Vertical Integration and Algorithm Bias*
Very Preliminary

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Abstract

This paper analyzes the incentives of the online platform such as Netflix and Spotify to bias its recommendation system. The platform has its own content and also rents content from another content provider for a fee. In the static setting the platform has no incentives to bias since the fee is fixed ex ante. In the dynamic setting, however, past consumers' experience affects their willingness to pay for both contents and hence, it influences the bargaining between the platform and the content provider. We also provide a version of the model where consumers do not pay for the main service which can be applied to the recent Google shopping case.

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1 Introduction

Recommendation algorithm is at the heart of the many online platforms who deals with a large amount of information or content or a large number of products. Google uses its recommendation algorithm to provide search results from 30 trillions web pages, Amazon uses its own algorithm to provide search results from 480 million products sold on its platform, Facebook uses newsfeed algorithm to select news from news shared by 2 billion users, Netflix uses its algorithm to recommend content from about 4000 movies and 2000 TV shows.

These algorithms are very powerful. A large majority people in many countries rely on Google's search algorithm to process information available on the Internet. Google's market shares in general search in most European and Latin American countries are about 90% and its market share in U.S. is between 60% and 70%. According to Pew Research Center (2016), two-thirds of Facebook users (i.e., 66% of US adults) get news from Facebook, whose newsfeed algorithm received a lot of criticism after the last presidential election in U.S. for its role in incentivizing people to produce and disseminate fake news, which are believed by many to have contributed to the election of Trump.

In this paper, we are interested in revisiting a classic question in competition policy from an angle of recommendation algorithm: does a vertically integrated firm or platform have an incentive to discriminate against its (upstream or downstream) rivals by manipulating its algorithm? Concerns about discrimination against the rival long distance call companies by the AT&T, which was vertically integrated into local call monopolies, led to its divestiture. Recently, the European Commission fined Google €2.42 billion for breaching EU antitrust rules. Bias against competitors in Google's algorithm is the core issue in the case. According to the European Commission (2017), Google has abused its market dominance as a search engine by giving an illegal advantage to another Google product, its comparison shopping service. On the one hand, "Google has systematically given prominent placement to its own comparison shopping service...". "They (Google's comparison shopping results) are placed above the results that Google's generic search algorithms consider most relevant." "On the other hand, rival comparison shopping services are subject to Google's generic search algorithms, including demotions...".

We consider two different platform business models. In the baseline model, the vertically integrated firm distributes content by selling subscription to consumers. This model can be applied to streaming sites such as Netflix but also to vertically integrated cable TV networks such as Comcast. Later, we provide an extension which can be applied to the Google shopping case.

Recommendations algorithms are opaque to outsiders: it is hard for them to know how algorithms function. According to the European Competition Commissioner Vestager (2017), "The trouble is, it's not easy to know exactly how those algorithms work. How they've decided what to show us, and what to hide."

We model such opaque nature of algorithms by assuming that algorithm is not contractible and hence chosen by its owner to maximize her payoff ex post after contracting stage.

In the baseline model, the vertically integrated platform (i.e. firm A) negotiates a fixed fee with firm B in order to distribute the latter's content. After the negotiation is done, firm A chooses the subscription price and consumers make subscription decisions after forming expectation about the bias in the recommendation algorithm of firm A. Finally, firm A chooses the level of bias in the algorithm. In the benchmark of a static model, we find that the platform has no strict incentive to introduce any bias in the algorithm as it has no effect on its payoff. However, when we consider a repetition of the model, we find that the platform has *a generic incentive to bias its algorithm* in that when variances of random terms are symmetric across content A and content B, the platform always has an incentive to bias the algorithm.

Our main focus is about how dynamic negotiation of the fixed fee affects the incentive to bias the algorithm. For this purpose, we consider a two-period repetition of the static model. Of course, in the second period, the platform has no strict incentive to bias the algorithm and hence we focus on the equilibrium with zero bias in the second period. However, in the first period, the platform very often has an incentive to bias the algorithm against firm B not because it affects its first period payoff but because it affects its second period payoff. More precisely, we assume that consumer surplus from each content is a random variable, which is composed of a random draw, which is constant across times after the first draw, and a period-specific noise. The bias increases the surplus from content A and decreases the surplus from content B although the total surplus is maximized at zero bias. Therefore, when the platform increases the bias, it induces consumers to believe that content A generates more surplus whereas content B generates less surplus. Hence, a positive bias increases firm A's disagreement payoff in period two at the same time reduces consumers' total willingness to pay for both content in period two. When firm B has some bargaining power, the platform typically has an incentive to introduce a bias. However, this bias is self-defeating as the platform's overall payoff in the equilibrium is lower than in the ideal case in which it can commit to no bias.

We also perform comparative statics. The amount of bias decreases with firm A's bargaining power and the variance of the noise in the surplus from A's content while increasing with the variance of the noise in the surplus from B's content. As A's bargaining power increases, it internalizes more the decrease in consumers' total willingness to pay. As the variance of the noise in the surplus from A's content increases, the bias is less effective in improving consumers' expectation about the surplus. We also find that the incentive to bias decreases with the degree of correlation between the surplus from content A and the surplus from content B.

Relation to prior literature

The closest paper is Bourreau, Doğan and Gaudin (2017). They analyze a static Hotelling-type model where the payment of the platform to the content provider is proportional to the consumption of the latter's content. This introduces the incentive for the platform to bias its recommendation towards its own content.

In terms of a general motivation, our paper is related to the literature on biased advice by the intermediaries. For example, in Hagiu and Jullien (2011) and Hagiu and Jullien (2014) the intermediary may divert consumer search while in Inderst and Ottaviani (2009) the intermediary may not recommend the best financial product. However, these papers are static and none of them looks at consumers' learning about their utility.

Finally, the model formally is a version of a career concerns model introduced by Holmström (1999).

The rest of the paper is organized as follows. Section 2 introduces the model and Section 3 solves the benchmark case with one period. The main model with two periods is analyzed in Section 4. Section 5 provides comparative statics results and introduces the correlation in the consumers' utilities from the two contents. Section 6 describes a version of the model that fits the Google shopping case. Section 7 concludes. All missing proofs are contained in the Appendix.

2 The baseline model

There are two risk-neutral firms: firm A and firm B. Each firm has its own content. In addition, firm A is vertically integrated and is a monopoly in the distribution of the content. There is a mass one of risk-neutral homogenous consumers. We consider a two-period model: let $t = 1, 2$ represent the time period.

To guide consumers' consumption of content, firm A uses an algorithm. The algorithm can be biased against firm B's content. Let $b_t \geq 0$ capture the degree of bias in the algorithm used in period t: if $b_t = 0$, the algorithm is unbiased and if

$b_t > 0$, it is biased against firm B. Conditional on that a consumer consumes both content, let $u_t^i(b_t)$ represent the surplus a consumer obtains from firm i's content for $i = A, B$ for period t. Let u_t^S represent the surplus a consumer obtains from firm A's content when she consumes solely A's content where the superscript S means "single". We assume that each realized surplus is a random variable such that for each $t = 1, 2$

$$\begin{aligned} u_t^A(b_t) &= u^A + B(b_t) + \varepsilon_t^A; \\ u_t^B(b_t) &= u^B - C(b_t) + \varepsilon_t^B; \\ u_t^S &= u^A + \Delta^S + \varepsilon_t^A \end{aligned}$$

where ε_t^i is independently (across $i = A, B$ and $t = 1, 2$) and identically distributed with a normal distribution with zero mean and variance equal to $(\sigma_\varepsilon^i)^2$. In addition, we assume that u^i is independently distributed across $i = A, B$ according to a normal distribution with mean μ^i and variance $\left(\sigma_\mu^i\right)^2$ for $i = A, B$. Let Δ^S be known. The sign of Δ^S depends on whether content B is a substitute or a complement to content A: $\Delta^S > 0$ if they are substitutes and $\Delta^S < 0$ if they are complements.

Regarding the benefit $B(b_t)$ and cost $C(b_t)$ of bias, we assume that the benefit is increasing and strictly concave and the cost is increasing and strictly convex. In addition, we assume the following.

Assumption 1 *$B(b_t) \leq C(b_t)$ for $b_t \geq 0$, $B(0) = C(0)$ and $B'(0) = C'(0)$.*

Assumption 1 implies that the total surplus (i.e. social welfare) $u_t^A(b_t) + u_t^B(b_t)$ is maximized at $b_t = 0$. When we need to compute the exact value of the bias, we will use the following specification:

$$B(b_t) = b_t, C(b_t) = b_t + \frac{b_t^2}{2}$$

We assume that at the end of period t, both firms and the market (including consumers) observe the realized $\left(u_t^A(b_t), u_t^B(b_t)\right)$ (respectively, u_t^S) if consumers consumed both content (respectively, content A only). However, we assume that $\left(u_t^A(b_t), u_t^B(b_t), u_t^S\right)$ is not contractible, which is a standard assumption in the career concern literature (Holmström (1999)).

In the beginning of each period, the two firms bargain over the fixed fee that firm A should pay to firm B in order to be able to distribute the latter's content. We consider the following stage game: In each period $t = 1, 2$

Stage 1 Firm A and firm B bargain over the price f_t that firm A should pay to firm B in order to distribute the latter's content in period t. If the bargaining fails, firm A distributes only its own content. Let $\beta \in (0,1)$ represent A's bargaining power.

Stage 2 Firm A sets the price p_t for consumer subscription. Consumers decide whether to subscribe or not to firm A.

Stage 3 Firm A chooses the bias b_t.

If there are multiple equilibria, we focus on the one that maximizes firm A's profit. Since the support of the normal distribution is unbounded it is possible that it is optimal not to show some content in period 2. By making the mean-to-variance ratio high enough, this possibility can be made arbitrarily small and hence, be ignored. This is standard in the literature; say, neither the original paper by Holmström (1999) nor its textbook exposition Bolton and Dewatripont (2005) does even mention this issue.

3 Benchmark: one period

Before analyzing the two-period model, let us analyze as a benchmark a single-period model. For simplicity, we do not write the time index. Then, at stage 3, Firm A's profit is equal to $p - f$. Hence, it is indifferent between any biases.

Let b^e be the bias expected by the firm B and the consumers. We study the equilibrium in which firm A indeed chooses $b = b^e$ and then study which b^e maximizes its profit. In equilibrium with $b = b^e$, firm A chooses p as follows

$$p = \mu^A + \mu^B + B(b) - C(b)$$

Firm A pays the following fee to B:

$$f = (1-\beta)\left(\mu^A + \mu^B + B(b) - C(b) - \mu^A - \Delta^S\right).$$

Hence, firm A's payoff is

$$p - f = \beta\left[\mu^A + \mu^B + B(b) - C(b)\right] + (1-\beta)(\mu^A + \Delta^S).$$

This payoff is maximized at $b = 0$.

Proposition 1 *In a one-period model, there is an equilibrium without bias. The payoff of firm A is higher in this equilibrium than in any other equilibrium with a positive bias.*

The proposition shows that if there is any bias in the algorithm, it must have to do with dynamics of bargaining.

4 Analysis of the baseline model

4.1 The second period

Suppose that $\left(u_1^A(b_1), u_1^B(b_1)\right)$ is realized. Let b_1^e denote the bias that the market (and firm B) expected A to choose at period one. Then, the updated expectation of the market about u^i is determined as follows:

Lemma 1 *Given $\left(u_1^A(b_1), u_1^B(b_1)\right)$ and b_1^e, the updated expectation of the market about u^i is determined as follows*

$$E_2\left[u^A\right] \equiv E\left[u^A \left| u_1^A(b_1), b_1^e\right.\right] = \mu^A \frac{\left(\sigma_\varepsilon^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2} + \left(u_1^A(b_1) - B(b_1^e)\right)\frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2}, \tag{1}$$

$$E_2\left[u^B\right] \equiv E\left[u^B \left| u_1^B(b_1), b_1^e\right.\right] = \mu^B \frac{\left(\sigma_\varepsilon^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2} + \left(u_1^B(b_1) + C(b_1^e)\right)\frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2}. \tag{2}$$

$$E_2\left[u^S\right] \equiv E\left[u^S \left| u_1^A(b_1), b_1^e\right.\right] = E\left[u^A \left| u_1^A(b_1), b_1^e\right.\right] + \Delta^S. \tag{3}$$

The updating formulae (1)-(2) are standard for the normal distribution. Indeed, the posterior is equal to a combination of the prior mean and the signal corrected according to the market expectation b_1^e with the weights equal to their relative precisions (inverse of variances), $\frac{1}{\left(\sigma_\mu^i\right)^2}/(\frac{1}{\left(\sigma_\mu^i\right)^2} + \frac{1}{(\sigma_\varepsilon^i)^2})$ and $\frac{1}{(\sigma_\varepsilon^i)^2}/(\frac{1}{\left(\sigma_\mu^i\right)^2} + \frac{1}{(\sigma_\varepsilon^i)^2})$, $i = A, B$, respectively.

Consider period 2. At stage 3, given f_2, p_2, from Proposition 1, firm A has no strict incentive to bias its algorithm. Hence, we focus on the equilibrium in which $b_2^e = 0$. Then,

$$p_2 = E_2\left[u^A\right] + E_2\left[u^B\right]$$

and

$$f_2 = (1-\beta)\left[E_2\left[u^B\right] - \Delta^S\right].$$

Hence, A's payoff in period 2 is given by

$$\pi_2(b_1, b_1^e) = p_2 - f_2 = E_2\left[u^A\right] + \beta E_2\left[u^B\right] + (1-\beta)\Delta^S.$$

4.2 The first period

Consider now period 1. At stage 3, given f_1, p_1, b_1^e, $b_2^e = 0$, firm A chooses b_1 to maximize $\pi_2(b_1, b_1^e)$, which is given by (up to a constant)

$$\begin{aligned}\max_{b_1} \pi_2(b_1, b_1^e) &= \mu^A \frac{\left(\sigma_\varepsilon^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2} + \left(\mu^A + B(b_1) - B(b_1^e)\right) \frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2} \\ &\quad + \beta \left[\mu^B \frac{\left(\sigma_\varepsilon^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2} + \left(\mu^B - C(b_1) + C(b_1^e)\right) \frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2} \right] + (1-\beta)\Delta^S.\end{aligned}$$

In other words, firm A chooses b_1 to maximize

$$\max_{b_1} \frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2} B(b_1) - \beta \frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2} C(b_1). \tag{4}$$

Let $\overline{\beta} \in (0, 1]$ be defined as

$$\overline{\beta} = \min \left\{ 1, \frac{\frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2}}{\frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2}} \right\} \tag{5}$$

For $\beta < \overline{\beta}$, let $b_1^*(\beta)$ be the solution of

$$\frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2} B'(b_1) = \beta \frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2} C'(b_1). \tag{6}$$

Hence, for any $\beta \in [0, \overline{\beta})$, $b_1^*(\beta) > 0$. In equilibrium, the market correctly anticipates $b_1^e = b_1^*$. In particular, in the symmetric case, $\sigma_\mu^A = \sigma_\mu^B$ and $\sigma_\varepsilon^A = \sigma_\varepsilon^B$, we have $\overline{\beta} = 1$: hence, there is a positive bias for any $\beta < 1$. This leads to the main result from the baseline model.

Proposition 2 *In the two-period model there are multiple equilibria which differ only with respect to the bias in period 2. In particular,*

(i) In any equilibrium, the first-period bias $b_1^*(\beta)$ is strictly positive for $\beta < \overline{\beta}$ and zero for $\beta \geq \overline{\beta}$. In the symmetric case, $\sigma_\mu^A = \sigma_\mu^B$ and $\sigma_\varepsilon^A = \sigma_\varepsilon^B$, the first-period bias $b_1^*(\beta)$ is strictly positive for any $\beta < 1$.

(ii) There is an equilibrium with zero bias in period 2. The payoff of firm A is higher in this equilibrium than in any other equilibrium.

We provide the intuition for the bias below. On the one hand, a positive bias b_1 increases, from (1) and (3), both $E_2\left[u^A\right]$ and $E_2\left[u^S\right]$ by $\frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2} B(b_1)$. In other words, it increases the expected value of content A, which means that it increases firm A's disagreement payoff. On the other hand, a positive bias b_1 decreases, from (2), $E_2\left[u^B\right]$ by $\frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2+\left(\sigma_\varepsilon^B\right)^2}C\left(b_1\right)$. In other words, it decreases the expected value of content B, which means a lower subscription price and a lower fixed fee paid by A to B. Firm A internalizes only β fraction of this lower surplus. This is why in the end A chooses the bias to maximize (4).

The prices in period 1 are given by

$$p_1 = \mu^A + \mu^B + B(b_1^*) - C\left(b_1^*\right);$$

$$f_1 = (1-\beta)\left[\mu^B + B(b_1^*) - C\left(b_1^*\right) - \Delta^S\right].$$

Therefore, firm A's first-period payoff is

$$\pi_1 = \mu^A + \beta\left[\mu^B + B(b_1^*) - C\left(b_1^*\right)\right] + (1-\beta)\Delta^S.$$

In the second period there is no bias and hence

$$\pi_2 = \mu^A + \beta\mu^B + (1-\beta)\Delta^S.$$

The overall payoff of firm A is

$$\Pi = \pi_1 + \delta\pi_2 = (1+\delta)\left[\mu^A + \beta\mu^B + (1-\beta)\Delta^S\right] + \beta\left[B(b_1^*) - C\left(b_1^*\right)\right].$$

where $\delta > 0$ is the discount factor. Hence, in the end, the bias hurts firm A since $B(b_1^*) - C\left(b_1^*\right) < 0$ for any $b_1^* > 0$. If firm A can commit to no bias, it wants to do that.

5 Comparative statics

5.1 Bargaining power and firm specific variances

From (4), we find:

Proposition 3 *The period-one equilibrium bias decreases with β, $\left(\sigma_\mu^B\right)^2$ and $\left(\sigma_\varepsilon^A\right)^2$ and increases with $\left(\sigma_\mu^A\right)^2$ and $\left(\sigma_\varepsilon^B\right)^2$.*

The bias decreases with firm A's bargaining power since firm A's internalizes more the loss in surplus resulting from the bias as its bargaining power increases.

In addition, the bias is an increasing function of $\frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2+\left(\sigma_\varepsilon^A\right)^2}$ since the observation u_1^A is more informative about μ^A as the ratio increases. For the same reason, the bias is a decreasing function of $\frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2+\left(\sigma_\varepsilon^B\right)^2}$.

For instance, when $B(b_t)$ and $C(b_t)$ are given as follows,

$$B(b_t) = b_t, C(b_t) = b_t + \frac{b_t^2}{2}, \tag{7}$$

we have for any $\beta \in [0, \overline{\beta})$

$$b_1^*(\beta) = \frac{\frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2+\left(\sigma_\varepsilon^A\right)^2} - \beta\frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2+\left(\sigma_\varepsilon^B\right)^2}}{\beta\frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2+\left(\sigma_\varepsilon^B\right)^2}} > 0.$$

5.2 Correlated utility from content

Suppose now that μ^A and μ^B are correlated with the correlation coefficient ρ. For example, sci-fi movies typically require high budgets and hence, only a handful of big studios can produce them while family and romance movies can be produced by independent directors and small studios. Watching an independent movie does not say anything about the next one while sci-fi movies are often quite predictable. As a result, the correlation will be higher for the funs of sci-fi movies than for stay-at-home mums. Note that the correlation does not imply anything about the substitutability or complementarity between the contents Δ^S since it does not change the expected utility from consuming them.

Consider the linear-quadratic specification in (7). Now, both u_1^A and u_1^B are used to update each of u^A and u^B. The posterior (1) becomes

$$\begin{aligned} E\left[u^A \left| u_1^A(b_1), u_1^B(b_1), b_1^e\right.\right] &= \mu^A + \frac{1+\frac{\left(\sigma_\varepsilon^B\right)^2}{\left(\sigma_\mu^B\right)^2} - \rho^2}{\left(1+\frac{(\sigma_\varepsilon^B)^2}{\left(\sigma_\mu^B\right)^2}\right)\left(1+\frac{(\sigma_\varepsilon^A)^2}{\left(\sigma_\mu^A\right)^2}\right) - \rho^2}\left(u_1^A(b_1) - b_1^e - \mu^A\right) \\ &\quad + \frac{\rho\frac{\left(\sigma_\varepsilon^A\right)^2}{\sigma_\mu^A\sigma_\mu^B}}{\left(1+\frac{(\sigma_\varepsilon^B)^2}{\left(\sigma_\mu^B\right)^2}\right)\left(1+\frac{(\sigma_\varepsilon^A)^2}{\left(\sigma_\mu^A\right)^2}\right) - \rho^2}\left(u_1^B(b_1) + b_1^e + \frac{\left(b_1^e\right)^2}{2} - \mu^B\right) \end{aligned} \tag{8}$$

The following proposition establishes the effect of the correlation in the symmetric case, $\sigma_\mu^A = \sigma_\mu^B = \sigma_\mu$ and $\sigma_\varepsilon^A = \sigma_\varepsilon^B = \sigma_\varepsilon$, which simplifies the expressions a lot.

Proposition 4 *In the symmetric case, $\sigma_\mu^A = \sigma_\mu^B = \sigma_\mu$ and $\sigma_\varepsilon^A = \sigma_\varepsilon^B = \sigma_\varepsilon$, the first-period bias equilibrium bias is equal to*

$$b_1 = \frac{1 + \frac{\sigma_\varepsilon^2}{\sigma_\mu^2} - \rho^2 + \frac{1}{2}\rho\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}}{\rho\frac{\sigma_\varepsilon^2}{\sigma_\mu^2} + \frac{1}{2}\left(1 + \frac{\sigma_\varepsilon^2}{\sigma_\mu^2} - \rho^2\right)} - 1.$$

It decreases with the correlation coefficient ρ.

Proof. See Appendix. ■

The intuition why a higher correlation decreases the equilibrium bias is the following. A higher value of u_1^A increases the expectation of u^B increasing the fee paid to firm B which makes bias b_1 less attractive for firm A. Also, a higher value of u_1^B increases the expectation of u^A improving the profits of firm A. Both effects are stronger when the correlation is higher. When $\rho = 1$, effects of b_1 on u^A and u^B are exactly the same making firm A effectively maximize the total surplus and hence yielding optimal $b_1 = 0$. When the contents are independent, $\rho = 0$, then optimal b_1 is 1. When the correlation is negative, a higher b_1 leads to a lower u_1^B which increases the expectation of u^A. When it is negative enough, the last effect becomes so strong that b_1 stops being costly and firm A wants to introduce infinite bias. Formally this means that the second-order condition of firm A's problem is not satisfied for ρ close to -1 (see the proof of Proposition 4 for details).

6 Extension to a different business model

In the Google shopping case, both Google and the competing comparison sites provide the service for free to consumers. They make money from merchants from which consumers buy the products. We extend here the baseline model to such a business model.

6.1 Model

Consider two firms, A and B. Firm A is vertically integrated with a general search service while firm B is a competing comparison site. Assume that A is monopoly in general search service: we do not explicitly model the surplus from the general search but the fact A is monopoly in general search gives A power to manipulate the algorithm displaying search results for the comparison sites. We assume that the comparison shopping service is provided for free to consumers who are homogenous in terms of the utility they obtain from the service up to a constant, which represents a random shock in terms of cost of participation.

As in the baseline model, we consider a two-period model. Conditional on that A shows both firms' comparison service, $\left(R_t^A(b_t), R_t^B(b_t), u_t^C(b_t)\right)$ represent A's revenue per consumer, B's revenue per consumer and consumer surplus from comparison shopping service, which depend on the bias $b_t \geq 0$ chosen by A. We assume

$$\begin{aligned} R_t^A(b_t) &= R^A + B(b_t) + \varepsilon_t^A, \\ R_t^B(b_t) &= R^B - C^B(b_t) + \varepsilon_t^B, \\ u_t^C(b_t) &= u^C - C^C(b_t) + \varepsilon_t^C, \end{aligned}$$

where ε_t^i is independently (across $i = A, B, C$ and $t = 1, 2$) and identically distributed with a normal distribution with zero mean and variance $(\sigma_\varepsilon^i)^2$. We assume a positive bias increases the revenue of A but decreases both B's revenue and consumer surplus.

If A shows only its own service, then we have

$$\begin{aligned} R_t^S(b_t) &= R^A + \Delta^A + \varepsilon_t^A, \\ u_t^S(b_t) &= u^C - \Delta^C + \varepsilon_t^C, \end{aligned}$$

where we assume $\Delta^A \gtrless 0$ and $\Delta^C > 0$. Let $n(u)$ be the number of consumers who use the service, which is assumed to be an increasing function of u. In addition, we assume that each of (R^A, R^B, u^C) is independently distributed across $i = A, B, C$ according to the normal distribution with mean μ^i and variance $\left(\sigma_\mu^i\right)^2$ for $i = A, B, C$.

Regarding the benefit $B(b)$ and cost $C^i(b)$ of bias (for $i = B, C$), we assume the following.

Assumption 2 *(i) $B(b) < C^B(b) + C^C(b)$ for $b \geq 0$, $B(0) = C^B(0) = C^C(0)$ and $B'(0) = C'^B(0) + C'^C(0) > 0$ where $C'^B(0) > 0$ and $C'^C(0) > 0$.*

(ii) $B(b)$ is strictly increasing and strictly concave and each cost function is strictly increasing and strictly convex.

The expected welfare in a given period is given by

$$W = n(\mu^C - C^C(b))\left[R^A + R^B + u^C + B(b) - C^B(b) - C^C(b)\right]. \tag{9}$$

Assumption 2 implies that the second term in (9) is maximized at $b = 0$. Hence, $\frac{\partial W}{\partial b}\mid_{b=0} < 0$ and $b = 0$ is the welfare-maximizing bias.

Each period $t = 1, 2$, the two firms engage in a two-part tariff bargaining where the two-part tariff is composed of a fixed fee $f_t \in R$ and a per-click fee w_t that firm B

pays to A. We consider a reduced form approach to w_t such that in a static one-period model, the bias chosen by firm A strictly decreases with w_t. In particular, as an upper bound on w_t, we assume that it is possible for both firms to agree on a short-term contract that makes firm A a residual claimant of $R_t^B(b_t)$: this is like renting B's shopping service to A. Such benchmark allows us to identify the source for the bias. We do not allow for a long-term contract making firm A residual claimant since such contract is equivalent to full vertical integration in which firm A owns B's shopping service. However, we allow for a succession of such short-term contracts in which the fixed fee in period two f_2 is negotiated in the end of period one.

The stage game in each period is:

Stage 1 Firm A and firm B bargain over the fixed price f_t and the per-click fee w_t the firm B should pay to firm A such that firm A makes B's service available to consumer. If the bargaining fails, firm 1 shows only its service to consumers. Let $\beta \in (0,1)$ represent A's bargaining power.

Stage 2 Consumers form expectations about the bias and make participation decision.

Stage 3 Firm A chooses the bias b_t.

6.2 Benchmark of one period

6.2.1 Vertical Integration

Consider as the benchmark the case in which firm A owns both comparison shopping services.

First, consider the case in which firm A can commit to b before consumers make participation decision. Then, firm A chooses the bias to maximize

$$E\pi^{VI} = n(\mu^C - C^C(b))\left[\mu^A + \mu^B + B(b) - C^B(b)\right].$$

As compared to the welfare in (9), firm A does not internalize the negative effect of the bias $C^C(b)$ on consumers that already participate and will then introduce a higher bias than socially optimal.

The first derivative with respect to b is

$$\begin{aligned}\frac{\partial E\pi^{VI}}{\partial b} &= n(\mu^C - C^C(b))\left[B'(b) - C'^B(b)\right] - \\ &\quad n'(\mu^C - C^C(b))C'^C(b)\left[\mu^A + \mu^B + B(b) - C^B(b)\right]. \end{aligned} \tag{10}$$

The vertically integrated monopolist compares the gain from the bias for a given measure of participating consumers with the loss from the bias resulting from a lower participation. When evaluated at $b = 0$ using $B'(0) = C'^B(0) - C'^C(0)$ from Assumption 2, (10) becomes

$$\frac{\partial E\pi^{VI}}{\partial b} |_{b=0} = n(\mu^C) - n'(\mu^C) \left[\mu^A + \mu^B\right].$$

Define the elasticity of participation as

$$\varepsilon_n(u) = \frac{\frac{dn}{du}}{\frac{n}{u}}.$$

If

$$\frac{\mu^C}{\mu^A + \mu^B} > \varepsilon_n(\mu^C), \tag{11}$$

then $\frac{\partial E\pi^{VI}}{\partial b} |_{b=0} > 0$ and hence, the vertically integrated monopolist has an incentive to introduce a bias against consumers.

The elasticity $\varepsilon_n(u)$ is likely to be very low. Most people use Google to search for whatever information on the net and search for products is just a minor part of such search. Although $n(u)$ is the number of consumers who use the service to search for products, it is hard to separate this participation from the general use of Google search. Therefore, even if Google introduces some bias in the algorithm affecting search results related to comparison websites, this is likely to have a very little impact on $n(u)$. In that case, the fully integrated platform will always introduce some bias.

Suppose now that the firm cannot commit to the bias and it chooses it after consumer participation decisions are made. Then, it chooses the bias to maximize

$$\mu^A + \mu^B + B(b) - C^B(b)$$

ignoring the negative effect $C^C(b_t)$ on the consumers both inframarginal (as in the commitment case) and marginal ones.

Proposition 5 *Consider a one-period model.*

(i) If the fully integrated monopolist can commit to the bias, it introduces a positive bias if and only if $\frac{\mu^C}{\mu^A+\mu^B} > \varepsilon_n(\mu^C)$. *Then, the equilibrium bias* b^{VI} *is determined by the solution of (10).*

(ii) If the fully integrated monopolist cannot commit to the bias, it always introduces a positive bias which is the solution of

$$B'(b) = C'^B(b). \tag{12}$$

In a two-sided market in which the platform generates a relatively large surplus on the merchants' side with respect to consumer side, $\frac{\mu^C}{\mu^A+\mu^B}$ is likely to be small. This can justify the platform's practice to charge zero price on the consumer side to boost consumer participation. In that case, $\frac{\mu^C}{\mu^A+\mu^B} > \varepsilon_n(\mu^C)$ is unlikely to be met. However, if the platform cannot commit to the algorithm, which is our main scenario, even a fully integrated platform has an incentive to create a bias.

6.2.2 Vertical separation

Now we consider the model in which B is separated from A. As it is obvious that in the commitment case, the efficient bargaining leads to the choice of the bias that takes place under vertical integration, we focus on the no-commitment case.

If the two firms can agree on a per-click fee w, let $c(b, w)$ represent the the opportunity cost of firm A from introducing a bias b. The opportunity cost represents the loss of revenue from per-click fee and hence increases with b and w. Then, firm A chooses the bias to maximize

$$B(b) - c(b, w),$$

of which the first-order condition is

$$B'(b) = \frac{\partial c(b, w)}{\partial b}.$$

We assume that $c(b, w)$ is convex in b and $\frac{\partial c(b,w)}{\partial b}$ increases with w. Hence, the bias chosen $b^{VS}(w)$ decreases with w. The two firms can agree on some w which leads to $b^{VS}(w^{VI}) = b^{VI}$, the level chosen by the fully integrated firm in the commitment case. Then, at the negotiation stage, they will agree to some fixed fee f and share the joint surplus.

Even if consumers may not observe w but form an expectation about it, which is correct in the equilibrium as we assume about the bias in the baseline model.

In what follows, we assume that $n(u)$ is a constant normalized to one. This implies that $b^{VI} > 0$ is found from (12) and there is no difference between commitment and no-commitment equilibria. We say that w^{VI} makes firm A residual claimant of firm B in the sense that it induces the former to behave as a fully integrated firm.

6.3 Two-period model

In this section, we analyze the two-period model. It is straightforward to see that when $n(u)$ is a constant as long as both services are provided by firm A, both firms will agree on $w_2 = w^{VI}$ in order to induce firm A to choose b^{VI}. As b^{VI} is determined by (12), this choice does not depend on the realized expectations of $E_2\left[R^A\right], E_2\left[R^B\right]$ and $E_2\left[u^C\right]$.

At the end of period 1, given $R_1^A(b_1), R_1^B(b_1), u_1^C(b_1)$, the posteriors are given by

$$\begin{aligned}
E_2\left[R^A\right] &= E\left[R^A \left| R_1^A(b_1), b_1^e\right.\right] = R^A \frac{\left(\sigma_\varepsilon^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2} + \left(R_1^A(b_1) - B(b_1^e)\right)\frac{\left(\sigma_\mu^A\right)^2}{\left(\sigma_\mu^A\right)^2 + \left(\sigma_\varepsilon^A\right)^2}, \\
E_2\left[R^B\right] &= E\left[R^B \left| R_1^B(b_1), b_1^e\right.\right] = R^B \frac{\left(\sigma_\varepsilon^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2} + \left(R_1^B(b_1) + C^B(b_1^e)\right)\frac{\left(\sigma_\mu^B\right)^2}{\left(\sigma_\mu^B\right)^2 + \left(\sigma_\varepsilon^B\right)^2}, \\
E_2\left[u^C\right] &= E\left[u^C \left| u_1^C(b_1), b_1^e\right.\right] = u^C \frac{\left(\sigma_\varepsilon^C\right)^2}{\left(\sigma_\mu^C\right)^2 + \left(\sigma_\varepsilon^C\right)^2} + \left(u_1^C(b_1) + C^C(b_1^e)\right)\frac{\left(\sigma_\mu^C\right)^2}{\left(\sigma_\mu^C\right)^2 + \left(\sigma_\varepsilon^C\right)^2}.
\end{aligned}$$

Consider the first period and suppose that the two firms agreed to $w_1 = w^{VI}$. Will it lead firm A to choose $b_1 = b^{IV}$? By definition of w^{IV}, this will lead to the choice of b^{VI} if there is no second period. But b_1 also affects firm A's second-period payoff.

Firm A's payoff at the time it chooses b_1 is (up to a constant)

$$\begin{aligned}
\Pi &= E_1\left[R^A\right] + B(b_1) - c(b_1, w^{IV}) + f_1 \\
&\quad + \delta E_1\left\{\left[\beta\left[E_2\left[R^A\right] + B(b^{VI}) + E_2\left[R^B\right] - C^B(b^{VI})\right] + (1-\beta)(E_2\left[R^A\right] + B(b^{VI}) + \Delta^A)\right]\right\}.
\end{aligned}$$

We consider the symmetric case, $\sigma_\varepsilon^i = \sigma_\varepsilon$ and $\sigma_\mu^i = \sigma_\mu$ for $i =$ A, B, C, and focus on the second-period payoff in Π. Let

$$\phi = \frac{\left(\sigma_\mu\right)^2}{\left(\sigma_\mu\right)^2 + \left(\sigma_\varepsilon\right)^2}.$$

Then, the expected value of A's second-period payoff is given by

$$\begin{aligned}
&\beta\left[\mu^A + (B(b_1) - B(b_1^e))\phi + B(b^{VI}) + \mu^B - (C^B(b_1) - C^B(b_1^e))\phi - C^B(b^{VI})\right] \\
&+(1-\beta)(\mu^A + (B(b_1) - B(b_1^e))\phi + B(b^{VI}) + \Delta^A).
\end{aligned}$$

The first-order condition is

$$\begin{aligned}
\frac{\partial \Pi}{\partial b_1} &= B'(b_1) - \frac{\partial c(b_1, w^{IV})}{\partial b} \\
&\quad + \delta\beta\left[B'(b_1) - C^{B\prime}(b_1)\right]\phi \\
&\quad + \delta(1-\beta)B'(b_1)\phi \\
&= 0
\end{aligned}$$

Therefore, we find that firm A chooses $b_1 > b^{VI}$ for any $\beta < 1$ and that firm A obviously chooses $b_1 = b^{VI}$ for $\beta = 1$.

The difference between this extension and the baseline model is that firm A has a strong incentive to bias the algorithm even in a static model. Therefore, we allowed the firms to use a two-part tariff contract in order to incentivize firm A to reduce the bias. In particular, the per-click fee which makes firm A a residual claimant will make firm A to behave like the vertically integrated firm. However, in a dynamic setting, even if the firms repeatedly sign a one-period contract that makes firm A a residual claimant, firm A has an incentive to bias the algorithm for a dynamic reason that we uncovered in the baseline model. Namely, the bias increases firm A's disagreement payoff by inducing consumers to believe that firm A's service better and therefore allow firm A's to capture a larger surplus when it bargains with firm B in second period.

Summarizing, we have:

Proposition 6 *Under Assumption 2 and when $n(u)$ is a constant,*

(i) A vertically integrated firm chooses the same bias $b^{VI} > 0$ in both periods.

(ii) When the two firms are separated,

a) In the second period, making firm A residual claimant of the revenue from firm B induces it to choose $b_2 = b^{VI}$.

b) In the first period, making firm A residual claimant of the revenue from firm B induces it to choose $b_1 > b^{VI}$.

In the Google shopping case, search results for the competing comparison sites were provided in the organic search. In that case, Google has a strong incentive to bias its algorithm. We envisioned a scenario in which Google and its competing comparison website engage in frictionless two-part tariff bargaining. We found that even if the first-period contract makes Google a residual claimant of the revenue of the competitor and Google anticipates that they will also sign a second-period contract which makes it a residual claimant again, Google has an incentive to bias algorithm above the level chosen by an integrated structure in first period.

7 Conclusion

References

Bolton, Patrick, and Mathias Dewatripont. 2005. *Contract Theory.* Cambridge:MIT Press.

Bourreau, Marc, Pinar Doğan, and Germain Gaudin. 2017. "Streaming Platforms and Recommendation Bias." working paper.

European Commission. 2017. "Antitrust: Commission fines Google €2.42 billion for abusing dominance as search engine by giving illegal advantage to own comparison shopping service - Factsheet." 27 June 2017, available at http://europa.eu/rapid/press-release_MEMO-17-1785_en.htm.

Hagiu, Andrei, and Bruno Jullien. 2011. "Why do intermediaries divert search?" *RAND Journal of Economics*, 42(2): 337–362.

Hagiu, Andrei, and Bruno Jullien. 2014. "Search diversion and platform competition." *International Journal of Industrial Organization*, 33(Supplement C): 48–60.

Holmström, Bengt. 1999. "Managerial Incentive Problems: A Dynamic Perspective." *Review of Economic Studies*, 66(1): 169–182.

Inderst, Roman, and Marco Ottaviani. 2009. "Misselling through Agents." *American Economic Review*, 99(3): 883–908.

Vestager, Margrethe. 2017. Speech at the Bundeskartellamt 18th Conference on Competition, Berlin, 16 March 2017, available at https://ec.europa.eu/commission/commissioners/2014-2019/vestager/announcements/bundeskartellamt-18th-conference-competition-berlin-16-march-2017_en.

Appendix

Proof of Proposition 4. Denote the coefficient of $\left(u_1^A\left(b_1\right)-b_1^e-\mu^A\right)$ in (8) by C_{own}^A and that of $\left(u_1^B\left(b_1\right)+b_1^e+\frac{\left(b_1^e\right)^2}{2}-\mu^B\right)$ by C_{other}^A. Similarly, the posterior (2) becomes

$$E\left[u^B\left|u_1^A\left(b_1\right),u_1^B\left(b_1\right),b_1^e\right.\right]=\mu^B+C_{own}^B\left(u_1^B\left(b_1\right)+b_1^e+\frac{\left(b_1^e\right)^2}{2}-\mu^B\right)+C_{other}^B\left(u_1^A\left(b_1\right)-b_1^e-\mu^A\right)$$

Firm A maximizes $\pi_2(b_1,b_1^e)$, which is given by

$$\begin{aligned}=\quad&\mu^A+C_{own}^A\left(\mu^A+b_1-b_1^e-\mu^A\right)+C_{other}^A\left(\mu^B-b_1-\frac{b_1^2}{2}+b_1^e+\frac{\left(b_1^e\right)^2}{2}-\mu^B\right)\\&+\frac{1}{2}\left[\mu^B+C_{own}^B\left(\mu^B-b_1-\frac{b_1^2}{2}+b_1^e+\frac{\left(b_1^e\right)^2}{2}-\mu^B\right)+C_{other}^B\left(\mu^A+b_1-b_1^e-\mu^A\right)+\Delta^S\right].\end{aligned}$$

Hence, firm A chooses b_1 to maximize

$$\max_{b_1}\left(C_{own}^A+\frac{1}{2}C_{other}^B\right)b_1-\left(C_{other}^A+\frac{1}{2}C_{own}^B\right)\left(b_1+\frac{b_1^2}{2}\right)\tag{13}$$

leading to

$$b_1=\frac{C_{own}^A+\frac{1}{2}C_{other}^B}{C_{other}^A+\frac{1}{2}C_{own}^B}-1=\frac{1+\frac{\left(\sigma_\varepsilon^B\right)^2}{\left(\sigma_\mu^B\right)^2}-\rho^2+\frac{1}{2}\rho\frac{\left(\sigma_\varepsilon^B\right)^2}{\sigma_\mu^A\sigma_\mu^B}}{\rho\frac{\left(\sigma_\varepsilon^A\right)^2}{\sigma_\mu^A\sigma_\mu^B}+\frac{1}{2}\left(1+\frac{\left(\sigma_\varepsilon^A\right)^2}{\left(\sigma_\mu^A\right)^2}-\rho^2\right)}-1.$$

In the symmetric case, $\sigma_\mu^A=\sigma_\mu^B=\sigma_\mu$ and $\sigma_\varepsilon^A=\sigma_\varepsilon^B=\sigma_\varepsilon$, this becomes

$$b_1=\frac{1+\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}-\rho^2+\frac{1}{2}\rho\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}}{\rho\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}+\frac{1}{2}\left(1+\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}-\rho^2\right)}-1.$$

Then, $\frac{\partial b_1}{\partial\rho}=-3\sigma_\varepsilon^2\frac{\sigma_\mu^2+\sigma_\varepsilon^2+\rho^2\sigma_\mu^2}{\left(-\sigma_\mu^2-\sigma_\varepsilon^2+\rho^2\sigma_\mu^2-2\rho\sigma_\varepsilon^2\right)^2}<0$.

The second-order condition of firm A's problem (13) is

$$C_{other}^A+\frac{1}{2}C_{own}^B\propto\rho\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}+\frac{1}{2}\left(1+\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}-\rho^2\right)>0$$

giving

$$\rho>\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}-\sqrt{1+\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}+\left(\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}\right)^2}\in(-1,0).$$

■