Intertwined network effects: theory and evidence*

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In many transaction two-sided markets, an "aggregator" platform (e.g., Trivago) gives buyers access to sellers (e.g., hotels) on a competing "source" platform (e.g., Booking), creating "intertwined network effects" (INE). Despite INE's prevalence, including multiple prominent mergers, its welfare implications remain understudied. Using a theoretical model, I show that INE increase consumer surplus but reduce seller surplus if platforms are sufficiently differentiated for sellers, and increase it otherwise. A non-consolidating merger with INE reduces double marginalization but increases market power, harming consumers if their network effects are sufficiently low. If platforms are sufficiently homogeneous to sellers, the merger reduces their surplus. I empirically validate these predictions by exploiting two cases where classified ads platforms introduced INE: Finn/Nettbil and Adverts/DoneDeal. Using event study designs, I show INE caused an increase in users on the aggregators. Using time series analysis, I provide descriptive evidence that INE increased the number of listings in the aggregators and decreased it in the source platforms. A difference-in-differences design confirms INE caused a drop in the number of listings in the source platforms. I discuss implications for merger control and asymmetric interoperability policies.

JEL Classification: D43, L13, L86, L41.

Keywords: intertwined network effects, platform compatibility, interoperability, platform competition, platform merger

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1 Introduction

In many two-sided transaction markets, aggregator platforms act as gateways to other matching platforms while always offering their own competing matching services. For example, the accommodation platform Trivago enables buyers to book a hotel directly on the hotel's website or via third-party platforms such as Booking, which are included in Trivago's search results. In such settings, buyers on an "aggregator" platform (e.g., Trivago) can access sellers listed on a separate "source" platform (e.g., Booking). As illustrated in Figure 1 below, this allows platform users on one side of the market to benefit from the presence of users on the other side of the market participating in a *competing* platform. In such instances, I say the two platforms set up *intertwined network effects* (INE).

Examples of aggregator platforms with INE include retail marketplaces (Google Shopping, Bing Shopping), price comparison sites (Price Runner), lodging metasearch platforms (Google Hotels, Trivago), real estate platforms (real estate online platforms hosting listings from real estate agencies, Jinka)[1] and digital wallets (Revolut/Bizum). Importantly, aggregators and source platforms are horizontally differentiated in at least one side of the market.[2] For example, generalist marketplaces and price comparison sites aggregate specialized marketplaces that only partially compete with them (e.g., Coches.net/Milanuncios, Adverts/DoneDeal, Idealo/Otto). Moreover, across multiple industries, it is common for platforms having set up intertwined network effects to merge while maintaining the two firms active.[3] Examples include eBay/Motors, Adevinta/Gumtree, Se Loger/Logic Immo, and eBay/StubHub.

Recent merger cases have featured intertwined network effects. Some of them involve aggregators merging with a source platform without consolidating. In others, platforms have (tried to) set up INE post-merger, typically arguing this practice constitutes a merger efficiency (e.g., Trade Me/Property NZ). Competition authorities' approaches to merger cases involving INE have been varied. Some have not factored INE into their assessment of the welfare effects of the merger (e.g., Schibsted/Nettbil, Trade Me/Property NZ, viagogo/Stubhub). Conversely, other competition authorities have deemed INE detrimental to specific user groups arguing that intensified network effects would increase the merged entity's market power (e.g., Booking/eTraveli, FDJ/Zeturf, Seek Asia Investments/JobStreet, Wedding Planner/Zank you).

The extant literature has investigated the effects of platform competition and mergers under various settings. However, little attention has been paid to how intertwined network effects

[1] The 2021 ImmoScout24 case in Germany provides a notable illustration of the relevance of INE: courts prohibited Berlin's dominant real estate platform from offering rebates to agents for seven-day exclusive listings, finding this arrangement would leverage cross-platform network effects to foreclose rivals (Kammergericht Berlin, 2022).

[2] Below I show that only if platforms are horizontal differentiation do platforms choose to implement INE.

[3] In this case, the merger is said to be "non-consolidating". For simplicity, I will hereafter say "merger" to refer to a *non-consolidating* merger.

could alter established results. This article studies the effect of intertwined network effects on platform demand, prices and the welfare of the two sides of the market. It also shows how a non-consolidating merger between the platforms affects these outcomes.

I develop a model in which two horizontally-differentiated platforms compete to facilitate transactions between two types of users benefiting from positive indirect network effects: buyers and sellers. I consider a competitive bottleneck setting in which buyers single-home and sellers partially multi-home. Sellers are charged a per-transaction price, while buyers interact for free. If the platforms decide to set up intertwined network effects, the buyers in the aggregator platform and the sellers in the source platform interact. For each of these cross-platform interactions, the aggregator platform charges an endogenously-determined per-transaction referral fee to the source platform. I first consider the scenario with intertwined network effects, and compare it to one without intertwined network effects when the platforms are legally independent. I then compare a scenario in which the platforms set INE and merge without consolidating with one in which legally-independent platforms set INE. In both comparisons, I characterize, for each platform and in the aggregate, the effects on the number buyers and sellers, prices and surpluses.

I show that INE benefit consumers on the aggregator platform and harm consumers of the source platform. However, the overall effect on consumer surplus is always positive. The gain in consumer surplus brought about by the aggregator platform consumers' capacity to reach all sellers always more than compensates the harm to the source platform's consumers triggered by an INE-driven drop in seller-side demand. As for sellers, if the platforms are sufficiently differentiated for them, INE decrease their surplus, and vice versa. The reason is that INE increase the number of sellers and their surplus in the aggregator platform, which is able to attract more consumers than without INE. Then, INE leads some single-homing sellers in the source platform to start multi-homing. Conversely, some multi-homing sellers leave the source platform to remain single-homed on the aggregator platform, thereby increasing seller-side demand there. With sufficiently low seller transportation costs, enough sellers stop multi-homing and start single-homing in the aggregator, leading to an increase in overall seller surplus, and vice versa.

If the network effects enjoyed by consumers are sufficiently low with respect to the stand-alone utility, the merger decreases their surplus, and vice versa. By eliminating the referral fee, the merger drives the aggregator platform to increase the per-transaction price it charges to sellers and the source platform to decrease it, while the average price drops. This leads some consumers to migrate from the aggregator to the source platform. On the seller side, the number of sellers increases in the source platform and decreases in the aggregator. If the stand-alone utility a consumer obtains from joining any of the two platforms is sufficiently low, network effects have a significant weight in her decision to join a platform over another. In this case, a sufficiently high amount of consumers switch from the aggregator (where the number of sellers decreases) to the source platform (where the number of sellers increases), which increases overall consumer surplus. The opposite happens if the stand-alone utility is sufficiently high.

A non-consolidating merger between two platforms having set up INE, in turn, reduces seller surplus if the platforms are sufficiently homogeneous to them. The decrease in the average price paid by sellers increases their surplus. However, the migration of buyers from the aggregator to the source platform, coupled with an increase in the number of sellers in the latter and an decrease in the former, reduces the overall number of interactions. Both single-homing sellers in the aggregator platform and multi-homing sellers who, because of INE, meet the aggregator platform's buyers in both platforms, have less interactions post-merger. This decreases seller surplus. The lower seller transportation costs are, the more sellers and thus consumers change their choice of platform post-merger; hence, the lower the number of post-merger interactions.

I then validate the model empirically by exploiting two cases in which classifieds ads platforms introduced INE: Finn/Nettbil and Adverts/DoneDeal. Using event study designs, I show that, as predicted by the theoretical model, INE caused an increase in the number of users in the aggregator. Using a time series approach, I provide descriptive evidence that INE increased the number of listings in the aggregator and decreased it in the source platform. A difference-in-differences design confirms the negative causal effect on the source platform DoneDeal.

The remainder of the article is organized as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 studies the impact of intertwined network effects on buyer- and seller-side demand for each platform, prices and the welfare of buyers and sellers when the platforms are legally independent. Section 5 analyzes how a non-consolidating merger affects these outcomes. Section 6 tests empirically the predictions of the model regarding the effect of INE on the number of consumers and sellers by studying two cases in which classifieds platforms set up intertwined network effects: Finn/Nettbil and Adverts/DoneDeal. Section 7 concludes and discusses the implications of the findings for merger control and asymmetric interoperability policies.

2 Related literature

This article relates to the vast literature on price competition between platforms, notably that focusing on a competitive-bottleneck setting (Belleflamme and Peitz, 2019; Armstrong, 2006). More precisely, given its focus on intertwined network effects, it contributes to the literature on interoperability with network effects (Bourreau et al., 2023; Bourreau and Krämer, 2022; Shekhar et al., 2022; Rasch and Wenzel, 2014; Doganoglu and Wright, 2006; Crémer et al., 2000; Katz and Shapiro, 1985; Farrell and Saloner, 1985). Within this literature, the article is particularly relevant to the work on interoperability between competing platforms with indirect network effects. This article also contributes to the growing literature on platform mergers (Ivaldi and Zhang, 2022; Farronato et al., 2020;

Correia-da Silva et al., 2019; Tan and Zhou, 2019; Baranes et al., 2014; Chandra and Collard-Wexler, 2009), notably to the strand focusing on non-consolidating horizontal mergers. Finally, this article relates to two papers studying analogous incentive problems in different settings. Lambin (2019) investigates the incentives and welfare effects of media platforms redirecting readers to competitors, while Hagiu et al. (2020) examine a firm's incentives to "host" a rival.

Lambin (2019) studies inter-platform referencing in digital media using a two-sided model analogous to this article's: users join a platform and "roam" to competitors for free to increase their utility, while advertisers pay for each interaction with users. Despite these similarities, there are key differences stemming from Lambin (2019)'s focus on media platforms, research questions and methods. His analysis differs in its core mechanism. Users value content quality, not the number of users on the other side of the market. Because platforms can reach users through referrals, these can harm users. If referral fees are sufficiently low, platforms' incentives to invest in content quality are weakened to the point that user welfare decreases. Referencing is reciprocal in equilibrium, with all users able to roam to the competing platform, and mediated by a third-party "go-between". By contrast, this article studies asymmetric platform cross-referencing without third-party mediation, where the aggregator's users access source platform sellers, but not reciprocally. Beyond these modeling differences, this article also differs in that it extends the analysis to seller welfare, merger effects, and an empirical validation of some of the model's predictions.

Hagiu et al. (2020) study the conditions under which a multiproduct firm profitably allows its consumers to purchase a product from a competing single-product firm, effectively "hosting" a rival. This arrangement turns the multiproduct firm into a platform and the single-product firm into its complementor. Their framework is related to the one in this article, in which intertwined network effects (INE) allow consumers from one platform to interact with sellers from a rival platform. However, in this article, both firms are platforms before and after the introduction of INE, each intermediating between buyers and sellers in the presence of network effects. In contrast, Hagiu et al. (2020) focus on the endogenous emergence of platforms and thus do not model network effects, which are essential to my analysis. Therefore, their motivating examples and foci differ from this articles'. Their study centers on the incentives to host a rival and its effect on firms' profits. This article extends the discussion by focusing on the welfare implications of intertwined network effects, both pre- and post-merger. This places this article within two strands of literature: one on interoperability between competing platforms with indirect network effects, and another on non-consolidating horizontal platform mergers. The remainder of this section examines this article's connections to each strand.

Interoperability between competing platforms with indirect network effects. This article relates to the literature studying the effect of interoperability (also referred to as "compatibility") in markets subject to network effects. Indeed, I model INE as an *asymmetric* form of compatibility between two platforms, also referred to as "one-way compatibility".

When INE are introduced, platform A consumers can interact with platform B sellers. However, platform A sellers cannot interact with platform B buyers. This asymmetry distinguishes INE from standard symmetric or "two-way" compatibility.

Specifically, this article relates to the strand of this literature that, building on the seminal contributions focusing on the compatibility between network goods subject to direct network effects (e.g., Katz and Shapiro, 1985; Farrell and Saloner, 1985; Farrell and Saloner, 1986; and Katz and Shapiro, 1994), shifted to studying the choice and welfare effects of compatibility between competing platforms enabling interactions between user groups subject to *indirect* network effects.

The closes contribution to this article's within the interoperability literature is Maruyama and Zennyo (2015). Building on Rasch and Wenzel (2014)'s results, they consider a setting in which two symmetric platforms intermediating between consumers and content providers can independently decide whether to be compatible with each other or not at a fixed cost. Consumers single-home while content providers might multi-home. When the fixed cost of interoperability is intermediate, the equilibrium is an "asymmetric case" in which one platform chooses compatibility while the other does not. This generates cross-platform network effects analogous to what I call "intertwined network effects" (INE) in this article. They find that the asymmetric case leads to a drop in consumer surplus. This contrasts with this article's findings, in which INE always benefit consumers. The reason is that, in my model, consumers are not charged, while INE allow them to interact with more sellers. As in this article, Maruyama and Zennyo (2015) show that if the network effects enjoyed by content providers are sufficiently small, content providers' surplus increases, and vice versa. While this article shares some similarities with Maruyama and Zennyo (2015), its setting differs in several key respects. Specifically, it focuses on transaction platforms that feature per-transaction pricing and an endogenous referral fee, which are characteristics not found in the content provision platforms studied by Maruyama and Zennyo (2015). In addition, this article examines the effects of a non-consolidating merger with INE, thereby extending the scope of the analysis. Most notably, in contrast to both Maruyama and Zennyo (2015) and prior studies on interoperability and indirect network effects, this article introduces an empirical test of the model's predictions.

To the best of my knowledge, other contributions to this literature have focused on cases in which interoperability is symmetric across platforms. An established result in this setting is that symmetric platforms serving single-homing users have an excessive incentive to be compatible with respect to a social planner. This is because, if users single-home, compatibility renders demand less elastic. This benefits platforms but might hurt users in absence of sufficient market expansion, as in Crémer et al. (2000). Doganoglu and Wright (2006) make this argument in their study of the interplay between platforms' compatibility decisions and users' decisions to multi-home. When users can multi-home, compatibility leads to market contraction because it makes multi-homing unnecessary, as users can already benefit from all the network externalities without joining more than one platform

(Doganoglu and Wright, 2006; Salim, 2010). Then, if users multi-home, compatibility makes demand more elastic (as consumers joining decisions are rival), which reduces prices and profits while increasing consumer surplus. Thus, platforms have insufficient incentives to be compatible with respect to the social optimum. Rasch and Wenzel (2014) encompass both cases in their study of a competitive-bottleneck model with single-homing users and multi-homing content developers, both subject to a membership fee charged by horizontally-differentiated symmetric platforms. They find that the private incentives to choose compatibility can be insufficient or excessive. The key mechanism here is the change in content provision. From a social-welfare perspective, compatibility is desirable if it increases content, which happens if content providers' network effects are sufficiently strong, and vice versa.

This article contributes to this literature by examining the relatively under-researched, yet prevalent, case of asymmetric compatibility (i.e., INE) between platforms. This focus on asymmetric compatibility in platforms facilitating interactions between two user groups experiencing indirect network effects sets it apart from previous studies (with the exception of Maruyama and Zennyo (2015)). The main departure from this literature lies not only in the focus on INE, but also on some key modeling choices, namely per-transaction prices and referral fees, as well as differential transportation costs across user groups. Although the focus of this article is not determining when the choice of (asymmetric) interoperability is socially-optimal, it shows that platforms might or might not choose to be interoperable; however, it always benefits consumers. Moreover, I provide a new allocative efficiency mechanism to explain the a priori ambiguous effect of compatibility on seller surplus. This mechanism relies on the extent to which platform differentiation allows sellers to reallocate across platforms and increase multi-homing. With sufficiently low seller transportation costs, the increase in number of sellers in the aggregator platform (where prices decrease and the number of buyers increases) surpasses the decrease in the number of sellers present in the source platform (where prices increase with respect to the increase in seller utility generated by more INE-driven interactions), leading to an increase in overall seller surplus, and vice versa

Non-consolidating horizontal platform mergers. The literature on non-consolidating horizontal platform mergers has explored the conditions under which the merged entity has incentives to lower prices and thus benefit at least one side of the market. Chandra and Collard-Wexler (2009) were the first to make this point in a model of mergers between newspapers intermediating between advertisers and readers. The latter are assumed to be heterogeneous and hence advertisers' value of being in the platform depends on the composition of consumers they can access. Because the platform cannot target consumers and price-discriminate, a marginal consumers' contribution might be negative. Therefore, price hikes might lead consumers to choose a newspaper to which their contribution to profit is negative. This can generate incentives to lower prices post-merger.

One of the main results that the literature subsequently established under various settings is that, if network effects are sufficiently strong, the merged entity has incentives to lower prices post-merger. Leonello (2010) makes this argument in a model of two platforms competing à la Hotelling on both sides of the market. If indirect network effects are sufficiently strong in side 2, the price decreases and demand increases in side 1. Important to this article, post-merger, the platforms also become interoperable in that side-1 users in platform A can access side-2 users of both platforms. This reinforces the merged entity's incentives to lower prices. Baranes et al. (2014) extend Leonello (2010)'s model to four platforms equidistantly located on a Salop circle with linear externalities and full market coverage. Similarly, they conclude that mergers between adjacent platforms may lead to lower prices if externalities are sufficiently strong. Tan and Zhou (2019) reverse this argument with a model that includes the possibility of non-linear externalities and in which consumers have a random utility function. Assuming full market coverage, they show that the merged entity always has incentives to increase prices unless there are strong cost-related efficiency gains. Finally, Garcia and Li (2024) study non-consolidating mergers between horizontally differentiated platform mergers with a focus on interplay between network effects, multihoming, and post-merger strategies (bundling and tying). In the competitive bottleneck case, they also show that, if network effects are sufficiently strong, users benefit from the merger, and lose surplus from it otherwise. Before the merger, multihoming users create positive externalities that benefit singlehoming users on the rival platform. After the merger, the monopolist internalizes these cross-platform spillovers, strengthening incentives to attract multihomers and raising overall welfare.

I contribute to this literature by exploring a new setting that generates an unstudied reason why a non-consolidating horizontal platform merger might harm users. In a competitive bottleneck model with INE and per-transaction pricing on the multi-homing side, the less differentiated the platforms are on the single-homing side, the more likely it is that the merger will harm both sides of the market. The reason is that, despite the overall increase in the number of sellers triggered by the merger, strong differentiation discourages single-homing users from sufficiently switching from the aggregator platform (where the number of sellers decreases) to the source platform (where the number of sellers increases).

3 The model

In this section I present the model in its two settings: with and without (i.e., the benchmark) intertwined network effects. In accordance with the observed features of matching platforms that set up INE, the model considers (i) a competitive bottleneck setting in which buyers single-home and sellers can multi-home[4]; (ii) a per-transaction price on the seller side and zero-pricing on the buyer side; and (iii) when INE are set up, an endogenously-determined

[4]Duch-Brown (2017) provides empirical evidence of the prevalence of seller multi-homing in Europe for a wider scope of platforms that includes the following categories: "marketplaces", "apps stores", "social networks" and "online advertising".

(per-transaction) referral fee charged by the aggregator to the source platform.

Model overview. There are two symmetric platforms in the market that compete to enable interactions between a unit mass of sellers and a unit mass of buyers. This interaction generates homogeneous, positive indirect network effects to both user groups. Platform $i \in \{0,1\}$ competes for sellers through the per-transaction price p_i^s it charges them. The platforms have constant marginal costs, which, for simplicity, are normalized to zero. Buyers and sellers perceive the platforms as horizontally differentiated. This horizontal differentiation, modeled through transportation costs, captures three non-mutually exclusive sources of platform heterogeneity. First, platforms may differ in their user experience or business models. For instance, some facilitate auctions while others enable direct sales. Second, users develop platform-specific behavioral or cognitive habits that make switching costly, even when platforms offer similar functionalities. Third, platforms may specialize in certain product categories, transaction types, or geographic markets, rendering them imperfect substitutes. I model horizontal differentiation à la Hotelling, where platform 0 is located at $x = 0$ and platform 1 at $x = 1$. Both consumers and sellers are uniformly distributed on a unit interval and face an opportunity cost of joining a platform that increases linearly over the distance at rates τ^b and τ^s, respectively. This cost translates consumer-specific preferences for one platform over the other that are unrelated to the number of sellers the platform gives access to or its standalone value. These include, inter alia, platform branding, user interface, integration with other tools or platforms, psychological switching costs, or loyalty programs.

Buyers single-home, whereas sellers partially multi-home. Buyers and sellers interact with a seller every time they meet. Each interaction generates a benefit α^b for the buyer and α^s for the seller. Buyers obtain the same stand-alone utility v^b from joining a platform. I assume this benefit is sufficiently high for the market to be covered on the buyer side. Let n_i^b and n_i^s denote the mass of buyers and sellers active on platform i, respectively.

In the following subsections, two settings of the model are described: the benchmark (denoted with the superscript B) and the intertwined network effects setting (denoted with the superscript *INE*).

3.1 Benchmark setting

Utility and profit functions. In the benchmark, the utility of a buyer located at $x \in [0,1]$ is:

$$U_i^{b^B}(x, p_0^s, p_1^s) := v^b + \alpha^b n_i^s(p_0^s, p_1^s) - \tau^b |x_i - x| \tag{1}$$

The utility of a seller located at $x \in [0,1]$ is:

$$U_i^{s^B}(x, p_0^s, p_1^s) := n_i^b(p_0^s, p_1^s)(\alpha^s - p_i^s) - \tau^s |x_i - x| \tag{2}$$

Thus, the utility of a multi-homing seller, indicated by superscript mhs, is:

$$U_i^{mhs^B} := n_0^b(\alpha^s - p_0^s) + n_1^b(\alpha^s - p_1^s) - \tau^s \tag{3}$$

The profit of platform $i \in \{0, 1\}$ is given by:

$$\Pi_i{}^B := n_i^b(p_0^s, p_1^s)n_i^s(p_0^s, p_1^s)p_i^s \tag{4}$$

Following the literature, Equations 1-4 assume consumers interact once with every seller accessible through their platform. Under the innocuous assumption that an agent's utility increases monotonically with the number of agents on the other side, this assumption allows to interpret payoffs expected utilities and profits.

Timing. In the first stage, platforms simultaneously set the per-transaction prices charged to sellers (p_0^s, p_1^s). In the second stage, consumers and sellers simultaneously choose which platform(s) to join.

3.2 Intertwined network effects setting

Utility and profit functions. In a setting in which platforms choose to set up intertwined network effects (INE, hereafter) at no cost, the buyers served by platform 0 can access all the sellers that have joined platform 1 in addition to the sellers having joined platform 0. Conversely, the sellers that decide to join platform 1 can access not only the buyers served by platform 1, but also those served by platform 0. For simplicity, we assume that neither buyers nor sellers face switching costs when interacting with a user from another platform.[5] Hereafter, I refer to platform 0 as the *aggregator* and to platform 1 as the *source platform*.

Sources of INE-generated surplus. Intertwined network effects generate surplus for consumers and sellers through two mutually inclusive channels. First, by expanding the range and diversity of potential matches between buyers and sellers, INE enhance match quality. For instance, since November 2014, a buyer seeking a second-hand car on the generalist classifieds platform Milanuncios can also view listings from the specialized car sales platform Coches.net. This broader selection increases the likelihood that consumers find a vehicle suited to their preferences while reducing search costs, thereby boosting consumer utility. Conversely, INE benefit car sellers even without increasing sales volume. By enlarging the pool of potential buyers with heterogeneous preferences, INE can facilitate faster sales at

[5] In the model, transportation costs (τ^b, τ^s) capture platform-affiliation frictions due to horizontal differentiation (e.g., business model, interface, habits, or specialization), rather than transaction-level frictions conditional on affiliation. Hence, when INE enable cross-platform interactions, users do not incur an additional transportation costs. If INE are implemented via click-out, cross-platform transactions may involve an additional interaction friction (e.g., redirection or re-authentication), which is abstracted from here. Equivalently, it can be interpreted as lowering the effective cross-platform match value.

lower search costs. For the same reasons, sellers may be able to raise prices either through negotiation or due to the heightened demand that INE generate.

Second, INE can raise the total number of transactions by connecting buyers with sellers they might not have encountered otherwise. This is common on platforms affected by INE where both the platforms and some sellers offer products across multiple categories. For example, Google Shopping and Amazon Marketplace both feature extensive, overlapping categories such as electronics, home & kitchen, fashion, sports, toys, and beauty. Since Google Shopping includes listings from Amazon Marketplace, a consumer browsing for a frying pan on Google Shopping might complete their purchase on Amazon Marketplace, buying, for example, a cooking knife from the same or a different seller. As a result, it is common to see some sellers multi-homing across platforms linked via INE, a phenomenon incorporated in my model.

Full double counting assumption. I capture the surplus generated by INE through the two channels discussed above by considering asymmetric cross-platform network effects (cf. terms in bold in Equations 5 and 6). To keep the model tractable, I assume "full double counting" of these network effects from overlapping agents. In other words, if a consumer and a multi-homing seller meet twice, they both obtain two times the benefit of the interaction. In the motivating examples of platforms that set up INE, it is likely that there is "partial double counting", whereby meeting an agent on the other side of the market a the second time yields partial additional network benefits. The results obtained through the *ad summum* assumption of full double counting provides an upper bound of the benefits of setting up INE for both user groups.

Transactions between a platform 0 buyer and a platform 1 seller take place in platform 1. platform 0 charges a referral fee f to platform 1 for each of these transactions. Therefore, in the INE setting, the utility of a buyer located at $x \in [0, 1]$ joining platform $i \in \{0, 1\}$ is given by:

$$U_i^{b\,INE} := \begin{cases} v^b + \alpha^b \left(n_0^s(p_0^s, p_1^s) + \mathbf{n_1^s}(\mathbf{p_0^s}, \mathbf{p_1^s})\right) - \tau^b|0 - x| & \text{if } i = 0 \\ v^b + \alpha^b\, n_1^s(p_0^s, p_1^s) - \tau^b|1 - x| & \text{if } i = 1 \end{cases} \tag{5}$$

The utility of a seller located at $x \in [0, 1]$ joining platform $i \in \{0, 1\}$ is in turn given by:

$$U_i^{s\,INE} := \begin{cases} n_0^b(p_0^s, p_1^s)\,(\alpha^s - p_0^s) - \tau^s|0 - x| & \text{if } i = 0 \\ \left(\mathbf{n_0^b}(\mathbf{p_0^s}, \mathbf{p_1^s}) + n_1^b(p_0^s, p_1^s)\right)(\alpha^s - p_1^s) - \tau^s|1 - x| & \text{if } i = 1 \end{cases} \tag{6}$$

And the profit of platform $i \in \{0, 1\}$ is given by:

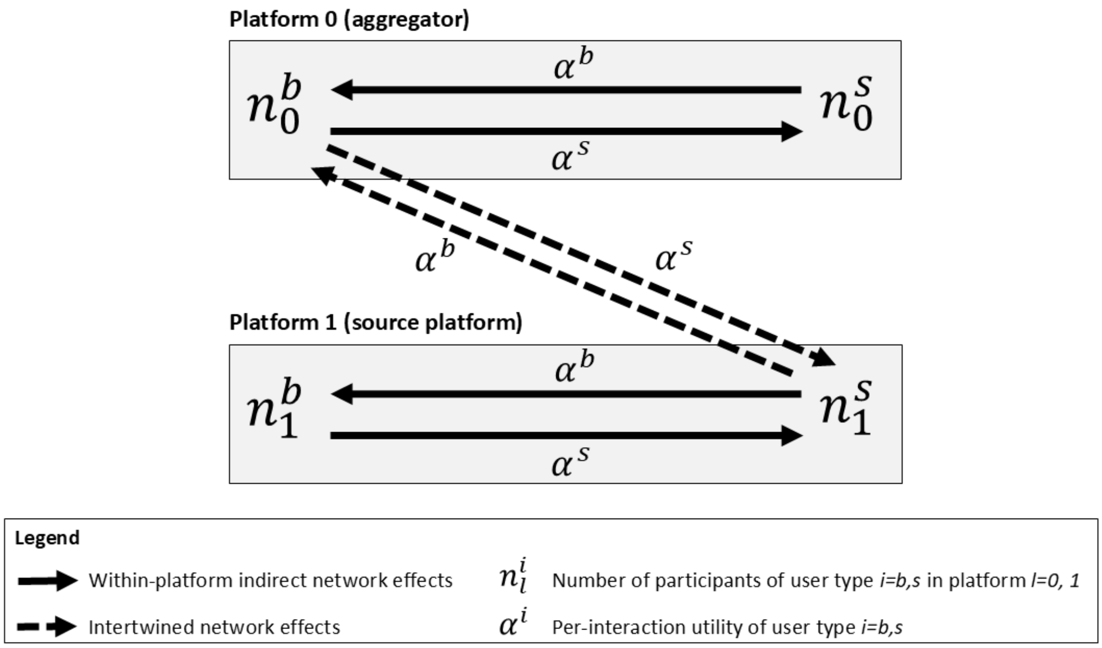
$$\Pi_i{}^{INE} := \begin{cases} n_0^b(p_0^s,p_1^s)\, n_0^s(p_0^s,p_1^s)\, p_0^s + \mathbf{n_0^b(p_0^s,p_1^s)\, n_1^s(p_0^s,p_1^s)\, f} & \text{if } i=0 \\ n_1^b(p_0^s,p_1^s)\, n_1^s(p_0^s,p_1^s)\, p_1^s + \mathbf{n_0^b(p_0^s,p_1^s)\, n_1^s(p_0^s,p_1^s)\, (p_1^s - f)} & \text{if } i=1 \end{cases} \tag{7}$$

In Equations 5- 7, the new terms with respect to the benchmark setting appear in bold. These correspond to the new interactions (Equations 5 and 6) and sources of revenue (Equation 7) that INE allow for.

Timing. In the first stage, platform 0 sets the per-transaction referral fee f. In the second stage, platform 1 decides whether to accept to set-up intertwined network effects or not. In the third stage, platforms simultaneously set the per-transaction prices charged to sellers (p_1^1, p_1^s). In the fourth stage, consumers and sellers simultaneously choose which platform(s) to join.

Table 6 in Appendix A summarizes the notation used throughout the article. Figure 1 illustrates the network effects that exist in each setting.

Figure 1: Indirect network effects with and without intertwined network effects



4 Intertwined network effects between legally-independent firms

In this section I characterize the equilibria of the two settings described in Section 3. I assume full information for all agents in the model, i.e., each agent observes all the price decisions and knows all the parameters of the model. The equilibrium concept is the Subgame Perfect Nash equilibrium.

4.1 Equilibrium without intertwined network effects (benchmark)

In this subsection I consider the benchmark setting described in Section 3.1. The aim is to characterize the equilibrium in which both platforms are active. To that end, I make the following assumptions.

Assumption 1 (Benchmark conditions)**.** *In the benchmark setting without intertwined network effects, the parameters satisfy the following conditions.*

$$\alpha^b\alpha^s < \tau^b\tau^s \tag{SOCs^B}$$

$$\alpha^b\alpha^s < \tau^b\tau^s \text{ and } \tau_b^2\tau^s < 4v_b^2\tau^s + \tau^b\alpha^b\alpha^s \tag{FPB^B}$$

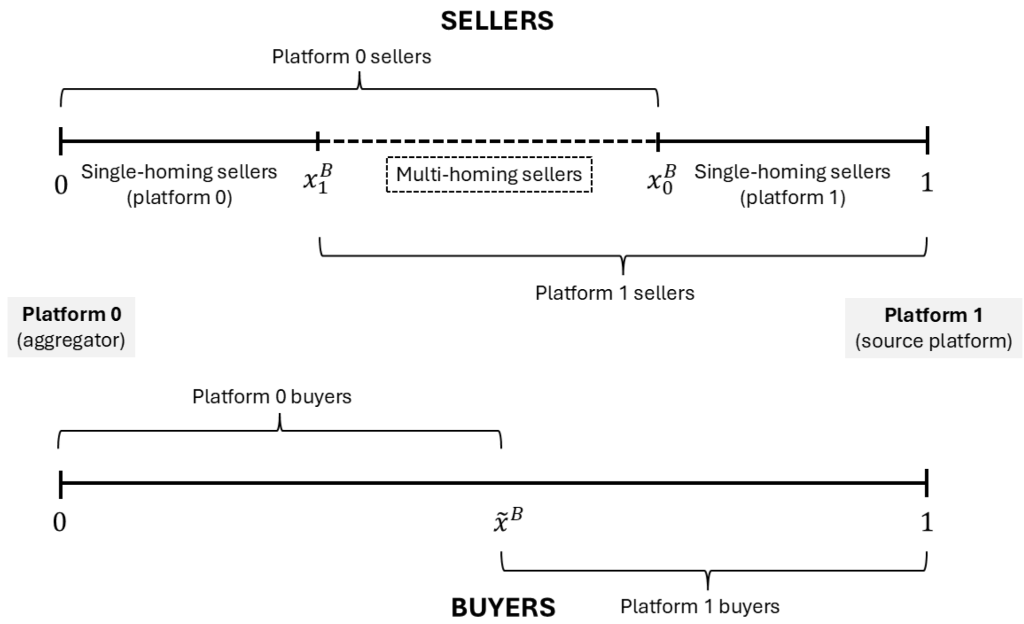
$$\begin{aligned} &-\tau^b\tau^s + \tau^s\alpha^b + \sqrt{-\tau^b\tau^s(\tau^b\tau^s - \alpha^b\tau^s)} < 0 \quad \text{and} \\ &-\tau^b\tau^s + 2\tau^s\alpha^b + \sqrt{-\tau^b\tau^s(\tau^b\tau^s - \alpha^b\tau^s)} > 0 \end{aligned} \tag{MHS^B}$$

The first condition requires that the second-order conditions of the platforms' maximization program are met. As is standard in platform competition models, this happens if indirect network effects are sufficiently small relative to platforms' horizontal differentiation. This also guarantees a stable and unique equilibrium in which both platforms are active. The second, more stringent condition ensures full market coverage on the buyer side by verifying that the consumer indifferent between the two platforms obtains positive surplus: $U_i^{b^B}(\tilde{x}^B) > 0$. The third condition imposes partial multi-homing by sellers in equilibrium, which in turn implies full coverage on the seller side. Then, in equilibrium, $0 < x_1^B < x_0^B < 1$.

Stage 2: demand configuration In the second stage, the consumer indifferent between joining platform 0 and 1 is located at $\tilde{x}^B$ such that $U_0^{b^B}(\tilde{x}^B) = U_1^{b^B}(\tilde{x}^B)$. Thus, the number of consumers buying from platform 0 is equal to $\tilde{x}^B$ and the number of consumers buying from platform 1 is equal to $1 - \tilde{x}^B$. Sellers are divided into three sub-intervals on the unit interval. Sellers located "on the left" only join platform 0. Those located "on the right" join only platform 1. Those located "in the middle" join both platforms and can thus interact with both platforms' buyers. The seller indifferent between joining platform 0 and not joining it is located at x_0^B such that $U_0^{s^B} = 0$. The seller indifferent between joining platform 1 and not joining it is located at x_0^B such that $U_1^{s^B} = 0$. Then, the sellers that single-home in platform 0 are located in the $[0, x_0^B]$ sub-interval, those who single-home in platform 1 in the $[x_0^B, 1]$ sub-interval and those who multi-home in the (x_1^B, x_0^B) sub-interval. To focus on the interesting case in which there is multi-homing in the benchmark, I assume for the time being that $0 < x_1^B < x_0^B < 1$ (I provide the necessary and sufficient conditions below), so that $n_0^s = x_0^B$ and $n_1^s = 1 - x_1^B$.[6] Figure 2 illustrates these intervals.

[6]Note that the existence of multi-homing on the seller side in the benchmark allows for an elastic demand on the money-making side of the market. This will be important to explain the effects of INE (cf. Section 4.3)

Figure 2: Demand configuration for buyers and sellers in the benchmark



Then, the number of buyers and sellers in each platform is found by solving the following system of four equations and four unknowns:

$$
\begin{aligned}
n_0^b &= \frac{\tau^b + n_0^s\alpha^b - n_1^s\alpha^b}{2\tau^b} \\
n_1^b &= 1 - \frac{\tau^b + n_0^s\alpha^b - n_1^s\alpha^b}{2\tau^b} \\
n_0^s &= \frac{n_0^b(\alpha^s - p_0^s)}{\tau^s} \\
n_1^s &= 1 - \frac{n_1^b(p_1^s + \tau^s - \alpha^s)}{\tau^s}
\end{aligned}
$$

Which yields buyer- and seller-side demand as a function of seller prices and the model's parameters.

$$
\begin{aligned}
n_0^b(p_0^s, p_1^s) &= \frac{\tau^b\tau^s + \alpha^b(p_1^s - \alpha^s)}{2\tau^b\tau^s + \alpha^b(p_0^s + p_1^s - 2\alpha^s)} \\
n_1^b(p_0^s, p_1^s) &= \frac{\tau^b\tau^s + \alpha^b(p_0^s - \alpha^s)}{2\tau^b\tau^s + \alpha^b(p_0^s + p_1^s - 2\alpha^s)} \\
n_0^s(p_0^s, p_1^s) &= \frac{(p_0^s - \alpha^s)(\alpha^b(\alpha^s - p_1^s) - \tau^b\tau^s)}{\tau^s(2\tau^b\tau^s + \alpha^b(p_0^s + p_1^s - 2\alpha^s))} \\
n_1^s(p_0^s, p_1^s) &= \frac{(p_1^s - \alpha^s)(\alpha^b(\alpha^s - p_0^s) - \tau^b\tau^s)}{\tau^s(2\tau^b\tau^s + \alpha^b(p_0^s + p_1^s - 2\alpha^s))}
\end{aligned}
\tag{8}
$$

and of a merger between platforms having set up INE (cf. Section 5.2).

Stage 1: platforms' choice of prices. In stage 1, each platform solves the maximization program $\max_{p_i^s} \Pi^i(p_0^s, p_1^s)$. Solving the system of first-order conditions yields only one set of equilibrium symmetric prices that satisfy the second-order conditions and for which $0 < x_1^B < x_0^B < 1$ (i.e., for which there is multi-homing on the seller side):

$$p_i^s = \alpha^s - \frac{\tau^b \tau^s \sqrt{\tau^b \tau^s (\tau^b \tau^s - \alpha^b \alpha^s)}}{\alpha^b} \tag{9}$$

The equilibrium per-interaction price charged to sellers depends positively on the benefit they obtain from an interaction with buyers on the platform, net of transportation costs. This is evident from (9), where the price increases with α^s and decreases with τ^s.[7] This price also depends positively on the per-interaction net benefit buyers obtain from participating in the platform. A higher net benefit for buyers increases their demand for the platform, making the platform more valuable for sellers and thus leading to a higher price.

Replacing these equilibrium prices in (8) gives the equilibrium buyer- and seller-side demand:

$$\begin{aligned} n_b^i &= \frac{1}{2}, \\ n_s^i &= \frac{\tau^b}{2\alpha^b} - \frac{\sqrt{\tau^b \tau^s (\tau^b \tau^s - \alpha^b \alpha^s)}}{2\tau^s \alpha^b}. \end{aligned}$$

Consumer and seller surplus are calculated respectively as:

$$\begin{aligned} CS^B &:= \int_0^{\tilde{x}^B} U_0^{b\,B}(x)dx + \int_{\tilde{x}^B}^1 U_1^{b\,B}(x)dx, \\ SS^B &:= \int_0^{x_0^B} U_0^{s\,B}(x)dx + \int_{1-x_1^B}^1 U_1^{s\,B}(x)dx. \end{aligned}$$

Then, the equilibrium values of the model in the benchmark setting (i.e., without intertwined network effects) are the following.

Lemma 1 (**Equilibrium without intertwined network effects**). *In the benchmark setting*

[7]The term $-\frac{\tau^b \tau^s \sqrt{\tau^b \tau^s (\tau^b \tau^s - \alpha^b \alpha^s)}}{\alpha^b}$ is strictly negative, as per the second-order condition on the seller side reported in Assumption 1.

equilibrium:

$$
\begin{aligned}
n_b^i &= \frac{1}{2}, \\
n_b^B &= 1, \\
n_i^s &= \frac{\tau^b}{2\alpha^b} - \frac{\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{2\tau^s\alpha^b}, \\
n_s^B &= \frac{\tau^b\tau^s - \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{\tau^s\alpha^b}, \\
p_s^i &= \alpha^s - \frac{\tau^b\tau^s\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{\alpha^b}, \\
\Pi_i^B &= \frac{\left(\tau^b\tau^s - \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)\left(\alpha^b\alpha^s + \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)} - \tau^b\tau^s\right)}{4\tau^s{\alpha^b}^2}, \\
\Pi^B &= \frac{\left(\tau^b\tau^s - \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)\left(\tau^b\tau^s - \alpha^b\alpha^s - \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)}{2\tau^s{\alpha^b}^2}, \\
CS^B &= v^b - \frac{\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{2\tau^s}, \\
SS^B &= \frac{\tau^b\left(4\tau^s\alpha^b + \alpha^b\alpha^s + 2\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right) - 2{\tau^b}^2\tau^s - 2\alpha^b\left(\tau^s\alpha^b + 2\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)}{4{\alpha^b}^2}.
\end{aligned}
$$

4.2 Equilibrium with intertwined network effects

I now turn to a setting in which platforms decide to set up intertwined network effects and characterize its equilibrium. In this setting, I make the assumptions below, which are analogous to those presented in Assumption 1.

Assumption 2 (Intertwined network effects conditions)**.** *In the intertwined network effects setting, the parameters satisfy the following conditions.*

$$\alpha^b\alpha^s < 2\tau^b\tau^s, \tag{SOCs^{INE}}$$

$$\frac{4(4v_B - 3\tau^b)\tau^b{\tau^s}^2 + (-8v_B + 11\tau^b)\tau^s\alpha^b\alpha^s - 2{\alpha^b}^2{\alpha^s}^2}{8\tau^s\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)} + \frac{\sqrt{\tau^b\tau^s\left(16{\tau^b}^3{\tau^s}^3 - 8{\tau^b}^2{\tau^s}^2\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2{\alpha^s}^2 - 2{\alpha^b}^3{\alpha^s}^3\right)}}{8\tau^s\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)} > 0, \tag{FPB^{INE}}$$

$$0 < 1 - \frac{\alpha^s}{4\tau^s} < \frac{\tau^b\tau^s\left(-4\tau^b\tau^s + 3\alpha^b\alpha^s\right)}{4\tau^s\alpha^b\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)}, \quad \frac{\sqrt{\tau^b\tau^s\left(16{\tau^b}^3{\tau^s}^3 - 8{\tau^b}^2{\tau^s}^2\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2{\alpha^s}^2 - 2{\alpha^b}^3{\alpha^s}^3\right)}}{4\tau^s\alpha^b\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)} < 1, \tag{MHS^{INE}}$$

To present shorter mathematical expressions, let me introduce the following additional notation:

$$\Omega := \sqrt{\tau^b\tau^s(16\tau^{b^3}\tau^{s3} - 8\tau^{b^2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b^2}\alpha^{s2} - 2\alpha^{b^3}\alpha^{s3})} \tag{10}$$

To avoid reverting to the benchmark case, I focus on the equilibrium in which, in stage 2, platform 1 accepts to set-up INE.[8] In the fourth stage, the consumer that is indifferent between joining platform 0 and 2 is located at $\tilde{x}^{INE}$ such that $U_0^{b^{INE}}(\tilde{x}^{INE}) = U_1^{b^{INE}}(\tilde{x}^{INE})$. Thus, the number of consumers served by platform 0 is equal to $\tilde{x}^{INE}$ and the number of consumers served by platform 1 is equal to $1 - \tilde{x}^{INE}$. The seller indifferent between joining platform 0 and not joining it is located at x_0^{INE} such that $U_0^{s^{INE}} = 0$. The seller indifferent between joining platform 1 and not joining it is located at x_1^{INE} such that $U_1^{s^{INE}} = 0$. Then, the sellers that single-home in platform 0 are located in the $[0, x_1^{INE}]$ sub-interval, those who single-home in platform 1 in the $[x_0^{INE}, 1]$ sub-interval and those who multi-home in the (x_1^{INE}, x_0^{INE}) sub-interval. As in the previous setting, I assume for the time being that $0 < x_1^{INE} < x_0^{INE} < 1$ (I provide the necessary and sufficient conditions below), so that $n_0^{s^{INE}} = x_0^{INE}$ and $n_1^{s^{INE}} = 1 - x_1^{INE}$.

Then, the number of buyers and sellers in each platform is found by solving the following system of four equations and four unknowns:

$$\begin{aligned}
n_0^{b^{INE}} &= \frac{\tau^b + n_0^{s^{INE}}\alpha^b}{2\tau^b} \\
n_1^{b^{INE}} &= 1 - \frac{\tau^b + n_0^{s^{INE}}\alpha^b}{2\tau^b} \\
n_0^{s^{INE}} &= \frac{n_0^{b^{INE}}(\alpha^b - p_0^s)}{\tau^s} \\
n_1^{s^{INE}} &= 1 - \frac{n_0^{b^{INE}}p_1^s + n_1^{b^{INE}}p_1^s + \tau^s - n_0^{b^{INE}}\alpha^s - n_1^{b^{INE}}\alpha^s}{\tau^s}
\end{aligned}$$

Which yields buyers- and seller-side demand a function of seller prices and the model's parameters.

$$\begin{aligned}
n_0^{b^{INE}}(p_0^s) &= \frac{\tau^b\tau^s}{2\tau^b\tau^s + p_0^s\alpha^b - \alpha^b\alpha^s} \\
n_1^{b^{INE}}(p_0^s) &= \frac{\tau^b\tau^s}{-2\tau^b\tau^s + \alpha^b(\alpha^b - p_0^s)} - 1 \\
n_0^{s^{INE}}(p_0^s) &= \frac{\tau^b(\alpha^s - p_0^s)}{2\tau^b\tau^s - \alpha^b(\alpha^s - p_0^s)} \\
n_1^{s^{INE}}(p_1^s) &= \frac{\alpha^s - p_1^s}{\tau^s}
\end{aligned} \tag{11}$$

[8] As illustrated by Figure 3, if platforms are sufficiently differentiated for buyers and sellers, setting-up INE is a dominant strategy for both.

Note that, while in (8) buyer- and seller-side demand depend on the price charged to sellers in both platforms, this is not the case in (11). With INE, the only price implicitly considered by buyers when deciding which platform to join is the per-transaction price charged to sellers in the aggregator (platform 0). Buyers know that, if they join platform 1, they can only interact with platform 1 sellers. If they join platform 0, they have access to both platforms' sellers and additional interactions with multi-homing platform 0 sellers. Hence, the difference in the number of sellers they can expect to interact with depends only on platform 0 prices to sellers, which determines the number of sellers joining platform 0.

As for sellers, under INE, their decision to join a platform does not depend on the rival platform's price charged to them. Sellers know that if they join platform 1 they will have access to all consumers at a p_1^s per-interaction price, and that if they (also) join platform 0 they will have (additional) interactions with consumers from platform 0 at p_0^s per interaction. This makes the decision to join each platform depend only of that platform's price. Additionally, note that, because platform 1 gives sellers access to all consumers, sellers' decision to join it do not depend on consumers' transportation costs in (11). This is not the case in absence of INE, as seen in (8).

In stage 3, each platform solves the maximization program $\max_{p_i^s} \Pi^i(p_0^s, p_1^s, f)$. Solving the system of first-order conditions yields a unique set of equilibrium prices:

$$
\begin{aligned}
p_0^{s*} &= \frac{f^2\tau^b\tau^s\alpha^b + 2\tau^b\tau^s\alpha^s(2\tau^b\tau^s - \alpha^b\alpha^s) + f\alpha^b(\alpha^b\alpha^s - 2\tau^b\tau^s)}{8\tau^{b^2}\tau^{s2} - 2\tau^b\tau^s\alpha^b\alpha^s + f\alpha^{b^2}\alpha^s} \\
p_1^{s*} &= \frac{f^2\alpha^{b^2}\alpha^s + 4\tau^b\tau^s(2\tau^b\tau^s - \alpha^b\alpha^s) + f\tau^b\tau^s(4\tau^b\tau^s - \alpha^b\alpha^s)}{f^2\alpha^{b^2} + 8\tau^b\tau^s(2\tau^b\tau^s - \alpha^b\alpha^s)}
\end{aligned}
\tag{12}
$$

In stage 1, platform 0 sets the optimal per-transaction referral fee that it charges platform 1 for every interaction between a platform 0 user and a platform 1 seller. To do so, it solves the maximization program $\max_{f} \Pi_0(p_0^{s*}, p_1^{s*}, f)$. The only value of f that satisfies the first and second-order condition of this maximization program is:

$$
f^* = \frac{2\big(\tau^b\tau^s(\alpha^b\alpha^s - 4\tau^b\tau^s) + \Omega\big)}{\alpha_b^2\alpha^s}
\tag{13}
$$

Replacing (12) and (13) in (11), I obtain the quantities on both sides of the market and the locations of the indifferent s in equilibrium. Consumer and seller surplus are calculated in the same way as in the benchmark setting and using the corresponding utility functions (cf. Section 3.2) and equilibrium threshold values.

Then, the equilibrium values of the model in the intertwined network effects setting are the following.

Lemma 2 (**Equilibrium with intertwined network effects**). *In the intertwined network* *effects setting equilibrium:*

$$
\begin{aligned}
n_0^{b\,INE} &= \frac{4\tau^{b2}\tau^{s2} - \tau^b\tau^s\alpha^b\alpha^s + \Omega}{16\tau^{b2}\tau^{s2} - 8\tau^b\tau^s\alpha^b\alpha^s},\\
n_1^{b\,INE} &= \frac{12\tau^{b2}\tau^{s2} - 7\tau^b\tau^s\alpha^b\alpha^s - \Omega}{16\tau^{b2}\tau^{s2} - 8\tau^b\tau^s\alpha^b\alpha^s},\\
{n_b}^{INE} &= 1,\\
n_0^{s\,INE} &= \frac{\tau^b\tau^s(3\alpha^b\alpha^s - 4\tau^b\tau^s) + \Omega}{4\tau^s\alpha^b(2\tau^b\tau^s - \alpha^b\alpha^s)},\\
n_1^{s\,INE} &= \frac{\alpha^s}{4\tau^s},\\
{n_s}^{INE} &= \frac{\alpha^s\left(2\tau^b\tau^s - \alpha^b\alpha^s\right) + \tau^b\tau^s\left(-4\tau^b\tau^s + 3\alpha^b\alpha^s\right)}{4\tau^s\alpha^b\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)} + \frac{\Omega}{4\tau^s\alpha^b\left(2\tau^b\tau^s - \alpha^b\alpha^s\right)},\\
p_0^{s\,INE} &= \frac{-16\tau^{b3}\tau^{s3} + 4\tau^{b2}\tau^{s2}\alpha^b\alpha^s - 2\tau^b\tau^s\alpha^{b2}\alpha^{s2} + \alpha^{b3}\alpha^{s3} + 4\tau^b\tau^s\Omega}{\alpha^{b3}\alpha^{s2}},\\
p_1^{s\,INE} &= \frac{3\alpha^s}{4},\\
f^{INE} &= \frac{2\big(\tau^b\tau^s(\alpha^b\alpha^s - 4\tau^b\tau^s) + \Omega\big)}{\alpha^{b2}\alpha^s},\\
{\Pi_0}^{INE} &= \frac{(4\tau^b\tau^s - \alpha^b\alpha^s)(-4\tau^{b2}\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s + \alpha^{b2} + \alpha^{s2} + \Omega)}{16\tau^s\alpha^{b2}(2\tau^b\tau^s - \alpha^b\alpha^s)},\\
{\Pi_1}^{INE} &= \frac{\alpha^{s2}}{16\tau^s},\\
\Pi^{INE} &= \frac{\alpha^{s2} + \left(-4\tau^b\tau^s + \alpha^b\alpha^s\right)\left(-4\tau^{b2}\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s + \alpha^{b2}\alpha^{s2}\right)}{16\tau^s\alpha^{b2}\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)} + \frac{\Omega}{16\tau^s\alpha^{b2}\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)},\\
CS^{INE} &= \frac{-16\tau^{b4}\tau^{s4} + 32\tau^{b3}\tau^{s3}\alpha^b\alpha^s - 2\alpha^{b2}\alpha^{s2}\Omega + 8v^b\tau^s(2\tau^b\tau^s - \alpha^b\alpha^s)\big(\tau^b\tau^s(4\tau^b\tau^s - \alpha^b\alpha^s) + \Omega\big)}{32\tau^b\tau^{s2}(\alpha^b\alpha^s - 2\tau^b\tau^s)}\\
&\quad + \frac{\tau^{b2}\tau^{s2}\big(13\alpha^{b2}\alpha^{s2} + 4\Omega\big) + \tau^b\tau^s\alpha^b\alpha^s\big(\alpha^{b2}\alpha^{s2} + 7\Omega\big)}{32\tau^b\tau^{s2}(\alpha^b\alpha^s - 2\tau^b\tau^s)},\\
SS^{INE} &= \frac{1}{32}\big(8\tau^s\left(\frac{\tau^{b2}}{\alpha^{b2}} - 2\right) - \frac{3\alpha^{s2}}{\tau^s} + \frac{16\alpha^{b4}\alpha^{s3} + 2\alpha^{b2}\alpha^s\big(\alpha^{b2}\alpha^s(32\tau^s + 3\alpha^s) - 4\Omega\big)}{\alpha^{b2}(\alpha^b\alpha^s - 2\tau^b\tau^s)^2}\\
&\quad + 2\tau^b\alpha^b\alpha^s\big(-\alpha^{b2}\alpha^s(32\tau^s + \alpha^s) + 3\Omega\big)\big).
\end{aligned}
$$

Note that, in the equilibrium with intertwined network effects, both platforms are active. There would be tipping on the buyer side in favor of the aggregator platform if the platforms are sufficiently homogeneous on the buyer side (i.e., if $\tau^b \leq \frac{3\alpha^b\alpha^s(3+\sqrt{11})}{8\tau^s}$). However, this condition is incompatible with the second order condition of the no-INE benchmark set out in Assumption 1.

4.3 Comparison between the equilibrium with and without intertwined network effects

In this subsection, I compare the equilibrium of the INE and the benchmark settings. Let me first introduce the following remark.

Remark 1. *Let $Cond^B$ and $Cond^{INE}$ be the parameter spaces defined by Assumptions 1 and 2, respectively.*

i) $\exists \quad S^B \quad \subseteq \quad Cond^B \quad | \quad S^B \cap Cond^{INE} = \varnothing$

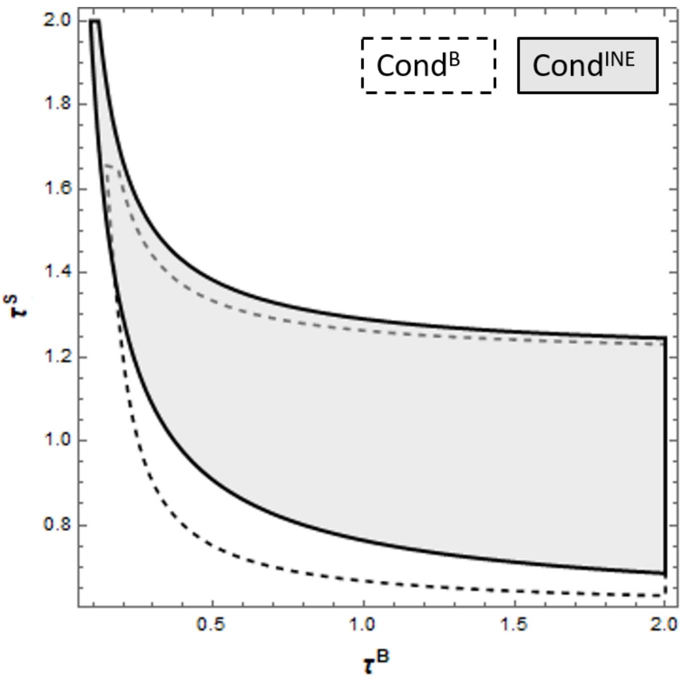
ii) $\exists \quad S^{INE} \quad \subseteq \quad Cond^{INE} \quad | \quad S^{INE} \cap Cond^B = \varnothing$

iii) $\exists \quad S^{B \cap INE} = Cond^B \cap Cond^{INE} \neq \varnothing$

Remark 1 characterizes three scenarios illustrated in Figure 3: parameters supporting only the benchmark equilibrium, only INE, or both. The overlap between these parameter spaces is considerable. Crucially, the non-overlapping regions—where only one equilibrium exists—correspond to parameter values with sufficiently low transportation costs that platforms become near-perfect substitutes. This reveals an important constraint: INE are only viable when platforms are horizontally differentiated enough to maintain distinct user bases, as excessive competition erodes the value of cross-platform access.

My analysis focuses on the $S^{B \cap INE}$ region (overlapping areas in Figure 3) where both equilibria are feasible. Within this region, Lemma 3 shows that platforms unilaterally prefer to implement INE.

Figure 3: Parameter spaces for the benchmark and the INE cases



Parameter setting: $\alpha^b = 0.1, \alpha^s = 2.4$ and $v^b = 1.3$.

Lemma 3 (**Platforms' incentives to set up intertwined network effects**)**.** *If Assumptions 1 and 2 hold, setting-up intertwined network effects is a dominant strategy for the aggregator and the source platforms. Formally,* $\Pi_0{}^{INE} > \Pi_0{}^{B}$ *and* $\Pi_1{}^{INE} > \Pi_1{}^{B}$. ***Proof***: *See Appendix B.1.*

Lemma 3 shows that, whenever both setting and not setting up intertwined network effects can be an equilibrium, setting INE is a dominant strategy for both platforms. The reason is that INE create new indirect network effects compared to the no-INE benchmark. With INE, sellers of the source platform (platform 1) can interact with consumers of the aggregator platform (platform 0). This generates additional surplus that the platforms can share among themselves. Note that, in this article's setting, part of this surplus stems from the assumption of full double counting of the network effects enjoyed by multi-homing sellers. Therefore, if double counting is partial, Lemma 3 might be softened and be valid only if partial double counting is sufficiently strong. In the remainder of this section, I study the effects of setting up INE on buyer- and seller-side demand, prices and each user group's welfare.

4.3.1 Prices

Lemma 4 shows how INE affect prices.

Lemma 4 (**Effect of intertwined network effects on prices charged to sellers**)**.** *When* *platforms set up intertwined network effects, the prices paid by sellers to the source platform increase ($p_1^{s\,INE} > p_1^{s\,B}$). In contrast, the prices paid by sellers to the aggregator platform can either increase or decrease depending on transportation costs. There exists a value $\tilde{\tau^b}(\alpha^b, \alpha^s, \tau^s)$ satisfying Assumptions 1 and 2 such that, if $\tau^b > \tilde{\tau^b}(\alpha^b, \alpha^s, \tau^s)$, they decrease ($p_0^{s\,INE} < p_0^{s\,B}$); otherwise (i.e., if $\tau^b < \tilde{\tau^b}(\alpha^b, \alpha^s, \tau^s)$), they increase ($p_0^{s\,INE} > p_0^{s\,B}$).*

Proof: *See Appendix B.2.*

platform 1 increases the price charged to sellers for two reasons. The first one is that the marginal cost of serving a seller that interacts with a consumer referred by platform 0 increases by f^{INE}. The second one is the increase in the network effects platform 1 offers to sellers given that post-INE, platform 1 sellers can interact with all buyers.

In platform 0, the price charged to sellers increases if consumers' transportation costs are sufficiently low, and decreases otherwise. The intuition is as follows. Post-INE, platform 0's quality to buyers increases, as they can now access both platforms' sellers through it. Hence, as shown in Lemma 5, post-INE, the number of buyers increases in platform 0. The lower transportation costs are, the stronger this increase is. With a sufficiently high increase in the number of buyers, platform 0 becomes so attractive to sellers that the platform maximizes profits by increasing the price it charges them. In this case, platform 0's price increases even if this reduces the number of sellers (and hence the number of consumers), leading to lower revenues through the referral fee. Conversely, if transportation costs are sufficiently low, the post-INE increase in the number of buyers in platform 0 is mild. Hence, the latter maximizes profit by lowering the price charged to sellers. This increases seller-side demand, and hence consumer-side demand, leading to higher revenues through the referral fee.

4.3.2 Demand configuration

Lemma 5 shows how INE affect demand configuration in equilibrium.

Lemma 5 (Effect of intertwined network effects on buyer- and seller-side demand). *When platforms set up intertwined network effects:*

i) The number of consumers increases in the aggregator platform ($n_0^{b\,INE} > n_0^{b\,B}$) and decreases in the source platform ($n_1^{b\,INE} < n_1^{b\,B}$)

ii) The number of sellers increases in the aggregator platform ($n_0^{s\,INE} > n_0^{s\,B}$) and decreases in the source platform ($n_1^{s\,INE} < n_1^{s\,B}$)

iii) The total number of sellers increases ($n_s^{INE} > n_s^B$) if and only if $\tau^b > \frac{(3+2\sqrt{3})\alpha^b\alpha^s}{6\tau^s}$ and decreases ($n_s^{INE} < n_s^B$) otherwise ($\tau^b < \frac{(3+2\sqrt{3})\alpha^b\alpha^s}{6\tau^s}$).

Proof: *See Appendix B.3.*

The results regarding buyer-side demand is intuitive. Post-INE, platform 0 becomes more attractive to buyers, as they can access all sellers through it. Given that buyers are assumed to single-home, this results in an increase in the number of buyers in platform 0 and a decrease in platform 1.

Regarding sellers, in the aggregator platform (platform 0), the number of consumers increases, while the price can increase or decrease depending on transportation costs. However, the net effect on a seller's utility is always positive. Hence, the number of sellers increases in platform 0. In the source platform (platform 1), post-INE, the number of buyers that can be reached increases by $\frac{1}{2}$, while the price also increases. The net effect on a platform 1 seller's utility is a priori ambiguous. However, the decrease in seller utility in platform 1 drives some multi-homing sellers to single-home in platform 0, leading to a decrease in the number of sellers in platform 1.

Hence, the overall number of sellers can only increase if seller multi-homing increases. This is the case if consumers' transportation costs are sufficiently high. As it can be analytically verified, the threshold of τ^b above which seller multi-homing increases is above that for which prices in platform 0 decreases prices. In other words, if seller prices decrease sufficiently in the aggregator platform, the number of sellers that switch from single-homing in platform 1 to multi-homing will exceed that of single-homing platform 1 sellers that start to single-home in platform 0, leading to an overall increase in seller multi-homing.

4.3.3 Buyer and seller surplus

Propositions 1 and 2 show how INE affects consumer and seller surpluses, respectively.

Proposition 1 (Effect of intertwined network effects on consumer surplus). *When platforms set up intertwined network effects, consumer surplus increases ($CS^{INE} > CS^{B}$).*

Proof: *See Appendix B.4.*

The introduction of intertwined network effects (INE) increases aggregate consumer surplus through two channels. For consumers on the **aggregator platform** (platform 0), surplus rises because consumers gain access to all sellers on both platforms, amplifying the intensive margin (more interactions) while the platform attracts more buyers (extensive margin). In contrast, consumers on the **source platform** (platform 1) experience a decline in surplus. Platform 1 loses buyers to the aggregator, reducing its extensive margin, and its diminished seller base curtails the intensive margin.

Critically, the aggregator's gains always outweigh the source platform's losses. The cross-platform interactions enabled by INE allow aggregator consumers to capture surplus from both platforms' sellers, whereas source platform consumers lose access only to their own sellers. This asymmetry ensures that the net effect on consumer surplus is unambiguously positive, regardless of platform differentiation or pricing strategies.[9]

Proposition 2 (**Effect of intertwined network effects on seller surplus**)**.** *When platforms set up intertwined network effects:*

i) If $\tau^s \leq \frac{5}{8}\alpha^s$, seller surplus increases ($SS^{INE} > SS^B$)

ii) If $\tau^s > \frac{5}{8}\alpha^s$, seller surplus decreases ($SS^{INE} < SS^B$)

Proof: *See Appendix B.5.*

Proposition 2 shows that if the platforms are sufficiently differentiated for sellers, INE decrease their surplus, and vice versa. As shown above, post-INE, the utility of a seller increases in platform 0 (the aggregator) and decreases in platform 1 (source platform). Hence, the number of single-homing sellers increases in the former and decreases in the latter. Thus, seller surplus increases in platform 0 and decreases in platform 1. The overall effect depends on which platforms' seller surplus increases the most, which in turn depends on whether the number of sellers increases more in platform 0 or in platform 1. With sufficiently low seller transportation costs, enough sellers stop multi-homing and start single-homing in platform 0, leading to an increase in overall seller surplus, and vice versa.

Finally, as shown in Corollary 1 below, setting-up INE is never Pareto-improving.

Corollary 1 (Impossibility of a Pareto improvement with intertwined network effects)**.** *When two legally-independent platforms set up intertwined network effects, the surplus of consumers and single-homing sellers in the source platform decrease. Hence, the introduction intertwined network effects is never Pareto-improving.*

Proof: *The result follows directly from Lemma 3 and Propositions 1 and 2.*

Corollary 1 highlights that, even when INE increase both platforms', consumers' and sellers' surpluses overall (i.e., when $\tau^s \leq \frac{5}{8}\alpha^s$, as per Proposition 2), this happens at the expense of the surpluses of single-homing sellers and consumers in the source platform.[10] After the introduction of INE, the number of sellers in the source platform decreases (cf. Lemma 5). This reduces the surplus of the consumers that remain in that platform post-INE. Moreover, as discussed above, INE reduce seller surplus in the source platform. The reason is that, while INE increase the utility of sellers in the source platform by giving them access to the

[9]This result does not depend on the full double counting assumption.

[10]Recall that, to keep the model tractable, I make the *ad summum* assumption of full double counting, which likely overestimates the additional surplus generated by INE. Hence, this result is robust to lower INE-driven surplus assumptions.

buyers of the aggregator, they also create incentives for the source platform to raise seller prices to a level that more than offsets this surplus increase.

5 Platform merger with intertwined network effects

In this section I analyze the impact of a non-consolidating merger between two platforms having set up intertwined network effects on prices and the demand and surpluses of buyers and sellers. As in the previous section, I assume full information for all agents, i.e., each agent observes all the price decisions and knows all the parameters of the model. The equilibrium concept is the Subgame Perfect Nash equilibrium.

5.1 Post-merger equilibrium with intertwined network effects

In this section I characterize the post-merger equilibrium in a setting with intertwined network effects. The superscript $INE-M$ is used to refer to this setting in which the platforms having set up intertwined network effects merged. In this setting, I make the following assumptions, which are analogous to Assumptions 1 and 2.

Assumption 3 (Platform merger with intertwined network effects conditions). *In the setting in which platforms having set up intertwined network effects merge, the parameters satisfy the following conditions.*

$$\alpha^b\alpha^s < \frac{4}{3}\tau^b\tau^s \qquad \textit{(NoTipping}^{\textit{INE-M}}\textit{)}$$

$$\frac{2\tau^b\tau^s}{\alpha^b} \neq \alpha^s \quad \textit{and} \quad 4v^b + \frac{2\alpha^b\alpha^s}{\tau^s} > \tau^b\left(3 + \frac{2\tau^b\tau^s}{-2\tau^b\tau^s + \alpha^b\alpha^s}\right) \qquad \textit{(FPB}^{\textit{INE-M}}\textit{)}$$

$$0 < 1 - \frac{\alpha^s}{2\tau^s} < \frac{\tau^b\alpha^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s} < 1 \qquad \textit{(MHS}^{\textit{INE-M}}\textit{)}$$

As in Section 3.2, the utility functions of buyers and sellers are given by (5) and (6), respectively. However, given that the platforms are under common ownership, the referral fee f represents an internal transfer, and it is therefore not charged. Then, the profit of platform $i \in \{0,1\}$ is given by:[11]

$$\Pi^{i^{INE-M}} := \begin{cases} n_0^b(p_0^s,p_1^s)n_0^s(p_0^s,p_1^s)p_0^s & \text{if } i=0 \\ p_1^s\left(n_1^b(p_0^s,p_1^s)n_1^s(p_0^s,p_1^s) + n_0^b(p_0^s,p_1^s)n_1^s(p_0^s,p_1^s)\right) & \text{if } i=1 \end{cases} \tag{14}$$

Thus, in this setting, the game has two stages. In the first stage, the platforms simultaneously set the prices charged to sellers (p_0^s, p_1^s). In the second stage, consumers and sellers

[11] Note that Equation 14 is equal to Equation 7 when $f = 0$.

simultaneously choose which platform(s) to join.

Given that their utility functions remain unchanged by the merger, in stage 2, consumer- and seller-side demand as a function of the prices charged to sellers and the model's parameters are given by (11). In stage 1, the platforms solve the joint maximization problem $\max_{p_0^s, p_1^s} \Pi(p_0^s, p_1^s)$, where $\Pi := \Pi^1 + \Pi^2$. Solving the system of first-order conditions yields a unique set of equilibrium prices:

$$\begin{aligned} p_0^{s*} &= \frac{\alpha^s \left(\alpha^b \alpha^s - 2\tau^b \tau^s\right)}{\alpha^b \alpha^s - 4\alpha^b \alpha^s} \\ p_1^{s*} &= \frac{\alpha^s}{2} \end{aligned} \tag{15}$$

Replacing (15) in (11) I obtain the quantities on both sides of the market and the locations of the indifferent consumer and sellers in equilibrium. Consumer and seller surplus are calculated in the same way as in the benchmark setting and using the corresponding utility functions and equilibrium threshold values.

Then, the post-merger equilibrium values of the model in the setting with intertwined network effects are the following.

Lemma 6 (**Post-merger equilibrium with intertwined network effects**). *Post-merger, in* *the intertwined network effects setting equilibrium:*

$$n_0^{b\,INE-M} = \frac{1}{4} + \frac{\tau^b\tau^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s}$$
$$n_1^{b\,INE-M} = \frac{3}{4} - \frac{\tau^b\tau^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s}$$
$$n^{b\,INE-M} = 1$$
$$n_0^{s\,INE-M} = \frac{\tau^b\alpha^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s}$$
$$n_1^{s\,INE-M} = \frac{\alpha^s}{2\tau^s}$$
$$n^{s\,INE-M} = \frac{\alpha^s}{2\tau^s} + \frac{\tau^b\alpha^s}{4\tau^b\tau^s - 2\alpha^b\alpha^s}$$
$$p_0^{s\,INE-M} = \frac{\alpha^s\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)}{-4\tau^b\tau^s + \alpha^b\alpha^s}$$
$$p_1^{s\,INE-M} = \frac{\alpha^s}{2}$$
$$\Pi_0{}^{INE-M} = \frac{\tau^b{\alpha^s}^2}{16\tau^b\tau^s - 8\alpha^b\alpha^s}$$
$$\Pi_1{}^{INE-M} = \frac{{\alpha^s}^2}{4\tau^s}$$
$$\Pi^{INE-M} = \frac{1}{4}{\alpha^s}^2\left(\frac{1}{\tau^s} + \frac{\tau^b}{4\tau^b\tau^s - 2\alpha^b\alpha^s}\right)$$
$$CS^{INE-M} = \frac{\left(4\tau^b\tau^s - \alpha^b\alpha^s\right)\left(4\left(4r_B - \tau^b\right)\tau^b{\tau^s}^2 + \left(-8r_B + 11\tau^b\right)\tau^s\alpha^b\alpha^s - 4{\alpha^b}^2{\alpha^s}^2\right)}{16\tau^s\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)^2}$$
$$SS^{INE-M} = \frac{1}{8}\left(\tau^s\left(-4 + \frac{{\tau^b}^2}{{\alpha^b}^2}\right) + 8\alpha^s - \frac{3{\alpha^s}^2}{\tau^s} + \frac{4{\tau^b}^4{\tau^s}^3}{{\alpha^b}^2\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)^2} + \frac{4{\tau^b}^3{\tau^s}^2}{{\alpha^b}^2\left(-2\tau^b\tau^s + \alpha^b\alpha^s\right)}\right)$$

For the equilibrium reported in Lemma 6 to be valid, a series of conditions summarized in Assumption 3 have to hold. These conditions are analogous to those presented in Assumption 1, with some caveats. First, the assumption that ensures the second order conditions hold ($\alpha^b\alpha^s < 2\tau^b\tau^s$) is replaced by the stricter NoTipping$^{\text{INE-M}}$ assumption ($\alpha^b\alpha^s < \frac{4}{3}\tau^b\tau^s$), which ensures there is no tipping on the buyer side in favor of the aggregator platform.[12] Second, the second order condition on the source platform's price ($-\frac{2}{\tau^s} < 0$) is not reported, as it is always met. In the same vein, the condition for the price vector that satisfies the first order conditions to constitute a maximum is always met when the second-order condition on the aggregator's price is verified. Hence, for the sake of simplification, I do not include it in Assumption 3.

5.2 Comparison between the pre- and post-merger equilibria

In this subsection, I compare the equilibrium with INE and legally-independent firms to a post-merger equilibrium with INE. Let me first introduce the following remark.

[12]This is a mild additional assumption. As illustrated in Figure 4, the tipping case is a corner case.

Remark 2. *Let $Cond^{INE}$ and $Cond^{INE-M}$ be the parameter spaces defined by Assumptions 2 and 3, respectively.*

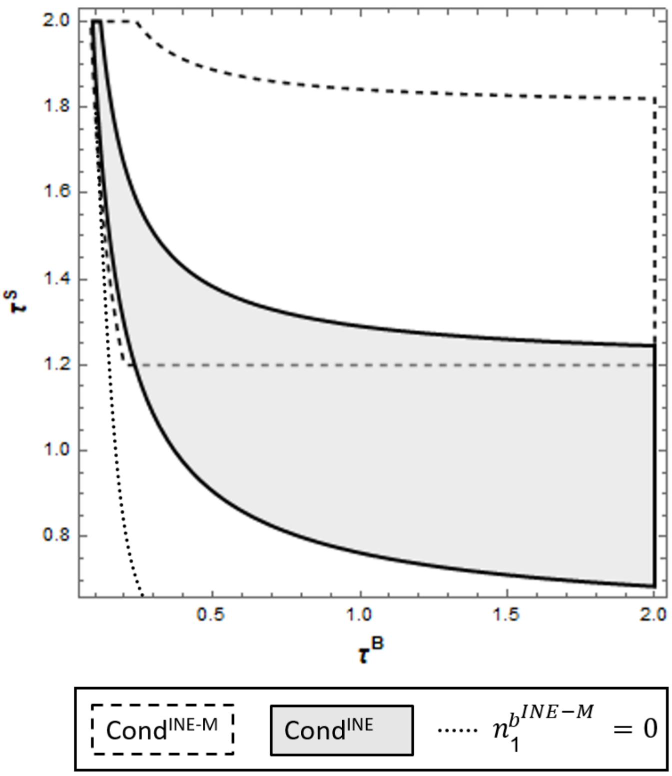
$$i)\ \exists \quad S^{INE} \quad \subseteq \quad Cond^{INE} \quad | \quad S^{INE} \cap Cond^{INE-M} = \varnothing$$

$$ii)\ \exists \quad S^{INE-M} \quad \subseteq \quad Cond^{INE-M} \quad | \quad S^{INE-M} \cap Cond^{INE} = \varnothing$$

$$iii)\ \exists \quad S^{INE \cap INE-M} = Cond^{INE} \cap Cond^{INE-M} \neq \varnothing$$

Remark 2 tells that there are admissible parameter spaces in which (i) only the case with INE and legally-independent firms can take place; (ii) only the case with INE and merged firms can take place; and (iii) both the cases with INE and either legally-independent or merged firms can take place. These are illustrated in Figure 4. Given that my focus is the effect of a non-consolidating merger relative to the case with INE and legally-independent firms, in the remainder of this section I will assume that Assumptions 2 and 3 hold, i.e., that the parameter space is the one defined as $S^{INE \cap INE-M}$ in Remark 2 (overlapping areas in Figure 4). As illustrated in Figure 4, the parameter spaces of the merger and INE cases overlap when transportation costs are sufficiently high for sellers. When this is the case, as per Lemma 7, platforms having set INE have an incentive to merge.

Figure 4: Parameter spaces for the merger and the INE cases



Parameter setting: $\alpha^b = 0.1, \alpha^s = 2.4$ and $v^b = 1.3$.

Lemma 7 (Incentives to merge in presence of intertwined network effects). *Consider* *two platforms having set up intertwined network effects. If Assumptions 2 and 3 hold, merging is a dominant strategy for both. Formally,* ${\Pi_0}^{INE-M} + {\Pi_1}^{INE-M} > {\Pi_0}^{INE} + {\Pi_1}^{INE}$.

***Proof**: See Appendix B.6.*

Lemma 7 shows that, whenever setting INE without common ownership and merging can be an equilibrium, the platforms always have incentives to merge. The reason is that this allows them to better internalize the cross-platform network effects by jointly maximizing profit. This provides a rationale of why it is common to see INE between transaction platforms belonging to the same group.

I now study how the merger affects prices, consumer- and seller-side demand and both user groups' surpluses. Note that a merger between two platforms having set up INE combines aspects of both horizontal and vertical mergers. On the one hand, the merger increases market power. On the other hand, it eliminates double marginalization, as the referral fee, which was an additional marginal cost for the source platform pre-merger, becomes an internal transfer post-merger.

5.2.1 Prices

Lemma 8 shows how the merger affects equilibrium prices.

Lemma 8 (**Merger effect on prices**). *Consider two platforms having set up intertwined network effects. After they merge, the price paid by sellers to the aggregator platform increases* (${p_0^s}^{INE-M} > {p_0^s}^{INE}$)*. In contrast, the price paid by sellers to the source platform decreases* (${p_1^s}^{INE-M} < {p_1^s}^{INE}$)*.*

***Proof**: See Appendix B.7.*

Lemma 8 shows how the change in the inter-platform pricing structure brought about by the merger affects the prices charged to sellers. When the platforms are under common ownership, they maximize profit jointly. Hence, the referral fee f disappears. This lowers the marginal per-interaction cost of the source platform (platform 1) and hence the price it charges to sellers. Conversely, in absence of the referral fee, the aggregator (platform 0) loses a revenue stream that was allowing it to set lower prices, attract more sellers and increase its revenue through referral fees. Hence, post-merger, platform 0 increases the price charged to sellers.

5.2.2 Demand configuration

Lemma 9 shows how the merger affects demand configuration in equilibrium.

Lemma 9 (**Merger effect on demand configuration**). *Consider two platforms having set up intertwined network effects. After they merge:*

i) The number of consumers decreases in the aggregator platform ($n_0^{b\,INE-M} < n_0^{b\,INE}$) and increases in the source platform ($n_1^{b\,INE-M} > n_1^{b\,INE}$)

ii) The number of sellers decreases in the aggregator platform ($n_0^{s\,INE-M} < n_0^{s\,INE}$) and increases in the source platform ($n_1^{s\,INE-M} > n_1^{s\,INE}$)

iii) The total number of sellers increases ($n_s^{INE-M} > n_s^{INE}$).

Proof: *See Appendix B.8.*

The changes in demand configuration induced by the merger follow from the change in prices analyzed in Lemma 8. Note that, contrary to what happens when comparing the effect of INE on demand configuration (cf. Lemma 5), the merger does not alter neither sellers' nor consumers' utility functions. Hence, the changes in their demand for each platform are only explained by their reaction to the price variation shown in Lemma 8. The price increase in platform 0 and the price decrease in platform 1 leads some platform 0 sellers to switch from single-homing to multi-homing, and some multi-homing sellers to start single-homing in platform 1. Hence, the number of consumers increases in the source platform and decreases the aggregator.

Interestingly, the increase in the number of sellers in platform 1 is always stronger than the decrease in platform 0, leading to an overall increase in the number of sellers led by more sellers multi-homing. This effect is analogous to the elimination of double marginalization effect in vertical mergers, which has been widely studied since Spengler (1950). Eliminating the referral fee results in a decrease in the price charged to sellers in the source platform (platform 1) that is stronger than the increase in the price charged to sellers in the aggregator platform (platform 0).[13] This expands overall sellers' demand for interactions.

5.2.3 Buyer and seller surplus

To present shorter mathematical expressions, let me introduce the following notation.

$$\tilde{v^b} := \frac{1}{16}\Bigg(2\tau^b + \frac{32{\tau^b}^2\tau^s}{\alpha^b\alpha^s} - \frac{4\alpha^b\alpha^s}{\tau^s} + \frac{\tau^b\alpha^b\alpha^s}{2\tau^b\tau^s - \alpha^b\alpha^s} + \sqrt{\frac{\tau^b\left(16\tau^b\tau^s - 5\alpha^b\alpha^s\right)^2\left(16{\tau^b}^3{\tau^s}^3 - 8{\tau^b}^2{\tau^s}^2\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2{\alpha^s}^2 - 2{\alpha^b}^3{\alpha^s}^3\right)}{\tau^s{\alpha^b}^2{\alpha^s}^2\left(-2\tau^b\tau^s + \alpha^b{\alpha^s}^2\right)}}\Bigg)$$

[13]It can be shown that $(p_0^{s\,INE-M} - p_0^{s\,INE}) + (p_1^{s\,INE-M} - p_1^{s\,INE}) < 0$ under Assumption 3.

Proposition 3 (**Merger effect on consumer surplus**). *Consider two platforms having set up intertwined network effects. After they merge, consumer surplus increases ($CS^{INE-M} > CS^{INE}$) if and only if $v^b < \tilde{v}^b(\tau^b, \tau^s, \alpha^b, \alpha^s)$, and decreases ($CS^{INE-M} < CS^{INE}$) otherwise (i.e., if $v^b > \tilde{v}^b(\tau^b, \tau^s, \alpha^b, \alpha^s)$).* ***Proof:*** *See Appendix B.9.*

The intuition is as follows. *Ceteris paribus*, the increase in the number of sellers in platform 1 (which benefits platform 1 buyers) and overall (which benefits the buyers served by both platforms) brought about by the merger benefits consumers in both platforms. This increases consumer welfare. However, due to the presence of INE, platform 0 consumers can interact with more sellers than platform 1 consumers irregardless of the ownership structure of the platforms. Hence, consumers switching from platform 0 to platform 1 decreases consumer surplus. If the stand-alone utility a consumer obtains from joining any of the two platforms (v^b) is sufficiently low, network effects have a significant weight in her decision to join a platform. In this case, a sufficiently high amount of consumers switch from platform 0 (where the number of sellers decreases) to platform 1 (where the number of sellers increases), which increases overall consumer surplus. The opposite happens if v^b is sufficiently high.

Proposition 4 (**Merger effect on seller surplus**). *Consider two platforms having set up intertwined network effects. After they merge, if $\tau^s \leq \frac{9\alpha^s}{16}$, seller surplus decreases.*

Proof: *See Appendix B.10.*

To analyze the merger's impact on seller surplus, I distinguish between *single-homing sellers* and *multi-homing sellers*. Post-merger, platform 0 (the aggregator) raises its price to sellers, while platform 1 (the source platform) lowers its price. For single-homing sellers in platform 0, this price hike reduces their surplus. Conversely, single-homing sellers in platform 1 benefit from the price reduction. Multi-homing sellers, however, face conflicting forces: they pay a higher price in platform 0 but a lower price in platform 1. Because multi-homing sellers interact with buyers from both platforms, the net effect depends on the balance between these price changes and the effect of the merger on the volume of interactions.

The merger also alters buyer-side demand: platform 0 loses buyers to platform 1. For multi-homing sellers, this reallocation reduces the number of interactions with platform 0's buyers, while increasing interactions with platform 1's buyers. However, due to intertwined network effects (INE), multi-homing sellers derive less value from platform 0's diminished buyer base. When seller transportation costs (τ^s) are sufficiently low, seller demand reacts significantly, amplifying the decline in overall interactions. This interaction loss dominates the price-driven benefits for sellers, leading to a net decrease in seller surplus.

6 Empirical results

In this section, I test the model's predictions regarding the impact of intertwined network effects on consumer- and seller-side demand. To do so, I exploit the introduction of INE between two pairs of platforms: Finn/Nettbil and Adverts/DoneDeal using event study designs and time series analysis.

Lemma 5 provides predictions on the effect of introducing INE on the number of buyers and sellers for the aggregator and the source platform. The setting in Lemma 5 assumes that INE predate common ownership; in contrast, the two cases studied in this section feature platforms already under common ownership before INE were introduced. It can be shown that, in the latter case, all the results in Lemma 5 but one hold.[14] The difference is that, when platforms are under common ownership, the total number of sellers unambiguously increases post-INE. Prediction 1 collects the expected effects of two merged platforms introducing INE on the number of buyers and sellers.

Prediction 1. *The introduction of intertwined network effects between two merged platforms leads to:*

1. *An increase in the number of buyers in the aggregator platform*
2. *A decrease in the number of buyers in the source platform*
3. *An increase in the number of sellers in the aggregator platform*
4. *A decrease in the number of sellers in the source platform*

In the next subsections, I test Prediction 1 on the two cases studied: Finn/Nettbil (Section 6.1) and Adverts/DoneDeal (Section 6.2).

6.1 Finn and Nettbil

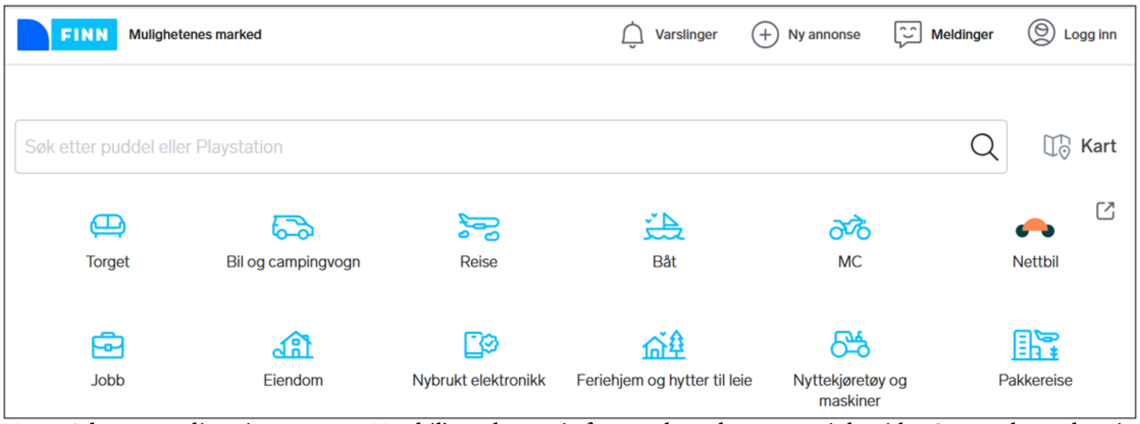
6.1.1 Case background

Finn is Norway's largest online marketplace, offering classified ads across various categories including real estate, jobs, and vehicles. Nettbil is a platform specializing in auction-based car sales from private sellers to dealers. In December 2019, Finn's parent company, Schibsted, acquired a majority stake (67%) in Nettbil. On December 15, 2020, Finn started providing

[14] To do so, one has to solve the baseline model presented in Section 3.1 with a caveat: in stage 1, the platforms solve the joint maximization program $\max_{p_0^s, p_1^s} \Pi(p_0^s, p_1^s)$. Then, one can compare the resulting equilibrium demand to the equilibrium demand of the post-merger with INE case reported in Lemma 6.

access to Nettbil's listings on its platform. As shown in Figure 5, since that day, users can click on a "Nettbil" icon located next to other thematic icons such as "Jobs" or "Housing". When users click on Nettbil's icon, they get redirected to Nettbil's webpage regardless of whether they entered through Finn's website or mobile app, as Nettbil does not have a mobile app. In other words, in December 2020 INE were introduced, with Finn being the aggregator and Nettbil being the source platform.

Figure 5: Finn's homepage



Note: A button redirecting users to Nettbil's webpage is featured on the upper-right side. Screenshot taken in April 23, 2025.

6.1.2 Data

To test Prediction 1, I retrieve data from Sensor Tower, one of the main providers of app usage data. The unit of observation is an app in a given country and week, with data available from October 2015 onward. For each app, I observe the number of weekly active users (WAU), which measures how many distinct devices (Android and iOS smartphones and tablets) have connected to the app. This metric does not capture user activity within the app and can therefore be interpreted as an extensive margin of consumer-side demand. Sensor Tower does not distinguish between business and end users. Nevertheless, this distinction is not sharp for the platforms studied, as they are classifieds platforms used primarily by individuals who can act as both buyers and sellers. Since Nettbil does not have a mobile app, I cannot observe the number of weekly active users on that platform.

The dataset spans from December 19, 2022 (the date in which Finn and Nettbil merged) and June 20, 2022. This year-and-a-half time window after the introduction of INE allows me to capture medium- to long-term causal effects of INE while voiding confounding influences from unrelated market changes and structural shifts that could arise in a longer observation period. Table 1 provides summary statistics of the data collected (1,048 observations) differentiating the pre- and post-INE periods.

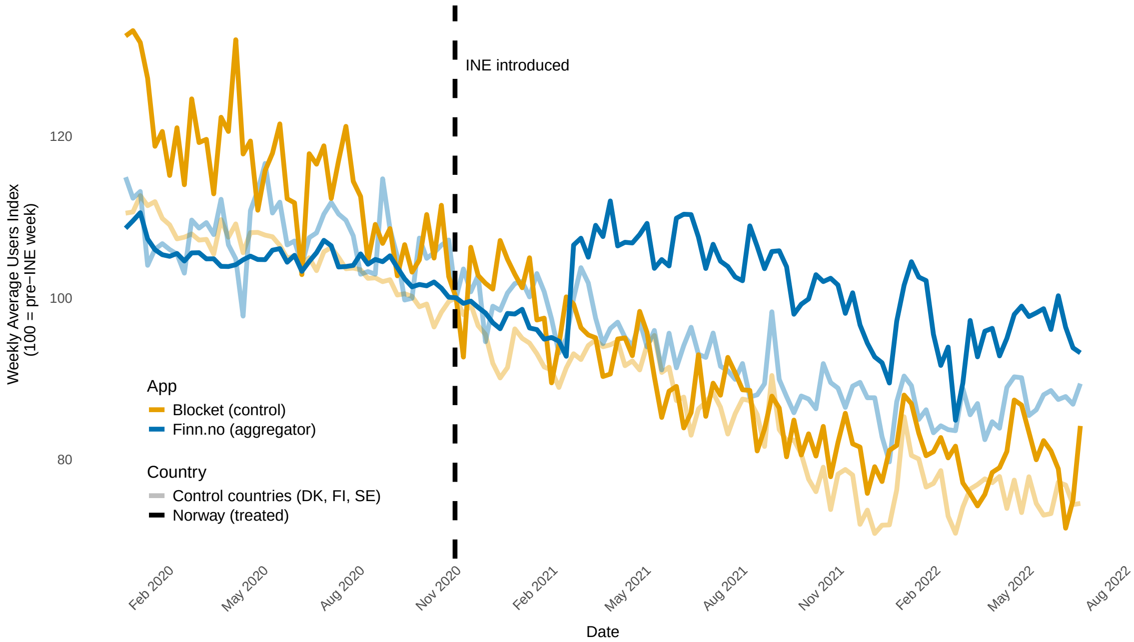
Table 1: Summary statistics by app, country, and period

Country	Metric	Finn		Blocket	
		Before	After	Before	After
NO	WAU (mean)	294,034	283,105	5,973	4,548
	WAU (s.d.)	7,248	16,447	477	422
	WAU (min)	276,975	239,534	4,852	3,744
	WAU (max)	311,987	316,099	6,966	5,607
	Number of weeks	50	81	50	81
DK	WAU (mean)	4,651	3,822	3,821	2,753
	WAU (s.d.)	273	357	265	362
	WAU (min)	4,164	3,245	3,243	2,130
	WAU (max)	5,326	4,542	4,329	3,433
	Number of weeks	50	81	50	81
FI	WAU (mean)	1,320	1,110	4,141	3,143
	WAU (s.d.)	67	87.7	237	272
	WAU (min)	1,171	943	3,534	2,672
	WAU (max)	1,502	1,372	4,556	3,888
	Number of weeks	50	81	50	81
SE	WAU (mean)	8,273	7,221	216,454	172,673
	WAU (s.d.)	475	406	9,021	16,222
	WAU (min)	7,215	6,160	197,808	146,400
	WAU (max)	9,256	8,358	233,231	199,470
	Number of weeks	50	81	50	81

6.1.3 Descriptive patterns

Figure 6 shows the evolution of Finn's and Blocket's WAU in Norway and other Nordic (control) countries during the period of analysis (December 2019 to June 2022).

Figure 6: Evolution of the number of monthly active users before and after the introduction of INE for Finn (treated) and Blocket (control) in treated (Norway) and control countries (Denmark, Finland and Sweden)



Note: the vertical dashed line indicates the month of introduction of intertwined network effects.

Before INE, Finn's WAU trend in Norway was similar to its trend in other Nordic countries, as well as to Blocket's in Norway and other Nordic countries. By the end of February 2021 (about 3 months after the introduction of INE), Finn's WAU saw a discrete jump that changed its trajectory with respect to other app-country pairs.

6.1.4 Empirical strategy

To provide causal evidence on the effect of INE on Finn's weekly active users (WAU), I employ an event study design. The model presented in Section 3 is agnostic as to whether INE generates a discrete or time-varying impact on platform demand. Accordingly, I estimate the following flexible event study specification:

$$\widetilde{\text{WAU}}_{act} = \sum_{t \neq -1} \beta_t \left(\text{Treated}_i \times t\right) + \gamma \cdot \text{CovidStr}_{ct} + \mu_{ac} + \delta_t + \varepsilon_{act} \tag{16}$$

where $\widetilde{\text{WAU}}_{act}$ denotes the z-score standardized number of weekly active users for app a in country c at week t, with t measured relative to the introduction of INE at $t = 0$. An observation i is considered treated if it corresponds to Finn in Norway at any post-treatment date ($t > 0$). I control for the COVID-19 Stringency Index (CovidStr_{ct}), which captures the intensity of government-imposed restrictions (e.g., lockdowns, school closures, travel bans) at the country-week level, as compiled by the Oxford COVID-19 Government Response Tracker developed by Hale et al. (2021). Given that the unit of observation is an app-country-week combination, the model includes app-country fixed effects (μ_{ac}) and time fixed effects (δ_t) to control for unobserved heterogeneity across these dimensions. The coefficients β_t capture the effect of INE introduction on WAU in week t, expressed in terms of pre-treatment standard deviations relative to the baseline period immediately preceding treatment ($t = -1$).

Counterfactual. The control group includes Finn in Denmark, Sweden, and Finland, as well as Blocket (a Swedish marketplace owned by Schibsted) in Norway, Denmark, Sweden, and Finland. These last three countries are socioeconomically similar and geographically close to Norway, but were not exposed to the INE between Finn and Nettbil, which only took place is Norway. Moreover, the shared ownership of Finn and Blocket by Schibsted helps reduce variation in weekly active users caused by parent-company-specific factors. This strengthens the validity of the counterfactual by minimizing confounding heterogeneity.

Addressing potential COVID-19 confounds. Although the study period overlaps with the COVID-19 pandemic, several features of the data and the empirical strategy suggest that causal identification is not compromised. First, the event study controls for the COVID-19 Stringency Index (CovidStr_{ct}), which measures government restrictions at the country-week level (Hale et al., 2021), mitigating bias from differential pandemic shocks. Second, other classifieds apps that compete with Finn display markedly different WAU trends before and after the event in Norway. This indicates that even if COVID-19 influenced app usage, its effects were heterogeneous across platforms rather than uniform, supporting the interpretation that observed WAU changes are mainly driven by platform-specific factors. Third, Finn's WAU display an abrupt jump after INE introduction (Figure 6), consistent with exogeneity of treatment timing given that COVID effects on app usage are expected to be gradual.

Standardization. I choose the z-score standardization as it effectively accounts for the considerable heterogeneity in variance across app-country units, as shown in Table 1, where variance ratios often exceed 5:1. This standardized mean difference approach centers and scales WAU within each app-country pre-treatment period. It thus ensures comparability across cases and stabilizes variance without relying on distributional assumptions inherent to other methods such as a log transformation. Moreover, a log transformation would not be appropriate here due to the extreme heterogeneity in scale. Combining log and z-score transformations would introduce over-standardization and complicate interpretation without additional inferential gains.

On the covered market assumption and empirical validity. The model assumes a covered market on the buyer side (Section 3), as is standard in the platform competition literature (Armstrong, 2006; Rochet and Tirole, 2006; Belleflamme and Peitz, 2019), whereas the empirical results (Sections 6.1.5 and 6.2.5) show that total users can vary with INE introduction. However, this does not invalidate the model's predictions or the empirical identification strategy. The covered market assumption does not impose a fixed market size in the relevant economic sense. Rather, it assumes that all potential buyers within the market join *some* platform, thereby focusing the analysis on cross-platform substitution patterns rather than on the size of the universe of potential platform users. Accordingly, my empirical strategies identify these *relative* effects across platforms rather than absolute market growth. The event study and difference-in-differences designs estimate within-market treatment effects using standardized outcomes (z-scores normalized by pre-treatment means and standard deviations), control groups and fixed effects that absorb common shocks, including potential market-wide expansions or contractions.

Importantly, seller-side demand remains fully elastic through multihoming (Section 3.1), allowing total seller participation to expand or contract endogenously. Since sellers constitute the money-making side subject to per-transaction pricing, this is precisely where platform competition operates and where INE generate welfare effects through allocative efficiency mechanisms (Propositions 1 and 2). The buyer-side coverage assumption thus isolates competitive reallocation effects—the focus of my theoretical predictions—while maintaining analytical tractability.

Randomization inference. The design features one treated unit (Finn in Norway) and seven control units: Finn in Denmark, Finland, and Sweden, plus Blocket in all four countries (Norway, Denmark, Finland and Sweden). With only eight total units providing identifying variation, standard asymptotic inference is unreliable. When the number of units is small, unit-robust variance estimators severely underestimate standard errors and conventional t-tests dramatically over-reject the null hypothesis (Conley and Taber, 2011; MacKinnon and Webb, 2020).

I therefore construct confidence intervals using randomization-based inference, which provides valid inference without relying on large-sample approximations for the treated units (Conley and Taber, 2011; MacKinnon and Webb, 2020). For each event time coefficient β_e, I compute randomization-based confidence intervals by inverting permutation tests (MacKinnon and Webb, 2020). Under the null hypothesis $H_0 : \beta_e = \beta_0$, I create an offset outcome $\text{WAU}^z_{igt} - \beta_0 \cdot \mathbb{1}[e_{gt} = e] \cdot \text{Treated}_{gt}$ and permute the treatment assignment across all eight platform-country combinations. For each permutation r, I estimate the model using the offset outcome and compute the t-statistic t_r. Following MacKinnon and Webb (2020), I use the t-statistic rather than the coefficient estimate as the test statistic to account for potential heterogeneity across units. The permutation p-value is the proportion of permutations where $|t_r| \geq |t_{\text{actual}}|$, providing an exact test under the sharp null hypothesis (Conley and Taber, 2011). The confidence interval consists of all values β_0 for which this p-value exceeds $\alpha = 0.05$.

Error clustering. Standard errors are clustered at the app-country level to account for serial correlation within each platform-country unit over time. Even after controlling for app-country and time fixed effects, the residuals may exhibit within-unit persistence due to platform-specific trends, user retention dynamics, or the gradual diffusion of platform features. Clustering at the app-country level yields standard errors that properly account for this within-unit correlation structure, producing more accurate t-statistics for the permutation distribution. This clustering choice aligns with the permutation procedure, which treats the eight app-country combinations as the fundamental units across which treatment is reassigned, ensuring consistency between the assumed dependence structure and the exchangeability assumption underlying randomization inference.

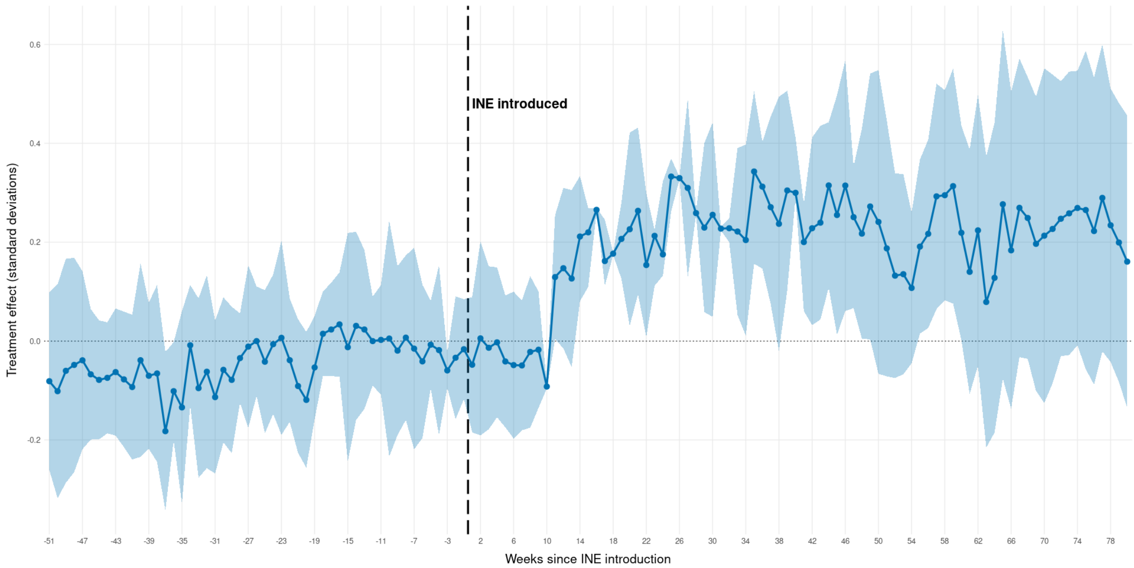
Rather than exhaustively testing every possible null value, I employ a binary search algorithm that efficiently locates the confidence interval bounds by iteratively narrowing the search range until convergence, with a tolerance of 0.01. This computational strategy substantially reduces the number of permutation tests required (MacKinnon and Webb, 2020). The initial search range is set to the point estimate $\pm$ 4 standard errors from the parametric fixed effects model. For each candidate null value, I adaptively refine the bounds: for the upper confidence limit, I search for the largest value not rejected by the permutation test; for the lower limit, I search for the smallest value not rejected.

The permutation distribution is constructed by assigning placebo treatment status to each of the eight platform-country combinations in turn while holding the timing and number of treated units constant (Conley and Taber, 2011; MacKinnon and Webb, 2020). This approach is valid under the assumption that the error term distribution is exchangeable across platform-country units conditional on the fixed effects, which is implied by random assignment of treatment conditional on platform-country and time fixed effects (Conley and Taber, 2011).

6.1.5 Results

Figure 7 presents the estimation results of Equation 16.

Figure 7: Effect of the introduction of intertwined network effects on Finn's monthly active users: Treated (Finn in Norway) vs. Control (Blocket in Denmark, Finland and Sweden)



Note: The vertical dashed line indicates week 0 (introduction of intertwined network effects). Shaded areas correspond to 95% randomization-based confidence intervals constructed by inverting permutation tests across all eight platform-country combinations.

Figure 7 depicts a treatment effect averaging 0.25 standard deviations in Finn's weekly active users from starting 11 weeks after the introduction of intertwined network effects and lasting up to week 48. This delayed causal response likely reflects a lag in measurement by Sensor Tower. The pre-treatment coefficients are small and statistically indistinguishable from zero, lending support to the parallel trends assumption. These results corroborate Prediction 1.1

6.2 Adverts and DoneDeal

6.2.1 Case background

Adverts and DoneDeal are two of Ireland's largest online classified advertising platforms, where users can buy and sell a wide range of goods and services. While both cover general categories like electronics, furniture, and vehicles, DoneDeal is particularly known for its strong focus on motor and agricultural listings. In July 2015, the two platforms merged. In December 2017, for some categories, whenever users would see less than 10 search results on Adverts, similar ads from DoneDeal started being displayed on Adverts.[15] In other words,

[15]See https://help.adverts.ie/hc/en-us/articles/360001288765-Ad-Sharing-from-to-DoneDeal-ie.

in December 2017 INE were introduced, with DoneDeal being the source platform and Adverts being the aggregator.

6.2.2 Data

If Predictions 1.1 and 1.3 hold, I should identify an increase in the number of WAU (i.e., consumers and sellers) in Adverts (the aggregator platform) caused by the introduction of INE. Second, as per Predictions 1.2 and 1.4, following INE, the number of weekly active users should decrease in DoneDeal, the source platform. To test these predictions, I use three datasets described below: the app usage dataset, the listings dataset, and the car imports dataset.

App usage dataset. As in the previous case, I use data from Sensor Tower to observe weekly active users. The unit of observation is an app within a specific country and week. For each app, I observe the number of weekly active users (WAU). The dataset spans from November 2016 (first date available in the data source) to April 2019.[16]

Table 2 provides summary statistics of the data collected (756 observations) differentiating the pre- and post-INE periods.

Table 2: Summary statistics by app, country, and period (Before/After intertwined network effects)

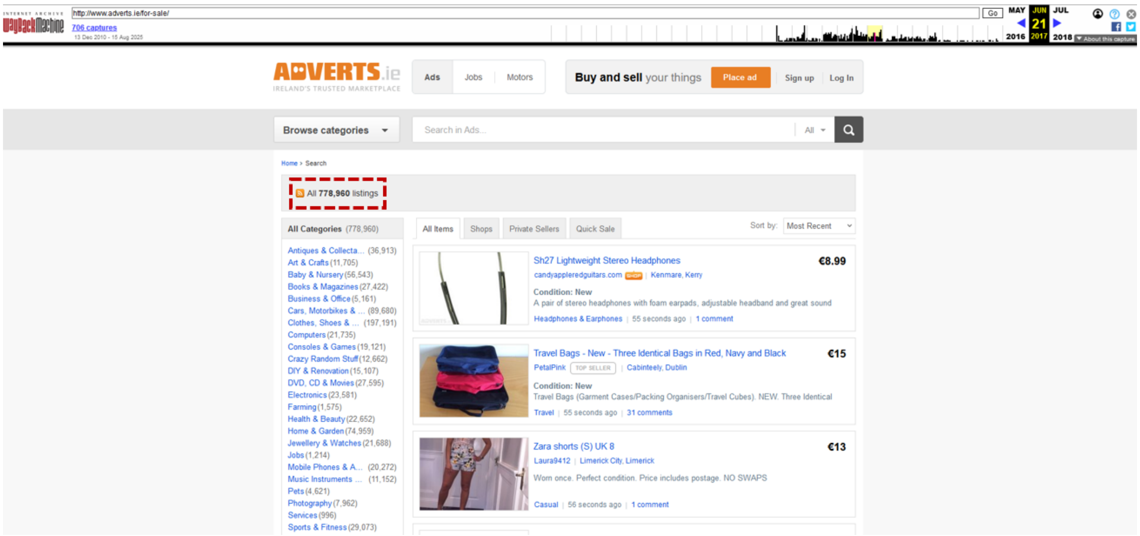
Country	Metric	Adverts		DoneDeal	
		Before	After	Before	After
IE	WAU (mean)	26,951	26,254	59,820	47,685
	WAU (s.d.)	1,769	1,025	4,466	2,453
	WAU (min)	23,442	21,891	51,268	42,769
	WAU (max)	30,879	27,978	69,379	54,155
	Number of weeks	55	71	55	71
GB	WAU (mean)	648	535	8,173	6,762
	WAU (s.d.)	51.2	55.4	411	496
	WAU (min)	514	426	7,350	5,961
	WAU (max)	788	688	9,235	8,298
	Number of weeks	55	71	55	71
US	WAU (mean)	281	176	1,091	480
	WAU (s.d.)	18.7	32.4	155	199
	WAU (min)	230	135	828	145
	WAU (max)	314	235	1,397	815

[16] Since May 2019, sellers on Adverts can opt-in to share their content with DoneDeal. As this introduces additional cross-platform network effects, I exclude data from May 2019 onward to isolate the effect of the INE introduced in December 2017.

Listings dataset. The second data source is Wayback Machine, a website developed by the nonprofit organization Internet Archive that aims to archive the entire internet. The Wayback Machine contains over 850 billion archived versions, or "snapshots", of web pages collected since May 12, 1996. It allows to observe how a specific URL appeared on a given date, provided that snapshots exist for that date.

I web-scraped the websites of three of the four platforms studied (Adverts, DoneDeal, and Finn) stored in Wayback Machine to create a time series of the total number of listings. The fourth platform of interest, Nettbil, does not show the total number of listings on its website, and therefore no data are available for it. In this preliminary version of the article, I only show results on listings for the Adverts/DoneDeal case. Figure 8 illustrates how the number of listings on a past date is displayed in the Wayback Machine for the classifieds platform Adverts.

Figure 8: Snapshot of Adverts in the Wayback Machine for July 21, 2017



Note: the red dashed rectangle highlights where the total number of listings is displayed.

For each platform, I scrape one snapshot per week over the same periods as the app usage data: December 2019 to June 2022 for the Finn/Nettbil case and November 2016 to April 2019 for the Adverts/DoneDeal case. To control for within-week seasonality, I select, for each platform, the most common day of the week with available snapshots. If a snapshot is missing for that day in a given week, I use the closest available day instead.

An observation in the listings dataset is the number of total listings on a platform The number of listings can be interpreted as an intensive-margin measure of seller-side demand, as it indicates how many items are on sale at a given time on a platform.

Table 3 provides summary statistics of the listings data collected for the Adverts/DoneDeal case (244 observations) differentiating the pre- and post-INE periods.

Table 3: Summary statistics by platform and period for Adverts and DoneDeal

Metric	Adverts		DoneDeal	
	Before	After	Before	After
Listings (mean)	768,583	826,724	315,756	296,102
Listings (s.d.)	35,140	9,698	15,544	13,735
Listings (min)	710,672	805,857	274,141	259,093
Listings (max)	827,797	846,702	332,149	312,372
Number of weeks	54	74	44	72

Car import dataset. I use data from the Irish motor industry's official statistics repository, SIMI Motorstats (Society of the Irish Motor Industry), to observe monthly used car import registrations in Ireland. SIMI Motorstats compiles vehicle registration data from the Department of Transport and provides comprehensive statistics on the Irish automotive market. The unit of observation is a month, and I observe the number of used vehicles imported to Ireland and registered for the first time in the Irish system. The dataset spans from January 2016 to April 2019, matching the temporal coverage of the listings dataset.

Table 4 provides summary statistics of the data collected (39observations) differentiating the pre- and post-INE periods.

Table 4: Summary statistics for used car imports in Ireland (Before/After indirect network effects introduction)

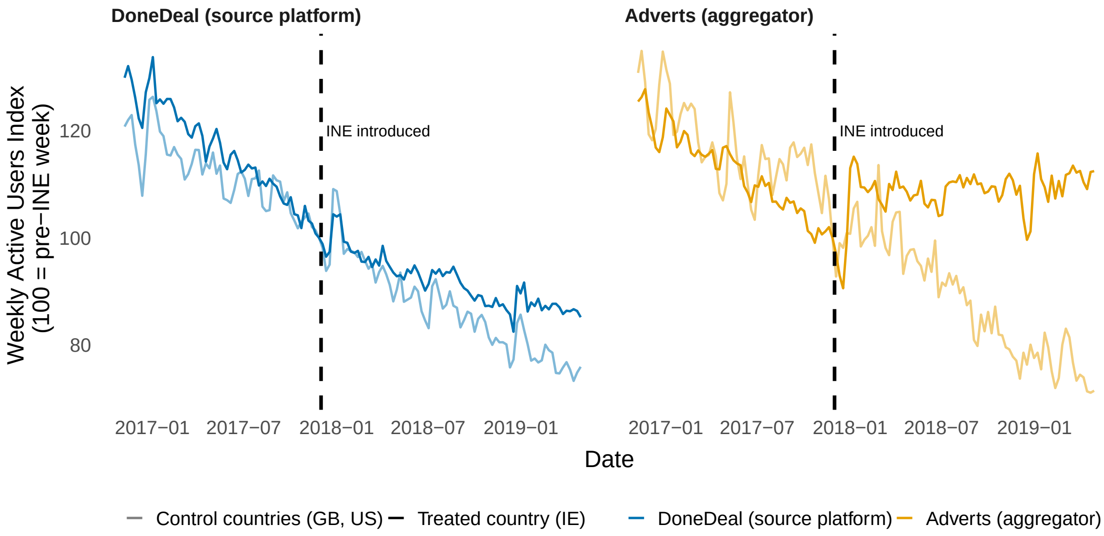
Metric	Before INE	After INE
Used imports (mean)	6,960	8,320
Used imports (s.d.)	1,270	1,125
Used imports (min)	4,509	5,526
Used imports (max)	8,662	9,139
Number of months	23	16

Note: Used car imports refer to vehicles imported to Ireland and registered for the first time in the Irish vehicle registration system. The pre-INE period covers January 2016 to November 2017 (23 months); the post-INE period covers December 2017 to April 2019 (16 months). Total observations: 39 months. Data source: SIMI Motorstats.

6.2.3 Descriptive patterns

Figure 9 shows the evolution of the number of monthly active users for Adverts (the aggregator) and DoneDeal (the source platform) during the period of analysis (November 2016 to April 2019).

Figure 9: Evolution of the number of monthly active users for Adverts (left axis) and DoneDeal (right axis) mobile apps (October 2015 to April 2019)



Note: The vertical dashed line indicates the month of introduction of intertwined network effects.

Figure 9 depicts a declining parallel trend between the two apps' usage in all countries until the introduction of INE in December 2017 (the treatment period). After that, there is a discrete increase in the number of users of Adverts in Ireland, followed by a flattening of the evolution of WAU over time. In the case of DoneDeal's Irish WAUs, the decline slightly decelerates around six months after the introduction of INE. In both apps, the trend of weekly active non-Irish users, in turn, does not seem to change substantially after INE were introduced.

6.2.4 Empirical strategy

Buyer- and seller-side demand: extensive margin. To provide causal evidence of the effect of INE on both buyer- and seller- side demand in Adverts and DoneDeal, I estimate the following two-way event study with 5-week event-time bins:

$$\begin{aligned}\widetilde{\text{WAU}}_{act} = &\sum_{b\neq-1} \beta_b^A \left(\text{Adverts} \times \text{Treated} \times \text{Bin}_b\right) \\ &+ \sum_{b\neq-1} \beta_b^D \left(\text{DoneDeal} \times \text{Treated} \times \text{Bin}_b\right) + \mu_{ac} + \delta_t + \varepsilon_{act}\end{aligned} \tag{17}$$

where $\widetilde{\text{WAU}}_{act}$ denotes the standardized number of weekly active users (WAU) for app a in country c and week t, normalized by the pre-treatment mean and standard deviation for each app-country pair. The subscript b indexes five-week event-time bins relative to the introduction of INE, with bin $b = -1$ denoting the reference bin corresponding to weeks

$[-5, -1]$. The coefficients β_b^A and β_b^D capture the differential change in WAU for the treated app-country pairs (Adverts and DoneDeal in Ireland, respectively) in each bin b relative to the reference bin. The indicator variable Treated equals 1 for observations from Ireland at any post-treatment date. I include app-country fixed effects μ_{ac} to control for time-invariant heterogeneity and date fixed effects δ_t to absorb aggregate shocks common to all units.

Counterfactual. To construct a counterfactual, I use the number of WAU for each app in two other countries where both Adverts and DoneDeal exhibit sizable user bases: Great Britain and the United States. These countries are appropriate controls for several reasons. First, they rank among the top three markets for both apps in terms of weekly active users (WAU).[17] Second, they were the two leading countries of origin for overseas visitors to Ireland in 2017 and 2018.[18] This overlap suggests that some app users in these countries may have economic or social ties to Ireland such as frequent travel or short-term housing needs, and may thus engage with Irish platforms. Yet, not being Irish residents, they were unlikely to benefit directly from the introduction of INE, which expanded Adverts' functionality for domestic users seeking agricultural and motor products. This makes them suitable counterfactuals for assessing the impact of INE. Third, both countries share strong linguistic and sociocultural affinities with Ireland, which are likely to influence digital consumption habits in similar ways. Finally, and crucially for the identification strategy, Great Britain and the United States are not exposed to the intertwined network effects between Adverts and DoneDeal, as these platforms operate exclusively in Ireland.

Binning the event-time into five-week intervals increases statistical power by aggregating weekly observations, thereby reducing noise and supporting more precise inference in the presence of limited treated units and weekly data spanning across 126 weeks.

Standardization. The choice of z-score standardization over alternative transformations is motivated by several methodological considerations specific to this setting. First, the data exhibit extreme heteroskedasticity, with variance ratios reaching 92:1 between countries in the post-treatment period, which violates the homoskedasticity assumptions required for valid statistical inference. Z-score normalization eliminates these variance differences by construction, ensuring that treatment effects are measured in comparable units across countries with vastly different user bases. Second, the standardization enables direct comparison of treatment effects across apps with heterogeneous response patterns: while Adverts exhibits a large discrete jump followed by trend stabilization, DoneDeal shows minimal response. Without standardization, the economic significance of small absolute changes in DoneDeal cannot be assessed relative to its typical variation pattern. Third, while log transformation followed by z-score standardization could theoretically combine

[17] During the period of analysis, Ireland, Great Britain, and the United States accounted for 95.4% of Adverts' total WAU, broken down as follows: Ireland (92.6%), Great Britain (2.0%), and United States (0.8%). For DoneDeal, these same countries accounted for 98.4% of total WAU: Ireland (85.3%), Great Britain (11.9%), and United States (1.2%).

[18] In 2017, Great Britain accounted for 40% of Ireland's overseas tourists, followed by the United States with 17%. In 2018, the shares were 36% and 18%, respectively, as reported by Fáilte Ireland (2019).

the advantages of both transformation methods, the additional complexity is unnecessary. Z-score normalization alone captures structural breaks and trend changes equally effectively while ensuring robust statistical inference in this few-treated-units setting.

Randomization inference. To conduct valid inference on the event-study coefficients in a setting with very few treated units and a long time series, I employ a randomization-based inference approach leveraging time permutation combined with a binary search for confidence interval estimation. Due to the limited number of treated units (one treated country per platform), traditional inference methods relying on asymptotic approximations are likely to be severely biased or invalid since they do not accurately estimate variability with so few units (Alvarez et al., 2025; Canay et al., 2017).

Unlike classical blocked or cluster randomization inference that permutes treatment assignments across units, the design here restricts permutations to the shock timing (event time) dimension because the small number of treated countries renders country-level permutations infeasible. This time permutation procedure preserves the panel structure and treatment assignment while generating a finite-sample randomization distribution of effects under the null hypothesis of no effect (Romano and Wolf, 2016; Hagemann, 2019). This approach aligns with recent advances advocating for randomization inference with few treated units and complex longitudinal data (Alvarez et al., 2025; Sant'Anna, 2024).

The method assigns event-time bins of five weeks to weekly observations relative to candidate placebo shock dates and estimates event-study coefficients within a fixed effects panel regression framework. Confidence intervals for treatment effects at each event-time bin are obtained through a binary search algorithm that iteratively tests null values against the permutation distribution of test statistics, calibrated at a conventional significance level of 0.05. This binary search enhances computational efficiency in deriving exact or near-exact randomization confidence intervals without relying on large sample normal approximations (MacKinnon and Webb, 2018).

By exploiting the known treatment assignment mechanism and leveraging a rich time-series dimension for permutation, this design-based inference provides robustness to heterogeneity, complex dependence structures, and small-sample biases intrinsic to model-based approaches. Consequently, it delivers theoretically grounded and practically reliable statistical inference on causal effects in settings characterized by very few treated units and extended temporal observations (Canay et al., 2017; Alvarez et al., 2025).

Seller-side demand: intensive margin. To assess how the introduction of indirect network effects (INE) affected seller-side demand through the intensive margin, proxied by the total number of listings, I employ two methods. First, a descriptive, time-series-based approach. Second, a difference-in-difference estimation. Below I describe each.

The **time series approach** compares observed listings after the introduction of INE to the expected trend based on pre-INE dynamics. Specifically, for each platform, I first selected the pre-INE period, which includes all dates prior to December 1, 2017. I then fit an asymmetric square-root cyclical model to the weekly number of listings during this pre-shock period. The model allows the peaks and troughs of the time series to grow at different rates and incorporates a sinusoidal component to capture the within-year seasonality observed in the data. Formally, the model is specified as:

$$\text{Listings}_t = \begin{cases} \left(a_{\text{peak}} + b_{\text{peak}}\sqrt{t}\right) \cdot |\sin(\omega t + \phi)| + C + \varepsilon_t, & \text{if } \sin(\omega t + \phi) \geq 0, \\ -\left(a_{\text{trough}} + b_{\text{trough}}\sqrt{t}\right) \cdot |\sin(\omega t + \phi)| + C + \varepsilon_t, & \text{if } \sin(\omega t + \phi) < 0. \end{cases} \tag{18}$$

where t indexes time in weeks, a_{peak} and b_{peak} control the growth of the peaks, a_{trough} and b_{trough} control the growth of the troughs, ω and ϕ capture seasonal cycles, C is a baseline level, and ε_t is the residual.

After fitting the model to the pre-INE data, I extrapolate the fitted trend into the post-INE period (December 2017 onward) and compare the observed number of listings to the model-predicted values. The models achieved an R-squared of 0.92 for Adverts and of 0.86 for DoneDeal. Deviations from the expected pre-INE trajectory are interpreted as descriptive evidence of the effect of INE on seller-side demand through the intensive margin.

To identify the causal effect of indirect network effects (INE) on seller-side demand, proxied by the number of listings, I implement a **difference-in-differences design** using monthly data from DoneDeal (treated group) and one-month lagged used car imports (control group). This control is appropriate for three reasons. First, motor listings account for approximately 45% of DoneDeal's total listings throughout the period of analysis. Second, DoneDeal operates exclusively in the Irish market, making it responsive to domestic automotive market conditions that also drive import registrations. Third, the one-month lag captures the operational delay between a vehicle's importation and registration and its subsequent listing on the platform, as imported used cars require documentation, inspection, and dealer preparation before advertisement. The estimation equation is:

$$\widetilde{Y}_{it} = \beta_1 \, \text{Treated}_i + \beta_2 \, \text{Post}_t + \beta_3 \, (\text{Treated}_i \times \text{Post}_t) + \delta_t + \varepsilon_{it} \tag{19}$$

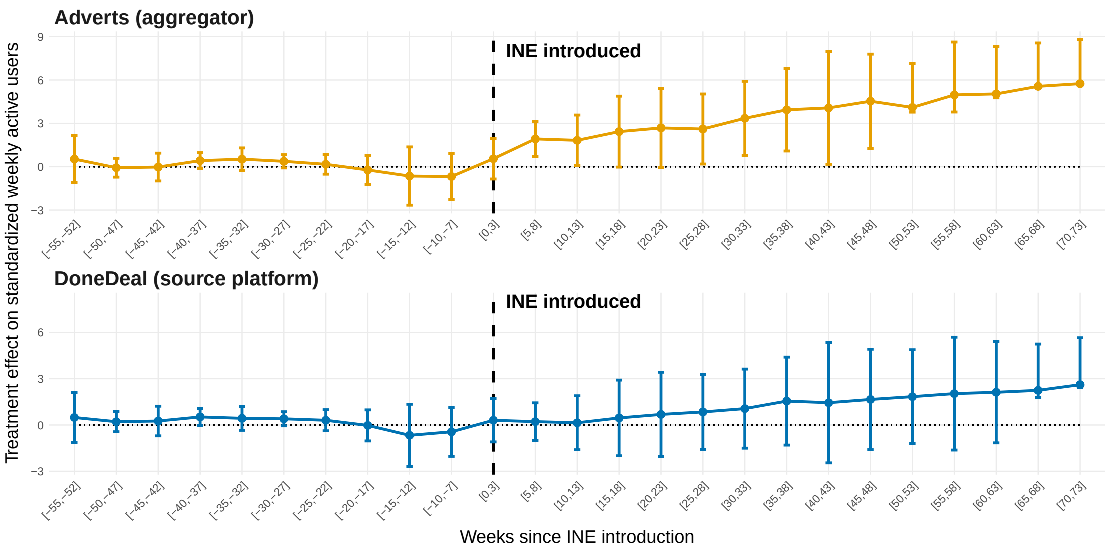
where $\widetilde{Y}_{it}$ denotes the z-score standardized outcome (listings or lagged used imports) for unit i in month t, Treated_i is an indicator that equals one for DoneDeal and zero for the control, Post_t indicates months on or after December 2017, and δ_t are time (month) fixed effects absorbing aggregate variation common to both units. The coefficient β_3 captures the difference-in-differences estimator: the average treatment effect of INE on DoneDeal listings, expressed in pre-treatment standard deviations relative to the trend in lagged used imports.

To obtain confidence intervals and significance tests robust to small-sample concerns and serial correlation, I employ randomization inference via time permutation. For each candidate placebo shock date within a feasible range, I re-estimate equation (19) and compute the test statistic t_{placebo}. The 95% confidence interval for β_3 is constructed using binary search: for each potential null hypothesis value β_3^0, I adjust the treated post-period observations by subtracting β_3^0, re-run the permutation procedure, and compute the p-value as the proportion of placebo test statistics exceeding the observed statistic in absolute value. The confidence interval bounds are the values of β_3^0 where this p-value equals 0.05. This procedure yields inference that is exact under the null hypothesis of no treatment effect and does not rely on distributional assumptions.

6.2.5 Results

Buyers and sellers app usage (extensive margin). Figure 10 shows the results of the estimation of Equation 17.

Figure 10: Effect of the introduction of intertwined network effects on Adverts' and DoneDeal's monthly active users



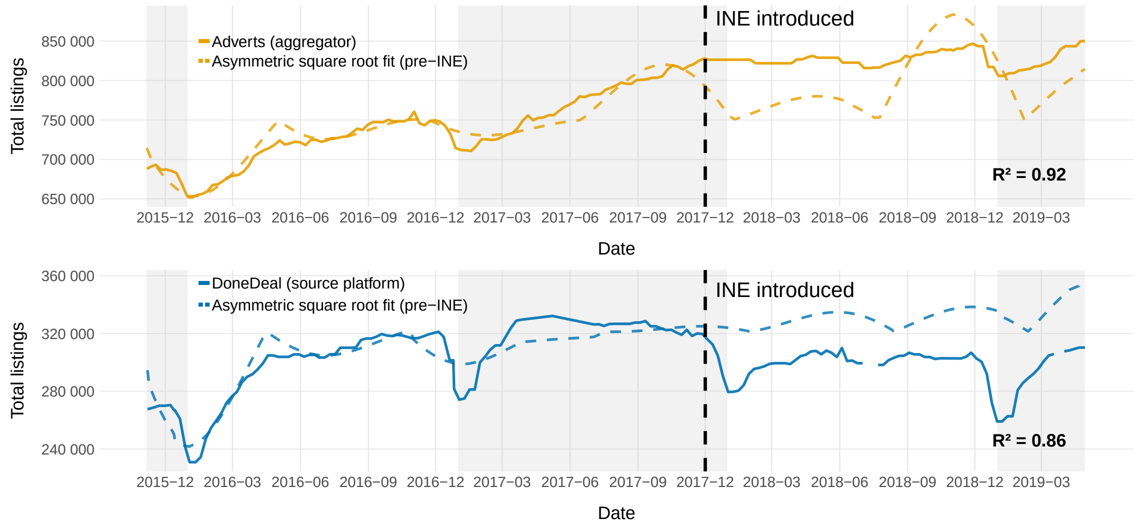
Note: The vertical dashed line indicates week 0 (introduction of intertwined network effects). Shaded areas correspond to 95% confidence intervals obtained from randomization inference via time permutation.

Figure 10 shows a positive and increasing treatment effect on the number of WAU for Adverts (the aggregator) starting five weeks after the introduction of INE. The effect ranges from 2.6 to 6 pre-shock standard deviations. This corroborates Predictions 1.1 and 1.3.

In contrast, the treatment effect is statistically indistinguishable from zero for DoneDeal (the source platform) up to 64 weeks after the introduction of INE. This suggest INE had no causal effect on buyer- and seller-side demand through the extensive margin channel for this platform. However, as shown below, and in line with Prediction 1.4, seller-side demand decreased in DoneDeal through the intensive margin channel.

Listings (intensive margin). Figure 11 shows the evolution of the total number of listings before and after the introduction of INE against an asymmetric square root function (cf. Equation 18) fitting each platform's pre-INE trend.

Figure 11: Evolution of the total number of listings before and after the introduction of INE for Adverts and DoneDeal vs asymmetric square root fits for both platforms' pre-INE values

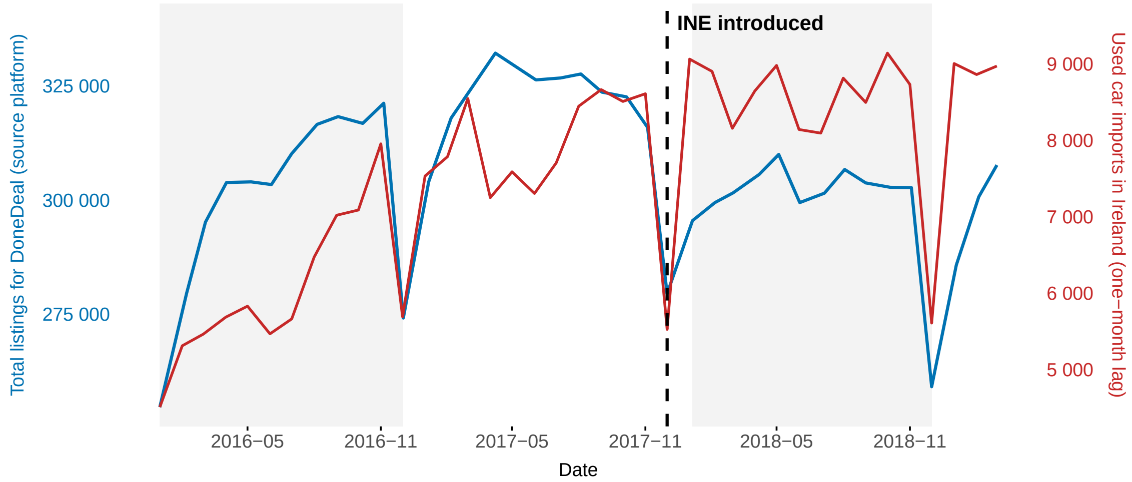


Note: the grayed areas indicate odd years. The R^2 displayed correspond to the fit of the asymmetric square root models on the observed pre-INE data.

Figure 11 shows that, after the introduction of INE, the number of total listings in Adverts (the aggregator) deviated upwards from its pre-INE trend. Conversely, the number of listings in DoneDeal (the source platform) decreased with respect to its pre-INE trend. This descriptive evidence supports the predictions of the model stated in Predictions 1.3 and 1.4.

Comparing the evolution of the total number of listings in DoneDeal with the imports of used cars in Ireland confirm these results. Figure 12 depicts the evolution of the total number of listings in DoneDeal against the number of imported second-hand cars to Ireland before and after the introduction of intertwined network effects. Both series exhibit parallel trends prior to December 2017, followed by a gradual post-treatment divergence. After INE are introduced, the growth rate of DoneDeal's listings slows down for approximately five months with respect to that of used car imports.

Figure 12: Evolution of the total number of listings and the number of used car imported to Ireland before and after INE introduction



The results of the differences-in-differences estimation of Equation 19 shown in Table 5 confirm negative causal effect of INE on the total number of listings of the source platform DoneDeal.

Table 5: Difference-in-Differences Estimates: Effect of INE on DoneDeal Listings

	Estimate	Std. Error	95% CI (Permutation)
Treatment Effect ($\hat{\beta}_3$)	-1.745^{***}	0.195	$[-1.748, -1.742]$
N		74	
Pre-period months		21	
Post-period months		16	
Time FE		Yes	

Notes: The table reports difference-in-differences estimates from equation (19). The treatment effect is measured in standardized units. Superscript *** indicates significance at the 1% level based on permutation-based inference.

Table 5 shows that the introduction of INE led to a statistically significant decline in DoneDeal listings relative to the counterfactual trend. DoneDeal listings decreased by approximately 1.75 standard deviations relative to used car imports following INE introduction. These results further confirm Prediction 1.4.

7 Conclusion and policy implications

This article studied how intertwined network effects influence buyer- and seller-side demand, pricing, and each user groups' welfare when platforms are independent and post-merger. It showed that while intertwined network effects increase consumer surplus, they reduce seller surplus if the platforms are sufficiently differentiated for sellers. In presence of INE, if the network effects they enjoy are sufficiently low, the merger harms consumers, and vice versa. If the platforms are sufficiently homogeneous to sellers, the merger reduces their surplus.

These results extend the literature on interoperability between platforms to an asymmetric case referred to as "intertwined network effects" in this article, a direction that had only been explored by Maruyama and Zennyo (2015). In the latter, when the network effects enjoyed by the multi-homing side of the market are sufficiently small, their surplus increases, and vice versa. However, given this article's focus on platforms applying per-transaction pricing to the multi-homing side of the market, I find that asymmetric interoperability (i.e., INE) benefits the single-homing side. The effect of INE on the multi-homing user group, in turn, depends on the reallocation of the multi-homing user group across platforms. With sufficiently low seller transportation costs, enough single-homing sellers in the source platform (where the quality-adjusted price increases) swithc from multi-homing to single-homing in the aggregator platform (where prices decrease and the number of buyers increases), leading to an increase in overall seller surplus, and vice versa.

This article also extends the growing literature on platform mergers. It introduces per-transaction pricing to the multi-homing side of the market to the canonical bottleneck setting to study a non-consolidating horizontal merger between platforms subject to INE. A novel mechanism through which a platform merger can harm users emerges from it: the reallocation of the single-homing user group across platforms. Despite the overall increase in the demand from the multi-homing user group triggered by the merger, if platform differentiation is sufficiently strong, single-homing users fail to sufficiently switch from the aggregator platform (where the number of sellers decreases) to the source platform (where the number of sellers increases). As a result, the multi-homing side is worse off post-merger. Moreover, if the network effects enjoyed by the users on the single-homing side are sufficiently low, post-merger, few consumers switch from the aggregator (where the number of sellers decreases) to the source platform (where the number of sellers increases). Hence, overall consumer surplus decreases.

The results of this article can help improving the analysis of mergers between transaction platforms subject to INE, or planning to set them up post-merger. Despite the existence of many platform mergers featuring INE, the literature on the topic remains scarce and competition authorities still lack clear guidance as to how they should be assessed. In that respect, Robertson provides a compelling case study in a note to the OECD (Robertson, 2023) and a subsequent article (Robertson, 2024). She shows how the ebay/Adevinta merger was differently analyzed in the eyes of the German (unconditional clearing), the Austrian (allowed with multiple structural and behavioral commitments) and the United Kingdom's (allowed with structural commitments) competition authorities. In these countries, the major concern was the overlap in the online classifieds market, in which both parties operate a transaction platform subject to per-transaction pricing on the seller side. Despite the different views expressed by these three competition authorities, an assessment of how post-merger intertwined network effects could affect consumers was absent in the three analyses. As in the ebay/Adevinta case, some competition authorities have not accounted for INE in their assessment of the merger's impact on welfare (e.g., Schibsted/Nettbil, Trade Me/Property NZ, viagogo/Stubhub). Other authorities, in turn, have judged them to be detrimental to at least one of the user groups (e.g., Booking/eTraveli, FDJ/Zeturf, Seek Asia Investments/JobStreet, Wedding Planner/Zank you). They typically argue that these effects would lessen competition by strengthening network effects.

This article shows that, in a competitive bottleneck setting with per-transaction pricing of multi-homing sellers, not only should INE be considered; they should be encouraged. Even in an extreme case in which the platforms do not face competition from legally-independent firms, INE benefit consumers. Then, competition authorities guided by a consumer welfare standard could use INE as remedies in merger cases and ask platforms for a commitment not to consolidate whenever the merger might significantly limit competition. An INE remedy would not be entirely novel. Booking and eTraveli proposed such a remedy in their blocked merger. The flight metasearch platform Kayak, owned by eTraveli, proposed to set up a choice screen that would have displayed accommodation offers from Booking and other competing hotel booking portals on the check-out page of the flight booking process.[19] The European Commission dismissed it arguing this would pose a risk of self-preferencing.

However, sometimes consumers, not sellers, are on the multi-homing side. This was the case in the merger between Française des Jeux (FDJ), France's national lottery, instant games and sports betting operator, and Zeturf, an online platform specialized in horse race betting. The analysis carried out by the Autorité de la Concurrence revealed that consumers (i.e., bettors) multi-home to different extents depending on the game. Moreover, the French competition authority noted that the two firms had started establishing INE pre-merger and would have incentives to continuing doing so post-merger.[20] Judging this would give the merged entity too much market power, it asked firms to undo INE as a remedy to authorize the merger. As per the results in this article, harm to consumers from such a merger should only arise when network effects play a minor role in consumers' decision to join one platform over another. It follows that, as a general principle, to allow the merger, competition authorities could ask the parties to undo (or commit not to set up) INE when consumers multi-home and network effects are weak, and ask them to set them up (or commit not to undo them) otherwise.

Moreover, as argued above, intertwined network effects are akin to asymmetric interoperability – as defined by Maruyama and Zennyo (2015) – between multi-sided platforms. In that respect, the results of this article can inform ongoing policy discussions about and the enforcement of asymmetric interoperability mandates imposed on some digital platforms. The European Commission's Digital Markets Act (DMA), currently under enforcement,

[19] See para. 1196 et seq. in European Commission (2023).
[20] See paras 49-56 in Autorité de la Concurrence (2023).

implemented interoperability obligations for platforms considered to be “gatekeepers” in any of the defined eight “core platform services”. Other major legislators and regulators are following suit within a narrower market scope. The United States’ proposed Open App Markets Act includes provisions for interoperability between app stores and operating systems. India’s Competition Commission has issued directives requiring Google to allow more interoperability for its services, including allowing third-party app stores and payment systems. If interoperability requirements to a specific platforms extend to transaction platforms charging per-transaction prices to the multi-homing side (which is possible under the DMA), the findings of this articles could inform the design of such requirements. In this respect, it should be noted that interoperability mandates usually intend to help both business and end users participating in a platform. In the setting analyzed in this article, INE might benefit one user group and harm the other. A possible solution could include regulating the per-transaction fee (if any is allowed) charged by the aggregator to make the user groups’ interest converge.

Although this article’s study of the effects of INE provides policy-relevant lessons for competition authorities and regulators, it leaves many questions open. Future work might complement this article’s results by extending the analysis of INE to other settings. How are results affected when there is multi-homing on both sides of the market? And how are results affected when the multi-homing side is also charged a participation fee? One of the main limitations of this article’s modelling is that it loses tractability when both sides of the market are charged a per-transaction price. It can therefore not provide insights in such cases, which are common in certain transaction platforms such as house rental platforms. Developing an alternative modelling of INE that allows to include per-transaction pricing to both sides of the market while preserving tractability might be a fruitful endeavour.

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Appendix A: Additional tables and figures

Table 6: Model notation and definitions

Symbol	Meaning	Category
x	Location of a buyer or seller on $[0, 1]$	Exogenous variable
v^b	Intrinsic value of participating in a platform for buyers	Parameter
α^b	Benefit to a buyer from interacting with a single seller (indirect network effect)	Parameter
α^s	Benefit to a seller from interacting with a single buyer (indirect network effect)	Parameter
τ^b	Transportation cost for buyers	Parameter
τ^s	Transportation cost for sellers	Parameter
p_i^{sL}	Per-transaction price charged to sellers by platform i in setting $L \in \{\text{B}, \text{INE}, \text{INE-M}\}$	Endogenous variable
f^L	Referral fee charged by platform 0 (aggregator) to platform 1 (source platform) in setting $L \in \{\text{INE}, \text{INE-M}\}$	Endogenous variable
n_i^{sL}	Mass of sellers joining platform i in setting $L \in \{\text{B}, \text{INE}, \text{INE-M}\}$	Endogenous variable
n_i^{bL}	Mass of buyers joining platform i in setting $L \in \{\text{B}, \text{INE}, \text{INE-M}\}$	Endogenous variable
U_i^{bL}	Utility of a buyer located at x from joining platform i in setting $L \in \{\text{B}, \text{INE}, \text{INE-M}\}$	Endogenous variable
U_i^{sL}	Utility of a seller located at x from joining platform i in setting $L \in \{\text{B}, \text{INE}, \text{INE-M}\}$	Endogenous variable
${\Pi_i}^L$	Profit of platform $i = 0, 1$ in setting $L \in \{\text{B}, \text{INE}, \text{INE-M}\}$	Endogenous variable

Appendix B: Proofs

B.1 Proof of Lemma 3

Subtracting the profits platforms 1 and 2 obtain in equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expressions, respectively.

$$\Pi_0{}^{INE} - \Pi_0{}^{B} = \frac{4\left(-\tau^b\tau^s + \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)\left(-\tau^b\tau^s + \alpha^b\alpha^s + \sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}\right)}{16\tau^s{\alpha^b}^2} + \frac{(4\tau^b\tau^s - \alpha^b\alpha^s)\left(-4{\tau^b}^2\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s + {\alpha^b}^2\alpha^{s2} + \sqrt{\tau^b\tau^s(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2\alpha^{s2} - 2{\alpha^b}^3\alpha^{s3})}\right)}{16\tau^s{\alpha^b}^2}$$

$$\Pi_1{}^{INE} - \Pi_1{}^{B} = \frac{(-2\tau^b\tau^s + \alpha^b\alpha^s + 2\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)})^2}{16\tau^s{\alpha^b}^2}$$

Given that all the parameters are positive by definition, the two expressions are non-negative. *Q.E.D.*

B.2 Proof of Lemma 4

Subtracting the prices platforms 1 and 2 charge sellers in equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expressions, respectively.

$$p_0^{s\,INE} - p_0^{s\,B} = -\frac{16{\tau^b}^3\tau^{s3} - 4{\tau^b}^2\tau^{s2}\alpha^b\alpha^s + {\alpha^b}^2\alpha^{s2}\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{u^3\alpha^{s2}} + \frac{\tau^b\tau^s(u^2\alpha^{s2} - 4\sqrt{\tau^b\tau^s(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s{\alpha^b}^2\alpha^{s2} - 2{\alpha^b}^3\alpha^{s3})})}{u^3\alpha^{s2}}$$

$$p_1^{s\,INE} - p_1^{s\,B} = \frac{4\tau^b\tau^s - \alpha^b\alpha^s - 4\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\alpha^s)}}{4\alpha^b}$$

$p_0^{s\,INE} - p_0^{s\,B}$ is positive if and only if $\frac{\alpha^b\alpha^s}{\tau^s} < \tau^b < \tilde{\tau^b}(\alpha^b, \alpha^s, \tau^s)$ and negative otherwise, where $\tilde{\tau^b}$ is the second root of the following polynomial:

$$P(\tau^b) = -{\alpha^b}^4\alpha^{s4} + 80\tau^{s2}{\alpha^b}^2\alpha^{s2}{\tau^b}^2 - 384\tau^{s3}\alpha^b\alpha^s{\tau^b}^3 + 256\tau^{s4}{\tau^b}^4$$

The expression for ${p_1^s}^{INE} - {p_1^s}^{B}$, in turn, is always positive given that all the parameters are strictly positive. *Q.E.D.*

B.3 Proof of Lemma 5

Platform 0 buyers. Subtracting the equilibrium number of buyers in platform 0 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$ {n_0^b}^{INE} - {n_0^b}^{B} = \frac{\tau^b\tau^s\left(-4\tau^b\tau^s+3\alpha^b\alpha^s\right)+\sqrt{\tau^b\tau^s\left(16\tau^{b3}\tau^{s3}-8\tau^{b2}\tau^{s2}\alpha^b\alpha^s+5\tau^b\tau^s\alpha^{b2}\alpha^{s2}-2\alpha^{b3}\alpha^{s3}\right)}}{8\tau^b\tau^s\left(2\tau^b\tau^s-\alpha^b\alpha^s\right)} $$

This expression is non-negative when the second-order conditions with intertwined network effects (cf. Assumption 2), i.e., $\alpha^b\alpha^s < 2\tau^b\tau^s$.

Platform 1 buyers. Subtracting the equilibrium number of buyers in platform 1 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$ {n_1^b}^{INE} - {n_1^b}^{B} = \frac{-2\tau^b\tau^s+\alpha^b\alpha^s+2\sqrt{\tau^b\tau^s(\tau^b\tau^s-\alpha^b\alpha^s)}}{4\tau^s\alpha^b} $$

Given that all the parameters are strictly positive by definition, this expression cannot be positive.

Platform 0 sellers. Subtracting the equilibrium number of sellers in platform 0 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$ {n_0^s}^{INE} - {n_0^s}^{B} = \frac{2\sqrt{\tau^b\tau^s(\tau^b\tau^s-\alpha^b\alpha^s)}+\frac{\tau^b\tau^s(-8\tau^b\tau^s+5\alpha^b\alpha^s)+\sqrt{\tau^b\tau^s(16\tau^{b3}\tau^{s3}-8\tau^{b2}\tau^{s2}\alpha^b\alpha^s+5\tau^b\tau^s\alpha^{b2}\alpha^{s2}-2\alpha^{b3}\alpha^{s3})}}{2\tau^b\tau^s-\alpha^b\alpha^s}}{4\tau^s\alpha^b} $$

Given that all the parameters are strictly positive by definition, this expression is non-negative.

Platform 1 sellers. Subtracting the equilibrium number of sellers in platform 1 with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

Given that all the parameters are strictly positive by definition, this expression cannot be positive.

$$ {n_1^s}^{INE} - {n_1^s}^{B} = \frac{-2\tau^b\tau^s+\alpha^b\alpha^s+2\sqrt{\tau^b\tau^s(\tau^b\tau^s-\alpha^b\alpha^s)}}{4\tau^s\alpha^b} $$

Platforms 0 and 1 sellers. Subtracting the equilibrium number of sellers present in both platforms with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$ {n_s}^{INE} - {n_s}^{B} = \frac{\alpha^s+\frac{4(-\tau^b\tau^s+\sqrt{\tau^b\tau^s(\tau^b\tau^s-\alpha^b\alpha^s)})}{\alpha^b}+\frac{\tau^b\tau^s(-4\tau^b\tau^s+3\alpha^b\alpha^s)+\sqrt{\tau^b\tau^s(16\tau^{b3}\tau^{s3}-8\tau^{b2}\tau^{s2}\alpha^b\alpha^s+5\tau^b\tau^s\alpha^{b2}\alpha^{s2}-2\alpha^{b3}\alpha^{s3})}}{\alpha^b(2\tau^b\tau^s-\alpha^b\alpha^s)}}{4\tau^s} $$

Bearing in mind that all the parameters are strictly positive by definition, ${n_s}^{INE} - {n_s}^{B} < 0 \iff \frac{\alpha^b\alpha^s}{\tau^s} \leq \tau^b < \frac{(3+2\sqrt{3})\alpha^b\alpha^s}{6\tau^s}$ and ${n_s}^{INE} - {n_s}^{B} > 0 \iff \tau^b > \frac{(3+2\sqrt{3})\alpha^b\alpha^s}{6\tau^s}$.

B.4 Proof of Proposition 1

Denote Δn_i^l the absolute value of the variation in demand of user group $i = b, s$ in platform $l = 0, 1$ after the introduction of intertwined network effects (INE), compared to the non-INE benchmark. Given the uniform distribution of the mass 1 total number of consumers, the absolute change in consumer surplus generated by INE in platforms 1 (denoted ΔCS^1) and 2 (denoted ΔCS^2) can be expressed as:

$$\Delta CS^1 = (\frac{1}{2} + \Delta n_0^b)({n_0^s}^B + \Delta n_0^s + {n_1^s}^B - \Delta n_1^s)$$
$$\Delta CS^2 = (\frac{1}{2} - \Delta n_1^b)\Delta n_1^s$$

Where the positive or negative signs are given by Lemma 5. The net effect of INE on consumer surplus on both platforms $\Delta CS := \Delta CS^1 + \Delta CS^2$ is hence:

$$\Delta CS = \Delta\frac{1}{2}{n_0^s}^B + \frac{1}{2}\Delta n_0^s + \frac{1}{2}{n_1^s}^B + \Delta n_0^b {n_0^s}^B + \Delta n_0^b \Delta n_0^s + \Delta n_0^b {n_1^s}^B - (\Delta n_0^b + \Delta n_1^b)\Delta n_1^s$$

Denote $\Delta n_0^b := \Delta n_b$. Given that the market is covered on the consumer side, $\Delta n_b = -\Delta n_1^b$, and hence $-\Delta n_1^b := -\Delta n_b$. Then,

$$\Delta CS = \left(\frac{1}{2} + \Delta n_b\right){n_0^s}^B + \left(\frac{1}{2} + \Delta n\right)\Delta n_0^s + \left(\frac{1}{2} + \Delta n_b\right){n_1^s}^B > 0.$$

Q.E.D.

B.5 Proof of Proposition 2

Subtracting seller surplus equilibrium with (cf. Lemma 2) and without (cf. Lemma 1) intertwined network effects yields the following simplified expression.

$$SS^{INE} - SS^B = \frac{1}{32}\Bigg(8\tau^s\left(-2 + \frac{{\tau^b}^2}{{\alpha^b}^2}\right) - \frac{3\pi_s^2}{\tau^s} - \frac{8}{{\alpha^b}^2}\Bigg(- 2{\tau^b}^2\tau^s$$
$$- 2\alpha^b\left(\tau^s\alpha^b + 2\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\pi_s)}\right) + \tau^b\left(4\tau^s\alpha^b + \alpha^b\pi_s + 2\sqrt{\tau^b\tau^s(\tau^b\tau^s - \alpha^b\pi_s)}\right)\Bigg)$$
$$+ \frac{1}{{\alpha^b}^2(-2\tau^b\tau^s + \alpha^b\pi_s)^2}\Bigg(16{\alpha^b}^4\pi_s^3 + 2{\tau^b}^2\tau^s\Big({\alpha^b}^2\pi_s\left(32\tau^s + 3\pi_s\right)$$
$$- 4\sqrt{\tau^b\tau^s\left(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\pi_s + 5\tau^b\tau^s{\alpha^b}^2\pi_s^2 - 2{\alpha^b}^3\pi_s^3\right)}$$
$$+ 2\tau^b\alpha^b\pi_s\Big(- {\alpha^b}^2\pi_s\left(32\tau^s + \pi_s\right) + 3\sqrt{\tau^b\tau^s\left(16{\tau^b}^3\tau^{s3} - 8{\tau^b}^2\tau^{s2}\alpha^b\pi_s + 5\tau^b\tau^s{\alpha^b}^2\pi_s^2 - 2{\alpha^b}^3\pi_s^3\right)}\Big)\Big)\Bigg)\Bigg)$$

To simplify the mathematical expressions, and without loss of generality in the proof[21], denote $R^n_{(d,r)}(x)$ a function describing the r^{th} root a polynomial of degree $d \in \mathbb{N}$ in variable x. Superscript $n = [I, II, ...]$ identifies the equation describing the polynomial. Roots are ordered in increasing order.

Bearing in mind that all the parameters are strictly positive by definition, $SS^{INE} - SS^{B} < 0$ if and only if one of the following sets of conditions, labelled as $C^{SS-}_{i \in \mathbb{N}}$ are met:

$$C_0^{SS-} = \left\{ \frac{5\alpha^s}{8} < \tau^s \leq \frac{17\alpha^s}{16} - \frac{1}{8}\sqrt{11}\alpha^s \right\} \text{ and } \left\{ \tau^b > R^I_{(9,3)} \left(\tau^b \alpha^b, \alpha^s, \tau^s) \right) \right\}$$

$$C_1^{SS-} = \left\{ \frac{17\alpha^s}{16} - \frac{1}{8}\sqrt{11}\alpha^s < \tau^s < R^{II}_{(4,2)} \left(\tau^s(\alpha^s) \right) \right\} \text{ and } \left\{ \tau^b \geq \frac{\alpha^s \alpha^b}{\tau^s}) \right\}$$

$$C_3^{SS-} = \left\{ R^{II}_{(4,2)}(\tau^s(\alpha^s)) \leq \tau^s < R^{III}_{(16,3)}(\tau^b(\alpha^b, \alpha^s, \tau^s)) \right\} \text{ and }$$
$$\left\{ \left(\frac{\alpha^s \alpha^s}{\tau^s} \leq \tau^b < R^I_{(9,1)} \left(\tau^b \alpha^b, \alpha^s, \tau^s) \right) \right) \vee \left(\tau^b > 2\alpha^s \right) \right\}$$

$$C_4^{SS-} = \left\{ \tau^s = R^{III}_{(16,3)} \left(\alpha^s \right) \right\} \text{ and } \left\{ \left(\frac{\alpha^s \alpha^s}{\tau^s} \leq \tau^b < R^I_{(9,1)} \left(\tau^b(\alpha^b, \alpha^s, \tau^s) \right) \right) \right\}$$

$$C_5^{SS-} = \left\{ \tau^s > R^{III}_{(16,3)} \left(\alpha^s \right) \right\} \text{ and } \left\{ \tau^b \geq \frac{\alpha^s \alpha^s}{\tau^s} \right\}$$

In the same vein, $SS^{INE} - SS^{B} > 0$ if and only if one of the following sets of conditions, labelled as $C^{SS+}_{i \in \mathbb{N}}$ are met:

$$C_0^{SS+} = \left\{ 0 < \tau^s \leq \frac{5\alpha^s}{8} \right\} \text{ and } \left\{ \tau^b \geq \frac{\alpha^s \alpha^s}{\tau^s} \right\}$$

$$C_1^{SS+} = \left\{ \frac{5\alpha^s}{8} < \tau^s \leq \frac{17\alpha^s}{16} - \frac{1}{8}\sqrt{11}\sqrt{\pi_s^2} \right\} \text{ and } \left\{ \frac{\alpha^b \alpha^s}{\tau^s} \leq \tau^b < R^I_{(9,1)} \left(\tau^b(\alpha^b, \alpha^s, \tau^s) \right) \right\}$$

[21] The full expressions are available upon request.

$$C_3^{SS+} = \left\{ \frac{17\alpha^s}{16} - \frac{1}{8}\sqrt{11}\sqrt{\pi_s^2} < \tau^s < R^{II}_{(4,2)} \right\} \text{and} \left\{ R^{I}_{(9,1)}(\tau^b\alpha^b, \alpha^s, \tau^s)) < \tau^b < R^{I}_{(9,3)}(\tau^b\alpha^b, \alpha^s, \tau^s)) \right\}$$

$$C_4^{SS+} = \left\{ \tau^s = R^{II}_{(4,2)} \right\} \text{and} \left\{ R^{I}_{(9,1)}(\tau^b\alpha^b, \alpha^s, \tau^s)) < \tau^b < 2\alpha^b \right\}$$

$$C_5^{SS+} = \left\{ R^{II}_{(4,2)} < \tau^s < R^{III}_{(16,3)}(\tau^s, \alpha^b, \alpha^s, u) \right\} \text{and} \left\{ R^{I}_{(9,1)}(\tau^b\alpha^b, \alpha^s, \tau^s)) < \tau^b < R^{I}_{(9,2)}(\tau^b\alpha^b, \alpha^s, \tau^s)) \right\}$$

Note that none of the C^{SS-} sets of conditions imply $\tau^s \leq \frac{5\alpha^s}{8}$. Moreover, condition C_0^{SS+} shows that $\tau^s \leq \frac{5\alpha^s}{8}$ is a sufficient condition for seller surplus to decrease, as the second part of the condition, $\tau^b \geq \frac{\alpha^s\alpha^b}{\tau^s}$, corresponds to the second order conditions in the pre-INE benchmark reported in Assumption 1. This proves point (i) of Proposition 2.

By comparing the C^{SS-} sets of conditions, it can be seen that they all imply $\tau^s > \tilde{\tau^s}(\alpha^s) > \frac{5}{8}\alpha^s$. This proves point (ii) of Proposition 2.

Q.E.D.

B.6 Proof of Lemma 7

Denote $\Pi^{INE-M} := {\Pi_0}^{INE-M} + {\Pi_1}^{INE-M}$ and $\Pi^{INE} := {\Pi_0}^{INE} + {\Pi_1}^{INE}$. Subtracting Π^{INE} from Π^{INE-M} and simplifying yields:

$$\Pi^{INE-M} - \Pi^{INE} = \frac{\left(-2\tau^b\tau^s + \alpha^b\alpha^s + 2\sqrt{\tau^b\tau^s\left(\tau^b\tau^s - \alpha^b\alpha^s\right)}\right)^2}{8\tau^s{\alpha^b}^2} > 0$$

Q.E.D.

B.7 Proof of Lemma 8

Subtracting the equilibrium price charged to sellers in platform 0 when the platforms are legally-independent (cf. Lemma 2) from the post-merger equilibrium price charged to sellers in platform 0 (cf. Lemma 6) and simplifying yields:

$$p_0^{s\,INE-M} - p_0^{s\,INE} = 2\tau^b\tau^s \left(\frac{1}{\alpha^b} - \frac{2\tau^b\tau^s}{\alpha^{b2}\alpha^s} + \frac{\alpha^s}{-4\tau^b\tau^s + \alpha^b\alpha^s} - \frac{2\left(-4\tau^{b2}\tau^{s2} + \sqrt{\tau^b\tau^s\left(16\tau^{b3}\tau^{s3} - 8\tau^{b2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b2}\alpha^{s2} - 2\alpha^{b3}\alpha^{s3}\right)}\right)}{\alpha^{b3}\alpha^{s2}} \right)$$

Which is strictly positive under the second order conditions of the merger case set out in Assumption 3, $\alpha^b\alpha^s < 2\tau^b\tau^s$

Subtracting the equilibrium price charged to sellers in platform 1 with INE (cf. Lemma 2) from the equilibrium price charged to sellers in platform 1 post-merger (cf. Lemma 6) yields:

$$p_1^{s\,INE-M} - p_1^{s\,INE} = -\frac{\alpha^s}{4} < 0.$$

Q.E.D.

B.8 Proof of Lemma 9

Platform 0 consumer-side demand. Subtracting the equilibrium demand on the consumer side in platform 0 with INE (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_0^{b\,INE-M} - n_0^{b\,INE} = \frac{4\tau^{b2}\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s - \alpha^{b2}\alpha^{s2} - \sqrt{\tau^b\tau^s(16\tau^{b3}\tau^{s3} - 8\tau^{b2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b2}\alpha^{s2} - 2\alpha^{b3}\alpha^{s3})}}{8\tau^b\tau^{s2}\alpha^b - 4\tau^s\alpha^{b2}\alpha^s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Platform 1 consumers-side demand. Subtracting the equilibrium number of consumers in platform 1 with INE (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_1^{b\,INE-M} - n_1^{b\,INE} = \frac{\tau^b\tau^s(-4\tau^b\tau^s + \alpha^b\alpha^s) + \sqrt{\tau^b\tau^s(16\tau^{b3}\tau^{s3} - 8\tau^{b2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b2}\alpha^{s2} - 2\alpha^{b3}\alpha^{s3})}}{8\tau^b\tau^s(2\tau^b\tau^s - \alpha^b\alpha^s)}$$

Which is strictly positive given that all the parameters are strictly positive by definition.

Platform 0 seller-side demand. Subtracting the equilibrium number of sellers in platform 0 when the platforms are legally independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_0^{s\,INE-M} - n_0^{s\,INE} = \frac{4\tau^{b2}\tau^{s2} - \tau^b\tau^s\alpha^b\alpha^s - \sqrt{\tau^b\tau^s(16\tau^{b3}\tau^{s3} - 8\tau^{b2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b2}\alpha^{s2} - 2\alpha^{b3}\alpha^{s3})}}{8\tau^b\tau^{s2}\alpha^b - 4\tau^s\alpha^{b2}\alpha^s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Platform 1 sellers-side demand. Subtracting the equilibrium number of sellers on the seller side in platform 1 when the platforms are legally independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_1^{s\,INE-M} - n_1^{s\,INE} = \frac{\alpha^s}{4\tau^s} > 0$$

Overall seller-side demand. Subtracting the equilibrium number of sellers in both platforms when the platforms are legally-independent (cf. Lemma 2) from the post-merger one (cf. Lemma 6) and simplifying yields:

$$n_s{}^{INE-M} - n_s{}^{INE} = \frac{-4\tau^{b^2}\tau^{s2} + \tau^b\tau^s\alpha^b\alpha^s + \sqrt{\tau^b\tau^s(16\tau^{b^3}\tau^{s3} - 8\tau^{b^2}\tau^{s2}\alpha^b\alpha^s + 5\tau^b\tau^s\alpha^{b^2}\alpha^{s2} - 2\alpha^{b^3}\alpha^{s3})}}{8\tau^b\tau^{s2}\alpha^b - 4\tau^s\alpha^{b^2}\alpha^s}$$

Which is strictly negative given that all the parameters are strictly positive by definition.

Q.E.D.

B.9 Proof of Proposition 3

Recalling that all the parameters are strictly positive, and assuming the second-order condition $SOCs^{INE}$ set out in Assumption 2 holds, $CS^{INE-M} > CS^{INE} \iff 2\tau^b\tau^{s2} > \tau^s\alpha^b\alpha^s$ and $v^b < \tilde{v^b}$, while $CS^{INE-M} < CS^{INE} \iff 2\tau^b\tau^{s2} > \tau^s\alpha^b\alpha^s$ and $v^b > \tilde{v^b}$.

Moreover:

$$CS^{INE-M} > CS^{INE} \cap v^b > \tilde{v^b} \cap Assumption\ 2 \cap Assumption\ 3 = \varnothing$$
$$CS^{INE-M} > CS^{INE} \cap v^b < \tilde{v^b} \cap Assumption\ 2 \cap Assumption\ 3 \neq \varnothing$$
$$CS^{INE-M} < CS^{INE} \cap v^b > \tilde{v^b} \cap Assumption\ 2 \cap Assumption\ 3 \neq \varnothing$$
$$CS^{INE-M} < CS^{INE} \cap v^b < \tilde{v^b} \cap Assumption\ 2 \cap Assumption\ 3 = \varnothing$$

Hence, $CS^{INE-M} > CS^{INE} \iff v^b < \tilde{v^b}$ and $CS^{INE-M} < CS^{INE} \iff v^b > \tilde{v^b}$.

Q.E.D.

B.10 Proof of Proposition 4

By comparing the region in which $SS^{INE-M} < SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3$ holds with that in which $SS^{INE-M} > SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3$ holds, it can be seen that

$$SS^{INE-M} > SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap \tau^b > \hat{\tau^b} = \varnothing$$
$$SS^{INE-M} < SS^{INE} \cap Assumption\ 2 \cap Assumption\ 3 \cap \tau^b > \hat{\tau^b} \neq \varnothing$$

Therefore, $\tau^b > \hat{\tau^b} \implies SS^{INE-M} < SS^{INE}$.