Private, social and self-insurance for long-term care in the presence of family help

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Abstract

We study the political determination of the level of social long-term care insurance when voters can top up with private insurance, saving and family help. Agents differ in income, probability of becoming dependent and of receiving family help, and amount of family help received. Social insurance redistributes across income and risk levels, while private insurance is actuarially fair.

The income-to-dependency probability ratio of agents determines whether they prefer social or private insurance. Family support crowds out the demand for both social and, especially, private insurance, as strong prospects of family help drive the demand for private insurance to zero. The availability of private insurance decreases the demand for social insurance but need not decrease its majority chosen level. A majority of voters would oppose banning private insurance.

Keywords: long-term care, political economy, social insurance, top up, familism, crowding out, weak and strong prospects of family help, voting.

JEL classification: D72, I13, J14
1 Introduction

While health care services aim at changing a health condition (from unwell to well), long-term care (hereafter LTC) merely aims at making the current condition (unwell) more bearable. Individuals need LTC due to disability, chronic condition, trauma, or illness, which limit their ability to carry out basic self-care or personal tasks that must be performed every day. Such activities are defined as activities of daily living (eating, dressing, bathing, getting in and out of bed, toileting and continence) or instrumental activities of daily living (preparing own meals, cleaning, laundry, taking medication, getting to places beyond walking distance, shopping, managing money affairs and using the telephone/Internet). A person is dependent if he or she has limitations in either type.

Dependent people can draw on four different types of resources to help alleviate their daily living problems. By far the most important quantitatively are their own resources (self-insurance\(^1\), or savings) and family help (mainly through informal help). Several countries also offer some form of social LTC insurance, although the size of these programs is usually low, especially compared to the pension programs. Finally, except for a handful of countries (such as the US and France), private insurance’s role is negligible, and in any case consistently smaller than that of the State.

Several articles have studied the interactions between some of these resources (see the literature section hereunder). The value added of our paper is that it considers simultaneously the four types of resources (family help, private, social and self-insurance) and that it studies the political choice of the level of social LTC insurance. More precisely, we study the determinants of the political support for social insurance in an environment where people differ in income, risk and availability of family help, and where they choose individually their private and self-insurance levels. As stated above, the availability of family help is of first importance for LTC, and distinguishes our approach from the literature studying the political support for other kinds of social insurance programs, such as health or social security.\(^2\)

We develop a framework where agents live two periods. They earn an income, pay taxes, save and buy private LTC insurance when young. Beyond income, they also differ in the probability of becoming dependent when old, in the probability of receiving help from their family if dependent, and in the amount received

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\(^1\) As other papers dealing with LTC (see Costa-Font 2010 for instance), we use the general definition of self-insurance (assets set aside to cover possible losses) rather than the more specific meaning used in insurance theory (prevention activities reducing the severity of the potential loss).

\(^2\) For instance, over 80% of dependent elderly live in their home or with their children, and for these people most of the care is informal. See Stone (2000).
from the family.³ They choose by majority voting the value of the proportional income tax rate that finances the lump sum social insurance transfer received if dependent. A crucial assumption is that social LTC insurance redistributes across income and (ex ante) across risk, while private insurance is actuarially fair and not redistributive.¹ To level the playing field, both forms of insurance are equally efficient (no loading factor, no cost of public fund or distortionary impact from taxation).⁵

While we endogenize the levels of the three types of insurance, family help is modelled as an exogenous norm. This corresponds to what Costa-Font (2010) calls familism, a short hand for “familism culture”, or “the embeddedness in a family’s social norms (family ties)” (p 2). Familism is exogenous because of the facts that “need of LTC is a non-repeated experience in human life, and that individuals are arguably prone to make decisions on the basis of intergenerational cultural values rather than repeatedly experienced actions. More specifically, parents tend to transfer their values to their children (...), which includes cultural norms towards funding and providing care in old age.” (p 3). For simplicity, we do not model the distinction between formal and informal help.

We first study the individually most-preferred (self, private and social) insurance allocation of agents. We show that agents most-prefer either social or private insurance, depending of their income-to-dependency probability ratio. Whatever this ratio, the support for their most-preferred insurance form decreases with the probability of receiving family help, and with the amount of this help. This is in line with the empirical results obtained by Costa-Font (2010). We then introduce the distinction between weak and strong prospect of family help: prospects are strong (resp., weak) for a given individual if her expected marginal utility is lower (resp., higher) when she is dependent with family as the only source of help (i.e., without any private or social insurance handouts) than when she remains autonomous. Social insurance happens to be more resilient than private insurance to family help, in the sense that no one with strong prospects of family help most

³Like most related papers, we assume that the state of the world for each individual is binary, with one being either dependent or not. In reality, there are several degrees of severity in the state of dependence. Public and private insurance schemes use scales such as the Katz or IADL (Independence in Activities of Daily Living) index to measure those degrees and provide compensations accordingly. Introducing such scales in our analysis would complicate matters a lot without bringing significant new insights.

⁴From now on, we slightly abuse terminology and refer to the risk of an agent as his probability of becoming dependent.

⁵Alternatively, we could have modelled public intervention as the subsidization of private insurance premia. Such a policy also benefits high-risk agents, provided that the demand for private insurance is increasing in individual risk. Its income redistribution component is less clear: richer agents pay more taxes, but may also consume more insurance and thus benefit more from the subsidy. We leave this analysis for future research.
prefers any private insurance, while some of these agents may still prefer a positive amount of social insurance. The main driving force of this result is that strong prospects of family help cancel out any need for an actuarially fair insurance like private insurance, while the (income and risk) redistribution embedded into social insurance induces a positive support from low income-to-dependency probability agents. The larger support for social insurance when prospects of family help are strong is in line with the stylized fact that social LTC insurance is more widespread worldwide and larger than private insurance in the countries where they coexist (although both are of a small size, which can be explained by our model as due to sizeable family help). Our model also generates a novel testable implication regarding the amount of self-insurance, which is not monotone in family help: it first increases with family help (as agents substitute more saving to less insurance) but then decreases with family help when agents prefer self-insurance to any other form of insurance.

We then show that there exists a Condorcet winning level of social LTC insurance (i.e., a level preferred by a majority of voters to any other level) even though voters differ in four dimensions. Voters’ characteristics that lead to a positive majority chosen level of social insurance are the existence of a large fraction of agents with low income-to-dependency probability ratios (so that the correlation between income and risk does matter), and with weak prospects of family help. We also study how the availability of private insurance impacts the support for social LTC insurance. This question is of interest as countries such as France are currently considering how to facilitate the access to private LTC insurance. Moreover, as explained below the literature has concentrated on the opposite relationship, namely the impact of social on private insurance. We obtain that, even though both types of insurance are substitutes, so that introducing private insurance decreases the support for social insurance by some agents, the majority chosen level of social insurance need not decrease. The intuition for this proposition is that agents prefer either social or private insurance, but never both, so that introducing private insurance either does not change an individual’s most-preferred social insurance level, or drives it to zero. We study several cases where individuals differ in one or at most two dimensions, and we obtain that the conditions required for private insurance to crowd out social insurance are empirically either not satisfied (when agents differ only in income) or very demanding. We finally obtain that a majority always opposes a ban on private insurance, even in the case where private insurance crowds out social insurance at the majority equilibrium.
1.1 Literature

As mentioned above, a rich literature has studied the interactions between private, social and self-insurance together with family help in the context of LTC. To the best of our knowledge, no paper takes the four forms of resources into account simultaneously.

Both Meier (1996) and Fabel (1996) develop a two-period overlapping generations model where young individuals save and choose how much private LTC insurance to buy. They study how these two decisions are affected by various institutional settings, such as pure private funding, social aid (where the public sector pays the part of the cost of LTC that an individual cannot afford), mandatory social insurance (funded or pay-as-you go) or obligatory minimum level of private coverage. Their focus is on aggregate saving and private insurance by a cohort, and they abstract both from family help and from the determination of the type/size of public policy. They often obtain ambiguous results as to the impact of the public policy on saving or private insurance.

Pauly (1990) studies various reasons why informed and risk averse agents do not buy actuarially fair private LTC insurance. His main argument is that Medicaid crowds out the demand for private insurance, because it reimburses all LTC costs (up to a threshold) once private funding is exhausted. Empirical evidence does not appear to confirm this phenomenon (Norton, 1995; Brown and Finkelstein, 2007). Pauly (1990) also investigates the interactions between family help and private insurance. He shows that the concerns for not impoverishing one’s spouse and for leaving a bequest do not always generate a positive demand for private LTC insurance. He also studies the manipulation of bequests in order to incentivize children to provide informal help.

Pauly (1990)’s seminal approach has given rise to several analytical analyses of this so-called “intra-family moral hazard problem”, by Courbage and Zweifel (2011) and Zweifel and Strüwe (1996, 1998) among others. There seems to be little empirical support for intra-family moral hazard: Sloan and Norton (1997) find that the bequest motive does not influence the demand for private insurance in the US, while Mellor (2001) finds no evidence consistent with the idea that the availability of caregivers discourages parents from obtaining market-purchased LTC insurance. Courbage and Roudaut (2008) rather find support in French data for an altruistic motive since LTC insurance is purchased not only to preserve bequests and to financially protect families in the event of disability, but also to reduce the burden of potential informal care givers.

All these papers either abstract from social insurance or take its characteristics as exogenous. There is surprisingly little literature on the determination of the (socially or individually) optimal level of social LTC insurance, especially when
compared with the related issues of health care, social security and annuities. On the normative side, Cremer and Pestieau (2013) use a model close to the one of this paper; they show that the case for social LTC insurance can only be defended when tax redistribution is restricted. On the positive side, Nuscheler and Roeder (2013) study how the heterogeneity in individual income and risk affects the preferences for redistributive income taxation versus public financing of LTC. Their model allows LTC to be provided by informal help received from the family, or through family transfers in cash and government’s transfers. Insurance (whether social, private or self-insurance in the form of saving) is not available since voters know whether the elderly in the family is dependent or not when taking their decisions. There is also no room for the correlation between income and risk, since the proportion of dependent elderly is the same in the two income classes considered. Their main result is the prediction of a negative correlation between income inequality and public LTC spending. De Donder and Leroux (2013) stress the behavioral biases exhibited by agents who vote for social LTC insurance and buy LTC annuities, a financial product that serves a higher transfer if dependent than if not. Agents all have the same income and risk (there is no family help) and differ in both the type and degree of myopia. They obtain that the low demand for private insurance is better explained by underestimation of the risk of becoming dependent than by procrastination.

Finally, our paper is also related to the literature on public provision of private goods when individuals can supplement public provision with private purchases—i.e., “top up” models such as Epple and Romano (1996) but especially Gouveia (1997). Gouveia (1997) models a setting where agents differ in income and probability of getting sick, and vote over (tax financed) public provision of health care that can be supplemented with private purchases. One important difference with our paper is that he models two goods (health care and a numeraire) and public provision of health care (rather than a monetary transfer as in our setting). Redistribution with the public scheme (from high to low income, and from low to high morbidity) then takes place through different (tax) prices of health care, with agents forbidden from selling the publicly provided amount of health care on the market. He obtains a majority-voting equilibrium which, under certain conditions, is of the “ends-against-the-middle” form, with low and high income voters favoring a lower amount of public provision than the one obtained at equilibrium. A common theme of Gouveia (1997) and of our paper is that a majority of voters would oppose banning the private sector (private LTC insurance in our setting, private health care sector in his) supplementing the public one.

The paper is organized as follows. Section 2 presents the model. Section 3
describes the most-preferred social, private and self-insurance allocation of agents. Section 4 studies the majority-chosen social insurance level. Section 5 concludes.

2 The Model

We consider a continuum of individuals living two periods. When young, they earn a wage, pay income taxes, save and buy private LTC insurance. When old, they live out of their saving, plus the social and private insurance transfers if they need LTC, plus a transfer from the family if they have caring children and they need LTC. There are four sources of heterogeneity among individuals $i$: their exogenous income, denoted by $w_i > 0$, their probability of needing LTC ($\pi_i \in [0, 1]$), their probability of having (caring and close) children when needing LTC ($p_i \in [0, 1]$), and the amount of help from the family the agent can count on if dependent, $f_i \geq 0$. An agent of type $i$ is thus characterized by the quadruplet $(w_i, \pi_i, p_i, f_i)$.

A young individual $i$’s lifetime utility function is given by

$$U_i = u(c_i) + (1 - \pi_i)u(s_i) + \pi_i [p_i H(d_i^c) + (1 - p_i) H(d_i^n)].$$  \hspace{1cm} (1)

The first term of (1) measures the instantaneous utility of individual $i$ when young, while the last two terms record his utility when old (for simplicity, we assume away any discounting of future utility). First-period consumption is denoted by

$$c_i = w_i (1 - \tau) - s_i - a_i,$$

where the first term measures disposable income when young, with $\tau$ a (proportional) contribution rate on labor income. The second term $s_i$ is private saving, while $a_i$ denotes the amount of private LTC insurance bought.$^8$

In the second period of life, we distinguish the utility function when autonomous (with probability $1 - \pi_i$), denoted by $u(.)$, from the utility when needing LTC (with probability $\pi_i$), denoted by $H(.)$. Agents have the same instantaneous

\footnote{There are many reasons why some parents cannot count on any assistance from their offspring: (i) they do not have children or their children prematurely died; (ii) their children are not altruistic; (iii) they migrated far away from each other; (iv) parents and children do not get along.}

\footnote{We do not model the transfer made by some young agents to their dependent parents in the first period of their life. This is consistent with the assumption that LTC transfers are tax deductible, or that they take the form of informal help that has as opportunity cost foregone income on the labor market. In both these cases, the income $w_i$ is income net of LTC transfers to parents. Introducing explicitly the LTC transfer to parents would add a fifth (and binary) dimension of heterogeneity to our already complex model. Moreover, the role played by this transfer would be very similar to the one played by income, so we would gain very little additional insight.}
utility function \( u(.) \) when young and when old but autonomous. Both \( u(.) \) and \( H(.) \) are increasing and concave functions of consumption. We assume that both satisfy the condition of infinite marginal utility for zero consumption levels. We also assume that \( u(c) > H(c) \) for any consumption level \( c \), but that \( u'(c) < H'(c) \) for all \( c \): people are happier if not in need of LTC, but have a higher marginal utility if dependent. Note that \( H(.) \) can be viewed as a reduced form of a utility comprising both standard consumption and LTC spending.\(^9\)

With probability \( 1 - \pi_i \), the individual remains autonomous and enjoys his saving (without loss of generality we posit a zero interest rate on savings). If the individual becomes dependent (with probability \( \pi_i \)), his consumption level depends on whether he receives help from his family. He does not receive such help with probability \( 1 - p_i \), in which case his consumption level is given by

\[
d_i^n = s_i + b + x_i,
\]

where \( b \) (resp., \( x_i \)) denotes the social (resp., private) insurance benefit. With probability \( p_i \), the dependent individual receives a transfer \( f_i \) from his family. As explained in the introduction when discussing familism, we assume that \( f_i \) is exogenously set by a social norm at the family level. We do not model the distinction between formal and informal help, but rather measure \( f_i \) as the monetary value of all the help received from the family. Consumption in that case is given by

\[
d_i^c = s_i + b + f_i + x_i.
\]

The social insurance lump sum transfer \( b \) paid to all dependent agents is financed by the proportional tax \( \tau \) on first-period labor income. For simplicity, we assume away demographic (and economic) growth, so that the social insurance program’s budget constraint is given by

\[
b = \tau \frac{\bar{w}}{\bar{\pi}}, \quad (2)
\]

where \( \bar{w} \) is the average income and \( \bar{\pi} \) is the average probability of needing LTC (and thus, by the law or large numbers, the proportion of old individuals who become dependent).

We model the private insurance scheme as actuarially fair: the premium does not depend on income but is based on the individual probability \( \pi_i \) (which is

\(^9\)The assumption that \( u'(c) < H'(c) \) is satisfied by definition if the advent of dependency is associated exclusively with financial costs. If dependency is also characterized by a change in preferences, we enter the realm of state-dependent utility functions. The assumption that \( u'(c) < H'(c) \) may then be disputed (see, for instance, Finkelstein et al. 2009, 2013), since some goods may substitute or complement good health. However, observe that if dependent people do not have higher marginal utility than when autonomous, then the lack of demand for (social and private) LTC insurance is not puzzling at all.
assumed to be observable by the insurer). Since LTC need is binary, there is no place for ex post moral hazard. Also, we assume that insurers do not condition their payment on the transfer made by children (for instance because they cannot observe it). Individuals can choose the quantity of private insurance that they buy, as measured by the insurance premium $a_i$ paid in the first period of life. In case they need LTC, they then receive an actuarially fair amount

$$x_i = \frac{a_i}{\pi_i}.$$  

The timing of the model runs as follows. Individuals first choose the value of $\tau$ by majority voting. We assume that only young agents vote\(^ {10}\) and that they vote as if the result of the vote would continue to hold in the next period.\(^ {11}\) They then observe the result of the vote, and decide privately how much to save and to privately insure against the risk of dependency. No decision is taken in the second period of life.

3 Most-preferred public, private and self-insurance allocation

We look for the most-preferred amounts of social, private and self insurance (respectively denoted by $\tau^*_i$, $a^*_i$ and $s^*_i$) of agents differing in income $w_i$, probability of dependency $\pi_i$ and family help $p_i$ and $f_i$. The respective first-order conditions (FOCs) are\(^ {12}\)

$$FOC_{\tau_i} : \bar{w} \frac{\pi_i}{\pi} \left[ -\frac{w_i}{\bar{w}} u'(c_i) + EH'_i \right] \leq 0,$$

$$FOC_{a_i} : -u'(c_i) + EH'_i \leq 0,$$

$$FOC_{s_i} : -u'(c_i) + (1 - \pi_i) u'(s_i) + \pi_i EH'_i = 0,$$

where

$$EH'_i = p_i H'(s_i + b + \frac{a_i}{\pi_i} + f_i) + (1 - p_i) H'(s_i + b + \frac{a_i}{\pi_i}).$$

\(^{10}\) In the absence of altruism, old agents are in favor of the value of $\tau$ which maximizes the transfer $b$ if they need LTC (or if they do not know yet whether they will be dependent later), and are indifferent as to the value of $\tau$ if not dependent. Their preference over $\tau$ thus does not depend on their $(w_i, p_i, \pi_i, f_i)$ characteristics, but simply on their dependency status. Allowing old people to vote would then complexify the analysis without bringing any novel insight.

\(^{11}\) This assumption is standard in the positive literature on pensions. See for instance Casamatta et al. (2000)

\(^{12}\) The fact that decisions are taken sequentially (first $\tau$ and then $a$ and $s$) does not matter in this section since we look at the most-preferred allocation of individual $i$. 

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is the expected marginal utility in case of LTC of an agent of type \( i \). Observe that the formulation of the FOC for saving holds with equality thanks to the assumption that \( \lim_{c \to 0} u'(c) = \infty \) (i.e., everyone saves a positive amount at his optimum).

The following definition will prove helpful throughout the paper.

**Definition 1** The prospect of family help of agent \( i \) is weak if

\[
(1 - p_i)H'(s) + p_i H'(s + f_i) > u'(s), \text{ for } s > 0,
\]

and is strong otherwise.\(^{13}\)

An agent faces a weak prospect of family help when his expected marginal utility if dependent is larger than his marginal utility if not in the absence of social and private insurance (i.e., when financial resources are the same in both states of the world, except for the family transfer received, with some probability, if dependent). Family help may be weak either because the transfer \( f_i \) is low or because the probability \( p_i \) to receive it is low.

We obtain the following proposition.

**Proposition 1** (i) Agents never most-prefer at the same time positive amounts of social and private insurance (i.e., \( \tau_i^* > 0, a_i^* > 0 \) is impossible), except if \( w_i/\pi_i = \bar{w}/\bar{\pi} \) in which case agent \( i \) is indifferent between the two forms of insurance, provided that they add to her most-preferred total insurance level.

(ii) Individuals with a weak prospect of family help prefer some social but no private insurance (i.e., \( \tau_i^* > 0, a_i^* = 0 \) if \( w_i/\pi_i < \bar{w}/\bar{\pi} \), and some private but no social insurance (i.e., \( \tau_i^* = 0, a_i^* > 0 \) if \( w_i/\pi_i > \bar{w}/\bar{\pi} \).

(iii) No individual with a strong prospect of family help prefers private insurance \( (a_i^* = 0 \text{ for all } i) \). Also, such agents prefer some social insurance (i.e., \( \tau_i^* > 0 \) if \( w_i/\pi_i < (\bar{w}/\bar{\pi})\mathbb{E}H'_i/u'(c_i) < \bar{w}/\bar{\pi} \), and no social insurance at all (i.e., \( \tau_i^* = 0 \)) otherwise.

**Proof:** See Appendix A

The intuition for this proposition runs as follows. Comparing the FOCs for social and private insurance ((3) and (4)), one immediately sees that agents prefer to use social insurance if \( w_i/\pi_i < \bar{w}/\bar{\pi} \), and private insurance otherwise. The intuition for this result is that social insurance redistributes across income and (ex ante) across risk levels while private insurance is actuarially fair and does not

\(^{13}\)We will assume for simplicity that the inequality has the same sign for all values of \( 0 < s < w_i \).
redistribute across income levels. Observe that the threshold $\bar{w}/\bar{\pi}$ is independent (i) of family help ($p_i$ and $f_i$), and (ii) of the correlation between income and risk in society (i.e., it is the ratio of average income to average dependency probability, rather than the average ratio of income-to-dependency probability, that matters). The latter observation comes from the fact that social insurance serves a lump sum transfer to dependent agents, with the transfer increasing in the tax base (as measured by average income) and decreasing in the proportion of recipients (which, by the law of large numbers, equals the average probability of becoming dependent).

Proposition 1 also shows that everyone with a weak prospect of family help most-prefers a positive amount of some form of insurance. Agents with a strong prospect of family help have no need for any actuarially fair and non-redistributive insurance, since their expected marginal utility if dependent is smaller than if autonomous in the absence of insurance transfers. Such agents then do not buy any private insurance ($a_i^* = 0$), but they favor a positive social insurance level if their income-to-dependency probability ratio is small enough, compared to the ratio of average income to average dependency probability, that they benefit a lot from the (risk or income) redistribution embedded in the social insurance program.

We now proceed to the comparative statics analysis of the most-preferred amount of insurance and of saving. We first study the group of agents who most-prefer a positive amount of social insurance.

We introduce the following assumption.

**Assumption 1** The coefficient of relative risk aversion $R(c_i) = -c_i u''(c_i)/u'(c_i)$ is low:

$$u'(c_i) + w_i u''(c_i) > 0$$

$$\iff R(c_i) < \frac{c_i}{w_i} < 1.$$  

Note that this assumption is slightly stronger than the assumption that $R(c_i) < 1$ since $w_i > c_i$. As we will see shortly, this assumption is used only as a sufficient (although not necessary) condition to ensure that the derivative of the most-preferred social insurance contribution rate with respect to income is negative.

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14 In a recent paper studying the LTC insurance market, Karagyozova and P. Siegelman (2012) surveys the empirical literature on relative risk aversion. They report very large ranges for empirically plausible individual values of $R$: from $[0.35, 1]$ for Hansen and Singleton (1983) to $[0.029, 680]$ for Halek and Eisenhauer (2001). Holt and Laury (2002) estimates that two thirds of respondents in their study have a value of $R$ between 0.15 and 0.93. Assumption 1 then seems reasonable.
Proposition 2 Take the individuals with $a_i^* = 0$ while $\tau_i^* > 0$ (and $s_i^* > 0$). We obtain that

(i) a larger income $w_i$ increases $s_i^*$ and decreases $\tau_i^*$; under Assumption 1;

(ii) a larger family help (either $p_i$ or $f_i$) increases $s_i^*$ and decreases $\tau_i^*$;

(iii) a larger dependency probability $\pi_i$ decreases $s_i^*$ and increases $\tau_i^*$.


To understand the intuition behind Proposition 2, observe first that saving and social insurance are (imperfect) substitutes (since they constitute two technologies to move resources from the present into the future), so that increasing one exogenously decreases the most-preferred level of the other, ceteris paribus. Then, each individual characteristic (income, family or dependency probability) impacts a decision ($\tau_i^*$ or $s_i^*$) directly, but also indirectly through its impact on the other decision.

We start with the impact of income, noting that $w_i$ plays a role only in the first period of life, since no transfer is conditioned on income when old. An increase in individual income has two direct effects on $\tau_i^*$: it makes social insurance more expensive (by increasing the tax payment), but it also decreases the marginal utility of first-period consumption (and thus the marginal utility cost of the social insurance transfer). If marginal utility does not decrease too fast (i.e., if Assumption 1 holds), the first impact is larger than the second and the net direct effect of an increase in income is to decrease $\tau_i^*$. The only direct impact of a larger income on the saving decision is the lower marginal utility from first-period consumption, which increases the optimal saving amount $s_i^*$. We then have that the direct and indirect effects of an increase in income reinforce each other: a larger income decreases $\tau_i^*$ directly but also indirectly because it makes saving more attractive. Likewise, a larger income increases saving directly but also indirectly by discouraging social insurance.

Increasing family help (either $p_i$ or $f_i$) decreases the expected marginal utility in case of dependency, which exerts a negative direct effect on both $\tau_i^*$ and $s_i^*$. Indirect effects have the opposite sign, as a lower $\tau$ induces a higher $s$, and a lower $s$ induces a higher $\tau$. We obtain that the direct effect is larger than the indirect one for social insurance, while the opposite occurs for saving. Intuitively, increases in $p_i$ or $f_i$ decrease the desire to insure as measured by $\tau_i^*$. This in turn decreases the marginal utility from first-period consumption, and pushes the individual to save more. In other words, more family help decreases the expected marginal utility when dependent compared to being non dependent, and leads the individual to reallocate his portfolio in favor of saving and against social insurance.

A larger dependency probability $\pi_i$ directly increases $\tau_i^*$ since it raises the probability to receive the social insurance transfer without affecting its tax price.
The direct effect of a larger $\pi_i$ on $s_i^*$ is more difficult to ascertain. It hinges on how the expected marginal utility when old varies when agents put more relative weight on being dependent. We know from the decision not to buy private insurance that expected marginal utility if dependent is lower than first-period marginal utility. Saving then ensures that marginal utility in the first period is a convex combination of marginal utility if dependent and if not. We then obtain that marginal utility if autonomous is larger than if dependent, so that increasing $\pi_i$ actually decreases the expected marginal utility in second period, inducing the agent to save less. Observe that the indirect effects then reinforce the direct effects: a larger probability pushes directly the agent to insure more and save less, the latter reinforcing his incentive to insure more, while the former reinforces his incentive to save less.

We now consider the agents who most-prefer a positive amount of private insurance.

**Proposition 3** Take the individuals with $\tau_i^* = 0$ while $a_i^* > 0$ (and $s_i^* > 0$). We obtain that

(i) a larger income $w_i$ increases $s_i^*$ and has an ambiguous effect on $a_i^*$;
(ii) a larger family help (either $p_i$ or $f_i$) increases $s_i^*$ and decreases $a_i^*$;
(iii) a larger dependency probability $\pi_i$ decreases $s_i^*$ and increases $a_i^*$.

**Proof:** Repeated use of Cramer’s rule - see working paper version available at http://idei.fr/doc/by/de_donder/private_social_june_2014.pdf

Agents with a larger income $w_i$ have a lower marginal utility from first-period consumption, which gives them more incentive to buy insurance and to save: the direct impact of $w_i$ on both $a_i^*$ and $s_i^*$ is positive. The indirect impact then goes into the opposite direction (since more saving induces to buy less insurance, while buying more insurance induces to save less). We obtain that the direct effect is unambiguously larger than the indirect one for saving, so that $s_i^*$ increases with $w_i$. As for insurance, the sign of the aggregate impact of $w_i$ depends on how saving affects the differential of second-period (expected) marginal utility according to dependency status. If more saving increases expected marginal utility when dependent compared to when autonomous, then richer agents buy more insurance. They buy less insurance in the opposite case.

The sign of the impact of family help (as measured by either $f_i$ or $p_i$) on the demand for saving and insurance is the same as in Proposition 2, and the intuition is similar.

By contrast, the channels through which a higher dependency probability $\pi_i$ impacts the most-preferred amount of private insurance and saving differ totally from the case studied in Proposition 2. Observe that, when $a_i^* > 0$ and $s_i^* > 0$,
we have that \( u'(s_i) = u'(c_i) = EH' \), so that putting more relative weight on the dependency state does not affect the expected marginal utility when old. At the same time, increasing \( \pi_i \) decreases the return from private insurance and thus lowers consumption levels (and increases marginal utility) when dependent. This direct impact of increasing \( \pi_i \) then increases the willingness both to insure privately and to save. On the other hand, indirect effects have the opposite sign. We obtain that the direct effect is larger than the indirect one for private insurance, while the opposite holds for saving. Intuitively, a larger dependency probability increases the expected marginal utility when dependent, and leads the individual to reallocate his portfolio in favor of private insurance and against saving.

We now put together the three propositions and summarize how individual characteristics affect separately the preferences for saving as well as for social and private insurance, starting with income. An individual with a very low income most-favors a positive amount of social insurance, because he benefits from the income redistribution, but no private insurance. He also saves a positive amount. As income increases, \( \tau^*_i \) decreases while \( s^*_i \) increases. If this individual has a weak prospect of family help, the decline in \( \tau^*_i \) as \( w_i \) increases continues up to the point where \( w_i/\pi_i = \bar{w}/\bar{\pi} \). At this point, the agent shifts his support from social to private insurance (i.e., \( \tau^*_i = 0 \) while \( a^*_i > 0 \)). From that point on, any increase in \( w_i \) has an ambiguous impact on \( a^*_i \). By continuity, \( s^*_i \) increases with \( w_i \) whether the agent prefers social or private insurance. If the agent rather enjoys a strong prospect of family help, his most-preferred value of \( \tau^*_i \) reaches zero for a value of \( w_i \) that is such that \( w_i/\pi_i < \bar{w}/\bar{\pi} \). From that point on, the individual favors no insurance whatsoever. His preferred amount of saving increases with income in all cases.

We perform the same exercise for the dependency probability. It will prove easier to treat separately the case of weak and strong family help prospects. With weak family help prospects, agents with very low values of \( \pi_i \) prefer some private insurance, with \( a^*_i \) increasing with \( \pi_i \). When \( w_i/\pi_i = \bar{w}/\bar{\pi} \) is reached, they stop buying private insurance and rather switch to a strictly positive amount of social insurance. As \( \pi_i \) further increases, \( \tau^*_i \) increases as well. Saving \( s^*_i \) decreases with \( \pi_i \) whether \( \tau^*_i > 0 \) (see Proposition 2) or \( a^*_i > 0 \) (see Proposition 3). With strong family help prospects, agents never buy private insurance whatever their individual probability \( \pi_i \). They most-prefer no social insurance as well, until their dependency probability is large enough that \( w_i/\pi_i \) is sufficiently small (and

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\(^{15}\)More precisely, an agent with \( w_i/\pi_i = \bar{w}/\bar{\pi} \) is indifferent between using private or social insurance, as long as the insurance transfer he receives when dependent corresponds to his optimal level.

\(^{16}\)This is reminiscent of the well known result (see Mossin (1968)) that risk-averse agents always wish to buy actuarially fair insurance.
definitely smaller than $\tilde{w}/\tilde{\pi}$) that they gain enough from the ex ante redistribution across risk levels to favor $\tau^*_i > 0$. From that point on, $\tau^*_i$ increases with $\pi_i$. Also, $s^*_i$ decreases with $\pi_i$ throughout with strong family help.

We now turn to the impact of family help, as measured by $p_i$ (we obtain similar results when varying $f_i$). Figure 1A illustrates the results when $w_i/\pi_i < \tilde{w}/\tilde{\pi}$ while Figure 1B assumes that $w_i/\pi_i > \tilde{w}/\tilde{\pi}$. A very low value of $p_i$ means that the prospects of family help are weak for the individual. He then favors $\tau^*_i > 0$ if $w_i/\pi_i < \tilde{w}/\tilde{\pi}$ or $a^*_i > 0$ if $w_i/\pi_i > \tilde{w}/\tilde{\pi}$. Increasing $p_i$ then decreases $\tau^*_i$ (Fig. 1A) or $a^*_i$ (Fig 1B), and increases $s^*_i$. If $w_i/\pi_i > \tilde{w}/\tilde{\pi}$, then $a^*_i$ becomes nil when $p_i$ is large enough that prospects of family help turn from weak to strong. If $w_i/\pi_i < \tilde{w}/\tilde{\pi}$, $\tau^*_i$ remains positive even for strong prospects of family help (thanks to income and risk redistribution), but decreases with $p_i$ until it becomes nil.\(^\text{17}\)

When $p_i$ is large enough that $a^*_i = \tau^*_i = 0$ (on either Fig. 1A or 1B), $s^*_i$ decreases with $p_i$ because a higher probability of family help decreases the expected marginal utility in the second period, and thus the expected benefit from saving. We then obtain that saving is not monotone in family help when agents endogenously switch from some (social or private) insurance to no insurance at all at their most-preferred allocation.

To summarize, we obtain that having weak prospects of family help is a necessary condition to most-prefer a positive amount of private insurance and that family support (as measured by either $f_i$ or $p_i$) decreases the support for both types of insurance.\(^\text{18}\) Such strong prospects for a large part of the polity may then explain the “puzzle” of the generalized lack of private insurance in OECD countries. Social insurance is less affected by family help thanks to the redistribution (across income and risk levels) that it entails. On a more prospective note, the decrease in family help that is widely expected to happen should give a boost to social and especially to private insurance, according to our model.

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\(^\text{17}\) As announced in the introduction, these results provide an analytical underpinning to those obtained by Costa-Font (2010): compare our Figures 1A and 1B with Costa-Font (2010)’s Figure 6.

\(^\text{18}\) When $p_i$ and $f_i$ are low enough that prospects of family help are weak, they crowd out the demand for social insurance by low $w_i/\pi_i$ types and for private insurance by large $w_i/\pi_i$ types. The crowding out is exclusively at the intensive margin, since the threshold value of $w_i/\pi_i$ (equal to $\tilde{w}/\tilde{\pi}$) which determines whether agents prefer social or private insurance is not affected by $f_i$ or $p_i$. When $p_i$ and $f_i$ are large enough that prospects of family help are strong, demand for private insurance disappears. From that point on, any increase in family help crowds out the demand for social insurance both at the intensive and at the extensive margins.
Before turning to the majority-chosen amount of social insurance, we should stress two limitations of the comparative static analysis we have performed. First, we have looked at variations of individual characteristics that affect a set of measure zero of individuals. This is of no import for family help, where the distribution of \( p_i \)'s and of \( f_i \)'s plays no role for individual preferences, so that the comparative static results we have obtained can be generalized when the characteristics of a set of positive measure of agents are changed. Unfortunately, we cannot proceed in the same way for variations of \( w_i \) and \( \pi_i \) for a group of agents when such variations affect \( \bar{w} \) and \( \bar{\pi} \), because of the role that average income and dependency probability play in the government’s budget balance equation (2). While we have shown that the individual impact of increasing \( w_i \) on \( \pi_i^* \) is negative, any increase in the income of a group of agents that raises the average income \( \bar{w} \) would add another effect in the opposite direction since a larger tax base would increase the return of the social insurance scheme. Likewise, the individual impact of increasing \( \pi_i \) on \( \pi_i^* \) is positive, while an increase in \( \pi_i \) for several agents that would raise \( \bar{\pi} \) adds a countervailing force on \( \pi_i^* \) by decreasing the return of the social insurance scheme.

The second limitation of Propositions 2 and 3 is that we assume that individual characteristics are modified one at a time (i.e., independently from one another). In reality, these individual characteristics are correlated. Observe that, if richer people tend to live longer and hence to have a larger probability of needing LTC (i.e., \( \text{cov}(w, \pi) > 0 \)), then the net impact of a higher \( w_i \) coupled with a higher \( \pi_i \) on \( s_i^* \) and \( \tau_i^* \) is ambiguous. Whether one impact is larger than the other one is essentially an empirical matter of both the intensity of the correlation and the amount of variance in the two characteristics. For instance, if (as we surmise), the variance in income levels is larger than the variance in the dependency probabilities (or if the covariance between both is low), then, under Assumption 1, richer people will favor a lower social insurance contribution rate (even though they may face a larger dependency probability than poorer people).

Income may also be correlated with the probability of receiving family help, but the sign of the correlation is far from clear, depending both on the type of data used (macro vs micro) and the type of help (formal vs informal). Using macro data in Europe, one observes a negative correlation between income and family support, with richer Northern European countries providing less informal family help, on average, than poorer Southern countries (the so-called “North-South gradient”, see SHARE (2005)). Focusing on micro data, Bonsang (2009) finds a positive correlation between income and financial help from the family. With a positive correlation, we obtain unambiguously that richer people prefer less social insurance, while the relationship between income and most-preferred social insurance can go both ways with a negative correlation between income and family support.

We now move to the majority-chosen level of social insurance.
4 The majority-chosen level of social insurance

Young agents vote first over $\tau$ and then choose $a$ and $s$. Differentiating (3) with respect to $\tau$ and using the envelope theorem for the choices of $a$ and $s$, it is straightforward to see that preferences over $\tau$ are concave (and thus single-peaked). We then apply the median voter theorem and obtain that there exists a value of $\tau$ that is preferred by a majority of voters to any other value of $\tau$. We denote this majority-chosen level of $\tau$ by $\tau^V$. It corresponds to the value of $\tau$ that is such that at least half the polity exhibits $\tau_i^* \geq \tau^V$ while $\tau_i^* \leq \tau^V$ for at least half as well. Since individuals differ in four dimensions, it is not possible at this level of generality to define the characteristics of the decisive voters. But this will not prevent us from obtaining several interesting results.

First, observe that the set of agents who favor $\tau_i^* > 0$ is made of all agents $i$ with $\bar{w}_i/\bar{\pi}_i < \bar{w}/\bar{\pi}$ and a weak prospect of family help together with all agents $i$ with a strong prospect of family help and a value of $\bar{w}_i/\bar{\pi}_i$ lower than some threshold that is itself strictly lower than $\bar{w}/\bar{\pi}$. If the mass of those two types of agents is at least equal to one half, then $\tau^V$ is strictly positive.

Second, from the discussion at the end of the previous section, we obtain that $\tau^V$ weakly decreases with family help (either $p_i$ or $f_i$). Unfortunately, as explained above, we can not draw similar inferences for variations in income and in dependency probability. Also, it is impossible at this level of generality to assess the impact of modifying the correlation between, say, income and risk, or income and family help, on $\tau^V$.

In the introduction, we mention that the literature has assessed how social insurance may decrease the demand for private insurance. Since several countries are currently considering facilitating access to private LTC insurance, we now study how the availability of private insurance impacts the support for social insurance. It is easy to see from the FOCs (3) and (4) that the two forms of insurance are imperfect substitutes. The intuition may then suggest that introducing the possibility to buy private insurance in a society where such insurance did not exist previously would always decrease the support for social insurance and result in a lower value of $\tau^V$. The next proposition shows that this need not be the case.

**Proposition 4** A sufficient (although not necessary) condition for the introduction of the possibility to buy private insurance not to affect the majority-chosen value of the social insurance contribution rate, $\tau^V > 0$, is that the proportion of individuals who face weak family prospects and are such that $\bar{w}_i/\bar{\pi}_i < \bar{w}/\bar{\pi}$ is larger than one half.

The intuition for this proposition is that agents prefer either social or private insurance, but never both, so that introducing private insurance either does not
change an individual’s most-preferred value of \( \tau \), or drives it to zero. The two categories of agents who prefer the same positive value of \( \tau \) whether private insurance is available or not are (i) those with weak prospects of family help and a ratio of income-to-probability lower than \( \bar{w}/\bar{\pi} \) (hence the sufficient condition above) and (ii) those with strong prospects and very low ratios of income to probability (so that the above condition is not necessary). The only agents who switch from social to private insurance are those with weak prospects of family help, and an income-to-probability ratio larger than \( \bar{w}/\bar{\pi} \) but not too large that they prefer to self-insure rather than buying any form of insurance.

To better understand how likely it is that private insurance crowds out social insurance, we now look at specific cases where individuals differ only in one or at most two dimensions. Observe first that, as long as agents do not differ in \( f_i \) or \( p_i \), they all face either weak or strong prospects of family help. We then focus on the more empirically relevant case where they all face weak prospects. If agents differ only in income \( w_i \) (and share the same \( \pi, p \) and \( f \)), then the sufficient condition in Proposition 4 is always satisfied, since all income distributions are such that \( w_{med} < \bar{w} \). Likewise, if agents differ only in the probability \( \pi_i \), the condition is satisfied provided that \( \pi_{med} > \bar{\pi} \). If agents differ in both income \( w_i \) and probability \( \pi_i \), then the sufficient condition remains empirically valid if the variance of the income distribution is, as we surmise, much larger than the variance of the probability distribution. If agents differ only in family help characteristics (\( p_i \) and \( f_i \)), then they are all indifferent between social and private insurance, as long as they are not forced to consume more (social) insurance than their most-preferred level. Private insurance would then totally crowd out social insurance, with everyone preferring to buy his most-preferred (private) insurance amount.

The role played by family help characteristics, and especially by whether prospects are weak or strong, is exemplified in Appendix B, where we study the case where agents differ in income \( w_i \) and belong to one of two groups, with one facing weak prospects (a low value of \( f \) and of \( p \)) and the other strong prospects (a

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19We have not been able to find empirical evidence regarding the distribution of the probability to become dependent.

20If agents only differ in either \( w_i \) or \( \pi_i \), then the only way that \( \tau^V \) can be affected by the availability of private insurance is by being driven down to zero. The reason is that the agents who switch to a most-preferred value of \( \tau \) of zero when private insurance is available are those who prefer a low value of \( \tau \) when private insurance is not available. This is no longer the case when agents differ in two non-exclusively family related dimensions (such as \( w_i \) and \( \pi_i \), \( w_i \) and \( p_i \) or \( w_i \) and \( f_i \), etc.) since there is no link between whether an agent most-prefers social or private insurance (driven entirely by whether \( w_i/\pi_i \lesssim \bar{w}/\bar{\pi} \)) and his most-preferred value of \( \tau \) (depending on \( w_i, \pi_i, p_i \) and \( f_i \)). In this latter case, the introduction of private insurance may decrease \( \tau^V \) without bringing it to zero.
large value of $f$ and of $p$).\footnote{A similar reasoning applies when agents also differ in $\pi_i$.} We obtain that the proportion of agents facing strong prospects of family help need to be neither too large nor too low for the introduction of private insurance to affect the majority-chosen level of social insurance. More precisely, this proportion has to be large enough that a large minority of the population most-prefers no social insurance even when private insurance is not available, but low enough for the weak prospect agents with larger-than-average income to be numerous enough to join this large minority, in order to form a majority of voters against social insurance when private insurance is available.

We now look at whether private insurance is supported by a majority of voters. Assume that agents vote first over whether to allow for private insurance, before voting over $\tau$ and finally choosing individually $a$ and $s$.

**Proposition 5** A majority always vote in favor of allowing private insurance.

**Proof:** See Appendix C

We show in the proof of Proposition 5 that a majority of voters favor the introduction of private insurance, whether such an introduction affects the majority chosen amount of social insurance or not. As we show in the proof, some voters may indeed vote in favor of the introduction of private insurance, even if they do not plan on using private insurance at all, in order to decrease (or even drive to zero) the majority-chosen amount of social insurance.

## 5 Conclusion

This paper has studied the determinants of the demand for private, social and self-insurance for LTC in an environment where individuals differ in earnings, family support and dependency risk. We can use the results of our analysis to try and shed light on the future development of the three types of insurance for LTC. The two main changes expected to affect LTC in the near future are (i) the doubling in the number of dependent individuals in the next twenty years within the OECD, associated with the rapid increase of very old (75+) people in the population, and (ii) the decline in family solidarity due to increased participation of women in the labor market, increased mobility and changing family values. The first effect can be modelled in our setting as an increase in the probability of becoming dependent of all agents. This higher probability will undoubtedly increase the needs when old, but we obtain that it does not necessarily imply an increase in the demand for social insurance, because a larger average probability of becoming dependent decreases the return of the social LTC insurance. Observe that the return of
the (actuarially fair) private insurance decreases with the individual dependency probability, while self-insurance return is not affected. The impact of a larger average dependency probability on the demand for social insurance thus depends on the probability distribution across people, and especially on its correlation with income. The impact of a diminishing family support is easier to ascertain: as we show, it unambiguously increases the demand for social insurance among agents with a low income-to-dependency probability ratio. As for individuals with a high ratio, a decrease in family help will first increase their self-insurance level, and then increase their demand for private insurance.

Our paper introduces an admittedly crude modelling of family help, in that the amount of help is dictated by a social norm, with no distinction between formal and informal help. The next step in our research agenda is to lift those two constraints in order to better understand the demand for social and private LTC insurance as a function, for instance, of the substitutability/complementarity between formal and informal help (see Van Houtven and Norton, 2008).
6 Appendix A

Proof of Proposition 1

(i) Observe from the FOC for \( a_i \) that \( a_i^* > 0 \) implies that \( EH_i' = u'(c_i) \). This in turn implies that \( FOC \tau_i > 0 \) if \( w_i/\pi_i < \bar{w}/\bar{\pi} \), an impossibility, and that \( FOC \tau_i < 0 \) if \( w_i/\pi_i > \bar{w}/\bar{\pi} \), so that \( \tau_i^* = 0 \). In the latter case, the FOC for saving implies that \( EH_i' = u'(s_i) = u'(c_i) \), which is compatible with the starting assumption that \( a_i^* > 0 \).

Similarly, observe from the FOC for \( \tau_i \) that \( \tau_i^* > 0 \) implies that \( EH_i' = \frac{u}{\pi_i} \frac{\bar{w}}{\bar{\pi}} u'(c_i) \). If \( w_i/\pi_i > \bar{w}/\bar{\pi} \), we then obtain that \( EH_i' > u'(c_i) \) and so that \( FOC a_i > 0 \), an impossibility. On the other hand, if \( w_i/\pi_i < \bar{w}/\bar{\pi} \), we have that \( EH_i' < u'(c_i) \) and that \( FOC a_i < 0 \), implying that \( a_i^* = 0 \). Finally, it is obvious that \( w_i/\pi_i = \bar{w}/\bar{\pi} \) is indifferent between \( a \) and \( \tau \), provided that \( EH_i' = u'(c_i) \)—i.e., that they obtain their most-preferred total insurance amount.

(ii) We first show that people buy either private or social insurance with weak family help—i.e., that \( a_i^* = \tau_i^* = 0 \) is impossible. In that case, with \( a_i^* = \tau_i^* = 0 \), by the FOC for saving, we would have \( EH_i' > u'(c_i) > u'(s_i) \), which in turn would imply that \( FOC a_i > 0 \), a contradiction with \( a_i^* = 0 \). The proof of part (i) has then shown that we have \( a_i^* > 0 \) and \( \tau_i^* = 0 \) when \( w_i/\pi_i > \bar{w}/\bar{\pi} \), and \( a_i^* = 0 \) and \( \tau_i^* > 0 \) when \( w_i/\pi_i < \bar{w}/\bar{\pi} \).

(iii) With strong family help, when \( a_i^* = \tau_i^* = 0 \), by the FOC for saving, we have \( EH_i' \leq u'(c_i) \leq u'(s_i) \), which in turn implies that \( FOC a_i \leq 0 \), consistent with \( a_i^* = 0 \). We then have that \( FOC \tau_i \leq 0 \) for \( \tau_i = 0 \) provided that \( w_i/\pi_i \geq x = (\bar{w}/\bar{\pi})EH_i'/u'(c_i) \), with \( x \leq \bar{w}/\bar{\pi} \) since \( EH_i' \leq u'(c_i) \). If \( w_i/\pi_i < x \), then we obtain that \( FOC \tau_i > 0 \) at \( \tau = 0 \), which is inconsistent with \( \tau_i^* = 0 \). Observe that \( EH_i' \) decreases with \( \tau_i \) while \( u'(c_i) \) increases with \( \tau_i \). Take then the value of \( \tau_i^* > 0 \) such that the FOC for \( \tau \) equals zero. Observe that \( EH_i' \leq u'(s_i) \) holds a fortiori when \( \tau > 0 \) while prospects of family help are strong. Hence, from the FOC for saving, we still have that \( EH_i' \leq u'(c_i) \leq u'(s_i) \) and thus that the FOC for \( a_i \) is negative: we have just shown that \( (\tau_i^* > 0, a_i^* = 0) \) is consistent with the three FOCs.
7 Appendix B

Assume that there are two groups of agents: those who have the same weak prospects of family help \((p_i = p_{\text{low}}\) and \(f_i = f_{\text{low}}\)) and those who have the same strong prospects \((p_i = p_{\text{high}}\) and \(f_i = f_{\text{high}}\)). Agents in both groups also differ in income \(w_i\), with the same distribution of income represented by the cdf \(F(w)\) in both groups. The fraction of agents with weak prospects in the population is denoted by \(\lambda\).

Figure 2 represents the most-preferred value of \(\tau\), as a function of income, for both groups, when private insurance is not available. We know from Propositions 2 and 3 that \(\tau^*\) is decreasing in \(w_i\), \(p_i\) and \(f_i\), so that agents with weak prospects of family help prefer a larger value of \(\tau\) than agents of the same income but with strong prospects. We denote by \(w_s\) (resp., \(w_w\)) the lowest income level of agents with strong (resp., weak) prospects of family help who most-prefer no social insurance. We know that \(w_s < \bar{w} < w_w\). We assume that \(p_{\text{high}}\) and \(f_{\text{high}}\) are large enough that \(w_s < w_{\text{med}}\).

Introducing private insurance means that agents with a weak prospect of family help and a larger-than-average income switch from social to private insurance. We concentrate on the conditions over \(\lambda\) under which \(V\) decreases (i.e., is driven to zero, as explained in footnote 20) when private insurance is introduced. The condition under which \(V > 0\) when private insurance is not available is

\[ \lambda F(w_w) + (1 - \lambda) F(w_s) > 1/2. \]

This condition is satisfied provided that \(\lambda\) is large enough. The condition under which \(V = 0\) when private insurance is available is

\[ \lambda F(\bar{w}) + (1 - \lambda) F(w_s) < 1/2. \]

This condition is satisfied provided that \(\lambda\) is low enough. Putting the two conditions together, we obtain that private insurance crowds out social insurance provided that the fraction of agents with strong prospects of family help is neither too high (so that \(V = 0\) without private insurance) nor too low (so that \(V\) remains positive and unchanged after private insurance is introduced).

\footnote{Otherwise, a majority always prefers a positive value of \(\tau\), whatever \(\lambda\), whether private insurance is available or not.}
8 Appendix C

Proof of Proposition 5

Three cases can occur.

(i) Allowing for private insurance does not change the majority chosen value of \( \tau \). In that case, allowing for private insurance increases the utility of the agents who buy some (i.e., top up) at equilibrium, and leaves the utility of others unchanged, so that nobody opposes the introduction of private insurance.

(ii) Allowing for private insurance totally crowds out social insurance (i.e., decreases \( \tau^V \) to zero). In that case, allowing private insurance results in a majority of agents enjoying their overall most-preferred option (no social insurance coupled with their individually most-preferred amount of private insurance). This majority then supports the introduction of private insurance.

(iii) Allowing for private insurance partially crowds out social insurance (i.e., decreases the value of \( \tau^V \) without driving it to zero). We add the subscript \( a \) (resp., \( \emptyset \)) to \( \tau^V \) and to \( \tau^* \) to describe the majority-chosen and individually most-preferred values of \( \tau \) when private insurance is (resp., is not) available. We show that all agents with \( \tau^*_a < \tau^V_a < \tau^\emptyset_V \) are better-off when private insurance is allowed (since \( \tau^V_a \) is the majority-winning value of \( \tau \) in that case, these agents then form a majority). If \( \tau^*_a = \tau^\emptyset_V > 0 \), then the agent prefers social to private insurance anyway, and thanks to the concavity of his utility function he prefers the value of \( \tau \) closest to \( \tau^*_a \) -- i.e., he prefers \( \tau^V_a \) to \( \tau^\emptyset_V \) -- and favors allowing private insurance as a way to decrease the equilibrium value of \( \tau \). If \( \tau^*_a = \tau^\emptyset_V = 0 \), then the agent wishes to buy neither social nor private insurance, and is better off with the lower value of \( \tau^V \) -- i.e., when private insurance is allowed. Finally, if \( \tau^a_V = 0 < \tau^\emptyset_V \), then allowing for private insurance has two beneficial impacts on this individual’s utility. First, keeping \( \tau = \tau^\emptyset_V \) and allowing for private insurance can only increase this agents’ utility (in the case he tops up social with private insurance). Second, once private insurance is allowed, this individual’s utility is decreasing in \( \tau \) (since \( \tau^a_V = 0 \) and since his utility is concave in \( \tau \)) so that he benefits from the reduction from \( \tau^\emptyset_V \) to \( \tau^a_V \), and votes in favor of allowing private insurance.

\[ \blacksquare \]
References


Figure 1 A: Social and self-insurance transfers as a function of $f_i$ or $p_i$ with $\frac{w_i}{\pi_i} < \frac{w}{\bar{\pi}}$.

Figure 1 B: Private and self-insurance transfers as a function of $f_i$ or $p_i$ with $\frac{w_i}{\pi_i} > \frac{w}{\bar{\pi}}$. 
Figure 2: Majority-chosen value of $\tau_i^*$ when agents differ only in income and belong to two groups differing in family help prospects.