t as a function of the difference between money supply m_t and expected money supply $E_{t-1}m_t$ (at equilibrium, money demand is equal to money supply) .

2. Using the monetary policy, determine the relationship between output and money growth ($m_t - m_{t-1}$).

3. Suppose that the BCE ignores that the economy is described by the AS curve and the central bank estimates a stable relationship from the data between output and money growth. The form of the estimated equation is the following:

$$y_t = \beta_0 + \beta_1(m_t - m_{t-1})$$

   The central would estimate a positive value of β_1. So, ECB would exploit this observed stable relationship because it states that output will increase if the growth rate of money is higher. So, ECB implements a new policy of the form:

$$\bar{m}_t = \mu_2 + \bar{m}_{t-1} + \varepsilon_t$$

   where $\mu_2 > \mu_1$. Show that this new monetary policy has no effect on output. In the light of the Lucas critique, interpret the policy mistake.

Exercice III: A Quantitative Evaluation of the Lucas Critique

We use the simple model of the course to quantitatively illustrate the Lucas critique. The model is described by the following two equations

$$y_t = aE_t y_{t+1} + bx_t$$

$$x_t = \rho x_{t-1} + \varepsilon_t$$

In what follows, we set $a = 0.9$ and $b = 1$. Use a (Excell) spreadsheet for the quantitative experiments

1. Solve the model theoretically, i.e. characterize the solution for any $|a| < 1$, $b \neq 0$ and $\rho \in [0, 1]$.
2. Set $\rho = 0$ and then compute the dynamic responses of y_t to a positive shock (equal to one) on the innovation ε_t. Do the figure.
3. Set $\rho = 0.5$ and redo the same quantitative exercise. Do the figure.
4. Suppose that we wrongly assume that the parameter of the solution is invariant to the parameter ρ. So, we use the first policy where $\rho = 0$ (that corresponds to past observations, before the implementation of the new policy where $\rho = 0.5$). Compute the the dynamic responses of y_t to a positive shock (equal to one) on the innovation ε_t when the change in ρ is omitted. Compare the dynamic responses to those obtained in question 3). Reports the responses on the same figure.
5. Now suppose that we wrongly assume naive expectations

   $$E_t y_{t+1} = y_t$$

   Redo the two previous quantitative exercises ($\rho = 0$ and $\rho = 0.5$) and compare on the same figure the rational expectations and the naive expectations.

Part II

Exercice I: The Granger Causality

The Granger causality is a statistical concept of causality (Granger, 1969). Broadly speaking, it states that a variable x *Granger causes* a variable y if the observation of x today helps to predict y tomorrow. We will use a simple model of asset pricing to illustrate the limits of this concept.

Let a standard asset pricing equation (this is a slight modification of the asset pricing equation in the course) that determines the price of a stock today (P_t) as an expected and discounted function of the price tomorrow (P_{t+1}) and the dividend tomorrow (D_{t+1}):

$$P_t = \beta E_t \left(P_{t+1} + D_{t+1} \right)$$

where $\beta \in (0,1)$ is a constant discount factor. E_t denote the rational expectations conditional on the information set in period t.

Let us assume a simple (exogenous) process for the dividend (Moving Average of order one, i.e. MA(1) process)

$$D_t = \bar{D} + \varepsilon_t + \theta \varepsilon_{t-1}$$

where $\theta \neq 0$ and $\bar{D}$ is a positive constant. The error term is zero mean and unpredictable ($E_t \varepsilon_{t+1} = 0$).

1. Determine the fundamental solution for the stock price.

2. Use the process of the dividend to solve the model.

3. Now, use the solution and the process of the dividends to show that the stock price *Granger causes* the dividends. Discuss this result in the light of this simple theoretical model.

Exercice II: The Cagan Model

Let us consider a log–linear money demand function:

$$m_t^d - p_t = -\alpha(E_t p_{t+1} - p_t)$$

where m is the quantity of money demand, p the price and E_t is the rational expectation conditional on the information set in period t. α is a positive parameter. We assume that the money supply m_t^s is exogenous (deterministic or stochastic).

1. Using the equilibrium of the money market, determine the forward looking representation for the price p_t.

2. Solve this equation forward and determine the fundamental equation.

3. Assume that the money supply is constant:

$$m_t^s = m, \ \ \forall t$$

   Determine the price p_t.

4. Assume a constant growth rate of money supply:

$$m_{t+j}^s = m_t + \mu j$$

   where μ is the rate of growth. Determine the price p_t.

5. Assume an anticipated increase in money supply at some future date $t+T$:

$$m_{t+j}^s = m_1 \quad \text{if } \ j < T \quad ; \quad m_{t+j}^s = m_2 \quad \text{if } \ j \geq T \quad (m_1 \neq m_2)$$

   Determine the price p_t.

6. Assume that the money supply is stochastic with an AR(1) process:

$$m_t^s = \rho m_{t-1}^s + \varepsilon_t$$

   where $\rho \in [0,1]$ and $E_t \varepsilon_{t+1} = 0$. Determine the price p_t. Discuss the solution with respect to the values of ρ.

Exercice III: Testing the Lucas Critique

We use again the simple model of the course with rational expectations. The model is described by the following two equations

$$y_t = aE_t y_{t+1} + bx_t + v_t$$

$$x_t = \rho x_{t-1} + \varepsilon_t$$

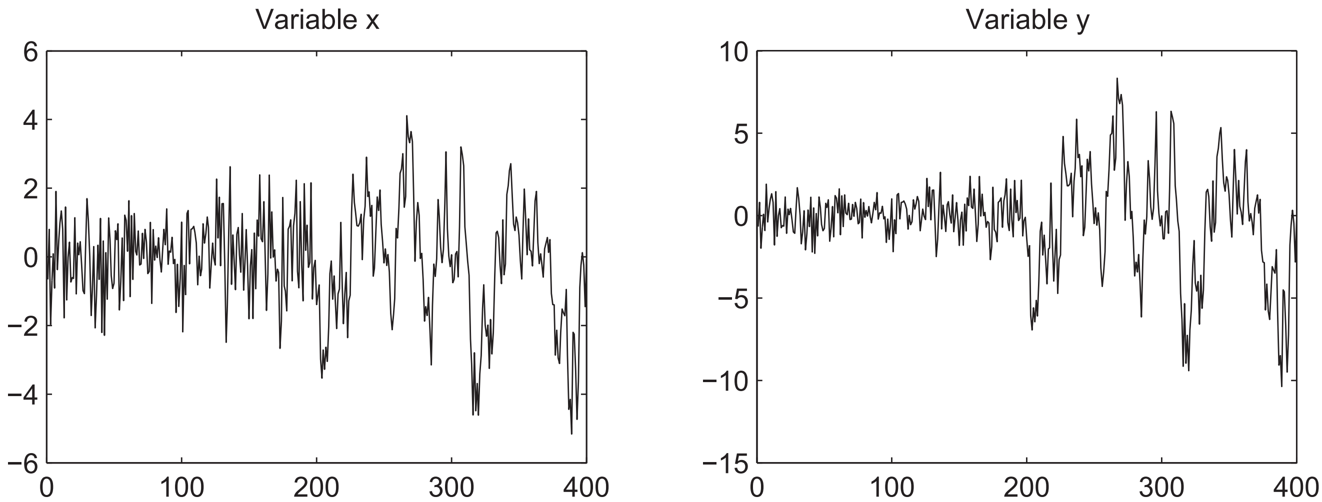
In what follows, we set $a = 0.5$ and $b = 1$. In addition, the first equation includes a shock v_t. This shock is a white noise, with a standard–error equals to 0.1 (this shock satisfies $E_t v_{t+1} = 0$).

We consider two policies:

- **Policy I:** $\rho = 0$.
- **Policy II:** $\rho = 1$.

For each policy, the standard error of the shock ε_t is set to 1. The two random variables ε_t and v_t are normally distributed and independent for all leads and lags ($E(\varepsilon_t, v_{t+j}) = 0, \forall j$). We assume a sample of size 400. For the periods $t = 1, ...N$, we simulate the model with the Policy I. For the periods $t = N+1, ..., T$ (with T=400), we simulate the model with the Policy II. The results of the simulation are reported in Figure 1. The left side of the figure includes the simulation of the variable x_t, whereas the right side of the figure reports the simulation of the variable y_t. The simulated data are available on my `Website` at TSE (this is an Excell file: `simuldatalucas.xls`).

Figure 1: Simulated Data



1. Compute the theoretical solution and then given the calibration, deduce the value of the parameter of the solution.
2. Concentrate first your analysis on the time series behavior of the variable x_t. From Figure 1 and the Excell file, propose a simple statistical diagnostic allowing to locate the change in the time series behavior of x_t. What is the date N of the change in regime for the variable x_t?
3. Now, concentrate your analysis on the time series behavior of the variable y_t. Propose again a simple statistical diagnostic to locate the change in the time series behavior of y_t. What is the date N of the change in regime for the variable y_t? Can we conclude that the Lucas critique holds?
4. Now consider the following reduced form equation for the variable y_t

$$y_t = a + bx_t + u_t$$

where u_t is an error term. What is the estimated values of a and b if we conduct a linear regression of y_t on a constant and the variable x_t for the whole sample? (i.e. for t=1,...,T). Redo the exercise for the two subsamples $t = 1, ..., N$ and $t = N+1, ..., T$. Compare the estimated parameters to the theoretical parameters, i.e. those obtained from the solution of the model and the "calibtrated" values for a, b and ρ.

Part III

Exercice I: News Shocks

We consider again our simple forward–looking model:

$$y_t = aE_t y_{t+1} + bx_t$$

where $|a| < 1$ and $b \neq 0$. Here we assume that the stochastic process for x_t is the following

$$x_t = \varepsilon_{t-1}$$

So, the realizations of x tomorrow (i.e. x_{t+1}) are known today. It follows tha $E_t x_{t+1} = x_{t+1}$. Keep this important property in mind for the sequel.

1. Solve the model.
2. Determine the dynamic responses of y_t to a positive shock (equal to one) on the innovation ε_t.
3. Now, assume a new process for x_t.

   $$x_t = \varepsilon_{t-2}$$

   What are the consequences for the expectations of the future values of x?
4. Solve the model.
5. Determine the dynamic responses of y_t to a positive shock (equal to one) on the innovation ε_t.
6. Now, assume a more general process for x_t.

   $$x_t = \varepsilon_{t-q}$$

   where q is a positive integer ($q \geq 0$). Solve the model for any q.
7. Determine the dynamic responses of y_t to a positive shock (equal to one) on the innovation ε_t.
8. **An Application** The Monetary Policy (Unexpected versus Expected). Consider an economy summarized by a Fisher Equation and a Taylor Rule. The Fisher equation is given by

   $$i_t = E_t \pi_{t+1}$$

   We assume that the real interest rate is constant and zero. The Taylor rule is given by

   $$i_t = \alpha \pi_t + s_t$$

   where $\alpha > 1$ and s_t is a shock to monetary policy. We assume two types of processes for s_t.

   Unexpected monetary policy shock: $s_t = \varepsilon_t$

   Expected monetary policy shock: $s_t = \varepsilon_{t-1}$

   Solve the model for the two types of monetary policy shocks and compare the dynamics of inflation after a shock to monetary policy.

Exercice II: An Extension of the Simple Dynamic Model

We now consider an extension of our simple forward–looking model by including a back–looking component (y_{t-1}) into the equation. The new model's version writes:

$$y_t = \frac{\lambda}{1+a\lambda} y_{t-1} + \frac{a}{1+a\lambda} E_t y_{t+1} + \frac{b}{1+a\lambda} x_t$$

where $|a| < 1$ and $|\lambda| < 1$. Notice that we retrieve the simple forward–looking model when we impose $\lambda = 0$.

We also assume that x_t is stochastic and follows an autoregressive process of order one (AR(1)):

$$x_t = \rho x_{t-1} + \varepsilon_t$$

where $\rho \in [0, 1]$.

1. Show that the model can be rewritten in the following form:

$$\tilde{y}_t = a E_t \tilde{y}_{t+1} + b x_t$$

   where $\tilde{y}_t = y_t - \lambda y_{t-1}$.

2. Determine $\tilde{y}_t$ as a expected and discounted sum of present and future values of x_t.

3. Using the AR(1) model for x_t, solve the model for $\tilde{y}_t$.

4. Solve the model for y_t.

Exercice III: Change in Monetary Policy and Aggregate Dynamics

Consider again an economy summarized by a Fisher Equation and a Taylor Rule. The Fisher equation is given by

$$r_t = i_t - E_t\pi_{t+1}$$

We assume that the real interest rate is exogenous and stochastic (MA(1) process):

$$r_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

where $\theta \neq 0$ and ε_t a zero mean and unpredictable random variable ($E_t\varepsilon_{t+i} = 0, \forall i > 0$).

The Taylor rule is given by

$$i_t = \alpha\pi_t$$

The parameter α can take two values (two monetary policies):

Policy I:

$$\alpha = \alpha_1 > 1$$

This corresponds to the case of an aggressive monetary policy rule, i.e. the nominal interest rate increase more than inflation.

Policy II:

$$\alpha = \alpha_2 < 1$$

This corresponds to the case of a passive monetary policy rule, i.e. the nominal interest rate increase less than inflation.

1. Solve the model when $\alpha = \alpha_1$ (**Policy I**).
2. Solve the model when $\alpha = \alpha_2$ (**Policy II**).
3. Compare these two economies (associated to two different monetary policies).
4. Show that the Lucas critique applies when $\alpha = \alpha_1$ when but does not apply when $\alpha = \alpha_2$.