Chap. II: Fiscal Multipliers

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MF 508

L3–TSE: Macroeconomics [redacted]

No Dynamic Optimization Problems in this chapter.

Just (repeated) static models

But some technicalities about general equilibrium models with endogenous labor supply.

Next year in Macro-M1 course (doctoral and standard track): explicit solutions to Dynamic Optimizing Problems (stochastic permanent income model, Two period model, infinite horizon, explicit representation of Ricardian equivalence, RBC models)

These notes are self contained!!!

Please redo all the models and problems that are expounded here!

Warning!!! This notes can contain many (Mathematical and English) typos!!!

References:

Robert Hall (2009) “By How Much Does GDP Rise if the Government Buys More Output”, *Brooking Paper on Economic Activity*, 2, pp. 183-249.

http://www.stanford.edu/ rehall/BPEA%20Fall%202009.pdf

ABSTRACT During World War II and the Korean War, real GDP grew by about half the increase in government purchases. With allowance for other factors holding back GDP growth during those wars, the multiplier linking government purchases to GDP may be in the range of 0.7 to 1.0, a range generally supported by research based on vector autoregressions that control for other determinants, but higher values are not ruled out. New Keynesian macroeconomic models yield multipliers in that range as well. Neoclassical models produce much lower multipliers, because they predict that consumption falls when government purchases rise. Models that deliver higher multipliers feature a decline in the markup ratio of price over cost when output rises, and an elastic response of employment to increased demand. These characteristics are complementary to another Keynesian feature, the linkage of consumption to current income. The GDP multiplier is higher—perhaps around 1.7—when the nominal interest rate is at its lower bound of zero.

Valerie Ramey, (2011) "Can Government Purchases Stimulate the Economy?" *Journal of Economic Literature*, 49(3), pp. 673-85.

http://econweb.ucsd.edu/~vramey/research/JEL_Fiscal_14June2011.pdf

Abstract This essay briefly reviews the state of knowledge about the government spending multiplier. Drawing on theoretical work, aggregate empirical estimates from the United States, as well as cross-locality estimates, I assess the likely range of multiplier values for the experiment most relevant to the stimulus package debate: a temporary, deficit-financed increase in government purchases. I conclude that the multiplier for this type of spending is probably between 0.8 and 1.5. (JEL E23, E62, H50)

Introduction

- Following large contractions in aggregate economic activity, expansionary government policy, i.e. large increase in government purchases of goods and services.
- See the US and Euro Area experiences in 2008.
- Other examples: "New Deal" policy in US during the 30', Military purchases before the WWII, Korean war, Vietnam war; Fiscal Stimulus in France: "Relance Chirac", "Relance Mitterand", "Relance Sarkozy".
- **The Question:** How much total output (and private consumption) increases when the government buys more goods and services???

I- Empirical Results

II- Results from AS-AD Model

III-Results from General Equilibrium Models

I- Empirical Results

I.1. Results from Simple Regressions on Military Spending

I.2. Results from Structural Vector Auto-Regressions (SVARs)

I.1. Results from Simple Regressions on Military Spending

The regression framework

$$\frac{y_t - y_{t-1}}{y_{t-1}} = \mu_0 + \mu_1 \frac{g_t - g_{t-1}}{y_{t-1}} + \varepsilon_t$$

The parameter μ_1 can be interpreted as the output multiplier.

$$\mu_1 \simeq \frac{\Delta y_t}{\Delta g_t}$$

The variable y is the real US GDP and g is the real military spending, considered as a "good" exogenous variable.

Why ? Military spending is associated to "rare" historical events (wars), and not related to economic activity (output, private consumption).

Note: if we consider the following regression framework

$$\frac{c_t - c_{t-1}}{y_{t-1}} = \mu_0 + \mu_1 \frac{g_t - g_{t-1}}{y_{t-1}} + \varepsilon_t$$

The parameter μ_1 can be interpreted as the (private) consumption multiplier.

The parameters μ_0 and μ_1 can be easily obtained by OLS regressions.

See the results below for the two types of multipliers

Figure 1: Results from Regressions on Military Spending

Table 1. Ordinary Least Squares Estimates of Government Purchases Multipliers for Military Spending[a]

Period	*GDP multiplier*	*Consumption multiplier*
1930–2008	0.55	–0.05
	(0.08)	(0.03)
1948–2008	0.47	–0.12
	(0.28)	(0.10)
1960–2008	0.13	–0.09
	(0.65)	(0.29)
1939–48	0.53	–0.05
	(0.07)	(0.02)
1949–55	0.48	–0.18
	(0.56)	(0.05)
1939–44	0.36	–0.11
	(0.10)	(0.03)
1945–49	0.39	–0.04
	(0.08)	(0.05)

Source: Author's calculations.
a. Numbers in parentheses are standard errors.

- The output multiplier is positive.
- Max for the output multiplier around 0.5; Min for the output multiplier around 0.13.
- The consumption multiplier is almost zero (negative).
- Min for the consumption multiplier around -0.18.

I.2. Results from Structural Vector Auto-Regressions (SVARs)

A Brief description of Structural Vector Auto-Regressions (SVARs).

Let Y_t a vector of variables of interest: GDP, private consumption, investment, public spending, ... (all the variables are in logs)

We assume that this vector follows a Vector Auto-Regression of the form

$$Y_t = A_0 + \sum_{i=1}^{p} A_i Y_{t-i} + u_t$$

where p is the number of lags (this number can be selected from statistical criteria, i.e. p is obtained such that the error term u_t is not serially correlated).

Notice that the model is dynamic and linear.

A_0 is a vector of constant terms and A_i $(i = 1, .., p)$ are matrices that represent the dynamic inter-actions between variables in the vector Y_t.

Without any restrictions on these matrices, this model can be easily estimated using OLS (equation by equation, [**Show this if necessary**]).

A Difficulty How to interpret the (estimated) error term u_t?

This error term is a "statistical" variable (residuals of the regression), without economic (structural) interpretation!!!

To properly interpret this term, we need to introduce (economic) restrictions on this random variable.

To see this easily, let us consider a VAR model with only 2 variables (in Y_t); this can be easily extend to more than two variables!

Let Y_t given by

$$Y_t = (\log(G_t), \log(Output_t))'$$

and we assume that Y_t follows a VAR(p) process.

To identify a government spending shock, we assume the following:

i) the statistical error term is linearly related to the structural shock ε_t

$$u_t = S\varepsilon_t$$

where S is a given (2×2) matrix.

ii) The covariance matrix of ε_t is an identity matrix.

$$E(\varepsilon_t \varepsilon_t') = I_2$$

We deduce

$$E(u_t u_t') = E(S \varepsilon_t \varepsilon_t' S') equiv SS'$$

We have

$$E(u_t u_t') = \Sigma$$

where Σ is the covariance matrix of the residuals. This matrix includes 3 unknown parameters (this matrix is symmetric).

The matrix S includes 4 unknown parameters. So we need to impose one additional restrictic to properly identify the structural shocks.

This can be achieved by imposing the following short-run restriction:

iii) The government spending shock is the only one shock that can modify government spending in the short-run.

This implies the following structure of the matrix S:

$$S = \begin{pmatrix} s_{11} & 0 \\ s_{21} & s_{22} \end{pmatrix}$$

Then

$$Y_t = A_0 + \sum_{i=1}^{p} A_i Y_{t-i} + S\varepsilon_t$$

with

$$\varepsilon_t = (\varepsilon_t^g, \varepsilon_t^{ng})'$$

where ε_t^g is the government spending shock and ε_t^{ng} represents other shocks.

We concentrate our analysis on the dynamics responses of output and consumption to the shock ε_t^g.

Results are reported below.

Figure 2: Results from SVARs

Table 2. Literature Estimates of Government Purchases Multipliers from Vector Autoregressions[a]

Source	Type of multiplier	Estimate: On impact	Estimate: After 4 quarters	Estimate: After 8 quarters
Blanchard and Perotti (2002, table 4)	Output	0.90	0.55	0.65
		(0.30)	(0.80)	(1.20)
Galí, López-Salido, and Vallés (2007, table 1)	Output	0.41	0.31	0.68
		(0.16)	(0.34)	(0.45)
	Consumption	0.07	0.11	0.49
		(0.10)	(0.19)	(0.28)
Perotti (2008, figure 3)	Output	0.70	1.00	1.20
		(0.20)	(0.50)	(0.50)
	Consumption	0.10	0.30	0.40
		(0.05)	(0.20)	(0.25)
Mountford and Uhlig (2008, table 4)	Output	0.65	0.27	–0.74
		(0.39)	(0.78)	(1.95)
Ramey (2008, figure 10a)	Output[b]	0.30	0.50	0.90
		(0.10)	(0.25)	(0.35)
Ramey (2008, figure 10b)	Consumption[c]	0.02	–0.17	–0.09

Source: Literature cited.

a. Numbers in parentheses are standard errors.

b. Ramey (2008) states results for both output and consumption as elasticities, which here have been converted to multipliers.

c. Separate elasticities were estimated for durables, nondurables, and services, so standard errors for total consumption are unavailable.

- On impact (very short–run), the output multiplier is positive.
- Max for the output multiplier around 0.9; Min for the output multiplier around 0.3.
- After some periods, the max for the output multiplier around 1.20
- The consumption multiplier is smaller but it is positive.
- The max for the consumption multiplier (after some periods) is around 0.5.

II- Results from AS-AD Model

II.1. The Keynesian Model

II.2. The IS-LM Model

II.3. The AS-AD Model

II.1. The Keynesian Model

A Simple Setup

Let us first consider the simplest version.

Only one behavior equation: the consumption function

Private investment is exogenous and constant

Government spending is exogenous

The aggregate resource constraint

$$Y_t = C_t + I_t + G_t$$

The consumption function

$$C_t = C_o + c(Y_t - T_t)$$

Here, in addition, we assume that taxes T_t are lump-sum and exogenous (or constant).

$$I_t = \bar{I} \quad , \quad G_t = \bar{G}_t \quad , \quad T_t = \bar{T}$$

The equilibrium is given by

$$Y_t = Y_t^D$$

where

$$Y_t^D = C_t + \bar{I} + G_t$$

or

$$Y_t^D = C_o + c(Y_t - \bar{T}) + \bar{I} + G_t$$

We obtain

$$Y_t = \frac{1}{1-c}\left(C_o - c\bar{T}) + \bar{I} + G_t\right)$$

So, the output multiplier is given by

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{1}{1-c}$$

We can also compute the consumption multiplier

$$\frac{\Delta C_t}{\Delta G_t} = \frac{c}{1-c}$$

<u>Numerical Application:</u> the Marginal Propensity to Consume is equal to 0.8 (saving

rate 20%)

So, the output multiplier is equal to 5!!!! Very large! (also very large for consumption, 4!!!)

A first extension: fiscal considerations

1. Balanced government budget.

Suppose that the government fully finance the spending G_t by taxes:

$$G_t = T_t$$

The aggregate demand is now given by

$$Y_t^D = C_o + c(Y_t - T_t) + \bar{I} + G_t \equiv C_o + c(Y_t - G_t) + \bar{I} + G_t$$

We obtain

$$Y_t = \frac{1}{1-c}\left(C_o + \bar{I} + (1-c)G_t\right)$$

So, the multiplier is given by

$$\frac{\Delta Y_t}{\Delta G_t} = 1$$

The multiplier is reduced in comparison to the previous case where government spending is financed by new public debt.

But, the output multiplier is still positive and relatively large (=1).

The consumption multiplier is equal to zero.

2. Distortionary taxation.

Suppose now that taxes T_t are proportional to income

$$T_t = tY_t$$

where $t \in [0, 1)$.

The aggregate demand is now given by

$$Y_t^D = C_o + c(Y_t - T_t) + \bar{I} + G_t \equiv C_o + c(1 - t)Y_t + \bar{I} + G_t$$

We obtain

$$Y_t = \frac{1}{1 - c(1 - t)} \left(C_o + \bar{I} + G_t \right)$$

So, the multiplier is given by

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{1}{1 - c(1 - t)}$$

The multiplier is reduced (when $t > 0$) in comparison to the benchmark case with lump-sum taxation.

Rmk notice that this property has some interesting features: distortionary taxes acts as "automatic stabilizers".

Rather than using discretionary government spending or taxes to stabilize the economy (in expansion, gov. increases taxes; in recession, gov. increases public spending), the government can just implement a distortionary taxation (on labor income).

This type of taxation automatically reduces fluctuations.

Numerical Application: the Marginal Propensity to Consume is equal to 0.8 (saving rate 20%). A reasonable number for the apparent marginal labor income tax (for Euro area) is around 40%.

So, the output multiplier is equal to 1.92!!!! Still large! (also positive for consumption!!!; $\simeq 0.92$)

A Second extension: small open economy

New aggregate resource constraint associated to an open economy

$$Y_t + Im_t = C_t + I_t + G_t + X_t$$

where Im_t represents the imports, and X_t the exports.

Let us assume that exports are exogenous, because they are function of the rest of the world: $X = \bar{X}$, or $X = X_o + \mu Y_t^\star$ ($\mu > 0$, i.e. when economic activity in the rest of world (foreign demand) increases, then exports increase), where $Y_t^\star$ is given for the small open economy.

Conversely, imports are related to domestic economic activity.

$$Im_t = Im_o + mY_t$$

where $m > 0$, i.e. when domestic activity (domestic demand) increases, then imports increase.

Let us assume that investment and taxes are exogenous and constant.

We obtain

$$Y_t = \frac{1}{1-c+m}\left(C_o - c\bar{T} + \bar{I} + \bar{X} - Im_o + G_t\right)$$

and the output multiplier is given by

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{1}{1-c+m}$$

Numerical Application: $c = 0.8$ and $m = 0.4$

$$\frac{\Delta Y_t}{\Delta G_t} = 1.6667$$

Still large, but if m increases, the ouptut multiplier deacreases (ex: $m = 0.8$, the output multiplier is equal to 1).

If we combine $G_t = T_t$ (balanced government budget) and small open economy, we obtain for $c = 0.8$ and $m = 0.4$,

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{1}{3}$$

II.2. The IS-LM Model

Here, we will proceed with two figures.

Figure 1: $i = \bar{i}$ the instrument of the monetary policy is the interest rate and money supply is endogenous; or a liquidity trap.

Figure on the blackboard

Figure 2: $M^s = \bar{M}$ the instrument of the monetary policy is the quantity of money and no speculative motive in money demand.

Figure on the blackboard

II.3. The AS-AD Model

Here, we will proceed with three figures.

Figure 1: Supply of goods and services displays an infinite elasticity to prices (discuss this case).

Figure on the blackboard

Figure 2: Supply of goods and services is totaly inelastic to prices (discuss this case).

Figure on the blackboard

Figure 3: Intermediate case (discuss this case).

Figure on the blackboard

Final remark We can also consider these findings in an open economy (fixed exchange rate, flexible exchange rate).

III-Results from General Equilibrium Models

III.1. A Simple Useful Framework

III.2. Extensions

III.1. A Simple Useful Framework

Three types of agents: households, firms and the government.

Closed economy.

Production economy: a homogenous good is produced with labor input.

Many periods (infinite horizon), but the problem is equivalent to a infinite sequence of static problems; so, we consider a static version of this model.

Households

The representative household seeks to maximize:

$$U(v_t, n_t)$$

where c_t denotes the real consumption and n_t is the labor supply (for example, the number of hours worked).

The utility satisfies $U_c > 0$ and $U_n \leq 0$ (desutility of labor).

We assume that $U(.,.)$ takes the following form

$$U(c_t, n_t) = \log(c_t) - \eta \frac{n_t^{1+\nu}}{1+\nu}$$

where the scale parameter η is positive and $\nu \geq 0$ is the inverse of the Frishean elasticity of labor.

$\nu = 0$: the elasticity of labor supply is infinite

$\nu \to \infty$: the labor supply is inelastic.

The utility function is separable between consumption and labor supply: $U_{cn} = 0$

In addition, we have

$$U_c = 1/c > 0 \text{ and } U_{cc} = -1/c^2 < 0$$

and

$$U_n = -\eta n^{\nu} < 0 \text{ and } U_{nn} = -\eta \nu n^{\nu - 1} \leq 0$$

The (sequence of) budget constraint

$$c_t + T_t \leq w_t n_t + \Pi_t$$

where c_t is real consumption,

T_t the level of taxes (lump-sum taxation),

w_t the real wage (the nominal wage divided by the price of consumption; the real wage measure the purchasing power of the nominal wage),

n_t the amount of labor supply (so $w_t \times n_t$ is the real labor income and $w_t \times n_t - T_t$ is the real disposable labor income),

and Π_t is the profit receives from the firm (the representative household owns the firm).

The maximization problem:

Form the Lagrangian $\mathcal{L}$

$$\mathcal{L} = \log(c_t) - \eta \frac{n_t^{1+\nu}}{1+\nu} - \lambda_t \left(c_t + T_t - w_t n_t - \Pi_t\right)$$

where λ_t is the Lagrange multiplier associated to the budget constraint in period t.

FOCs (w.r.t.) consumption and labor supply

$$\frac{1}{c_t} = \lambda_t$$

$$\eta n_t^{\nu} = \lambda_t w_t$$

We deduce that $\lambda_t > 0$ $(\forall t)$ since $c_t > 0$, so the the budget constraint binds.

$$c_t + T_t = w_t n_t + \Pi_t$$

every period.

Now, substitute the FOC on consumption into the FOC on labor supply

$$\eta n_t^\nu = \frac{1}{c_t} w_t$$

This equation represents the Marginal Rate of Substitution (**MRS**) between Lesiure (labor supply) and consumption. The ratio of the two marginal utilities is equal to the real wage (the opportunity cost of leisure).

Firms

The firms produce an homogenous good (denoted y_t) on a competitive market.

Only one input: labor

Technology: constant return to scale (see the problem set for an example with decreasing return)

$$y_t = an_t$$

where $a > 0$ is a scale parameter (a represents the level of the technology)

Profit function:

$$\Pi_t = y_t - w_t n_t \equiv (a - w_t)\, n_t$$

Profit maximization yields:

$$w_t = a$$

The real wage is constant and is equal to the level of the technology (the marginal productivity of labor)

Notice that the profit Π_t is zero.

Government

The government collects taxes T_t (lump-sum taxation) every period.

These taxes are used to finance government spending g_t, with a balanced government budget $g_t = T_t$.

An important remark: This is equivalent to finance government spending by public debt

$$B_{t+1} = (1 + r_t)B_t + g_t - T_t$$

General Equilibrium

The final good y_t can be used to private consumption c_t and public spending g_t

$$y_t = c_t + g_t$$

Now use the MRS between consumption and leisure

$$\eta n_t^\nu = \frac{1}{c_t} w_t \quad ,$$

the production function

$$y = a n_t \Longleftrightarrow n_t = (y_t/a)$$

and the FOC of the firm

$$w_t = a$$

We deduce

$$\eta \frac{y_t^\nu}{a^\nu} = \frac{a}{c_t}$$

Now, use the resources constraint

$$y_t = c_t + g_t \Longleftrightarrow c_t = y_t - g_t$$

and then obtain

$$\eta \frac{y_t^\nu}{a^\nu} = \frac{a}{y_t - g_t}$$

This equation define the general equilibrium of this economy.

A difficulty: this equation is highly non-linear in y_t and then it is difficult to determine the solution, i.e. an explicit form for the function

$$y = f(g_t)$$

A Rmk: One exception: linear utility (disutility) in leisure (labor supply): $\nu = 0$

$$U(c_t, n_t) = \log(c_t) - \eta n_t$$

FOC

$$\eta = \frac{1}{c_t} w_t \quad ,$$

and using the previous equilibrium conditions

$$\eta = \frac{1}{y_t - g_t} a$$

or equivalently

$$y_t = \frac{a}{\eta} + g_t$$

This implies that y_t is an increasing function of g_t and the multiplier, defined as

$$\frac{\Delta y_t}{\Delta g_t}$$

is equal to unity. Note that the private consumption is constant and is given by

$$c_t = \frac{a}{\eta} \quad \forall t$$

So the multiplier for the private consumption

$$\frac{\Delta c_t}{\Delta g_t}$$

is zero.

End of the Rmk

We go back to the equilibrium condition.

$$\eta \frac{y_t^\nu}{a^\nu} = \frac{a}{y_t - g_t}$$

A simple way to study the properties of this economy is to compute an approximation of this condition.

More precisely, we will consider a log-linearization around the deterministic steady state of the economy.

We need to determine two objects:

1) the steady-state (or long–run) value of y_t

2) the relative change in y_t in the neighborhood of the steady state.

Step 1. The steady-state. Let $\bar{y}$ and $\bar{g}$ the steady state values of y_t and g_t. These steady-state values must solve

$$\eta\frac{\bar{y}^{\nu}}{a^{\nu}} = \frac{a}{\bar{y}-\bar{g}}$$

Still highly non-linear, but we can easily prove that the steady-state exists:

Let denote F_1, where

$$F_1(\bar{y}) = \eta\frac{\bar{y}^{\nu}}{a^{\nu}}$$

and F_2

$$F_2(\bar{y}) = \frac{a}{\bar{y}-\bar{g}}$$

for $\bar{g}$ given. The equilibrium steady-state value of y exist if there exists a value of $\bar{y}$ such that

$$F_1(\bar{y}) = F_2(\bar{y})$$

The function F_1 verifies

$$F_1(0) = 0 \qquad F_1' > 0 \qquad F_1'' <=> 0 (\text{depending on the value of } \nu)$$

The function F_2 verifies

$$\lim_{\bar{y} \to \bar{g}^+} F_2 = +\infty$$

$$F_2' < 0$$

$$\lim_{\bar{y} \to +\infty} F_2 = 0$$

Show a figure

So, it exists a unique value of $\bar{y}$ that satisfies

$$F_1(\bar{y}) = F_2(\bar{y})$$

Step 2. The log-linearization around the (deterministic and unique) steady-state.

A Simple presentation

Log–linear approximation (extensively used for more complex models in the recent and modern DSGE literature)

Let the following function

$$y = f(x_1, x_2, ..., x_n)$$

Linear approximation:

$$y \approx f(\bar{x}_1, \bar{x}_2, ..., \bar{x}_n) + \sum_{i=1}^{n} \left.\frac{\partial y}{\partial x_i}\right|_{x=\bar{x}} (x_i - \bar{x}_i)$$

where

$$\bar{y} = f(\bar{x}_1, \bar{x}_2, ..., \bar{x}_n)$$

In what follows we will replace $\approx$ by the the equality $=$.

The previous equation rewrites

$$y - \bar{y} = \sum_{i=1}^{n} \left.\frac{\partial y}{\partial x_i}\right|_{x=\bar{x}} (x_i - \bar{x}_i)$$

Now divide both side by $\bar{y}$:

$$\frac{y-\bar{y}}{\bar{y}} = \sum_{i=1}^{n} \left.\frac{\partial y}{\partial x_i}\right|_{x=\bar{x}} \frac{1}{\bar{y}}(x_i - \bar{x}_i)$$

or equivalently

$$\frac{y-\bar{y}}{\bar{y}} = \sum_{i=1}^{n} \left.\frac{\partial y}{\partial x_i}\right|_{x=\bar{x}} \frac{\bar{x}_i}{\bar{y}}\frac{x_i - \bar{x}_i}{\bar{x}_i}$$

The log–linear approximation of y is then given by

$$\hat{y} = \sum_{i=1}^{n} \left.\frac{\partial y}{\partial x_i}\right|_{x=\bar{x}} \frac{\bar{x}_i}{\bar{y}}\hat{x}_i$$

where

$$\hat{y} = (y-\bar{y})/\bar{y} \simeq \log(y) - \log(\bar{y})$$

and

$$\hat{x}_i = (x_i - \bar{x}_i)/\bar{x}_i \simeq \log(x_i) - \log(\bar{x}_i)$$

Examples:

1) Production function

$$y_t = a_t n_t^{\alpha}$$

y_t is the output, a_t the TFP (Total Factor Productivity, a measure of the efficiency of the technology, or technical progress) and n_t the labor input.

The parameter $\alpha \in (0, 1]$ allows to measure the return of the input.

The log–linear approximation

$$\hat{y}_t = n^{\alpha}(a/y)\hat{a}_t + a\alpha n^{\alpha-1}(n/y)\hat{n}_t$$

From $y = an^{\alpha}$, this reduces to:

$$\hat{y}_t = \hat{a}_t + \alpha\hat{n}_t$$

2) Equilibrium condition

Closed economy

$$y_t = c_t + i_t + g_t$$

The log–linear approximation

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{i}}{\bar{y}}\hat{i}_t + \frac{\bar{g}}{\bar{y}}\hat{g}_t$$

$\bar{c}/\bar{y}$, $\bar{i}/\bar{y}$ and $\bar{g}/\bar{y}$ represent the average shares ($\bar{c}/\bar{y}+\bar{i}/\bar{y}+\bar{g}/\bar{y} = 1$ by construction) or steady–state values in the model's language.

3) Euler equation on consumption

log utility and stochastic returns (see asset pricing models)

$$\frac{1}{C_t} = \beta E_t R_{t+1} \frac{1}{C_{t+1}}$$

where R is the gross return on assets between periods t and $t+1$.

Steady state

$$\beta R = 1$$

Log–linearization:

$$-\hat{C}_t = E_t \hat{R}_{t+1} - E_t \hat{C}_{t+1}$$

or

$$\hat{C}_t = -E_t \hat{R}_{t+1} + E_t \hat{C}_{t+1}$$

Application

Let us consider again the equilibrium condition

$$\eta \frac{y_t^\nu}{a^\nu} = \frac{a}{y_t - g_t}$$

and the notations

$$F_{1,t} = F_{2,t}$$

where

$$F_{1,t} = \eta \frac{y_t^\nu}{a^\nu}$$

and

$$F_{2,t} = \frac{a}{y_t - g_t}$$

First consider the steady state of $F_{1,t}$

$$\bar{F}_1 = \eta \frac{\bar{y}^\nu}{a^\nu}$$

We know that the steady state of y exist and is unique. So, this is the case for F_1.

Now, log-linearize the function $F_{1,t}$ around $\bar{F}_1$

$$\hat{F}_{1,t} = \eta \frac{\bar{y}^{\nu-1}}{a^\nu} \frac{\bar{y}}{\bar{F}_1} \hat{y}_t \equiv \nu \hat{y}_t$$

Second, consider the steady state of $F_{2,t}$

$$\bar{F}_2 = \frac{a}{\bar{y} - \bar{g}}$$

We know that the steady state of y exist and is unique. So, this is the case for F_2.

We log-linearize the function $F_{2,t}$ around $\bar{F}_2$

$$\hat{F}_{2,t} = -\frac{a}{(\bar{y} - \bar{g})^2} \frac{\bar{y}}{\bar{F}_2} \hat{y}_t + \frac{a}{(\bar{y} - \bar{g})^2} \frac{\bar{g}}{\bar{F}_2} \hat{g}_t$$

Using the value of $\bar{F}_2$, this reduces to

$$\hat{F}_{2,t} = -\frac{\bar{y}}{(\bar{y} - \bar{g})} \hat{y}_t + \frac{\bar{g}}{(\bar{y} - \bar{g})} \hat{g}_t$$

or equivalently

$$\hat{F}_{2,t} = -\frac{1}{1 - s_g} \hat{y}_t + \frac{s_g}{1 - s_g} \hat{g}_t$$

where

$$s_g = \bar{g}/\bar{y}$$

Finally, from the identify

$$F_{1,t} = F_{2,t}$$

we immediately deduce

$$\hat{F}_{1,t} = \hat{F}_{2,t}$$

Now, we replace $\hat{F}_{1,t}$ and $\hat{F}_{2,t}$ in the previous equation

$$\nu \hat{y}_t = -\frac{\bar{1}}{1-s_g}\hat{y}_t + \frac{s_g}{1-s_g}\hat{g}_t$$

We obtain

$$\left(\nu + \frac{\bar{1}}{1-s_g}\right)\hat{y}_t = \frac{s_g}{1-s_g}\hat{g}_t$$

or equivalently

$$\left(1+\nu(1-s_g)\right)\hat{y}_t = s_g\hat{g}_t$$

or

$$\hat{y}_t = \frac{s_g}{1+\nu(1-s_g)}\hat{g}_t$$

The parameter

$$\frac{s_g}{1+\nu(1-s_g)}$$

is the elasticity of GDP (output) to public spending.

This elasticity is a decreasing function of ν (the inverse of the elasticity of labor supply).

The multiplier:

Note that

$$\hat{y}_t = \frac{y_t - \bar{y}}{\bar{y}}$$

and

$$\hat{g}_t = \frac{g_t - \bar{g}}{\bar{g}}$$

We deduce

$$\frac{y_t - \bar{y}}{\bar{y}} = \frac{s_g}{1 + \nu(1 - s_g)} \frac{g_t - \bar{g}}{\bar{g}}$$

or equivalently

$$\Delta y_t = \frac{s_g}{1+\nu(1-s_g)}\frac{\bar{y}}{\bar{g}}\Delta g_t$$

where

$$\Delta y_t = y_t - \bar{y} \quad \text{and} \quad \Delta g_t = g_t - \bar{g}$$

We obtain

$$\frac{\Delta y_t}{\Delta g_t} = \frac{1}{1+\nu(1-s_g)}$$

The multiplier verifies:

i) $\Delta y_t/\Delta g_t = 1$ when $\nu = 0$ (infinite elasticity of labor supply)

ii) $\partial(\Delta y_t/\Delta g_t)/\partial\nu < 0$ (the multiplier is higher when labor supply is more elastic)

iii) When $\nu \to \infty$ (inelastic labor supply), $\Delta y_t / \Delta g_t \to 0$.

When we consider private consumption

$$\hat{y}_t = \frac{s_g}{1 + \nu(1 - s_g)} \hat{g}_t$$

and

$$\hat{y}_t = (1 - s_g)\hat{c}_t + s_g \hat{g}_t$$

This last equation is obtained from the log-linearization of the aggregate resource constraint

$$y_t = c_t + g_t$$

Combining the two previous equations yields

$$(1 - s_g)\hat{c}_t + s_g\hat{g}_t = \frac{s_g}{1 + \nu(1 - s_g)}\hat{g}_t$$

and we deduce

$$\hat{c}_t = -\left(\frac{s_g}{1 - s_g}\right)\left(\frac{\nu(1 - s_g)}{1 + \nu(1 - s_g)}\right)\hat{g}_t$$

Note again that

$$\hat{c}_t = \frac{c_t - \bar{c}}{\bar{c}}$$

and

$$\hat{g}_t = \frac{g_t - \bar{g}}{\bar{g}}$$

So, we deduce

$$\Delta c_t = -\frac{\bar{c}}{\bar{g}} \left(\frac{s_g}{1 - s_g} \right) \left(\frac{\nu(1 - s_g)}{1 + \nu(1 - s_g)} \right) \Delta g_t$$

$$\Delta c_t = -\frac{\bar{c}/\bar{y}}{\bar{g}/\bar{y}} \left(\frac{s_g}{1 - s_g} \right) \left(\frac{\nu(1 - s_g)}{1 + \nu(1 - s_g)} \right) \Delta g_t$$

$$\Delta c_t = -\frac{1 - s_g}{s_g} \left(\frac{s_g}{1 - s_g} \right) \left(\frac{\nu(1 - s_g)}{1 + \nu(1 - s_g)} \right) \Delta g_t$$

$$\Delta c_t = -\left(\frac{\nu(1 - s_g)}{1 + \nu(1 - s_g)} \right) \Delta g_t$$

So we obtain

$$\frac{\Delta c_t}{\Delta g_t} = -\left(\frac{\nu(1-s_g)}{1+\nu(1-s_g)}\right)$$

We have the following properties

i) the (private) consumption multiplier is negative.

ii) it is equal to zero when the elasticity of labor supply is infinite ($\nu = 0$), so no-crowding out effect.

iii) it is equal to -1 when the elasticity of labor supply is zero ($\nu \to \infty$, inelastic labor supply), so perfect crowding out effect.

Discussion The key parameter is the (inverse of) elasticity of labor supply (ν).

When ν is small (large elasticity of labor supply), the output multiplier is larger.

Conversely, when ν is large, the output multiplier is almost zero.

The positive effect of government spending on output comes from the negative wealth (income) effect of government spending.

If G increases, the households will anticipate a decrease in their wealth (income), because the government must satisfy the budget constraint.

So, their consumption will decrease.

However, because households can use their labor supply to smooth consumption and then to reduce the negative wealth effect on consumption of an increase in G, aggregate labor and output will increase.

If labor supply is sufficiently elastic, the output multiplier is equal to one and the effect on private consumption is zero.

Conversely, if labor supply is inelastic, aggregate labor and output are unaffected.

III.2. Extensions

1. Other representation of preferences
2. Minimal consumption
3. Balanced government budget
4. Edgeworth Complementarity
5. Productive government spending
6. Externality (in preferences or in production)
7. Dynamic Models

1. Other representation of preferences

Let us consider another representation of households' preferences.

The specification of utility is now given by

$$U(c_t, n_t) = \log\left(c_t - \eta\frac{n_t^{1+\nu}}{1+\nu}\right)$$

The household' s budget constraint is the same as before, so the maximisation problem is very similar, except for the specification of the utility function.

The Lagrangian

$$\mathcal{L} = \log\left(c_t - \eta\frac{n_t^{1+\nu}}{1+\nu}\right) - \lambda_t\left(c_t + T_t - w_t n_t - \Pi_t\right)$$

where λ_t is the Lagrange multiplier associated to the budget constraint in period t.

FOCs (w.r.t.) consumption and labor supply

$$\frac{1}{c_t - \eta\frac{n_t^{1+\nu}}{1+\nu}} = \lambda_t$$

$$\frac{\eta n_t^{\nu}}{c_t - \eta\frac{n_t^{1+\nu}}{1+\nu}} = \lambda_t w_t$$

We deduce that $\lambda_t > 0$ since $c_t > 0$, so the the budget constraint binds.

$$c_t + T_t = w_t n_t + \Pi_t$$

Now, substitute the FOC on consumption into the FOC on labor supply

$$\frac{\eta n_t^{\nu}}{c_t - \eta\frac{n_t^{1+\nu}}{1+\nu}} = \frac{1}{c_t - \eta\frac{n_t^{1+\nu}}{1+\nu}} w_t$$

or equivalently

$$\eta n_t^{\nu} = w_t$$

We obtain a labor supply that depends only on the real wage (no wealth effect, through the marginal utility of consumption).

All the results about the equilibrium can be easily deduced.

From the firms'optimization problem (with constant return to scale on the labor input), we have

$$w_t = a$$

So, we obtain

$$\eta n_t^{\nu} = a$$

It follows that labor supply (and thus labor at equilibrium) is constant (or hours worked).

If labor is constant, then output (y_t) is constant and independent from government spending.

So, the output multiplier is zero:

$$\frac{\Delta y_t}{\Delta g_t} = 0$$

and the consumption multiplier is

$$\frac{\Delta c_t}{\Delta g_t} = -1$$

Discussion Here, the specification of utility eliminate the wealth (income) effect in labor supply.

Only, the effect of real wage on labor supply (only the substitution effect).

2. Minimal consumption

Let us now consider a new specification of utility

$$U(c_t, n_t) = \log(c_t - c_m) - \eta \frac{n_t^{1+\nu}}{1+\nu}$$

where c_m is the minimal consumption (subsistence level).

The household' s budget constraint is the same as before, so the maximisation problem is very similar, except for the specification of the utility function.

The Lagrangian

$$\mathcal{L} = \log(c_t - c_m) - \eta \frac{n_t^{1+\nu}}{1+\nu} - \lambda_t (c_t + T_t - w_t n_t - \Pi_t)$$

where λ_t is the Lagrange multiplier associated to the budget constraint in period t.

FOCs (w.r.t.) consumption and labor supply

$$\frac{1}{c_t - c_m} = \lambda_t$$

$$\eta n_t^{\nu} = \lambda_t w_t$$

(Again, we deduce that $\lambda_t > 0$ since $c_t > 0$, so the the budget constraint binds,

$c_t + T_t = w_t n_t + \Pi_t$)

Now, substitute the FOC on consumption into the FOC on labor supply

$$\eta n_t^{\nu} = \frac{1}{c_t - c_m} w_t$$

and then use the production function

$$y = a n_t \Longleftrightarrow n_t = (y_t/a)$$

and the FOC of the firm

$$w_t = a$$

We deduce

$$\eta \frac{y_t^{\nu}}{a^{\nu}} = \frac{a}{c_t - c_m}$$

Now, use the resources constraint

$$y_t = c_t + g_t \Longleftrightarrow c_t = y_t - g_t$$

and then obtain

$$\eta \frac{y_t^\nu}{a^\nu} = \frac{a}{y_t - g_t - c_m}$$

This equation define the general equilibrium of this economy.

A difficulty: this equation is highly non-linear in y_t and then it is difficult to determine the solution.

We have the following properties

Property 1: The steady-state output exists and is unique.

Property 2: The log–linear approximation for output is given

$$\hat{y}_t = \frac{s_g}{1 + \nu(1 - s_g - s_m)}\hat{g}_t$$

where $s_g = \bar{g}/\bar{y}$ and $s_m = c_m/\bar{y}$.

Property 3: The output multiplier is given by

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{1}{1 + \nu(1 - s_g - s_m)}$$

Property 4: The output multiplier is an increasing function of the minimal consumption (s_m).

Property 5: The consumption multiplier is still negative but minimal consumption mitigates the crowding-out effect.

[REDO ALL THE COMPUTATIONS]

Numerical Application:

Case 1. (benchmark case) $\nu = 1$, $s_g = 0.2$ and $s_m = 0$

$$\frac{\Delta Y_t}{\Delta G_t} = 0.5556 \qquad \frac{\Delta C_t}{\Delta G_t} = -0.4445$$

Case 2. $\nu = 1$, $s_g = 0.2$ and $s_m = 0.5$

$$\frac{\Delta Y_t}{\Delta G_t} = 0.7692 \qquad \frac{\Delta C_t}{\Delta G_t} = -0.2307$$

3. Balanced government budget

In the case of lump-sum taxation, it is equivalent to finance public spending either by taxes or by public debt (Ricardian equivalence theorem, next year in M1, a formal proof in dynamic stochastic general equilibrium models)

[If necessary, show the result on the blackboard]

We now investigate the case of two distortionary taxes: labor income tax and consumption tax (VAT)

i) Labor income tax

Same utility as in the benchmark case

$$U(c_t, n_t) = \log(c_t) - \eta \frac{n_t^{1+\nu}}{1+\nu}$$

But the budget constraint is now given by

$$c_t + T_t \leq (1 - \tau_{w,t}) w_t n_t + \Pi_t$$

where $\tau_{w,t}$ is the labor income tax.

FOCs of the household (w.r.t.) consumption and labor supply

$$\frac{1}{c_t} = \lambda_t$$

$$\eta n_t^{\nu} = \lambda_t w_t (1 - \tau_{w,t})$$

(Again, we deduce that $\lambda_t > 0$ since $c_t > 0$, so the the budget constraint binds)

Now, substitute the FOC on consumption into the FOC on labor supply (we obtain the MRS)

$$\eta n_t^{\nu} = \frac{1}{c_t} w_t (1 - \tau_{w,t})$$

Concerning the firm, we have the same optimality condition

$$w_t = a$$

Finally, the government will perfectly balanced public spending g_t by labor income taxation

$$g_t = \tau_{w,t} w_t n_t$$

Consider the MRS with $w_t = a$

$$\eta n_t^{\nu} = \frac{a}{c_t}(1 - \tau_{w,t})$$

Rmk: We see that labor taxes create a (fiscal) wedge in the MRS. Absent distortionary taxation, the MRS will be

$$\eta n_t^{\nu} = \frac{a}{c_t}$$

Now use the government budget constraint

$$\tau_{w,t} = \frac{g_t}{w_t n_t}$$

From the firm'side, we know that $w_t n_t = a n_t \equiv y_t$. We deduce

$$\tau_{w,t} = \frac{g_t}{y_t}$$

So, we obtain

$$1 - \tau_{w,t} = 1 - \frac{g_t}{y_t} \equiv \frac{y_t - g_t}{y_t}$$

or

$$1 - \tau_{w,t} = \frac{c_t}{y_t}$$

If we substitute into the MRS

$$\eta n_t^{\nu} = \frac{a}{c_t}\frac{c_t}{y_t}$$

or equivalently

$$\eta y_t^{1+\nu} = a^{1+\nu}$$

So output is constant and unaffected by an increase in government spending. This is because when g increases, $\tau_{w,t}$ must increase to balance the government budget constraint, thus reducing the incentive of supplying more labor. This leads to perfectly offset the increase in labor supply after an increase in government spending.

ii) Consumption tax

Same utility as in the benchmark case

$$U(c_t, n_t) = \log(c_t) - \eta \frac{n_t^{1+\nu}}{1+\nu}$$

But the budget constraint is now given by

$$(1 + \tau_{c,t})c_t + T_t \leq w_t n_t + \Pi_t$$

where $\tau_{c,t}$ is the consumption tax (VAT).

FOCs of the household (w.r.t.) consumption and labor supply

$$\frac{1}{c_t} = \lambda_t(1 + \tau_{c,t})$$

$$\eta n_t^{\nu} = \lambda_t w_t$$

(Again, we deduce that $\lambda_t > 0$ since $c_t > 0$, so the the budget constraint binds)

Now, substitute the FOC on consumption into the FOC on labor supply (we obtain the MRS)

$$\eta n_t^{\nu} = \frac{1}{c_t} w_t \frac{1}{1 + \tau_{c,t}}$$

Rmk: very similar to the previous case (effect of labor income tax or consumption tax on labor supply). Taxes on consumption creates a wedge in the MRS.

Concerning the firm, we have the same optimality condition

$$w_t = a$$

Finally, the government will perfectly balanced public spending g_t by taxing consumption

$$g_t = \tau_{c,t} c_t$$

Consider the MRS with $w_t = a$

$$\eta n_t^{\nu} = \frac{a}{c_t} \frac{1}{1 + \tau_{c,t}}$$

Now use the government budget constraint

$$\tau_{c,t} = \frac{g_t}{c_t}$$

$$1 + \tau_{c,t} = 1 + \frac{g_t}{c_t} \equiv \frac{y_t}{c_t}$$

If we substitute into the MRS

$$\eta n_t^{\nu} = \frac{a}{c_t}\frac{y_t}{c_t}$$

or equivalently

$$\eta y_t^{1+\nu} = a^{1+\nu}$$

So output is constant and unaffected by an increase in government spending. This is because when g increases, $\tau_{c,t}$ must increase to balance the government budget constraint, thus reducing the incentive of supplying more labor. This leads to perfectly offset the increase in labor supply after an increase in government spending.

4. Edgeworth Complementarity

Let us now consider a new specification of utility

$$U(c_t, n_t) = \log(c_t + \alpha_g g_t) - \eta \frac{n_t^{1+\nu}}{1+\nu}$$

where α_g allows to measure the degree of complementarity/substitutability between private consumption c_t and public spending g_t.

$\alpha_g = 0$: standard case.

$\alpha_g > 0$: substitutability between private consumption c_t and public spending g_t.

$\alpha_g < 0$: complementarity between private consumption c_t and public spending g_t.

Discuss examples of complementarity/substitutability between pri-

vate goods and public goods.

The household' s budget constraint is the same as before, so the maximization problem is very similar, except for the specification of the utility function.

The Lagrangian

$$\mathcal{L} = \log\left(c_t + \alpha_g g_t\right) - \eta \frac{n_t^{1+\nu}}{1+\nu} - \lambda_t \left(c_t + T_t - w_t n_t - \Pi_t\right)$$

Notice that g_t is given for the household.

λ_t is the Lagrange multiplier associated to the budget constraint in period t.

FOCs (w.r.t.) consumption and labor supply

$$\frac{1}{c_t + \alpha_g g_t} = \lambda_t$$

$$\eta n_t^{\nu} = \lambda_t w_t$$

(Again, we deduce that $\lambda_t > 0$ since $c_t > 0$, so the the budget constraint binds, $c_t + T_t = w_t n_t + \Pi_t$)

Now, substitute the FOC on consumption into the FOC on labor supply (we obtain the MRS)

$$\eta n_t^{\nu} = \frac{1}{c_t + \alpha_g g_t} w_t$$

and then use the production function

$$y = an_t \Longleftrightarrow n_t = (y_t/a)$$

and the FOC of the firm

$$w_t = a$$

We deduce

$$\eta \frac{y_t^\nu}{a^\nu} = \frac{a}{c_t + \alpha_g g_t}$$

Now, use the resources constraint

$$y_t = c_t + g_t \Longleftrightarrow c_t = y_t - g_t$$

and then obtain

$$\eta \frac{y_t^\nu}{a^\nu} = \frac{a}{y_t - g_t + \alpha_g g_t}$$

or

$$\eta \frac{y_t^\nu}{a^\nu} = \frac{a}{y_t - (1 - \alpha_g) g_t}$$

This equation define the general equilibrium of this economy.

A difficulty: this equation is highly non-linear in y_t and then it is difficult to determine the solution, except if $\nu = 0$.

We have the following properties

Property 1: The steady-state output exists and is unique.

Property 2: The log–linear approximation for output is given

$$\hat{y}_t = \frac{s_g(1-\alpha_g)}{1+\nu(1-s_g(1-\alpha_g))}\hat{g}_t$$

where $s_g = \bar{g}/\bar{y}$.

Property 3: The output multiplier is given by

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{1-\alpha_g}{1+\nu(1-s_g(1-\alpha_g))}$$

Property 4: The output multiplier is a decreasing function of α_g.

Property 5: The consumption multiplier can be positive if private consumption and public spending display a sufficient degree of complementarity , i.e. α_g is sufficiently negative.

[REDO ALL THE COMPUTATIONS]

Numerical Application:

Case 1. (benchmark case) $\nu = 1$, $s_g = 0.2$ and $\alpha_g = 0$

$$\frac{\Delta Y_t}{\Delta G_t} = 0.5556 \qquad \frac{\Delta C_t}{\Delta G_t} = -0.4445$$

Case 2. $\nu = 1$, $s_g = 0.2$ and $\alpha_g = 1$

$$\frac{\Delta Y_t}{\Delta G_t} = 0 \qquad \frac{\Delta C_t}{\Delta G_t} = -1$$

Case 3. $\nu = 1$, $s_g = 0.2$ and $\alpha_g = -1$

$$\frac{\Delta Y_t}{\Delta G_t} = 1.25 \qquad \frac{\Delta C_t}{\Delta G_t} = 0.25$$

5. Productive government spending

For the household, same utility as before

$$U(c_t, n_t) = \log(c_t) - \eta \frac{n_t^{1+\nu}}{1+\nu}$$

and same budget constraint

$$c_t + T_t \leq w_t n_t + \Pi_t$$

The main difference with the benchmark case is the specification of the production function.

We assume that public spending is productive, i.e. an increase in g will increase output and (marginal) labor productivity.

A shortcut, because this is not the flow of purchases g that can increase the private output, but rather the public capital (public infrastructures)

The production function takes the form

$$y = a n_t g_t^{\varphi}$$

If $\varphi = 0$, we retrieve the benchmark case.

If $\varphi > 0$, public spending is productive.

FOCs of households: same as before.

MRS:

$$\eta n_t^{\nu} = \frac{1}{c_t} w_t$$

Firms.

Profit function:

$$\Pi_t = y_t - w_t n_t \equiv \left(a g_t^{\varphi} - w_t\right) n_t$$

Profit maximization yields (g_t are given for the firm):

$$w_t = a g_t^{\varphi}$$

The real wage is equal to the marginal productivity of labor (affected by g_t if $\varphi \neq 0$).

Now, we replace the real wage into the MRS:

$$\eta n_t^\nu = \frac{1}{c_t} a g_t^\varphi$$

and again, we use the aggregate constraint $y_t = c_t + g_t$:

$$\eta n_t^\nu = \frac{1}{y_t - g_t} a g_t^\varphi$$

This equation define the general equilibrium of this economy.

A difficulty (again): this equation is highly non-linear in y_t and then it is difficult to determine the solution, except if $\nu = 0$.

We have the following properties

Property 1: The steady-state output exists and is unique.

Property 2: The log–linear approximation for output is given

$$\hat{y}_t = \frac{s_g + \varphi(1 - s_g)}{1 + \nu(1 - s_g(1 - \alpha_g))}\hat{g}_t$$

where $s_g = \bar{g}/\bar{y}$.

Property 3: The output multiplier is given by

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{1 + \varphi(1 - s_g)/s_g}{1 + \nu(1 - s_g(1 - \alpha_g))}$$

Property 4: The output multiplier is an increasing function of φ.

Property 5: The consumption multiplier can be positive if public spending is sufficiently productive, i.e. φ is sufficiently positive.

[REDO ALL THE COMPUTATIONS]

Numerical Application:

Case 1. (benchmark case) $\nu = 1$, $s_g = 0.2$ and $\varphi = 0$

$$\frac{\Delta Y_t}{\Delta G_t} = 0.5556 \quad \frac{\Delta C_t}{\Delta G_t} = -0.4445$$

Case 2. $\nu = 1$, $s_g = 0.2$ and $\varphi = 0.2$

$$\frac{\Delta Y_t}{\Delta G_t} = 1 \quad \frac{\Delta C_t}{\Delta G_t} = 0$$

Case 3. $\nu = 1$, $s_g = 0.2$ and $\varphi = 0.4$

$$\frac{\Delta Y_t}{\Delta G_t} = 1.4444 \quad \frac{\Delta C_t}{\Delta G_t} = 0.4444$$

6. Externality (in preferences or in production)

Discussion + on the blackboard

Discussion + on the blackboard