Vertical Control Change and Platform Organization under Network Externalities *

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March 31, 2022

Abstract

In this paper, we examine how the introduction of network externalities impact an open and vertically integrated platform's post-merger contractual relationship with third-party sellers distributing through its marketplace. Regardless of whether the platform uses linear contracts or two-part tariffs, we find that, provided these contracts are public, the platform has no incentive to exclude its non-integrated rivals and that the latter's market share rises as network effects gain importance. Vertical integration serves as a commitment device that open platforms can use to convince potential users (e.g., consumers and developers) that their ecosystem will be large and compelling. Interestingly, when the open platform competes with a closed rival, i.e., with a fully integrated ecosystem, it may find it profitable to subsidize independent third-party sellers to strategically steer demand away from the competing ecosystem. These results have novel managerial implications on the incentives of a platform to open up its ecosystem to third-party sellers, as well as for the regulation of vertical integration in the presence of network effect and when different platforms operate alternative business models.

Keywords: Open Ecosystems, Network Externalities, Platforms, Vertical Integration

JEL Codes: L22, L41, L51

*We would like to thank the Editor (Josua Gans), an Associate Editor, and three anonymous referees for helpful comments and suggestions. We are also indebted to Daniele Condorelli, Miguel De la Mano, Guillaume Duquesne, Dan O'Brien, and Frederic Palomino. The views expressed in this paper are the authors' sole responsibility and cannot be attributed to Compass Lexecon or its clients.

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1 Introduction

Network externalities are a fundamental pillar of the aggregative role played by digital platforms. Early platform adopters enjoy direct and indirect benefits as new users join their network. This is because trading and diversification opportunities expand with the network size. Platforms with a large user base may also design more efficient ecosystems that minimize buyers' search costs — e.g., by targeting quality and offering customized services — so to better match them with the supply side. Nevertheless, network externalities may also favor the emergence of dominant networks, thereby consolidating their gatekeeper power. Concerns about so-called 'self-preferencing' strategies are behind several prominent antitrust cases all over the world and motivate recent proposals for ex-ante regulation, such as the Digital Markets Act (DMA) in the European Union and related initiatives in Korea, Japan, the US and, more recently, in China.[1]

Scholars in economics and management have extensively acknowledged the importance of network externalities. Stemming from Katz & Shapiro (1985), many models have examined different and important aspects of network externalities related to inter- and intra-platform competition both from a static and a dynamic perspective (see, e.g., Argenziano & Gilboa (2012), Caillaud & Jullien (2001), Caillaud & Jullien (2003), Halaburda et al. (2020), Halaburda & Yehezkel (2013), Hagiu (2006) , Mitchell & Skrzypacz (2006), among many others). But, these models have systematically overlooked the potential impact of the existence of material network effects on the competitive conduct of vertically integrated platforms and their incentives to foreclose or marginalize non-integrated competitors.[2]

Absent efficiencies, vertical mergers traditionally raise competitive concerns since they may result in the anticompetitive foreclosure of unintegrated rivals: the so-called 'foreclosure doctrine' (e.g., Hart et al. (1990), Bolton & Whinston (1991), Rey & Tirole (2007), among many others). Are network externalities likely to change this view? How do these externalities shape the competitive conduct of vertically integrated platforms? What type of industry characteristics weaken, or even revert, the standard foreclosure logic? Can consumers benefit from vertical integration in these contexts?

To answer these questions, we introduce direct and indirect network externalities in a canonical vertical integration model. A platform (upstream supplier) provides an essential input (or, equivalently, the right to access its network) to third-party sellers competing in the downstream market. Following industry practice, we assume that the listing contracts charged to third-party sellers are public and can influence participants' expectations about the network size.[3] Many B2C and B2B platforms (e.g., such as Alibaba, Amazon and the Apple App Store) publicly disclose the listing fees required to third-party sellers and

[1] See e.g., Halaburda et al. (2020), Jullien & Sand-Zantman (2020) among many others.

[2] There is a growing empirical literature that discusses such issues but from the perspective of platform entry into complementors' product space. For instance, Zhu (2019), He et al. (2020), Li & Agarwal (2017), Wen & Zhu (2019), Zhu & Liu (2018), Foerderer et al. (2018), among others.

[3] For example, Microsoft announced lowering its price to 12% following Epic. Similarly in 2021, Google announced the lowering of its commission rates to 15% following Apple. In the IoT platform market, platforms such as AWS, Amazon, Google and IBM publicly publish their price. See https://iskerrett.medium.com/price-comparison-of-iot-platform-vendors-b07ab4bbf0e

App developers who join their marketplaces.[4] Public contracts also capture the idea that in some highly digitalized industries, such as e-commerce, hardware devices, payment systems, health services etc, listing contracts are long term and hard to change or renegotiate (see, e.g., Karle et al. (2020) and Condorelli & Padilla (2021), for models with similar assumptions).[5]

Within this context, we first study how a vertical merger changes the platform's incentive to contract with, and possibly marginalize, independent third-party sellers operating in its network, and how such a change is shaped by network externalities. Second, we assess the impact of vertical integration on total welfare, industry profits and consumer surplus, depending on the magnitude of such externalities.

Basic insights. We start by considering a stylized model à la Katz & Shapiro (1985) in which only direct network externalities are present. In this model, each consumer's utility increases with the number of other consumers purchasing from sellers, including the vertically integrated one, operating in the same ecosystem. We find that, regardless of whether listing contracts are linear or require a fixed fee, the vertically integrated platform has no incentive to fully foreclose the non-integrated third-party sellers if there are positive (even small) network effects. Moreover, the incentive to marginalize rivals falls with the extent of these effects. In a nutshell, the vertically integrated platform anticipates that if consumers observe a too high listing fee (which marginalizes the rivals' output) they will form low expectations on the network size. Hence, their willingness to pay will drop at the expense of the platform itself.

Furthermore, we establish that vertical integration is profitable, benefits consumers, and is thus total welfare-enhancing under linear contracts. However, it is profit, consumer surplus, and total welfare neutral when the platform offers two-part tariffs. Of course, as standard in the literature, vertical integration is profitable, and consumers benefit when contracts are linear because it mitigates double marginalization. These two positive effects magnify as network externalities tighten since the platform has an incentive to reduce the unit fees charged to its intra-platform rivals to increase consumer demand.

The neutrality result arising under two-part tariffs can also be easily understood. Even if the platform does not integrate, it fully internalizes downstream profits via the fixed fee and controls output decisions by setting appropriately the linear component of their listing contracts (recall that with public contracts, there is no opportunism problem à la Hart et al. (1990)). As a result, a vertically separated platform can always replicate the outcome of a vertically integrated one and vice-versa. In addition, since aggregate output must be the same with and without vertical integration, consumers are indifferent too and, therefore, vertical integration is neutral from a total welfare point of view.

A two-sided market perspective. Having uncovered the basic logic behind the impact of network externalities on foreclosure incentives in a model with direct network externalities only, we then show that the same reasoning applies when considering indirect network effects. Specifically, we introduce

[4]See report Borck et al. (2020) funded by Apple. This report provides a detailed overview of the public information regarding prices charged to third party sellers on digital platforms.

[5]Further, it is a reasonable assumption in literature. See Casner & Teh (2021), Tremblay (2016), Jullien & Pavan (2019), Tremblay (2020), Chellappa & Mukherjee (2021), Bakos & Halaburda (2020) , Tan et al. (2020). We discuss secret contracts in Section 4.

developers in the baseline model to capture a two-sided market perspective. These players (e.g., App developers) provide services (e.g., Apps) that consumers enjoy when joining the platform through the sellers' products (e.g., when they buy an electronic device, such as a smartphone or a tablet, compatible with the platform's OS). In this model, developers decide whether or not to join the platform given their expectations on the number of consumers that (in equilibrium) join the platform. At the same time, as in the baseline model, consumers decide whether and from which seller they buy, given their expectations on the number of developers that adopt the platform. We argue that, under the hypothesis of rational expectations, the demand function has precisely the same features as with direct network externalities: its intercept indirectly depends on the expected network size through the developers' participation decision. Therefore, all the above conclusions apply irrespective of whether network externalities are direct or indirect.

Contract disclosure. A key hypothesis for the results illustrated above is that listing contracts charged to third-party sellers are observed by the market and can influence participants' expectations about the network size. A natural question then is whether the platform has an incentive to disclose these contracts (or credibly commit to making them observable). We show that contract disclosure is always in the platform's best interest. While vertical integration is always profitable with and without public contracts, the platform can influence participants' expectations in the former case. Under secret agreements, instead, the platform always has an incentive to foreclose the rival by a standard logic — i.e., whatever expectations consumers have about the non-integrated sellers' output, the vertically integrated entity has an incentive to foreclose those sellers and monopolize the market. Consumers will thus rationally anticipate this behavior and revise their willingness to pay downward. This implies that platforms can use contract disclosure as a commitment device to keep their networks large and competitive.

Competing networks. Finally, to better understand the industry characteristics that may impact the link between vertical integration and the foreclosure incentives, we also consider a version of the model in which the platform faces competition from an integrated rival operating a closed ecosystem. It turns out that, with inter-platform competition, the platform has an even stronger incentive to open its ecosystem and accommodate its intra-platform rivals. The open but vertically integrated platform has a strategic incentive to commit to lowering the fees charged to its non-integrated sellers to reduce the rival ecosystem's output. Interestingly, when network effects are large, the open platform may even subsidize intra-platform competitors, by setting fees below marginal costs, to steal additional business from the competing ecosystem. Intra-platform rivals are kept active and effectively used as 'pirates' or 'fighting brands' to capture market share.

Managerial takeaways. Summing up, our analysis delivers the following novel managerial insights.

First, it shows that a vertically integrated platform may gain from a strategy to open up its ecosystem to competing third-party sellers when demand features network effects. By doing so, the platform can favorably influence consumer expectations on the network effects in the market. A monopolist (closed) platform (ecosystem) cannot influence consumers' expectations as these beliefs are formed before the output is set. This simple logic may contribute to explaining why some integrated platforms keep their ecosystems open and accommodate entry by rivals instead of foreclosing them as in traditional retailing industries where network externalities are by and large absent.

Second, when an entrant platform competes with a closed (vertically integrated) incumbent, it can strategically set the fees offered to third-party sellers to gain market share by influencing its users' and the rival's expectations. Specifically, as network effects gain importance in the market, the entrant platform may be better off by reducing fees and, in some cases, even subsidizing its third-party sellers. Disclosing fees acts as a signalling device to consumers and the closed ecosystem incumbent. A lower fee generates favorable consumer expectations regarding ecosystem size and thus increases the aggregate output supplied by the open ecosystem at the expense of the closed one. Notably, this is consistent with the smartphone OS market's entry sequence. Apple's iOS was the closed ecosystem incumbent platform that was vertically integrated. Google, an incumbent to encourage consumer adoption of its platform, publicly offered its Android OS to phone manufacturers at zero cost.[6] Since Google invests heavily in the Android OS every year through updates, the average price being charged to third party manufacturers that sell phones could be seen as negative (or below cost).

Third, it may be profitable for the platform to invest more aggressively in increasing consumer benefit from network effects. This is particularly the case if the platform's revenue generation arises from a fee charged to third-party sellers in the market. This increased investment will promote the participation of third-party sellers in the market and further expand the ecosystem size. The reason is that under vertical integration, platforms obtain higher gains per unit sold due to the elimination of double marginalization. Therefore, these platforms are willing to make a greater investment in the network effects. Furthermore, when facing an integrated incumbent, this incentive to invest is expected to be much higher as it can provide the platform with another signalling instrument that the platform can use to expand the output of its ecosystem.

Finally, our model also shows that platforms operating in markets where network externalities are particularly significant will profit from disclosing listing fees to the market to influence participants' expectations favorably and attract a large user base. As argued above, this practice is quite common in the digital industry, and our model offers a possible explanation for this phenomenon.

2 Related Literature

Vertical integration is a recursive theme in IO and management. The classical 'foreclosure doctrine' first formulated by Ordover et al. (1990) argues that vertical mergers are likely to have anticompetitive effects because merged sellers may commit to foreclose partially or in full non-integrated rivals to relax downstream competition. Hart et al. (1990) and Bolton & Whinston (1991) bolstered these arguments

[6]"Free" Android OS was accompanied by some conditions regarding pre-installation of Google Apps.

by confirming their validity in a more general setting encompassing cases overlooked by Ordover et al. (1990).[7] Hart et al. (1990) conclude that foreclosure can indeed be an important consequence of vertical mergers even if one drops the restrictive assumptions in Ordover et al. (1990). Contrasting with these seminal works, we show that even when the platform can commit to fully foreclosing a rival, it has no incentive to do so in the presence of network externalities. Further, as network externalities rise, partial foreclosure becomes a lesser concern.

Our paper also adds to the literature on markets featuring network externalities (seminal works include Caillaud & Jullien (2003), Rochet & Tirole (2003), Parker & Van Alstyne (2005), Armstrong (2006), among others). Our contribution here is twofold.

First, we enrich the strand of literature on the effects of price announcements by platforms under network effects. Hagiu & Hałaburda (2014) find that a monopolist platform always prefers to reveal pricing information. Instead, when symmetric platforms compete, they prefer not to reveal information.[8] Belleflamme & Peitz (2019*b*) generalize the model of Hagiu & Hałaburda (2014) and additionally find that results depend on the single- or multi-homing decisions of the two sides. Similarly, Chellappa & Mukherjee (2021) and Jullien & Pavan (2019) find that pre-announcement to inform market expectations can be profitable for platforms. Our results show that public contracts can be interpreted as a commitment device by a platform to influence users' expectations regarding the ecosystem size when their utility function features (direct and/or indirect) network externalities.[9] This is more so the case when the platform is an entrant and faces a vertically integrated incumbent platform.[10]

Second, we contribute to the recent literature that focuses on hybrid platforms, such as hybrid marketplaces.[11] Some works find that platform entry in competition with third party sellers can be pro-competitive.[12] Hagiu & Spulber (2013) suggest that platforms facing unfavorable demand conditions have more incentive to enter into the seller product space. Hagiu, Teh & Wright (2020) find that platform entry constraints third-party seller pricing and might be welfare-enhancing. Similar pro-competitive effects are documented by Dryden et al. (2020), Etro (2021) and Tremblay (2020). However, entry in competition with third-party sellers also opens the channel for well known negative effects such as foreclosure of rivals.[13] De Corniere & Taylor (2014) show that entry through a vertical merger between a

[7]The results of Ordover et al. (1990) were based on two simplifying assumptions: integrated entities can commit to foreclose and only offer per-unit fee to the non-integrated rivals.

[8]One must note that they assume that platforms can coordinate not to reveal information. Instead, if platforms unilaterally chose to reveal pricing information, they would do so and be in a prisoner's dilemma like situation.

[9]For instance, ARM a semiconductor IP platform based on the RISC architecture, ubiquitously found in mobile devices, announces its per-unit license fees publicly for SoC and IoT hardware firms. Similarly, in the gaming industry, Sony announced reduction in its price to developers in 2009.

[10]In a similar spirit, Hagiu, Jullien & Wright (2020) show that hosting a rival can be beneficial as it makes them complementors for the core product. In our setting, hosting a rival benefits the product of the platform through increased willingness to pay arising from the expected demand of the rival.

[11]Recent empirical works include Zhu (2019), He et al. (2020), Li & Agarwal (2017), Wen & Zhu (2019), Zhu & Liu (2018), Foerderer et al. (2018), among others.

[12]Gautier et al. (2021) show that platform bundled entry can be a tool to resolve the inefficiency arising from Cournot complementarity. However, this entry can lead to fragmentation of network effects and harm consumers.

[13]Anderson & Bedre-Defolie (2020) considers the decision of a platform to compete with third party sellers. They find

search engine platform and a publisher can lead to full foreclosure of rival publishers. Padilla et al. (2020) study a dynamic framework to understand the incentive of a platform to abuse its gatekeeper role by privileging its own products. They found that the incentive to foreclose third party sellers arise when the gatekeepers face saturated demand and this may be detrimental to consumers.[14]

The closest paper to ours is Pouyet & Trégouët (2021) that considers the impact of a vertical merger between a platform and one of its downstream third-party sellers in the presence of network effects. As in our paper, they find that network effects dampen the incentive to foreclose. However, in contrast to us, they allow third-party sellers to access another platform. Hence, in their model foreclosure is not a primary concern by construction. In our setting, instead, the platform is a monopolist and can credibly commit to induce market exit of rivals. We find that the presence of network effects ensures that full foreclosure is not a profitable strategy irrespective of inter-platform competition. Another paper that is close to ours is Economides (1996). He also finds results that are consistent with ours. However, Economides (1996) is concerned with the incentive of a platform to invite entry while we are interested in the welfare effects of vertical integration in platform markets. There are three main differences between our and this paper. First, while Economides (1996) considers the incentive of a market leader to invite entry, he does not consider vertical integration, which is a building block of our analysis. Second, he examines separately fixed and linear listing fees, while we also examine the case of two-part tariffs. Third, we also extend the analysis to the case of competing ecosystems, which is neglected in his model.

The rest of the paper is organized as follows. In Section 3 we lay down the baseline model and characterize the equilibrium with vertical separation and with vertical integration. Then we compare consumer surplus, total welfare and profits under the two regimes. In Section 4 we extend the baseline model in several directions — i.e., we illustrate how results change with endogenous and indirect network externalities, with competing ecosystems, non-linear contracts, and discuss the role of investments in quality improvements and public contracts. Section 5 concludes. All proofs are in the Appendix. Additional material and proofs not contained in the paper's main body can be found in the online Appendix.

3 The baseline model

To gain insights on the link between vertical integration and foreclosure in platform markets, we first lay down a stylized model where only direct network externalities are present. Specifically, we consider consumers whose utility increases with the number of other consumers purchasing from sellers operating in the same ecosystem. The classic illustration of such same-sided network effects is the telephone industry: the more people have a phone line, the more valuable it is to have a phone. Platforms in which users buy and sell at the same time also feature prominent direct network externalities. For example, in the stock

that entry of a platform increases seller fees which reduces seller participation thereby hurting consumers. van den Boom & Samranchit (2020) show that conglomerate mergers can lead to reduced entry which can hurt consumers.

[14] Instead, Carroni et al. (2021) consider a vertical merger between an important third-party seller and a platform. They find that exclusive contracts (foreclosure of a rival platform) are less likely post-merger.

exchange and in platforms intended to promote the exchange of second-hand items, users do not know in advance whether they will buy or sell; hence, they value the presence of other users. More recent examples of industries where direct network externalities play a prominent role are those in which platforms exploit their customer base to improve the quality of their services, expand the variety of products available on their marketplaces and customize offers. In these cases, a growing user base enables a platform to acquire more accurate customer information and make investments more tailored to their needs and tastes, benefiting all participants.

Once we have illustrated the fundamental forces that shape platforms' foreclosure incentives with only direct network externalities, we show in Section 4.1 that these forces are also present in a two-sided industry with indirect network effects. In these cases, which are common in many modern two-sided platforms, the value of the service increases for one user group (e.g., consumers) when a new user of a different group (e.g., App developers) joins the network. However, this indirect mechanism does not add substantial new insights to the baseline analysis because, as we shall argue, what matters is the effect of network externalities on the total amount of transactions taking place in the platform, which affects users' willingness to pay through their expectations, and thus aggregate demand.

Players and environment. Consider an ecosystem (distribution network) formed by an upstream monopolistic platform (hereafter U) and two downstream independent sellers (each denoted by D_i, with $i = 1, 2$).[15] U charges sellers D_1 and D_2 fees w_1 and w_2 respectively for access to its platform (e.g., marketplace) where they compete to attract final consumers.

The demand side features direct network externalities and is modeled à la Katz & Shapiro (1985). Consumers are heterogeneous in their basic willingness to pay and are homogeneous in their valuation for the network externality. We assume that the basic willingness to pay, denoted by r, is uniformly distributed over the support $\left[\mu - \frac{\sigma}{2}, \mu + \frac{\sigma}{2}\right]$, where $\mu \geq 0$ is the average willingness to pay and $\sigma > 2\mu$ is a measure of its volatility (heterogeneity). The expected utility of a consumer of type r buying from D_i is

$$u\left(X^e, P_i\right) \triangleq r + \theta X^e - P_i, \quad i = 1, 2,$$

where $X^e \triangleq \sum_{i=1,2} x_i^e$ is the aggregate output that consumers expect D_1 and D_2 to distribute. The parameter $\theta \geq 0$ measures the strength of the direct network externality — i.e., the more users on the network, the more likely is that other users will be interested in joining the network, as reflected by a higher willingness to pay (demand intercept as shown below).

Under the above specification, D_1 and D_2 have positive demand only if the following 'no arbitrage condition' holds

$$P_1 - \theta X^e = P_2 - \theta X^e. \tag{1}$$

Since D_1 and D_2 are perfectly compatible, we must have $P_1 = P_2 = P$. Therefore, consumers buy the

[15] We consider a duopoly only for illustrative purposes. It can be shown that our results remain true qualitatively with $N > 2$ third party sellers (proofs are available upon request).

product only if

$$r \geq r^{\star} \triangleq \underbrace{P - \theta X^{e}}_{\text{Reservation price}}.$$

Hence, the total demand for the product distributed within the network is

$$X \triangleq 1 - \Pr\left[r \leq r^{\star}\right] = 1 - \frac{P - \theta X^{e} - (\mu - \frac{\sigma}{2})}{\sigma},$$

with $X \triangleq \sum_{i=1,2} x_i$ being the sum of D_1 and D_2's outputs. Notice that the parameter σ is an inverse measure of the responsiveness of demand to price — i.e., as σ falls, demand becomes more responsive to price (e.g., because goods outside U's ecosystem are closer substitutes to those distributed by D_1 and D_2).

Following Katz & Shapiro (1985), we assume that D_1 and D_2 compete by setting quantity. Bisceglia et al. (2021) consider a model with differentiated price competition and show that, in contrast to quantity competition, in that case foreclosure does not arise even without network externalities (for completeness, we examine the case of price competition in the online Appendix). Hence, the inverse demand function is

$$P(X^{e}, X) \triangleq \max\left\{0, \mu + \frac{\sigma}{2} + \theta X^{e} - \sigma X\right\}.$$

We assume that consumers form expectations before third-party sellers choose their output, but after listing contracts have been offered — i.e., contracts have a 'signalling' content to the extent that consumers infer outputs from these deals (more below).[16]

We assume that $\theta \leq \sigma$ — i.e., the market price is more responsive to actual output than consumer expectations — to guarantee a downward sloping demand function. Marginal costs, upstream and downstream, are normalized to 0 without loss of generality.

Industry structure, contracts and payoffs. We compare two alternative industry structures:

(a) *Vertical separation*: D_1 and D_2 are separated from U.

(b) *Vertical integration:* U vertically integrates with D_1, while D_2 remains an independent (non-integrated) unit.[17]

In the baseline version of the model, we assume linear listing contracts — i.e., U offers each third-party seller D_i a fee w_i that D_i has to pay for each transaction made on the platform. Hence, when D_i sells x_i units of product, the payment collected by U is $w_i x_i$ (alternatively w_i can be interpreted as a linear

[16]For example, Belleflamme & Peitz (2019*b*) and Hagiu & Hałaburda (2014) show that it is profitable for platforms to disclose listing fees when platforms compete less fiercely. For similar results, see also Tremblay (2020), Casner & Teh (2021), Tremblay (2016), Jullien & Pavan (2019), Chellappa & Mukherjee (2021), and Suleymanova & Wey (2012).

[17]For simplicity, here we do not consider partial vertical integration (see, e.g., Spiegel et al. (2013)).

wholesale price). Hence, while under vertical separation D_i's payoff is

$$(P(X^e, X) - w_i)\, x_i, \quad \forall i = 1, 2,$$

and P's payoff is

$$\sum\nolimits_{i=1,2} w_i x_i,$$

under vertical integration the merged-entity's payoff is

$$P(X^e, X)x_1 + w_2 x_2,$$

while D_2's payoff is the same as with vertical separation.

Notice that, once integrated with D_1, U has the option to foreclose D_2 by setting a sufficiently large per-unit fee w_2 (i.e., such that $x_2 = 0$). Focusing on the conservative case in which the benefits associated with a foreclosure strategy are maximized, we assume that D_2 has no outside option, meaning that it has no access to an alternative (even inferior) platform.[18] Therefore, if U decides to (fully) foreclose, D_2 exits the market and U-D_1 is a monopolist.

Timing and equilibrium concept. The timing of the game is as follows:

$t = 1$ U decides whether to merge with D_1.

$t = 2$ U publicly sets the listing fees charged to its downstream unit(s). Consumers observe contracts and form an expectation X^e.

$t = 3$ Sellers choose outputs, profits materialize and payments are made.

As in Katz & Shapiro (1985), the solution concept in the downstream competition game is Fulfilled Expectations Cournot Equilibrium. Specifically, each seller chooses its output level taking as given consumers' expectations X^e under the assumption that these expectations are consistent with the equilibrium outcome — i.e., rational expectations — and are formed at the interim stage after contracts have been offered but before output is set.[19] Throughout, we will focus on symmetric equilibria such that, under vertical separation, both third party sellers produce the same output and receive the same contract.

Technical assumptions. To guarantee that $\Pr[r \geq r^\star] \in (0, 1)$ we impose the following technical requirement:

A1 The dispersion index $d \triangleq \frac{\sigma^2}{\mu}$ of the consumers' willingness to pay is sufficiently large — i.e.,

$$d > \overline{d} \triangleq \frac{2\sigma^2 (5\sigma - 2\theta)}{15\sigma^2 + 4\theta^2 - 18\theta\sigma}.$$

[18] See Pouyet & Trégouët (2016) for a model in which the platform is competitively constrained by the presence of inefficient entrants.

[19] See also, Belleflamme & Peitz (2019*a*) for a related paper that models competition between sellers in a Cournot setting.

This condition simply implies that consumer preferences are sufficiently dispersed to guarantee that a positive mass of consumers, but not all of them, buy the product.

Finally, we assume that the platform does not charge consumers a price for using its services. We explore this possibility in the online Appendix and show that our conclusions remain valid even under this circumstance — i.e., the presence of an access price charged to consumers has only the effect of reducing consumers' reservation price but it does not matter for the indifference condition (1).

3.1 Equilibrium under vertical separation

First, consider the pre-merger regime — i.e., the case in which U is vertically separated and contracts with D_1 and D_2.

Quantity setting stage. For given w_1 and w_2, D_i $(i = 1, 2)$ solves

$$\max_{x_i \geq 0} \left(P\left(X^e, X\right) - w_i\right) x_i. \tag{2}$$

Recalling that consumers' expectations X^e have already been formed in stage 2, when setting output each seller takes as given those expectations on the ecosystem's size. Then, differentiating with respect to x_i (holding X^e constant) and then imposing rational expectations — i.e., $X^\star = X^e$ — it is easy to show that sellers' first-order conditions imply

$$x_i^\star\left(w_i, w_{-i}\right) \triangleq \frac{2w_{-i}(\sigma-\theta) - 2w_i(2\sigma-\theta) + \sigma(2\mu+\sigma)}{2\sigma(3\sigma-2\theta)}, \quad \forall i = 1, 2, \tag{3}$$

$$X^\star\left(w_1, w_2\right) \triangleq \sum_{i=1}^{2} x_i^\star\left(w_i, w_j\right) = \frac{2\mu + \sigma - \sum_{i=1,2} w_i}{3\sigma - 2\theta}, \tag{4}$$

and

$$P^\star\left(w_1, w_2\right) \triangleq P\left(X^\star\left(w_1, w_2\right), X^\star\left(w_1, w_2\right)\right) = \mu + \frac{\sigma}{2} - (\sigma-\theta)\, X^\star\left(w_1, w_2\right). \tag{5}$$

The interpretation of the above expressions is rather simple. The quantity set by each seller is decreasing with its own fee and increasing in the rival's fee. Interestingly,

$$\frac{\partial^2 x_i^\star\left(w_i, w_{-i}\right)}{\partial w_{-i}\partial\theta} = -\frac{1}{\left(2\theta - 3\sigma\right)^2} \leq 0, \quad \forall i = 1, 2.$$

meaning that D_i's incentive to expand output in response to an increase in the fee charged to the rival is mitigated by the presence of relatively stronger network effects. This is because a reduction in the rival's output lowers consumers' willingness to pay for both products and, therefore, also lowers the (equilibrium) final price. In turn, this makes it unprofitable for D_i to expand demand by as much as when network effects are absent. Thus, the traditional Cournot externality arising from outputs being strategic substitutes is dampened in the presence of network externalities.

Simple inspection of aggregate output reveals that, holding fees constant, $X^*(w_1, w_2)$ is increasing in θ. The intuition is that an increase in θ is akin to improving the quality of the network, which increases the consumers' willingness to pay, thereby inducing D_1 and D_2 to expand their sale volumes.

Moreover, differentiating $P^\star(w_1, w_2)$ with respect to θ, we get

$$\frac{\partial P^\star(\cdot)}{\partial \theta} = X^\star(\cdot) - (\sigma - \theta)\underbrace{\frac{\partial X^\star(\cdot)}{\partial \theta}}_{(+)} = \frac{\sigma(2\mu + \sigma - \sum_{i=1,2} w_i)}{(3\sigma - 2\theta)^2}.$$

This equation reflects two opposing forces. The first term reflects that as θ increases, consumers buy more for a given price and, hence, the equilibrium price will increase. The second term captures the impact of an increase in θ on the elasticity of demand: as θ grows, $X^*(\cdot)$ also grows and the elasticity of demand increases. This effect is increasing in σ. Specifically, as σ increases, price becomes more responsive to output. Under the assumption that

$$\mu + \frac{\sigma}{2} \geq \frac{1}{2}\sum_{i=1,2} w_i,$$

which ensures that aggregate output is positive (a conjecture that will be verified ex-post), the first effect dominates. Hence, for given listing contracts, stronger network externalities increase the market price in addition to increasing output.

Contracting stage. We can now turn to characterize equilibrium fees under vertical separation. U chooses w_1 and w_2 to maximize its profit — i.e.,

$$\max_{w_1, w_2} \sum_{i=1,2} w_i x_i^\star(w_i, w_{-i}).$$

Notice that, having imposed the rational expectations equilibrium requirement at the downstream quantity-setting stage, at the contract-setting stage U internalizes the effect of increasing the fees w_1 and w_2 on these expectations that are indeed correct at equilibrium. Differentiating with respect to w_i, we have

$$\underbrace{x_i^\star(w_i, w_{-i}) + w_i \frac{\partial x_i^\star(w_i, w_{-i})}{\partial w_i}}_{\text{Margin + Volume effects}} + \underbrace{w_{-i}\frac{\partial x_{-i}^\star(w_{-i}, w_i)}{\partial w_i}}_{\text{Strategic Effect } (+)} = 0, \quad \forall i = 1, 2. \tag{6}$$

This condition reflects the impact of higher fees on U's profit. There is a trade-off between upstream margins and downstream volumes: for given D_i's output, a higher w_i increases the revenue earned by U on each unit of sale made by D_i. At the same time, by increasing w_i, U exerts downward pressure on D_i's output, thereby reducing its revenue. In addition to these two standard effects, by increasing w_i, U also positively impacts D_{-i}'s demand because outputs are strategic substitutes and contracts are public, thereby increasing the fee that U collects from D_{-i}.

Solving the above condition, we obtain the equilibrium fee under vertical separation.

Proposition 1 *With linear contracts and vertical separation, there exists a unique, symmetric equilibrium in which each seller is charged*

$$w_L^\star \triangleq \frac{2\mu+\sigma}{4}.$$

In this equilibrium, the individual output, the aggregate output and the market price are, respectively,

$$x_L^\star \triangleq \frac{2\mu+\sigma}{4(3\sigma-2\theta)}, \quad X_L^\star \triangleq \frac{2\mu+\sigma}{2(3\sigma-2\theta)}, \quad P_L^\star \triangleq \frac{(2\mu+\sigma)(2\sigma-\theta)}{2(3\sigma-2\theta)}.$$

All these variables and U's profit are increasing in θ.

As explained above, an increase in θ is akin to improving the quality of the network which increases the consumers' willingness to pay, thereby increasing individual and aggregate output, retail price and upstream profits. Interestingly, the equilibrium fee is independent of the degree of network externalities. This result is the resultant of two opposing forces. On the one hand, when θ increases, consumers' expectations rise, which (other things being equal) leads to a higher equilibrium price, and thus to a higher fee because U has an incentive to extract the expanded downstream margin. On the other hand, a higher fee tends to create double marginalization, which lowers output and thus reduces U's profit. On the net, these two effects compensate each other under our linear-quadratic specification.

Finally, notice that

$$X_L^\star \geq X^M \triangleq \frac{2\mu+\sigma}{2(2\sigma-\theta)},$$

where X^M is the output that would be chosen by a fully integrated monopolist who is therefore unable to use listing contracts as a communication device to influence consumers' expectations and exploit network externalities at its own advantage.

3.2 Equilibrium under vertical integration

We now consider the case in which U and D_1 merge. Throughout, we conjecture and verify ex-post that foreclosure does not occur at equilibrium.

Quantity setting stage. In the second period, for given w_2, the merged entity U-D_1 and its non-integrated rival D_2 set outputs to maximize their profits. Once again, consumers' expectations are taken as given at the quantity-setting stage. Specifically, U-D_1 solves

$$\max_{x_1\geq 0} \underbrace{P(X^e, X)\, x_1}_{\text{Direct sales}} + \underbrace{w_2 x_2}_{\text{Indirect revenue}},$$

which is the sum of the profit made through the direct sales channel (i.e., the integrated unit) and the revenue collected from the independent seller. D_2 solves

$$\max_{x_2\geq 0} \left(P(X^e, X) - w_2\right) x_2.$$

As before, recall that consumer's expectations X^e have already been formed when the merged entity sets output. Differentiating with respect to x_i (holding X^e constant) and then imposing rational expectations — i.e., $X^\star = X^e$ — it is easy to show that sellers' first-order conditions imply Differentiating with respect to x_1 and x_2, respectively, and imposing rational expectations — i.e., $X^e = \sum_{i=1,2} x_i^{VI}$ — it is easy to show (see the Appendix) that under vertical integration

$$x_1^{VI}(w_2) \triangleq \frac{2w_2(\sigma-\theta)+\sigma(2\mu+\sigma)}{2\sigma(3\sigma-2\theta)} > x_2^{VI}(w_2) \triangleq \max\left\{0, \frac{\sigma(2\mu+\sigma)-2w_2(2\sigma-\theta)}{2\sigma(3\sigma-2\theta)}\right\}.$$

The output of the integrated entity rises in the fee charged to the non-integrated rival. However, as explained before, the rate at which a higher w_2 tends to increase x_1 falls with the extent of network effects — i.e., $x_1^{VI}(w_2)$ is increasing in w_2, but

$$\frac{\partial^2 x_1^{VI}(w_2)}{\partial w_2 \partial \theta} = -\frac{1}{(2\theta-3\sigma)^2} < 0.$$

Furthermore, D_2's output falls as the fee rises and the rate at which it does so, decreases with network externalities — i.e., $x_2^{VI}(w_2)$ is decreasing in w_2, and

$$\frac{\partial^2 x_2^{VI}(w_2)}{\partial w_2 \partial \theta} = -\frac{1}{(2\theta-3\sigma)^2} < 0.$$

This is because the reaction functions become relatively less steep when network externalities become stronger. Hence, the traditional Cournot price-externality (arising from outputs being strategic substitutes) is less significant in the presence of network effects. Any reduction in x_2 due to a higher w_2 triggers a lower increase in output by the rival for large values of θ, which in turn makes $x_2^{VI}(w_2)$ less responsive to w_2.

Finally, notice that $x_2^{VI}(w_2) > 0$ if and only if

$$w_2 < \bar{w} \triangleq \frac{\sigma(2\mu+\sigma)}{2(2\sigma-\theta)}. \tag{7}$$

That is, full foreclosure occurs if and only if $w_2 \geq \bar{w}$. The 'choke price' $\bar{w}$ is increasing in θ — i.e., foreclosure is less likely when network externalities are relatively stronger — which is a direct consequence of what we explained above. Moreover, $\bar{w}$ is increasing in μ since there is less incentive to foreclose in a larger market. The impact of σ on the choke price is ambiguous: a higher σ increases the demand intercept, thereby making foreclosure less likely, but it also makes aggregate output less responsive to price, which means that there is less competition from products distributed outside the ecosystem, which expands the merged entity's incentive to monopolize the market.

In an interior solution — i.e., assuming that $w_2 < \bar{w}$ — aggregate output is

$$X^{VI}(w_2) \triangleq \sum_{i=1,2} x_i^{VI}(w_2) = \frac{2\mu + \sigma - w_2}{3\sigma - 2\theta}.$$

As before, holding w_2 constant, a higher θ expands aggregate output. The market price is

$$P^{VI}(w_2) \triangleq P\left(X^{VI}(w_2), X^{VI}(w_2)\right) = \mu + \frac{\sigma}{2} - (\sigma - \theta)\frac{2\mu + \sigma - w_2}{3\sigma - 2\theta},$$

which, is again increasing in θ under the assumption that aggregate output is positive, i.e.,

$$2\mu + \sigma - w_2 \geq 0,$$

since the direct effect of a higher θ on consumers' willingness to pay more than compensates the indirect (positive) effect of θ on aggregate output.

Contracting stage. U maximizes the sum of D_1's direct sales profit and the revenue collected from D_2 — i.e.,

$$\max_{w_2} \underbrace{P^{VI}(w_2)x_1^{VI}(w_2)}_{D_1\text{'s sales profit}} + \underbrace{w_2 x_2^{VI}(w_2)}_{\text{Downstream revenue}}.$$

Again, in the contracting stage, U internalizes the effect of w_2 on $X^{VI}(w_2)$ which is the intuitive reason why, as it will be clear below, it benefits from dealing with D_2; namely, to enhance consumers' beliefs concerning the total quantity. Differentiating with respect to w_2, by the Envelope Theorem, we obtain

$$\underbrace{\left[x_2^{VI}(\cdot) + w_2\frac{\partial x_2^{VI}(\cdot)}{\partial w_2}\right]}_{\text{Marginal downstream revenue}} + \underbrace{\frac{\partial P(\cdot)}{\partial X}x_1^{VI}(\cdot)\frac{\partial x_2^{VI}(\cdot)}{\partial w_2}}_{\text{Strategic effect }(+)} + \underbrace{\frac{\partial P(\cdot)}{\partial X^e}\frac{\partial X^{VI}(\cdot)}{\partial w_2}x_1^{VI}(\cdot)}_{\text{Network externalities }(-)} = 0. \tag{8}$$

The three terms in the above condition reflect the following forces. First, a higher fee has a volume and a margin effect on the downstream revenue that the integrated entity extracts from the non-integrated rival. Second, by reducing D_2 's output, a higher fee impacts the retail market price and the integrated entity's direct sale profit, a strategic effect echoing the Stackelberg first-mover advantage. Finally, there is a network externality effect, which characterizes how a higher fee influences the formation of consumer expectations on the ecosystem size. By increasing the fee charged to D_2, the merged entity reduces the aggregate output, and hence (in a fulfilled expectations equilibrium), it also reduces the positive demand intercept generated by consumers' expectations. We elicit the existence of the network externality effect, in the absence of this effect, w_2 would be set very high. The trade-off between these effects determines the equilibrium fee with vertical integration, which is characterized in the next proposition.

Proposition 2 *Under vertical integration, the equilibrium contract that U-D_1 offers to D_2 features*

$$w_L^{VI} \triangleq \bar{w} - \underbrace{\frac{\theta\sigma(3\sigma - 2\theta)(2\mu + \sigma)}{4(2\sigma - \theta)\left(\theta^2 + 5\sigma\left(\sigma - \theta\right)\right)}}_{(+)} \leq \bar{w}, \quad \forall \theta \in [0, \sigma]\,. \tag{9}$$

Foreclosure occurs only when there are no network effects — i.e., $\theta = 0$. For every $\theta > 0$, U-D_1 has no incentive to foreclose D_2. Moreover, w_L^{VI} is inverted-U shaped with respect to θ and features a maximum at

$$\theta^\star \triangleq \frac{(5 - \sqrt{5})\sigma}{4}.$$

while D_2's equilibrium output is increasing in θ. In this equilibrium, individual outputs, aggregate output and the market price are, respectively,

$$x_2^{VI} \triangleq \frac{\theta(2\mu + \sigma)}{4\left(\theta^2 + 5\sigma\left(\sigma - \theta\right)\right)} < x_1^{VI} \triangleq \frac{(2\mu + \sigma)(5\sigma - 3\theta)}{4\left(\theta^2 + 5\sigma\left(\sigma - \theta\right)\right)},$$

$$X_L^{VI} \triangleq \frac{(5\sigma - 2\theta)(2\mu + \sigma)}{4\left(\theta^2 + 5\sigma\left(\sigma - \theta\right)\right)},$$

and

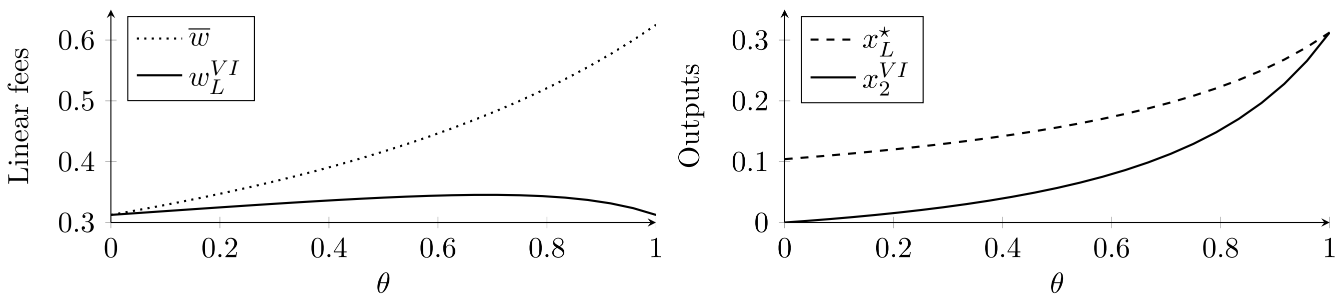
$$P_L^{VI} \triangleq \frac{\sigma\left(2\mu + \sigma\right)\left(5\sigma - 3\theta\right)}{4\left(\theta^2 + 5\sigma\left(\sigma - \theta\right)\right)}.$$

The merged entity U-D_1 has no incentive to fully foreclose its rival when network effects are in place — i.e., $w_L^{VI} < \bar{w}$ for every positive, even negligible, θ. To gain insights, suppose that $\theta = 0$, by the Envelope Theorem, equation (8) can be rewritten as follows.

$$\frac{\partial x_2^{VI}(\cdot)}{\partial w_2} \underbrace{\left[P(\cdot) + \frac{\partial P(\cdot)}{\partial X} X^{VI}(\cdot)\right]}_{\text{Monopoly rule}} + x_2^{VI}(\cdot) = 0,$$

implying that industry profit maximization (i.e., the monopoly outcome) requires $x_2^{VI}(\cdot) = 0$ and thus $w_L^{VI} = \bar{w}$. In the presence of network effects, the fee is always set lower than the level that fully forecloses D_2 as the negative network externalities effects kicks in, ensuring that the independent unit is never fully foreclosed. The optimal fee w_L^{VI} is shaped by two opposing forces. First, by a standard monopolization logic, for given consumers' expectations, the integrated platform would like to marginalize D_2, by charging a high w_2, as consumers' willingness to pay increases (i.e., as θ grows large). Second, increasing the rival's fee depresses consumers' expectations since they anticipate that D_2's output will drop, which in turn reduces the equilibrium price and U's profit. These two effects are affected by changes in θ. An increase in θ makes foreclosure less likely ($\partial\overline{w}/\partial\theta > 0$), which implies that a higher w_L is needed to secure monopoly rents. However, an increase in θ also makes network effects more important which exerts a downward pressure on w_L.

As intuition suggests, w_L^{VI} is increasing in θ for low values of this parameter because consumers' expectations are relatively less relevant than the standard marginalization logic. On the contrary, for θ large enough, w_L^{VI} decreases in θ because marginalizing the rival is relatively less profitable than increasing the market price via a higher ecosystem size. As an illustrative example, Figure 1 plots the equilibrium fee and D_2's output pre- and post-merger as functions of θ.



(a) Post-merger wholesale price and choke wholesale price

(b) Post-merger and pre-merger output of D_2.

Figure 1: Equilibrium wholesale prices and outputs for $\mu = 1/8$ and $\sigma = 1$.

Interestingly, $\theta^\star$ is increasing in σ. Hence, the less (resp. more) heterogenous consumer preferences are, the larger (resp. smaller) the region of parameters in which stronger network externalities reduce the equilibrium fee, thereby promoting market participation of the independent unit.

Finally, notice that

$$x_1^{VI} - X^M = -\frac{\theta(\sigma + 2\mu)(\sigma - \theta)}{4(2\sigma - \theta)(\theta^2 + 5\sigma(\sigma - \theta))} \leq 0,$$

but

$$X_L^{VI} - X^M = \frac{\theta\sigma(2\mu + \sigma)}{4(2\sigma - \theta)(\theta^2 + 5\sigma(\sigma - \theta))} \geq 0.$$

Although the merged entity U has a first-mover's advantage vis-à-vis D_2— i.e., setting w_2 is de facto equivalent to set $x_2^{VI}(w_2)$ — unlike the traditional Stackelberg leader it prefers to reduce its output below the monopoly level, by setting w_2 below $\bar{w}$ in order to foster D_2's output and trigger consumer's expectations (only when $\theta = 0$ the merged entity behaves as a Stackelberg leader and sets output to the monopoly level).

3.3 Competitive and welfare effects

We are now in the position to assess the competitive and welfare effects of the merger. A useful first step is to compare equilibrium fees and market shares pre- and post-merger.

Proposition 3 *The equilibrium fee is higher under vertical integration than vertical separation — i.e., $w_L^{VI} > w_L^\star$. Moreover, D_1 's market share increases under vertical integration while D_2's market share drops — i.e.,*

$$x_1^{VI} > x_L^\star > x_2^{VI} \geq 0.$$

Moreover, D_2's output is always positive in presence of network effects — i.e., $x_2^{VI} > 0$ for all $\theta > 0$.

The reason why the fee increases after the merger is simple: by increasing the fee charged to its independent unit the merged entity is able to divert demand from the rival to its own direct channel, whose production cost is zero. This increase in D_2's fee lowers its output in the vertical integration case in comparison to the vertical separation case. Furthermore, the output of D_1 increases as the marginal cost of production of D_1 falls along with a diversion of output away from D_2.

Corollary 1 *The following comparative statics holds:*

- $\frac{\partial(w_L^{VI}-w_L^\star)}{\partial\theta} \leq 0$ *if and only if* $\theta \geq \theta^\star$;
- $\frac{\partial(x_1^{VI}-x_L^\star)}{\partial\theta} > 0$ *for every* $\theta \in [0,\sigma)$;
- $\frac{\partial|x_2^{VI}-x_L^\star|}{\partial\theta} \leq 0$ *if and only if* $\theta \geq \frac{4\sigma}{25}$.

The first result is immediate from the fact that w_L^{VI} is inverted-U shaped in θ, while $w_L^\star$ is constant. The intuition for the second result is also simple: by cashing directly the downstream revenues, the vertically integrated platform internalizes the effect of consumers' expectations more than under vertical separation. Finally, regarding the effect of θ on the difference between D_2's output with and without vertical integration, the result shows that such difference falls in θ when this parameter is not too small (and vice-versa). The intuition is as follows. In contrast to the vertical separation case, the equilibrium fee is inverted U-shaped with respect to θ under vertical integration. Hence, θ must have a non-monotone effect on D_2's output too. Specifically, an increase in θ impacts D_2 's output as follows

$$\underbrace{\frac{\partial x_2^{VI}(w_L^{VI})}{\partial\theta}}_{\text{Direct effect }(+)} + \underbrace{\frac{\partial x_2^{VI}(w_L^{VI})}{\partial w_2}\frac{\partial w_L^{VI}}{\partial\theta}}_{\text{Indirect effect }(\ ?)}.$$

The direct effect is always positive as it increases the demand intercept, thereby accommodating an expansion of D_2's output, and this effect is always larger under vertical integration. Instead, the indirect effect is positive for θ being sufficiently large and negative otherwise. Therefore, when θ is small, the indirect effect has a different sign than the direct effect. As a result, D_2's output increases with θ at a slower rate under vertical integration than under vertical separation for θ small, and the opposite holds otherwise.

We can now turn to study the effects of vertical integration on profits.

Proposition 4 *With linear contracts, vertical integration is profitable to U-D_1 but detrimental to D_2. The loss incurred by D_2 becomes negligible when θ is sufficiently large.*

Under linear contracts, the merger is profitable because it enables U to eliminate one layer of double marginalization — i.e., D_1's unit cost drops to zero. Yet, the merger reduces D_2's profit. This is because, being less exposed to double marginalization, the integrated supplier extracts a larger fraction of D_2's profit via a higher fee — i.e., vertical integration solves the double marginalization problem with D_1.while it makes it worse with D_2. This loss, however, becomes negligible when network externalities are sufficiently large since extracting more rents from D_2 depresses consumers expectation on the ecosystem size, thereby reducing demand.

We finally study the effects of the merger on consumer surplus and total welfare.

Proposition 5 *With linear contracts, the merger always increases consumer surplus, industry profit and thus total welfare. The gain in consumer surplus and total welfare is increasing in θ.*

The reason why the merger increases consumer surplus is standard: it reduces double marginalization while retaining competition in the product market. The reason why total industry profit increases is also intuitive: the loss of D_2 induced by a higher fee post merger is more than compensated by the increase in profits of the merged entity because more production in the industry is allocated to the most efficient firm. The effect on total welfare then follows immediately: since products are homogenous and sellers compete by setting quantity, D_2's loss is internalized by its rivals, whose profit increases. Hence, in addition to increasing consumer surplus, the merger also increases industry profit, thereby increasing total welfare. Obviously, when network externalities increase, the intra-platform rival's participation in the market increases as well as consumers benefit. An increase in θ, lowers the negative implications of vertical integration on the rival while the benefits from the elimination of double marginalization remain. Thus, consumer benefits from more competition over and above the elimination of double marginalization. Instead, in the vertical separation case, the welfare benefits of network externalities are partially dissipated by excessive double marginalization under vertical separation. By contrast, by eliminating one layer of double marginalization, vertical integration allows consumers and the integrated entity to appropriate the benefits of network externalities better.

4 Extensions and further remarks

This section shows that the analysis developed above and its main insights remain true and, in some cases, even strengthen when most of its underlying assumptions are relaxed.

4.1 Indirect network externalities

So far we have only considered direct network externalities. We now take a two-sided market perspective and assume that network externalities are indirect rather than direct. To this purpose, we consider a game with four types of players: the platform, the consumers, the sellers, and the developers. Compared to the previous model, the novelty here is the presence of developers. These players provide additional (complementary) services that consumers enjoy when joining the platform through the product purchased from the sellers. A common example of such an environment is the modern electronic device industry.

The value that consumers attribute to an electronic device (e.g., a tablet or a smart-phone) is a function of the number of Apps available on that device, which is in turn determined by the operating system managed by the platform. On the other hand, an App developer's incentives to develop an App to use on a specific type of device depend on the number of users who adopt compatible devices. This implies that there are indirect network externalities in the adoption of electronic devices.

We argue that this additional layer of complexity in the market structure does not alter the baseline model and that the two settings are de facto equivalent. Suppose that the inverse demand function (derived as in the baseline model) is as follows:

$$P(\cdot) \triangleq \max\left\{0, \mu + \frac{\sigma}{2} + \gamma\Delta^e - \sigma X\right\},$$

where Δ^e denotes the consumers' belief regarding the total mass of developers (apps) active within the ecosystem, and γ is the marginal utility from the presence of an additional developer.

Developers' are heterogenous and their participation is modeled as follows. They are distributed according to their random outside option $k \sim \mathcal{U}[0,1]$.[20] A developer of type k obtains the following utility from participating in the platform

$$v(\cdot) \triangleq \phi X^e - k,$$

where X^e denotes the mass of expected consumers on the platform. As standard in the literature we assume common beliefs on the two-sided of the market — i.e., all consumers hold the same expectation on the number of developers joining the platform and vice-versa. The parameter ϕ is the value that developers attribute to each additional (expected) consumer on the platform. For simplicity, we assume that developers are not charged an entry price by the platform for offering their products on the platform.[21]

Developers will be active in the ecosystem as long as they make positive expected profits given their belief on the total consumer demand on the network — i.e.,

$$v(\cdot) \geq 0 \quad \Leftrightarrow \quad k \leq \underline{k}(X^e) \triangleq \phi X^e.$$

The above condition determines the mass of developers $\Delta(X^e) = \phi X^e$ that will join the platform conditional on their (common) belief X^e.

The timing of the (modified) game is as follows:

$t = 1$ U decides whether to merge with D_1.

$t = 2$ U publicly sets the listing fee(s) charged to its downstream unit(s). Consumers and developers observe contracts. Consumers form expectations on the mass of developers affiliating with platforms Δ^e and developers form expectations on the consumer demand X^e.

[20]We assume the standard regularity conditions for well behaved demands.

[21]The results do not change with the inclusion of a positive ecosystem **participation** fee charged to the developers (see the online Appendix).

$t = 3$ Sellers choose outputs, profits materialize and payments are made.

Under the hypothesis that beliefs are correct in equilibrium and that contracts are public to all players, this model produces the same results as in the baseline model with θ being replaced by the product $\gamma\phi$ — i.e.,

$$P(\cdot) = \max\left\{0, \mu + \frac{\sigma}{2} + \gamma\phi X^e - \sigma X\right\}.$$

As a result, when U vertically integrates with a seller, it will not foreclose its non-integrated rivals provided that γ and ϕ are both positive. The solution of the model is, therefore, exactly the same as in the baseline model with $\theta = \phi\gamma$.

Of course, the introduction of access prices on both sides of the market may slightly change the equilibrium values, but the link between network externalities and foreclosure will not be altered by the fact that U can charge developers and consumers an access price.[22]

4.2 Endogenous network effects

Up to this point, we have assumed that the parameter θ capturing network effects is exogenous. However, one could imagine that platforms can engage in costly activities to improve their ecosystems' quality and foster network externalities — e.g., advertising and promotional campaigns, R&D activities aimed at strengthening the compatibility standards between the different products belonging to the ecosystem, etc. In the following, we show that the incentive to invest in θ is higher under vertical integration case than in the no integration case. We impose the conservative assumption that there are no other integration efficiencies to this purpose. Therefore, we examine the determinants of U's marginal benefit from an expansion of θ with and without vertical integration. In the Appendix we show that the following holds:

Proposition 6 *If U can invest to enhance network externalities, it invests more under vertical integration than vertical separation — i.e.,*

$$\frac{\partial(P_L^{VI} x_1^{VI} + w_L^{VI} x_2^{VI})}{\partial\theta} > \frac{\partial(w_L^\star X_L^\star)}{\partial\theta} > 0.$$

This result strengthens our previous conclusions. With vertical integration U directly internalizes the impact of θ on the equilibrium price through the downstream unit's profit. By contrast, under vertical separation the effect is only indirect through the equilibrium values $w_L^\star$ and $X_L^\star$.

4.3 Endogenous contract disclosure

The results illustrated above hold under the hypothesis that listing contracts charged to third-party sellers are observed by the market and can influence participants' expectations about the network size. A natural question is whether the platform has an incentive to disclose these contracts (or credibly commit to

[22] This extension is available in the online Appendix.

making them observable). In this section, we show that contract disclosure is always in the platform's best interest. While vertical integration is always profitable with and without public contracts, the platform can influence participants' expectations in the former case. By contrast, under secret contracts — i.e., when the fee that is offered to a seller is unknown to consumers and the rival — the vertically integrated platform always has an incentive to foreclose the downstream rival. The reason is simple: for every expectation $x_2^e > 0$, the integrated entity has always an incentive to (secretly) charge a high enough fee to foreclose D_2 — i.e., such that $x_2 = 0$. Consumers will rationally anticipate this behavior and accordingly reduce their willingness to pay — i.e., the only credible expectation is $x_2^e = 0$.

Therefore, when U integrates with D_1 and forecloses D_2, and consumers anticipate a monopoly in the downstream market, U solves

$$\max_{x_1 \geq 0} \left(\mu + \frac{\sigma}{2} + \theta x_1^e - \sigma x_1\right) x_1,$$

whose solution, assuming rational expectations ($x_1^e = x_1$) yields

$$x^M \triangleq \frac{\sigma + 2\mu}{2\,(2\sigma - \theta)},$$

which as expected is increasing in θ. Substituting this expression into U's monopoly profit we have:

$$\pi^M \triangleq \frac{\sigma\,(2\mu + \sigma)^2}{4\,(2\sigma - \theta)^2}.$$

Comparing this expression with U's profits under public linear contracts and vertical integration, i.e.,

$$\pi_L^{VI} \triangleq P_L^{VI} x_1^{VI} + w_L^{VI} x_2^{VI} = \frac{5\sigma(2\mu + \sigma)^2}{16(\theta^2 + 5\sigma\,(\sigma - \theta))},$$

we have

$$\pi_L^{VI} - \pi^M \triangleq \frac{\theta^2\sigma\,(\sigma + 2\mu)^2}{16(\theta^2 + 5\sigma\,(\sigma - \theta))\,(\theta - 2\sigma)^2},$$

which is always positive and equal to zero at $\theta = 0$. Therefore, U always has an incentive to disclose its contract rather than making them secret and monopolize the market post merger.

4.4 Two-part tariffs

Suppose now that U offers two-part tariffs — i.e., D_i is offered a contract $\mathcal{C}_i \triangleq (w_i, F_i)$ specifying a linear fee w_i, to be paid for each unit of sale made, and a fixed (lump sum) fee F_i. Hence, when D_i sells x_i units of final product, the payment collected by U is $w_i x_i + F_i$. The rest of the assumptions are as in the baseline model. Once again, we impose that consumer preferences are sufficiently dispersed to guarantee that only a positive mass of consumers, but not all, buy the product.

A2 The dispersion index $d \triangleq \frac{\sigma^2}{\mu}$ of the consumers' willingness to pay is sufficiently large — i.e.,

$$d \geq d^{\star\star} \triangleq \max\left\{0, \frac{2\sigma^2}{3\sigma - 4\theta}\right\}.$$

Since fixed fees are sunk when outputs are chosen, the equilibrium of the quantity-setting subgame is the same as in the baseline model. Hence, we can directly focus on the contracting stage.

Vertical separation. U chooses $\mathcal{C}_1$ and $\mathcal{C}_2$ to maximize its profits — i.e.,

$$\max_{(\mathcal{C}_i)_{i=1,2}} \sum_{i=1,2} \left[w_i x_i^\star \left(w_i, w_j\right) + F_i\right],$$

subject to D_i's participation constraint

$$F_i \leq \left[P^\star \left(w_1, w_2\right) - w_i\right] x_i^\star \left(w_i, w_j\right), \quad \forall i = 1, 2.$$

Substituting D_i's binding participation into U's maximization problem and rearranging terms, the above maximization problem rewrites as

$$\max_{w_i} \underbrace{P^\star \left(w_1, w_2\right) X^\star \left(w_1, w_2\right)}_{\text{Ecosystem profit}}.$$

Differentiating with respect to w_i, we obtain the following first-order conditions

$$\underbrace{\frac{\partial P^\star(\cdot)}{\partial X^e}\frac{\partial X^\star(\cdot)}{\partial w_i} X^\star(\cdot)}_{\text{Network externality effect (-)}} + \underbrace{\frac{\partial X^\star(\cdot)}{\partial w_i}\left(\frac{\partial P^\star(\cdot)}{\partial X} X^\star(\cdot) + P^\star(\cdot)\right)}_{\text{Industry profit maximization w/o network effects}} = 0, \quad \forall i = 1, 2. \tag{10}$$

As standard in a model with (vertical) public contracts, there is a trade-off between volumes and profit margin. By increasing w_i, U exerts downward pressure on the aggregate output. At the same time, a lower aggregate output increases the market price that, in turn, expands U's profit margin.

The expression above clearly shows that network externalities reduce the linear component of the contract offered by U to D_i. At $\theta = 0$ the network externality effect vanishes; as a result, the equilibrium fees are set to maximize industry profit (monopoly outcome). Instead, when $\theta > 0$, network effects reduce the incentive to implement the monopoly outcome since the first term in (10) is negative.

Solving the first-order condition (10), we can show the following proposition.

Proposition 7 *With two-part tariffs and vertical separation, there is a unique symmetric equilibrium such that*

$$w_T^\star \triangleq \frac{(\sigma - 2\theta)(2\mu + \sigma)}{8(\sigma - \theta)}, \quad \forall i = 1, 2. \tag{11}$$

with $w_T^\star$ being decreasing in θ.

The equilibrium fee charged under vertical separation is decreasing in θ. Interestingly, for $\theta > \frac{\sigma}{2}$ the equilibrium fee is negative (i.e., below marginal cost) even with vertical separation. The platform finds it profitable to subsidize firm outputs to expand its ecosystem size which increases consumer demand without hurting margins. This is because under two-part-tariffs U is able to internalize all the benefit of increasing consumers' expectations on the ecosystem size through the fixed fee.

Vertical integration. The merged entity U-D_1 solves the following maximization problem

$$\max_{\mathcal{C}_2} P^{VI}(w_2)\, x_1^{VI}(w_2) + w_2 x_2^{VI}(w_2) + F_2 \tag{12}$$

subject to D_2's participation constraint

$$F_2 \leq \left[P^{VI}(w_2) - w_2\right] x_2^{VI}(w_2).$$

Substituting D_2's binding participation constraint into U-D_1's maximization problem and rearranging terms, the above maximization problem rewrites as

$$\max_{w_2} \underbrace{P^{VI}(w_2) X^{VI}(w_2)}_{\text{Ecosystem profit}}.$$

Differentiating with respect to w_2, by the Envelope Theorem, we obtain

$$\underbrace{\frac{\partial P^{VI}(\cdot)}{\partial X^e} \frac{\partial X^{VI}(\cdot)}{\partial w_2} X^{VI}(\cdot)}_{\text{Network externality effect }(-)} - \underbrace{(P^{VI}(\cdot) - w_2) \frac{\partial X^{VI}(\cdot)}{\partial w_2}}_{\text{Strategic effect }(+)} = 0. \tag{13}$$

Hence, we can show the following.

Proposition 8 *With two-part tariffs and vertical integration, the equilibrium fee charged to D_2 is*

$$w_T^{VI} \triangleq \frac{(\sigma - 2\theta)(2\mu + \sigma)}{4(\sigma - \theta)} \tag{14}$$

$$= \bar{w} - \frac{\theta(3\sigma - 2\theta)(2\mu + \sigma)}{4(2\sigma - \theta)(\sigma - \theta)} \leq \bar{w}, \tag{15}$$

with w_T^{VI} decreasing in θ. Moreover, for every $\theta > 0$, the merged entity has no incentive to foreclose the non-integrated rival — i.e., $w_T^{VI} < \bar{w}$ so that $x_2^{VI} > 0$.

This proposition demonstrates that the logic of the no foreclosure result discussed in the baseline model holds even under two-part tariffs. Once more, for θ sufficiently large — i.e., $\theta > \frac{\sigma}{2}$ — the equilibrium fee is negative. Notice, however, that the equilibrium fee under vertical integration is more responsive to θ

than under vertical separation — i.e.,

$$\left|\frac{\partial w_T^{VI}}{\partial \theta}\right| > \left|\frac{\partial w_T^{\star}}{\partial \theta}\right|.$$

The reason is that, with vertical integration, U has only one instrument to influence consumers' expectations. Hence, D_2's fee must react more to θ under integration than separation.

4.4.1 Competitive and welfare effects

We now turn to study the competitive and the welfare effects of the merger under two-part tariffs.

Proposition 9 *The equilibrium linear fee is higher under vertical integration than vertical separation — i.e.,* $w_T^{VI} > w_T^{\star}$*. Moreover,* D_1*'s output increases post-merger while* D_2*'s output drops post-merger — i.e.,* $x_1^{VI} > x^{\star} > x_2^{VI}$*.*

The reason why the fee increases after the merger is simple: for given retail price, by increasing the fee the merged entity is able to divert demand form the rival to its own direct channel, whose production cost is zero. However, as discussed above, in order to exploit network effects, the merged entity has no incentive to fully foreclose the rival — i.e., $w_T^{VI} < \bar{w}$ when $\theta > 0$.

Corollary 2 *With two-part tariffs,* θ *unambiguously reduces the difference between fees and outputs with and without vertical integration.*

The above corollary shows that as network effects rise, the incentive of the merged entity to increase the rival's cost mitigates. Hence, the negative impact on D_2's output levels is lower for (relatively) large network effects.

Finally, the following neutrality result holds.

Proposition 10 *Vertical integration is welfare neutral — i.e., it does not impact industry profits and aggregate output.*

The reason why, with public two-part tariffs, vertical integration does not impact profits and aggregate output is as follows. With two-part tariffs and vertical separation, U fully internalizes downstream profits via the fixed fee while influencing output decisions and consumers' expectations through the choice of the per-unit fee. Essentially, when contracts are public, the vertically separated supplier can always replicate the market outcome obtained under vertical integration — i.e., by setting $w_1 = 0$ and $w_2 = w_T^{VI}$. Hence, U's profit is the same before and after the merger, implying that vertical integration does not impact aggregate output either.

Finally, notice that while in general, the shares of the aggregate surplus accruing to each party in a vertical relationship always depend on the attractiveness of outside options available to sellers, in this paper, we assumed a monopolist platform to derive conservative results concerning its incentives to foreclose. In this respect, the presence of an outside option for the sellers can only reinforce the non-foreclosure result.

4.5 Competing (two-sided) ecosystems

In this section, we introduce competition between two-sided ecosystems. Specifically, we consider a vertically integrated platform (hereafter the incumbent, I) that competes with D_1 and D_2 in the downstream market. Following Katz & Shapiro (1985), we assume that the two ecosystems are not compatible. Furthermore, we posit that I offers a developer network value of γ_I to consumers while U offers a different network value to consumers γ_U. Specifically, the utility of consumers that joining ecosystem $j \in \{I, U\}$ is

$$U_j(\cdot) \triangleq r + \gamma_j \Delta_j^e - P_j,$$

where, as before, Δ_j^e is the consumers' (common) expectation on the mass of developers joining ecosystem j. The hedonic prices must be such that

$$P_U - \gamma_U \Delta_U^e = P_I - \gamma_I \Delta_I^e,$$

Let the common reservation value be

$$r^{\star\star} \triangleq P_I - \gamma_I \Delta_I^e = P_U - \gamma_U \Delta_U^e,$$

consumers buy on either network if and only if $r \geq r^{\star\star}$. Assuming again that $r \sim \mathcal{U}\left[\mu - \frac{\sigma}{2}, \mu + \frac{\sigma}{2}\right]$, aggregate demand is

$$\Pr\left[r \geq r^{\star\star}\right] = X \triangleq \sum_{i=1,2} x_i + x_I.$$

Hence, the inverse demand function for the products distributed within U's ecosystem is

$$P_U(\cdot) \triangleq \max\left\{0, \mu + \frac{\sigma}{2} + \gamma_U \Delta_U^e - \sigma X\right\},$$

while the inverse demand function for the products distributed by I is

$$P_I(\cdot) \triangleq \max\left\{0, \mu + \frac{\sigma}{2} + \gamma_I \Delta_I^e - \sigma X\right\}. \tag{16}$$

As before, we assume that developers are heterogenous with respect to their fixed cost k, which is uniformly distributed on the unit interval. Developers of type k joining platform j obtain utility $v_j(k) \triangleq \phi_j X_j^e - k$. Developers are active on a platform if they get positive utility from interacting with consumers on that platform. Therefore, the total mass of developers on platform U and I are given as

$$\Delta_U \triangleq \phi_U X_U^e, \qquad \Delta_I \triangleq \phi_I x_I^e,$$

with $X_U^e \triangleq \sum_{i=1,2} x_i^e$.

The structure of the game and the solution procedure is then as before. As before, we focus on equilibria such that consumers' expectations are fulfilled and, under vertical separation, D_1 and D_2 produce the same output and receive the same contract offer. We restrict attention to the region of parameters in which equilibrium outputs are positive, second-order conditions are satisfied and $r^{\star\star}$ is interior. To obtain interior solutions, we impose that network externalities are not too high. That is, normalizing $\theta_I \triangleq \gamma_I \phi_I$ and $\theta_U \triangleq \gamma_U \phi_U$, we assume the following:

A3 $\theta_I < \sigma$ and

$$0 < \theta_U < \hat{\theta}_U \triangleq \frac{\sigma\,(4\theta_I - 7\sigma)}{2(\theta_I - 2\sigma)} - \frac{\sigma}{2}\sqrt{\frac{6\theta_I^2 - 22\theta_I\sigma + 21\sigma^2}{(\theta_I - 2\sigma)^2}}.$$

The restriction on θ_I ensures that pre-merger outputs are positive. The restriction on θ_U ensures that the post-merger market price of the incumbent is positive.

The key difference with the previous analysis is the presence of I, whose maximization problem is

$$\max_{x_I} P_I\,(\cdot)\,x_I.$$

Differentiating with respect to x_I, the first-order condition yields

$$x_I^\star\,(\Delta_I^e, X_U) \triangleq \frac{\mu + \frac{\sigma}{2} + \gamma_I \Delta_I^e - \sigma X_U}{2\sigma},$$

which, as expected, is increasing in I's network size and decreasing in the aggregate output distributed within U's network.

Then, the next result summarizes the equilibrium characterization under vertical separation.

Proposition 11 *When U competes with an integrated rival I, the symmetric equilibrium under vertical separation is such that*

$$w_U^\star \triangleq \frac{(\sigma - \theta_I)(2\mu + \sigma)}{4(2\sigma - \theta_I)}.$$

The equilibrium linear fee $w_U^\star$ is decreasing in θ_I and constant in θ_U. $X_U^\star$ is increasing in θ_U, while $x_I^\star$ is increasing in θ_I and decreasing in θ_U. Interestingly, $X_U^\star$ rises with θ_I if and only if $\theta_I \leq \tilde{\theta}_I \triangleq \frac{2\sigma}{3}$ and $\theta_U \geq \tilde{\theta}_U \triangleq \frac{\sigma}{2}$.

As intuition suggests, the equilibrium fee offered by U to its sellers is decreasing with the strength of network effects of the rival's ecosystem. Moreover, as the value of own network benefits on an ecosystem increase, the total output increases on the respective ecosystem. Further as θ_U increases, the output of the integrated ecosystem falls. Interestingly, the total output of ecosystem U also increases in the network effects of the rival integrated ecosystem (θ_I) when θ_U is sufficiently large and θ_I small. Intuitively, the output expansion effect arising from a fall in w_U^* with θ_I dominates the output reduction effect from an increase in the attractiveness of ecosystem I when θ_U is large and θ_I is small.

Proposition 12 *When competing with an integrated rival, the merged entity U- D_1 has no incentive to foreclose D_2 even if $\theta_U = 0$. The equilibrium fee is*

$$w_U^{VI} \triangleq \frac{\sigma(2\mu+\sigma)(\sigma-\theta_I)(4\theta_U(\theta_I-2\sigma)+\sigma(6\sigma-5\theta_I))}{4\theta_U^2(\theta_I-2\sigma)^2-4\theta_U\sigma(8\sigma-5\theta_I)(2\sigma-\theta_I)+4\sigma^2(5\theta_I^2-15\theta_I\sigma+11\sigma^2)},$$

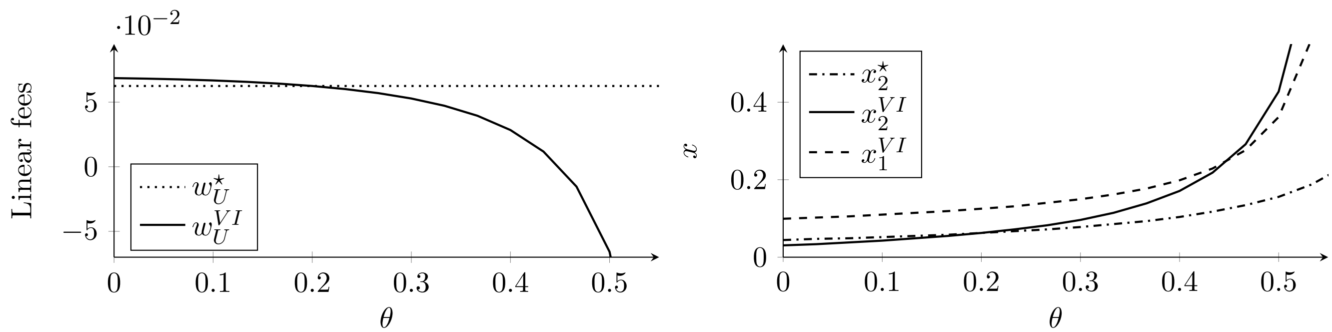
with $w_U^{VI} < 0$ if and only if $\theta_U > \theta_U^{\star\star} \triangleq \frac{\sigma(6\sigma-5\theta_I)}{4(2\sigma-\theta_I)}$ and $\theta_I > \theta_I^{\star\star} \triangleq \frac{2\sigma}{3}$.

In line with the baseline model, the integrated supplier U-D_1 does not foreclose D_2. Interestingly, with competing ecosystems, this is true even in the absence of network effects — i.e., at $\theta_U = 0$.[23] The reason is that dealing with D_2 is a commitment device vis-à-vis the incumbent to expand output. Indeed, w_U^{VI} can be even negative (below marginal cost) for θ_U and θ_I sufficiently large: in this parameter region, U commits to subsidize D_2 to lower I's output.

Hence, we can state the following.

Proposition 13 *D_2's output is larger than the output of U-D_1 if and only if $\theta_U > \theta_U^{\star\star}$ and $\theta_I > \theta_I^{\star\star}$.*

The interesting result here is that, when network effects are sufficiently large, the output of the non-integrated unit can be higher than that of the vertically integrated seller. Hence, with network externalities and competing ecosystems, the standard foreclosure logic not only fails, but it is even reversed: the integrated platform has an incentive to subsidize its non-integrated units to squeeze the incumbent's market share. As an illustrative example, Figure 2 plots the equilibrium fee and D_2's output pre- and post-merger as functions of θ_U and θ_I.



(a) Post-merger wholesale price and choke wholesale price

(b) Post-merger and pre-merger output of D_2.

Figure 2: Equilibrium wholesale prices and outputs for $\mu = 1/8$, $\sigma = 1$ and $\theta_I = 0.75$.

[23] This result is also confirmed in a differentiated Bertrand competition setting, see, e.g., Condorelli & Padilla (2021).

4.5.1 Competitive and welfare effects

We can finally characterize the competitive and the welfare effects of the merger with competing ecosystems. Let

$$\underline{\theta} \triangleq \frac{\sigma(\sigma - \theta_I)}{2\sigma - \theta_I},$$

then:

Proposition 14 *Comparing the fee charged to D_2 and its output pre- and post-merger, we can state the following:*

- *Under strong network externalities ($\theta_U > \underline{\theta}$): $w_U^{VI} < w_U^\star$ and $x_2^{VI} > x_2^\star$;*
- *Under weak network externalities ($\theta_U \leq \underline{\theta}$): $w_U^{VI} \geq w_U^\star$ and $x_2^{VI} \leq x_2^\star$.*

This proposition shows that, with competing ecosystems, the standard foreclosure logic is reversed provided network externalities are strong enough — i.e., D_2 produces more under vertical integration than under vertical separation.

We can finally state the welfare effects of the merger with competing ecosystems.

Proposition 15 *When U competes with an integrated rival, the merger is always profit-increasing and consumer surplus increasing. Moreover, D_2's profit is higher under vertical integration than under vertical separation if and only if $\theta_U > \underline{\theta}$.*

As in the baseline model, the merger is profitable because it allows U to eliminate one source of double marginalization while controlling I 's output with the fee charged to its non-integrated unit; and it is also consumer surplus increasing because, in addition to eliminating double marginalization, when θ_U is sufficiently large, it also expands D_2's output via a lower fee. The effect on D_2's profit follows immediately Proposition (14). When $\theta_U > \underline{\theta}$, the fee charged to D_2 is lower than the pre-merger level. As a result, D_2's output, and thus its profit, increases in the post-merger regime.

5 Conclusions

In this paper, we argued that the traditional vertical foreclosure logic fails when considering downstream markets featuring direct and indirect network effects. In particular, we have shown that when the utility that each consumer derives from consumption increases with the number of other people consuming the same good (direct network externalities) or with the number of developers in the platform (indirect network externalities) and vice versa, vertically integrated platforms have no incentive to foreclose their downstream nonintegrated rivals fully. Moreover, the incentive to marginalize opponents falls with the extent of network externalities — i.e., the market participation (output) of the nonintegrated seller(s) increases with the degree of network externalities. We have also argued that while vertical integration is profitable and consumer surplus enhancing when the supplier can offer linear listing contracts, it is profit and consumer surplus neutral with two-part tariffs. Interestingly, results are robust when introducing competing networks.

These results have several managerial implications and shed new light on the business conduct of modern platforms. In particular, we have uncovered a positive link between platforms' propensity to open their ecosystems to third-party sellers and network externalities and publicly disclose contracts to influence market participants' expectations about networks' size. In addition, our analysis also provides a conceptual framework and a set of tools that can help policymakers assess the social desirability of vertical mergers in the digital industry.

6 Appendix

Proof of Proposition 1. Assuming a symmetric equilibrium, the expression in equation (6) can be rewritten as

$$\frac{2\mu+\sigma-4w_L^\star}{6\sigma-4\theta}=0,$$

whose solution yields

$$w_L^\star=\frac{2\mu+\sigma}{4}.$$

Substituting $w_L^\star$ into the expression for aggregate output, we obtain

$$X_L^\star \triangleq X^\star(w_L^\star)=\frac{2\mu+\sigma}{2(3\sigma-2\theta)}.$$

Differentiating $X_L^\star$ with respect to θ

$$\frac{\partial X_L^*}{\partial\theta}=\frac{2\mu+\sigma}{(3\sigma-2\theta)^2},$$

which is always positive.

Finally, the equilibrium market price is

$$P_L^\star \triangleq \frac{(2\mu+\sigma)(2\sigma-\theta)}{2(3\sigma-2\theta)}.$$

which is also increasing in θ.

It can be verified that U's profit is

$$\pi_U^\star \triangleq \frac{(2\mu+\sigma)^2}{8(3\sigma-2\theta)},$$

which is clearly increasing in θ. ■

Proof of Proposition 2. The first-order condition (8) can be rewritten as

$$\frac{(5\sigma-4\theta)\sigma(2\mu+\sigma)-4w_L^{VI}(5\sigma^2+\theta^2-5\theta\sigma)}{2\sigma(3\sigma-2\theta)^2}=0,$$

whose solution yields

$$w_L^{VI} \triangleq \frac{\sigma(2\mu+\sigma)(5\sigma-4\theta)}{4(\theta^2+5\sigma\,(\sigma-\theta))},$$

with $w_L^{VI}<\bar{w}$. Therefore, D_2 is never fully foreclosed for every $\theta\in[0,\sigma)$.

Differentiating w_L^{VI} with respect to θ

$$\frac{\partial w_L^{VI}}{\partial\theta}=\frac{\sigma(2\mu+\sigma)(5\sigma\,(\sigma-2\theta)+4\theta^2)}{4(\theta^2+5\sigma\,(\sigma-\theta))^2}\geq 0 \quad\Leftrightarrow\quad \theta\leq\theta^\star \triangleq \frac{(5-\sqrt{5})\sigma}{4}.$$

Substituting the equilibrium fee into the outputs and the market price, we have

$$x_2^{VI} = \frac{\theta(2\mu+\sigma)}{4(\theta^2+5\sigma(\sigma-\theta))} < x_1^{VI} = \frac{(5\sigma-3\theta)(2\mu+\sigma)}{4(\theta^2+5\sigma(\sigma-\theta))},$$

$$X_L^{VI} = \frac{(5\sigma-2\theta)(2\mu+\sigma)}{4(\theta^2+5\sigma(\sigma-\theta))}, \quad P_L^{VI} = \frac{\sigma(5\sigma-3\theta)(2\mu+\sigma)}{4(\theta^2+5\sigma(\sigma-\theta))}.$$

It is immediate to see that $x_2^{VI} > 0$ if and only if $\theta > 0$. Finally, differentiating with respect to θ, we get

$$\frac{\partial x_2^{VI}}{\partial \theta} = \frac{(5\sigma^2-\theta^2)(2\mu+\sigma)}{4(\theta^2+5\sigma(\sigma-\theta))^2} > 0,$$

which concludes the proof. ■

Proof of Proposition 3. Taking the difference between the equilibrium fee under vertical integration and vertical separation, we have

$$w_L^{VI} - w_L^{\star} = \frac{\theta(2\mu+\sigma)(\sigma-\theta)}{4(\theta^2+5\sigma(\sigma-\theta))} \geq 0.$$

Moreover,

$$x_1^{VI} - x_L^{\star} = \frac{\left(5\theta^2-14\theta\sigma+10\sigma^2\right)(2\mu+\sigma)}{4(3\sigma-2\theta)(\theta^2+5\sigma(\sigma-\theta))} > 0. \tag{17}$$

$$x_2^{VI} - x_L^{\star} = -\frac{(5\sigma-3\theta)(\sigma-\theta)(2\mu+\sigma)}{4(3\sigma-2\theta)(\theta^2+5\sigma(\sigma-\theta))} < 0, \tag{18}$$

which concludes the proof. ■

Proof of Corollary 1. The sign of the derivative with respect to θ of the difference $w_L^{VI} - w_L^{\star}$ is immediate since $w_L^{\star}$ is constant in θ. Moreover, it is easy to show

$$\frac{\partial(x_1^{VI} - x_L^{\star})}{\partial \theta} = \frac{(2\sigma-\theta)\left(20\sigma^3+36\theta^2\sigma-10\theta^3-45\theta\sigma^2\right)(2\mu+\sigma)}{4(2\theta-3\sigma)^2(\theta^2+5\sigma(\sigma-\theta))^2} > 0,$$

and that

$$\frac{\partial(x_2^{VI} - x_L^{\star})}{\partial \theta} = -\frac{\left(6\theta^4-32\theta^3\sigma+59\theta^2\sigma^2-40\theta\sigma^3+5\sigma^4\right)(2\mu+\sigma)}{4(2\theta-3\sigma)^2(\theta^2+5\sigma(\sigma-\theta))^2} \geq 0 \quad \Leftrightarrow \quad \theta \geq 0.16\sigma,$$

which concludes the proof. ■

Proof of Proposition 4. The post-merger profit of U is

$$\pi_L^{VI} \triangleq P_L^{VI} x_1^{VI} + w_L^{VI} x_2^{VI} = \frac{5\sigma(2\mu+\sigma)^2}{16(\theta^2+5\sigma(\sigma-\theta))}.$$

Comparing profits with and without vertical integration

$$\pi_L^{VI} - \pi_U^\star = \frac{\left(5\sigma^2 - 2\theta^2\right)(2\mu + \sigma)^2}{16(3\sigma - 2\theta)(\theta^2 + 5\sigma(\sigma - \theta))} > 0.$$

The profit of each downstream firm under vertical separation is

$$(P_L^\star - w_L^\star)x_L^\star = \frac{\sigma(2\mu + \sigma)^2}{16(2\theta - 3\sigma)^2}.$$

D_2's profit under vertical integration is

$$(P_L^{VI} - w_L^{VI})x_2^{VI} = \frac{\theta^2\sigma(2\mu + \sigma)^2}{16(\theta^2 + 5\sigma(\sigma - \theta))^2}.$$

Taking the difference between theese expressions we have

$$(P_L^\star - w_L^\star)x_L^\star - (P_L^{VI} - w_L^{VI})x_2^{VI} = \frac{\sigma(5\sigma - 3\theta)(\sigma - \theta)\left(5\sigma^2 - \theta^2 - 2\theta\sigma\right)(2\mu + \sigma)^2}{16(2\theta - 3\sigma)^2(\theta^2 + 5\sigma(\sigma - \theta))^2} > 0.$$

At the $\theta = \sigma$, the above profit difference is equal to 0.

Finally, we can compute the difference between total industry profit with and without vertical integration

$$P_L^{VI}X_L^{VI} - P_L^\star X_L^\star = \frac{\left(2\theta^2 + 5\sigma^2 - 6\theta\sigma\right)\left(2\theta^3 + 5\sigma\left(\sigma^2 - \theta^2\right) + \theta\sigma(\sigma - \theta)\right)(\sigma + 2\mu)^2}{16\left(2\theta - 3\sigma\right)^2\left(\theta^2 + 5\sigma(\sigma - \theta)\right)^2} > 0,$$

which positive since $2\theta^2 + 5\sigma^2 - 6\theta\sigma$ is decreasing in θ and is positive at $\theta = 0$. ■

Proof of Proposition 5. Before comparing consumer surplus, it is useful to compare the aggregate output under vertical integration and vertical separation — i.e.,

$$X_L^{VI} - X_L^\star = \frac{\left(2\theta^2 - 6\theta\sigma + 5\sigma^2\right)(2\mu + \sigma)}{4(3\sigma - 2\theta)\left(\theta^2 - 5\theta\sigma + 5\sigma^2\right)} > 0.$$

Next, notice that the expression for consumer surplus is given as

$$CS(X) = \frac{X^2}{2}.$$

Therefore, the difference between consumer surplus with vertical integration and vertical separation is

$$CS_L^{VI} - CS_L^\star = \frac{\left(2\theta^2 - 6\theta\sigma + 5\sigma^2\right)\left(6\theta^2 - 26\theta\sigma + 25\sigma^2\right)(2\mu + \sigma)^2}{32(2\theta - 3\sigma)^2(\theta^2 + 5\sigma(\sigma - \theta))^2} > 0$$

which is always positive.

Differentiating the above expression with respect to θ yields

$$\frac{\left(24\theta^6-264\theta^5\sigma+1216\theta^4\sigma^2-3008\theta^3\sigma^3+4230\theta^2\sigma^4-3210\theta\sigma^5+1025\sigma^6\right)(2\mu+\sigma)^2}{16(3\sigma-2\theta)^3(\theta^2+5\sigma\left(\sigma-\theta\right))^3}>0,$$

which is always positive for $\theta<\sigma$.

Total welfare in the pre-merger case is

$$TW_L^{\star}=\pi_U^{\star}+2(P_L^{\star}-w_L^{\star})x_L^{\star}+CS_L^{\star}=\frac{(4\sigma+1-2\theta)(2\mu+\sigma)^2}{8(2\theta-3\sigma)^2}.$$

Total welfare in the post-merger case is

$$TW_L^{VI}=\pi_L^{VI}+(P_L^{VI}-w_L^{VI})x_2^{VI}+CS_L^{VI}=\frac{(5\sigma-2\theta)(5\sigma(2\sigma+1)-\theta(6\sigma+2))(2\mu+\sigma)^2}{32(\theta^2+5\sigma\left(\sigma-\theta\right))^2}.$$

THence, the difference between total welfare with vertical integration and vertical separation is

$$\frac{1}{32}\left(\frac{(2\theta-5\sigma)(\theta(6\sigma+2)-5\sigma(2\sigma+1))}{(\theta^2+5\sigma\left(\sigma-\theta\right))^2}+\frac{4(2\theta-4\sigma-1)}{(2\theta-3\sigma)^2}\right)(2\mu+\sigma)^2>0$$

which is always positive.

Differentiating the above expression with respect to θ yields

$$\frac{\Gamma(2\mu+\sigma)^2}{16(3\sigma-2\theta)^3\left(\theta^2-5\theta\sigma+5\sigma^2\right)^3}>0,$$

where

$$\begin{aligned}\Gamma\triangleq\ &(8\theta^6(\theta+3)-44\theta^5(\theta+6)\sigma+4\theta^4(304-3\theta)\sigma^2+4\theta^3(167\theta-752)\sigma^3\\&+18\theta^2(235-123\theta)\sigma^4+5(205-541\theta)\sigma^6+15\theta(229\theta-214)\sigma^5+875\sigma^7)\end{aligned}$$

is positive for $\theta\leq\sigma$. ■

Proof of Proposition 6. Differentiating U's equilibrium profit with respect to θ under vertical separation and vertical integration, respectively, we obtain

$$\frac{\partial(w_L^{\star}X_L^{\star})}{\partial\theta}=\frac{(2\mu+\sigma)^2}{4(3\sigma-2\theta)^2}>0,\tag{19}$$

$$\frac{\partial(P_L^{VI}x_1^{VI}+w_L^{VI}x_2^{VI})}{\partial\theta}=\frac{5\sigma(5\sigma-2\theta)(2\mu+\sigma)^2}{16\left(\theta^2+5\sigma\left(\sigma-\theta\right)\right)^2}>0.\tag{20}$$

Taking the difference between these expressions, it holds that

$$\frac{\partial(P_L^{VI}x_1^{VI}(w_L^{VI})+w_L^{VI}x_2^{VI}(w_L^{VI}))}{\partial\theta}-\frac{\partial(w_L^{\star}X_L^{\star})}{\partial\theta}=\frac{(2\mu+\sigma)^2}{16}\frac{125\sigma^4-4\theta^4+80\theta^2\sigma^2-190\theta\sigma^3}{(3\sigma-2\theta)^2\left(\theta^2+5\sigma\left(\sigma-\theta\right)\right)^2}.$$

The numerator of this expression is strictly decreasing in θ (for $\sigma \geq \theta$) and it is positive at $\theta = 0$. Hence, the result. ■

Proof of Proposition 7. Solving the first-order conditions in (10) for a symmetric equilibrium, we immediately obtain the fee described in (11), which is clearly decreasing in θ. ■

Proof of Proposition 8. Solving the first-order condition in (13), we obtain

$$w_T^{VI} = \frac{(\sigma - 2\theta)(2\mu + \sigma)}{4(\sigma - \theta)},$$

which is clearly decreasing in θ. Recall that

$$\bar{w} = \frac{\sigma(2\mu + \sigma)}{2(2\sigma - \theta)},$$

is the choke price such that $x_2^{VI}(\bar{w}) = 0$. Hence, every $w_2 > \bar{w}$ implies that D_2 is fully foreclosed. It is immediate to verify that, for any $\theta > 0$, $w_T^{VI} < \bar{w}$ and thus $x_2^{VI}(w_T^{VI}) > 0$. ■

Proof of Proposition 9. To begin with, notice that

$$w_T^{VI} - w_T^{\star} = \frac{(\sigma - 2\theta)(2\mu + \sigma)}{8(\sigma - \theta)}, \tag{21}$$

which is positive under **A2**. Similarly, comparing outputs, we have

$$\begin{aligned} x_1^{VI} - x_T^{\star} &= \frac{(\sigma - 2\theta)(2\mu + \sigma)}{8(\sigma - \theta)\sigma} > 0, \\ x_2^{VI} - x_T^{\star} &= -\frac{(\sigma - 2\theta)(2\mu + \sigma)}{8(\sigma - \theta)\sigma} < 0, \end{aligned} \tag{22}$$

which concludes the proof. ■

Proof of Corollary 2. Considering equations (21) and (22), notice that

$$w_T^{VI} - w_T^{\star} = \sigma(x_1^{VI} - x_T^{\star}), \quad (x_1^{VI} - x_T^{\star}) = -(x_2^{VI} - x_T^{\star}).$$

Therefore, to study the impact of θ on these differences it is enough to compute

$$\frac{\partial(w_T^{VI} - w_T^{\star})}{\partial\theta} = -\frac{\sigma(2\mu + \sigma)}{8(\theta - \sigma)^2} < 0,$$

which yields immediately the result stated in the corollary. ■

Proof of Proposition 10. It is immediate to verify that $X_T^{\star} = X^{VI}$ and that $P_T^{\star} = P^{VI}$. Now, recall that U's profit is the industry profit, which is thus the same pre- and post-merger. By the same token, it is immediate to verify that consumer surplus is the same pre- and post-merger. ■

Proof of Proposition 11. Solving the first-order conditions associated with D_1, D_2 and I's maximization problems, and imposing rational expectations, we have

$$x_i(w_i, w_{-i}) = \frac{\sigma(2\mu+\sigma)(\sigma-\theta_I) + 2w_{-i}(\sigma^2+\theta_I\theta_U - \sigma(\theta_I+2\theta_U)) - 2w_i(3\sigma^2+\theta_I\theta_U - 2\sigma(\theta_I+\theta_U))}{2\sigma(4\sigma(\sigma-\theta_U) - \theta_I(3\sigma-2\theta_U))}, \quad \forall i = 1,2,$$

with

$$X_U(w_1, w_2) = \frac{(\sigma-\theta_I)(\sigma+2\mu-w_1-w_2) - \sigma(w_1+w_2)}{4\sigma(\sigma-\theta_U) - \theta_I(3\sigma-2\theta_U)},$$

$$x_I(w_1, w_2) = \frac{\sigma(\sigma+2(w_1+w_2+\mu)) - 2\theta_U(\sigma+2\mu)}{8\sigma(\sigma-\theta_U) - \theta_I(6\sigma-\theta_U)}.$$

U's profit is therefore

$$\sum_{i=1,2} w_i x_i(w_i, w_{-i}).$$

Differentiating with respect to w_i yields

$$\frac{\sigma(8\mu-\theta_I+4\theta_U-16w_{-i}) - 8w_i(2\sigma-\theta_I) - 2\theta_I(4\mu+\theta_U-4w_{-i})}{8(4\sigma(\sigma-\theta_U) - \theta_I(3\sigma-2\theta_U))} + \frac{w_{-i}-w_i}{\sigma} + \frac{1}{8} = 0, \quad \forall i = 1,2.$$

Imposing symmetry, it follows

$$w_U^\star \triangleq \frac{(\sigma-\theta_I)(2\mu+\sigma)}{4(2\sigma-\theta_I)}. \tag{23}$$

Differentiating $w_U^\star$ with respect to θ_I yields

$$\frac{\partial w_U^\star}{\partial \theta_I} = -\frac{\sigma(2\mu+\sigma)}{4(2\sigma-\theta_I)^2} < 0.$$

Further, it is immediate that $\frac{\partial w_U^\star}{\partial \theta_U} = 0$.
Substituting the equilibrium fee into the outputs and market price, yields

$$X_U^\star \triangleq X_U(w_U^\star, w_U^\star) = \frac{(2\mu+\sigma)(\sigma-\theta_I)}{8(\sigma-\theta_U) - \theta_I(6\sigma-4\theta_U)}, \tag{24}$$

$$x_I^\star \triangleq x_I(w_U^\star, w_U^\star) = \frac{(\sigma-\theta_I)(\sigma+2\mu)}{4(4\sigma(\sigma-\theta_U) - \theta_I(3\sigma-2\theta_U))}, \tag{25}$$

$$P_U^\star \triangleq \frac{(2\mu+\sigma)(\sigma-\theta_I)(3\sigma^2+\theta_I\theta_U - 2\sigma(\theta_I+\theta_U))}{2(2\sigma-\theta_I)(4\sigma(\sigma-\theta_U) - \theta_I(3\sigma-2\theta_U))}, \tag{26}$$

$$P_I^\star \triangleq \frac{\sigma(2\mu+\sigma)(\sigma-\theta_I)(3\sigma^2+2\theta_I\theta_U - 2\sigma(\theta_I+2\theta_U))}{2(2\sigma-\theta_I)(4\sigma(\sigma-\theta_U) - \theta_I(3\sigma-2\theta_U))}. \tag{27}$$

Differentiating $X_U^\star$ with respect to θ_U and θ_I, respectively, yields

$$\begin{aligned}\frac{\partial X_U^\star}{\partial \theta_U} &= \frac{(2\sigma-\theta_I)(\sigma-\theta_I)(2\mu+\sigma)}{(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))^2} > 0,\\ \frac{\partial X_U^\star}{\partial \theta_I} &= -\frac{\sigma(\sigma-2\theta_U)(2\mu+\sigma)}{2(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))^2}.\end{aligned}$$

Notice that the $\frac{\partial X_U^\star}{\partial \theta_U}$ is unambiguously positive as $\sigma > \theta_I$. Interestingly, the expression for $\frac{\partial X_U^\star}{\partial \theta_I}$ is unambiguously positive if and only if $\theta_U > \tilde{\theta}_U \triangleq \sigma/2$ and $\theta_I < \tilde{\theta}_I \triangleq 2\sigma/3$ else it is negative. The condition $\theta_I < \tilde{\theta}_I$ ensures that $\tilde{\theta}_U \in [0, \hat{\theta}_U)$ is within the relevant parameter space as defined under **A3**.

Next differentiating $x_I^\star$ with respect to θ_U and θ_I, respectively, yields

$$\begin{aligned}\frac{\partial x_I^\star}{\partial \theta_U} &= -\frac{\sigma(\sigma-\theta_I)(2\mu+\sigma)}{(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))^2} < 0,\\ \frac{\partial x_I^\star}{\partial \theta_I} &= \frac{(2\mu+\sigma)(\sigma^2(7\sigma^2+8\theta_U^2-16\theta_U\sigma)-\sigma\theta_I(9\sigma^2+8\theta_U^2-18\theta_U\sigma)+\theta_I^2(3\sigma-2\theta_U)(\sigma-\theta_U))}{(2\sigma-\theta_I)^2(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))^2}.\end{aligned}$$

Notice that the $\frac{\partial x_I^\star}{\partial \theta_U}$ is unambiguously negative since $\sigma > \theta_I$. The sign of the expression for $\frac{\partial x_I^\star}{\partial \theta_I}$ depends on the sign of the following polynomial

$$\mathcal{G} \triangleq (\sigma^2(7\sigma^2+8\theta_U^2-16\theta_U\sigma)-\sigma\theta_I(9\sigma^2+8\theta_U^2-18\theta_U\sigma)+\theta_I^2(3\sigma-2\theta_U)(\sigma-\theta_U)).$$

The second derivative of $\mathcal{G}$ with respect to θ_U is

$$\frac{\partial^2 \mathcal{G}}{\partial \theta_U^2} = 4(2\sigma-\theta_I)^2 > 0.$$

This implies that $\mathcal{G}$ is convex in θ_U. Solving for $\mathcal{G} = 0$ for θ_U, yields the two solutions

$$\theta_U^A \triangleq \frac{\sigma(8\sigma-5\theta_I-\sqrt{8\sigma^2+\theta_I^2-8\theta_I\sigma})}{4(2\sigma-\theta_I)}, \qquad \theta_U^B \triangleq \frac{\sigma(8\sigma-5\theta_I+\sqrt{8\sigma^2+\theta_I^2-8\theta_I\sigma})}{4(2\sigma-\theta_I)},$$

with $0 < \theta_U^A < \theta_U^B$. Further, comparing θ_U^A with $\hat{\theta}_U$, we observe that

$$\theta_U^A - \hat{\theta}_U = \frac{\sigma\left(2\sqrt{21\sigma^2+6\theta_I^2-22\theta_I\sigma}+3\theta_I-6\sigma-\sqrt{8\sigma^2+\theta_I^2-8\theta_I\sigma}\right)}{4(2\sigma-\theta_I)} > 0 \;\forall \sigma > \theta_I.$$

Thus, implying that the expression $\mathcal{G}$ is always positive since under **A3** our relevant parameter range is restricted to $\theta_U < \hat{\theta}_U$. Thus, recalling the convexity of $\mathcal{G}$ with respect to θ_U, we can conclude that $\frac{\partial x_I^\star}{\partial \theta_I} > 0$. ■

Proof of Proposition 12. Solving the first-order conditions associated with D_1's, D_2's and I's maximization problems, and imposing rational expectations, we have

$$x_1^{VI}(w_2) = \frac{\sigma(2\mu+\sigma)(\sigma-\theta_I)+2w_2(\sigma^2+\theta_I\theta_U-\sigma(\theta_I+2\theta_U))}{2\sigma(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))}, \tag{28}$$

$$x_2^{VI}(w_2) = \frac{\sigma(2\mu+\sigma)(\sigma-\theta_I)-2w_2(3\sigma^2+\theta_I\theta_U-2\sigma(\theta_I+\theta_U))}{2\sigma(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))}, \tag{29}$$

$$X_U^{VI}(w_2) = \frac{(\sigma-\theta_I)(\sigma+2\mu-w_2)-\sigma w_2}{4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U)}, \tag{30}$$

$$x_I^{VI}(w_2) = \frac{\sigma(\sigma+2(w_2+\mu))-2\theta_U(\sigma+2\mu)}{8\sigma(\sigma-\theta_U)-\theta_I(6\sigma-\theta_U)}. \tag{31}$$

Setting $x_2^{VI}(w_2)=0$ and solving for w_2 yields the choke price

$$\overline{w}_U \triangleq \frac{\sigma(\sigma-\theta_I)(2\mu+\sigma)}{6\sigma^2+2\theta_I\theta_U-4\sigma(\theta_I+\theta_U)} > 0.$$

The above choke price is always positive as $\sigma > \max\{\theta_I, \theta_U\}$.

The profit of the merged entity U-D_1 is

$$P^{VI}(\cdot)\, x_1^{VI}(w_2) + w_2 x_2^{VI}(w_2).$$

Differentiating with respect to w_2 and solving the corresponding first-order condition we have

$$w_U^{VI} \triangleq \frac{\sigma(2\mu+\sigma)(\sigma-\theta_I)(6\sigma^2+4\theta_I\theta_U-\sigma(5\theta_I+8\theta_U))}{44\sigma^4+4\theta_I^2\theta_U^2-4\theta_I\theta_U\sigma(5\theta_I+4\theta_U)+4\sigma^2(5\theta_I^2+18\theta_I\theta_U+4\theta_U^2)-4\sigma^3(15\theta_I+16\theta_U)}.$$

Solving $w_U^{VI}=0$ with respect to θ_U yields a unique solution

$$\theta_U^C \triangleq \frac{\sigma(6\sigma-5\theta_I)}{4(2\sigma-\theta_I)}.$$

Further, the slope of $\frac{\partial w_U^{VI}}{\partial \theta_U}|_{\theta_U=\theta_U^C} = -\frac{16(\sigma-\theta_I)(2\mu+\sigma)}{5\sigma(2\sigma-\theta_I)} < 0$. Thus, we can conclude that $w_U^{VI} < 0$ if and only if $\theta_U \geq \frac{3(6\sigma-5\theta_I)}{4(2\sigma-\theta_I)}$ and $\theta_I > 2\sigma/3$. The lower bound on $\theta_I > 2\sigma/3$ ensures that $\theta_U^C \in (0, \hat{\theta}_U)$ which is our relevant parameter space under **A3**.

Next, we comparie $\bar{w}_U$ with w_U^{VI}. Solving $\bar{w}_U - w_U^{VI} = 0$ with respect to θ_I, we obtain the following three solutions.

$$\theta_I^F \triangleq \sigma, \quad \theta_I^G \triangleq \frac{\sigma(\sigma+2\theta_U)}{\theta_U} > \sigma > 0, \quad \theta_I^H \triangleq \frac{4\sigma(\sigma-\theta_U)}{3\sigma-2\theta_U} > 0$$

The first and the second solutions are outside the relevant parameter range. The third solution is within our relevant parameter range when $\theta_U < \sigma/2$. Further, notice that the slope of $\bar{w}_U - w_U^{VI})$ with respect to θ_I at this feasible solution is given as

$$\frac{\partial(\bar{w}_U - w_U^{VI})}{\partial\theta_I})|_{\theta_I=\theta_I^H} = -\frac{3(3\sigma-2\theta_U)^2(\sigma-2\theta_U)(2\mu+\sigma)}{4\sigma^4} < 0 \text{ for } \theta_U < \sigma/2.$$

Thus, we can conclude that $\bar{w}_U > w_U^{VI}$.

Finally, comparing w_U^{VI} with the choke price $\bar{w}_U$ at $\theta_U = 0$ yields

$$\left(\bar{w}_U - w_U^{VI}\right)|_{\theta_U=0} = \frac{(4\sigma - 3\theta_I)(\sigma - \theta_I)(2\mu + \sigma)}{4(3\sigma - 2\theta_I)(11\sigma^2 + 5\theta_I^2 - 15\theta_I\sigma)} > 0.$$

Thus, even at $\theta_U = 0$, D_2 is not foreclosed. ■

Proof of Proposition 13. Substituting w_L^{VI} into the equations (28)-(31) and the (equilibrium) market prices, we obtain

$$\begin{aligned}
x_1^{VI} &\triangleq \frac{(2\mu + \sigma)(\sigma - \theta_I)(7\sigma^2 + 3\theta_I\theta_U - \sigma(5\theta_I + 6\theta_U))}{44\sigma^4 + 4\theta_I^2\theta_U^2 - 4\theta_I\theta_U\sigma(5\theta_I + 4\theta_U) + 4\sigma^2(5\theta_I^2 + 18\theta_I\theta_U + 4\theta_U^2) - 4\sigma^3(15\theta_I + 16\theta_U)}, \\
x_2^{VI} &\triangleq \frac{(2\mu + \sigma)(\sigma - \theta_I)(\sigma(\sigma + 2\theta_U) - \theta_I\theta_U)}{44\sigma^4 + 4\theta_I^2\theta_U^2 - 4\theta_I\theta_U\sigma(5\theta_I + 4\theta_U) + 4\sigma^2(5\theta_I^2 + 18\theta_I\theta_U + 4\theta_U^2) - 4\sigma^3(15\theta_I + 16\theta_U)}, \\
x_I^{VI} &\triangleq \frac{(2\mu + \sigma)(7\sigma^3 - 2\theta_I\theta_U^2 + 4\theta_U\sigma(2\theta_I + \theta_U) - \sigma^2(5\theta_I + 14\theta_U))}{44\sigma^4 + 4\theta_I^2\theta_U^2 - 4\theta_I\theta_U\sigma(5\theta_I + 4\theta_U) + 4\sigma^2(5\theta_I^2 + 18\theta_I\theta_U + 4\theta_U^2) - 4\sigma^3(15\theta_I + 16\theta_U)},
\end{aligned}$$

so that

$$X_U^{VI} \triangleq \frac{(2\mu + \sigma)(\sigma - \theta_I)(4\sigma(2\sigma - \theta_U) - \theta_I(5\sigma - 2\theta_U))}{44\sigma^4 + 4\theta_I^2\theta_U^2 - 4\theta_I\theta_U\sigma(5\theta_I + 4\theta_U) + 4\sigma^2(5\theta_I^2 + 18\theta_I\theta_U + 4\theta_U^2) - 4\sigma^3(15\theta_I + 16\theta_U)}.$$

As a result,

$$\begin{aligned}
P^{VI} &\triangleq \frac{(2\mu + \sigma)(\sigma - \theta_I)(7\sigma^2 + 3\theta_I\theta_U - \sigma(5\theta_I + 6\theta_U))}{44\sigma^4 + 4\theta_I^2\theta_U^2 - 4\theta_I\theta_U\sigma(5\theta_I + 4\theta_U) + 4\sigma^2(5\theta_I^2 + 18\theta_I\theta_U + 4\theta_U^2) - 4\sigma^3(15\theta_I + 16\theta_U)}, \\
P_I^{VI} &\triangleq \frac{\sigma(2\mu + \sigma)(7\sigma^3 - 2\theta_I\theta_U^2 + 4\theta_U\sigma(2\theta_I + \theta_U) - \sigma^2(5\theta_I + 14\theta_U))}{44\sigma^4 + 4\theta_I^2\theta_U^2 - 4\theta_I\theta_U\sigma(5\theta_I + 4\theta_U) + 4\sigma^2(5\theta_I^2 + 18\theta_I\theta_U + 4\theta_U^2) - 4\sigma^3(15\theta_I + 16\theta_U)}.
\end{aligned}$$

Comparing the outputs, we note that

$$x_2^{VI} - x_1^{VI} = -\frac{w_U^{VI}}{\sigma}.$$

From the above, when $w_U^{VI} < 0$ yields the result that $x_2^{VI} - x_1^{VI} > 0$. Recall from Proposition (7) that $w_U^{VI} < 0$ if and only if $\theta_U > \frac{\sigma(6\sigma - 5\theta_I)}{4(2\sigma - \theta_I)}$ and $\theta_I > \frac{2\sigma}{3}$. This concludes the proof. ■

Proof of Proposition 14. Comparing the fees under vertical integration and under vertical separation, we obtain

$$w_U^{VI} - w_U^{\star} = \frac{(\sigma - \theta_I)(2\mu + \sigma)(\sigma^2 - \theta_I\theta_U + 2\sigma\theta_U)(\sigma^2 + \theta_I\theta_U - \sigma(2\theta_U + \theta_I))}{(44\sigma^4 + 4\theta_I^2\theta_U^2 - 4\theta_I\theta_U\sigma(5\theta_I + 4\theta_U) + 4\sigma^2(5\theta_I^2 + 18\theta_I\theta_U + 4\theta_U^2) - 4\sigma^3(15\theta_I + 16\theta_U))(2\sigma - \theta_I)}.$$

Solving the above difference with respect to θ_U yields two solutions

$$\theta_U^L \triangleq -\frac{\sigma^2}{2\sigma - \theta_I} < 0, \quad \theta_U^M \triangleq \frac{\sigma(\sigma - \theta_I)}{2\sigma - \theta_I} > 0.$$

The first solution is outside the relevant parameter space. The second solution is in our relevant parameter space. Next, consider the slope of $w_U^{VI} - w_U^\star$ with respect to θ_U at $\theta_U = \theta_U^M$, this yields

$$\frac{\partial(w_U^{VI} - w_U^\star)}{\partial\theta_U}|_{\theta_U=\theta_U^M} = -\frac{(\sigma-\theta_I)(2\mu+\sigma)}{4\sigma(2\sigma-\theta_I)} < 0.$$

Thus, we can conclude that $w_U^{VI} - w_U^\star > 0$ if and only if $\theta_U \leq \frac{\sigma(\sigma-\theta_I)}{2\sigma-\theta_I}$. Similarly, comparing D_2's output under vertical integration and under vertical separation $x_2^{VI}(w_U^{VI}) - x_2^\star(w_U^\star)$, yields

$$x_2^{VI} - x_2^\star = -(w_U^{VI} - w_U^\star)\underbrace{\frac{(2\sigma-\theta_I)(7\sigma^2+3\theta_I\theta_U-\sigma(6\theta_U+5\theta_I))}{(\sigma^2+2\theta_U\sigma-\theta_I\theta_U)(4\sigma^2+2\theta_I\theta_U-3\theta_I\sigma-4\theta_U\sigma)}}_{(+)}.$$

The expression in the fraction is always positive and thus the sign of $x_2^{VI} - x_2^\star$ is inverse of the sign of $w_U^{VI} - w_U^\star$. Specifically for the case when $w_U^{VI} - w_U^\star > 0$ holds, we must have $x_2^{VI} - x_2^\star < 0$ and vice-versa. This concludes the proof. ■

Proof of Proposition 15. U's profit in the vertical separation case is

$$w_U^\star X_U^\star = \frac{(\sigma-\theta_I)^2(2\mu+\sigma)^2}{8(2\sigma-\theta_I)(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))}.$$

Similarly, the profit of the vertically integrated platform is

$$P^{VI}x_1^{VI} + w_U^{VI}x_2^{VI} = \frac{5\sigma(2\mu+\sigma)^2(\sigma-\theta_I)^2}{16(11\sigma^4+\theta_I^2\theta_U^2-\theta_I\theta_U\sigma(5\theta_I+4\theta_U)+\sigma^2(5\theta_I^2+18\theta_I\theta_U+4\theta_U^2)-\sigma^3(15\theta_I+16\theta_U))}.$$

Dividing the profit of the vertically integrated platform with the profit of U under vertical separation yields

$$\frac{P^{VI}x_1^{VI} + w_U^{VI}x_2^{VI}}{w_U^\star X_U^\star} = \frac{5\sigma(2\sigma-\theta_I)(4\sigma^2+2\theta_I\theta_U-\sigma(3\theta_I+4\theta_U))}{2((11\sigma^4+\theta_I^2\theta_U^2-\theta_I\theta_U\sigma(5\theta_I+4\theta_U)+\sigma^2(5\theta_I^2+18\theta_I\theta_U+4\theta_U^2)-\sigma^3(15\theta_I+16\theta_U)))}.$$

In the following, we prove that the above profit ratio is greater than 1. For this, we show that the numerator of the above fraction is unambiguously larger than the denominator. We denote the expression in the numerator as $\mathcal{S} \triangleq (5\sigma(2\sigma-\theta_I)(4\sigma^2+2\theta_I\theta_U-\sigma(3\theta_I+4\theta_U)))$ and denote the expression in the denominator as $\mathcal{T}$. Taking the difference of the numerator with the denominator — $\mathcal{S} - \mathcal{T} = 0$, and solving for θ_U yields the following two solutions.

$$\theta_U^N \triangleq -\sigma\sqrt{\frac{5}{2}} - \frac{\sigma^2}{2\sigma-\theta_I} < 0, \quad \theta_U^Q \triangleq \sigma\sqrt{\frac{5}{2}} - \frac{\sigma^2}{2\sigma-\theta_I} > 0.$$

We discard the first solution as it is outside the relevant parameter space. The second solution θ_U^Q is positive, however, $\theta_U^Q > \hat{\theta}_U$. Next, considering the slope of $\mathcal{S} - \mathcal{T}$ with respect to θ_U at $\theta_U = \theta_U^Q$ yields

$$\frac{\partial(\mathcal{S}-\mathcal{T})}{\partial\theta_U}|_{\theta_U=\theta_U^Q} = -2\sqrt{10}\sigma(2\sigma-\theta_I)^2 < 0.$$

Thus, we show that for all $\theta_U < \theta_U^Q$ it must be that $\mathcal{S} > \mathcal{T}$. Recall that the upperbound of $\hat{\theta}_U$ as defined under **A3** is lower than θ_U^Q. Thus, it is immediate that the profit of U under vertical integration is higher than under vertical separation.

D_2's profit under vertical separation is

$$(P_U^\star - w_U^\star)x_2^\star = \frac{\sigma(2\mu+\sigma)^2(\sigma-\theta_I)^2}{16(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))^2}.$$

Similarly, the profit of D_2 under vertical integration is

$$(P_U^{VI} - w_U^{VI})x_2^{VI} = \frac{\sigma(2\mu+\sigma)^2(\sigma-\theta_I)^2(\sigma(2\theta_U+\sigma)-\theta_I\theta_U)^2}{16(11\sigma^4+\theta_I^2\theta_U^2-\theta_I\theta_U\sigma(5\theta_I+4\theta_U)+\sigma^2(5\theta_I^2+18\theta_I\theta_U+4\theta_U^2)-\sigma^3(15\theta_I+16\theta_U))^2}.$$

Dividing the profit of D_2 under vertical integration with the profit under vertical separation yields the following expression

$$\frac{(P_U^{VI} - w_U^{VI})x_2^{VI}}{(P_U^\star - w_U^\star)x_2^\star} = \frac{(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))^2(\sigma(2\theta_U+\sigma)-\theta_I\theta_U)^2}{(11\sigma^4+\theta_I^2\theta_U^2-\theta_I\theta_U\sigma(5\theta_I+4\theta_U)+\sigma^2(5\theta_I^2+18\theta_I\theta_U+4\theta_U^2)-\sigma^3(15\theta_I+16\theta_U))^2}.$$

In the following, we derive the conditions when the above profit ratio is greater than 1. For this, we obtain the conditions under which the numerator of the above fraction is unambiguously larger than the denominator. We denote the expression in the numerator as

$$\mathcal{Y} \triangleq (4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))^2(\sigma(2\theta_U+\sigma)-\theta_I\theta_U)^2,$$

and denote the expression in the denominator as

$$\mathcal{Z} \triangleq (11\sigma^4+\theta_I^2\theta_U^2-\theta_I\theta_U\sigma(5\theta_I+4\theta_U)+\sigma^2(5\theta_I^2+18\theta_I\theta_U+4\theta_U^2)-\sigma^3(15\theta_I+16\theta_U))^2.$$

Solving $\mathcal{Y} - \mathcal{Z} = 0$for θ_U yields the following positive solution within the relevant parameter space[24]

$$\theta_U^R \triangleq \frac{\sigma(\sigma-\theta_I)}{2\sigma-\theta_I} > 0.$$

Next, computing the slope of $\mathcal{S} - \mathcal{T}$ with respect to θ_U at $\theta_U = \theta_U^Q$, we have

$$\frac{\partial(\mathcal{S}-\mathcal{T})}{\partial\theta_U}|_{\theta_U=\theta_U^Q} = 4\sigma^3(2\sigma-\theta_I)^4 > 0.$$

Hence, for all $\theta_U > \theta_U^R$ it must that $\mathcal{Y} > \mathcal{Z}$, so that D_2's profit under vertical integration is higher than under vertical separation when $\theta_U > \frac{\sigma(\sigma-\theta_I)}{2\sigma-\theta_I}$.

To show that consumer surplus is higher under vertical integration, it is sufficient to show that aggregate output is higher under vertical integration than under vertical separation. Then, $X_U^{VI} + x_I^{VI} -$

[24]There are 4 possible solutions. For brevity, we discard the negative valued solutions as they are outside the relevant parameter space. We also discard the positive valued solution that is above the upperbound imposed on θ_U under Assumption **A2**.

$(X_U^\star + x_I^\star)$ yields

$$\frac{\mathcal{H}^{VI}(2\mu+\sigma)(\sigma-\theta_I)^2(10\sigma^4+2\theta_I^2\theta_U^2-2\sigma\theta_I\theta_U(3\theta_I+4\theta_U)+\sigma^2(5\theta_I^2+20\theta_I\theta_U+8\theta_U^2)-2\sigma^3(7\theta_I+8\theta_U)}{4(2\sigma-\theta_I)(11\sigma^4+\theta_I^2\theta_U^2-\theta_I\theta_U\sigma(5\theta_I+4\theta_U)+\sigma^2(5\theta_I^2+18\theta_I\theta_U+4\theta_U^2)-\sigma^3(15\theta_I+16\theta_U))}$$

where

$$\mathcal{H}^{VI} \triangleq \frac{1}{(4\sigma(\sigma-\theta_U)-\theta_I(3\sigma-2\theta_U))} > 0.$$

The sign of the difference in outputs depends on the sign of the term

$$\zeta \triangleq 10\sigma^4+2\theta_I^2\theta_U^2-2\sigma\theta_I\theta_U(3\theta_I+4\theta_U)+\sigma^2(5\theta_I^2+20\theta_I\theta_U+8\theta_U^2)-2\sigma^3(7\theta_I+8\theta_U).$$

Differentiating with respect to θ_I, it can be shown that $\frac{\partial\zeta}{\partial\theta_I} < 0$. Hence, evaluating ζ at $\theta_I = \sigma$ we have

$$\zeta|_{\theta_I=\sigma} = \sigma^2(\sigma^2+2\theta_U^2-2\theta_U\sigma) > 0,$$

implying that the difference in market outputs is always positive under **A3**. ■

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Online Appendix

In this online Appendix we present further extensions of the analysis developed in the main body of the paper. Specifically, we show that the results of the baseline model extend to the case of price competition, the scenario where there are N competing firms downstream and to the case in which the platform charges positive access prices to consumers and developers.

1 Differentiated price competition model

In order to study price competition we modify the utility function in Singh & Vives (1984) to account for cross-side network effects.[1] The utility function of the representative buyer is then given by

$$U(\cdot) = (1 + \gamma\Delta^e) \sum_{i=1,2} x_i - \sum_{i=1,2} \frac{x_i^2}{2} - \beta x_1 x_2 - p_1 x_1 - p_2 x_2,$$

where network effects enter the utility function as a quality shifter — i.e., a component that increases willingness to pay. Recall that Δ^e is the expected mass of developers on the ecosystem, the parameter $\gamma \geq 0$ represents the benefit that consumers obtain from interacting with each additional developer on the platform. The parameter $\beta \in (0, 1)$, instead, denotes an inverse measure of the degree of product differentiation — i.e., the higher this parameter, the closer substitutes products. The associated demand for the two retailers are

$$x_i(\Delta^e, p_i, p_{-i}) = \frac{1 + \gamma\Delta^e}{1 + \beta} - \frac{p_i - \beta p_{-i}}{1 - \beta^2} \qquad \forall i = 1, 2.$$

As in the benchmark model, developers are heterogeneous in their outside options k which is distributed uniformly on the unit interval and obtain value ϕ on every consumer joining the platform. The utility of a developer of type k is then

$$v(\cdot) = \phi X^e - k,$$

where, as in the baseline model, $X^e \triangleq \sum_{i=1,2} x_i^e$ is the expected total demand, ϕ is the network benefit enjoyed by developers. Hence, all developers such that $k \leq \Delta(X^e) = \phi X^e$ join the platform.

The timing of the game is as follows.

[1] See Pouyet & Trégouët (2021) for a model that employs the Shubik & Levitan (2013) utility functions and appends cross-sided network effects on it.

$t = 1$ U decides whether to merge with D_1.

$t = 2$ U publicly sets the listing fees charged to its downstream unit(s).

$t = 3$ Sellers sets prices, consumers form expectations on the mass of developers on the platform Δ^e and developers form expectations on the mass of consumers on the platform X^e. They simultaneously decide wether to join the platform. Profits materialize and payments are made.

As in the main body of the paper, we use the transformation $\theta \triangleq \gamma\phi$ to represent the net network effect.

A1 $\theta < \frac{1+\beta}{2}$.

Imposing rational expectations in stage 3 — i.e., $X^e = X^\star = \sum_{i=1,2} x_i(\Delta^\star, p_i, p_{-i})$ and $\Delta^e = \Delta^\star = \phi X^\star$ — we obtain the system of direct demand functions

$$x_i^\star(p_i, p_{-i}) \triangleq \frac{1 - \beta - (1-\theta)p_1 + (\beta - \theta)p_2}{(1-\beta)(1+\beta-2\theta)} \qquad \forall i = 1, 2.$$

In the following subsections, we characterize the equilibrium with and without vertical integration and compare consumer surplus and profits pre- and post-merger.

1.1 Equilibrium under vertical separation

Each firm D_i $(i = 1, 2)$ solves

$$\max_{p_i \geq 0} (p_i - w_i) x_i^\star(p_i, p_{-i}),$$

Differentiating with respect to p_i and solving the system of first-order conditions, yields

$$p_i^\star(w_i, w_{-i}) \triangleq \frac{2 - \beta - \beta^2 - 3\theta(1-\beta) + 2w_i(1-\theta)^2 + w_{-i}(1-\theta)(\beta-\theta)}{(2+\beta-3\theta)(2-\beta-\theta)} \qquad \forall i = 1, 2.$$

For given w_1 and w_2, the individual demand for each firm D_i is therefore

$$x_i^\star(p_i^\star(w_i, w_{-i}), p_{-i}^\star(w_{-i}, w_i)) \triangleq \frac{1}{6}\left(\frac{w_{-i} - w_i}{1-\beta} + \frac{w_{-i} - w_i}{2+\beta-3\theta} + \frac{2 - w_i - w_{-i}}{1+\beta-2\theta} + \frac{2 - w_i - w_{-i}}{2-\beta-\theta}\right) \qquad \forall i = 1, 2.$$

We can now characterize equilibrium fees under vertical separation. U chooses w_1 and w_2 to maximize its profits

$$\max_{w_1, w_2} \sum_{i=1,2} w_i x_i^\star(p_i^\star(w_i, w_{-i}), p_j^\star(w_{-i}, w_i)).$$

Differentiating with respect to w_i and solving the two first-order conditions simultaneously, yields

$$w_L^\star = \frac{1}{2}.$$

Substituting this value into the prices, individual and aggregate demand yields

$$\begin{aligned}
x_L^\star &\triangleq x_i^\star(p_i^\star(w_L^\star,w_L^\star),p_j^\star(w_L^\star,w_L^\star)) = \frac{1-\theta}{2(1+\beta-2\theta)(2-\beta-\theta)},\\
p_L^\star &\triangleq p_i^\star(w_L^\star,w_L^\star) = \frac{3-2\beta-\theta}{2(2-\beta-\theta)},\\
\Pi_L^U &\triangleq w_L^\star \sum_{i=1,2} x_L^\star = \frac{1-\theta}{2(1+\beta-2\theta)(2-\beta-\theta)}.
\end{aligned}$$

1.2 Equilibrium under vertical integration

As in the baseline model, we conjecture (and verify ex-post) that the merged entity U-D_1 does not foreclose the rival at equilibrium. In the second stage, the merged entity $U-D_1$ and its nonintegrated rival D_2 set prices to maximize their profits. Specifically, $U-D_1$ solves

$$\max_{p_1\geq 0} \underbrace{p_1 x_1^\star(p_1,p_2)}_{\text{Direct sale revenue}} + \underbrace{w_2 x_2^\star(p_2,p_1)}_{\text{Wholesale revenue}},$$

D_2, instead, solves

$$\max_{p_2\geq 0}(p_2-w_2)x_2(p_2,p_1).$$

Differentiating with respect to p_1 and p_2 and solving the corresponding first-order conditions yields

$$\begin{aligned}
p_1^{VI}(w_2) &\triangleq w_2\left(1-\frac{1-\beta}{2(2+\beta-3\theta)}\right)+\frac{(2-3w_2)(1-\beta)}{2(2-\beta-\theta)},\\
p_2^{VI}(w_2) &\triangleq w_2\left(1+\frac{1-\beta}{2(2+\beta-3\theta)}\right)+\frac{(2-3w_2)(1-\beta)}{2(2-\beta-\theta)}.
\end{aligned}$$

Taking the difference between these expressions, it straightforward to show that

$$p_2^{VI}(w_2)-p_1^{VI}(w_2) = \frac{w_2(1-\beta)}{2+\beta-3\theta} > 0.$$

Hence, The associated aggregate and individual demand functions are respectively

$$\begin{aligned}
X^{VI}(w_2) &\triangleq \frac{2(1-\theta)-w_2(1+\beta-2\theta)}{(1+\beta-2\theta)(2-\beta-\theta)},\\
x_1^{VI}(w_2) &\triangleq x_1^\star(p_1^{VI}(w_2),p_2^{VI}(w_2)) = \frac{1-\theta}{(1+\beta-2\theta)(2-\beta-\theta)}+\frac{w_2}{2}\left(\frac{1}{2+\beta-3\theta}-\frac{1}{2-\beta-\theta}\right),\\
x_2^{VI}(w_2) &\triangleq x_2(X^{VI},p_2^{VI}(w_2),p_1^{VI}(w_2)) = \frac{(1-\theta)(2+\beta-3\theta-2w_2(1+\beta-2\theta))}{(2+\beta-3\theta)(1+\beta-2\theta)(2-\beta-\theta)}.
\end{aligned}$$

Taking the difference between U-D_1's an D_2's output, we have

$$x_1^{VI}(w_2) - x_2^{VI}(w_2) = \frac{w_2}{2+\beta-3\theta} > 0.$$

Furthermore, it directly follows that $x_2^{VI}(w_2) \geq 0$ if and only if

$$w_2 \leq \overline{w} \triangleq \frac{2+\beta-3\theta}{2\left(1+\beta-2\theta\right)}.$$

That is, full foreclosure occurs if and only if $w_2 > \overline{w}$.

Moving backward to the contracting stage, U maximizes the sum of D_1's direct sales profit and the revenue collected from D_2 — i.e.,

$$\max_{w_2} p_1^{VI}(w_2)x_1^{VI}(w_2) + w_2 x_2^{VI}.$$

The first-order condition with respect to w_2 yields

$$w_2^{VI} \triangleq \frac{(2+\beta-3\theta)(4-\beta(2-\beta)-3\theta(2-\theta)}{2(1-\theta)\left(8+\beta^2-2\theta(8+\beta)+9\theta^2\right)} > w_2^*.$$

Then, taking the difference between the equilibrium fee and the above choke price yields

$$w_2^{VI} - \overline{w} = -\frac{(2+\beta-3\theta)}{2}\left(\frac{1}{\beta-2\theta+1} - \frac{4-(2-\beta)\beta+3(2-\theta)\theta}{(1-\theta)\left(\beta^2-2(\beta+8)\theta+9\theta^2+8\right)}\right) < 0.$$

This result shows that under price competition with differentiated products, foreclosure of a rival post-merger is never a concern. Equilibrium prices, individual demands and profit of the platform are, respectively,

$$\begin{aligned}
p_1^{VI} &= \frac{(\beta-3\theta+2)(4-\beta-3\theta)}{2\left(\beta^2-2(\beta+8)\theta+9\theta^2+8\right)}, \\
p_2^{VI} &= \frac{1}{2}\left(1+\frac{1-\beta}{1-\theta} - \frac{4(1-\beta)(1-\theta)}{\beta^2-2(\beta+8)\theta+9\theta^2+8}\right), \\
x_1^{VI} &= \frac{1}{6}\left(\frac{2}{1+\beta-2\theta} + \frac{3}{1-\theta} - \frac{2(8+\beta-9\theta)}{\beta^2-2(\beta+8)\theta+9\theta^2+8}\right), \\
x_2^{VI} &= \frac{\beta^2-2(\beta+2)\theta+3\theta^2+2}{(\beta-2\theta+1)\left(\beta^2-2(\beta+8)\theta+9\theta^2+8\right)}, \\
\Pi_U^{VI} &= \frac{1}{36}\left(\frac{8}{\beta-2\theta+1} + \frac{9}{1-\theta} - \frac{4(2\beta-9\theta+7)}{\beta^2-2(\beta+8)\theta+9\theta^2+8}\right) > \Pi_L^U.
\end{aligned}$$

Comparing the platform's profit pre- and post-merger, we have

$$\Pi_U^{VI} - \Pi_U^{\star} = \frac{(1-\beta)(8+4\beta+\beta^3-\theta(28+\beta(8+3\beta))+\theta^2(32+7\beta)-13\theta^3)}{4(1+\beta-2\theta)(1-\theta)(2-\beta-\theta)(8+\beta^2-16\theta-2\beta\theta+9\theta^2)} > 0,$$

showing that the merger is always profitable.

Consumer surplus. We can now study the impact of the merger on consumer surplus. Substituting the equilibrium values into the representative consumer's utility function, CS under vertical separation is

$$U_L^\star \triangleq U(\phi X_L^\star, p_L^\star, p_L^\star, x_L^\star, x_L^\star) = \frac{(\beta+1)(\theta-1)^2}{4(\beta-2\theta+1)^2(\beta+\theta-2)^2}.$$

Under vertical integration, instead, we have

$$\begin{aligned} U^{VI} &\triangleq U(\phi X^{VI}(w_2^{VI}), p_1^{VI}(w_2^{VI}), p_2^{VI}(w_2^{VI}), x_1^{VI}(w_2^{VI}), x_1^{VI}(w_2^{VI})) \\ &= \frac{1}{648}\left(\frac{72(\beta+1)}{(\beta-2\theta+1)^2} + \frac{40(\beta+1)}{(1-\beta)(\beta-2\theta+1)} + \frac{81}{(\theta-1)^2} - \frac{324\beta}{(1-\beta)(1-\theta)}\right) \\ &\quad -\frac{1}{648}\left(\frac{716-536\beta^2-2736\beta(1-\theta)-180\theta}{(1-\beta)\left(\beta^2-2(\beta+8)\theta+9\theta^2+8\right)} - \frac{144(1-\beta)(\beta(\beta-9\theta+9)-1)}{\left(\beta^2-2(\beta+8)\theta+9\theta^2+8\right)^2}\right). \end{aligned}$$

Comparing these expressions, we have

$$\begin{aligned} \Delta U \triangleq U^{VI} - U_L^\star = \frac{1}{648}&\left(\frac{54(\beta+1)}{(\beta-2\theta+1)^2} + \frac{16(\beta+1)}{(1-\beta)(\beta-2\theta+1)} + \frac{81}{(\theta-1)^2} - \frac{324\beta}{(\beta-1)(\theta-1)} + \frac{12(\beta+1)}{(1-\beta)(2-\beta-\theta)}\right) \\ -\frac{1}{648}&\left(\frac{18(\beta+1)}{(2-\beta-\theta)^2} + \frac{716-536\beta^2+2736\beta(\theta-1)-180\theta}{(1-\beta)\left(\beta^2-2(\beta+8)\theta+9\theta^2+8\right)} + \frac{144(1-\beta)(1-\beta(\beta-9\theta+9))}{\left(\beta^2-2(\beta+8)\theta+9\theta^2+8\right)^2}\right). \end{aligned}$$

In the next figure, we plot this difference in the the relevant space for (θ, β) to show that the above utility difference is always positive.

The figure is equivalent to a proof given the restrictions on θ and β.

2 N downstream firms

We now modify the baseline model by assuming that there are N downstream firms (each denoted by D_i, with $i = 1, .., N$) to whom U supplies its input. For simplicity, let us consider linear fees as in the baseline model. Following the same methodology of Katz & Shapiro (1985), it can be shown that the (inverse) demand function is

$$P(X, X^e) \triangleq \max\left\{0, \mu + \frac{\sigma}{2} + \theta X^e - \sigma X\right\}$$

where, as in the baseline model,

$$X \triangleq \sum_{i=1}^{N} x_i,$$

denotes aggregate output, and

$$X^e \triangleq \sum_{i=1}^{N} x_i^e,$$

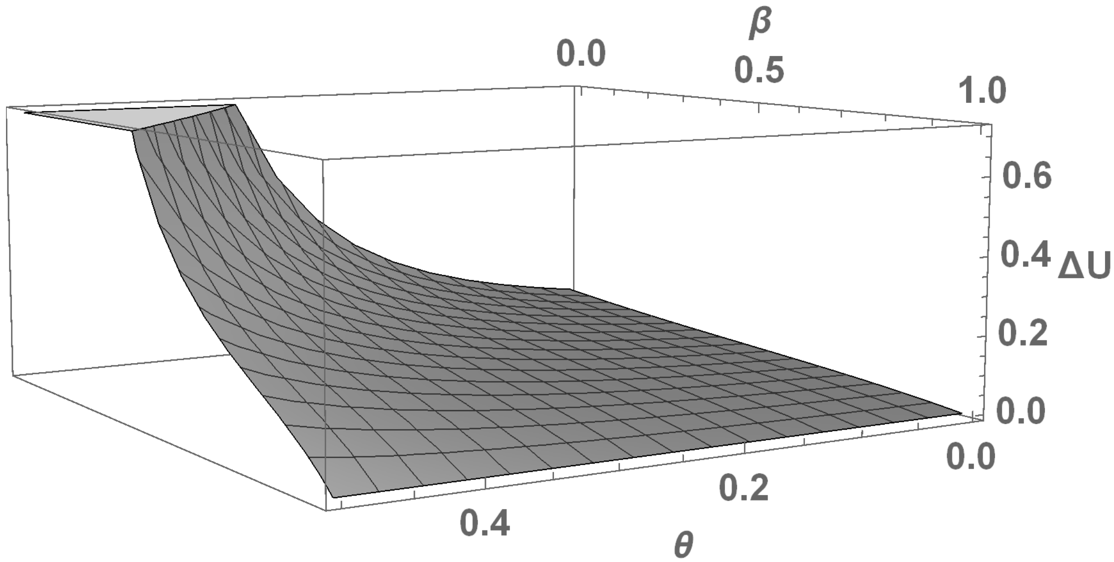

Figure 1: Utility difference in the vertical integration case with the vertical separation case: $\Delta U \triangleq U^{VI} - U_L^{\star}$

denotes the aggregate output that consumers expect downstream firms to distribute.

2.1 Equilibrium under vertical separation

For given wholesale prices w_i, each downstream firm D_i $(i = 1, .., N)$ solves

$$\max_{x_i \geq 0} \left(P\left(X^e, X\right) - w_i\right) x_i.$$

Differentiating with respect to x_i yields the following system of N first order conditions

$$\mu + \frac{\sigma}{2} + \theta X^e - \sigma \sum_{i=1}^{N} x_i - w_i - \sigma x_i = 0 \qquad \forall i = 1, .., N.$$

Summing up these N FOCs and imposing rational expectations, yields the industry output — i.e.,

$$X^{\star}\left(w_1, .., w_N\right) \triangleq \sum_{i=1}^{N} x_i^{\star}\left(w_i, w_{-i}\right) = \frac{N(\mu + \frac{\sigma}{2}) - \left(\sum_{i=1}^{N} w_i\right)}{N(\sigma - \theta) + \sigma}.$$

Hence,

$$x_i^{\star}\left(w_i, w_{-i}\right) \triangleq \frac{P(X^{\star}\left(\cdot\right), X^{\star}\left(\cdot\right)) - w_i}{\sigma},$$

and

$$P^{\star}(\cdot) \triangleq P\left(X^{\star}(\mathbf{w}), X^{\star}(\mathbf{w})\right) = \mu + \frac{\sigma}{2} - (\sigma - \theta)\left(\frac{N(\mu + \frac{\sigma}{2}) - \left(\sum_{i=1}^{N} w_i\right)}{N(\sigma - \theta) + \sigma}\right).$$

Under linear wholesale prices and vertical separation, U's maximization problem is

$$\max_{w_1,\ldots,w_N} \sum_{i=1}^{N} w_i x_i^{\star}\left(w_i, w_{-i}\right).$$

Differentiating with respect to w_i, we have

$$\underbrace{x_i^{\star}\left(w_i, w_{-i}\right) + w_i \frac{\partial x_i^{\star}\left(w_i, w_{-i}\right)}{\partial w_i}}_{\text{Margin + Volume effects}} + \sum_{j \neq i}^{N} \underbrace{w_j \frac{\partial x_j^{\star}\left(w_{-i}, w_i\right)}{\partial w_i}}_{\text{Strategic Effect }(+)}, \quad \forall j \neq i = 1, ..N. \tag{1}$$

Computing the derivative of quantities with respect to w_i yields

$$\begin{aligned} \frac{\partial x_i^{\star}(\cdot)}{\partial w_i} &= \frac{\frac{\partial P(X^{\star}(\cdot),,X^{\star}(\cdot))}{\partial w_i} - 1}{\sigma} \qquad \forall i = 1, .., N, \\ \frac{\partial x_j^{\star}(\cdot)}{\partial w_i} &= \frac{\frac{\partial P(X^{\star}(\cdot),X^{\star}(\cdot))}{\partial w_i}}{\sigma} \qquad \forall i, j = 1, .., N,\ j \neq i. \end{aligned}$$

Substituting these expressions into (1) and simplifying after imposing symmetry, we get

$$\frac{2\mu + \sigma - 4w^{\star}}{2(N(\sigma - \theta) + \sigma)} = 0 \quad \Rightarrow \quad w_L^{\star} = \frac{\sigma + 2\mu}{4}.$$

Substituting $w_L^{\star}$ in the expression for aggregate output, we get

$$X^{\star} \triangleq \frac{N(2\mu + \sigma)}{4(N(\sigma - \theta) + \sigma)}.$$

Then

$$x^{\star} \triangleq \frac{(2\mu + \sigma)}{4(N(\sigma - \theta) + \sigma)}$$

and the associated market price is

$$P^{\star} \triangleq \mu + \frac{\sigma}{2} - (\sigma - \theta) X_L^{\star}.$$

2.2 Equilibrium under vertical integration

We now consider the case in which U and D_1 merge. In the second period, for given fee w_j offered by U-D_1 to D_j ($j \in 2,..,N$), the merged entity solves

$$\max_{x_1 \geq 0} \underbrace{P\left(X^e, X\right) x_1}_{\text{Direct sales}} + \underbrace{\sum_{j=2}^{N} w_j x_j}_{\text{wholesale revenue}},$$

which consists of the profit earned through the direct sales channel and the wholesale revenue.

By contrast, D_j ($j = 2,..,N$) solves

$$\max_{x_j} \left(P\left(X^e, X\right) - w_j\right) x_j.$$

Differentiating U-D_1's profit and D_j's profit with respect to x_1 and each x_j, respectively, yields the following system of N equations

$$P\left(\cdot\right) - \sigma x_1^{VI} = 0,$$

$$P\left(\cdot\right) - w_j - \sigma x_j^{VI} = 0 \quad \forall j = 2,..,N.$$

As before, summing the above N first order conditions while imposing rational expectations yields the aggregate output — i.e.,

$$X^{VI}(w_2,..,w_N) \triangleq \sum_{i=1}^{N} x_i^{VI}(w_2,..,w_N) = \frac{N(\mu + \frac{\sigma}{2}) - \sum_{j=2}^{N} w_j}{N(\sigma - \theta) + \sigma}.$$

Further, substituting $X^{VI}\left(\cdot\right)$ in the market price yields

$$P^{VI}\left(w_2,..,w_N\right) \triangleq P\left(X^{VI}(w_2,..,w_N), X^{VI}(w_2,..,w_N)\right) = \mu + \frac{\sigma}{2} - \left(\sigma - \theta\right) X^{VI}(w_2,..,w_N).$$

Hence,

$$x_1^{VI}(w_2,..,w_N) \triangleq \frac{P^{VI}\left(w_2,..,w_N\right)}{\sigma},$$

and

$$x_j^{VI}(w_2,..,w_N) \triangleq \frac{P^{VI}\left(w_2,..,w_N\right) - w_j}{\sigma}.$$

It is immediate that $x_1^{VI}(w_2,..,w_N) > x_j^{VI}(w_2,..,w_N)$.

Further, for given w_k with $k \neq j$, we solve $x_j^{VI}(w_2,.,\overline{w}_j,.,w_N) = 0$ to obtain the choke wholesale price given as

$$\overline{w}_j \triangleq \frac{\sigma(\sigma + 2\mu) + 2(\sigma - \theta)\sum_{k \neq j}^{N} w_k}{N(\sigma - \theta) + \sigma} \text{ for } k \neq j \in 2,..,N. \tag{2}$$

Hence, exclusion of firm j occurs if and only if $w_j > \overline{w}_j$.

Moving backward to the contracting stage, U maximizes the sum of D_1's direct sales profit and the wholesale revenue collected from D_j — i.e.,

$$\max_{w_2,..,w_N} \underbrace{P^{VI}(w_2,..,w_N)x_1^{VI}(w_2,..,w_N)}_{\text{Sale profit}} + \underbrace{\sum_{j=2}^{N} w_j x_j^{VI}(w_2,..,w_N)}_{\text{Wholesale revenue}}.$$

Differentiating with respect to w_j ($j = 2,..,N$) by the Envelope Theorem, we obtain

$$\underbrace{\frac{\partial P^{VI}(\cdot)}{\partial X^e}\frac{\partial X^{VI}(\cdot)}{\partial w_j}x_1^{VI}(\cdot)}_{\text{Network externalities }(-)} + \underbrace{\frac{\partial P^{VI}(\cdot)}{\partial X}x_1^{VI}(\cdot)\sum_{k=2}^{N}\frac{\partial x_k^{VI}(\cdot)}{\partial w_j}}_{\text{Strategic effect }(+)} + \underbrace{\left[x_j^{VI}(\cdot) + \sum_{k=2}^{N} w_k \frac{\partial x_k^{VI}(\cdot)}{\partial w_j}\right]}_{\text{Marginal wholesale revenue }(?)} = 0. \qquad (3)$$

Taking the derivative of outputs with respect to w_j, we get

$$\begin{aligned}
\frac{\partial x_1^{VI}(\cdot)}{\partial w_j} &= \frac{\frac{\partial P^{VI}(\cdot)}{\partial w_j}}{\sigma}, \qquad \forall j = 2,..,N\\
\frac{\partial x_j^{VI}(\cdot)}{\partial w_j} &= \frac{\frac{\partial P^{VI}(\cdot)}{\partial w_j} - 1}{\sigma}, \qquad \forall j = 2,..,N\\
\frac{\partial x_k^{VI}(\cdot)}{\partial w_j} &= \frac{\frac{\partial P^{VI}(\cdot)}{\partial w_j}}{\sigma} \qquad \forall j \neq k.
\end{aligned}$$

Substituting these comparative statics into equation (3) and simplifying under the hypothesis of symmetry of the non-integrated firms — i.e., $w_j^{VI} = w^{VI}$ for every $j = 2,..,N$ — yields

$$w^{VI} \triangleq \frac{\sigma(2\mu+\sigma)((N+3)\sigma - \theta(N+2))}{4\left(\theta^2 - \theta(N+3)\sigma + (N+3)\sigma^2\right)}.$$

Substituting w_L^{VI} into equation (2), we get the (symmetric) choke price as

$$\overline{w}^{VI} = \frac{\sigma(2\mu+\sigma)\left(\theta^2\left(N^2-2\right) + \theta(4 - N(2N+3))\sigma + N(N+3)\sigma^2\right)}{4(N\sigma - \theta(N-1))\left(\theta^2 - \theta(N+3)\sigma + (N+3)\sigma^2\right)}.$$

Taking the difference between w^{VI} and $\overline{w}^{VI}$, we get

$$w^{VI} - \overline{w}^{VI} = -\frac{\theta\sigma(2\mu+\sigma)(N(\sigma-\theta)+\sigma)}{4(N\sigma - \theta(N-1))\left(\theta^2 - \theta(N+3)\sigma + (N+3)\sigma^2\right)} < 0.$$

The above expression is negative for all $\sigma > \theta > 0$. Hence, even with N downstream firms U has no incentive to foreclose.

It then follows that

$$X^{VI} \triangleq \frac{(2\mu+\sigma)((N+3)\sigma-2\theta)}{4\left(\theta^2-\theta(N+3)\sigma+(N+3)\sigma^2\right)}.$$

The equilibrium post-merger market price is

$$P^{VI} \triangleq \mu+\frac{\sigma}{2}-(\sigma-\theta)\left(\frac{(2\mu+\sigma)((N+3)\sigma-2\theta)}{4\left(\theta^2-\theta(N+3)\sigma+(N+3)\sigma^2\right)}\right).$$

Competitive and welfare effects. We are now in the position of assessing the competitive and welfare effects of the merger.

U's pre-merger profit is

$$\pi^\star \triangleq \frac{N(2\mu+\sigma)^2}{16(N(\sigma-\theta)+\sigma)}.$$

U's post-merger profit is, instead,

$$\pi^{VI} \triangleq \frac{(N+3)\sigma(2\mu+\sigma)^2}{16\left(\theta^2-\theta(N+3)\sigma+(N+3)\sigma^2\right)}.$$

Taking the difference of post-merger profit with pre-merger profit levels, we get

$$\pi^{VI}-\pi^\star = \frac{(2\mu+\sigma)^2\left((N+3)\sigma^2-\theta^2N\right)}{16((N+1)\sigma-\theta N)\left(\theta^2-\theta(N+3)\sigma+(N+3)\sigma^2\right)},$$

which is positive for all $\sigma>\theta>0$.

Moreover,

$$X^{VI}-X^\star = \frac{(2\mu+\sigma)\left(\sigma(3\sigma-2\theta)+N(\theta-\sigma)^2\right)}{4((N+1)\sigma-\theta N)\left(\theta^2-\theta(N+3)\sigma+(N+3)\sigma^2\right)} > 0,$$

which is positive for all $\sigma>\theta>0$.

Next, since in a Cournot model with linear demand the expression for consumer surplus is $CS(\cdot) = X^2/2$. Therefore, the merger benefits consumers.

Remark on developers. As discussed in the benchmark model, our model can be interpreted as a two-sided industry with indirect network externalities where developers hold expectations about consumers and vice-versa. Hence, the mass of active developers in the vertical separation case and in the vertical integration case is then given as $\Delta^\star \triangleq \phi X^\star$ and $\Delta^{VI} \triangleq \phi X^{VI}$ with

$$\Delta^{VI}-\Delta^\star = \phi\left(X^{VI}-X^\star\right) > 0,$$

since $X^{VI} > X^\star$. In the following, we detail how N impacts mass of developers in each regime.

Proposition 1 *In both regimes, the mass of active developers increases with the number of sellers —*

i.e., $\frac{\partial \Delta^\star}{\partial N} > 0$ *and* $\frac{\partial \Delta^{VI}}{\partial N} > 0$. *Moreover,*

$$\frac{\partial \left(\Delta^{VI} - \Delta^\star\right)}{\partial N} < 0.$$

Proof. Differentiating $\Delta_L^\star$ and Δ_L^{VI} with respect to N, we get

$$\begin{aligned}
\frac{\partial \Delta^\star}{\partial N} &= \phi \frac{\sigma(2\mu+\sigma)}{4(N(\sigma-\theta)+\sigma)^2} > 0, \\
\frac{\partial \Delta^{VI}}{\partial N} &= \phi \frac{\theta\sigma(2\sigma-\theta)(2\mu+\sigma)}{4\left(\theta^2-\theta(N+3)\sigma+(N+3)\sigma^2\right)^2} > 0.
\end{aligned}$$

Moreover, taking the difference we have

$$\frac{\partial \Delta^{VI}}{\partial N} - \frac{\partial \Delta^\star}{\partial N} = \frac{\phi\sigma\left(\sigma-\theta\right)\left(\sigma+2\mu\right)\varkappa\left(N,\sigma,\theta\right)}{4\left(\theta^2-\theta(N+3)\sigma+(N+3)\sigma^2\right)^2\left(N(\sigma-\theta)+\sigma\right)^2}$$

where

$$\begin{aligned}
\varkappa\left(N,\sigma,\theta\right) \triangleq\; & N^2\theta^3 - N^2\sigma^3 + \theta^3 - 9\sigma^3 - 6N\sigma^3 \\
& +11\theta\sigma^2 - 5\theta^2\sigma + 3N^2\theta\sigma^2 - 3N^2\theta^2\sigma + 10N\theta\sigma^2 - 4N\theta^2\sigma,
\end{aligned}$$

which is decreasing in N and negative at $N = 2$. Hence, $\frac{\partial \Delta^{VI}}{\partial N} < \frac{\partial \Delta^\star}{\partial N}$. ■

As the number N of sellers increases, consumers and developers expect fiercer competition, increasing their expectations of the ecosystem's size. This increases the value for the ecosystem for both developers and consumers, and thus, their participation of developers increases. Interestingly, although an increase in the number of sellers tends to spur developers in both regimes, it does more so under vertical separation — i.e., aggregate output rises faster with N under vertical separation than vertical integration. The reason is that since the aggregate output is lower under vertical separation than vertical integration and outputs are strategic substitutes, an entrant can produce more when its rivals produce relatively less. Hence, the impact of an additional competitor in the downstream market is stronger under vertical separation than vertical integration. This means that the difference between the mass of developers pre- and post-merger falls as the number of sellers rises.

3 Participation fee to developers

In this section, we introduce an access price for developers — i.e., in the two-sided market version of the model, we allow the platform to charge a fee to each developer that joins it.

As in the benchmark model, the demand side features network externalities and is modeled à la Katz & Shapiro (1985).

The inverse demand function is therefore

$$P(\Delta^e, X) \triangleq \max\left\{0, \mu + \frac{\sigma}{2} + \gamma\Delta^e - \sigma X\right\}.$$

The utility of a developer of type k is

$$v(k) = \phi X^e - l - k$$

where ϕ is the benefit that developers obtain from interacting with every additional consumer, X^e is the total mass of expected consumers on the platform and l is the fee charged by the platform for developers joining its ecosystem. Developers affiliate with the platform only if they obtain positive value from doing so. Therefore, the mass of developers affiliating with the platform is $\Delta(X^e, l) = \phi X^e - l$.

The timing of the game is as in the baseline model, with the only caveat that when U sets contracts for sellers it also sets the fee to developers. As in the baseline model, we make the following assumption.

A2 $\max\{\gamma, \phi\} < \sigma < 1$.

3.1 Equilibrium under vertical separation

For given w_1, w_2 and l, seller D_i $(i = 1, 2)$ solves

$$\max_{x_i \geq 0} \left(P\left(\Delta^e, X\right) - w_i\right) x_i.$$

Differentiating with respect to x_i, and imposing rational expectations — i.e., $\Delta^e = \Delta^\star = \phi X^\star - l$ and $X^e = X^\star$— it is easy to show that sellers' first-order conditions imply

$$x_i^\star\left(w_i, w_{-i}, l\right) \triangleq \frac{1}{2}\left(\frac{w_{-i} - w_i}{\sigma} - \frac{(w_i + w_{-i} - 2\mu + 2\gamma l - \sigma)}{3\sigma - 2\gamma\phi}\right), \quad \forall i = 1, 2,$$

$$\begin{aligned} X^\star\left(w_1, w_2, l\right) &\triangleq \sum_{i=1}^{2} x_i^\star\left(w_i, w_{-i}, l\right) = \frac{2\mu + \sigma - \sum_{i=1,2} w_i - 2\gamma l}{3\sigma - 2\gamma\phi}, \\ \Delta^\star(w_1, w_2, l) &\triangleq \phi X^\star(w_1, w_2, l) - l, \end{aligned}$$

and

$$P^\star\left(w_1, w_2, l\right) \triangleq P\left(\Delta^\star\left(w_1, w_2, l\right), X^\star\left(w_1, w_2, l\right)\right) = \mu + \frac{\sigma}{2} + \gamma\Delta^\star(w_1, w_2, l) - \sigma X^\star\left(w_1, w_2, l\right).$$

We can now turn to characterize equilibrium fees under vertical separation. U solves

$$\max_{w_1, w_2, l} \underbrace{l\Delta^\star(w_1, w_2, l)}_{\text{Developer revenue}} + \underbrace{\sum_{i=1,2} w_i x_i^\star\left(w_i, w_{-i}, l\right)}_{\text{Wholesale revenue}}.$$

Differentiating with respect to w_i and l, we have

$$\underbrace{l\frac{\partial\Delta^{\star}(w_1,w_2,l)}{\partial w_i}}_{\text{Volume effect on the developer side }(-)} + \underbrace{x_i^{\star}(w_i,w_{-i})+w_i\frac{\partial x_i^{\star}(w_i,w_{-i},l)}{\partial w_i}}_{\text{Margin + Volume effects on the consumer side}} \underbrace{+w_{-i}\frac{\partial x_{-i}^{\star}(w_{-i},w_i)}{\partial w_i}}_{\text{Strategic Effect }(+)} = 0 \quad \forall i=1,2.$$

and

$$\underbrace{l\frac{\partial\Delta^{\star}(w_1,w_2,l)}{\partial l}}_{\text{Volume effect on the developer side }(-)} + \underbrace{\Delta^{\star}(w_1,w_2,l)}_{\text{Margin effect on the developers side }(+)} + \underbrace{\sum_{i=1,2} w_i\frac{\partial x_i^{\star}(w_i,w_{-i},l)}{\partial l}}_{\text{Volume effect on the consumer side }(-)} = 0.$$

The above first-order conditions reflect the impact of higher fees (w_1, w_2 and l) on U's profit. There is a trade-off between upstream margins and downstream volumes, and the impact of the fee under consideration (either w_i or l) on the other side of the market. For given D_i's output, a higher w_i increases the revenue earned by U on each unit of sale made by D_i. At the same time, by increasing w_i, U exerts downward pressure on D_i's output, thereby reducing its revenue. In addition to these two effects, by increasing w_i, U also positively impacts D_{-i}'s demand because outputs are strategic substitutes and contracts are public, thereby increasing the fee that U collects from D_{-i}. Finally, because of indirect network externalities there is an additional negative effect triggered by an increase of w_i. Specifically, a higher w_i lowers developers' demand as they expect less consumer participation on the platform: a cross-side effect. By the same token, when considering the derivative of the platform's profit with respect to l, we observe that apart from the standard margin and volume effect, there is also an impact on consumer participation. Specifically, an increase in l lowers consumer participation on the platform as they expect lower developer participation.

Solving the above conditions we obtain the equilibrium fee charged to sellers and developers under vertical separation — i.e.,

$$w^{\star} \triangleq \frac{(2\mu+\sigma)(3\sigma-\phi(\gamma+\phi))}{2\left(6\sigma-(\gamma+\phi)^2\right)}, \qquad l^{\star} \triangleq \frac{(\phi-\gamma)(2\mu+\sigma)}{2\left(6\sigma-(\gamma+\phi)^2\right)}.$$

The equilibrium output, the mass of developers that joins the platform and the market price are, respectively,

$$x^{\star} \triangleq \frac{2\mu+\sigma}{2\left(6\sigma-(\gamma+\phi)^2\right)}, \quad \Delta^{\star} \triangleq \frac{(2\mu+\sigma)(\gamma+\phi)}{2\left(6\sigma-(\gamma+\phi)^2\right)}, \quad P^{\star} \triangleq \frac{(2\mu+\sigma)\left(4\sigma-\phi(\gamma+\phi)\right)}{2\left(6\sigma-(\gamma+\phi)^2\right)}.$$

The equilibrium profit of the platform is

$$2w^{\star}x^{\star}+l^{\star}\Delta^{\star} \triangleq \frac{(2\mu+\sigma)^2}{4\left(6\sigma-(\gamma+\phi)^2\right)},$$

which is increasing in both consumer and developer network benefits — i.e., γ and ϕ.

3.2 Equilibrium under vertical integration

The merged entity U-D_1 solves

$$\max_{x_1} \underbrace{P\left(\Delta^e, X\right) x_1}_{\text{Direct sales}} + \underbrace{w_2 x_2}_{\text{Wholesale revenue}} + \underbrace{l\Delta(X^e, l)}_{\text{Developer revenue}},$$

which is the sum of the profit made through the direct sales channel (i.e., the integrated unit), the revenue collected from the independent seller D_2 and the fees collected from developers. The non-integrated seller D_2 solves

$$\max_{x_2 \geq 0} \left(P\left(\Delta^e, X\right) - w_2\right) x_2.$$

Differentiating with respect to x_1 and x_2, respectively, and imposing rational expectations — i.e., $X^e = X^{VI} = \sum_{i=1,2} x_i^{VI}$ and $\Delta^e = \Delta^\star(X^{VI}, l)$— it is easy to show that under vertical integration

$$\begin{aligned}
x_1^{VI}(w_2, l) &\triangleq \frac{2w_2(\sigma - \gamma\phi) + \sigma(2\mu + \sigma) - 2l\gamma\sigma}{2\sigma(3\sigma - 2\gamma\phi)}, \\
x_2^{VI}(w_2, l) &\triangleq \frac{\sigma(2\mu + \sigma) - 2w_2(2\sigma - \gamma\phi) - 2l\gamma\sigma}{2\sigma(3\sigma - 2\gamma\phi)}, \\
\Delta^{VI}(w_2, l) &\triangleq \phi(x_1^{VI}(w_2, l) + x_2^{VI}(w_2, l)) - l.
\end{aligned}$$

From the above it is immediate that $x_1^{VI}(w_2, l) > x_2^{VI}(w_2, l)$. Further, the output of the integrated entity rises in the fee charged to the non-integrated rival while D_2's output falls as the fee rises.

Finally, notice that $x_2^{VI}(w_2, l) \geq 0$ if and only if

$$w_2 \leq \bar{w}(l) \triangleq \frac{\sigma(2\mu + \sigma - 2l\gamma)}{2(2\sigma - \gamma\phi)},$$

which, as expected, is decreasing in l — i.e., the high l, the lower the mass of developers and the less worthwhile is to keep the non-integrated rival alive.

In an interior solution, aggregate output, market price and developer demand is given as

$$\begin{aligned}
X^{VI}(w_2, l) &\triangleq \sum_{i=1,2} x_i^{VI}(w_2, l) = \frac{2\mu + \sigma - w_2 - 2l\gamma}{3\sigma - 2\gamma\phi}, \quad \Delta^{VI}(w_2, l) \triangleq \phi X^{VI}(w_2, l) - l, \\
P^{VI}(w_2, l) &\triangleq P\left(\Delta^{VI}(w_2, l), X^{VI}(w_2, l)\right) = \mu + \frac{\sigma}{2} - (\sigma - \gamma\phi) X^{VI}(w_2, l),
\end{aligned}$$

Moving to the contracting stage, U solves

$$\max_{w_2, l} \underbrace{P^{VI}(w_2, l) x_1^{VI}(w_2, l)}_{\text{Direct sale profit}} + \underbrace{w_2 x_2^{VI}(w_2, l)}_{\text{Wholesale revenue}} + \underbrace{l\Delta^{VI}(w_2, l)}_{\text{Developer revenue}}.$$

Differentiating with respect to w_2 and l, by the Envelope Theorem, we obtain

$$\underbrace{\left[x_2^{VI}(\cdot)+w_2\frac{\partial x_2^{VI}(\cdot)}{\partial w_2}\right]}_{\text{Marginal downstream revenue}}+\underbrace{\frac{\partial P(\cdot)}{\partial X}x_1^{VI}(\cdot)\frac{\partial x_2^{VI}(\cdot)}{\partial w_2}}_{\text{Strategic effect }(+)}+$$
$$\underbrace{\frac{\partial P(\cdot)}{\partial \Delta^e}\frac{\partial \Delta(\cdot)}{\partial X^e}\frac{\partial X^{VI}(\cdot)}{\partial w_2}x_1^{VI}(\cdot)}_{\text{Network externalities }(-)}+\underbrace{l\frac{\partial \Delta(\cdot)}{\partial X^e}\frac{\partial X^{VI}(\cdot)}{\partial w_2}}_{\text{Cross-side effect }(-)}=0,$$

and

$$\underbrace{\left(w_2\frac{\partial x_2^{VI}(\cdot)}{\partial l}+\frac{\partial P(\cdot)}{\partial X}x_1^{VI}(\cdot)\frac{\partial x_2^{VI}(\cdot)}{\partial l}\right)}_{\text{Cross-side effect (?)}}+\underbrace{\frac{\partial P(\cdot)}{\partial \Delta^e}\frac{\partial \Delta(\cdot)}{\partial X^e}\frac{\partial X^{VI}(\cdot)}{\partial l}x_1^{VI}(\cdot)}_{\text{Network externalities }(-)}$$
$$+\quad\underbrace{\Delta^{VI}(\cdot)+l\left(\frac{\partial \Delta(\cdot)}{\partial X^e}\frac{\partial X^{VI}(\cdot)}{\partial l}+\frac{\partial \Delta}{\partial l}\right)}_{\text{Volume + margin effects on the developers' side (?)}}\quad=0.$$

Solving the above first-order conditions simultaneously, yields the equilibrium contract that U-D_1 offers to D_2 and the fee charged to developer

$$w^{VI}\triangleq\frac{(2\mu+\sigma)(5\sigma-\phi(2\gamma+\phi))}{20\sigma-5\gamma^2-10\gamma\phi-\phi^2}\le\bar{w},\quad l^{VI}=\frac{(2\mu+\sigma)(5\sigma(\phi-\gamma)-2\gamma\phi^2)}{2\sigma(20\sigma-5\gamma^2-10\gamma\phi-\phi^2)}.$$

In this equilibrium, individual outputs are

$$x_2^{VI}\triangleq\frac{\phi(\gamma+\phi)(2\mu+\sigma)}{2\sigma(20\sigma-5\gamma^2-10\gamma\phi-\phi^2)}<x_1^{VI}\triangleq\frac{(2\mu+\sigma)(10\sigma-\phi(3\gamma+\phi))}{2\sigma(20\sigma-5\gamma^2-10\gamma\phi-\phi^2)},$$

Hence, foreclosure occurs only when there are no network effects — i.e., $\phi=0$. For every $\phi>0$, U-D_1 has no incentive to foreclose D_2.

The merged entity U-D_1 has no incentive to fully foreclose its rival when network effects are in place — i.e., $w_L^{VI}<\bar{w}$ for every positive, even negligible, ϕ. The remaining equilibrium outcomes are

$$X^{VI}\triangleq\frac{(2\mu+\sigma)(5\sigma-\gamma\phi)}{\sigma(20\sigma-5\gamma^2-10\gamma\phi-\phi^2)},\quad\Delta^{VI}\triangleq\frac{5(\gamma+\phi)(2\mu+\sigma)}{2\left(20\sigma-5\gamma^2-10\gamma\phi-\phi^2\right)}$$

and

$$P^{VI}\triangleq\frac{(2\mu+\sigma)(10\sigma-\phi(3\gamma+\phi))}{2(20\sigma-5\gamma^2-10\gamma\phi-\phi^2)}.$$

Competitive and welfare effects. We can now assess the effects of the merger. First, recalling that consumer surplus is increasing in the aggregate output, we can show that

$$X^{VI} - X^{\star} = \frac{(2\mu + \sigma)\left(10\sigma^2 + \gamma^3\phi + 2\gamma^2\phi^2 - 6\gamma\sigma\phi + \gamma\phi^3 - 4\sigma\phi^2\right)}{\sigma\left(20\sigma - 5\gamma^2 - 10\gamma\phi - \phi^2\right)\left(6\sigma - (\gamma + \phi)^2\right)},$$

which is always positive since $\sigma > \max\{\gamma, \phi\}$. Hence, consumers benefit from vertical integration.

Next, consider developer surplus. Under vertical separation, developers obtain

$$DS^{\star} \triangleq \int_0^{\phi X^{\star} - l^{\star}} (\phi X^{\star} - l^{\star} - k)dk = \frac{(\gamma + \phi)^2(2\mu + \sigma)^2}{8\left(6\sigma - (\gamma + \phi)^2\right)^2},$$

while under vertical integration they obtain

$$DS^{VI} \triangleq \int_0^{\phi X^{VI} - l^{VI}} (\phi X^{VI} - l^{VI} - k)dk = \frac{25(\gamma + \phi)^2(2\mu + \sigma)^2}{8\left(20\sigma - 5\gamma^2 - 10\gamma\phi - \phi^2\right)^2}.$$

Comparing these expressions, we have

$$DS^{VI} - DS^{\star} = \frac{(\gamma + \phi)^2(2\mu + \sigma)^2\left(5\sigma - 2\phi^2\right)\left(25\sigma^2 - 5\gamma^2 - 10\gamma\phi - 3\phi^2\right)}{2\left(20\sigma - 5\gamma^2 - 10\gamma\phi - \phi^2\right)^2\left(6\sigma - (\gamma + \phi)^2\right)^2} > 0.$$

Since $\sigma > \max\{\gamma, \phi\}$, this difference is always positive. Hence, developers also benefit from the merger.

4 Participation fee to consumers

Finally, we modify the baseline model by allowing U to charge a participation fee (hereafter denoted by T) to consumers that buy from a seller operating within its network. The rest of the assumptions are as in the baseline model with the only caveat that in the second stage U also sets T (as seen above).

The expected utility of a consumer of type r buying from D_i is

$$u\left(X^e, P_i\right) \triangleq r + \theta X^e - P_i - T, \quad i = 1, 2.$$

Under the above specification, D_1 and D_2 have positive demand only if the following 'no arbitrage condition' holds

$$P_1 - \theta X^e + T = P_2 - \theta X^e + T.$$

As a result, it must be $P_1 = P_2 = P$. Consumers, therefore, patronize either D_1 or D_2 if and only if

$$r \geq r^{\star} \triangleq P - \theta X^e + T.$$

The total demand for the product distributed within U's network is

$$X\left(P, T, X^{e}\right) \triangleq 1-\operatorname{Pr}\left[r \leq r^{\star}\right]=1-\frac{P+T-\theta X^{e}-\left(\mu-\frac{\sigma}{2}\right)}{\sigma},$$

whose inverse is

$$P(X^{e}, X, T) \triangleq \max \left\{0, \mu+\frac{\sigma}{2}+\theta X^{e}-T-\sigma X\right\}.$$

It is immediate to see that the fee T charged to consumers reduces their willingness to pay. Therefore, as we will argue below, its effect will be equivalent to an increase in X (i.e., as reflected by a reduction of the fees charged to the sellers).

4.1 Equilibrium under vertical separation

For given w_1 and w_2, D_i $(i=1,2)$ solves

$$\max_{x_i \geq 0}\left(P\left(X^{e}, X, T\right)-w_{i}\right) x_{i}.$$

Differentiating with respect to x_i (holding X^e constant) and then imposing rational expectations — i.e., $X^\star = X^e$ — it is easy to show that sellers' first-order conditions imply

$$x_{i}^{\star}\left(w_{i}, w_{-i}, T\right) \triangleq \frac{2 w_{-i}(\sigma-\theta)-2 w_{i}(2 \sigma-\theta)+\sigma(2 \mu+\sigma)-2 T \sigma}{2 \sigma(3 \sigma-2 \theta)}, \quad \forall i=1,2,$$

$$X^{\star}\left(w_{1}, w_{2}, T\right) \triangleq \sum_{i=1}^{2} x_{i}^{\star}\left(w_{i}, w_{-i}, T\right)=\frac{2 \mu+\sigma-T-\sum_{i=1,2} w_{i}}{3 \sigma-2 \theta},$$

and

$$P^{\star}\left(w_{1}, w_{2}, T\right) \triangleq P\left(X^{\star}\left(w_{1}, w_{2}\right), X^{\star}\left(w_{1}, w_{2}, T\right)\right)=\mu+\frac{\sigma}{2}-T-(\sigma-\theta) X^{\star}\left(w_{1}, w_{2}\right).$$

Moving backward to the contracting stage, U chooses w_1, w_2 and T to maximize

$$\max_{w_1, w_2, T} \underbrace{\sum_{i=1,2} w_{i} x_{i}^{\star}\left(w_{i}, w_{-i}, T\right)}_{\text{Wholesale revenue}}+\underbrace{T X^{\star}(w_{1}, w_{2}, T)}_{\text{Consumers' participation fees}}.$$

Differentiating with respect to w_i, we have

$$\underbrace{x_{i}^{\star}\left(w_{i}, w_{-i}, T\right)+w_{i} \frac{\partial x_{i}^{\star}\left(w_{i}, w_{-i}, T\right)}{\partial w_{i}}+w_{-i} \frac{\partial x_{-i}^{\star}\left(w_{-i}, w_{i}, T\right)}{\partial w_{i}}}_{\text{Margin + volume + strategic effects on the wholesale revenue (?)}}+\underbrace{T \frac{\partial X^{\star}(w_{1}, w_{2}, T)}{\partial w_{i}}}_{\text{Effect on participation fees }(-)}=0, \quad \forall i=1,2,$$

which simply reflects the negative impact of the sellers' fees on the direct revenue made by the platform on its customer base, over and above the effects identified in the baseline model.

Differentiating with respect to T, we have

$$\underbrace{\sum_{i=1,2} w_i \frac{\partial x_i^\star (w_i, w_{-i}, T)}{\partial T}}_{\text{Wholesale volume effect } (-)} + \underbrace{X^\star(w_1, w_2, T) + T\frac{\partial X^\star(w_1, w_2, T)}{\partial T}}_{\text{Margin + volume effects on participation fees (?)}} = 0, \quad \forall i = 1, 2.$$

As expected, a higher T has an indirect effect on U's wholesale revenue since it reduces the individual output of each seller, but it also impacts the participation fees collected by the platform directly from consumers.

Solving the above first-order conditions, the following holds.

Proposition 2 *With linear contracts and vertical separation, there is a continuum of equilibria — i.e., every pair $T^\star$ and $w^\star$ is optimal as long as*

$$T^\star + w^\star = \frac{2\mu + \sigma}{4}.$$

All these equilibria are equivalent in terms of individual and aggregate output — i.e.,

$$x^\star \triangleq \frac{2\mu + \sigma}{4(3\sigma - 2\theta)}, \quad X^\star \triangleq \frac{2\mu + \sigma}{2(3\sigma - 2\theta)}.$$

The multiplicity of equilibria can be easily understood: U can extract surplus from consumers either by increasing T and gain on the participation fees, or by increasing w_1 and w_2 so to extract a higher wholesale revenue. In a symmetric equilibrium, these instruments have the same marginal benefit as this benefit is equal to the aggregate output, and the same marginal cost since from the expression of $X^\star(w_1, w_2, T)$ it is easy to see that T and w_i have the same marginal impact on aggregate output.

4.2 Equilibrium under vertical integration

Under vertical integration, the merged entity U-D_1 solves

$$\max_{x_1 \geq 0} \underbrace{P(X^e, X, T)\, x_1}_{\text{Direct sale revenue}} + \underbrace{w_2 x_2}_{\text{Wholesale revenue}} + \underbrace{TX}_{\text{Consumers' fees}},$$

D_2 solves

$$\max_{x_2 \geq 0} \left(P(X^e, X, T) - w_2\right) x_2.$$

Differentiating with respect to x_1 and x_2, respectively, and imposing rational expectations — i.e., $X^e = \sum_{i=1,2} x_i^{VI}$ — it can be shown that

$$x_1^{VI}(w_2, T) \triangleq \frac{2(w_2 + T)(\sigma - \theta) + \sigma(2\mu + \sigma)}{2\sigma(3\sigma - 2\theta)} > x_2^{VI}(w_2, T) \triangleq \max\left\{0, \frac{\sigma(2\mu + \sigma) - 2(w_2 + T)(2\sigma - \theta)}{2\sigma(3\sigma - 2\theta)}\right\}.$$

In an interior solution, aggregate output is

$$X^{VI}(w_2,T) \triangleq \sum_{i=1,2} x_i^{VI}(w_2,T) = \frac{2\mu+\sigma-w_2-T}{3\sigma-2\theta},$$

and the market price is

$$P^{VI}(w_2,T) \triangleq P\left(X^{VI}(w_2,T), X^{VI}(w_2,T), T\right) = \mu + \frac{\sigma}{2} - (\sigma-\theta)\frac{2\mu+\sigma-w_2}{3\sigma-2\theta},$$

Moving backward to stage $t=2$, the platform solves

$$\max_{w_2,T} P^{VI}(w_2,T)x_1^{VI}(w_2,T) + w_2 x_2^{VI}(w_2,T) + TX^{VI}(w_2,T).$$

Differentiating with respect to w_2, by the Envelope Theorem, we obtain

$$\underbrace{\left[x_2^{VI}(\cdot) + w_2\frac{\partial x_2^{VI}(\cdot)}{\partial w_2}\right]}_{\text{Marginal wholesale revenue}} + \underbrace{\frac{\partial P(\cdot)}{\partial X}x_1^{VI}(\cdot)\frac{\partial x_2^{VI}(\cdot)}{\partial w_2}}_{\text{Strategic effect }(+)} + \underbrace{\frac{\partial P(\cdot)}{\partial X^e}\frac{\partial X^{VI}(\cdot)}{\partial w_2}x_1^{VI}(\cdot)}_{\text{Network externalities }(-)} + \underbrace{T\frac{\partial x_2^{VI}(w_2,T)}{\partial w_2}}_{\text{Effect on participation fees }(-)} = 0.$$

The intuition for this condition is, mutatis mutandis, similar to that discussed in the vertical separation case, and will be omitted for brevity.

Differentiating with respect to T, by the Envelope Theorem, we obtain

$$\underbrace{\left(\underbrace{\frac{\partial P(\cdot)}{\partial T}} + \underbrace{\frac{\partial P(\cdot)}{\partial X}\frac{\partial x_2^{VI}(\cdot)}{\partial T}}\right)x_1^{VI}(\cdot)}_{\text{Margin effect on direct sales (?)}} + \underbrace{\frac{\partial P(\cdot)}{\partial X^e}\frac{\partial X^{VI}(\cdot)}{\partial T}x_1^{VI}(\cdot)}_{\text{Network externalities }(-)} + \underbrace{w_2\frac{\partial x_2^{VI}(\cdot)}{\partial T}}_{\text{Effect on wholesale revenue}}$$

$$+ \underbrace{X^{VI}(w_2,T) + T\frac{\partial x_2^{VI}(w_2,T)}{\partial T}}_{\text{Margin and volume effects on participation fees (?)}} = 0.$$

The access fee charged to consumers has a volume and a margin effect on the revenue collected by the platform from its final users, and indirect effects on the wholesale and the direct sale revenues. The following then holds:

Proposition 3 *Under vertical integration, there is a continuum of equilibria — i.e., every pair T^{VI} and w^{VI} is optimal as long as*

$$T^{VI} + w^{VI} \triangleq \bar{w} - \underbrace{\frac{\theta\sigma(3\sigma-2\theta)(2\mu+\sigma)}{4(2\sigma-\theta)\left(\theta^2+5\sigma(\sigma-\theta)\right)}}_{(+)} \leq \bar{w}, \quad \forall\theta\in[0,\sigma].$$

These equilibria are equivalent for what concerns individual and aggregate output — i.e.,

$$x_2^{VI} \triangleq \frac{\theta(2\mu+\sigma)}{4\left(\theta^2+5\sigma\left(\sigma-\theta\right)\right)} < x_1^{VI} \triangleq \frac{(2\mu+\sigma)(5\sigma-3\theta)}{4\left(\theta^2+5\sigma\left(\sigma-\theta\right)\right)},$$

$$X^{VI} \triangleq \frac{(5\sigma-2\theta)(2\mu+\sigma)}{4\left(\theta^2+5\sigma\left(\sigma-\theta\right)\right)}.$$

Moreover, U-D_1 has no incentive to foreclose D_2 for every $\theta > 0$.

The intuition behind the multiplicity of equilibria is as in the case of vertical separation and hinges on the substitutability between T and w_2. Notice that individual and aggregate outputs in both regimes — i.e., with and without vertical integration — are as in the benchmark model. Hence, the competitive and welfare effects of the merger are identical to the ones discussed in the benchmark model with $T = 0$.

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