The design of long term care insurance contracts[1]

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Abstract

This paper studies the design of long term care (LTC) insurance contracts in the presence of *ex post* moral hazard. While this problem bears some similarity with the study of health insurance (Blomqvist, 1997) the significance of informal LTC affects the problem in several crucial ways. First, there is the potential crowding out of informal care by market care financed through insurance coverage. Second, this adds a third player to the process, namely the informal caregiver. Third, the information structure becomes more intricate. Informal care is not likely not to be publicly observable and one can expect the caregivers to have better information on the true needs of their relatives than the insurer. We determine the optimal second-best contract and show that the optimal reimbursement rate can be written as an A-B-C expression à la Diamond (1998). Interestingly, informal care directly affects only the first term. More precisely the first term *decreases* with the presence and significance of informal care. Roughly speaking this means that the efficient LTC insurance contract should offer lower (marginal) reimbursement rates than its counterpart in a health insurance context.

1 Introduction

Long-term care (LTC) expenditures represent a significant source of financial uncertainty for elderly households. For example, 70 percent of those who turned 65 need some long-term care before they die. Those 65 and older require assistance for an average of three years over their remaining lifetimes. Although a third will require no long-term services, 20 percent will need care for between two and five years, and another 20 percent will require this assistance for five years or more. Moreover, while these services remain overwhelmingly "low-tech," they are nonetheless extremely expensive. The average "private pay" rate for a single room in a nursing home exceeds \$75,000 per year. Home health aides cost an average of \$18 per hour.[1] Let us add that 18% of those aged 65 or plus live in an institution.[2] This type of care costly, and exceed average pensions by far. The average pension of a French household is about 1,200€ a month, while the average cost of institutional long-term care for in France is currently at 35,000€ per year (see OECD, 2006). The yearly price of a nursing home in the US ranges between \$40,000 and \$75,000 (see Taleyson, 2003).[3]

Standard, insurance theory suggests that the random and costly nature of long-term care makes it precisely the type of risk that would make insurance attractive for risk-averse individuals. Yet in reality, most of the expenditure risk is uninsured. For instance in the US, only 4 percent of long-term care expenditures are paid for by private insurance, while one third are paid for out of pocket. By contrast, in the health sector as a whole, private insurance pays for 35 percent of expenditures and only 17 percent are paid for out of pocket.[4]

There are a host of potential theoretical explanations for the limited size of the private long-term care insurance market, what has been called the LTC insurance puzzle. These explanations have been developed in a number of papers (see, *e.g.*, Pestieau and Ponthiere (2012), Brown and Finkelstein (2007)). Amongst these is the argument that

[1] See Howard Gleckman, H. (2009).

[2] See CBO (2013).

[3] For a more detailed descirbtion of the extent and nature of LTC expenses see Cremer et *al.* (2012).

[4] See Brown and Finkelstein (2007) who argue that *"Long-term care represents one of the largest uninsured financial risks facing the elderly in the United States."*

insurance contracts are badly designed and perceived as "too expensive". While the high price of LTC insurance is in part explained by significant loading costs, the presence of *ex post* moral hazard which makes it difficult for insurers to control costs appears to be another contributing factor.

The limited insurance coverage for long-term care expenditures has important implications for the welfare of the elderly, and potentially for their adult children as well. These implications will only become more pronounced as society ages and as family solidarity is subject to an array of new challenges related to the decline of family norm, the increasing labor participation of middle-age women, the number of childless families and the mobility of children. Consequently, offering an adequately designed insurance contract is a crucial step towards the financing of LTC need during the decades to come.

The design of LTC insurance contracts bears some similarity with that of health insurance. In both cases *ex post* moral hazard appears to be pervasive. This phenomenon refers to the increase of health expenditures, which is due to health insurance coverage; Arrow (1963) and Pauly (1968). Health insurance typically reduces the share of health expenditures that is paid by policyholders. This tends to increase total expenditures, as long as patients' demand is not totally price inelastic. To mitigate this moral hazard problem, insurance coverage is typically less than 100%. Copayments for medical products take various forms. On top of (possibly capped) deductible, patients may face various types of copayments.

The optimal design of the reimbursement scheme has been studied in the health insurance literature, mainly by concentrating on linear (affine) contracts.[5] The efficient nonlinear contract has been studied by Blomqvist (1997). Formally, this question is similar to the Mirrleesian type optimal income tax problem, with the (marginal) reimbursement rate being the counterpart to the marginal tax rate in the optimal tax literature.

The nature of *ex post* moral hazard in the LTC context is of slightly different nature. In that respect, Kessler (2007) identifies three types of *ex post* uncertainty in case of dependency. These are *(i)* the point at which one is considered to have lost autonomy,

[5] See for instance Spence and Zeckhauser (1971) and Besley (1988).

(ii) how severe the loss of autonomy is considered to be and *(iii)* the level of assistance considered to be normal in relation to a certain degree of loss of autonomy. He argues that while the first two risks can easily be dealt with, thanks to the current disability tests, the problem is with the third type of risk that entails a lot of subjectivity. This in turn implies that the problem of *ex post* moral hazard appears to be more "severe" for LTC insurance than for health insurance. Kessler (2007) argues that this pleads for lower (marginal) reimbursement rates within the context of LTC which is in line with the common practice in the French insurance market where payments are typically "flat". In other words, the insured individuals are entitled to a (periodic) lump-sum payment conditional on their (observable) degree of dependency. While this argument affects the form of the efficient contract, the underlying theoretical approach remains essentially identical to that of Blomqvist (1997). Put differently the difference between long-term care and health insurance would hinge essentially on empirical issue and particularly the distribution of "types" and the specification of preferences (which determines the price elasticity of individual demand for formal care).

However, there is another major difference between health and long-term care namely the significance of informal care in the latter case. While medical acts *per se* require the intervention of professionals, a great deal of long-term care is provided informally, by family members, mainly spouses and children. The exact extent of family care is extremely difficult to quantify, precisely because it is by definition informal and not traded in a market. Still, it is widely acknowledged that it represent an important part of total care services. For instance, according to Norton (2000) "*A general rule of thumb is that about two-thirds of care for the elderly is informal care*".

The objective of our paper is to study how the efficient insurance contract should be designed when informal care is accounted for. We show that even from a purely methodological perspective this implies several interesting departures from Blomqvist's analysis. First, there is the potential crowding out of informal care by market care financed through insurance coverage. Second, and even more fundamentally, this adds a third player into the process namely the informal caregiver. Third, the information structure becomes more intricate. Informal care is not likely not to be observable (or at the very least not verifiable). Moreover, one can expect the caregivers to have better information on the true needs of their relatives than the insurer.

We consider an economy where individuals are identical *ex ante.* With some probability, they are healthy and autonomous; otherwise, they become dependent and have LTC needs determined by a parameter θ, which is also used to identify their *ex post* type. Dependent individuals benefit from informal care which depends on their needs which are observable to the caregivers, and on the formal care they receive. Informal care is in part crowded out by formal care. Observe that the variable θ does *not* represent the degree (or severity) of dependency but the variations in individual needs *within* a given class of dependency, as defined for instance by the Katz scale (or Index of Independence in Activities of Daily Living, ADL).[6] In other words we consider only one level of dependency (of the IADL index).[7]

The insurer (which may be private or public) offers a contract which specifies a premium and a reimbursement depending on expenditures on formal care. No *ad hoc* restrictions, such as linearity, on the contract are imposed. However, the insurer's policy in constrained by the information available. It observes neither individual needs nor the level of informal care they receive. Consequently, a first-best outcome with full insurance and reimbursements conditioned on true needs is not feasible.

We determine the optimal second-best contract and show that the optimal reimbursement rate can be written as an A-B-C expression à la Diamond (1998), a decomposition which has been widely used in the optimal tax literature since. Interestingly, informal care directly affects only the first term. More precisely the first term *decreases* with the presence and significance of informal care. Roughly speaking this means that the efficient LTC insurance contract should offer lower reimbursement rates than its counterpart in a health insurance context (for given preferences and distribution of types). We show that this result follows from incentive considerations. The larger this first term, the larger will be (roughly speaking) the difference in the slope of the indifference curve between mimicked and mimicking individuals. Lowering the reimbursement

[6] Which relies on number the basic daily activities for which the persons needs assistance.

[7] Our model could easily be generalized to account for several levels of dependency. Since the IADL index is observable the insurer can use "tagging" and condition the reimbursement on its level.

rate is then a more effective device to relax an otherwise binding incentive constraint.

The paper is organized as follows. In Section 2, we introduce the model. In particular, we define preferences, specify the determination of informal care and the timing of the game. We pay particular attention to the way individual needs affect the willingness to pay for market care, and how this relationship is affected by the presence of informal care. This turns out to be the key factor explaining our results. In Section 3, we consider the case where insurers have full information and can observe the *ex post* level of dependent individuals' needs. In Section 4, we determine the second best contract under asymmetric information when the insurer does not observe needs nor informal care. We concentrate on the way the reimbursement rate is affected by informal care. Section 5 presents some numerical simulations which illustrate how the impact of informal care on the reimbursement *rules* translates into actual *levels*.

2 Model

2.1 Policyholders

Individuals are *ex ante* identical and endowed with a disposable income w. With probability π, they are healthy and autonomous and have utility $v(c_0)$, where c_0 is net consumption; assume $v' > 0$ and $v' < 0$. With probability $(1-\pi)$ they become dependent and have LTC needs determined by a parameter θ, which is also used to identify the *ex post* type. Individual θ has preferences

$$U(\theta) \equiv u(c) + H(z, \theta), \tag{1}$$

where c is net consumption of a numeraire good. The variable $z = m+a$, represents total LTC services consisting of informal care, a, provided by the family and expenditures on market care m. The random variable θ is distributed over $\left[\underline{\theta}, \bar{\theta}\right]$ with a density $f(\theta)$ and a distribution function. We further assume that for every vector (c, z, θ) we have $v(c) > u(c) + H(z, \theta)$: for a given consumption level, healthy individuals are always better off than those suffering from an impairment, irrespective of the level of care that the latter may receive. Using subscripts to denote partial derivative, we have $u_c > 0$, $H_z > 0$, $u_{cc} < 0$, $H_{zz} < 0$, $H_{z\theta} > 0$, $H_\theta < 0$. The representative individual's expected utility is given by

$$EU = \pi v(c_0) + (1-\pi) \int_{\underline{\theta}}^{\bar{\theta}} \left[u(c(\theta)) + H(z(\theta), \theta) \right] dF(\theta)$$

2.2 Insurance market

We consider an insurance market with identical insurers, perfect competition and free entry. In equilibrium, profits are zero; there is no loading factor. Under these assumptions the problem of an insurer is to maximize the expected utility of the representative individual under a zero profit constraint.[8] We first study the case where the insurers can observe the LTC needs θ of each individual *ex post*. Then we turn to the case where LTC needs are not observable by the insurers.

2.3 The timing

The timing of the model is as follows. First, insurance companies choose a menu $\{P, I(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$, specifying the premium P, paid *ex ante* and the reimbursement, $I(\theta)$, as a function of the (observed or declared) state of nature. Second, the state of nature is realized and the variable θ is drawn for the dependent individuals; who choose their LTC expenses $m(\theta)$. Third, the dependent's person family observes the quadruplet $\{\theta, P, I(\theta), m(\theta)\}$, and chooses the level of a to provide. In particular, children observe their parent's θ. In words, unlike the insurance company, they know their parent "true" LTC needs. For the moment we just treat aid as a "black box" and posit that a is determined by the function $a(m, \theta)$. In words, informal aid depends on the needs and on market care. It does *not* depend directly on insurance protection. We shall show below through an example how the "black box" can be opened and the function $a(m, \theta)$ derived. Note that the insurance protection has an indirect impact on a via m. We assume that $-1 < a_m < 0$, and $a_\theta > 0$. The first property means that market care does not completely crowd out informal care. The second one stipulates that informal care increases with dependency needs.

[8] A necessary condition for this to hold is that is that health insurance is exclusive in the sense that policy holders can only be covered by one insurer (e.g see Pauly, 1974).

2.4 Implementation

The insurance company offers individuals a contract at a premium P which specifies a reimbursement $I(m)$. In other words the reimbursement depends on the individuals expenditures on market care. The zero profit condition requires that the premium covers expected reimbursements:

$$P = (1-\pi) \int_{\underline{\theta}}^{\bar{\theta}} I[m(\theta)] f(\theta)\, d\theta. \tag{2}$$

For the subsequent analysis it is convenient to think about, the contract as specifying a reimbursement net of premium equal to $I(m) - P = R(m)$ where $R'(m)$ is the marginal reimbursement rate of the insurance contract. Our objective is to characterize the properties of this reimbursement schedule and, in particular the marginal reimbursement rate. As discussed in the introduction, the typical French LTC insurance contract represents a special and extreme case in that it implies a flat reimbursement so that $R' = 0$. In other countries we observe $R' > 0$ with various reimbursement rates (and deductibles).

Once the state of nature is realized, a dependent individual θ chooses the amount of market care to solve

$$\max_m u(w + R(m) - m) + H(m + a(m, \theta), \theta) \tag{3}$$

Note that insurance protection is given and represented by $R(m)$, while the parents anticipate the level of informal aid $a(m, \theta)$ they will receive. Differentiating (3) yields the following first order condition:

$$-\big(1 - R'(m)\big) u_c + (1 + a_m) H_z = 0 \tag{4}$$

Rearranging yields:

$$MRS_{cm} = \frac{(1 + a_m) H_z}{u_c} = 1 - R'(m) \tag{5}$$

where MRS_{cm} denotes the absolute value of the marginal rate of substitution between consumption and LTC expenses evaluated at the point (m, c). As usual we can think about this MRS as marginal willingness to pay. Observe that when $R'(m) > 0$, LTC expenses are subsidized at the margin and this translates into a *MRS* which is smaller than one. With the subsidy, the price of m (relative to c) is smaller than one.

2.5 Indifference curve properties

For future reference, the following lemma studies some properties of the MRS_{cm} the marginal rate of substitution between consumption and private LTC expenditures.

Lemma 1 *Assume $a_{m\theta} \leq 0$ and $H_{zz\theta} < 0$; then for any given levels of (m, c), we have*[9]

$$\left(\frac{dMRS_{cm}}{d\theta}\right)_{family} < \left(\frac{dMRS_{cm}}{d\theta}\right)_{no\ family}.$$

Proof. Differentiating MRS_{cm} given by the LHS of (5) with respect to θ yields:

$$\begin{aligned} \frac{dMRS_{cm}}{d\theta} &= \frac{(a_{m\theta}H_z + (1+a_m)\,a_\theta H_{zz} + (1+a_m)\,H_{z\theta})}{u_c} \\ &= \frac{1}{u_c}\left[a_{m\theta}H_z + (1+a_m)\,a_\theta H_{zz} + (1+a_m)\,H_{z\theta}\right] \end{aligned}$$

without family help, one has

$$\left(\frac{dMRS_{cm}}{d\theta}\right)_{\text{no family}} = \frac{H_{z\theta}}{u_c} > 0 \tag{6}$$

$$\left(\frac{dMRS_{cm}}{d\theta}\right)_{\text{family}} = \frac{H_{z\theta}}{u_c} + \frac{1}{u_c}\left[a_{m\theta}H_z + (1+a_m)\,a_\theta H_{zz} + a_m H_{z\theta}\right] \tag{7}$$

If $H_{zz\theta} < 0$, $H_{z\theta}$ is higher without family help (for a given level of m, z is necessarily lower without family help). Moreover, $a_{m\theta} \leq 0$ guarantees that the second term in brackets in the RHS of (7) is negative so that:

$$\left(\frac{dMRS_{cm}}{d\theta}\right)_{\text{family}} < \left(\frac{dMRS_{cm}}{d\theta}\right)_{\text{no family}} \quad \text{for any } c, m$$

if $a_{m\theta} \leq 0$ and $H_{zz\theta} \leq 0$ (sufficient conditions). ■

Without informal care, the marginal willingness to pay increases with the dependency needs, since θ increases the marginal of utility of total LTC consumption as measured by H_z. Recall that we assume $H_{z\theta} > 0$. Informal care mitigates this effect because it implies that θ has several negative effects on the *MRS*. First, there is the

[9] When c and m are given, the level of net insurance coverage is fully determined and given by $R(m) = c - w + m$.

crowding out which is measured by a_m. Second, informal care increases by an amount equal to a_θ which decreases the marginal utility of z by $(1+a_m)\,a_\theta H_{zz}$. Finally, informal care affects the value of the crowding out effect, a_m, by $a_{m\theta}$ which is assumed to be negative in the lemma.[10]. Lemma 1 tells that the impact of LTC needs on the marginal willingness to pay for m (in terms of private consumption c) is thus higher when there is no informal care as long as the crowding out effect as measured by a_m is decreasing with dependency needs; this does not appear to be an unreasonable assumption. Put differently, the marginal willingness to pay for private LTC consumption increases less with dependency needs in the presence of informal care.

In the remainder of the paper, we assume that even though the presence of informal care reduces the value of the derivative, it continues to be positive so that

$$\left(\frac{dMRS_{cm}}{d\theta}\right)_{\text{family}} > 0. \tag{8}$$

In words, the willingness to pay for m is continues to be positive when there is family aid. Intuitively, this is the case if informal care from relatives is not crowded out too much by private LTC expenses. The next section gives an example where $a_{m\theta} < 0$ and $H_{zz\theta} < 0$; it also provides a necessary and sufficient condition for condition (8) to hold.

2.6 Example

Assume that preferences of dependent parents are represented by

$$u(c) + H(z-\theta). \tag{9}$$

Their children are altruistic, for given levels of $d = w + R(m)$ and m, they provide a level of informal care determined by

$$a(d,m,\theta,\gamma) = \arg\max_a \left\{\gamma\left[u(d-m) + H(m+a-\theta)\right] - \phi(a)\right\},$$

where γ is the degree of altruism, while $\phi(a)$ represents the disutility of caregiving, with $\phi'(a) > 0$ and $\phi''(a) > 0$. Recall that children observe their parent's needs θ. The level of care is implicitly defined by the following FOC

$$\gamma H_z(m+a-\theta) - \phi'(a) = 0. \tag{10}$$

[10] This is just a sufficient condition.

The SOC is satisfied because

$$SOC = \gamma H_{zzz}\left(m+a-\theta\right) - \phi''\left(a\right) < 0. \tag{11}$$

Differentiating (10) and making use of (11) yields

$$\begin{aligned} a_d &= 0, & (12)\\ a_m &= -a_\theta = -\frac{\gamma H_{zz}\left(m+a-\theta\right)}{SOC} \in]-1,0[. & (13)\end{aligned}$$

This is consistent with the "black box" assumption we made on the function $a\left(m,\theta\right)$ specifying informal aid; see Subsection 2.3. In particular (12) shows that a does not depends on d (given our separability assumption). Furthermore we obtain $-1 < a_m < 0$; informal care decreases with market care, but there is no full crowding out, as long as $\phi\left(a\right)$ is strictly convex. Differentiating

$$MRS_{cm} = \frac{\left(1+a_m\right)H_z(m+a-\theta)}{u_c},$$

with respect to θ and rearranging yields

$$\left(\frac{dMRS_{cm}}{d\theta}\right)_{\text{family}} = \frac{1}{u_c}\left[-\left(1+a_m\right)^2 H_{zz} + a_{m\theta}H_z\right] \tag{14}$$

where we used $a_\theta = -a_m$. Differentiating (13) with respect to θ yields

$$a_{m\theta} = -\frac{\gamma H_{zzz}\phi''\left(a\right)}{\left(\gamma H_{zz} - \phi''\left(a\right)\right)^2},$$

where $\gamma H_{zz} - \phi''\left(a\right) = SOC$ <0, so that $a_{m\theta} \leq 0$ if and only if $H_{zzz} \geq 0$ which also guarantees that $H_{zz\theta} \leq 0$. Since

$$\left(1+a_m\right)^2 = \frac{\left(\phi''\left(a\right)\right)^2}{\left(\gamma H_{zz} - \phi''\left(a\right)\right)^2}$$

one has

$$\left(\frac{dMRS_{cm}}{d\theta}\right)_{\text{family}} = -\frac{\left(\phi''\left(a\right)\right)^2 H_{zz}}{u_c\left(\gamma H_{zz} - \phi''\left(a\right)\right)^2}\left[1 + \frac{\gamma H_{zzz}H_z}{\phi''\left(a\right)H_{zz}}\right]. \tag{15}$$

The first factor in this expression is positive so that the sign depends on the term in brackets. Consequently, expression (15) is positive if and only if $-\gamma H_{zzz}H_z/\phi''\left(a\right)H_{zz} < 1$. As an example, take $H\left(.\right) = \log\left(.\right)$ and $\phi\left(a\right) = \left(1/2\right)a^2$ then $a_{m\theta} \leq 0$, $H_{zz\theta} \leq 0$ and $\left(dMRS_{cm}/d\theta\right)_{\text{family}} > 0$ if and only if $\gamma < 1/2$. In words, this condition requires that the degree of altruism is not too large.

3 Full information insurance scheme

Assume first that each insurer can observe LTC needs θ. Define $d(\theta) = c(\theta) + m(\theta) = w + R(\theta)$, which represents income including net insurance benefits (benefits minus premium). The problem of each insurer is given by

$$\max_{c_0, d(\theta), m(\theta)} EU = \pi v(c_0) + (1-\pi) \int_{\underline{\theta}}^{\bar{\theta}} \left[u(d(\theta) - m(\theta)) + H(m(\theta) + a(m(\theta), \theta), \theta) \right] f(\theta) \, d\theta$$

$$\text{s.t. } w - \pi c_0 - (1-\pi) \int_{\underline{\theta}}^{\bar{\theta}} d(\theta) f(\theta) \, d\theta \geq 0$$

Denoting $\mathcal{L}$ the Lagrangian expression and μ the multiplier associated to the resource constraint, the first-order conditions with respect to $c_0, d(\theta)$ and $m(\theta)$ are given by

$$\frac{\partial \mathcal{L}}{\partial c_0} = v_{c_0} - \mu = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial d(\theta)} = u_c(c(\theta)) - \mu = 0 \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial m(\theta)} = (1 + a_m(m(\theta), \theta)) H_z(z(\theta), \theta) - u_c(c(\theta)) = 0 \tag{18}$$

Equations (16) and (17) yield

$$v_{c_0} = u_c(c(\theta)) = u_c(c(\theta')) \text{ for any } \theta \neq \theta'. \tag{19}$$

Consequently, marginal utility of net consumption is equalized across all states of nature. In other words, and not surprisingly, there is full insurance. Moreover, substituting (16) into (18) yields

$$MRS_{cm}(c(\theta), z(\theta), \theta) = MRS_{cm}(c(\theta'), z(\theta'), \theta') = 1 \tag{20}$$

so that $R'(m(\theta)) = R'(m(\theta')) = 0$ for any θ, θ'. This result is also not surprising. It says that under full information the solution involves no distortion in the tradeoff between c and m. When there is no asymmetry of information and in particular no *ex post* moral hazard, nothing is gained by introducing distortions in the individual choices. To sum up we have a first-best allocation with full insurance.

4 Insurance scheme under asymmetric information

Assume now that insurers cannot observe LTC needs θ. The problem of an insurer is the same as under full information except that the profile $(d(\theta), m(\theta))$ should also satisfy the following incentive constraint

$$\begin{aligned} & u(d(\theta) - m(\theta)) + H(m(\theta) + a(m(\theta), \theta), \theta) \geq \\ & u\left(d\left(\theta'\right) - m\left(\theta'\right)\right) + H\left(m\left(\theta'\right) + a\left(m\left(\theta'\right), \theta\right), \theta\right) \end{aligned} \tag{21}$$

for any θ, θ'. In words, a type θ individual should be at least as well off with the bundle $(d(\theta), m(\theta))$ intended for him than with any of the bundles designed for the other types. Note that when mimicking an individual of another type, informal care continues to be determined by the true type; recall that family members do observe their relative's LTC needs. The corresponding local incentive compatibility constraint is

$$\begin{aligned} \dot{U} &= a_\theta((m(\theta), \theta)) H_z(m(\theta) + a(m(\theta), \theta) \\ &\quad + H_\theta(m(\theta) + a(m(\theta), \theta). \end{aligned} \tag{22}$$

We assume that this constraint is monotonic and that $\dot{U} < 0$ is satisfied.[11] In the absence of informal care this would require no further restriction given that $u_\theta < 0$. However, informal care explains the first term in (22) which is of opposite sign: higher needs lead to a higher level of aid. Monotonicity applies as long as the increase in aid does not overcompensate for the increased needs. One can easily verify that $\dot{U} = H_z(a_\theta - 1) < 0$ in the example described in section 2.6. The program of the insurer can now be written

[11] Equation (8) along with $\dot{m} > 0$ are *sufficient* conditions which ensure that the local incentive constraint (22) implies the global incentive constraint (21).

as follows

$$\max_{c_0,d(\theta),m(\theta)} EU = \pi v\left(c_0\right) + (1-\pi)\int_{\underline{\theta}}^{\bar{\theta}} U\left(\theta\right) f\left(\theta\right) d\theta$$

$$w - \pi c_0 - (1-\pi)\int_{\underline{\theta}}^{\bar{\theta}} d\left(\theta\right) f\left(\theta\right) \geq 0$$

$$U\left(\theta\right) = u\left(d\left(\theta\right) - m\left(\theta\right)\right) + H\left(m\left(\theta\right) + a\left(m\left(\theta\right),\theta\right),\theta\right)$$

$$\dot{U} = a_\theta\left(\left(m\left(\theta\right),\theta\right)\right) H_z\left(m\left(\theta\right) + a\left(m\left(\theta\right),\theta\right),\theta\right)$$

$$+H_\theta\left(m\left(\theta\right) + a\left(m\left(\theta\right),\theta\right),\theta\right)$$

The solution to this problem is derived in Appendix A, where we establish the following proposition

Proposition 1 *The optimal marginal reimbursement rates can be written as*

$$R'\left(m\left(\theta\right)\right) = \mathcal{A}\left(\theta\right)\mathcal{B}\left(\theta\right)\mathcal{C}\left(\theta\right), \tag{23}$$

where

$$\mathcal{A}\left(\theta\right) = \frac{dMRS_{cm}}{d\theta}, \tag{24}$$

$$\mathcal{B}\left(\theta\right) = \frac{\left(1 - F\left(\theta\right)\right)}{f\left(\theta\right)}, \tag{25}$$

$$\mathcal{C}\left(\theta\right) = \int_{\theta}^{\bar{\theta}} \frac{\left(1 - \frac{\mu}{u_c(x)}\right) f\left(x\right) dx}{\frac{\mu}{u_c(\theta)}\left(1 - F\left(\theta\right)\right)}, \tag{26}$$

where $u_c\left(x\right) \equiv u_c\left(d\left(x\right) - m\left(x\right)\right)$ *for any* $x \in \left[\underline{\theta}, \bar{\theta}\right]$, *while* μ *is the Lagrange multiplier associated to the zero profit condition.*

A similar decomposition can be found in the optimal taxation literature, where the marginal income tax is the counterpart to the marginal reimbursement rate in our setting.[12] To our knowledge this decomposition has not been used in the health economics context. In particular it has not been adapted to the (very thin) literature

[12] Except for the sign; we are dealing with a reimbursement rate which is by definition a negative tax.

on optimal non linear health insurance contracts which is formally a special case of our problem. In the context of optimal tax literature the terms, $\mathcal{A}(\theta)$, $\mathcal{B}(\theta)$ and $\mathcal{C}(\theta)$ are often respectively referred to as the "efficiency" term, the shape of the distribution of types, and the preference for redistribution. The latter is to be interpreted here as the preference for risk sharing; our individuals are identical *ex ante* but redistribution occurs between states of nature.[13] While the efficiency term is often presented in terms of elasticities, we use an alternative formulation which brings out the impact of family care more visibly.[14] Observe that all three terms are strictly positive for all $\theta \in]\underline{\theta}, \bar{\theta}[$ Specifically, $\mathcal{A}(\theta) > 0$ follows from equation (8); $\mathcal{B}(\theta)$ is simply the inverse of the hazard rate, which is strictly positive by definition, except for $\underline{\theta}$ where it is equal to zero. Finally, the appendix shows that the transversality conditions imply $\mathcal{C}(\theta) > 0$ for $\theta \in]\underline{\theta}, \bar{\theta}[$ while $\mathcal{C}(\underline{\theta}) = \mathcal{C}(\bar{\theta}) = 0$. Consequently, we obtain that the reimbursement rate R' is strictly positive everywhere except for $\underline{\theta}$ and $\bar{\theta}$ where it is equal to zero. This is in line with the traditional results pertaining to marginal tax rates obtained in the optimal tax literature, which have been shown to apply to health care reimbursement rates by Blomqvist (1997). While these results are not new, it is interesting to note that they remain valid within the context of LTC, when informal care is accounted for.

The advantage of the decomposition presented in expression that it brings out the impact of informal care in a simple and intuitive way. The point it that informal care only directly affects the efficiency term $\mathcal{A}(\theta)$. Put differently, this is the only term which is affected for a given level of m and an insurance schedule $R(m)$ when family help is introduced. Moreover, we know from Lemma 1, that the presence of informal care tends to decrease this term if $a_{m\theta} \leq 0$ and $H_{zz\theta} \leq 0$. We thus may expect that

[13] e.g. see Diamond (1998).

[14] One can easily show that

$$\frac{dMRS}{d\theta} = -\frac{(1 - R'(m))}{\theta}\frac{\alpha_\theta}{\eta_\theta}$$

where α_θ is the elasticity of $m(\theta)$ with respect to θ and η_θ is the compensated elasticity of m with respect to its price $(1 - R'(m))$. As a consequence, (23) can be rewritten as:

$$\frac{R'(m(\theta))}{1 - R'(m(\theta))} = -\frac{\alpha_\theta}{\eta_\theta}\frac{1 - F(\theta)}{\theta f(\theta)} C(\theta)$$

which is the counterpart to the marginal income tax formula that appears for example in Jacquet *et al.* (2013).

introducing family help calls for a lower reimbursement rate if these two last conditions are fulfilled.

Intuitively this result can be understood as follows. The first term measures how the slope of the indifference curves in the (m, c) space is affected by θ. The larger this term, the larger will be (roughly speaking) the difference in the slope of the indifference curve between mimicked and mimicking individuals. And we know from the optimal tax literature that it's precisely the difference in slope of indifference curves which allows to relax an otherwise binding incentive constraint by creating a distortion (which here translates into $R'(m) > 0$). Recall that in a context where the policy is designed to maximize the objective function given the information structure distortions are desirable only when they relax an otherwise binding incentive constraint. In our setting $\dot{U} < 0$ effectively implies that incentive constraints bind from low to high θ's (from "good" to "bad" types). Now, more needy individuals (the mimicked ones) tend to have a higher willingness to pay for m and the upwards distortion which is implied by $R' > 0$ is effective because it hurts the mimicker more than the mimicked individual. This explains why we get a positive reimbursement rate in an optimal health insurance contract. Now, within the context of LTC the "story" is slightly different. More needy individuals can now count on informal care, which in turn reduces their willingness to pay for market care (because they know that market care crowds out informal care). Consequently, the slopes of the indifference curve of mimicked and mimicking individuals become more similar and the benefits of a distortion are mitigated. This in turn explain that informal care will result in "flatter" optimal contracts.

Now, all this is of course based on an expression which is not effectively a closed form solution so it is only valid *ceteris paribus.* While this is common practice in optimal tax models, it does imply that the results have to be interpreted with care. In particular, the levels of m differ when family helps is introduced and the insurance schedule is endogenous. This is why we turn to numerical simulations in the next section. Though of course less general, this allows us to obtain an explicit (though numerical) solution for the optimal insurance scheme and compare it with and without informal care.

5 Simulation results

These simulations are purely illustrative and not meant to rely on a realistic estimation or even calibration. The only realistic feature we have integrated is that informal care represents about two thirds of total care. We represent the dependency needs distribution by a discrete binomial distribution with parameter 5 and 1/3 distributed on the interval $[1, 25]$ using 25 types; see Figure 1. We make the following parametric assumptions: (a) $v(c_0) = -\exp(-c_0)$, (b) $u(c) = -\exp(-c)$, (c) $H(z, \theta) = \log(1 + \theta) * \log(z - \theta)$, (d) $\phi(a) = a^2/2$, and (e) $w = 15$ and $\pi = 0.6$.

We run two simulations. In the benchmark case, there is no family help. The second scenario is calibrated so that family help represents around 60% of total LTC expenditures. Figure 2 illustrates the marginal reimbursement rates in each scenario, the one with informal care being represented by the dashed curve. Comparing the two solutions shows that informal care decreases reimbursement rates for all levels of dependency needs quite substantially. This is perfectly in line with the results presented in Proposition 1. The value added of the example is that the conclusion is based on a full solution, rather than the *ceteris paribus* argument given in the previous section. In other words, we provide an example where the results pertaining to optimal reimbursement rules translate into properties of the effective *levels* of the marginal reimbursement rate.

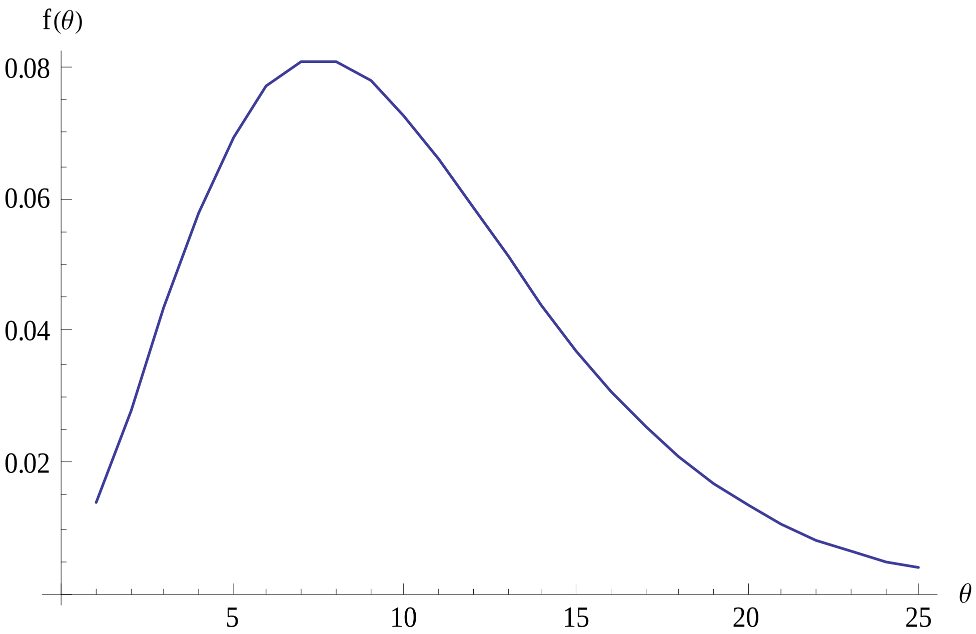

Figure 1: The distribution of types

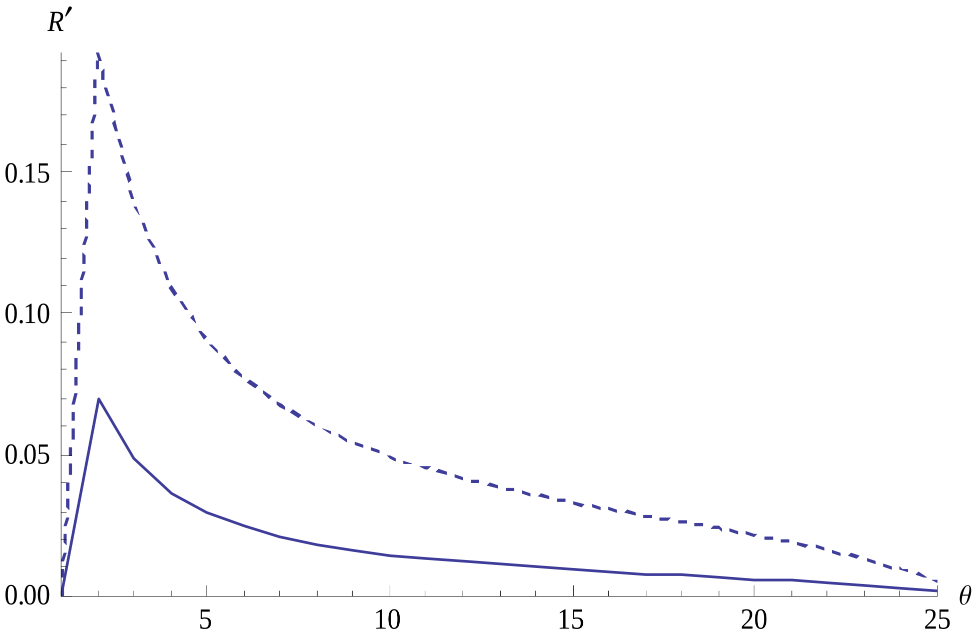

Figure 2: The marginal reimbursement rates as a function of LTC needs.

The numerical application also allows us to make welfare comparisons. Specifically, we can compare the levels of expected utility achieved with the optimal contract EU^* to that achieved under the optimal flat (or pooling) contract, EU^p, where all dependent persons receive the same transfer, irrespective of their LTC expenditures. In other words the pooling contract is such that individuals receive a payment of R^p with probability $(1-\pi)$, that is if they are dependent and no payment otherwise. As mentioned in the introduction, this is the type of LTC contract typically offered by French insurers.

Absent of informal care we have $EU^* = 3.05915$ and $EU^p = 3.05897$. With informal care, on the other hand, we obtain $EU^* = 3.88519$ and $EU^p = 3.88518$. Observe that welfare variations are "small", which is not unusual in optimal tax models. In either case we have of course $EU^* > EU^p$. Not surprisingly also utility is always larger with informal care than without it. Recall that the levels of utility are those of the parents so that informal care can only be beneficial. While all these results are trivial, the figures reveal nevertheless one interesting property, namely that the welfare loss of using a pooling contract rather than the optimal contract is smaller with informal care than without it. Though intuitive, and merely based on an example, this property is interesting because the analytical results, which pertain to marginal reimbursement rates, do not allow for welfare comparisons.

6 Summary and conclusions

This paper has studied the design of the efficient LTC insurance contract. The setting accounts for what is probably the main difference between health and long-term care, namely the role played by informal care. In the case of LTC the problem of *ex post* moral hazard is exacerbated by the possibility that insurance coverage may lead to crowding out of informal care. In addition, the information structure differs from the one considered for instance by Blomqvist (1997) for health insurance, in that informal caregivers have better information about the needs of their relatives than the insurer.

We have derived the expression for the optimal reimbursement rate and shown that it can be decomposed into three terms which resemble the terms that determine the marginal income tax rate in the optimal tax literature. We have shown that only one of these terms directly depends on informal care and that it will tend to reduce the marginal reimbursement rate for all levels of θ. We have illustrated these results by numerical simulations. They have shown that the properties obtained for the optimal reimbursement *rules,* also translate into lower *levels* of the reimbursement rate, at least for the considered specifications. The example also shows that the welfare loss of using a pooling contract rather than the optimal contract is smaller with informal care than without it.

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Appendix

A The derivation of expression (23)

Treating $U(\theta)$ as the state variable and control variables as $d(\theta)$ and $m(\theta)$ and c_0, the Hamiltonian of this problem is

$$\begin{aligned}\mathcal{H} =&\pi v(c_0) + (1-\pi)U(\theta)f(\theta)\\ &-\alpha(\theta)(1-\pi)\left[U(\theta) - u(d(\theta)-m(\theta)) - H(m(\theta)+a(m(\theta),\theta),\theta)\right]\\ &+\mu\left(w-\pi c_0-(1-\pi)d(\theta)f(\theta)\right)\\ &+\lambda(\theta)\left\{a_\theta((m(\theta),\theta))H_z(d(\theta)-m(\theta),m(\theta)+a(m(\theta),\theta),\theta)\right.\\ &\quad\left.+H_\theta(d(\theta)-m(\theta),m(\theta)+a(m(\theta),\theta),\theta)\right\}\end{aligned}$$

where $\lambda(\theta)$ is the costate variable associated with equation (22), $\alpha(\theta)$ is the shadow value of the constraint on $U(\theta)$ and μ is the Lagrange multiplier associated with the resource constraint. From the Pontryagin principle:

$$\dot{\lambda}(\theta) = -\frac{\partial\mathcal{H}}{\partial U(\theta)} = \alpha(\theta) - f(\theta). \tag{27}$$

Differentiating $\mathcal{H}$ with respect to $d(\theta)$ and $m(\theta)$ yields

$$\alpha(\theta)\,u_c - \mu f(\theta) = 0, \tag{28}$$

$$\alpha(\theta)\left[(1+a_m)\,H_z - u_c\right] + \lambda(\theta)\left[a_{\theta m}H_z + a_\theta\,(1+a_m)\,H_{zz} + (1+a_m)\,H_{z\theta}\right] = 0, \tag{29}$$

while the transversality conditions are

$$\lambda(\underline{\theta}) = \lambda\left(\bar{\theta}\right) = 0. \tag{30}$$

Combining equations (28) and (29), we obtain

$$\mu f(\theta)\left[MRS_{cm} - 1\right] = -\lambda(\theta)\left[a_{\theta m}H_z + a_\theta\,(1+a_m)\,H_{zz} + (1+a_m)\,H_{\theta z}\right],$$

so that

$$MRS_{cm} = 1 - \frac{\lambda(\theta)}{\mu f(\theta)}\left[a_{\theta m}H_z + a_\theta\,(1+a_m)\,H_{zz} + (1+a_m)\,H_{\theta z}\right]. \tag{31}$$

Substituting (31) into (5) yields

$$R'(m(\theta)) = \frac{\lambda(\theta)}{\mu f(\theta)}\left[a_{\theta m}H_z + a_\theta\,(1+a_m)\,H_{zz} + (1+a_m)\,H_{\theta z}\right],$$

Using (7), this can be rewritten as

$$R'(m(\theta)) = \frac{\lambda(\theta)}{f(\theta)}\frac{u_c}{\mu}\frac{dMRS_{cm}}{d\theta}. \tag{32}$$

Substituting (28) into (27) yields

$$\dot{\lambda}(\theta) = \left(\frac{\mu}{u_c} - 1\right) f(\theta). \tag{33}$$

Using the transversality conditions, equation (30), we obtain

$$\lambda(\theta) = \int_{\theta}^{\bar{\theta}} \left(1 - \frac{\mu}{u_c(x)}\right) f(x)\,dx, \tag{34}$$

where $u_c(x) = u_c(d(x) - m(x))$. Since $u_c(x)$ is an increasing function of x, equations (30) and (33) imply $\lambda(\theta) > 0$. This establishes that $\mathcal{C}(\theta)$ is positive, except at $\underline{\theta}$ and $\bar{\theta}$. Substituting (34) into (32) and rearranging terms finally yields (23).