Toulouse School of Economics

Allocating essential inputs

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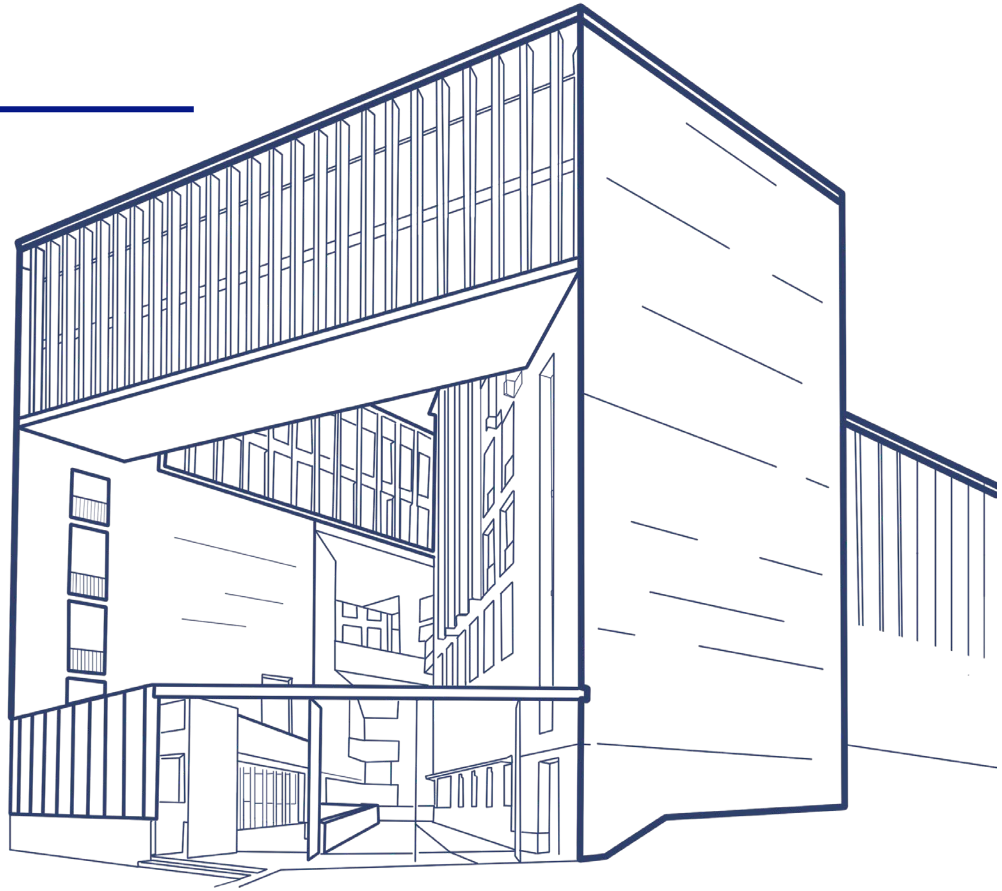

Spectrum Auctions and Market Structure Conference
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Spectrum allocation

- Historically: first come, first served; beauty contest
- Since the 90s: switch to auctions
  - Multiple lots, possibly multiple bands and multiple regions
  - Multiple formats: seq. or simultaneous, 1st- or 2nd-price, …
- Objective: social welfare → trade-off
  - Auction revenue: competition in the auction
  - Consumer surplus: competition in the downstream market

Spectrum auctions

- Motivation
  - Active involvement of stakeholders, transparency / fairness
  - Generating revenue (caps and set-asides to maintain competition)
  - *Eliciting information / efficient use of spectrum*
- Track record
  - Increased concentration

What do we do

- Study optimal input allocation
  - Simple duopoly setting
    - Having more bandwidth reduces cost of service
    - One firm is initially ahead of the other
  - Additional bandwidth becomes available
    - Regulator allocates bandwidth
    - Can also tax firms
    - [No price regulation]
  - Consumer surplus / social welfare (weight on revenue / profits)
  - Complete / incomplete information

Findings

- [Always optimal to allocate all bandwidth]
- Complete information
  - Consumer surplus: minimize cost asymmetry
  - Social welfare: maintains some asymmetry
- Incomplete information
  - Spence-Mirrlees condition cannot hold → *bunching*
  - If uncertainty is large and focus mostly on consumers

full bunching → no role for auctions

Baseline setting (complete information)

- Bertrand duopoly (with possibly asymmetric costs)
  - Two firms: I and E
  - Unit costs: $c_i = C(B_i)$, with $C' < 0 < C''$
  - Initially, $B_I > B_E$; E thus obtains zero profit, and I obtains:

$$\pi(B_I, B_E) = [C(B_E) - C(B_I)]D\big(c(B_E)\big)$$

  → increases with the bandwidth advantage:

$$\partial_1\pi > 0 > \partial_2\pi$$

- Additional amount of bandwidth Δ to be shared

$$b_I \geq 0, b_E \geq 0, b_I + b_E \leq \Delta$$

Consumer surplus

- **Proposition** $[\max S(p) \equiv \int_p^{+\infty} D(x)dx]$
  - Allocate all the additional bandwidth Δ
  - Minimize cost asymmetry
- Intuition: minimize the higher of the two costs
- Resulting market price
  - $p^S = \underline{c}_E \equiv C(B_E + \Delta)$ if $\Delta < B_I - B_E$
  - $p^S = \hat{c} \equiv C\left(\frac{B_I + B_E + \Delta}{2}\right)$ if $\Delta \geq B_I - B_E$

Social welfare

- **Proposition** $[\max W(p) \equiv S(p) + \lambda(t_I + t_E)$ s.t. $t_i \leq \pi_i]$
  - Allocate all the additional bandwidth Δ
  - Tax all profits: $t_i = \pi_i$ for $i = I, E$
  - There exists $\bar{\lambda} > \underline{\lambda} \geq \frac{1}{2}$ such that
    - $p^W(\lambda) = p^S$ for $\lambda \leq \underline{\lambda}$
    - $p^W(\lambda)$ is continuous and strictly increasing for $\lambda \in [\underline{\lambda}, \bar{\lambda}]$
    - $p^W(\lambda) = \bar{c}_E \equiv C(B_E)$ if $\lambda \geq \bar{\lambda}$

Robustness check

- Horizontal differentiation à la Hotelling
  - Linear transportation cost t
  - Individual consumer demand $d(p)$, surplus $s(p)$
- As t tends to zero
  - Optimal allocation tends to that of pure Bertrand competition
- Unit demand
  - Unit cost $C(B) = \alpha/B$ or $C(B) = \alpha - \beta B - \gamma B^2$
  - Optimal to limit cost difference whenever $\lambda \leq \underline{\lambda}$, for some $\underline{\lambda} \leq \frac{1}{2}$

Incomplete information

- Uncertainty about E's handicap: $B_I = B$ and $B_E = B - \gamma$
  - Still optimal to allocate all additional bandwidth
  - Let $\pi_E(b;\gamma) \equiv \max\{\pi(B - \gamma + b, B + \Delta - b), 0\}$ (and $\pi_I(b;\gamma)$…)
- Revelation principle
  - Direct incentive compatible mechanism

$$\{(b_I(\gamma), b_E(\gamma)), (t_I(\gamma), t_E(\gamma))\}$$

  - Individual rationality

$$\pi_I(b_I(\gamma);\gamma) - t_I(\gamma) \geq 0 \text{ and } \pi_E(b_E(\gamma);\gamma) - t_E(\gamma) \geq 0$$

  - Incentive compatibility

$$\pi_E(b_E(\gamma);\gamma) - t_E(\gamma) \geq \pi_E(b_E(\gamma');\gamma) - t_E(\gamma')$$

Incomplete information

- Transfers
  - Still optimal to tax I's profit: $t_I(\gamma) = \pi_I(b(\gamma); \gamma)$
  - If uncertainty large enough, needs to leave rents to E
- Spence-Mirrlees monotonicity?
  - Willingness to pay to increase bandwidth from b to $b' > b$

$$\delta(\gamma) \equiv \pi_E(b'; \gamma) - \pi_E(b; \gamma) (\geq 0)$$

  - Suppose $\pi_E(b'; \gamma) > 0$ (otherwise, $\delta(\gamma) = 0$)
    - if $\pi_E(b; \gamma) < 0$, then $\delta'(\gamma) > 0$
    - if instead $\pi_E(b; \gamma) < 0$, then $\delta'(\gamma) < 0$

Bunching

- **Lemma**
  - Suppose $\lambda \leq 1/2$
  - If optimal to equalize costs for some γ, full bunching:
$$b^*(\gamma) = b$$
- **Proposition**
  - Suppose demand is inelastic, and $\lambda \leq 1/2$
  - If $\min\{\gamma\} < \Delta$, full bunching: $b^*(\gamma) = b$

Discussion and extensions

- **Standard auctions**
  - Auction formats: sequential, VCG (CCAs), ascending (SMRAs)
  - I gets all blocks
  - With sequential auctions, I may pays nothing
- **Two-sided incomplete information**
  - Simple binary setting
  - **Proposition** If large enough heterogeneity, full bunching

Thank you !

Additional material

- Standard auctions (complete information)

- Two-sided incomplete information

Standard auctions

- Setting
  - [Complete information]
  - Additional bandwidth Δ divided in k blocks of Δ/k
  - Auction formats
    - sequential (one block at a time)
    - clock and combinatorial clock auctions (CCAs): VCG
    - simultaneous multi-round ascending auctions (SMRAs): ascending

Standard auctions

- Sequential auctions
  - With any classic format (FP/SP sealed bids, ↗/↘), I gets all blocks
  - If in addition $B_I - B_E > \Delta/k$, then I pays nothing
- Simultaneous auctions (all blocks)
  - Same outcome with VCG and ascending: I gets all blocks
  - However, needs to pay E's profit from winning all blocks

Two-sided incomplete information

- Simple binary setting
  - Each firm's initial bandwidth can be high or low
    $$B_I^H - B_I^L = B_E^H - B_E^L = \delta \text{ and } B_I^H - B_E^H = B_I^L - B_E^L = \gamma$$
  - Inelastic demand
  - Notation: $\gamma_i^{hk} \equiv B_i^h - B_j^k$ for $i \neq j \in \{I, E\}$ and $h, k \in \{H, L\}$
  - Similarly for t_i^{hk}, π_i^{hk} and $\tilde{\pi}_i^{hk}$ (profit from misreporting h)
- Ex post incentive implementation: for $i = I, E$ and $k = H, L$
  - (IR_i^{hk}) $\pi_i^{hk} - t_i^{hk} \geq 0$
  - (IC_i^{hk}) $\pi_i^{hk} - t_i^{hk} \geq \tilde{\pi}_i^{hk} - t_i^{\tilde{h}k}$ for $\tilde{h} \neq h$

Two-sided incomplete information

- Preliminaries
  - Optimal DICM satisfies, for $i = I, E$ and $k = H, L$
    - (IC_i^{Lk}) and (IR_i^{Lk}) are both binding
    - $\pi_i^{Lk} - \tilde{\pi}_i^{Hk} \geq \tilde{\pi}_i^{Lk} - \pi_i^{Hk}$

    Conversely, any DICM satisfying the above is IC

  - Still optimal to allocate all additional bandwidth
  - There exists $\hat{\lambda} > 0$ such that, for any $\lambda \leq \hat{\lambda}$, moving towards cost equalization always enhances expected welfare

Bunching

- Assumption: H and L are so different that a firm of type L necessarily wins against a rival of type H

$$\delta > \gamma + \Delta$$

- **Proposition** Under this Assumption, full bunching:

$$\left(b_I^{hk*}, b_E^{kh*}\right) = \left(\frac{\Delta-\gamma}{2}, \frac{\Delta+\gamma}{2}\right) \text{ for any } h,k \in \{H,L\}$$

- Implication: no need to elicit firms' information; optimal allocations based on regulator's prior belief