8.9 Online Appendix for "Making auctions for transport capacity to roll": Simulations

8.9.1 A model for railway slots allocation

To adjust the model to study quality improvement rather than cost reduction, we make the following assumptions. We assume a continuum of potential passengers, predominantly commuters or frequent riders, on the particular route. Passengers decide on their preferred train operator in advance, based on the train frequencies and fares of each operator. When a passenger chooses an operator for a period, such as a monthly or annual pass, she does not know her arrival time at the station and must consider the expected "schedule delay cost." This is the cost associated with the need to wait for a later train. Other things being equal, riders won't switch operators if there is a more convenient departure time.[32]

With a slight misuse of terminology, and for the sake of brevity, these costs are also referred to as expected "waiting costs". These waiting costs are represented by a decreasing function $H(K_j)$ of the number of slots allocated of firm j, K_j. These costs are equal to the value of time multiplied by the expected schedule delay, which we assume to be proportional to the inverse of frequency.[33]

Under these assumptions, the total cost a passenger incurs when choosing to purchase from operator j is $f_j + H(K_j)$ where f_j is the fare charged by the operator and K_j is the number of slots of firm j. This cost is also commonly referred to in the transport literature as the "generalized price." The net benefit of consumer i from purchasing from firm j is:

$$U_{ij} = u_i - f_j - H(K_j) \tag{12}$$

where u_i is the consumer's valuation for a train trip at her ideal departure time, and is identical for both firms. Consumers have heterogeneous valuations. We assume that consumer valuations are uniformly distributed on the interval $[a - b, a]$, for some real numbers a, b, with $a \geq b \geq 0$. The utility associated to not traveling is normalized to zero. Equilibrium price, p, is determined by

$$p = f_j + H(K_j) = a - b(q_I + q_E), \tag{13}$$

where q_i is the quantity of seats offered by operator i. So, the fare, f_j, firm j receives is

$$f_j = a - b(q_I + q_E) - H(K_j) \tag{14}$$

Equations (14) for $j = I, E$ state that active consumers are indifferent between buying from firm I and firm E, and prefer buying than not buying. Indeed, the net benefit of consumer i from purchasing from firm j is given by (12). Conditions (14) for $j = I, E$ state that the "generalized price" facing consumer buying from firm j , $f_j + H(K_j)$, should be identical for both firms. If both firms have the same generalized price p, consumer i is better-off buying (from either firm) than not buying iff $u_i > p$. The right-hand side of (13) comes from the assumption of a uniform distribution of u_i.

Lastly, we assume that the two operators have identical and constant marginal cost of γ for serving a passenger.[34] Firm j's profit is

$$\pi_j = (f_j - \gamma)q_j$$

Using (13), given K_I and K_E each firm j will want to choose q_j so as to maximize

$$\pi_j = \Big(p - H(K_j) - \gamma)\Big)q_j$$

This model is then equivalent to that developed in Section 3: once we set $C(K) = H(K) + \gamma$, the previous relation becomes equivalent to equation (1).

8.9.2 Calibration of the model

Consumers are characterized by their maximal departure time, i.e., the latest time they would be ready to leave. We assume that arrival times are uniformly distributed on the rush hour period.[35] This implies that each train operator will put a train at the end of the period and sets equal spacing between its departure slots to minimize expected waiting cost, which is then given by the formula $L/2K$.[36] We assume the value of business travelers' time is $w = 46.8$€ per hour; this value is based on a public French report on cost-benefit analysis (Quinet et al., 2014). This results in an expected waiting cost of $H(K) = wL/2K = \frac{140.4}{K}$.

SCNF reports that it incurs an incremental cost of 39 euros per rider, not including tolls it must pay to the infrastructure manager.

The annual peak hour ridership on the Paris-Lyon route is approximately 3.94 million, the average fare is 90€. As previously noted, we assume that the inverse demand function is linear. We use the price elasticity estimate of business passenger demand from Wardman (2014). This implies that the price intercept of the demand curve of is $a = 276$, and the slope is $b = 43.23$. The calibrated parameter a can be viewed as the maximal willingness to pay of the peak-hour riders on the Paris-Lyon route.

Figure 5 shows total profits and new entrant's profit depending on the number of slots allocated to the new entrant (horizontal axis). We see from this figure that the new entrant is not active when it is allocated only one slot: riders waiting costs would be too high. However, when the new entrant is allocated 2 or more slots, the entrant is active. Conditional on the entrant being active in the market, the sum of the two firms' profits is maximized when the entrant gets all the 8 additional slots. In other words, our Maximal Profit condition holds.

Equations (3) and (9) which determine the thresholds characterizing the optimal allocations in Lemmas 2 and 3 imply values for $\tilde{a}^{CS}$ of 80 and for $\tilde{a}^{W}$ of 149. Both are strictly lower than the demand intercept $a = 276$. This implies that the optimal allocation is to give all new slots to the entrant whether the objective is to maximize total welfare or consumer surplus.

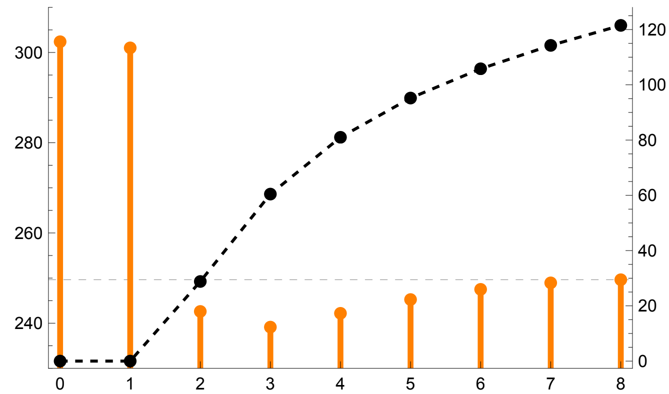

Figure 5: Total profits (orange vertical bars, left axis) and new entrant's profit (dotted curve, right axis) depending on the number of slots allocated to the new entrant (horizontal axis). The incumbent begins with 9 slots and 8 additional slots are available to be allocated between the entrant and the incumbent.

8.9.3 Additional simulation results

Consider the five scenarios described in section 6.3.

Figure 6 illustrates the main results, using the same outcome trees as in Figures 1 and 2. With use the same notation as in Table 2 to describe the scenarios. For example, $1 + \{4, 3\}$ represents two sequential auctions for packages of size 4 and 3, with one slot set-aside for the entrant. At each node the left branch denotes the incumbent winning the auction, the right branch denotes the entrant winning the auction and a pair of numbers represents the net profit for the incumbent and the new entrant, respectively, net of what will be paid in the subsequent auctions.[37] A solid line indicates the actual winner of the auction.

For example, consider Scenario (b) after the incumbent wins the first auction. If it loses the second auction, its operating profits will be 161 and the entrant will have an operating profit of 81. In contrast, the incumbent will have an operating profit of 302 if it also wins the second auction (and the entrant gets a zero payoff). Thus, the incumbent's willingness to pay for the second package is $302 - 161 = 141$ and the entrant would be willing to pay 81 for the second package. The incumbent would then win the second auction, pay 81 and get a net profit equal to $302 - 81 = 221$. Graph (b) of Figure 6 shows that in a two-stage auction when each package has 4 slots, the entrant would not win any slots. Graph (c) shows that entry is possible if the slots are sold in a sequence of three auctions, with 2 slots in each of the first two auctions and 4 slots in the third auction.

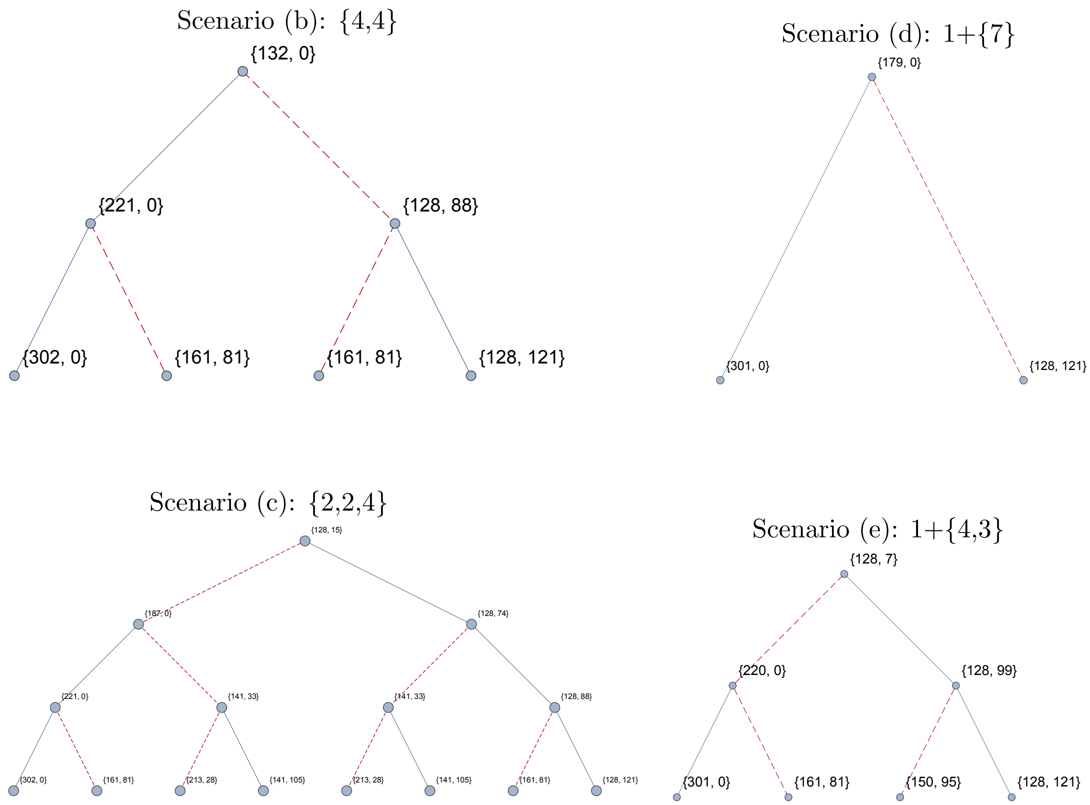

Figure 6: Decision tree for sequential auctions for 8 slots with and without set-aside. Incumbent has already 9 slots. A solid line indicates the actual winner of the auction. See main text for more explanation.

This example shows that dividing the auction into smaller tranches can allow entry. This is not the case for all sequences of three auctions though. Figure 7 presents the results for all sequences of three auctions. It shows that only 5 of them allow entry. The revenue that the infrastructure operator gets from the auction varies greatly, from 89 to 166. Remember that with a one-shot auction for all the slots, the auction revenue is 122 (see Table 2).

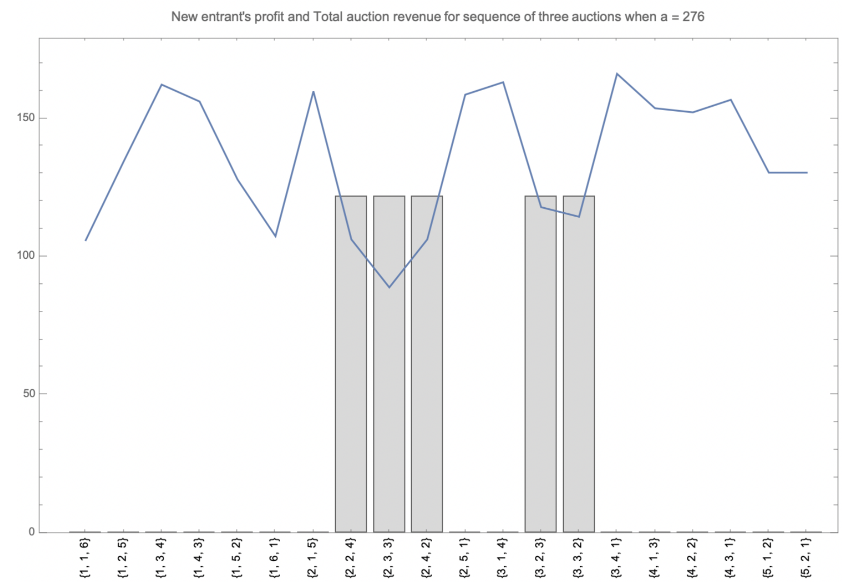

Figure 7: The sequence of three auctions are ordered in the lexicographic order. Of the 20 possible auction sequences, only 5 allow entry, and the new entrant then wins all the auctions. Its profit on the downstream market is then equal to 121. The auction revenue (which goes to the infrastructure operator) varies greatly.

Lastly, to study more extensively the impact on the auction format on entry, Table 3 shows the minimum value of a that guarantees that the new entrant wins at least one of these auctions, for all auction format involving two sequential auctions (left column), three sequential auctions (second column), and four sequential actions (last two columns).[38]

When considering two-stage auctions, this coincides with the threshold $\tilde{a}^{seq}$ defined in the core text (and the new entrant then wins both auctions). In that case, we see that the split that maximizes the likelihood of entry (the one with the smallest $\tilde{a}^{seq}$), is an equal division of the slots ($\Delta_1 = \Delta_2 = 4$), in which case $\tilde{a}^{seq} = 287$, as was discussed in section 6.3.

Consider now auctions with three or four stages. The sequence with the smallest threshold for a is $\{2, 3, 3\}$ (the threshold in that case is 246), although packages with fewer slots in each package are possible. For example, if the sequence is $\{2, 3, 2, 1\}$, the critical threshold is 262.

Additionally, we see that the ordering of the sequence of auctions matters, and simple reverse ordering of the auction of the packages can affect the outcome; for instance, the sequence $\{2,3,3\}$ has a lower threshold than the sequence $\{3,3,2\}$, which has a lower threshold than the sequence $\{3,2,3\}$.

{1, 7}	547.591
{2, 6}	346.164
{3, 5}	294.406
{4, 4}	287.475
{5, 3}	312.755
{6, 2}	390.651
{7, 1}	657.606

{1, 1, 6}	380.107
{1, 2, 5}	315.723
{1, 3, 4}	290.274
{1, 4, 3}	298.909
{1, 5, 2}	328.654
{1, 6, 1}	398.753
{2, 1, 5}	309.348
{2, 2, 4}	261.103
{2, 3, 3}	246.109
{2, 4, 2}	261.103
{2, 5, 1}	304.072
{3, 1, 4}	303.119
{3, 2, 3}	272.195
{3, 3, 2}	267.502
{3, 4, 1}	295.068
{4, 1, 3}	327.816
{4, 2, 2}	301.688
{4, 3, 1}	317.557
{5, 1, 2}	383.467
{5, 2, 1}	373.761
{6, 1, 1}	509.893

{1, 1, 1, 5}	319.392
{1, 1, 2, 4}	270.544
{1, 1, 3, 3}	256.273
{1, 1, 4, 2}	274.646
{1, 1, 5, 1}	325.319
{1, 2, 1, 4}	270.544
{1, 2, 2, 3}	249.273
{1, 2, 3, 2}	249.273
{1, 2, 4, 1}	272.522
{1, 3, 1, 3}	276.249
{1, 3, 2, 2}	266.487
{1, 3, 3, 1}	273.542
{1, 4, 1, 2}	304.573
{1, 4, 2, 1}	304.479
{1, 5, 1, 1}	348.352
{2, 1, 1, 4}	318.162
{2, 1, 2, 3}	278.815
{2, 1, 3, 2}	275.205

{2, 1, 3, 2}	275.205
{2, 1, 4, 1}	310.156
{2, 2, 1, 3}	251.533
{2, 2, 2, 2}	254.626
{2, 2, 3, 1}	253.711
{2, 3, 1, 2}	261.702
{2, 3, 2, 1}	261.702
{2, 4, 1, 1}	298.144
{3, 1, 1, 3}	332.833
{3, 1, 2, 2}	308.978
{3, 1, 3, 1}	329.737
{3, 2, 1, 2}	278.488
{3, 2, 2, 1}	281.653
{3, 3, 1, 1}	310.156
{4, 1, 1, 2}	359.963
{4, 1, 2, 1}	359.963
{4, 2, 1, 1}	343.783
{5, 1, 1, 1}	426.055

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Table 3: Minimum level required for parameter a (passenger's highest willingness to pay) for the new entrant to win (in the considered cases, either the incumbent or the new entrant wins all the auction). Sequences of auction are listed in lexicographic order and grouped in tables for sequence of two, three and four auctions. See main text for details about the parameters.