Market Design for Distributional Objectives in (Re)assignment: An Application to Improve the Distribution of Teachers in Schools

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Market Design and Human Capital

- Teachers' distribution is important for students' human capital
  - $\Rightarrow$ How to attract them where they are the most needed?
- In most countries: teachers' assignment is not a standard market
  - Sometimes centralized assignment with priorities
  - Public sector: budget constraint for wages
  - Amenities of schools can be important
- The big question: what is the optimal policy mix to attract/retain teachers in disadvantaged areas?
  - Wages
  - Matching procedure: priorities, algorithm
  - Amenities of schools (e.g. class size...etc)

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- Government Sector: Police officers (e.g. Chicago), doctors (many countries)

⇒ Common features: centralized employer with distribution objectives

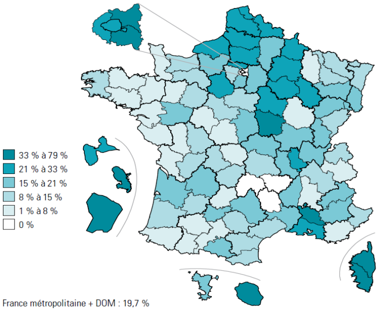
  - Main application in this paper:

    Disadvantaged regions have relatively more inexperienced teachers; to decrease the education achievement gap in the country, more experienced teachers are needed in these regions.

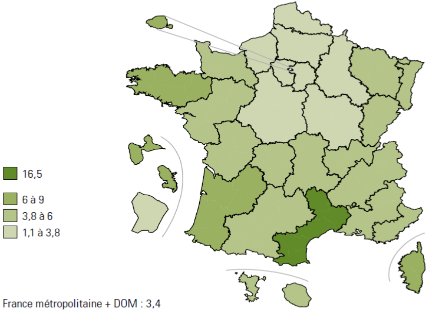
    (Chetty, Friedman, Rockoff, 2014, Allen, Mian, Sims, 2016)

Example: Teacher Distribution in France

Share of students in a disadvantaged school



Ratio of teachers age 50+ to age 30–



Contribution

- **Contribution 1**: We propose a new mechanism which incentivizes truthful reports from teachers and improves both schools and teachers with respect to a status-quo matching.
    - A school improvement is measured by a (Lorenz) shift of the type distribution of its assigned teachers following a priority ordering over types (e.g., a ranking over experience levels).

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- **Contribution 2**: In a large market setting, we show how a global objective of decreasing inequality across schools can be achieved by designing priorities for schools and using our proposed mechanism to shift their teacher type distribution.
- **Contribution 3**: Using French data, we conduct empirical simulations:
  - Our mechanism achieves a decrease in inequality while other benchmarks do not, notably those without distributional objectives.

The Model

- T: set of teachers
  N: set of new teachers
  $T \setminus N$: set of tenured teachers
  Types: t has a teacher type $\theta(t) \in \Theta$ finite
  - Example: $\theta(t)$ = experience in years

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- S: set of schools, each school s with a quota q_s
- ω: status-quo matching is the initial allocation
  - $\omega_t \in S \cup \{\emptyset\}$: the initial school of teacher t
    $\omega_t = \emptyset \iff t$ is a new teacher
  - $\omega_s \subseteq T$: the initial employees of school s

The Model

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- Allocation: A matching is an assignment of teachers to schools so that
  - each teacher is matched with at most one school, and
  - each school does not exceed its quota.

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- φ is strategy-proof (SP) if for every profile P, teacher t, and manipulation $\hat{P}_t$,

$$\varphi_t(P_t, P_{-t})\ R_t\ \varphi_t(\hat{P}_t, P_{-t}).$$

Inequality Reduction: Priorities vs. Global Objective

- Main objective: reduction of inequality of types across schools (e.g. experience)

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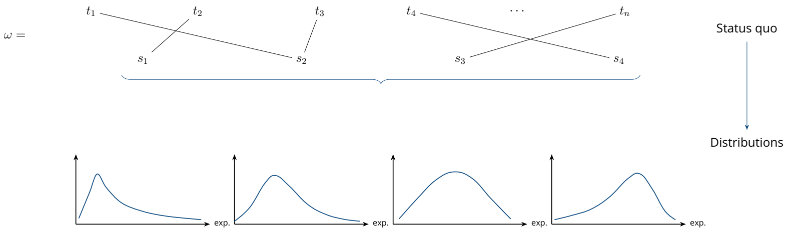
  - Simpler:
    Improve each school's distribution of experience according to some fixed priority order over experience levels

Type Rankings and Status-quo Improvement



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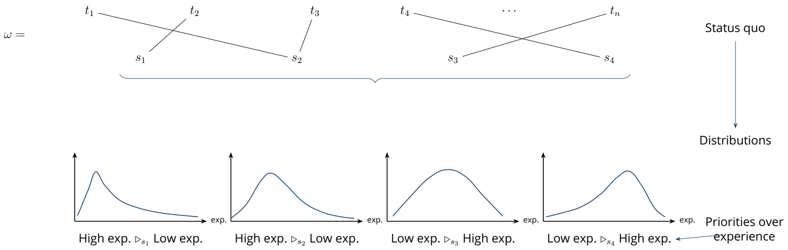

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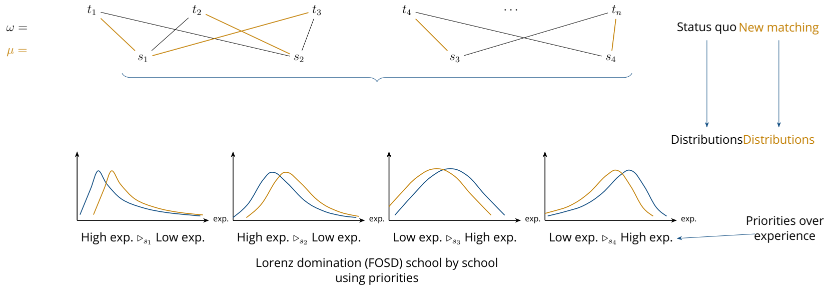

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  For any two sets of teachers $\bar{T}, \hat{T}$ if

  $$\forall \ \theta \quad \sum_{\theta' \trianglerighteq_s \theta} \# \text{ type-}\theta' \text{teachers in } \bar{T} \ \geq \sum_{\theta' \trianglerighteq_s \theta} \# \text{ type-}\theta' \text{teachers in } \hat{T},$$

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- A mechanism φ is $\rhd$-status-quo improving ($\rhd$-SI) if for every profile P and school s,

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- Where do we use it?
  - This approach will be justified by type ranking design for inequality reduction.

Constrained Efficient Mechanisms

- A mechanism φ is $\triangleright$-constrained efficient if
  - it is IR and $\triangleright$-SI, and
  - for every profile P, $\varphi(P)$ is not Pareto dominated (for teachers) by another IR and $\triangleright$-SI matching.

SI Cycles and Chains (SI-CC) Mechanism

- Algorithm based on directed graphs with nodes being teachers and schools

⇒ Implement cycles and chains to achieve the objectives

- School pointing rule: Pointing to lower-ranked teachers first: important for strategy-proofness.
- Teacher pointing rule: Need a counter at each school to keep track of improvements to determine whether a teacher can point.
- Chain construction & selection rule: Ensure SI by not leaving occupied seats empty.

SI-CC Results

Theorem

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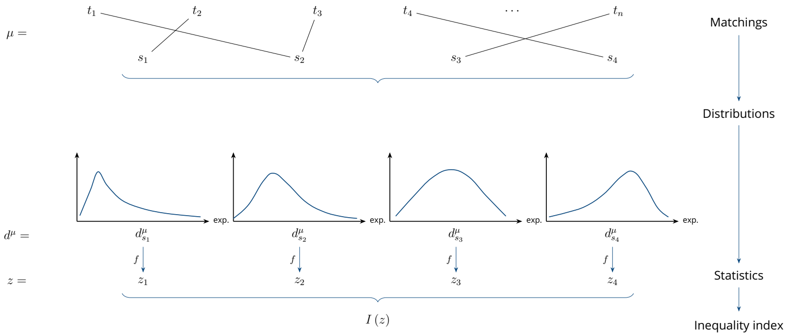
Remark

Any change in pointing rules in SI-CC (except tie-breaking) may lead to a violation in either $\triangleright$-SI, $\triangleright$-constrained efficiency, or strategy-proofness.

Details

Type Ranking Design for Inequality Reduction



Type Ranking Design for Inequality Reduction

- Teacher types are assigned values in $\mathbb{R}_+$ such that

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- Statistic vector $f(d^\mu) \Rightarrow$ inequality index $I\big(f(d^\mu)\big) \in \mathbb{R}$

Type Ranking Design for Inequality Reduction

- Inequality index $I(z_1, \ldots, z_m)$

  Properties:

  - continuously differentiable almost everywhere
  - symmetric for schools with the same weight:
    w_s: ratio of the quota of school s to total quota of schools
  - satisfies a weak form of convexity:
    $\nearrow$ in the statistic of the worst school
    or
    $\searrow$ in the statistic of the best school

    $\Rightarrow \searrow$ in inequality

Type Ranking Design for Inequality Reduction

- Examples:
  - Weighted Gini Index:

$$I(z) = \frac{1}{2\sum_s w_s z_s} \sum_s \sum_{s'} w_s w_{s'} |z_s - z_{s'}|$$

  - T20/B20 Ratio:

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- Matching μ reduces inequality below the status quo if

$$I\big(f(d^\mu)\big) \leq I\big(f(d^\omega)\big)$$

Large Market and Inequality Reduction

- Fix a (re)assignment market $\langle T, \Theta, S, q, P, \omega \rangle$, statistic f, and inequality index I.

- $E^n = \left(E_s^{\theta,n}\right)_{\theta\in\Theta, s\in S}$: sets of non-participating employees of each type for each school (indexed by n).

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  $\Rightarrow$ Base economy E_s^0 induces replica economies $\{E_s^n\}_{n=0}^{\infty}$.

  $\Rightarrow$ Sets of non-participating employees become dominant in the large.

  - Example: In France, 96.5% of teachers were non-participating in 2013.

Status-quo improvement $\Rightarrow$ Inequality reduction

Proposition

For any base economy for the replica economies, there exists a type ranking profile for schools $\triangleright^*$ such that for a large enough market size n, if μ is $\triangleright^*$-status-quo improving then μ reduces inequality below the status quo.

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    $\Rightarrow$ H schools rank low experience over high experience

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Sketch: Constructing the Type Ranking Profile $\rhd^*$

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  $\Rightarrow$ Inequality index may increase with $\rhd^*$-status-quo improvement.

- Closing the proof: In a large market, the signs of partial derivatives do not change.
  $\Rightarrow$ Inequality index will decrease with status-quo improvement for $\rhd^*$.

Doing better than SI-CC?

- With a strictly FOSD-increasing statistic:

Proposition

There exists some base economy for the replica economies such that for n large enough, μ is $\triangleright^*$-status-quo improving only if μ reduces inequality below the status quo.

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Proposition

No individually rational and strategy-proof mechanism generates less inequality whenever possible than the SI-CC mechanism induced by $\triangleright^*$.

Related Literature

- Centralized teacher (re)assignment, and other jobs:

  Pereyra (2013); Combe, Tercieux, Terrier (2021); Dur & Kesten (2019); Agarwal (2015); Thakur (2020); Sidibe et al., (2021) . . .

- Efficient matching and constraints:

  Shapley & Scarf (1974); Abdulkadiroğlu & Sönmez (1999); Papai (2000); Roth, Sönmez, Ünver (2004); Dur, Kesten, Ünver (2015); Pycia & Ünver (2017); Takamasa, Tamura, Yokoo (2018); Dur & Ünver (2019); Hafalır, Kojima, Yenmez (2022). . .

- Unequal distribution of teachers across schools:

  Bobba et al. (2021); Bates et al. (2021); Biasi et al. (2021); Tincani (2021); Laverde et al. (2024) . . .

Skip to the Conclusion

French teacher assignment

- Data on French centralized assignment of teachers to regions in 2013

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- Estimation of teachers' pref over regions: $u_{t,R}$ Details
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- Estimation on each of 8 fields (Maths, History, Sport...)
- Final sample: 10,460 teachers: 5,833 tenured teachers (55.8%) and 4,627 new teachers

Skip the result

Counterfactual Analysis

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Aims to quantify the algorithms performance in a real-life setting:

- SI-CC (our SI constrained efficient mechanism)
- Benchmark for SI-CC: TTC*
  - As SI-CC but does not impose status-quo improvement for schools ($\sim$ school choice TTC with IR).
- Current French system: DA*
  - The currently used mechanism which is a variation of Deferred Acceptance with IR.

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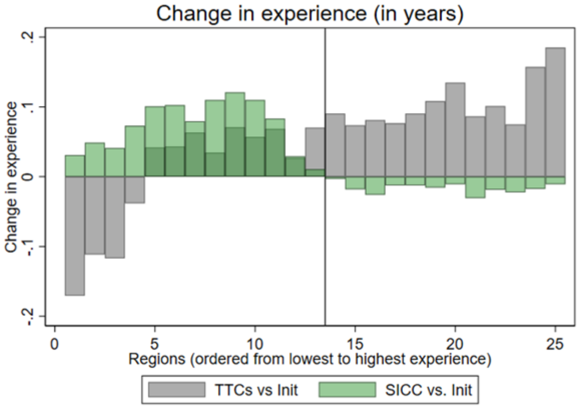
- Inequality index and schools' priorities
  - We use the Gini index together with the mean experience statistic in each region
  - We follow the priority design construction from the theory

		Regions	Teacher-Type Ranking ▷
1	L = Gini derivative < 0	Créteil Versailles Amiens...	High exp ▷ . . . ▷ Low exp ▷ $\emptyset$
2	H = Gini derivative > 0	Bordeaux Rennes Lyon...	$\emptyset$ ▷ Low exp ▷ . . . ▷ High exp

Empirical Results: SI-CC decreases inequality

	SI-CC	TTC*	SI-CC*	Current French
	(1)	(2)	(3)	(4)
Panel A. Inequality Index				
T20/B20 ratio (at status quo = 1.3579)	1.3579	1.3760	1.3786	1.3909
Panel B. Teacher mobility				
Total moved/newly assigned	4,977	5,765	5,853	5,646
Tenured teachers moved from B20 regions	190	787	1,142	915
Average rank of region assigned				
All teachers	9.4	9.8	10.6	10.3
Tenured teachers	7.7	6.7	6.8	6.8
New teachers	11.5	13.7	15.3	14.6

Empirical results: mean variations for each region



Empirical results: Mobility in the poorest regions

	SI-CC	TTC*	SI-CC*	Current French
	(1)	(2)	(3)	(4)
Panel A. Inequality Index				
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- Under SI-CC: decrease of inequalities is achieved at the cost of less mobility in the least experienced regions

⇒ Can we mitigate this cost?

SI-CC*: relaxed version of SI-CC

- Current algorithm in France: Deferred Acceptance (DA) with priorities given by law mainly based on experience
- 3 features in the current system
  1) Newcomers have to rank all regions
  2) Newcomers have low priority and end up in 3 least exp. regions
  3) No constraints on experience: least exp. regions loose experienced teachers

SI-CC*: relaxed version of SI-CC

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- 3 features in the current system
  1) Newcomers have to rank all regions
  2) Newcomers have low priority and end up in 3 least exp. regions
  3) No constraints on experience: least exp. regions loose experienced teachers

- Variation of SI-ICC:
  - Change newcomers lists to make them rank the (three) most disadvantaged regions first
  - Prioritize some high exp. teachers from B20 in other regions
  - Run SI-CC

$\Rightarrow$ Strategy-proof (bec. of 1)

Empirical results: Mobility in the poorest regions

	SI-CC	TTC*	SI-CC*	Current French
	(1)	(2)	(3)	(4)
Panel A. Inequality Index				
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Concluding Remarks

- We design schools' priorities that reflect a central authority's objective of reducing inequality.
  - School-specific priorities achieve an interlinked, global objective in a large market.
- We design a second-best mechanism, SI-CC, that improves both teachers' welfare and reduces inequality compared to an initial allocation.
- SI-CC can be used instead of the current French mechanism (a DA-variant based on reducing "justified envy").
  - In France, a TTC-variant mechanism is used for teacher assignment.
- Our counterfactual analysis using French data shows that
  - SI-CC reduces inequality across regions, unlike its benchmarks,
  - while ensuring a high teacher mobility (especially its variant, SI-CC*).

Appendix

SI-CC Mechanism

Given type ranking profile $\triangleright$:

- Step k:
  - Each remaining school s points to its remaining lowest type status-quo employee under $\triangleright_s$ (if there are many, it uses a fixed tie-breaker).
  - Each remaining teacher t points to her top choice among $\emptyset$ and all remaining schools s that satisfy:

Type (1) School improvement by replacement: if s points to a teacher t' and replacing her with t will make the current match of s to $\triangleright_s$-Lorenz dominate ω_s,

or

Type (2) School improvement by addition: if t is acceptable for s under $\triangleright_s$ and s has a vacant seat.

  - $\emptyset$ points to every teacher pointing to it.

SI-CC Mechanism

- Step k continued:

  Two cases:

  (i) There exists a cycle in which
  either every teacher's pointing satisfies (1)
  or there are only one teacher and option $\emptyset$

SI-CC Mechanism

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  Each teacher is assigned to the school/option she is pointing to, go to Step k+1.

SI-CC Mechanism

- Step k continued:

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  (ii) There exists a chain and (i) does not hold.

SI-CC Mechanism

- Step k continued:

  Two cases:

  (i) There exists a cycle in which
  either every teacher's pointing satisfies (1)
  or there are only one teacher and option $\emptyset$

  Each teacher is assigned to the school/option she is pointing to, go to Step k+1.

  (ii) There exists a chain and (i) does not hold.

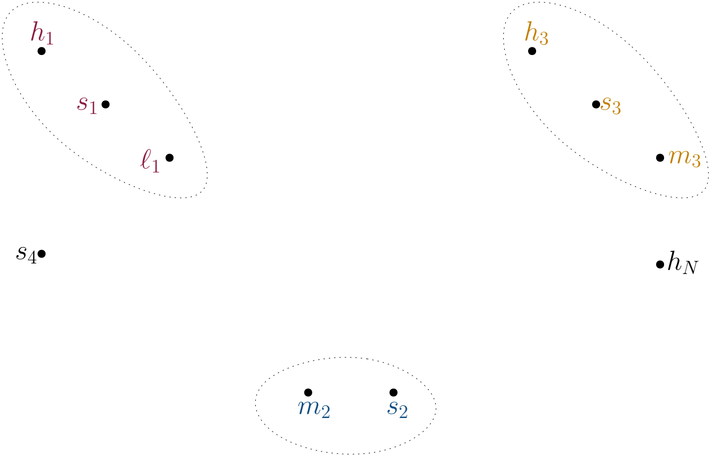
  - If there is a remaining new teacher: we select a chain starting with a new teacher (using a fixed tiebreaker) and ending with a school with a vacant position
  - Otherwise: we remove each school s whose all status-quo employees are assigned, go to Step k+1.

SI-CC Example

- 4 schools: s_1, s_2, s_3, s_4
- $q_{s_1} = q_{s_3} = 2$ and $q_{s_2} = q_{s_4} = 1$
- 3 teacher types: high (h), medium (m), low (ℓ) experiences
- 6 teachers: 3 high , 2 medium , 1 low type
- status-quo matching:
  - h_1 and ℓ_1 at s_1
  - m_2 at s_2
  - h_3 and m_3 at s_3
  - h_N new teacher
  - no teacher at s_4

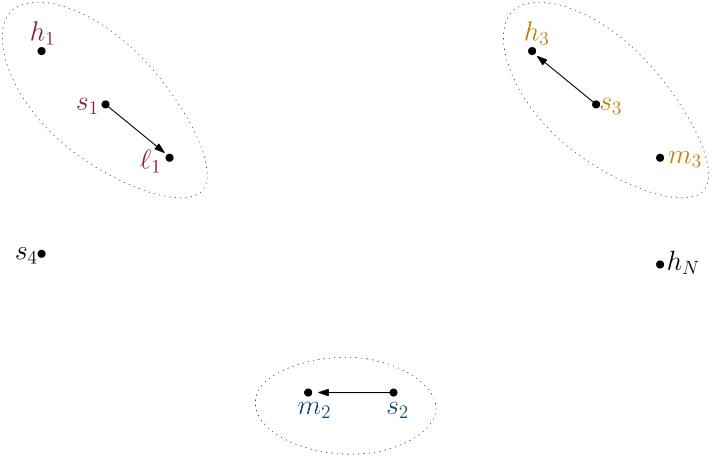
h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	s_2	s_4	s_1	s_2	s_2
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	h_1, ℓ_1	m_2	m_3, h_3	$\emptyset$
current:	h_1, ℓ_1	m_2	m_3, h_3	$\emptyset$



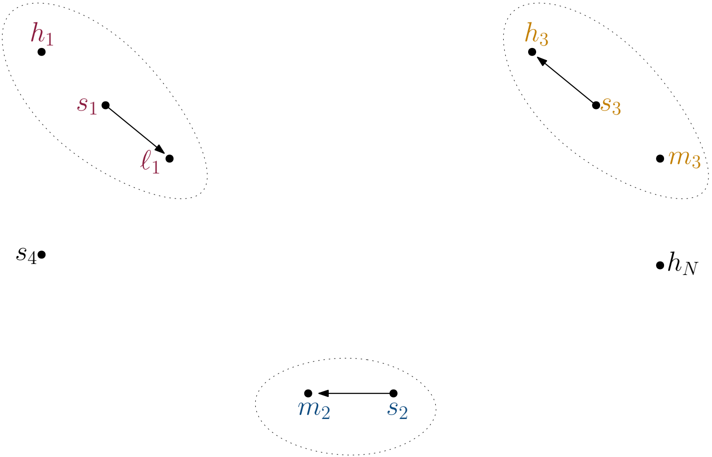
h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	s_2	s_4	s_1	s_2	s_2
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	$h_1, \underline{\ell_1}$	$\underline{m_2}$	$m_3, \underline{h_3}$	$\emptyset$
current:	h_1, ℓ_1	m_2	m_3, h_3	$\emptyset$



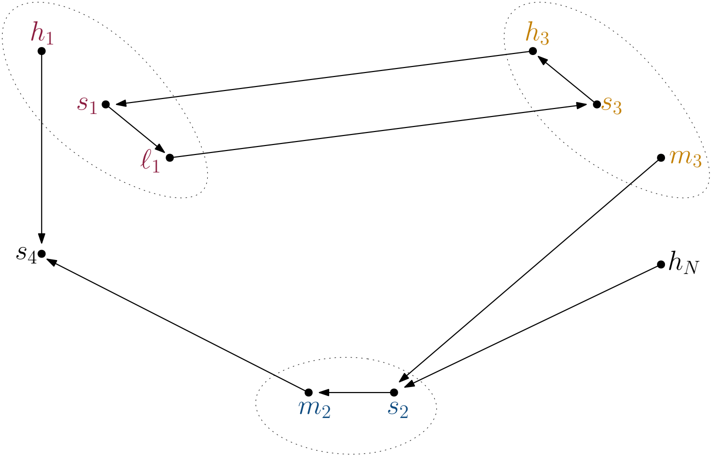
h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	~~s_2~~	s_4	s_1	s_2	s_2
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	$\boxed{m}$	m	m
	ℓ	$\boxed{\ell}$	h	h
status quo:	$h_1, \underline{\ell_1}$	$\underline{m_2}$	$m_3, \underline{h_3}$	$\emptyset$
current:	h_1, ℓ_1	m_2	m_3, h_3	$\emptyset$



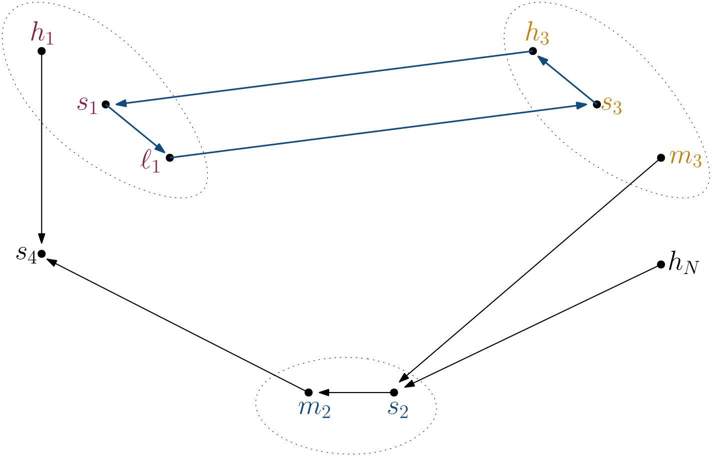
h_1	ℓ_1	m_2	h_3	m_3	h_N
$\underline{s_4}$	~~s_2~~	$\underline{s_4}$	$\underline{s_1}$	$\underline{s_2}$	$\underline{s_2}$
s_2	$\underline{s_3}$	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	$h_1, \underline{\ell_1}$	$\underline{m_2}$	$m_3, \underline{h_3}$	$\emptyset$
current:	h_1, ℓ_1	m_2	m_3, h_3	$\emptyset$



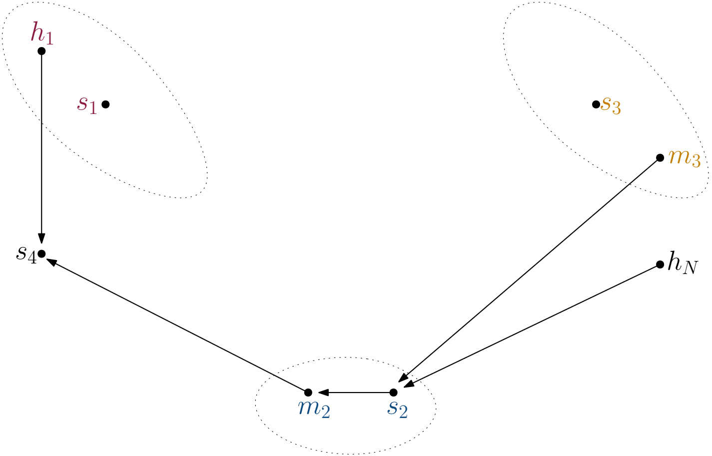
h_1	ℓ_1	m_2	h_3	m_3	h_N
$\underline{s_4}$	~~s_2~~	$\underline{s_4}$	$\underline{s_1}$	$\underline{s_2}$	$\underline{s_2}$
s_2	$\underline{s_3}$	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	$h_1, \underline{\ell_1}$	$\underline{m_2}$	$m_3, \underline{h_3}$	$\emptyset$
current:	h_1, ℓ_1	m_2	m_3, h_3	$\emptyset$



h_1	ℓ_1	m_2	h_3	m_3	h_N
$\underline{s_4}$	~~s_2~~	$\underline{s_4}$	s_1	$\underline{s_2}$	$\underline{s_2}$
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	h_1, ℓ_1	$\underline{m_2}$	m_3, h_3	$\emptyset$
current:	h_1, h_3	m_2	ℓ_1, m_3	$\emptyset$



h_1	ℓ_1	m_2	h_3	m_3	h_N
$\underline{s_4}$	~~s_2~~	$\underline{s_4}$	s_1	$\underline{s_2}$	$\underline{s_2}$
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	$\underline{h_1}, \ell_1$	$\underline{m_2}$	$\underline{m_3}, h_3$	$\emptyset$
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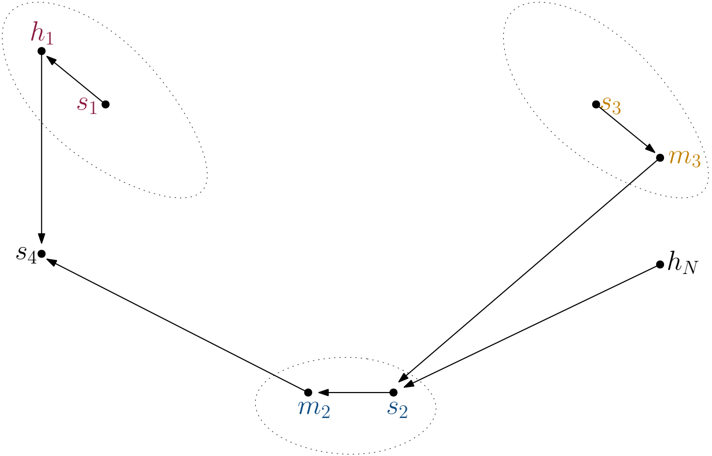
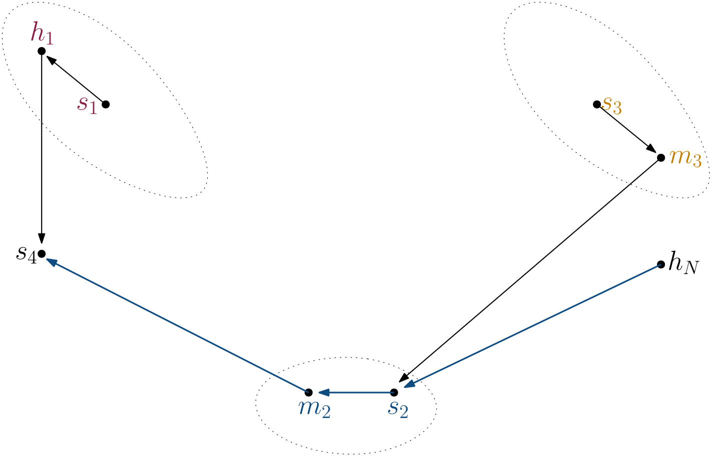


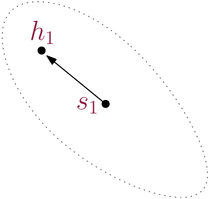
h_1	ℓ_1	m_2	h_3	m_3	h_N
$\underline{s_4}$	~~s_2~~	$\underline{s_4}$	s_1	$\underline{s_2}$	$\underline{s_2}$
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

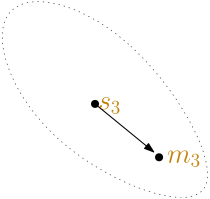
	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	$\underline{h_1}, \ell_1$	$\underline{m_2}$	$\underline{m_3}, h_3$	$\emptyset$
current:	h_1, h_3	m_2	ℓ_1, m_3	$\emptyset$



h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	~~s_2~~	s_4	s_1	s_2	s_2
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

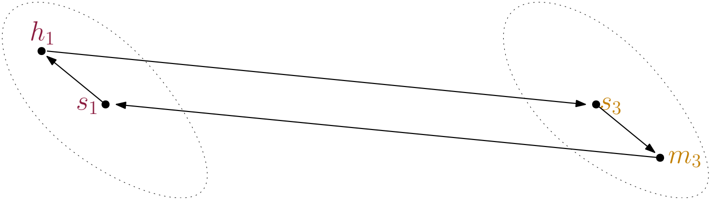
	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	$\underline{h_1}, \ell_1$	m_2	$\underline{m_3}, h_3$	$\emptyset$
current:	h_1, h_3	h_N	ℓ_1, m_3	m_2





h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	~~s_2~~	s_4	s_1	s_2	s_2
s_2	s_3	s_3	s_3	$\underline{s_1}$	s_1
$\underline{s_3}$	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	$\underline{h_1}, \ell_1$	m_2	$\underline{m_3}, h_3$	$\emptyset$
current:	h_1, h_3	h_N	ℓ_1, m_3	m_2



h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	~~s_2~~	s_4	s_1	s_2	s_2
s_2	s_3	s_3	s_3	$\underline{s_1}$	s_1
$\underline{s_3}$	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	$\underline{h_1}, \ell_1$	m_2	$\underline{m_3}, h_3$	$\emptyset$
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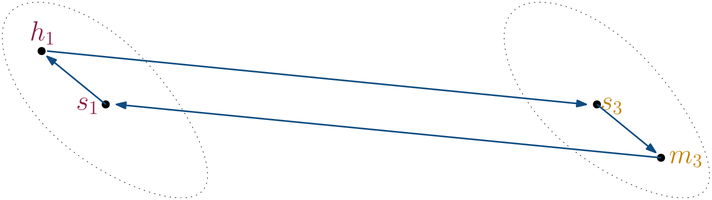

h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	~~s_2~~	s_4	s_1	s_2	s_2
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	ℓ
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	h_1, ℓ_1	m_2	m_3, h_3	$\emptyset$
current:	h_3, m_3	h_N	ℓ_1, h_1	m_2

Demand estimation

Random utility model estimated as a function of teachers' and regions' characteristics

$$u_{t,R} = \delta_R + Z'_{t,R}\beta + \varepsilon_{t,R} \tag{1}$$

δ_R	region fixed effect
$Z_{t,R}$	teacher-region-specific observables
$\varepsilon_{t,R}$	random shock i.i.d. over t and R
	type-I extreme value distribution, Gumbel(0,1)

Goal: Estimate the model and run counter-factuals

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Goal: Estimate the model and run counter-factuals

- Separate estimation for tenured teachers and newcomers
- Estimation on each of our 8 fields Details
- Final sample: 10,460 teachers: 5,833 tenured teachers (55.8%) and 4,627 new teachers

Demand estimation

Teacher characteristics:

- Qualification
- Experience
- Family status

Teacher-region specific characteristics:

- Birth region
- Current region

(Interacted) Region characteristics:

- Socio-economic measure
- Academic performance measure
- ...

Preference estimation

- **Identifying assumption based on stability** (Chiappori-Salanié (16), Akyol-Krishna (17), Artemov-Che-He (19), Fack-Grenet-He (19))
  - Teachers might skip unreachable regions from their ranking
    but
  - Assignment = most preferred region within feasible regions
- Logit choice probabilities. Estimate β via ML.

Fit is a lot better than assuming truthtelling Back.

Eight markets

	All teachers	Newcomers	Initially assigned	Vacant positions
	(1)	(2)	(3)	(4)
All subjects	10460	4627	5833	3912
Sport	2066	568	1498	475
French	1645	786	859	663
English	1374	746	628	640
Mathematics	1563	958	605	824
Spanish	999	316	683	248
History-Geography	1230	657	573	562
Biology	746	286	460	246
Physics-Chemistry	837	310	527	254

Teachers' characteristics

	Tenured			Newcomers		
	French (1)	Math (2)	English (3)	French (4)	Math (5)	English (6)
% Female	76.1	47.0	85.4	80.3	41.7	80.4
% Maried	48.5	45.0	46.8	41.1	39.4	40.9
% In disadvantaged school	10.4	13.2	4.4	0.0	0.0	0.0
Experience (in years)	7.48	7.23	7.18	2.76	2.24	2.30
% Advanced teaching qualif	7.9	29.1	8.8	16.8	31.7	15.2
Observations	859	605	628	786	958	746

Back