Competition and Screening with Motivated Workers*

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Abstract

We study optimal contracts offered by two firms competing for the exclusive services of workers, who are privately informed about their ability and motivation. Firms differ in their technology and in the mission they pursue, and motivated workers are keen to be hired by the mission-oriented firm. In equilibrium, sorting of workers between firms is independent of the distribution of types and is almost always efficient. When no single firm can employ all workers, sorting is ability-neutral. A compensating wage differential might emerge: the mission-oriented firm offers lower wages and lower returns to ability with respect to the standard firm.

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1 Introduction

There exists a well-established empirical evidence on compensating wage differentials that are uniquely generated by differences in job characteristics or attributes for which heterogeneous workers have different willingnesses to pay. For instance, an earnings penalty has been documented for public firms as opposed to private ones and for not-for-profit firms relative to for-profit organizations.¹

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¹For compensating wage differentials see Rosen (1986). The case of public versus private firms has been studied by Disney and Gosling (1998) and Melly (2005), among others. Lower average wages in not-for-profit firms relative to for-profit ones

Among the prominent differences in jobs and/or firms, determining wage gaps, Besley and Ghatak (2005) consider that some firms are often identified as mission-oriented because of the sector they operate in (education, health and defence), whereas, for Bénabou and Tirole (2010), mission-orientation stems from firms' explicit strategies, for example, in terms of corporate social responsibility: some firms take employee-friendly or environment-friendly actions, some employers are mindful of ethics, or they even have an investor-friendly behavior (as ethical banks). Those organizations have in common the pursuit of a mission or goal that is valuable for some workers, precisely those who share such objectives and who are characterized by non-pecuniary motivations, together with the standard extrinsic ones.

The idea that intrinsic motivation for being employed by mission-oriented firms might be the source of wage gaps has been first proposed by Heyes (2005), for caring vs non-caring jobs in the health sector, and by Delfgaauw and Dur (2007) who analyze applicants' tastes for being employed at a specific firm. These studies predict that relatively low pay and weak monetary incentives endogenously emerge in jobs where intrinsic motivation matters.

However, another strand of empirical work points out that the wage differential might arise because of a selection bias, given that a wage gap can also reflect *unobservable* differences in workers' ability across sectors or firms.²

Therefore, an open question still remains. Suppose that a wage penalty for workers employed in mission-oriented sectors or firms is observed, although neither workers' intrinsic motivation nor ability can be directly measured: then, wages can be lower either because of the lower reservation wages of motivated workers or because of the lower productivity of workers self-selecting into such sectors or firms (or because of a combination of these two effects). In other words, when workers' productivity and motivation are the workers' private information, is it possible to disentangle the pure compensating wage differential from the selection effect of ability?

To this respect, Delfgaauw and Dur (2008) characterize the optimal incentive schemes offered by a public, cost-minimizing agency that faces a perfectly competitive private sector and wants to hire workers with unknown laziness (the opposite of productive ability) and public service motivation. They find that, when the public institution has to produce a sufficiently high output, then it attracts all dedicated workers (i.e. individuals characterized by high ability and high public service motivation) as well as the laziest workers (i.e. the ones characterized by low ability and no public service motivation). Lazy workers work less and are paid less than in the private sector whereas dedicated workers are offered higher wages by the public agency. However, the model cannot account for the distribution of workers' laziness (ability) between the two sectors and is therefore not informative about the selection effect of ability.

In our paper, we consider a labor market characterized by two profit-maximizing firms, a missionhave been found by Preston (1989) and Gregg et al. (2011).

²See Goddeeris (1988), Hwang et al. (1992), Gibbons and Katz (1992), Goux and Maurin (1999).

oriented and a standard firm. The two firms compete to attract workers who are heterogeneous with respect to both their skills and their intrinsic motivation. These two characteristics are the workers' private information and are discretely and independently distributed. The two firms simultaneously offer screening contracts defined by a task level (the observable effort) and a non-linear wage rate which depends on effort. Because of the strategic interaction between the two firms, the workers' outside options are type-dependent and endogenous and thus the analysis of a multi-principal framework with bidimensional screening is called for.

Motivated workers care about the mission pursued by the firm which employs them. More precisely, the payoff of motivated agents depends on their own type but also on the type of firm hiring them. When motivated workers are hired by the mission-oriented employer, and only by him, they benefit from intrinsic motivation and enjoy (at least to a certain extent) their personal contribution to the output produced by the firm. Conversely, all workers experience a cost from effort provision, which can differ across workers types but which does not depend on the possible mission of the employer.³ The framework we describe is relevant, for instance, when a teacher (nurse) is choosing between a standard school (hospital) and a religious one, when a manager is facing the choice between working at a standard or at an environmental-friendly company, when a financial expert can apply for a job either at a commercial bank or at an ethical bank.

Therefore, firms' heterogeneity stems from workers' motivation, which has a positive impact on the output produced by the mission-oriented firm, although it has no impact on the output of the standard firm. Moreover, the two firms are heterogeneous in their technologies because their marginal productivity of labor is different. Importantly, we take a general perspective in dealing with the differences in firms' technologies and study all possible environments: the ones where either the mission-oriented firm or the standard firm is overall more efficient and the one in which the mission-oriented firm is more efficient in hiring motivated workers whereas the standard firm is more efficient in employing non-motivated workers. The relevance of the various instances is discussed in the Concluding Section of the paper.

Taking into account the combined effect of the two sources of firms heterogeneity (workers' motivation and firms' technology), we say that one firm is fully dominant when it succeeds in hiring all types of workers even when the rival firm offers the highest possible utility to all potential applicants.

When there does not exist a fully dominant organization, then the mission-oriented firm is more efficient in hiring motivated workers, because of the labor-donation aspect inherent in intrinsic motivation, although the standard firm is more efficient in hiring non-motivated employees. This represents the most interesting case to analyze where, in equilibrium, workers sort themselves by motivation: the mission-

³Thus, a peculiarity of our model is that the mission-oriented firm will have to design screening contracts based on both dimensions of private information, ability and motivation, while the standard firm will have to offer the same contract to workers with the same ability, taking into account that their outside options differ depending on their intrinsic motivation.

oriented firm hires motivated types while the standard firm employs non-motivated agents. Hence, we show that workers' self-selection does not depend on ability, i.e. it is ability-neutral, and it is efficient. This result is general because sorting of worker types according to motivation is independent of the distribution of types. Thus we can conclude that, when skills and intrinsic motivation are independently distributed (which is the case that we analyze in detail in the paper), ability-neutrality implies that average ability is the same between firms. When instead skills and intrinsic motivation are positively (respectively, negatively) correlated, ability-neutrality implies that average ability is higher (respectively lower) for the mission-oriented firm than for the standard firm.

When the market is segmented according to motivation, the degree of competition between employers influences the importance of outside options relative to internal incentive compatibility in the screening contracts and this is turn determines the level of distortions in optimal allocations, i.e. in effort levels. In particular, if competition is harsh, because firms are similar in technology and agents' motivation is not too high, then both firms ask first-best effort levels to hired workers. In this case, outside options dominate incentive compatibility and screening contracts resemble the ones arising in equilibrium with duopolistic competition and full information. If instead competition is mild, because firms' technologies are sufficiently different from each other and agents' motivation is relevant, then internal incentive compatibility is the driving force and effort levels are the ones we observe under monopsony and asymmetric information.⁴ In-between harsh and mild competition, firms might optimally impose to low-ability workers effort distortions which are in-between the ones observed in the two other cases. In a nutshell, distortions are lower the higher the degree of competition between the two firms.

As for the non-linear wages offered by the two firms, under marker segmentation we find that, for a wide range of parameter configurations, a wage differential emerges because the total salary gained by motivated workers at the mission-oriented firm is lower than the salary that the same worker would gain if employed by the standard firm.⁵ Such a wage gap is always associated with higher effort provision: motivated workers are committed to exert higher effort at the mission-oriented firm, where they choose to work, than at the standard firm. The result that sorting is ability-neutral and that average ability is the same across firms (a consequence of the independent distribution of skills and motivation) allows us to conclude that the earnings penalty experienced by motivated workers is due to a true compensating wage differential and is not driven by adverse selection with respect to ability. However, workers' ability does play a role in that the earnings penalty is increasing in ability. This fact is consistent with the empirical

⁴The case of a monopsonistic, mission-oriented firm has been analyzed in Barigozzi and Burani (2013).

⁵Note that each firm always offers a contract to each type of worker, even though in equilibrium a worker accepts to work for only one firm and even though some contracts will never be chosen in equilibrium. So our framework with competing principals is particularly useful to study the wage differential because it allows us to compare the salary offered to the same worker by different firms.

evidence on the public-private wage gap documented in Roomking and Weisbrod (1999) and Bargain and Melly (2008), among others. Interestingly, a compensating wage differential, which is increasing in ability, implies that contracts offered by the mission-oriented firm are characterized by lower returns to ability than contracts designed by the standard firm. In this sense, our model with (bidimensional) adverse selection confirms results from the personnel economics focusing mainly on moral hazard and showing that workers in mission-oriented sectors are generally offered low-powered incentives (see, among others, Besley and Gathak 2005 and Makris 2009).

Finally, our results suggest a possible explanation for the rise of firms' social-mission and for the coexistence of standard and mission-oriented firms in the market. Suppose that firm A is initially endowed with a production technology which is very similar to that of firm B so that competition in the market is very tough. Moreover, consider that firm A decides to become corporate socially responsible by sacrificing some of its profits to better preserve the environment (see Bénabou and Tirole 2010). In terms of our framework, this can be interpreted as firm A moving to a more costly but 'green' technology. This possibly enables firm A to loosen competition and improve its performance thanks to the labor donations of those workers who are environmental friendly. In particular, firm A can survive and make positive profits if the gain from labor donations by motivated workers more than compensates both the costs of the adoption of the green technology and the additional information rents that have to be paid because of the increased heterogeneity in the workers' types. In the end, in the competition for attracting the best workers, being differentiated with respect to a mission allows a firm to extract more surplus from dedicated employees, who might be willing to work more in exchange for a lower salary. In Section 6 of the paper we discuss the empirical relevance of the different situations analyzed in the model and the insights provided by our results.

The rest of the paper is organized as follows. In the following subsection we describe the related literature. In Section 2, we set up the model; in Section 3, as benchmark cases, we present the first-best and the equilibrium with perfectly informed competing firms. Section 4 introduces asymmetric information and describes the equilibrium screening strategies of the two firms when one of them is fully dominant. In Section 5, the equilibrium assignment of workers to firms is considered together with its efficiency. Under market segmentation, we also discuss how average ability of workers employed by the two firms changes with the distribution of ability and motivation. In Subsection 5.3, the full

⁶The idea that firms can attract motivated workers by becoming socially responsible is not new. Brekke and Nyborg (2008) consider a perfectly competitive labor market with full employment where individual wages are equal for all workers within a given firm. Thus, workers have no pecuniary incentive to work hard, and the firm faces a moral hazard problem. However, some workers strictly prefer socially responsible employers and this results in lower equilibrium wage in socially responsible firms than is standard firm. Moreover, workers' preferences are such that individuals who self-select into socially responsible firms also work harder (because providing higher effort improves one's self-image) and this increases socially responsible firms' productivity. As a consequence, socially responsible and standard firms might coexists in equilibrium.

characterization of the optimal contracts is then provided when workers sort themselves by motivation and when ability and motivation are uniformly distributed. Subsection 5.4 comments on the existence of both wage differences and different returns to ability for the two firms. Finally Section 6 concludes by discussing the various scenarios analyzed in the model and interpreting the results in terms of the coexistence of standard and mission-oriented firms in the market.

1.1 Related literature

Our work contributes to two different strands of literature: from an economic point of view, it adds to the recent and rapidly growing literature on the self-selection of workers with intrinsic motivation into different firms/sectors of the labor market; from a technical point of view, it explicitly solves a multiprincipal game in a labor market where two firms compete to attract workers who are characterized by two different dimensions of private information.

The problem of the design of optimal incentive schemes for intrinsically motivated workers has been tackled by Murdock (2002), Besley and Gathak (2005) and Prendergast (2007), whose attention has primarily been devoted to moral hazard, while we consider the screening problem. Heyes (2005) and Delfgaauw and Dur (2007) are the first papers that address the issue of the selection of workers who are heterogeneous with respect to their motivation. They show that, as a worker's motivation increases, the worker's reservation wage decreases. Therefore, as the wage increases, the average motivation of the workers who are willing to accept the job deteriorates. But workers' heterogeneity in ability is not considered.

Previous results from theoretical literature admitting for workers' private information are ambiguous on whether mission-oriented firms or sectors are characterized by lower or higher workers' productivity on average. In particular, Handy and Katz (1998) consider the selection of managers who differ in terms of ability and devotion to the non-profit firm. They impose an exogenously given ranking of both effort provisions and reservation wages for different types of managers and they find that lower wages are effective in attracting managers that are more committed to the cause of the non-profit firm. But this comes at the cost of selecting less able managers who are unable to command higher wages in standard sectors. More importantly, Delfgaauw and Dur (2010) consider a perfectly competitive economy consisting of the public and the private sector. Workers are heterogeneous with respect to both productivity and motivation and firms can perfectly observe both workers' characteristics. Thus workers are paid their full marginal product in both sectors. Moreover, output prices are such that the return to managerial ability is lower in the public than in the private sector. Hence, when workers' intrinsic motivation is independent of output, a public-private earnings differential exists, which is caused partly by a compensating wage differential (motivated workers evaluate more being employed in the public sector) and partly by selection

arising endogenously from the adjustment in prices to differences in job attributes (on average more productive workers enter the private sector where remuneration is higher). When instead public service motivation is output-oriented, the selection into the public sector is ability-neutral. Our model extends the setup in Delfgaauw and Dur (2010), given that we consider bidimensional adverse selection rather than full information about the workers' characteristics and given that we consider strategic interaction between the two firms. We confirm the result of ability-neutrality of sorting of workers between firm. Moreover, for a large set of parameters, we also document a wage gap, which increases in ability and which penalizes workers employed at the mission-oriented firm; but this gap arises despite the fact that workers with the same ability exert higher effort at the mission-oriented firm than at the standard firm.⁷,⁸

Our paper is also related to Delfgaauw and Dur (2008), where, again, the problem of workers' self-selection into public vs private sectors is considered and the screening problem of the governmental agency is tackled. As for the setup, we depart from Delfgaauw and Dur (2008) in two main respects: first, their private sector is perfectly competitive and therefore sectors do not interact strategically. Second, their screening mechanism is simplified because the public agency is constrained to hire at most two types of agents. As for the results, we find a different selection pattern of workers to firms and we are able to compare average ability of workers between the two firms, while Delfgaauw and Dur (2008)'s framework does not allow for such a comparison. Indeed, they underline that their "model does not necessarily imply that workers in the public sector are on average more lazy than workers in the private sector; nor does it imply that lazy workers are always more numerous in the public sector than in the private sector" (see page 173).

More recently, DeVaro et al. (2015) consider a non-profit firm that faces a non-distribution constraint and that is bound to offer flat wages to its employees. The non-profit firm competes with perfectly competitive for-profit rivals in hiring a worker who is heterogeneous in skills and who derives intrinsic motivation from the non-profit social mission. It is shown that the worker is hired by the non-profit firm if intrinsic motivation is sufficiently high and that a wage differential favoring for-profit firms emerges when the latter are more effective than the nonprofit firm in training workers.⁹

⁷In our setup, the main determinant of the wage gap is not the difference in output prices (as for Delfgaauw and Dur 2010) but the superior technology of the standard firm. Indeed, when both firms are assumed to have the same production function then, fixing ability, motivated types always provide more effort and always earn more at the mission-oriented firm than at the standard firm.

⁸Barigozzi et al. (2014) and Barigozzi and Turati (2012) consider labor supply in a market where workers have private information on both productive ability and motivation and where the wage rate is flat. They show that the lemons' problem might be exacerbated by the presence of bidimensional asymmetric information because an increase in the market wage can determine a simultaneous decrease in both average vocation and average productivity of applicants.

⁹The paper also tests the theoretical results with data on California establishments showing that for-profits firms offer higher wages and higher incentive pay with respect to non-profits.

Finally, Bénabou and Tirole (2013) analyze a model where firms compete to attract workers that are heterogeneous with respect to their productivity and their work ethics, i.e. the extent to which agents "do the right thing" beyond what their material self-interest commands. In a framework with multitasking and moral hazard, they show how competition for the most productive workers interacts with the incentive structure inside firms to undermine work ethics. Besides the different focus of the two papers, Bénabou and Tirole (2013) assumes an affine compensation scheme with incentive power and a fixed wage, we instead consider non-linear contracts. Moreover, their screening is not bidimensional but it is performed by firms with respect to one dimension at a time (either productivity or work ethics).

From a technical point of view, our paper draws both from the literature on multidimensional screening and from the literature on multi-principals. Models where both problems are simultaneously considered are very few and tend to rely on simplifying assumptions, as we explain below.

Screening when agents have several unobservable characteristics has been analyzed by some important papers that deal with continuous distributions of types: Armstrong and Rochet (1999), Armstrong (1996), Rochet and Chonè (1998), Basov (2001, 2005) and Deneckere and Severinov (2011). They all show that it is almost impossible to extend to the multidimensional environment the qualitative results and the regularity conditions of the unidimensional case. Our model is characterized by a discrete type space, and by one screening instrument available to the principal (namely the contractible effort level) so that the closest paper to ours is Armstrong (1999), which considers optimal price regulation of a monopoly that is privately informed about both its cost and demand function. Barigozzi and Burani (2013) considers the screening problem of a mission-oriented monopsonist willing to hire a worker of unknown ability and motivation. The present paper adds the important dimension of competition between two differentiated firms, a mission-oriented and a standard firm. Investigating a model with multi-principals brings into the analysis many important and new results which are consistent with some stylized facts and which are useful to interpret the empirical findings. In particular, we show that: (i) competition reduces effort distortions although it increases workers' rewards (via outside options) with respect to monopsony; (ii) competition reduces the pervasiveness of pooling contracts with respect to monopsony; (iii) under monopsony, the wage rate of motivated workers is always higher than the wage rate of non-motivated workers with the same ability (this can also be true irrespective of ability), because of the cumulative effect of information rents, although under competition this is not necessarily the case (if a compensating wage differential penalizing workers hired by the mission-oriented firm emerges, for instance).

The multi-principal literature with asymmetric information was initiated by the seminal contributions of Martimort (1992) and Stole (1992). Within this literature, the paper that is most closely related to ours is Biglaiser and Mezzetti (1993), which studies two principals competing for the exclusive services of an agent in the presence of both adverse selection and moral hazard. The two principals have different technologies and one principal is more efficient in hiring low-skilled types whereas the other is more

efficient in employing high-skilled types. Intermediate types are the ones for whom competition is harsher: both principals make zero profits on these types, who get the same contract and are indifferent between working for either principal. Besides the difference between the continuous and the discrete setup, we depart from this work because we consider bidimensional rather than unidimensional screening.

Another related model is Rochet and Stole (2002) which extends the analysis carried out in Stole (1995) and studies duopolists competing in nonlinear prices in the presence of both vertical and horizontal preference uncertainty. In particular, it is assumed that consumers are heterogeneous and privately informed about their preference for quality and about their outside opportunity cost. Importantly, the outside option enters the consumer's utility function independently of quality, therefore the good's quality is the only screening instrument and contracts consist of quality-price pairs that only depend on consumers' (unidimensional) preference for quality. The other unknown characteristic, the outside opportunity cost, only affects the consumers' participation constraints but not the incentive compatibility constraints. The same restriction is imposed by Lehmann et al. (2014) in analyzing optimal nonlinear income taxes between two competing governments. In that model, individuals differ in both skills and migration costs but the latter enter agents' utility independently of earnings. Therefore, optimal nonlinear taxes only depend on a unidimensional characteristic, namely the skill level, through observable earnings, and the single crossing condition still holds. The other characteristic, the cost of migration, only influences the agents' decision about where to reside. We depart from both Rochet and Stole (2002) and Lehmann et al. (2014) because we consider a setup with full-fledged bidimensional screening, where both our dimensions of private information (skills and intrinsic motivation) enter the workers' preference in association with the screening instrument (effort) and where the single crossing condition does not hold. Thus, both characteristics have to be taken into account when examining incentive compatibility and participation constraints and both characteristics determine optimal screening contracts.

2 The model

We consider a multi-principal setting with bidimensional adverse selection. Two principals (firms) compete to hire an agents (workers). Each agent (she) can work exclusively for one principal (he). Principals and agents are risk neutral.¹⁰

Effort supplied by the agent is the only input the two firms need in order to produce. We call e the observable and measurable effort (task) level that the agent is asked to provide. ¹¹ Both principals'

 $^{^{10}}$ One could rephrase the whole setup considering two sectors populated by a monopsonistic firm each.

 $^{^{11}}$ In particular, the variable e can be interpreted as a job-specific requirement like the amount of hours of labor the agent is asked to devote to production.

production functions display constant returns to effort in such a way that

$$q^{P}(e) = k^{P}e,$$

where k^P denotes the marginal productivity of effort for principal $P \in \{MO, S\}$, with MO referring the mission-oriented firm and S referring to the standard firm. We normalize the marginal productivity of effort for the mission-oriented principal to $k^{MO} = 1$ and set $k^S = k$. Importantly k^S can be smaller or greater than $k^{MO} = 1$. We will discuss in detail the economic interpretations of the different cases in Section 6.

The principals' profit functions are given by

$$\pi^{P}(e) = q^{P}(e) - w^{P} = k^{P}e - w^{P},$$

where the price of output is assumed to be exogenous and set equal to 1 in both sectors, and where w^P is the total salary paid to workers hired by principal P.

Suppose that a unit-mass population of agents differ in two characteristics, ability and intrinsic motivation, that are independently distributed and can take two values each.¹² In order to make notation less cumbersome, we use upper-case letters to denote high values of workers' characteristics and lower-case letters to denote low values.

A worker characterized by high ability incurs in a low cost of providing a given effort level. Ability is denoted by $\theta_i \in \{\theta_A, \theta_a\}$ where $\theta_a > \theta_A > 0$. A fraction ν of employees has high ability (i.e. a low cost of effort) θ_A , the fraction $1 - \nu$ is instead characterized by low ability (i.e. a high cost of effort) θ_a . Ability is the only relevant workers' characteristic for the standard firm, although the benefit from intrinsic motivation can only be enjoyed when motivated workers are employed by the mission-oriented firm. Indeed, we assume that workers, to a certain extent, derive utility from exerting effort for the mission-oriented firm. Since there exists a one-to-one relationship between effort exerted and output produced by the mission-oriented principal, this interpretation is equivalent to considering intrinsic motivation as the enjoyment of one's personal contribution to the mission-oriented principal's output.¹³ Paralleling ability, we assume that motivation takes two possible values $\gamma_j \in \{\gamma_m, \gamma_M\}$, with $\gamma_M > \gamma_m \geq 0$. A faction μ of workers is characterized by high motivation γ_M , the fraction $1 - \mu$ has instead low motivation γ_m .

So there are four types of agents, denoted as $ij = \{AM, Am, aM, am\}$, where the first index represents ability and the second index represents motivation. In what follows, we will refer to the fraction of a given agent's type or to its probability interchangeably.

¹²In Section 5.3, in order to fully characterize the optimal contracts, we will restrict attention to a uniform distribution.

¹³The same interpretation of intrinsic motivation can be found in Besley and Ghatak (2005) and Delfgaauw and Dur (2007, 2008, 2010-only as for Section 5) and traces back to the "warm-glow giving" or impure altruism theory in Andreoni (1990).

For simplicity, we set the lower bounds of the support of the distribution for both attributes at $\theta_A = 1$ and $\gamma_m = 0$. We will thus focus on situations in which the agent can be either intrinsically motivated, with motivation parameter taking value $\gamma_M = \gamma$ or not motivated at all. Our results will depend on how the difference or heterogeneity in motivation $\Delta \gamma = \gamma_M - \gamma_m = \gamma$ relates to the difference in ability $\Delta \theta = \theta_a - \theta_A = \theta - 1$. Furthermore, we impose that $\Delta \gamma \leq 1$, or else that $0 < \gamma \leq 1$, in order to prevent the mission-oriented firm from paying negative salaries to motivated workers at the first-best. Finally, it is assumed that $\Delta \theta \leq 1$, or else that $1 < \theta \leq 2$, which allows the mission-oriented firm to rank workers' types differently according to their effort provision again at the first-best (see equation 2).

When a worker is not hired by any principal, we assume that her utility is zero. If a worker is hired by one principal, her *reservation utility* is endogenous and it depends on the contract offered by the rival principal.

When a worker is hired by the standard principal, her utility is

$$U_{ij}^S = w_{ij} - \frac{1}{2}\theta_i e_{ij}^2.$$

In fact, motivated workers do not enjoy any benefit from motivation when hired by the standard firm. As a consequence, from the point of view of the standard principal, workers AM and Am on one side and workers aM and am on the other side are equally productive. However, agents with the same ability potentially benefit from different outside options. In fact, given ability, motivated workers are valued more than non-motivated workers by the mission-oriented firm because they provide more effort and contribute more to the firm's output. Thus intrinsic motivation positively affects motivated workers' outside options even though it does not alter effort provision for workers employed by the standard firm.

When a worker is hired by the mission-oriented principal, her utility takes the form

$$U_{ij}^{MO} = w_{ij} - \frac{1}{2}\theta_i e_{ij}^2 + \gamma_j e_{ij},$$

where both productivity θ_i and motivation γ_i are related to effort exertion. 15,16

¹⁴Given this simplification, we will refer to the difference in motivation $\Delta \gamma$ and to the level of motivation γ interchangeably. ¹⁵This linear-quadratic specification of the utility function is widely used in the literature on workers' intrinsic motivation (see Besley and Ghatak 2005 and Delfgaauw and Dur 2008, 2010). The same objective function for the agent is also considered in the literature on multidimensional screening with a continuum of types (see Laffont *et al.* 1987, Basov 2005,

and Deneckere and Severinov 2011), where solutions are found imposing a uniform distribution and a unit square type space.

¹⁶Our setting shares some similarities with Gomes et al. (2014) where workers, characterized by sector-specific productivity, choose the sector to work in and the effort to supply when a social planner optimally sets non-linear income taxes. Our framework is also related to agency models with adverse selection and altruistic agents, as Choné and Ma (2004), Makris and Siciliani (2013) and Bassi et al. (2014).

The marginal rate of substitution between effort and wage is given by

$$MRS_{e,w}^{MO} = -\frac{\partial U_{ij}^{MO}/\partial e_{ij}}{\partial U_{ij}^{MO}/\partial w_{ij}} = \theta_i e_{ij} - \gamma_j,$$

which is always positive for non-motivated workers with $\gamma_j = 0$. When the effort required by the firm is sufficiently low, i.e. when $e_{ij} < \frac{\gamma_j}{\theta_i}$ and j = M, motivated workers' indifference curves have a negative slope in the space (e, w) and effort is a 'good'. Note that providing effort represents a net cost to the agent when

$$-\frac{1}{2}\theta_i e_{ij}^2 + \gamma_j e_{ij} < 0.$$

Thus, if the effort required by the mission-oriented principal is sufficiently low, motivated workers could perform their task when receiving a non-positive reward. In other words, they would be ready to volunteer to be hired by the mission-oriented firm. Finally, notice that the indifference curves of 'intermediate' types aM and Am cross twice: for the mission-oriented firm, the single-crossing property does not hold.

The timing of the game is as follows. The two principals simultaneously offer a menu of contracts of the form $\{e_{ij}^P, w^P\}$, with $P \in \{MO, S\}$. Workers observe the contracts, choose which principal (if any) to work for and select a contract. Then workers exert the effort level specified by the chosen contract, output is produced and the contracted wage is paid.

An equilibrium is such that each principal chooses a menu of contracts that maximizes his expected profit, given the contracts offered by the rival principal and given the equilibrium choice of workers. Workers choose the contracts that maximize their utility. Principals are bound to offer contracts that make non-negative profit. If a worker is indifferent between working for the two principals, it is assumed that with probability one she will choose to work for the principal making the higher profit on that type. In fact, the principal with the higher, strictly positive payoff is able to raise her reward by $\varepsilon > 0$ and break the tie.

In Sections 4 and 5 we will study competition with (bidimensional) adverse selection. Importantly, our framework originates from the combination of two simpler environments: (i) the case of two firms competing to attract heterogeneous workers under full information; (ii) the case of a monopsonistic firm designing screening contracts under asymmetric information. Case (i) will be shortly examined in Section 3 while Case (ii), first analyzed in Barigozzi and Burani (2013), will be discussed in the sequel. In the qualitative description of the equilibrium contracts in Sections 4 and 5, we will refer to "first-best" effort levels as the effort levels that are relevant in Case (i) and to "second-best" effort levels as the solutions to the screening programs in Case (ii).

3 Benchmark cases

In this section, we illustrate the first-best solution of the model and then the equilibrium when principals compete with each other under full information.

3.1 The first-best

The first-best effort levels are obtained by maximizing total surplus (sum of the agent's utility and the principal's profit) with respect to the worker's effort for each type of worker and for each type of principal. They have the following expressions

$$e_{AM}^{FB,S} = e_{Am}^{FB,S} = e_{A.}^{FB,S} = k \quad e_{aM}^{FB,S} = e_{a.}^{FB,S} = e_{a.}^{FB,S} = \frac{k}{\theta}$$
 (1)

for the standard principal, and

$$e_{AM}^{FB,MO} = 1 + \gamma$$
 $e_{Am}^{FB,MO} = 1$ $e_{aM}^{FB,MO} = \frac{1+\gamma}{\theta}$ $e_{am}^{FB,MO} = \frac{1}{\theta}$ (2)

for the mission-oriented principal, where $e_{aM}^{FB,MO} > e_{Am}^{FB,MO}$ if and only if $\gamma > \theta - 1 = \Delta\theta$. Workers are efficiently assigned to the principal for whom the highest first-best effort is provided. Thus, the efficient assignment only depends on the relative magnitude of the numerators of first-best efforts, namely on the relative magnitude of $1 + \gamma$, k, and 1.

Remark 1 The efficient assignment of workers to firms is such that: (a) when k < 1, all workers' types are allocated to the mission-oriented firm; (b) when $k > 1 + \gamma$, all workers are allocated to the standard firm; (c) when $1 < k < 1 + \gamma$, motivated workers are assigned to the mission-oriented firm and non-motivated types are assigned to the standard firm.

In the boundary case in which k = 1 (respectively, $k = 1+\gamma$), motivated (resp. non-motivated) workers are allocated to the mission-oriented (resp. standard) firm while non-motivated (resp. motivated) ones are randomly assigned to the two firms.

3.2 Competition under full information

Suppose now that the two principals observe the worker's type and simultaneously offer her a contract (e_{ij}^P, w_{ij}^P) , with $P \in \{MO, S\}$. The best strategy for each principal is to ask each agent the first-best effort level: this allows the two principals to generate the highest revenue to be used to attract the worker (notice that the game describes a situation where two firms characterized by different efficiency levels compete à la Bertrand to attract a worker of known type). The principal that is less efficient in hiring some workers' types offers them a wage that drives his profits to zero.¹⁷ The more efficient principal,

¹⁷Notice that any lower wage possibly offered by the less efficient principal would generate profitable deviations and thus cannot be part of an equilibrium strategy.

instead, offers a wage which exactly meets the best possible offer of the competitor and is thus able to attract the workers.

The allocation of workers to principals is the efficient one, as described in Remark 1. When $1 < k < 1 + \gamma$, the standard firm will make its highest offer to motivated workers, and the mission-oriented firm will meet that offer attracting motivated workers. In the same way, the mission-oriented principal will make his best offer to non-motivated workers and the standard principal will meet that offer attracting these workers. When k < 1 or $k > 1 + \gamma$ the more efficient principal is hiring all the workers.

In Appendix A.1, we compute the wages offered by the two principals in equilibrium, we further discuss the cases with k = 1 and $k = 1 + \gamma$ and characterize the properties of the allocation.

4 Fully dominant principals

We start tackling the issue of competition between the two non-informed principals in the case in which, in equilibrium, all worker's types are hired by one principal only and the other principal remains inactive. Following Biglaiser and Mezzetti (1993), we call such situations of 'deterred competition' equilibria with a fully dominant principal.

When a firm is fully dominant, it is able to hire all types of workers and to make non-negative profits on all workers, even when the rival principal, the dominated principal, offers them their first-best total surplus. To be more precise, suppose that principal $P \in \{MO, S\}$ is the dominated principal. Then he unsuccessfully competes with the dominant principal by offering each type of worker a contract such that:

(i) the effort level is the first-best effort $e_{ij}^{FB,P}$, and (ii) the total wage is obtained imposing zero profits from that type, i.e.

$$\pi_{ij}^{P} = k^{P} e_{ij}^{FB,P} - w_{ij}^{P} = 0 \iff w_{ij}^{P} = k^{P} e_{ij}^{FB,P}.$$
 (3)

In this way, the dominated principal offers each type of agent the maximal possible utility, which is given by

$$U_{ij}^{TS,P} = k^{P} e_{ij}^{FB,P} - \frac{1}{2} \theta_{i} \left(e_{ij}^{FB,P} \right)^{2} + \gamma_{j} e_{ij}^{FB,P}, \tag{4}$$

where the superscript TS stands for total surplus and where the term $\gamma_j e_{ij}^{FB,P}$ is equal to zero when the dominated principal is the standard one.¹⁸ Conversely, the fully dominant principal succeeds in attracting all types of worker by offering them at least $U_{ij}^{TS,P}$.

We expect that a principal is fully dominant when he is more efficient than the other in hiring workers, as already suggested by Remark 1.

¹⁸Observe that the contracts offered by the dominated firm are always incentive compatibile since the workers are the recipients of all the surplus.

4.1 Fully dominant standard principal

The standard principal is only able to screen applicants on the basis of their ability, whereas intrinsic motivation does not affect the contracted effort and the firm's output. In other words, any incentive compatible contract that the standard principal might offer must be such that workers with the same ability are offered the same contract, whereby

$$e_{AM}^{S} = e_{Am}^{S} = e_{A}^{S}$$
 and $e_{aM}^{S} = e_{am}^{S} = e_{a}^{S}$

and

$$w_{AM}^S = w_{Am}^S = w_{A}^S$$
 and $w_{aM}^S = w_{am}^S = w_a^S$.

But types characterized by the same ability and different intrinsic motivation are not identical from the standard principal's viewpoint, because they enjoy different outside options. Indeed, when the standard principal is fully dominant, each worker's outside option is equal to her first-best total surplus offered by the mission-oriented firm and given by

$$U_{AM}^{TS,MO} = \frac{(1+\gamma)^2}{2} \quad U_{aM}^{TS,MO} = \frac{(1+\gamma)^2}{2\theta} \quad U_{Am}^{TS,MO} = \frac{1}{2} \quad U_{am}^{TS,MO} = \frac{1}{2\theta} \quad .$$
 (5)

Note that $U_{AM}^{TS,MO} > U_{Am}^{TS,MO}$ and that $U_{aM}^{TS,MO} > U_{am}^{TS,MO}$: the mission-oriented principal is always able to leave a strictly higher utility to motivated types than to non-motivated types with the same ability. Thus, in order to be able to hire all workers, the standard firm must offer them the motivated workers' outside options. In other words, the relevant participation constraints will be those of motivated workers.

Then, the fully dominant standard principal's program corresponds to a two-types screening problem with type-dependent but exogenous outside options and is as follows

$$\max_{(e_{i.}^{S}, w_{i.}^{S})} E\left(\pi^{S}\right) = \nu \left(ke_{A.}^{S} - w_{A.}^{S}\right) + (1 - \nu) \left(ke_{a.}^{S} - w_{a.}^{S}\right)$$
 (FDSP)

subject to the two participation constraints of motivated types¹⁹

$$w_{i\cdot}^{S} - \frac{1}{2}\theta_i \left(e_{i\cdot}^{S}\right)^2 \ge U_{iM}^{TS,MO} \tag{PC_{iM}^S}$$

for every i = a, A and two incentive compatibility constraints

$$w_{i\cdot}^{S} - \frac{1}{2}\theta_{i} \left(e_{i\cdot}^{S}\right)^{2} \ge w_{i'\cdot}^{S} - \frac{1}{2}\theta_{i} \left(e_{i'\cdot}^{S}\right)^{2} \tag{IC_{i\cdot vsi'\cdot}^{S}}$$

for every i = a, A and $i' \neq i$. Adding the two incentive compatibility constraints, one can easily check that implementability requires that

$$e_{AM}^S = e_{Am}^S = e_{A.}^S > e_{aM}^S = e_{am}^S = e_{a.}^S$$

As said, given the magnitudes of $U_{ij}^{TS,MO}$, only the participation constraints of motivated types matter. Indeed, once PC_{AM}^S is satisfied, then PC_{Am}^S is slack and, similarly, once PC_{AM}^S holds, then PC_{Am}^S is slack.

In order to solve problem FDSP, we build on the analysis of Laffont and Martimort (2002, Chapter 3.3, pages 101-105). They study type-dependent participation constraints and countervailing incentives when there are two types of agent and the inefficient type's outside option is zero although the one of the efficient type is strictly positive. As in Laffont and Martimort (2002), the presence of type-dependent participation constraints alters the natural ordering of incentive and participation constraints. As a consequence the solution to problem FDSP exhibits five different regimes according to which participation and incentive compatibility constraints are binding. In particular, which regime is in place depends on the magnitude of the difference in outside options for motivated types $U_{AM}^{TS,MO} - U_{aM}^{TS,MO}$. Observe that the analysis is relatively easy here because the difference in outside options is fixed. The analysis becomes more complex in the case of marker segmentation, because outside options become endogenous.²⁰

Moreover, the result obtained by Rochet and Stole (2002) in their Lemma 1 (see page 285) holds, whereby the upward incentive constraint (requiring that the low-ability worker does not choose the contract designed for the high-ability one) can never be binding. Thus, only the first three out the five possible regimes are relevant in our setup, and the standard principal never resorts to countervailing incentives.

The Lemma that follows summarizes the main findings and Appendix A.2 contains the detailed analysis.

Lemma 1 The standard principal is fully dominant only if $k > 1 + \gamma$. The optimal allocations are such that effort for high-ability workers is not distorted (the 'no distortion at the top' property holds) whereas effort distortions for low-ability workers are increasing in k. In particular: (1) when $k > k_1 = \frac{(1+\gamma)(\theta-\nu)}{\sqrt{\theta}(1-\nu)} > 1+\gamma$, outside options are irrelevant and effort for low-ability workers is set at the second-best; (2) when $k_2 = (1+\gamma)\sqrt{\theta} < k \le k_1$, the effort of low-ability types is distorted downwards (but less than at the second-best); and (3) when $1+\gamma < k \le k_2$, all effort levels are set at the first-best.

When $k=1+\gamma$, regime (3) of the above Lemma would hold but the standard firm could not be fully dominant in this case. Indeed, in regime (3), both participation constraints of high- and low-skilled motivated workers are binding, while all incentive constraints are slack. Both principals offer the same payoff $U_{iM}^{TS,MO}$ to all workers i=a,A and set first-best effort levels for all workers. But then, because $k=1+\gamma$ and because the standard principal is bound by incentive compatibility to offer the same contract to workers with the same ability, the standard principal makes zero profits form all types. Such a situation cannot be an equilibrium because the standard principal can profitably deviate by 'leaving' motivated workers to the mission-oriented firm. When $k=1+\gamma$, or when k approaches $1+\gamma$ from above, Section 5.3 shows that the unique equilibrium is such that workers sort themselves by motivation and

 $^{^{20}\}mathrm{See}$ Section 5.1.

the standard principal only hires non-motivated types making strictly positive profits on these types of applicant.

4.2 Fully dominant mission-oriented principal

When principal MO fully dominates, the equilibrium strategy of principal S is to provide the workers with their total surplus utilities

$$U_{A}^{TS,S} = \frac{k^2}{2} \text{ or } U_{a}^{TS,S} = \frac{k^2}{2\theta}.$$

which only differ according to ability. Now, the mission-oriented principal must offer each type of agent a level of utility at least as high as $U_i^{TS,S}$ for i=a,A.

Thus, the mission-oriented principal solves a screening problem with bidimensional adverse selection and type-dependent but exogenous outside options, which is as follows

$$\max_{\left(e_{ij}^{MO}, w_{ij}^{MO}\right)} E\left(\pi^{MO}\right) = \nu \mu \left(e_{AM}^{MO} - w_{AM}^{MO}\right) + \nu \left(1 - \mu\right) \left(e_{Am}^{MO} - w_{Am}^{MO}\right) + \left(1 - \nu\right) \mu \left(e_{aM}^{MO} - w_{aM}^{MO}\right) + \left(1 - \nu\right) \left(1 - \nu\right) \left(e_{am}^{MO} - w_{am}^{MO}\right) + \left(1 - \nu\right) \left(1 - \nu\right) \left(e_{am}^{MO} - w_{am}^{MO}\right)$$

$$\left(FDMOP\right)$$

subject to four participation constraints whose generic form is

$$w_{ij} - \frac{1}{2}\theta_i \left(e_{ij}^{MO}\right)^2 + \gamma_j e_{ij}^{MO} \ge U_{i}^{TS,S}$$
 (PC_{ij}^{MO})

and twelve incentive compatibility constraints that are such that

$$w_{ij}^{MO} - \frac{1}{2}\theta_{i} \left(e_{ij}^{MO}\right)^{2} + \gamma_{j}e_{ij}^{MO} \geq w_{i'j'}^{MO} - \frac{1}{2}\theta_{i} \left(e_{i'j'}^{MO}\right)^{2} + \gamma_{j}e_{i'j'}^{MO} \qquad (IC_{ijvsi'j'}^{MO})^{2} + \gamma_{j}e_{i'j'}^{MO} + \gamma_{j}e$$

with ij different from i'j'.

The solution to this program is found extending the analysis of a companion paper, Barigozzi and Burani (2013), where the problem of bidimensional screening is considered for type-independent reservation utilities, which are normalized to zero.

In order to characterize optimal contracts with full separation and full participation of types, we add incentive compatibility constraints two by two, and find the following implementability condition

$$e_{AM}^{MO} > \max \left\{ e_{Am}^{MO}; e_{aM}^{MO} \right\} \ge \min \left\{ e_{Am}^{MO}; e_{aM}^{MO} \right\} > e_{am}^{MO}$$

For the mission-oriented firm, the combined impact of ability and motivation on the workers' effort and on the output produced is as follows: the most productive type is worker AM, who will be asked to exert the highest effort, whereas the least productive type is worker am, who will be asked to provide the lowest effort. Worker types Am and aM are in-between and their required effort levels cannot be ordered unambiguously.²¹ Two possible states of the word must then be considered. If motivation has a

²¹The existence of two possible orderings of effort levels is a consequence of the bidimensionality of our problem and of the failure of the single-crossing condition. It would not be observed in a unidimensional set-up with different types of employees characterized by a single summary statistic, like the overall cost of providing effort.

As for participation constraints, note that outside options are the same for types with the same ability. So one can show that once IC_{iMvsim}^{MO} and PC_{im}^{MO} are both satisfied, then PC_{iM}^{MO} is slack, with i=a,A. In other words, when considering types with the same ability but different motivation, one can disregard the participation constraint of motivated types because it is implied by the participation constraint of non-motivated workers. The same conclusion cannot be drawn for workers with the same motivation but different ability. Thus both PC_{Am}^{MO} and PC_{am}^{MO} might be relevant and the latter implies the former only when $e_{am}^{MO} > \frac{k}{\sqrt{\theta}}$, or when the marginal productivity of labor in the standard sector k is sufficiently low, as shown in Appendix A.3.1.

We omit here a detailed description of the equilibrium contracts that solve program FDMOP, and we only state a result that parallels the one obtained in the preceding Section 4.1. When motivation prevails (Case \mathcal{M}), the procedure to obtain a solution and the optimal contracts are presented in Appendix A.3.2; when ability prevails (Case \mathcal{A}), the solution is available upon request to the authors.

When k=1, the two firms are equally efficient in hiring non-motivated workers, who are in turn indifferent between accepting the contracts offered by the two principals. Hence, when k=1, motivated types prefer to be hired by the mission-oriented principal, while non-motivated types randomize between principals.²² There exist different payoff-equivalent equilibria according to the actual choice of non-motivated types. At one extreme we have the fully dominant mission-oriented principal; at the other extreme we have segmentation according to motivation and we refer the reader to Section 5.3 (and to

²²Our tie-breaking rule does not help to select one of the two equilibria since both principals are already making zero profits on non-motivated types.

Situation (ii) with principal S in Case 3, in particular) for a description of such an instance.

5 Competing principals

Suppose that $1 \le k \le 1 + \gamma$, whereby neither principal fully dominates and both principals are active in equilibrium. Still, each principal is dominant relative to a subset of types. In particular, the mission-oriented firm is dominant relative to motivated workers (and it is dominated with respect to non-motivated workers) whereas the standard firm is dominant relative to non-motivated workers (and it is dominated with respect to motivated types). Notice that the subsets of types, relative to whom each principal is dominant, are disjoint, except for the boundary cases of k = 1, when both principals are weakly dominant relative to non-motivated workers, or $k = 1 + \gamma$, when both principals are weakly dominant relative to motivated types.

As in the situations in which one principal is fully dominant, in equilibrium, each firm offers four potentially different contracts that must always satisfy internal incentive compatibility, independently of the fact that some contracts will not be chosen and will remain out-of-equilibrium contracts. Moreover, each principal forms a conjecture about the assignment of workers to firms and this will help each firm define which are the relevant outside options and thus the possible binding participation constraints. In equilibrium, firms' conjectures about the sorting of workers are correct and are such that the principal, who is dominated relative to a given subset of types, will expect these types to be hired by the rival principal and will offer these types out-of-equilibrium contracts. However, differently from Section 4, when principals compete against each other, the relevant outside options are endogenous.

In equilibrium only the participation constraints of non-motivated workers will be relevant for each principal. To understand why, notice that, for the standard firm, out-of-equilibrium contracts are the same as the contracts offered to hired types with the same ability. The relevant participation constraints for the standard firm, that is dominated relative to motivated types and expects to hire non-motivated types only, are precisely those of non-motivated types. Similarly, for the mission-oriented firm, the relevant participation constraints are the ones of non-motivated types (relative to whom the mission-oriented firm is dominated), as in the case in which firm MO is fully dominant. Its out-of-equilibrium contracts are such that, when the participation constraint of a non-motivated type is binding, then this type is offered the first-best total surplus contract, subject to the constraint imposed by the incentive scheme offered to the remaining types. This comes from the fact that Bertrand competition in the utility space drives outside options to their highest possible levels. Leaving to a non-motivated type, whose participation constraint is binding, a lower utility would trigger profitable deviations because each firm would have an incentive to increase such utility in order to hire that type of worker. And this would upset the equilibrium.

In equilibrium, the assignment of workers to firm is based on motivation.²³

Proposition 1 (i) When $1 < k < 1 + \gamma$, in equilibrium the unique assignment of workers to firms is such that motivated workers are hired by the mission oriented firm and non-motivated workers are hired by the standard firm: sorting is ability-neutral. (ii) When k = 1 multiple payoff-equivalent assignments are possible in equilibrium. (iii) When $k = 1 + \gamma$ the only equilibrium assignment is market segmentation, as in (i).

Proof. (i) When $1 < k < 1 + \gamma$, in equilibrium the unique possible matching of workers to firms is such that motivated workers are hired by the mission-oriented firm and non-motivated workers are hired by the standard firm. This occurs because it is always optimal for a firm to hire workers relative to whom it is dominant and to offer out-of-equilibrium contracts to types relative to whom it is dominated. Indeed, consider non-motivated workers. Take any contract offered by the mission-oriented firm to non-motivated workers. Then the standard firm is always able to offer precisely the same contract while making strictly higher profits from these types (because of its superior technology, i.e. k > 1). The standard firm could then use these higher profits to raise all workers' rewards without violating incentive compatibility, and make non-motivated workers strictly prefer its contract. Consider now motivated workers. Take any contract offered by the standard firm to such workers. Then the mission-oriented firm is always able to offer a contract characterized by the same effort level but lower wage, such that $w^{MO} = w^S - \gamma e$, and make strictly higher profits from these types (given that $1 + \gamma > k$). Again, the mission-oriented firm could use these higher profits and raise all workers' rewards without violating incentive compatibility, making motivated workers strictly better-off.

- (ii) When k = 1, both principals are weakly dominant relative to non-motivated workers and competition drives profits from these types to zero for both principals, who offer the same utility to all non-motivated workers. Hence, the mission-oriented firm is still dominant relative to motivated types, whereas non-motivated types randomize between principals.
- (iii) When $k = 1 + \gamma$, both principals are weakly dominant relative to motivated workers and competition drives profits from these types to zero for both principals, who offer the same utility to all motivated workers. Hence, the standard firm is still dominant relative to non-motivated types, whereas motivated workers are indifferent between firms. But the situation in which the standard firm is hiring all workers cannot be an equilibrium because the standard principal is actually making zero profits from all types (because incentive compatibility forces him to offer the same contract to equally able types) and he can

²³Point (i) of the Proposition below extends to our setting with asymmetric information the result obtained by Delfgaauw and Dur (2010) who find that, in case of output-oriented motivation, sorting is ability-neutral.

²⁴Observe that profits to the standard firm are equal to $\pi^S = ke - w^S$ while profits to the mission-oriented firm are given by $\pi^{MO} = e - w^{MO}$. Setting $w^{MO} = w^S - \gamma e$ yields $\pi^{MO} = (1 + \gamma) e - w^S > \pi^S$.

profitably deviate by renouncing to hire motivated types and by making strictly positive profits from non-motivated workers, who have strictly lower outside options than motivated workers.

Notice that there's an asymmetry between cases (ii) and (iii) of the above Proposition, which reflects the difference in the first statement of Lemmas 1 and 2, and which has the following implications for the efficiency of the equilibrium assignment of workers to firms.

Corollary 1 The equilibrium assignment of workers to firms is efficient when $k \leq 1 + \gamma$ and when k is sufficiently higher than $1 + \gamma$.

From Remark 1 and Lemma 2 the equilibrium assignment of workers to firms is efficient when k < 1; also from Remark 1 and Proposition 1 sorting is efficient in equilibrium when $1 \le k \le 1 + \gamma$. However, when k approaches $1+\gamma$ from above, efficiency would require the standard firm to hire all types of worker although in equilibrium we still observe market segmentation. The reason is the same as in part (iii) of the Proof of Proposition 1, which extends to values of k slightly above $1+\gamma$. Indeed, when hiring all workers the standard firm is bound to offer at least U_{aM}^{MO} to low-ability workers and U_{AM}^{MO} to high-ability workers and this drives its profits from all workers very close to zero. Conversely, when renouncing to motivated workers, the standard firm is saving $\left(U_{aM}^{MO}-U_{am}^{MO}\right)$ from low-ability workers and $\left(U_{AM}^{MO}-U_{Am}^{MO}\right)$ from high-ability workers and this more than compensates the very low profits from motivated workers. As k increases the two forces balance each other until when efficiency is restored.²⁵

In the next Subsections, we first describe the procedure followed in order to find candidate equilibria and then we provide the full characterization of equilibrium contracts. Before doing so let us mention some general insights that Proposition 1 allows us to gain concerning the average level of ability of workers hired by the two firms. In effect, the results contained in Proposition 1 do not depend on the assumptions made about the distribution of types and are thus robust to changes in such distribution.

Corollary 2 The equilibrium sorting of workers types according to motivation is independent of the distribution of types. (a) When skills and intrinsic motivation are independently distributed, ability-neutrality implies that average ability is the same across firms. (b) When skills and intrinsic motivation are positively (respectively, negatively) correlated, ability-neutrality implies that average ability is higher (respectively lower) for the mission-oriented firm than for the standard firm.

We can thus foresee the answer to our research question concerning the determinants of wage differentials in markets where mission-oriented and standard firms compete. Suppose that a wage penalty for workers hired by the mission-oriented firm exists and suppose that one wants to disentangle the pure compensating differential effect caused by workers' motivation from the negative selection effect of ability.

²⁵Biglaiser and Mezzetti (1993) also find an inefficient assignment of workers to principals under some parameter configurations. This result which is even more pervasive, given the continuity of the types space in their model.

Then, when skills and intrinsic motivation are independently distributed, the wage gap is totally driven by motivation and mission-oriented firms are not affected by adverse selection with respect to ability. When instead skills and intrinsic motivation are negatively correlated, then the wage penalty would partly be explained by a true compensating wage differential and it would partly be caused by adverse selection with respect to ability; finally, when skills and intrinsic motivation are positively correlated, the wage gap would only arise because of motivation and it would partially be offset by a propitious selection effect with respect to ability.

5.1 The standard principal

Remind that principal S offers the same contract to workers with the same ability. He is dominated with respect to motivated workers, so he anticipates that in equilibrium he is going to attract non-motivated types only. Thus, principal S designs his contracts considering the outside options of non-motivated workers only (which are lower than the outside options of motivated types). In other words, in order to succeed in hiring non-motivated types Am and am, principal S must be able to leave them at least U_{Am}^{MO} and U_{am}^{MO} , respectively. Such reservation utilities are endogenous but, because of the simultaneity of moves, are taken as given by principal S.

Then, the standard principal's program is as FDMOP in Section 4.1 except that the relevant participation constraints are those of non-motivated workers

$$w_{i.}^{S} - \frac{1}{2}\theta_{i} \left(e_{i.}^{S}\right)^{2} \ge U_{im}^{MO}$$
 (PC_{im})

for every i = a, A.

One can replicate the analysis which has been carried out in Section 4.1 and in Appendix A.2, substituting the total surplus utilities $U_{AM}^{TS,MO} - U_{aM}^{TS,MO}$ which mattered there with $U_{Am}^{MO} - U_{am}^{MO}$ and, accordingly, substituting the participation constraints PC_{AM}^{S} and PC_{aM}^{S} with the now relevant PC_{Am}^{S} and PC_{am}^{S} , respectively. The five different regimes are still in place and so are the optimal effort levels associated with each regime. Figure 1 represents the reaction function of principal S who is interested in hiring non-motivated types only.

Insert Figure 1 around here

Notice that the two firms' programs are now interdependent. Indeed, when PC_{im}^{S} is binding for the type with ability i=a,A, then it must necessarily be the case that PC_{im}^{MO} is binding as well. In other words, $U_{iM}^{S}=U_{im}^{S}=U_{im}^{MO}$ and type im is indifferent between working for either firm (the tie-breaking rule mentioned at the end of Section 2 might then apply). Conversely, when PC_{im}^{S} is slack, then it must be the case that $U_{im}^{S}>U_{im}^{MO}$ and that type im strictly prefers to work for the standard firm rather than for the mission-oriented principal.

5.2 The mission-oriented principal

Remind that, as opposed to the standard principal, the mission-oriented principal (when possible) offers four different contracts. In equilibrium, principal MO will expect to hire motivated agents only and will offer out-of-equilibrium contracts to non-motivated types so as to satisfy internal incentive compatibility.

In order to solve principal's MO program, we start by taking as given each one of the possible five regimes in which principal S can find himself. This allows to single out which participation constraint between PC_{Am}^S and PC_{am}^S is binding. When PC_{im}^S , with i=a,A, is binding, it means that PC_{im}^{MO} is binding as well and that type im is indifferent between the two principals. Then, the dominated principal MO will offer this type her first-best total surplus and will make zero profits from that type. In particular, if PC_{Am} is binding, then $U_{Am}^S = U_{Am}^{TS,MO} = \frac{1}{2}$ and the mission-oriented firm obtains zero profit on the out-of-equilibrium contract for worker Am. In the same way, if PC_{am} is binding then $U_{am}^S = U_{am}^{TS,MO} = \frac{1}{2\theta}$ and the mission-oriented principal earns zero profit from worker am.²⁶

For the sake of concreteness, let us suppose that the standard principal is in the situation where irrelevance of outside options holds (Case 1 in Figure 1). Then PC_{Am}^S is slack (i.e. $U_{Am}^S > U_{Am}^{MO}$) while PC_{am}^{S} is binding (i.e. $U_{am}^{S}=U_{am}^{MO}$). Thus, we study a program for the mission-oriented principal which is similar to FDMOP, but which is such that PC_{Am}^{MO} is slack (as $U_{Am}^S > U_{Am}^{MO}$) and PC_{am}^{MO} is binding, whereby the contract offered to type am satisfies $e^{MO}_{am}=e^{FB,MO}_{am}=\frac{1}{\theta}=w^{MO}_{am}$ and $U^{MO}_{am}=U^{TS,MO}_{am}=\frac{1}{2\theta}$. The solution will clearly depend on whether motivation prevails, ability prevails, or neither motivation nor ability prevail. In case of multiple solutions, we take the one guaranteeing the highest profits to principal MO. Once the bidimensional screening problem of the mission-oriented firm is solved, the utility U_{Am}^{MO} is also well defined so that the value $U_{Am}^{MO}-U_{am}^{TS,MO}$, which enters the solution of principal S's program, is fully determined. The last step consists in checking whether the difference $U_{Am}^{MO} - U_{am}^{TS,MO}$, that has been found solving the mission-oriented principal's program, is compatible with the bounds defining Case 1 for principal S. If so, then the solution obtained is an equilibrium, otherwise it must be discarded. We repeat the same procedure for all the other regimes for principal S, from 2 to 5. Notably, the difference in reservation utilities $U_{Am}^{MO}-U_{am}^{MO}$ is never too big so as to yield Cases 4 and 5 for the standard principal. Therefore, countervailing incentives are never observed at equilibrium. This analysis is relegated to Appendix A.4.

Least, but not last, consider that the standard principal is constrained to offer only two contracts and that non-motivated types are worse-off when mimicking motivated workers employed by the missionoriented firm. Thus, one may easily check that incentive compatibility *between* principals is always

²⁶Note that, because of competition, the mission-oriented principal is able to screen applicants at a higher cost than when he does not face any rival principal. In particular, by increasing (with respect to the monopsony case with zero outside options for all types) the utility that non-motivated workers obtain out of equilibrium, the mission-oriented principal must pay larger information rents to motivated workers.

satisfied in equilibrium.

5.3 Sorting according to motivation

In what follows, we characterize the optimal incentive schemes offered by the two competing principals when $1 \le k \le 1 + \gamma$ and when workers optimally sort themselves by motivation. We simplify the analysis by restricting attention to a uniform distribution of types, whereby the probability of each type is set equal to 1/4.

Different situations emerge according to the magnitude of k, which governs principal S's regimes, and according to the magnitudes of γ and θ , that influence the states of the world for principal MO, as we explain in Appendix A.4.

Situation (i) When k is high and γ is not too low, i.e. when $\overline{k} = \frac{(2\theta - 1)}{\theta} \le k \le 1 + \gamma$ and $\underline{\gamma} = \frac{(\theta - 1)}{\theta} \le \gamma < 1$, then the optimal incentive scheme is such that:

Principal S is always in Case 1 (irrelevance of outside options) and sets the second-best effort levels $e_{AM}^{FB,S} = e_{Am}^{FB,S} = k$ and $e_{aM}^{SB,S} = e_{am}^{SB,S} = \frac{k}{(2\theta-1)}$.

Principal MO is such that motivation always prevails: employed types AM and aM are required to make efforts $e^{FB,MO}_{AM}=1+\gamma$ and $e^{SB,MO}_{aM}=\frac{1+\gamma}{(2\theta-1)}$, types Am and am are offered out-of-equilibrium pooling contracts with effort $e^{MO}_{Am}=e^{FB,MO}_{am}=\frac{1}{\theta}$, respectively.

Situation (ii) When k is not high and $1 \le k < \overline{k}$, the equilibrium is in dominant strategies and optimal incentive schemes are as follows:

Principal S is such that:

- when $1 \le k < \underline{k} = \sqrt{\theta}$, Case 3 holds and first-best effort levels are set for all workers $e_{AM}^{FB,S} = e_{Am}^{FB,S} = k$ and $e_{aM}^{FB,S} = e_{am}^{FB,S} = \frac{k}{\theta}$,
- when $\underline{k} \leq k < \overline{k}$, Case 2 holds and optimal effort levels $e_{AM}^{FB,S} = e_{Am}^{FB,S} = k$ and $e_{aM}^{*,S} = e_{am}^{*,S} = \frac{1}{\sqrt{\theta}}$ are required.

Principal MO is such that:

• when $0 < \gamma < \gamma^{\mathcal{A}} = (\theta - 1)$, ability prevails (Case \mathcal{A}), motivated types are asked to provide first-best effort levels $e^{FB,MO}_{AM} = 1 + \gamma$ and $e^{FB,MO}_{aM} = \frac{1+\gamma}{\theta}$ and non-motivated types are offered out-of-equilibrium contracts with first-best effort levels $e^{FB,MO}_{am} = \frac{1}{\theta}$ and $e^{FB,MO}_{Am} = 1$ and all the surplus.

²⁷Observe that $\overline{k} \le 1 + \gamma$ if and only if $\gamma \ge \underline{\gamma}$ therefore it cannot simultaneously be that $\gamma < \underline{\gamma}$ and $k > \overline{k}$.

- for $\gamma^A \leq \gamma \leq \gamma^M = 2 (\theta 1)$, neither ability nor motivation prevail, intermediate types' effort levels are pooled and the first-best total surplus is offered out-of-equilibrium to non-motivated types, whereby $e_{AM}^{FB,MO} = 1 + \gamma$, $e_{aM}^{MO} = e_{Am}^{FB,MO} = 1$ and $e_{am}^{FB,MO} = \frac{1}{\theta}$, with $e_{aM}^{SB,MO} < e_{aM}^{MO} = 1 < e_{aM}^{FB,MO}$.
- for $\gamma^{\mathcal{M}} < \gamma \leq 1$ and $\theta < \frac{3}{2}$ (ensuring that $\gamma^{\mathcal{M}} < 1$), motivation prevails (Case \mathcal{M}), motivated types are required to provide effort levels $e_{AM}^{FB,MO} = 1 + \gamma$ and $e_{aM}^{SB,MO} = \frac{1+\gamma}{2\theta-1}$ and non-motivated types are offered out-of-equilibrium the first-best total surplus, whereby $e_{am}^{FB,MO} = \frac{1}{\theta}$ and $e_{Am}^{FB,MO} = 1$.

As mentioned at the end of Section 4.1 and in the proof of Proposition 1, when $k = 1 + \gamma$, the unique equilibrium is the one described in Situation (i).

Also observe that in Situation (ii) both firms have dominant strategies and their optimal contracts are independent of what the rival proposes. This feature of the equilibrium depends on the fact that, when principal S is in Cases 2 or 3, both participation constraints of non-motivated types are binding and thus the mission-oriented principal always offers to these types their first-best total surplus. Hence, the difference in outside options for non-motivated types is fixed and does not depend on γ . In addition, the standard principal will find himself in Case 2 or 3 depending on the magnitude of k, while the mission-oriented principal will choose his optimal contracts not according to k but only according to the relative magnitudes of γ and $\Delta\theta$, which in turn determine whether motivation or ability prevail.

Finally, it can be checked that, given ability, motivated types hired by the mission-oriented firm always provide higher effort than non-motivated types hired by the standard firm, with the exception of low-ability types in Situation (ii) when motivation prevails for principal MO and principal S is in Case 3.

We further propose a taxonomy of the above-mentioned equilibria with respect to the degree of competition between principals, which in turn depends on whether principals are sufficiently different both in technology and in the impact of the workers' motivation.

Proposition 2 Neither firm distorts effort provided by high-ability workers. As for the effort provided by low-ability workers, the following happens: (a) If competition is mild, i.e. if k is high and γ is not too low, then both firms ask low-ability workers to provide the second-best effort levels, and Situation (i) holds. (b) If competition is harsh, i.e. if both k and γ are low, then both firms ask low-ability workers to provide first-best effort levels and Situation (ii) holds with principal S being in Case 3 and principal MO being in Case A. (c) Otherwise, firms might ask low-ability workers that they hire to provide an effort which is in-between the first- and second-best level.

The intuition behind these results is the following. When competition is harsh, because firms are similar to each other, then outside options are the determinant of equilibrium effort levels for hired

workers, whereas internal incentive compatibility only plays a minor role. This outcome is the one that most resembles the full information equilibrium corresponding to Bertrand competition, with each firm requiring first-best effort levels and offering wages such that the best offer of the competitor is met. Conversely, when competition is mild, because firms are sufficiently differentiated from each other, then outside options are not particularly relevant and internal incentive compatibility is the driving force in determining equilibrium effort levels for hired workers, which are the same as under monopsony. Also note that, when competition is mild, the so called *separation property* is satisfied, whereby competition among principals only affects the agents' compensation schemes but not the optimal allocation, that is the effort levels (see Biglaiser and Mezzetti 2000, and the references therein). Finally, when the degree of competition is neither harsh nor mild, then the *separation property* does not hold because, at least for principal S, distortions in the agents' effort levels are either reduced with respect to the second-best contracts or vanish.

We can conclude that distortions in effort provision are always decreasing in the level of competition and that competition between principals never leads to *countervailing incentives*, i.e. upward distortions in effort levels never occur.

5.4 Market segmentation, wage differentials and returns to ability

In this section, we compare the wage schemes offered by the two firms. In particular, it is interesting to consider the model's predictions as for the wage differential, if any, between the mission-oriented firm and the standard one. We focus on the case of workers sorting according to their motivation, i.e. $1 \le k \le 1+\gamma$, and we first compare the wage rate offered by the two principals to motivated and non-motivated workers, fixing the level of ability. Then, we compare the return to ability across firms, that is we consider the wage increase that employees receive in response to an increase in their level of ability. This concept bears some similarity to the power of incentives studied in a moral hazard framework (see Besley and Gathak 2005, where it is suggested that mission-oriented firms offer low-powered incentives to their employees, and Bénabou and Tirole 2013) and transposed in an adverse selection framework by Delfgaauw and Dur (2008) and Makris (2009).

For a wide range of parameter configurations, it can be shown that

$$w_{AM}^{MO} < w_{AM}^S = w_{Am}^S \tag{6}$$

and also that

$$w_{aM}^{MO} < w_{aM}^S = w_{am}^S. (7)$$

These results hold when competition is mild, i.e. when Situation (i) occurs and k is sufficiently high (higher than the relevant threshold $\overline{k} = \frac{(2\theta-1)}{\theta}$), or when Situation (ii) holds, ability prevails for the

mission-oriented principal (meaning that γ must be low) and k is still sufficiently high (higher than the threshold $\underline{k} = \sqrt{\theta}$). The intuition is the following: when Situation (i) holds and the mission-oriented firm offers out-of-equilibrium a pooling contract to non-motivated types, or when Situation (ii) holds and ability prevails for principal MO (i.e. γ is low), then motivated agents do not cumulate large information rents because they are unable to mimic many other types of workers. These agents are thus offered low wages. This fact depressed the left-hand side of the above inequalities. On the other hand, when k is sufficiently high, principals are sufficiently differentiated in terms of technology and this raises the wages that are paid by the standard firm, thus raising the right-hand side of the above inequalities.

So we can indeed observe a true compensating wage differential between the two firms, which is entirely driven by intrinsic motivation and which does not depend on the differences in workers' ability, given that average ability is the same for both firms (as discussed below Corollary 2). However, ability does matter in that inequality (6) is easier to be satisfied than inequality (7). In other words, it might be the case that the wage differential exists for high-ability types but not for low-ability workers or that it is larger for high-ability workers than for low-skilled employees. This supports the empirical findings that the wage differential is increasing in ability and that the wage penalty is more severe at the top of the wage ladder rather than at the bottom. The fact that the public sector wage penalty is higher for managers and top executives with respect to lower levels in the hierarchy is documented by Preston (1989), Roomking and Weisbrod (1999) and Bargain and Melly (2008), among others.²⁸

Finally, note that when the wage differential is in place, it is always the case that $e_{AM}^{MO} > e_{AM}^S = e_{Am}^S$ and that $e_{aM}^{MO} > e_{aM}^S = e_{am}^S$. Hence, equally skilled workers provide higher effort when hired by the mission-oriented firm that offers lower wage rates. This is not sufficient to generate higher profits for the mission-oriented principal, because his technology is inferior to that of the standard principal.

The fact that a compensating wage differential emerges for a wide range of parameter configurations, coupled with the observation that such wage differential is increasing in ability, has immediate implications on the return to ability provided by the two firms. Let us then consider the difference between the return to ability for workers hired by the mission-oriented firm, that is $w_{AM}^{MO} - w_{aM}^{MO}$, and the return to ability for workers hired by the standard firm, i.e. $w_{Am}^S - w_{am}^S$. In particular, if

$$w_{AM}^{MO} - w_{aM}^{MO} < w_{Am}^{S} - w_{am}^{S},$$

then in equilibrium, the gain from increased ability is lower for workers employed by the mission oriented firm. So, our results show that, when the wage differential exists, it is always the case that the mission-oriented firm provides lower returns to ability relative to the standard firm.

The remark that follows fixes the main ideas illustrated in this section.

²⁸We refer the reader to the excellent review of the literature contained in Delfgaauw and Dur (2010)

Remark 2 When k is sufficiently high and either competition is mild or ability prevails for the mission-oriented firm (γ is low) then: (i) a compensating wage differential exists because, fixing ability, motivated workers employed by the mission-oriented firm earn less, although exerting more effort, than if they were employed by the standard firm; (ii) the mission-oriented firm provides lower returns to ability relative to the standard firm.

Conversely, we always observe a wage premium for workers hired by the mission-oriented firm when k is not high and γ is high, namely in Situation (ii) with motivation prevailing for the mission-oriented firm and firm S being in Cases 2 or 3. More generally, a wage premium can still be in place for motivated workers when k is sufficiently low while γ is sufficiently high.

To conclude, our model can accommodate and explain both the empirical evidence showing the existence of a wage differential in favor of workers employed at standard firms (as an example, Roomking and Weisbrod 1999, DeVaro et al. 2015) and the evidence of a wage gap favoring instead workers employed at mission-oriented firms/sectors (see, for example, the works on the non-profit sector by Mocan and Tekin 2003, Borjas et al. 1983, and Preston 1988).

6 Concluding remarks

In our model, only when a firm is mission-oriented, can it generate a non-monetary benefit that motivated workers enjoy when they exert effort and contribute to the firm's output and goal. In different words, the interaction between a mission-oriented firm and a motivated worker increases the total surplus that employer-employee pairs can obtain.

Although firms' differentiation according to a mission is exogenously given in our framework, the model is sufficiently rich to provide some interesting insights concerning the conditions that allow mission-oriented and standard firms to coexist in the market.²⁹ Moreover, the comments and examples that follow represent a motivation for the different parameter configurations analyzed in the model (that is k < 1, and $1 \le k \le 1 + \gamma$ or $k > 1 + \gamma$)

Start with the case in which k = 1, namely consider the instances in which the two firms are endowed with the same technology. Then, without the choice of a mission by one of them, the two firms would be identical and competition would be so harsh as to drive their profits from each worker type to zero. By choosing a mission, instead, a firm is able to obtain positive profits from motivated workers, although competition still drives profits from non-motivated workers to zero.³⁰ Thus mission-orientation

²⁹In their Section C, Besley and Ghatak (2005) discuss how to extend their moral-hazard model to the situation where the choice of the mission by principals is endogenous.

³⁰Mission-orientation, although, has the drawback of enlarging the type space, so that the firm is forced to pay more information rents to screen workers.

provides a way out of the Bertrand paradox and it is similar to horizontal differentiation heavily analyzed in Industrial Organization; the difference is that, in our model, mission-orientation aims at increasing the willingness to pay of workers, who are ready to accept lower salaries, rather than the consumers' willingness to pay for final goods; moreover, in our model, firms' differentiation is asymmetric, in the sense that only one firm benefits from mission-orientation.

Our model can also accommodate for mission-orientation stemming from corporate social responsibility. The standard view of corporate social responsibility is that it is about sacrificing profits in the social interest (see the discussion in Bénabou and Tirole 2010). In our model, the mission-oriented firm has a positive impact on society because of the additional surplus generated through motivated workers. Moreover, the fact that a firm is ready to sacrifice some profits in order to pursue its mission corresponds to the instance in which the mission-oriented firm suffers from some disadvantage with respect the standard firm. And this happens when the difference between the two firms' technologies is such that k > 1. In this situation, our model predicts that a mission-oriented firm can survive as long as the benefit from attracting motivated workers, and improving the firm's performance because of their labor donation, more than compensates the opportunity-cost of being mission-oriented, i.e. as long as $1 + \gamma \geq k$. Note that the coexistence of mission-oriented and standard firms in the market not only increases overall efficiency because of the surplus generated by the premium for intrinsic motivation but also because competition reduces the distortions in the optimal allocations, i.e. effort levels (see Proposition 2). When $k > 1 + \gamma$, instead, the opportunity-cost of being mission-oriented is so high that the standard firm is fully dominant and the mission-oriented firm only acts as a potential entrant offering outside-options that increase the workers' outside options and, consequently, the workers' salary.

Finally, as Bénabou and Tirole (2010) point out, overall there seems to be no or a slightly positive correlation between socially responsible behaviour and corporate returns (see references therein for further discussion), so that also the case where k < 1 is worthy to be studied. Now, the mission-oriented firm does not face any trade-off and, rather, it enjoys a double advantage: it has a superior technology and it benefits from workers' intrinsic motivation. This case mirrors the one with $k > 1 + \gamma$ because the standard firm plays the role of a potential entrant offering outside options that reduce the market power of the monopsonist, mission-oriented firm.

A Appendix

A.1 Competition under full information

When k < 1, the mission-oriented principal is able to hire all workers, who are asked to provide the first-best effort (see equation 2) and receive a payoff equal to the best offer of the standard principal. Note

that workers are indifferent between accepting the contracts proposed by the two firms: the tie-breaking rule applies in favor of the mission-oriented principal who makes positive profits on all types. In this case, wages are given by

$$w_{AM}^{MO} = \underbrace{\frac{k^2}{2}}_{+} + \underbrace{\frac{(1+\gamma)^2}{2} - \gamma (1+\gamma)}_{-}$$
(8)

$$w_{aM}^{MO} = \underbrace{\frac{k^2}{2\theta}}_{\text{outside option}} + \underbrace{\frac{(1+\gamma)^2}{2\theta} - \frac{\gamma(1+\gamma)}{\theta}}_{\text{net cost of effort}}$$
(9)

$$w_{Am}^{MO} = \underbrace{\frac{k^2}{2}} + \underbrace{\frac{1}{2}} \tag{10}$$

$$w_{Am}^{MO} = \frac{k^2}{2} + \frac{1}{2}$$
outside option cost of effort
$$w_{am}^{MO} = \frac{k^2}{2\theta} + \frac{1}{2\theta}$$
(10)
$$w_{am}^{MO} = \frac{k^2}{2\theta} + \frac{1}{2\theta}$$
outside option cost of effort

where the first term in each line covers the outside option (the best offer of the competitor) while the second part rewards the (net) cost of the first-best effort.

When $k > 1 + \gamma$, the standard principal is able to hire all workers by asking them to provide the first-best effort (see equation 1 now) and by offering them the same payoff as the competitor. Wages are given by

$$w_{AM}^S = \underbrace{\frac{(1+\gamma)^2}{2}}_{\text{outside option}} + \underbrace{\frac{k^2}{2}}_{\text{cost of effort}}$$
 (12)

$$w_{aM}^{S} = \underbrace{\frac{(1+\gamma)^{2}}{2\theta}}_{\text{outside option}} + \underbrace{\frac{k^{2}}{2\theta}}_{\text{cost of effort}}$$
(13)

$$w_{Am}^{S} = \underbrace{\frac{1}{2}}_{\text{outside option}} + \underbrace{\frac{k^{2}}{2}}_{\text{cost of effort}}$$
(14)

$$w_{Am}^{S} = \underbrace{\frac{1}{2\theta}}_{\text{outside option}} + \underbrace{\frac{k^{2}}{2\theta}}_{\text{cost of effort}}$$
(15)

Finally, when $1 < k < 1 + \gamma$, there is segmentation in that motivated workers are hired by the missionoriented principal at wages (8) and (9), whereas non-motivated workers are hired by the standard principal at wages (14) and (15).

We can summarize the equilibrium allocation under full information as follows.

Remark 3 Competition under full information. Optimal contracts are such that all effort levels are set at the first-best. Both principals earn positive profits on the types they hire and workers receive a positive reservation utility corresponding to the best offer that the less efficient firm is able to make.

(a) When k < 1, all worker's types are hired by the mission-oriented principal. (b) When $k > 1 + \gamma$, all worker's types are hired by the standard principal. (c) When $1 < k < 1 + \gamma$ the market is fully segmented: the mission-oriented principal hires motivated workers whereas the standard principal hires non-motivated ones.

When k=1 or when $k=1+\gamma$ principals are equally efficient in hiring non-motivated or motivated types, respectively. The tie-breaking rule does not apply in these cases, because both principals earn zero profits on contested types. Thus, when k=1, principal MO could hire all worker's types although earning strictly positive profits from motivated types only or he could hire motivated workers only and full segmentation would remain the equilibrium. A symmetric argument applies to principal S when $k=1+\gamma$.

A.2 Fully dominant standard principal

Suppose that the standard principal is able to hire all types of workers when the mission-oriented rival is giving them the first-best total surplus. The fully dominant standard principal's program is FDSP given in the main text. Five different regimes are possibly relevant depending on which (motivated types') participation and incentive constraints are binding.

A.2.1 Case 1: Irrelevance of outside options

Suppose that PC_{aM}^S and $IC_{A\cdot vsa}^S$ are the binding constraints as in the standard two-types adverse selection problem. Solving the binding constraints for wages (and omitting both the superscript S referring to the standard principal and the second subindex referring to motivation, when no confusion arises) one obtains

$$w_a = \frac{1}{2}\theta e_a^2 + U_{aM}^{TS,MO} \tag{16}$$

and

$$w_A = \frac{1}{2}e_A^2 + \frac{1}{2}(\theta - 1)e_a^2 + U_{aM}^{TS,MO}.$$
(17)

Substituting such wages into the standard principal's programme yields

$$E\left(\pi^{S}\right) = \nu \left(ke_{A} - \left(\frac{1}{2}e_{A}^{2} + \frac{1}{2}\left(\theta - 1\right)e_{a}^{2} + U_{aM}^{TS,MO}\right)\right) + (1 - \nu)\left(ke_{a} - \left(\frac{1}{2}\theta e_{a}^{2} + U_{aM}^{TS,MO}\right)\right)$$

and maximizing with respect to effort levels gives

$$e_A = k = e_A^{FB}$$

and

$$e_a = \frac{k(1-\nu)}{\theta-\nu} = e_a^{SB} < e_A.$$

Let us then check ex-post that omitted constraints are indeed satisfied. Participation constraint PC_{AM} is slack iff

$$\frac{1}{2}e_{A}^{2}+\frac{1}{2}\left(\theta-1\right) e_{a}^{2}+U_{aM}^{TS,MO}-\frac{1}{2}e_{A}^{2}>U_{AM}^{TS,MO}$$

that is iff

$$e_a > \sqrt{\frac{2\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right)}{\left(\theta - 1\right)}}$$

or, substituting for the optimal value of e_a iff

$$\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right) < \frac{(\theta - 1)k^2(1 - \nu)^2}{2(\theta - \nu)^2} = \Delta U_1.$$
(18)

Since $U_{AM}^{TS,MO} - U_{aM}^{TS,MO}$ is known to the standard principal and is equal to $\frac{(\theta-1)(1+\gamma)^2}{2\theta}$, condition (18) can be rewritten, solving explicitly for k, as

$$k > \frac{(1+\gamma)(\theta-\nu)}{\sqrt{\theta}(1-\nu)} = k_1$$

where $k_1 > 1 + \gamma$ always holds.

The payoff to the standard principal from hiring high-ability workers is equal to

$$\pi_A = ke_A - w_A = ke_A - \left(\frac{1}{2}e_A^2 + \frac{1}{2}(\theta - 1)e_a^2 + U_{aM}^{TS,MO}\right),$$

which, substituting for optimal effort levels and for $\boldsymbol{U}_{aM}^{TS,MO},$ amounts to

$$\pi_A = \frac{k^2}{2} - \frac{k^2 (1 - \nu)^2 (\theta - 1)}{2 (\theta - \nu)^2} - \frac{(1 + \gamma)^2}{2\theta},$$

where $\pi_A > 0$ is always true when $k > k_1$. Similarly, the payoff to the standard principal from hiring low-ability workers is equal to

$$\pi_a = ke_a - w_a = ke_a - \left(\frac{1}{2}\theta e_a^2 + U_{aM}^{TS,MO}\right),$$

which, substituting for optimal effort e_a^{SB} and for $U_{aM}^{TS,MO}$, amounts to

$$\pi_a = \frac{k^2 \left(1 - \nu\right) \left(\theta - 2\nu + \theta\nu\right)}{2 \left(\theta - \nu\right)^2} - \frac{\left(1 + \gamma\right)^2}{2\theta},$$

where $\pi_a > 0$ is always true when $k > k_1$.

Summarizing, Case 1 is characterized by PC_{aM} and IC_{Avsa} holding with equality and by effort levels $e_A^{FB} = k$ and $e_a^{SB} = \frac{k(1-\nu)}{\theta-\nu}$; it holds for $k > \frac{(1+\gamma)(\theta-\nu)}{\sqrt{\theta}(1-\nu)} = k_1 > 1 + \gamma$.

A.2.2 Case 2: Both PCs and the high-ability workers' IC are binding

Suppose now that both participation constraints PC_{aM} and PC_{AM} are binding and that IC_{Avsa} binds as well. Solving the binding constraints for wages one obtains expressions (16),

$$w_A = \frac{1}{2}e_A^2 + U_{AM}^{TS,MO} \tag{19}$$

and (17), respectively, whereby, equating (19) and (17) one gets

$$U_{AM}^{TS,MO} - U_{aM}^{TS,MO} = \frac{1}{2} \left(\theta - 1\right) e_a^2$$

or

$$e_a = \sqrt{\frac{2\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right)}{(\theta - 1)}} = e_a^*.$$

Finally, maximizing the principal's objective function with respect to e_A only yields

$$e_A = k = e_A^{FB}$$
.

Note that effort for the low-ability type is less downward distorted than in Case 1 iff $e_a^* \ge e_a^{SB}$ or else iff

$$\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right) \ge \frac{k^2 (1-\nu)^2 (\theta-1)}{2 (\theta-\nu)^2} = \Delta U_1.$$

Moreover, consider the incentive compatibility constraint IC_{avsA} that was ignored in the reduced programme: it is satisfied when

$$e_A > \sqrt{\frac{2\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right)}{(\theta - 1)}}, \label{eq:eA}$$

that is when $e_A^{FB} = k > e_a^*$, hence it is verified when

$$\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right) < \frac{k^2 (\theta - 1)}{2\theta^2} = \Delta U_2,$$
 (20)

where $\Delta U_2 > \Delta U_1$. So this case holds for $\Delta U_1 \leq \left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right) < \Delta U_2$. Alternatively, replacing $\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right)$ for its value $\frac{(\theta-1)(1+\gamma)^2}{2\theta}$ one gets $e_a^* = \frac{(1+\gamma)}{\sqrt{\theta}}$ and solving condition (20) explicitly for k one obtains

$$k > (1 + \gamma)\sqrt{\theta} = k_2$$

where $1 + \gamma < k_2 < k_1$. Hence, Case 2 holds for $k_2 < k \le k_1$.

The payoff to the standard principal from hiring high-ability workers is equal to

$$\pi_A^S = ke_A - w_A = ke_A - \left(\frac{1}{2}e_A^2 + U_{AM}^{TS,MO}\right) = \frac{k^2}{2} - \frac{(1+\gamma)^2}{2}$$

and it is strictly positive when $k > 1 + \gamma$. Similarly, the payoff to the standard principal from hiring low-ability workers is equal to

$$\pi_a = ke_a - w_a = ke_a - \left(\frac{1}{2}\theta e_a^2 + U_{aM}^{TS,MO}\right) = \frac{k(1+\gamma)}{\sqrt{\theta}} - \frac{(\theta+1)(1+\gamma)^2}{2\theta}$$

and, again, it is such that $\pi_a > 0$ when $k > k_2$.

In short, Case 2 is characterized by PC_{aM} , PC_{AM} and IC_{Avsa} all holding with equality and by effort levels $e_A^{FB} = k$ and $e_a^* = \frac{(1+\gamma)}{\sqrt{\theta}}$; it holds for $1+\gamma < (1+\gamma)\sqrt{\theta} = k_2 < k \le k_1$.

A.2.3 Case 3: Both PCs are binding

Suppose now that participation constraints of both types AM and aM are binding and that the low-ability agents' incentive compatibility constraint is slack. Then, effort levels are the efficient ones, namely

$$e_A^{FB} = k$$

and

$$e_a^{FB} = \frac{k}{\theta}.$$

Examining the incentive compatibility constraint IC_{Avsa} one finds that it is satisfied if and only if

$$e_a \leq \sqrt{\frac{2\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right)}{(\theta - 1)}}$$

which is true for $\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right) \ge \Delta U_2$. As far as the incentive compatibility constraint IC_{avsA} is concerned, it is slack iff

$$e_A > \sqrt{\frac{2\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right)}{(\theta - 1)}}$$

holds. So this case occurs if and only if

$$\Delta U_2 \le \left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO} \right) < \frac{k^2 (\theta - 1)}{2} = \Delta U_3$$

or iff

$$k_3 = \frac{(1+\gamma)}{\sqrt{\theta}} < k \le k_2$$

where $k_3 < 1 + \gamma$.

The payoff to the standard principal from hiring high-ability workers is the same as in Case 2 and it is non-negative iff $k \ge 1 + \gamma$. Similarly, the payoff to the standard principal from hiring low-ability workers is equal to

$$\pi_a^S = ke_a - w_a = ke_a - \left(\frac{1}{2}\theta e_a^2 + U_{aM}^{TS,MO}\right) = \frac{k^2}{2\theta} - \frac{(1+\gamma)^2}{2\theta}$$

which is non-negative for $k \geq 1 + \gamma$. Hence Case 3 is only valid when $1 + \gamma \leq k < k_2$ otherwise the principal makes negative profits on all workers' types.

Summarizing, Case 3 is characterized by PC_{aM} and PC_{AM} holding with equality and by effort levels $e_A^{FB} = k$ and $e_a^{FB} = \frac{k}{\theta}$. It holds for $k_3 = \frac{(1+\gamma)}{\sqrt{\theta}} < k \le k_2$, but, because the standard principal is making strictly negative profits for $k < 1 + \gamma$, then Case 3 is only relevant when $1 + \gamma \le k \le k_2$.

A.2.4 Case 4: Both PCs and low-ability workers' IC are binding

Suppose that both participation constraints remain binding but, because the low-ability types are attracted by the contract offered to the high-ability types, low-ability agents' incentive constraint is binding

as well. Solving the binding constraints for wages one obtains expressions (16) and (19) together with

$$w_a = \frac{1}{2}\theta e_a^2 - \frac{1}{2}(\theta - 1)e_A^2 + U_{AM}^{TS,MO}.$$
 (21)

Equating expressions (16) and (21) yields

$$e_A^* = \sqrt{\frac{2\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right)}{(\theta - 1)}} = \frac{(1 + \gamma)}{\sqrt{\theta}}$$

and maximizing the principal's programme with respect to e_a only one gets

$$e_a^{FB} = \frac{k}{\theta}.$$

Note that the incentive compatibility constraint IC_{Avsa}^S that was ignored is slack if and only if $\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right) > \Delta U_3$. Precisely the same condition ensures that the high-ability worker's effort is distorted upwards with respect to its first-best level. After the discussion of Case 5 below, it will be clear that Case 4 arises when

$$\Delta U_3 < \left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO} \right) < \frac{\nu^2 k^2 (\theta - 1)}{2 (1 - \theta (1 - \nu))^2} = \Delta U_4$$

or, in terms of k when

$$k_4 = \frac{(1+\gamma)(1-\theta(1-\nu))}{\nu\sqrt{\theta}} < k < k_3 < (1+\gamma).$$

The payoff to the standard principal from hiring high-ability workers is the same as the payoff from low-ability workers in Case 2 and it is equal to

$$\pi_A = ke_A - w_A = ke_A - \left(\frac{1}{2}e_A^2 + U_{AM}^{TS,MO}\right) = \frac{k(1+\gamma)}{\sqrt{\theta}} - \frac{(\theta+1)(1+\gamma)^2}{2\theta},$$

where $\pi_A < 0$ is always the case when $k < 1 + \gamma$. Similarly, the payoff to the standard principal from hiring low-ability workers is the same as in Case 3 and it is strictly negative iff $k < 1 + \gamma$. Hence Case 4 can be discarded because it yields strictly negative profits to the principal for all workers' types.

A.2.5 Case 5: Countervailing incentives

Finally, suppose that participation constraint PC_{AM} and incentive constraint IC_{avsA} are both binding. Wages must then satisfy conditions (19) and (21). Substituting these expressions into the principal's profit function one obtains

$$\max_{e_A;e_a} E\left(\pi^S\right) = \nu \left(ke_A - \left(\frac{1}{2}e_A^2 + U_{AM}^{TS,MO}\right)\right) + (1-\nu)\left(ke_a - \left(\frac{1}{2}\theta e_a^2 - \frac{1}{2}\left(\theta - 1\right)e_A^2 + U_{AM}^{TS,MO}\right)\right);$$

the solutions to the above programme are

$$e_A^{CI} = \frac{\nu k}{1 - \theta \left(1 - \nu\right)},$$

where the superscript CI stands for countervailing incentives, and

$$e_a^{FB} = \frac{k}{\theta}.$$

Note that $e_A^{CI} > 0$ if and only if $\theta < \frac{1}{(1-\nu)}$ and that $e_A^{CI} > e_a^{FB}$ always holds. The incentive compatibility constraint IC_{Avsa} that was ignored is always satisfied while participation constraint PC_{aM} is satisfied for

$$e_A < \sqrt{\frac{2\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right)}{(\theta - 1)}}$$

or else for

$$\left(U_{AM}^{TS,MO} - U_{aM}^{TS,MO}\right) > \frac{\nu^2 k^2 (\theta - 1)}{2 (1 - \theta (1 - \nu))^2} = \Delta U_4.$$

Alternatively, the above condition can be expressed in terms of k as

$$k < \frac{(1+\gamma)\left(1-\theta\left(1-\nu\right)\right)}{\nu\sqrt{\theta}} = k_4.$$

The payoff to the standard principal from hiring high-ability workers is equal to

$$\pi_{A} = ke_{A} - w_{A} = ke_{A} - \left(\frac{1}{2}e_{A}^{2} + U_{AM}^{TS,MO}\right) = \frac{\left(2\left(1 - \theta\left(1 - \nu\right)\right) - \nu\right)\nu k^{2}}{2\left(1 - \theta\left(1 - \nu\right)\right)^{2}} - \frac{\left(1 + \gamma\right)^{2}}{2}$$

which is always negative for $k < 1 + \gamma$. Hence Case 5 can be discarded because it yields strictly negative profits to the principal.

A.3 Fully dominant mission-oriented principal

When principal MO fully dominates, optimal contracts are the solution to program FDMOP, which is given in the main text. The constraints that such program has to satisfy are fully displayed below.

A.3.1 Constraints

Participation constraints are the following: for type AM

$$w_{AM}^{MO} - \frac{1}{2} \left(e_{AM}^{MO} \right)^2 + \gamma e_{AM}^{MO} \ge \frac{k^2}{2},$$
 (PC_{Am})

for type Am

$$w_{Am}^{MO} - \frac{1}{2} \left(e_{Am}^{MO} \right)^2 \ge \frac{k^2}{2},$$
 (PC_{Am})

for type aM

$$w_{aM}^{MO} - \frac{1}{2}\theta \left(e_{aM}^{MO}\right)^2 + \gamma e_{aM}^{MO} \ge \frac{k^2}{2\theta}$$
 (PC_{aM})

and finally for type am one has

$$w_{am}^{MO} - \frac{1}{2}\theta \left(e_{am}^{MO}\right)^2 \ge \frac{k^2}{2\theta}.$$
 (PC_{am})

The incentive compatibility constraints are the following: for type AM

$$w_{AM}^{MO} - \frac{1}{2} \left(e_{AM}^{MO} \right)^2 + \gamma e_{AM}^{MO} \ge w_{Am}^{MO} - \frac{1}{2} \left(e_{Am}^{MO} \right)^2 + \gamma e_{Am}^{MO}, \qquad (IC_{AMvsAm}^{MO})$$

$$w_{AM}^{MO} - \frac{1}{2} \left(e_{AM}^{MO} \right)^2 + \gamma e_{AM}^{MO} \ge w_{aM}^{MO} - \frac{1}{2} \left(e_{aM}^{MO} \right)^2 + \gamma e_{aM}^{MO}, \qquad (IC_{AM vsaM}^{MO})$$

$$w_{AM}^{MO} - \frac{1}{2} \left(e_{AM}^{MO} \right)^2 + \gamma e_{AM}^{MO} \ge w_{am}^{MO} - \frac{1}{2} \left(e_{am}^{MO} \right)^2 + \gamma e_{am}^{MO}; \qquad (IC_{AMvsam}^{MO})$$

for type Am

$$w_{Am}^{MO} - \frac{1}{2} \left(e_{Am}^{MO} \right)^2 \ge w_{AM}^{MO} - \frac{1}{2} \left(e_{AM}^{MO} \right)^2,$$
 (IC_{AmvsAM}^{MO})

$$w_{Am}^{MO} - \frac{1}{2} \left(e_{Am}^{MO} \right)^2 \ge w_{aM}^{MO} - \frac{1}{2} \left(e_{aM}^{MO} \right)^2,$$
 (IC_{AmvsaM}^{MO})

$$w_{Am}^{MO} - \frac{1}{2} \left(e_{Am}^{MO} \right)^2 \ge w_{am}^{MO} - \frac{1}{2} \left(e_{am}^{MO} \right)^2;$$
 (IC_{Amvsam}^{MO})

for type aM

$$w_{aM}^{MO} - \frac{1}{2}\theta \left(e_{aM}^{MO} \right)^2 + \gamma e_{aM}^{MO} \ge w_{AM}^{MO} - \frac{1}{2}\theta \left(e_{AM}^{MO} \right)^2 + \gamma e_{AM}^{MO}, \tag{IC_{aMvsAM}^{MO}}$$

$$w_{aM}^{MO} - \frac{1}{2}\theta \left(e_{aM}^{MO}\right)^2 + \gamma e_{aM}^{MO} \ge w_{Am}^{MO} - \frac{1}{2}\theta \left(e_{Am}^{MO}\right)^2 + \gamma e_{Am}^{MO}, \qquad (IC_{aM vsAm}^{MO})$$

$$w_{aM}^{MO} - \frac{1}{2}\theta \left(e_{aM}^{MO}\right)^{2} + \gamma e_{aM}^{MO} \ge w_{am}^{MO} - \frac{1}{2}\theta \left(e_{am}^{MO}\right)^{2} + \gamma e_{am}^{MO}; \qquad (IC_{aMvsam}^{MO})$$

and finally for type am one has

$$w_{am}^{MO} - \frac{1}{2}\theta \left(e_{am}^{MO}\right)^2 \ge w_{AM}^{MO} - \frac{1}{2}\theta \left(e_{AM}^{MO}\right)^2,$$
 (IC_{amvsAM}^{MO})

$$w_{am}^{MO} - \frac{1}{2}\theta \left(e_{am}^{MO}\right)^2 \geq w_{Am}^{MO} - \frac{1}{2}\theta \left(e_{Am}^{MO}\right)^2, \tag{IC_{amvsAm}^{MO}}$$

$$w_{am}^{MO} - \frac{1}{2}\theta \left(e_{am}^{MO}\right)^2 \ge w_{aM}^{MO} - \frac{1}{2}\theta \left(e_{aM}^{MO}\right)^2.$$
 (IC_{amvsaM}^{MO})

As mentioned in the main text, omitting the superscript relative to the type of principal and considering incentive compatibility constraints, implementability requires that condition

$$e_{AM} \ge \max\{e_{Am}; e_{aM}\} \ge \min\{e_{Am}; e_{aM}\} \ge e_{am}$$

be satisfied. Furthermore, concerning intermediate types Am and aM, one has that either

$$e_{aM} > e_{Am} \text{ and } e_{aM} + e_{Am} \le \frac{2\gamma}{\theta - 1},$$
 (22)

or

$$e_{Am} > e_{aM}$$
 and $e_{aM} + e_{Am} \ge \frac{2\gamma}{\theta - 1}$, (23)

or that $e_{aM} = e_{Am}$. When $e_{aM} > e_{Am}$, we will say that motivation prevails, whereas, when $e_{Am} > e_{aM}$, we will say that ability prevails. These implementability conditions allow to disregard some global downward incentive constraints and to focus on local ones.

Considering now the participation constraints, one can show that

$$\underbrace{w_{aM} - \frac{1}{2}\theta e_{aM}^2 + \gamma e_{aM} \ge w_{am} - \frac{1}{2}\theta \left(e_{am}\right)^2 + \gamma e_{am}}_{IC_{aMvsam}} > \underbrace{w_{am} - \frac{1}{2}\theta e_{am}^2 \ge \frac{k^2}{2\theta}}_{PC_{am}}$$

implying that

$$w_{aM} - \frac{1}{2}\theta e_{aM}^2 + \gamma e_{aM} > \frac{k^2}{2\theta}$$

so the participation constraint PC_{aM}^{MO} for type aM is automatically satisfied when PC_{am}^{MO} holds. Also

$$\underbrace{w_{AM} - \frac{1}{2}e_{AM}^2 + \gamma e_{AM} \ge w_{Am} - \frac{1}{2}e_{Am}^2 + \gamma e_{Am}}_{IC_{AMvsAm}} > \underbrace{w_{Am} - \frac{1}{2}e_{Am}^2 \ge \frac{k^2}{2}}_{PC_{Am}}$$

thus the participation constraint PC_{Am}^{MO} for type AM is automatically satisfied when PC_{Am}^{MO} is. So PC_{aM}^{MO} and PC_{Am}^{MO} can be discarded because they are implied by PC_{am}^{MO} and PC_{Am}^{MO} , respectively. Finally, one can write

$$\underbrace{w_{Am} - \frac{1}{2}e_{Am}^2 \ge w_{am} - \frac{1}{2}e_{am}^2}_{IC_{Amvs_{am}}} > \underbrace{w_{am} - \frac{1}{2}\theta e_{am}^2 \ge \frac{k^2}{2\theta}}_{PC_{am}}$$

In order for PC_{Am}^{MO} to be satisfied when PC_{am}^{MO} is, assume first that PC_{am}^{MO} is binding and then substitute the corresponding expression for w_{am} into the right hand side of IC_{Amvsam} . Thus one obtains

$$w_{Am} - \frac{1}{2}e_{Am}^2 \ge \frac{1}{2}(\theta - 1)e_{am}^2 + \frac{k^2}{2\theta} > \frac{k^2}{2},$$

where the last inequality is satisfied if and only if

$$e_{am} > \frac{k}{\sqrt{\theta}}.$$

But note that the highest possible value that e_{am} can take is $e_{am}^{FB}=\frac{1}{\theta}$ and $e_{am}^{FB}>\frac{k}{\sqrt{\theta}}$ holds for

$$k < \frac{1}{\sqrt{\theta}} < 1.$$

So, for k sufficiently low, PC_{Am} can be discarded, otherwise PC_{Am} must also be taken into account as relevant. In other words, when all worker types are offered a different contract by the principal, it is necessary to consider the participation constraint of the worst type am together with the one for type Am.

Let us then consider a reduced programme where one guesses which are the constraints that are binding, finds the solution, and ex-post checks that the solution is such that the neglected constraints are satisfied as well. First of all, one has to assume which condition between (22) or (23) holds. Let us analyze the case where motivation prevails and condition (22) holds; we leave it to the reader to consider the other cases (pooling of intermediate types and ability prevails).

A.3.2 Motivation prevails

Suppose that motivation prevails, whereby $e_{aM} > e_{Am}$ and $e_{aM} + e_{Am} \le \frac{2\gamma}{\theta - 1}$.

Full participation and full separation Start considering fully separating and fully participating contracts whereby optimal effort levels are such that $e_{AM} > e_{aM} > e_{am} > 0$. There are different regimes to be considered according to which constraints one assumes to be binding.

Case $\mathcal{M}.1$ (Irrelevance of outside options) Let us impose that all downward local incentive constraints IC_{AMvsaM}^{MO} , IC_{aMvsam}^{MO} , IC_{Amvsam}^{MO} bind and that PC_{am}^{MO} is also binding. Solving for the wage levels yields

$$w_{am} = \frac{1}{2}\theta e_{am}^2 + \frac{k^2}{2\theta},\tag{24}$$

$$w_{Am} = \frac{1}{2}e_{Am}^2 + \frac{1}{2}(\theta - 1)e_{am}^2 + \frac{k^2}{2\theta},$$
(25)

$$w_{aM} = \frac{1}{2}\theta e_{aM}^2 - \gamma \left(e_{aM} - e_{Am}\right) - \frac{1}{2}\left(\theta - 1\right)\left(e_{Am}^2 - e_{am}^2\right) + \frac{k^2}{2\theta}$$
 (26)

and finally

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma \left(e_{AM} - e_{Am}\right) + \frac{1}{2}\left(\theta - 1\right)e_{aM}^2 - \frac{1}{2}\left(\theta - 1\right)\left(e_{Am}^2 - e_{am}^2\right) + \frac{k^2}{2\theta}$$
(27)

Note that PC_{Am} is slack iff

$$e_{am} > \frac{k}{\sqrt{\theta}}.$$

Substituting the above wages into the principal's programme and maximizing for effort levels one gets

$$e_{AM} = 1 + \gamma = e_{AM}^{FB}, \tag{28}$$

$$e_{aM} = \frac{(1-\nu)(1+\gamma)}{(\theta-\nu)} = e_{aM}^{SB},$$
 (29)

$$e_{Am} = \frac{\nu (1 - \mu) - \mu \gamma}{(1 - (1 - \nu) (1 - \mu)) - \mu \theta} = e_{Am}^{SB}$$
(30)

and

$$e_{am} = \frac{(1-\nu)(1-\mu)}{\theta - (1-(1-\nu)(1-\mu))} = e_{am}^{SB}.$$
 (31)

Considering the monotonicity conditions, one can check that this solution exists if and only if $\theta < \min \left\{ \overline{\theta}_1^{\mathcal{M}}, \overline{\theta}_2^{\mathcal{M}} \right\}$ and $\underline{\gamma}^{\mathcal{M}} < \gamma < \overline{\gamma}^{\mathcal{M}}$ with

$$\underline{\gamma}^{\mathcal{M}} \equiv \frac{(\mu(1-\nu)+\nu(1-\mu))(\theta-1)}{(\nu\mu(\theta-1)+(1-\nu)(1-(1-\nu)(1-\mu)))}
\overline{\gamma}^{\mathcal{M}} \equiv \frac{(1-\mu)(1-(1-\nu)(1-\mu))(\theta-1)}{\mu(\theta-(1-(1-\nu)(1-\mu)))}
\overline{\theta}_{1}^{\mathcal{M}} \equiv \frac{(1-(1-\nu)(1-\mu))}{\mu}
\overline{\theta}_{2}^{\mathcal{M}} \equiv \frac{((\mu+\nu-3\mu\nu)+(1-\nu)(1-(1-\nu)(1-\mu)))}{(\mu+\nu-3\mu\nu)}$$

where $\gamma > \underline{\gamma}^{\mathcal{M}}$ is equivalent to $e_{aM} > e_{Am}$, whereas $\gamma < \overline{\gamma}^{\mathcal{M}}$ holds iff $e_{Am} > e_{am}$ and where $e_{Am} > 0$ is true iff $\theta < \overline{\theta}_1^{\mathcal{M}}$.³¹ Finally, PC_{Am} is slack when $e_{am}^{SB} > \frac{k}{\sqrt{\theta}}$ holds and the latter inequality amounts to

$$k < \frac{(1-\nu)(1-\mu)\sqrt{\theta}}{\theta - (1-(1-\nu)(1-\mu))} = k_1^{\mathcal{M}},$$

with $k_1^{\mathcal{M}} < 1$.

Case $\mathcal{M}.2$ Following a procedure similar to the one in Laffont and Martimort (2002) which we already applied for the fully dominant standard principal, suppose now that IC_{AMvsaM}^{MO} , IC_{aMvsAm}^{MO} , IC_{Amvsam}^{MO} bind and that both PC_{am}^{MO} and PC_{Am}^{MO} are binding.

Now expressions from (24) to (27) are still relevant together with

$$w_{Am} = \frac{1}{2}e_{Am}^2 + \frac{k^2}{2}. (32)$$

Then, equating (32) and (25) and solving for e_{am} yields

$$e_{am} = \frac{k}{\sqrt{\theta}} = e_{am}^*$$

whereas other effort levels are the same as in Case $\mathcal{M}.1$. Note that $e_{am}^* \leq e_{am}^{FB} = \frac{1}{\theta}$ if and only if

$$k \leq \frac{1}{\sqrt{\theta}} = k_2^{\mathcal{M}}$$

with $k_1^{\mathcal{M}} < k_2^{\mathcal{M}} < 1$. Moreover, the monotonicity condition $e_{Am}^{SB} > e_{am}^*$ is satisfied iff

$$k < \frac{\left(\nu \left(1 - \mu\right) - \mu\gamma\right)\sqrt{\theta}}{\left(1 - \left(1 - \nu\right)\left(1 - \mu\right)\right) - \mu\theta} = k_3^{\mathcal{M}}.$$

Observe that both inequalities $k_3^{\mathcal{M}}>k_2^{\mathcal{M}}$ and $e_{Am}^{SB}>e_{am}^{FB}=\frac{1}{\theta}$ hold iff

$$\gamma < \frac{\left(1 - \left(1 - \nu\right)\left(1 - \mu\right)\right)\left(\theta - 1\right)}{\theta\mu} = \gamma_1^{\mathcal{M}}$$

where $\gamma_1^{\mathcal{M}} < \overline{\gamma}^{\mathcal{M}}$ always holds and $\gamma_1^{\mathcal{M}} > \underline{\gamma}^{\mathcal{M}}$ occurs for $(\mu + \nu - 3\mu\nu - \nu^2 + \mu\nu^2) > 0$, which is always the case for $\nu \le \frac{3-\sqrt{5}}{2}$ although for $\nu > \frac{3-\sqrt{5}}{2}$ it holds when $\mu < \frac{\nu(1-\nu)}{(3\nu-\nu^2-1)} = \mu_0$ with $\mu_0 \ge 1$ iff $\nu \le \frac{1}{2}$. Moreover, $k_3^{\mathcal{M}} > k_1^{\mathcal{M}}$ iff $\gamma < \overline{\gamma}^{\mathcal{M}1}$, which must be the case.

So Case $\mathcal{M}.2$ holds for $k_1 < k < \min\{k_2, k_3\}$. In particular, suppose that $\gamma_1^{\mathcal{M}} > \underline{\gamma}^{\mathcal{M}}$ holds. When $\underline{\gamma}^{\mathcal{M}} < \gamma < \gamma_1^{\mathcal{M}}$ then $k_1^{\mathcal{M}} < k_2^{\mathcal{M}} < k_3^{\mathcal{M}}$ and Case $\mathcal{M}.2$ holds for $k_1 < k < k_2$. When $\gamma_1^{\mathcal{M}} < \gamma < \overline{\gamma}^{\mathcal{M}}$ then $k_1^{\mathcal{M}} < k_2^{\mathcal{M}} < k_2^{\mathcal{M}}$ and thus Case $\mathcal{M}.2$ holds for $k_1 < k < k_3$. If instead $\gamma_1^{\mathcal{M}} \le \underline{\gamma}^{\mathcal{M}}$ then, because $\gamma > \underline{\gamma}^{\mathcal{M}}$ must hold, it is always the case that $\gamma > \gamma_1^{\mathcal{M}}$ and the second sub-case holds.

³¹This result is taken directly from Barigozzi and Burani (2013).

Case $\mathcal{M}.3.a$ Suppose now that $\underline{\gamma}^{\mathcal{M}} < \gamma < \gamma_1^{\mathcal{M}}$ and that IC_{AMvsaM}^{MO} and IC_{aMvsAm}^{MO} bind and that both PC_{am}^{MO} and PC_{Am}^{MO} are binding. Then expressions (24) and (32) hold and from IC_{aMvsAm}^{MO} one obtains

$$w_{aM} = \frac{1}{2}\theta e_{aM}^2 - \gamma \left(e_{aM} - e_{Am}\right) - \frac{1}{2}\left(\theta - 1\right)e_{Am}^2 + \frac{k^2}{2}$$
(33)

and finally from IC_{AMvsaM}^{MO} one has

$$w_{AM} = \frac{1}{2}e_{AM}^2 - \gamma \left(e_{AM} - e_{Am}\right) + \frac{1}{2}\left(\theta - 1\right)\left(e_{aM}^2 - e_{Am}^2\right) + \frac{k^2}{2}.$$
 (34)

Substituting in the firm's expected profits and maximizing with respect to e_{ij} yields $e_{AM} = e_{AM}^{FB}$, e_{aM} and e_{Am} equal to their second-best levels as in the preceding solutions (see expressions from 28 to 30) and

$$e_{am} = \frac{1}{\theta} = e_{am}^{FB}.$$

A necessary condition for this solution to hold is that $e_{Am}^{SB} > e_{am}^{FB}$ which occurs iff $\gamma < \gamma_1^{\mathcal{M}}$ and which is precisely the case at hand. So Case $\mathcal{M}.3.a$ with $e_{am} = e_{am}^{FB}$ and $e_{Am} = e_{Am}^{SB} > e_{am}^{FB}$ exists for $\underline{\gamma}^{\mathcal{M}} < \gamma < \gamma_1^{\mathcal{M}}$ and $k_2 < k < k_3$.³²

Finally notice that profits to the mission-oriented firm from worker am are given by

$$\pi_{am} = e_{am} - w_{am} = e_{am}^{FB} - \frac{1}{2}\theta \left(e_{am}^{FB}\right)^2 - \frac{k^2}{2\theta} = \frac{(1-k)(1+k)}{2\theta}$$

and they are non-negative as long as $k \leq 1$. So, any other regime having $e_{am} = e_{am}^{FB}$ is only relevant for $k \leq 1$.

Case $\mathcal{M}.3.b$ In the cases in which either $\underline{\gamma}^{\mathcal{M}} < \gamma_1^{\mathcal{M}}$ and $\gamma_1^{\mathcal{M}} < \gamma < \overline{\gamma}^{\mathcal{M}}$ or $\gamma_1^{\mathcal{M}} < \underline{\gamma}^{\mathcal{M}}$ and $\underline{\gamma}^{\mathcal{M}} < \gamma < \overline{\gamma}^{\mathcal{M}}$ then $k_3 < k_2$ and, for $k_3 < k < k_2$, there must be pooling between types am and Am at

$$e_{Am}^* = e_{am}^* = \frac{k}{\sqrt{\theta}}$$

where both effort levels are in-between the second and the first-best.

But note that profits from worker Am are given by

$$\pi_{Am} = e_{Am} - w_{Am} = e_{Am}^* - \frac{1}{2} (e_{Am}^*)^2 - \frac{k^2}{2} = \frac{\left(2\sqrt{\theta} - k(\theta + 1)\right)k}{2\theta}$$

which are non-negative for

$$k \le \frac{2\sqrt{\theta}}{(\theta+1)} = k_4^{\mathcal{M}} < 1,$$

with $k_4^{\mathcal{M}} > k_2^{\mathcal{M}}$ and $k_4^{\mathcal{M}} > k_3^{\mathcal{M}}$ iff

$$\gamma > \frac{\left(2\mu + \nu \left(1 - \mu\right)\right)\left(\theta - 1\right)}{\left(\theta + 1\right)\mu} = \gamma_3^{\mathcal{M}},$$

where $\gamma_3^{\mathcal{M}} < \gamma_1^{\mathcal{M}}$: hence, when $k_3^{\mathcal{M}}$ is relevant because $\gamma > \gamma_1^{\mathcal{M}}$, then it is always the case that $k_4^{\mathcal{M}} > k_3^{\mathcal{M}}$ and profit are positive.

³²Observe that
$$k_3^{\mathcal{M}} < 1$$
 iff $\gamma > \frac{\nu(1-\mu)\sqrt{\theta}-(1-(1-\nu)(1-\mu))+\mu\theta}{\mu\sqrt{\theta}} = \gamma_2^{\mathcal{M}}$ with $\gamma_2^{\mathcal{M}} < \gamma_1^{\mathcal{M}}$ iff $\theta < \frac{(1-(1-\nu)(1-\mu))}{\mu} = \overline{\theta}_1^{\mathcal{M}}$.

Case $\mathcal{M}.4$ Suppose that both participation constraints PC_{am}^{MO} and PC_{Am}^{MO} are binding and that the binding incentive compatibility constraints are now IC_{AMvsaM}^{MO} , IC_{aMvsAm}^{MO} and IC_{amvsAm}^{MO} . Then, expressions (24) and (32) hold, from IC_{amvsAm}^{MO} one obtains

$$w_{am} = \frac{1}{2}\theta e_{am}^2 - \frac{1}{2}(\theta - 1)e_{Am}^2 + \frac{k^2}{2}$$
(35)

and finally IC_{aMvsAm}^{MO} and IC_{AMvsaM}^{MO} yield expressions (33) and (34) respectively. Equating the two expressions in w_{am} and solving for e_{Am} yields

$$e_{Am} = \frac{k}{\sqrt{\theta}} = e_{Am}^*$$

where $e_{Am}^* > e_{Am}^{SB}$ iff $k > k_3^{\mathcal{M}}$, which is the case, and $e_{Am}^* < e_{Am}^{FB} = 1$ iff $k < \sqrt{\theta} > 1$. The optimal allocation for the remaining types is as in Case $\mathcal{M}.3.a$ with $e_{AM} = e_{AM}^{FB}$, $e_{aM} = e_{aM}^{SB}$ and finally $e_{am} = e_{am}^{FB}$.

Necessarily this regime is relevant when the implementability condition $e_{Am}^* < e_{aM}^{SB}$ holds, which is true for

$$k < \frac{(1-\nu)(1+\gamma)\sqrt{\theta}}{(\theta-\nu)} = k_5^{\mathcal{M}}$$

with $k_5^{\mathcal{M}} < 1$ iff

$$\gamma < \frac{(\theta - \nu) - (1 - \nu)\sqrt{\theta}}{(1 - \nu)\sqrt{\theta}} = \gamma_4^{\mathcal{M}}$$

and $k_5^{\mathcal{M}} > k_2^{\mathcal{M}}$ iff

$$\gamma > \frac{(\theta - 1)\nu}{(1 - \nu)\theta} = \gamma_5^{\mathcal{M}}$$

where $\gamma_1^{\mathcal{M}} > \gamma_5^{\mathcal{M}} > \underline{\gamma}^{\mathcal{M}}$ iff $(\mu + \nu - 3\mu\nu - \nu^2 + \mu\nu^2) > 0$ which is precisely the same condition guaranteeing that $\underline{\gamma}^{\mathcal{M}} < \gamma_1^{\mathcal{M}}$. Hence $k_5^{\mathcal{M}} > k_2^{\mathcal{M}}$ always holds when $\gamma > \max\left\{\underline{\gamma}^{\mathcal{M}}, \gamma_1^{\mathcal{M}}\right\}$. Finally, $k_5^{\mathcal{M}} > k_3^{\mathcal{M}}$ for $\gamma > \underline{\gamma}^{\mathcal{M}}$, which is always the case. Note that the optimal contract also depends on whether $e_{aM}^{SB} < e_{Am}^{FB} = 1$ which is true iff

$$\gamma < \frac{(\theta - 1)}{(1 - \nu)} = \gamma_6^{\mathcal{M}}$$

with $\gamma_6^{\mathcal{M}} > \gamma_5^{\mathcal{M}} > \underline{\gamma}^{\mathcal{M}}$. In particular, if $\gamma > \gamma_6^{\mathcal{M}}$ then $k_5^{\mathcal{M}}$ is always higher than 1 and this case only holds for $\max\{k_2, k_3\} < k < 1$. Conversely, if $\gamma_5^{\mathcal{M}} < \gamma < \gamma_6^{\mathcal{M}}$ and $k_5^{\mathcal{M}} < 1$, then this case holds for $\max\{k_2, k_3\} < k < k_5^{\mathcal{M}}$. But above $k_5^{\mathcal{M}}$ it becomes impossible to separate intermediate types and a pooling contract must be offered to Am and aM.

Pooling When motivation prevails, there are also optimal pooling contracts. In particular, the mission-oriented principal might want to offer the same contract to non-motivated workers Am and am when γ is sufficiently high, i.e. when $\gamma > \overline{\gamma}^{\mathcal{M}}$, or he might want to offer the same contracts to intermediate types Am and aM when γ is sufficiently low, i.e. when $\gamma < \underline{\gamma}^{\mathcal{M}}$. The special regime of irrelevance of outside options has been studied in Barigozzi and Burani (2013) whereas the remaining regimes can be obtained following the same logic used so far and are then omitted.

A.4 Optimal contracts with competing principals

As mentioned in the main text, when $1 \leq k \leq 1 + \gamma$ and none of the principals is fully dominant, we proceed by taking one of the regimes in which principal S might find himself (starting from Case 1 and moving to Case 5) as given. By so doing, we are imposing that the difference $U_{Am}^{MO} - U_{am}^{MO}$, which is still not know at this stage, belongs to a certain interval. The relevant thresholds are the ones computed for the five regimes in Appendix A.2 and they depend on the magnitude of k. We then solve for the mission-oriented principal's optimal incentive schemes and find the actual value of $U_{Am}^{MO} - U_{am}^{MO}$. Finally, we check whether the latter is compatible with the selected regime for principal S.

Assume that the distribution of types be not only independent but uniform, with 1/4 being the probability that any type of worker realizes.

A.4.1 Principal S is in Case 1

When the standard principal is in Case 1, it must be the case that $U_{Am}^{MO} - U_{am}^{MO} < \frac{k^2(\theta-1)}{2(2\theta-1)^2}$ (see Figure 1). In this regime, the only binding participation constraint is PC_{am}^S . Therefore, type am must be indifferent between the two firms and PC_{am}^{MO} must be binding as well. The mission-oriented principal offers to this type the first-best effort level and makes zero profits from this type of agent, whereby $e_{am}^{FB,MO} = \frac{1}{\theta}$ and $U_{am}^{TS,MO} = \frac{1}{2\theta}$.

Motivation prevails Suppose further that motivation prevails for the mission-oriented principal (Case \mathcal{M}), whereby optimal effort levels must be ordered as $e_{AM} > e_{aM} > e_{Am} \ge e_{am}$.³³

Full separation of types One could solve a problem in which each type of worker gets a different contract and in which the binding constraints are the downward incentive compatibility ones IC_{AMvsaM} , IC_{aMvsAm} and IC_{Amvsam} together with PC_{am} . Solving the binding constraints for the wage rates, substituting them into the principal's objective function and maximizing it with respect to effort levels (omitting e_{am} which is already fixed at e_{am}^{FB}) yields

$$e^{FB}_{AM} = 1 + \gamma \quad e^{SB}_{aM} = \frac{1+\gamma}{2\theta-1} \quad e^{SB}_{Am} = \frac{1-2\gamma}{3-2\theta} \quad e^{FB}_{am} = \tfrac{1}{\theta} \ .$$

This candidate solution with full separation of types exists for $\theta < \frac{3}{2}$ (ensuring that $e_{Am} > 0$) and for $\frac{4(\theta-1)}{(2\theta+1)} = \underline{\gamma}^{\mathcal{M}} < \gamma < \overline{\gamma}^{\mathcal{M}} = \frac{3(\theta-1)}{2\theta}$, where inequalities $\underline{\gamma}^{\mathcal{M}} < \gamma$ and $\gamma < \overline{\gamma}^{\mathcal{M}}$, respectively, are equivalent to the monotonicity conditions $e_{aM} > e_{Am}$ and $e_{Am} > e_{am}$.³⁴ Profits to the mission-oriented principal

³³From now on, when no confusion arises, we omit the superindex relative to the type of principal considered.

³⁴All omitted participation and incentive compatibility constraints have been checked to hold ex-post. The same is true for all subsequent problems so that we avoid repeating a similar statement each time.

from hired types AM and aM are equal to

$$\pi^{\mathcal{M},FS} = \frac{1}{4} \left(\frac{\theta(1+\gamma)^2}{(2\theta-1)} + \frac{(1-2\gamma)(\theta-4\gamma+2\theta\gamma-1)}{(3-2\theta)^2} - \frac{(2\theta-1)}{\theta^2} \right) , \tag{36}$$

where the superscript \mathcal{M}, FS stands for Motivation prevails, Full Separation of types. There remains to compute the outside option left by principal MO to type Am, which is given by $U_{Am}^{MO} = w_{Am}^{MO} - \frac{1}{2}e_{Am}^2$; substituting for $w_{Am}^{MO} = \frac{1}{2}e_{Am}^2 + \frac{(2\theta-1)}{2\theta^2}$ (which has been found imposing that IC_{Amvsam} binds) yields $U_{Am}^{MO} = \frac{(2\theta-1)}{2\theta^2}$ and thus $U_{Am}^{MO} - U_{am}^{MO} = \frac{(\theta-1)}{2\theta^2}$. Such difference in reservation utilities is compatible with principal S being in Case 1 if and only if $\frac{(\theta-1)}{2\theta^2} < \frac{k^2(\theta-1)}{2(2\theta-1)^2}$ or else if and only if

$$k > \frac{(2\theta - 1)}{\theta} = \overline{k},$$

where $\overline{k} > 1$ always holds while $\overline{k} < 1 + \gamma$ iff

$$\gamma > \frac{\theta - 1}{\theta} = \underline{\gamma}.$$

Note that $\underline{\gamma} < \underline{\gamma}^{\mathcal{M}}$ always holds, so the condition $\gamma > \underline{\gamma}$ is always verified when motivation prevails, and in turn $\overline{k} < 1 + \gamma$ is true in this case.

Pooling between non-motivated types Am and am Suppose that PC_{am}^{MO} is still binding but that a pooling contract is offered to non-motivated types whereby effort levels are ordered as $e_{AM} > e_{AM} = e_{am} = \frac{1}{\theta}$, and wages are such that $w_{am} = w_{Am} = \frac{1}{\theta}$ (again, principal MO makes zero profits on types that he is not able to hire). Optimal effort levels are given by

$$e^{FB}_{AM}=1+\gamma$$
 $e^{SB}_{aM}=\frac{1+\gamma}{2\theta-1}$ $e_{Am}=e^{FB}_{am}=\frac{1}{\theta}$.

This solution exists when the monotonicity condition $e_{aM} > e_{Am}$ is satisfied, which is equivalent to $\gamma > \frac{\theta-1}{\theta} = \underline{\gamma}$. This solution thus holds for a larger set of parameter configurations relative to the fully separating solution when motivation prevails. Profits for principal MO from the hired types AM and aM are given by

$$\pi^{\mathcal{M}, Pool_{Am+am}} = \frac{1}{8} \left((1+\gamma)^2 + \frac{(1+\gamma)^2}{(2\theta-1)} - \frac{2(2\gamma+1)}{\theta} \right) ,$$
 (37)

where the superscript now stands for Motivation prevails, Pooling between types Am and am. It can be checked that $\pi^{\mathcal{M},Pool_{Am+am}} > \pi^{\mathcal{M},FS}$ iff $\gamma > \frac{(\theta-1)(3-\theta)}{2\theta(2-\theta)} = \gamma_1$ where $\gamma_1 < \underline{\gamma}^{\mathcal{M}}$ always holds for $\theta < \frac{3}{2}$. Hence, when motivation prevails and both solutions with full separation and pooling between non-motivated types are in place, then principal MO strictly prefers pooling to full separation, meaning that the latter solution can be discarded. Finally, note that outside options for non-motivated types are the same as in the previous case with full separation of types, whereby $U_{Am}^{MO} - U_{am}^{MO} = \frac{(\theta-1)}{2\theta^2}$ and compatibility with Case 1 for principal S is still given by the condition $k > \overline{k}$.

Pooling between intermediate types Suppose now that effort levels offered by principal MO are ordered as $e_{AM} > e_{aM} = e_{Am} > e_{am}$. There are two possible types of solutions with pooling of intermediate types, depending on whether IC_{aMvsam} or IC_{Amvsam} binds first. In particular, IC_{aMvsam} binds first if and only if $e_{aM} = e_{Am} + e_{am} > \frac{2\gamma}{\theta-1}$ holds, whereas IC_{Amvsam} binds first if and only if $e_{aM} = e_{Am} + e_{am} < \frac{2\gamma}{\theta-1}$ holds.

Case $\mathcal{P}(1)$ Suppose that IC_{aMvsam} is binding while IC_{Amvsam} is slack: we call this situation Case $\mathcal{P}(1)$ and denote it with the superscript $\mathcal{P}1$. Consider further PC_{am} and IC_{AMvsaM} as binding constraints so that optimal effort levels are given by

$$e_{AM}^{FB} = 1 + \gamma$$
 $e_{aM}^{SB} = e_{Am} = \frac{1+\gamma}{2\theta-1}$ $e_{am}^{FB} = \frac{1}{\theta}$.

Monotonicity condition $e_{aM} = e_{Am} > e_{am}$ holds iff $\gamma > \underline{\gamma}$ and IC_{aMvsam} is binding while IC_{Amvsam} is slack iff $e_{aM} = e_{Am} + e_{am} > \frac{2\gamma}{\theta - 1}$ or else iff $\gamma < \underline{\gamma}$. Since these two conditions are incompatible, Case $\mathcal{P}(1)$ can be discarded.

Case $\mathcal{P}(2)$ Suppose now that IC_{Amvsam} is binding while IC_{aMvsam} is slack: we call this situation Case $\mathcal{P}(2)$ and denote it with the superscript $\mathcal{P}2$. Consider further PC_{am} and IC_{AMvsAm} as binding constraints so that optimal effort levels are given by

$$e^{FB}_{AM} = 1 + \gamma \quad e_{aM} = e_{Am} = \frac{2-\gamma}{2} \quad e^{FB}_{am} = \tfrac{1}{\theta} \enspace . \label{eq:empty}$$

Monotonicity condition $e_{aM} = e_{Am} > e_{am}$ holds iff $\gamma < \frac{2(\theta-1)}{\theta} = \overline{\gamma}^{\mathcal{P}2} = 2\underline{\gamma}$. Moreover, IC_{Amvsam} is binding while IC_{aMvsam} is slack iff $e_{aM} = e_{Am} + e_{am} < \frac{2\gamma}{\theta-1}$ or else iff $\gamma > \frac{2(\theta-1)(\theta+1)}{\theta(\theta+3)} = \underline{\gamma}^{\mathcal{P}2}$. Hence Case $\mathcal{P}(2)$ exists iff $\underline{\gamma}^{\mathcal{P}2} < \gamma < \overline{\gamma}^{\mathcal{P}2}$. Since $\underline{\gamma}^{\mathcal{P}2} > \underline{\gamma}$, Case $\mathcal{P}(2)$ coexists with the solution that is in place when motivation prevails and there is pooling between non-motivated types. Profits to principal MO in the present case are equal to

$$\pi^{\mathcal{P}2} = \frac{1}{8} \left((1+\gamma)^2 + \frac{(2-3\gamma)(2-\gamma)}{4} - \frac{2(2\theta-1)}{\theta^2} \right)$$

and it possible to show that $\pi^{\mathcal{P}2} < \pi^{\mathcal{M}, Pool_{Am+am}}$ whenever the two solutions coexist. So Case $\mathcal{P}(2)$ can be discarded.

Ability prevails Suppose now that ability prevails for the mission-oriented principal, whereby the solution to principal MO's program must be such that effort levels are ordered as $e_{AM} > e_{Am} > e_{aM} \ge e_{am}$. Here we distinguish between two possible solutions with full separation of types: Case $\mathcal{A}.a$ that holds when IC_{AMvsAm} , IC_{AmvsaM} and IC_{aMvsam} are binding, which is equivalent to $e_{aM} + e_{am} > \frac{2\gamma}{\theta-1}$, and

Case $\mathcal{A}.b$ that holds when IC_{AMvsAm} , IC_{Amvsam} and IC_{aMvsAm} are binding, or else when $e_{Am} + e_{am} < \frac{2\gamma}{\theta - 1} < e_{Am} + e_{aM}$.

Case $\mathcal{A}.a$ In Case $\mathcal{A}.a$ the binding constraints are the downward incentive compatibility constraints IC_{AMvsAm} , IC_{AmvsaM} and IC_{aMvsam} together with participation constraint PC_{am} . Optimal effort levels are given by

$$e_{AM}^{FB} = 1 + \gamma$$
 $e_{Am}^{SB} = 1 - \gamma$ $e_{aM}^{SB} = \frac{(1+3\gamma)}{3\theta-2}$ $e_{am}^{FB} = \frac{1}{\theta}$.

Monotonicity condition $e_{Am} > e_{aM}$ holds iff $\gamma < \frac{3(\theta-1)}{(3\theta+1)} = \overline{\gamma}_1^{Aa}$ while $e_{aM} > e_{am}$ holds iff $\gamma > \frac{2(\theta-1)}{3\theta} = \underline{\gamma}^{Aa}$. Moreover the requirement $e_{aM} + e_{am} \geq \frac{2\gamma}{\theta-1}$ is satisfied iff $\gamma \leq \frac{2(2\theta-1)(\theta-1)}{\theta(3\theta-1)} = \overline{\gamma}_2^{Aa}$, but $\overline{\gamma}_1^{Aa} < \overline{\gamma}_2^{Aa}$ and so this candidate solution exists for $\underline{\gamma}^{Aa} < \gamma < \overline{\gamma}^{Aa} = \overline{\gamma}_1^{Aa}$. Now, $\overline{\gamma}^{Aa} < \underline{\gamma}$ so this case A.a does not coexist with the case in which motivation prevails and there is pooling of non-motivated types. Reservation utilities for non-motivated types in this case are equal to $U_{am}^{MO} = \frac{1}{2\theta}$ and $U_{Am}^{MO} = w_{Am} - \frac{1}{2}e_{Am}^2$. Substituting for w_{Am} as given by IC_{AmvsaM} binding one has $U_{Am}^{MO} = \frac{1}{2}(\theta-1)e_{aM}^2 - \gamma e_{aM} + \gamma e_{am}$; finally, considering optimal effort levels, the latter expression becomes $U_{Am}^{MO} = \frac{8\gamma - \theta - 26\theta\gamma + \theta^2 + 3\theta\gamma^2 + 18\theta^2\gamma - 9\theta^2\gamma^2}{2\theta(3\theta-2)^2}$ and the difference in reservation utilities is equal to $U_{Am}^{MO} - U_{am}^{MO} = \frac{11\theta + 8\gamma - 26\theta\gamma - 8\theta^2 + 3\theta\gamma^2 + 18\theta^2\gamma - 9\theta^2\gamma^2 - 4}{2\theta(3\theta-2)^2}$. Now, Case A.a is compatible with principal S being in Case 1 if and only if $U_{Am}^{MO} - U_{am}^{MO} < \frac{k^2(\theta-1)}{2(2\theta-1)^2}$. Solving for k, the latter inequality becomes

$$k > \frac{(2\theta - 1)}{(3\theta - 2)} \sqrt{\frac{(11\theta + 8\gamma - 26\theta\gamma - 8\theta^2 + 3\theta\gamma^2 + 18\theta^2\gamma - 9\theta^2\gamma^2 - 4)}{\theta(\theta - 1)}} = k_4$$

but note that $k_4 > 1 + \gamma$ always holds and hence Case $\mathcal{A}.a$ can be discarded because it can never be compatible with principal S being in Case 1.

Case A.b In Case A.b, the binding incentive compatibility constraints are IC_{AMvsAm} , IC_{Amvsam} and (upward) IC_{aMvsAm} together with participation constraint PC_{am} . Optimal effort levels are given by

$$e^{FB}_{AM}=1+\gamma \quad e^{SB}_{Am}=\frac{(1-2\gamma)}{(2-\theta)} \quad e^{FB}_{aM}=\frac{(1+\gamma)}{\theta} \quad e^{FB}_{am}=\frac{1}{\theta} \ . \label{eq:eradiation}$$

Monotonicity condition $e_{Am} > e_{aM}$ is satisfied iff $\gamma < \frac{2(\theta-1)}{(\theta+2)} = \overline{\gamma}^{Ab}$ while condition $e_{Am} + e_{am} < \frac{2\gamma}{\theta-1}$ holds iff $\gamma > \frac{(\theta-1)}{\theta} = \underline{\gamma}$ where $\overline{\gamma}^{Ab} < \underline{\gamma}$. So the above conditions are not compatible with each other and Case A.b can be discarded because it exists for an empty set of parameters.

Pooling between motivated types Suppose now that a pooling contract is offered by principal MO to motivated types whereby effort levels are ordered as $e_{AM} > e_{Am} > e_{aM} = e_{am} = \frac{1}{\theta}$. The

³⁵ Case $\mathcal{A}.a$ corresponds to Case $\mathcal{A}.1$ and Case $\mathcal{A}.b$ corresponds to Case $\mathcal{A}.3$ in the companion paper. When the distribution of types is not uniform another case emerges, called Case $\mathcal{A}.2$, which is such that the binding constraints are IC_{AMvsAm} , IC_{Amvsam} and IC_{aMvsam} and which holds for $e_{aM} + e_{am} < \frac{2\gamma}{\theta - 1} < e_{Am} + e_{am}$.

incentive compatibility constraints that one assumes to be binding are IC_{AMvsAm} , IC_{AmvsaM} together with participation constraint PC_{am} . Optimal effort levels are

$$e^{FB}_{AM}=1+\gamma$$
 $e_{Am}=1-\gamma$ $e_{aM}=e^{FB}_{am}=\frac{1}{\theta}$.

This solution exists iff $\gamma < \underline{\gamma}$ or else iff the monotonicity condition $e_{Am} > e_{aM}$ holds. Reservation utilities are such that $U_{Am}^{MO} - U_{am}^{MO} = \frac{(2\theta-1)}{2\theta^2} - \frac{1}{2\theta} = \frac{(\theta-1)}{2\theta^2}$ as in the previous regimes and compatibility with principal S being in Case 1 occurs for $k > \frac{(2\theta-1)}{\theta} = \overline{k}$. But note that $\overline{k} > 1 + \gamma$ holds whenever $\gamma < \underline{\gamma}$ so that the condition $k > \overline{k}$ can never be satisfied in this case and this candidate solution must be discarded.

A.4.2 Principal S is in Cases from 2 to 4

When the standard principal is in Cases from 2 to 4, the binding participation constraints are both PC_{am}^S and PC_{Am}^{NO} . Therefore, both PC_{am}^{NO} and PC_{Am}^{NO} must be binding as well and both types am and Am must be indifferent between the two firms. The mission-oriented principal offers them first-best effort levels and makes zero profits from these types of agent, whereby $e_{am}^{NO} = \frac{1}{\theta}$ and $U_{am}^{TS,MO} = \frac{1}{2\theta}$ together with $e_{Am}^{NO} = 1$ and $U_{Am}^{TS,MO} = \frac{1}{2}$. Now the difference in reservation utilities for non-motivated types is fully determined and is equal to $U_{Am}^{TS,MO} - U_{am}^{TS,MO} = \frac{1}{2} - \frac{1}{2\theta} = \frac{(\theta-1)}{2\theta}$.

Motivation prevails Suppose that motivation prevails for the mission-oriented principal, whereby effort levels must be ordered as $e_{AM} > e_{aM} > e_{Am} = 1 > e_{am} = \frac{1}{\theta}$. The binding constraints are the downward incentive compatibility IC_{AMvsaM} and IC_{aMvsAm} , together with PC_{Am} and PC_{am} . Solving for the wage rates, substituting them into the principal's objective function and maximizing with respect to effort levels (omitting e_{Am} and e_{am} which are already determined) yields

$$e^{FB}_{AM} = 1 + \gamma \quad e^{SB}_{aM} = \tfrac{1+\gamma}{2\theta-1} \quad e^{FB}_{Am} = 1 \quad e^{FB}_{am} = \tfrac{1}{\theta} \ .$$

This candidate solution exists for $\theta < \frac{3}{2}$ and $\gamma > \gamma^{\mathcal{M}} = 2 (\theta - 1)$, where inequality $\gamma > \gamma^{\mathcal{M}}$ is equivalent to the monotonicity condition $e_{aM} > e_{Am}$, and where $\gamma^{\mathcal{M}} < 1$ whenever $\theta < \frac{3}{2}$. Also, condition $\gamma > \gamma^{\mathcal{M}}$ is sufficient for the requirement $e_{aM} + e_{Am} < \frac{2\gamma}{\theta - 1}$ being satisfied. Finally, profits to the mission-oriented principal from hired types AM and AM are equal to

$$\pi_{\mathcal{M}} = \frac{\theta(1+\gamma)^2 - (2\gamma - \theta + 2)(2\theta - 1)}{4(2\theta - 1)} . \tag{38}$$

The difference in reservation utilities $U_{Am}^{TS,MO}-U_{am}^{TS,MO}=\frac{(\theta-1)}{2\theta}$ is compatible with principal S being in Case 2 if and only if $\frac{k^2(\theta-1)}{2(2\theta-1)^2} < U_{Am}^{MO}-U_{am}^{MO}=\frac{(\theta-1)}{2\theta} \leq \frac{k^2(\theta-1)}{2\theta^2}$. As for the lower bound, it is satisfied when

$$k < \frac{(2\theta - 1)}{\sqrt{\theta}} = \overline{\overline{k}},$$

where $\overline{\overline{k}} > 1$ always holds and $\overline{\overline{k}} < 1 + \gamma$ is true iff

$$\gamma > \frac{(2\theta - 1) - \sqrt{\theta}}{\sqrt{\theta}} = \overline{\gamma},$$

where $\overline{\gamma} < \gamma^{\mathcal{M}}$. So $\overline{\overline{k}}$ is always included in the interval $(1; 1 + \gamma)$ when motivation prevails. As for the upper bound, it is satisfied iff

$$k \ge \sqrt{\theta} = k$$
,

with $\underline{k} > 1$ and $\underline{k} < 1 + \gamma$ iff $\gamma > \sqrt{\theta} - 1 = \underline{\gamma}$, where $\underline{\gamma} < \gamma^{\mathcal{M}}$. Hence, \overline{k} is also included in the interval $(1; 1 + \gamma)$ when motivation prevails. Finally note that

$$\underline{k} < \overline{k} < \overline{\overline{k}} \tag{39}$$

always holds.

Conversely, principal S is in Case 3 for $\frac{k^2(\theta-1)}{2\theta^2} < U_{Am}^{MO} - U_{am}^{MO} = \frac{(\theta-1)}{2\theta} \le \frac{k^2(\theta-1)}{2}$. The lower bound is satisfied for $k < \underline{k}$ while the upper bound holds iff

$$k \ge \frac{1}{\sqrt{\theta}} = k_5$$

where $k_5 < 1$ always holds. So $k \ge k_5$ is always satisfied and principal S is in Case 3 for $1 \le k < \underline{k}$, whereas Case 4 cannot be compatible with motivation prevailing for principal MO.

Pooling between intermediate types Suppose that the ordering of effort levels is such that $e_{AM} > e_{aM} = e_{Am} = 1 > e_{am} = \frac{1}{\theta}$. Now the binding constraints are IC_{AMvsAm} , PC_{Am} and PC_{am} . Optimal effort levels are

$$e_{AM}^{FB}=1+\gamma$$
 $e_{aM}=e_{Am}^{FB}=1$ $e_{am}^{FB}=\frac{1}{\theta}$

and this solution exists iff

$$\gamma \geq \frac{\theta-1}{2} = \gamma^{\mathcal{P}},$$

which ensures that IC_{aMvsam} is satisfied, with $\gamma^{\mathcal{P}} < \gamma^{\mathcal{M}}$. Note that at this solution principal MO is making positive profits from type AM only, which are equal to

$$\pi_{\mathcal{P}} = \frac{\gamma^2}{8} \tag{40}$$

and which are always smaller than the profits when motivation prevails. So this solution only holds for $\gamma^{\mathcal{P}} \leq \gamma < \gamma^{\mathcal{M}}$.

This solution is compatible with Case 2 for principal S iff $\underline{k} < k \leq \overline{k}$, where $\overline{k} < 1 + \gamma$ when $\gamma > \overline{\gamma}$, with $\overline{\gamma} > \gamma^{\mathcal{P}}$. Hence when $\gamma^{\mathcal{P}} < \gamma < \overline{\gamma}$ we have $\overline{k} > 1 + \gamma$, so the condition $k \leq \overline{k}$ is always satisfied. The solution is also compatible with Case 3 holding for principal S when $k \geq \underline{k}$, where $\underline{k} < 1 + \gamma$ iff $\gamma > \underline{\gamma}$ and $\underline{\gamma} < \gamma^{\mathcal{P}}$. Thus, $\underline{k} < 1 + \gamma$ is always true when $\gamma^{\mathcal{P}} \leq \gamma < \gamma^{\mathcal{M}}$ and the pooling solution holds. Conversely, Case 4 can be neglected because the difference in reservation utilities is incompatible with values of k such that $k \geq 1$.

Ability prevails Suppose now that ability prevails for principal MO and that the ordering of effort levels is such that $e_{AM} > e_{Am} = 1 > e_{aM} > e_{am} = \frac{1}{\theta}$. Again one has to distinguish between Case $\mathcal{A}.a$ and Case $\mathcal{A}.b$

Case $\mathcal{A}.a$ In Case $\mathcal{A}.a$, the binding incentive compatibility constraints are IC_{AMvsAm} and IC_{aMvsam} together with participation constraints PC_{Am} and PC_{am} . Optimal effort levels are given by

$$e_{AM}^{FB} = 1 + \gamma \quad e_{Am}^{FB} = 1 \quad e_{aM}^{FB} = \frac{(1+\gamma)}{\theta} \quad e_{am}^{FB} = \frac{1}{\theta}$$
 (41)

The monotonicity condition $e_{Am} > e_{aM}$ holds when $\gamma < (\theta - 1) = \gamma^A$. This solution exists when IC_{aMvsam} binds before IC_{aMvsAm} , which occurs when $\gamma < \frac{(\theta - 1)}{2} = \gamma^{\mathcal{P}} < \gamma^A$. Compatibility conditions are the same as before, namely this solution is compatible with principal S being in Case 3 for $1 \le k \le \underline{k}$ or in Case 2 for $\underline{k} < k \le \overline{k}$. But note that $\underline{k} < 1 + \gamma$ iff $\gamma > \underline{\gamma}$ where $\underline{\gamma} < \gamma^{\mathcal{P}}$. Then, if $0 < \gamma \le \underline{\gamma}$, this solution is compatible with principal S being in Case 3 only. Conversely, when $\underline{\gamma} < \gamma < \gamma^{\mathcal{P}}$, this solution is compatible with principal S being in Case 3 for $1 \le k \le \underline{k}$ or with principal S being in Case 2 for $\underline{k} < k \le 1 + \gamma$, as $\overline{k} > 1 + \gamma$ when $\gamma < \gamma^{\mathcal{P}}$.

Case A.b In Case A.b, the binding incentive compatibility constraints are IC_{AMvsAm} and IC_{aMvsAm} together with participation constraints PC_{Am} and PC_{am} . Optimal effort levels are the same as in (41) and this solution exists for $\gamma^{\mathcal{P}} \leq \gamma < \gamma^{A}$. Within these bounds, the monotonicity condition $e_{Am} > e_{aM}$ is satisfied and IC_{aMvsAm} binds before IC_{aMvsam} . This solution coexists with pooling between intermediate types, therefore a comparison between profits associated with the two solutions is called for. Profits in this case are given by

$$\pi_{\mathcal{A}b} = \frac{1}{8} \left((1+\gamma)^2 + \frac{(1+\gamma)^2}{\theta} - (4\gamma + 3 - \theta) \right)$$
 (42)

and they are always higher than profits given by expression (40). Therefore Case $\mathcal{A}.b$ is chosen for $\gamma^{\mathcal{P}} \leq \gamma < \gamma^{\mathcal{A}}$, although pooling between intermediate types will be the solution only when $\gamma^{\mathcal{A}} \leq \gamma \leq \gamma^{\mathcal{M}}$. Compatibility of this solution with Case 3 for principal S is ensured when $1 \leq k \leq \underline{k}$ and with Case 2 when $\underline{k} < k \leq 1 + \gamma$, being $\overline{\gamma} > \gamma^{\mathcal{A}}$. Again, Case 4 can be discarded.

Before turning to Case 5 for the standard principal, straightforward computations lead us to observe that profits which principal MO makes when motivation prevails and he offers a pooling contract to non-motivated types, and when principal S is in Case 1, are always strictly higher than profits accruing to principal MO given that the rival principal S is in Cases 2-4. In other words, profits given by expression (37) are always strictly higher than those in expressions (38), (40) and (42).

 $^{^{36}}$ Profits associated with Case A.a are not displayed here but they are lower than those in (37) too.

A.4.3 Principal S is in Case 5

When the standard principal is in Case 5, the only binding participation constraint is PC_{Am}^S . Type Am is indifferent between the two firms and PC_{Am}^{MO} must be binding as well. The mission-oriented principal offers the first-best effort level and makes zero profits from type Am, whereby $e_{Am}^{MO} = 1$ and $U_{Am}^{TS,MO} = \frac{1}{2}$. Conversely, type am strictly prefers the standard principal and is such that $U_{am}^S > U_{am}^{MO}$.

Motivation prevails Suppose that motivation prevails for the mission-oriented principal, whereby effort levels are ordered as $e_{AM} > e_{AM} > e_{Am} = 1 \ge e_{am}$.

Full separation of types Assume that each type of agent is offered a different contract and that the binding constraints are the downward incentive compatibility ones IC_{AMvsaM} , IC_{aMvsAm} , the upward IC_{amvsAm} , together with PC_{Am} . Solving for the wage rates, substituting them into the principal's objective function and maximizing with respect to effort levels (omitting e_{Am} which is already determined) yields

$$e^{FB}_{AM} = 1 + \gamma$$
 $e^{SB}_{aM} = \frac{1+\gamma}{2\theta-1}$ $e^{FB}_{Am} = 1$ $e^{FB}_{am} = \frac{1}{\theta}$.

This candidate solution exists for $\theta < \frac{3}{2}$ and $\gamma > \gamma^{\mathcal{M}} = 2 \left(\theta - 1\right)$, where inequality $\gamma > \gamma^{\mathcal{M}}$ is equivalent to the monotonicity condition $e_{aM} > e_{Am}$ and where $\gamma^{\mathcal{M}} < 1$ whenever $\theta < \frac{3}{2}$. The outside option of type am is $U_{am}^{MO} = w_{am} - \frac{1}{2}\theta e_{am}^2$. Substituting for w_{am} from the binding constraint IC_{amvsAm} one gets $U_{am}^{MO} = \frac{1}{2}\theta e_{am}^2 + \frac{(2-\theta)}{2} - \frac{1}{2}\theta e_{am}^2 = \frac{(2-\theta)}{2}$. Hence the difference in reservation utilities for non-motivated types is equal to $U_{Am}^{TS,MO} - U_{am}^{MO} = \frac{1}{2} - \frac{(2-\theta)}{2} = \frac{(\theta-1)}{2}$ and this solution is compatible with principal S being in Case 5 for $U_{Am}^{TS,MO} - U_{am}^{MO} = \frac{(\theta-1)}{2} > \frac{k^2(\theta-1)}{2(2-\theta)^2}$ or else for

$$k < (2 - \theta) = k_6$$

where $k_6 < 1$ always holds. So this solution can be discarded.

Pooling between non-motivated types Suppose that effort levels are such that $e_{AM} > e_{aM} > e_{Am} = 1 = e_{am}$. The binding constraints are the downward incentive compatibility ones IC_{AMvsaM} , IC_{aMvsAm} , together with PC_{Am} . Optimal effort levels are given by

$$e^{FB}_{AM} = 1 + \gamma \quad e^{SB}_{aM} = \tfrac{1+\gamma}{2\theta-1} \quad e^{FB}_{Am} = e_{am} = 1 \ .$$

The outside option for type am is equal to $U_{am}^{MO} = w_{am} - \frac{1}{2}\theta e_{am}^2 = \frac{(2-\theta)}{2}$ and is the same as in the previous case. Hence, as before, the difference in reservation utilities $U_{Am}^{MO} - U_{am}^{MO} = \frac{(\theta-1)}{2}$ is not compatible with the bounds that define Case 5.

Pooling between intermediate types Suppose that effort levels are ordered as $e_{AM} > e_{aM} = e_{Am} = 1 > e_{am}$. Now the constraints that one assumes to be binding are IC_{AMvsaM} , PC_{Am} and IC_{amvsAm} yielding optimal effort levels

$$e^{FB}_{AM}=1+\gamma \quad e_{aM}=e^{FB}_{Am}=1 \quad e^{FB}_{am}=\tfrac{1}{\theta} \ .$$

But the difference in reservation utilities $U_{Am}^{MO} - U_{am}^{MO}$ is still the same as in the preceding regimes and thus this solution can be discarded because it is not compatible with the bounds delimiting Case 5 for principal S.

Ability prevails Suppose that, in the mission-oriented sector, ability prevails and that the ordering of effort levels is such that $e_{AM} > e_{Am} = 1 > e_{aM} \ge e_{am}$.

Full separation of types Now the only possible set of binding constraints is IC_{AMvsAm} , PC_{Am} , IC_{aMvsAm} and finally IC_{amvsaM} . Optimal effort levels are given by

$$e^{FB}_{AM} = 1 + \gamma \quad e^{FB}_{Am} = 1 \quad e_{aM} = \frac{(2\gamma + 1)}{2\theta} \quad e^{FB}_{am} = \tfrac{1}{\theta} \ ,$$

where e_{aM} is upward distorted. This solution exists when the monotonicity condition $e_{Am} > e_{aM}$ is satisfied, namely when $\gamma < \frac{2\theta-1}{2}$. The reservation utility of type am is equal to $U_{am}^{MO} = \frac{(1-2\gamma)(1+2\gamma)+4\theta(2\gamma-\theta+2)}{8\theta}$ and thus the difference in reservation utilities becomes $U_{Am}^{MO} - U_{am}^{MO} = \frac{1}{2} - \frac{(1-2\gamma)(1+2\gamma)+4\theta(2\gamma-\theta+2)}{8\theta} = \frac{4\theta(\theta-1-2\gamma)-(1-2\gamma)(1+2\gamma)}{8\theta}$ which is lower than in the preceding cases and thus not compatible with the bounds delimiting Case 5 for principal S.

Finally note that, when principal S is in Case 5, it is never optimal for the mission-oriented principal to offer the null contract to type am. Indeed, this type would always have an incentive to take the contract offered by principal MO to type Am and then IC_{amvsAm}^{MO} would always be violated.

Therefore, Case 5 for principal S can never be attained in equilibrium when principals compete and $1 \le k \le 1 + \gamma$.

A.5 Wage differentials and returns to ability

Depending on the different combinations of states of the world for the two principals, different wages characterize the optimal contracts. Let us consider each possible combination in turn.

Let us start with Situation (i) of Section 5.3. The standard principal is in Case 1 and offers wages

$$w_{Am}^S = w_{AM}^S = \frac{(2\theta-1)^2 + \theta^2 k^2 (4\theta-3)}{2\theta (2\theta-1)^2} \quad w_{am}^S = w_{aM}^S = \frac{(2\theta-1)^2 + k^2 \theta^2}{2\theta (2\theta-1)^2}$$

although the mission-oriented principal offers pooling contracts to non-motivated types and optimal wages offered to motivated types are

$$w_{AM}^{MO} = \frac{\theta(2\theta-1)^2(1-\gamma)(1+\gamma) + \theta(\theta-1)(1+\gamma)^2 + (1+2\gamma)(2\theta-1)^2}{2\theta(2\theta-1)^2} \quad w_{aM}^{MO} = \frac{\theta^2(1+\gamma)^2 - 2\gamma\theta(1+\gamma)(2\theta-1) + (1+2\gamma)(2\theta-1)^2}{2\theta(2\theta-1)^2} \quad .$$

Then type aM gets a lower wage from the mission-oriented firm if and only if $w_{aM}^{MO} < w_{aM}^{S}$ that is if and only if

$$k > \frac{\sqrt{(2\gamma - 6\theta\gamma + \theta^2 + 2\theta\gamma^2 + 6\theta^2\gamma - 3\theta^2\gamma^2)}}{\theta} = k_7 ,$$

where $\overline{k} < k_7 < 1 + \gamma$. As for type AM we have $w_{AM}^{MO} < w_{AM}^S$ if and only if

$$k > \sqrt{\frac{(2\gamma - 10\theta\gamma - 3\theta^2 + 4\theta^3 - 2\theta\gamma^2 + 10\theta^2\gamma + 5\theta^2\gamma^2 - 4\theta^3\gamma^2)}{\theta^2(4\theta - 3)}} = k_8$$

with $k_8 < k_7$. Hence, it is easier to observe the wage gap for motivated workers with high-ability rather than with low-ability. Moreover, $k_8 < \overline{k}$ for $\gamma < \frac{\left(5\theta^2 - 5\theta + 1\right) - \left(2\theta - 1\right)\sqrt{28\theta^3 - 16\theta^2 - 12\theta^4 + 1}}{\theta(4\theta^2 - 5\theta + 2)} = \gamma_2$ where $\gamma_2 > \underline{\gamma}$. Then, for sufficiently low motivation, that is for $\underline{\gamma} \le \gamma < \gamma_2$, high-ability motivated workers always experience an earnings penalty, independently of k. As for the returns to ability, we have $w_{AM}^{MO} - w_{aM}^{MO} < w_{AM}^{S} - w_{am}^{S}$ iff $k > \sqrt{\frac{(1+\gamma)(\theta - \gamma(\theta - 1))}{\theta}} = k_9$, where $k_9 < \overline{k}$ always holds. Hence we always observe lower returns to ability for the mission-oriented firm in Situation (i).

Suppose now that we are in Situation (ii) of Section 5.3.

When ability prevails for principal MO and Case $\mathcal{A}.a$ holds, although principal S is in Case 3, then wages at the standard firm are such that

$$w_{Am}^S = w_{AM}^S = \frac{k^2 + 1}{2} \quad w_{am}^S = w_{aM}^S = \frac{k^2 + 1}{2\theta}$$
 (43)

although wages at the mission-oriented firm are equal to

$$w_{AM}^{MO} = \frac{2\gamma + 2 - \gamma^2}{2} \quad w_{aM}^{MO} = \frac{2\gamma + 2 - \gamma^2}{2\theta} \quad .$$
 (44)

Then, motivated types earn less at the mission-oriented firm where they choose to work (irrespective of their ability) if and only if

$$k > \sqrt{1 + \gamma \left(2 - \gamma\right)} = k_{10},$$

where $k_{10} < \underline{k}$ for $\gamma < 1 - \sqrt{(2-\theta)} = \gamma_3$, with $\gamma^A > \gamma_3 > \gamma^P$. Hence, when principal MO is in Case A.a, one observes the wage differential for $k_{10} < k < \underline{k}$. As for the returns to ability, one has $w_{AM}^{MO} - w_{aM}^{MO} < w_{Am}^S - w_{am}^S$ iff $k_{10} < k < \underline{k}$, namely lower returns to ability are offered by the mission-oriented firm precisely under the same conditions under which an earnings penalty emerges.

When principal S is in Case 2 and $\underline{k} \leq k < \overline{k}$ whereas principal MO is still in Case $\mathcal{A}.a$, the only wage that changes with respect to expressions (43) and (44) is w_{aM}^S which becomes lower and equal to $w_{aM}^S = \frac{\theta+1}{2\theta}$. Now, motivated types always earn less at the mission-oriented firm and the wage differential is always in place.

Lower returns to ability are also offered by the mission-oriented firm, because $w_{AM}^{MO} - w_{aM}^{MO} < w_{Am}^{S} - w_{am}^{S}$ holds iff $k > \sqrt{\frac{(\theta-1)(2\gamma+2-\gamma^2)+1}{\theta}} = k_{11}$ but $k_{11} < \underline{k}$, so inequality $k > k_{11}$ is always satisfied in this case.

Suppose now that ability prevails for principal MO and Case A.b holds whereas principal S is in Case 3, then wages are the same as in expressions (43) and (44) except for w_{aM}^{MO} which increases to $w_{aM}^{MO} = \frac{2\theta+1-(\theta-\gamma)^2}{2\theta}$. We observe a wage gap for type AM only when $\gamma^{\mathcal{P}} < \gamma < \gamma_3$ and $k_{10} < k < \underline{k}$ but the wage gap never exists for type aM. Lower returns to ability are offered by the mission-oriented principal iff $\sqrt{(\theta-\gamma^2)} = k_{12} < k < \underline{k}$. If instead principal S is in Case 2 then the pay penalty is in place for type AM when $\gamma^{\mathcal{P}} < \gamma < \gamma_3$, or when $\gamma_3 \le \gamma < \gamma^A$ and $k_{10} < k < \overline{k}$ occur whereas the pay gap exists for type aM when $\gamma^{\mathcal{P}} < \gamma < \theta - \sqrt{\theta} = \gamma_4 < \gamma_3$. And lower returns to ability are offered by the mission-oriented principal iff $k > \sqrt{\frac{(\theta^2-(\theta-1)\gamma^2)}{\theta}} = k_{13}$; but $k_{13} < \underline{k}$ therefore lower returns to ability are always offered when principal S is in Case 2 and principal MO in Case A.b.

When principal MO offers pooling contracts to types Am and aM, wages at the mission-oriented firm are

$$w_{AM}^{MO} = \frac{2\gamma + 2 - \gamma^2}{2} \quad w_{aM}^{MO} = 1$$
 .

Then, irrespective of whether principal S is in Case 2 or 3, type aM is always paid more by the mission-oriented firm, whereas the wage differential still exists for type AM provided that $k_{10} < k < \overline{k}$. As for the returns to ability, lower returns always exist when principal S is in Case 2 because the necessary and sufficient condition is $k > \sqrt{\frac{1+(2-\gamma)\theta\gamma}{\theta}} = k_{14}$ and $k_{14} < \underline{k}$. Finally, lower returns to ability exist when principal S is in Case 3 iff $\sqrt{\frac{(2-\gamma)\gamma\theta-(\theta-1)}{(\theta-1)}} = k_{15} < k < \underline{k}$.

To conclude, suppose that motivation prevails for principal MO so that wages at the mission-oriented firm are

$$w_{AM}^{MO} = \tfrac{(1-\gamma)(1+\gamma)(2\theta-1)^2 + (\theta-1)(1+\gamma)^2 + (2\gamma+2-\theta)(2\theta-1)^2}{2(2\theta-1)^2} \quad w_{aM}^{MO} = \tfrac{(1+\gamma)(\theta+2\gamma-3\theta\gamma) + (2\gamma+2-\theta)(2\theta-1)^2}{2(2\theta-1)^2}$$

Again, irrespective of whether principal S is in Case 2 or 3, both types aM and AM are always paid more by the mission-oriented firm, and the wage differential does not exist. Now, lower returns to ability are never offered by the mission-oriented firm when principal S is in Case 2, although they do arise for $k_{15} = \frac{\sqrt{4\theta(\gamma+\theta-\theta\gamma)(\gamma+1)-(2\theta-1)^2}}{(2\theta-1)} < k < \underline{k}$ when principal S is in Case 3.

References

- [1] Andreoni J. (1990), "Impure Altruism and Donations to Public Goods: a Theory of Warm-Glow Giving", *Econometrica*, 64, 51-75.
- [2] Armstrong M. (1996), "Multiproduct nonlinear pricing", Econometrica, 64, 51-75.
- [3] Armstrong M. and J-J. Rochet (1999), "Multi-dimensional Screening: A User's Guide", European Economic Review, 43, 959-979.
- [4] Armstrong M. (1999), "Optimal Regulation with Unknown Demand and Cost Functions", Journal of Economic Theory 84, 196-215.

- [5] Bargain, O. and B. Melly (2008): "Public Sector Pay Gap in France: New Evidence Using Panel Data", IZA Discussion Paper n. 3427.
- [6] Barigozzi F. and N. Burani (2013), "Screening Workers for Ability and Motivation", working paper n. 866, Department of Economics, University of Bologna.
- [7] Barigozzi F., N. Burani and D. Raggi (2014), "Politicians, Civil Servants, and Lemons", working paper n. 883, Department of Economics, University of Bologna.
- [8] Barigozzi F. and G. Turati (2012), "Human Health Care and Selection Effects. Understanding Labour Supply in the Market for Nursing", Health Economics 21(4), 477-483.
- [9] Basov S. (2005), Multidimensional Screening, Springer-Verlag.
- [10] Basov S. (2001), "Hamiltonian Approach to Multidimensional Screening" Journal of Mathematical Economics, 36, 77-94.
- [11] Bassi, M., Pagnozzi, M. and S. Piccolo (2014) "Optimal Contracting with Altruism and Reciprocity" Research in Economics, 68(1), 27-38.
- [12] Bénabou R. and J. Tirole (2010), "Individual and Corporate Social Responsibility", Economica, 77, 1-19.
- [13] Bénabou R. and J. Tirole (2013), "Bonus Culture: Competitive Pay, Screening, and Multitasking", NBER Working Paper No. 18936.
- [14] Besley T. and M. Ghatak (2005), "Competition and Incentives with Motivated Agents", American Economic Review 95(3), 616-636.
- [15] Biglaiser G. and C. Mezzetti (1993), "Principals Competing for an Agent in the Presence of Adverse Selection and Moral Hazard", Journal of Economic Theory 61, 309-330.
- [16] Biglaiser G. and C. Mezzetti (2000), "Incentive Auctions and Information Revelation", RAND Journal of Economics 31(1), 145-164.
- [17] Borjas, G.J., H.E. III Frech and P.B. Ginsburg (1983), "Property Rights and Wages: The Case of Nursing Homes", Journal of Human Resources, 18, 231-246.
- [18] Brekke, K. A. and K. Nyborg (2008), "Moral hazard and moral motivation: corporate social responsibility as
- [19] labor market screening", Resource and Energy Economics 30(4), 509-516.
- [20] Choné P. and C.A. Ma (2004), "Asymmetric Information from Physician Agency: Optimal Payment and Healthcare Quantity", Annales d'Economie et de Statistique 101-102, 229–256.
- [21] Delfgaauw J. and R. Dur (2007), "Signaling and screening of workers' motivation", Journal of Economic Behavior and Organization 62, 605-624.

- [22] Delfgaauw J. and R. Dur (2008), "Incentives and workers' motivation in the public sector", *The Economic Journal* 118, 171-191.
- [23] Delfgaauw J. and R. Dur (2010), "Managerial talent, motivation, and self-selection into public management", *Journal of Public Economics* 94(9-10), 654-660.
- [24] Deneckere R. and S. Severinov (2011), "Multi-Dimensional Screening: A Solution to a Class of Problems", *mimeo*, econ.ucsb.edu.
- [25] DeVaro J., N. Maxwell and H. Morita (2015), "Compensation and Intrinsic Motivation in Nonprofit and For-Profit Organizations", Mimeo.
- [26] Disney R. and A. Gosling (1998), "Does it Pay to Work in the Public Sector?" Fiscal Studies 19, 347-374.
- [27] Gibbons R. and L. Katz (1992), "Does Unmeasured Ability Explain Inter-Industry Wage Differentials?", Review of Economic Studies 59, 515-535.
- [28] Goddeeris J.H. (1988), "Compensating Differentials and Self-Selection: An Application to Lawyers", Journal of Political Economy 96(2), 411-428.
- [29] Goux D. and E. Maurin (1999), "Persistence of Interindustry Wage Differentials: A Reexamination Using Matched Worker-Firm Panel Data", Journal of Labor Economics 17(3), 492-533.
- [30] Gregg, P., P. Grout, A. Ratcliffe, S. Smith, and F. Windmeijer, (2011), "How important is pro-social behaviour in the delivery of public services?", *Journal of Public Economics* 95(7-8), 758-766.
- [31] Handy F. and E. Katz (1998), "The wage differential between nonprofit institutions and corporations: getting more by paying less?", *Journal of Comparative Economics* 26, 246-261.
- [32] Heyes A. (2005) "The economics of vocation or -why is a badly paid nurse a good nurse-?", Journal of Health Economics 24, 561-569.
- [33] Hwang H., W.R. Reed and C. Hubbard (1992), "Compensating Wage Differentials and Unobserved Productivity", Journal of Political Economy 100(4), 835-858.
- [34] Laffont J.-J. and D. Martimort (2002), The theory of Incentives: The Principal-Agent Model, Princeton University Press.
- [35] Laffont J-J., E. Maskin and J.-C. Rochet (1987), "Optimal Non-Linear Pricing with Two-dimensional Characteristics" in T. Groves, R. Radner and S. Reiter, eds., Information, Incentives and Economic Mechanisms, University of Minnesota Press, Minneapolis, 256-266.
- [36] Lehmann E., L. Simula and A. Tannoy (2014), "Tax Me if You Can! Optimal Nonlinear Income Tax Between Competing Governments", Quarterly Journal of Economics, 129(4), 1995-2030.

- [37] Makris M. (2009), "Incentive for Motivated Agents Under an Administrative Constraint", Journal of Economic Behavior and Organization, 71, 428-440.
- [38] Makris, M. and L. Siciliani (2013), "Optimal Incentive Schemes for Altruistic Providers", Journal of Public Economic Theory, 15(5), 675-699.
- [39] Margolis J. and H. Elfenbein (2007), "Do Well by Doing Good: Don't Count on It", Harward Business Review, 86(1), 19.
- [40] Martimort, D. (1992), "Multi-principaux avec anti-selection", Annales d'Economie et de Statistique, 28, 1-37.
- [41] Melly, B. (2005), "Public-private wage differentials in Germany: evidence from quantile regression", Empirical Economics 30(2), 505-520.
- [42] Mocan N. and E. Tekin (2003), "Nonprofit sector and part-time work: an analysis of employer—employee matched data of child care workers", *Review of Economics and Statistics* 85(1), 38-50.
- [43] Murdock K. (2002), "Intrinsic Motivation and Optimal Incentive Contracts", RAND Journal of Economics, 33(1), 650-671.
- [44] Prendergast C. (2007), "The Motivation and Bias of Bureaucrats", American Economic Review 97(1), 180-196.
- [45] Preston A. (1988). "The Effects of Property Rights on Labor Costs of Nonprofit Firms: An Application to the Day Care Industry." Journal of Industrial Economics, 36, 337-350.
- [46] Preston, A. (1989), "The nonprofit worker in a for-profit world", Journal of Labor Economics 7, 438-463.
- [47] Rochet J.-C. and P. Chonè (1998), "Ironing, Sweeping, and Multidimensional Screening" Econometrica, 66, 783-826.
- [48] Rochet J.-C. and L.A. Stole (2002), "Nonlinear Pricing with Random Participation" Review of Economic Studies, 69, 277-311.
- [49] Roomking M. and B. Weisbrod (1999): "Managerial Compensation and Incentives in For-Profit and Nonprofit Hospitals", Journal of Law, Economics and Organization, 15(3), 750-781.
- [50] Rosen, S. (1986), "The Theory of Equalizing Differences", in: Ashenfelter, O., Layard, R. (Eds.), Handbook of Labor Economics, vol. 1, North Holland, pp. 641-692.
- [51] Stole, L.A. (1992), "Mechanism Design Under Common Agency", mimeo, University of Chicago.
- [52] Stole L.A. (1995), "Nonlinear pricing and oligopoly", Journal of Economics & Management Strategy 4, 529-562.

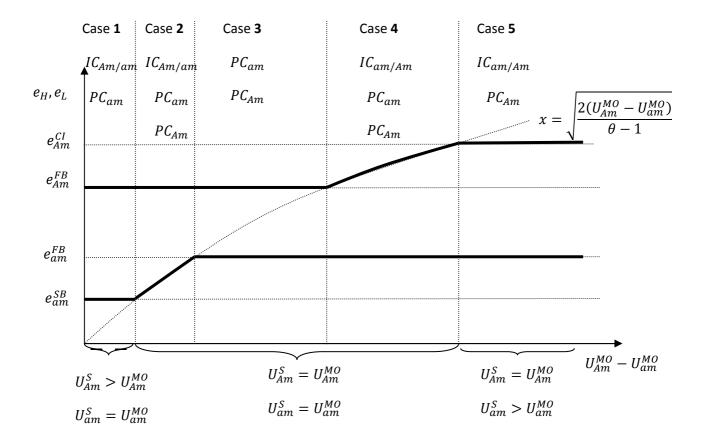


Figure 1: Reaction function of firm S when $1 \le k \le 1 + \gamma$.