Conflict in Unified Growth Theory

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Abstract

History is plagued with wars, domination, and conflict over resources unaccounted for by mainstream growth theory. We build a model of long-run growth in which two countries compete for resources in fixed supply. Countries allocate their income between consumption, fertility, education, and military spending. Strategic complementarities between competing countries affect the child quantity/quality trade-off and the demographic transition, and therefore the exit from the Malthusian trap. Small differences in initial productivity lead to growing inequality in access to resources which accelerates the transition out of the Malthusian regime of the advantaged country at the expense of the other. The differential take-off to a modern growth regime accentuates imbalances in military power and amplifies differences in GDP per capita. The disadvantaged country reacts by strategically increasing fertility. Although it comes at the expense of education, population growth reduces the gap in military capabilities. This eventually allows the disadvantaged country to take off in turn, but differences in GDP per capita and population size persist. We discuss how the model captures the historical dynamics of divergence and reconvergence between the North and the Global South. However, we characterize conditions under which a region converges to a different balanced growth path in which one country becomes a local hegemon and the other gradually disappears.

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1 Introduction

History is plagued with wars, episodes of domination, and imbalances in the control of resources unaccounted for by mainstream growth theory. This paper precisely considers a world where production and long-run development are contingent to the outcome of conflict, a non-cooperative game in which countries fight for the control of resources in fixed supply. We introduce competition for the control of land, a fixed factor of production, in a two-country long-run growth model, and show how the rich and non-linear dynamics it generates may shed light on human history over the *longue durée*. We hereby propose a formal, quantitative theory of conflict and how it interacts with classical mechanisms such as demography and production in subtle ways, that we see as a powerful grid of analysis for long-run economic growth.

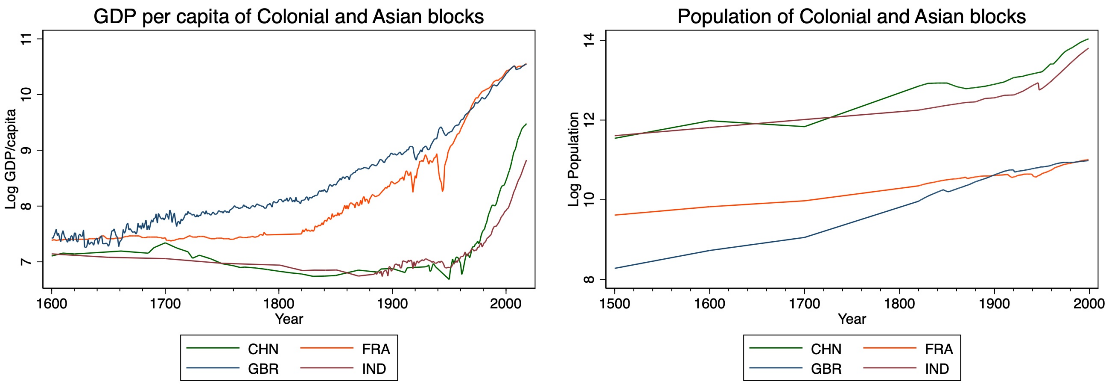
The defining feature of long-run economic development probably is the stagnation of income per capita for several thousands of years, before the fairly recent take-off to a modern regime of economic growth in the 19th century. Although population had always caught up with and offset the benefits of technological innovation in a Malthusian fashion, a demographic transition that occurred first in the Old World eventually allowed a sustained rise in the standards of living.[1] A unified theory of growth emerged from efforts to account for transition from stagnation to growth, pioneered by Galor and Weil (2000) and followed by a number of theorists putting emphasis on a variety of factors.[2] In Unified Growth Theory (Galor, 2011), land is a fixed factor of production over which control is implicit. Competing societies have always fought for resources, from the Neolithic Revolution to this day. Empires have expanded and collapsed, territories have been conquered and defended, resources have been seized and recaptured. In other words, development over the long-run was never a smooth process.

Conflict over fixed resources makes the process of growth and development a potential zero-sum game, with clear winners and losers. An illustration can be found in Figure 1, in which we plot the evolution of GDP per capita and the dynamics of population for two groups of countries: the main colonial powers of Western Europe (Great Britain and France) and the two corresponding Asian powers (China and India). Not only did the European take-off start earlier, resulting in what

[1]That the world was Malthusian before the Industrial Revolution is generally accepted by economic historians and growth theorists alike. The adjective refers to the famous book of Thomas Malthus, *An Essay on the Principle of Population* published in 1798, in which he argued that improvements in standards of living were only temporary as they translated into population growth, reducing future income per capita.

[2]Notable examples include Jones (2001); Hansen and Prescott (2002); Doepke (2004); Weisdorf (2004); Cervellati and Sunde (2005); Boucekkine et al. (2007)

Figure 1: **GDP per capita and population, selected countries, log**



Sources: Maddison Project database, 2020 (Bolt and Van Zanden, 2020; Broadberry et al., 2010, 2015, 2018; Wu et al., 2014; Xu et al., 2017; Ridolfi, 2017) and authors' calculation.

Pomeranz (2000) labeled the Great Divergence, it also seems it occurred *at the expense* of Asia in this case, and the Global South more generally.

To address the matter at hand, we adopt and adapt the canonical unified growth model (Galor and Weil, 2000; Galor, 2011) in which land is a fixed factor of production. To this model, we add two main ingredients. First, we consider two countries and thus allow for conflict. Conflict between the two countries takes place for the control of land, an essential input in production in limited supply. The outcome is determined through a Tullock contest success function (Tullock, 1980) that allocates this fixed resource according to the relative *military capabilities* of countries.[3] Second, those military capabilities are endogenous and are determined by utility maximization in each country. We assume that a country's military capabilities depend on two inputs: military spending and population size. A representative agent in each country therefore uses their income to consume, raise a number children and invest in their education as in a standard unified growth model, but also allocates resources to military investments.[4] Importantly, because the size of the

[3]The term military capabilities comes from Dal Bó et al. (2022), in which they argue that investments in *defense capabilities* where key to the emergence of the first civilizations.

[4]There are therefore two representative agents that one can also interpret as either social planners or sovereigns in both countries. This representative agent assumption allows us to focus on the non-cooperative game *between* rather than *within* countries. The assumption of a representative agent is however only a shortcut leading to a conveniently simple allocation of resources, in particular between military spending and demography. The latter decision can however be decentralized to individual agents by a ruler, and guided by the amount of transfers to individual agents it provides, an alternative modelling assumption explored in a companion paper and discussed later in this text.

population matters for the appropriation of resources, fertility decisions, which are at the heart of the transition from stagnation to growth in unified growth theory, now have an inherent strategic component that plays a crucial role in long-run dynamics.

Countries therefore build military capabilities through both military investments and fertility decisions, in order to secure a fraction of the fixed supply of land that can be used for economic production. Each country takes the actions of the its opponent as given and essentially behaves *à la* Cournot. Although our model does not have closed-form solutions, we can characterize best response functions and prove the existence of a Nash equilibrium. We show how the introduction of conflict over fixed resources yields strategic complementarities between countries that divert resources away from consumption and channel them towards military spending and greater fertility. We highlight in particular how strategic considerations in fertility decisions bring two contradictory forces to the dynamics of unified growth models. On the one hand, by increasing the opportunity cost of education, they bias the child quantity/quality trade-off towards quantity. On the other hand, the resulting faster population growth accelerates the pace of development through a positive externality of population size on the rate of technological progress.

We then study how conflict amplifies small, exogenous differences in technology between countries. We solve the model numerically, and show how a simple asymmetric technology shock generates rich, non-linear long-run dynamics that replicate features of comparative development. A small technological advantage in the Malthusian era allows a country to gradually capture a greater fraction of resources. The resulting gains in income per capita translate into increases in fertility, but the greater military capabilities brought by a larger population partly offset the Malthusian check on income per capita. Faster population growth also accelerates the pace of technological progress and allows the dominating country to take-off earlier. The great divergence in GDP per capita that ensues aggravates the imbalances in military capabilities between the two countries and the inequality in the fraction of total resources controlled. However, strategic complementarities make the dominated country drastically increase fertility in response to the demographic transition of the other. Although this is done at the expense of education and delays the transition from child quantity to quality, the surge in population progressively restores equality in military capabilities. Technological progress through the scale effect of population size in the dominated country eventually allows it to take-off out of Malthusian stagnation in turn.

The introduction of conflict in unified growth theory therefore amplifies small, initial differences in GDP per capita. We quantify the magnitude of the divergence between countries, and compare with a counterfactual with the same technology shock but in which we shut down the possibility of conflict and appropriation of resources. More importantly however, strategic complementarities may generate the endogenous switch from divergence to reconvergence that can be seen in Figure 1. Through the gradual recapture of lost resources, the dominated country eventually catches up, even if only partially as differences in GDP per capita and population size persist in the long-run. We argue that the appropriation of resources could be seen as an important component of the rise of Western Europe and the ensuing Great Divergence, and emphasize how this non-cooperative game for the control of resources also helps explain the relatively recent period of reconvergence by part of the Global South.

However, this reconvergence process is not guaranteed. We run a large number of simulations, using our baseline calibration but varyting both the size and the timing of the asymmetric technology shock, and show that if the shock is either too big or occurs too early in development, the world economy embarks on a different balanced growth path. The cumulative advantage in the military competition is such that the advantaged country appropriates the totality of fixed resources in the long-run. The dominated country, deprived of too many resources too quickly cannot bear the cost of raising fertility and military spending to reverse the balance of power when its opponent takes-off. It eventually cannot sustain a reproduction rate above one and its population begins to gradually decline until it disappears. We show how this multiplicity of long-run growth paths also arises with variations on the relative returns to scale of military expenditure and population size in military capabilities, as well as the returns to land in aggregate production.

The rest of the paper is organized as follows: Section 2 provides a broad overview on various scientific literatures on which we build and attempt to contribute. Section 3 sets up the model, describes both countries' optimal decisions, and characterize the Nash equilibrium. In Section 4, we calibrate the model to solve it numerically, and discuss the effect of conflict on long-run dynamics. Section 5 concludes.

2 Literature review

Our paper is related to various strands of literature to which we aim to contribute in various manners.

Conflict and population dynamics. The canonical model that we use in this paper has been established by Galor (2005, 2011) and at the heart of the transition from stagnation to growth lie fertility decisions. Of particular importance is the quantity-quality trade-off emphasized by Gary Becker. When the pace of technological progress provides enough incentives to acquire human capital, parents switch from quantity to quality by investing in their children's education, triggering the demographic transition and the escape from the Malthusian trap. Population dynamics therefore play a crucial role in the transition to sustained economic growth, and we emphasize how the introduction of conflict brings strategic considerations to fertility decisions. Strategic fertility decisions are the crux of de la Croix and Dottori (2008), in which the ensuing population race provoked the exhaustion of natural resources explains the collapse of the Easter Island. Strategic fertility and the effect of conflict on the child quantity-quality trade-off is also considered by Bezin et al. (2018) who argue that with imperfect property rights, increasing fertility to build bargaining power is done at the expense of education. In our model, agents also take strategic complementarities into account when making fertility decisions, as population size increases military capabilities. The introduction of conflict in a unified growth model therefore biases the child quantity-quality trade-off in favor of quantity and thus influences the timing of the escape of Malthusian stagnation. In a related fashion, Galor and Mountford (2008) as well as O'Rourke et al. (2019) consider how comparative economic development shapes different terms for the child quantity/quality trade-off and show how international trade induced some regions to channel resources to either education or population growth. Finally, Acemoglu et al. (2020) build a simple Malthusian model in which population growth may intensify conflict by creating pressure over scarce resources. On the other hand, Levine and Modica (2013) emphasize the role of *free resources*, the surplus available once subsistence needs have been met, and argue that population attains a level that is lower than the Malthusian subsistence level to allow resources to be devoted to conflict. In our model, the Malthusian dynamics of population matters for conflict because population size plays a key role in military capabilities and as such, territorial expansion counteracts some of the negative effect of population growth on income per capita.

Conflict and long-run growth. To our knowledge, attempts to introduce the possibility of conflict in unified growth theory have been scarce. A notable exception is Lagerlöf (2010), who studies how the exogenous transition from Malthusian stagnation to Solovian prosperity led to a decline in warfare between countries. Our paper, by contrast, investigates how conflict influences the endogenous transition to the modern growth regime, but also how the take-off of a country may impede the development of others. However, the contribution of conflict and warfare on a variety of prerequisites to long-run economic growth has been the subject of a vast literature at the intersection of history, economics, and political science. Lagerlöf (2014) again investigates how competition over fixed resources within a region can induce investments in new technologies and argues that political fragmentation in Europe may explain why it was ahead of China at the start of the Industrial Revolution. Dincecco and Onorato (2016) link military conflict in Europe to the rise of urbanization which indirectly laid the foundations for economic growth, and Aghion et al. (2019) find that military threats spur investments in education. Tilly (1990) emphasized the role of war in the making of modern states. Gennaioli and Voth (2015) build on his idea to argue that the need to finance war was a key driver in the emergence of powerful and centralized states. Besley and Persson (2010) also study the effects of conflict on state capacity and in turn on property rights and prosperity through well-functioning markets. Dal Bó et al. (2022) go further back in time and argue that the need for defense capabilities was crucial in the making of the two first civilizations, Sumer and Egypt. We nevertheless abstract from such political economy considerations about the role of state making by focusing on the optimal decisions of representative agents in different countries, who could be interpreted as already established states and/or rulers. Finally and from a different perspective, Voigtländer and Voth (2013) paint a bleak picture of pre-industrial Europe in which the prevalence of war and poor sanitary conditions in urban centers reinforced the disastrous population shock that was the Black Death, pushed up death rates, and eventually led to a transition to a high wage equilibrium.

The economics of conflict. Conflict itself has been extensively studied theoretically by economists and political scientists alike. Seminal contributions include Hirshleifer (1995a,b) who thought of conflict as an economic activity and analyzed the corresponding individual incentives and their effects on social order, and Grossman and Kim (1995) who consider the trade-off between production and predation in a general equilibrium model and ponder on the implications in terms of property rights. This idea of a choice between *guns and butter* has since been a staple in the economics of conflict and is extensively reviewed by Garfinkel and Skaperdas (2007). Levine and Modica (2021) take an evolutionary approach to conflict between different societies to study how it dynamically shaped social institutions over the long-run, and focus in particular on the emergence of potential hegemons. Dziubiński et al. (2021) also investigate the possibility of hegemony in a network of heterogeneous kingdoms fighting for resources. Conflict between competing societies has also been introduced in a variety of contexts and it is impossible to set up an exhaustive list of important contributions.

It is worth mentioning however Alesina and Spolaore (2005, 2006), who model the endogenous determination of both the number and the size of countries through conflict and war, and Findlay et al. (2017), who develop an extensive theory of frontier formation based on the need for territorial expansion. Our contribution here is to embed the strategic considerations behind resource appropriation in a long-run, endogenous growth model to study how conflict influences the transition to sustained prosperity and shapes comparative long-run development.

Comparative long-run development. Finally, although we let theory guide our analysis, this paper is ultimately about comparative economic development. It touches notably on the fundamental debate on the rise of the West. Why the take-off to sustained economic prosperity occurred in Western Europe and not elsewhere is one of the fundamental questions of the social sciences to which economic historians have long attempted to provide an answer. The influence of deep-rooted factors has recently gained traction and is surveyed in Spolaore and Wacziarg (2013). The role of institutions that secure property rights, foster individual economic incentives, and constrain predation have long been proposed as a potential explanation, notably by North and Thomas (1973). The nature of institutions is also at the heart of Acemoglu et al. (2005), Robinson and Acemoglu (2012) and Acemoglu and Robinson (2022) in which a subtle balance of power between the state and society, itself a result of structural and historical forces, is required for sustained prosperity. Institutions are also key in igniting scientific research and emulation, as emphasized by Mokyr (1992) in particular. Allen (2009) also puts technology at the forefront, but investigates the combination of factors that made the Industrial Revolution take place in Britain. Among those factors is the British Empire, and the benefited it provided the Island through the appropriation of resources. Related to this paper, empires play a central part in Findlay and O'rourke (2009) who tell a history of globalization driven by war, and in particular conflict for access not only to resources, but also to precious markets and trade routes. As put by Federico (2021), *"very few authors doubt that in the long run, the evolution of markets was mostly shaped by the visible hand of the state"*, sometimes with military force in the hands of private interests.[5] There is ample evidence that access to international resources required investments in military capabilities as much as into research and development and pure international exchange over goods and energy.

[5]This expression seems to originate from the 1977 award-winning book by Alfred Chandler, "The Visible Hand: The Managerial Revolution in American Business", although it referred to the organizational structure within businesses.

3 The model

We build a unified growth model in which two countries evolve in the same region and compete for a resource in fixed supply, interpreted as land, denoted by X. Each country is an overlapping generations economy in which a single good is produced using land, labor, and human capital. Those inputs are determined by optimal decisions with respect to both the quantity and quality of children, and investments in military capabilities.

3.1 Production

The production side of the economy is essentially the same as in Galor and Weil (2000) and Galor (2011), except countries can only use for production the fraction of land they control, which we denote by δ_t. Aggregate output is a function of labor L_t, of efficiency units of labor h_t, the level of technology A_t and the fraction δ_t of the fixed supply of land in the region. It is defined by the following production function:

$$Y_t = (h_t L_t)^{\alpha} (A_t \delta_t X)^{\chi} \tag{1}$$

which gives, in per capita terms:

$$y_t = h_t^{\alpha} L_t^{\alpha-1} (A_t \delta_t X)^{\chi} \tag{2}$$

Note that we do not restrict ourselves to constant returns to scale in production as we do not impose *a priori* $\alpha + \chi = 1$.

3.2 Military capabilities and conflict over land

Land is allocated between the two countries according to a Tullock contest success function (Tullock, 1980), the outcome of which is interpreted not as a probability of victory and thus of control over the whole land, but as the share of that land that can be used for production by each country. The inputs of this Tullock function are both countries' military capabilities $\mathcal{M}$. Note that throughout the paper, we take the perspective of one of the two countries, and denote the variable of the other with an asterisk. The fraction of the land controlled by each country in period $t+1$ is denoted by δ_{t+1} and $\delta^{\star}_{t+1}$ respectively, and is given by:

$$\delta_{t+1} = \frac{\psi \mathcal{M}_{t+1}^{\pi}}{\psi \mathcal{M}_{t+1}^{\pi} + \psi^{\star} \mathcal{M}_{t+1}^{\star\pi}} \tag{3}$$

$$\delta^{\star}_{t+1} = \frac{\psi^{\star} \mathcal{M}_{t+1}^{\star\pi}}{\psi \mathcal{M}_{t+1}^{\pi} + \psi^{\star} \mathcal{M}_{t+1}^{\star\pi}} \tag{4}$$

where ψ and $\psi^\star$ are parameters governing the efficacy of each country's military capabilities. π is a parameter that we introduce to act as a shifter of the marginal efficiency of military capabilities. Setting $\pi = 0$ will allow us to compare our model with a canonical unified growth model without conflict.

Military capabilities are therefore assumed to be a function of military investments undertaken by each country in the previous period M_{t+1}, as well as the size of the adult population of each country in the current period L_{t+1}, such that:

$$\mathcal{M}_{t+1} = \mathcal{M}(M_{t+1}, L_{t+1}) \tag{5}$$

We assume military capabilities are determined by a Cobb-Douglas function of those two inputs. Denoting per capita military investments by m_{t+1} and the rate of fertility by n_t such that $M_{t+1} = m_{t+1}L_t$ and $L_{t+1} = n_t L_t$, military capabilities in $t+1$ are determined in t and are equal to:

$$\begin{aligned} \mathcal{M}_{t+1} &= M_{t+1}^{\eta_M} L_{t+1}^{\eta_L} \\ &= m_{t+1}^{\eta_M} n_t^{\eta_L} L_t^{\eta_M+\eta_L} \end{aligned} \tag{6}$$

Note that we also do not *a priori* restrict the function to have constant returns to scale. Because military capabilities depend on the size of the population in addition to military expenditure, fertility decisions are going to play a key role in the conflict. This notion of strategic fertility is at the heart of de la Croix and Dottori (2008). We derive the marginal effects of both fertility and military expenditures on the share of land controlled by a country in Appendix A.1.

3.3 Preferences and budget constraint

Each country is populated by a number of identical individuals who live for two periods.[6] In the first period, childhood, agents do not take any decision and consume a fraction of their parents' income, denoted by y_t and defined in the next sub-section. In the second period, adulthood, they inelastically supply one unit of labor, decide how much children they have n_t, and allocate their resources between consumption c_t, investment in military capabilities m_{t+1} which increases the share of land that can be used in production and therefore affects output next period, as well as investment in their children's education e_{t+1}.

[6]Because we focus on strategic complementarities *between* rather than *within* countries, we study the optimization problem of a representative agent in each country. Those representative agents can be interpreted as social planners who ensure the first best optimum is attained through a variety of policy instruments acting on military spending and fertility decisions in particular.

Raising a child has a cost $\mathcal{C}$. We assume that it requires both paying an incompressible fixed cost τ_0 and devoting a fraction $\tau_1 + e_{t+1}$ of total resources to their upbringing, where e_{t+1} is a direct investment in education, such that $\mathcal{C}(e_{t+1}, y_t) = \tau_0 + (\tau_1 + e_{t+1})y_t$. The introduction of a fixed cost of raising each child makes the demand for children increase with income, which yields a Malthusian flavor in the earlier stage of development during which increasing fertility offsets gains in productivity and keeps output per capita from taking-off. There are various ways of generating this Malthusian mechanism, from assuming non-homothetic preferences to the presence a subsistence consumption constraint, but we choose this specification for its flexibility.[7]

Because there are no property rights to land within a country, income is simply equal to the output per capita of the country. The budget constraint is therefore:

$$y_t = c_t + \underbrace{[\tau_0 + (\tau_1 + e_{t+1})y_t]}_{\mathcal{C}(e_{t+1}, y_t)} n_t + m_{t+1} \tag{7}$$

Agents care about their own consumption level c_t, but also about the number of children they have n_t as well as the future income of those children y_{t+1}. Their preferences are given by the following utility function:

$$u_t = (1 - \gamma)\log(c_t) + \gamma \log(n_t y_{t+1}) \tag{8}$$

Note that the utility they derive from both military and education investments comes from their effect on the future output per capita of the next generation.

3.4 Efficiency units of labor

We assume that in a given period, the level of efficiency units of labor h_{t+1}, called hereafter human capital for simplicity, is an increasing function of education investments made in the previous period and denoted by e_{t+1}, and a decreasing function of the growth rate of technological progress g_{t+1}. The underlying argument is that new technologies depreciate the existing stock of human capital by making old techniques and knowledge obsolete. Rapid technological growth therefore requires

[7] Another minor difference with the canonical model, is that contrary to Galor (2011), investment costs in fertility and education are proportional to GDP per capita whereas in Galor (2011), the costs are proportional to potential output, that is output when all workers are employed fully. This has no deep consequence here and the assumption is only for convenience.

greater investments in education to keep a constant level of human capital. Formally:

$$h_{t+1} = h(e_{t+1}, g_{t+1}) \tag{9}$$

In the canonical model [Galor (2011), chapter 5], the function $h(.)$ satisfies the following assumptions:

$$h_e(e_{t+1}, g_{t+1}) > 0 \qquad h_{ee}(e_{t+1}, g_{t+1}) < 0$$
$$h_g(e_{t+1}, g_{t+1}) < 0 \qquad h_{gg}(e_{t+1}, g_{t+1}) > 0$$

The returns to education in human capital production are therefore concave, while the depreciation effect of technological progress is convex. It is also assumed that $h_{eg}(e_{t+1}, g_{t+1}) > 0$ such that the rate of technological progress raises the returns to education, or that technology and education are complementary in the formation of human capital. Finally, the level of technology A_t grows at the rate $g_{t+1} = \dfrac{A_{t+1} - A_t}{A_t}$. This rate of technological progress between periods t and $t+1$ is endogenous and assumed to be a function of the level of education as well as the size of the adult population in t, e_t and L_t. Again, we do not depart from the canonical unified growth model and assume the following conditions:

$$g_e(e_t, L_t) > 0 \qquad g_L(e_t, L_t) > 0 \qquad g(0, L_t) > 0$$

The rate of technological progress is therefore increasing with the level of education of the population, but also through its size via a scale effect. The third condition says that there can be potentially slow technological progress in the absence of education, precisely driven by the growth of population that eventually matters for the emergence and the diffusion of new ideas. To prevent the model from exploding, we assume that this scale effect of population shuts off when its size reaches some threshold.

3.5 Maximization problem

A representative agent in each country solves the following maximization problem:

$$\max_{c_t, n_t, m_{t+1}, e_{t+1}} u_t = (1-\gamma)\log(c_t) + \gamma \log(n_t y_{t+1}) \tag{10}$$
$$\text{s.t.} \quad y_t = c_t + \mathcal{C}(e_{t+1}, y_t) n_t + m_{t+1} \tag{11}$$

The Lagrangian writes as follows:

$$\begin{aligned} \mathcal{L} &= u_t(c_t, n_t, e_{t+1}, m_{t+1}) \\ &+ \lambda_t(y_t - c_t - \mathcal{C}(e_{t+1}, y_t) n_t - m_{t+1}) \\ &+ \mu_t^e e_{t+1} \end{aligned} \tag{12}$$

Let us denote $\mathcal{E}^n_{t+1} = \dfrac{n_t}{y_{t+1}}\dfrac{\partial y_{t+1}}{\partial n_t}$, $\mathcal{E}^m_{t+1} = \dfrac{m_{t+1}}{y_{t+1}}\dfrac{\partial y_{t+1}}{\partial m_{t+1}}$, and $\mathcal{E}^e_{t+1} = \dfrac{e_{t+1}}{y_{t+1}}\dfrac{\partial y_{t+1}}{\partial e_{t+1}}$[8] the elasticities of future per capita output with respect to fertility, military spending, and education respectively, and write the first order conditions of one of the two countries:

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \quad \Leftrightarrow \quad \frac{1-\gamma}{c_t} = \lambda_t \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \quad \Leftrightarrow \quad \frac{\gamma}{n_t}\left(1 + \mathcal{E}^n_{t+1}\right) = \mathcal{C}(e_{t+1}, y_t)\lambda_t \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial m_{t+1}} = 0 \quad \Leftrightarrow \quad \frac{\gamma}{m_{t+1}}\mathcal{E}^m_{t+1} = \lambda_t \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial e_{t+1}} = 0 \quad \Leftrightarrow \quad \frac{\gamma}{e_{t+1}}\mathcal{E}^e_{t+1} = \lambda_t n_t y_t \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \quad \Leftrightarrow \quad y_t = c_t + \mathcal{C}(e_{t+1}, y_t)n_t + m_{t+1} \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial \mu^e_t} \geq 0 \quad \Leftrightarrow \quad e_{t+1} \geq 0 \tag{18}$$

with the slackness condition from Karish-Kuhn-Tucker (KKT hereafter): $\mu^e_t \dfrac{\partial \mathcal{L}_t}{\partial \mu^e_t} = 0$. Note that there is a symmetric problem in the other country, with variables indexed by asterisks.

The first condition states that the marginal utility of consumption must be equal, out of the Malthusian regime, to the multiplier of the budget constraint. The second shows that the marginal utility of fertility is the sum of the direct utility derived from the number of children and the effect of fertility on future GDP per capita and must equal the rearing cost per children $\mathcal{C}(e_{t+1}, y_t)$ per unit of marginal consumption. The third condition says that the marginal utility of investment in military capabilities comes from the change in output next period (thus the change in children's future income) and must equal the opportunity cost of such spending. The fourth and the fifth are the budget constraint and the positivity constraint for education respectively.

3.6 The military investments – fertility trade-off

The incentives of the representative agents in both countries to invest in military capabilities come from their effect on the future output per capita of their children's generation through the fraction of total resources captured. Using Appendix equations (34-39), we can write the elasticity of future

[8] Note that this elasticity is the product of the elasticity $\mathcal{E}^h_{t+1} = \dfrac{h_{t+1}}{y_{t+1}}\dfrac{\partial y_{t+1}}{\partial h_{t+1}}$ and $\mathcal{E}^{h,e} = \dfrac{e_{t+1}}{h_{t+1}}\dfrac{\partial h_{t+1}}{\partial e_{t+1}}$ that is, $\mathcal{E}^e_{t+1} = \mathcal{E}^{h,e} \times \mathcal{E}^h_{t+1}$

output per capita with respect to military spending:

$$
\begin{aligned}
\mathcal{E}^m_{t+1} &= \frac{m_{t+1}}{y_{t+1}}\frac{\partial y_{t+1}}{\partial m_{t+1}} \\
&= \chi\frac{m_{t+1}}{\delta_{t+1}}\frac{\partial \delta_{t+1}}{\partial m_{t+1}} \\
&= \chi\pi\eta_M\delta^\star_{t+1} > 0 \qquad (19)
\end{aligned}
$$

Fertility decisions are driven by the desire to have children *per se*, but also by their effect on future output per capita. In a standard growth model, although an increase in fertility has a positive effect on production through labor supply, it is counteracted by the resulting increase in population. In the case of diminishing returns to labor that is usually considered, output per capita declines with fertility. In our framework however, because the size of the population improves military capabilities, fertility has another positive effect on output per capita through an increase in the fraction of fixed resources that can be used for production. Using Appendix equations (34-39) again, let us decompose the effect of fertility on output per capita:

$$
\begin{aligned}
\mathcal{E}^n_{t+1} &= \frac{n_t}{y_{t+1}}\frac{\partial y_{t+1}}{\partial n_t} \\
&= \underbrace{\chi\frac{n_t}{\delta_{t+1}}\frac{\partial \delta_{t+1}}{\partial n_t}}_{\text{positive effect of military capabilities}} + \underbrace{\alpha}_{\text{positive effect of labor supply}} - \underbrace{1}_{\text{negative effect of population growth}} \\
&= \chi\pi\eta_L\delta^\star_{t+1} + \alpha - 1 \qquad (20)
\end{aligned}
$$

Even in the presence of diminishing returns to labor $\alpha < 1$, an increase in fertility may therefore increase future output per capita when the returns of population size on the fraction of fixed resources controlled are high enough. The returns to fixed resources in aggregate production χ as well as the prominence of population size in military capabilities η_L play an important role in determining the effect of fertility on output per capita.

We denote by $\mathcal{S}_M(\delta^\star_{t+1}) = \chi\pi\eta_M\delta^\star_{t+1}$ and $\mathcal{S}_L(\delta^\star_{t+1}) = \chi\pi\eta_L\delta^\star_{t+1}$ the returns to military spending and fertility to production through the control of fixed resources respectively. Notice that, because the Tullock contest success function implies diminishing returns to military capabilities, a greater fraction of total resources controlled by the enemy country $\delta^\star$ raises the returns to both military spending and fertility. Combining the first order conditions (14) and (15) we get the equation governing the trade-off between military investment and fertility:

$$
\frac{m_{t+1}}{n_t} = \frac{\mathcal{S}_M(\delta^\star_{t+1})}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})} \cdot \mathcal{C}(e_{t+1}, y_t) \qquad (21)
$$

In our framework, the love for children *per se* exactly offsets the negative effect of population growth on output per capita in individuals' valuation of fertility. The presence of the returns to labor α on the denominator above indicates that fertility therefore increases utility through future labor supply in addition to strategic considerations. Because of decreasing marginal utility, an increase in the fraction of land controlled by the enemy country $\delta^\star$ thus makes military expenditure increase more than fertility.

3.7 The optimal education level

Taking the first order condition (16) and substituting for λ_t using (14), we get:

$$\underbrace{\frac{e_{t+1}}{y_{t+1}}\frac{\partial y_{t+1}}{\partial e_{t+1}}}_{\mathcal{E}^e_{t+1}} = \underbrace{\frac{e_{t+1}y_t}{\mathcal{C}(e_{t+1},y_t)}}_{\text{share of child rearing exp. devoted to educ.}} \cdot \left(1+\underbrace{\frac{n_t}{y_{t+1}}\frac{\partial y_{t+1}}{\partial n_t}}_{\mathcal{E}^n_{t+1}}\right) \tag{22}$$

This equation says that the elasticity of future per capita output with respect to investments in education must be greater than or equal to the share of per capita children expenditure devoted to education, times the marginal benefits of fertility.[9] The complementary slackness condition from KKT then gives us the (EM) – education/military capabilities – curve:

$$\mathcal{E}^e_{t+1} - \frac{e_{t+1}y_t}{\mathcal{C}(e_{t+1},y_t)}\cdot\left(\alpha+\mathcal{S}_L(\delta^\star_{t+1})\right)\begin{cases} = 0 & \text{if} \quad e_{t+1} > 0 \\ < 0 & \text{if} \quad e_{t+1} = 0 \end{cases} \tag{23}$$

Strategic considerations in fertility decisions captured in $\mathcal{S}_L(\delta^\star_{t+1})$, through the effect of population size on military capabilities, will influence the child quantity/quality trade-off and thus the optimal education level. In particular, an increase in the share of resources controlled by the enemy country $\delta^\star_{t+1}$ raises the marginal benefits of fertility and thus the opportunity cost of education, and therefore skews the trade-off towards quantity rather than quality.

The main feature of unified growth theory is the presence of a long period of stagnation in which the economy is stuck in a Malthusian trap and does not invest in human capital. This is ensured by the existence of a corner solution for the optimal education level $e_{t+1} = 0$ in the early stage of development. In Galor and Weil (2000), individuals do not invest in education because the returns

[9] It is frequent to have a link between two elasticities and a factor's share. For instance, the Marshall-Allen laws of demand states that the demand elasticity of labor is a function of the product of the share of labor in total costs and of the demand elasticity of output (Hamermesh, 1996).

to human capital are not enough to offset the costs of acquiring it until the rate of technological progress reaches a threshold that eventually makes education attractive. We follow the same logic here and establish the following Lemma:

Lemma 1.
Investments in education occur only when the rate of technological progress g_{t+1} surpasses some threshold $\hat{g}(\delta^{\star}_{t+1}, y_t) > 0$:

$$e_{t+1} = \begin{cases} 0 & \text{if} \quad g_{t+1} \leq \hat{g}(\delta^{\star}_{t+1}, y_t) \\ e(g_{t+1}, \delta^{\star}_{t+1}, y_t) & \text{if} \quad g_{t+1} > \hat{g}(\delta^{\star}_{t+1}, y_t) \end{cases} \tag{24}$$

The level of education chosen by adults in period t for their children e_{t+1} is an increasing function of the rate of technological progress g_{t+1}:

$$\frac{\partial e_{t+1}}{\partial g_{t+1}} > 0$$

Proof: *see Appendix A.2.*

3.8 Equilibrium

The equilibrium of the model consists of the optimal decision rules of each country evolving in the region (two in our case, but it can easily be generalized to N countries) for c_t, m_{t+1}, n_t and e_{t+1}, the equation characterizing each country's military capabilities, and the Tullock contest success function for the allocation of resources, in addition to production functions and laws of motion of population and technology. We cannot express the equilibrium as a system of closed form solutions, but let us denote by $\Gamma = (1-\gamma)/\gamma$ the relative taste for consumption, and use the first order conditions to get:

$$c_t = \frac{\Gamma}{\Gamma + \alpha + \mathcal{S}_L(\delta^{\star}_{t+1}) + \mathcal{S}_M(\delta^{\star}_{t+1})} \cdot y_t \tag{25}$$

$$m_{t+1} = \frac{\mathcal{S}_M(\delta^{\star}_{t+1})}{\Gamma + \alpha + \mathcal{S}_L(\delta^{\star}_{t+1}) + \mathcal{S}_M(\delta^{\star}_{t+1})} \cdot y_t \tag{26}$$

$$n_t = \frac{\alpha + \mathcal{S}_L(\delta^{\star}_{t+1})}{\Gamma + \alpha + \mathcal{S}_L(\delta^{\star}_{t+1}) + \mathcal{S}_M(\delta^{\star}_{t+1})} \cdot \frac{y_t}{\mathcal{C}(e_{t+1}, y_t)} \tag{27}$$

$$e_{t+1} = \begin{cases} 0 & \text{if} \quad g_{t+1} \leq \hat{g}(\delta^{\star}_{t+1}, y_t) \\ e(g_{t+1}, \delta^{\star}_{t+1}, y_t) & \text{if} \quad g_{t+1} > \hat{g}(\delta^{\star}_{t+1}, y_t) \end{cases} \tag{28}$$

$$\mathcal{M}_{t+1} = m_{t+1}^{\eta_M} n_t^{\eta_L} L_t^{\eta_M + \eta_L} \tag{29}$$

$$\delta^{\star}_{t+1} = \frac{\psi^{\star} \mathcal{M}^{\star}_{t+1}{}^{\pi}}{\psi \mathcal{M}^{\pi}_{t+1} + \psi^{\star} \mathcal{M}^{\star\pi}_{t+1}} \tag{30}$$

Note that a corresponding system of equations hold for the other country and characterizes $c_t^\star$, $m_{t+1}^\star$, $n_t^\star$ and $e_{t+1}^\star$, as well as $\mathcal{M}_{t+1}^\star$ and δ_{t+1}. Representative agents in both countries behave *à la* Cournot, that is, they take enemy military capabilities ($\mathcal{M}_{t+1}^\star$ in this case) as given. Despite the lack of a closed form solution to the equilibrium, we can characterize the best response functions of agents in each country and study how consumption, military spending, fertility, and education vary with enemy military capabilities. We do so in the following proposition:

Proposition 1.
In the Malthusian regime (if $g_{t+1} \leq \hat{g}_{t+1}$):

$$\frac{\partial m_{t+1}}{\partial \mathcal{M}_{t+1}^\star} > 0 \quad ; \quad \frac{\partial n_t}{\partial \mathcal{M}_{t+1}^\star} > 0 \quad \text{iff} \quad \eta_M/\eta_L < \Gamma/\alpha \quad ; \quad \frac{\partial c_t}{\partial \mathcal{M}_{t+1}^\star} < 0$$

In the modern growth regime (if $g_{t+1} > \hat{g}_{t+1}$):

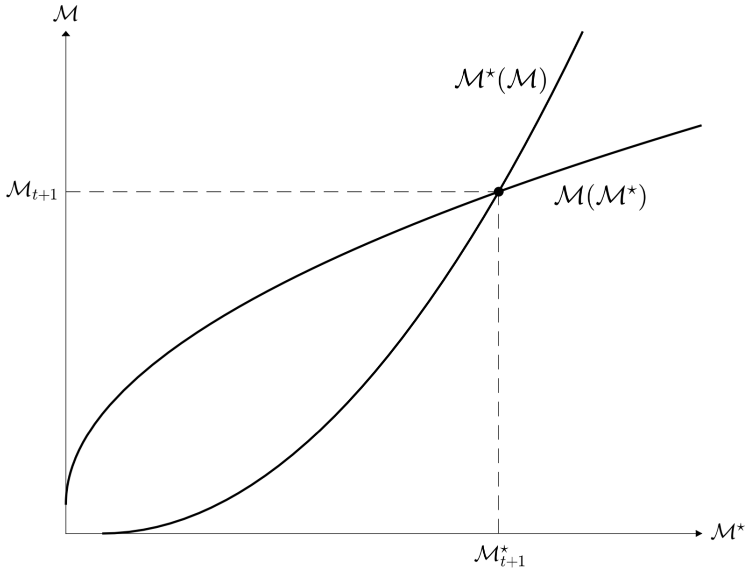
$$\frac{\partial m_{t+1}}{\partial \mathcal{M}_{t+1}^\star} > 0 \quad ; \quad \frac{\partial n_t}{\partial \mathcal{M}_{t+1}^\star} > 0 \quad ; \quad \frac{\partial c_t}{\partial \mathcal{M}_{t+1}^\star} < 0 \quad ; \quad \frac{\partial e_{t+1}}{\partial \mathcal{M}_{t+1}^\star} < 0$$

Proof: *see Appendix A.3.*

Unsurprisingly, military spending unambiguously increase with enemy military capabilities. The rate of fertility in the Malthusian regime only increases if the relative importance of population size in military capabilities is large enough ($\eta_M/\eta_L < \Gamma/\alpha$). When the returns to military spending are much larger than the returns to population size, a country responds to an increase in enemy military capabilities by lowering fertility to devote more resources to military investments. In the modern growth regime, fertility increases with enemy military capabilities, irrespective of those relative returns as the representative agent reduces investments in education to free up more resources to the building of military capabilities. The intensity of conflict therefore biases the child quantity/quality trade-off towards quantity. Furthermore, we can also show that $\partial \hat{g}_{t+1}/\partial \mathcal{M}_{t+1}^\star > 0$ such that an increase in enemy military capabilities, all else equal, makes the transition from the Malthusian to the modern growth regimes less likely.

To illustrate the strategic complementarities brought about by the introduction of conflict over fixed resources, it is useful to build the best response functions of each country in terms of military capabilities, $\mathcal{M}(\mathcal{M}^\star)$ and $\mathcal{M}^\star(\mathcal{M})$. Those complementaries come from the fact that military capabilities in a country are increasing in enemy military capabilities, as can be seen in Figure 2 describing the case when $g_{t+1} \leq \hat{g}_{t+1}$.

Figure 2: **Best response functions in the Malthusian regime**



The introduction of conflict makes each country behave according to the actions of the other, competing countries. Strategic considerations force them to devote resources to investments in military capabilities and increase the number of children they have, at the expense of both consumption and investments in education. As a result, the more powerful the competing countries are (that is, the greater is $\mathcal{M}^\star$), the more countries have to divert resources away from other use and channel them into their military capabilities. The following corollary stems from Proposition 1:

Corollary 1.
In the Malthusian regime, there exists a unique stable Nash equilibrium.
Proof: *see Appendix A.4.*

Strategic complementarities remain out of the Malthusian regime, as the military capabilities best response functions still have positive slopes. Proving the uniqueness of the Nash equilibrium however is less straightforward, but we establish in Appendix A.4 a condition that ensures that the two curves intersect only once.

4 Long-run dynamics

Because the dynamics of our two-country set-up are too complicated to be characterized analytically, in this section we solve the model numerically. We first describe our calibration strategy as well as functional forms we choose for functions that were only implicitly defined until now. We next describe long-run dynamics when the two countries are strictly identical and the equilibrium is therefore symmetric, and discuss how introducing conflict over fixed resources influences an otherwise standard unified growth model. We then introduce some asymmetry between the two countries and show how small differences in technology can generate rich, non-linear dynamics that we believe can shed light on long-run economic development, and we quantify the role of conflict in differences in GDP per capita and population size. Finally, we emphasize the existence of multiple balanced growth paths the model may converge to, depending on the magnitude and the timing of exogenous technology shocks.

4.1 Calibration strategy

Parameter values. The value of the labor share, α is set equal to 0.6. We begin with the case of constant returns to scale in the production function, the returns to land χ are therefore 0.4. The returns to labor α is our only free parameter. Our strategy to calibrate the model is then to consider a symmetric equilibrium in which the two countries are identical and thus follow the same development path, controlling the same constant share of total resources $\delta = \delta^\star = 1/2$, and set the remaining parameters to match some targets on education and fertility in the long-run.

We start from a benchmark case with constant returns to scale in military capabilities: $\eta_M + \eta_L = 1$. We calibrate η_L such that the share of military spending is equal to 4% of output in the long-run, and η_M is set accordingly. We impose the rate of fertility to be equal to one in the long-run such that population converges to a constant level for simplicity. We set γ such that the total child-rearing expenditure, including investments in education, is equal to 20% of output in the long-run. We target those education expenditure to be 7.5% of output. In the next subsection, we define functional forms for the human capital production function as well as and the growth rate of technology, and calibrate their parameters to match their respective targets. The long-run growth rate of technology is assumed to converge to $g = g^\star = 2$. Finally, we set $\tau_0 = 0.1$ and calibrate τ_1 to ensure $n = 1$ is compatible with the education level.

Table 1: Benchmark parameter values

Name	Symbol	Value(s)	Source/Target
Benchmark case			
Utility and individual parameters			
Relative taste for children	γ	0.2830	$\mathcal{C}/y \to 20\%$
Fixed costs of fertility	τ_0, τ_1	$0.10, 0.125$	$n \to 1$
Macroeconomic parameters			
Returns to scale in production	α, χ	0.6, 0.4	
Human capital production	ρ	0.7295	$e \to 0.075$
Growth rate of technology	$\tilde{a}$, θ	12.0344, 1	$g \to 2$,
Military capabilities parameters			
Exponent of total military capabilities	π	1	
Returns to military expenditure	η_1	2/3	$m/y \to 4\%$
Returns to population size	η_2	1/3	$\eta_2 = 1 - \eta_1$

Initialization targets. We need to choose a set of initial conditions for the share of land controlled by each country δ_0 and $\delta_0^\star$, population sizes L_0 and $L_0^\star$, and levels of technology A_0 and $A_0^\star$. We consider that the two countries are identical initially, such that $\delta_0 = \delta_0^\star$, $L_0 = L_0^\star$ and $A_0 = A_0^\star$. We choose an arbitrary level of population size, the only constraint is that it is small enough to ensure the existence of a Malthusian regime for a decent number of periods. We then set the initial level of technology such that fertility in both countries is equal to one in the first period.

Functional forms. Finding an adequate functional form for the human capital production function that satisfies those conditions, gives us a corner solution such that $e_{t+1} = 0$ in the initial stage of development, and actually makes economic sense is not an easy task. Lagerlöf (2006) proposes a parametric functional form where the fraction of parental resources devoted to child rearing acts as input in the production of human capital, but the part that is not directly related to investments in education are less effective. We adopt and adapt this idea here. Recall that a child's upbringing requires a fraction $\tau_1 + e_{t+1}$ of parental resources in addition to the fixed physiological cost τ_0. We therefore have, with $\rho < 1$:

$$h_{t+1} = \frac{e_{t+1} + \rho(\tau_0/y_t + \tau_1)}{e_{t+1} + \rho(\tau_0/y_t + \tau_1) + g_{t+1}} \tag{31}$$

Note that because the fixed cost τ_0 makes the total fraction of resources devoted to children decrease with parental income y_t, effectively acting as a subsistence constraint, income per capita will have a small negative effect on the input in the human capital production function.

Education depends on technological progress, but technological progress in turn depends on education. In particular, the growth rate of technology from t to $t+1$ denoted by g_{t+1} is an increasing function of the education of adult individuals in t (such that agents do not internalize the effect of their children's education on technological progress when trading off between quantity and quality), as well as the population size in t. This scale effect of population vanishes at some point to prevent the model from exploding. Again, Lagerlöf (2006) proposes a functional form in which the rate of technological progress depends on the input of the human capital production function that we slightly modify:

$$g_{t+1} = (e_t + \rho(\tau_0/y_{t-1} + \tau_1))a(L_t) \tag{32}$$

where $a(L_t)$ characterizes the scale effect:

$$a(L_t) = \min\{\theta L_t, \tilde{a}\} \tag{33}$$

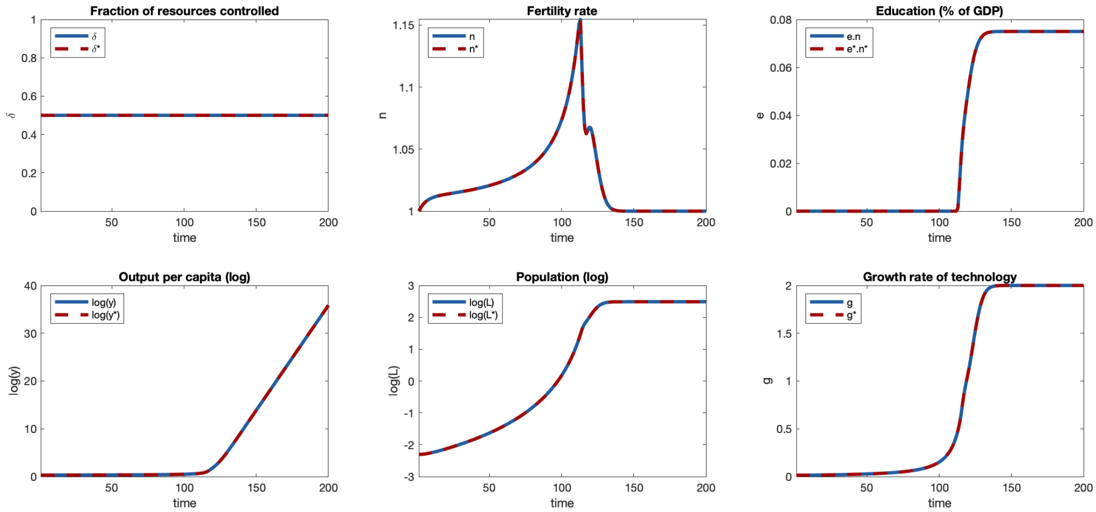
We show in Appendix A.5 that those functional forms indeed yield the same results on the optimal education decision described in the general case characterized in Lemma 1.

4.2 Long-run dynamics in the symmetric equilibrium

We start by considering two exactly identical countries, which naturally leads to a symmetric equilibrium in which $\delta_{t+1} = \delta^\star_{t+1} = 1/2$ at all times. As shown in Figure 3, in this baseline scenario, the long-run path of development of both countries, in addition to being identical, resembles that of a standard unified growth model. There is a long initial phase of stagnation because of a Malthusian check on income per capita through increases in fertility.[10] Therefore, in the early stage of economic development, when the rate of technological progress is too low to make investing in education attractive, any incremental increase in technology is offset by a rise in fertility which makes income per capita stagnate or increase at a very slow pace. The driving force behind the transition from Malthusian stagnation to a regime of sustained economic growth in the canonical model is the positive externality of population size on technological progress. Even in the absence of education, this scale effect of population allows the growth rate of technology to gradually increase in the background. Although this does not translate into sizable improvements in standards of living precisely

[10] Galor and Weil (2000) model this Malthusian mechanism with a subsistence constraint on consumption, which makes fertility rise with income as long as the constraint is binding. Because we abstract from such a constraint, this positive relationship between income and population growth comes from the shape of the cost function for child-rearing. More precisely, all else equal, fertility increases with income if and only if $\mathcal{C}(y,e) - y \cdot \partial\mathcal{C}(y,e)/\partial y > 0$ which holds with the introduction of our fixed rearing cost per child.

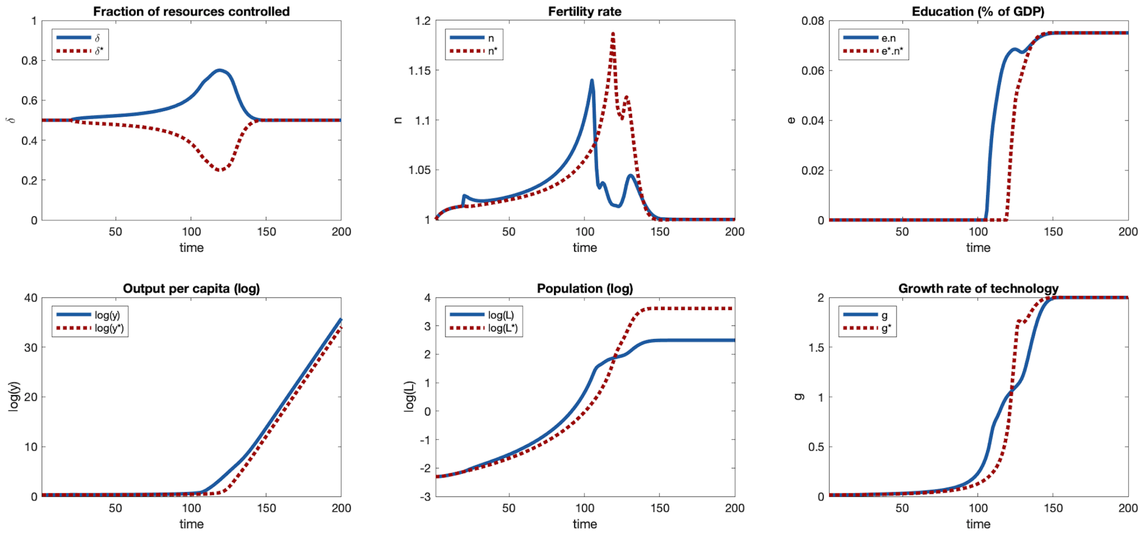
Figure 3: **Long-run dynamics in the symmetric equilibrium**



because of this Malthusian check, it eventually raises the returns to human capital enough to make parents switch from child quantity to child quality and start investing in education. The resulting demographic transition allows an economy to escape the Malthusian trap, and output per capita can finally take-off.

The effect of conflict on long-run dynamics. The introduction of conflict brings two contradicting forces to the dynamics of long-run development in unified growth theory. On the one hand, a higher rate of fertility accelerates the pace of development through the positive externality of population size on technological progress. This does not necessarily mean an earlier escape from Malthusian stagnation however, because conflict reduces incentives to invest in education. Strategic considerations in fertility decisions indeed bias the child quantity/quality trade-off towards quantity, which could potentially hinder the demographic transition. Which of the two effects dominates the other ultimately determines whether conflict over resources delayed or accelerated the take-off to a modern growth regime characterized by sustained investments in education.

Figure 4: **Long-run dynamics with an asymmetric technology shock**



Effect of a 7.5% positive technology shock in the Blue country in $t = 20$.

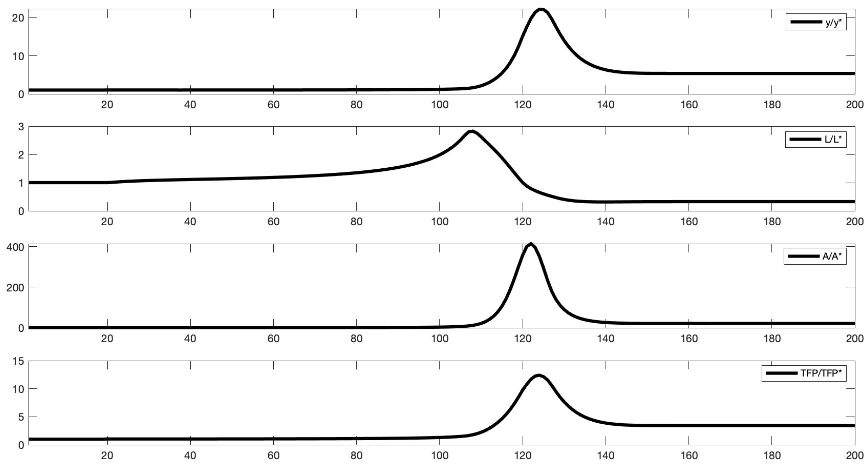
4.3 Long-run dynamics with a small asymmetric technology shock: The Great Divergence and Reconvergence

We now investigate the effect of a small, temporary, positive technology shock experienced by only one of the two countries in the Malthusian regime. More precisely, we take our baseline calibration described above and impose a shock that raises the level of technology A_t by 7.5% at time t=20, but does not affect level of technology of the other country, $A_t^{\star}$. We display the results in Figure 4.

It immediately appears that the effect of a small technology shock early in development are non-linear, substantial, and permanent. In the Malthusian regime, an improvement in the level of technology does not have much effect on standards of living, because fertility increases and this quickly offsets the gains in terms of income per capita. However, even a small advantage in technology allows a country to capture a slightly greater fraction of resources that can be used for production. The resulting gains in income from appropriation increases fertility even further, allowing the population of the advantaged country to grow faster than that of the other country, thus consolidating the advantage in military capabilities.

Importantly, this greater population growth also translates into a faster pace of technological

Figure 5: **Long-run dynamics with an asymmetric technology shock – persistent differences**



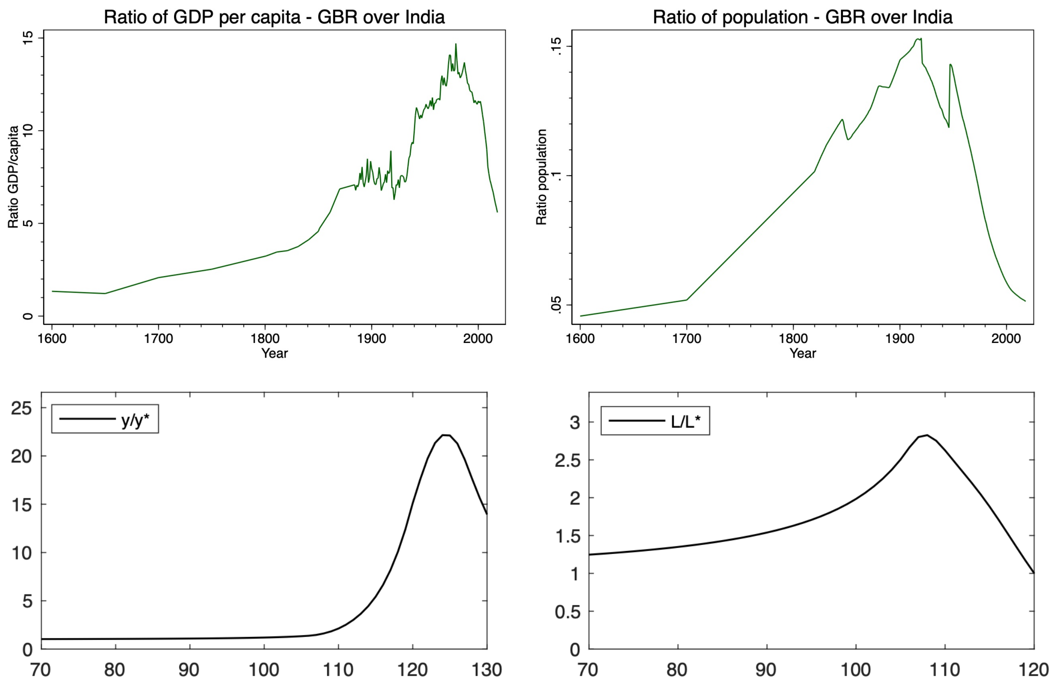
Effect of a 7.5% positive technology shock in the Blue country in $t = 20$ on differences in GDP/capita, population, technology and TFP (ratios).

progress, through the positive scale effect of population on innovation. A faster growth rate of technology coupled to an increasing share of resources, which raises the benefits of investing in child quality relative to quantity, triggers an early escape out of the Malthusian trap for the advantaged country. The advantaged country starts to drastically reduce fertility and invest in education instead, which puts an end to the Malthusian check and allows income per capita to rise quickly. This take-off to sustained economic growth also leads to an increase in military spending, and the share of resources appropriated increases even more despite population growing at a much slower pace.

However, the disadvantaged country responds to this loss of resources by increasing fertility and its population starts to grow very fast, catching up and eventually surpassing that of the advantaged country, restoring part of the disadvantage in military capabilities. Furthermore, as population grows in the dominated country, the growth of technology also accelerates and this triggers the demographic transition in turn. The resulting growth in income per capita coupled to the greater population size allows the disadvantaged country to eventually catch up until resources are equally shared again.

The possibility of conflict over the appropriation of resources in fixed supply therefore brings rich non-linear dynamics to unified growth theory. In particular, small shocks in technology are amplified and yield substantial divergence between countries. The strategic nature of the game between two countries however leaves room for the dominated country to react accordingly, and the model endogenously gives rise to a process of reconvergence. It is interesting to note however that despite a return to an equal fraction of fixed resources controlled by both countries, there remain persistent differences between the two: as is depicted in Figure 5, the initially advantaged country has a permanently higher GDP per capita (by a factor of 5.4), and the initially disadvantaged country has a permanently larger population, by a factor of 3. In Appendix A.7, we also consider alternative asymmetric shocks to either the size of the population or the relative efficiency of military capabilities in the Tullock contest.

Figure 6: **Ratios of GDP per capita and population, GBR and India**



Notes: Maddison project database, 2020 data (Bolt and Van Zanden, 2020; Broadberry et al., 2010, 2015) and authors' calculation (1600 – 2020) vs. benchmark calibration of the model (t=70;130).

Comparison with historical data. Although our numerical exercise is not an attempt to rigorously take our stylized model to the data, we believe the mechanism depicted here shed light on historical long-run dynamics in which conflict over the appropriation of resources played a central role. The Great Divergence that occurred between Western Europe and the rest of the world has been the subject of an extensive literature spanning various disciplines. However, while historians and economic historians have long emphasized the importance of military conflict on comparative development, most theoretical models of long-run growth fail to account for this zero-sum game that is the competition for fixed resources. Introducing conflict in an otherwise standard unified growth model allows to capture those dynamics of divergence and reconvergence between colonial powers and the countries they engaged with observed in the last centuries.

Taking the relationship between England and India as an illustrative case study, our model accounts remarkably well for the comparative long-run dynamics of the two countries: the slow but gradual divergence in GDP per capita accompanied by the faster growth of the English population, the take-off of England following the demographic transition, and the growth of the Indian population in response, planting the seed for reconvergence.[11] In Figure 6, we plot the ratios of GDP per capita and population size between England and India observed in the data against the same ratios given by the benchmark calibration of the model. Note that this exercise does not aim at making our theory fit the data.[12] It is rather conducted to illustrate how introducing conflict in an otherwise standard unified growth model allows to capture salient features of long-run comparative dynamics, at least qualitatively.

Figure 6 shows how conflict for the control of fixed resources substantially amplifies the effect of a small asymmetric technology shock. In Table 2, we compare our baseline simulation with an alternative scenario in which the possibility of conflict is shut down by setting $\pi = 0$ in the Tullock contest success function. We consider the same small asymmetric technology shock, but countries now share an equal and fixed fraction of total resources that no investments in military capabilities can change. Of particular interest are two moments in the data generated by the model: the peak of the divergence between the two countries, and the long-run GDP per capita ratio towards which the model converges. Looking at the peak of the divergence, the maximum difference in GDP per capita observed between the two countries over the course of development, the result is striking. The appropriation of resources allows the advantaged country to attain a level of GDP per capita 22 times greater than the disadvantaged country. Without conflict, this same small technology shock naturally has persistent effect but only lead to a 4-fold difference. More importantly, notice how without conflict, the maximum differences in GDP per capita coincides with the long-run ratio. In

[11] See also Appendix A.6 Figure 12 for a a comparison of a set of colonial powers with China, India and Japan.

[12] Our calibration strategy indeed takes the two countries to be initially identical, which England and India were obviously not, especially in terms of population. Furthermore, the pace of development is not calibrated to match the period 1600–2000 we look at, and we therefore zoom in on when the divergence and the reconvergence are observed in our baseline simulation. Finally, our model converges to a balanced growth path in which ratios of GDP per capita and population are constant, a path the real world has obviously not yet converged to.

Table 2: Comparison data vs. benchmark model

Techn. shock	Model Benchmark	Data I GBR/India	Data II Col./Asia
Cross-country GDP per capita ratios			
With conflict ($\pi = 1$)			
$y/y^\star(+\infty)$	5.35	5.59 in 2018	3.51 in 2018
$\max y/y^\star$	22.2	14.1 in 1973	8.16 in 1961
Without conflict ($\pi = 0$)			
$y/y^\star(+\infty)$	3.88		
$\max y/y^\star$	3.88		

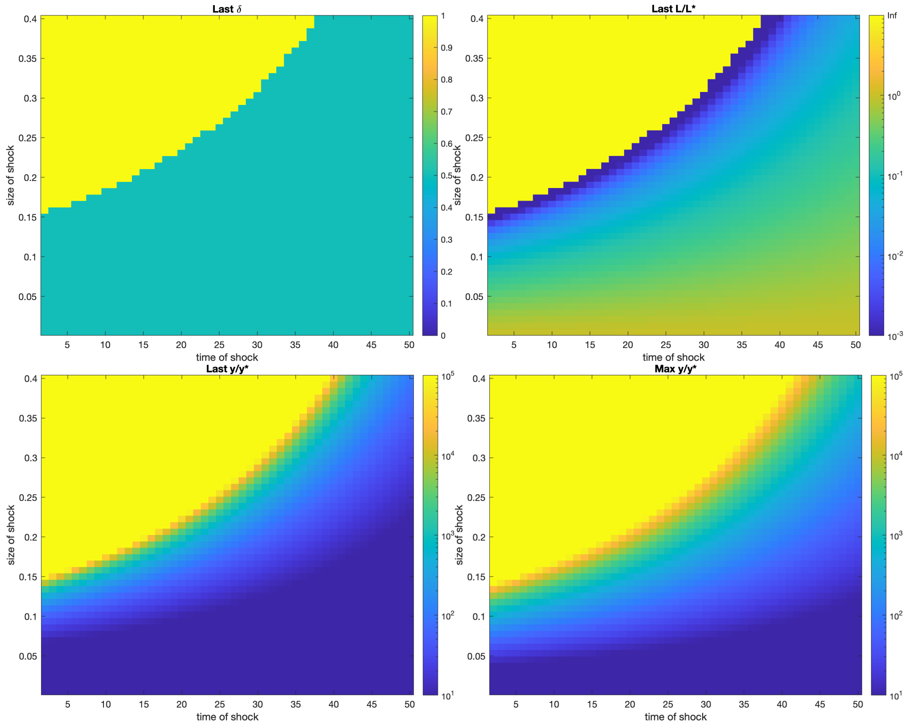
Source: Maddison Project database, 2020 (Bolt and Van Zanden, 2020) and authors' calculation.

the case with conflict, the long-run difference in GDP per capita is much lower than at the peak of the divergence. Strategic complementarities between the two countries, and in particular the strategic considerations in fertility decisions, yield a reversal of the dynamics which a standard unified growth model is unable to generate. This endogenous switch from divergence to reconvergence is a salient feature observed in the data, with varying magnitudes that a more thorough calibration exercise could attempt to match.

4.4 Larger asymmetric shocks: Regional hegemony

Increasing the magnitude of the asymmetric technology shock or imposing it at an earlier stage may both lead to a new convergence regime. We run a large number of additional simulations, using our baseline calibration but we vary both the magnitude of the shock and the period at which the shock occurs. The results are displayed in Figure 7. The top left corner shows the fraction of resources controlled by the advantaged country in the long-run, and a clear frontier separating two regimes appears. The region in green depicts simulations in which both δ_{t+1} and $\delta^\star_{t+1}$ converge to $1/2$ in the long-run, such that both countries control an equal share of resources. As in the previous section, this does not mean that differences in GDP per capita disappear. The advantaged country still benefits from a greater level of GDP per capita, while the other has permanently higher population levels because it caught up, albeit partially, through strategic increases in fertility.

Figure 7: **Phase diagram: variations around the size and timing of the technological shock**

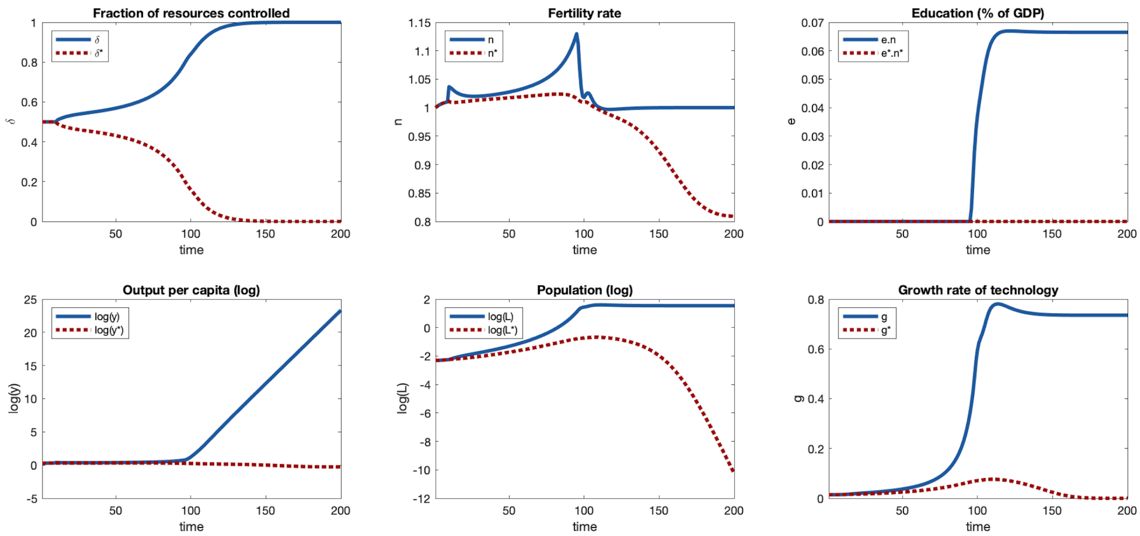


2500 simulations with various values for selected parameters. No recalibration, initial conditions calculated to reach a fertility rate of 1.

However, reconvergence towards this equal sharing of resources is not guaranteed. The phase diagrams in Figure 7 reveals the existence of a clear frontier. The zone in yellow in the top left corner characterizes simulations in which δ_{t+1} converges to one and $\delta^{\star}_{t+1}$ to zero. If the asymmetric technology shock is either too big or occurs too early in development, the region embarks on a very different balanced growth path. The cumulative advantage in the military competition is such that the favored country ends up controlling the totality of fixed resources in the long-run. This occurs because the dominated country, deprived of too many resources too quickly, cannot bear the cost of raising fertility and military spending to reverse the balance of power when the dominating country takes-off. The dominated country becomes so poor that it cannot sustain a reproduction rate above one, its population begins to gradually decline until it disappears. The dynamics are illustrated in Figure 8.

More specifically, the appropriation of resources by the technologically favored country during

Figure 8: **Long-run dynamics with a large asymmetric technology shock**

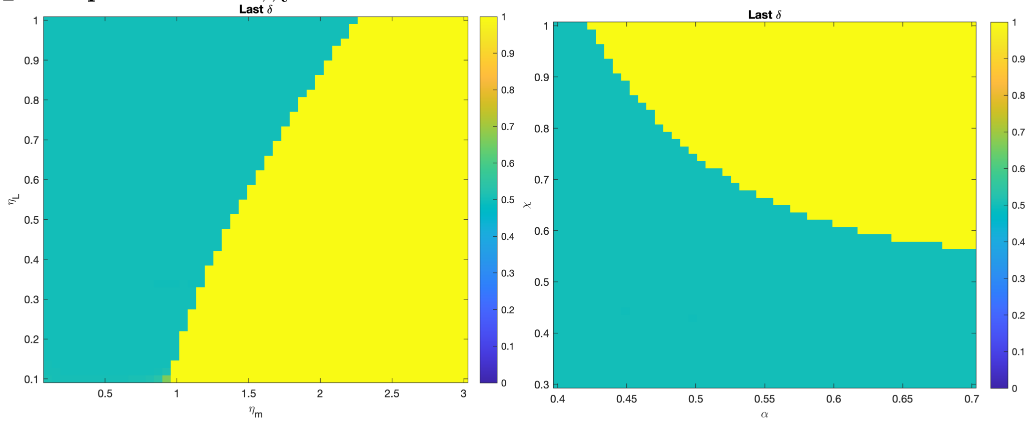


Effect of a 20% positive technology shock in the blue country in $t = 10$.

the Malthusian era is too quick. The resulting increases in income per capita from a greater fraction of resources controlled allows fertility to increase gradually. Population growth, through both its effect on military capabilities and the positive externality on the pace of technological progress, allows the initially advantaged country to consolidate its dominance and sets up a virtuous circle. Meager technological improvements are not enough to offset the accelerating loss of resources for the other country, and income growth is too slow to permit increases in fertility though the standard Malthusian mechanism. The very modest growth of population in the disadvantaged country do not allow technological progress to accelerate through those scale effects benefiting the other country. When the advantaged country experiences the demographic transition and takes-off out of the Malthusian trap, the other country does not have enough resources to react by raising fertility for strategic reasons. As the advantaged country captures all resources, the other goes extinct.

Notice however that, because the advantaged country never really faces a challenge for the control of resources, the effect of strategic complementarities in fertility are attenuated. A positive effect is that the lack of need for a large population size reduces the opportunity cost of investing in education, and accelerates the transition out of the Malthusian period. However, an indirect effect is that the country does not benefit as much from the positive externality of rapid population growth

Figure 9: **Phase diagram: variations around the returns to scale in military capabilities η_M, η_L and production α, χ**



2500 simulations with various values for parameters. No recalibration, initial conditions calculated to reach a fertility rate of 1.

on the rate of technological progress. As a result, the growth rate of technology in the long-run is lower than in the case where strategic interactions between the two countries lead to a race for the control of resources, a race that is wasteful but potentially indirectly beneficial.

Different regimes in the parameter space. The magnitude and the timing of the technology shock are not the only parameters that determine on which of those two balanced growth paths the two countries can converge to. A clear frontier also appears if, for a given technology shock, we allow the relative returns to scale in military capabilities, η_M and η_L, and production α, χ to vary. As can be seen in the left paniel of Figure 9, departing from constant returns to scale in military capabilities leaves the door open to the total domination of the advantaged country. Increasing returns to scale in military spending makes an early transition out of the Malthusian regime a substantial advantage as income starts to grow rapidly. The relative returns to population size in military capabilities must therefore be high enough to allow the dominated country to not go extinct and catch-up. The right panel of Figure 9 indicates that returns to scale in production also matters. In particular, substantial returns to land χ, by making the fixed factor of production more important for aggregate production, lead countries to devote more resources to the conflict. This naturally gives the edge to the advantaged country and makes it more likely to control the totality of resources in the long-run.

5 Concluding comments

Our objective in this paper was to introduce the possibility of conflict over fixed resources in unified growth theory. The strategic complementarities between the decisions of competing countries yield rich, non-linear dynamics of divergence and convergence that an otherwise standard model cannot account for. In particular, the strategic considerations in fertility that stem from the role of population size in military capabilities are a remarkable source of catching up for initially disadvantaged countries. Although we let theory guide us, we believe the model matches relatively well the long-run comparative dynamics of Western European powers that were the first to take-off to prosperity in the 19th century, and some of the countries in the Global South that have been able to embark on a path of reconvergence in more recent years.

In addition to this benchmark case that is remarkably robust to a wide range of parameters, we also identify another potential balanced growth path the model may converge too, especially if the asymmetry between the two countries is substantial. The appropriation of resources by a country following a positive technology shock may not only slow down, but also prevent the development of the other country. If the disadvantaged country gets deprived of resources too suddenly and becomes too poor to bear the cost of fertility and sustain a positive growth rate of population, strategic complementarities are not enough to allow a reconvergence process to set in. The dominating countries then gradually extends its control over all resources, while the population of the dominated country goes extinct. Contrary to a standard unified growth model, a take-off to sustained economic prosperity is therefore not inevitable.

Investigating the way conflict over resources interacts with the dynamics of population in a Malthusian setting is a fascinating area of future research. The fundamental role demography played in long-run economic development over the last few millenia has long been recognized by economists, economic historians and growth theorists alike. Jones (2022) has recently begun to ponder how population dynamics will shape the future, and we hope that the theory we propose in this paper can also shed light on this pressing question.

A Appendix

A.1 Derivatives of the Tullock contest success function

First, let us note the partial derivatives of military capabilities $\mathcal{M}_{t+1}$ with respect to military investment m_{t+1} and fertility n_t:

$$\begin{aligned}
\frac{\partial \mathcal{M}_{t+1}}{\partial m_{t+1}} &= \eta_M m_{t+1}^{\eta_M - 1} n_t^{\eta_L} L_t^{\eta_M+\eta_L} = \eta_M \mathcal{M}_{t+1}/m_{t+1} && (34)\\
\frac{\partial \mathcal{M}_{t+1}}{\partial n_t} &= \eta_L m_{t+1}^{\eta_M} n_t^{\eta_L - 1} L_t^{\eta_M+\eta_L} = \eta_L \mathcal{M}_{t+1}/n_t && (35)
\end{aligned}$$

Now let us consider the general case in which military capabilities are raised to the power *pi* in the Tullock contest success function:

$$\delta_{t+1} = \frac{\psi \mathcal{M}_{t+1}^{\pi}}{\psi \mathcal{M}_{t+1}^{\pi} + \psi^{\star} {\mathcal{M}_{t+1}^{\star}}^{\pi}} \tag{36}$$

The partial derivative of the above equation with respect to a variable denoted by x is:

$$\begin{aligned}
\frac{\partial \delta_{t+1}}{\partial x} &= \pi \psi \frac{\partial \mathcal{M}_{t+1}}{\partial x} \mathcal{M}_{t+1}^{\pi-1} \frac{\psi^{\star} {\mathcal{M}_{t+1}^{\star}}^{\pi}}{\left(\psi \mathcal{M}_{t+1}^{\pi} + \psi^{\star} {\mathcal{M}_{t+1}^{\star}}^{\pi}\right)^2}\\
&= \frac{\pi}{\mathcal{M}_{t+1}} \frac{\partial \mathcal{M}_{t+1}}{\partial x} \delta_{t+1} \delta_{t+1}^{\star} && (37)
\end{aligned}$$

Using the partial derivatives with respect to military spending and fertility, we eventually obtain:

$$\begin{aligned}
\frac{\partial \delta_{t+1}}{\partial m_{t+1}} &= \pi \eta_M \frac{\delta_{t+1}\delta_{t+1}^{\star}}{m_{t+1}} && (38)\\
\frac{\partial \delta_{t+1}}{\partial n_t} &= \pi \eta_L \frac{\delta_{t+1}\delta_{t+1}^{\star}}{n_t} && (39)
\end{aligned}$$

A.2 Proof of Lemmma 1

Note that $\partial y_{t+1}/\partial e_{t+1} = \alpha y_{t+1} h_e(e_{t+1}, g_{t+1})/h(e_{t+1}, g_{t+1})$, the optimal level of education is given by the following equation:

$$F^e \equiv \alpha \frac{\partial h(e_{t+1}, g_{t+1})}{\partial e_{t+1}} \frac{\mathcal{C}(e_{t+1}, y_t)}{y_t} - (\alpha + \mathcal{S}_L(\delta_{t+1}^{\star})) h(e_{t+1}, g_{t+1}) \begin{cases} = 0 & \text{if} \quad e_{t+1} > 0 \\ < 0 & \text{if} \quad e_{t+1} = 0 \end{cases}$$

The function $F^e(e_{t+1}, g_{t+1}, \delta^\star_{t+1}, y_t)$ implicitly defines the relationship between the optimal education level e_{t+1} and the other endogenous variables. Because g_{t+1} does not affect military investments nor fertility decisions, we can study its effect on the optimal education independently. It is easy to check that $\partial F^e/\partial e_{t+1} < 0$ and $\partial F^e/\partial g_{t+1} > 0$. We make the assumption that $F^e(0, 0, \delta^\star_{t+1}, y_t) < 0$ to ensure the existence of a corner solution $e_{t+1} = 0$ for a positive rate of technological progress g_{t+1}. To see that it is the case, note that $F^e(0, g_{t+1}, \delta^\star_{t+1}, y_t)$ is monotically increasing in g_{t+1} all else equal, and $\lim_{g_{t+1}\to\infty} F^e(0, g_{t+1}, \delta^\star_{t+1}, y_t) > 0$. The assumption that $F^e(0, 0, \delta^\star_{t+1}, y_t) < 0$ thus ensures that there exists a unique $\hat{g}_{t+1} \equiv \hat{g}(\delta^\star_{t+1}, y_t) > 0$ such that $F^e(0, \hat{g}_{t+1}, \delta^\star_{t+1}, y_t) = 0$. Therefore, $e_{t+1} = 0$ for $g_{t+1} \leq \hat{g}_{t+1}$ and $\partial e_{t+1}/\partial g_{t+1} = -\dfrac{\partial F^e/\partial g_{t+1}}{\partial F^e/\partial e_{t+1}} > 0$. **QED.**

A.3 Proof of Proposition 1.

In this section, we characterize how the optimal decisions of a given country vary with enemy military capabilities. We focus on the three main variables m_{t+1}, n_t and e_{t+1}, and for that we define three implicit functions connecting them to $\delta^\star_{t+1} = \delta^\star(m_{t+1}, n_t, L_t, \mathcal{M}^\star_{t+1})$, the last choice (consumption) being the residual variable obtained by the budget constraint. Let us define the following functions:

$$\begin{aligned}
F^m(.) &= \frac{m_{t+1}}{y_t} - \underbrace{\frac{\mathcal{S}_M(\delta^\star_{t+1})}{\Gamma + \alpha + \mathcal{S}_L(\delta^\star_{t+1}) + \mathcal{S}_M(\delta^\star_{t+1})}}_{=f^m(m_{t+1}, n_t, L_t, \mathcal{M}^\star_{t+1})} \\
F^n(.) &= \frac{\mathcal{C}(e_{t+1}, y_t) n_t}{y_t} - \underbrace{\frac{\alpha + \mathcal{S}_L(\delta^\star_{t+1})}{\Gamma + \alpha + \mathcal{S}_L(\delta^\star_{t+1}) + \mathcal{S}_M(\delta^\star_{t+1})}}_{=f^n(m_{t+1}, n_t, y_t, L_t, \mathcal{M}^\star_{t+1})} \\
F^e(.) &= \alpha \frac{\partial h(e_{t+1}, g_{t+1})}{\partial e_{t+1}} \frac{\mathcal{C}(e_{t+1}, y_t)}{y_t} - (\alpha + \mathcal{S}_L(\delta^\star_{t+1})) h(e_{t+1}, g_{t+1})
\end{aligned}$$

Using equations (26), (27) and (28), we get the following system of three equations and three unknowns m_{t+1}, n_t and e_{t+1}:

$$\begin{aligned}
F^m(m_{t+1}, n_t, y_t, L_t, \mathcal{M}^\star_{t+1}) &= 0 \\
F^n(m_{t+1}, n_t, e_{t+1}, y_t, L_t, \mathcal{M}^\star_{t+1}) &= 0 \\
F^e(m_{t+1}, n_t, e_{t+1}, y_t, L_t, \mathcal{M}^\star_{t+1}, g_{t+1}) &= 0
\end{aligned}$$

Note that agents take as given y_t, L_t, g_{t+1}, as well as $\mathcal{M}^\star_{t+1}$. With a slight abuse of notation, we abstract from L_t, y_t and g_{t+1}, because we want to study how a country reacts to changes in enemy military capabilities $\mathcal{M}^{\star}_{t+1}$. The system above therefore writes as:

$$F^m(m_{t+1}, n_t, \mathcal{M}^{\star}_{t+1}) = 0 \quad (40)$$

$$F^n(m_{t+1}, n_t, e_{t+1}, \mathcal{M}^{\star}_{t+1}) = 0 \quad (41)$$

$$F^e(m_{t+1}, n_t, e_{t+1}, \mathcal{M}^{\star}_{t+1}) = 0 \quad (42)$$

A.3.1 The case without education $e_{t+1} = 0$

We start by studying the case where $e_{t+1} = 0$. We therefore have a system of two equations, two endogenous variables m_{t+1} and n_t, and one exogenous variable $\mathcal{M}^{\star}_{t+1}$. We drop the time subscript for clarity:

$$F^m(m, n, \mathcal{M}^{\star}) = 0 \quad (43)$$

$$F^n(m, n, \mathcal{M}^{\star}) = 0 \quad (44)$$

The implicit function theorem says that, at values of m_{t+1} and n_t that satisfies the two equations above, there exist two differentiable functions $m(\mathcal{M}^{\star})$ and $n(\mathcal{M}^{\star})$ such that:

$$F^m(m(\mathcal{M}^{\star}), n(\mathcal{M}^{\star}), \mathcal{M}^{\star}) = 0 \quad (45)$$

$$F^n(m(\mathcal{M}^{\star}), n(\mathcal{M}^{\star}), \mathcal{M}^{\star}) = 0 \quad (46)$$

Using the chain rule for the derivatives, we get:

$$\frac{\partial F^m}{\partial m} m'(\mathcal{M}^{\star}) + \frac{\partial F^m}{\partial n} n'(\mathcal{M}^{\star}) + \frac{\partial F^m}{\partial \mathcal{M}^{\star}} = 0 \quad (47)$$

$$\frac{\partial F^n}{\partial m} m'(\mathcal{M}^{\star}) + \frac{\partial F^n}{\partial n} n'(\mathcal{M}^{\star}) + \frac{\partial F^n}{\partial \mathcal{M}^{\star}} = 0 \quad (48)$$

Rewriting:

$$m'(\mathcal{M}^{\star}) = \frac{-\left(\frac{\partial F^m}{\partial \mathcal{M}^{\star}} + \frac{\partial F^m}{\partial n} n'(\mathcal{M}^{\star})\right)}{\frac{\partial F^m}{\partial m}} \quad (49)$$

$$n'(\mathcal{M}^{\star}) = \frac{-\left(\frac{\partial F^n}{\partial \mathcal{M}^{\star}} + \frac{\partial F^n}{\partial m} m'(\mathcal{M}^{\star})\right)}{\frac{\partial F^n}{\partial n}} \quad (50)$$

Substituting back:

$$\frac{\partial F^m}{\partial m} m'(\mathcal{M}^{\star}) - \frac{\frac{\partial F^m}{\partial n}}{\frac{\partial F^n}{\partial n}} \left(\frac{\partial F^n}{\partial \mathcal{M}^{\star}} + \frac{\partial F^n}{\partial m} m'(\mathcal{M}^{\star}) \right) + \frac{\partial F^m}{\partial \mathcal{M}^{\star}} = 0 \quad (51)$$

$$-\frac{\frac{\partial F^n}{\partial m}}{\frac{\partial F^m}{\partial m}} \left(\frac{\partial F^m}{\partial \mathcal{M}^{\star}} + \frac{\partial F^m}{\partial n} n'(\mathcal{M}^{\star}) \right) + \frac{\partial F^n}{\partial n} n'(\mathcal{M}^{\star}) + \frac{\partial F^n}{\partial \mathcal{M}^{\star}} = 0 \quad (52)$$

Rearranging to get the derivatives we are looking for:

$$m'(\mathcal{M}^\star) = \frac{\frac{\partial F^m}{\partial n}\frac{\partial F^n}{\partial \mathcal{M}^\star} - \frac{\partial F^m}{\partial \mathcal{M}^\star}\frac{\partial F^n}{\partial n}}{\frac{\partial F^n}{\partial n}\frac{\partial F^m}{\partial m} - \frac{\partial F^n}{\partial m}\frac{\partial F^m}{\partial n}} \tag{53}$$

$$n'(\mathcal{M}^\star) = \frac{\frac{\partial F^n}{\partial m}\frac{\partial F^m}{\partial \mathcal{M}^\star} - \frac{\partial F^n}{\partial \mathcal{M}^\star}\frac{\partial F^m}{\partial m}}{\frac{\partial F^n}{\partial n}\frac{\partial F^m}{\partial m} - \frac{\partial F^n}{\partial m}\frac{\partial F^m}{\partial n}} \tag{54}$$

Intermediate step. The objective is now to study the sign of those two derivatives. Recall:

$$F^m(m,n,\mathcal{M}^\star) = \frac{m}{y} - \underbrace{\frac{S_M(\delta^\star)}{\Gamma+\alpha+\mathcal{S}_L(\delta^\star_{t+1})+\mathcal{S}_M(\delta^\star_{t+1})}}_{=f^m(\delta^\star)}$$

$$F^n(m,n,\mathcal{M}^\star) = \frac{\mathcal{C}n}{y} - \underbrace{\frac{\alpha+S_F(\delta^\star)}{\Gamma+\alpha+\mathcal{S}_L(\delta^\star_{t+1})+\mathcal{S}_M(\delta^\star_{t+1})}}_{=f^n(\delta^\star)}$$

Let us establish a few intermediate results that will prove useful. First, the derivatives of functions f^m and f^n with respect to $\delta^\star$ are respectively:

$$\frac{\partial f^m}{\partial \delta^\star} = \frac{\chi\pi\eta_M(\Gamma+\alpha)}{[\Gamma+\alpha+\mathcal{S}_L(\delta^\star_{t+1})+\mathcal{S}_M(\delta^\star_{t+1})]^2} > 0$$

$$\frac{\partial f^n}{\partial \delta^\star} = \frac{\chi\pi(\Gamma\eta_L-\alpha\eta_M)}{[\Gamma+\alpha+\mathcal{S}_L(\delta^\star_{t+1})+\mathcal{S}_M(\delta^\star_{t+1})]^2} \begin{cases} >0 & \text{iff} \quad \frac{\eta_M}{\eta_L} < \frac{\Gamma}{\alpha} \\ <0 & \text{iff} \quad \frac{\eta_M}{\eta_L} > \frac{\Gamma}{\alpha} \end{cases}$$

This allows us to sign the partial derivatives that we need. Note that the derivatives with respect to F^n are ambiguous which will make proofs slightly less straightforward.

$$\frac{\partial F^m}{\partial m} = \frac{1}{y} - \frac{\partial \delta^\star}{\partial m}\frac{\partial f^m}{\partial \delta^\star} > 0 \quad ; \quad \frac{\partial F^n}{\partial m} = -\frac{\partial \delta^\star}{\partial m}\frac{\partial f^n}{\partial \delta^\star} \lesseqgtr 0$$

$$\frac{\partial F^m}{\partial n} = -\frac{\partial \delta^\star}{\partial n}\frac{\partial f^m}{\partial \delta^\star} > 0 \quad ; \quad \frac{\partial F^n}{\partial n} = \frac{\mathcal{C}}{y} - \frac{\partial \delta^\star}{\partial n}\frac{\partial f^n}{\partial \delta^\star} \lesseqgtr 0$$

$$\frac{\partial F^m}{\partial \mathcal{M}^\star} = -\frac{\partial \delta^\star}{\partial \mathcal{M}^\star}\frac{\partial f^m}{\partial \delta^\star} < 0 \quad ; \quad \frac{\partial F^n}{\partial \mathcal{M}^\star} = -\frac{\partial \delta^\star}{\partial \mathcal{M}^\star}\frac{\partial f^n}{\partial \delta^\star} \lesseqgtr 0$$

Sign of the denominator. Notice the denominator of both derivatives is the same. Let us start by showing it is always negative. Denote:

$$\begin{aligned} \text{den} &= \frac{\partial F^n}{\partial n}\frac{\partial F^m}{\partial m} - \frac{\partial F^n}{\partial m}\frac{\partial F^m}{\partial n} \\ &= \left(\frac{1}{y} - \frac{\partial \delta^\star}{\partial m}\frac{\partial f^m}{\partial \delta^\star}\right)\left(\frac{\mathcal{C}}{y} - \frac{\partial \delta^\star}{\partial n}\frac{\partial f^n}{\partial \delta^\star}\right) - \left(\frac{\partial \delta^\star}{\partial n}\frac{\partial f^m}{\partial \delta^\star}\right)\left(\frac{\partial \delta^\star}{\partial m}\frac{\partial f^n}{\partial \delta^\star}\right) \\ &= \frac{1}{y}\left\{\frac{\mathcal{C}}{y} - \left(\mathcal{C}\frac{\partial \delta^\star}{\partial m}\frac{\partial f^m}{\partial \delta^\star} + \frac{\partial \delta^\star}{\partial n}\frac{\partial f^n}{\partial \delta^\star}\right)\right\} \end{aligned}$$

Using the expressions for the derivatives of the Tullock contest function defined earlier in the appendix $\partial\delta^\star/\partial m = -\pi\eta_M\delta\delta^\star/m$ and $\partial\delta^\star/\partial n = -\pi\eta_L\delta\delta^\star/n$, and then substituting for $\partial f^n/\partial\delta^\star$ and $\partial f^m/\partial\delta^\star$, we get:

$$\text{den} = \frac{1}{y}\left\{\frac{\mathcal{C}}{y} + \pi\delta\delta^\star\left(\mathcal{C}\frac{\eta_M}{m}\frac{\partial f^m}{\partial\delta^\star} + \frac{\eta_L}{n}\frac{\partial f^n}{\partial\delta^\star}\right)\right\}$$

A sufficient condition for the denominator of both derivatives to be positive is:

$$\left(\mathcal{C}\frac{\eta_M}{m}(\eta_M\Gamma + \eta_M\alpha) + \frac{\eta_L}{n}(\Gamma\eta_L - \alpha\eta_M)\right) > 0$$

which is always true as shown below:

$$\begin{aligned}
&\Leftrightarrow \left(\mathcal{C}\frac{\eta_M}{m}(\eta_M\Gamma + \eta_M\alpha) + \frac{\eta_L}{n}\underbrace{(\Gamma\eta_L - \alpha\eta_M)}_{\lesseqgtr 0}\right) > 0 \\
&\Leftrightarrow \eta_M(\Gamma+\alpha) + \underbrace{\frac{m}{\mathcal{C}n}}_{=\mathcal{E}^m/(1+\mathcal{E}^n)}\frac{\eta_L}{\eta_M}(\Gamma\eta_L - \alpha\eta_M) > 0 \\
&\Leftrightarrow \Gamma + \alpha + \frac{\chi\pi\eta_M\delta^\star}{\alpha + \chi\pi\eta_L\delta^\star}\frac{\eta_L}{\eta_M}(\Gamma\frac{\eta_L}{\eta_M} - \alpha) > 0 \\
&\Leftrightarrow [\alpha + \chi\pi\eta_L\delta^\star](\Gamma+\alpha) + \chi\pi\eta_M\delta^\star\frac{\eta_L}{\eta_M}(\Gamma\frac{\eta_L}{\eta_M} - \alpha) > 0 \\
&\Leftrightarrow [\alpha + \chi\pi\eta_L\delta^\star]\Gamma + \alpha^2 + \cancel{\chi\pi\eta_L\delta^\star\alpha}... \\
&\qquad ... + \chi\pi\eta_L\delta^\star\frac{\eta_L}{\eta_M}\Gamma - \cancel{\chi\pi\eta_L\delta^\star\alpha} > 0
\end{aligned}$$

Because all remaining terms are positive, the condition is satisfied and the denominator is always positive:

$$\text{den} > 0$$

It follows that both derivatives are of the sign of their respective numerator, which we study next.

Sign of the numerator of $m'(\mathcal{M}^\star)$. Let us first study the numerator of the derivative of military spending with respect to enemy military capabilities. Denote:

$$\begin{aligned}
\text{num}_m &= -\frac{\partial F^m}{\partial\mathcal{M}^\star}\frac{\partial F^n}{\partial n} + \frac{\partial F^m}{\partial n}\frac{\partial F^n}{\partial\mathcal{M}^\star} \\
&= \frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^m}{\partial\delta^\star}\left\{\frac{\mathcal{C}}{y_t} - \frac{\partial\delta^\star}{\partial n}\frac{\partial f^n}{\partial\delta^\star}\right\} + \left(-\frac{\partial\delta^\star}{\partial n}\frac{\partial f^m}{\partial m}\right)\left(-\frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^n}{\partial\delta^\star}\right) \\
&= \frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^m}{\partial\delta^\star}\left\{\frac{\mathcal{C}}{y_t} - \cancel{\frac{\partial\delta^\star}{\partial n}\frac{\partial f^n}{\partial\delta^\star}} + \cancel{\frac{\partial\delta^\star}{\partial n}\frac{\partial f^n}{\partial\delta^\star}}\right\} \\
&= \frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^m}{\partial\delta^\star}\frac{\mathcal{C}}{y_t} > 0
\end{aligned}$$

The numerator of $m'(\mathcal{M}^\star)$ being positive, military spending always increase with enemy military capabilities:

$$m'(\mathcal{M}^\star) = \frac{\frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^m}{\partial\delta^\star}\mathcal{C}}{\frac{\mathcal{C}}{y} + \pi\delta\delta^\star\left(\mathcal{C}\frac{\eta_M}{m}\frac{\partial f^m}{\partial\delta^\star} + \frac{\eta_L}{n}\frac{\partial f^n}{\partial\delta^\star}\right)} > 0 \tag{55}$$

Sign of the numerator of $n'(\mathcal{M}^\star)$. Let us now study the numerator of the derivative of fertility with respect to enemy military capabilities. Denote:

$$\begin{aligned}
\text{num}_n &= -\frac{\partial F^n}{\partial\mathcal{M}^\star}\frac{\partial F^m}{\partial m} + \frac{\partial F^n}{\partial m}\frac{\partial F^m}{\partial\mathcal{M}^\star} \\
&= -\left(-\frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^n}{\partial\delta^\star}\right)\left(\frac{1}{y} - \frac{\partial\delta^\star}{\partial m}\frac{\partial f^m}{\partial\delta^\star}\right) + \left(-\frac{\partial\delta^\star}{\partial m}\frac{\partial f^n}{\partial\delta^\star}\right)\left(-\frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^m}{\partial\delta^\star}\right) \\
&= \frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^n}{\partial\delta^\star}\left\{\frac{1}{y} - \cancel{\frac{\partial\delta^\star}{\partial m}\frac{\partial f^m}{\partial\delta^\star}} + \cancel{\frac{\partial\delta^\star}{\partial m}\frac{\partial f^m}{\partial\delta^\star}}\right\} \\
&= \frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^n}{\partial\delta^\star}\frac{1}{y}
\end{aligned}$$

Because both y and $\partial\delta^\star/\partial\mathcal{M}^\star$ are both positive, num_n is of the sign of $\partial f^n/\partial\delta^\star$:

$$\text{num}_n \begin{cases} > 0 & \text{iff} \quad \frac{\eta_M}{\eta_L} < \frac{\Gamma}{\alpha} \\ < 0 & \text{iff} \quad \frac{\eta_M}{\eta_L} > \frac{\Gamma}{\alpha} \end{cases}$$

Again, because the derivative of the rate of fertility with respesct to enemy capabilities is of the sign of its numerator, we have:

$$n'(\mathcal{M}^\star) = \frac{\frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\frac{\partial f^n}{\partial\delta^\star}}{\frac{\mathcal{C}}{y} + \pi\delta\delta^\star\left(\mathcal{C}\frac{\eta_M}{m}\frac{\partial f^m}{\partial\delta^\star} + \frac{\eta_L}{n}\frac{\partial f^n}{\partial\delta^\star}\right)} \begin{cases} > 0 & \text{iff} \quad \frac{\eta_M}{\eta_L} < \frac{\Gamma}{\alpha} \\ < 0 & \text{iff} \quad \frac{\eta_M}{\eta_L} > \frac{\Gamma}{\alpha} \end{cases} \tag{56}$$

What happens to consumption? Finally, we can use the budget constraint and the results obtained above to investigate how consumption varies with enemy military capabilities:

$$c(\mathcal{M}^\star) = y - \mathcal{C}n(\mathcal{M}^\star) - m(\mathcal{M}^\star)$$

Differentiating the budget constraint gives:

$$\begin{aligned}
c'(\mathcal{M}^\star) &= -[\mathcal{C}n'(\mathcal{M}^\star) + m'(\mathcal{M}^\star)] \\
&= -\frac{\mathcal{C}\frac{\partial\delta^\star}{\partial\mathcal{M}^\star}\left[\frac{\partial f^n}{\partial\delta^\star} + \frac{\partial f^m}{\partial\delta^\star}\right]}{\frac{\mathcal{C}}{y} + \pi\delta\delta^\star\left(\mathcal{C}\frac{\eta_M}{m}\frac{\partial f^m}{\partial\delta^\star} + \frac{\eta_L}{n}\frac{\partial f^n}{\partial\delta^\star}\right)}
\end{aligned}$$

It is straightforward to show that $\dfrac{\partial f^n}{\partial\delta^\star} + \dfrac{\partial f^m}{\partial\delta^\star} > 0$. Therefore:

$$c'(\mathcal{M}^\star) < 0 \tag{57}$$

A.3.2 The case with education $e_{t+1} > 0$

We now turn to the case where $e_{t+1} > 0$. We have a system of three equations, three endogenous variables m_{t+1}, n_t and e_{t+1}, and one exogenous variable $\mathcal{M}^\star_{t+1}$. We drop the time subscript for clarity:

$$\begin{aligned} F^m(m, n, \mathcal{M}^\star) &= 0 && (58) \\ F^n(m, n, e, \mathcal{M}^\star) &= 0 && (59) \\ F^e(m, n, e, \mathcal{M}^\star) &= 0 && (60) \end{aligned}$$

Using the implicit function theorem again, there exist three differentiable functions $m(\mathcal{M}^\star)$, $n(\mathcal{M}^\star)$ and $e(\mathcal{M}^\star)$ such that:

$$\begin{aligned} F^m(m(\mathcal{M}^\star), n(\mathcal{M}^\star), \mathcal{M}^\star) &= 0 && (61) \\ F^n(m(\mathcal{M}^\star), n(\mathcal{M}^\star), e(\mathcal{M}^\star), \mathcal{M}^\star) &= 0 && (62) \\ F^e(m(\mathcal{M}^\star), n(\mathcal{M}^\star), e(\mathcal{M}^\star), \mathcal{M}^\star) &= 0 && (63) \end{aligned}$$

Using the chain rule for the derivatives, we get:

$$\frac{\partial F^m}{\partial m} m'(\mathcal{M}^\star) + \frac{\partial F^m}{\partial n} n'(\mathcal{M}^\star) + \frac{\partial F^m}{\partial \mathcal{M}^\star} = 0 \quad (64)$$

$$\frac{\partial F^n}{\partial m} m'(\mathcal{M}^\star) + \frac{\partial F^n}{\partial n} n'(\mathcal{M}^\star) + \frac{\partial F^n}{\partial n} e'(\mathcal{M}^\star) + \frac{\partial F^n}{\partial \mathcal{M}^\star} = 0 \quad (65)$$

$$\frac{\partial F^e}{\partial m} m'(\mathcal{M}^\star) + \frac{\partial F^e}{\partial n} n'(\mathcal{M}^\star) + \frac{\partial F^e}{\partial n} e'(\mathcal{M}^\star) + \frac{\partial F^e}{\partial \mathcal{M}^\star} = 0 \quad (66)$$

Rearranging, we can first express the derivatives of military spending and fertility as a function of the derivative of education investments:

$$m'(\mathcal{M}^\star) = \frac{\frac{\partial \delta^\star}{\partial \mathcal{M}^\star} \frac{\partial f^m}{\partial \delta^\star} \mathcal{C} + y \frac{\partial F^m}{\partial n} \frac{\partial F^n}{\partial e} e'(\mathcal{M}^\star)}{\frac{\mathcal{C}}{y} + \pi \delta \delta^\star \left(\mathcal{C} \frac{\eta_M}{m} \frac{\partial f^m}{\partial \delta^\star} + \frac{\eta_L}{n} \frac{\partial f^n}{\partial \delta^\star} \right)} \quad (67)$$

$$n'(\mathcal{M}^\star) = \frac{\frac{\partial \delta^\star}{\partial \mathcal{M}^\star} \frac{\partial f^n}{\partial \delta^\star} - y \frac{\partial F^m}{\partial m} \frac{\partial F^n}{\partial e} e'(\mathcal{M}^\star)}{\frac{\mathcal{C}}{y} + \pi \delta \delta^\star \left(\mathcal{C} \frac{\eta_M}{m} \frac{\partial f^m}{\partial \delta^\star} + \frac{\eta_L}{n} \frac{\partial f^n}{\partial \delta^\star} \right)} \quad (68)$$

Plugging this into equation (66), we can solve for the derivative of education with respect to enemy military capabilities:

$$e'(\mathcal{M}^\star) = \frac{\frac{\partial \delta^\star}{\partial \mathcal{M}^\star} \frac{\partial \mathcal{S}_L}{\partial \delta^\star} \frac{\mathcal{C}}{y} h}{\frac{\partial F^n}{\partial e} \frac{\partial \delta^\star}{\partial n} \frac{\partial \mathcal{S}_L}{\partial \delta^\star} h + \frac{\partial F^e}{\partial e} \left[\frac{\mathcal{C}}{y} + \pi \delta \delta^\star \left(\mathcal{C} \frac{\eta_M}{m} \frac{\partial f^m}{\partial \delta^\star} + \frac{\eta_L}{n} \frac{\partial f^n}{\partial \delta^\star} \right) \right]} \quad (69)$$

Recall that $F^e = \alpha \frac{\partial h(e,g)}{\partial e} \frac{\mathcal{C}(e,y)}{y} - (\alpha + \mathcal{S}_L(\delta^\star))h(e,g)$. Taking the partial derivative with respect to e and noting that $\partial \mathcal{C}/\partial e = y$, we have:

$$\frac{\partial F^e}{\partial e} = \alpha \frac{\partial^2 h}{\partial e^2} \frac{\mathcal{C}}{y} - \mathcal{S}_L(\delta^\star) \frac{\partial h}{\partial e} < 0 \tag{70}$$

Noting that $\partial \delta^\star / \partial n < 0$ and $\partial F^n / \partial e = n$, it is straightforward to see that the denominator of $e'(\mathcal{M}^\star)$ is negative and since the numerator is positive, we have:

$$e'(\mathcal{M}^\star) < 0 \tag{71}$$

Because $e'(\mathcal{M}^\star) < 0$, the expressions for $m'(\mathcal{M}^\star)$ and $n'(\mathcal{M}^\star)$ above immediately tell us that the derivative of military spending is lower than when education is at a corner solution, and that of fertility is greater than when education is null. Plugging the expression for $e'(\mathcal{M}^\star) < 0$ and substituting, it is possible to show that $m'(\mathcal{M}^\star) > 0$ and $n'(\mathcal{M}^\star) > 0$.

A.4 Proof of Corollary 1.

In this, we prove the existence, uniqueness, and stability of the Nash equilibrium in the Malthusian regime and derive a necessary condition for the same results when education is positive.

Existence, uniqueness and stability of the Nash equilibrium in the Malthusian regime. Both countries essentially choose their own military capabilities - via military spending and fertility choices – taking enemy military capabilities as given. Using the expression from military capabilities and the results above:

$$\mathcal{M}(\mathcal{M}^\star) = m(\mathcal{M}^\star)^{\eta_M} n(\mathcal{M}^\star)^{\eta_L} L^{\eta_M + \eta_L}$$

Taking the derivative with respect to enemy military capabilities:

$$\begin{aligned} \mathcal{M}'(\mathcal{M}^\star) &= \left(\eta_M \frac{m'(\mathcal{M}^\star)}{m(\mathcal{M}^\star)} + \eta_L \frac{n'(\mathcal{M}^\star)}{n(\mathcal{M}^\star)} \right) \mathcal{M}(\mathcal{M}^\star) \\ &= \frac{1}{\text{den}} \left(\frac{\eta_M}{m(\mathcal{M}^\star)} \text{num}_m + \frac{\eta_L}{n(\mathcal{M}^\star)} \text{num}_n \right) \mathcal{M}(\mathcal{M}^\star) \end{aligned}$$

Using expressions for den, num_m and num_n, we obtain after some straightforward algebra and substitutions:

$$\mathcal{M}'(\mathcal{M}^\star) = \underbrace{\frac{\pi \delta \delta^\star \left\{ \frac{\eta_M}{m} \frac{\partial f^m}{\partial \delta^\star} \mathcal{C} + \frac{\eta_L}{n} \frac{\partial f^n}{\partial \delta^\star} \right\}}{\mathcal{C}/y + \pi \delta \delta^\star \left\{ \frac{\eta_M}{m} \frac{\partial f^m}{\partial \delta^\star} \mathcal{C} + \frac{\eta_L}{n} \frac{\partial f^n}{\partial \delta^\star} \right\}}}_{\equiv \phi(\mathcal{M}^\star) < 1} \frac{\mathcal{M}(\mathcal{M}^\star)}{\mathcal{M}^\star} > 0$$

Note that we proved earlier that the term in curly bracket is always strictly positive, such that $\mathcal{M}'(\mathcal{M}^\star) > 0$, and $\phi(\mathcal{M}^\star) < 1$. Military capabilities therefore increase unambiguously in response to an increase of enemy military capabilities. Let us now study the sign of the second derivative and show it is negative, such that the function $\mathcal{M}(\mathcal{M}^\star)$ is strictly concave. To simplify notation, let us define:

$$\xi(\mathcal{M}^\star) = \pi\delta\delta^\star \left\{ \frac{\eta_M}{m} \frac{\partial f^m}{\partial \delta^\star} \mathcal{C} + \frac{\eta_L}{n} \frac{\partial f^n}{\partial \delta^\star} \right\}$$

such that:

$$\mathcal{M}'(\mathcal{M}^\star) = \frac{\xi(\mathcal{M}^\star)}{\mathcal{C}/y + \xi(\mathcal{M}^\star)} \frac{\mathcal{M}(\mathcal{M}^\star)}{\mathcal{M}^\star}$$

Taking the second derivative gives:

$$\mathcal{M}''(\mathcal{M}^\star) = \frac{\xi'(\mathcal{M}^\star) \cdot \mathcal{C}/y}{(\mathcal{C}/y + \xi(\mathcal{M}^\star))^2} \cdot \frac{\mathcal{M}(\mathcal{M}^\star)}{\mathcal{M}^\star} + \frac{\xi(\mathcal{M}^\star)}{\mathcal{C}/y + \xi(\mathcal{M}^\star)} \cdot \frac{\mathcal{M}'(\mathcal{M}^\star)\mathcal{M}^\star - \mathcal{M}(\mathcal{M}^\star)}{(\mathcal{M}^\star)^2}$$

Using the fact that

$$\begin{aligned} \mathcal{M}'(\mathcal{M}^\star)\mathcal{M}^\star - \mathcal{M}(\mathcal{M}^\star) &= (\phi(\mathcal{M}^\star) - 1)\mathcal{M}(\mathcal{M}^\star) \\ &= -\frac{C/y}{C/y + \xi(\mathcal{M}^\star)} \mathcal{M}(\mathcal{M}^\star) \end{aligned}$$

and substituting back into the second derivative and rearranging:

$$\mathcal{M}''(\mathcal{M}^\star) = \underbrace{\frac{C/y}{(C/y + \xi(\mathcal{M}(\mathcal{M}^\star))^2} \frac{\mathcal{M}(\mathcal{M}^\star)}{\mathcal{M}^\star}}_{>0} \left\{ \xi'(\mathcal{M}^\star) - \frac{\xi(\mathcal{M}^\star)}{\mathcal{M}^\star} \right\}$$

To show that $\mathcal{M}(\mathcal{M}^\star)$ is concave, we simply need to show that:

$$\xi'(\mathcal{M}^\star) - \frac{\xi(\mathcal{M}^\star)}{\mathcal{M}^\star} < 0 \Leftrightarrow \frac{\xi'(\mathcal{M}^\star)}{\xi(\mathcal{M}^\star)} < \frac{1}{\mathcal{M}^\star}$$

Recall that $\xi(\mathcal{M}^\star) = \pi\delta\delta^\star \left\{ \frac{\eta_M}{m} \frac{\partial f^m}{\partial \delta^\star} \mathcal{C} + \frac{\eta_L}{n} \frac{\partial f^n}{\partial \delta^\star} \right\}$. Let us use $\frac{\partial f^m}{\partial \delta^\star}$ and $\frac{\partial f^n}{\partial \delta^\star}$ and denote the term in curly bracket as:

$$u(\mathcal{M}^\star) = \frac{\mathcal{C}\frac{\eta_M}{m}\eta_M(\Gamma + \alpha) + \frac{\eta_L}{n}(\eta_L\Gamma - \eta_M\alpha)}{[\Gamma + \alpha + \mathcal{S}_L(\delta^\star) + \mathcal{S}_M(\delta^\star)]^2}$$

Because we showed above that $m'(\mathcal{M}^\star) > 0$ and $n'(\mathcal{M}^\star) > 0$ iff $\eta_L\Gamma - \eta_M\alpha > 0$, it is straightforward to see that $u'(\mathcal{M}^\star) < 0$. Using this and noting that $\delta\delta^\star = (1 - \delta^\star)\delta^\star$ we have:

$$\xi'(\mathcal{M}^\star) = \pi \frac{\partial \delta^\star}{\partial \mathcal{M}^\star} (1 - 2\delta^\star) u(\mathcal{M}^\star) + \pi(1 - \delta^\star)\delta^\star u'(\mathcal{M}^\star)$$

Let us finally show that:

$$
\begin{aligned}
\frac{\xi'(\mathcal{M}^\star)}{\xi(\mathcal{M}^\star)} &< \frac{1}{\mathcal{M}^\star} \\
\Leftrightarrow \frac{\pi\frac{\partial\delta^\star}{\partial\mathcal{M}^\star}(1-2\delta^\star)u(\mathcal{M}^\star) + \pi(1-\delta^\star)\delta^\star u'(\mathcal{M}^\star)}{\pi\delta\delta^\star u(\mathcal{M}^\star)} &< \frac{1}{\mathcal{M}^\star}
\end{aligned}
$$

Using the derivative of the Tullock contest success function, we get:

$$
\begin{aligned}
\Leftrightarrow \frac{\pi(1-2\delta^\star)}{\mathcal{M}^\star} + \frac{u'(\mathcal{M}^\star)}{u(\mathcal{M}^\star)} &< \frac{1}{\mathcal{M}^\star} \\
\Leftrightarrow \pi(1-2\delta^\star) + \underbrace{\frac{u'(\mathcal{M}^\star)}{u(\mathcal{M}^\star)}}_{<0} &< 1
\end{aligned}
$$

Because $1-\delta^\star < 1$ by definition and the term $u'(\mathcal{M}^\star)/u(\mathcal{M}^\star)$ is negative, a sufficient condition for the function $\mathcal{M}(\mathcal{M}^\star)$ to be strictly concave is therefore $\pi = 1$, which we assume in our baseline case. Because $\mathcal{M}(\mathcal{M}^\star)$ is strictly increasing and concave, the function $\mathcal{M}^\star(\mathcal{M})$ is also strictly increasing and concave by symmetry. Since $\mathcal{M}(0) = \mathcal{M}^\star(0) > 0$ by design of the Tullock contest, both curves intersect only once in the $(\mathcal{M}^\star, \mathcal{M})$ schedule as shown in Figure 2. There is therefore a unique Nash equilibrium. Furthermore, because $\mathcal{M}(\mathcal{M}^\star)$ intersects $\mathcal{M}^\star(\mathcal{M})$ from above, the Nash equilibrium is stable. **QED.**

Existence, uniqueness and stability out of the Malthusian regime. The expression for military capabilities remain unchanged out of the Malthusian regime:

$$
\mathcal{M}(\mathcal{M}^\star) = m(\mathcal{M}^\star)^{\eta_M} n(\mathcal{M}^\star)^{\eta_L} L^{\eta_M+\eta_L}
$$

Again, taking the derivative with respect to enemy military capabilities, we obtain after some substitutions:

$$
\begin{aligned}
\mathcal{M}'(\mathcal{M}^\star) &= \left(\eta_M \frac{m'(\mathcal{M}^\star)}{m(\mathcal{M}^\star)} + \eta_L \frac{n'(\mathcal{M}^\star)}{n(\mathcal{M}^\star)}\right)\mathcal{M}(\mathcal{M}^\star) \\
&= \frac{\xi(\mathcal{M}^\star) - \eta_L e'(\mathcal{M}^\star)\mathcal{M}^\star}{\mathcal{C}(\mathcal{M}^\star)/y + \xi(\mathcal{M}^\star)} \frac{\mathcal{M}(\mathcal{M}^\star)}{\mathcal{M}^\star}
\end{aligned}
$$

Compared to the Malthusian regime, notice the introduction of the derivative of education with respect to enemy military capabilities, and the fact that the cost of child rearing $\mathcal{C}$ is now function of enemy military capabilities as well. This complicates substantially the analysis of the concavity of the best response function $\mathcal{M}(\mathcal{M}^\star)$. After some further substitutions, we can write the above equation as:

$$\mathcal{M}'(\mathcal{M}^\star) = \frac{\frac{\partial \mathcal{S}_L}{\partial \delta^\star}\pi\eta_L\delta\delta^\star h - \frac{\partial F^e}{\partial e}\xi(\mathcal{M}^\star)}{\frac{\partial \mathcal{S}_L}{\partial \delta^\star}\pi\eta_L\delta\delta^\star h - \frac{\partial F^e}{\partial e}\left(\mathcal{C}(\mathcal{M}^\star)/y + \xi(\mathcal{M}^\star)\right)}\frac{\mathcal{M}(\mathcal{M}^\star)}{\mathcal{M}^\star}$$

Denote by $\mathcal{A}(\mathcal{M}^\star) = \frac{\partial \mathcal{S}_L}{\partial \delta^\star}\pi\eta_L\delta\delta^\star h - \frac{\partial F^e}{\partial e}\xi(\mathcal{M}^\star)$ and $\mathcal{B}(\mathcal{M}^\star) = -\frac{\partial F^e}{\partial e}\mathcal{C}(\mathcal{M}^\star)/y$. A necessary condition for the existence, uniqueness and stability of the Nash equilibrium is that the best response function of military capabilities is concave: $\mathcal{M}''(\mathcal{M}^\star) < 0$, or:

$$\frac{\mathcal{A}'(\mathcal{M}^\star)}{\mathcal{A}(\mathcal{M}^\star)} - \frac{\mathcal{B}'(\mathcal{M}^\star)}{\mathcal{B}(\mathcal{M}^\star)} < \frac{1}{\mathcal{M}^\star}$$

A.5 Optimal education level with functional forms

Using the functional form for the human capital production function defined in equation (31) and the equation characterizing the optimal education level, we get:

$$\alpha(\tau_0/y_t + \tau_1 + e_{t+1})g_{t+1} - (\alpha + \mathcal{S}_L(\delta^\star_{t+1}))(e_{t+1} + \rho(\tau_0/y_t + \tau_1))(e_{t+1} + \rho(\tau_0/y_t + \tau_1) + g_{t+1}) = 0$$

After rearranging, we obtain the following second-order polynomial:

$$e_{t+1}^2 + \left[2(\tau_0/y_t + \tau_1) + \left(1 - \frac{\alpha}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})}\right) g_{t+1}\right] e_{t+1} + (\rho(\tau_0/y_t + \tau_1))^2$$
$$+ \left(\rho - \frac{\alpha}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})}\right)(\tau_0/y_t + \tau_1)g_{t+1} = 0$$

Let us first write down the determinant:

$$\begin{aligned}
\Delta &= \left(2\rho(\tau_0/y_t + \tau_1) + \left(1 - \frac{\alpha}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})}\right) g_{t+1}\right)^2 \\
&\quad -4\left((\rho(\tau_0/y_t + \tau_1))^2 + \left(\rho - \frac{\alpha}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})}\right)(\tau_0/y_t + \tau_1)g_{t+1}\right) \\
&= \left(1 - \frac{\alpha}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})}\right)^2 g_{t+1}^2 + 4(\tau_0/y_t + \tau_1)(1-\rho)\frac{\alpha}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})} g_{t+1} > 0 \\
&= \Delta(\delta^\star_{t+1}, g_{t+1}, y_t) > 0
\end{aligned}$$

Because the determinant is always positive, there are two solutions to the above polynomial. Because $\left[2\rho(\tau_0/y_t + \tau_1) + \left(1 - \frac{\alpha}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})}\right) g_{t+1}\right] > 0$, the following root is always negative and is ruled out:

$$\frac{-\left[2\rho(\tau_0/y_t + \tau_1) + \left(1 - \frac{\alpha}{\alpha + \mathcal{S}_L(\delta^\star_{t+1})}\right) g_{t+1}\right] - \sqrt{\Delta(\delta^\star_{t+1}, g_{t+1}, y_t)}}{2} < 0$$

The optimal education level is therefore:

$$\frac{-\left[2\rho(\tau_0/y_t+\tau_1)+\left(1-\frac{\alpha}{\alpha+\mathcal{S}_L(\delta^\star_{t+1})}\right)g_{t+1}\right]+\sqrt{\Delta(\delta^\star_{t+1},g_{t+1},y_t)}}{2}\equiv e(\delta^\star_{t+1},g_{t+1},y_t)$$

Note that this level may also be negative, in which case the positivity constraint is binding such that:

$$e_{t+1}=\max\left\{0,e(\delta^\star_{t+1},g_{t+1},y_t)\right\}$$

Finally, we can characterize when $e(\delta^\star_{t+1},g_{t+1},y_t)$ turns positive:

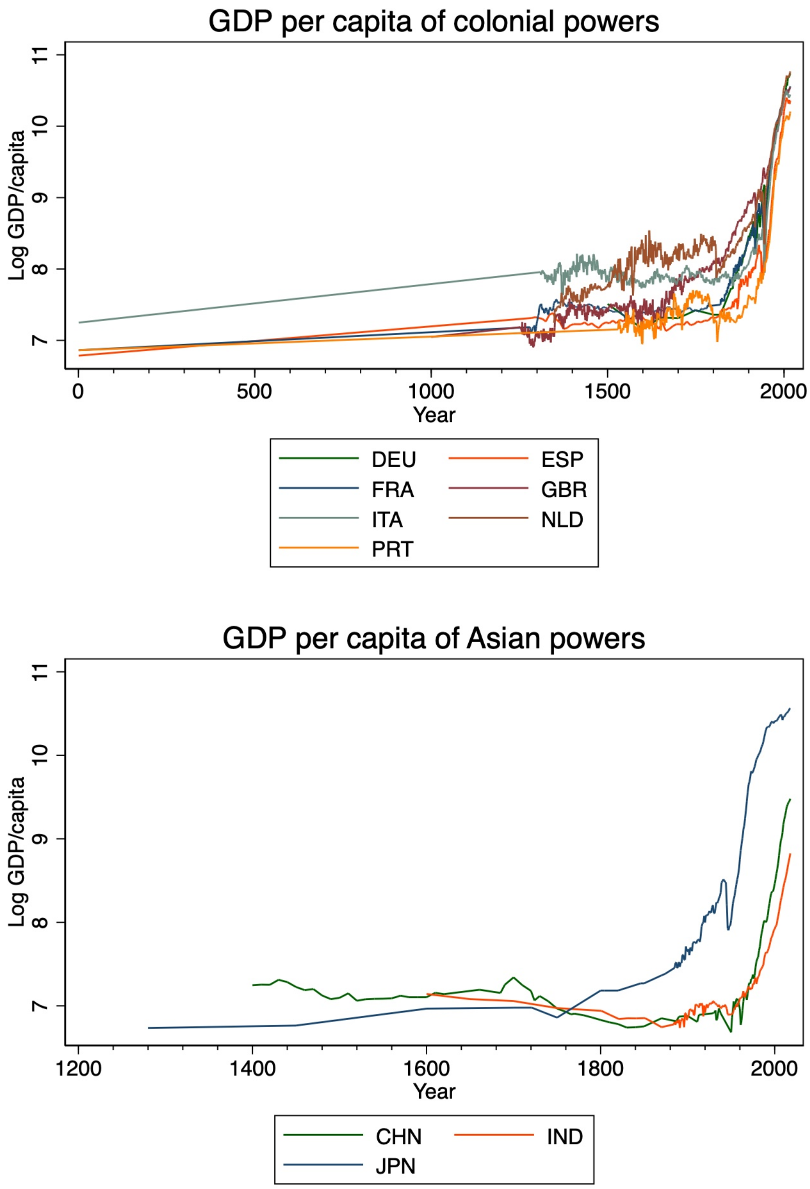
$$\begin{aligned} e(\delta^\star_{t+1},g_{t+1},y_t) &> 0 \\ (\rho(\tau_0/y_t+\tau_1))^2+\left(\rho-\frac{\alpha}{\alpha+\mathcal{S}_L(\delta^\star_{t+1})}\right)(\tau_0/y_t+\tau_1)g_{t+1} &< 0 \end{aligned}$$

which after some algebra gives us the following condition on the rate of technological progress:

$$g_{t+1}>\frac{\rho^2(\alpha+\mathcal{S}_L(\delta^\star_{t+1}))}{(1-\rho)\alpha-\rho\mathcal{S}_L(\delta^\star_{t+1})}(\tau_0/y_t+\tau_1)\equiv\hat{g}(\delta^\star_{t+1},y_t)$$

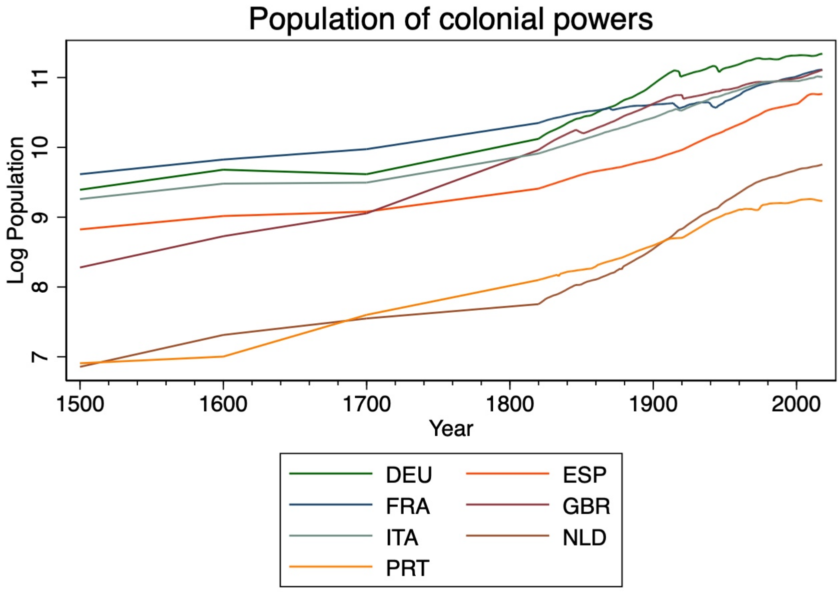
A.6 Additional data from the Maddison database

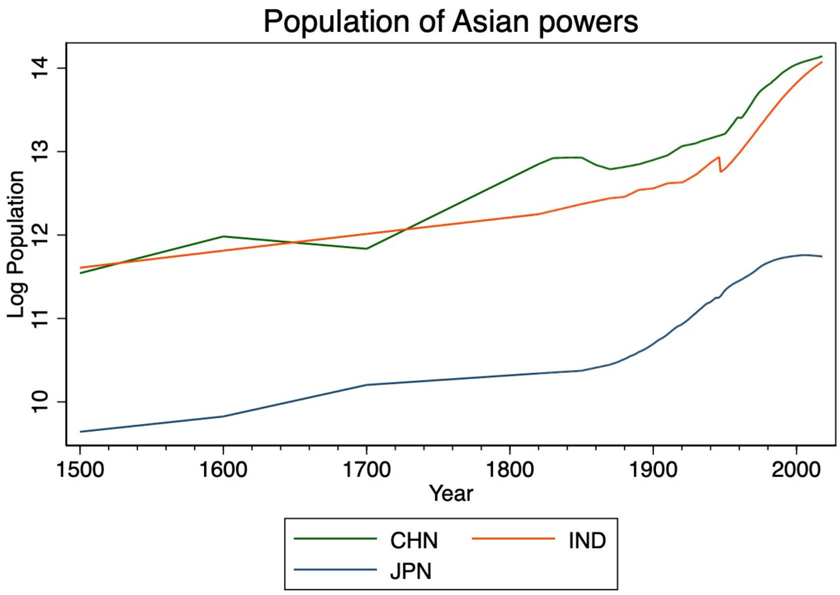
Figure 10: **GDP per capita, selected countries, log**



Source: Maddison Project database, 2020 (Bolt and Van Zanden, 2020; Palma and Reis, 2019; Pfister, 2022; Malanima, 2011; Baffigi, 2011; Alvarez-Nogal and De La Escosura, 2013; Prados De La Escosura, 2017; Van Zanden and Van Leeuwen, 2012; Smits et al., 2000; Bassino et al., 2019; Fukao et al., 2015; Ridolfi, 2017; Broadberry et al., 2010, 2015, 2018; Xu et al., 2017; Wu et al., 2014; Broadberry et al., 2015) and authors' calculation. .

Figure 11: **Population, selected countries, log**





Source: Maddison Project database, 2020 (Bolt and Van Zanden, 2020; Palma and Reis, 2019; Pfister, 2022; Malanima, 2011; Baffigi, 2011; Alvarez-Nogal and De La Escosura, 2013; Prados De La Escosura, 2017; Van Zanden and Van Leeuwen, 2012; Smits et al., 2000; Bassino et al., 2019; Fukao et al., 2015; Ridolfi, 2017; Broadberry et al., 2010, 2015, 2018; Xu et al., 2017; Wu et al., 2014; Broadberry et al., 2015) and authors' calculation.

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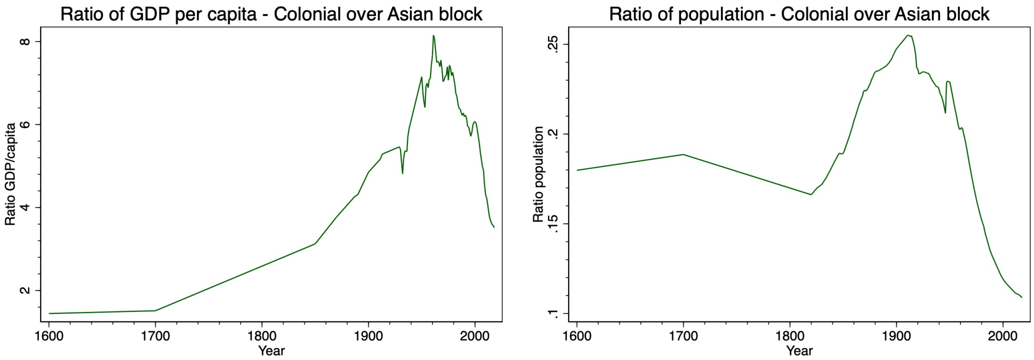
Figure 12: **Ratios of GDP per capita and population, Colonial powers (GBR, France, Germany, Portugal, The Netherlands and Italy) vs Asian block (India, China, Japan)**



Source: Maddison Project database, 2020 (Bolt and Van Zanden, 2020; Palma and Reis, 2019; Pfister, 2022; Malanima, 2011; Baffigi, 2011; Alvarez-Nogal and De La Escosura, 2013; Prados De La Escosura, 2017; Van Zanden and Van Leeuwen, 2012; Smits et al., 2000; Bassino et al., 2019; Fukao et al., 2015; Ridolfi, 2017; Broadberry et al., 2010, 2015, 2018; Xu et al., 2017; Wu et al., 2014; Broadberry et al., 2015) and authors' calculation.

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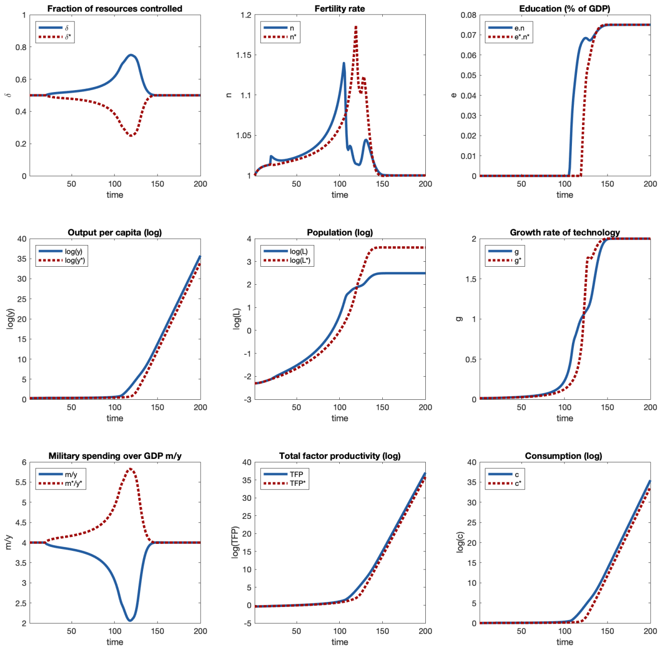
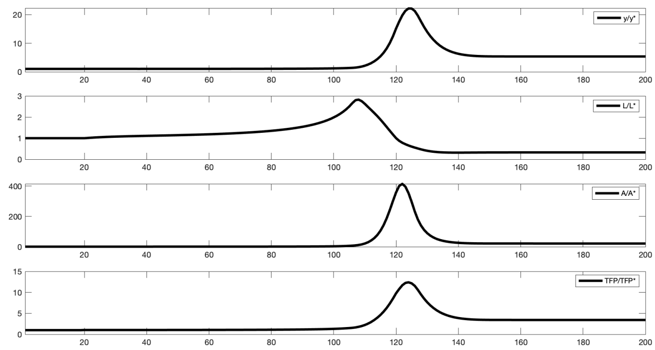
A.7 Additional simulations

Figure 13: **Long-run dynamics with an asymmetric technology shock (benchmark)**

Top chart: A 7.5% improvement of technology in country A in $t = 20$. (Scenario 1)



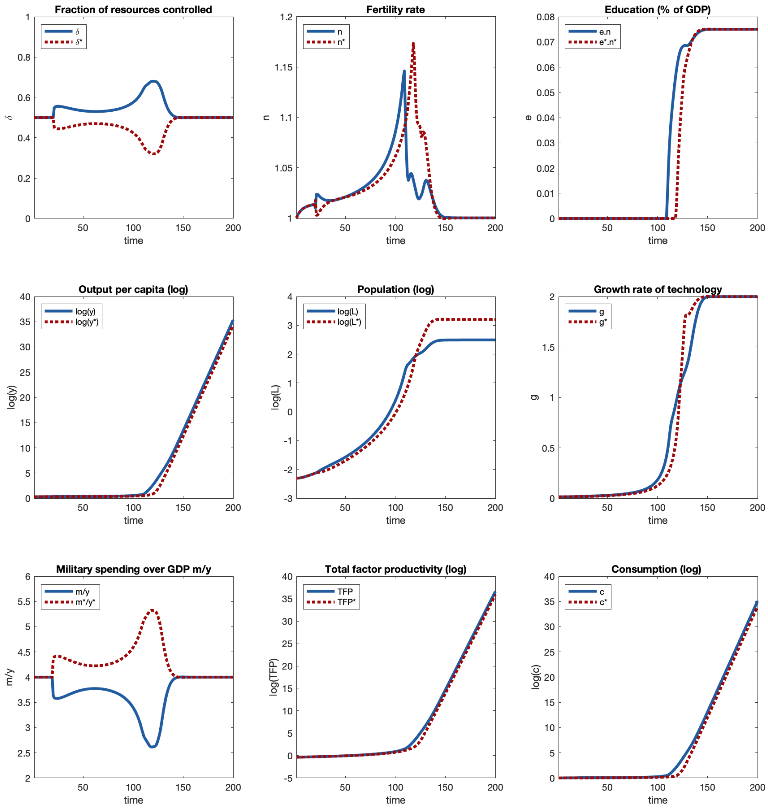
Bottom chart: ratios (Scenario 1)



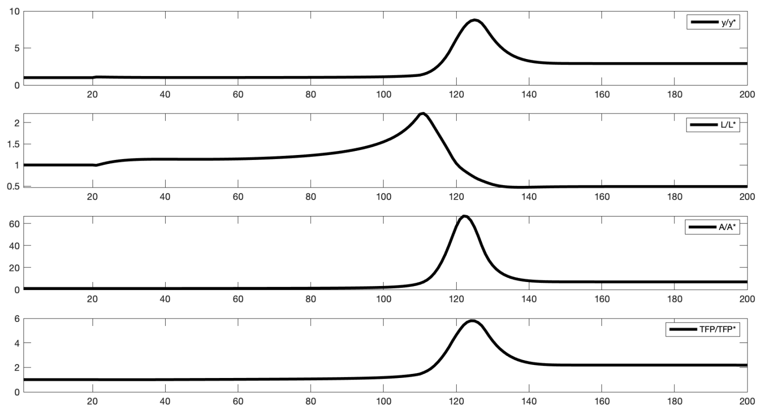
Notes. We plot e_{t+1} at the time of decision, at t

Figure 14: **Long-run dynamics with an asymmetric military shock**

Bottom chart: A 10% decline in military capacity of country B. (Scenario 3)



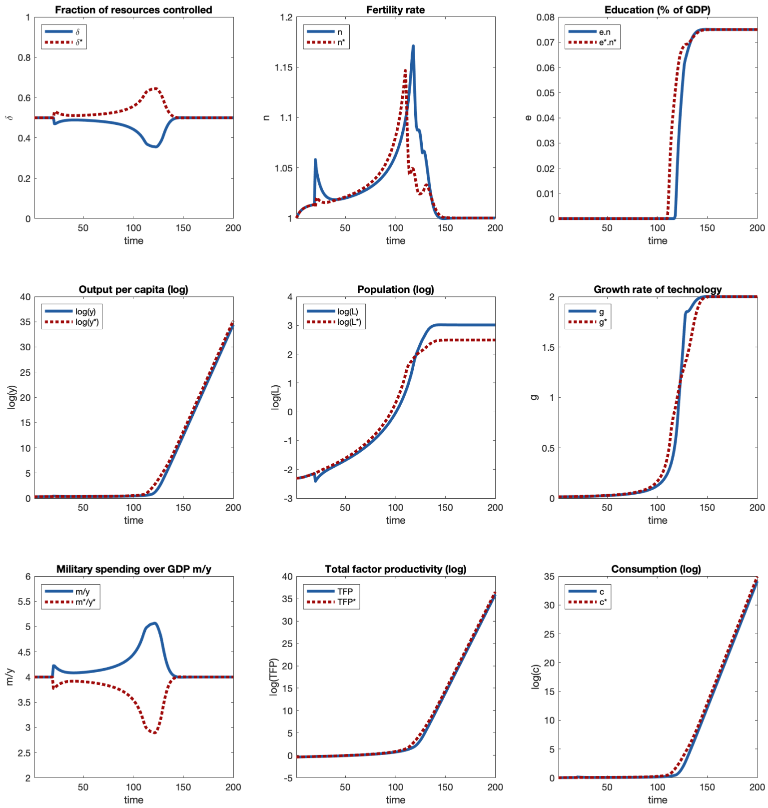
Bottom chart: ratios (Scenario 3)



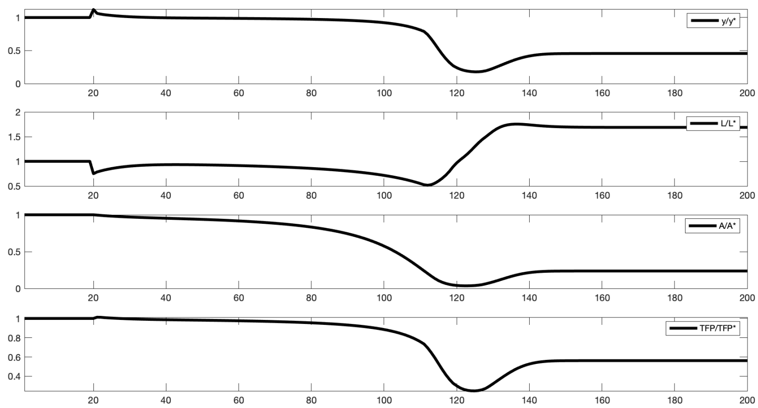
Notes. We plot e_{t+1} at the time of decision, at t

Figure 15: **Long-run dynamics with an asymmetric population shock**

Bottom chart: A 10% increase in population of country A. (Scenario 2)

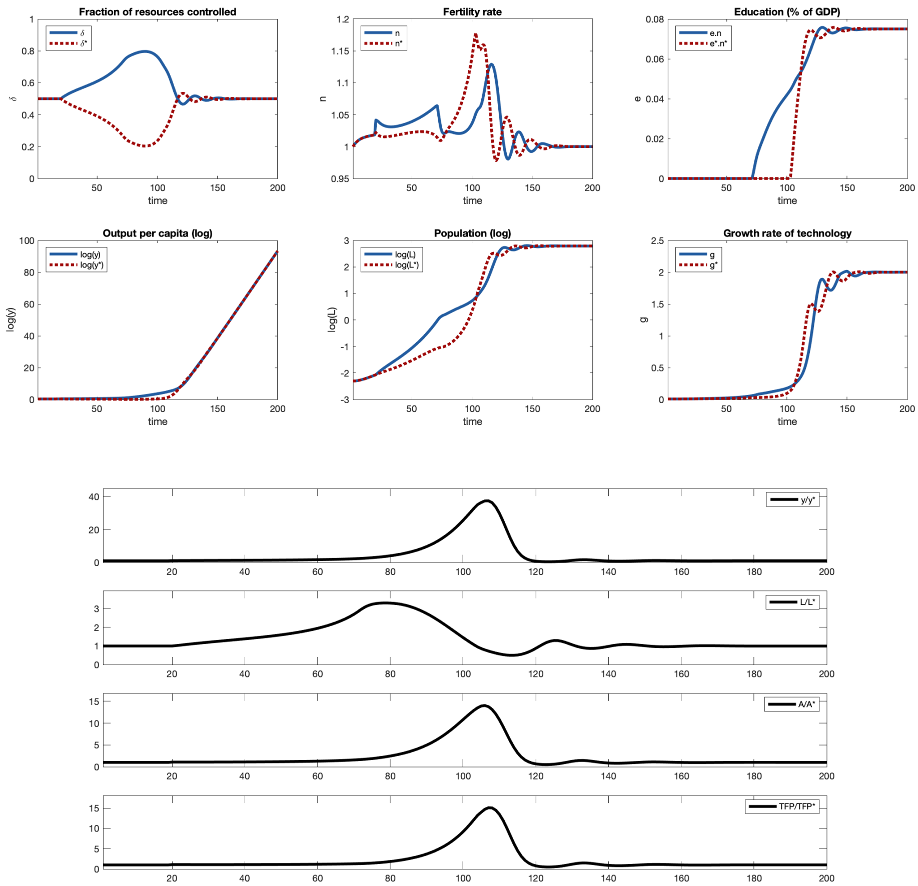


Bottom chart: ratios (Scenario 2)



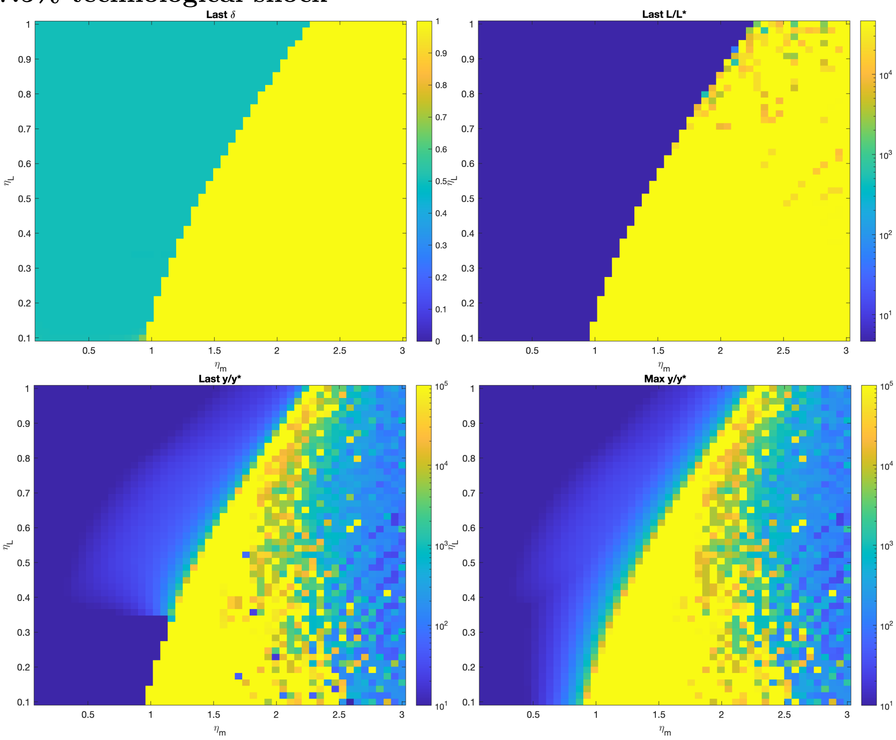
Notes. We plot e_{t+1} at the time of decision, at t

Figure 16: **Long-run dynamics with an asymmetric technology shock: attenuating cycles**



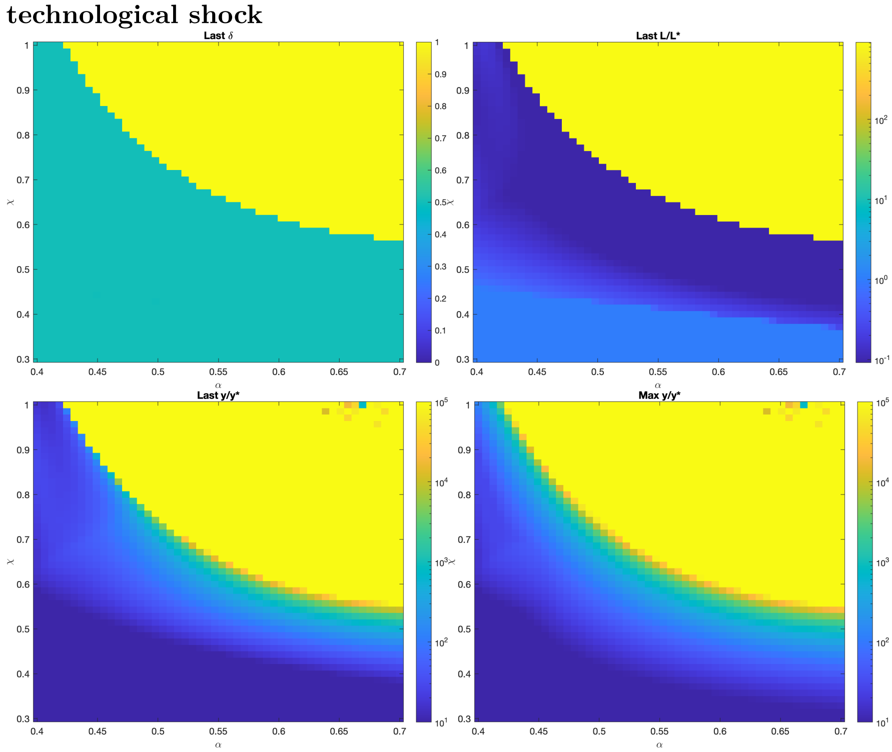
A 7.5% improvement of technology in country 'Blue' in $t = 20$, with $\alpha = .6$ and $\chi = 1$, recalibration of parameters and initial conditions to match moments of interest.

Figure 17: **Phase diagram: variations around the military capability coefficients η_M, η_L, case of a 7.5% technological shock**



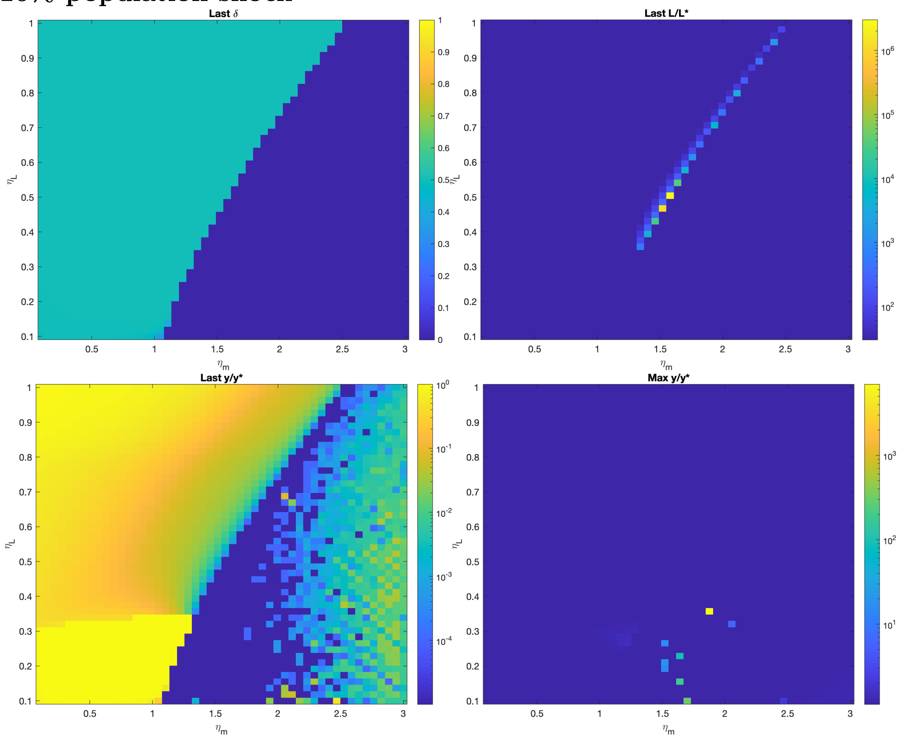
2500 simulations with various values for parameters. No recalibration, initial conditions calculated to reach a fertility rate of 1.

Figure 18: **Phase diagram: variations around the returns to scale coefficients α, χ, case of a 7.5% technological shock**



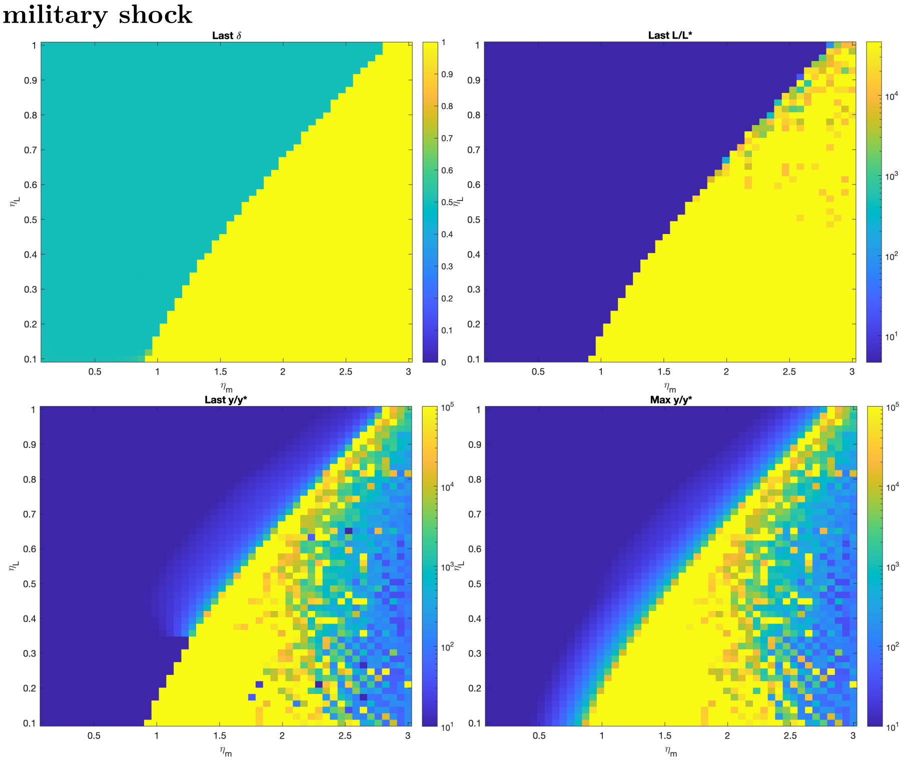
2500 simulations with various values for parameters. No recalibration, initial conditions calculated to reach a fertility rate of 1.

Figure 19: **Phase diagram: variations around the military capability coefficients η_M, η_L, case of a 10% population shock**



2500 simulations with various values for parameters. No recalibration, initial conditions calculated to reach a fertility rate of 1.

Figure 20: **Phase diagram: variations around the military capability coefficients η_M, η_L, case of a military shock**



2500 simulations with various values for parameters. No recalibration, initial conditions calculated to reach a fertility rate of 1.

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