Pro-competitive rationing in multi-unit auctions

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Multi-unit auctions

Each year multi-unit auctions trade divisible-goods worth trillions of dollars, for example in wholesale electricity markets and treasury bond auctions. How can competitiveness of such auctions be improved?

I consider a procurement auction, where each producer submits a stepped supply curve.
Rationing

The market is often cleared in the middle of a step.
Rationing methods in prior-art

- Pro-rata on the margin rationing: Completely accept infra-marginal volume. Ration marginal volume proportionally; each bidder gets the same share of its marginal volume accepted. Often used in single round auctions.

- Time-priority: Completely accept infra-marginal volume and give early marginal bids priority to late marginal bids. Often used in financial exchanges.


- Gresik (2001) and Saez et al. (2007) introduce various rationing rules, where rationing is disproportionate and infra-marginal volumes are often completely accepted.

- Simon and Zame (1990), Jackson and Swinkels (1999) analyze rationing rules from an existence perspective.
Influence of rationing rule

Field and Large (2012) empirically observe that rationing rule (time priority or pro-rata on the margin) influences bidding behaviour in financial exchanges.

Rationing rule matters more when volume of marginal bids is large relative to infra-marginal volume, as in security auctions, financial exchanges and frequent batch auctions.
New rationing rule

- Infra-marginal volume is completely accepted.
- Disproportionate rationing on the margin.
- Rationing rule depends on clearing price.
Pro-competitive rationing

New rationing rule makes bidding more competitive; producers provide commodity at lower price.

New rationing rule gives producers with large marginal volumes priority at low prices and producers with small marginal volumes priority at high prices.
Model: new rationing rule

1) Accept all infra-marginal bids.
2) Accept marginal bids little by little. Each increment in the accepted volume is split according to the following rule:

\[
\text{producer i's share} = \frac{\left(\text{Excess supply}_i(\varepsilon)\right)^{\mu_j}}{\sum_{k=1}^{N} \left(\text{Excess supply}_k(\varepsilon)\right)^{\mu_j}}
\]

3) Disproportionality of the rule is determined by \(\mu_j\), which depends on the clearing price.

\(\mu_1 = \infty \Rightarrow\) Maximum priority to bidders with large excess supply at lowest price

\(\mu_M = 0 \Rightarrow\) Maximum priority to bidders with small excess supply at highest price

\(\mu_j\) decreases with \(j\) such that competition is boosted by the same factor at each price level.
Model: Bidding format

Similar to Holmberg, Newbery and Ralph (2013)
Theoretical model: SFE assumptions

Consider uniform-price auction with $N$ producers

Auctioneer’s demand is uncertain. It is announced ex-post.

Costs are common knowledge.

One shot game.

Solve for Nash equilibrium where each producer maximizes its profit given supply functions of competitors.

SFE assumptions (Klemperer and Meyer, 1989) have been empirically verified (Hortacsu and Puller, 2008; Wolak, 2007).
Result for optimal rationing rule

Optimal rationing gives auctioneer approximately same total procurement cost as an auction with standard rationing and 

$$(1+1/(v-1))(N-1)+1>N$$

producers with same total production cost.

Pro-competitive effect is larger when bids accumulate at a few price levels, $v$.

If producers bid at only two price levels, introducing pro-competitive rationing is equivalent to increasing the number of producers from $N$ to $2(N-1)+1$. 
Example with two price levels

Aggregate supply curves

Pro-rata, 2 firms

Pro-rata, 3 firms

Optimal rationing, 2 firms

Aggregate marginal cost

Total output relative to total production capacity
Reverse ascending bid auction

Bid price increments tend to get denser towards the clearing price in clock auctions and other multi-round auctions where bidding starts at the reservation price.

In comparison to standard rationing, auctioneer’s payoff is larger if the rationing rule gives small marginal volumes maximum priority at all price levels. But an optimal rationing rule, where rationing depends on the clearing price, is even better.
Conclusions

Optimal rationing rule is disproportionate and depends on the clearing price.

Optimal rationing rule gives producers with large marginal volumes priority at low clearing prices and bidder’s with small marginal volumes priority at high clearing prices.

Pro-competitive rule has larger effect when bids accumulate at a few prices. Under beneficial circumstances, an optimal rationing rule has the same effect as a doubling of the number of bidders.

In clock auctions and similar multi-round auctions, competitiveness is improved by simply giving maximum priority to small marginal volumes at all price levels.