Policy tradeoffs under risk of abrupt climate change

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Abstract

By now it is widely recognized that the more serious threats of climate change are associated with abrupt events capable of inflicting losses on a catastrophic scale. Consequently, the main role of climate policies is to balance between mitigation efforts, aimed at delaying (or even preventing) the occurrence of such events, and adaptation actions, aimed at minimizing the damage inflicted upon occurrence. The former affects the accumulation of greenhouse gases in the atmosphere; the latter determines the impact of loss once the event occurs. This work examines the tradeoffs associated with these two types of policy measures by characterizing the optimal mitigation-adaptation mix in the long run.

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Keywords: Climate policy, catastrophic events, mitigation, adaptation, steady state, multi state.

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1 Introduction

It is widely recognized by now that the more serious threats of climate change are associated with abrupt changes capable of inflicting losses on a catastrophic scale (Alley et al. 2003, Field et al. 2012). Each link in the chain leading from anthropogenic emission of greenhouse gases (GHG) to the abrupt change in climate and the ensuing damage involves uncertain elements (Schelling 2007, Tol 2012). An appropriate framework to analyze such situations involves discrete events triggered by conditions that are either imperfectly understood (e.g., include unknown parameters) or involve genuine stochastic elements. Any climate change-induced event can be categorized as one or a combination of these two types.

Tsur and Zemel (1996), for example, studied the first type of climate events – those triggered when a certain threshold is crossed (i.e., tipping point events). While the threshold itself does not change (hence crossing it is a deterministic event), its location depends on parameters that are unknown or only partially known to modelers and policymakers. In contrast, the events analyzed in Tsur and Zemel (1998) are triggered by genuinely stochastic conditions. It turns out that the method of analysis as well as the ensuing optimal policies differ between these two types of events (see discussion in Tsur and Zemel 2007). Here we consider the latter type of climate events – those triggered by stochastic conditions.

Policy measures for dealing with threats of abrupt climate change can be categorized into two types. The first includes measures aimed at delaying or even preventing the event occurrence by reducing emission of GHG or by capturing (sequestering) carbon and storing it at harmless locations. Such measures are commonly referred to as *mitigation* policies. Measures of the second type are aimed at reducing, or even eliminating, the damage caused by the event once it occurs, e.g., building levees to prevent flooding, developing a cure or a vaccine for diseases that are likely to spread due to the arrival of certain pathogens, or developing crop varieties that can better sustain a range of climate conditions. These measures are commonly referred to as *adaptation* policies. A comprehensive climate policy contains measures of both types and characterizing the optimal policy requires evaluating the tradeoffs between them (Tol 2005, Bréchet et al. 2013). In this work we present a framework for accomplishing this goal, focusing on the long run.

To that end, we use the mitigation-adaptation framework offered by Zemel (2015), which combines mitigation policies affecting the random occurrence date of a detrimental event (such as in Tsur and Zemel 1998) with adaptation policies affecting the damage inflicted upon occurrence (such as in Tsur and Withagen 2013). By assuming that the costs and effects of adaptation investments are linear, Zemel (2015) was able to characterize the entire time profile of the optimal mitigation-adaptation policy.

In this work we relax this linearity assumption and focus on characterizing the optimal steady state, i.e., the optimal adaptation-mitigation policy in the long run. We do this by extending the method of Tsur and Zemel (2014c) for characterizing optimal steady states of multi-state dynamic systems to situations involving random events. In the present context the model contains two state variables: an atmospheric GHG stock, affecting the occurrence probability of a detrimental event, determined by the mitigation policy; and an adaptation capital stock whose role is to reduce the damage inflicted upon occurrence.

We provide necessary conditions for the location and stability of optimal steady states. These conditions give rise to a simple method for characterizing the optimal mitigation-adaption mix in the long run. An example illustrates how the method works in a particular setting.

2 Setup

An abrupt climate-change induced event, capable of inflicting a severe damage, may occur at some uncertain future date T. The distribution of T is governed by a hazard rate function $h(Q)$ that depends on the atmospheric GHG stock Q. The event inflicts a damage $\psi(k)$ that depends on the adaptation capital k available at T. The climate policy consists of mitigation efforts to curb the accumulation of GHG and of investment in adaptation capital. The policymaker task is to set the optimal mix of these two activities over time. The model described below addresses this problem.

2.1 Climate policy

Production activities at time t generate emissions at the rate $m(t)$ that accumulate to form the GHG stock $Q(t)$ according to

$$\dot{Q}(t) = m(t) - \gamma Q(t), \tag{2.1}$$

where γ is the natural GHG removal rate. Emission is bounded above by a finite maximal rate $\bar{m}$ and mitigation at time t is measured as the difference $\bar{m} - m(t)$ between the maximal and actual rates. The upper bound on m implies the maximal feasible GHG stock $\bar{Q} = \bar{m}/\gamma$. Given that no event has occurred by time t_0, the GHG stock process $Q(\cdot)$ affects the distribution of the random occurrence date T of the event through the hazard rate function

$h(Q)$ according to

$$S(t|\,t_0) \equiv Pr\{T > t|\,T > t_0\} = e^{-\int_{t_0}^{t} h(Q(s))ds} \qquad (2.2)$$

for $t \geq t_0 \geq 0$. The corresponding conditional distribution and density functions of T are, respectively,

$$F_T(t|\,t_0) = 1 - S(t|\,t_0) \text{ and } f_T(t|\,t_0) = F_T'(t|\,t_0) = h(Q(t))e^{-\int_{t_0}^{t} h(Q(s))ds}. \qquad (2.3)$$

We assume that $h(0) = 0;\;\; h'(Q) > 0;\; h''(Q) \geq 0$.

Occurrence at time T inflicts the damage $\psi(k(T))$, where $\psi(\cdot)$ decreases in the adaptation capital k at a diminishing rate: $\psi(k) > 0,\; \psi'(k) < 0,\; \psi''(k) > 0$. The adaptation capital accumulates according to

$$\dot{k}(t) = a(t) - \delta k(t), \qquad (2.4)$$

where $a(\cdot)$ is the investment in adaptation capital and δ is a depreciation rate.

The production activities associated with emission give rise to the instantaneous utility $u(m, a)$ which increases with the emission rate m and decreases with adaptation investment rate a (since the latter comes at the expense of consumption). More specifically, we assume[1]

$$u_m(\cdot,\cdot) > 0 \text{ for } 0 < m < \bar{m};\;\; u_a(\cdot,\cdot) < 0 \text{ for } a > 0;$$
$$u_{mm}(\cdot,\cdot) < 0;\;\; u_{aa}(\cdot,\cdot) < 0;\;\; u_{mm}(\cdot,\cdot)u_{aa}(\cdot,\cdot) > u_{am}^2(\cdot,\cdot). \qquad (2.5)$$

A climate policy consists of the action processes $\{m(t), a(t), t \geq 0\}$. A policy is feasible if $m(t) \in [0, \bar{m}]$ and $a(t) \geq 0$ for all $t \geq 0$.

[1]Subscripts denote partial derivatives. In typical applications, the contributions of m and a to the utility are independent, $u_{am} = 0$ and the last condition is trivially satisfied. The assumption on u_{aa} represents the point of departure from the model of Zemel (2015), since it rules out a linear dependence on the control variable a.

2.2 Payoff

We consider recurrent events, i.e., events that may occur again and again, where the distribution of the next occurrence date is determined by the hazard process $h(Q(t))$, as defined in (2.2), when t_0 is the previous occurrence date or zero if no event has yet occurred (see Tsur and Zemel 1998, for a distinction between single occurrence and recurrent events). Each time the event occurs it inflicts the penalty $\psi(k)$ corresponding to the adaptation capital k at the occurrence date. Apart from inflicting the penalty, occurrence does not change the flow of utility or the dynamics of the stock variables, nor the probability distribution of yet another occurrence.

Let $v(Q,k)$ denote the value function (i.e., the value of the objective obtained with the optimal policy when the initial stocks are Q and k). Assuming, without loss of generality, that $t_0 = 0$ and the first event occurs at T, the payoff at $t_0 = 0$ is

$$\int_0^T u(m(t), a(t))e^{-\rho t}dt + e^{-\rho T}[v(Q(T), k(T)) - \psi(k(T))].$$

Taking expectation with respect to the distribution T, using (2.2)-(2.3), gives the expected payoff

$$\int_0^\infty [u(m(t), a(t)) + h(Q(t))\varphi(Q(t), k(t))]\, e^{-\int_0^t [\rho + h(Q(s))]ds} dt, \tag{2.6}$$

where

$$\varphi(Q,k) \equiv v(Q,k) - \psi(k) \tag{2.7}$$

is the continuation value at the time of occurrence. The optimal policy is the feasible process $\{m(t), a(t),\, t \geq 0\}$ that maximizes (2.6) subject to (2.1) and (2.4), given $Q(0) = Q_0$, $k(0) = k_0$. The value $v(Q_0, k_0)$ is obtained by evaluating the objective (2.6) at the optimal policy. Note that (2.6) contains the value function $v(\cdot,\cdot)$ via the continuation value $\varphi(\cdot,\cdot)$, implying that $v(\cdot,\cdot)$ is only implicitly defined. For long run analysis aimed at characterizing the steady states, the implicit definition poses no difficulty, as shown below.

3 Long run properties

Let $X = (Q, k)'$ and $C = (m, a)'$ denote, respectively, the state and action vectors (a prime over a vector or a matrix indicates the transpose operator). For any state X, let $\hat{C}(X)$ denote the adaptation-mitigation actions that maintain the state fixed at X indefinitely. Noting (2.1) and (2.4),

$$\hat{C}(X) = (\gamma Q, \delta k)'. \tag{3.1}$$

Let $W(X)$ denote the expected payoff obtained when the (not necessarily optimal) steady state policy $\hat{C}(X)$ is maintained indefinitely (*before* and *after* occurrences). Under this policy, the state process remains fixed at X and the T distribution (2.3) reduces to exponential (with the parameter $h(Q)$). Evaluating (2.6) under the steady state policy gives

$$W(X) = \frac{u(\gamma Q, \delta k) + h(Q)\varphi(X)}{\rho + h(Q)}. \tag{3.2}$$

Since the steady state policy proceeds also after occurrence, the continuation value $\varphi(X)$ reduces to $W(X) - \psi(k)$ and (3.2) becomes

$$W(X) = \frac{u(\gamma Q, \delta k) + h(Q)[W(X) - \psi(k)]}{\rho + h(Q)}.$$

Solving for $W(X)$, we find

$$W(X) = \frac{u(\gamma Q, \delta k) - h(Q)\psi(k)}{\rho}. \tag{3.3}$$

The first term $u(\gamma Q, \delta k)/\rho$ is the steady state value *without catastrophic risk.* The second term describes the expected cumulative loss from a Poisson series of events when the penalty ψ associated with each event is weighted at the discount factor corresponding to its time of occurrence.

Let $\hat{X}$ denote an optimal steady state. Since $W(X) \le v(X)$ with equality holding at $\hat{X}$, it follows that

$$v(\hat{X}) = \frac{u(\gamma\hat{Q}, \delta\hat{k}) - h(\hat{Q})\psi(\hat{k})}{\rho} \tag{3.4}$$

and

$$v_X(\hat{X}) = W_X(\hat{X}), \tag{3.5}$$

where $v_X(X)$ and $W_X(X)$ are, respectively, the gradient vectors of $v(X)$ and $W(X)$ with respect to $X = (Q, k)'$.

Let $f(X, C) \equiv u(m, a) + h(Q)\varphi(X)$ be the instantaneous utility associated with the expected payoff (2.6). The gradient vector of f with respect to C, evaluated at $(X, \hat{C}(X))$ is given by

$$f_C = \begin{pmatrix} u_m(\gamma Q, \delta k) \\ u_a(\gamma Q, \delta k) \end{pmatrix}. \tag{3.6}$$

The state-dynamics equations (2.1) and (2.4) can be jointly expressed as $\dot{X} = G(X, C)$, where

$$G(X, C) \equiv C - \begin{pmatrix} \gamma & 0 \\ 0 & \delta \end{pmatrix} X = \begin{pmatrix} m - \gamma Q \\ a - \delta k \end{pmatrix}.$$

Let J_X^G and J_C^G denote the Jacobian matrices with respect to X and C, respectively. Then,

$$J_X^G = \begin{pmatrix} -\gamma & 0 \\ 0 & -\delta \end{pmatrix}, \quad J_C^G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.7}$$

Next, following Tsur and Zemel (2014c) we introduce the function[2]

$$L(X) \equiv \begin{pmatrix} l_1(X) \\ l_2(X) \end{pmatrix} = (\rho + h(Q)) \left([J_C^{G\prime}]^{-1} f_C + W_X(X) \right), \tag{3.8}$$

[2] J_C^G is given in (3.7) as the identity matrix, hence including its inverse in the definition of L appears redundant. We keep it here for consistency with the general theory (see Appendix) and to allow extensions with more complicated state-dynamics equations.

where

$$W_X(X) = \frac{1}{\rho}\begin{pmatrix} \gamma\ u_m(\gamma Q, \delta k) - h'(Q)\psi(k) \\ \delta\ u_a(\gamma Q, \delta k) - h(Q)\psi'(k) \end{pmatrix}$$

was defined above as the gradient vector of $W(X)$. In the present setting, noting (3.6) and (3.7), $L(\cdot)$ specializes to

$$L(X) = \frac{\rho + h(Q)}{\rho}\begin{pmatrix} (\rho+\gamma)u_m(\gamma Q, \delta k) - h'(Q)\psi(k) \\ (\rho+\delta)u_a(\gamma Q, \delta k) - h(Q)\psi'(k) \end{pmatrix}. \tag{3.9}$$

The significance of $L(\cdot)$ is manifest in:

Property 1. *Assume the state bounds* $0 \le Q \le \bar{Q}$ *and* $0 \le k \le \bar{k}$. *The following conditions hold at an optimal steady state* $\hat{X} = (\hat{Q}, \hat{k})'$*:*

(i) If $\hat{Q} \in (0, \bar{Q})$ *and* $\hat{k} \in (0, \bar{k})$ *then* $L(\hat{X}) = 0$.

(ii) If $\hat{Q} = \bar{Q}$, *then* $l_1(\hat{X}) \ge 0$; *if* $\hat{k} = \bar{k}$, *then* $l_2(\hat{X}) \ge 0$.

(iii) If $\hat{Q} = 0$, *then* $l_1(\hat{X}) \le 0$; *if* $\hat{k} = 0$, *then* $l_2(\hat{X}) \le 0$.

Property 2. *If a steady state* $\hat{X}$ *at which* $L(\hat{X}) = 0$ *is locally stable,*[3] *then* $\det(J_X^L(\hat{X})) > 0$.

With some modifications to account for the presence of a Q-dependent hazard, the proofs of the properties proceed along the steps of the proofs of Propositions 1 and 2 in Tsur and Zemel (2014c). The proof of Property 1 is outlined in the appendix; the proof of Property 2 is omitted.

For the model at hand, Property 1 implies, noting (3.9), that at an internal steady state (where $L(\hat{X}) = 0$) the following conditions hold

$$u_m(\gamma\hat{Q}, \delta\hat{k}) = \frac{h'(\hat{Q})\psi(\hat{k})}{\rho+\gamma}\ ; \qquad u_a(\gamma\hat{Q}, \delta\hat{k}) = \frac{h(\hat{Q})\psi'(\hat{k})}{\rho+\delta}\ .$$

The first condition defines the optimal steady state for the Q process when the adaptation capital is constrained at $\hat{k}$. The second condition defines the

[3] $\hat{X}$ is locally stable if there exists some $\epsilon > 0$ such that (along the optimal trajectory) $\|X(t_0) - \hat{X}\| < \epsilon$ at some t_0 implies $X(t) \to \hat{X}$.

optimal steady state for the k process when the GHG stock is constrained to be fixed at $\hat{Q}$. When both stocks are free to vary, an optimal steady state requires both conditions to hold.

Evaluating the Jacobian of L at the internal steady state, we find

$$J_X^L(\hat{X}) = \frac{\rho + h(\hat{Q})}{\rho} \times$$
$$\begin{pmatrix} \gamma(\rho+\gamma)u_{mm}(\gamma\hat{Q},\delta\hat{k}) - h''(\hat{Q})\psi(\hat{k}) & \delta(\rho+\gamma)u_{ma}(\gamma\hat{Q},\delta\hat{k}) - h'(\hat{Q})\psi'(\hat{k}) \\ \gamma(\rho+\delta)u_{ma}(\gamma\hat{Q},\delta\hat{k}) - h'(\hat{Q})\psi'(\hat{k}) & \delta(\rho+\delta)u_{aa}(\gamma\hat{Q},\delta\hat{k}) - h(\hat{Q})\psi''(\hat{k}) \end{pmatrix}. \tag{3.10}$$

The two diagonal elements of J_X^L are negative. Condition (2.5) regarding u_{ma} ensures that the presence of this term in the off-diagonal elements cannot, on its own, reverse the sign of the determinant (i.e., $h \equiv 0$ implies that $\det(J_X^L)$ is positive). The stability condition of Proposition 2, then, depends essentially on the magnitude of $h'(\hat{Q})\psi'(\hat{k})$ vis-à-vis the other elements of the matrix. The stability property is examined in more detail in terms of a specific example in the following section.

4 Example

We follow Zemel's (2015) example, modifying the adaptation policy to allow for nonlinear effects. Table 1 presents the specifications and parameter values used in the numerical solution. Using Table 1, $L(X)$, defined in (3.9), specializes to

$$l_1(Q,k) = \frac{\rho + \beta Q}{\rho}\left((\rho+\gamma)(\alpha - \gamma Q) - \frac{\beta K_m \psi_0}{k + K_m}\right) \tag{4.1a}$$

and

$$l_2(Q,k) = \frac{\rho + \beta Q}{\rho}\left(-(\rho+\delta)(1+\mu)(k\delta)^{\mu} + \frac{Q K_m \beta \psi_0}{(k + K_m)^2}\right). \tag{4.1b}$$

Table 1: Function specifications and parameter values.

Function	Specification	Parameter value	Description
$u(m,a)$	$\alpha m - m^2/2 - a^{1+\mu}$	$\alpha = 2,\ \mu = 1$	utility
$h(Q)$	βQ	$\beta = 0.005$	hazard rate
$\psi(k)$	$\psi_0 K_m/(K_m + k)$	$\psi_0 = 10,\ K_m = 50$	damage function
		$\rho = 0.03$	discount rate
		$\gamma = 0.01$	GHG decay rate
		$\delta = 0.03$	capital depreciation rate
		$\bar{Q} = 200$	maximal GHG stock
		$\bar{k} = 33.33$	maximal adaptation capital

Viewing the solution Q of $l_1(Q,k) = 0$ as a function of k, i.e. $Q = Q_1(k)$ we find

$$Q_1(k) = R_1 - \frac{R_2}{(k + K_m)} \quad \text{where} \quad R_1 \equiv \frac{\alpha}{\gamma} = \bar{Q} \quad \text{and} \quad R_2 \equiv \frac{\beta K_m \psi_0}{\gamma(\rho+\gamma)}.$$

The function $Q_1(\cdot)$ is increasing. The higher is the adaptation capital k, the lower is the inflicted damage hence the planner can increase the GHG stock and the associated hazard. This observation reflects an important adaptation-mitigation tradeoff: a higher adaptation capital provides a higher insurance coverage against the perils of a climate change catastrophe, hence reduces the incentive to exert mitigation efforts in order to avoid or delay occurrence.

Viewing the solution Q of $l_2(Q,k) = 0$ as a function of k, i.e. $Q = Q_2(k)$ we find

$$Q_2(k) = R_3 k^{\mu} (k + K_m)^2 \quad \text{where} \quad R_3 \equiv \frac{\delta^{\mu}(\rho+\delta)(1+\mu)}{\beta K_m \psi_0}.$$

The function $Q_2(\cdot)$ is also increasing. The larger is the GHG stock, the higher the risk and the incentive to invest in increasing k. The condition that $L(\cdot)$ must vanish at an internal steady state clearly displays the tradeoffs between the two responses to the hovering risk. Indeed, imposing this condition entails

$Q_1(k) = Q_2(k)$ which reduces to a polynomial equation in k. Using the values in Table 1, we find that the conditions $l_1 = l_2 = 0$ reduce in this example to

$$0.00144 \times k(k+50)^3 - 200 \times (k+50) + 6250 = 0. \tag{4.2}$$

This 4^{th} order polynomial equation in k admits two real roots, of which one is negative hence infeasible, while the other corresponds to the feasible steady state $\hat{X} \equiv (\hat{Q}, \hat{k}) = (106.178, 16.616)$. At this state, the Jacobian matrix (3.10) reduces to

$$J_X^L(\hat{X}) = \frac{\rho + h(\hat{Q})}{\rho} \begin{pmatrix} -0.0004 & 0.000563 \\ 0.000563 & -0.0054 \end{pmatrix}. \tag{4.3}$$

which has a positive determinant, as required by Property 2 for $(\hat{Q}, \hat{k})$ to be locally stable.

While these considerations leave a unique candidate for an internal steady state, one needs to investigate also the possibility of corner steady states. This requires to check the sign of the relevant component of $L(\cdot)$ at each possible corner. To consider the possibility that an optimal steady state falls on a boundary $\hat{Q} = \bar{Q} = 200$ or $\hat{Q} = 0$, Figure 1 depicts the functions $l_1(\bar{Q}, k)$ and $l_1(0, k)$ for $k \in [0, \bar{k}]$. It is seen that the upper curve is negative and the lower curve is positive for all $k \in [0, \bar{k}]$, ruling out, by virtue of Property 1, the possibility that a steady state falls on one of the Q boundaries. This means that the occurrence hazard is insufficient to drive the GHG stock (and the corresponding emission rate) all the way down to zero, but on the other hand, it does not allow this stock to reach the maximal level $\bar{Q}$ that would have been obtained if the emission rate m were chosen so as to maximize the utility u at all times.

To check the possibility that the steady state falls on a k boundary ($k = 0$ or $k = \bar{k} = 33.33$), Figure 2 shows $l_2(Q, \bar{k})$ and $l_2(Q, 0)$ for $Q \in [0, \bar{Q}]$. The

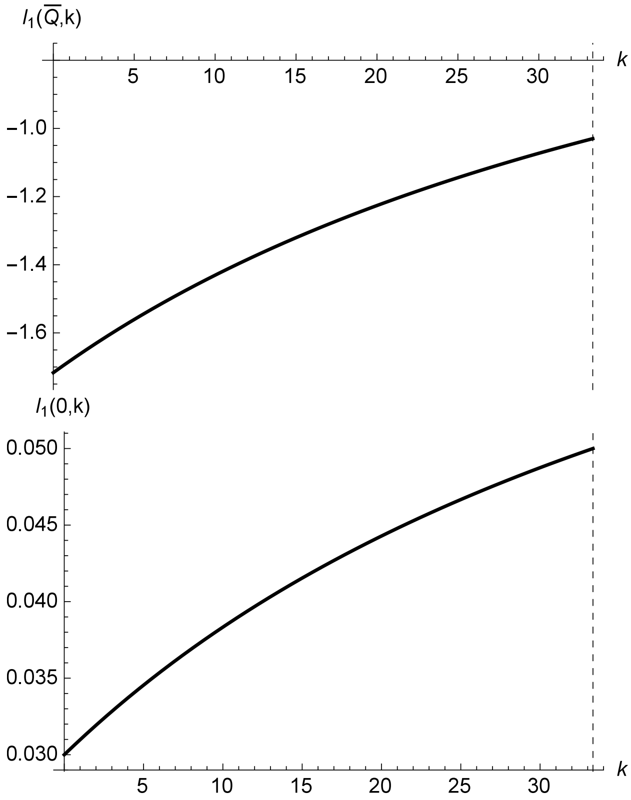

Figure 1: The upper panel shows $l_1(\bar{Q}, k)$ vs. k. The lower panel shows $l_1(0, k)$ vs. k. The upper curve is always negative and the lower curve is always positive in the feasible range for k, ruling out a steady state with either $Q = \bar{Q}$ or $Q = 0$.

upper curve is negative for all $Q \in [0, \bar{Q}]$, ruling out the possibility of a steady state falling at $k = \bar{k}$ (Property 1). The lower curve is positive for all $Q > 0$ and crosses zero at $Q = 0$. Thus the same Property allows a steady state with $\hat{k} = 0$ only if $\hat{Q} = 0$. However, $\hat{Q} = 0$ was ruled out above, implying that the steady state cannot fall on a k boundary: in the long run some adaptation is desirable but not at the full feasible rate. This leaves the internal state $(\hat{Q}, \hat{k}) = (106.178, 16.616)$ as the unique optimal steady state in this case.

Increasing the hazard sensitivity parameter β from 0.005 to 0.01 changes

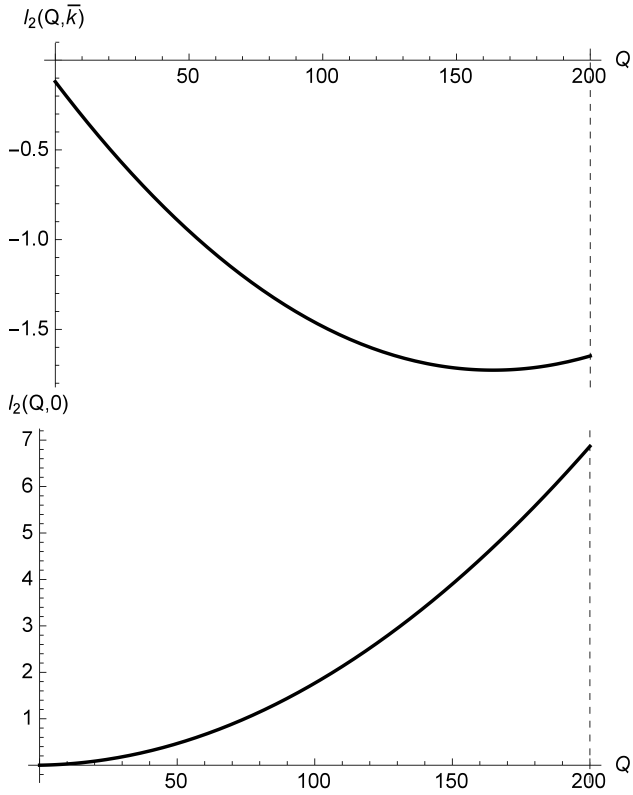

Figure 2: The upper panel shows $l_2(Q, \bar{k})$ vs. Q. The lower panel shows $l_2(Q, 0)$ vs. Q. The upper curve is always negative and the lower curve is always positive in the feasible range for Q, ruling out a steady state with either $k = \bar{k}$ or $k = 0$.

the polynomial equation (4.2) to

$$0.00072 \times k(k + 50)^3 - 200 \times (k + 50) + 12500 = 0$$

and this equation does not admit any real root. It follows from Property 1 that no internal steady state (with both Q and k away from their respective corners) can be optimal and at least one state must lie on a corner. We investigate the various possibilities with the help of Figures 3 - 5.

Observing the upper panel of Figure 3, we see that Property 1-(ii) rules out the possibility that $\hat{Q} = \bar{Q}$ (because $l_1(\bar{Q}, k) < 0$ for all $k \in [0, \bar{k}]$). In contrast, the lower panel leaves open the possibility that $\hat{Q} = 0$ as long as

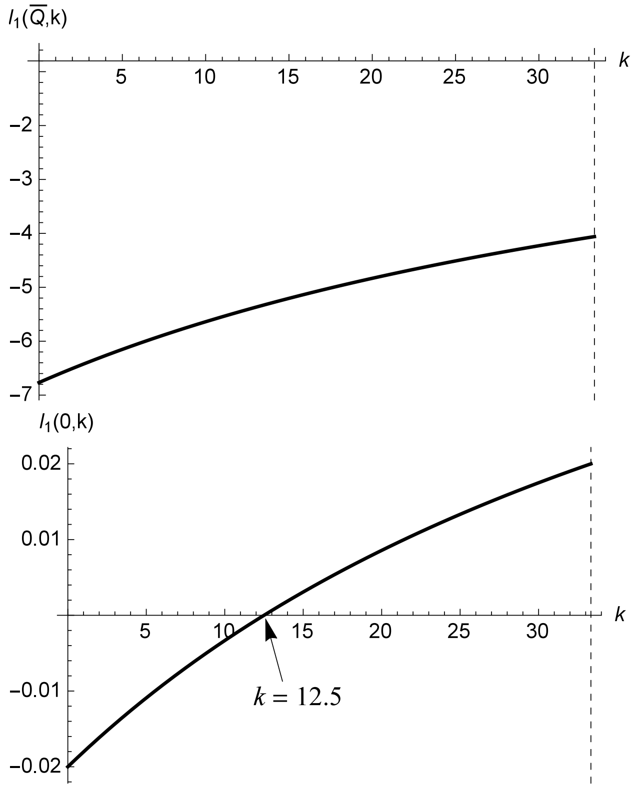

Figure 3: $l_1(\bar{Q}, k)$ (upper panel) and $l_1(0, k)$ (lower panel) vs. k under high hazard sensitivity $\beta = 0.01$. The negative values of $l_1(\bar{Q}, k)$ rule out a steady state with $\hat{Q} = \bar{Q}$. The negative values of $l_1(0, k)$ leave open the possibility of a steady state with $\hat{Q} = 0$ and $\hat{k} \leq 12.5$.

$\hat{k} \leq 12.5$ (since $l_1(0, k) \leq 0$ for $k \in [0, 12.5]$). Setting $Q = 0$ in (4.1b) yields a negative value for $l_2(0, k)$ for all $k > 0$ hence Property 1-(i) implies that no internal k state can couple with $\hat{Q} = 0$ to form an optimal steady state. However, with $k = 0$, we obtain from the same equation $l_2(0, 0) = 0$ hence the double-corner state $(\hat{Q}, \hat{k}) = (0, 0)$ meets the conditions of Property 1 for an optimal steady state.

Turning to Figure 4, the lower panel shows that Property 1-(iii) rules out the possibility that $\hat{k} = 0$ and $\hat{Q} > 0$ (since $l_2(Q, 0) > 0$ for $Q > 0$) but leaves open the possibility $\hat{Q} = \hat{k} = 0$, discussed above. The interpretation of the

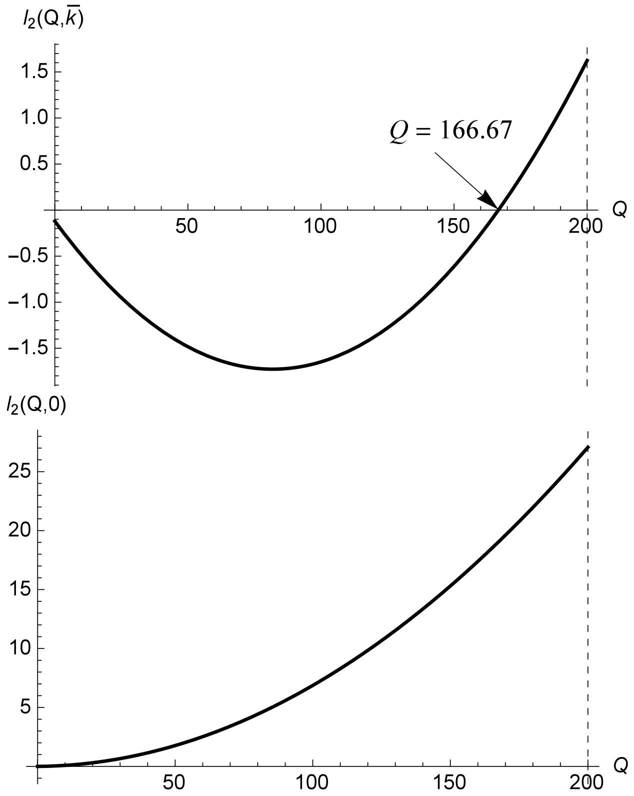

Figure 4: $l_2(Q, \bar{k})$ (upper panle) and $l_2(Q, 0)$ (lower panel) vs. the GHG stock Q under high hazard sensitivity $\beta = 0.01$. The positive values of $l_2(Q, 0)$ rule out a steady state with $\hat{k} = 0$ and $\hat{Q} > 0$. The positive values of $l_2(Q, \bar{k})$ leave open the possibility of a steady state with $\hat{k} = \bar{k}$ and $\hat{Q} \geq 167.67$. However, the latter possibility is ruled out by considerations involving $l_1(Q, \bar{k})$ (see Figure 5 below).

upper panel is somewhat more complex. First, it rules out the possibility that $\hat{k} = \bar{k}$ for $Q < 166.67$ (by virtue of Property 1-(ii), since $l_2(Q, \bar{k}) < 0$ for $Q < 166.67$). Thus, if a steady state with $\hat{k} = \bar{k}$ is optimal, it must have $\hat{Q} \geq 166.67$. However, Figure 5 shows that $l_1(Q, \bar{k}) < 0$ in this Q range, hence Property 1 implies that such a steady state cannot be optimal.

The above considerations leave $(\hat{Q}, \hat{k}) = (0, 0)$ as the unique optimal steady state in the case of $\beta = 0.01$. The result can be attributed to the high hazard sensitivity. The strong dependence of the hazard rate on the GHG

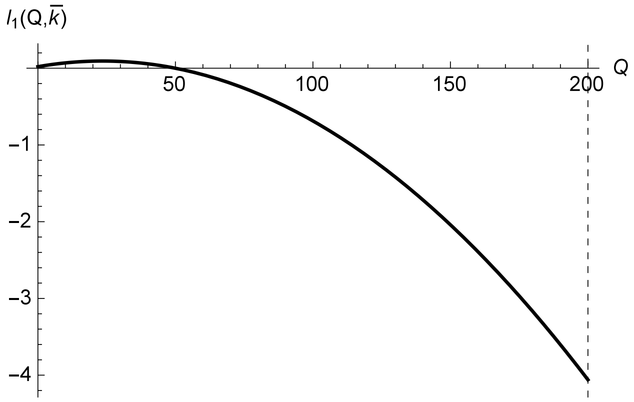

Figure 5: $l_1(Q, \bar{k})$ vs. the GHG stock Q under high hazard sensitivity $\beta = 0.01$. The negative values of $l_1(Q, \bar{k})$ at the higher Q states rule out (by virtue of Property 1-(ii)) a steady state with $\hat{k} = \bar{k}$ and $\hat{Q}$ in this range.

stock provides a strong incentive to reduce emissions and bring the occurrence probability down to zero. However, eliminating the risk also removes the motivation to invest in adaptation, hence the adaptation capital stock k is also driven down to its lowest feasible level. The tradeoffs between mitigation and adaptation measures are evident in this case.

5 Concluding comments

We study long-term adaptation-mitigation tradeoffs in situations involving risk of catastrophic climate events, where the mitigation policy influences the event occurrence probability and the adaptation policy affects the severity of damage upon occurrence. The analysis extends Zemel (2015) to non-linear policies and is based on the multi state $L-$method of Tsur and Zemel (2014c), appropriately modified to allow for uncertain discrete events. We find that the method can identify a unique candidate for the optimal two-dimensional steady state, both when this state is interior, determined by the adaptation/mitigation tradeoffs, as well as when it is a corner state depending on some feasibility constraint. In both cases, the eventual steady state reflects the strong interaction between the adaptation and mitigation responses to the catastrophic risk.

Although the model is presented in the context of a climate change problem, the framework can be used, with obvious modifications, in other multi-dimensional resource situations involving uncertain discrete events, such as an abrupt regime shift in the dynamics of exploited ecosystems and other regenerating resources (see examples in Dasgupta and Mäler 2003, Tsur and Zemel 2007, Polasky et al. 2011, de Zeeuw and Zemel 2012).

Appendix

Proof of Property 1: The following derivation combines the arguments of Tsur and Zemel (2014a,b) to show how the properties of the L-method presented in Tsur and Zemel (2014c) extend to the case of a multi-state system evolving under event uncertainty. We recall the notation $f(X, C) = u(C) + h(Q)\varphi(X)$ and

$$W(X) = \int_0^\infty f(X, \hat{C}(X))e^{-[\rho+h(Q)]t}dt = \frac{f(X, \hat{C}(X))}{\rho + h(Q)},$$

and note that although the simple form adopted here for the state equation $\dot{X} = G(X, C)$ reduces the Jacobian J_C^G to the identity matrix (see 3.7), the formulation holds for more general specifications hence we refer to this Jacobian in its general form.

For any feasible X, we compare the payoff $W(X)$ obtained under the steady state policy $C = \hat{C}(X)$ with the payoff obtained from a small feasible variation of this policy. If the variation policy yields a payoff that exceeds $W(X)$, then the steady-state policy is not optimal at X and this state does not qualify as an optimal steady state. For small $\varepsilon > 0$ and $\Delta = (\delta_1, \delta_2)'$, the variation policy is defined by

$$C^{\varepsilon\Delta}(t) \equiv \begin{cases} \hat{C}(X) + [J_C^G(X, \hat{C}(X))]^{-1}\Delta & \text{if } t < \varepsilon \\ \hat{C}(X(\varepsilon)) & \text{if } t \geq \varepsilon \end{cases}.$$

While $t < \varepsilon$, $C^{\varepsilon\Delta}(t)$ deviates slightly from the steady-state policy $\hat{C}(X)$, then it enters a steady state at $X(\varepsilon)$. During the first period when $t < \varepsilon$,

$$\dot{X} = G(X, \hat{C}(X)) + J_C^G(X, \hat{C}(X))[J_C^G(X, \hat{C}(X))]^{-1}\Delta + o(\delta) = \Delta + o(\delta),$$

which brings the state at $t = \varepsilon$ to $X(\varepsilon) = X + \varepsilon\Delta + o(\varepsilon\delta)$.

Let $\Gamma(t) \equiv \int_0^t [\rho + h(Q(s))]ds$. The contribution to the objective under the variation policy $C^{\varepsilon\Delta}(t)$ during $t < \varepsilon$ is evaluated, up to $o(\varepsilon\delta)$ terms, by

$$\begin{aligned} &\int_0^\varepsilon f\left(X(t), \hat{C}(X) + [J_C^G(X, \hat{C}(X))]^{-1}\Delta\right) e^{-\Gamma(t)} dt \quad = \\ &\int_0^\varepsilon f\left(X(t), \hat{C}(X) + [J_C^G(X, \hat{C}(X))]^{-1}\Delta\right) e^{-[\rho+h(Q)]t} dt \; + \\ &\int_0^\varepsilon f\left(X(t), \hat{C}(X) + [J_C^G(X, \hat{C}(X))]^{-1}\Delta\right) [e^{-\Gamma(t)} - e^{-[\rho+h(Q)]t}] dt. \end{aligned}$$

The first integral in the right can be expressed as

$$\begin{aligned} &\int_0^\varepsilon f(X, \hat{C}(X)) e^{-[\rho+h(Q)]t} dt + [f_C(X, \hat{C}(X))]'[J_C^G(X, \hat{C}(X))]^{-1}[\varepsilon\Delta] + o(\varepsilon\delta) = \\ &\quad W(X)\left[1 - e^{-[\rho+h(Q)]\varepsilon}\right] + [f_C(X, \hat{C}(X))]'[J_C^G(X, \hat{C}(X))]^{-1}[\varepsilon\Delta] + o(\varepsilon\delta), \end{aligned}$$

and the second integral is $o(\varepsilon\delta)$.

The contribution of $C^{\varepsilon\delta}$ during the infinite period $t \geq \varepsilon$ is evaluated, up to $o(\varepsilon\delta)$ terms, by

$$\begin{aligned} &\int_\varepsilon^\infty f(X(\varepsilon), \hat{C}(X(\varepsilon))) e^{-[\rho+h(Q(\varepsilon))]t} dt = \int_\varepsilon^\infty [\rho + h(Q(\varepsilon))] W(X(\varepsilon)) e^{-[\rho+h(Q(\varepsilon))]t} dt = \\ &\int_\varepsilon^\infty [\rho + h(Q(\varepsilon))] W(X) e^{-[\rho+h(Q(\varepsilon))]t} dt + \int_\varepsilon^\infty [\rho + h(Q(\varepsilon))][W_X(X)]'[\varepsilon\Delta] e^{-[\rho+h(Q(\varepsilon))]t} dt. \end{aligned}$$

The first integral on the second line can be expressed as

$$W(X)\int_{\varepsilon}^{\infty}[\rho+h(Q(\varepsilon))]e^{-[\rho+h(Q(\varepsilon))]t}dt = W(X)e^{-[\rho+h(Q(\varepsilon))]\varepsilon} = W(X)e^{-[\rho+h(Q)]\varepsilon}+o(\varepsilon\delta)$$

and the second integral is approximated by $[W_X(X)]'[\varepsilon\Delta] + o(\varepsilon\delta)$.

Summing the contributions of the two periods gives the payoff $V^{\varepsilon\Delta}(X)$ obtained under the variation policy:

$$V^{\varepsilon\Delta}(X) = W(X) + \Big[[J_C^G(X,\hat{C}(X))]'^{-1}f_C(X,\hat{C}(X)) + W_X(X)\Big]'[\varepsilon\Delta] + o(\varepsilon\delta).$$

Thus, noting (3.8),

$$V^{\varepsilon\Delta}(X) - W(X) = [L(X)]'[\varepsilon\Delta]/[\rho + h(Q)] + o(\varepsilon\delta).$$

The signs of the elements of Δ can be freely chosen, while $\varepsilon > 0$. Now, if $L(X) \neq \mathbf{0}$ we can set $\Delta = \delta L(X)$, where δ is a small positive constant, hence $[L(X)]'\Delta > 0$. This implies $V^{\varepsilon\Delta}(X) > W(X)$ and X is not an optimal steady state. Thus, only the roots of $L(\cdot)$ qualify as legitimate candidates for an optimal steady state. The only possible exceptions are the feasibility bounds of Q and k. Choosing $\delta_1 > 0$ is not feasible at $\bar{Q}$ because this policy would drive the $Q(\cdot)$ process outside the feasible domain. It follows that $X = (\bar{Q}, k)'$ cannot be excluded as an optimal steady state if $l_1(X) > 0$. A similar argument implies that $X = (0, k)'$ cannot be excluded as an optimal steady state if $l_1(X) < 0$. Analogous constraints on the sign of $l_2(X)$ apply at the corner states $X = (Q, \bar{k})'$ and $X = (Q, 0)'$. □

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