Multi-attribute quality competition with imperfect signals

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May 2015

Preliminary

Abstract

We model competition for a multi-attribute service, like health care services, where consumers observe attribute quality imprecisely before deciding on a provider. High quality in one attribute is more important in terms of ex post utility. Attribute quality is stochastic, providers can shift resources in order to increase expected quality in some attributes. Consumers rationally focus on attributes depending on signal precision and beliefs about the providers' resource allocations. When signal precision is such that consumers focus weakly on the less important attribute, any Perfect Bayesian Nash Equilibrium is inefficient. Increasing signal precision can reduce welfare, as the positive effect of better provider selection is overcompensated by the negative effect that a shift in consumer focusing has on provider quality choice. We discuss the providers' incentives for information disclosure.

Keywords: multi-attribute good, quality signals, focusing

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1 Introduction

Many goods and services have multiple relevant quality dimensions, some of which are imperfectly observable before consumption. A prime example are health care services: When choosing doctors, hospitals or taking decisions about nursing homes, patients care about clinical quality, which is difficult to measure and observe, and non-clinical quality factors such as general appeal of the doctor's office or hospital environment, short waiting times and interpersonal skills of the staff. Interestingly, empirical research found that publicized clinical quality scores, primarily in the form of hospital mortality rates, have a positive but only weak effect on hospital choice.[1] Other quality dimensions however seem to play an important role for the choice of health care providers. Goldman and Romley (2008) analyze the role of amenities alongside treatment quality measures on hospital choice for Californian data. They show that various measures of treatment quality of hospitals (e.g. mortality rates) have only a small effect on patient demand while improvements in amenities strongly raise demand. Furthermore, consumers' perceptions of reputation and specialty medical services as well as satisfaction with a prior hospital stay significantly affect hospital choice. Satisfaction with a prior stay may thereby be driven partly by non-clinical factors. Fornara, Bonaiuto, and Bonnes (2006) e.g. show that hospital users' perceived quality improves when the humanization degree of the hospital environment increases.

An important concern in this context is whether a potentially strong demand response to non-clinical quality attributes such as amenities, interpersonal skills or perceived high quality environment leads to a suboptimal quality of care. This would be the case if clinical quality is more important to generate consumer welfare than all other dimensions of care - such that quality should be high on the clinical quality dimension -, but health care providers do not provide sufficiently high quality in the clinical dimension as consumer demand is more responsive to quality differences in other dimensions. However, why should consumers respond more to quality differences in other dimensions than medical quality if medical quality is the important dimension in terms of their realized utility? Generally, why would consumers focus

[1] See e.g. Dranove and Sfekas (2008) and the discussion therein. Jung, Feldman, and Scanlon (2011) analyze the effects of different dimensions of hospital quality on hospital choice for a surgical procedure. They find that consumers tend to use hospitals with better clinical quality scores, even before the scores are publicized, however the effect of clinical quality on hospital choice is again relatively small. Interestingly, public reporting of clinical quality scores did not imply a change in the consumers' informal information about clinical quality with a significant effect on hospital choice.

on an attribute that is less important in terms of consumption utility? A crucial observation is that many quality dimensions can only be observed imperfectly ex ante, and that the precision of information varies across dimensions. In health care, the signals that patients receive on medical treatment quality in hospitals are often weak or imprecise, as it is difficult to effectively measure medical treatment quality beyond mortality rates. However, information about the general appeal of the doctor's office, amenities in hospitals, and interpersonal skills is often available with fairly high precision.

We model provider competition in a market where consumers observe attribute quality of a two-attribute service only imperfectly. Providers can shift resources in order to increase expected quality in either one or the other attribute. A consumer's utility gain from an increase in quality in one attribute is larger than in the other attribute, thus representing the situation where e.g. high quality in the medical treatment dimension is more important for consumer welfare. Consumers receive a two-dimensional signal about realized quality from each provider.

We first define rational focusing on attributes: A consumer focuses on an attribute if a high quality signal in this attribute drives consumer choice. We say that focusing is strong if this holds for any combination of beliefs about the underlying expected quality, whereas focusing is weak if this holds only for symmetric beliefs. With this definition, we can describe consumers' focus on attributes depending on the precision of quality signals in the attributes.

We show that equilibria exist in which providers concentrate resources to the less important attribute. This occurs if, for a given difference in utility gain from increases in quality in the attributes, the quality signal in this attribute is more precise than in the other attribute to the extent that the consumers focus on this attribute. Equilibrium is unique under strong focusing. If signal precisions are such that consumers' focus is even only weakly on the less important attribute, all Perfect Bayesian Nash equilibria are inefficient. Increasing signal precision, e.g. by introducing a signal in the less important attribute, can reduce welfare, as the positive effect of better provider selection due to higher signal precision can be overcompensated by the negative effect that the shift in consumer focusing, induced by the change in signal precision, has on provider quality choice. We show that if the difference in importance of high quality of the two attributes is large enough but not too large, there exist areas of signal precision such that increasing signal precision induces a shift in resources from the more important to the less important attribute induces in the unique equilibrium, which in turn leads to an unambiguous welfare loss despite a positive selection effect from higher signal precision.

We then analyze the market when disclosing information is a strategic choice of providers. Disclosing information is modeled as sending informative signals by e.g. taking part in quality reports or on feedback platforms. The precision of the signals is again exogenous to capture the inherent difficulty of measuring quality in some attributes. We show that there exist equilibria under voluntary information disclosure in which providers concentrate resources on the less important attribute and only publish signals in this attribute. Thus, not only resource allocation, but also signal disclosure might be inefficient. However, there also exist equilibria where providers concentrate resources on the important attribute and only publish information on the important attribute although consumers would focus on the less important attribute if they received signals in both attributes. Then, mandating full disclosure might be welfare-reducing. We discuss disclosure policies like information mandates or a ban on information in some attributes in section 6.

Related Literature

We define rational focusing via the precision of signals that consumers receive about attributes in an environment with imperfect quality information. A consumer, evaluating according to expected utility, focuses on an attribute if, for given ranges in feasible outcomes, the difference in the precision of signals is such that the difference between signal value and expected outcome in this attribute is, compared to the other attribute, low. Focusing here is thus different from focusing models that assume that there is an exogenous difference between decision utility and consumption utility. In Koszegi and Szeidl (2013) e.g., under perfect information, focus weights of attributes in decision utility depend positively on the range of feasible outcomes in attributes.

The literature on markets with multi-attribute competition and quality investment is scarce. Closest to our work are Dranove and Satterthwaite (1992) and Bar-Isaac, Caruana, and Cuñat (2012). In Dranove and Satterthwaite (1992), competing manufacturers sell goods through retailers where retail price is random and customers evaluate quality idiosyncratically. Customers observe prices and quality only with noise and search retailers using an optimal sequential search rule. An increase in the precision of the price observation may then decrease welfare through the indirect effects of a change in the customers' search: Prices fall, but quality is reduced as well. If the latter effect is stronger, increasing precision of the price observation reduces consumer welfare. In contrast, we model a market where homogeneous consumers have a higher ex post utility from one attribute. Instead of searching, customers receive signals from all providers. We show under what conditions on signal precision and beliefs the customers' focus is on the less important attribute and derive the welfare consequences. We also discuss information disclosure choice of providers. Bar-Isaac et al. (2012) analyze monopoly provision of a two-attribute good where quality is imperfectly observable. Contrary to our set-up, they consider active consumers who choose which information to acquire. Customers are heterogenous in their valuation for attributes and can assess quality at a cost. The monopolist can invest in an increase of the probability of high quality in one attribute. A reduction in the customers' costs of acquiring information on the other attribute may then reduce quality investment, the decrease in costs of assessment shifts the consumer that is indifferent between assessing one or the other dimension towards the first attribute, reducing demand and thereby quality investment. In terms of welfare, the direct positive effect of reduction in assessment costs may then be dominated by the negative investment effect to reduce overall consumer welfare.

While the workings in our model show some analogy to the logic of the multitasking literature as Holmstrom and Milgrom (1991), the modelling and conclusions are however quite different. In the multitasking literature, effort substitutability implies complementarity of the optimal (linear) incentive pay for tasks.[2] Better information in the sense of a reduction in the noise of the performance improves the tailoring of incentive pay and does not have a negative value for the principal. In contrast, we consider a market for a multi-attribute service where consumers receive noisy signals about realized quality by competing providers. The key contractual incompleteness in this market is that attributes cannot be separately priced such that consumers do not separately evaluate expected quality and utility differences in each attribute and that consumers cannot commit to ignore signals. Better information in the sense of increasing signal precision may then decrease welfare, as it is individually rational for customers to focus too strongly on high signals in the less important attribute.

[2]Kaarboe and Siciliani (2011) analyze optimal contracting between a purchaser and a partly altruistic provider of health services within the multitasking framework where one quality dimension is verifiable whereas the second is not. Kaarboe and Siciliani (2011) show that provider altruism with respect to health benefit can lead to overall complementarity of qualities even if they are substitutes on the effort cost side such that high powered incentives may be optimal.

2 Model

Consider a two-attribute good or service $q = (q_1, q_2)$ with $q_i \in \{h, l\}$ for $i = 1, 2$ where h stands for high quality and l for standard quality respectively. Two providers A and B provide the service. Quality cannot be contracted on. The provider compensation is a uniform, exogenously set fee $P > 0$ per unit of service provided.[3] Each provider $j \in \{A, B\}$ has fixed resources, which are symmetric across providers, and makes a resource allocation decision $a^j \in [0, 1]$ that specifies how resources are distributed across the two attributes. Quality realization is stochastic with

$$\mathbb{P}(q_1^j = h|a^j) = a^j(1-p) + (1-a^j)p = \mathbb{P}(q_2^j = h|1-a^j) \text{ and } p \in (0, \frac{1}{2})$$

p describes the impact of the resource allocation a^j on quality realization. The lower p the larger the impact of a^j on quality realization. The probability that high quality in the first attribute is provided is the highest for $a^j = 1$ and the lowest for $a^j = 0$ ($P(q_1^j = h|a^j = 1) = 1 - p$ and $P(q_1^j = h|a^j = 0) = p$). For the second attribute it is the other way around. We say that provider j concentrates resources in attribute 1 (2) if he sets $a^j = 1$ ($a^j = 0$). The quality level is realized independently for each attribute.

The assumption how quality realization depends on the resource allocation implicitly incorporates two symmetries: a symmetric impact of resource allocation on quality realization across providers and that quality realization is symmetrically spread around $\frac{1}{2}$. We later discuss how both symmetries might be removed and why then our qualitative results do not change. Variable costs of providing the service are set to 0. Providers maximize expected profit. There is a continuum of consumers K in the market with mass 1. Each consumer $k \in K$ has utility $u(q)$ from consuming a good with quality attributes $q = (q_1, q_2)$ that is additively separable in attributes, i.e. $U(q) = \sum_{i=1}^{2} u_i(q_i)$. Utility gain from high quality versus standard quality is higher for the first attribute than for the second, i.e.

$$\theta = \frac{u_1(q_1 = h) - u_1(q_1 = l)}{u_2(q_2 = h) - u_2(q_2 = l)} > 1.$$

In the health context, for example, attribute 1 is the medical quality of a hospital visit, with attribute 2 the friendliness and attentiveness of the staff and comfort of the rooms. Standard quality in the attribute medical quality could then be interpreted

[3] Fees cannot be set separately for attributes.

with a cure of the health problem with a certain probability of adverse side or medium term effects from the service, whereas high quality is cure of the health problem with a lower associated probability of adverse side or medium term effects from the service.

Without loss of generality we normalize consumption utility of standard quality in both attribute to zero ($u_1(q_1 = l) = u_2(q_2 = l) = 0$) and high quality in the second attribute to 1 ($u_2(q_2 = h) = 1$). This implies $u_1(q_1 = h) = \theta > 1$. As the price P is an exogenously given set fee we interpret $u(q)$ as the net utility of consuming a good, i.e. after paid the price P (or any fraction of it). This includes settings where the consumer might not pay herself for the good like for health services when any health insurer bears the costs. Each consumer's utility from abstaining from consuming the service is $\underline{u} < 0$.

Consumers cannot perfectly observe the quality levels q^A and q^B of provider A and B respectively. Consumers receive signals about realized quality in the attributes of the service from each provider before deciding on a provider. Each consumer receives signals $s^j = (s^j_1, s^j_2) \in \{ll, lh, hl, hh\}$, $j \in \{A, B\}$. Signals s^j_i are generated with error ϵ_i with $\epsilon_i = \mathbb{P}(s_i = h \mid q_i = l) = \mathbb{P}(s_i = l \mid q_i = h) < \frac{1}{2}$, we write $\epsilon = (\epsilon_1, \epsilon_2)$. For better readability we write s^j for the signal a consumer k receives instead of s^j_k. We furthermore might use $s^j = s$ as long as it is clear from the context. We assume that signals are independently distributed, however, the results are the same if this is not the case and for instance all consumers receive the same signal that is generated as described above.

Consumers do not observe the providers' resource allocation decisions. To evaluate signals from providers, consumers have belief $b^j \in [0, 1]$ about the resource allocation a^j, $j \in \{A, B\}$. Given the belief, consumers update about the quality of the service from providers according to Bayes' rule.

We denote the expected utility of a consumer when selecting provider j after receiving a signal $s^j = (s^j_1, s^j_2)$ with errors $\epsilon = (\epsilon_1, \epsilon_2)$ and underlying belief b^j by $U(s^j, b^j, \epsilon)$. When receiving signal s^A from provider A and signal s^B from provider B a consumer then chooses provider A if

$$U(s^A, b^A, \epsilon) > U(s^B, b^B, \epsilon)$$

Ties are broken equally. For ϵ fixed we write $(s|b) \succ (s'|b')$ if $U(s, b, \epsilon) > U(s', b', \epsilon)$, i.e. when observing signal s with underlying belief b a consumer faces a higher expected utility than when observing signal s' with underlying belief b'.

The timing of the game is as follows:

Stage 1: Provider A and provider B simultaneously decide on their resource allocation a^A and a^B, respectively. Consumers do not observe resource allocations.

Stage 2: For each provider the quality level in both attributes is realized.

Stage 3: Each consumer receives iid signals $s_i^j \in \{h, l\}$ on q_i^j for all $i \in \{1, 2\}$ and $j \in \{A, B\}$ on realized quality.

Stage 4: Each consumer chooses a provider.

Stage 5: Consumption utility is realized.

Given the set-up, maximizing profits for providers corresponds to maximizing the probability of being selected as provider. In the following, we analyze perfect Bayesian Nash equilibria (PBE) of the game. We require consumer beliefs to be consistent with provider resource allocations in equilibrium.

3 Focusing on attributes

A consumer receives two signals s, one from each provider. Assume that one of the signals, say from provider A, indicates standard quality in the first and high quality in the second, i.e. $s^A = lh$. The other one indicates high quality in the first and standard quality in the second, i.e. $s^B = hl$. Whether the signal of high quality in the first or in the second attribute is decisive for the consumer's provider choice now does not only depend on the relative ex-post importance of high quality but also on the relative signal precision, for given beliefs and p. If the consumer prefers signal lh over signal hl and therefore picks provider A, high quality in the second attribute drives consumer choice and we say that the consumer focuses on attribute 2.
This is generalized and formalized in the following definition of focusing, where we differentiate between weak and strong focusing.

Definition 1. (Focusing on Attributes) Fix ϵ, p and θ. A consumer...

- ...*strongly focuses* on attribute i if for any two signals $s = (s_1, s_2)$ and $s' = (s'_1, s'_2)$ with $s_i = h$ and $s'_i = l$ signal s is yields higher expected utility independent of the belief, i.e. $(s|b) \succ (s'|b')$ for all beliefs $b, b' \in [0, 1]$.

- ...*weakly focuses* on attribute i if for any two signals $s = (s_1, s_2)$ and $s' = (s'_1, s'_2)$ with $s_i = h$ and $s'_i = l$ signal s yields higher expected utility for symmetric beliefs, i.e. $(s|b) \succ (s'|b)$ for all symmetric beliefs $b \in [0, 1]$.

The definition implies that strong focusing on attribute 1 is equivalent to $(hl|b) \succ (lh|b')$ for all beliefs b, b' and strong focusing on attribute 2 is equivalent to $(lh|b) \succ (hl|b')$ for all beliefs b, b'. It analogously holds with symmetric beliefs for weak focusing. Note that for any given p and θ, whether a consumer focuses on an attribute or not only depends on the signal technology. The definition of focusing is independent of providers' actual actions since only a consumer's beliefs enter the focusing definition.

Naturally, focusing on attributes depends on the signal error $\epsilon = (\epsilon_1, \epsilon_2)$, the realization probability $1 - p$ and the utility weight θ of attribute 1. Intuitively, the smaller the error in one attribute keeping the signal precision in the other attribute fixed, the more informative the signals are in this attribute and the more likely it is that there is focusing on this attribute. The utility factor $\theta > 1$ implies that high quality provided in attribute 1 is more important than high quality provided in attribute 2. Hence, if signal precision in attribute 1 is not lower than in attribute 2, consumers - at least weakly - focus on attribute 1. However, conversely, if signal precision in attribute 2 is higher than in attribute 1, consumers might (weakly) focus on attribute 2 if θ is small enough.
Generally, we can divide the signal error space into focusing areas for given p and θ. The following lemma describes the separating lines for the focusing areas.

Lemma 1. Fix p and $\theta > 1$. Then there exist continuous and increasing functions $f^{s1} \leq f^{w1} \leq f^{w2} \leq f^{s2}$ with $f^i : [0, \frac{1}{2}] \to [0, \frac{1}{2}]$ that divide the signal error space $[0, \frac{1}{2}]^2$ into focusing areas. A consumer...

- *...strongly focuses on attribute* 2 iff $\epsilon_1 > f^{s2}(\epsilon_2)$. There is $\epsilon_2^* < \frac{1}{2}$ such that f^{s2} strictly increases on $[0, \epsilon_2^*]$ and $f^{s2}(\epsilon_2) = \frac{1}{2}$ for all $\epsilon_2 \geq \epsilon_2^*$. $\epsilon_2^* > 0$ iff $\theta < \frac{1}{1-2p}$.

- *...weakly focuses on attribute* 2 iff $\epsilon_1 > f^{w2}(\epsilon_2)$. f^{w2} strictly increases in ϵ_2. Furthermore, $0 < f^{w2}(0) < f^{w2}(\frac{1}{2}) = \frac{1}{2}$.

- *...strongly focuses on attribute* 1 iff $\epsilon_1 < f^{s1}$. f^{s1} strictly increases in ϵ_2 and $0 < f^{s1}(0) < p < f^{s1}(\frac{1}{2}) < \frac{1}{2}$.

- *...weakly focuses on attribute* 1 iff $\epsilon_1 < f^{w1}(\epsilon_2)$. f^{w1} strictly increases in ϵ_2. Furthermore, $0 < f^{w1}(0) < f^{w1}(\frac{1}{2}) = \frac{1}{2}$.

For $\theta \to 1$ all lines converge to the 45-degree-line . For $\theta \to \infty$ the separating line of strict focusing on attribute 1 converges to p and all other lines converge to $\frac{1}{2}$.

Proof. See appendix. □

Figure 1 illustrates the separating lines for $p = 0.25$ and $\theta = 2$. Figure 2 illustrates the separating lines for again $p = 0.25$ but $\theta = 1.4$.

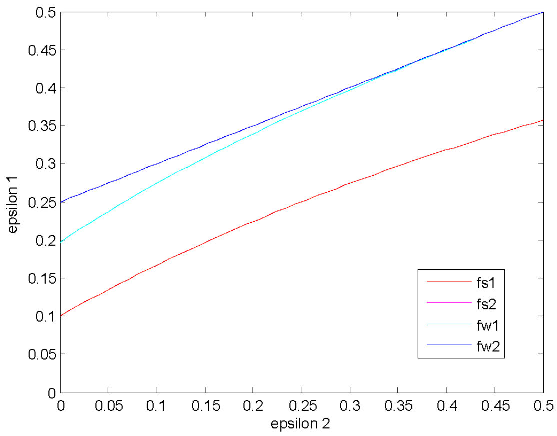

Figure 1: $p = 0.25$ and $\theta = 2$

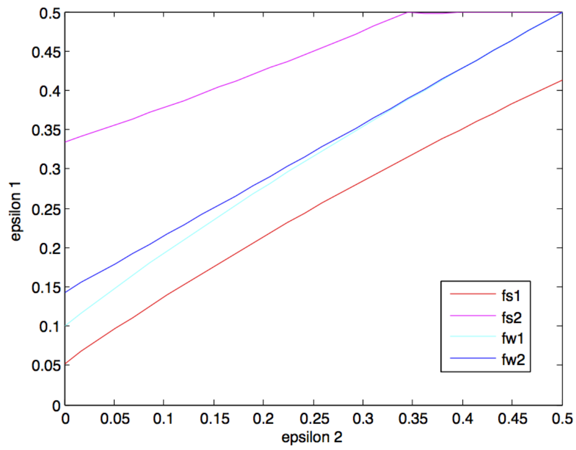

Figure 2: $p = 0.25$ and $\theta = 1.4$

The two figures visualize how the focusing areas change when θ is varied. $\theta > 1$ implies that the area of focusing on attribute 1 is larger than the area of focusing on attribute 2. For large θ ($\theta > \frac{1}{1-2p}$, which is the case in figure 1), attribute 1 is important enough such that the area of strong focusing on attribute 2 vanishes completely. An area of weak focusing exists independent of the magnitude of θ. However, this area becomes arbitrarily small for θ converging to infinity. For $\theta \to 1$, all separating lines converge to the 45-degree-line.

The lemma shows that for a fixed error in one attribute, lowering the error in the other attribute makes the signals in this attribute more important and might shift the focus of a customer towards this attribute. For any $\theta > 1$ and p we can choose ϵ_1 large enough such that lowering ϵ_2 results in a shift from weak focusing on attribute 1 to weak focusing on attribute 2.

For the further analysis, we will be interested in the conditions under which ϵ_1 can be chosen such that there is a shift from strong focusing on attribute 1 to weak focusing on attribute 2 when lowering ϵ_2. Graphically, this translates to finding a horizontal line such that this line crosses both the area of strict focusing on 1 and the area of weak focusing on 2. In our examples, for instance, this is the case for $\epsilon_1 = 0.3$. The following corollary shows that $\theta < \frac{1}{1-2p}$ is a sufficient condition such that for intermediate ϵ_1 the shift occurs from strong focusing on attribute 1 to weak focusing on attribute 2.

Corollary 1. *Fix p and $\theta < \overline{\theta} = \frac{1}{1-2p}$. There exist ϵ_1 such that by varying ϵ_2 the consumers' focus shifts from strong focusing on attribute 1 to weak focusing on attribute 2. This means that for $\epsilon = (\epsilon_1, \epsilon_2)$ with*

- *ϵ_2 large enough consumers strongly focus on attribute 1.*
- *ϵ_2 small enough consumers weakly focus on attribute 2.*

If θ is close enough to 1, even a shift from strong focusing on 1 to strong focusing on 2 can be performed by varying ϵ_2.

Proof. See appendix. □

4 Provider quality incentives and equilibria

With the analysis of consumer focusing, we can examine the providers' incentives to allocate resources between attributes. We say that a strategy of a provider is strictly (weakly) dominant if for any combination of the consumers' beliefs this strategy is strictly (weakly) better than any other strategy. We can show that once consumers strongly focus on one attribute and the signal error in this attribute is lower than the signal error in the other attribute, it is a strictly dominant strategy for the provider to concentrate all resources on this attribute. If focusing is weak but not strong, it is a weakly dominant strategy to concentrate resources in the respective attribute.

Proposition 1. *Let θ, p and $\epsilon = (\epsilon_1, \epsilon_2)$ be such that consumers...*

- *....strongly (weakly) focus on attribute 2. Then it is a strictly (weakly) dominant strategy for any provider j to concentrate resources on attribute 2, i.e. $a^j = 0$.*
- *...strongly (weakly) focus on attribute 1 and $\epsilon_1 \leq \epsilon_2$. Then it is a strictly (weakly) dominant strategy for any provider j to concentrate resources on attribute 1, i.e. $a^j = 1$.*

Proof. See appendix. □

The main idea of the proof is that for fixed beliefs of consumers the resource allocation of the provider does not influence the expected utility of any consumer when receiving a specific signal. This is because consumers cannot observe the investment but perform the Bayesian updating when receiving the signal based on their beliefs. However, what changes when the provider selects a different resource allocation is the probabilities with which the signals are generated. If consumers weakly focus on one attribute and the signal error in this attribute is lower than in the other attribute, concentrating resources on this attribute generates "better" signals with higher probability than any other strategy. While weak focusing on attribute 2 already implies $\epsilon_2 \leq \epsilon_1$, we have to require $\epsilon_1 \leq \epsilon_2$ when considering weak focusing on attribute 1.

The proposition implies that for strong focusing on attribute 2 it is a strictly dominant strategy for the providers to concentrate resources on attribute 2. This holds independent of the beliefs of consumers. For weak focusing, providers might be indifferent between different resource allocations. This crucially depends on the beliefs of consumers. For symmetric beliefs about the providers' resource allocations it is still a strictly dominant strategy for the providers to concentrate resources on attribute 2 when consumers weakly focus on attribute 2. However, if consumers have asymmetric beliefs, selection of the provider might be based only on the beliefs ignoring the signals. Then providers are indifferent between different resource allocations. This occurs, for instance, if consumers belief that provider A concentrated resources on attribute 1 and provider B on attribute 2 and the parameters are such that consumers choose provider A independent of the signal. For instance, $\epsilon = (\epsilon_1, \epsilon_2) = (\frac{1}{2}, 0)$ and $\theta > \frac{1}{1-2p}$ satisfy $(ll|b^A = 1) \succ (hh|b^B = 0)$ from which follows that A is chosen independent of the signal.

Proposition 1 directly implies that if consumers focus on one attribute and the signal error in this attribute is lower than in the other attribute, concentrating resources on this attribute and corresponding beliefs is a Perfect Bayesian Nash Equilibrium.

Corollary 2. *Let θ, p and $\epsilon = (\epsilon_1, \epsilon_2)$ be such that consumers...*

- *... weakly focus on attribute 2. Then it is a PBE that both providers concentrate their resources on attribute 2 and consumers have corresponding beliefs, i.e. $a = b = (0, 0)$.*

- *...weakly focus on attribute 1 and $\epsilon_1 \leq \epsilon_2$. Then it is a PBE that both providers concentrate their resources on attribute 1 and consumers have corresponding* *beliefs, i.e.* $a = b = (1, 1)$.

The symmetric equilibria described in the corollary above might not be unique. In the following we show that strong focusing on an attribute (and $\epsilon_1 \leq \epsilon_2$ for focusing on attribute 1) implies uniqueness of the respective symmetric equilibrium. However under weak focusing further equilibria might exist. We show that if consumers weakly focus on attribute 2, the only further equilibria that might exist are asymmetric equilibria in which consumers select the provider solely based on the beliefs and signals are irrelevant. The same holds for weak focusing on attribute 1 if the signal error in attribute 1 is lower than the one in attribute 2.

Proposition 2. *Let* θ, p *and* $\epsilon = (\epsilon_1, \epsilon_2)$ *be such that consumers...*

- *... weakly focus on attribute* 2*. Any PBE with* $a = b \neq (0, 0)$ *is asymmetric, i.e.* $a^A \neq a^B$ *and consumers select provider A if and only if* $a^A > a^B$*. The symmetric PBE* $a = b = (0, 0)$ *is unique if one of the following conditions holds*
  - *consumers even strongly focus on attribute* 2
  - ϵ_2' *exist such that for* $(\epsilon_1, \epsilon_2')$ *consumers strongly focus on attribute* 1
  - ϵ_1' *exist such that for* $(\epsilon_1', \epsilon_2)$ *consumers strongly focus on attribute* 2
- *... weakly focus on attribute* 1 *and* $\epsilon_1 \leq \epsilon_2$*. Any PBE with* $a = b \neq (1, 1)$ *is asymmetric, i.e.* $a^B \neq a^B$ *and consumers select provider A if and only if* $a^A > a^B$*. Strong focusing on* 1 *implies uniqueness of the symmetric PBE* $a = b = (1, 1)$*.*

For weak focusing multiple equilibria might exist. Note that for weak focusing on attribute 2 however, equilibrium is unique if for the given ϵ_1 some ϵ_2' exists such that for this combination of errors consumers would strongly focus on attribute 1. This is because, if an asymmetric equilibrium exists, with consistent beliefs signal *ll* from the provider with higher a is preferred to signal *hh* from the other provider. This remains to hold when increasing ϵ_2. However, then there is a contradiction with strong focusing, where *hh* is preferred to *ll* for any symmetric or asymmetric beliefs. The intuition for the third condition for uniqueness is the same.
For the cases where multiple equilibria might exist, note that only the symmetric equilibrium where both providers concentrate their resource on the attribute consumers' focus is an equilibrium in dominant strategies of the providers and therefore

also robust to perturbations in consumers' beliefs. Assume that consumers weakly focus on attribute 2 and assume that multiple equilibria exists. As seen in Proposition 1 it is a dominant strategy to concentrate resources on attribute 2 independent of the consumers' beliefs. Thus, even for beliefs outside the equilibrium path (e.g. the consumers' beliefs are not consistent with the equilibrium strategy of the providers) concentrating resources attribute 2 is a dominant strategy. For instance, for symmetric beliefs it is strictly dominant for each provider concentrate resources on attribute 2. Therefore, the equilibrium where both provider concentrate resources on attribute 2 is the only one that is robust with respect to perturbations in the beliefs. All other strategies are weakly dominated for all beliefs of the consumers. This might serve as a selection criteria when consumers weakly focus on attribute 2. The argument holds analogously for equilibrium selection for focusing on attribute 1 when $\epsilon_1 \leq \epsilon_2$ (and therefore consumers weakly focus on attribute 1).

Observable resource allocation

So far, we assumed that the consumers have beliefs about the providers' resource allocation. In the following, we investigate how our results change if consumers can observe the resource allocation, but still do not observe the realization of quality and again receives signals about it. The main difference to the case where the resource allocation is unobservable is that by choosing a particular a the providers now send additional information. This has the following effect: Under unobservable provider choice in Proposition 1, for a certain belief of a consumer a change in a provider's action did not change the expected utility of a signal, but only the probabilities with which the signals are generated. However now, when a is observable, a change in a provider's actions also changes the expected utility of a particular signal.

Then, for parameter constellations where concentrating resources in attribute 2 is a strictly dominant strategy under non-observability of provider choice, putting resources into attribute 1 might be a strictly dominant strategy once resource allocations are observable. If this is case, the inefficiency from the too low expected quality in attribute 1 in equilibrium disappears once the resource allocations are observable. Whether this change occurs depends on the probability $e_2 = \epsilon_2(1-p) + p(1-\epsilon_2)$ that a low signal for attribute 2 is generated if the provider concentrates resources on attribute 2. For low e_2, i.e. if the probability that a high signal is generated in attribute 2 remains high, observability of investments does not influence the equilibrium outcome as concentrating resources in attribute 2 remains more profitable.

However, for large e_2 the equilibrium might differ.

Proposition 3. *(Observable Resource Allocation) Fix* $\theta < \frac{1}{1-2p}$. *Let* $\epsilon = (\epsilon_1, \epsilon_2)$ *be such that consumers strongly focus on attribute* 2. *Define* $e_2 = \epsilon_2(1-p) + p(1-\epsilon_2)$. *If* $e_2 < 1 - \sqrt{\frac{1}{2}}$ *concentrating resources on attribute* 2 *is a strictly dominant strategy such that the corresponding symmetric PBE is unique.*

If $e_2 > \frac{3-\sqrt{5}}{2}$ *concentrating resources on attribute* 1 *is a strictly dominant strategy such that the corresponding symmetric PBE is unique.*

Proof. See appendix. □

The intuition behind this Proposition is that if p or ϵ_2 are rather large (which implies that e_2 is rather large), concentrating resources on attribute 2 does not payoff for the provider as the probability that only a low signal in attribute 2 is generated is high. On the other hand, for non-observable resource allocations with given consumers' beliefs, concentrating resources on attribute 2 might be a dominant strategy as for this only that ϵ_2 is small enough is crucial.

If e_2 is intermediate such that it is not covered by the bounds presented in the proposition, it depends on the specific combination of the parameters whether concentrating resources on attribute 1 or concentrating resources on attribute 2 is strictly dominant.

5 Welfare and comparative statics

We now discuss welfare consequences of consumers focusing on attributes. Note that in the model, total provider surplus is fixed. For the welfare analysis, we will not consider the distribution of producer surplus between provider and henceforth concentrate on consumer welfare. Thus, we will use the terms welfare synonymous to consumer welfare. Denote by $W[a, b, \epsilon]$ (consumer) welfare if providers' resource allocations are $a = (a^A, a^B)$, consumers have belief $b = (b^A, b^B)$ and receive quality signals with error $\epsilon = (\epsilon_1, \epsilon_2)$. Then

$$W[a, b, \epsilon] = \sum_{q^B} \sum_{q^A} \mathbb{P}(q^A|a^A)\mathbb{P}(q^B|a^B)U[(q^A, q^B), b, \epsilon] \tag{1}$$

where $\mathbb{P}(q^j|a^j)$ is the probability that q^j is realized for resource allocation a^j and $U[(q^A, q^B), b, \epsilon]$ is the expected utility of a consumer with belief b if quality (q^A, q^B) is realized and signals are generated with error $\epsilon = (\epsilon_1, \epsilon_2)$.

There are two key drivers of welfare in the market: Firstly, a pure quality aspect, i.e. the expected consumption utility without considering signals, which is determined by the resource allocations. Secondly, a provider selection effect which works through signal precision. This last one is important when considering the welfare effect of changes in signal precision, where a lower error c.p. improves selection based on true underlying quality. Before analyzing changes in the precision of the signals, we first look at welfare for a given signal precision.

Proposition 4. *Fix p and θ. For all $\epsilon = (\epsilon_1, \epsilon_2)$ concentrating resources on 1 and corresponding beliefs yields higher welfare than concentrating resources on 2 and corresponding beliefs, i.e.*

$$W[(1,1),(1,1),(\epsilon_1,\epsilon_2)] > W[(0,0),(0,0),(\epsilon_1,\epsilon_2)]$$

Proof. See appendix. □

Thus, independent of ϵ, i.e. even if ϵ_2 were low and ϵ_1 such that a "correct" selection is more likely on attribute 2, if both providers concentrate resources on attribute 1 and consumers had the corresponding belief, welfare is higher than if both providers concentrate resources on attribute 2 and consumers had the corresponding belief. This is because, for a *given* ϵ, shifting from $a = 0$ to $a = 1$ always (weakly) improves expected consumption utility in the market since $\theta > 1$. Adjusting beliefs correctly from $b = 0$ to $b = 1$ then improves selection.
From Corollary 2 we know that if ϵ is such that consumers strongly focus on attribute 2, the unique PBE has both providers concentrating resources in attribute 2 with corresponding consumer belief. Thus, Proposition 3 implies that when consumers focus on attribute 2, the unique PBE is inefficient. Under weak focusing on attribute 2, from Proposition 2 any equilibrium that is not the one in which both providers choose $a = 0$ is asymmetric and provider j is chosen if and only if $a^j > a^{-j}$. I.e., except for the symmetric equilibrium, in equilibrium a provider is chosen with probability 1, independently of the signals that the customer receives. Then welfare in these equilibria is again lower compared to the situation where both providers concentrate resources on attribute 1 and consumers hold the corresponding belief, as quality provision is inefficient, and there is no selection based on signals.
The above discussion is summarized in the next corollary.

Corollary 3. *Let θ, p and $\epsilon = (\epsilon_1, \epsilon_2)$ be such that consumers focus weakly on attribute 2. Then any PBE is inefficient.*

Proof. For strong focusing, this follows directly from Corollary 2 and Proposition 3. For weak focusing, first consider the symmetric BNE where both providers invest in attribute 2. Then quality provision is inefficient by Proposition 4. Second, consider any other BNE with $a = b = (x^A, x^B)$ and $x^A > x^B$. Proposition 2 showed that consumers then choose provider A ignoring the signals sent. Thus, expected utility is $(1-p)\theta+p$ (if $x^A = 1$) or even smaller (if $x^A < 1$). If both providers invest in attribute 1 welfare is strictly higher as signals are then valuable to consumers and by selection based on the signals they receive an expected utility higher than $(1-p)\theta + p$. □

The interesting question is whether increasing signal precision increases welfare. For ϵ_1 large enough we saw that by increasing the precision in the second attribute we might move from an equilibrium where both provider invest in attribute 1 to an equilibrium where both invest in attribute 2. From above, the latter is not efficient. The welfare effect when increasing the precision is not obvious as there are two opposed effects. On the one hand, increasing the precision might lead to a "worse" provision of quality. On the other hand, consumers are able to make more sophisticated choices and can better select the providers with high quality realizations. In the following we show that there exist parameter ranges such that increasing signal precision in attribute 2 for given ϵ_1 unambiguously leads to a reduction in welfare if it shifts both providers from investing in attribute 1 to both providers investing in attribute 2.

Proposition 5. *Fix $\theta > \underline{\theta} = \frac{1-p-p^2}{1-2p}$, p and ϵ_1. Consider any ϵ_2 and ϵ_2' such that consumers weakly focus on attribute 2 for $\epsilon = (\epsilon_1, \epsilon_2')$. Then the following holds*

$$W[(0,0),(0,0),(\epsilon_1,\epsilon_2')] < W[(1,1),(1,1),(\epsilon_1,\epsilon_2)]$$

Proof. See appendix. □

There is a lower bound on θ which ensures that, even for a maximal improvement in welfare from increasing signal precision – which would the case for a change from $\epsilon_2 = 1/2$ to $\epsilon_2 = 0$ –, the effect of reducing expected quality in attribute 1 with the shift in resource allocation, dominates. Proposition 4 implies particularly that if a change in ϵ_2 causes a shift from an equilibrium where both providers invest in attribute 1 to an equilibrium where both providers invest in attribute 2, there is an unambiguous welfare loss. This is made precise in the following corollary.

Corollary 4. *Fix p and $\theta > \underline{\theta}$. Consider $\epsilon = (\epsilon_1, \epsilon_2)$ with $\epsilon_1 \leq \epsilon_2$ and $\epsilon' = (\epsilon_1, \epsilon_2')$ such that for ϵ consumers weakly focus on attribute 1 and for ϵ' consumers weakly* *focus on attribute 2. Then an increase in the signal precision of attribute 2 from ϵ_2 to ϵ_2' results in a welfare loss in the unique PBE in dominant strategies.*

If, furthermore, $\underline{\theta} < \theta < \overline{\theta}$, ϵ and ϵ' can be chosen such that for ϵ consumers strongly focus on attribute 1 and for ϵ' consumers weakly focus on attribute 2. Then an increase in the signal precision of attribute 2 from ϵ_2 to ϵ_2' results in a welfare loss in the unique PBE.

For $\epsilon = (\epsilon_1, \epsilon_2)$ and $\epsilon' = (\epsilon_1, \epsilon_2)$ such that consumers weakly focus on attribute 1 for $\epsilon = (\epsilon_1, \epsilon_2)$ and weakly on attribute 2 for $\epsilon' = (\epsilon_1, \epsilon_2)$ multiple equilibria might exist. Therefore it is a priori not clear which equilibria are selected and thus whether a reduction in welfare occurs when lowering ϵ_2 to ϵ_2'. However, as discussed the symmetric equilibria stands out as it is an equilibrium in dominant strategies and robust with respect to perturbations in the beliefs. When only concentrating on equilibria in dominant strategies, for any $\theta > \underline{\theta}$ the welfare loss occurs when lowering ϵ_2 such that it induces a switch from weak focusing on attribute 1 to weak focusing on attribute 2.

From Corollary 1 we know that for $\theta < \overline{\theta} = \frac{1}{1-2p}$ there exist ϵ_1 such that for $\epsilon = (\epsilon_1, \frac{1}{2})$ consumers strongly focus on attribute 1 and for $\epsilon = (\epsilon_1, 0)$ consumers weakly focus on attribute 2. Thus, we indeed can choose ϵ and ϵ' as described above. Furthermore, Corollary 2 and Proposition 2 showed that in this case the equilibria are unique. $\theta > \underline{\theta}$ ensures that there is a welfare loss.

6 Information disclosure

So far we assumed that each consumer receives informative signals for both attributes, for instance by quality reports of an institute where participation is mandatory for providers. However, it might be a choice of providers to send quality signals, e.g. via quality reports, participation in evaluation, or an establishment of an online feedback platform. To incorporate information provision by the providers, we now change the game in the following way: The providers' resource allocations are as in the baseline model not observable. Whether consumers receive signals now depends on an information disclosure decision by providers. Before quality is realized, each provider can decide for each attribute whether to disclose information in terms of a signal or not. We assume that the provider, when deciding about disclosure, cannot influence the precision of the signal. I.e., when disclosing information in attribute 1 he sends a signal about this attribute with error ϵ_1 and when disclosing information in attribute

2 he sends a signal about this attribute with error ϵ_2. The reason that he cannot influence the signal precision might, for instance, be due to a general difficulty in measuring quality in the attribute. In terms of our leading example of hospital quality think of consumers that rate experienced quality in a hospital on a platform. A strategy of a provider thus now consists of the tuple of a resource allocation a and a disclosure decision $d \in \{none, s_1, s_2, s_1 s_2\}$ where s_1 (s_2) stands for disclosing only on attribute 1 (2) and $s_1 s_2$ for disclosure in both attributes. Providers simultaneously choose the disclosure and resource allocation strategy. Consumers now might not receive signals in some attribute, but they update their beliefs about resource allocations depending on whether they receive signals in attributes.
We will first analyze voluntary information provision by providers. Then we proceed to analyze the welfare effects of information disclosure policies such as making information disclosure mandatory or even prohibiting information disclosure in some attributes. In this section, we will restrict attention to PBE in dominant strategies.

PBE in dominant strategies under voluntary information disclosure then can be one of two categories: Either information provision only in one attribute with concentration of resources in this attribute, or full information provision, with concentration of resources on the attribute that the consumers focus on when they receive informative signals in both attributes.[4]

A crucial consideration will be how consumers choose providers when one sends only a signal in attribute 1 and the other one only in attribute 2. Once $(h\cdot|1) \succ (\cdot h|0)$ (i.e. a signal of high quality in attribute 2 and no signal in attribute 1 under belief 1 yields higher expected utility than a high quality signal in attribute 1 and no signal in attribute 2 under belief 0), the provider only sending a signal in attribute 1 is selected with probability greater than $\frac{1}{2}$. Then both sending only signal 2 cannot be an equilibrium as sending a signal only in attribute1 is a profitable deviation. Once $(h\cdot|0) \succ (\cdot h|1)$ the provider only sending signal 2 is selected with probability greater than $\frac{1}{2}$ and both sending only signal 1 cannot be an equilibrium. It can be straight forward shown that

$$(h\cdot|1) \succ (\cdot h|0) \forall \epsilon \;\Leftrightarrow\; \theta > \theta^c = \frac{1-p}{1-2p}.$$

This particularly also says that if $\theta < \theta^c$ there exist ϵ (e.g. $\epsilon = (\frac{1}{2}, 0)$ such that $(\cdot h|0) \succ$

[4]I.e., no information provision, or concentration of resources on an attribute for which information is not sent cannot occur in equilibrium.

$(h \cdot |1))$. We can now describe equilibria under voluntary information depending on the level of θ.

Proposition 6. *(i) Fix p and θ. For any ϵ such that consumers weakly focus on attribute 2, it is never an equilibrium that both providers voluntarily disclose both signals. For any ϵ such that $\epsilon_1 < \epsilon_2$ (and therefore consumers weakly focus on attribute 1) it is a unique PBE in dominant strategies that providers disclose information only in attribute 1 and concentrate resources on attribute 1.*

(ii) Fix p and $\theta > \theta^c$. Then there exist ϵ such that consumer weakly focus on attribute 2 when receiving both signals and, however, under voluntary information disclosure in the unique PBE in dominant strategies, providers concentrate resources on attribute 1 and disclose information only on attribute 1. Furthermore, for any ϵ such that consumers weakly focus on attribute 2 when receiving both signals, it is not an equilibrium that both providers disclose information only in attribute 2.

(iii) Fix p and $\theta < \theta^c$. Then there exist ϵ such that in the unique PBE in dominant strategies, providers concentrate resources on attribute 2 and disclose information only on attribute 2 under voluntary information disclosure.

Proof. See appendix. □

Particularly, the proof of the proposition shows that $\epsilon = (\frac{1}{2}, 0)$ satisfies the first claim of (ii) for any $\theta > \theta^c$ and p. Furthermore, once $p > \frac{1}{3}$, it also holds for any ϵ such that the consumers weakly focus on attribute 2 as long as $\epsilon_1 > p$. The claim (iii) is satisfied once ϵ is such that $(\cdot h|0) \succ (h \cdot |1)$. For $\theta < \theta^c$ this is the case, for instance, for $\epsilon = (\frac{1}{2}, 0)$.

By using the results of the last section, welfare of these potential equilibrium outcomes can be partially ranked. For this, recall that $W[a, b, \epsilon]$ denotes the expected (consumer) welfare if providers' resource allocations are $a = (a^A, a^B)$, consumers have belief $b = (b^A, b^B)$ and receive quality signals with errors $\epsilon = (\epsilon_1, \epsilon_2)$. Receiving no signal in an attribute i then corresponds to $\epsilon_i = \frac{1}{2}$.

Keeping the resource allocation constant and only improving precision, we have, by the simple selection effect,

$$W[(1,1),(1,1),(\epsilon_1,\epsilon_2)] > W[(1,1),(1,1),(\epsilon_1,1/2)],$$
$$W[(0,0),(0,0),(\epsilon_1,\epsilon_2)] > W[(0,0),(0,0),(1/2,\epsilon_2)].$$

From a selection and resource allocation effect going in the same direction,

$$W[(1,1),(1,1),(\epsilon_1,\epsilon_2)] > W[(0,0),(0,0),(1/2,\epsilon_2)]$$

From the equilibrium consideration in Section 4 we know that if ϵ is such that consumers weakly focus on attribute 2 and consumers receive both signals, the unique PBE in dominant strategies is that resources are concentrated on attribute 2 with corresponding beliefs. Therefore, in equilibrium, provision of a signal only in attribute 2 yields always less expected welfare than provision of both signals. Furthermore, we know from Proposition 5, that if $\theta > \underline{\theta}$, signal provision only in attribute 1 yields higher welfare than signal provision in both attributes, i.e.

$$W[(1,1),(1,1),(\epsilon_1,1/2)] > W[(0,0),(0,0),(\epsilon_1,\epsilon_2)].$$

In the following we determine equilibria under voluntary information provision. The welfare discussion above will then help to compare voluntary information disclosure with different possible information disclosure policies in terms of welfare.

The proposition implies that, under voluntary information disclosure, there exist equilibria in which providers concentrate resources on an attribute and only publish quality signals in that respective attribute. Thus, it might be the case that not only resource allocation, but also information provision is inefficient. However, as the second part of the proposition and the welfare considerations before show, once θ is such that $\theta > \theta^c$, voluntary information disclosure might even result in providers voluntarily withholding information in attribute 2 and, with it, inducing equilibria with higher welfare compared to forced information disclosure.

From the proposition we can deduce what type of policies regarding information disclosure can improve upon the market outcome. Policies might thereby mandate full disclosure, i.e. in both attributes, or e.g. mandate disclosure on attribute 1, without regulation of information on attribute 2. Another option would be to ban disclosure on attribute 2.

Corollary 5. *(i) Fix p and θ. For any ϵ such that consumers weakly focus on attribute* 1 *and $\epsilon_1 < \epsilon_2$, mandatory information disclosure in attribute* 2 *strictly increases welfare compared to voluntary information disclosure and is an optimal policy.*

(ii) Fix p and $\theta > \theta^c$. Let ϵ be such that consumer weakly focus on attribute 2 *when receiving both signals, and disclosing information only in attribute* 1 *is the unique PBE in dominant strategies under voluntary information disclosure. Volun-*

tary information disclosure is already an optimal policy. The same equilibrium is reached by banning information disclosure in attribute 2 *and by mandating information disclosure in attribute* 1*. Mandating disclosure in all attributes decreases welfare.*

(iii)Fix p *and* $\theta < \theta^c$*. Let* ϵ *be such that disclosing only in attribute* 2 *is the unique equilibrium in dominant strategies under voluntary information disclosure. For* $\theta > \underline{\theta}$*, mandating information disclosure in attribute* 1 *strictly increases welfare but is not necessarily an optimal policy. Banning information disclosure in attribute* 2 *strictly increases welfare and is optimal. For* $\theta < \underline{\theta}$ *mandating information disclosure is optimal for some* ϵ *while banning information disclosure in* 2 *is not. Banning information disclosure in attribute* 2 *might even reduce welfare compared to voluntary disclosure.*

The corollary is based on the results of the proposition above combined with what we know about welfare that was discussed in the beginning of this section. Part (i) of the corollary is straight forward to see as mandating information disclosure in attribute 2 ensures that information is disclosed in both attributes which in turn is optimal.

To discuss parts (ii) and (iii) we can limit our attention to signal errors ϵ such that if receiving both signals consumers weakly focus on attribute 2. An optimal disclosure policy is then such that it induces that in equilibrium either both providers invest in attribute 1 and information is disclosed only in attribute 1 or both providers invest in attribute 2 and information is disclosed in both attributes. The first is desired if

$$W[(1,1),(1,1),(\epsilon_1,1/2)] > W[(0,0),(0,0),(\epsilon_1,\epsilon_2)],$$

the latter if the reverse holds.

First, consider $\theta > \theta^c$ and ϵ such disclosing information only in attribute 1 and is the unique PBE in dominant strategies. From the proposition above we already know that this is the case if ϵ_1 is high enough and ϵ_2 is low enough since it holds for $\epsilon = (\frac{1}{2}, 0)$. Furthermore, it holds for all ϵ that imply weak focusing as long as $\epsilon_1 > p$ and $p > \frac{1}{3}$. $\theta > \theta^c$ implies that particularly $\theta > \underline{\theta}$ and therefore the optimal policy has to induce an equilibrium where both providers invest in 1 and disclose only in 1. Thus, voluntary information disclosure is already optimal while any policy involving mandating disclosure in attribute 2 is harmful to welfare.

Second, consider $\theta < \theta^c$ and ϵ such that disclosing only in attribute 2 is the unique PBE in dominant strategies. Again, by the proposition above, this holds if ϵ_1 is high enough and ϵ_2 is low enough.

For $\underline{\theta} < \theta < \theta^c$, it is desired that information is disclosed only in attribute 1. This can be achieved once banning information disclosure in attribute 2. For ϵ_1 high enough and ϵ_2 low enough, mandatory information disclosure in 1 is not an optimal policy since then providers would additionally disclose information about attribute 2. However, mandatory information disclosure in 1 yields higher expected welfare than complete voluntary disclosure.

For $\theta < \underline{\theta}$ it might be desirable that information about both attributes is available. For ϵ_1 high enough and ϵ_2 low enough, mandating information only in attribute 1 is an optimal policy since then, providers voluntarily disclose information about attribute 2 yielding the desirable outcome. In this case, banning information disclosure in attribute 2 is not optimal. Furthermore, as $\theta < \underline{\theta}$ it might even yield higher welfare if information is disclosed only in attribute 2 than only in 1 which implies that banning disclosure in attribute 2 might even be worse than voluntary information disclosure.

7 Discussion

Symmetries in quality realization.

To make our model tractable it incorporates two symmetries in how the resource allocation impacts quality realization which we will shortly discuss in the following. Both arise from our assumption

$$\mathbb{P}(q_1 = h|a^j) = a^j(1-p) + (1-a^j)p = \mathbb{P}(q_2 = h|1-a^j).$$

We do not need the symmetries for our qualitative results - the symmetries rather shift boarderlines but do not change the qualitative claims. In the following we explain how symmetries can be removed and what implications this has.

Symmetric impact of resource allocation on quality realization. We assume that for any resource allocation decision a^j the probability that high quality is realized in attribute 1 equals the probability of high quality realization in attribute 2 if resources are allocated according to $1-a^j$, i.e. $\mathbb{P}(q_1 = h|a^j) = \mathbb{P}(q_2 = h|1-a^j)$. The parameter p can be interpreted as a measure of how effective resources in both attributes are for quality realization. Our assumption implies that resources have the same impact of quality realization for both attributes.

One way to give up this assumption is to consider different parameters p_1 and p_2

for the effectiveness of the resource allocation, particularly

$$\mathbb{P}(q_1 = h|a^j) = a^j(1 - p_1) + (1 - a^j)p_1$$

$$\mathbb{P}(q_2 = h|1 - a^j) = a^j(1 - p_2) + (1 - a^j)p_2.$$

Once p_1 is smaller than p_2 resources are more effective in attribute 1 than in attribute 2 on quality realization and the other way around. This additional asymmetry does not qualitatively change our results but would only add one additional asymmetry across attributes in addition to signal errors ϵ and relevance θ to our model. Thus, $p_1 < p_2$ would additionally favor investments in attribute 1 while $p_1 > p_2$ would favor investments in attribute 2. This produces a shift in the boarders of focusing as well as when concentrating resources in one attribute is a dominant strategy. The smaller the difference between p_1 and p_2 the closer we come to the presented results. However, since this source of asymmetry across attributes is not focus of our work we do not include it into our basic model while being aware that distortion due to further asymmetries across attributes might occur in applications.

Quality realization probabilities symmetrically spread around $\frac{1}{2}$. A second symmetry behind our assumption on how a^j impact quality realization is that it implies $\mathbb{P}(q_1 = h|a^j) = 1 - \mathbb{P}(q_1 = h|1 - a^j)$. Particularly, if resources are equally split among attributes, the probability of high quality realization is $\frac{1}{2}$ in both attributes. This symmetry can be given up by assuming instead

$$\mathbb{P}(q_1 = h|a^j) = a^j\overline{p} + (1 - a^j)\underline{p} = \mathbb{P}(q_2 = h|1 - a^j) \text{ for } 0 < \underline{p} < \overline{p}.$$

Then, resources still are equally effective in both attributes (see discussion point above), but probabilities of high quality realization are not any more symmetrically spread around $\frac{1}{2}$. Under this assumption, an equal split of resources among attributes implies that the probability of high quality realization is $\frac{\underline{p}+\overline{p}}{2}$ for both attributes. In the following we argue that our qualitative results do not change but only critical values for θ or ϵ might change.

For this, we first consider how the error space is divided into focusing areas (see Lemma 1). For any fixed $(\underline{p}, \overline{p})$ and θ, we again can describe separating lines by monotonically increasing functions. And, again, an area of weak focusing on attribute 2 and attribute 1 and an area of strict focusing on attribute 1 always exist. The area of strict focusing on attribute 2 exists if and only if $\theta < \frac{1}{\overline{p}-\underline{p}}$. Thus, the qualitative picture remains the same.

The incentives for the providers do not change and thus, Proposition 1 can be formulated in the same way. Particularly, once the consumer weakly focus on one attribute and the signal error in this attribute is smaller than in the other attribute, it is a weakly dominant strategy to concentrate resources on this attribute. Strict focusing implies strict dominance.

Considering welfare implications what has to be adjusted is the critical value $\overline{\theta}$ from which on the negative welfare effect of a shift in resources from attribute 1 to attribute 2 dominates the positive welfare effects from selection improvements when increasing signal precision in attribute 2. Particularly, $\underline{\theta} = \frac{1-(1-\overline{p})^2-\underline{p}}{\overline{p}-\underline{p}}$.

Symmetric providers.

We consider symmetric providers in the sense that both face the same signal errors and the same realization probabilities for a resource allocation decision a^j. If we assumed asymmetric provider in the sense that they might differ in ϵ and p the main drivers of the model are the same. What changes is that the focusing areas of consumers look differently for both providers. However, if for both providers consumers (strongly) focus on the same attribute there is no qualitative difference in the results expect that the bounds for critical θ might change. If for one provider the consumer focuses on one attribute and for the other provider on the other attribute, there might be no symmetric equilibrium but only asymmetric ones.

Assumption of homogeneous θ

We set-up the model to particularly look at a situation where consumers are homogeneous and have a higher valuation for some attribute, i.e. where results are not driven by heterogeneous consumer valuations for attributes. However, consumers might of course differ in the consumption utility θ of high quality in the first attribute compared to high quality in the second attribute.[5] In health, for different clinical areas different θ hold. For instance, θ for consumers suffering from cancer should be rather high as clinical factors are much more important than amenities. On the other hand, for births θ might be rather low as generally not many complications are expected. Our results than can be applied for each health area separately. In areas with a high θ concentrating resources on attribute 1 is an equilibrium while in areas with a low

[5]Note that ex-post differences in θ, i.e. differences that occur after the decision for a provider, can be considered as being already incorporated in θ when interpreting utilities for each quality state as expected utilities. Reasons for ex-post heterogeneity includes e.g. differences in quality perception.

θ concentrating resources on attribute 2 might be an equilibrium.
Even within one area θ might differ among consumers. Reasons might be differences in individual preferences or the severity of the individual consumer's health case. Consider any signal error $\epsilon = (\epsilon_1, \epsilon_2)$ with $\epsilon_2 < \epsilon_1$. This implies that there is a threshold θ_2 such that for $\theta < \theta_2$ the consumers strongly focus on attribute 2. It is clear that if for each consumer $k \in K$, $\theta_k < \theta_2$ holds, concentrating resources on attribute 2 is a dominant strategy. Analogously, there is a threshold θ_1 such that if for each consumer $k \in K$, $\theta_k > \theta_1$ holds, concentrating resources on attribute 1 is a dominant strategy. Generally, which effect dominates depends on the distribution of θ in the population. If the mass of consumers whose θ is below (above) the respective critical thresholds is sufficiently large, then concentrating resources on attribute 2 (1) is an equilibrium outcome.

8 Conclusion

We model provider competition in a market where consumers observe attribute quality of a two-attribute service only imperfectly. A consumer focuses on an attribute if a high quality signal in this attribute drives consumer choice. Focusing is strong if this is the case for all combinations of beliefs about underlying expected quality, whereas it is weak for symmetric beliefs. We show that, even if one attribute is less important in terms of consumption utility, the consumer might focus on this attribute such that providers concentrate their resources on quality improvement in this attribute. If signal precision is such that consumers focus weakly on this attribute, any equilibrium is inefficient. An increase in signal precision can lead to a welfare reduction as the positive effect of better selection of the provider from an increase in signal precision is overcompensated by the negative effect that a shift in consumer focusing has on provider quality choice. Furthermore, we discuss providers' incentives to voluntarily disclose or withhold information and the implications for optimal information disclosure policies. In many relevant cases, mandatory disclosure of information in more important attributes leads to unambiguous welfare gains. Mandatory full disclosure, i.e. in all attributes, might however not be an optimal policy. Importantly, it might even be necessary to ban information disclosure in less important attributes, once their precision is high and the negative effect of a shift in resources to those attribute might dominate positive selection effects.

In health care, there has been an increase in the availability of information about provider quality via e.g. quality reporting requirements or public feedback platforms.[6] For hospital report cards, most empirical literature finds positive but small consumer reactions to publicized quality information. Our model is fully consistent with the positive demand effect: if quality reporting reduces signal error only in the medical attribute, it unambiguously increases welfare if the effect is strong enough. However, reporting requirements or the increasing availability of public feedback platforms often also improve the precision of information about other dimensions. Better overall information about health care providers might however imply a higher relative precision of information in the less important quality attributes, with adverse effects on quality. For overall welfare, the disclosure policy is critical. Mandatory information disclosure in more important attributes rarely harms, and banning information disclosure in less important attributes might even be necessary within an optimal disclosure policy.

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[6] An example is the Arztnavigator in Germany. Patients rate their doctor visits on scales for satisfaction with attributes such as the doctor's practice and staff and communication besides treatment.

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Appendix

Proof of Lemma 1. First, we consider an auxiliary function to describe difference in expected utilities when observing a signal hl versus a signal lh for certain beliefs. Then we define the separating lines for the focusing areas by using the characteristics of this auxiliary function.

Auxiliary functions. Consider the following function

$$f(y,z) := \frac{yz}{yz + (1-y)(1-z)} \quad \text{for} \quad y \in [0,1], z \in (0,1)$$

The function $f(y,z)$ has the following properties

- $f(y,z) = f(y,z)$ and $f(y,z)$ is increasing in y and in z (rewrite $f(y,z) = \frac{1}{1+(\frac{1}{y}-1)(\frac{1}{z}-1)}$).
- $f(y,z) + f(1-y, 1-z) = 1$
- $f(y, 1-z) - f(y,z) = f(1-y, 1-z) - f(1-y, z)$ is decreasing in z (follows by monotonicity). For $z < \frac{1}{2}$ it is furthermore increasing in $y \in (0, \frac{1}{2})$ and decreasing in $y \in (\frac{1}{2}, 1)$, analogously for $z > \frac{1}{2}$ it is decreasing in $y \in (0, \frac{1}{2})$ and increasing in $y \in (\frac{1}{2}, 1)$ (follows by symmetry around $y = \frac{1}{2}$ and consideration of the partial derivative with respect to y).

With the help of the function f we can describe the expected utilities of a consumer that performs Bayesian updating when receiving a signal s_i in attribute i from provider j for whom the consumer beliefs that the provider makes an investment decision $a^j = b$. For this we define $x = b(1-p) + (1-b)p$ which is the probability that high quality in attribute 1 is provided. Thus, beliefs $b \in [0,1]$ uniquely correspond to probabilities $x \in [p, 1-p]$ of high quality served in attribute 1. Then, we can describe the expected utilities in the following way.

$$\begin{aligned}
U(s_i, b, \epsilon_i) &= \mathbb{P}(q_i = h|s_i, x)u(q_i = h) \\
&= \frac{\mathbb{P}(s_i|q_i = h)\mathbb{P}(q_i = h|x)}{\mathbb{P}(s_i|x)} u(q_i = h) \\
&= \frac{\mathbb{P}(s_i|q_i = h)\mathbb{P}(q_i = h|x)}{\mathbb{P}(s_i|q_i = h)\mathbb{P}(q_i = h|x) + \mathbb{P}(s_i|q_i = l)\mathbb{P}(q_i = l|x)} u(q_i = h) \\
&= f(a_i, x_i)u(q_i = h) \text{ with } a_i = \mathbb{P}(s_i|q_i = h) \text{ and } x_i = \mathbb{P}(q_i = h|b)
\end{aligned}$$

For $s_i = h$ we have $a_i = 1 - \epsilon_i$ and for $s_i = l$ analogously $a_i = \epsilon_i$. x_i is the probability that high quality is served in attribute i, therefore $x_i = x$ for $i = 1$ and $x_i = 1 - x$ for $i = 2$.

The difference in expected utilities when observing signal hl with underlying belief x and signal lh with underlying belief x' can be as follows.

$$\begin{aligned}
g(\epsilon_1, \epsilon_2, x, x') &= U(hl, x, \epsilon) - U(lh, x', \epsilon) && (2) \\
&= [f(1-\epsilon_1, x) - f(\epsilon_1, x')]\theta - [f(1-\epsilon_2, 1-x') - f(\epsilon_2, 1-x)] && (3) \\
&= [f(1-\epsilon_1, x) - f(\epsilon_1, x')]\theta - [f(1-\epsilon_2, x) - f(\epsilon_2, x')] && (4)
\end{aligned}$$

The last step follows by the characteristics of f. Then $(hl|x) \succ (lh|x') \Leftrightarrow g(\epsilon_1, \epsilon_2, x, x') > 0$ and $(lh|x') \succ (hl|x) \Leftrightarrow g(\epsilon_1, \epsilon_2, x, x') < 0$. Thus, the sign of g will be important for the focusing of the consumers.

g is strictly decreasing in ϵ_1 and strictly increasing in ϵ_2. Therefore, if for $(\epsilon_1^*, \epsilon_2^*)$ a consumer (strictly) focuses on attribute 2 he (strictly) focuses on attribute 2 for all $(\epsilon_1, \epsilon_2^*)$ with $\epsilon_1 > \epsilon_1^*$ and $(\epsilon_1^*, \epsilon_2)$ with $\epsilon_2 < \epsilon_2^*$ as well. the same holds for (strict) focusing on attribute 1 with reversed signs.

Definition of the separating lines We use the function g to describe the separating lines of the four possible focusing areas. For a fixed ϵ_2 define $\epsilon_1^*(x, x')$ as the unique root of $g(\cdot, \epsilon_2, x, x')$ if existent and $\frac{1}{2}$ otherwise. If existent, the root is unique because of the monotonicity characteristics.

- $f^2(\epsilon_2) = \max_{x,x'}\{\epsilon_1^*(x, x')|x, x' \in [p, 1-p]\}$
- $f^{w2}(\epsilon_2) = \max_x\{\epsilon_1^*(x, x)|x \in [p, 1-p]\}$

- $f^{w1}(\epsilon_2) = \min_x\{\epsilon_1^*(x,x)|x \in [p, 1-p]\}$
- $f^1(\epsilon_2) = \min_{x,x'}\{\epsilon_1^*(x,x')|x, x' \in [p, 1-p]\}$

Compactness of $[p, 1-p]$ and continuity of g ensure that both maximum and minimum do exist. The focusing behavior as described in the lemma follows by the definitions of the functions. For this note that $\epsilon_1 = 0$ implies $g = \theta - [f(1-\epsilon_2), x) - f(\epsilon_2, x')] > 0$ independent of the beliefs.

Characteristics of the separating lines. As g is continuous and monotonically decreasing in ϵ_1 and increasing in ϵ_2 the functions f^2 are continuous and increasing in ϵ_2. The more specific characteristics are as follows.

$f^2(\epsilon_2)$: Consider $\theta = \frac{1}{1-2p}$. Then, for any beliefs x, x' we get $g(\frac{1}{2}, 0, x, x') = (x - x')\frac{1}{1-2p} - 1 \leq 0$ with equality $x = 1-2p$ and $x' = p$. Therefore, for $\theta > \frac{1}{1-2p}$ there always exist belief such that no root exist and therefore $f^2(\epsilon_2) = \frac{1}{2}$ for all ϵ_2. No assume $\theta < \frac{1}{1-2p}$. Particularly, $0 < f^2(0) < \frac{1}{2}$. and $f^2(\frac{1}{2}) = \frac{1}{2}$. Define ϵ_2^* such that $g(\frac{1}{2}, \epsilon_2^*, 1-p, p) = 0$. Then the following holds $0 < \epsilon_2^* < \frac{1}{2}$ and for all $\epsilon_2 > \epsilon_2^*$ not root of $g(\cdot, \epsilon_2^*, 1-p, p)$ exists and therefore $f^2(\epsilon_2) = \frac{1}{2}$ for all $\epsilon_2 > \epsilon_2^*$.

$f^{w1}(\epsilon_2)$ and $f^{w2}(\epsilon_2)$: For symmetric beliefs g has the form

$$g(\epsilon_1, \epsilon_2, x, x') = [f(1-\epsilon_1, x) - f(\epsilon_1, x)]\theta - [f(1-\epsilon_2, x) - f(\epsilon_2, x)]$$

$g(0, \epsilon_2, x) = \theta - [f(1-\epsilon_2, x) - f(\epsilon_2, x)] > 0$ and $g(\frac{1}{2}, \epsilon_2, x) = 0 - [f(1-\epsilon_2, x) - f(\epsilon_2, x)] \leq 0$. Due to monotonicity for all beliefs x and all ϵ_2 there exists a unique ϵ_1^* such that $g(\epsilon_1^*, \epsilon_2, x) = 0$. For $\epsilon_2 = \frac{1}{2}$ we have $f^{w1}(\frac{1}{2}) = f^{w2}(\frac{1}{2}) = \frac{1}{2}$. Furthermore, only for $\epsilon_2 = \frac{1}{2}$ the root of g equals $\epsilon_1 = \frac{1}{2}$.

Note that $f^{w1}(\epsilon_2) \geq \epsilon_2$ (g is always negative) for all $\epsilon_1 \leq \epsilon_2$ which means that the consumer weakly focuses on attribute 2. Both functions therefore lie above the 45-degree line.

$f^1(\epsilon_2)$: For $\epsilon_1 = 0$ the function g is always larger than zero (independent of the belief and ϵ_2). For $\epsilon_1 = \frac{1}{2}$ we have $g(\frac{1}{2}, \epsilon_2, p, 1-p) < 0$ independent of ϵ_2. Therefore, the minimum root of $g(;\epsilon_2, x, x')$ is always larger than zero and smaller than $\frac{1}{2}$ which shows $0 < f^1(\epsilon_2) < \frac{1}{2}$ for all ϵ_2. For $\epsilon_2 = 0$ the function g has the form

$$g(\epsilon_1, \epsilon_2, x, x') = [f(1-\epsilon_1, x) - f(\epsilon_1, x')]\theta - 1$$

The smallest root ϵ_1^* occurs for beliefs that minimize g. This is the case for $x = p$ and $x' = 1-p$. Therefore, $f^1(0) = \epsilon_1^*$ with ϵ_1^* being the root of $g = f(1-\epsilon_1, p) -$

$f(\epsilon_1, 1-p)\theta - 1$. This show that $f^1(0) < p$ because for $\epsilon_1 = p$ it is still negative. The same argument holds to show that $f^1(\frac{1}{2}) > p$.

Dependence on θ. First, consider $\theta \to 1$. Then the function g converges to

$$g(\epsilon_1, \epsilon_2, x, x') = [f(1-\epsilon_1, x) - f(\epsilon_1, x')] - [f(1-\epsilon_2, 1-x') - f(\epsilon_2, 1-x)]$$

We show that for any beliefs x and x' the function g is zero if and only if $\epsilon_1 = \epsilon_2$. It can be easily seen that it holds for $x = x'$. Analogously, it holds for $x = p$ and $x' = 1-p$ as well as $x = 1-p$ and $x' = p$. Thus

$$(hl|0) = (lh|1) = (hl|1) = (lh|0).$$

For arbitrary b and b' we then can write the expected utilities of the signals as linear combinations of the expected utilities for the cases $x, x' \in \{p, 1-p\}$. Particularly,

$$\begin{aligned} U(hl, b=1)) &= yU(hl, b=1)) + (1-y)U(hl, b=0) && (5)\\ &= U(hl, b=1)) && (6)\\ &= zU(Q(lh, b=1)) + (1-z)U(lh, b=0) && (7)\\ &= U(lh, b') && (8) \end{aligned}$$

Here, y is the probability that the provider concentrated resources on attribute 1 conditioned on the consumer observed signal hl and has a belief b. z is defined analogously. Therefore, for $\theta \to 1$ the expected utilities when observing hl or lh are the same independent of the underlying beliefs and thus all separating functions converge to

$$f^2(\epsilon_2) = f^{w1}(\epsilon_2) = f^{w2}(\epsilon_2) = f^1(\epsilon_2) = \epsilon_2.$$

Second, consider $\theta \to \infty$. For f^2 we have already seen that $f^2 = \epsilon_2$ for all $\theta > \frac{1}{1-2p}$. If $x = x'$ and θ is arbitrary high, the function g is always positive except for the case that $\epsilon_1 = \frac{1}{2}$. Therefore, $f^{w1} = f^{w2} = \frac{1}{2}$ as well. For arbitrary beliefs, the minimum ϵ_1 for which the function g with $\theta \to \infty$ is zero, is $\epsilon_1 = p$ (and beliefs $x = p$ and $x' = 1-p$. Therefore, f^1 converges to $f^1(\epsilon_1) = p$. □

Proof of Corollary 1. For $\theta \to 1$ all lines converges to the 45-degree-line which implies that for θ small enough a shift from strong focusing on attribute 1 to weak or strong focusing on attribute 2 by lowering ϵ_2 can be performed. It remains to show that for $\theta < \frac{1}{1-2p}$ an error ϵ_1 such that $\epsilon_1 = f^{w2}(0)$ (i.e. $\epsilon = (\epsilon_1, 0)$ is on the separating line for weak focusing on attribute 2) implies $\epsilon_1 < f^{s1}(\epsilon_2)$ (i.e. the error $\epsilon = (\epsilon_1, \frac{1}{2})$ is in the area of strong focusing on attribute 1).

For this, we have to show that for $\epsilon = (p, 0)$ and $\theta < \frac{1}{1-2p}$ the consumer weakly focuses on attribute 2. This is sufficient because the Lemma implies that for $\epsilon = (p, \frac{1}{2})$ the consumer strongly focuses on attribute 1.

For $\epsilon = (p, 0)$ and any belief x we have to show that $(lh|x) > (hl|x)$ for $\theta < \frac{1}{1-2p}$. $(lh|x) > (hl|x)$ is equivalent to $f(p, x)\theta + 1 < f(1 - p, x)\theta$. This is equivalent to $\theta < \frac{1}{f(1-p,x)-f(p,x)}$. From the proof of Lemma 1 we know that $f(1 - p, x) - f(p, x)$ attains its maximum at $x = \frac{1}{2}$. Therefore, $\frac{1}{1-2p} < \frac{1}{f(1-p,x)-f(p,x)}$. As $\theta < \frac{1}{1-2p}$, $\theta < \frac{1}{f(1-p,x)-f(p,x)}$ holds as well and the consumer weakly focuses on attribute 2. □

Proof of Proposition 1. First we show weak focusing of the consumers on attribute i and $\epsilon_i \leq \epsilon_{-i}$ implies that concentrating resources on i weakly dominates concentrating resources on the other attribute - independent of the consumers' beliefs. Second, we show that this implies that concentrating resources on i is a weakly dominant strategy. Third, we show that strong focusing on i and $\epsilon_i \leq \epsilon_{-i}$ implies that concentrating resources on attribute i is a strictly dominant strategy.

Concentrating resources on attribute i weakly dominates concentrating resources on the other attribute. The main idea is that independent of whether the provider concentrates resources on attribute 1 or attribute 2, the same signals are generated. The expected utility of each possible signal does not depend on the allocation decision for given beliefs. What does depend on the allocation decision is the probability of each signal. For weak focusing on attribute i concentrating resources on i generates "more preferred signals" with higher probability compared to concentrating resources on the other attribute.

$\epsilon_1 < \epsilon_2$ and weak focusing on attribute 1: Assume that consumers have any belief b about the providers' strategy (possibly not the same for each provider). Let provider B have any strategy (possibly not known to provider A). We have to show that it is a weakly dominant strategy for provider A to concentrate resources on attribute 1, i.e. $a^A = 1$. A consumer either receives signal ll, lh, hl or hh from provider A. Independent on her belief b^A, a consumer faces the following preference ordering of signals if received from provider A.

$$(hh|b^A) \succ (hl|b^A) \succ (lh|b^A) \succ (ll|b^A).$$

The expected utility a consumer receiving s and having belief b is

$$U(s|b) = \sum_q u(q)\mathbb{P}(q|s,b).$$

Thus, the allocation decision of the providers does not influence the expected utilities consumers with belief b are facing when receiving a signal s. Therefore, the ordering of signals remains the same for A playing $a^A = 1$ and $a^A = 0$. However, the probabilities of the signals depend on the allocation decision of the provider. For playing $a^A = 1$ and $\epsilon_1 \leq \epsilon_2$ the ordering is as follows

$$\mathbb{P}(s = lh|1) < \mathbb{P}(s = ll|1) \leq \mathbb{P}(s = hh|1) < \mathbb{P}(s = hl|1)$$

For playing $a^A = 0$ this ordering is reversed with $\mathbb{P}(s = lh|0) = \mathbb{P}(s = hl|1)$, $\mathbb{P}(s = hh|0) = \mathbb{P}(ll|1)$, $\mathbb{P}(s = ll|0) = \mathbb{P}(s = hh|1)$ and $\mathbb{P}(s = hl|0) = \mathbb{P}(s = lh|1)$. Thus, the probabilities when playing $a^A = 1$ instead of $a^A = 0$ some of probability of $s = ll$ is shifted to hh, and from $s = lh$ to $s = hl$ (better signals have more weight). Thus, for any allocation strategy of B, provider A is selected by any consumer with weakly higher probability when playing $a^A = 1$ instead of $a^A = 0$. This holds independent of the beliefs b about the allocation decision of A and B. Therefore $a^A = 1$ is a weakly dominant strategy.

$\epsilon_1 > \epsilon_2$ and weak focusing on attribute 2: The approach is the same as above. If consumers weakly focus on attribute 2 the signal ordering for any belief b^A is the following

$$(hh|b^A) \succ (lh|b^A) \succ (hl|b^A) \succ (ll|b^A).$$

The signal probabilities for playing $a^A = 1$ have the ordering

$$\mathbb{P}(s = lh|1) < \mathbb{P}(s = hh|1) \leq \mathbb{P}(s = ll|1) < \mathbb{P}(s = hl|1)$$

Here we used that $\epsilon_1 > \epsilon_2$ if the consumer weakly focuses on attribute 2. For playing $a^A = 0$ the ordering reverses with $\mathbb{P}(s = lh|0) = \mathbb{P}(s = hl|1)$, $\mathbb{P}(s = hh|0) = \mathbb{P}(ll|1)$, $\mathbb{P}(s = ll|0) = \mathbb{P}(s = hh|1)$ and $\mathbb{P}(s = hl|0) = \mathbb{P}(s = lh|1)$. Thus, in this case $a^A = 0$ influences the signal probabilities such that better signals have higher probabilities and $a^A = 0$ is a weakly dominant strategy.

Concentrating resources on attribute i is a weakly dominant strategy.

Above we showed that concentrating resources on i weakly dominates concentrating resources on the other attribute. Fix now the belief b of consumers and the allocation strategy q^B of provider B. Then the probability that provider A is selected by the consumers given that provider A chooses $a^j = x \in [0,1]$, i.e. $\mathbb{P}(A|a^A, a^B, b)$, is a linear combination of the respective probabilities for $a^A = 1$ and $a^A = 0$:

$$\mathbb{P}(A|a^A = x, a^B, b) = x\mathbb{P}(A|a^A = 1, a^B, b) + (1-x)\mathbb{P}(A|a^A = 0, a^B, b) \text{ with } x \in [0,1].$$

Therefore, $\mathbb{P}(A|a^A = 1, a^B, b) > \mathbb{P}(A|a^A = x, a^B, b)$ is equivalent to $\mathbb{P}(A|a^A = 1, a^B, b) > \mathbb{P}(A|a^A = 0, a^B, b)$ which show that the dominance of $a^A = 1$ over $a^A = 0$ directly implies the dominance of $a^A = 1$ over $a^A = x$. Thus, concentrating resources on i is a weakly dominant strategy independent of the consumers' beliefs. Therefore, it especially holds if the consumer have consistent beliefs meaning they belief that resources are concentrated on attribute i. Thus $(a^A, a^B) = (b^A, b^B) = (1,1)$ and $(a^A, a^B) = (b^A, b^B) = (0,0)$ describe PBE in the respective cases of errors and weak focusing.

□

Proof of Proposition 2. We first show the parts of the proposition that claim that strong focusing on an attribute implies uniqueness of the PBE. Then we show that any further equilibria are asymmetric and who is selected in asymmetric equilibria. Finally we show that the last two conditions of the first part of the proposition also imply uniqueness. **Strong focusing implies uniqueness.** To show the uniqueness of the equilibrium it is sufficient to show that for strong focusing (and $\epsilon_1 < \epsilon_2$ in case of strong focusing on 1) concentrating resources on this attribute is a strict dominant strategy for the providers. First, assume that consumers strongly focus on attribute 1 and $\epsilon_1 < \epsilon_2$. Furthermore, assume consumers have belief $b = (b^A, b^B)$ and provider B has chosen any strategy.

We already saw that for provider A it is a weakly dominant strategy to choose $a^A = 1$. To show that the equilibrium is unique we first show that either $(lh|b^B) \succeq (lh|b^A)$ or $(hl|b^A) \succeq (hl|b^B)$. This will help us to show the strict dominance.

We now show that either $(lh|b^B) \succeq (lh|b^A)$ or $(hl|b^A) \succeq (hl|b^B)$ indeed hold. For any belief b^j and any signal s there is $x^j(s) \in [0,1]$ such that

$$U(s|b^j) = x^j(s)U(s|1) + (1 - x^j(s))U(s|0) \text{ with } x^j(s) \in [0,1]$$

$x^j(s)$ is strictly monotonically increasing in b^j (the higher the probability that j invested in attribute 1, the higher the weight on $\mathbb{E}u(s|1)$). Therefore, $x^A(s) > x^B(s) \Leftrightarrow x^A(s') > x^B(s')$.

Assume that $(lh|b^A) \succ (lh|b^B)$. Then

$$x^A(lh)U(lh|1) + (1 - x^A(lh))U(lh|0) > x^B(lh)U(lh|1) + (1 - x^B(lh))U(lh|0)$$

We are done if this implies that $(hl|b^A) \succeq (hl|b^B)$, i.e.

$$x^A(hl)U(hl|1) + (1 - x^A(hl))U(hl|0) > x^B(hl)U(hl|1) + (1 - x^B(hl))U(hl|0)$$

This inequality is indeed implied by the first inequality because if $U(lh|0) > U(lh|1)$ it implies $U(hl|0) > U(hl|1)$[7] and by the first inequality $x^A(lh) < x^B(lh)$. This implies $x^A(hl) < x^B(hl)$ and therefore the second inequality holds as well. If on the other hand $U(lh|b^j = 0) < U(lh|1)$, it follows that $U(hl|0) < U(hl|1)$, $x^A(lh) > x^B(lh)$ and $x^A(hl) > x^B(hl)$ which again gives the second inequality.

To see that this implies strict dominance assume that $(hl|b^A) \succeq (hl|b^B)$. Then the probability of A winning if both providers send the signal hl is larger than zero. If A now shifts from playing $a^j = 0$ to playing $a^j = 1$ the probability of sending hl raises on the cost of the probability of send signal lh. But, signal lh sent by A looses versus signal hl sent by B (strong focusing). Thus, the by combining with the results from the first part, the expected probability of player A winning strictly raises when playing $a^j = 1$ and therefore it is a strictly dominant strategy. If instead $(lh|b^B) \succeq (lh|b^A)$ the argumentation is the same.

For strong focusing on attribute 2 the same argument applies. Here we show with the same methods, that either $(lh|b^A) \succeq (lh|b^B)$ or $(hl|b^B) \succeq (hl|b^A)$ holds. In combination with the first part of the proof it implies that the expected probability of A winning strictly raises when playing $a^j = 0$. Therefore, $a^j = 0$ is a strictly dominant strategy.

Asymmetric equilibria.

Assume that consumers weakly focuses on attribute 2 and $a = b = (x^A, x^B) \neq (0, 0)$ is an equilibrium.

First, the equilibrium is not symmetric, i.e. $x^A \neq x^B$. This is because for symmetric beliefs concentrating resources on attribute 2 is a strictly dominant strategy.

[7] $U(lh|0) > U(lh|1)$ is equivalent to $(f(\epsilon_1, 1-p) - f(\epsilon_1, p))\theta > f(1-\epsilon_2, 1-p) - f(1-\epsilon_1, p)$. By the characteristics of f discussed in the lemma about the focusing this is equivalent to $(f(1-\epsilon_1, 1-p) - f(1-\epsilon_1, p))\theta > f(\epsilon_2, 1-p) - f(\epsilon_1, p)$ which is equivalent to $U(hl|0) > U(hl|1)$

Second, we want to show that provider A is selected with probability one if and only if $x^A > x^B$. Without loss of generality assume that $x^A > x^B$. We want to show that this implies already $(ll|x^A) \succ (hh|x^B)$ which means that provider A is selected independent of the signal. Assume the contrary, i.e. $(hh|x^B)$ yields at least the same expected utility as $(ll|x^A)$. Then provider B has an incentive to deviate by choosing $a^B = 0$ instead of $a^B = x^B$: From the proof of Proposition 1 we know $a^B = 0$ is weakly dominant. If $(ll|x^A) \succ (hh|x^B)$ does not holds, it is also strictly dominant because if B send hh and A sends ll provider B is selected with strictly positive probability. If B send ll and A send ll, on the other hand, provider B is never chosen. As a shift from $a^B = x^B$ to $a^B = 0$ generates signal hh with higher probability on the cost of sending signal ll and all other shifts in probabilities are weakly better as well it is strictly dominant for B to concentrate resources on attribute 2.

Thus, if $a = b = (x^A, x^B)$ is a PBE and $x^A > x^B$ provider A is selected with probability one.

The part for weak focusing on attribute 1 follows by the same arguments.

Further conditions for uniqueness.

Assume that for $\epsilon = (\epsilon_1, \epsilon_2)$ consumers weakly focuses on attribute 2 and strictly focuses on attribute 1 for $\epsilon' = (\epsilon_1, \frac{1}{2})$. Assume that for $\epsilon = (\epsilon_1, \epsilon_2)$ the equilibrium is not unique. Particularly, this implies that $(ll|1) \succ (hh|0)$ is equivalent to

$$[f(\epsilon_1, 1-p) - f(1-\epsilon_1, p)]\theta > f(1-\epsilon_2, 1-p) - f(\epsilon_2, p) = 2f(1-\epsilon_2, 1-p) - 1.$$

The right hand side is decreasing in ϵ_2, therefore if $(ll|x^A) \succ (hh|x^B)$ holds for $\epsilon = (\epsilon_1, \epsilon_2)$ it holds as well when ϵ_2 increases and particularly for $\epsilon' = (\epsilon_1, \frac{1}{2})$. If the consumer strongly focuses on attribute 1 for $\epsilon' = (\epsilon_1, \frac{1}{2})$ it is a contradiction because then $(hh|0) \succ (ll|1)$.

Now assume that ϵ_1' is such that for $(\epsilon_1', \epsilon_2)$ consumers strongly focus on attribute 2. No assume that for (ϵ_1, ϵ_2) the equilibrium is not unique. This implies particularly $\epsilon_1 < \epsilon_1'$ and that $(ll|1) \succ (hh|0)$ which is again equivalent to

$$[f(\epsilon_1, 1-p) - f(1-\epsilon_1, p)]\theta < f(1-\epsilon_2, 1-p) - f(\epsilon_2, p)$$

The left hand side is increasing in ϵ_1. Thus, if it holds for any ϵ_1, it also holds for $\epsilon_1' > \epsilon_1$. This contradicts that for $(\epsilon_1', \epsilon_2)$ the consumer strongly focuses on attribute 2 since then $(hh|0) \succ (ll|1)$. □

Proof of Proposition 3. Fix p, θ and ϵ as considered in the proposition. We are interested in the winning probability of provider A if B concentrates resources on attribute 1 and A concentrates resources on attribute 2 when investments are observable (i.e. the consumer has also the corresponding beliefs). If the probability of A winning is larger than one half, it is a strict dominant strategy to concentrate resources on attribute 2. If it is smaller than one half it is a strict dominant strategy to concentrate resources on attribute 1.

To assess the winning probabilities we explicitly consider for which signal combinations A wins. If B sends a signal with $s_2 = l$ and A sends a signal with $s_2 = h$ (which occurs with probability $(1 - e_2)^2$ the consumer selects provider A as she strongly focuses on attribute 2. The only other cases where A might win are the signal combinations $(s^A, s^B) = (hh, lh)$ and $(s^A, s^B) = (hl, ll)$ (whether or not A wins depends again on the parameters). In all other cases B is selected. This follows by the fact that if the same signals are generated provider B is selected and all other remaining signal combinations are implied either by strong focusing or by B winning for the same signals.

Therefore, provider A is selected at least with probability $(1 - e_2)^2$ and at most with probability $(1 - e_2)^2 + 2e_1^2 e_2(1 - e_2)$.

Thus, if $(1-e_2)^2 > \frac{1}{2}$ concentrating resources on attribute 2 is a strictly dominant strategy which is equivalent to $e_2 < 1 - \sqrt{\frac{1}{2}}$.

If $(1 - e_2)^2 + 2e_1^2 e_2(1 - e_2) < \frac{1}{2}$ concentrating resources on attribute 1 is a strictly dominant strategy which is equivalent to $e_2 > \frac{3-\sqrt{5}}{2}$. □

Proof of proposition 4. We will first show that for given symmetric consumers' beliefs with $b^A = b^B = b'$ of the providers' resource allocation,

$$W[(1, x), (b', b'), (\epsilon_1, \epsilon_2)] > W[(0, x), (b', b'), (\epsilon_1, \epsilon_2)]$$

for all $x \in [0, 1]$, i.e. in particular $W[(1, 0), (b', b'), (\epsilon_1, \epsilon_2)] > W[(0, 0), (b', b'), (\epsilon_1, \epsilon_2)]$ and $W[(1, 1(, (b', b'), (\epsilon_1, \epsilon_2)] > W[(0, 1), (b', b'), (\epsilon_1, \epsilon_2)]$, from symmetry of $W[.]$ with respect to providers it follows that $W[(1, 1), (b', b'), (\epsilon_1, \epsilon_2)] > W[(0, 0), (b', b'), (\epsilon_1, \epsilon_2)]$. Note that the only variables in

$$W[(a^A, a^B), (b^A, b^B), (\epsilon_1, \epsilon_2)] = \sum_{q^B} \sum_{q^A} \mathbb{P}(q^A|a^A)\mathbb{P}(q^B|a^B)U(q^A, q^B|b, \epsilon) \tag{9}$$

that depend on the resource allocation decision of provider A are $\mathbb{P}(q^A|a^A)$ for $q^A = hl$ and $q^A = lh$ ($\mathbb{P}(hh|a^A) = \mathbb{P}(ll|a^A) = (1-p)p$ for all a^A). Thus, we need to show that for $b = b', b'$

$$\begin{aligned}
& \sum_{q^B} \mathbb{P}(q^B|a^B)[\mathbb{P}(hl|1)U(hl, q^B|b,\epsilon) + \mathbb{P}(lh|1)U(lh, q^B|b,\epsilon)] && (10)\\
& > \sum_{q^B} \mathbb{P}(q^B|a^B)[\mathbb{P}(hl|0)U(hl, q^B|b,\epsilon) + \mathbb{P}(lh|0)U(lh, q^B|b,\epsilon)] && (11)\\
\Leftrightarrow \quad & \sum_{q^B} \mathbb{P}(q^B|a^B)[(1-p)^2 U(hl, q^B|b,\epsilon) + p^2 U(lh, q^B|b,\epsilon)] && (12)\\
& > \sum_{q^B} \mathbb{P}(q^B|a^B)[p^2 U(hl, q^B|b,\epsilon) + (1-p)^2 U(lh, q^B|b,\epsilon)] && (13)\\
\Leftrightarrow \quad & \sum_{q^B} \mathbb{P}(q^B|a^B)U(hl, q^B|b,\epsilon) && (14)\\
& > \sum_{q^B} \mathbb{P}(q^B|a^B)U(lh, q^B|b,\epsilon) && (15)
\end{aligned}$$

For $q^B = lh$ and $q^B = hl$ we have $U(hl, q^B|b,\epsilon) \geq U(lh, q^B|b,\epsilon)$. Furthermore, $\mathbb{P}(hh|a^b) = \mathbb{P}(ll|a^b) = p(1-p)$ independent of a^b. Thus we have to show that

$$U(hl, hh|b,\epsilon) + U(hl, ll|b,\epsilon) > U(lh, hh|b,\epsilon) + U(lh, ll|b,\epsilon) \tag{16}$$

Note that $U(q^A, q^B|b,\epsilon) = u(q^A)\mathbb{P}(q^A|q^A, q^B, b, \epsilon) + u(q^b)(1 - \mathbb{P}(q^A|q^A, q^B, b, \epsilon))$ where $\mathbb{P}(q^A|q^A, q^B, b, \epsilon)$ is the probability that q^A is chosen by the consumer if quality levels q^A and q^B are realized, consumer has belief b and the signal error is ϵ. Thus the previous inequality is equivalent to

$$\begin{aligned}
& (u(hl) - u(hh))\mathbb{P}(hl|hl, hh, b, \epsilon) + (u(hl) - u(ll))\mathbb{P}(hl|hl, ll, b, \epsilon) && (17)\\
> \quad & (u(lh) - u(hh))\mathbb{P}(lh|lh, hh, b, \epsilon) + (u(lh) - u(ll))\mathbb{P}(lh|lh, ll, b, \epsilon) && (18)\\
\Leftrightarrow \quad & -\mathbb{P}(hl|hl, hh, b, \epsilon) + \theta\mathbb{P}(hl|hl, ll, b, \epsilon) && (19)\\
> \quad & -\theta\mathbb{P}(lh|lh, hh, b, \epsilon) + \mathbb{P}(lh|lh, ll, b, \epsilon) && (20)
\end{aligned}$$

For symmetric belief $b = b', b'$ the following holds:

$$\begin{aligned}
\mathbb{P}(hl|hl, hh, b, \epsilon) = \quad & \mathbb{P}(ll|ll, lh, b, \epsilon) && = 1 - \mathbb{P}(lh|lh, ll, b, \epsilon) && (21)\\
\mathbb{P}(hl|hl, ll, b, \epsilon) = \quad & \mathbb{P}(hh|hh, hl) && = 1 - \mathbb{P}(hl|hl, hh, b, \epsilon) && (22)
\end{aligned}$$

Inserting this into the above inequality reduces the inequality to $\theta > 1$ which holds by definition of θ in our model.

$\square$

Proof of Proposition 5. First note that $W(a, b, (\epsilon_1, \epsilon_2))$ is decreasing in both ϵ_1 and ϵ_2 (the more precise signals the better the consumer can select). Therefore, for any ϵ_1 fixed it is sufficient to show the inequality for $\epsilon_2 = \frac{1}{2}$ and $\epsilon_2' = 0$ because this then implies that the inequality holds for any other ϵ_2 and ϵ_2'.
Denote

$$\Delta W_{10}(\epsilon_1) = W[(1,1),(1,1),(\epsilon_1, \frac{1}{2})) - W((0,0),(0,0),(\epsilon_1, 0)].$$

We first show that $\Delta W_{10}(\frac{1}{2}) > 0$ and then show that this implies the inequality for all other ϵ_1.

To show that $\Delta W_{10}(\frac{1}{2}) > 0$ we explicitly calculate the expected utilities. For $a = (1,1)$, $\epsilon_2 = \frac{1}{2}$ and corresponding beliefs the signal are of no value for the consumer and therefore

$$W[(1,1),(1,1),(\frac{1}{2}, \frac{1}{2})] = (1-p)\theta + p.$$

For $a = (0,0)$, $\epsilon_2 = 0$ and corresponding beliefs $b = (0,0)$ the consumer receives no signal in the first attribute and a precise signal in the second attribute. Thus, in the first attribute high quality is realized with probability p while in the second attribute high quality is realized with probability $1-p^2$ (the consumer weakly focuses on attribute 2 and therefore she only picks low quality in the second attribute if both providers realize low quality). Therefore

$$W[(0,0),(0,0),(\frac{1}{2}, 0)] = p\theta + 1 - p^2.$$

This implies that $\Delta W_{10}(\frac{1}{2}) > 0$ is equivalent to $\theta > \frac{1-p-p^2}{1-2p}$.

Now we show that $\Delta W_{10}(\epsilon_1)$ decreases in ϵ_1 which then implies that $\Delta W_{10}(\epsilon_1) > 0$ for all ϵ_1 as long as the consumer weakly focuses on attribute 2 for $(\epsilon_1, 0)$, i.e. $\frac{\partial}{\partial \epsilon_1} \Delta W_{10}(\epsilon_1) < 0$. The intuition of $\Delta W_{10}(\epsilon_1)$ decreasing in ϵ_1 is as follows: An improvement of the signal quality in the first attribute as a larger effect on expected utility if there is no signal in the second attribute ($\epsilon_2 = \frac{1}{2}$) compared to a precise signal ($\epsilon_2 = 0$). Thus, the welfare difference increases when ϵ_1 decreases.

For explicit calculation we calculate the partial derivative of the expected utilities separately. First, consider $W[(1,1),(1,1),(\epsilon_1, \frac{1}{2})]$. Signals in the second attribute have no value for the consumer. As quality is realized independently for both attributes the consumer's expected utility in the second attribute is p. For the first attribute there are four different combinations of quality realization of the two providers. The consumer faces high quality in the first attribute if both providers realize high quality (occurs with probability $(1-p)^2$) or if one of the providers realizes high quality and the other one standard quality (occurs with probability $2(1-p)p$) and the consumer chooses correctly the provider with the high quality realization (which she does with probability $(1-\epsilon_1)^8$. Thus, for the expected utility the following holds

$$W((1,1),(1,1),(\epsilon_1,\frac{1}{2})) = [2(1-\epsilon_1)(1-p)p + (1-p)^2]\theta + p.$$

Second, consider $W[(0,0),(0,0),(\epsilon_1,0)]$. For this we consider all possible realizations of quality in the second attribute separately. $q_2 = (h,l)$ or $q_2 = (l,h)$ is realized with probability $2p(1-p)$ in both cases the signal of attribute 1 is irrelevant as the consumer weakly focuses on attribute 2 and has a precise signal in attribute 2. Thus the expected utility given realizations $q_2 = (h,l)$ or $q_2 = (l,h)$ is $\theta p + 1$ as utility in the first attribute is realized independent of quality in the second attribute.

If $q_2 = (h,h)$ or $q_2 = (l,l)$ is realized the selection of the provider is only based on the signal in the first attribute. If $q_1 = (h,l)$ or $q_1 = (l,h)$ high quality is selected with probability $(1-\epsilon_1)$. For $q_1 = (h,h)$ the consumer selects high quality in attribute 1 with probability 1 and for $q_1 = (l,l)$ standard quality is selected.

Consolidation of those considerations gives

$$\begin{aligned} W((0,0),(0,0),(\epsilon_1,0)) &= 2(1-p)p(\theta p+1) && (23)\\ &+ (1-p)^2(\theta(p^2+2(1-p)p(1-\epsilon_1))+1) && (24)\\ &+ p^2(\theta(p^2+2(1-p)p(1-\epsilon_1)) && (25) \end{aligned}$$

where the first term represents expected utility of the consumer if $q_2 = (h,l)$ or $q_2 = (l,h)$ is realized, the second if $q_2 = (h,h)$ is realized and the third if $q_2 = (hl,l)$ is realized.

Now we can calculate $\frac{\partial}{\partial \epsilon_1}\Delta W_{10}(\epsilon_1)$ as

$$\frac{\partial}{\partial \epsilon_1}\Delta W_{10}(\epsilon_1) = -2(1-p)p + 2(1-p)^3 p + 2(1-p)p^3$$

[8] If A realizes h and B realizes l, A is chosen with probability $\frac{1}{2}$ if both send the same signal and with probability 1 if A sends h and B sends l. The overall probability that A is choose is the $2\frac{1}{2}\epsilon_1(1-\epsilon_1) + (1-\epsilon_1)^2$

This is always negative as $-2(1-p)p + 2(1-p)^3 p + 2(1-p)p^3 < 0$ is equivalent to $p - 1 < 0$ which always holds.

□

Proof of Proposition 6. (i) For the first part, consider ϵ such that consumers weakly focus on attribute 2 when they receive informative signals in both attributes. To show that in equilibrium providers never disclosure information on both attributes, assume to the contrary that an equilibrium exists with disclosure in both attributes by both providers. Since ϵ is such that consumers weakly focus on attribute 2 when they receive informative signals in both attributes, it is a weakly dominant strategy to concentrate resources on attribute 2.[9] If one provider is disclosing information only in attribute 2 and concentrates resources on attribute 2 and the other discloses information in both attributes and concentrates resources on attribute 2, the first is selected with a higher probability than the latter. Then, disclosing only in attribute 2 is a profitable deviation.

If ϵ is such that $\epsilon_1 < \epsilon_2$ and it can be shown with the same arguments that if one provider is disclosing information only in attribute 1 and concentrates resources on attribute 1 and the other discloses information in both attributes and concentrates resources on attribute 1, the first is selected with a higher probability than the latter. Furthermore, disclosing information only in attribute 1 and concentrating resources on attribute 1 also yields higher selection probability if the other provider does not send any signal or discloses information only in attribute 2 and concentrates resources on attribute 2. This implies that concentrating resources on attribute 1 and disclosing information only in attribute 1 is a unique PBE in dominant strategies.

(ii) Consider any ϵ such that consumers weakly focus on attribute 2 when receiving both signals. Once $\theta > \theta^c$, $(h \cdot |1) \succ (\cdot h|0)$ holds.

We first show the second part of the proposition. If provider B discloses information only in attribute 2 and concentrates resources on attribute 2, provider A is selected with probability greater than $\frac{1}{2}$ if disclosing information only in attribute 1 and concentrating resources on 1. This shows it is not an equilibrium that both providers disclose information only in attribute 2 and concentrate resources on attribute 2.

Now we show the first part. We have already seen that disclosing information only in attribute 1 and concentrating resources on this attribute yields a selection

[9] Note that this remains the same when consumers can update their beliefs about resource allocations depending on receiving signals.

probability greater than $\frac{1}{2}$ if the other provider discloses information only in attribute 2 and concentrates resources on 2. The same holds if the other provider does not disclose any information. If it also holds if the other provider discloses information in both attributes and concentrates resources on attribute 2, then only disclosing information in attribute 1 and concentrating resources on 1 can be a PBE in dominant strategies.

To show that there exist ϵ such that this holds, assume that provider A discloses information only in attribute 1 and concentrates resources on attribute one and provider B discloses information in both attributes and concentrates resources on attribute 2. We know want to know for which ϵ the probability that A is selected is greater than $\frac{1}{2}$. Note that A is always selected when sending h in attribute 1. and B either sends ll, hl or hl. For $\epsilon = (\frac{1}{2}, 0)$ provider A is also selected when B sends hh and A send h in attribute 1. Therefore, for $\epsilon = (\frac{1}{2}, 0)$ provider A is selected with probability greater than $\frac{1}{2}$. Note that there are several other ϵ for which this holds. For instance, once $\epsilon_1 > p$, provider A also is selected when sending l in attribute 1 and provider B sends hl or ll. Then, once $p > \frac{1}{3}$ the total probability that A is selected is greater than $\frac{1}{2}$ which can be shown by explicit calculation.

(iii) The proof is organized as follows. First, we show that if $\theta < \theta^c$ we can choose $\epsilon = (\epsilon_1, \epsilon_2)$ such that $(\cdot h|0) \succ (h \cdot |1)$. Second, we show that $(\cdot h|0) \succ (h \cdot |1)$ implies that consumers strongly focus on attribute 2. Third, we show that $(\cdot h|0) \succ (h \cdot |1)$ is sufficient such that disclosing information only on attribute 2 and concentrating resources on attribute 2 with corresponding beliefs is a PBE. Finally, we show that it is a unique PBE in dominant strategies.

1. Choice of ϵ: For $(\cdot h|0) \succ (h \cdot |1)$ and $x, x' \in [p, 1-p]$ the following holds

$$\begin{aligned} & (\cdot h|0) \succ (h \cdot |1) \\ \Leftrightarrow \quad & p\theta + f(1-\epsilon_2, 1-p) > f(1-\epsilon_1, p)\theta + (1-p) \end{aligned}$$

The left hand side is decreasing in ϵ_2 and the right hand side is decreasing in ϵ_1. So the error for which the inequality is the easiest to fulfill is $\epsilon = (\frac{1}{2}, 0)$. For this error the inequality transfers to $\theta < \frac{1-p}{1-2p}$. Thus, only if $\theta < \frac{1-p}{1-2p}$ there exist an $\epsilon = (\epsilon_1, \epsilon_2)$ such that $(\cdot h|0) \succ (h \cdot |1)$. We just showed that at least for $\epsilon = (\frac{1}{2}, 0)$ it is the case.

2. $(\cdot h|0) \succ (h \cdot |1)$ **implies strong focusing on** 2**:** Consider some ϵ such that $(\cdot h|b = 0) \succ (h \cdot |b = 1)$ holds. We show that this implies that consumers strongly focus on attribute 2 for ϵ. To see this, note that by the considerations above $(\cdot h|b =$

$0) \succ (h \cdot |b = 1)$ is equivalent to

$$f(1-\epsilon_1, 1-p)\theta - f(1-\epsilon_2, 1-p) < p(\theta - 1).$$

On the other hand, if $(lh|0) \succ (hl|1)$, consumers strongly focus on attribute 2[10]. $(lh|b = 0) \succ (hl|b = 1)$ is equivalent to

$$[f(1-\epsilon_1, 1-p)\theta - f(1-\epsilon_2, 1-p) < \frac{1}{2}(\theta - 1).$$

Because $p < \frac{1}{2}$, $(\cdot h|b = 0) \succ (h \cdot |b = 1)$ implies strong focusing on attribute 2.

3. $a = (0, 0)$ **and disclosing information only in attribute** 2 **with consumer belief** $b = (0, 0)$ **forms a PBE:** Consider ϵ such that $(\cdot h|0) > (h \cdot |1)$ holds and assume that both providers concentrate resources on attribute 2 and disclose signals only in attribute 2. Consumers have corresponding beliefs. We show that no provider has an incentive to deviate.

First, concentrating resources on attribute 2 is a weakly dominant strategy, thus there is no incentive to deviate for the providers. Furthermore, any deviation would is weakly dominated. We now show that there is no profitable deviation on resource allocation and disclosure, which we order below by disclosure.

Deviation to some resource allocation and disclosure s_1: Assume that provider A deviates by not disclosing s_2 but s_1. Note, that in this case the belief of the consumer is not necessary constant as it was the case when we considered dominant strategies in Proposition 1. This is because disclosing information is itself a signal about the resource allocation. If A discloses information on s_1 and B on s_2 then by $(\cdot h|0) \succ (h \cdot |1)$, B wins whenever generating a signal h in the second attribute the probability of which is larger than $\frac{1}{2}$ for $a^B = 0$. This however holds for any belief b^A about provider A's resource allocation (and thus for any change in provider A's resource allocation). Thus, provider A does not have any incentive to deviate.

Deviation to some resource allocation and disclosure $s_1 s_2$: Assume that provider A deviates by disclosing both s_1 and s_2. Again, B wins whenever generating signal h in the second attribute - except for A generating hh. On the other hand, B also wins when generating l in the second attribute and A generates ll (this holds again for any beliefs about A's resource allocation). Thus, B wins with probability $(1-e_2)-(1-e_2)^2 e_1+e_2^2(1-e_1)$. Here e_i is the probability that an l signal is generated if

[10]This holds because for $\epsilon_2 \leq \epsilon_1$ and any signal $(s|0) \succ (s|1)$ holds. Thus, $(lh|0) \succ (hl|1)$ is a sufficient condition for strong focusing on attribute 2.

resources are concentrated on attribute i. The term decreases in e_1. Inserting $e_1 = \frac{1}{2}$ then shows that B wins with at least a probability of $(1-e_2)-(1-e_2)^2e_1+e_2^2\frac{1}{2} = \frac{1}{2}$. Therefore A has no incentive to deviate.

Deviation to some resource allocation and disclosure *none*: Assume that provider A deviates by not disclosing any signal. Then again, B wins whenever B sends signal h in the second attribute by $(\cdot h|0) \succ (h\cdot|1)$. Therefore, A has no incentive to deviate.

4. Uniqueness of the PBE: Assume that there exists another PBE. This PBE then has to by symmetric. If it is not symmetric the provider that wins with probability smaller than $\frac{1}{2}$ could just imitate the other provider and with it beliefs turn symmetric and both provider win with probability $\frac{1}{2}$.[11] By the same line of arguments as above, neither disclosing only s_1 nor disclosing both s_1 and s_2 nor disclosing no signal can be a PBE as then one of the providers had an incentive to deviate to disclosing only 2.

□

[11] Note that this is e.g. different from the baseline model without voluntary information disclosure as there is no updating of the belief based disclosure such that there, asymmetric equilibria might exist.