Ad-platform competition under endogenous multihoming at both sides of the market*

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Abstract

Standard media economics models assume that consumers singlehome (they patronize a single platform). This assumption is questionable, not least since digitalization of media has made multihoming more prevalent than in the analog area. In a Hotelling model with two media platforms, we allow consumers as well as advertisers to multihome. In this framework, we analyze media platform competition for heterogeneous advertisers when consumers dislike ads. First, we show that the standard assumption of singlehoming consumers is dubious, since consumers will not singlehome in equilibrium unless competition on the consumer side of the market is weak (i.e., when transportation costs are high). Second, we show that even intrinsically symmetric platforms may have incentives to differentiate vertically, in the sense that they may choose different advertising levels when we open for multihoming consumers. If this constitutes an equilibrium behavior, there will be partial (incomplete) multihoming on both sides of the market. Remarkably, we find that advertising prices as well as platform profits may increase with the consumer disutility of ads in this equilibrium. The intuition is that the more consumers dislike ads, the higher is the number of singlehoming consumers. Since each platform has monopoly power over its singlehoming consumers in the advertising markets, these consumers are more valuable than those who multihome. Finally, we show that if platform competition is sufficiently intensive, then all consumers will multihome and all advertisers singlehome.

JEL Classification: D11, D60, L13.

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1 Introduction

Consumers choose whether to access media content from one or more platforms (TV-and radio-channels, online and printed newspapers, streaming services like podcasts, and so forth). Not surprisingly, a fraction, but not all, consumers are frequently observed to patronize more than one platform. Such partial multihoming on the consumer side of the market is not a new phenomenon. In the early twentieth century, 15% of US households read two or more newspapers on daily basis (see Gentzkow et al., 2014, who use survey data from 1917-1924). However, the development of digital media platforms has made multihoming much more attractive and accessible for consumers. Previously, consumers were for instance typically restricted to choose among a few printed newspapers distributed locally, but digitalization has in principle made it possible for consumers to access all media platforms worldwide by downloading an app or website (see e.g., discussion by Bakos and Halaburda, 2021).

Ad-financed platforms sell eyeballs to advertisers, and the value of an ad depends on the number of consumers it attracts and on whether the consumers can be reached elsewhere. For singlehoming consumers, each platform has monopoly power in offering these consumers to advertisers. In contrast, for multihoming consumers, platforms can at most charge advertisers the incremental value of the platform over the rival platform(s).[1] In the literature, this is labelled the incremental pricing principle (Ambrus et al., 2016; Anderson et al., 2012, 2018; Athey et al., 2018). The incremental value is smaller than the singlehoming price if, in the words of Gentzkow et al. (2021), there are diminishing returns with respect to duplication of impressions; if this is the case, media platforms charge advertisers less for multihoming consumers than for singlehoming consumers.[2] Empirical evidence indicates that this is the case. Gentzkow et al. (2014) estimate that advertising rates were lower for multihoming than singlehoming consumers in the twentieth century US newspaper market.[3] Using more recent data on US magazines, Shi (2016) estimates that an exclusive reader is worth twice as much as a multihomer.[4]

Gentzkow et al. (2021) generalize the incremental pricing principle and predict that a platform's ad price per consumer is lower the more "active" are the platform's consumers (where the activity level is assumed to be positively correlated with the extent to which the consumers visit other outlets). They find

[1] We consider purely ad-financed platforms, but many digital platforms, e.g., streaming platforms like HBO and Netflix earn their revenues from consumers. The principle of incremental pricing is also important in such one-sided markets. When consumers are multihoming, subscribing e.g., to both HBO and Netflix, the price that can be charged from a multihoming consumer is the incremental value of each platform (Anderson et al., 2017; Kim and Serfes, 2006). If HBO slightly reduces its price, some previously exclusive consumers of Netflix are turned into multihomers.

[2] See also Jeitschko and Tremblay (2020), Bakos and Halaburda (2021) and Belleflamme and Peitz (2019).

[3] The model of advertising competition used by Gentzkow et al. (2014) draws on Armstrong (2002) and unpublished versions of Ambrus et al. (2016), Anderson et al. (2018).

[4] Another recent empirical paper allowing for multihoming is Affeldt et al. (2021), analyzing the Italian newspaper market.

support for this prediction in data on television and social media advertising. On TV there is a premium ad-price for younger viewers, since these typically watch less TV (are less active on this media platform) than older viewers. On social media there is a premium price for older viewers, who are less active on social media. Consequently, the increase in multihoming consumers may be a crucial threat for ad-financed platforms (see e.g., discussion by Athey and Scott Morton, 2021).

We consider a model set-up where two ad-financed platforms compete for heterogeneous advertisers. On the consumer side, we follow Anderson et al. (2018). In contrast to the standard Hotelling (1929) set-up, consumers may multihome. Furthermore, we open for heterogeneous advertisers, which may either specialize on one or the other platform, depending on the strength of the individual advertiser's willingness to pay for communication, or else to buy ads on both platforms. Hence, we allow for singlehoming or multihoming on both sides of the market, with the outcome determined endogenously in the model.[5]

Mussa and Rosen (1978) develop a framework with singlehoming consumers, where competing firms offer products of different qualities. The consumers are assumed to differ by their willingness to pay for quality (see Anderson, de Palma, and Thisse, 1992, for a review). Gabszewicz and Wauthy (2003) extend the framework by allowing for "multihoming" consumers. The set-up of Gabszewicz and Wauthy is very naturally adapted to describe advertiser demand for reaching consumers across platforms delivering different numbers of singlehoming and multihoming consumers.

By combining the frameworks of Anderson et al. (2018), as described above, and Gabszewicz and Wauthy (2003), we have at the outset four possible equilibrium constellations in our model: both consumers and advertisers (partly or fully) multihome, only the consumers multihome, only the advertisers multihome, or both parties singlehome. We slim these down. First, if no consumers want to multihome, the only equilibrium has full multihoming among advertisers. This is a unique equilibrium if competition on the consumer side is weak (i.e., the distance disutility, the transportation cost, is sufficiently high).

Second, if the degree of competition on the consumer side is sufficiently strong - low transportation costs - there exists a pure strategy equilibrium where all consumers multihome and advertisers singlehome. If both platforms can deliver all eyeballs, there is no need for advertisers to multihome. All profit erodes for the platforms if the value of a second impression is zero. In summary, if on one side of the market all agents singlehome, agents on the other side will multihome.

Interestingly, if competition for consumers is relatively strong, there may also exist an asymmetric equilibrium with partial multihoming on both sides of the market. Advertising prices, and consequently platform profits, are strictly pos-

[5] One argument used for not allowing for multihoming on both sides of the market is that as long as all agents at one side multihome, there is no gain from multihoming for agents at the other side. However, this argument does not hold under partial multihoming, as emphasized by Bakos and Halaburda (2021), and we show that partial multihoming at both sides of the market may arise in equilibrium.

itive in the asymmetric equilibrium. The asymmetric equilibrium arises despite an assumption that platforms are symmetric with respect to intrinsic quality levels. From the consumers perspective, one of the platforms has "high quality" (because it has a low ad volume), while the other platform has "low quality" (a high ad volume). If the platforms had the same ad levels, they would compete profits down to zero (ref the Bertrand paradox), but through choosing different ad levels (quality levels) they are able to earn positive profit. This resembles the mechanism in the seminal paper by Shaked and Sutton (1982).

When characterizing the asymmetric equilibrium with partial multihoming on both sides, we find that a higher disutility of ads need not have negative impact on platform profit. Indeed, for the low-quality platform (the one with the higher ad volume) both profit and ad prices increase with disutility of ads (the same is true for the high-quality platform if the disutility of ads is sufficiently large). This is in sharp contrast to predictions in standard models of media economics (that do not allow for consumer multihoming). The intuition is, however, straightforward: A higher level of disutility of ads leads to less consumer multihoming. Other things equal this is an advantage for the platforms, since they can charge more for exclusive than for multihoming eyeballs on the advertising market. By the same token, a lower incremental value of the second good to consumers may enhance platform profits because the number of singlehoming consumers increases.

The rest of this article is organized as follows. In Section 2 we present the formal model and specify the consumer and advertising side of the market. In Section 3 we describe an equilibrium where all consumers single home, and in Section 4 we describe an equilibrium where all consumers multihome. In the former equilibrium, the advertisers will multihome, while they will singlehome in the latter. In Section 5 we derive and characterize the asymmetric equilibrium. Finally, in Section 6 we offer some concluding remarks.

2 The model

There are two advertising-financed media platforms, 1 and 2. Each chooses a price to charge to advertisers to display the ads that are included in its content. Ads are a nuisance to media consumers, who choose either one or both platforms (i.e., consumers *may* choose to multihome). A given set of ad levels on platforms might generate a base of exclusive consumers for each platform as well as a base of consumers common to both.

Throughout we assume that there is no benefit from reaching a particular consumer more than once; this implies that platforms are most interested in exclusive consumers. Athey et al. (2018) make a similar assumption, while Ambrus et al. (2016) and Anderson et al. (2018) allow for a positive incremental value from a second impression.[6]

[6] We believe that our qualitative results hold as long as the marginal advertiser benefit of an ad is decreasing in the number of impressions.

Let r_i denote the number of *exclusive* consumers on Platform i, and let r_c be the *common* (or shared) consumers across both platforms. The *total* number of consumers on platform i is $D_i = r_i + r_c$, $i = 1, 2$. The novelty of the present paper is to endogenize multihoming behavior on *both* sides of the market. We show that while some consumers and advertisers might prefer to singlehome, others could find it optimal to multihome. For what follows, we shall sometimes find it convenient to write out the consumer demand function as $r_i(a_1, a_2)$, $i = 1, 2, c$, where a_1 and a_2 denote the ad levels on platforms 1 and 2 respectively.[7] We describe the specific consumer multihoming model below.

2.1 Advertisers

The advertiser model follows the set-up of the model of Gabszewicz and Wauthy (2003) who extend the consumer model of vertical differentiation to include joint purchase. We purloin the model to describe advertiser demand across different platforms which may have some consumers in common.

Assume that advertisers are vertically differentiated with respect to their willingness to pay to contact consumers (as per Anderson and Coate, 2005, for example).[8] Let $\theta(r_i + r_c) - P_i$ denote advertiser θ's value from buying an ad slot on platform i alone when platform i sets ad price P_i. The parameter θ is uniformly distributed on the unit interval, $\theta \in [0, 1]$, so advertiser demand for each platform is linear. Allowing too for multihoming advertisers, $\theta(r_i + r_j + r_c) - P_i - P_j$ is the value of advertising on both platforms.

2.2 Consumers

We introduce a specific consumer model; a simplified version of the extended Hotelling model from Anderson et al. (2018) in the way that we do not consider location incentives. A crucial feature of the present model is that we allow for disutility of ads, while Anderson et al. (2018) assume that consumers are ad-neutral. The surplus of a consumer located at x from accessing only platform 1 or only platform 2 are given by respectively

$$u_1 = \Psi - tx - \gamma a_1 \text{ and} \tag{1}$$

$$u_2 = \Psi - t(1 - x) - \gamma a_2. \tag{2}$$

Here t is the "transportation" cost, Ψ is the reservation price, and γ is the nuisance per ad. All these parameters are positive. Platform 1 and 2 are located in 0 and 1, respectively, and we assume market coverage and market participation from both platforms. Consumers are uniformly distributed over the Hotelling line. If a consumer is multihoming the incremental surplus from

[7] Armstrong (2002) briefly deploys such a model, although without drawing out its broader conclusions for media economics.

[8] Athey et al. (2018) allow for heterogenous advertisers in a set up that allows for multihoming consumers, while Anderson et al. (2018) assume that all advertisers have an identical willingness to pay for ads.

the other product is $(\Psi - t|x - x_i|)\,\delta - \gamma a_i$ where $\delta \in [0,1]$. Here, δ reflects the incremental value for consumers of having a second variant.

When all consumers singlehome, $r_c = 0$, the location of the indifferent consumer defines the consumer demand for platform i:

$$r_i = \frac{1}{2} - \gamma\frac{a_i - a_j}{2t} \text{ where } i,j = 1,2;\ i \neq j;\ r_1 + r_2 = 1. \tag{3}$$

Note that the size of the audience on platform i is increasing in the rival's advertising volume ($dr_i/da_j > 0$). This is a common feature for media economics models with singlehoming consumers.

In contrast, if at least some of the consumers access both platforms ($r_c > 0$), the utility of a consumer who consume good 2 due to its incremental value over good 1 equals

$$u_{12} = u_1 + \{[\Psi - t(1-x)]\,\delta - \gamma a_2\}\,. \tag{4}$$

Clearly, consumer x will not access the second platform unless $[\Psi - t(1-x)]\,\delta \geq \gamma a_2$. The utility of a consumer who reads/views good 1 due its incremental value over good 2 is analogously given by $u_{21} = u_2 + \{[\Psi - tx]\,\delta - \gamma a_1\}$. We can then derive demand for good i from the location of the consumer who is indifferent between buying both goods and only that of the rival, i.e. $u_{ji} = u_i$:

$$D_i = r_i + r_c = \frac{1}{t}\left[\Psi - \kappa a_i\right] < 1, \tag{5}$$

where we make the following definition

$$\kappa \equiv \gamma/\delta$$

It is then straightforward to show that:

$$r_i = 1 - \frac{1}{t}\left(\Psi - \kappa a_j\right) \text{ and } r_c = \frac{1}{t}\left(2\Psi - \kappa\left(a_1 + a_2\right)\right) - 1. \tag{6}$$

Under both SHC (singlehoming consumers) and MHC (multihoming consumers), it is clear that the platform with more ads has fewer consumers. This follows from (3) and (6) which show that $r_i > r_j$ if $a_i < a_j$. Equation (5) reveals $dD_i/da_i < 0$ and $dD_i/da_j = 0$, such that the number of consumers patronizing platform i is decreasing in its own advertising level but independent of the rival's advertising level. Equation (6) further reveals that $dr_i/da_j > 0$ and $dr_i/da_i = 0$. The fact that r_i only depends on the rival's advertising level might seem surprising. However, it is a fundamental feature when demands are based on incremental value (see Anderson et al., 2018).

Equations (3) and (6) reflect that the platforms provide symmetric intrinsic quality levels from the consumers' perspective. This is why $r_i = r_j$ if the platforms have equal numbers of ads. As soon as one platform has more ads, $a_i > a_j$, its rival will have more consumers, $r_j > r_i$. The intuition is straightforward: if media consumers dislike ads, they will (other things equal) perceive the platform with the lower advertising volume as more attractive than its rival. Without loss of generality, we shall henceforth assume:

Assumption 1: *If the platforms have different ad levels, then platform 2 will be the one with the greater number of exclusive consumers and the smaller ad level:*

$$r_2 > r_1 \text{ if } a_2 < a_1 \text{ and } r_1 = r_2 \text{ if } a_1 = a_2.$$

2.3 Equilibrium concept

The equilibrium concept is this. First, platform set prices *per ad*, and an advertiser paying that ad price accesses all consumers on the platform. We further make the (in our view) reasonable assumption that consumers do not observe ad prices. This implies that consumers do not investigate ad prices in order to deduce the ad level in a specific program before choosing whether to watch. Instead, the ad levels are rationally anticipated. Advertisers do (obviously) observe the ad price, and they rationally and correctly anticipate the consumers on each channel/platform.

The equilibrium concept is showcased next for the relatively simple and most familiar case with full singlehome among consumers (FSHC).

3 singlehoming consumers (FSHC)

To illustrate how the equilibrium works – and also to start the analysis with a central case in the literature – suppose that consumers all single home (they choose the better channel for them but none choose both). Later, we will derive the conditions that ensure that this constitutes an equilibrium.

Under FSHC – either by assumption or as an equilibrium outcome – competition for advertisers is closed down, and we have the monopoly bottleneck problem (Armstrong, 2002; 2006). The assumption of singlehoming consumers is made in most of the media economics literature, and is consistent with standard discrete choice models of consumer choice, for example, including the linear city of Hotelling (1929) and circle models (Salop, 1979; Vickrey, 1964).

Suppose that r_1 consumers are expected on Platform 1 and r_2 on Platform 2. Given these expectations, advertisers will buy access to either or both platforms as long as their value per consumer times the expected number of consumers are at least equal to the price per ad charged by the platform. When all consumers singlehome, the decision on each platform is independent of the decision on the other. Therefore the platforms will charge monopoly access prices. That is, the monopoly quantity – number of advertisers – will be attained by setting the ad price to the number of consumers times the willingness to pay of the marginal advertiser (the monopoly "quantity"). This latter calculus ties down how many ads per channel there are.

More formally, the profit to an advertiser of type θ is $\theta r_i - P_i$ if it places an advert on platform i, which has set a per ad price P_i, and is expected to deliver r_i media consumers. The advertiser nets $\theta(r_i + r_j) - P_i - P_j$ if it advertises on both platforms. Since the advertising decision is separable and independent across platforms, the advertiser will place its ad on platform i if and only if $\theta r_i \geq P_i$ (where we break indifference in favor of placing an ad). The problem facing Platform i is then to maximize the revenue from ads, which is the product of the mass of advertisers it attracts and the price per ad:

$$\max_{P_i} \pi_i = P_i \left(1 - \frac{P_i}{r_i}\right),$$

where $\frac{P_i}{r_i}$ is simply the price paid per (expected) media consumer. For given r_i, this is a simple monopoly problem. Thus each platform sets a monopoly price per consumer, and the ad price is this times the mass of consumers so $\{P_i, a_i\} = \left\{\frac{r_i}{2}, \frac{1}{2}\right\}$. This means that ad levels and prices are strategically independent and also independent of consumer attitudes to ads. This is to be contrasted with the results in, e.g., Anderson and Coate (2005) and Armstrong (2006), where advertising prices are observed not only by the advertisers but also by the consumers. The latter means that if a platform increases its advertising price, the consumers will be aware of this and can deduce that the platform will have fewer ads. Then the platforms will trade-off the effect of a lower advertising volume with that of attracting a larger number of consumers (if consumers dislike ads). This trade-off does not exist in our case, since we assume that consumers do not observe ad prices.

To verify FSHC as an equilibrium we need to check that no consumer wants to access a second platform. The consumer who benefits most from multihoming is the one at location $x = \frac{1}{2}$. From (4), her incremental value is $[\Psi - t/2]\,\delta - \gamma/2$ when $a = 1/2$, so the condition for the FSHC-case to be an equilibrium is that this incremental value be negative. We can state:

Lemma 1: *If $t \geq t^{FSHC} \equiv 2\left(\Psi - \frac{\kappa}{2}\right)$ then there exists a symmetric FSHC equilibrium ($r_c = 0$). All advertisers of types $\theta \geq 1/2$ are multihoming (MHA, where $a_1 = a_2 = 1/2$)* and *ad prices are $P_i = \frac{r_i}{2}$, with $r_i = 1/2$ for $i = 1, 2$.*

4 Full multihoming consumers (FMHC)

The opposite (book-end) extreme case is when all consumers multihome (which we will call FMHC). When there are no singlehoming consumers, the advertisers can reach any given consumer on either of the channels. This situation generates Bertrand competition between platforms and so the ad price goes down to 0 as the service offered by each platform is exactly the same. Taken at face value, this implies that advertisers do not care whether they buy ads from only one or from both platforms because the price is nothing. This is an artefact of the assumption that the marginal cost of inserting an ad for a platform is zero. To generalize, we shall therefore assume that the advertisers split (and an equal split is natural) as this would be the case at any positive and equal ad price. We thus arrive at the result that all advertisers singlehome (FSHA). It therefore book-ends the first case we presented, where consumers are singlehoming and (all active) advertisers are multihoming. Put another way, the insight is that if one side singlehomes, the other side multihomes.

We now need to find on the consumer side when it is that they all want to multihome at the purported equilibrium, which has half the advertisers on each platform. The consumer who benefits least from multihoming is the one at location $x = 0$. From (4), her incremental value is $[\Psi - t]\,\delta - \gamma/2$ when $a = 1/2$, so the condition for the FMHC-case to be an equilibrium is that this incremental value be positive:

Lemma 2: *If $t < t^{FMHC} \equiv \Psi - \frac{\kappa}{2}$ then there exists a symmetric FMHC equilibrium ($r_c = 1$). All advertisers advertise, half of them on each platform ($a_1 = a_2 = 1/2$)* and *ad prices are $P_i = 0$.*

From Lemmas 1 and Lemma 2 we can conclude that for $t \in \left(t^{FMHC}, t^{FSHC}\right)$ there neither exists an equilibrium where all consumers multihome nor one where all consumers singlehome. The length of this interval is $\Psi - \frac{\kappa}{2}$.

5 Partial multihoming consumers (PMHC)

The remaining equilibrium type involves some (but not all) consumers multihoming. On the advertiser side, we shall show that there can be only one equilibrium configuration. This we do by first ruling out all the other possible advertiser configurations, and by deriving conditions which ensure that the remaining configuration constitutes an equilibrium. At the outset we have four possible pure-strategy equilibria on the advertiser side of the market:

◦ $\{B, 0\}$: the highest θ-types advertise on both platforms and the rest do not advertise at all

◦ $\{B, i, 0\}$: the highest θ-types advertise on both platform, the next θ-types only on platform i, and the rest do not advertise at all

◦ $\{B, 1, 2, 0\}$ or alternatively $\{B, 2, 1, 0\}$: the highest θ-types advertise on both platform, the next θ-types advertises only on platform 1 (resp. 2), the next only on platform 2 (resp. 1), and the rest do not advertise at all.

From Lemma 1 above, we know that under FSHC we have an $\{B, 0\}$ equilibrium candidate where $r_1 = r_2$ and $a_1 = a_2$. We now show the following:

Lemma 3: *Assume that consumer demand satisfies $r_2 > r_1$ as $a_2 < a_1$, and $r_2 = r_1$ as $a_2 = a_1$. (i) There can be no symmetric $\{B, 0\}$ equilibrium if* $0 < r_c < 1$ (PMHC); (ii) *there can be no $\{B, i, 0\}$ equilibrium; (iii) there can be no* $\{B, 2, 1, 0\}$) equilibrium.

Proof. (i) We prove by contradiction that there can be no $\{B, 0\}$ equilibrium with $r_c > 0$. Suppose there were such a $\{B, 0\}$ equilibrium. The advertiser who is indifferent between advertising at both platforms and only at platform j (θ_{Bj}) is defined by $\theta_{Bj} r_i - P_i = 0$. However, if this advertiser only advertises on platform i, its profits will be $\theta_{Bj}(r_i + r_c) - P_i > 0$. Thus, it cannot be optimal to advertise on both platforms, so $\{B, 0\}$ cannot be an equilibrium with $r_c > 0$.

(ii) We prove by contradiction that there can be no $\{B, i, 0\}$ equilibrium for $r_c \geq 0$. Suppose that there were an equilibrium where the top θ-types advertised on both platforms, the next tranche advertises only on platform i, and the lowest not at all. The advertiser which is indifferent between advertising on Platform i and not advertising at all is given by $\theta_{0i}(r_i + r_c) - P_i = 0$, while the advertiser which is indifferent between buying at only platform i and at both platforms is given by $\theta_{iB} r_j - P_j = 0$ (the marginal benefit from also advertising at platform j is equal to the advert price). We then have $a_i = 1 - \frac{P_i}{r_i + r_c}$ and $a_j = 1 - \frac{P_j}{r_j}$. Solving $P_i = \arg\max \pi_i$ yields $\{P_i, a_i\} = \left\{\frac{r_i + r_c}{2}, \frac{1}{2}\right\}$. For Platform j we likewise find $\{P_j, a_j\} = \left\{\frac{r_j}{2}, \frac{1}{2}\right\}$. We thus have $a_i = a_j$, such that all active advertisers would choose to advertise at both platforms, which is a contradiction for any $r_c \geq 0$.

(iii) From the analysis above there remain only two possible equilibrium candidates, $\{B, 2, 1, 0\}$ and $\{B, 1, 2, 0\}$. Both imply that the lower-type advertisers singlehome. However, the latter cannot be an equilibrium because then the media product with the larger number of consumers would have the lower advertising price. ■

Therefore all other types are ruled out and we have the result:

Lemma 4: *Suppose that consumer demand satisfies $r_2 > r_1$ as $a_2 < a_1$ and $r_2 = r_1$ as $a_2 = a_1$. For $r_c \in (0, 1)$ (PMHC) the only candidate equilibrium is* $\{B, 2, 1, 0\}$.

5.1 Asymmetric equilibria with partial multihoming consumers

Given Lemma 4, we are left with (at most) $\{B, 2, 1, 0\}$ as the candidate equilibrium profile for advertisers consistent with some consumers multihoming. If this equilibrium exists, it is necessarily asymmetric. We now characterize the candidate equilibrium and then show parameters for which it exists.

The lowest marginal advertiser, which is indifferent between buying from 1 and not at all, is at

$$\theta_{01} = \frac{P_1}{r_1 + r_c}.$$

The next marginal type is indifferent between buying only from platform 1 and buying only from platform 2,

$$\theta_{12} = \frac{P_2 - P_1}{r_2 - r_1}.$$

Finally, the type indifferent between buying 2 alone and buying on both platforms follows the incremental pricing condition, so

$$\theta_{2B} = \frac{P_1}{r_1}.$$

From these expressions the demand for each platform - the advertising levels - are readily derived as

$$a_1 = (1 - \theta_{2B}) + (\theta_{12} - \theta_{01}),$$

while platform 2 has ads from all types down to θ_{12} so that

$$a_2 = 1 - \theta_{12}.$$

Platform profit is $\pi_i = P_i a_i$, $i = 1, 2$. Since $a_2 = 1 - \theta_{12}$, platform 2's profit may be rewritten as

$$\pi_2 = P_2 \left(1 - \frac{P_2 - P_1}{r_2 - r_1}\right)$$

The first-order condition, $\partial\pi_2/\partial P_2 = 0$, is given by

$$1 - \frac{2P_2 - P_1}{r_2 - r_1} = 0.$$

We find the reaction function (the second-order condition is negative) as

$$P_2 = \frac{r_2 - r_1}{2} + \frac{P_1}{2}. \tag{7}$$

The last term is the classic 50 cents on the dollar property in the reaction function. This reaction function for platform 2 resembles the reaction function for the top firm without multihoming. This makes sense because the multihomers are simply in the top of 2's range, and are not marginal.

Proceeding likewise for Platform 1 we have

$$a_1 = (1 - \theta_{2B}) + (\theta_{12} - \theta_{01}),$$

and profit then becomes

$$\pi_1 = P_1 \left(1 - \frac{P_1}{r_1} + \frac{P_2 - P_1}{r_2 - r_1} - \frac{P_1}{r_1 + r_c}\right).$$

From the first-order condition, $\partial\pi_1/\partial P_1 = 0$, we have (the second-order condition is negative)

$$1 - \frac{2P_1}{r_1} + \frac{P_2 - 2P_1}{r_2 - r_1} - \frac{2P_1}{r_1 + r_c} = 0.$$

This reaction function is more elaborate because of the extra margins at which consumers are picked up. Inserting the expression for P_2 from (7) yields the equilibrium prices as

$$P_1 = \frac{3}{\Omega} \tag{8}$$

$$P_2 = \frac{1}{2}P_1 + \frac{r_2 - r_1}{2} = \frac{3}{2\Omega} + \frac{r_2 - r_1}{2}, \tag{9}$$

where $\Omega = \left(\frac{4}{r_1} + \frac{3}{r_2 - r_1} + \frac{4}{r_1 + r_c}\right)$. This last term reflects the various margins of demand pick-up. Clearly, both prices are homogenous of degree 1: doubling all consumer segments simply doubles equilibrium prices. The composition effects of consumer bases can best be understood by normalizing $r_2 + r_1 + r_c = 1$. We can then substitute out r_c and perform comparative statics on the equilibrium ad prices.

Inserting for (8) and (9) into the demand functions for ads we have

$$a_1 = 1 - \frac{3}{\Omega r_1} + \frac{1}{2} - \frac{3}{2\Omega\left(r_2 - r_1\right)} - \frac{3}{\Omega\left(1 - r_2\right)} \tag{10}$$

$$a_2 = \frac{1}{2} + \frac{3}{2\Omega\left(r_2 - r_1\right)} \tag{11}$$

Hence, we can find the advertising difference as

$$a_1 - a_2 = 1 - \frac{3}{\Omega r_1} - \frac{3}{\Omega\left(r_2 - r_1\right)} - \frac{3}{\Omega\left(1 - r_2\right)} \in (0, 1).$$

The sign of $a_1 - a_2$ follows from the sign of $\left(\frac{4}{r_1} + \frac{3}{r_2 - r_1} + \frac{4}{1 - r_2}\right) - \left(\frac{3}{r_1} + \frac{3}{r_2 - r_1} + \frac{3}{1 - r_2}\right)$ and so must be positive, as indeed is stipulated in Assumption 1.

By inserting (10) and (11) into (6) (and recalling that $\kappa \equiv \gamma/\delta$) we find:

$$r_1 = \frac{3t - \Psi + S}{10t} \text{ and } r_2 = \frac{r_1}{2} + \frac{t - (\Psi - \kappa)}{2t} = \frac{13t - 11\Psi + 10\kappa + S}{20t}, \tag{12}$$

with $S \equiv \sqrt{20\kappa\left(3\Psi - t - \kappa\right) - 6\Psi t - 39\Psi^2 + 49t^2}$, where the root is a real number if $t \geq t^L$, where

$$t^L \equiv \frac{\left(8\sqrt{30} + 3\right)\Psi - \left(6\sqrt{30} - 10\right)\kappa}{49}. \tag{13}$$

The restriction $t > t^L \approx 0.96\Psi - 0.47\kappa$ reflects the fact that if t is sufficiently small, competition will be so fierce that all consumers multihome. From (12) we further find that $r_1|_{t=t^L} = \frac{1}{5t^S}\left(\Psi - \frac{3}{4}\kappa\right)\left(12\sqrt{30} - 20\right)$ and $r_2|_{t=t^L} = 1 - \frac{7\left(\Psi - \frac{3}{4}\kappa\right)\left(4\sqrt{30}+40\right)}{10t^S}$. At $t = t^L$ and $\Psi = \frac{3}{4}\kappa$ we thus have $r_1 = 0$ and $r_2 = 1$. This constitutes the lower bound for the $\{B, 2, 1, 0\}$ area.

For $\{B, 2, 1, 0\}$ to be an equilibrium, we must further have $\theta_{01} < \theta_{12} < \theta_{2B} < 1$. Inserting for (8) and (9) into the expressions for the θ's we find

$$\begin{aligned}
\theta_{01}-\theta_{12} &= -\frac{(r_2-r_1)(2-r_1-2r_2)}{5r_1r_2-r_1+4r_2-4r_1^2-4r_2^2}<0 \\
\theta_{12}-\theta_{2B} &= -\frac{(r_2-r_1)(r_c-r_1)}{5r_1r_2-r_1+4r_2-4r_1^2-4r_2^2}<0 \text{ for } r_c>r_1 \\
\theta_{2B}-1 &= -\frac{2r_1(1-r_1)+r_2(1-r_2)+2r_1(r_2-r_1)}{5r_1r_2-r_1+4r_2-4r_1^2-4r_2^2}<0.
\end{aligned}$$

We thus see that $\theta_{01}<\theta_{12}$ and $\theta_{2B}<1$ are always satisfied. The condition $\theta_{12}<\theta_{2B}$, however, requires that

$$r_c-r_1=\frac{(3\Psi-t-2\kappa)-S}{4t}>0. \tag{14}$$

It can be shown that (14) holds if $t<t^H$, where

$$t^H\equiv\Psi-\frac{\kappa}{2}.$$

The final requirement for $\{B,2,1,0\}$ to be an equilibrium candidate is that $r_2>r_1$ (or equivalently $a_2<a_1$) at $t=t^H$, and from equation (12) we find that this is true if $\Psi<2\kappa$.

We can state:

Lemma 5: *Necessary and sufficient conditions for the existence of an asymmetric equilibrium are that* $t\in\left(t^L,t^H\right)$ *and* $\Psi\in\left(\frac{3}{4}\kappa,2\kappa\right)$. *Some, but not all, of the agents on each side of the market multihome in the asymmetric equilibrium (PMHC-PMHA).*

Remarkably, t^H coincides with t^{FMHC}. From Lemma 1-5, we can thus conclude:

Proposition 1: *Suppose that*

- $t>t^{FSHC}$. *Then there exists a unique equilibrium in pure strategies where all consumers singlehome and all advertisers multihome (FSHC-FMHA)*
- $t\in\left(t^H,t^{FSHC}\right)$. *Then there does not exist any equilibrium in pure strategies. The length of this interval is equal to* $\left\|t^{SHV}-t^H\right\|=\Psi-\frac{\kappa}{2}$.
- $t<t^H$. *Then there exists an equilibrium in pure strategies with full multihoming on the consumer side and singlehoming on the advertiser side (FMHC-FSHA). This equilibrium is unique if* $\Psi\notin\left(\frac{3}{4}\kappa,2\kappa\right)$. *If* $\Psi\in\left(\frac{3}{4}\kappa,2\kappa\right)$, *then there also exists an equilibrium in pure strategies where some consumers and some advertisers multihome (PMHC-PMHA);* $\{B,2,1,0\}$.

Advertising prices are strictly positive in the asymmetric equilibrium, and so are consequently profits. The area where we have can have an asymmetric equilibrium is restricted. This follows naturally from our assumption that the platforms provide symmetric intrinsic quality levels in the consumers perspective. What is surprising is that we in fact find such an equilibrium with this strong restriction, and that it partly overlaps with the zero profit equilibrium where all consumers multihome.

Figure 1, which measures the number of exclusive consumers on the vertical axis and transportation costs on the horizontal axis, illustrates Proposition 1. To ensure that there exists an asymmetric equilibrium for some levels of transportation costs, we have chosen parameter values such that $\Psi \in \left(\frac{3}{4}\kappa, 2\kappa\right)$. More precisely, we have set $\kappa = 2.5$ (with $\delta = 0.4$ and $\gamma = 1$) and Ψ at the upper limit $\Psi = 2\kappa - \varepsilon = 5 - \varepsilon$. With these parameter values, we have a unique equilibrium where all consumers multihome ($r_1 = r_2 = 0$) for $t < t^L \approx 3.60$. This equilibrium holds up to $t = t^H \approx 3.75$, but for $t \in (3.60, 3.75)$ there also exists an asymmetric equilibrium with $r_2 > r_1 > 0$. The asymmetric equilibrium yields positive profits for the platforms, and so they clearly prefer this one to the FMHC-SHA-equilibrium, where the advertising price (and thus the profit level for the platforms) is equal to zero.

Figure 1 also illustrates that there does not exist any equilibrium in pure strategies for $t \in (3.75, 7.50)$. Only for $t > 7.50$ do we have an equilibrium in pure strategies under FSHC, with $r_1 = r_2 = 1/2$ (provided that $t < 9.60$; otherwise the market is not covered).

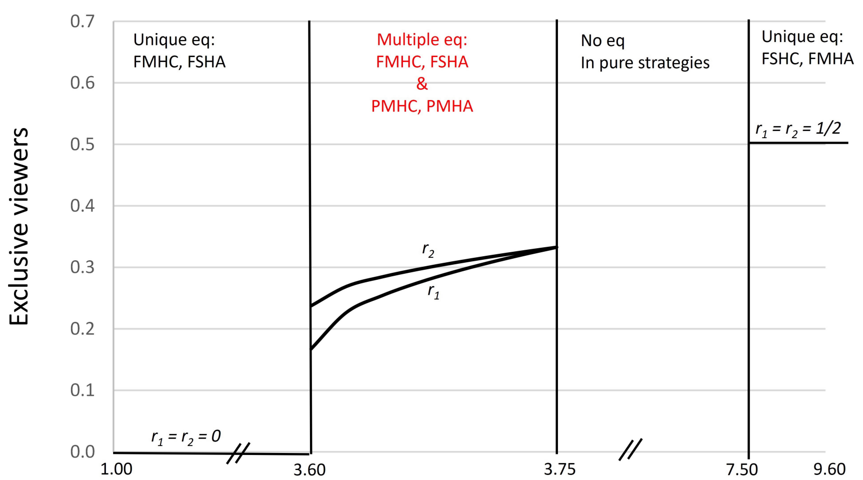

Figure 1: *Symmetric and asymmetric equilibria* ($\Psi = 5.0$, $\kappa = 2.5$)

From Figure 1 we note that $r_1 \to r_2$ as $t \to t^H$. However, it follows from equations (6) that this only holds if $\Psi/\kappa \approx 2$. For lower values of Ψ/κ, we have $r_1 < r_2$ also as $t \to t^H$. This is illustrated in Figure 2, where $\Psi = 4.0$ and $\kappa = 2.5$ (again with $\delta = 0.4$ and $\gamma = 1$). Note also that the range where we have multiple equilibria is smaller in Figure 2 than in Figure 1. Indeed, it can be shown that the range where the $\{B, 2, 1, 0\}$ equilibrium exists approaches zero as $\Psi/\kappa \to 3/4$.

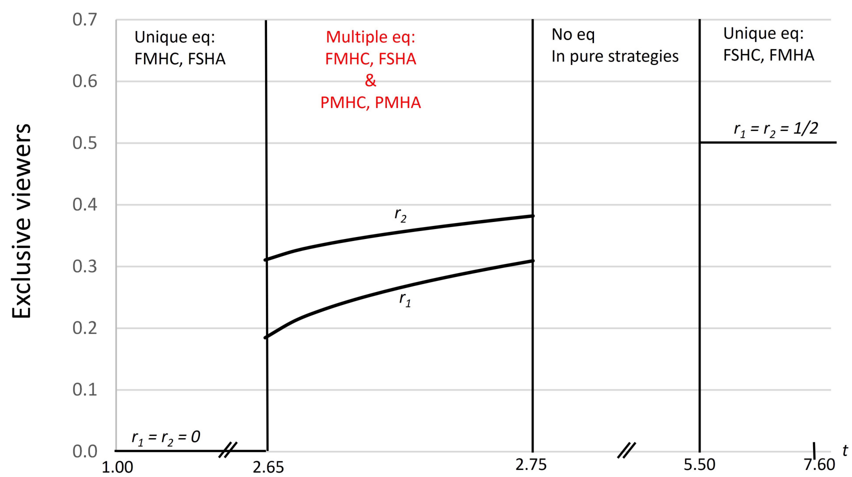

Figure 2: *Symmetric and asymmetric equilibria* ($\Psi = 4.0,\ \kappa = 2.5$)

5.2 Characterizing the equilibrium with partial asymmetric multihoming equilibrium (PMHC-PMHA)

We are now ready to characterize the asymmetric equilibrium. We start out by stating the following result:

Proposition 2: *Equilibrium with PMHC-PMHA. A higher disutility of ads (γ) or a lower incremental value of the second good to media consumers (δ) increases the number of exclusive consumers on each platform, and makes the platforms less asymmetric in size; $dr_1/d\kappa > dr_2/d\kappa > 0$.*

Proof. Recalling that $\kappa = \gamma/\delta$ we can use equation (12) to derive

$$\frac{dr_1}{d\kappa} = \frac{3\Psi - t - 2\kappa}{tS} > 0 \text{ and } \frac{dr_2}{d\kappa} = \frac{3\Psi - t - 2\kappa + S}{2tS} > 0. \tag{15}$$

The signs on the derivatives in (15) follow from (14), since $(3\Psi - t - 2\kappa) > 0$ is a necessary (though not sufficient) condition to ensure that $r_c > r_1$. For the size difference between the platforms we find $\frac{dr_1}{d\kappa} - \frac{dr_2}{d\kappa} = \frac{(3\Psi - t - 2\kappa) - S}{2St} > 0$, where the sign follows from equation (14), which ensures that $r_c - r_1 > 0$. ■

The intuition for the result in Proposition 2 is that the greater is the disutility of ads, the less attractive it is for the consumers to attend both platforms. In particular will the marginal consumers that connect to a second platform be more annoyed, such that the number of multihoming consumers falls.

At the outset it might seem surprising that $\frac{dr_1}{d\kappa} > \frac{dr_2}{d\kappa}$, since $a_1 > a_2$. Other things equal, a higher disutility of ads should increase the relative attractiveness of platform 2. The intuition for why it nonetheless is platform 1 which attracts the larger number of "new" exclusive consumers hinges on the following striking result:

Proposition 3: *Equilibrium with PMHC-PMHA. The advertising volume* increases *with the consumers' disutility of ads, and more so for the larger platform; $da_2/d\gamma > da_1/d\gamma > 0$.*

Proof. As noted above, each platform's number of exclusive consumers depends on the rival's advertising level. More precisely, equation (6) tells us that $r_i = 1 - \frac{1}{t}\left(\Psi - \frac{\gamma a_j}{\delta}\right)$. Since equation (15) shows that the number of exclusive consumers for both platforms is increasing in κ, it follows immediately that the same is true for the advertising level, and that a_2 increases more than a_1 (because r_1 increases more than r_2). ■

An interesting implication of Propositions 2 and 3 is that higher disutility of ads need not have a negative impact on platform profits, since it increases the number of exclusive consumers on each channel. Indeed, we can prove the following result:

Proposition 4: *Equilibrium with PMHC-PMHA. Suppose that the disutility of ads increases. Then both profit and the advertising prices for*

- *the smaller platform 1 increase*
- *the larger platform 2 increase in the neighborhood where $r_c = r_1$.*

Proof. See Appendix ■

Figure 3 illustrates the results in Proposition 4; the profit level of platform 1 is strictly increasing in γ, while the profit level of platform 2 is a U-shaped function of γ.[9] Even though this result is in sharp contrast to standard results in media economics, the intuition why profit may increase in γ is straight forward: a higher disutility of ads leads to less consumer multihoming. Other things equal, this is an advantage for the platforms if it is more profitable to sell exclusive eyeballs than shared ones on the advertising market.

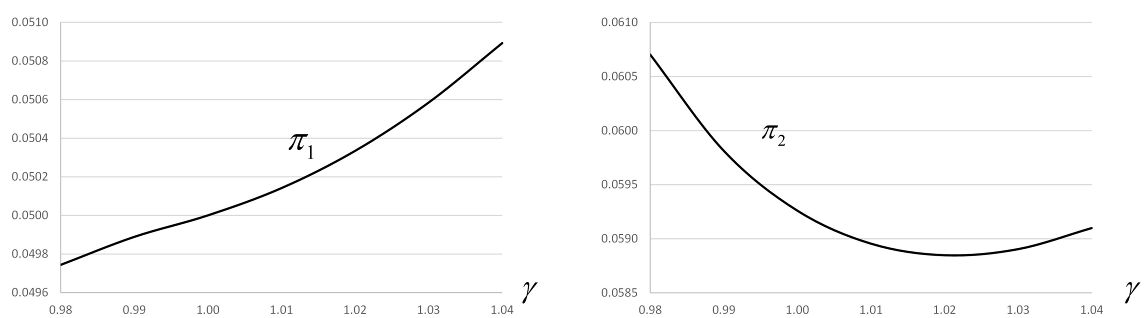

Figure 3: *Partial multihoming. Profit as a function of disutility of ads.*

[9] Similar to in Figure 2, we have set $\Psi = 4.0$ and $\delta = 0.4$. Transportation costs are fixed at $t = 2.7$, .

6 Concluding Remarks

Previously, singlehoming on the consumer side of the market could be an outcome of limited abilities to choose. The option to choose printed newspapers was restricted to those distributed locally. With singlehoming consumers, media platforms have monopoly power over the eyeballs in the advertising marked. This market power has eroded in the digital era, and tougher competition for advertisers may explain the fall in ad-revenues for mainstream media the recent decades. The eyeballs of multihoming consumers cannot be sold for a higher price than the incremental value reaching consumers more than once. Digital platforms from the whole world can compete in selling ads in a local market that previously was reserved for a local newspaper.

We have presented a model with endogenous multihoming at both side of the market. Full singlehoming on the consumer side of the market, as assumed in standard (two-sided) media models, does not arise in equilibrium unless competition for consumers is weak. When competition for consumers is sufficiently strong, there exists an equilibrium with full consumer multihoming (advertisers are then singlehoming, since all consumers can be reached on both platforms). In this case, there may also exist an asymmetric equilibrium with partial multihoming on both sides of the market. The asymmetric equilibrium has a Shaked and Sutton (1982) feature. One of the platforms has a higher ad price, such that its ad volume is lower than the rival. From consumers' perspective the platform with low ad-volume has higher quality than the rival, since consumers dislike ads. Characterizing the asymmetric equilibrium, we show higher disutility of ads reduce consumer multihoming (more exclusive eyeballs). Hence, ad prices increase in disutility of ads. This result contrast standard media models.

Our results provide important managerial and policy insights. As noted above, full consumer singlehoming does not arise in equilibrium unless competition on the consumer side of the market is weak. This is a cautionary tale for using lessons from standard models of media economics (Anderson and Coate, 2005; Armstrong, 2006; and subsequent papers), where competition for advertisers is closed by assuming that all consumers are singlehoming.

7 Appendix

Proof of Proposition 4:

We now want to show that $dP_1/d\kappa = (dP_1/dr_1)\,(dr_1/d\kappa) + (dP_1/dr_2)\,(dr_2/d\kappa) > 0$. In order to do so, we first use equations (8) and (9)to find

$$\frac{dP_1}{dr_1} = \left(\frac{4}{r_1^2} - \frac{3}{(r_2 - r_1)^2}\right)\frac{P_1^2}{3} < 0 \text{ and} \tag{16}$$

$$\frac{dP_1}{dr_2} = \left(\frac{3}{(r_2 - r_1)^2} - \frac{4}{(1 - r_2)^2}\right)\frac{P_1^2}{3} > 0. \tag{17}$$

From Proposition 2 we know that $dr_1/d\kappa \geq dr_2/d\kappa$. A sufficient condition for

$dP_1/d\kappa$ to be positive, is thus that $(dP_1/dr_1)+(dP_1/dr_2)>0$. Adding (16) and (17) we find

$$\frac{dP_1}{dr_1}+\frac{dP_1}{dr_2}=\frac{4P_1^2}{3}\left(\frac{1}{r_1^2}-\frac{1}{(1-r_2)^2}\right)=\frac{4P_1^2}{3}\left(\frac{1}{r_1^2}-\frac{1}{(1-r_2)^2}\right)=\frac{(r_1+1-r_2)(1-r_1-r_2)}{r_1^2(1-r_2)^2}>0.$$

Since both a_2 and P_2 are increasing in κ, it follows that $d\pi_2/d\kappa>0$.
Proof. To show that $d\pi_1/d\kappa>0$ at the boundary where $r_c=r_1$, we differentiate P_2 with respect to r_1 and r_2 around $t^H=\Psi-\frac{\kappa}{2}$. This yields

$$\begin{aligned}\left.\frac{dP_2}{dr_1}\right|_{t^H} &= -\frac{21\left(\Psi^2+t^2\right)-2t\left(t+18\Psi\right)}{75\left(\Psi-t\right)^2} \text{ and}\\ \left.\frac{dP_2}{dr_2}\right|_{t^H} &= \frac{69\left(\Psi^2+t^2\right)+17t^2-144t\Psi}{150\left(\Psi-t\right)^2}\end{aligned}$$

Evaluating (15) around yields $dr_1/d\kappa=dr_2/d\kappa=1/t$. Using $dP_2/d\kappa=(dP_2/dr_1)(dr_1/d\kappa)+(dP_2/dr_2)(dr_2/d\kappa)$ we thus find

$$\left.\frac{dP_2}{d\kappa}\right|_{t^H}=\frac{(4t-3\Psi)^2}{50\left(\Psi-t\right)^2}>0.$$

Since $da_2/d\kappa>0$ and $dP_2/d\kappa|_{t^H}>0$ it follows that $d\pi_2/d\kappa|_{t^H}>0$. ■

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9 Scraps (wording odds unused)

9.0.1 singlehoming advertisers (SHA)

Let us now investigate the possibility of an endogenous asymmetry, such that $r_1 \neq r_2$ assume that platform 2 has the larger audience; $r_2 > r_1$.

Without loss of generality, we assume that if there is an asymmetry, then platform 2 has the larger audience; $r_2 > r_1$. Let us first ask whether there can exist such an equilibrium if the advertisers singlehome. If so, the top θ-type advertisers will be on platform 2, followed by a next tranche of them on Platform 1, and the lowest θ-types not advertising. This is the classic vertical differentiation model (as usually applied to consumer choice of exclusive product, as in Mussa and Rosen, 1978). The lowest marginal advertiser, denoted by θ_{01}, is indifferent between buying a slot on platform 1 and not buying any slot at all. This advertiser type is consequently found by solving $\theta_{01}(r_1 + r_c) - P_1 = 0$. The next marginal type, θ_{12}, is indifferent between buying only from platform 1 and buying only from platform 2: $\theta_{12}(r_1 + r_c) - P_1 = \theta_{12}(r_2 + r_c) - P_2$. From these expressions we find that demand for advertising is given by $a_1 = (\theta_{12} - \theta_{01}) = \frac{P_2 - P_1}{r_2 - r_1} - \frac{P_1}{r_1 + r_c}$ and $a_2 = (1 - \theta_{12}) = 1 - \frac{P_2 - P_1}{r_2 - r_1}$. Platform profit is $\pi_i = P_i a_i$.