

An Analysis of the Last Mile Using a Simple Monte Carlo Model¹

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Draft version

Abstract - The paper presents a simulation model with input parameters that include (among others) grid size and shape (e.g., square or rectangular), number of stops, and location of the entry point to the grid. Each intersection on the grid is a possible stop. By altering input parameters the paper explores: the relationship between density and total distance traveled; the effect of clustering stops; the savings in distance traveled from merging two or more routes; the savings in travel distance from e-commerce versus consumers shopping at a store; the effect on the travel distance using different entry points; the effect of one-way streets, the effect of barriers (like rivers, railway tracks, closed streets, etc.) requiring the use of a bridge, a tunnel, or a single street to cross the barrier; and the savings in travel distance by allowing some pieces to be deferred to the next day. ⁶

Key Words – Monte Carlo, delivery, simulation, travel distance, e-commerce

1. Introduction

The paper is organised as follows: Section 2 provides a background of modeling of delivery networks; Section 3 describes the model; Section 4 presents the relationship between the number of delivery points and the distance travelled generated by the model. Section 5 explores the effects of differing variables and scenarios on TDT and ADT; section 6 evaluates the savings from allowing pieces to be deferred; and section 7 provides a conclusion.

¹ The opinions expressed in this paper are those of the authors and not the Postal Regulatory Commission or the Department of Education.

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⁶ The other input parameters are the number of duplicate stops shared by more than one route, the location of a central pickup and delivery point, and the location of a barrier and passageway.

2. Background

Residential parcel delivery had its origins in the U.S. with the development of department stores towards the end of the 19th Century. Shoppers would use public transportation to visit downtown stores and instead of carrying their purchases home, they had them delivered by the store. Many delivery firms were started to serve this market including the now-global United Parcel Service (UPS) which had its origin as a department store delivery firm in Seattle in 1907 as the "American Messenger Company." In 1919 it expanded to Oakland California and changed its name to the United Parcel Company.

In order to participate in the growing parcel market, the U.S. Post Office Department introduced a new service called Parcel Post in 1913. This was at a time when most city delivery postal routes were on foot and so motorized vehicles were used for parcel delivery. While today the overwhelming majority of delivery routes are motorized and deliver both parcels and letters, the Postal Service still relies on parcel routes to deliver many packages.⁷

As the number of parcels grew and especially with their explosive growth as the Internet developed, much attention was given to designing parcel delivery routes so that they were efficient (i.e. tried to keep delivery cost as low as possible.) The so-called vehicle routing problem was initially concerned with developing the shortest route to serve a collection of points. UPS (and assumingly others) have moved away from that simple concept to include other was of optimizing routes. For example, UPS "gives each driver a specific route to follow and that includes a policy that drivers should never turn through oncoming traffic... unless absolutely necessary."⁸ UPS also uses a proprietary system called ORION (On-Road Integrated Optimization and Navigation) to optimize routes based on extensive data analysis.⁹ Amazon has moved from the simplistic concept of minimizing costs as it is common to see more than one Amazon delivery vehicle on the same neighborhood on a given day. We must conclude that

⁷ When this happens, of course, there are few, if any, economies of scope as compared with delivering letters and parcels on the same route.

⁸ Kendall, G. "Why UPS Drivers Don't Turn Left And You Probably Shouldn't Either" www.ge.com, February 6, 2017.

⁹ "How Does USPS Plan Its Delivery Routes" [eLogii blog](https://www.logijournal.com), July 22, 2024.

logistical and service considerations, at least with respect to Amazon, are paramount to simply minimizing last mile delivery costs.

Modeling of delivery networks can be very sophisticated and tailored to specific populations, geography, delivery modes, and products. For example, Arnold, et al. (2018) simulated various delivery scenarios using demand and geographic parameters that are specific to Antwerp; the paper also provided an overview of other studies of the urban logistics and last mile distribution of e-commerce.¹⁰

In contrast, this paper seeks to create a simple model that can be used to measure the effects of a variety of parameters on the distance travelled. Transportation models typically incorporate as many real-world details affecting delivery time and distance travelled as possible, but the intent of the model in this paper is to estimate the effects of parameter changes in a generalized way to illustrate the relative sensitivities of distance travelled within and between each of the scenarios created using different values for the various parameters.

3. Description of the Model

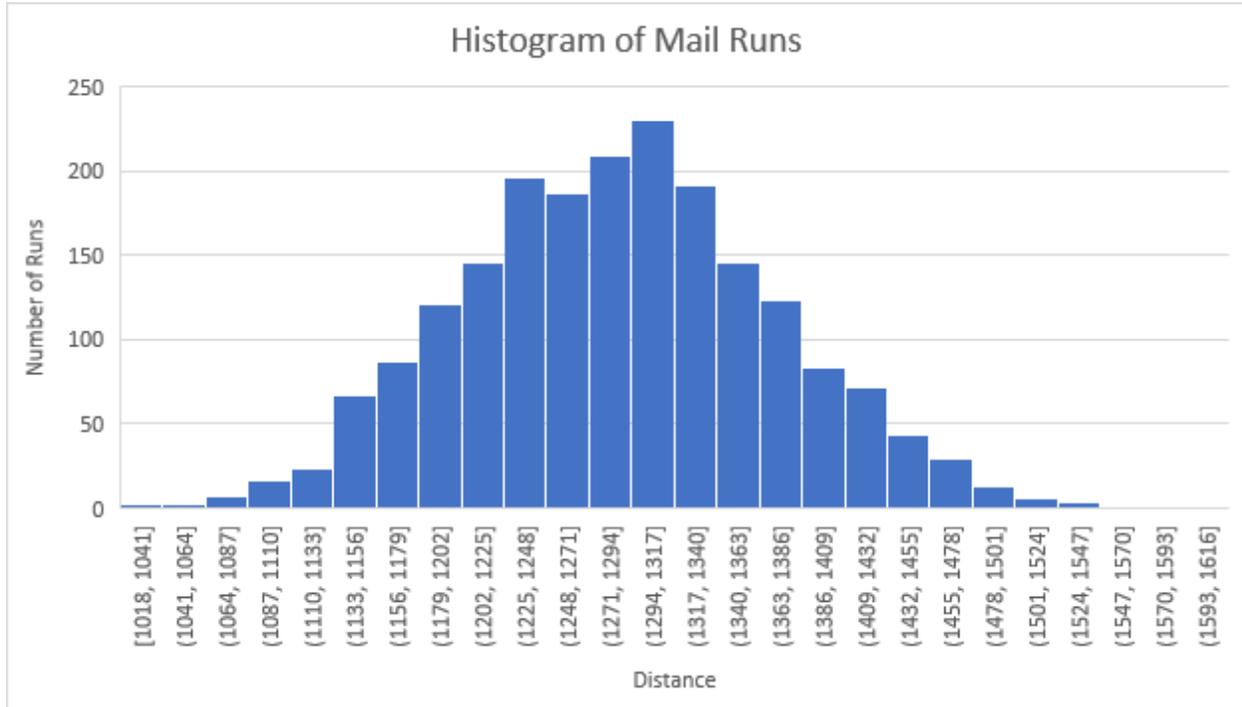
The model used is a modification of the simulation from Cohen, et al. (2026). In this simulation, a customizable area is populated with delivery points before a delivery vehicle travels to each delivery point and then back to the entry point using a greedy algorithm. The model calculates the total distance traveled (TDT) and the average distance traveled (ADT) for the delivery vehicle. As a Monte Carlo model, the simulation is repeated thousands of times. The resulting mean values of TDT and ADT are designated as the values for that particular set of input parameters. Figure 1 displays a histogram of the TDT for each of the 1,000 runs of the model for a 100x100 matrix¹¹ with 100 randomly populated delivery points and with the entry point in the middle of the bottom of the grid. As expected, it displays a normal distribution.

Figure 1

¹⁰ The authors believe that much of the analysis of parcel delivery route structure is considered proprietary and thus, not in the public literature.

¹¹ The terms “matrix” and “grid” are used interchangeably in this paper.

**Histogram of TDT Values for 1,000 Runs of the Model
(100x100 grid with 100 stops randomized for each run)**



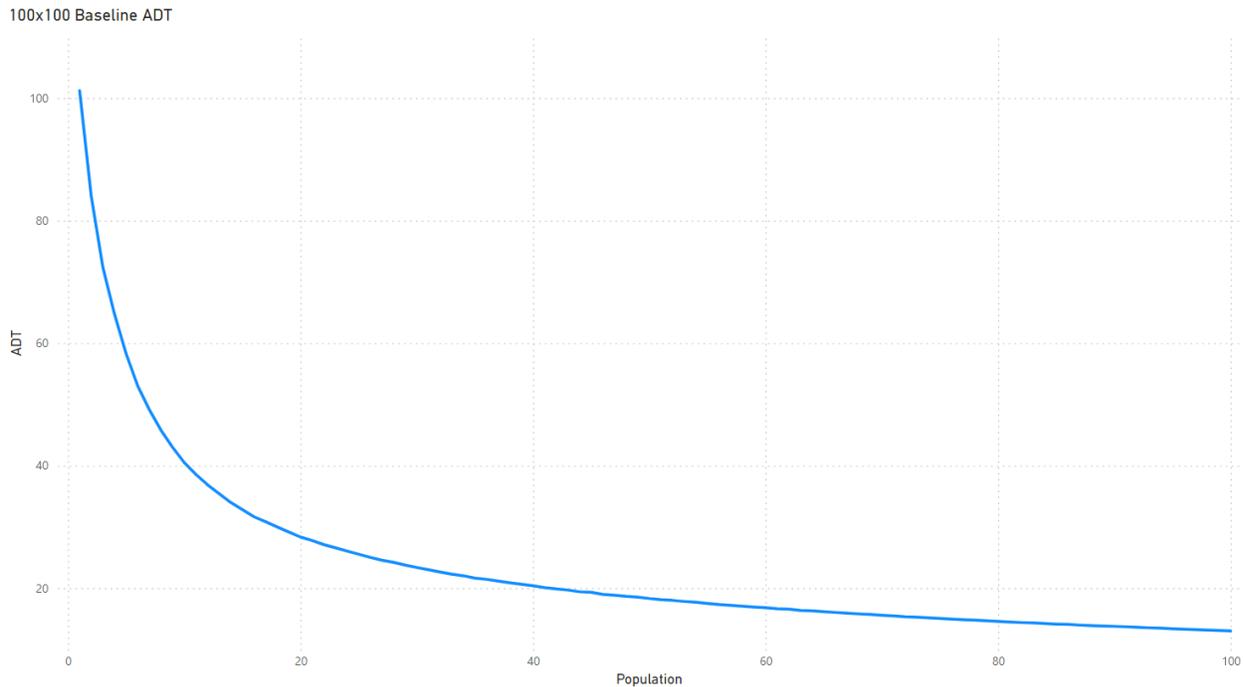
Modeling of delivery networks can be very sophisticated and tailored to specific populations, geography, delivery modes, and products. For example, Arnold, et al. (2018) simulated various delivery scenarios using demand and geographic parameters that are specific to Antwerp; the paper also provided an overview of other studies of the urban logistics and last mile distribution of e-commerce. This paper is not empirical and addresses its topics using a simple model with the goal of helping the reader to gain intuitive insights into the topics addressed.

4 Relationship of Delivery Points to Average Distance Traveled (ADT)

This issue is examined by using a 100x100 matrix. Between 1 and 100 delivery points are randomly assigned, and the ADT is calculated for a van moving through the assigned delivery points or stops. The result is the exponential graph shown in Figure 2, in which the ADT drops swiftly until the number of stops reaches about 40. After that, the advantage of increased

delivery points (or increased density) diminishes.¹² As the savings from increased density diminishes, other factors, such as service, may become more important.

Figure 2
ADT vs Number of Delivery Points

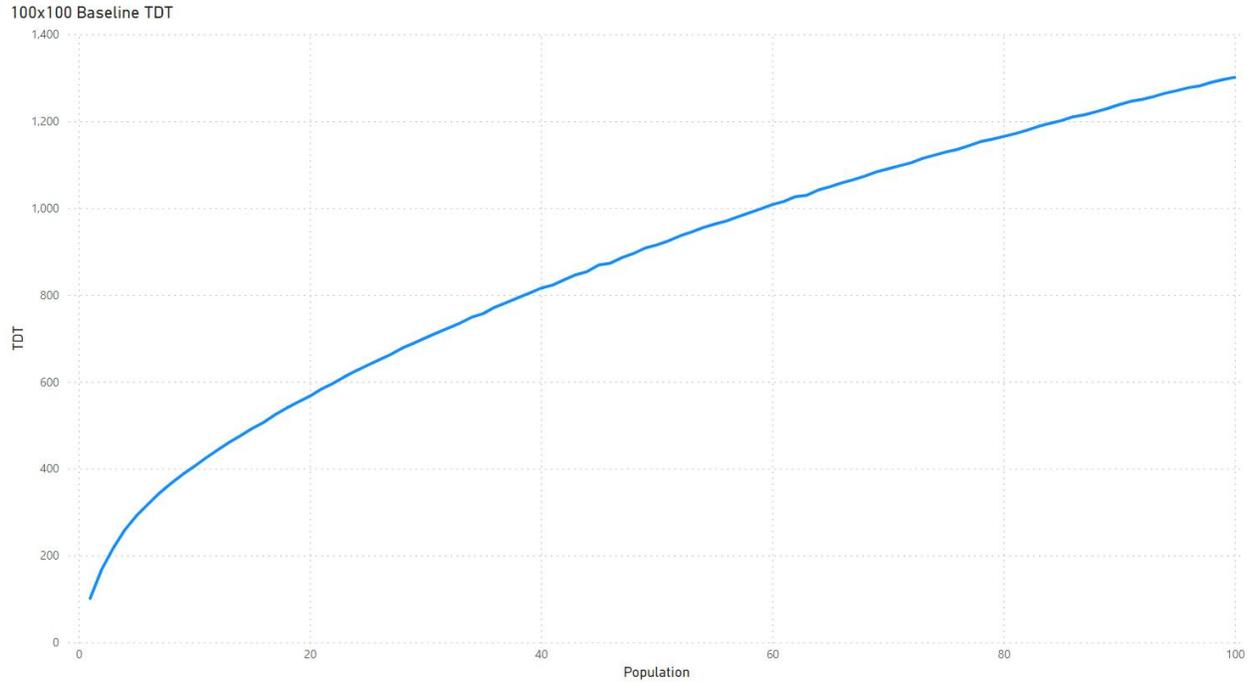


For the convenience of the reader a table displaying the relationship of ADT to the number of delivery points can be found in the Appendix.

The effect of increasing delivery points (or density) on TDT can be seen in Figure 3. Increasing the number of delivery points initially causes the curve to be non-linear, but adding additional delivery points results in the curve being close to linear.

Figure 3
TDT vs Number (Density) of Delivery Points

¹² The shape of this curve is consistent with a similar graph in Arnold, et al. (2018) that shows the decrease in costs per delivery as deliveries per square kilometer increase.



5.1 Effects of Clustering of Delivery Points

To examine the effects of clustering, 4 different scenarios were evaluated. Two of the scenarios divide 100 delivery points into four clusters, with the center point of each cluster randomly selected and 25 delivery points normally distributed around the center point.¹³ The density of the clusters is varied, such that the standard deviation of the clusters is 5 in one scenario, and 10 in another. The other two scenarios create 10 clusters of 10 delivery points each, also with a standard deviation of 5 or 10. Table 1 presents the Total Distance Traveled (TDT) for each of the clustering scenarios.

Table 1
Effect of Delivery Point Clustering on TDT

	Standard Deviation = 5	Standard Deviation = 10
4 Clusters of 25 Delivery Points	531	852
10 Clusters of 10 Delivery Points	721	1,007

¹³ The x and y coordinates of each center point serve as the mean values of x and y for the delivery points in the cluster, whose coordinates are randomly selected assuming a normal distribution.

A fully random distribution of delivery points results in a TDT of 1290, demonstrating that clustering of delivery points reduces the distance traveled.

5.2 Savings from Merging Routes

By comparing the results of routes with different numbers of deliveries, the model provides insight into the potential savings from merging routes. Table 2 shows the effect on Total Distance Traveled (TDT) and ADT of merging two routes with 10 deliveries each into a single route with 20 deliveries, and of merging two routes with 50 deliveries each into a single route with 100 deliveries.

Table 2
Savings from Merged Routes

	TDT	ADT
2 Routes of 10	803	40.1
1 Route of 20	563	28.1
Savings	240	12.0
Percent Savings	29.9%	29.9%
2 Routes of 50	1,817	18.2
1 Route of 100	1,289	12.9
Savings	528	5.3
Percent Savings	29.1%	29.1%

In percentage terms, the savings in both TDT and ADT are the same for both scenarios (the difference is due to rounding and statistical variation in the simulations). But in absolute terms, the scenarios behave differently. While the total distance saved by combining routes with 10 deliveries is relatively small, it is much larger on a per-delivery basis. Conversely, combining routes with 50 deliveries saves a relatively large total distance, but the savings per delivery is much smaller.

5.3 Effect of Different Entry Points on Total Distance Traveled

To explore this effect, we have used a 100 x 100 grid and placed the entry points at y=50, x=50 (viz. middle of the grid), y=1, x=50 (viz. middle of the bottom of the grid), and y=1, x=1 (viz. corner). The results are shown in Table 3.

**Table 3
TDT for Selected Entry Points**

Stops	Y=50, X=50 Center	Y=1, X=50 Mid-Edge	Y=1, X=1 Corner
50	908	928	948
100	1,289	1,309	1,328
200	1,744	1,760	1,777

5.4 Effect of E-Commerce on Total Distance Traveled

E-commerce involves last mile delivery, eliminating the necessity for each customer to travel separately to a store to make purchases. Thus, there are Total Distance Traveled (TDT) savings to society when e-commerce is used by shoppers. To get a sense of the savings the model is used with the central shopping point put in the center of the grid, in the middle of the edge, and in the corner. Table 4 shows the percentage savings in TDT for e-commerce delivery using 50, 100, and 200 different delivery points in a 100x100 grid.

**Table 4
TDT Savings (%) from E-commerce Shopping vs Physical Shopping**

(100x100 grid)

	Starting Point	Starting Point	Starting Point
# of stops	Center	Mid-Edge	Corner
50	81.8%	87.6%	90.5%
100	87.1%	91.3%	93.4%
200	91.3%	94.1%	95.6%

The relationship between the number of stops and the reduction in TDT can also be expressed mathematically by fitting a logarithmic function to a set of modeled results, as shown in Figure 4.

Figure 4
Retail TDT as a Percentage of E-Commerce TDT

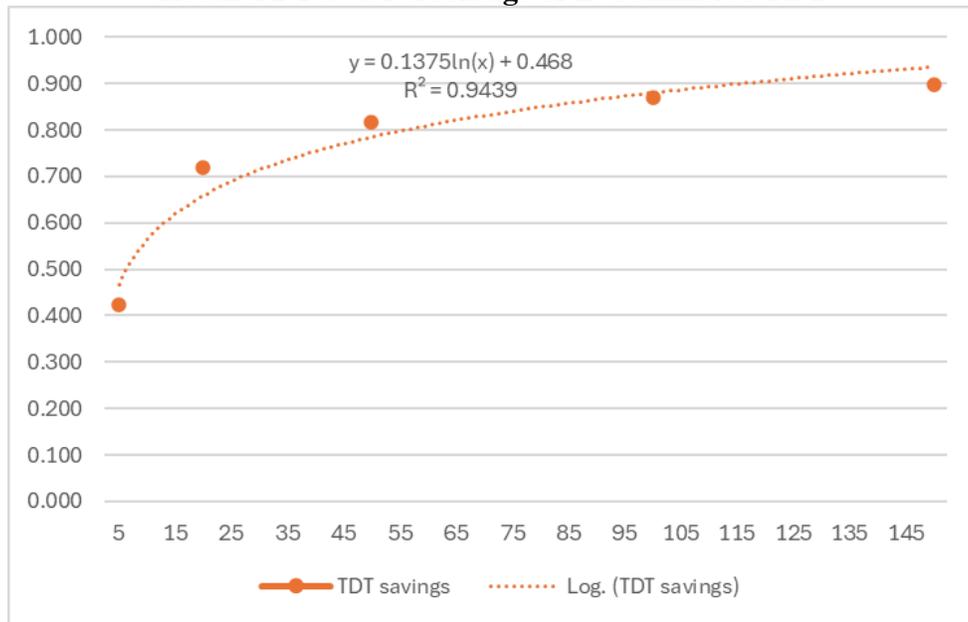
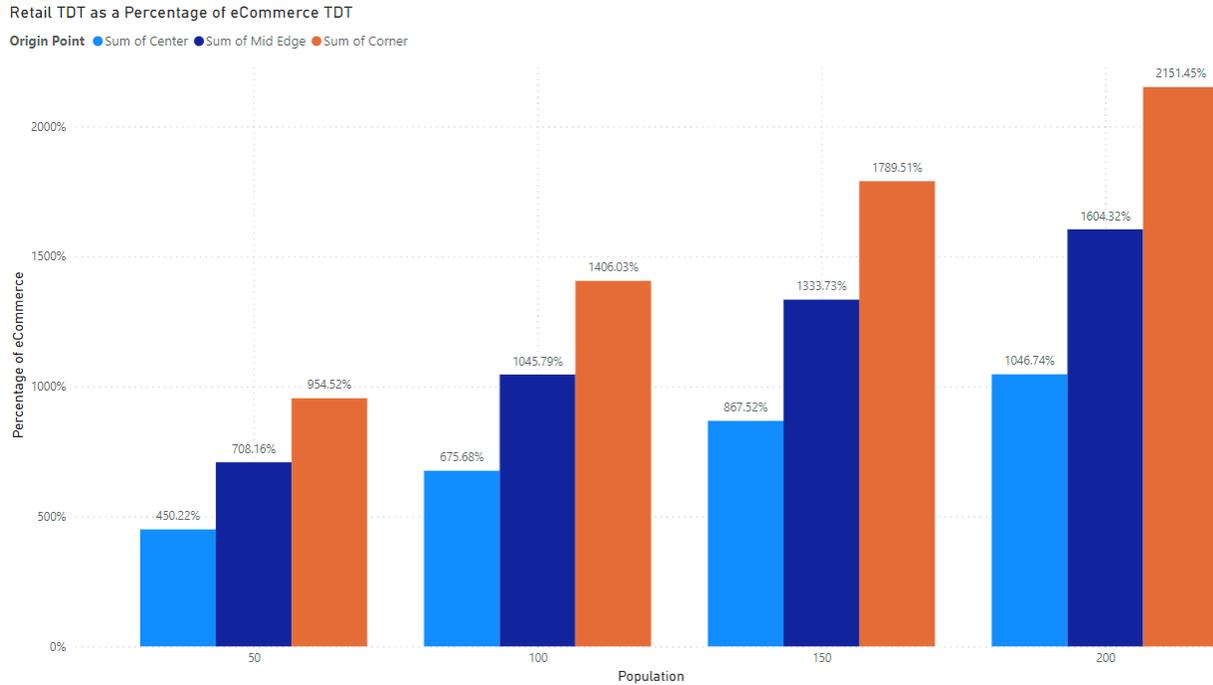


Figure 5 shows the sum of distances traveled by customers picking up from a retail location as a percentage of the TDT of an eCommerce delivery alternative. This illustrates that the location of the retail or delivery starting point is as important to the comparison as the number of stops.

Figure 5
Retail TDT as a Percentage of E-Commerce TDT



5.5 Effect of Shape on Total Distance Traveled

This subject is examined by using rectangles of different dimensions all having the same area. The model is used to generate the Total Distance Traveled (TDT) for 100 stops in the following rectangles: 100x100, 200x50, 400x25 and 800x12 ½.¹⁴ The results are shown in Table 5. The 100x100 grid is used as the base of comparison in percentage terms.

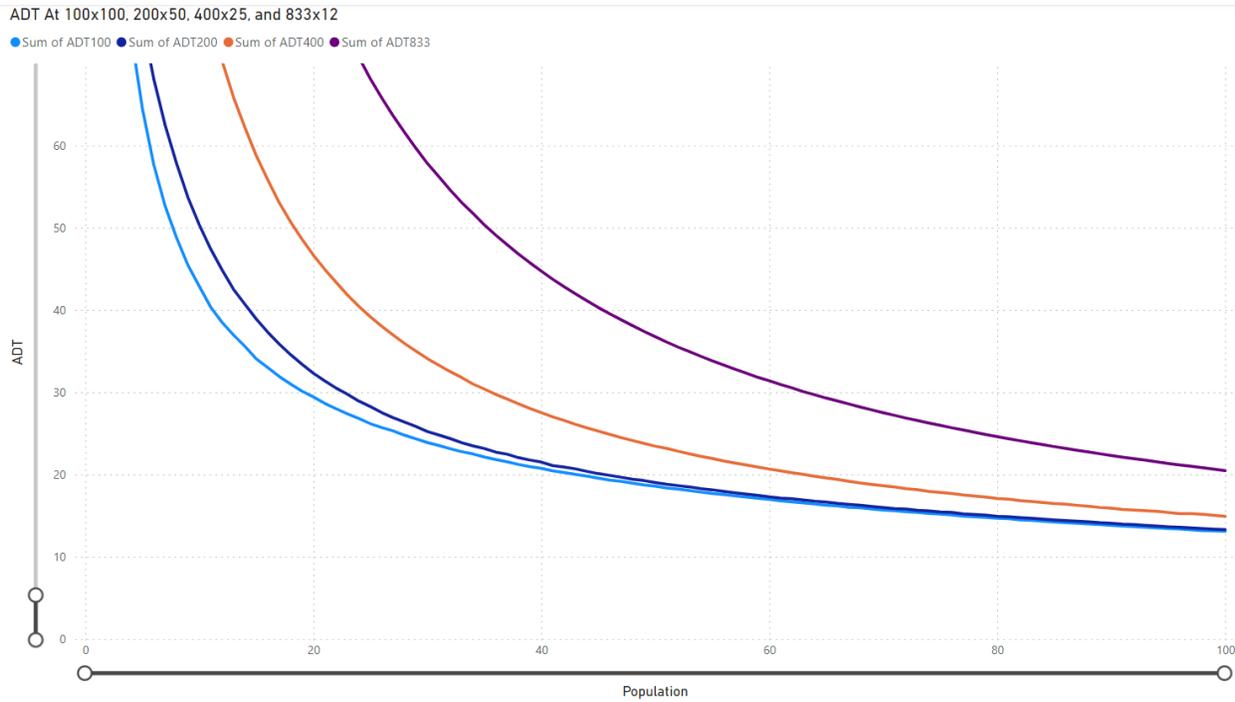
Table 5
Total Distance Traveled for Several Shapes and % Difference from Base Case

¹⁴ Because of the design of the model the sides of the rectangles must be whole numbers, so 12.5 cannot be used. Instead 833x12 is used here but this does not affect the TDT calculation significantly.

# stops	100x100 (Base)	200x50	% from Base	400x25	% from Base	800x2.5	% from Base
50	929	951	2.3%	1172	26.2%	1338	44.0%
100	1309	1329	1.5%	1491	13.9%	2947	25.1%

Figure 6 graphically displays Average Distance Traveled (ADT) for each of the four shapes, illustrating the higher costs of delivering to a given number of stops in more oblong rectangles of the same total area. As the area is more restricted in one dimension, the ADT increases.

Figure 6
Average Distance Traveled for Several Different Shapes



5.6 Effect of Barriers

This section examines scenarios with a barrier (such as a river) with one point of crossing (such as a bridge) inserted in a 100 x 100 grid. The barrier is placed in the middle of the grid where $x=50$ with one point where a delivery van can pass, in the middle where $y=50$, or at the edge where $y=0$. Table 6 displays the percentage change with the barriers vs. a barrier free grid.

Table 6
Effect of barriers on Total Distance Traveled for 50, 100, and 200 stops

Stops	No Barrier	Crossing at $x=50$ and $Y=50$	% change from no barrier	Crossing at $y=0$ and $x=50$	% change from no barrier
50	928	985	6.2%	1,075	15.8%
100	1,309	1,364	4.2%	1,451	10.8%
200	1,760	1,863	5.9%	1,954	11.0%

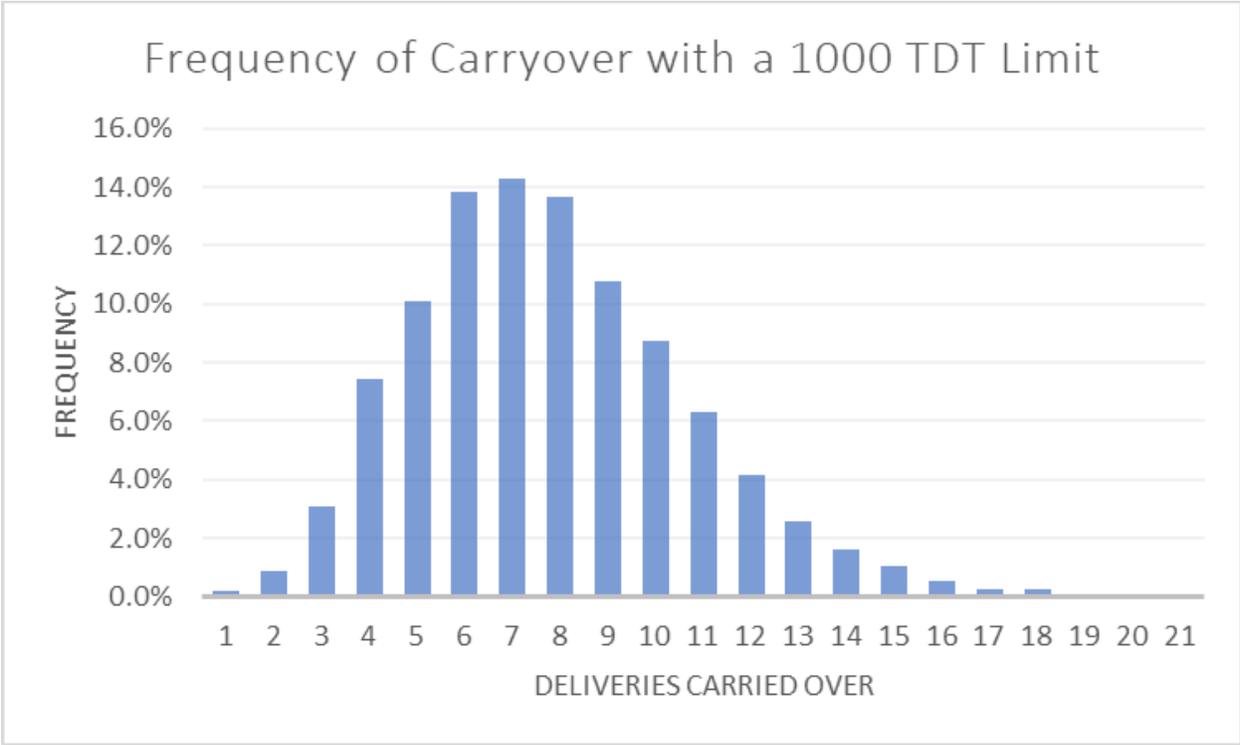
We see that the effect of a crossing in the center of the barrier is a lower Total Distance Traveled (TDT) than a crossing at the bottom of the barrier. Consistent with the effects of narrower-shaped areas in the previous section, at 100 stops the effect of a barrier is smaller than at 50 stops. However, the increase in the effect of the barrier for a route with 200 stops is counterintuitive, and likely is a result of backtracking caused by the greedy algorithm.

6. TDT Savings from Allowing Delivery to be Deferred for the Purpose of Eliminating Overtime

The representation of deferrals in the model is achieved by running the model with a limit on Total Distance Traveled (TDT). If this limit is exceeded, the delivery point that contributed the most to TDT is identified by running the model multiple times removing a single delivery point each time, one by one. The model thus determines which delivery point saves the most time when deferred. If the limit is still exceeded even when deferring deliveries to this delivery point, the process repeats, removing additional delivery points until the limit is no longer exceeded. Delivery points that are deferred are then added back into the model run for the next day. The model can provide insight into cost savings provided by deferrals by comparing the TDT of the

model with and without deferrals. Limiting the TDT in the model is analogous to limiting the number of hours a delivery driver is able to work in a day, so limiting TDT could provide a disproportionate reduction in costs. For instance, limiting a delivery driver to 8 hours of labor would prevent overtime costs or accelerated depreciation of a vehicle from overuse. Figure 7 is a histogram showing the frequency of carryover over the course of 5000 simulated days. TDT is limited to 1000 in a 100x100 grid with 100 delivery points.

Figure 7
Frequency of Carryover with a 1000 TDT Limit



We see that the 1000 TDT limit applied to the model with an expected TDT of approximately 1300 (see Appendix Table) caused at least one delivery point being carried over to the next day of the model in all 5000 iterations. At a ~23 percent reduction in expected TDT, the proportion of deliveries postponed stabilizes at approximately 8 pieces per iteration, or 8 percent of all pieces. It should be noted that with this carryover limit there will almost never be an instance where there are no deliveries carried over in an iteration.

The TDT limit can be adjusted so the amount of carryover remains within reasonable boundaries while maximizing TDT savings. Figures 8 and 9 show TDT limits of 900 and 1100, which result in TDT reductions of ~31 percent and ~15 percent, respectively.

Figure 8
Frequency of Carryover with a 1100 TDT Limits

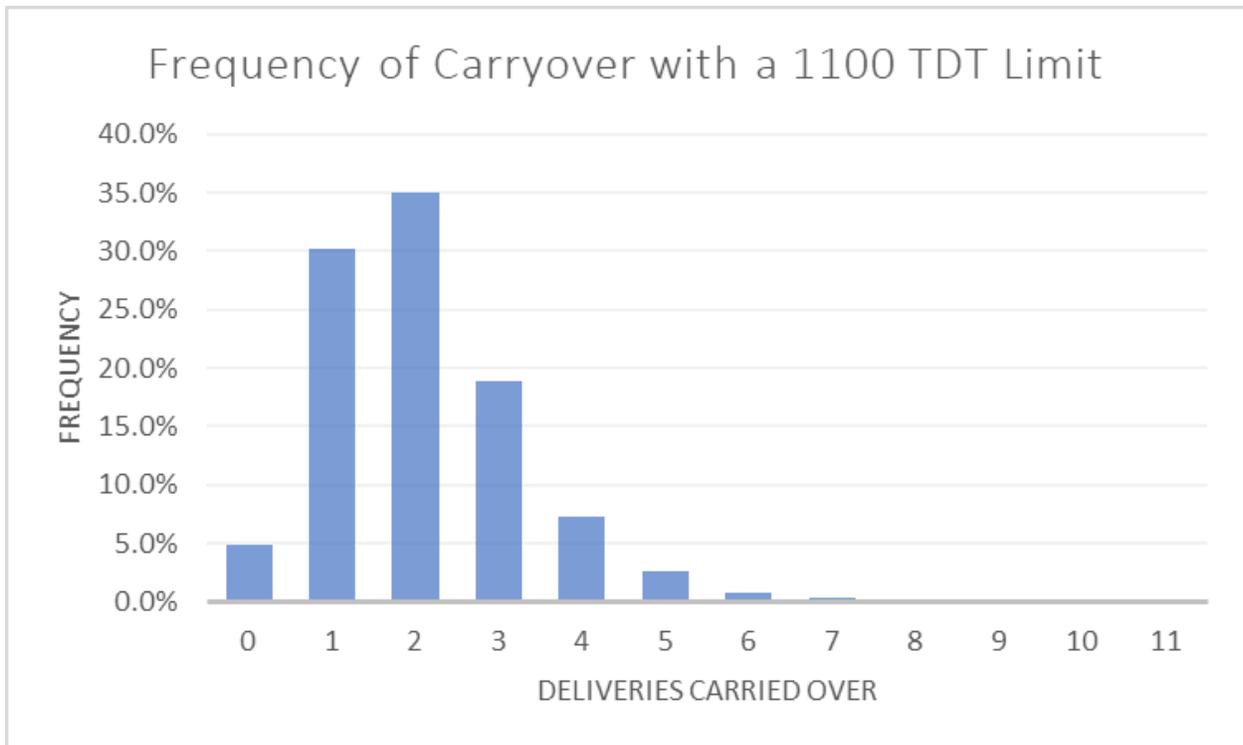
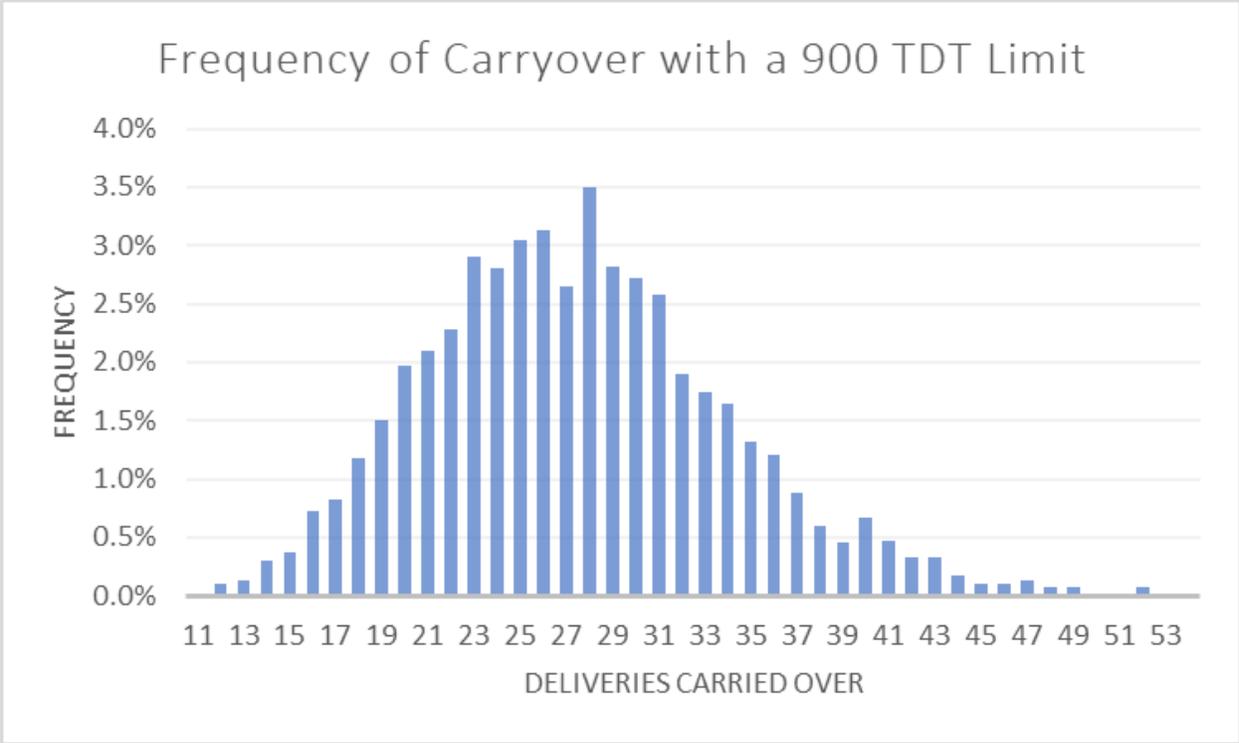


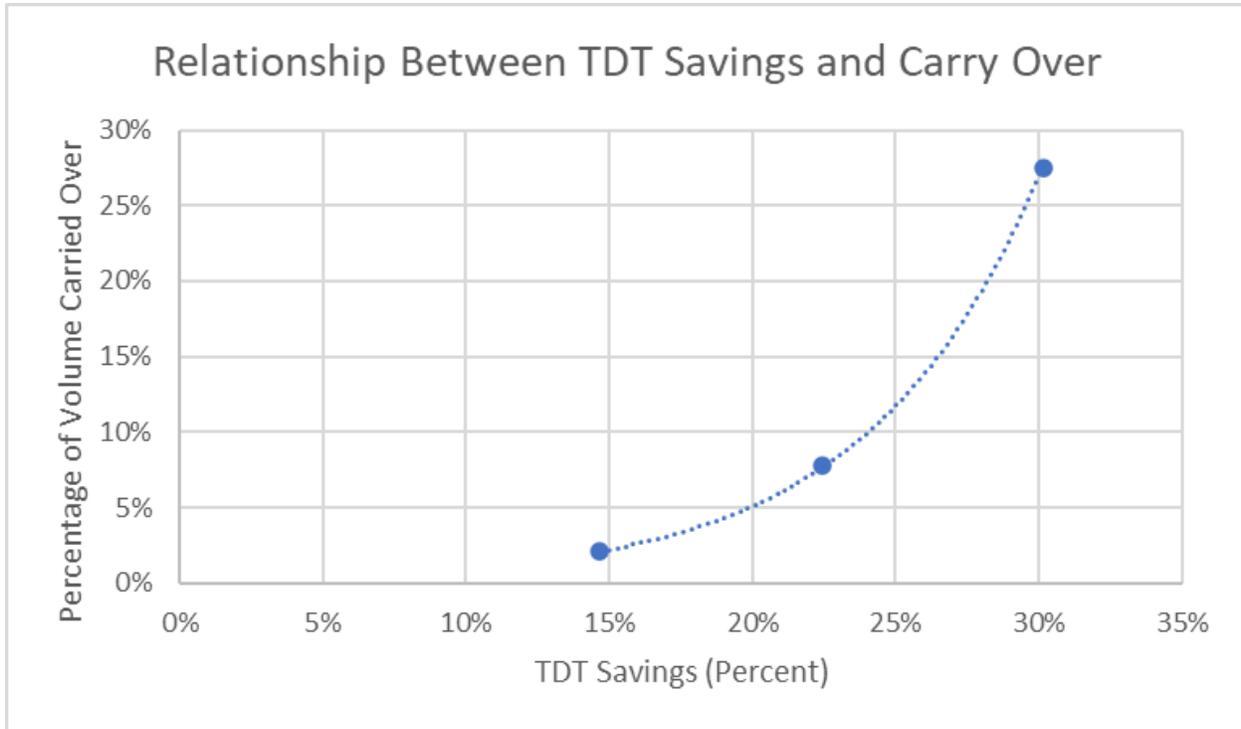
Figure 9
Frequency of Carryover with a 900 TDT Limits



A TDT limit of 900 has an average of 27 deliveries carried over to future days. A model run with a TDT limit of 1100 has an average of approximately two deliveries carried over each day, and in 5 percent of days there are no deliveries carried over. This relationship between reduction in TDT and percentage of deliveries deferred is nonlinear, with TDT reductions over 30 percent resulting in very dramatic increases to the required amount of carry over. As such, pursuing TDT reductions of over 30 percent quickly results in overwhelming amounts of undelivered volume. The figure below depicts this nonlinear relationship.

Figure 10

Relationship Between Total Distance Traveled Savings and Carry Over



This nonlinear relationship impacts the processing speed of the model. As more volume is carried over to future days, the model requires exponentially more time to process routing to an increasing number of delivery points. As such, the model has limitations when examining extreme cases of carryover in which the TDT is limited to less than 70 percent of its expected value.

7. Conclusion

The studies on residential delivery that we have located are based on specific geographies. In contrast, this paper treats residential delivery in the abstract with no reference to any specific geography. The results of the modeled scenarios show that all the examined parameters are important contributors to the cost of residential delivery parcels routes. This is particularly the case for the number of stops on a route and the extent of clustering of the stops. The model also demonstrates the considerable savings achievable through deferring delivery, and the limits

imposed by costs to performance. Most importantly, the model gives researchers and others studying last mile route design a tool to trade off the parameters that are applicable to their particular situations. Further research may allow the application of the model to situations where several routes must be established, and to use actual data about the distribution of stops to identify the ranges of parameters and scenarios on which to focus the analysis.¹⁵

¹⁵ The model code is available by contacting Samuel Robinson via his email address (above).

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Appendix Table of TDT and ADT for 100 x 100 Grid

Volume	TDT	ADT
1	99.37	99.367
2	165.99	82.99475
3	216.59	72.1955
4	255.50	63.873875
5	290.13	58.0251
6	316.63	52.77166667
7	340.37	48.62421429
8	363.70	45.462875
9	383.12	42.5685
10	401.43	40.1425
11	418.68	38.06136364
12	437.19	36.43225
13	453.97	34.92046154
14	471.09	33.64907143
15	487.84	32.52276667
16	502.68	31.417625
17	518.50	30.49991176
18	532.31	29.57277778
19	549.26	28.90818421
20	562.72	28.136125
21	578.07	27.52695238
22	590.97	26.86247727
23	604.65	26.28923913
24	617.18	25.71577083
25	630.65	25.22618
26	643.20	24.73826923
27	656.80	24.3257963
28	667.66	23.84489286
29	681.86	23.51243103

30	693.18	23.10611667
31	706.32	22.78466129
32	715.97	22.37395313
33	729.37	22.1019697
34	742.37	21.8345
35	750.74	21.44971429
36	764.83	21.24536111
37	774.41	20.93008108
38	783.81	20.62651316
39	796.64	20.42655128
40	805.70	20.14245
41	815.77	19.89681707
42	825.29	19.64985714
43	836.00	19.44177907
44	848.58	19.28580682
45	858.21	19.07141111
46	866.73	18.84193478
47	878.25	18.68611702
48	884.13	18.41928125
49	896.43	18.29454082
50	908.73	18.17456
51	914.21	17.92567647
52	924.03	17.76980769
53	933.19	17.60733962
54	943.09	17.46469444
55	952.38	17.31598182
56	962.06	17.17955357
57	969.65	17.01138596
58	978.91	16.87778448
59	986.30	16.71688983
60	995.90	16.59828333

61	1003.72	16.4545
62	1013.52	16.34704032
63	1020.71	16.20176984
64	1030.11	16.0954375
65	1037.03	15.95436154
66	1047.25	15.86742424
67	1054.51	15.73889552
68	1061.88	15.61584559
69	1070.51	15.51458696
70	1078.60	15.40855
71	1087.70	15.31974648
72	1097.88	15.24834722
73	1101.69	15.09160274
74	1107.58	14.96733108
75	1116.67	14.88894
76	1124.90	14.80125658
77	1130.79	14.68551948
78	1139.47	14.60857051
79	1147.66	14.52738608
80	1152.52	14.40655625

81	1160.01	14.32109877
82	1170.28	14.27166463
83	1174.71	14.15318072
84	1184.17	14.0972619
85	1191.57	14.01852353
86	1197.93	13.92946512
87	1201.84	13.81428736
88	1208.23	13.72986364
89	1217.08	13.67508427
90	1225.87	13.62076111
91	1232.56	13.54464286
92	1239.34	13.47109239
93	1243.41	13.36996774
94	1249.12	13.28847872
95	1255.50	13.21577895
96	1261.09	13.13633854
97	1274.71	13.14131959
98	1274.53	13.00540816
99	1280.27	12.9320404
100	1289.20	12.891955