Source taxes versus end-of-chain taxes in General Equilibrium

Reyer Gerlagh, Etienne Lorang, Aude Pommeret, Antonin Pottier

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Work in Progress: comments welcome. Note that future revisions and corrections may affect findings and lead to different notation.

Abstract

We propose a general equilibrium model to analyze the taxation of an externality, where only the aggregate amount matters. In this common situation, regulation is traditionally considered upstream at the source of the externality. We rigorously define regulation downstream at the end-of-chain, through taxation of the embodied externality, as in the case of greenhouse gas emissions footprinting. We identify conditions for equivalence between source-based and end-of-chain taxes. We show that implementation via end-of-chain taxes requires the existence of equilibrium price schedules under which goods are traded as bundles of quantities and embodied externalities. Our results characterize the informational and pricing structures required, and provide a unified general equilibrium framework for the analysis of environmental policies.

1 Introduction

Externalities violate the structural assumption in general equilibrium that production sets and preferences are independent[1] hence invalidating the two welfare theorems. Such a market failure calls for government intervention

[1] Debreu (1959), Chapter 3, Note 2.

and since Pigou (1920, Chap. VI, §8-11), economists have long advocated externality pricing as an effective and efficient policy to address unintentional spillover effects of economic transactions. However, the price instrument still requires careful selection. For example, there is a broad literature focusing on differences in general equilibrium, between taxes and prices resulting from tradable quotas. Weitzman (1974) pointed to differences in their capacity to absorb uncertainty, while recently Anderson and Duanmu (2025) demonstrate that even under certainty the two are not equivalent. In this paper, we investigate a different variation of pricing policies: the point along the value chain *where* an externality tax is levied. The key contribution of this paper is to identify the assumptions and market mechanisms under which the point of taxation, at the source or at the end of the value chain, does not affect allocation outcomes.[2] Our framework is generically applicable to any impersonal producer externality (Starrett, 1973; Laffont, 1977), where only the aggregate amount of the externality matters. Such a situation opens the way to shifting the point of taxation along the value chain. Thanks to accounting rules, the externality can be followed and cumulated through the value chain, and ultimately be attributed to the end-of-chain. We can thus define the intensity of embodied externality for a good as the externality generated all along the value chain to produce one unit of that good[3].

The location of an externality tax in the value chain is relevant as it affects its salience, implied responsibility, and political acceptance. A common example is the case of carbon prices. As illustrated by the French yellow vest movement in 2018, the public is not always convinced that carbon prices are a good instrument to mitigate climate change, while the average worldwide price of carbon falls short of the required levels by a wide margin (OECD, 2021).[4] Therefore, a shift in the point of taxation, from the source (producer) to the end of chain (consumer), can be relevant for policy design. In particular, improving the political acceptability of carbon pricing may

[2]The consequences of distorting the value chain at different levels has also been studied in Antràs and Chor (2013) in the context of contractual frictions and Antràs and De Gortari (2020) in the context of costly trade.

[3]This is inspired by, and applies to, to CO_2 footprinting (or consumption-based accounting) which follows emissions from their source to final consumption goods (see Peters (2008) in an input-output framework)

[4]By contrast, there is no opposition to subsidies (Douenne and Fabre, 2022; Dechezleprêtre et al., 2025), but these are costly both in terms of welfare and public finances (Schubert et al., 2025).

involve relocating taxes along the value chain, which would involve develcopping accounting rules at the corporate level to track carbon across the value chain (Kaplan and Ramanna, 2021). As a matter of fact, the Carbon Pricing Leadership Coalition advocates "to price carbon further downstream nearer to the point of final consumption" (CPLC, 2018). At a more operational level, CE-Delft (2015) recommended to tax the carbon embodied in products (they called this end-of-chain taxation Carbon Added tax). Maier et al. (2024) argue that such a tax, as an alternative design, could enhance public acceptability and policy feasibility, based on simulations for 27 EU Member States, whereas Grubb et al. (2020) (who call it carbon embodied charges) emphasize the implementation challenges it faces. However, none of these papers provides a theoretical setting to analyze how rational producers would respond to end-of-chain taxation.

From an economic theory perspective, it is natural to ask what stage or event most closely resembles the Pigouvian spirit for a tax. For carbon pricing, this is usually the point where CO_2 is emitted; however, viewing climate change through the fossil-fuel value chain reverses this interpretation, making extraction the source stage and emissions the end-of-chain outcome.[5] For non-toxic material waste, whether mining impacts or waste disposal constitute the relevant externality similarly affects where a Pigouvian tax should be applied. In practice, identifying a single stage is challenging, and Pigouvian policy design may require combining upstream and downstream instruments. When pollution or landscape destruction stem from mining, source taxation is closer to the Pigouvian benchmark; when waste disposal is the relevant externality, end-of-chain taxation is. Given that both stages are relevant, the precise point of Pigouvian taxation is ambiguous (Palmer et al., 1997; Palmer and Walls, 1997).[6]

Beyond acceptability and proximity to the Pigouvian spirit, the relevant question is where price signals most efficiently incentivize product-design changes (Walls and Palmer, 2001). We propose an extended general competitive equilibrium model where externality taxes are calculated on the basis of the source of the externality,[7] or of the end-of-chain externality embodied in

[5] Hobbs et al. (2010) emphasizes that the interpretation of upstream versus downstream is context-dependent.

[6] In some cases, directly taxing the externality can be impossible (Sandmo, 1978). Knittel and Sandler (2018) show that taxing a proxy, e.g., gasoline for CO_2 is a non efficient second-best policy.

[7] Source taxation refers to taxing the externality where it is generated and is not

final goods. Importantly, the theory allows the externality and its intensity in goods to vary endogenously. The embodied externality (carbon footprints or material intensities of goods) will respond to policies. We show that embodied externalities in equilibrium are uniquely well-defined by the allocation of factors and goods. We analyze what (non-standard) assumptions are needed to guarantee the equivalence between source and end-of-chain taxation. In particular, we find one key required mechanism: rational producers must face a price schedule for their demand and supplies, meaning that prices for products depend on the embodied externalities. Intuition runs as follows. For final goods, producers compete to offer the good at the lowest consumer price *inclusive of the end-of-chain tax*. That is, demand by consumers is characterized by a price schedule. Profit maximization runs a recursive procedure; faced with price schedules for output, firms set price schedules for their own inputs. In equilibrium, all agents are confronted with the same (consistent) price schedules.

With this paper, we generalize several strands of field literature. We first contribute to the literature on vertical targeting, i.e. the point of regulation of an externality (Bushnell and Mansur, 2011; Mansur, 2012; Aldy et al., 2010; Goulder and Schein, 2013). We also add to the literature about production-based versus consumption-based policies (Jakob et al., 2014). Steckel et al. (2010); Jakob et al. (2013) show that the way of accounting and taxing (source or end-of-chain) has neither efficiency nor distributive effects in the presence of a global cap-and-trade regime, if goods have constant intensities so that the externality is simply proportional to output. Our analysis allows externality intensities to adjust endogenously to policy, as in Lininger (2015). He however adopts a partial equilibrium framework that presumes, rather than demonstrates, the equivalence between consumption-based and energy-input taxation. Empirically, Shapiro and Walker (2018) show how US emissions reduction coming from environmental regulation is driven by emission intensity changes. Emissions intensities are measured as direct emissions per unit of output at the product or industry level. While they capture the externality generated at a single stage of production, not the cumulative externality embodied through the value chain.[8] We generalize these results,

necessarily related to the start of the value chain.

[8] Their work echoes the broader decomposition literature initiated by Copeland and Taylor (1994) and Grossman and Krueger (1995), and extended by Antweiler et al. (2001) and Levinson (2009), which separates pollution changes into scale, composition, and technique channels.

working in a general equilibrium setting and allowing producers to substitute between inputs and thus adapt the intensity of the externality embodied in their product. We show that equivalence between production-based and consumption-based taxation (i.e. between source and end-of-chain taxation) is robust to this more complex setting, and thus there are multiple instruments to decentralize an optimal solution.

Finally, we establish a methodological connection with the literature on product quality and product differentiation, i.e. hedonic prices. In our project, the intensity of the embodied externality, that is the embodied externality per unit of good, resembles the concept of "quality" of a good; in addition, we rely on the concept of price schedules. While these two features bring our work into contact with a different body of literature, the underlying economic foundations remain fundamentally distinct.

Starting with Lancaster (1966), followed by Rosen (1974); Leland (1977); Drèze and Hagen (1978), this literature characterizes goods by their attributes, valued by the consumer in their utility function. In our setting, the externality, that is the carbon or material intensity, does not appear in the consumers' preferences. Consumers only value the quantities of goods consumed, whatever the attributes. Their reason to distinguish the different attributes arrives through the end-of-chain tax consumers have to pay. In addition, the properties of price schedules are rather different in our setting, as compared to the literature on good qualities.

2 The structure of an economy with end-of-chain-taxation

2.1 Primitives

There is a finite set of consumers, labeled by the subscript $h \in \mathcal{H} = \{1, \ldots, H\}$. There is a finite set of production factors that cannot be produced, labeled by the subscript $k \in \mathcal{K} = \{1, \ldots, K\}$. Finally there is a finite set of produced goods, labeled by $i \in \mathcal{I} = \{1, \ldots, I\}$. We use subscripts i, j to index produced goods; we frequently need to denote both sending and receiving goods for intermediate deliveries between firms . We use the terms goods and commodities interchangeably.[9]

The distinction between produced goods and endowed factors is primarily one of analytical convenience. In principle, a factor and a good may denote the same physical product providing identical services to its user; such a product may be producible, while also being available through a positive endowment. In that sense, the good/factor classification is not intrinsic to the product itself. However, within the context of our analysis, the two are treated as conceptually distinct. Our focus is on externalities arising from production. For produced goods, it is therefore meaningful to consider the externalities embodied in the good, as these are endogenous and vary with economic decisions. By contrast, when the same product is available as an endowed factor, its availability is not the result of production and hence does not generate production-related externalities; the embodied externality is therefore zero. This allows for allocations that include produced goods with strictly positive embodied externalities alongside endowed factors with zero embodied externalities. For clarity and tractability, we find it convenient to capture this distinction directly in the primitives of the model.

2.2 Firms: production relations

Firms are specialized, i.e. a given firm produces only one commodity. At start, we simplify the production side of the economy by assuming that there is only a representative firm that produces good j (we relax later this assumption, see 7.3). We have a natural identification $\mathcal{J} \simeq \mathcal{I}$. In the sequel, $\mathcal{J}$ and $\mathcal{I}$ could be used interchangeably, but we keep the not arbitrary distinction between the two, to prepare the case with multiple firms per commodity (see Appendix Section 7.3).

For firm $j \in \mathcal{J}$ producing commodity j, the production set is a subset of factor and commodity space $F_j \subset \mathbb{R}^{\mathcal{K} \cup \mathcal{I}}$. We only require F_j to be convex and contains 0, so that the firm can choose not to operate at all. A production plan $z_j \in F_j$ can be decomposed into $l_j := (-z_{jk})_k \in \mathbb{R}_+^{\mathcal{K}}$ the production factors used as inputs, $x_j := (-z_{ji})_i \in \mathbb{R}_+^{\mathcal{I}}$ (with $x_{jj} = 0$) the goods used as inputs[10], and $y_j := z_{jj} \in \mathbb{R}_+$ the output of commodity j. Slightly abusing

[9] For convenient notation, throughout the analysis, we will multiply vectors and matrixes without using an apostroph or superscript T as transposition symbols to avoid too much clutter in superscripts. That is we write xA instead of $x'A$ or x^TA. We use $v \cdot z \equiv \sum_i v_i z_i$ to denote the inner (dot) product of two vectors v, z in $\mathbb{R}^n$,.

[10] The input demand of firm j for the good i is x_{ji}. When writing x_{ji} as matrix, the

notations, we have $z_j = (-l_j, -x_j, y_j)$.

2.3 Externality

In the systematic exposition of Arrow (1969), an externality is the consequence that an action (like consumption of commodity i) of agent j has on agent k. To deal with them with the framework of general equilibrium, Arrow supposed that externalities are measurable. A further simplification arises with impersonal externalities, when the effects of externality do not depend on the agent producing it, but only on the sum of amounts generated by several agents (Bergstrom, 1976; Starrett, 1973). Although personal externalities have been extensively discussed in the follow-up of Coase (1960), impersonal externalities have been considered standard to model environmental problems since at least Sandmo (1975), so that the term "impersonal" is often omitted in the literature. The classical example is air pollution, when only the total amount matters, irrespective of who discharged the pollutant. This is commonly termed a "public bad" (Shitovitz and Spiegel, 2003), however this terminology[11] describes the nature of the externality from the side of who is impacted: every agent is affected by the externality at the same level (no one can exclude oneself from the exposure, and being exposed to the externality does not reduce the level others are exposed to). Air pollution is an externality that is actually both public and impersonal. What matters for our results is the impersonal character of the externality, which emphasized the symmetrical role played by those who cause the externality. We therefore retain this wording[12].

Our model has one impersonal externality[13]. The externality is generated by producers and its effect is left unspecified: it could affect the utilities of (some) consumers or the production sets of (some) producers. The impor-

first index (rows) labels the demanding firm, the second index (columns) the supplying firm. This matrix is transposed compared to the common notations used in input-output models.

[11] Meade (1952) called this type "atmosphere" externalities, whereas Shapley and Shubik (1969) labeled externalities of this type "diffusive", that they opposed to "directed". Mas-Colell et al. (1995, 11.D) use "non-depletable".

[12] Mas-Colell et al. (1995, 11.D) call it an "homogeneous" externality, Anderson and Duanmu (2025, Definition 3.9) an externality that "arises from the total net emissions of the regulated commodities".

[13] The extension to a finite number of impersonal producer externalities is left to the reader.

tant thing is that only the total amount of externality generated matters. As Baumol (1972) noted, the optimal level of an externality is very hard to determine, and for economists the implementation of a "more or less arbitrary" quantity is of most interest. Therefore the government would regulate the total amount of externality[14] and we do not need to explicitly introduce how utility and production sets would depend on it[15]. Indeed, as we work with a constant level of externality, be it optimal or not, indicating dependence on its level is irrelevant for our purposes and would only clutter notations.

Specifically, each firm produces an amount of the externality, depending on its production plan. The amount of externality r_j produced by firm j for production plan z_j is specified with an externality function (possibly null) $G_j : F_j \subset \mathbb{R}^{\mathcal{K}\cup\mathcal{I}} \to \mathbb{R}_+, z_j \mapsto r_j$. We only require G_j to be convex and $G_j(0) = 0$ (no externality produced when the firm does not operate)[16]. [17]

In the physical (material) interpretation of our framework, r_j represents the amount of material extracted from the environment by the firm and physically becoming part of the output product. In the emissions interpretation, r_j represents the amount of GHG emitted by the firm (in the framework of the Greenhouse Gas (GHG) Protocol used for auditing, this correspond to Scope 1 emissions).

Our externality function is similar to, and more general than, the emissions function of King et al. (2019). The framework is sufficiently flexible to accommodate end-of-pipe abatement as the next illustration shows.

Illustration 1 (End-of-pipe abatement). *Consider a firm j producing electricity, with coal and labor as inputs. Reducing emissions by Carbon Capture and Storage (CCS) is represented through CCS services as a third input in the production set, and $G_j(.) = e.coal - CCS$, with e the unabated CO_2-coefficient of coal, and CCS measured in tons of CO_2.*

[14]For the application to GHG emissions, this would be the carbon budget as in Chakravorty et al. (2006) or van der Ploeg (2020).

[15]We however specify in footnotes how we could explicitly introduce the externality.

[16]Alternatively, we could define production possibilities as a subset $\hat{F}_j$ of $\mathbb{R}^{\mathcal{K}\cup\mathcal{J}} \times \mathbb{R}$, the space of production factors, commodities and produced externality, we switch from one description to another with $\hat{F}_j = \{(z, -r) | z \in F_j, -r \leqslant -G_j(z)\}$. Introducing the effect of externality on production is then simply a matter of defining the production set for each value of the total externality, that is a production set correspondence: $R \mapsto \hat{F}_j(R)$.

[17]In traditional settings, the externality is simply one of the inputs or outputs of the firm, so that our externality function would simply select a component of z_j to reproduce this setting.

From that, we can construct the externality embodied in a good, which is inherited from the embodied externality of inputs used in production in addition to the externality added at the production stage. There is here an analogy with Antràs and De Gortari (2020) where costs compound at each stage of the value chain (in our case: externality). This operation is standard in environmental economics where it is known as footprinting or consumption-based accounting (Wiedmann et al., 2006; Peters, 2008). However, it is usually applied at a macro level and, to the best of our knowledge, we are the first to formulate it in a micro-setting. More precisely, we introduce the intensity of embodied externality of a good (in short intensity) as the amount of embodied externality per unit of the good. Consider firm j that chooses good inputs x_{ji} of intensity θ_{ji}. As production factors do not embody any externality per assumption, the commodity j produced by firm j has intensity θ_{jj} such that:

$$\theta_{jj} y_j = G_j(z_j) + \sum_{i \in \mathcal{J}} \theta_{ji} x_{ji} \tag{1}$$

The equation epitomizes the cumulative nature of the embodied externality and the balance it must obey: the embodied externality that comes out of the firm is equal to the embodied externality that comes in plus the externality added in the production process. One way to read it is that the intensity of output is a mere consequence of the choices of inputs and their intensities. Once the firm has chosen its production plan and the intensities of its inputs, it is committed to what the intensity of the output will be.

To maintain a notation that collects factors and produced goods in one vector, we extend the intensity θ_j to production factors, specifying that production factors have no embodied externality $\theta_{jk} = 0$ for $k \in \mathcal{K}$. The set of possible vectors of intensities is denoted $\mathbb{T} = \{0\}^{\mathcal{K}} \times \mathbb{R}_+^{\mathcal{I}}$.

Introducing θ_j, the vector of intensities for production factors and goods chosen by firm j, equation (1) for firm j can thus be rewritten as:

$$\theta_j \cdot z_j = G_j(z_j) \tag{2}$$

The production possibilities for firm j in the economy with intensities now apply conjointly to production plans and intensities. This subset F_j^E of $F_j \times \mathbb{T}$ can then be described as:

$$F_j^E := \{(z_j, \theta_j) | z_j \in F_j,\ \theta_j \in \mathbb{T},\ \theta_j \cdot z_j = G_j(z_j)\} \tag{3}$$

Once we introduce intensities, there are thus two constraints on firm's operations, which are of different nature. The first is that the production plan is feasible ($z_j \in F_j$), the second is that the intensity of production plan balances exactly to match the externality added by the firm ($\theta_j \cdot z_j = G_j(z_j)$). Whereas the first is enforced by the technology available to the firm, the second is an accounting balance that needs to be monitored. We come back to this difference in the discussion (section 6).

As is standard in a general equilibrium setting, each firm chooses the intensities of the inputs it demands in a decentralized manner, only taken into account given prices (to be introduced below). At equilibrium however, the demanded input must be supplied: hence we will require that firms agree on intensities[18], *i.e.* there exists an equilibrium intensity vector such that $\theta_j = \theta$ for all j. To reduce clutter in equations, we may anticipate the equilibrium and remove the subscript j for intensities writing θ instead of θ_j for the chosen intensity by firm j. From now on, when we write θ_i, it is a scalar, the i-th component of θ, the vector of intensities common to all economic agents at equilibrium.

Illustration 2 (End-of-pipe abatement: intensity)**.** *Coming back to the exemple of the electricity produced by firm j with coal, labor and CCS services, we note that the emission intensity θ_j of electricity incorporates the abated emissions G_j, as well as the emissions of the production of the CCS infrastructure, added through $\theta_{CCS}.CCS$ in equation (1).*

2.4 Government

The introduction of the intensity of embodied externality is the key step for regulating the externality at the end-of-chain, whereas the source regulation only requires the knowledge of externality directly generated at the firm level. We describe both taxes below as part of the government's actions, and explicate the resulting equilibria in Definition 6 and 8.

There is a government that can raise taxes and rebate tax revenues to consumer. We introduce λ the rebate scheme with $\sum_{h \in \mathcal{H}} \lambda_h = 1$. The government rebates to consumer h a share λ_h of the tax revenue to be collected. This rebate scheme is fixed.

The government introduces a taxation system in order to act on the aggregate quantity, or total amount, of the externality in the economy:

[18] Compare (2) with Definition 4.

$R := \sum_{j \in \mathcal{J}} r_j = \sum_{j \in \mathcal{J}} G_j(z_j)$. It can use two different taxes: a source tax t that taxes at the firm level the externality added by the firm and an end-of-chain tax τ that taxes at the consumer level the embodied externality in the goods consumed. As consumers can be taxed on the externality embodied in the good they consume, firms can compete on embodied externality by changing their production plan z_j.

2.5 Prices

Introducing prices of goods is more complicated because a given good can have different levels of its attribute (the intensity). We borrow from the product quality literature that if attributes matter to buyers (for us, consumers and other firms), prices of produced goods depend on the level of their attributes. However, in the product quality literature, characteristics on which prices depend are directly valued by consumers whereas here the characteristic on which prices depend (the intensity) arises from the regulation of the externality, i.e the imposition of an end-of-chain tax based on this characteristic. We will offer later insights to discuss this modelling choice (see subsection 2.8 below).

Specifically, price of good i does not exist per se but is replaced by a price schedule, that is a price for good i at every possible level of intensity. The price schedule $P_i : \mathbb{R}_+ \to \mathbb{R}_+ : \theta_i \mapsto p_i$ continuously maps the externality embodied in one unit of commodity i, to the price $p_i = P_i(\theta_i)$ of good i with intensity θ_i.

Prices for factors of production are standard prices, and $p_k \in \mathbb{R}_+$ is the price of factor k. To homogenize notations, we extend the price schedule to production factors $P_k(\theta_k) = p_k$, remembering that only $\theta_k = 0$ is possible, hence the domain of P_k is restricted to $\{0\}$.

Thus, the vector price schedule is in shorthand described through $P : \mathbb{T} \to \mathbb{R}_+^{\mathcal{K} \cup \mathcal{I}} : \theta \mapsto p$. Note that the price schedule for good i does not depend on the attributes of other goods, $\theta_i = \tilde{\theta}_i \Rightarrow P_i(\theta) = P_i(\tilde{\theta})$.

2.6 Firms: maximization behavior

We can now specify firms' behavior. Consider a firm $j \in \mathcal{J}$ and $(z, \theta) \in F_j^E$. When firm j has production plan z with intensity θ, its profits are:

$$\pi_j^E(z, \theta; P, t) := P(\theta) \cdot z - tG_j(z)\,. \tag{4}$$

Externality produced at the firm level is directly taxed by the government through t. Furthermore, firm pays prices depending on the chosen intensities θ. However, intensities have to follow the externality balance at the firm level (equation (1)). An interpretation is the following: the firm j chooses the intensities θ of all its inputs, for which it has to pay $P(\theta)$ in (4). In turn, the θ determines the output intensity θ_j by equation (1), which determines the price at which it can sell its output in (4).

In a competitive (price-taker) setting, firms take price schedules as given, and maximize their profits defined in (4). That is firms endogenize the way prices vary with intensities but consider price for a given intensity as given. We will note $\mathcal{P}_j^E(P, t)$ the set[19] of combinations of production plans and intensities that maximize the profits function, given price schedules P and source tax t:

$$\mathcal{P}_j^E(P, t) := \underset{(z,\theta) \in F_j^E}{\arg\max}\, \pi_j^E(z, \theta; P, t) \tag{5}$$

When $\mathcal{P}_j^E(P, t)$ is non-empty, we note $\pi_j^E(P, t)$ the maximum profits attained for any element of $\mathcal{P}_j^E(P, t)$:

$$\pi_j^E(P, t) := \pi_j^E(z, \theta; P, t),\ for\ (z, \theta) \in \mathcal{P}_j^E(P, t) \tag{6}$$

Because $(0, \theta) \in F_j^E$, we have $\pi_j^E(P, t) \geqslant 0$.

2.7 Consumers

For every $h \in \mathcal{H}$, the individual consumption set is $X_h \subset \mathbb{R}_+^{\mathcal{K} \cup \mathcal{I}}$, the positive orthant of the production factor and commodity space. Individual h's consumption is c_h. Individual h's preferences are represented by a utility function

[19] If profits are unbounded, $\mathcal{P}_j^E(P, t)$ is empty.

$u_h : X_h \to \mathbb{R}$. For a given level of aggregate externality[20] R, utility explicitly depends on the quantity of commodities consumed and nothing else. In particular, it does not depend on the intensity of the externality embodied in the goods consumed: contrary to Kaufmann et al. (2024), consumers do not care themselves about the externality generated by their purchase.

Each consumer is endowed with a vector of production factors $\omega_h \in \mathbb{R}_+^{\mathcal{K}}$. By extending the endowment to commodities with zero, we can naturally view ω_h as an element of $\mathbb{R}_+^{\mathcal{K}\cup\mathcal{J}}$. The consumer has a contractual claim to the share $\alpha_{hj} \geqslant 0$ of the profit π_j of the firm j (with for each $j \in \mathcal{J}, \sum_h \alpha_{hj} = 1$). Finally, the consumer receives a lump sum income transfer from the government T_h, according to the rebate scheme it has chosen. Her total income[21] is $I_h = P(\theta) \cdot \omega_h + \alpha_h \cdot \pi + T_h$.

The government raises the end-of-chain tax at the consumer level. Formally, for a good i of intensity θ_i, the price is $P_i(\theta_i)$ and the government levies a tax proportional to the externality embodied in this good $\tau\theta_i$, hence the price per unit of good for the consumer becomes $P_i(\theta_i) + \tau\theta_i$. The rationale behind the end-of-chain tax is as follows. In the emission interpretation, the government taxes the carbon footprint of the goods finally consumed. In the material interpretation, consumption represents the end of the life cycle of a product and thus its material content becomes waste, and this waste is taxed by the government.

Given price schedules and the tax τ on embodied externality, consumer h maximizes her utility given income I_h. We will note $\mathcal{C}_h^E(P, \tau, I_h)$ the set of elements of the consumption set and their intensities that maximize the utility subject to the budget constraint, given price schedules P, end-of-chain tax τ, and income I_h, i.e.

$$\mathcal{C}_h^E(P, \tau, I_h) := \underset{(c_h, \theta) \in X_h \times \mathbb{T}}{\arg\max}\ u_h(c_h) \text{ subject to } (P(\theta) + \tau\theta) \cdot c_h \leqslant I_h \qquad (7)$$

In what follows, $P(\theta) + \tau\theta$ will be called the 'total consumer price', which is the price that matters for consumers. The budget constraint in (7) allows for

[20] As said above, introducing explicitly the dependence on it would only clutter notations. We have actually a consumption set correspondence $R \mapsto X_h(R)$ which defines the consumption set of consumer h for each value R of the total amount of externality. The preferences are defined by a preordering on the graph of this correspondence. This preordering induces a preordering on $X_h(R)$ that we represent by a utility function $u_h(R)$ for each value R.

[21] Because the consumer is not endowed in goods but only in production factor, her income actually does not depend on the intensities: $P(\theta) \cdot \omega_h = \sum_{k \in \mathcal{K}} p_k \omega_{hk}$.

two different interpretations of this total consumer price. The end-of-chain tax may be included in the consumer price paid at the time of sale, at the counter, in a sort of Externality Added Tax similar to a Value Added Tax. Alternatively, the end-of-chain tax may be levied after sale, for instance when the good is disposed of. These two interpretations are further discussed in Section 6.

2.8 Justification of price schedules

Having introduced the whole structure of our economy, we can now offer a more substantial justification of the necessity to introduce price schedules. Price schedules simply mean that the price of good i is not unique but that there is actually a price (noted $P_i(\theta_i)$) for each intensity θ_i of good i.

Consider the perspective of a consumer purchasing good i and suppose that we have a traditional price p_i, independant of the intensity of the good. The consumer derives utility independently of the intensity θ_i of the good: intensity does not enter the utility function. Yet the total consumer price is $p_i^c = p_i + \tau\theta_i$, so that demand does depend on the intensity through the budget constraint. In the language of Little (1979), intensity is irrelevant for direct preferences over commodity bundles, but relevant for indirect preferences over budget sets, since it affects the terms on which goods can be purchased. This distinction has direct implications for equilibrium pricing. Between all possible varieties of the good with their own intensities and prices, at equilibrium the consumer will purchase the good with the intensity that yields the lowest total consumer price p_i^{c*}. Now, consider the perspective of the suppliers of good i. Rational producers actually face a demand schedule for their product that depends not only on the price they offer, but also on the intensity: demand is zero for a firm that supplies the good with intensity θ_i and price $p_i > p_i^{c*} - \tau\theta_i$, and the firm can win the entire market if she can offer the good with intensity θ_i at $p_i < p_i^{c*} - \tau\theta_i$. Rational producers in competition will therefore not take as given the price they received, irrespective of the intensity of the good they provide, but rather the total consumer price. Hence they will assume that the price at which they could sell their product depends on its intensity, that there is a price schedule. Hence starting from traditional prices, we are bound to introduce price schedules, i.e. prices that depend on the intensity. In the context of an economy with end-of-chain taxation, traditional prices, independant of the intensity of the good, are inconsistent with rationality of producers and consumers (i.e. their optimizing behaviors).

This is the reason why we have to introduce price schedules in this kind of economy.

Our approach resembles Calcott and Walls (2000) who directly assume a price schedule in a setting where consumers face different prices across product varieties due to differences in recycling and disposal fees. Price schedules are already present in the product quality literature (Rosen, 1974; Leland, 1977), where goods with different attributes are valued differently by the consumers, hence leading the producers to set different prices that fundamentally depend on the attributes. In our model, the intensity of a good is an attribute. There is however a key difference with the product quality literature: in that framework, attributes enter the utility function directly, whereas here, as explained above, intensity affects only the budget constraint. Price schedules arise thus not from heterogeneous tastes over attributes, but from the interaction between a uniform end-of-chain tax and the arbitrage behavior of rational agents.

3 General competitive equilibrium of an economy with end-of-chain taxation

3.1 Allocations of a production economy

Having laid out all agents and their behavior, we can now define a production economy, its allocations, their feasibility and their consistent intensities.

Definition 1. *A* ***production economy***

$$\mathfrak{E} := ((u_h, \omega_h)_{h \in \mathcal{H}}, (F_j, G_j)_{j \in \mathcal{J}})$$

is the list of a set of consumers $\mathcal{H}$ with their utility functions u_h and endowments ω_h; and, a set of producing firms $\mathcal{J}$ (one for each commodity $i \in \mathcal{I}$), with their production sets F_j and externality functions G_j.

Definition 2. *An* ***allocation***

$$\mathfrak{a} := ((c_h)_{h \in \mathcal{H}}, (z_j)_{j \in \mathcal{J}})$$

of a production economy $\mathfrak{E}$ is a list of consumptions c_h for each consumer, production plans z_j for each firm.

The allocation is feasible when, for each production factor, demand (for consumption and intermediate inputs) does not exceed supply (endowment), and when, for each good, demand (for consumption and intermediate inputs) equals supply (production). Endowments can be freely disposed of, but produced goods have to be used by an economic agent. We introduce the set $D = \mathbb{R}_-^{\mathcal{K}} \times \{0\}^{\mathcal{I}}$.

Definition 3 (Feasibility; no free disposal for produced goods)**.** *An allocation* $\mathfrak{a} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}})$ *of a production economy* $\mathfrak{E}$ *is* **feasible** *when:*

$$\sum_{h\in\mathcal{H}} c_h - \sum_{j\in\mathcal{J}} z_j - \sum_{h\in\mathcal{H}} \omega_h \in D \tag{8}$$

Equation (8) can be unpacked differently for production factors and goods, given our notations:

$$\forall k \in \mathcal{K} \qquad \sum_{j\in\mathcal{J}} l_{jk} + \sum_{h\in\mathcal{H}} c_{hk} \leqslant \sum_{h\in\mathcal{H}} \omega_{hk} \tag{9}$$

$$\forall i \in \mathcal{I} \qquad \sum_{j\in\mathcal{J}} x_{ji} + \sum_{h\in\mathcal{H}} c_{hi} = y_i \tag{10}$$

We now build a base example for an economy and its set of feasible allocations that we will use throughout the manuscript to illustrate consistent intensities, the role of price schedules, and end-of-chain and source taxes in equilibrium, etc.
We illustrate the theory through a two-sector example referring to the sectors as metals and manufacturing. The illustration shows the generic model features and its dealing with an externality. It translates the above generic formalisms in easily understandable conditions for feasibility of an allocation and consistency of intensities.

On the interpretation of externalities; one interpretation considers material extraction (‘minerals’) from the environment, by the metals sector, as causing an externality; think of pollution from mining. In that setting, embodied externalities represent minerals embodied in goods. The second interpretation uses the same variables and outcomes, but re-interprets what is resource extraction in the first interpretation as an external effect of CO_2 emissions, with carbon footprinting as the embodied external effect.

Illustration 3 (Feasibility)**.** *The production economy is defined as follows. There is one production factor ’labor’,* $\mathcal{K} = \{0\}$ *and two commodities ’metal’,* *and 'manufactured goods', labeled by the subscript* $i \in \mathcal{I} = \{1, 2\}$. *There are external effects, which are* CO_2 *emissions from the production of metal. We describe the agents below.*

(i) The unique household h *is endowed with one unit of labor only,* $\omega_h = (1, 0, 0)$ *and values consumption of manufactured goods:* $u_h(c_h) = c_{h,2}$.

(ii) The metal-producing firm $j = 1$ *takes labor* l_1 *as input and produces metal* y_1, *with production set* $F_1 = \{(-l_1, y_1, 0) | l_1, y_1 \geqslant 0,\ y_1 \leqslant l_1\}$. *Metal production emits* CO_2 *proportionally to output* $G_1((-l_1, y_1, 0)) = y$.

(iii) The manufacturing firm $j = 2$ *takes labor* l_2 *and metal* x_{21} *as inputs, producing manufactured goods* y_2, *with production set* $F_2 = \{(-l_2, -x_{21}, y_2) | l_2, x_{21}, y_2 \geqslant 0, y_2 \leqslant 2\sqrt{l_2 x_{21}}\}$. *The manufacturing sector does not emit* CO_2: $G_2 = 0$.

When we restrict to "efficient" allocations, in the sense that production factor is fully employed, $l_2 = 1 - l_1$ *and firms are active and operate at their production frontiers, it is straightforward to verify that an allocation* $\mathfrak{a} = (c_h, z_1, z_2)$ *of the production economy is feasible, when for any labor used in the metal-producing sector* $l_1 \in (0, 1)$, $c_h = (0, 0, 2\sqrt{l_1(1 - l_1)}$, $z_1 = (-l_1, l_1, 0)$, $z_2 = (-(1 - l_1), -l_1, 2\sqrt{l_1(1 - l_1)})$. *Note that the aggregate externality generated by this allocation is* l_1.

Definition 4 (Consistent intensities)**.** *Intensities* $\theta \in \mathbb{T}$ *are* **consistent** *with an allocation* $\mathfrak{a} = ((c_h)_{h \in \mathcal{H}}, (z_j)_{j \in \mathcal{J}})$ *of a production economy* $\mathfrak{E}$ *when they satisfy the embodied externality balances* (2): $\forall j \in \mathcal{J}, \quad \theta \cdot z_j = G_j(z_j)$.

Given the above definitions and properties of allocations, we can now prove a basic property. The level of externality that enters the economy (at the firm level) also leaves the economy (at the consumer level): nothing is lost, nothing is added. Seen in the light of taxation (to be introduced below), this property establishes that source taxation and end-of-chain taxation have the same tax base, paving the way for their equivalence.

Lemma 1 (Aggregate level of externality)**.** *Given a feasible allocation* $\mathfrak{a}$ *and consistent intensities* θ, *the aggregate level of externality* $R(\mathfrak{a})$ *produced by* *firms is the same as the aggregate level of externality embodied in consumption:*

$$R(\mathfrak{a}) := \sum_{j \in \mathcal{J}} r_j = \theta \cdot \sum_{h \in \mathcal{H}} c_h \tag{11}$$

where $r_j = G_j(z_j)$.

Proof. Given the definitions of sets, for any $\tilde{\theta} \in \mathbb{T}$ and any $d \in D$, we have $\tilde{\theta} \cdot d = 0$. Furthermore, for any endowment vector ω, we have $\tilde{\theta} \cdot \omega = 0$.

Now, the allocation satisfies equation (8), so $\theta \cdot (\sum_h c_h - \sum_j z_j - \sum_h \omega_h) = 0$, which reduces to $\theta \cdot \sum_h c_h = \theta \cdot \sum_j z_j$. Given that intensities are consistent and satisfy equation (2), the balance (11) follows. □

The illustration shows how material intensity adjusts endogenously to allocative choices on factor use and intermediate inputs. We can how the two interact to verify Lemma 1.

Illustration 4 (Consistent intensities)**.** *For the allocations found in illustration 3, we can compute the consistent intensities. The balance (1) writes for firm 1:* $\theta_1 l_1 = l_1$ *(externality embodied in metal is equal to externality emitted directly by the firm, since there is no good input). For firm 2, it writes:* $\theta_2 2\sqrt{l_1(1-l_1)} = \theta_1 l_1$ *(externality embodied in manufactured good is equal to externality embodied in metal input, since no externality is directly emitted). This gives*

$$\theta_1 = 1 \; ; \theta_2 = \sqrt{l_1/(4-4l_1)} \tag{12}$$

With θ_1, θ_2*, we have a consistent accounting of the externality embodied in each good traded in the economy. Aggregate externality is* $R = l_1$ *and we verify that the externality embodied in final consumption* $\theta_2 2\sqrt{l_1(1-l_1)} = l_1$ *is equal to aggregate externality (Lemma 1).*

3.2 Equilibrium of a production economy with end-of-chain taxation

Definition 5. *A* ***production economy with end-of-chain (and source) taxation***

$$\mathfrak{E}^E := ((u_h, \omega_h, \alpha_h, \lambda_h)_{h \in \mathcal{H}}, (F_j, G_j)_{j \in \mathcal{J}}, t, \tau)$$

is a production economy $\mathfrak{E} = ((u_h, \omega_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{J}})$ as in definition 1 plus a list of contractual shares α_h and a government with an end-of-chain tax $\tau > 0$, a source tax $t \geqslant 0$, and a rebate scheme $(\lambda_h)_{h\in\mathcal{H}}$.

We use a superscript E to denote economies with end-of-chain and source taxation, to distinguish them from economies with source taxation only, that we introduce later and that will be denoted by a superscript S. An economy with end-of-chain (and source) taxation has necessarily an end-of-chain tax (i.e. $\tau > 0$), but the source tax is only a possibility (i.e. $t \geqslant 0$). Hence its characteristic feature is the end-of-chain taxation. Because of this reason and in order to shorten the language, economies with end-of-chain and source taxation are simply called in the sequel economies with end-of-chain taxation. A production economy with end-of-chain taxation has an obvious underlying production economy, and we can therefore extend definition of allocation and their properties to production economies with end-of-chain economy. Slightly abusing language, we will also speak of an allocation of a production economy with end-of-chain taxation

We have now all definitions and notation in place to define equilibrium. Lemma 1 helps to write revenues raised by the government more shortly, $t\sum_j r_j + \tau\theta \cdot \sum_h c_h = (t+\tau)R(\mathfrak{a})$.

Definition 6 (Equilibrium)**.** *A competitive equilibrium $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, \theta, P)$ of an economy $\mathfrak{E}^E$ with end-of-chain taxation is a list of an allocation $\mathfrak{a}(\mathfrak{e}) = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}})$, intensities $\theta \in \mathbb{T}$, and price schedules P, such that the allocation is feasible, intensities are consistent with it, consumption maximizes utility, production plan maximizes profits, and tax revenues are rebated according to λ. More formally:*

1. *allocation $\mathfrak{a}(\mathfrak{e})$ follows (8) (feasibility);*
2. *intensities θ follow (2) for each $j \in \mathcal{J}$ (consistency);*
3. *$\forall j \in \mathcal{J} : (z_j, \theta) \in \mathcal{P}_j^E(P, t)$ (producer's maximisation);*
4. *$\forall h \in \mathcal{H} : (c_h, \theta) \in \mathcal{C}_h^E(P, \tau, I_h)$ with $I_h = P(\theta)\cdot\omega_h + \alpha_h \cdot \pi^E(P, t) + \lambda_h(t + \tau)R(\mathfrak{a})$ (consumer's maximisation).*

The next illustration presents the equilibrium concept. It adds an end-of-chain tax to Illustration 3 where we described the set of feasible allocations, and Illustration 4 that added consistent intensities. Here we add an end-of-chain tax $\tau > 0$, no source tax $t = 0$, with redistributed revenues $T_h = \tau R$.

After the illustration, we verify that for any feasible allocation of Illustration 3 and consistent intensities of Illustration 4, there are factor prices q, good price schedules $P_i(\theta_i)$, and an end-of-chain tax level τ such that they implement the allocation.

Illustration 5 (Equilibrium with end-of-chain taxation)**.** *We describe consumer and producer behavior given price schedules $P_j(\theta_j)$, factor prices q, and resulting equilibrium conditions.*

(i) Given prices $q, P_2(\theta_2)$ for labor and manufactured goods, consumer h maximizes her utility given her revenues from endowments and lump sum transfers:

$$\max_{c_{2,h},\theta_3} c_{2,h} \ \textit{ subject to} \, (P_2(\theta_2) + \tau\theta_2)c_{2,h} \leqslant q + T_h$$

FOCs result in:

$$\mathcal{C}_h^E(P, \tau, I_h) = \left\{ \begin{array}{c} c_{2,h} = \frac{I_h}{P_2(\theta_2)+\tau\theta_2} \\ P_2'(\theta_2) = -\tau \end{array} \right\} \tag{13}$$

That is, the consumer exhausts her budget and chooses an intensity for the consumption good such that its marginal cost in terms of higher tax payment equals its marginal gains in terms of lower price of the good.

(ii) Metal production $i = 1$ solves

$$\max_{l_1} P_1(\theta_1)l_1 - ql_1$$

(substituting $r_1 = y_1 = l_1$). Note that the intensity is constant, $\theta_1 = 1$. The price schedule at other intensity levels is irrelevant to Firm 1. FOCs result in $x_{11} = x_{12} = 0$ and:

$$\mathcal{P}_1^E(P, t) = \left\{ \begin{array}{c} \theta_1 = 1 \\ P_1(\theta_1) \leqslant q \perp y_1 \geqslant 0 \end{array} \right\} \tag{14}$$

Technology being linear for firm 1, it produces if and only if the price of the good equals the production costs, ie. labor cost. Note that the optimal production set is empty for $P_1(\theta_1 = 1) > q$ as in that case the optimal output level is unbounded.

(iii) *Production of firm 2 satisfies* $y_2 = 2\sqrt{l_2 x_{21}}$ *and* $\theta_2 y_2 = \theta_1 x_{21}$*, so that profits it maximizes can be rewritten as follows:*

$$\max_{l_2;x_{21};\theta_1} P_2\left(\frac{1}{2}\theta_1\sqrt{x_{21}/l_2}\right)2\sqrt{l_2 x_{21}} - ql_2 - P_1(\theta_1)x_{21}$$

The FOCs result in:

$$\mathcal{P}_2^E(P,t) = \left\{ \begin{array}{c} l_2 = \sqrt{\frac{P_1(\theta_1)-P_2'(\theta_2)\theta_1}{q}}\frac{y_2}{2} \\ x_{21} = \sqrt{\frac{q}{P_1(\theta_1)-P_2'(\theta_2)\theta_1}}\frac{y_2}{2} \\ P_2 \leqslant \sqrt{q/(P_1(\theta_1) - P_1'(\theta_1)\theta_1)}(P_1(\theta_1) - P_1'(\theta_1)\theta_1/2) \perp y_2 \geqslant 0 \\ P_1'(\theta_1) = P_2'(\theta_2) \end{array} \right\} \tag{15}$$

Optimality conditions for inputs x_{21} *and* l_2 *balance their contribution to the production of* y_2 *and there relative costs, ie. labor cost and* x_{21} *cost, that includes its contribution to the intensity of good 2.*

We can now construct an equilibrium taking all conditions together. Consumer's FOCs transmit the end-of-chain taxation τ *into a local property of the manufacturing good price schedule:* $P_2'(\theta_2) = -\tau$*. The manufacturing firm's FOCs then transmit this information into the metal price schedule* $P_1'(\theta_1) = -\tau$*. Metal production technology imposes* $\theta_1 = 1$ *and prices* $P_1(\theta_1) = q$ *and manufacturing FOCs then provide the inputs and intensity* $\theta_2 = 1/(2\sqrt{1+\tau/q})$ *with price level* $P_2(\theta_2) = (q+\tau/2)/\sqrt{1+\tau/q}$*. The factor and goods balances then provide the input and output levels* $l_1 = 2q/(2q+\tau)$*,* $l_2 = 2(q+\tau)/(2q+\tau)$*,* $y_2 = c = 4q\sqrt{(1+\tau/q)}/(2q+\tau)$*. The equilibrium has money neutrality: the allocation is not affected if taxes and prices rise by the same factor.*

We add four remarks. First, for any $0 < R < 1$ from Illustration 3, we can implement the efficient allocation by setting taxes such that $\tau/q = 2(1-R)/R$. Second, the above equilibrium identifies unique local properties for the price schedules. The global properties at out-of-equilibrium intensity levels θ_j are not unique; there are infinitely many price schedules all supporting the same allocation. Below we will establish some global conditions for $P_2(\theta_2)$ but these do not completely identify it.

Third it is a key feature that the manufacturing firm does not minimize production costs, which would have resulted in unit prices for manufactured goods when one unit of metals plus one unit of labor produces two units of manufactured goods, and $\theta_2 = 1/2$. If a firm were to produce such a good the consumer would still prefer the more expensive manufactured good produced with less metals and more labor per output, because total consumer price $(P_2 + \tau\theta_2)$ are less for the equilibrium outcome $(\sqrt{1+\tau})$ compared to the costs of consumption of the cheapest manufactured good $(1+\tau/2)$, for $\tau > 0$. The example thereby illustrates how price schedules naturally arise through demand. A firm that supplies the good with variety θ_2 and price $p_2 > \sqrt{1+\tau} - \tau\theta_2$ (which is the case if minimizing production costs) will face zero demand. The firm can win the entire market if she can offer the good at $p_2 < \sqrt{1+\tau} - \tau\theta_2$. As a result, competition leads to the constraint for the price schedule: $P_2(\theta_2) \geqslant \sqrt{1+\tau} - \tau\theta_2$. Any price schedule $P_2(\theta_2)$ that sits between the manufacturer's production costs (for θ_2) and the linear extrapolation that satisfies the local properties will support the same allocation.

Fourth, by adding multiple households, and setting differentiated tax redistribution shares, we can construct an outcome such that the agent with sufficiently large λ_h strictly benefits from low levels of taxation compared to no taxation. We refer to Anderson and Duanmu (2025) for various examples where the distribution of tax revenues has substantial effects on the distribution of welfare among agents.

We are now interested in properties of competitive equilibria of an economy with end-of-chain taxation. To that end, we will not proceed directly but construct a mapping between an equilibrium of an economy with end-of-chain taxation and an equilibrium of an economy with source taxation, that is a traditional economy where the externality is regulated through Pigouvian (source) taxation of the direct externalities caused by producer's decisions.

4 The economy with source taxation

The economy with source taxation extends a production economy by taxing externality at source taxation and redistributing tax revenues.

Definition 7. *A* ***production economy with source taxation***

$$\mathfrak{E}^S := ((u_h, \omega_h, \alpha_h, \lambda_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{J}}, t)$$

is a production economy $((u_h, \omega_h, \alpha_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{J}})$ *as in Definition 1 plus a government with a source tax* $t > 0$ *and a rebate scheme* $(\lambda_h)_{h\in\mathcal{H}}$*.*

The use of Pigovian taxes is classic. An allocation for such economy is a list of inputs and consumption; there is no need for specifying intensities as the externalities are covered at the source. The structure is mostly as developed in section 2, but prices of goods are traditional prices, i.e. prices constitute a vector $p \in \mathbb{R}_+^{\mathcal{K}\cup\mathcal{I}}$.

The behavior of producers and consumers is therefore simplified. More precisely, instead of maximising (4), firm j maximises its profits (with superscript S for source taxation) defined, for a production plan $z \in F_j$, as:

$$\pi_j^S(z; p, t) := p \cdot z - tG_j(z) \tag{16}$$

Corresponding to (5), we define $\mathcal{P}_j^S(p, t)$ the set of combinations of inputs that maximize the profit function:

$$\mathcal{P}_j^S(p, t) := \arg\max_{z\in F_j} \pi_j^S(z; p, t) \tag{17}$$

Corresponding to (6), when $\mathcal{P}_j^S(p, t)$ is non-empty, we note $\pi_j^S(p, t)$ the maximum profits attained for any element of $\mathcal{P}_j^S(p, t)$:

$$\pi_j^S(p, t) := \pi_j^S(z; p, t),\ for\, z \in \mathcal{P}_j^S(p, t) \tag{18}$$

Given prices p and her revenues $I_h = p \cdot \omega_h + \alpha_h \cdot \pi^S(p, t) + T_h$, consumer h maximizes her utility derived from consumption c_h. We will note $\mathcal{C}_h^S(p, I_h)$ the set of elements of the consumption set X_h that maximize the utility subject to the budget constraint at prices p, i.e.

$$\mathcal{C}_h^S(p, I_h) := \arg\max_{c_h\in X_h} u_h(c_h) \text{ subject to } p \cdot c_h \leqslant I_h \tag{19}$$

An equilibrium of that economy is a list of an allocation, prices such that the allocation is feasible, consumers maximize utility given prices and tax rebates, producers maximize profits given prices. The formal definition of an equilibrium with source taxation is the standard one. One can see how close and how different it is from equilibrium with end-of-chain taxation, defined in Section 3.

Definition 8 (Equilibrium)**.** *A competitive equilibrium* $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, p)$ *of an economy* $\mathfrak{E}^S$ *with source taxation is a list of an allocation* $\mathfrak{a}(\mathfrak{e}) = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}})$*, prices* p*, such that the allocation is feasible, consumption maximizes utility, production maximizes profits, and tax revenues are rebated according to* λ*. More formally:*

1. *allocation* $\mathfrak{a}(\mathfrak{e})$ *follows* (8) *(feasibility);*
2. $\forall j \in \mathcal{J} : z_j \in \mathcal{P}_j^S(p,t)$ *(producer's maximisation);*
3. $\forall h \in \mathcal{H} : c_h \in \mathcal{C}_h^S(p, I_h)$ *with* $I_h = p \cdot \omega_h + \alpha_h \cdot \pi^S(p,t) + \lambda_h t R(\mathfrak{a})$ *(consumer's maximisation).*

We now transfer the previous illustration for an end-of-chain taxation equilibrium to see how source taxation reproduces identical allocations, given tax levels, but for different prices. The illustration uses the same template describing consumption, production, and the equilibrium allocation.

Illustration 6 (Equilibrium with source taxation)**.** *We consider a source tax* $t \geqslant 0$ *with redistributed revenues* $T_h = tR$*.*

(i) Given prices q, p_2 *for labor and manufactured goods, consumer* h *maximizes her utility given its revenues from endowments and lump sum transfers:*

$$\max_{c_{2,h},\theta_3} c_{2,h} \ \ \textit{subject to}\, p_2 c_{2,h} \leqslant q + T_h$$

FOCs result in:

$$\mathcal{C}_h^E(p,t,\tau,T_h) = \left\{ \ c_{2,h} = \tfrac{q+T_h}{p_2} \ \right\} \tag{20}$$

That is, the consumer exhausts her budget and is indifferent with respect to the intensity for the consumption good.

(ii) Metal production $i = 1$ *solves*

$$\max_{l_1} p_1 l_1 - (q+t) l_1$$

after substituting $r_1 = y_1 = l_1$*. FOCs result in* $x_{11} = x_{12} = 0$ *and:*

$$\mathcal{P}_1(p,t) = \left\{ \ p_1 \leqslant q + t \perp y_1 \geqslant 0 \ \right\} \tag{21}$$

Technology being linear for firm 1, it produces if and only if the price of the good equals the production costs, ie. labor cost and tax.

(iii) Production of firm 2 satisfies $y_2 = 2\sqrt{l_2 x_{21}}$, so that profits it maximizes can be rewritten as follows:

$$\max_{l_2;x_{21};\theta_1} 2p_2\sqrt{l_2 x_{21}} - ql_2 - p_1 x_{21}$$

The FOCs result in:

$$\mathcal{P}_2(p,t) = \left\{ \begin{array}{c} l_2 = \sqrt{\frac{p_1}{q}}\frac{y_2}{2} \\ x_{21} = \sqrt{\frac{q}{p_1}}\frac{y_2}{2} \\ p_2 \leqslant \sqrt{qp_1} \perp y_2 \geqslant 0 \end{array} \right\} \tag{22}$$

Optimality conditions for inputs x_{21} and l_2 balance their contribution to the production of y_2 and there relative costs, ie. labor cost and x_{21} cost.

We normalize prices to $q = 1$. The resulting equilibrium allocation is:

$$p_1 = 1 + t; p_2 = \sqrt{1+t} \tag{23}$$

$$l_1 = \frac{2}{2+t} \tag{24}$$

$$l_2 = \frac{2(1+t)}{2+t} \tag{25}$$

$$y_2 = \frac{4\sqrt{1+t}}{2+t} \tag{26}$$

$$c_h = \frac{1}{\sqrt{1+t}} + \lambda_h \frac{2t}{(2+t)\sqrt{1+t}} \tag{27}$$

Note that the solution satisfies $c_{12} + c_{22} = y_2$.

As in the previous illustration, we see from (24) that the source tax can reproduce all feasible efficient outcomes of the economy. More specifically, the source and end-of-chain taxes are interchangeable for their effect on the allocation l_i, x_i, y_i, c_h. Yet, they have different effects on prices p_i and $P_i(.)$. Below we will identify conditions under which allocative equivalence holds for any production economy, and when (any combination of) the two types of externality taxes can implement the efficient allocation.

5 The equivalence between source taxation and end-of-chain taxation

To see how equilibria of an economy with end-of-chain taxation look like, we state our first result that an equilibrium of an economy with source taxation can be mapped to an equilibrium of an economy with end-of-chain taxation.

For the analysis, it is useful to distinguish those firms that directly or indirectly contribute to the utility of consumers from those that do not. For an allocation, the set of firms that do not participate in the production chain of goods consumed is called the isolated part. In the illustration in the appendix, we show that such an isolated set of firms add no value and can be scaled down without changing what can be considered the substance of the economy. Thus we first show that any equilibrium can be chosen such that its isolated part is inactive.

Lemma 2. *Let* $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, p)$ *be an equilibrium of an economy with source taxation. Then we can construct an equilibrium* $\mathfrak{e}' = ((c_h)_{h\in\mathcal{H}}, (z'_j)_{j\in\mathcal{J}}, p)$*, that has the same consumptions as* $\mathfrak{e}$*, same total externality* $R(\mathfrak{a}(\mathfrak{e}')) = R(\mathfrak{a}(\mathfrak{e}))$*, but whose isolated part is inactive (all firms in the isolated part have production plans equal to zero).*

The definition of the isolated part is made precise in the proof (see appendix 7.1), but the economic reasons behind it is clear: in a way, the isolated part of the equilibrium functions as a closed system, does not contribute to the utility of the consumers and so is spurious. It remains to show how it can be removed. The idea is just to shut down the firms of the isolated part to have a new equilibrium with same consumptions.

Thanks to the previous lemma, we can assume from now on, without loss of generality, that the isolated part of the equilibrium is inactive. We can then compute consistent intensities from this type of equilibrium of an economy with source taxation.

Lemma 3. *Let* $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, p)$ *be an equilibrium for an economy with source taxation* $\mathfrak{E}^S$*. For firms with positive output, we can construct unique consistent intensities* $\theta(\mathfrak{e})$*.*

Proof. See appendix 7.1. □

Proposition 1. *Let $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, p)$ be an equilibrium for an economy with source taxation $\mathfrak{E}^S = ((u_h, \omega_h, \alpha_h, \lambda_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{J}}, t)$ and $\theta(\mathfrak{e})$ the consistent intensities defined in Lemma 3. Take any end-of-chain tax level $0 < \tau' \leqslant t$. Then $\mathfrak{e}' = ((c_h)_{h\in\mathcal{H}}, (z_j,)_{j\in\mathcal{J}}, \theta(\mathfrak{e}), P)$, where $P_j(\theta) = \max(p_j - \tau'\theta_j, 0)$, is an equilibrium for an economy with end-of-chain taxation $\mathfrak{E}^E = ((u_h, \omega_h, \alpha_h, \lambda_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{I}}, t', \tau')$ with $t' = t - \tau'$.*

Proof. See appendix 7.1. □

Let $\mathfrak{e}'$ be the equilibrium constructed in the proposition above. Then $P_j(\theta(\mathfrak{e})) = p_j - \tau'\theta_j(\mathfrak{e})$, and $P_j'(\theta(\mathfrak{e})) = -\tau_j'$.

The second result investigates the equivalence in the reverse direction: an equilibrium for an economy with end-of-chain taxation can be mapped to an equilibrium of an economy with source taxation. As above, we start with a lemma to get rid of possible activities in the isolated part of the equilibrium (where the isolated part is still define as the set of firms that do not participate in the production chain of goods consumed).

Lemma 4. *Let $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j,)_{j\in\mathcal{J}}, \theta, p)$ be an equilibrium of an economy with end-of-chain taxation, we can construct an equilibrium $\mathfrak{e}' = ((c_h)_{h\in\mathcal{H}}, (z_j')_{j\in\mathcal{J}}, \theta, p)$, that has the same consumptions as in $\mathfrak{e}$, same total externality, but whose isolated part is inactive (all firms in the isolated part have production plans equal to zero).*

Without loss of generality, we now assume that the equilibrium has an inactive isolated part. We can now state how our second main proposition.

Proposition 2. *Let $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, \theta, P)$ be an equilibrium for an economy with end-of-chain taxation $\mathfrak{E}^E = ((u_h, \omega_h, \alpha_h, \lambda_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{J}}, t, \tau)$. Then $\mathfrak{e}' = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, p')$ where $p' = P(\theta) + \tau\theta$ is an equilibrium of the economy with source taxation $\mathfrak{E}^S = ((u_h, \omega_h, \alpha_h, \lambda_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{J}}, t')$ with $t' = t + \tau$.*

Proof. See appendix 7.2. □

The above proposition informs us that, under the conditions stated, equilibrium with end- of-chain taxation can also be realized as equilibrium with source taxation. Prices in the equilibrium with source taxation bear a simple relationship with price schedules of the equilibrium with end-of-chain taxation: they are simply the price at which goods are traded by firms ($P(\theta)$)

plus the cost of the externality embodied in the good ($\tau\theta$). End-of-chain taxation does not lead to equilibria different from the ones that can be obtained through source taxation. Thus the two regulatory schemes achieve the same outcome, although through different means.

Call two equilibria equivalent when they share the same allocation.

Corollary 1. *The above 2 propositions imply:*

1. *any equilibrium with end-of-chain (and source) taxation is equivalent to a well-defined (Pigovian) equilibrium with source taxation;*
2. *any equilibrium with end-of-chain (and source) taxation is equivalent to another equilibrium with end-of-chain (and source) taxation, provided the sum of taxes ($t + \tau$) are the same;*
3. *any equilibrium with end-of-chain taxation is equivalent to an equilibrium with end-of-chain taxation that has linear price schedules.*

Our constructive equivalence between equilibrium with source taxation and equilibrium with end-of-chain taxation

Corollary 2. *If, under assumptions A on the economy with source taxation, there is a general equilibrium of this economy that has property P, then under the corresponding assumptions A on the economy with end-of-chain taxation, there is also a general equilibrium of that economy that has property P.*

For several assumptions and properties (mainly related to decentralization of Pareto-optimal allocations), see Anderson and Duanmu (2025).

The Arrow-Debreu theorem establishes that an equilibrium exists absent externality taxes, $t = \tau = 0$. Let us denote this reference externality by R_0. There may be multiple equilibria, in which case we take the maximum externality level. Then Arrow-Debreu establishes the following.[22]

[22] See also Anderson and Duanmu (2025) Theorem 2. Note that the corollary allows for multiple equilibria. Use Anderson 2025 to have existence for any R of an equilibrium with a quota economy, then equivalence between equilibria of economy with source taxation; alternatively, use Arrow-Debreu on an auxiliary economy to have existence of equilibrium with source taxation directly. In both cases, it is unclear to me where the condition $R < R_0$ would comes from.

Corollary 3. *For any $R < R_0$, there exists an equilibrium of the regulated economy with either source taxes, $t > 0, \tau = 0$, or end-of-chain taxes $t = 0, \tau > 0$ with aggregate resource use $R = R(\mathfrak{a})$. The equilibrium is R-constrained Pareto-efficient.*

6 Discussion

The theorems established above demonstrate that the social optimum can be decentralized either through taxation of the externality at its source or through taxation of the same externality embodied in final consumption. This formal equivalence grants policymakers considerable freedom in the choice of instrument. A social planner seeking to reduce the environmental footprint of the economy may, in principle, rely indistinctly on upstream resource taxes or on end-of-chain consumption-based instruments. Indeed, policies seek a richer set of externality taxation as witnessed by the EU Carbon Border Adjustment Mechanism (CBAM), which can be interpreted as in-between source and end-of-chain regulation through intermediate-chain implementation.

Yet, the equivalence pertains to allocations and not to the informational or institutional requirements of the corresponding equilibria. Implementing the latter typically entails a richer informational structure and, consequently, distinct feasibility constraints. End-of-chain taxation presupposes that the externality intensity of each good is observable and verifiable at all stages of production. The planner must be able to index tax rates, and hence equilibrium prices, by product attributes such as embodied emissions or material content. In addition, even though profit maximization has to obey material constraint, public authorities have to monitor the coherence of the flows of externalities in the sense that externalities should be perfectly preserved along the value chain. The informational burden of measuring and recording products' attributes The informational burden of constructing such a tax base increases with the complexity of production networks. For relatively simple sectors, such as primary energy, tracing embodied externalities may be straightforward; for complex inputs—particularly in the chemical or intermediate-goods industries—the feasibility of assigning a precise footprint to each product is questionable. Existing evidence in industrial ecology points to both measurement costs and uncertainty regarding footprint data (Pauliuk et al., 2017; Tisserant et al., 2017; Liu et al., 2013). The effective implementation of end-of-chain instruments thus requires a costly informational infrastructure that can support Measurement, Reporting, and Verification (MRV) of product characteristics across value chains (Bellassen and Stephan, 2015; Bellassen et al., 2015).

Challenge for the implementation of end-of-chain taxation may go beyond the information requirement. If the consumer price includes the tax, as would be the case for embodied emissions, the buyer only faces a price schedule for a generic good, which requires no particular information or ability to proceed the information. However, the tax might have to be paid after the sale, for instance at the time of disposal, as would be the case for material content as an externality. In such a case, the buyer must decide while facing menus of prices for goods that differ across their attributes, which may be demanding in terms of information, cognitive ability or rationality. In both cases, the (lack of) salience of the tax for the consumer remains extensively documented, whereas the tax is included in the price (Chetty et al., 2009; Taubinsky and Rees-Jones, 2018) or the buyer will have to pay the tax, or an extra cost, ex post (Sallee, 2014; Allcott and Wozny, 2014).[23]

Additional complexity is brought by uncertainty. It is unrealistic to assume that producers know their technology and consumers know their marginal utility, as both are subject to shocks. Facing non-linear price schedules, agents shall respond to the expected marginal prices rather than their exact price but empirical evidence by Ito (2014) indicates that households respond imperfectly to such prices, displaying systematic biases in marginal price perception. While the present model abstracts from bounded rationality and treats pricing as informationally neutral, these findings call attention to behavioral frictions that can compromise the practical validity of the end-of-chain approach. Source-based instruments, in contrast, internalize the externality upstream and therefore circumvent the behavioral complexity that arises when consumers must interpret multidimensional price signals.

Distributional considerations further distinguish the two forms of regulation. Anderson and Duanmu (2025) emphasize that taxation is often complemented with rebate schemes that effectively allocate property rights among agents, generating distributional implications that vary between source and end-of-chain taxation.

These insights naturally connect to the literature on second-best environ-

[23]See Farhi and Gabaix (2020) for a detailed analysis of taxation with misperception of agents.

mental regulation. When market imperfections, informational frictions, or jurisdiction constraints prevent the implementation of the first best, mixed or sequential instruments may enhance welfare. Calcott and Walls (2000) argue that efficiency in such contexts requires a combination of end-of-chain fees and upstream measures, while empirical work by Knittel and Sandler (2018) provides evidence of implicit externality taxation in transportation markets. These findings underscore that the theoretical equivalence between upstream and downstream approaches should not be interpreted as policy indifference: practical efficacy depends on institutional capacity, behavioral constraints, and the informational structure of production.

Strategic considerations play a role at the international macro level. Downstream instruments have been criticized for creating free-riding incentives in open economies (Harstad, 2012), while resource-based taxation aligns better with rent-sharing in resource-rich countries and may thus be politically more acceptable. Asheim (2013) shows that such policies allow resource exporters to appropriate scarcity rents without altering the global distribution of income, a mechanism consistent with broader distributional analyses (Asheim et al., 2019). Subsequent contributions extend these insights to unilateral settings and propose complementary arrangements combining demand- and supply-side intervention to foster cooperation (Fæhn et al., 2017; Asheim and Harstad, 2025).

Overall, our analysis operates in a first-best environment and abstracts from both strategic and distributional frictions. In practice, such considerations crucially determine which equilibrium is implementable. When the informational conditions underpinning the equivalence fail—because emissions are unobservable, MRV is incomplete, or behavioral frictions distort responses—source-based taxation remains implementable, while end-of-chain taxation may not. These limits delineate the boundary between formal equivalence and effective feasibility.

References

Aldy, J. E., Krupnick, A. J., Newell, R. G., Parry, I. W. H., and Pizer, W. A. (2010). Designing Climate Mitigation Policy. Journal of Economic Literature, 48(4):903–934.

Allcott, H. and Wozny, N. (2014). Gasoline prices, fuel economy, and the energy paradox. Review of Economics and Statistics, 96(5):779–795.

Anderson, R. M. and Duanmu, H. (2025). Cap-and-Trade and Carbon Tax Meet Arrow–Debreu. Econometrica, 93(2):357–393. Publisher: The Econometric Society.

Antràs, P. and Chor, D. (2013). Organizing the global value chain. Econometrica, 81(6):2127–2204.

Antràs, P. and De Gortari, A. (2020). On the geography of global value chains. Econometrica, 88(4):1553–1598.

Antweiler, W., Copeland, B. R., and Taylor, M. S. (2001). Is free trade good for the environment? American economic review, 91(4):877–908.

Arrow, K. J. (1969). The organization of economic activity: Issues pertinent to the choice of market versus nonmarket allocation. In Committee, J. E., editor, Analysis and Evaluation of Public Expenditures: The PPB System, pages 47–64. U.S. Govt. Printing Office, Washington, D.C.

Asheim, G. B. (2013). A Distributional Argument for Supply-Side Climate Policies. Environmental and Resource Economics, 56(2):239–254.

Asheim, G. B., Fæhn, T., Nyborg, K., Greaker, M., Hagem, C., Harstad, B., Hoel, M. O., Lund, D., and Rosendahl, K. E. (2019). The case for a supply-side climate treaty. Science, 365(6451):325–327. Publisher: American Association for the Advancement of Science.

Asheim, G. B. and Harstad, B. (2025). Complementary climate policies for supply and demand.

Baumol, W. J. (1972). On taxation and the control of externalities. The American Economic Review, 62(3):307–322.

Bellassen, V. and Stephan, N. (2015). Accounting for Carbon. Cambridge University Press.

Bellassen, V., Stephan, N., Afriat, M., Alberola, E., Barker, A., Chang, J.-P., Chiquet, C., Cochran, I., Deheza, M., Dimopoulos, C., et al. (2015). Monitoring, reporting and verifying emissions in the climate economy. Nature Climate Change, 5(4):319–328.

Bergstrom, T. C. (1976). Collective Choice and the Lindahl Allocation Method. In Lin, S. A. Y., editor, Theory and measurement of economic externalities, pages 111–131. Academic Press, New York.

Bushnell, J. B. and Mansur, E. T. (2011). Vertical targeting and leakage in carbon policy. American Economic Review, 101(3):263–267.

Calcott, P. and Walls, M. (2000). Can downstream waste disposal policies encourage upstream "design for environment"? American Economic Review, 90(2):233–237.

CE-Delft (2015). Carbon added tax as an alternative climate policy instrument. Technical report, CE-delft.

Chakravorty, U., Magné, B., and Moreaux, M. (2006). A hotelling model with a ceiling on the stock of pollution. Journal of Economic Dynamics and Control, 30:2875–2904.

Chetty, R., Looney, A., and Kroft, K. (2009). Salience and taxation: Theory and evidence. American economic review, 99(4):1145–1177.

Coase, R. H. (1960). The Problem of Social Cost. The Journal of Law & Economics, 3:1–44.

Copeland, B. and Taylor, M. (1994). North-south trade and the environment. Quarterly Journal of Economics, 109(3):755–787.

CPLC (2018). How can consumption-based carbon pricing address carbon leakage and competitiveness concerns? Technical report, Carbon Pricing Leadership Coalition.

Debreu, G. (1959). Theory of value: An axiomatic analysis of economic equilibrium, volume 17. Yale University Press.

Dechezleprêtre, A., Fabre, A., Kruse, T., Planterose, B., Sanchez Chico, A., and Stantcheva, S. (2025). Fighting climate change: International attitudes toward climate policies. American Economic Review, 115(4):1258–1300.

Douenne, T. and Fabre, A. (2022). Yellow vests, pessimistic beliefs, and carbon tax aversion. American Economic Journal: Economic Policy, 14(1):81–110.

Drèze, J. H. and Hagen, K. P. (1978). Choice of Product Quality: Equilibrium and Efficiency. Econometrica, 46(3):493–513.

Farhi, E. and Gabaix, X. (2020). Optimal taxation with behavioral agents. American Economic Review, 110(1):298–336.

Fisher, F. M. (1962). An Alternate Proof and Extension of Solow's Theorem on Nonnegative Square Matrices. Econometrica, 30(2):349–350.

Fæhn, T., Hagem, C., Lindholt, L., Mæland, S., and Rosendahl, K. E. (2017). Climate Policies in a Fossil Fuel Producing Country: Demand versus Supply Side Policies. The Energy Journal, 38(1):77–102. Publisher: SAGE Publications.

Goulder, L. H. and Schein, A. R. (2013). Carbon taxes versus cap and trade: a critical review. Climate Change Economics, 04(03):1350010.

Grossman, G. M. and Krueger, A. B. (1995). Economic growth and the environment. The quarterly journal of economics, 110(2):353–377.

Grubb, M., Crawford-Brown, D., Neuhoff, K., Schanes, K., Hawkins, S., and Poncia, A. (2020). Consumption-oriented policy instruments for fostering greenhouse gas mitigation. Climate Policy, 20(sup1):S58–S73.

Harstad, B. (2012). Buy Coal! A Case for Supply-Side Environmental Policy. Journal of Political Economy, 120(1):77–115.

Hobbs, B. F., Bushnell, J., and Wolak, F. A. (2010). Upstream vs. downstream CO2 trading: A comparison for the electricity context. Energy Policy, 38(7):3632–3643.

Ito, K. (2014). Do consumers respond to marginal or average price? evidence from nonlinear electricity pricing. American Economic Review, 104(2):537–563.

Jakob, M., Marschinski, R., and Hübler, M. (2013). Between a Rock and a Hard Place: A Trade-Theory Analysis of Leakage Under Production- and Consumption-Based Policies. Environmental and Resource Economics, 56(1):47–72.

Jakob, M., Steckel, J. C., and Edenhofer, O. (2014). Consumption- Versus Production-Based Emission Policies. Annual Review of Resource Economics, 6(1):297–318.

Kaplan, R. S. and Ramanna, K. (2021). Accounting for climate change. Harvard Business Review, 99(6):120–131.

Kaufmann, M., Andre, P., and Kőszegi, B. (2024). Understanding markets with socially responsible consumers. The Quarterly Journal of Economics, 139:1989–2035.

King, M., Tarbush, B., and Teytelboym, A. (2019). Targeted carbon tax reforms. European Economic Review, 119:526–547.

Knittel, C. R. and Sandler, R. (2018). The welfare impact of second-best uniform-pigouvian taxation: evidence from transportation. American Economic Journal: Economic Policy, 10(4):211–242.

Laffont, J.-J. (1977). Effets externes et théorie économique. Éditions du Centre national de la recherche scientifique, Paris.

Lancaster, K. J. (1966). A New Approach to Consumer Theory. Journal of Political Economy, 74(2):132–157.

Leland, H. E. (1977). Quality Choice and Competition. The American Economic Review, 67(2):127–137.

Levinson, A. (2009). Technology, international trade, and pollution from us manufacturing. American economic review, 99(5):2177–2192.

Lininger, C. (2015). Consumption-Based Approaches in International Climate Policy. Springer International Publishing, Cham.

Little, J. T. (1979). Indirect preferences. Journal of Economic Theory, 20(2):182–193.

Liu, Z., Xi, F., and Guan, D. (2013). Tie carbon emissions to consumers. Nature, 493(7432):304–305.

Maier, S., De Poli, S., and Amores, A. (2024). Carbon taxes on consumption: distributional implications for a just transition in the eu. Technical report, European Commission and Joint Research Centre.

Mansur, E. T. (2012). Upstream versus Downstream Implementation of Climate Policy. In Fullerton, D. and Wolfram, C. D., editors, The design and implementation of US climate policy, pages 179–193. The University of Chicago Press, Chicago.

Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). Microeconomic Theory. Oxford University Press, New York, N.Y.

Meade, J. E. (1952). External Economies and Diseconomies in a Competitive Situation. The Economic Journal, 62(245):54–67.

OECD (2021). Effective Carbon Rates 2021: Pricing Carbon Emissions through Taxes and Emissions Trading. OECD Publishing, Paris.

Palmer, K., Sigman, H., and Walls, M. (1997). The cost of reducing municipal solid waste. Journal of environmental economics and management, 33(2):128–150.

Palmer, K. and Walls, M. (1997). Optimal policies for solid waste disposal taxes, subsidies, and standards. Journal of Public Economics, 65(2):193–205.

Pauliuk, S., Arvesen, A., Stadler, K., and Hertwich, E. G. (2017). Industrial ecology in integrated assessment models. Nature Climate Change, 7(1):13–20.

Peters, G. P. (2008). From production-based to consumption-based national emission inventories. Ecological Economics, 65(1):13–23.

Pigou, A. C. (1920). The economics of welfare. Macmillan, London, 2 edition.

Rosen, S. (1974). Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition. Journal of Political Economy, 82(1):34–55.

Sallee, J. M. (2014). Rational inattention and energy efficiency. The Journal of Law and Economics, 57(3):781–820.

Sandmo, A. (1975). Optimal taxation in the presence of externalities. The Swedish Journal of Economics, 77(1):86–98.

Sandmo, A. (1978). Direct versus indirect pigovian taxation. European Economic Review, 7(4):337–349.

Schubert, K., Pommeret, A., and Ricci, F. (2025). Confronting the carbon pricing gap: Second best climate policy. Journal of the Association of Environmental and Resource Economists.

Shapiro, J. S. and Walker, R. (2018). Why is pollution from us manufacturing declining? the roles of environmental regulation, productivity, and trade. American economic review, 108(12):3814–3854.

Shapley, L. S. and Shubik, M. (1969). On the Core of an Economic System with Externalities. The American Economic Review, 59(4):678–684.

Shitovitz, B. and Spiegel, M. (2003). Cournot-Nash and Lindahl equilibria in pure public "bad" economies. Economic Theory, 22(1):17–31.

Solow, R. M. (1952). On the Structure of Linear Models. Econometrica, 20(1):29–46.

Starrett, D. A. (1973). Externalities and the Core. Econometrica, 41(1):179–183.

Steckel, J. C., Kalkuhl, M., and Marschinski, R. (2010). Should carbon-exporting countries strive for consumption-based accounting in a global cap-and-trade regime? Climatic Change, 100(3):779–786.

Taubinsky, D. and Rees-Jones, A. (2018). Attention variation and welfare: theory and evidence from a tax salience experiment. The Review of Economic Studies, 85(4):2462–2496.

Tisserant, A., Pauliuk, S., Merciai, S., Schmidt, J., Fry, J., Wood, R., and Tukker, A. (2017). Solid waste and the circular economy: a global analysis of waste treatment and waste footprints. Journal of Industrial Ecology, 21(3):628–640.

van der Ploeg, R. Rezai, A. (2020). The risk of policy tipping and stranded carbon assets. Journal of Environmental Economics and Management, 100.

Walls, M. and Palmer, K. (2001). Upstream pollution, downstream waste disposal, and the design of comprehensive environmental policies. Journal of Environmental Economics and Management, 41(1):94–108.

Weitzman, M. (1974). Prices vs. quantities. The review of economic studies, 41(4).

Wiedmann, T., Minx, J., Barrett, J., and Wackernagel, M. (2006). Allocating ecological footprints to final consumption categories with input-output analysis. Ecological Economics, 56(1):28–48.

7 Appendixes with proofs

We start with some notations about firms. Recall that $\mathcal{J}$ is the set of firms. Given an allocation, we note $\mathcal{J}_{y>0}$ the set of firms with positive output. Given an equilibrium, we note $\mathcal{J}_{p>0}$ the set of firms whose output has positive price, and $\mathcal{J}_{p=0}$ the set of firms whose output has null price. As the allocation or equilibrium considered will always be clear from the context, we do not explicitly indicate the dependence of these sets on it.

Given a feasible allocation $\mathfrak{a}$, we construct the correspondence $\Delta : (\mathcal{J} \cup \mathcal{H}) \twoheadrightarrow \mathcal{J}$ by $\Delta(h) = \{j \in \mathcal{J} | c_{hj} > 0\}$ for $h \in \mathcal{H}$ and $\Delta(i) = \{j \in \mathcal{J} | x_{ji} > 0\}$ for $i \in \mathcal{J}$. Given a $\mathcal{G} \subset (\mathcal{J} \cup \mathcal{H})$, $\Delta(\mathcal{G})$ is simply the set of firms that make (positive) deliveries to economic agents (firms or consumers) in $\mathcal{G}$. For a subset of firms $\mathcal{J}' \subset \mathcal{J}$, we note $\Delta^{|\mathcal{J}'}$ when the co-domain of Δ is restricted to $\mathcal{J}'$: $\Delta(\mathcal{G})$ is simply the set of firms of $\mathcal{J}'$ that make (positive) deliveries to economic agents in $\mathcal{G}$. Slightly abusing notations, we do not indicate when the domain of Δ is restricted (especially to $\mathcal{J}$). Note that the image of the correspondence is included in $\mathcal{J}_{y>0}$.

We can iterate Δ and define for a $\mathcal{G} \subset (\mathcal{J} \cup \mathcal{H})$, $\bar{\Delta}(\mathcal{G}) = \bigcup_{n=1}^{+\infty} \Delta^n(\mathcal{G})$. The economic interpretation is straightforward: it is the set of firms that appears at some stage in the production chain of goods delivered to agents of $\mathcal{G}$. Correspondingly, we define $\bar{\Delta}^{|\mathcal{J}'}(\mathcal{G}) = \bigcup_{n=1}^{+\infty} \left(\Delta^{|\mathcal{J}'}\right)^n(\mathcal{G})$.

Given a feasible allocation $\mathfrak{a}$ and a $\mathcal{J}' \subset \mathcal{J}_{y>0}$, we define the intertrade matrix between firms of $\mathcal{J}'$ as $A(\mathcal{J}') = (x_{ij}/y_j)_{i,j \in \mathcal{J}'}$. Element a_{ij} represents the proportion of output of firm j delivered to firm i. $A(\mathcal{J}')$ is a square non-negative matrix. We caution readers familiar with input-output notation as we deviate in two ways from common practice in IO analyses. First, our matrix A associates rows, the first index, with use of intermediates, and columns with supplying the intermediate, inversely from the IO literature. Second, we divide intermediate deliveries by the supplier, not by the firm using the deliveries. With these two differences, the notation $a_{ij} = (x_{ij}/y_j)_{ij}$ looks identical to the common input-output intensity matrix, but, with these two differences, has a very different meaning: it is not a matrix of technical coefficients. The reason for considering this matrix is that it offers a natural approach to prove regularity and uniqueness of the intensity vector, see the proof of Lemma 3 below.

We recall the Brauer-Solow conditions (Solow, 1952; Fisher, 1962) applying to a square non-negative matrix A for $I - A$ to be invertible. Renumber rows and columns to write A as a decomposable matrix[24]:

$$A = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ 0 & A_{22} & \dots & A_{2n} \\ 0 & 0 & \ddots & \vdots \\ 0 & \dots & 0 & A_{nn} \end{pmatrix}$$

Proposition 3. *(Brauer-Solow conditions) Suppose no column sum of A be greater than unity. If each diagonal submatrix $A_{11}, \dots, A_{nn}$ have at least one column sum strictly less than unity, then $(I - A)$ is invertible and if $f \geqslant 0$ then $(I - A)^{-1} f \geqslant 0$.*

We can now state a lemma that will prove useful in our proofs.

Lemma 5. *Take a feasible allocation $\mathfrak{a}$, a $\mathcal{G} \subset (\mathcal{J} \cup \mathcal{H})$ and a $\tilde{\mathcal{J}} \subset \mathcal{J}$. If $\tilde{\mathcal{J}} \cap \mathcal{G} = \varnothing$, then $A\left(\bar{\Delta}^{|\tilde{\mathcal{J}}}(\mathcal{G})\right)$ satisfies the Brauer-Solow conditions.*

The economic interpretation and reason for the lemma is as follows. The set $\bar{\Delta}^{|\tilde{\mathcal{J}}}(\mathcal{G})$ represents the firms that belong to production chains that stay in $\tilde{\mathcal{J}}$ and finally make deliveries to $\mathcal{G}$. The intertrade matrix within $\bar{\Delta}^{|\tilde{\mathcal{J}}}(\mathcal{G}) \subset \tilde{\mathcal{J}}$ must have some leakage because there are deliveries outside $\tilde{\mathcal{J}}$, i.e. to agents in $\mathcal{G}$. Hence, it must satisfy the Brauer-Solow conditions.

Proof. The matrix $A = A\left(\bar{\Delta}^{|\tilde{\mathcal{J}}}(\mathcal{G})\right)$ is non-negative and the sum of its columns is no greater than 1. Suppose the Brauer-Solow conditions are not satisfied. Then we have a non-empty subset of indices $\mathcal{J}'$ such that for any $j \in \mathcal{J}'$, $\sum_{i \in \mathcal{J}'} a_{ij} = 1$, or equivalently $y_j = \sum_{i \in \mathcal{J}'} x_{ij}$. This means that firms in $\mathcal{J}'$ makes only deliveries to firms in $\mathcal{J}'$: as deliveries to firms in $\mathcal{J}'$ already exhaust the output of a firm in $\tilde{\mathcal{J}}'$, no delivery can be made outside. Formally, this writes[25] $(\Delta^{|\tilde{\mathcal{J}}})^{-1}(\mathcal{J}') \subset \mathcal{J}'$.

[24]If A is indecomposable, $A = A_{11}$.

[25]Note the difference between $\Delta(\mathcal{J}')$, the set of firms that deliver to $\mathcal{J}'$, and $\Delta^{-1}(\mathcal{J}')$, the set of firms that firms of $\mathcal{J}'$ deliver to.

Now, firms in $\mathcal{J}'$ only deliver to firms in $\mathcal{J}'$. It is therefore not possible to build a production chain starting from a firm in $\mathcal{J}'$ that finally ends up in $\mathcal{G}$, which is a contradiction given that $\mathcal{J}' \subset \bar{\Delta}^{|\tilde{\mathcal{J}}}(\mathcal{G})$. Formally, given that $\mathcal{J}'$ is non-empty, there is a n such that $\mathcal{J}' \cap (\Delta^{|\tilde{\mathcal{J}}})^n(\mathcal{G}) \neq \varnothing$. Hence $(\Delta^{|\tilde{\mathcal{J}}})^{-n}(\mathcal{J}') \cap \mathcal{G} \neq \varnothing$. However, given $(\Delta^{|\tilde{\mathcal{J}}})^{-1}(\mathcal{J}') \subset \mathcal{J}'$, we have $(\Delta^{|\tilde{\mathcal{J}}})^{-n}(\mathcal{J}') \subset \mathcal{J}' \subset \tilde{\mathcal{J}}$. Given the assumption that $\tilde{\mathcal{J}} \cap \mathcal{G} = \varnothing$, we have a contradiction. Thus $\mathcal{J}'$ is actually empty and the matrix A follows the Brauer-Solow conditions. □

Finally, there is a link between profits in the economy with end-of-chain taxation and profits in the economy with source taxation. This would prove useful to investigate the equilibria of the two economies.

Lemma 6. *Take any* $(z, \theta) \in F_j^E$ *and any* $\tau \geqslant 0$*, then:*

$$\pi_j^E(z, \theta; P, t) = \pi_j^S(z; P(\theta) + \tau\theta, t + \tau). \tag{28}$$

Proof. Take any $(z, \theta) \in F_j^E$ and any $\tau \geqslant 0$. By definition (see eq. (4)), we have $\pi_j^E(z, \theta; P, t) = P(\theta) \cdot z - tG_j(z)$ and (see eq. (16)), $\pi_j^S(z; p, t) = p \cdot z - tG_j(z)$ Hence,

$$\begin{aligned} \pi_j^S(z; P(\theta) + \tau\theta, t + \tau) &= (P(\theta) + \tau\theta) \cdot z - (t + \tau)G_j(z) && (29)\\ &= P(\theta) \cdot z - tG_j(z) + \tau(\theta \cdot z - G_j(z)) && (30)\\ &= \pi_j^E(z, \theta; P, t) + \tau(\theta \cdot z - G_j(z)) && (31)\\ & && (32) \end{aligned}$$

The relation follows because $\theta \cdot z = G_j(z)$, since $(z, \theta) \in F_j^E$ (see eq. (3)). □

7.1 Proof that an equilibrium of the economy with source taxation can be mapped to an equilibrium of the economy with end-of-chain taxation.

Proof of Lemma 2. Consider $\bar{\Delta}(\mathcal{H})$ the set of firms that participate in production chains to deliver goods to consumers. The complementary set $\mathcal{J}_{iso} = \mathcal{J} \backslash \bar{\Delta}(\mathcal{H})$ is what we called the isolated part of the economy, in the sense that these firms do not participate in production chain of consumption goods and only deliver goods to this isolated part. So for any $j \in \mathcal{J}_{iso}$, $y_j = \sum_{i \in \mathcal{J}_{iso}} x_{ij}$. It is also isolated in a further sense. Recall that profits are $\pi_j^S = p \cdot z_j - t.G_j(z_j) =$

$p_j y_j - \sum_{i \in \mathcal{J}} p_i x_{ji} - t.r_j - \sum_{k \in \mathcal{K}} p_k l_{jk}$. Summing profits over all firms in $\mathcal{J}_{iso}$ and inserting market clearing conditions for output of firms in $\mathcal{J}_{iso}$, one gets $\sum_{j \in \mathcal{J}_{iso}} \pi_j = -\sum_{j \in \mathcal{J}_{iso}} \left(\sum_{i \in \bar{\Delta}(\mathcal{H})} p_i x_{ji} + t.r_j + \sum_{k \in \mathcal{K}} p_k l_{jk} \right)$. The l.h.s is non-negative and the r.h.s is non-positive, so all terms are null.

So firms in $\mathcal{J}_{iso}$ make no payment outside $\mathcal{J}_{iso}$: nor to government ($t.r_j = 0$, hence $r_j = 0$ because $t > 0$), neither to firms outside the subset ($p_i x_{ji} = 0$, $i \in \bar{\Delta}(\mathcal{H})$), neither to consumers, whether in the form of dividends ($\pi_j = 0$) or of location of production factors ($p_k l_{jk} = 0$).

This implies that firms in $\mathcal{J}_{iso}$ can receive no goods from firms in $\bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p>0}$ and use it as input . They can however receive deliveries from firms in $\bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0}$. The firms in $\mathcal{J}_{iso}$ are therefore isolated from the rest of the economy, but not completely. Were it not for this peculiarity, it would be very easy to set to zero all production plans of firms in $\mathcal{J}_{iso}$: it would not change anything for the rest of economic agents, except possibly relinquishing production factors that have however zero prices.

We define the new allocation $\mathfrak{e}' = ((c_h)_{h \in \mathcal{H}}, (z'_j)_{j \in \mathcal{J}}, p)$ as follows. For $j \in \bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p>0}$, production plans stay the same: $z'_j = z_j$; for $j \in \mathcal{J}_{iso}$, production plans are set to zero: $z'_j = 0$; for $j \in \bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0}$, production plans are simply scaled down: $z'_j = \lambda_j z_j$ with $\lambda_j \in [0, 1]$ (because the production set F_j is convex, this is a valid production plan). The λ_j are chosen so that feasibility conditions is satisfied: for any $j \in \bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0}$, $\lambda_j y_j = \sum_{i \in \bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p>0}} x_{ij} + \sum_{i \in \bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0}} \lambda_i x_{ij}$. Firms with zero-price take no input from firms with positive prices, hence $A(\bar{\Delta}(\mathcal{H}))$ is decomposable (although maybe not completely: firms with zero-price can make deliveries to firms with positive prices), with $A(\bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0})$ being a principal submatrix. Because of Lemma 5, $A(\bar{\Delta}(\mathcal{H}))$ satisfies the Solow-Brauer conditions, and so does $A(\bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0})$. So we can find $\lambda_j \geqslant 0$ satisfying the feasibility conditions (feasibility conditions for production factors are satisfied because we release some quantity of them).

Because we have $y_j = \sum_{i \in \bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p>0}} x_{ij} + \sum_{i \in \bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0}} x_{ij} + \sum_{i \in \mathcal{J}_{iso}} x_{ij}$, λ_j also satisfy $(1-\lambda_j) y_j = \sum_{i \in \bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0}} (1-\lambda_i) x_{ij} + \sum_{i \in \mathcal{J}_{iso}} x_{ij}$ and so $1 - \lambda_j \geqslant 0$. The production plans of firms in $\bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0}$ are thus well defined.

It is straightforward to verify that $\mathfrak{e}'$ is an equilibrium. By construction, the allocation $\mathfrak{a}(\mathfrak{e}')$ is feasible. Nothing has changed for consumers nor for firms in $\bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p>0}$. For firms in $\bar{\Delta}(\mathcal{H}) \cap \mathcal{J}_{p=0}$ or in $\mathcal{J}_{iso}$, zero was the maximum attainable profits in $\mathfrak{e}$. Prices have not changed, so the new production plans also maximize profits for these firms. □

Proof of Lemma 3. Without loss of generality thanks to Lemma 2, we have assumed that the isolated part of the equilibrium is inactive. Hence, we have $\bar{\Delta}(\mathcal{H}) = \mathcal{J}_{y>0}$ and all firms with zero output have zero production plans. For the latter, the consistency conditions are void, so that their intensities are indeterminate, but they are also useless. For the former, the consistency conditions write $\theta_j y_j = r_j + \sum_{i \in \mathcal{J}_{y>0}} \theta_i x_{ji}$. Introduce $\Theta_j = \theta_j y_j$, because $j \in \mathcal{J}_{y>0}$, there is a one-to-one correspondence between the vector Θ and θ. And Θ verifies $\Theta_j = r_j + \sum_{i \in \mathcal{J}_{y>0}} \Theta_i x_{ji}/y_i$, or in matrix form $\Theta = r + A\Theta$ where $A = A(\bar{\Delta}(\mathcal{H}))$. By lemma 5, A satisfies the Brauer-Solow conditions and Θ is uniquely defined and so is θ.

□

We state proof of Property 5 before proof of Proposition 1, as it will be needed therein.

Proof of Property 5. We need to show that consistent intensities $\theta(\mathfrak{e})$ are no greater than p/τ'. Given that $\tau' \leqslant t$, it suffices to show that $\theta_j \leqslant p_j/t$.

Recall from proof of Lemma 3, that, $\Theta = \theta y$ verifies $\Theta = r + A.\Theta$. For all $j \in \mathcal{J}_{y>0}$, by definition of profits $\pi_j^S(p,t)$, we have $p_j y_j = \sum_i p_i x_{ji} + \sum_k p_k l_{jk} + \pi_j^S + t.r_j$. This writes in matrix form, with obvious notations: $py = A.py + pl + \pi^S + t.r$. Thus $(py - t\Theta) = A.(py - t\Theta) + pl + \pi^S$. The vector $(pl + \pi^S)$ is non-negative $((pl + \pi^S)_j = \sum_k p_k l_{jk} + \pi_j^S)$ and the matrix A satisfies the Brauer-Solow conditions. Hence $py - t\Theta$ is non-negative and $p_j \geqslant t\theta_j$ as claimed. □

Proof of Proposition 1. It is quite direct to prove that $\mathfrak{e}'$ is an equilibrium of the economy with end-of-chain taxation $\mathfrak{E}^E$. Before examining the four conditions of Definition 6, we have however to take care of the kink in price schedules. For $\theta \in \mathbb{R}_+$ an intensity of good i, we introduce the capped intensity $\theta^c = \min(\theta, p_i/\tau')$ (note that this maps depends on i, although we do not explicitly state this dependence, which would be clear from the context). For $\theta \in \mathbb{T}$, we simply extend it component-wise: $\theta^c = \min(\theta, p/\tau')$. Given the definition of price schedules of $\mathfrak{e}'$, we have the following property: $P(\theta^c) + \tau'\theta^c \leqslant P(\theta) + \tau'\theta$: total consumer price at the capped intensity is not greater than total consumer price at the original intensity. Note that because of property 5, we have $\theta(\mathfrak{e})^c = \theta(\mathfrak{e})$.

1. Note that the allocation does not change: $\mathfrak{a}(\mathfrak{e}) = \mathfrak{a}(\mathfrak{e}')$. Because $\mathfrak{e}$ is an equilibrium of the economy with source taxation $\mathfrak{E}^S$ by assumption, the allocation $\mathfrak{a}(\mathfrak{e})$ is feasible. The first condition is satisfied.

2. by construction of Lemma 3, the intensities $\theta(\mathfrak{e})$ are consistent with the allocation $\mathfrak{a}(\mathfrak{e}) = \mathfrak{a}(\mathfrak{e}')$. The second condition is satisfied.

3. We need to show that $(z_j, \theta(\mathfrak{e}))$ deliver the highest profits in F_j^E. Take a $(\tilde{z}, \tilde{\theta}) \in F_j^E$. First, to better understand the logic, take the simple case when $\tilde{\theta}^c = \tilde{\theta}$, so that $P(\tilde{\theta}) = p - \tau'\tilde{\theta}$. With equation (28), we verify that, in that case, $\pi_j^E(\tilde{z}, \tilde{\theta}; P, t') = \pi_j^S(\tilde{z}; p, t)$. But, because $z_j \in \mathcal{P}_j^S(p, t)$, this is not greater than $\pi_j^S(z_j; p, t) = \pi_j^E(z_j, \theta(\mathfrak{e}); P, t')$.

   To show, that $(z_j, \theta) \in \mathcal{P}_j^E(P, t')$, we only need to extend the results to any $(\tilde{z}, \tilde{\theta}) \in F_j^E$, so also when $\tilde{\theta}^c \neq \tilde{\theta}$. First if the intensity of output $\tilde{\theta}_j$ is above p_j/τ', then $P_j(\tilde{\theta}_j) = 0$, profits for $(\tilde{z}, \tilde{\theta})$ are non positive and thus no greater than $\pi_j^E(z_j, \theta(\mathfrak{e}); P, t')$. So we can assume that $\tilde{\theta}_j$ is below p_j/τ', and thus $P_j(\tilde{\theta}_j) = p_j - \tau'\tilde{\theta}_j$. The idea is to show that reducing the intensity of inputs will increase profits but these are still not greater than $\pi_j^E(z_j, \theta(\mathfrak{e}); P, t')$. Construct the intensity vector $\hat{\theta}$ by reducing intensities of inputs to their capped level $\hat{\theta}_{j'} = \tilde{\theta}_{j'}^c$ (for $j' \neq j$) and adapt the intensity of output accordingly so that $\hat{\theta} \cdot \tilde{z} = G(\tilde{z})$. Note that because we have reduced the intensity of inputs, the intensity of output is also reduced and thus $\hat{\theta}_j \leqslant \tilde{\theta}_j$, so that $P_j(\hat{\theta}_j) = p_j - \tau'\hat{\theta}_j$, and more generally $\hat{\theta}^c = \hat{\theta}$. Given that $-P_{j'}(\tilde{\theta}_{j'}) \leqq -P_{j'}(\hat{\theta}_{j'}) + \tau'(\tilde{\theta}_{j'} - \hat{\theta}_{j'})$ for $j' \neq j$, we have $\pi_j^E(\tilde{z}, \tilde{\theta}; P, t') \leqslant \pi_j^E(\tilde{z}, \hat{\theta}; P, t')$, which by the analysis of the simple case, is not greater than $\pi_j^E(z_j, \theta(\mathfrak{e}); P, t')$.

   No $(\tilde{z}, \tilde{\theta}) \in F_j^E$ can deliver greater profits than $(z_j, \theta(\mathfrak{e}))$, so $(z_j, \theta(\mathfrak{e})) \in \mathcal{P}_j^E(P, t')$ and we also have $\pi_j^E(P, t') = \pi_j^S(p, t)$.

4. For consumers, the same logic applies as for firms. As in the proof of Proposition 2, because the tax is levied on the aggregate externality $R(\mathfrak{a})$ (Lemma 1), which does not change, government revenues do not change, profits do not change either, so consumer (effective) income does not change, $I_h' = I_h$. So, the same consumption bundle c_h with the externality $\theta(\mathfrak{e})$ can be bought, and total expenditures by consumers (including end-of-chain taxes) are p_i for good i, independently of the chosen externality intensity $\tilde{\theta}_i$ (provided $\tilde{\theta}_i^c = \tilde{\theta}_i$, but the consumer will

chose it that way as choosing a greater intensity only raises the total consumer price, without providing more utility). Thus the consumer cannot improve its consumption bundle relative to $c_h, \theta(\mathfrak{e})$: $(c_h, \theta(\mathfrak{e})) \in \mathcal{C}_h^E(P, \tau, I'_h)$. The fourth condition is satisfied.

The defined $\mathfrak{e}'$ is thus an equilibrium of the economy $\mathfrak{E}^E$ with end-of-chain taxation. □

7.2 Proof that an equilibrium of the economy with end-of-chain taxation can be mapped to an equilibrium of the economy with source taxation.

Proof of Lemma 4. We proceed as in the proof of Lemma 2. The isolated part consists of firms outside $\bar{\Delta}(\mathcal{H})$, i.e. in $\mathcal{J}_{iso} = \mathcal{J} \backslash \bar{\Delta}(\mathcal{H})$. For the same reasons as in the economy with source taxation, firms in $\mathcal{J}_{iso}$ make no payment outside $\mathcal{J}_{iso}$: nor to government ($t.r_j = 0$), neither to firms outside the subset ($P_i(\theta_i)x_{ji} = 0$, $i \in \bar{\Delta}(\mathcal{H})$) neither to consumers, whether in the form of dividends ($\pi_j = 0$) or of location of production factors ($p_k l_{jk} = 0$). Firms in $\mathcal{J}_{iso}$ can however use inputs from firms in $\bar{\Delta}(\mathcal{H})$ with zero prices (i.e., with $P_i(\theta_i) = 0$) and, because t can be zero, we are however not sure that r_j is null.

To see why we still have $r_j = 0$, sum the balance (1) over all $j \in \mathcal{J}_{iso}$, and introduce the market clearing conditions for these firms $y_j = \sum_{i \in \mathcal{J}_{iso}} x_{ij}$, because firms in $\mathcal{J}_{iso}$ only deliver goods to $\mathcal{J}_{iso}$. One gets $0 = \sum_{j \in \mathcal{J}_{iso}} r_j + \sum_{j \in \mathcal{J}_{iso}, i \in \bar{\Delta}(\mathcal{H})} \theta_i x_{ji}$. Because all terms of the rhs are non negative, they are all null. Firms in $\mathcal{J}_{iso}$ do not produce any externality but may use goods with zero intensities (and zero prices) as inputs.

We are now in a position to shut down firms in $\mathcal{J}_{iso}$, imposing zero production plans, and scale down, if necessary, productions of firms with zero prices. As in the proof of Lemma 2, this allocation is feasible and change nothing to prices and to the rest of the economy, so that we have a new equilibrium whose isolated part is inactive. □

To prove Proposition 2, we first establish some useful properties of price schedules in equilibria for an economy with end-of-chain taxation. Specifically, to avoid complications in later proofs, we have to deal with inactive firms (zero output): we show that intensities of the goods produced by these firms are irrelevant at equilibrium.

Lemma 7. *Let* $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, \theta, P)$ *be an equilibrium for an economy with end-of-chain taxation* $\mathfrak{E}^E$*. Take* $\ell \in \mathcal{J}_{y=0}$*, then one can arbitrarily choose the intensity* θ_ℓ *of firm* ℓ *and still stay at equilibrium. Formally, choose any* θ'_ℓ *in the admissible range, and denote by* θ' *the intensity vector that copies* θ *but with* ℓ*-th component being* θ'_ℓ*, then* $\mathfrak{e}' = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, \theta, P)$ *is an equilibrium.*

Proof. It is actually straightforward to verify the four conditions of Definition 6. The allocation does not change so it is feasible (Def 3); when the firm has zero output, the consistency condition is void for that firm and so θ' is still consistent with the allocation (Def 4). Because ℓ is not produced, it is not consumed nor used as inputs, so changing its intensity does not change the profits of firms (5) nor the budget constraint for consumers (7). So for all $h \in \mathcal{H}$, when $(c_h, \theta) \in \mathcal{C}_h^E(P, \tau, I_h)$ then also $(c_h, \theta') \in \mathcal{C}_h^E(P, \tau, I_h)$, and for all $j \in \mathcal{J}$, when $(z_j, \theta) \in \mathcal{P}_j^E(P, t)$, then also $(z_j, \theta') \in \mathcal{P}_j^E(P, t)$. So $\mathfrak{e}'$ is an equilibrium. □

We point out a subtlety implicit in the above proof. When varying the intensity θ_ℓ, we do not change the price schedule $P_\ell(.)$, and thus the price level varies with the intensity; in general $P_\ell(\theta'_\ell) \neq P_\ell(\theta_\ell)$. That is, it is essential that the P in $\mathcal{P}_j^E(P, t)$ and $\mathcal{C}_h^E(P, \tau, I_h)$ is not a price level vector, but the full price schedule vector, the levels of which are different in the baseline and counterfactual scenario for θ. The above lemma implies that we can assume, without loss of generality, that the intensity minimizes 'user costs' as stated in the corollary below.

Corollary 4. *Let* $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, \theta, P)$ *be an equilibrium for an economy with end-of-chain taxation* $\mathfrak{E}^E$*. We can always choose the intensities of firms with zero output* ($j \in \mathcal{J}_{y=0}$) *such that* θ_j *minimizes the total consumer price* $P_j(\theta_j) + \tau\theta_j$*.*

Proof. Given that $P_j(\theta) \geqslant 0$, $P_j(\theta) + \tau\theta \underset{\theta\to\infty}{\longrightarrow} \infty$, by continuity of price schedule, we can always find a θ that minimizes the total consumer price. □

For the sequel, without loss of generality, we therefore assume that the equilibrium is in such a form. From Lemma 4, we also assume that the isolated part of the equilibrium is inactive. We can now state the property of price schedule.

Lemma 8. *Let $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j, \theta_j)_{j\in\mathcal{J}}, p)$ be an equilibrium for an economy with end-of-chain taxation $\mathfrak{E}^E$. Then price schedules are such that for all $j \in \mathcal{J}$, for all $\tilde{\theta} \in \mathbb{R}_+$,*

$$P_j(\theta_j) + \tau\theta_j \leqslant P_j(\tilde{\theta}) + \tau\tilde{\theta} \tag{33}$$

This lemma is natural when applied to goods consumed by consumers: it states that the total consumer price $P_j(\tilde{\theta}) + \tau\tilde{\theta}$, i.e. the price paid by consumers for good j inclusive of end-of-chain taxes, is minimal at equilibrium intensity θ_j of good j. Hence the total consumer price of good j is not greater than other possible total consumer prices. Competition between firms producing good j make them produce goods with the intensity that minimizes the price paid by consumers. The lemma extends the property to all goods. We can also rewrite the lemma as $P_j(\theta_j) - P_j(\tilde{\theta}) \leqslant \tau(\tilde{\theta} - \theta_j)$, and say that, when moving to the equilibrium situation, increases in prices is lower than the value of the reduction of intensities of embodied externality.

Proof. Given the assumptions on equilibrium derived from Corollary 4, the property is granted for firms with zero output. We need to prove it for firms with positive output. Recall that, because the isolated part is inactive, we have $\mathcal{J}_{y>0} = \bar{\Delta}(\mathcal{H}) = \bigcup_{n=1}^{+\infty} \Delta^n(\mathcal{H})$.

First, we show that the property (33) holds for $\Delta(\mathcal{J}')$ when it holds for $\mathcal{J}' \subset \mathcal{J}$. Take a $j \in \Delta(\mathcal{J}')$, we want to show that for any $\tilde{\theta}_j$, we have $P_j(\theta_j) + \tau\theta_j \leqslant P_j(\tilde{\theta}_j) + \tau\tilde{\theta}_j$. There exists $i \in \mathcal{J}'$ for which $x_{ij} > 0$. It is immediate that firm i can swap input j with intensity θ_j for inputs with intensity $\tilde{\theta}_j$ and change the intensity of its output from θ_i to $\tilde{\theta}_i$ with $\tilde{\theta}_i = \theta_i + (\tilde{\theta}_j - \theta_j)x_{ij}/y_i$, or equivalently $(\tilde{\theta}_i - \theta_i)y_i = (\tilde{\theta}_j - \theta_j)x_{ij}$.

Because (z_i, θ) maximises i's profits, when moving from $(z_i, \tilde{\theta})$ to (z_i, θ) change in revenues exceed change in costs. Thus $(P_j(\theta_j) - P_j(\tilde{\theta}_j))x_{ij} \leqslant (P_i(\theta_i) - P_i(\tilde{\theta}_i))y_i$. Because property (33) holds for i, this is not greater than $\tau(\tilde{\theta}_i - \theta_i)y_i = \tau(\tilde{\theta}_j - \theta_j)x_{ij}$. Because $x_{ij} > 0$, property (33) also holds for $j \in \Delta(\mathcal{J}')$.

We then show recursively that the property (33) holds for any $\Delta^n(\mathcal{H})$. The set $\Delta(\mathcal{H})$ is the set of firms whose output is consumed by at least one consumer. Because the consumer only cares of the type of the goods and not its intensity, she will choose to consume the version of the good with the lowest total consumer price, hence $P_j(\theta_j) + \tau\theta_j$ is indeed minimal for $j \in \Delta(\mathcal{H})$. By the property just proven, if property (33) holds for $\Delta^n(\mathcal{H})$, it holds for $\Delta^{n+1}(\mathcal{H})$. Finally, the property (33) holds for $\bar{\Delta}(\mathcal{H})$. □

The proof of Proposition 2 is now a matter of straightforward verification of the three conditions for Definition 8: the allocation is feasible, production maximizes profits, and consumption maximizes utility.

Proof of Proposition 2. We convert the equilibrium $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, \theta, P)$ of the economy with end-of-chain taxation into an equilibrium of the economy with source taxation, denoted by a prime: $\mathfrak{e}' = ((c_h)_{h\in\mathcal{H}}, (z_j)_{j\in\mathcal{J}}, p')$ by setting, for the latter, constant total consumer prices $p' = P(\theta) + \tau\theta$ and source taxes $t' = t + \tau$.

1. Note that the allocation does not change: $\mathfrak{a}(\mathfrak{e}) = \mathfrak{a}(\mathfrak{e}')$. Because $\mathfrak{e}$ is an equilibrium of the economy with end-of-chain taxation $\mathfrak{E}^E$ by assumption, the allocation $\mathfrak{a}(\mathfrak{e})$ is feasible. The first condition is satisfied.

2. To start with, for any $\tilde{z} \in F_j$, construct the intensity $\tilde{\theta}$, such that intensities of inputs are equilibrium intensities $\tilde{\theta}_{j'} = \theta_{j'}$, for $j' \neq j$, and adapt intensity $\tilde{\theta}_j$ of output, so that $\tilde{\theta} \cdot \tilde{z} = G_j(\tilde{z})$.

   With equation (28), it is then straightforward to verify that: $\pi_j^S(\tilde{z}; p', t') = \pi_j^E(\tilde{z}, \tilde{\theta}; P, t) + ((P_j(\theta_j) + \tau\theta_j) - (P_j(\tilde{\theta}_j) + \tau\tilde{\theta}_j))y_j$. Because of Lemma 8, we have $\pi_j^S(\tilde{z}; p', t') \leqslant \pi_j^E(\tilde{z}, \tilde{\theta}; P, t)$ (profits in the economy with source taxation are no greater than profits in the economy with end-of-chain taxation, with same equilibrium intensities of inputs). Note that at equilibrium allocation z_j, this reduces to: $\pi_j^S(z_j; p', t') = \pi_j^E(z_j, \theta; P, t)$ (profits are equal).

   We need to prove that for each firm j, $z_j \in \mathcal{P}_j^S(p', t')$. Because $(z_j, \theta) \in \mathcal{P}_j^E(P, t)$, we have for any $(\tilde{z}, \tilde{\theta}) \in F_j^E$, $\pi_j^E(\tilde{z}, \tilde{\theta}; P, t) \leqslant \pi_j^E(z_j, \theta; P, t)$, and thus $\pi_j^S(\tilde{z}; p', t') \leqslant \pi_j^S(z_j; p', t')$.

3. Because the tax is levied on the aggregate externality $R(\mathfrak{a})$ (Lemma 1), which does not change, government revenues do not change. By previous item, profits of firms do not change, and thus household (effective) income does not change, $I'_h = I_h$. Note that in the economy with source taxation, consumers do not choose the intensity of consumer goods. They consume the good with intensity as offered in equilibrium. Given $(c_h, \theta) \in \mathcal{C}_h^E(P, \tau, I_h)$, we only need to establish that there is no other c'_h that provides higher utility, while satisfying the budget constraint for the economy with source taxation. Indeed, because of the construction of prices p', for any consumption bundle c'_h that satisfy

the budget constraint of the economy with source taxation, the consumption bundle c'_h with the equilibrium intensities also remains in the budget constraint of the original economy with end-of-chain taxation. Thus if c_h maximizes utility for $\mathfrak{E}^E$ (i.e. $(c_h, \theta) \in \mathcal{C}_h^E(P, \tau, I_h)$), it also does for $\mathfrak{E}^S$, that is, $c_h \in \mathcal{C}_h^S(p', I'_h)$.

The defined $\mathfrak{e}'$ is thus an equilibrium of the economy $\mathfrak{E}^S$ with source taxation.

□

7.3 Settings with multiple firms

We investigate here how our framework can accommodate multiple firms that produce a commodity. When we have multiple firms per commodity, the set of firms $\mathcal{J}$ is larger than the set of commodities $\mathcal{I}$. There is no longer a natural identification $\mathcal{J} \simeq \mathcal{I}$ but we have however a surjective function $\iota : \mathcal{J} \to \mathcal{I}$ that maps each firm j to the commodity it produces $\iota(j)$.

For the economy with source taxation, there is nothing to change in its description: consumers and firms only care about the type of the commodities, not which firm produced it, so consumption c_h and input x_j bundles are elements of the commodity set $\mathbb{R}_+^{\mathcal{I}}$.

For the economy with end-of-chain taxation, heterogeneous firms per good add a level of complexity. Different firms producing the same commodity can embody different level of externality in the commodity. We therefore have to keep track not only of the commodity type, as for the economy with source taxation, but also of the embodied externality, hence of the firm that produced the commodity. That is consumption c_h and input x_j bundles are elements of the expanded commodity space $\mathbb{R}_+^{\mathcal{J}}$ that keeps the information of the origins of the commodity (i.e. there is one coordinate per firm). Actually, there are as many commodities as there are producing firms. However, utility functions, production sets or externality functions do not depend on the firm of origin, (i.e. there are functions ($u : \mathbb{R}_+^{\mathcal{I}} \to \mathbb{R}$, $F : \mathbb{R}_+^{\mathcal{K}} \times \mathbb{R}_+^{\mathcal{I}} \to \mathbb{R}$) over $\mathbb{R}_+^{\mathcal{I}}$, not over $\mathbb{R}_+^{\mathcal{J}}$). We therefore introduce the aggregation map $\alpha : \mathbb{R}_+^{\mathcal{J}} \to \mathbb{R}_+^{\mathcal{I}}$ that aggregates commodity per type, regardless of their firm of origin. Specifically, for $x \in \mathbb{R}_+^{\mathcal{J}}$, we have $\alpha(x)_i = \sum_{j \in \iota^{-1}(\{i\})} x_j$. We can now extend the domain of utility functions, ctions, or externality functions to $\mathbb{R}_+^{\mathcal{J}}$, as follows, where, with slight abuse of notation, we keep the same letter for extended functions: for $c_h \in \mathbb{R}_+^{\mathcal{J}}$, we define $u(c_h) = u(\alpha(c_h))$, and for $l \in \mathbb{R}_+^{\mathcal{K}}$ and $x \in \mathbb{R}_+^{\mathcal{J}}$, $F(l, x) = F(l, \alpha(x))$, and similarly for G.

In the main text (see p. 11), the vector of price schedule $P(\theta)$ is defined as a vector in $\mathbb{R}^{\mathcal{I}}$. While we keep the schedules over $\mathcal{I}$, we extend its level to a vector in $\mathbb{R}^{\mathcal{J}}$ through $P(\theta)_j = P_{\iota(j)}(\theta_j)$. With these notations, for $x \in \mathbb{R}^{\mathcal{J}}_+$, $P(\theta) \cdot x$ is well defined and means what is expected: the total costs of inputs, each valued at its own, producer-dependent, price.

Having extended notations when there is more than one firm per commodity, we can now see that the Definition 6 of an equilibrium of an economy with end-of-chain taxation is still meaningful with multiple firms, as is the Definition 8 of the equilibrium for an economy with source taxation. We can now state the equivalent of Proposition 2:

Proposition 4. *Let* $\mathfrak{e} = ((c_h)_{h\in\mathcal{H}}, (z_j, \theta_j)_{j\in\mathcal{J}}, P)$ *be an equilibrium for an economy with end-of-chain taxation* $\mathfrak{E}^E = ((u_h, \omega_h, \lambda_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{J}}, t, \tau)$. *Then* $\mathfrak{e}' = ((\alpha(c_h))_{h\in\mathcal{H}}, (l_j, \alpha(x_j))_{j\in\mathcal{J}}, p')$ *where* $p'_{\iota(j)} = P_j(\theta_j) + \tau\theta_j$ *is an equilibrium of the economy with source taxation* $\mathfrak{E}^S = ((u_h, \omega_h, \lambda_h)_{h\in\mathcal{H}}, (F_j, G_j)_{j\in\mathcal{J}}, t')$ *with* $t' = t + \tau$.

Proof. To see that $\mathfrak{e}'$ is well defined, we need to show that $p'_{\iota(j)}$ univocally described a vector in $\mathbb{R}^{\mathcal{I}}$. ι being surjective, every component is defined, but not necessarily uniquely when two different firms j, j' produce the same commodity $\iota(j) = \iota(j')$. To that end, we remark that Lemma 7 is readily extended to the setting with multiple firms, so is Corollary 4. So, without loss of generality, we assume that firms j with zero output have intensities minimizing $P_j(\theta_j) + \tau\theta_j$. Lemma 8 is also readily extended to the setting with multiple firms so that all firms have θ_j at the minimum. Meaning that all price schedules for all $i \in \mathcal{I}$, and intensities for all firms $j \in \mathcal{J}$ are such that all firms producing the same commodity $i = \iota(j)$ maps to the same total consumer price $p'_i = P_j(\theta_j) + \tau\theta_j$ (a real number, not a vector).

The proof of Proposition 4 is a slight variation of the proof of Proposition 2. We straightforwardly adapt the verification of the three conditions for Definition 8.

1. If $\mathfrak{a}(\mathfrak{e})$ is feasible (firm by firm), then $= \mathfrak{a}(\mathfrak{e}')$ is also feasible (commodity per commodity).

2. We only need to notice that, because of the construction of prices p', for any consumption bundle c'_h that satisfy the budget constraint of the economy with source taxation, any consumption bundle c''_h such that $\alpha(c''_h) = c'_h$ with the equilibrium intensities also remains in the budget constraint of the original economy with end-of-chain taxation.

3. For any $\tilde{z}$, we have $\pi_j^S(\tilde{l}, \alpha(\tilde{x}); p', t') \leqslant \pi_j^E(\tilde{z}, \theta; p, t)$, with equality for z_j. The proof that $(l_j, \alpha(x_j)) \in \mathcal{P}_j^S(p', t')$ then follows as above, remarking that for any $x \in \mathbb{R}_+^{\mathcal{I}}$, we can find a $x' \in \mathbb{R}_+^{\mathcal{J}}$, such that $\alpha(x') = x$ (α is surjective).

The defined $\mathfrak{e}'$ is thus an equilibrium of the economy $\mathfrak{E}^S$ with source taxation.
□

As the economy with end-of-chain taxes can be converted in an economy with source taxes, it is then easily shown that adding firms above the number of goods and production technologies, does not lead to a more diverse economy. One representative firm per good suffices.

Corollary 5. *Take an economy with end-of-chain taxation with multiple firms, such that firms producing the same commodity have the same production and externality functions. Then, at equilibrium, without loss of generality, we can assume that there is only one active ("representative") firm per commodity.*

Proof. Take an equilibrium $\mathfrak{e}$ and map it with proposition 4 to an equilibrium of an economy with source taxation. We know that in this economy, at equilibrium, firms producing the same commodity operate with the same input ratio. Then, the firms in the economy with end-of-chain taxation operate at the same intensity, so that we can forget the distinction between firms. □

7.4 Economy with an isolated part

To build our equivalence of source taxation and end-of-chain taxation, we took some care to rule out the case where the economy has an isolated part with non-zero activity. Below we present a few illustrations that make the isolated part and the problems it could raise more tangible.

The basic idea for an isolated part of an economy is illustrated by two firms who mutually exchange their products, without any connection to the remainder of the economy. Our assumptions allow for such firms. That is, we have a firm producing 'widget A' from 'widget B' and selling its output to the second firm that produces 'widget B' from 'widget A'. This part of the economy is isolated from the rest of the economy: it does not use other products as inputs and its products ('widget A' and 'widget B') are neither used as inputs by the rest of the economy nor are they consumed. This isolated part is totally irrelevant for welfare of consumers, but it may nonetheless exist; it shapes a kind of economic 'perpetuum mobile'.

If this isolated part could produce some externality, an end-of-chain taxation could not reach it. There is no final consumer (who pays the end-of-chain tax) of the goods produced in the isolated part. The equivalence between source taxation and end-of-chain taxation (Proposition 1 and Proposition 2) would then be broken. Illustration 7 demonstrates the mechanism that rules this out. An isolated part that produces an externality is always inactive *in equilibrium*. The reason for inactivity in an equilibrium with source taxation is subtly different from the reason for inactivity in an equilibrium with end-of-chain taxation, so that the illustration discusses both mechanisms separately. It builds on the two-firm economy of Illustration 4 and extends it with two auxiliary firms that together form the isolated part: strictly positive net output without factor inputs, causing externalities alongside activity. At equilibrium with source taxation or with end-of-chain taxation, the isolated part is inactive.

Illustration 7 (An isolated part with externality is inactive)**.** *There are two consumers labeled by the superscript $h \in \mathcal{H} = \{1, 2\}$. There is one factor 'labor', $\mathcal{K} = \{1\}$ and four commodities 'metal', 'manufactured goods', 'widget A' and 'widget B' , labeled by the subscript $i \in \mathcal{I} = \{1, 2, 3, 4\}$. Households, metal, manufacturing, and external effects are described in Illustration 4 (i), (ii), (iii). The two auxiliary firms are described by:*

- *(iv) Firm producing widget A, $j = 3$, takes widget B as input, producing widget A goods through the technology (where we use shorthand notation)[26] $F_3 = \{y_3 \leqslant x_{3,4}\}$. The firm does not emit, $r_3 = G_3 = 0$.*
- *(v) Firm producing widget B, $j = 4$, takes widget A as input, producing widget B goods through the technology $F_4 = \{y_4 \leqslant x_{4,3}\}$. This sector produces externality proportionally to its output: $G_4 = y_4$.*

Take a feasible allocation, balance between supply and demand requires that $x_{4,3} = y_3$ and $x_{3,4} = y_4$ (see Definition 3). Clearly, given the productions sets, there are feasible allocations with y_3 and y_4 positive. However, at equilibrium, this is not possible.

[26] All variables not listed are zero.

Consider an equilibrium of the economy with source taxation $t > 0$. The allocation is feasible and we have $\pi_3 + \pi_4 = -t.y_4$. Profits at the left-hand side must be non-negative in equilibrium, but the right-hand side is non-positive. Hence we must have $y_4 = 0$. At the equilibrium of the economy with source taxation, the isolated part is actually inactive.

Consider an equilibrium of the economy with end-of-chain taxation. The allocation is feasible and, per the definition of equilibrium, there exist intensities consistent with the allocation. Thus we have $\theta_3 y_3 - \theta_4 x_{3,4} = 0$ for firm 3 and $\theta_4 y_4 - \theta_3 x_{4,3} = y_4$. Summing the two equations, the terms of the left-hand side cancel out because of the feasibility conditions, yielding $0 = y_4$. At the equilibrium of the economy with end-of-chain taxation, the possible isolated part is inactive.

Our next illustration exemplifies the type of isolated parts that can be active in equilibirum (in constrast to the previous illustration), yet that can be made inactive as in Lemma 2 and Lemma 4 without affecting the equilibrium in any meaningful way.

Illustration 8 (The isolated part can be made inactive)**.** *We keep the same set of firms and production sets as above, but only change the externality functions of firm producing widget B to $G_4 = 0$. Clearly, nothing precludes an equilibrium to have positive productions of widget A and widget B. In this illustration, and contrary to the previous one, the isolated part is not necessarily inactive. Furthermore, intensities of the widgets are consistently but not uniquely defined (Definition 4) as long as we have $\theta_3 = \theta_4$.*

For any equilibrium allocation with positive productions for widget A and widget B, we can scale down activity in parallel (i.e. $y_3 = y_4$) to zero. This has no consequences for regulatory purposes as the isolated part does not produce any externality nor has it consequences for welfare considerations as its production is not consumed outside the isolated part.

In the previous illustration, the isolated part was fully isolated from the rest of the economy, hence making it inactive was simple. It would be possible to provide a more complex example where the isolated part uses a factor input with zero price, which is also used in the non-isolated part. In such a situation, it can be shown that the isolated part can also be scaled down to zero.