Does Fiscal Policy Matter?

Tax, Transfer and Spend in a Macro ABM

with Capital and Credit

T. Assenza[a,b]   P. Colzani [a,c]   D. Delli Gatti[a,d]   J. Grazzini[a]

December 23, 2017

[a] *Complexity Lab in Economics,*

Department of Economics and Finance, Catholic University, Milan,

[b] *CeNDEF, University of Amsterdam,*

[c] *University of Lausanne*

[d] *CESifo Group Munich*

Abstract

We investigate, compare and contrast the emerging properties of a macroeconomic agent based model along the lines of Assenza, Delli Gatti, and Grazzini (2015) when the Government experiments with different policy configurations: (1) Tax and Transfer (TT), (2) Tax, Transfer and Spend (TTS), (3) the implementation of a fiscal rule, such as a stylized Stability and Growth Pact (SGP). In some of the scenarios considered, a remarkable property can be detected, which we label the Balanced Budget Emergent Property (BBEP): the scale of activity in the aggregate (GDP, employment, unemployment rate) is such that a balanced budget emerges spontaneously. The strong implication of this property is that the fiscal authority is able to target GDP and the unemployment rate, a result reminiscent of the Blinder-Solow framework. It is worthnoting, however, that there are many

departures from the rule, which we have detected by carrying out the sensitivity analysis.

Keywords: agent based macroeconomics, fiscal policy

JEL codes:

1 Introduction

"Does Fiscal Policy Matter?" Blinder and Solow's (BS) answer to this fundamental question 40 years ago was a resounding "Yes!" (Blinder and Solow (1973)) This conclusion was based on the macroeconomic properties of an aggregative IS-LM framework enriched by a *wealth effect* due to the accumulation of Government bonds. In a nutshell, in their model the accumulation of public debt due to expansionary fiscal policy reverbated positively on the aggregate scale of activity because of the households' positive reaction – in terms of additional consumption – to the accumulation of private wealth in the form of Government bonds.

Is this wealth effect always at work? In other words "Are Government Bonds Net wealth?". Barro's answer to this question – published almost simultaneously to BS – was nuanced but has been generally interpreted as a clear "No" (Barro (1974)). Inasmuch as households take into account the future tax liabilities that the current expansionary fiscal policy engender, the accumulation of Government bonds will not produce a positive wealth effect on consumption. Ultra-rational *Ricardian* households – i.e. forward looking households who take into account the change in future fiscal stance due to the current policy – would not behave as Blinder and Solow predicted. In the most extreme scenario, a fiscal expansion in t – e.g. an increase in Government expenditure – will imply a reduction of consumption in the same period as households increase savings to face the increase in taxes required to balance the budget in the future: Government expenditure and household consumption are negatively correlated so that a fiscal expansion has no real effects.

The line of reasoning inaugurated by Barro has dominated research on the real effects of fiscal policy thereafter. This is true both for the New Classical and the New Keynesian macroeconomic frameworks. Only when *Limited Asset Market Participation* is taken into account, i.e. when a fraction of the population ("rule of thumb" consumers) is assumed to have no access to financial markets and therefore to be unabe to smooth consumption, fiscal policy "matters" in a New

Keynesian framework (Gali' J. and Valles (2004, 2007)).

In this paper we explore the effects of fiscal policy in a Macroeconomic Agent Based Model in which, by construction, agents are endowed with limited cognitive capabilities (bounded rationality). In this context, therefore, agents are far from being able to foresee the future as implied by Ricardian ultra-rationality. Moreover, in our model financial markets are not developed enough to allow household to effectively smooth consumption. In an Agent Based context, therefore, fiscal policy matters. This is indeed the case in the fiscal policy experiments put forward so far in this literature.

Fiscal policy matters so much that the public sector can somehow steer the economy toward a target of the unemployment rate by tuning appropriately fiscal parameters avoiding persistent fiscal imbalances. In the long run, in fact, i.e., abstracting from short run fluctuations, tax revenues tend to fund Government outlays. This is very much in the spirit of the original BS model even if we embed this result in a theoretical framework which is not at all aggregative and the mechanisms by which we reach this result is different.

We carry out fiscal policy experiments in a variant of the macroeconomic agent based model with capital and credit (CC-MABM hereafter) developed by Assenza, Delli Gatti, and Grazzini (2015). The original model features a closed economy populated by households, firms (producing capital and consumption goods) and banks. In this paper we enrich the model by introducing a Government which collects taxes on income, provides unemployment subsidies and carries out public expenditures. In case of a deficit, the Government issues bonds that are sold to the private sector.

We consider three policy experiments. First, we analyze the emerging properties of the model when the fiscal policy makers *Tax and Transfer (TT)*, i.e., they collect taxes on wage income to fund unemployment subsidies. The Government engages therefore in "within workers redistribution". Second, we introduce Government expenditure. This is the *Tax, Transfer and Spend (TTS)* configuration of fiscal policy. Finally we investigate the effects of a "fiscal rule" such as the *Stability and Growth Pact (SGP)*: the policy maker must comply with constraint on the deficit/GDP ratio. In all the experiments we carry out a thorough sensitivity analysis to assess the robustness of our results.

In order to get a flavour of the results, let's consider the TT experiment. Simulations show that the aggregate scale of activity generated by the CC-MABM (measured by GDP,

employment or the unemployment rate) tends to a long run average level (a statistical quasi-equilibrium) which is consistent with a balanced Government budget. In other words, in the long run tax revenues in the aggregate tend to be equal to total payments for unemployment subsidies. This phenomenon occurs *spontaneously*, it is not forced by the modeler (we do not impose a balanced budget constraint). In this sense, the balanced Government budget is an *emerging property* of the model. We label this phenomenon the *Balanced Budget Emerging Property (BBEP)*.

From BBEP follows that in the long run employment, GDP and the unemployment rate generated by the model are functions of the *tax rate* on wage income and the *replacement rate*, i.e. the fraction of the wage which is paid out by the Government as a subsidy to unemployed workers. The unemployment rate, for instance, is increasing in the replacement rate and decreasing in the tax rate. A fiscal expansion (a cut in the tax rate or an increase of the replacement rate) unambiguously reduces the unemployment rate. A *policy frontier* can be analytically determined which associates a specific unemployment rate to each tax rate, given the replacement rate. This is a menu for the policy maker: the fiscal authority may drive the scale of activity of the macroeconomy towards a target unemployment rate by choosing the appropriate tax rate.

We run a large number of Montecarlo simulations to check the robusness of this result. The analysis of sensitivity shows, indeed, that *the BBEP is not a universal law.* For instance, in the TT experiment, when the tax rate is "too low" – i.e., smaller than a numerical threshold – the BBEP does not hold exactly, it provides only a fairly reasonable approximation to the empirical evidence generated by the artificial data.

In the TTS experiment, given the tax rate and the replacement rate, by increasing Government expenditure the unemployment rate falls below the level reachable in the TT experiment. Sensitivity shows, however, that when Government expenditure is "too high" the BBEP does not hold any more and any attempt to increase GDP by further increasing Government expenditure is counterproductive: the unemployment rate may indeed increase.

The paper is organized as follows. Section 2 is devoted to a review of the literature on fiscal policy in MABMs. In section 3 we describe the basic features of the CC-MABM we use for simulations. In section 4 we describe the results of simulations in model 0, without public sector. In section 5 we describe the results of the fiscal policy experiments. Section6 concludes.

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2 Related literature

Agent based macroeconomics has grown rapidly in the last decade. Medium sized MABMs consider at least three types of agents –households, firms and banks – interacting on five markets: consumption goods, capital or investment goods, labour, credit, deposits. There are essentially six computational frameworks: (1) the model developed by Ashraf, Gershman and Howitt (Ashraf, Gershman, and Howitt (2016, 2017)); (2) the group of models developed by Delli Gatti, Gallegati and co-authors (the so-called CATS model, see, for example, Delli Gatti, Desiderio, Gaffeo, Cirillo, and Gallegati (2011)); (3) the framework developed by Dawid and co-authors, known as Eurace@Unibi (Dawid, Gemkow, Harting, van der Hoog, and Neugart (2018); Dawid, Harting, van der Hoog, and Neugart (2018)); (4) the Eurace simulator developed by Cincotti and co-authors (Cincotti, Raberto, and Teglio (2012)),[1] (6) the "Keynes meeting Schumpeter" (KS) framework developed by Dosi and co-authors (Dosi, Fagiolo, and Roventini (2010a,b); Napoletano, Dosi, Fagiolo, and Roventini (2012); Dosi, Fagiolo, Napoletano, and Roventini (2013); Dosi, Fagiolo, Napoletano, Roventini, and Treibich (2015); Dosi, Pereira, Roventini, and Virgillito (2017)).

The model we use to carry out policy experiments (Assenza et al. (2015) is a variant with capital of the model presented in chapter 3 of Delli Gatti, Desiderio, Gaffeo, Cirillo, and Gallegati (2011), in which production is carried out by mens of labour alone.

A number of fiscal policy experiments have been carried out in these models. One early example is the analysis of the effects of fiscal policy in a framework put forward by Russo, Catalano, Gaffeo, Gallegati, and Napoletano (2007) which focuses on technical progress. In their model, firms spend a fraction of their profits in R&D which feeds the adoption of innovative techniques. An emergent property of the model, therefore, is the positive relationship between the firm's scale of activity and technical progress.

In this model the Government levies taxes on profits. They carry out two experiments, both of the Tax and Transfer (TT) type. In the first one tax revenues finance unemployment subsidies; in the second one they are redistributed to firms so that they can increase R&D expenditure. In the first experiment redistribution goes from firms to unemployed workers and therefore boosts

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[1]The Eurace project, funded by the European Commission in 2006-2009, (see Deissenberg, van der Hoog, and Dawid (2008)) has generated two lines of research: the Eurace@unibi model with an emphasis on techological innovation and labour market performance and the Eurace simulator which has a specia role for financial markets.

consumption (of the people on the dole). In the second experiment redistribution occurs "within firms". Simulations show that in the first experiment, i.e., when redistributed tax revenues eventually boost demand, the effect on growth is negative while in the second experiment, when tax revenues are redistributed to firms, there is a positive effect on growth because this TT experiment end up boosting R&D.

Neveu (2013) adopts a framework similar to Russo et al. (2007) but carries out a different experiment. The Government channels tax revenues partly to fund transfers to the unemployed and partly to subsidise R&D expenditure. A fiscal boost of demand and R&D expenditure mitigates the amplitude and duration of the recession.

Using a KS model, Dosi, Fagiolo, and Roventini (2010a) perform a TT policy experiment which redistributes income from firms to unemployed workers, as in the first experiment carried out by Russo et al. (2007). Contrary to the results of Russo et al. (2007), in the KS framework, an increase of the replacement rate has a sizable positive impact on productivity and output, because it boosts demand, firms' sales and therefore R&D expenditure, which is assumed to be a fraction of sales. The Keynesian (demand driven) engine of the KS model is complementary to the Schumpeterian (technology driven) engine. Moreover an increase in the generosity of the unemployment benefit mitigates macroeconomic volatility.[2]

Dosi, Fagiolo, Napoletano, Roventini, and Treibich (2015) and Teglio, Mazzocchetti, Ponta, Raberto, and Cincotti (2018) explore the effects of stylized fiscal rules reminiscent of the Stability and Growth Pact (SGP) and the Fiscal Compact in a MABM of the KS and Eurace type respectively. Not surprisingly, both papers find that these fiscal rules depress growth and employment and increase output volatility. The introduction of escape clauses mitigates the contractionary bias of the fiscal rules.

Napoletano, Roventini, and Gaffard (2015) show that a an expansionary fiscal policy financed in deficit mitigates the negative macroeconomic impact of bankruptcy and speeds up the recovery, a result in line with Minsky's policy recommendation in the aftermath of a financial crisis. Moreover, the fiscal multiplier is sensitive to the stage of the business cycle: it is large in the downturn well after the shock.

Harting (2015) focuses on the implications of different TT experiments on growth rate and macroeconomic volatility. In the first one transfers are channeled to households, whose dispos-

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[2]In Dosi, Fagiolo, Napoletano, and Roventini (2013) the volatility-mitigating effect of TT redistribuiton from firms to unemployed workers is increasing with the firms' market power.

able income and consumption increase. In the second experiment, the Government subsidises firms' purchase of capital goods, irrespective of their quality. Finally, in the third experiment the Government subsidises only the purchase of high quality machine tools. All the TT policies mitigate macroeconomic volatilty but the first one (the subsidization of households' consumption) yields a much larger budget imbalance.

So far limited work has been done on fiscal policy in the context of the CATS model. This paper is a first step in filling this gap.

3 The Model

3.1 The environment

In this section we will describe only the main features of the model, which is based – as we already pointed out – on Assenza, Delli Gatti, and Grazzini (2015). We refer the interested reader to this paper for details. This subsection provides an overview of the model, focusing on agents' interactions.

The economy consists of four sectors, the production or corporate sector, the household sector, the banking system and the public sector.

The corporate sector is populated by $N_F = N_F^c + N_F^k$ firms. N_F^k capital goods producing firms (K-firms) employ labour to provide capital goods (K-goods) to N_F^c consumption goods producing firms (C-firms). We therefore model a stylized *supply chain*: upstream K-firms provide capital to downstream C-firms in order to satisfy the latter's production needs. C-firms employ labour and capital to produce and sell consumption goods (C-goods) to households and to the public sector (Government expenditure).[3]

The household sector consists of $N_H = N_W + N_F$ households who can be either workers or "capitalists", i.e., firm owners. We assume that every firm is owned by a capitalist, hence there are N_F capitalists. The owner of a firm earns dividends proportional to the firm's profit if the latter is positive.

N_W workers supply labor to the production sector. If employed, each worker earns a wage; if unemployed, she will receive an unemployment subsidy equal to a fraction (the replacement rate) of the wage of the employed. The market for labour is characterized by *search and matching*:

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[3]We will elaborate on this in section 3.7.

unemployed workers search for a job at firms and stop searching when a match occurs.

In this paper we assume that only wages are taxed. Hence the disposable income of the employed worker is a fraction of the wage. Unemployed workers and capitalists do not pay taxes [4] so that their disposable income coincides with the unemployment subsidies and dividends respectively.

Both workers and capitalists are consumers, i.e., buyers on the market for C- goods. They devote a *consumption budget* to purchase C-goods. The market for C-goods is characterized by search and matching: households search for trading opportunities at C-firms and stop searching when a match occurs. Also the markets for K-goods is characterized by search and matching: C-firms search for trading opportunities at K-firms and stop searching when a match occurs.

For simplicity, the banking sector consists of only one bank. Households and firms hold deposits at the bank, which, for simplicity, are not remunerated. The bank also extends loans to firms which need to fill the financing gap (production costs net of internally generated funds). Production costs coincide with the wage bill in the case of K-firm; it includes also capital goods expenditure in the case of C-firms.

Since there is only one bank, by construction there cannot be search and matching on the market for credit. The bank sets the price (interest rate on loans) and the quantity of credit supplied to firms. The price/quantity decision of the bank is based on the assessment of the borrowing firm' s financial fragility, which is a proxy of the credit risk run by the lender. In this context, a firm may well face a limit on the amount of credit it can get (credit rationing).

In such a framework we introduce a stylized public sector: the fiscal authority collects taxes on wage income, provides unemployment subsidies and carries out public expenditures. In case of a public sector deficit, bonds are issued and sold to the bank.[5] The interest rate on Government bonds is equal to the risk free interest rate. Figure 1 depicts agents interactions on the five markets included in our model: deposits, credit, labor, K-goods and C-goods. The way in which markets work will be described in the following.

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[4]This assumption will be relaxed in the future.

[5]This is another restrictive assumption which will be relaxed in the future.
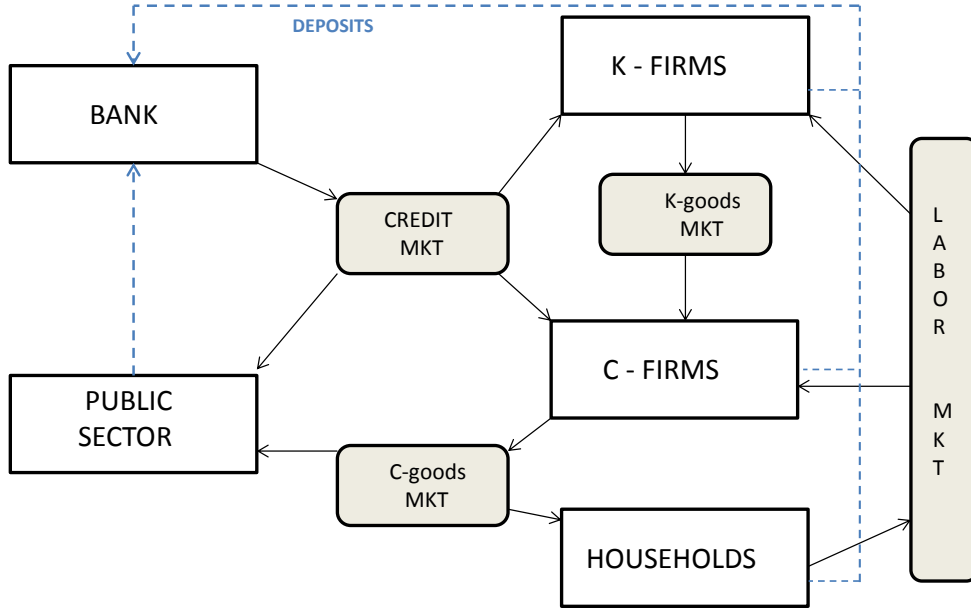
Figure 1: Agents and markets

3.2 Households

3.2.1 Workers

There are N_W workers. Each worker supplies 1 unit of labour inelastically. If employed, she receives the nominal wage w_t and pays out a fraction t_w of this wage to the Government. $t_w \in (0,1)$ is the *tax rate*. If unemployed, the worker searches for a job visiting a subset Z_e of firms (chosen at random among the population of firms) and applies to the first one who has posted vacancies. Since the wage is uniform across firms and labour is homogeneous, once an unemployed worker finds a firm with an unfilled vacancy she stops searching and the match occurs. Unemployed workers who have not succeeded in finding a job (because firms in their subset did not post vacancies or because they have already filled all the vacancies), receive an unemployment subsidy from the Government equal to a fraction z of the nominal wage. $z \in (0,1)$ is the *replacement rate*. Hence, a worker's (disposable) income is $w_t(1 - t_w)$ if employed, zw_t if unemployed.

3.2.2 Capitalists

There are N_F "capitalists", one for each firm. The owner of the f-th firm, $f = 1, 2, ...N_F$, receives a fraction τ (the pay-out ratio) of the current profit π_{ft} if the latter is positive. In the following, for simplicity, we will assume that dividends are not taxed. If a firm faces a loss, it will not distribute dividends and will reduce its net worth correspondingly. Whenever a firm goes bankrupt, another one will replace it. We assume that the initial equity of the entrant firm is provided by the capitalist that owned the bankrupt firm. The capitalis' wealth, therefore, will be reduced correspondingly.

3.2.3 Consumption

Households (workers and capitalists) are consumers/savers. Each consumer sets her *consumption budget* equal to the sum of her *permanent income* – a weighted average of current and past incomes (wages and unemployment subsidies in the case of workers, dividends in the case of firm owners) – and a fraction of her wealth.[6]

Once a consumer has defined her consumption budget, she visits a subset Z_c of C-firms chosen at random and ranks them in ascending order of price: the consumer starts purchasing goods from the firm which posts the lowest price; if she still has resources to be spent on consumption goods, she buys from the second one (the firm which posts the second lowest price) and so on until the consumption budget is exhausted. Therefore the search and matching process implies a negative price elasticity of demand for the C-goods.

If the consumer's demand has not been completely satisfied after Z_c visits, she is forced to save the unspent portion of the consumption budget. Hence, savings are equal to the difference between actual disposable income and the budget allocated to consumption plus the involuntary savings eventually deriving from unsatisfied demand.

Savings are deposited at the bank. By assumption households do not hold Government bonds. Therefore households' wealth takes the form only of deposits held at the bank.

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[6]This rule of thumb is reminiscent of Carroll's model of consumption, Carroll (2009), Carroll (1997), Carroll (1994)

3.3 C-firms

3.3.1 Price and quantity setting

Since each household visits only a (small) subset of firms – i.e. households do not explore the entire space of purchasing opportunities – each C-firm has some *market power* on its own local market (i.e. there are as many local C-markets as there are C-firms).

The firm has to set individual price and quantity under uncertainty. It knows from experience that if it charges higher prices it will get smaller demand but it does not know the actual demand schedule (i.e., how much the consumers would buy at any given price). The firm, in fact, observes only the current willingness to pay of the visiting consumers, who change from time to time. Hence the firm is unable to maximize profits setting the marginal cost (which is known) equal to the marginal revenue (unknown). The best the firm can do consists in setting the price as close as possible to the average price level – a proxy of the price set collectively by its competitors[7] – and production as close as possible to (expected) demand in order to minimize involuntary inventories (in case of excess supply) or the queue of unsatisfied customers (in case of excess demand).

In t, the i-th C-firm, $i = 1, 2..., N_F^c$, must choose the price and desired output for t+1 $\left(P_{it+1}, Y_{it+1}^*\right)$. Desired output is anchored to expected demand $Y_{it+1}^* = Y_{it+1}^e$. The firm's information set in t consists of (i) the average price level P_t and (ii) excess demand

$$\Delta_{it} := Y_{it}^d - Y_{it} \tag{1}$$

where Y_{it}^d is actual demand and Y_{it} is actual output in t. Δ_{it} shows up as a queue of unsatisfied customers if positive; as an inventory of unsold goods if negative.

Notice that Δ_{it} is a proxy of the *forecasting error* $\epsilon_{it} := Y_{it}^d - Y_{it}^e$ where Y_{it}^e is expected demand formed in t-1 for t. Δ_{it} coincides with ϵ_{it} iff production plans are fulfilled, i.e. $Y_{it}^* = Y_{it}$. Production plans, however, may not be fulfilled: actual production Y_{it} can differ from desired quantity Y_{it}^* if constraints on the availability of capital, labour and funding inhibit the attainment of the desired scale of activity (more on this below). In symbols: $Y_{it} \leq Y_{it}^e$. Therefore $\Delta_{it} = \epsilon_{it} + (Y_{it}^e - Y_{it})$ where the expression in parentheses is a non-negative discrepancy between

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[7]As usual in a monopolistic competition setting, the firm assumes its price has a negligible weight in the average price level

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expected demand and actual production.

If there is a positive forecasting error (i.e. under-estimation of demand), then there will be *a fortiori* excess demand (a queue of unsatisfied customers). If there is a negative forecasting error (i.e. over-estimation of demand), then there will be excess supply (involuntary inventories) only if the negative error is greater in absolute value than the discrepancy between expected demand and actual production. This will be case, of course, if the discrepancy is "sufficiently small" which is generally the case.

By assumption C-goods are not storable: involuntary inventories of perishable consumption goods cannot be carried over from one period to the next. Therefore they cannot be employed to satisfy future demand. They play, however, the very useful role of an element of the information set available to the firm when setting the price and the quantity for the future.

Given this information set, a firm can decide either to update the current price or to vary the quantity to be produced.

The decision process is based on two *rules of thumb* which govern price changes and quantity changes respectively. These updating rules are represented by simple adaptive algorithms. The *price adjustment* rule is:

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P_{it+1} = \begin{cases} P_{it}(1 + \eta_{it}) & \text{if } \Delta_{it} > 0; \ P_{it} < P_t \\ P_{it}(1 - \eta_{it}) & \text{if } \Delta_{it} \leq 0; \ P_{it} > P_t \end{cases} \tag{2}
$$

where η_i is a random positive parameter drawn from a distribution with support $(0, \bar{\eta})$.[8]

The signs of Δ_{it} and of the difference $P_{it} - P_t$ dictate the direction of price adjustment but the *magnitude* of the adjustment is stochastic and bounded by the width of the support of the distribution. This is one of the main sources of randomness in the model. We also assume that the firm will never set a price lower than the average cost (which includes not only the cost of labour and capital goods but also interest payments).[9]

As said above, the firm sets the desired quantity Y_{it+1}^* at the level of expected demand Y_{it+1}^e. Hence the *quantity adjustment* rule can be conceived also as an updating algorithm for demand

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[8]The distribution from which the idiosyncratic parameter is drawn is the same for all the firms and is time-invariant.

[9]While the attainment of the desired scale of activity is constrained by lack of capital, labour or finance, there are no obstacles to setting the desired price provided the price emerging from (2) is greater than the average cost.

expectations:

$$Y_{it+1}^* = Y_{it+1}^e = \begin{cases} Y_{it} + \rho\mathbf{1}_{[P_{it}>P_t]}\Delta_{it} & \text{if } \Delta_{it} > 0 \\ \\ Y_{it} + \rho\mathbf{1}_{[P_{it}<P_t]}\Delta_{it} & \text{if } \Delta_{it} \leq 0 \end{cases} \tag{3}$$

where ρ is a positive parameter, smaller than one.

$\mathbf{1}_{[P_{it}>P_t]}$ is an *indicator function* equal to 1 if $P_{it} > P_t$, 0 otherwise. Analogously, $\mathbf{1}_{[P_{it}<P_t]}$ is an *indicator function* equal to 1 if $P_{it} < P_t$, 0 otherwise.

In the case of quantity adjustment, the signs of Δ_{it} and of the difference $P_{it} - P_t$ dictate the direction of quantity adjustment. The *magnitude* of the adjustment, however, is not stochastic but determined by excess demand. If we assume that the discrepancy between expected demand and desired production is negligible, so that excess demand coincides with the forecasting error, we can interpret (3) as a standard *adaptive mechanism* to update demand expectations. By iteration, as it is well known, desired production in t+1 will be determined by the weighted average of past quantities with exponentially decaying weights.

3.3.2 Labor and Capital Requirements

Once a decision has been taken on desired output in t+1, the firm determines how much capital (capital requirement) and how much labour (labour requirement) it needs to reach that level of activity.

Capital in t+1 will be determined by adding investment in t (more on this later) to undepreciated capital in t (law of motion of capital). Actual capital in t+1 may or may not satisfy the capital requirement. If the capital requirement is satisfied – i.e., if actual capital is greater than the capital requirement – only a fraction of the available capital will be utilized: the desired *rate of capacity utilization* will be smaller than one. Employment will be adjusted to reach the desired rate of capacity utilization.

If the capital requirement is not satisfied – i.e., if actual capital is smaller than the capital requirement – available capital will be utilized at full capacity (the rate of capacity utilization will be one) but desired output will not be reached. Employment will be adjusted to reach full capacity utilization.

We assume the i-th firm is endowed with a Leontief technology: $Y_{it} = \min(\alpha N_{it}, \kappa\omega_{it}K_{it})$ where α and κ represent labor and capital productivity respectively (constant and uniform across firms) and $\omega_{it} \in (0, 1]$ is the rate of capacity utilization at firm i. In the absence of

labour constraints $Y_{it} = \kappa \omega_{it} K_{it}$ and employment is $N_{it} = \frac{\kappa}{\alpha} \omega_{it} K_{it}$ where $\frac{\kappa}{\alpha}$ is the reciprocal of capital intensity. When capital is employed at full capacity – i.e. when $\omega_{it} = 1$ – output will be $\hat{Y}_{it} = \kappa K_{it}$. This is "full capacity" output.

The capital stock available in t+1 K_{it+1} is determined by investment activity carried out in t I_{it} (to be discussed in the next section) and cannot be modified in t+1. Hence in period t+1 the maximum attainable output is $\hat{Y}_{it+1}$.

Once the target quantity Y^*_{it+1} has been set, the firm determines the labor and capital requirements. The desired stock of capital $K^*_{it+1} = Y^*_{it+1}/\kappa$ defines a cut-off value for the capital stock. There are two scenarios.

(a) If the actual (pre-determined) capital stock K_{it+1} is "large enough" – i.e. greater than the threshold K^*_{it+1} – the firm can reach the desired scale of production by setting the *rate of capacity utilization* at the appropriate (desired) level. In fact $K^*_{it+1} = \omega^*_{it+1} K_{it+1}$ where ω^*_{it+1} is the desired rate of capacity utilization. In this case the capital requirement corresponds to a fraction ω^*_{it+1} of the actual capital stock. Hence labor requirement (which coincides with desired employment) is $N^d_{it+1} = N^*_{it+1} = \omega^*_{it+1} \frac{\kappa}{\alpha} N_{it+1}$. This scenario occurs if the firm is downsizing (which of course means that the rate of capacity utilization will go down and some workers will be fired) or if the firm is increasing production but to a limited extent, i.e. within the limits of available capital.

(b) If the desired output turns out to be greater than full capacity output, i.e. $Y^*_{it+1} > \hat{Y}_{it+1} = \kappa K_{it+1}$, the actual capital stock is not large enough to allow the firm to attain the target level of output: $K_{it+1} < K^*_{it+1}$. In this case, the firm faces a constraint due to the availability of capital – i.e. a *capital constraint* – on the scale of activity. Hence it has to give up the target level of output and set production at the maximum level attainable given the available stock of capital $\hat{Y}_{it+1} = \kappa K_{it+1}$. Therefore labour requirement will be $N^d_{it+1} = \frac{\kappa}{\alpha} K_{it+1}$. This scenario occurs if the firm would like to increase production beyond the limits of available capital. This attempt, as we have shown, cannot be successful.

Whatever the scenario, if the actual employment level N_{it} is smaller than labor requirement the firm will post vacancies $V_{it+1} = N^d_{it+1} - N_{it}$ where $N^d_{it+1} = N^*_{it+1}$ in case (a) and $N^d_{it+1} = \frac{\kappa}{\alpha} K_{it+1}$ in case (b). If the opposite holds true – i.e., if the actual employment level is greater than labor requirement – the firm will fire $N_{it} - N^d_{it+1}$ workers.

It is worth noting that, if the firm is planning to increase its scale of activity but "too

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few" unemployed workers visit the firm, not all the vacancies will be filled. In this case, labour becomes the scarce factor and the firm has to give up the target level of output. Production will be set at the maximum level attainable given available employment. This will force the firm also to revise the capital utilization rate. In this case, the firm faces a constraint due to the availability of labour – i.e. a *labour constraint* – on the scale of activity it wants to attain.

3.3.3 Wage setting

The firm sets the nominal wage on the basis of labour market conditions captured by the distance between the current unemployment rate and a threshold unemployment rate, $\hat{u}$. Whenever the unemployment rate u_t is above the threshold firms cut wages. On the contrary, whenever the unemployment rate is below the threshold, wages are increased.

Formally, the wage updating mechanism can written as follows:

$$w_{t+1} = \begin{cases} w_t \left[1 + u_{up} \left(\hat{u} - u_t\right)\right]; & \hat{u} - u_t > 0 \\ w_t \left[1 + u_{down} \left(\hat{u} - u_t\right)\right] & \hat{u} - u_t < 0 \end{cases} \tag{4}$$

where u_{up} and u_{down} are parameters determining the reaction of nominal wages to deviations of the unemployment rate from the threshold. We will assume that $u_{up} > u_{down}$ to capture the downward stickiness of nominal wages. The value of $\hat{u}$ captures the structure of the labor market and can be interpreted as the natural rate of unemployment.

3.3.4 Investment

As mentioned above, the firm determines in t the capital stock which will be available for use in production in t+1 by means of investment I_{it}. By assumption, in planning the expansion of the capital stock, the firm adopts a long run perspective. We assume in fact that, in deciding investment, the firm sets a *benchmark* equal to the capital stock used in production "on average" since the beginning of activity $\bar{K}_{it}$. This, in turn, is computed by means of the following adaptive algorithm: $\bar{K}_{it} = v\bar{K}_{it-1} + (1-v)\omega_{it-1}K_{it-1}$ where $v \in (0,1)$ is a memory parameter.[10] By iteration, $\bar{K}_{it}$ turns out to be the weighted average of past utilized capital from the beginning

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[10]In the current period – say t – at the moment of deciding investment, the firm does not know actual demand and therefore the current rate of capital utilization. In fact the decision on the quantity to be produced which will yield capacity utilization is taken at the same time as investment decisions. Hence the latest information available concerning utilized capital is $\omega_{it-1}K_{it-1}$.

of activity until t-1 with exponentially decreasing weights.

Capital depreciates at the rate δ (if used in production).[11] Moreover we assume that capital adjustment is costly so that only a fraction γ of C-firms is able to invest in each period. Hence investment necessary "on average" to replace worn out capital is $\frac{\delta}{\gamma}\bar{K}_{it}$.

We assume, moreover, that the firm plans to maintain, in the long run, a capital stock buffer. Therefore the *target* capital stock is equal to $K_{it+1}^T = \frac{1}{\bar{\omega}}\bar{K}_{it}$ where $\bar{\omega} \in (0,1)$ is the desired long run capital utilization rate. Net investment is $K_{it+1}^T - K_{it-1}$.

Therefore gross investment in t is:

$$I_{it} = \left(\frac{1}{\bar{\omega}} + \frac{\delta}{\gamma}\right)\bar{K}_{it} - K_{it-1} \tag{5}$$

Once investment has been determined, the i-th C-firm visits a subset Z_k of K-firms chosen at random to purchase capital goods. Visited K-firms are ranked in ascending order of price and the C-firm starts buying capital goods from the K-firm which has posted the lowest price. If this purchase does not exhaust planned investment, the C-firm will purchase capital goods also at the second firm in the ranking and so on. If the C-firm's demand for K-goods has not been completely satisfied after Z_k visits, it is forced to "save" the unspent portion of the investment budget. Therefore actual investment may turn out to be lower than planned investment.

3.4 K-firms

In setting the price and the quantity, K-firms follow essentially the same heuristics adopted by C-firms and described in subsection 3.3. To be precise, while the price adjustment rule adopted by K-firms is the same as that of C-firms (see (2)), *mutatis mutandis*, the quantity adjustment rule departs from the one adopted by C- firms (see (3)) to take into account the fact that K-goods are durable and therefore storable: inventories of capital goods can be carried on from one period to another and sold in the future.

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[11]Of course, idle capital does not depreciate.

The quantity adjustment rule of the j-th K-firm, $j = 1, 2, ..., N_F^k$ therefore is:

$$Y_{jt+1}^* = Y_{jt+1}^e - Y_{jt}^k = \begin{cases} Y_{jt} + \rho \mathbf{1}_{[P_{jt} > P_t^k]} \Delta_{jt} - Y_{jt}^k & \text{if } \Delta_{jt} > 0 \\ Y_{jt} + \rho \mathbf{1}_{[P_{jt} < P_t^k]} \Delta_{jt} - Y_{jt}^k & \text{if } \Delta_{jt} < 0 \end{cases} \tag{6}$$

where Y_{jt+1}^* is the desired scale of activity, Y_{jt+1}^e is expected demand, Δ_{jt} excess demand, P_{jt} is the individual price and P_t^k is the average price of capital goods. Y_{jt}^k is the inventory of capital goods produced and held by firm j at time t which can be used to face demand in t+1.

Since K-firms are endowed with a linear production function whose only input is labour, once the price-quantity configuration has been set, a K-firm may post vacancies or fire workers in order to fulfill labor requirements.

3.5 Credit

3.5.1 Financing gap

Once the quantity to be produced has been set and the cost of inputs determined, the firm has to deal with financing. Consider a generic firm, indexed by $f = 1, 2..., N_F$. If the firm's internal liquidity (i.e. the current deposits held at the bank) M_{ft-1} is "abundant", i.e., greater than the costs to be incurred, the firm can self-finance production. If, on the other hand, liquidity is not sufficient to carry out production up to the desired level, the firm applies for a loan to fill its financing gap

$$F_{ft} = \min(wN_{ft} + \mathbf{1_c}P_{t-1}^k I_{ft} - M_{ft-1}, 0) \tag{7}$$

where $\mathbf{1_c}$ is an indicator function which assigns value 1 to C-firms and 0 to K-firms. In fact only C-firms purchase capital goods.[12] By definition, the financing gap is the demand for loans of the f-th firm. [13]

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[12]When the firm goes to the bank to ask for a loan, it has determined investment but has not carried out the search for trading opportunities on the K-goods market yet. Therefore it does not know the actual price of the capital goods it will purchase. We suppose therefore that the firm estimates the cost of investment goods by multiplying I_{ft} times the past average price of investment goods P_{t-1}^k.

[13]Of course, if the firm can self-finance production, the financing gap is zero, i.e., the firm does not apply for a loan.

3.5.2 Interest rates and credit supply

For simplicity we reduce the cardinality of the set of banks to one. The bank collects deposits from all the firms and households, supplies credit to firms and purchases Government bonds. In every period a firm with a positive financing gap will ask for a loan of the same size. The bank decides (i) the size of the loan that will be actually extended and (ii) the interest rate on that loan. In both decisions, the borrower's leverage will play a key role. The leverage λ_{ft} of the f-th firm is defined as:

$$\lambda_{ft} = \frac{L_{ft}}{E_{ft} + L_{ft}} \tag{8}$$

where L_{ft} is the firm's debt and E_{ft} is the firm's equity or net worth.

The interest rate charged by the bank to the f-th firm r_{ft} is determined as a mark up on the risk free interest rate r. The mark up is increasing with the probability of default p_{ft} which in turn is increasing with financial fragility, represented by the firm's leverage: $p_{ft} = p(\lambda_{ft})$. In the end, therefore:

$$r_{ft} = \mu(r, \lambda_{ft}) \tag{9}$$

where the function $\mu(.)$ is increasing with both arguments. Adopting the expression pioneered by Bernanke and Gertler, also in this model the firm is charged an *external finance premium* proportional to its financial fragility.

In order to determine the size of the loan to be extended to the f-th firm, the bank (i) determines the potential loss Γ_b on credit extended as a fraction $\phi \in (0, 1)$ of its net worth E_{bt} and (ii) decides the new credit line Φ_{ft} to be supplied to the f-th firm as a function of potential loss and credit already extended to the firm. The potential loss is defined as $\Gamma_b = \phi E_{bt}$. Credit extended to firm f is $\Phi_{ft} + L_{ft-1}$ where L_{ft-1} is debt of the f-th firm towards the bank inherited from the past. Therefore $(\Phi_{ft} + L_{ft-1})p_{ft} = \phi E_{bt}$. Rearranging, the new credit line turns out to be:

$$\Phi_{ft} = \frac{\phi}{p(\lambda_{ft})} E_{bt} - L_{ft-1} \tag{10}$$

Given the current exposure of the bank to the firm, the new credit line is increasing with the bank's net worth and decreasing with the firm's leverage.

The size of the loan actually granted to firm f at time t will be

$$\hat{L}_{ft} = \min\left(\Phi_{ft}; F_{ft}\right) \tag{11}$$

18

i.e., the minimum between new credit line and the financing gap. If the latter is greater than the former the firm will face a *borrowing constraint* and therefore will be forced to scale down production.

3.6   Net Worth Updating

In every period, the firm's net worth E_f is updated by means of retained profits:

$$E_{ft+1} = E_{ft} + (1 - \tau)\pi_{ft} \tag{12}$$

where τ is the dividend payout ratio and π_{ft} is the firm's profit:

$$\pi_{ft} = P_{ft}\min(Y_{ft}, Y_{ft}^d) - (wN_{ft} + \mathbf{1_c}\omega_{ft}\delta K_{ft} + r_{ft}L_{ft}) \tag{13}$$

Whenever the firm's equity turns negative, the firm goes bankrupt and is replaced by a new one. The owner of the bankrupt firm confers the initial net worth of the entrant firm (out of her own private wealth). Hence, the population of firms is kept constant.

Also the bank's net worth is updated by means of retained profits:

$$E_{bt+1} = E_{bt} + (1 - \tau)\pi_{bt} - BD_t \tag{14}$$

where π_{bt} is the bank's profit and BD_t is *bad debt*, i.e., the book value of non-performing loans. We assume that the bank does not remunerate firms' and households' deposits while it earns interests on loans (if borrowers are solvent) and the risk free interest rate on Government bonds. Hence, in every period the bank's profit is

$$\pi_{bt} = \sum_{s=1}^{N_F^s} r_{st}L_{st} + rB_{t-1} \tag{15}$$

where N_F^s is the cardinality of the subset of borrowing firms which are solvent. Hence

$$BD_t = \sum_{n=1}^{N_F^n} L_{nt} \tag{16}$$

where N_F^n is the cardinality of the subset of borrowing firms which are not solvent.

3.7 The Public Sector

In every period, the public sector (or Government) raises tax revenues on wage income $TA_t = t_w w_t N_t$ where N_t is total employment. Total Government outlays consist of Government expenditure G_t and transfers.

We do not model the *production* of public goods and services (justice, defence, etc.). As a proxy for the provision of public goods we use Government *purchases of C-goods*, assuming that these goods will then be made available to the population at large (collective consumption). Government expenditure is defined as $G_t = g w_t N_W$ where g – i.e., per-worker Government expenditure (expressed in wage unit) – will be one of our three key fiscal parameters. For the sake of brevity, we will refer to g as the Government expenditure parameter.

The i-th C-firm receives (additional) demand from the public sector equal to a fraction σ_i of Government expenditure. This fraction is equal to the firm's market share of households' consumption. In a sense, the Government sprinkles additional public demand on C-firms in proportion to each firm's share of private demand.

We also assume, for simplicity, that the Government is the first agent to visit each firm. A physical "crowding out" phenomenon therefore may take place: private consumers visiting a firm may not find enough goods to buy due to the previous purchasing activity of the Government. Notice, finally, that the i-th firm may not be able to satisfy entirely Government demand $G\sigma_i$ due to lack of sufficient supply. In this case, we assume that the government cannot re-allocate demand to the other firms: only actual expenditures are recorded in the books. Hence actual Government expenditure may be lower than the budget G allocated to collective consumption.

Transfers consist of: (i) unemployment subsidies and (ii) interest payments on outstanding Government bonds. Total unemployment subsidies are $US_t = z w_t (N_W - N_t)$, where the unemployment subsidy zw which each unemployed worker receives is a fraction z (the "replacement rate") of the current wage w. Interest payments are $INT_t = r B_{t-1}$ where r is the cost of public debt, equal, by assumption, to the risk free interest rate.[14] Notice that the the risk free interest rate is exogenous. The fixity of the cost of debt implies the absence of a financial "crowding out" effect. By construction, an expansionary fiscal policy (e.g., an increase of g) does not affect the risk free rate. Since the latter is the baseline rate for the interest rate on loans (see equation

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[14]The risk free rate may be under the control of monetary authorities. Monetary policy experiments, however, are outside the scope of the present analysis.

(9)), an expansionary fiscal policy does not affect directly the cost of private debt either.[15]

While Government expenditure G is constant by assumption, tax revenues and unemployment subsidies are time varying as they are proportional to the scale of aggregate activity measured by total employment. Employment and the unemployment rate are linked as follows: $N_t = (1 - u_t)N_W$. Therefore $TA_t = t_w w_t (1 - u_t) N_W$ and $US_t = z w_t u_t N_W$. Other things being equal, tax revenues (resp. unemployment benefits) decrease (increase) with the unemployment rate.

A public sector deficit occurs when the resources from income taxation turn out to be lower than total outlays: $DEF_t = (US_t + G_t + rB_{t-1}) - TA_t$. In this case, the Government will issue new bonds $B_t - B_{t-1} = DEF_t$. For simplicity, we assume that the Government sells its bonds only to the bank. We assume moreover that regulation (a portfolio constraint) forces the bank to purchase Government bonds. [16] Since it is the bank which purchases Government bonds, interest payments can be conceived as Government transfers to the bank.

As already pointed out above, we assume that neither households nor firms can be bondholders. This is of course a restrictive assumption which we will relax in the future, introducing the possibility for these agents to choose the portfolio of their assets. Notice that , in principle, since (i) the interest rate on Government bonds is certain and fixed and (ii) deposits are not remunerated, households (and firms) have an incentive to hold all the Government bonds they are offered.

The purchase of Government bonds translates into an increase of bank's deposits and total liquidity available to the economy.

If in a certain period tax revenues turn out to be higher than Government outlays, the Government records a surplus which will take the form of deposits at the bank. Net Government debt therefore decreases.

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[15]The cost of private debt, however, is affected by the borrowing firms' leverage – as shown by equation (9) – so that there may be an indirect effect of an increase in Government expenditure on the cost of private debt inasmuch as an expansionary fiscal policy raises the firms' leverage.

[16]This assumption rules out the problem of the portfolio choice on the part of the bank. Absent a portfolio constraint, in fact, a risk neutral bank could decide how much Government debt to hold by comparing the interest rate on Government bonds with the expected interest rate on its portfolio of loans to firms.

4 Simulations

We assume the economy is populated by 1120 households (1000 workers and 120 capitalists) and 120 firms, of which 1/6 in the K-sector. Overall there are more than 20 parameters, which have been generally calibrated using the empirical evidence, when available. Table 1 reports the values assigned to the most important parameters of the *private sector*. They are constant across experiments and across scenarios. The parameters concerning the activity of the *public sector* are specific to each policy experiment and will be described below.

Transaction costs are high in markets for goods: buyers visit only $Z_c = Z_k = 2$ sellers. We assume that they are lower on the labor market: an unemployed worker, in fact, can visit $Z_e = 5$ firms.

The productivity of capital κ is the reciprocal of the capital/output ratio, which is set at 3 according to a well known stylized fact. The productivity of labor α is set at 1/2. Therefore full employment GDP is equal to $1000 \times (1/2) = 500$. Since the price level in the base year is $P_0 = 3$ GDP at full employment at the prices of the base year is 1500.

The long run desired rate of capital utilization is set at $\overline{\omega} = 85\%$ which is roughly in line with average capacity utilization in the USA before the Global Financial Crisis.

We will generate n variants of the model by tuning the fiscal parameters: the tax rate on wages t_w, the replacement rate z and the government expenditure parameter g.

For each model, we run S simulations, each simulation lasting for T periods. For each variable of interest, therefore, we generate a cross section of S data in each period. For the generic variable x in model n, the cross section in period t is represented by the vector $x_{n,t,s}$ where $s = 1, 2, ..., S$. The cross sectional mean will be denoted with $x_{n,t} = \frac{1}{S} \sum_{s=1}^{S} x_{n,t,s}$. Volatility will be measured by the cross sectional standard deviation $\sigma_{n,t}$. For each model and each variable of interest we can compute the *long run mean* $\bar{x}_{(n)}$ of the cross-sectional mean $x_{n,t}$: $\bar{x}_{(n)} = \frac{1}{t_1 - t_0} \sum_{t=t_0}^{t_1} x_{n,t}$ where t_0 and t_1 do not coincide necessarily with the beginning and the end of the time horizon considered in the simulation. We generally discard a transient consisting of a certain number of periods.

Our *benchmark* is model 0, i.e. the model without public sector, characterized by $t_{w(0)} = z_{(0)} = g_0 = 0$. We run the benchmark model S=192 times, each simulation lasting for T=3000 periods.[17] For the generic variable x in model 0, the cross section in period t is the vector $x_{0,t,s}$

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[17]Why exactly 192? Given the computational power of our server, by running 192 simulations we are

| Parameter	Description	Value
T	Number of periods	3000
N_W	Number of workers	1000
N_F^c	Number of C-firms	100
N_F^k	Number of K-firms	20
Z_e	Number of firms visited by an unemployed worker	5
Z_c	Number of C-firms visited by a consumer	2
Z_k	Number of K-firms visited by a C-firm	2
τ	Dividend pay out ratio (firms)	0.2
τ_b	Dividends pay out ratio (bank)	0.3
r	Risk free interest rate	0.01
ρ	Quantity adjustment parameter	0.9
η	Price adjustment random parameter	U(0,0.1)
α	Productivity of labor	0.5
κ	Productivity of capital	1/3
ϕ	Bank's maximum loss parameter	0.002
γ	fraction of C-firms which invest in each period	0.25
δ	Depreciation of capital	0.02
$\bar{\omega}$	Desired capacity utilization rate	0.85
w	Initial wage	1
u_{up}	upward wage adjustment	0.1
u_{down}	downward wage adjustment	0.01
u^T	unemployment threshold	0.1

Table 1: Parameters
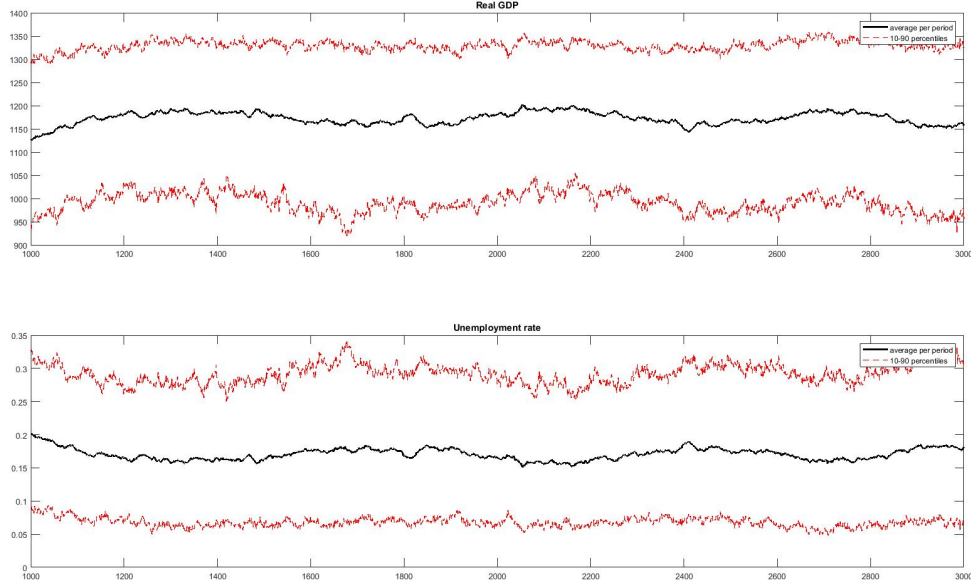
Figure 2: Simulations of model 0: $t_{w(0)} = z_{(0)} = g_0 = 0$.

where $s = 1, 2, ..., 192$. The cross sectional mean will be denoted with $x_{0,t} = \frac{1}{192} \sum_{s=1}^{192} x_{0,t,s}$.

Volatility will be measured by the cross sectional standard deviation $\sigma_{0,t}$. The *long run mean*

$\bar{x}_{(0)}$ of the cross-sectional mean $x_{0,t}$ will be $\bar{x}_{(0)} = \frac{1}{2000} \sum_{t=1000}^{3000} x_{0,t}$.

In figure 2, for each of the last 2000 periods of our time horizon we report the cross sectional

mean of two variables: (a) GDP at constant prices [18] $Q_{0,t}$ and (b) the unemployment rate

$u_{0,t}$.[19] Both GDP and the unemployment rate fluctuate irregularly around the long run mean:

$\bar{u}_{(0)} = \frac{1}{2000} \sum_{t=1000}^{3000} u_{0,t} = 0.16$ and $\bar{Q}_{(0)} = \frac{1}{2000} \sum_{t=1000}^{3000} Q_{0,t} = 1200$. [20]. The long run average

volatility of the unemployment rate is $\bar{\sigma}_{(0)} = 0.029$.

5   Fiscal policy experiments

In this section we will investigate the macroeconomic effects of three fiscal policy experiments:

(1) *Tax and Transfer (TT)* – i.e., pure "within workers redistribution" consisting of taxing

workers' wage income and spending only on subsidies for the unemployed – (subsection 5.1); (2)

Tax, Transfer and Spend (TTS), in which the Government also spends on C-goods (subsection

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using efficiently our facilities.

[18]$Q_{0,t}$ denotes GDP of period t at the prices of the base year, set conventionally at period 0, in the benchmark. Hence, $Q_{0,t} = P_0 Y_{0,t}$. The price level of period 0 is set to $P_0 = 3$.

[19]The unemployment rate is defined as usual: $u_t = (N_W - N_t)/N_W$

[20]Full employment GDP at the prices of the base year is $Q^f = 1500$

5.2); (3) the implementation of a fiscal rule such as a stylized *Stability and Growth Pact (SGP)*: in this case, the Government must comply with a constraint on the deficit/GDP ratio (subsection 5.3).

5.1 Tax and Transfer

In this experiment the Government raises taxes on wage income and implements transfers in the form of unemployment subsidies.[21] There is no Government expenditure, i.e. no collective consumption. Under pure TT, therefore: $t_w > 0, z > 0, g = 0$

5.1.1 The emergence of the balanced budget property

Let's consider two TT scenarios:

1. the tax rate on wage income is $t_{w(1)} = 9\%$ and the replacement rate is $z_{(1)} = 50\%$;

2. the tax rate goes down to $t_{w(2)} = 6\%$ while the replacement rate is $z_{(2)} = 50\%$.

   In symbols: $t_{w(2)} < t_{w(1)}$ and $z_{(2)} = z_{(1)}$.

   Figure 3 shows *one (representative) simulation* of the model under scenario (1).[22]

We report the time series of the data generated by this simulation for: (i) the budget deficit-to-GDP ratio $d_{1,t}$ ("deficit ratio" hereafter)[23] in the top left panel; (ii) the unemployment rate $u_{1,t}$ in the top right panel; (iii) GDP at the prices of the base year $Q_{1,t}$ in the bottom left panel and (iv) the public sector debt-to-gdp ratio $b_{1,t}$ ("debt ratio" hereafter) in the bottom right panel. The deficit ratio fluctuates around a long run mean of approximately $\bar{d}_{(1)} = 0$. In every period, the public sector can record a surplus or a deficit (we will discuss this issue momentarily) but in the long run, on average, the budget is balanced. The unemployment rate fluctuates around a long run average $\bar{u}_{(1)} = 15\%$. The long run mean of GDP is approximatelyn $\bar{Q}_{(1)} = 1200$.

Notice that the public sector budget is balanced only as a long term average. In every period the public sector can record a surplus or a deficit of non negligible magnitude. For instance, in scenario (1), as shown by the top left panel of figure 3, the deficit ratio can be as high as 14% of GDP while surpluses can reach a peak of 6%. This is mirrored in the dynamics of the public

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[21]As we already pointed out, we do not consider the taxation of dividends at this stage of the analysis. Of course, if dividends were taxed we could also analyze the effects of redistribution from capitalists to unemployed workers.

[22]Results are robust to changes of the random seed, as we shall see.

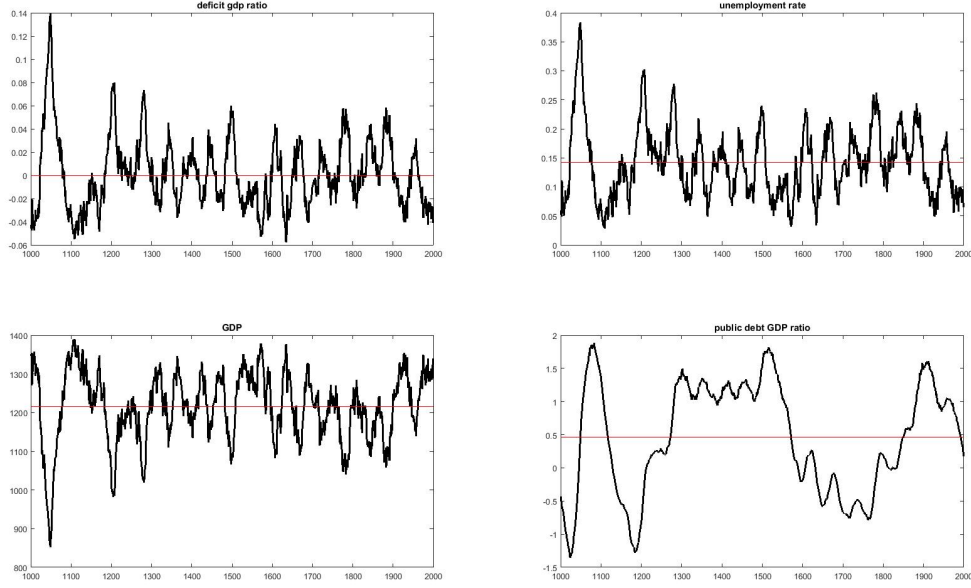[23]In TT experiments, the budget deficit is $DEF_t = (US_t + rB_{t-1}) - TA_t$

Figure 3: TT, scenario 1: $t_{w(1)} = 9\%$, $z_{(1)} = 50\%$

sector debt. As shown by the bottom right panel of figure 3, the stock of debt indeed goes up sharply – reaching 200% of GDP – around period $t_0 = 1100$ (due to a spike of the deficit ratio which occurs slightly earlier) but then goes down very rapidly and falls into negative territory (see for instance period $t_1 = 1200$) and then up again. A negative value for public debt must be interpreted as the accumulation of assets on the part of the public sector. Assets in the hands of the public sector are deposits at the bank (see discussion in section 3.7).

It seems therefore that one of the emergent properties of the model is the balanced budget. Is this a fluke? In order to check, consider scenario (2), with $t_{w(2)} = 6\%$ and $z_{(2)} = 50\%$. Figure 4 reports data generated by one representative simulation in this scenario. The figure is built following the same criteria of figure 3. Once again, the deficit ratio seems to fluctuate around a long run mean approximately equal to 0: $\bar{d}_{(2)} = 0$. The long run unemployment rate is now $\bar{u}_{(2)} = 8\%$ and $\bar{Q}_{(2)} = 1330$.

We therefore can tentatively put forward the following empirical regularity (extracted from the artificial data generated by our simulations):

Remark 1 (Balanced Budget Emergent Property (BBEP)) *With TT, the unemployment rate settles on a long run mean such that on average unemployment subsidies are fully financed by tax revenues.*
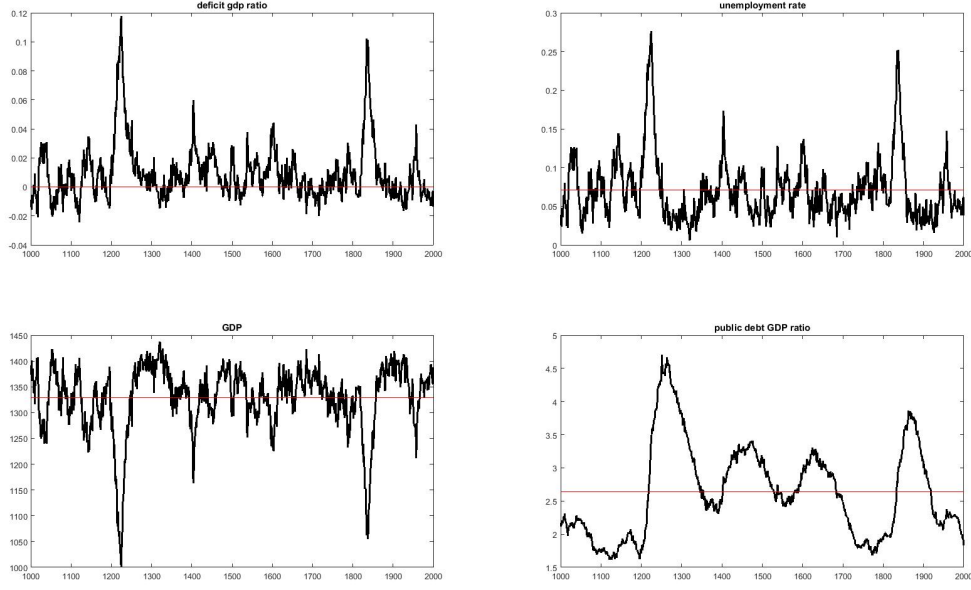
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Figure 4: TT, scenario (2): $t_{w(2)} = 6\%$, $z_{(2)} = 50\%$

5.1.2 TT in a toy model of the public sector

The essence of the BBEP of the macroeconomic agent based model with TT can be captured by the following simple aggregative model of the public sector. The level of tax revenues in every period is linearly decreasing with the unemployment rate:

$$TA = t_w w (1 - u) N_W + \varepsilon_\tau \tag{17}$$

where ε_τ is a white noise which captures random shocks to tax revenues.

On the other hand, the level of unemployment subsidies is linearly increasing with the unemployment rate:

$$US = zwuN_W + \varepsilon_\upsilon \tag{18}$$

where ε_υ is a is a white noise Government transfers to the unemployed. These schedules are represented in figure 5 (where random shocks are switched off).

The budget is balanced when

$$TA = US \tag{19}$$

Plugging equations (17) and (18) into (19) and solving for u we get the *Balanced Budget*
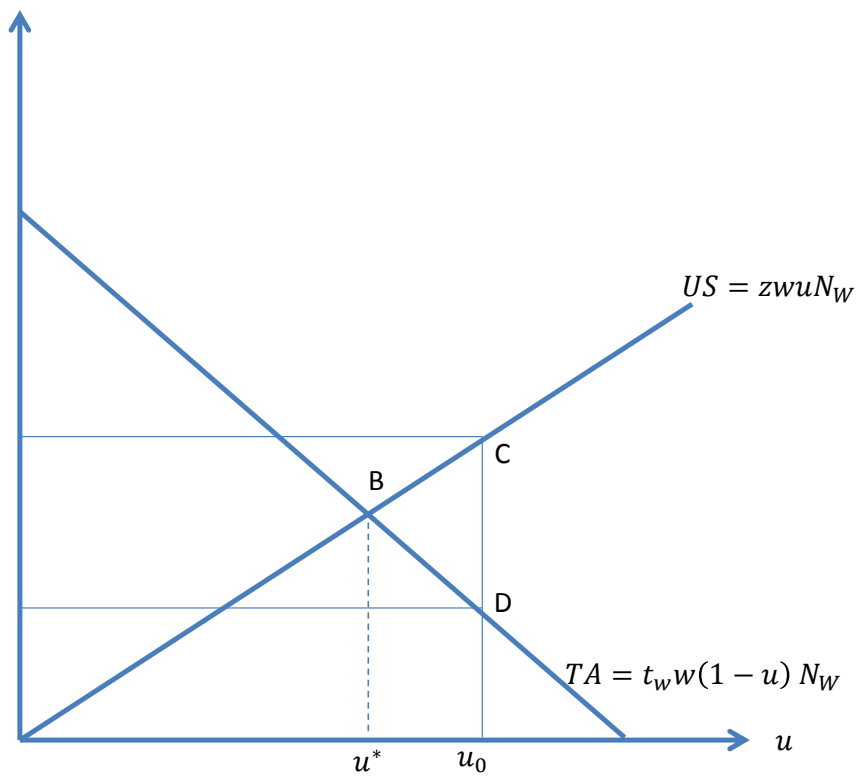
27

Figure 5: A toy model of the public sector

Unemployment Rate (BBUR):

$$u^* = \frac{t_w}{t_w + z} + \frac{\varepsilon_\tau - \varepsilon_v}{(t_w + z)wN_W} \tag{20}$$

The expected value of the BBUR is

$$E(u^*) = \frac{t_w}{t_w + z} \tag{21}$$

Graphically, the expected value of the BBUR is the coordinate on the x-axis of the point at which the TA and US schedules intersect.

The long run average unemployment rate obtained by simulating the macro ABM in scenario (1) is indeed almost the same as the expected BBUR obtained from (21) using the numerical values of the replacement rate and the tax rate of scenario (1), i.e. $E(u^*_{(1)}) = \bar{u}_{(1)} = 15\%$.[24]

In scenario (2) the expected BBUR is $E(u^*_{(2)}) = 10\%$ while $\bar{u}_{(2)} = 8\%$. In this second scenario there is a slight difference between the two.

For the moment, let's assume that the BBEP property holds. Why does the macro-economy self-organizes around an aggregate scale of activity such that the public sector is balanced? The reason is a strong *crowding in* effect of redistribution. Consider again figure 5. Let's abstract from the random disturbances. Suppose that at time t_0 the unemployment rate u_0 is "high", i.e. greater than the BBUR. Within workers redistribution consists in subsidies going to the unemployed workers (which translate into additional consumption) equal to zwu_0N_W funded by taxes on wages (which depress consumption) equal to $t_w w(1 - u_0)N_W$. The latter, however, are not sufficient to completely finance the former. The segment CD measures the public sector deficit in t_0. From the picture it is clear that the net effect of redistribution on consumption must be positive (somehow proportional to the distance CD). The increase in demand of C-goods then trickles up into an increase in demand for K-goods that are necessary to produce C-goods. Hence the unemployment rate goes down – this is the short run development – until the BBUR is reached "in the long run".[25] Of course, if the unemployment rate were initially "low", i.e.

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[24]To be precise, absent Government expenditure, the following definition holds $GB = TA - (US + rB)$ A balanced budget occurs when $US + rB = TA$ Notice that we do not consider interest payments in computing u^* as in the long run there is no deficit and therefore no debt accumulation, i.e. $B = 0$.

[25]In the transition, there can be also a wealth effect, which takes two forms. First, both employed and unemployed people who got a subsidy will accumulate wealth through savings. This increase in wealth translates into additional consumption (see section 3.2.3). Second, interest payments on Government debt adds to the bank's revenues, which translate into an increase of net worth and therefore of credit

smaller than the BBUR, redistribution would have a negative net effect on consumption so that the unemployment rate would go up. Point B therefore is a stable equilibrium of this simple model.

From (21) we infer that the expected BBUR is increasing with the tax rate and decreasing with the replacement rate.[26] If the policy maker sets the replacement rate at a constant level, say $\bar{z}$, from (21) we derive a a *policy frontier* which associates an expected BBUR $E(u^*)$ to each t_w:

$$E(u^*) = \frac{t_w}{t_w + \bar{z}} \tag{22}$$

It is easy to see that this frontier is increasing and concave. The expected BBUR is equal to zero when the tax rate is zero and reaches a maximum equal to $u_M^* = 1/(1 + \bar{z})$ when the tax rate is 1.

Remark 2 (Policy frontier) *With TT, the Government can set a target for the unemployment rate and steer the economy towards that target by choosing the appropriate tax rate.*

5.1.3 TT sensitivity analysis: experimenting with different t_W.

In order to understand better the properties of the model, we fix the replacement rate at $\bar{z}$ and consider $t_w \in (t^m, t^M)$. For each tax rate in this interval, given z, we have a TT scenario. To be specific, we set $t^m = 0.01$, $t^M = 0.2$, z=50%. Scenarios (1) and (2) of the previous section are special cases of this set up with $t_w = 0.09$ and $t_w = 0.06$ respectively.

For each tax rate, we run 192 simulations and generate a cross section of 192 data for feach variable of interest in each period. For example, the cross section of the generic variable x in period t in the scenario characterized by the tax rate t_w can be represented by the vector $x_{t_w,t,s}$ where $s = 1, 2, ..., 192$. The cross sectional mean will be denoted with $x_{t_w,t} = \frac{1}{192}\sum_{s=1}^{192} x_{t_w,t,s}$. For each tax rate and each variable of interest we compute the long run mean across 1000 periods in the middle range of our time horizon $\bar{x}_{(t_w)} = \frac{1}{1000}\sum_{t=1000}^{2000} x_{t_w,t}$.

In each panel of figure 6, for each variable x we report the plot generated by the simulations of the long run mean $\bar{x}_{(t_w)}$ as a function of the tax rate. We consider 8 variables. In the first row: (i) GDP, (ii) inflation, (iii) unemployment rate (iv) credit. In the second row: (v) deficit

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extended to firms. Notice that in the transition the deficit shrinks and tends asymptotically to zero.

[26]It is easy to see that in the long run $TA^* = US^* = \frac{wt_w z}{t_w + z}$ which is increasing in both t_w and z.
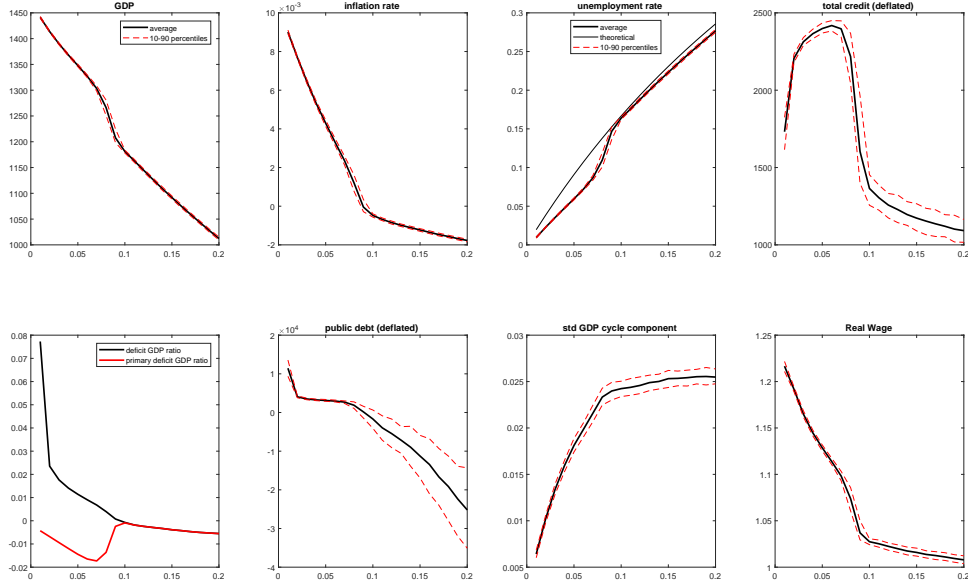
Figure 6: TT: sensitivity analysis. In each panel we show the long run mean of each variable generated by the simulations for each level of the tax rate, given the replacement rate z=50%

ratio, (vi) public sector debt, (vii) standard deviation of the cyclical component of HP-filtered GDP, (viii) real wage.

From panel (v) we infer that the BBEP holds for relatively high tax rates, i.e. for $t_w > 0.1$. For tax rates below 10% the public setor runs a primary surplus but the budget is in the red if we include interest payments. Panel (iii) traces the upward relationship between the unemployment rate and the tax rate retrieved from the simulations. As expected, the unemployment rate is increasing with the tax rate. Moreover, from panels (i) (vii) we infer that GDP is decreasing with the tax rate and macroeconomic volatility is increasing with the tax rate.

The thin line in panel (iii) is the *policy frontier* (22) derived in the previous section. When $t_w = 0.09$ (TT: scenario 1) the actual unemployment rate is essentially on the policy frontier. When $t_w = 0.06$ (TT: scenario 2) the unemployment rate generated by the simulations is smaller than the theoretical value.

5.2 Tax, Transfer and Spend

We now introduce Government expenditure. Government expenditure is kept constant during the whole time window; it is allocated to C-firms in relation to their market shares as shown above.

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5.2.1 TTS in the toy model

Before analyzing the output of simulations, let's slightly modify the simple model of the public sector to introduce Government spending, along with taxes and transfers. Revenues TA are represented by equation (17) as in section 5.1.2. Total Government outlays consist now of Government expenditure G (exogenous) and unemployment subsidies US, represented by equation (18). As mentioned above, we write Government spending as follows: $G = gwN_W$. Therefore the equation of total Government outlays is

$$OU = (zu + g)wN_W + \varepsilon_v \tag{23}$$

Equation (23) is represented by the OU schedule in figure (7) (ignoring the random shock). The OU schedule is obtained by shifting the US schedule up appropriately. The balanced budget condition is

$$TA = OU \tag{24}$$

Plugging (17) and (23) into (24) and solving the associated equation we get the BBUR in the presence of Government expenditure

$$u_G^* = \frac{t_w - g}{t_w + z} + \frac{\varepsilon_\tau - \varepsilon_v}{(t_w + z)wN_W} \tag{25}$$

The expected BBUR $E(u_G^*) = \frac{t_w - g}{t_w + z}$ is decreasing with the replacement rate and Government expenditure and increasing with the tax rate.[27]

It is straightforward to infer that *Government expenditure magnifies the crowding in effect* generated by TT. Consider figure 7. Suppose there areno shocks and the initial unemployment rate is u^*. Suppose now Government expenditure occurs: this translates into additional expenditure on final goods somehow proportional to the distance BF. Hence the unemployment rate goes down until the BBUR is reached.

From (25) we can predict that the equilibrium unemployment rate is *decreasing monotonically and linearly* with g.

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[27]It is easy to see that when the unempyment rate reaches the expected BBUR, $TA^* = OU^* = \frac{t_w(z+g)}{t_w+z}wN_W$.
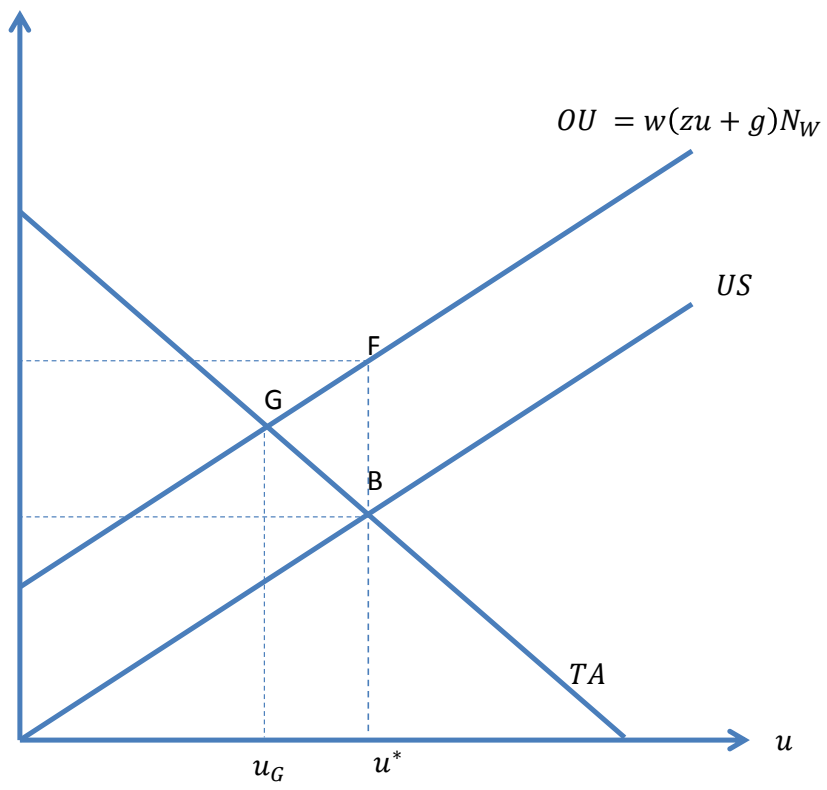
Figure 7: The augmented toy model with Government expenditure

$OU = w(zu + g)N_W$

US

TA

u

u_G u^*

F

G

B

5.2.2 TTS sensitivity analysis: experimenting with different g.

We borrow the numerical values of t_w and z from scenario (2) of TT, i.e., the tax rate is equal to 6% and the replacement rate is 50%. Then we consider different $g \in (g^m, g^M)$. For each g in this interval, therefore, we have a TTS scenario. To be specific, we set $g^m = 0.01$, $g^M = 0.2$. For each g, we generate a cross section of 192 data for each variable of interest in each period, which can be represented by the vector $x_{g,t,s}$. The cross sectional mean will be denoted with $x_{g,t} = \frac{1}{192} \sum_{s=1}^{192} x_{g,t,s}$. The long run mean will be denoted with $\bar{x}_{(g)} = \frac{1}{1000} \sum_{t=1000}^{2000} x_{g,t}$.

In each panel of figure 8, we report the long run mean of each variable associated to different levels of g. The variables are the same as in the previous sensitivity analysis.

A quick look at the plots shows that the *BBEP holds for low g* only, say $g < 0.06$. In this interval, the unemployment rate is linearly decreasing with g (as predicted). Moreover, GDP is linearly increasing and volatility is decreasing with g. Moreover, inflation and the real wage are increasing.

Once the bottom of the unemployment rate (around zero) and the peak of GDP have been reached, a further increase of g has a negative effect on GDP and employment. The unemployment rate, in fact, increases (slightly), GDP falls and volatility increases. We conclude therefore that *the response of the unemployment rate, GDP and volatility to increases in g is nonmonotonic.*

Why is it so? An increase of g boosts demand directly. Firms benefit from extra demand which translates into additional sales and profits. Hence their net worth steps up and credit goes down. The crowding in effect already discussed under TT is enhanced.

But this is not the end of the story. While both prices and the nominal wages go up, the former grow more rapidly than the latter so that real wages go down. Beyond a certain threshold, the increase in g crowds consumption out through the depressing effect of inflation on real wages.

5.3 Stability and Growth Pact

Suppose now the policy maker adopts a *fiscal rule* of the SGP type: the policy mix changes in response to the violation of a deficit/GDP threshold. In normal times, i.e. when the deficit ratio is below $\hat{d}$, say 3%, the Government regularly carries out its transfer and spending programs.
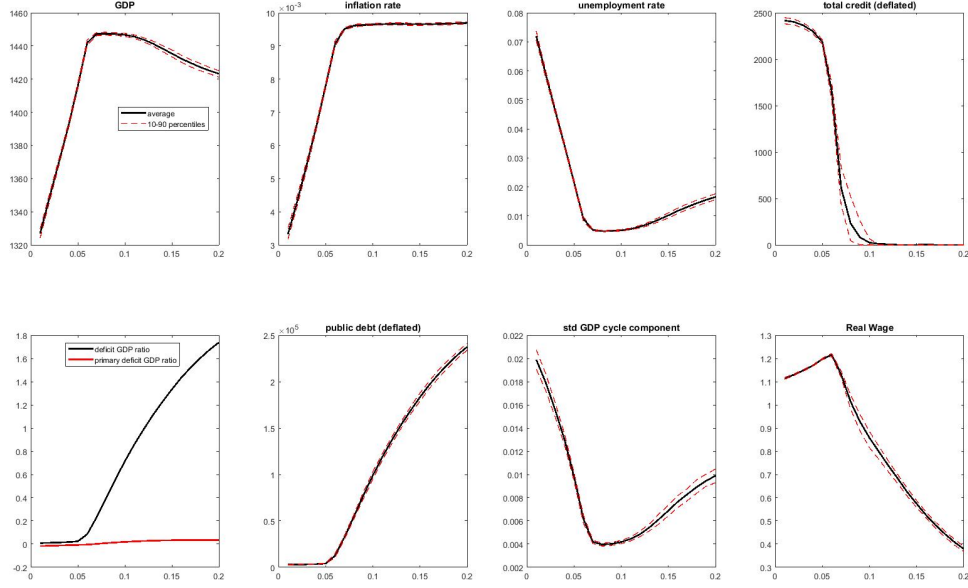
Figure 8: TTS: sensitivity analysis. In each panel we show the long run mean of each variable generated by the simulations for each level of Government eexpenditure g, given the tax rate $t_w = 6\%$) and the replacement rate $z = 50\%$

Should the ratio exceed $\hat{d}$, a restrictive policy is applied: the Government downsizes these programs "appropriately". This is a policy feedback rule which makes Government programs conditional on macroeconomic conditions captured by the threshold deficit/GDP ratio.

5.3.1 SGP sensitivity analysis: experimenting with different $\hat{d}$

We borrow the numerical values of t and z from scenario (2) of TT, i.e., the tax rate is equal to 6% and the replacement rate is equal to 50%. We set also $g = 0$. We assume that when the deficit ratio violates the ceiling $(d > \hat{d})$ the Government revises the replacement rate down in order to bring back the deficit ratio to $\hat{d}$.

We consider different $\hat{d} \in (\hat{d}^m, \hat{d}^M)$. To be specific, we set $\hat{d}^m = 0.01$, $\hat{d}^M = 0.2$. For each $\hat{d}$ in this interval, therefore, given t and z, we have a SGP scenario. For each scenario we generate a cross section of 192 data for each variable of interest in each period, represented by the vector $x_{\hat{d},t,s}$. The cross sectional mean will be denoted with $x_{\hat{d},t} = \frac{1}{192} \sum_{s=1}^{192} x_{\hat{d},t,s}$. The long run mean across 1000 periods in the middle range of our time horizon is $\bar{x}_{(\hat{d})} = \frac{1}{1000} \sum_{t=1000}^{2000} x_{\hat{d},t}$.

In figure 9, we report the the long run values of the usual variables associated to different levels of the threshold $\hat{d}$

The BB property does not hold in the SGP setting: if we take into account interest payments,
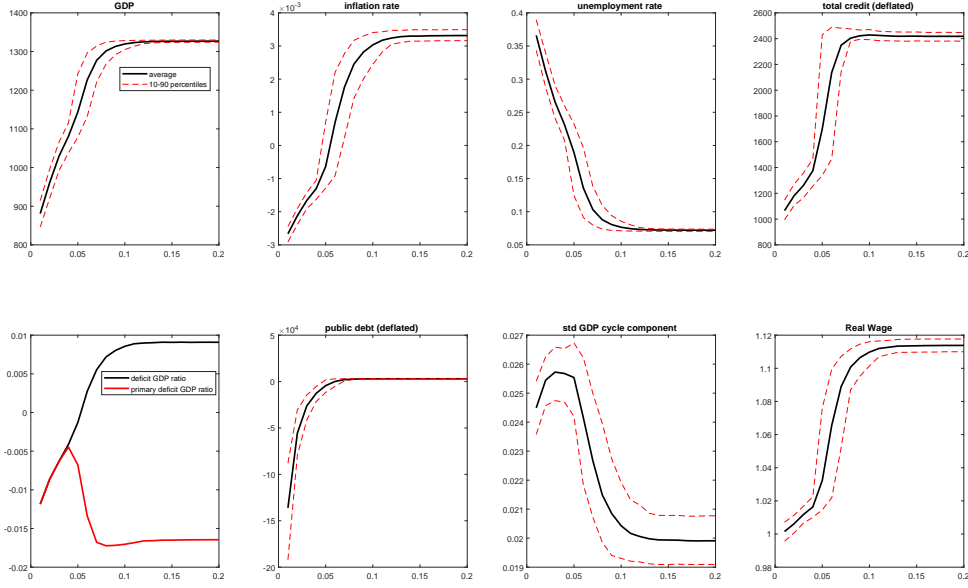
Figure 9: SGP: sensitivity analysis. In each panel we show the long run mean of each variable generated by the simulations for each level of $\hat{d}$, given $t_w = 6\%$, $z = 50\%$ and $g = 0$

there is a deficit for the vast majority of thresholds. The primary budget, on the other hand, shows a surplus. The SGP has a contractionary effect for levels of the threshold below 8% or so. The lower the ceiling, the higher the unemployment rate and the lower GDP. The unemployment rate is decreasing in $\hat{d}$, as expected, but it bottoms out around $\hat{d} = 0.08$. This is due to the fact that a high ceiling is not actually constraining the fiscal space of Government.

6 Conclusions

In this paper we have carried out fiscal policy experiments in a MABM with capital and credit along the lines of Assenza et al. (2015). We consider three settings: (1) *Tax and Transfer (TT)* in which the Government collects taxes on wage income to fund unemployment subsidies. (2) *Tax, Transfer and Spend (TTS)*,i.e., TT cum Government expenditure; (3) *Stability and Growth Pact (SGP)* in which, if the deficit/GDP ratio violates a pre-specified ceiling the Government reduces the geneorosity of unemployment benefits. In all the experiments we carry out a thorough sensitivity analysis to assess the robustness of our results.

In a large numebr of scenarios, the unemployment rate tends to long run level (a quasi-equilibrium) which is consistent with a balanced Government budget. We label this phenomenon the *Balanced Budget Emerging Property (BBEP)*. This result is reminiscent of the Blinder-Solow

framework.

From BBEP follows that we can detect a *policy frontier* which associates a specific unemployment rate to each configuration of fiscal parameters. This is a menu for the policy makers, who may drive the macroeconomy to a target unemployment rate by choosing the appropriate fiscal parameters.

From the analysis of sensitivity however, we conclude that *the BBEP is not a universal law.* For instance, in the TTS experiment, sensitivity shows that when Government expenditure is "too high" the BBEP does not hold any more and any attempt to increase GDP by further increasing Government expenditure is counterproductive: the unemployment rate may indeed increase.

Our exploration can be further extended to analyze: (a) the effects of different policy scenarios, for example TT experiments with redistribution from firm owners to workers, both employed and unemployed; (b) the effects of different arrangements as far as the purchase of Government bonds are concerned (in this paper only the bank purchases Government bonds); (c) the interaction of fiscal and monetary policy.

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