Strategic data sales to competing firms*

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Abstract

The unprecedented access of firms to consumer level data facilitates more precisely targeted individual pricing. We study the incentives of a data broker to sell data about a segment of the market to three competing firms. The segment only includes a share of the consumers in the market around one of the firms. Data are never sold exclusively. Despite the data are particularly tailored to the potential clientele of one of the firms, we show that the data broker has incentives to sell the list to its competitors. Such market outcome is not socially optimal, and a regulator considering to mandate data sharing can shift the surplus from the data broker to downstream firms.
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1 Introduction

Data gathering, sharing and usage is widespread in today's digital economy. The use of mobile phones and other connected devices has resulted into the continuous generation of massive amount of data. Many businesses are demanding access to harvest the data and exploit their potential. Data intermediaries as data brokers and marketing agencies have experienced sustained success. For instance, estimates suggest that the data brokerage sector is expected to grow at an annual rate of 11.5 percent until 2026 (Transparency Market Research, 2017).

The contribution of data to the economy goes beyond the size of the sector: for example, the total impact of the data market on the EU's economy in 2017 was estimated to be 335.6 billion euros, corresponding to 2.4 per cent of total GDP (Frontier Technology Quarterly, 2019). More generally, data are mostly non-rival and, as such, their use and re-use can generate positive externalities and boost growth (Jones and Tonetti, 2020). As a result, data sharing is strongly incentivised by policy makers and, in certain circumstances, it may even be mandated. The European Commission, for example, announced the EU Data Strategy (European Commission, 2020) to boost data sharing among firms. This is achieved through both the proposed Data Governance Act and the complementary Data Act. The Digital Market Act regulating large platforms also includes mandatory data sharing as a crucial tool.

Beyond the positives, data sharing also poses risks. There are well known individual privacy concerns, but data transfers can also negatively affect market competition. The recent report by the UK Financial Conduct Authority (2019) highlights how the use of data for price discrimination is an established practice in the insurance market. Moreover, they show how insurance companies charge different prices to consumers in the same risk class, depending on other individual characteristics, such as the likelihood to switch the service provider.

An important issue is whether the access to data from upstream firms can advantage some competing firms that have access to it. For example, Martens and Mueller-Langer (2020) point out how sharing real-time digital car data between manufacturers and a network of official dealers can lead to price discrimination and potential foreclosure of independent downstream competitors.

Data do not always uniformly cover all consumers in the market. In the previous example, access to real-time vehicles' data may not be provided to all dealers and garages. In the health sector, some retail pharmacies or insurance companies may benefit from data shared by digital platforms gathering health information through wearables and other devices (Apple Watch, FitBit). In these examples and in other sectors (e.g., finance and banking, hospitality), due to the recent progresses in AI systems, it is likely that a segment of consumers is profiled and only some firms can access this information to personalise prices (Acemoglu, 2021).

This paper studies the competitive issues related to data selling or sharing by focusing on a data broker that possesses information about *one segment* of the market. This information can be thought of as being the result of a marketing study on a particular segment of the market or, alternatively, as data gathered on the previous or potential clientele of one of the firms competing in the downstream market. In this context, we ask how is the data broker selling this information, and to whom. Are the data sold exclusively or to more than one of the market competitors? Which of the downstream firms ends up buying the data, and what are the implications for the market outcome?

We tackle the previous questions in a simple model with one data broker and three firms that compete in prices. The firms and consumers are located in a circular city (Salop, 1979). The data broker has information on the location of consumers in the arc of the city around one of the firms. As location captures the preference of the consumers, the data can be used to personalise prices and, hence, price discriminate. Clarity of exposition motivates the choice of presenting the three firms setting as the main model. However, all results are robust to any number of firms, as we show in section 6.

There are a number of insights provided by our analysis. First, the selling mechanism influences the outcome of the game. Second, consider the case in which the data broker chooses to auction the data. Then, the firm whose arc of consumers is included in the data broker's dataset does not purchase it. Instead, the data broker has an incentive to sell it to the two competing firms. Even if the data appear tailored for the firm whose consumers have been profiled, such firm does not have the highest willingness to pay for the dataset.

The intuition for this finding lies in the *strategic reaction* of competing firms to the use of data. The possession of data for consumers close to the firm and the ability to personalise the price offers, make rival firms particularly aggressive in pricing. This limits the benefit of obtaining the data for the firm. The strategic price reaction of competing firms is less pronounced when the data are handed to the two competing firms neighbouring the one whose market arc has been profiled by the data broker. This implies that the willingness to bid of the two rivals is higher than the one of the firm for which the data seem tailored.

Finally, assume the selling mechanism is a take-it-or-leave-it (TIOLI) offer. Then the profit maximising choice of the data broker is to sell the consumer information to all the firms in the market. This result confirms that the exclusive purchase of consumer information by the firm for which the data would seem tailored is not an equilibrium.

Recent years have witnessed a growing interest in the economic impact of data in markets and, in particular, data sharing and trading. A number of studies have concentrated on issues such as privacy and its market implications (Conitzer *et al.*, 2012; Casadesus-Masanell and Hervas-Drane, 2015; Choe *et al.*, 2017; Choi *et al.*, 2019; Ichihashi, 2020), the impact of data-driven mergers (Chen *et al.*, 2020; De Cornière and Taylor, 2020; Prat and Valletti, 2021), and data ownership (Dosis and Sand-Zantman, 2019). Bergemann *et al.* (2021), Gu *et al.* (2021), and Ichihashi (2021) analyse upstream competition (or lack of) between data brokers which can then be sold downstream.

Some articles, like ours, have modelled the asymmetric access of firms to consumer information. Gu *et al.* (2019) consider the effect of exclusive information that enables personalised on the incentives to act as price leader in the market. Belleflamme *et al.* (2020) study the impact of asymmetric precision in the information held for the profitabilty of price discrimination. They find that as long as the two firms are not identically able to profile consumers, they can both charge price above the marginal cost. Our work also models personalised pricing but the asymmetric access to the information is endogenous, as it is sold by the data broker. Further, the information only covers a segment of consumers that have an innate preference for a specific firm.

This paper also contributes to the literature on data brokers incentives. Montes *et al.* (2019) model privacy concerned consumers and finds that a data broker always has an incentive to sell data exclusively to a competing duopoly firm. Bounie *et al.* (2021) also study a duopoly and characterises the optimal partition of a consumer database. Through partitioning, the data broker always sells non-overlapping information to both firms. Finally, Kim *et al.* (2019) and Martens *et al.* (2021) also study data sharing in a Salop model with three firms: the former in the context of data-driven mergers, whereas the latter focuses on platforms. Martens *et al.* (2021) assume that only the platform knows the locations of the firms and, as a result, may bias consumer recommendations. Instead, in Kim *et al.* (2019), like in our paper, the relevant information is the location of consumers. In their article, all consumers in the market are profiled and in a pre-merger equilibrium the data are sold exclusively. Instead, we focus on situation in which the information held by the data broker only covers a particular segment of the market. The main implication is that exclusive selling of the non-divisible information is never the optimal strategy for a data broker.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 focuses on price competition. Section 4 presents the equilibrium prices, profits and welfare. Section 5 studies the data broker's sale of the dataset. Section 6 extends our results and test their robustness in a number of directions, e.g., a higher number of firms and different sizes of the profiled arc of consumers. Section 7 discusses the results and their managerial and policy implications. Unless otherwise stated, the proofs are in Appendix A.

2 The framework

The market. One data broker and three competing retailers $i = 1, 2, 3$. Consumers are uniformly distributed on the unit circle (Salop, 1979) and their location is denoted as x. Firms are located equidistantly at $y_1 = 0$, $y_2 = 1/3$, and $y_3 = 2/3$. Consumers can demand at most one unit of the good. The utility of a consumer x for the good of firm i is:

$$U(x, y_i) = v - t|x - y_i| - p_i, \tag{1}$$

where v is the good's valuation, t is the unit transport cost, and p_i is the price. For simplicity, there are no variable or fixed costs.

Consumer information and data selling. The data broker can collect information only on some of the consumers in the market but not all. For example, the data broker may collect data on consumers located in the segment between firm $i-1$ and firm $i+1$. Without loss of generality, we assume that the data broker has information about consumer located between firm 3 and firm 2. In other words, the data broker has information on consumers *on the arc around firm 1*, i.e., $x \in [2/3, 1]$ and $x \in [0, 1/3]$, respectively. For sake of clarity, we will refer to this arc as to the *profiled segment* of the market. Instead, we will refer to the non-profiled arc between firm 2 and firm 3 - i.e., $x \in [1/3, 2/3]$ - as to the *anonymous segment* (Figure 1). The valuable information in this model is the location of consumers x. The data broker sells the data by auction. In Section 5, we show that an auction is a more profitable strategy for the data broker than making a *take-it-or-leave-it-offer* to a subset of the firms in the market (Montes *et al.*, 2019; Bounie *et al.*, 2021).

Timing. At Stage 0 the data broker costlessly gathers information about consumers on one of the segments of the market; in our case, the segment between firm 2 and firm 3, i.e., the segments $x \in [2/3, 1]$ and $x \in [0, 1/3]$. At Stage 1, the firms bid for the consumer information in data broker's possession. At Stage 2, firms engage in price competition. As we look for the Subgame Perfect Nash Equilibrium, the game is solved by backward induction.

3 Price competition

There are several possible subgames to be considered at Stage 2. We start from two benchmark cases (Section 3.1): (i) no firm has access to consumer information, (ii) all firms have access to consumer information. We then consider the case in which one firm has exclusive access to the list (Section 3.2): this firm can be firm 1, whose market segment has been profiled, or one between firm 2 and firm 3. Finally, we consider the case in which two firms get the consumer information: in one case, the two firms include firm 1, in the other, firm 2 and firm 3 have access to the information (Section 3.3).

3.1 Benchmark cases

3.1.1 No firm has access to consumer information

If no firm has access to consumer information, each firm simultaneously sets their prices to maximise profits. In other words, there is price competition *à la* Salop with three firms. For given prices, each firm's demand depends on the consumers indifferent between buying from the firm or one of its two neighbours, i.e.:

$$U(x, y_i) = U(x, y_{i-1}) \text{ and } U(x, y_i) = U(x, y_{i+1}),$$

where the utility functions are defined as in equation (1). As a result, the profit function of, for example, firm 1 is:

$$\pi_1 = p_1 \left[\left(\frac{t}{3} + \frac{p_2 - p_1}{2t} \right) + \left(\frac{t}{3} + \frac{p_3 - p_1}{2t} \right) \right].$$

Standard profit maximization leads to the following result (proof omitted):

Proposition 1. (***Salop, 1979***) *The unique equilibrium in a pricing subgame in which no firm has access to consumer information is characterised by the following prices and profits:*

$$p_i = \frac{t}{3}, \;\; \pi_i = \frac{t}{9}, \;\; i = 1, 2, 3.$$

3.1.2 All firms have access to consumer information

If all firms have access to the information on consumers on the profiled segment of the market, firms will use the information to condition price offers to the consumers location, and price discriminate. In other words, firms can send personalised offers to consumers at each location x on the arc.

This implies that firms are competing fiercely at each location x: as the distance of each firm is the only source of differentiation, Bertrand competition with heterogeneous costs (due to the distance) takes place at each location. Firms charge a non-negative price at each location, as otherwise they would make a loss and decrease their profit. Hence, the closest firm attracts the consumers, and it can charge a non-negative price that exactly matches the offer of the second closest firm (Thisse and Vives, 1988; Taylor and Wagman, 2014; Montes *et al.*, 2019). For example, considering the sub-arc between firm 1 and firm 2, firm 1 can attract all consumers located between $x = 0$ and $x = 1/6$. On that sub-arc, firm 2 cannot offer any price lower than $p_2(x) = 0$. The price schedule for firm 1 can be found by solving for $p_1(x)$ the following:

$$U(x, y_1) = v - tx - p_1(x) = v - t(1/3 - x) = U(x, y_2),$$

leading to: $p_1(x) = t/3 - 2tx$. On the sub-arc between $x = 1/6$ and $x = 1/3$, a similar argument establishes that $p_1(x) = 0$ as the non-negativity constraint binds.

Following a similar reasoning, the firms' price schedules on the arc $x \in [2/3, 1]$ and $x \in [1/3, 2/3]$ are as follows:

$$p_1(x) = \begin{cases} t\,(1/3 - 2x)\,, & \text{if } 0 \leq x < 1/6 \\ t\,(2x - 5/6)\,, & \text{if } 5/6 \leq x < 1\,; \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$p_2(x) = \begin{cases} t\,(2x - 1/3)\,, & \text{if } \frac{1}{6} \le x < \frac{1}{3} \\ 0, & \text{otherwise} \end{cases}; \tag{3}$$

$$p_3(x) = \begin{cases} t\,(5/6 - 2x)\,, & \text{if } 2/3 \le x < 5/6 \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

Despite the access to the data is symmetric, the price schedules (2)-(3)-(4) are clearly different and firms face an asymmetric situation. In particular, firm 1 price discriminates consumers both on its left and its right, whereas firms 2 and 3 can apply personalised schedules only on one side. This feature will play a notable role in the following analysis. The remaining consumers on the anonymous segment, i.e., between firm 2 and firm 3, are offered a uniform price. The indifferent consumer is identified by solving $U(x, y_2) = U(x, y_3)$. Solving the profit-maximisation problem leads to:

Proposition 2. *If all firms have access to consumer information, the equilibrium consists of the price schedules* (2)-(3)-(4) *and the prices:*

$$p_2 = p_3 = \frac{t}{3}.$$

The firms' profits are, respectively,

$$\pi_1 = \frac{t}{18},\;\; \pi_2 = \pi_3 = \frac{t}{12}.$$

Proposition 2 illustrates the asymmetric profit impact of the possession of the consumer information. Indeed, all firms compete more fiercely for the profiled segment and, as a result, they make less profit than in the no information benchmark (Proposition 1). However, firm 1 is more damaged than firms 2 and 3, as its potential customers are profiled on both sides. The rivals' customers are only profiled on one of their two market segments. The uniform prices paid by the non profiled consumers on the anonymous segment are relatively high: in fact, they are the same as in the no information benchmark.

3.2 Exclusive access to consumer information

3.2.1 Firm 1 has access to consumer information

If firm 1 has exclusive access to the list, it will use it to personalise offers to the consumers on the profiled segment. Firm 2 and firm 3, instead, can only set uniform prices, p_2 and p_3, for all consumers. Given those prices, firm 1 price schedule is:

$$p_1(x) = \begin{cases} \max\{p_2 + t\,(1/3 - 2x)\,, 0\}\,, & \text{if } 0 \le x < 1/3 \\ \max\{p_3 + t\,(2x - 5/6)\,, 0\}\,, & \text{if } 2/3 \le x < 1 \end{cases}. \tag{5}$$

Denote the consumers for which the price schedule of firm 1 is zero, i.e., $p_1(\tilde{x}_{12}) = p_1(\tilde{x}_{13}) = 0$, as $\tilde{x}_{12} = 1/6 + p_2/2t$ and $\tilde{x}_{13} = 5/6 - p_3/2t$. Assume these consumers lie on the profiled segment. Then, the following proposition summarises our main findings in the pricing subgame if firm 1 has exclusive access to information about consumers on its own arc.

Proposition 3. *If firm 1 has exclusive access to consumer information, the equilibrium consists of the price schedules* (5) *and the prices:*

$$p_2 = p_3 = \frac{2}{9}t.$$

The firms' profits are, respectively,

$$\pi_1 = \frac{25}{162}t, \;\; \pi_2 = \pi_3 = \frac{4}{81}t.$$

As a result of firm 1 having exclusive access to the consumer information, firm 2 and firm 3 become more aggressive in pricing. The equilibrium prices, in fact, reflect the trade-off between the usual uniform price competition on the anonymous segment and the need to match firm 1's personalised prices on the profiled segment. Firm 1 makes more profit than the competitors thanks to the exclusive information. Its profits are even higher than in the two benchmark cases. Firm 2 and firm 3, instead, make less profits than in the cases of Section 3.1.

3.2.2 Firm 2 or firm 3 have access to consumer information

Consider the case of either firm 2 or firm 3 having exclusive access to information about consumers on the arc around the rival (firm 1), i.e., the profiled segment. Assume that firm 2 has access to the consumers' information without loss of generality. In this case, firm 2 sets a price schedule for the profiled consumers ($x \in [2/3, 1]$ and $x \in [1/3, 2/3]$) and a price p_2 for non-profiled consumers on the anonymous segment. Firm 1 and firm 3 set uniform prices p_1 and p_3. Given these prices, firm 2 personalised price schedule is:

$$p_2(x) = \max\left\{p_1 + t\left(2x - 1/3\right), 0\right\}. \tag{6}$$

Denote the consumers for which the price schedule of firm 2 is zero, i.e., $p_2(\tilde{x}_{21}) = 0$, as $\tilde{x}_{21} = 1/6 - p_1/2t$, and assume that these consumers lie on the profiled segment. The equilibrium in the pricing subgame if firm 2 has exclusive information on consumers on firm 1's arc can be characterised as follows.

Proposition 4. *If firm 2 has exclusive access to consumer information, the equilibrium consists of* *the price schedule* (6) *and the prices*

$$p_1 = \frac{19}{78}t, \;\; p_2 = \frac{25}{78}t, \;\; p_3 = \frac{4}{13}t.$$

The firms' profits are, respectively,

$$\pi_1 = \frac{361}{6084}t, \;\; \pi_2 = \frac{3275}{24336}t, \;\; \pi_3 = \frac{16}{169}t.$$

Firm 1 suffers the competition of firm 2's personalised prices on its own arc and, as a result, decreases its price, which is the lowest. This affects firm 3, who posts a higher price but lower than firm 2 in response. The pricing rankings reflect those of profits: firm 2 benefits the most from exclusive information about firm 1's arc of consumers. Firm 1, in turn, is the most damaged by firm 2 having information about its own market segment.

3.3 Two firms access consumer information

The final subgames to consider are when a subset of more than one firm but not all have access to the information on firm 1 arc of consumers. The subset can include firm 1 or not, and we will analyse these two cases in turn in what follows.

3.3.1 Firm 1 and 2 have access to consumer information

If firm 1 and firm 2 have access to the information, they can offer personalised prices to consumers on the profiled segment ($x \in [2/3, 1]$ and $x \in [0, 1/3]$). There will be intense competition between firm 1 and firm 2 for the profiled consumers lying on the sub-arc between them. In particular, neither firm can offer a price lower than its cost or it would make losses, i.e., $p_i(x) \geq 0, \;\; \forall x[0, 1/3], i = 1, 2$. This allows identifying the price schedule and the indifferent consumer on that arc. Moreover, firm 2 and firm 3 also offer posted prices p_2 and p_3. Given the price of firm 3 and the previous observations, the price schedules of firm 1 and firm 2 are, respectively:

$$p_1(x) = \begin{cases} \max\left\{t\left(1/3 - 2x\right), 0\right\} & \text{if } \; x \in [0, 1/3] \\ \max\left\{p_3 + t\left(2x - 5/3\right), 0\right\} & \text{if } \; x \in [2/3, 1] \end{cases}, \tag{7}$$

$$p_2(x) = \max\left\{t\left(2x - 1/3\right), 0\right\}. \tag{8}$$

The consumers for which the price schedule of firm 1 and firm 2 are zero are located $\tilde{x}_{12} = 1/6$. Denote also the consumers for which the price schedule of firm 1 is zero, i.e., $p_1(\tilde{x}_{31}) = 0$, as $\tilde{x}_{31} = 5/6 - p_3/2t$. As a result, the equilibrium in the pricing subgame if firm 1 and firm 2 have information on the consumers on firm 1's arc can then be characterised as follows.

Proposition 5. *If firm 1 and firm 2 have access to consumer information, the equilibrium consists of the price schedules* (7)-(8) *and the prices*

$$p_2 = \frac{2}{7}t, \;\; p_3 = \frac{5}{21}t.$$

The firms' profits are, respectively,

$$\pi_1 = \frac{193}{1764}t, \;\; \pi_2 = \frac{121}{1764}t, \;\; \pi_3 = \frac{25}{441}t.$$

In case the firm whose arc is profiled and a rival have the information, the third firm with no information is the most damaged. Firm 3, in fact, faces fierce competition from the personalised offers of firm 1 and, as a result, its price is lower than the one of firm 2. Firm 3 also gets the lowest profit, whereas firm 1 benefits from personalised pricing and has the highest profit.

3.3.2 Firm 2 and firm 3 have access to consumer information

If firm 2 and firm 3 have access to the information, they can offer personalised prices to consumers on the profiled segment ($x \in [2/3, 1]$ and $x \in [1/3, 2/3]$). All three firms will also offer posted prices p_i. Given these prices, the schedules for firm 2 and firm 3 are, respectively:

$$p_2(x) = \max\left\{p_1 + t\left(2x - 1/3\right), 0\right\}, \tag{9}$$

$$p_3(x) = \max\left\{p_1 + t\left(5/3 - 2x\right), 0\right\}. \tag{10}$$

Denote the consumers for which the price schedule of firm 2 and firm 3 are zero, i.e., $p_2(\tilde{x}_{21}) = p_2(\tilde{x}_{31}) = 0$, as $\tilde{x}_{21} = 1/6 - p_1/2t$ and $\tilde{x}_{31} = 5/6 + p_1/2t$, respectively. Assume that these consumers lie on the profiled arc. Then, the equilibrium in the pricing subgame if firm 2 and firm 3 have information on the consumers on firm 1's arc can then be characterised as follows.

Proposition 6. *If firm 2 and firm 3 have access to consumer information, the equilibrium consists of the price schedules* (9) - (10) *and the prices*

$$p_1 = \frac{t}{6}, \;\; p_2 = \frac{t}{3}, \;\; p_3 = \frac{t}{3}.$$

The firms' profits are, respectively,

$$\pi_1 = \frac{t}{36}, \;\; \pi_2 = \frac{17}{144}t, \;\; \pi_3 = \frac{17}{144}t.$$

The equilibrium prices for non profiled consumers are the same as in the benchmark: the competition between firm 2 and firm 3 for the anonymous segment is not affected by the information. The profiled segment, in fact, is served by both firms through personalised offers. Firm 1 suffers the consequences of this information allocation, as it has to decrease its price to compete with personalised pricing on its own market arc. The lower price of firm 1 is also reflected in much lower profit than the two informed competitors.

4 Prices, profits, and welfare

We start with a recap of the results of the pricing stage. Table 1 reports the equilibrium posted prices, firms' and industry profits in all the pricing subgames. Each subgame's label is used as superscript in the ensuing comparisons and analysis. The table highlights one interesting feature of the presence of personalised pricing on posted prices: no matter what subgame is reached, posted prices are never higher than in the no information benchmark ($t/3$). This underlines the pro-competitive effect of personalised prices, which induces rivals to be more competitive and best respond with lower posted prices.

Proposition 7 provides a comparison of the firm's profits in each of the possible pricing subgames. It is important to recall that if one of the firms whose consumers are not all profiled gets the information exclusively, this is firm 2 and not firm 3.

Proposition 7. *The equilibrium profit of each firm in the pricing subgames compare as follows:*

$$\pi_1^1 > \pi_1^{NI} > \pi_1^{12} > \pi_1^2 > \pi_1^{AI} > \pi_1^{23},$$

$$\pi_2^2 > \pi_2^{23} > \pi_2^{NI} > \pi_2^{AI} > \pi_2^{12} > \pi_2^1,$$

$$\pi_3^{23} > \pi_3^{NI} > \pi_3^2 > \pi_3^{AI} > \pi_3^{12} > \pi_3^1.$$

Proof: Follows from Table 1.

The proposition makes clear that firm 1, whose segment of nearby consumers is profiled, benefits from exclusive use of the list, despite the consequent increase of competition intensity. Interestingly, its second best would be that no information is shared or sold. This outcome would be better than sharing the data with firm 2, as it drives all firms to set the highest possible price, whereas sharing the list would entail a competitive pressure that is detrimental to profits. In detail, by sharing data with firm 2 rather than have them alone, firm 1 would not be able to fully exploit the potential of the list when competing against firm 2. Ultimately, this negative effect more than compensate the relatively softer competitive pressure exerted by firm 3.

Similarly, firm 2, greatly benefits from having exclusively access to consumers' information. Intuitively, exclusive access to data means that firm 2 can price discriminate one segment of the market. Firm 1's best reply is to lower her price and be more aggressive against both firm 2's prices schedule and firm 3's price. However, price competition does not propagate as if firm 1 had the data, since firm 3 faces competition on just one sub-segment of her market.

Finally, it is interesting to notice that the profit of firm 2 when all firms buy the data is higher than its profit when it buys it jointly with firm 1, i.e., $\pi_2^{AI} > \pi_2^{12}$. At same time, the profit of firm 3 is higher when firms 1 and 2 have both access to the list than when firm 1 has it exclusively, i.e., $\pi_3^{12} > \pi_3^{1}$. When all firms have access to the information, there is no impact of the list on pricing in the non-profiled segment. Then, this leads to standard Salop competition between firm 2 and firm 3. When, instead, only firm 1 and 2 have access to the list, firm 3 needs to price more aggressively to match the personalised offers of firm 1. This more intense price competition also affects firm 2 and negatively impacts its profit. At the same time, competition is even fiercer when firm 1 has exclusive access to the list. In that case both firms 2 and 3 have to react aggressively to the personalised offers of firm 1 on its arc.

4.1 Welfare analysis

The previous analysis has important implications. From the industry perspective, no information maximises the joint profits whereas the most competitive subgame is when all firms have access to the list of profiled consumers. When the firm whose arc is profiled has access to the information, either exclusively or jointly, the industry profits decrease compared to the case when the rivals do. Exclusive information (for example, to firm 2 or firm 1) leads to higher industry profits than if the same firms share the information with one of the rivals.

As expected, the consumer surplus displays an almost perfectly inverse order. The best scenario is when all firms have access to the list, whereas no information is the less desirable subgame. This result is in line with Parker *et al.* (2020), who call for a regulatory intervention that facilitates data sharing mechanisms to benefit consumers. In our setting, this can be explained as a consequence of the intense price competition when all firms have access to the information. Interestingly, from a consumer's perspective we note that the exclusive availability of the information to firm 1 is equivalent to the case in which both firm 2 and 3 access it. Indeed, the different allocation of the information does not affect the intensity competition in each sub-segment of the market.

Finally, the total surplus is maximised in the two benchmark cases of no information and when all firms have access to it. The only difference is that in the former case the allocation is biased towards the firms, whereas in the latter towards consumers. Moreover, the subgame in which the information is held by the firm whose arc of consumers has been profiled (firm 1) is the least desirable from a welfare perspective. As there are no demand expansion effects and prices are transfers, all the total surplus results are driven by the overall transport costs and the symmetry of the location of the indifferent consumers.

Proposition 8. *The industry profits in the pricing subgames compare as follows:*

$$\Pi^{NI} > \Pi^2 > \Pi^{23} > \Pi^1 > \Pi^{12} > \Pi^{AI}.$$

As for consumer surplus:

$$CS^{AI} > CS^{12} > CS^{23} = CS^1 > CS^2 > CS^{NI},$$

and total surplus:

$$TS^{NI} = TS^{AI} > TS^{23} > TS^{12} > TS^2 > TS^1.$$

5 Data sales

We finally focus on the data broker decision. There are several mechanisms that the data broker can employ to sell the data. In particular, building upon Jehiel and Moldovanu (2000), we consider an auction in which the firms can bid to ensure access to the data. In the next subsection we will analyse how the findings are affected if the data broker uses other selling mechanisms.

We design the auction as follows. First, the data broker chooses how many "contracts" to sell. Then, given the number of contracts available, firms place their bids. We define their willingness to pay as the difference in the firms' revenues if they obtain the list and the counterfactual case in which a rival company purchases the data in their place. This leads to the following data broker profits:

$$\pi^1_{DB} = \pi^1_1 - \pi^2_1 = 0.095t, \qquad \pi^2_{DB} = \pi^2_2 - \pi^1_2 = 0.086t,$$
$$\pi^{12}_{DB} = (\pi^{12}_1 - \pi^{23}_1) + (\pi^{12}_2 - \pi^{12}_3) = 0.093t, \qquad \pi^{23}_{DB} = 2(\pi^{23}_2 - \pi^{12}_3) = 0.122t,$$
$$\pi^{AI}_{DB} = (\pi^{AI}_1 - \pi^{23}_1) + 2(\pi^{AI}_2 - \pi^{12}_3) = 0.080t$$

Proposition 9. *Assume the selling mechanism is an auction offer. Then, the profits of the data broker in the pricing subgames compare as follows:*

$$\pi^{23}_{DB} > \pi^1_{DB} > \pi^{12}_{DB} > \pi^2_{DB} > \pi^{AI}_{DB}.$$

Proof: Follows from the above derivations.

5.1 Other selling mechanisms

Another mechanism that the data broker could use to sell data is through a TIOLI offer. That way, the data broker can extract all the willingness to pay of the firms for the list. In particular, we define the willingness to pay as the difference in the profits if they buy the list and the counterfactual case in which they do not. This leads to the following data broker profits:

$$\pi_{DB}^{1} = \pi_1^1 - \pi_1^{NI} = 0.043t, \qquad \pi_{DB}^{2} = \pi_2^2 - \pi_2^{NI} = 0.024t,$$
$$\pi_{DB}^{12} = (\pi_1^{12} - \pi_1^2) + (\pi_2^{12} - \pi_2^1) = 0.070t, \qquad \pi_{DB}^{23} = 2(\pi_2^{23} - \pi_3^2) = 0.046t$$
$$\pi_{DB}^{AI} = (\pi_1^{AI} - \pi_1^{23}) + 2(\pi_3^{AI} - \pi_3^{12}) = 0.080t$$

Proposition 10. *Assume the selling mechanism is a TIOLI offer. Then, the profits of the data broker in the pricing subgames compare as follows:*

$$\pi_{DB}^{AI} > \pi_{DB}^{12} > \pi_{DB}^{23} > \pi_{DB}^{1} > \pi_{DB}^{2}.$$

Proof: Follows from the above derivations.

Finally, we consider the possibility that the data are sold through sequential bargaining (Rubinstein, 1982; Sobel and Takahashi, 1983). Under this mechanism, the data broker makes an offer for the whole list to each firm sequentially. The game stops when one firm acquires information and we assume there is no discounting. At each stage, the data intermediary proposes the list to one firm or to a group of firms and nothing to the others. In this case, we define the willingness to pay as the difference in the profits if they buy the list and the counterfactual case in which they do not but one (or more) of their rivals does. The results of the analysis are qualitatively identical to the case of an auction (see Proposition 9).

Propositions 9, 10 and, more generally, our findings on data selling provide interesting insights. A data broker that has profiled one arc of consumers around a firm never sells the consumers information *exclusively* to the firm whose market segment has been profiled.

If the data broker adopts an auction or a sequential bargaining selling method, the optimal choice is to sell consumers information not to firm 1, but to the two rivals together, firms 2 and 3. The only scenario in which firm 1 obtains the list is when the data broker chooses a TIOLI offer and sells the data to all firms in the market. This is also the only scenario in which private incentives are aligned with the social optimum (see Section 4.1).

6 Extensions and robustness

In this section, we test our main findings by considering various robustness checks and extensions to our baseline model. First, we extend the analysis to different competition intensities, as captured by the number of firms (i.e., $n = 4, 5, 10$). Second, we allow for different levels of the coverage of the list. In other words, we are interested to know whether firms 2 and 3 are still interested in obtaining the list even if it includes consumers located further away from them. More details can be found in the Web Appendices B and C.

6.1 A higher number of firms

A market with three firms is undoubtedly a special case, and we would like to gain insights on cases in which more firms, potentially $n\ (\geq 3)$ of them are active in the market. Regrettably, a full generalization to n firms proves to be complex. This is due to asymmetric shock on the prices of different firms, which are then asymmetrically transmitted to all other firms.[1] In the Web Appendix B, we setup the problem of all the firms and identify the first-order conditions in all subgames of the firm pricing stage.

The model, however, can be fully solved numerically for any number of firms.[2] We can then provide results and confirm the validity of our previous insights for a given number of competing firms: $n = 4$, $n = 5$, and $n = 10$. Given the monotonicity of the results, we can conclude that these cases are sufficient to infer trends for the problem in object.

To begin with, some notes on the pricing subgames are in order. First, it is important to establish that *no firm other than 1, 2 and n* has an incentive to buy the list. As those firms are located far from the consumers that the data broker has profiled, they cannot extract profits through personalised pricing and, as such, their willingness to pay for the list is zero in all pricing subgames.

Second, all segments apart from those with profiled consumers are characterised by first order conditions typical of competition *à la* Salop. In particular, in subgames where the list affects *symmetrically* the firms on both sides of firm 1, but not their posted prices (i.e., when firms 2 and n have the list or all three firms have it), all competitors except firm 1 pick in equilibrium Salop prices .

Further, in case firm 1 holds the list exclusively, the price impact of it propagates symmetrically through both firm 2 and firm n and then through to the other competitors. The more challenging cases, instead, are the ones where the list affects firms that do not own it *asymmetrically*: this is the case for subgames where firm 2 has the list exclusively or when both firms 1 and firm 2 acquire it.

What are then the implications for the data broker? As in our benchmark model with three firms, we find that the DB has no incentives to sell the data exclusively. Moreover, our numerical results confirm that, in equilibrium, information is sold symmetrically to the two firms located at the extremes of the list, jointly. In other words, Propositions 9 and 10 hold regardless of the number of competitors in the market. Indeed, all variables change smoothly and monotonically with the number of firms: Table 2 provides a summary of the firms prices and profits and the data brokers profit in case of sale through an auction.[3]

Finally, on the basis of Table 2, we can note that the market structure induced by the data broker is such that firms 2 and n have the list. This implies that all $n - 3$ firms located

[1] We note that the problem has similarities with the case of asymmetric costs shock in the Salop model, addressed by Syverson (2004) and solved under fixed locations and complete information by Alderighi and Piga (2012). Yet, there are further types of asymmetry that makes it even more complicated.

[2] In the case of asymmetric subgames, the procedure can be quite tedious as n grows large.

[3] The results for a TIOLI selling mechanism are equally robust.

away from the arc of profiled consumers are *unaffected* by the price competition in the profiled segment of the market. As noticed above, these firms play the usual Salop strategy, i.e. their price is t/n. Indeed, the firm suffering from the enhanced price competition induced by the list is firm 1, which ends up charging its customers $t/2n$.

6.2 The coverage of the list

A special assumption of the benchmark model is that all consumers located in the arc between firm 3 and firm 2 and centered around firm 1 are profiled. Here, we extend the model to consider a symmetric arc, of length A on each side of firm 1. The details of the setup and the analysis can be found in Web Appendix C.

Intuitively, the analysis suggests that, as long as the arc of profiled consumers is sufficiently large to allow firms 2 and 3 to directly benefit from it, the data broker never sells the list exclusively to one firm. Indeed, we identify two main thresholds of the length of the profiled arc that play an important role for the robustness our main findings: these are $A^* = 1/6$, and $A^+ = 5/24$.

The first threshold (A^*) separates the case where the data broker sells the list *exclusively to firm 1* ($A < A^*$) and the ones where more than one firm obtains the list in equilibrium. In fact, above the threshold ($A > A^*$), *all firms* obtain the list in equilibrium. This holds true only up to the second threshold ($A = A^+$): if the profiled arc is wider, a data broker sells the information only to the two rivals of the firm around which the data have been gathered (i.e., firms 2 and 3).

Figure 2 provides a graphical representation of the aforementioned results. In other words, the main finding is that as long as $A \geq 1/6$, selling the information to the firm whose arc is profiled is never part of the data broker profit maximizing strategy as in our benchmark model. Indeed, the list is sold to either all the firms in the market ($A \in [1/6, 5/24]$), or to both firm 2 and firm 3 ($A \in [5/24, 1/3]$). The latter result confirms that the results in 9 holds as long as the profiled arc is sufficiently wide. Finally, it is only when the arc of profiled consumers is relatively small, $A < 1/6$, that the data broker finds selling the information exclusively to firm 1 more convenient.

Interestingly, we identify a global maximum in the innovator's revenues that coincides with an arc's length larger than but close to $A = 1/6$. This result suggests that, if the data broker has information about all the consumers on the potential market of firm 1, her incentive is to sell only a half of it to all the competing firms. Consequently, the data broker is willing to throw away a share of the list and sell the "damaged" version to all the firms in the market. Intuitively, this result implies that if collecting data is costly, the data broker would never gather information about all the potential customers of one firm, but only a fraction of them.

7 Discussion

Not always detailed information is available about all consumers in the market, and often the potential clientele of one firm is better profiled than others. This paper has studied the strategic incentives of a data broker to sell this type of information to competing firms. In particular, we considered a database of consumers that covers only the potential customers of one of the competing firms. The data can be used to implement personalised pricing.

In this setting, we find that each of the three firms in the market would benefit from the exclusive use of the data. Interestingly, however, the second-best outcome for the firm whose segment of potential consumers is profiled would be that no information is shared nor sold. In particular, we find that if the access to the information creates an asymmetry between the competitors, the defensive response of firms without the list can be particularly aggressive, with a negative impact on profitability.

At the aggregate level, firms would be better off when no information is shared, as this is the scenario in which price competition is as soft as possible. Conversely, when all three firms have access to consumer data, price competition in the profiled segment is very fierce. As all firms can set a competitive personalised price for the consumers in the profiled segment, surplus extraction is minimal.

Consumer surplus ordering reflects the mirror image of the above results. When all firms have access to data, the intensity of competition in the profiled segment makes consumers better off on average. Although non-profiled consumers face the usual Salop price, the gains for those on the profiled sub-arc is so high that they outweigh any other scenarios.

Interestingly, total surplus represents a synthesis of the opposite results for industry profits and consumers surplus. From a welfare perspective, there is no difference between all firms having access to data or no firm, as they both represent the first best. However, the two cases are not equivalent, as the former favours consumers, whereas the latter would be preferred by firms. In other words, a policy-maker considering to mandate data-sharing or not faces a choice between which side of the market to back.

The most important finding, however, regards the sale of data: the data broker's incentives are not aligned with those of the policy-maker. In fact, the data broker's strategy depends on the selling mechanism adopted. In case data are auctioned, it sells the consumer list to both the firms located at the extremes of the profiled segment. Instead, in case of a TIOLI offer, the list is sold to all firms in the market. Contrary to previous findings in the literature (e.g., Montes *et al.*, 2019; Kim *et al.*, 2019), an implication of our findings is that the data broker *never* has an incentive to sell the data exclusively.

Morever, as an auction provides the data broker with higher profits than a TIOLI offer, we shall identify this scenario as the expected equilibrium of the game. Thus, in such equilibrium, the second best is realised and the firm whose potential consumers are profiled needs to price aggressively in order to best respond to the personalised price of the two neighbouring rivals. On the other hand, consumers on the anonymous sub-arc are not affected by the list and, as a result, they do not suffer from possible brand mismatches.

These findings carry important managerial and policy implications. Consider a data broker that is in possession of data on the potential clientele of a firm. A somewhat counter-intuitive implication of our findings is that the data broker should not approach such a firm first. Instead, the maximum willingness to pay could be extracted by auctioning off the data. In that case, as long as the data cover a sufficient part of the arc around the firm, the closest rivals would acquire the list.

A firm whose potential clientele is profiled faces a profitability risks, and needs to adopt defensive strategies. For example, if the data are not already available, one option could be to make it harder for a data broker to profile the consumers. For example, these could entail making privacy salient on their website, to enhance their consumers attention in releasing data. Another, more costly, option could be to vertically integrate with the data broker, in order to internalise the impacts of data selling. Finally, a regulation mandating data sharing would be in the interest of such a firm.

From a policy perspective, we note that the data broker is not spontaneously willing to sell the information to all the firms in the market. Thus, the welfare maximising policy-maker should consider either a ban on data collection and sale (if the goal is to favour aggregate profits over consumer surplus) or a mandatory data sharing regulation, which would not only achieve the maximum exploitation of data but also induce pro-competitive market outcomes.

We shall note, however, that second policy option changes firms incentives substantially, and the stakeholders should be aware of it. Whereas selling data to all three firms can be an optimal strategy under certain circumstances, if the data broker *must* sell consumer information to all the firms, her *bargaining power* collapses to the minimum.

Such a regulatory intervention would be welcomed downstream but is likely to face hostile reactions by data holders. Thus, a policy-maker that aims to support this policy should design a tax on data usage to competing firms to redistribute the revenues with the data broker, particularly if the latter has to recover from the costs of collecting data, and needs to be incentivised to do so.

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Table 1: Summary of the prices and profits in each subgame of the pricing stage.

	No info (NI)	All info (AI)	Excl 1 (1)	Excl 2 (2)	Both 1 & 2 (12)	Both 2 & 3 (23)
p_1	0.333 t	-	-	0.244 t	-	0.167 t
p_2	0.333 t	0.333 t	0.222 t	0.321 t	0.286 t	0.333 t
p_3	0.333 t	0.333 t	0.222 t	0.308 t	0.238 t	0.333 t
π_1	0.111 t	0.056 t	0.154 t	0.059 t	0.109 t	0.028 t
π_2	0.111 t	0.083 t	0.049 t	0.135 t	0.069 t	0.118 t
π_3	0.111 t	0.083 t	0.049 t	0.095 t	0.057 t	0.118 t
Π	0.333 t	0.222 t	0.253 t	0.289 t	0.235 t	0.264 t
CS	v - 0.417 t	v - 0.306 t	v - 0.361 t	v - 0.388 t	v - 0.333 t	v - 0.361 t
TS	v - 0.084 t	v - 0.084 t	v - 0.108 t	v - 0.099 t	v - 0.098 t	v - 0.097 t

Table 2: The number of firms, prices and profits.

No info	p_1	p_2	p_n	π_1	π_2	π_n	π_{DB}^{Auct}
$n=3$	0.333t	0.333t	0.333t	0.111t	0.111t	0.111t	0
$n=4$	0.250t	0.250t	0.250t	0.063t	0.063t	0.063t	0
$n=5$	0.200t	0.200t	0.200t	0.040t	0.040t	0.040t	0
$n=10$	0.100t	0.100t	0.100t	0.010t	0.010t	0.010t	0
1, 2 and n	p_1	p_2	p_n	π_1	π_2	π_n	π_{DB}^{Auct}
$n=3$	-	0.333t	0.333t	0.056t	0.083t	0.083t	0.080t
$n=4$	-	0.250t	0.250t	0.031t	0.469t	0.469t	0.043t
$n=5$	-	0.200t	0.200t	0.020t	0.030t	0.030t	0.027t
$n=10$	-	0.100t	0.100t	0.005t	0.008t	0.008t	0.0068t
Excl 1	p_1	p_2	p_n	π_1	π_2	π_n	π_{DB}^{Auct}
$n=3$	-	0.222t	0.222t	0.154t	0.049t	0.049t	0.095t
$n=4$	-	0.179t	0.179t	0.092t	0.032t	0.032t	0.058t
$n=5$	-	0.145t	0.145t	0.060t	0.021t	0.021t	0.038t
$n=10$	-	0.073t	0.073t	0.015t	0.005t	0.005t	0.0096t
Excl 2 (n)	p_1	p_2	p_n	π_1	π_2	π_n	π_{DB}^{Auct}
$n=3$	0.244t	0.321t	0.308t	0.059t	0.135t	0.095t	0.086t
$n=4$	0.183t	0.247t	0.232t	0.033t	0.077t	0.054t	0.046t
$n=5$	0.146t	0.199t	0.186t	0.021t	0.050t	0.034t	0.029t
$n=10$	0.073t	0.100t	0.093t	0.005t	0.013t	0.009t	0.0071t
Both 1 & 2 (n)	p_1	p_2	p_n	π_1	π_2	π_n	π_{DB}^{Auct}
$n=3$	-	0.286t	0.238t	0.109t	0.069t	0.057t	0.093t
$n=4$	-	0.240t	0.183t	0.062t	0.045t	0.034t	0.058t
$n=5$	-	0.197t	0.146t	0.040t	0.030t	0.021t	0.038t
$n=10$	-	0.099t	0.073t	0.010t	0.007t	0.005t	0.0096t
Both 2 & n	p_1	p_2	p_n	π_1	π_2	π_n	π_{DB}^{Auct}
$n=3$	0.167t	0.333t	0.333t	0.028t	0.118t	0.118t	0.122t
$n=4$	0.125t	0.250t	0.250t	0.016t	0.066t	0.066t	0.066t
$n=5$	0.100t	0.200t	0.200t	0.010t	0.043t	0.043t	0.042t
$n=10$	0.050t	0.100t	0.100t	0.003t	0.011t	0.011t	0.0105t

Figure 1: The Salop model with three firms. The dashed line represents the anonymous segment and the full line the profiled one.

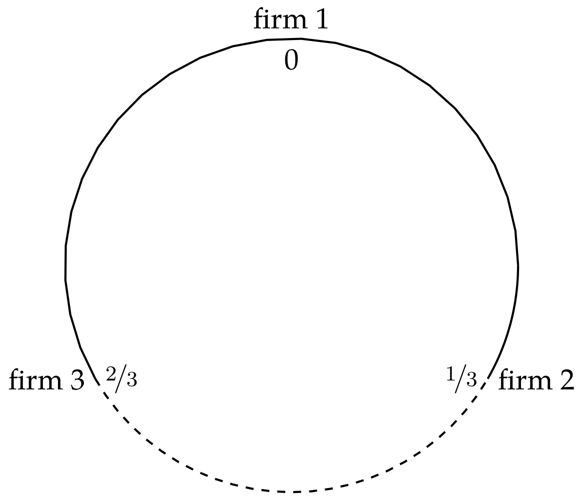
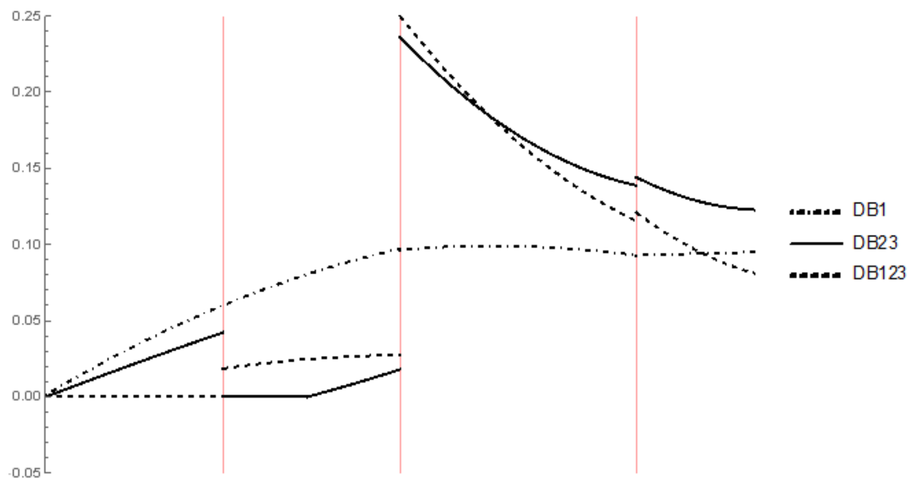


Figure 2: Willingness to pay for and length of the list



A Appendix

A.1 Proof of Proposition 2

As a result of the pricing results (2)-(3)-(4), the profits on the profiled segment of the market are:

$$\begin{aligned}\pi_1^d &= \int_0^{1/6} p_1(x)dx + \int_{5/6}^1 p_1(x)dx = \frac{t}{18} \\ \pi_2^d &= \int_{1/6}^{1/3} p_2(x)dx = \frac{t}{36} \\ \pi_3^d &= \int_{2/3}^{5/6} p_3(x)dx = \frac{t}{36}\end{aligned}.$$

The remaining consumers on the anonymous segment, i.e., between firm 2 and firm 3, are offered a uniform price. The indifferent consumer is identified by solving $U(x, y_2) = U(x, y_3)$. The firms' profit functions are:

$$\pi_2 = p_2 \left(\frac{1}{6} + \frac{p_3 - p_2}{2t} \right) + \frac{t}{36}, \qquad \pi_3 = p_3 \left(\frac{1}{6} + \frac{p_2 - p_3}{2t} \right) + \frac{t}{36}.$$

Standard calculations lead to the profit-maximising anonymous prices

$$p_2 = p_3 = \frac{t}{3}.$$

Using the price schedules (2)-(3)-(4), and the prices p_2 and p_3, the profits of the firms can be written as

$$\pi_1 = \frac{t}{18}, \ \pi_2 = \pi_3 = \frac{t}{12}.$$

Q.E.D.

A.2 Proof of Proposition 3

From the price schedule (5), the profit function of the firms are:

$$\pi_1 = \int_0^{\tilde{x}_{12}} [p_2 + t(1/3 - 2x)]\, dx + \int_{\tilde{x}_{13}}^1 [p_3 + t(2x - 5/6)]\, dx.$$

$$\pi_2 = p_2 \left[\left(\frac{1}{6} + \frac{p_3 - p_2}{2t} \right) + \left(\frac{1}{3} - \tilde{x}_{12} \right) \right].$$

$$\pi_3 = p_3 \left[\left(\frac{1}{6} + \frac{p_2 - p_3}{2t} \right) + \left(\tilde{x}_{13} - \frac{2}{3} \right) \right].$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_2 = p_3 = \frac{2}{9}t.$$

Using these prices and the price schedule (5), it is possible to derive the profits of the firms:

$$\pi_1 = \frac{25}{162}t,\ \pi_2 = \pi_3 = \frac{4}{81}t.$$

Q.E.D.

A.3 Proof of Proposition 4

From the price schedule (6), the profit function of the firms can be written as:

$$\pi_1 = p_1 \left[\tilde{x}_{21} + \left(\frac{1}{6} + \frac{p_3 - p_1}{2t} \right) \right].$$

$$\pi_2 = p_2 \left(\frac{1}{6} + \frac{p_3 - p_2}{2t} \right) + \int_{\tilde{x}_{21}}^{1/3} \left[p_1 + t\left(2x - 1/3\right) \right] dx.$$

$$\pi_3 = p_3 \left[\left(\frac{1}{6} + \frac{p_2 - p_3}{2t} \right) + \left(\frac{1}{6} + \frac{p_1 - p_3}{2t} \right) \right].$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{19}{78}t,\ p_2 = \frac{25}{78}t,\ p_3 = \frac{4}{13}t.$$

Using these prices and the price schedule (5), it is possible to derive the profits of the firms:

$$\pi_1 = \frac{361}{6084}t,\ \pi_2 = \frac{3275}{24336}t,\ \pi_3 = \frac{16}{169}t.$$

Q.E.D.

A.4 Proof of Proposition 5

From the price schedules (7)-(8), the profit function of the firms are:

$$\pi_1 = \int_0^{1/6} \left[t\left(1/3 - 2x\right) \right] dx + \int_{\tilde{x}_{31}}^{1} \left[p_3 + t\left(5/3 - 2x\right) \right] dx.$$

$$\pi_2 = p_2 \left(\frac{1}{6} + \frac{p_3 - p_2}{2t} \right) + \int_{1/6}^{1/3} \left[t\left(2x - 1/3\right) \right] dx.$$

$$\pi_3 = p_3 \left[\left(\frac{1}{6} + \frac{p_2 - p_3}{2t} \right) + \left(\tilde{x}_{31} - 2/3 \right) \right].$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_2 = \frac{2}{7}t,\ p_3 = \frac{5}{21}t.$$

Using these prices and the price schedules (7)-(8), it is possible to derive the profits of the firms:

$$\pi_1 = \frac{193}{1764}t,\ \pi_2 = \frac{121}{1764}t,\ \pi_3 = \frac{25}{441}t.$$

Q.E.D.

A.5 Proof of Proposition 6

From the price schedules (9)-(10), the profit function of the firms are: The profit function of the firms are:

$$\pi_1 = p_1 \left[\tilde{x}_{21} + (1 - \tilde{x}_{31})\right].$$

$$\pi_2 = p_2 \left(\frac{1}{6} + \frac{p_3 - p_2}{2t}\right) + \int_{\tilde{x}_{21}}^{1/3} \left[p_1 + t\left(2x - 1/3\right)\right] dx.$$

$$\pi_3 = p_3 \left(\frac{1}{6} + \frac{p_2 - p_3}{2t}\right) + \int_{\tilde{x}_{31}}^{1/3} \left[p_1 + t\left(5/3 - 2x\right)\right] dx.$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{t}{6},\ p_2 = p_3 = \frac{t}{3}t.$$

Using these prices and the price schedules (7)-(8), it is possible to derive the profits of the firms:

$$\pi_1 = \frac{t}{36},\ \pi_2 = \pi_3 = \frac{17}{144}t.$$

Q.E.D.

B Web Appendix (not for publication): A generic number of firms

In this Appendix we set up the game for a generic number of firms n. We analyse the firms' pricing choices for a given allocation of the list covering the arc around firm 1 and spanning between firms n and 2. As per Table 2, there are six pricing subgames to consider: no firm has access to the list (No info), all relevant firms have the list (1,2 and n), exclusive access of firm 1 (Excl 1) or one of its neighbours firm 2 or firm n (Excl 2(n)) and, finally, when the list is acquired by firm 1 and firm 2 (or firm n, labelled Both 1 & 2) or by firm 2 and firm n (Both 2 & n). We consider each of these subgames in turn. Whereas finding a general solution for any number of firms n is beyond the scope, we identify the asymmetries in pricing that make such a problem challenging in a number of subgames.

B.1 No information

If no firm has access to the list, each firm simultaneously sets posted prices to maximise profits and price competition *à la* Salop takes place. The indifferent consumers between buying from any firm i and its left or its right neighbour (i.e., firms $i-1$ or $i+1$) are:

$$U(x, y_i) = U(x, y_{i-1}) \text{ and } U(x, y_i) = U(x, y_{i+1}),$$

where the utility functions are defined as in equation (1). As a result, the profit function of, for example, firm i is:

$$\pi_i = p_i \left[\left(\frac{t}{n} + \frac{p_{i+1} - p_i}{2t} \right) + \left(\frac{t}{n} + \frac{p_{i-1} - p_i}{2t} \right) \right]. \qquad (11)$$

The first order conditions are obtained by taking the first derivative of (11) with respect to p_i and setting it equal to zero. In this subgame, symmetry allows to easily find the equilibrium. Indeed, the unique pricing equilibrium if no firm has a list is:

$$p_i = \frac{t}{n}, \ \ \pi_i = \frac{t}{n^2}, \ \ i = 1, \ldots, n.$$

Note that this benchmark is equivalent to the case in which the list is held by any firm that is not on or around the profiled arc (i.e., firms 1, 2 and n). This is because the distance from the firm and the profiled consumers would be too high to try offer personalised prices.

B.2 Firms 1, 2 and n have the list

Suppose all the relevant firms on and around the profiled arc have access to the list. In this case, firm 1 posts a price schedule using the list, whereas firm 2 and firm n post both a personalised schedule and a posted price. All other firms set posted prices. Between the firms that set posted prices, standard price competition *à la* Salop takes place.

Consider first the list segments. The price schedules are identified as follows:

$$U(x, y_1) = v - tx - p_1(x) = v - t(1/n - x) - p_2(x) = U(x, y_2) \tag{12}$$
$$U(x, y_n) = v - t(x - n-1/n) - p_n(x) = v - t(1 - x) - p_1(x) = U(x, y_1) \tag{13}$$

Firm 1 personalised price schedule on the $x \in [0, 1/n]$ is obtained by solving (12) for $p_1(x)$ and setting $p_2(x) = 0$. Similarly, the price schedule on the $x \in [n-1/n, 1]$ is obtained by solving (13) for $p_1(x)$ and setting $p_n(x) = 0$.

Following the above procedure in this case, we obtain this schedule for firm 1:

$$p_1(x) = \begin{cases} t\left(1/n - 2x\right), & \text{if } 0 \leq x < 1/2n \\ t\left(2x - 2n-1/n\right), & \text{if } 2n-1/2n \leq x < 1 \\ 0, & \text{otherwise} \end{cases} ;$$

and the schedules for firms 2 and n can be derived similarly. The indifferent consumers between firms 1 and 2 are located at $\tilde{x}_{12} = 1/2n$, and between 1 and n are at $\tilde{x}_{n1} = 2n-1/2n$. They are exactly halfway between firm 1 and its rivals on both sides.

Consider firms k and k-1 ($k = 3, \ldots, n$) not using the list on a segment. The indifferent consumers between such firms is identified by setting:

$$U(x, y_k) = v - t(k-1/n - x) - p_k = v - t(x - k-2/n) - p_{k-1} = U(x, y_{k-1})$$

so that:

$$x_{kk-1} = \left[\frac{2k-3}{2n} + \left(\frac{p_k - p_{k-1}}{2t}\right)\right] \tag{14}$$

As a result, the profit functions for firms 1, 2, n and k ($k \neq 1, 2, n$) are, respectively:

$$\begin{aligned}
\pi_1 &= \int_{\tilde{x}_{n1}}^{1} \left[t\left(2x - 2n-1/n\right)\right] dx + \int_{0}^{\tilde{x}_{12}} \left[t\left(1/n - 2x\right)\right] dx \\
\pi_2 &= \int_{\tilde{x}_{12}}^{1/n} \left[t\left(2x - 1/n\right)\right] dx + p_2\left(\frac{1}{2n} + \frac{p_3 - p_2}{2t}\right) \\
\pi_n &= p_n\left(\frac{1}{2n} - \frac{(p_n - p_{n-1})}{2t}\right) + \int_{n-1/n}^{\tilde{x}_{n1}} \left[t\left(2n-1/n - 2x\right)\right] dx \\
\pi_k &= p_k\left(x_{k+1k} - x_{k-1k}\right) = p_k\left[\frac{1}{n} + \frac{p_{k+1} - 2p_k + p_{k-1}}{2t}\right].
\end{aligned}$$

The first order conditions for profit maximisation are exactly the same as in the Salop model for firms k, $k \neq 1, 2, n$, and the equivalent but for only one segment for the extreme firms 2 and n:

$$i = 2: \ \frac{1}{2n} + \frac{p_3 - 2p_2}{2t} = 0 \qquad\qquad i = n: \ \frac{1}{2n} + \frac{p_{n-1} - 2p_n}{2t} = 0$$

$$i = k: \ \frac{1}{n} + \frac{p_{k+1} - 4p_k + p_{k-1}}{2t} = 0.$$

Symmetry allows solving this subgame analytically, and fully characterise the equilibrium. In particular:

$$p_2 = p_k = p_n = \frac{t}{n} \ \text{ and } \ \pi_1 = \frac{t}{2n^2}, \ \pi_2 = \pi_n = \frac{3t}{4n^2}, \ \pi_k = \frac{t}{n^2}.$$

B.3 Exclusive list to firm 1

Consider now if firm 1 has acquired the list exclusively. In this scenario, only firm 1 chooses a price schedule using the list, whereas all the remaining firms post a uniform price. The price schedule of firm 1 for the list segment are identified using (12) and (13), but noting that p_2 and p_n are posted prices, and not price schedules this time. Following the outlined procedure, the schedule of firm 1 is:

$$p_1(x) = \begin{cases} p_2 + t\left({}^1\!/\!_n - 2x\right), & \text{if } 0 \leq x < \tilde{x}_{12} \\ p_n + t\left(2x - {}^{2n-1}\!/\!_n\right), & \text{if } \tilde{x}_{n1} \leq x < 1 \\ 0, & \text{otherwise} \end{cases} .$$

The indifferent consumers between firms 1 and 2 are located at $\tilde{x}_{12}(p_2) = {}^{p_2}\!/\!_{2t} + {}^1\!/\!_{2n}$, and between 1 and n are at $\tilde{x}_{n1}(p_n) = {}^{p_n}\!/\!_{2t} + {}^{2n-1}\!/\!_{2n}$. Their location depends on the posted prices of the rival firms on both sides. This crucially affects the pricing incentives.

Consider firms k and k-1 ($k = 3, \dots, n$). The indifferent consumers between such firms and the neighbouring firms are still identified by (14) above. As a result, the profit functions for firms 1, 2, n and k ($k \neq 1, 2, n$) are, respectively:

$$\pi_1 = \int_{\tilde{x}_{n1}}^{1} \left[p_n + t\left(2x - {}^{2n-1}\!/\!_n\right)\right] dx + \int_0^{\tilde{x}_{12}} \left[p_2 + t\left({}^1\!/\!_n - 2x\right)\right] dx$$

$$\pi_2 = p_2 \left[\left(\frac{1}{2n} + \frac{p_3 - p_2}{2t}\right) + \left({}^1\!/\!_n - \tilde{x}_{12}(p_2)\right)\right]$$

$$\pi_n = p_n \left[\left(\frac{1}{2n} - \frac{(p_n - p_{n-1})}{2t}\right) + \left(\tilde{x}_{n1}(p_n) - {}^{n-1}\!/\!_n\right)\right]$$

$$\pi_k = p_k\left(x_{k+1k} - x_{k-1k}\right) = p_k\left[\frac{1}{n} + \frac{p_{k+1} - 2p_k + p_{k-1}}{2t}\right].$$

The first order conditions for profit maximisation are:

$$i=2: \ \frac{1}{2n}+\frac{p_3-2p_2}{2t}-p_2=0 \qquad\qquad i=n: \ \frac{1}{2n}+\frac{p_{n-1}-2p_n}{2t}-p_n=0$$
$$i=k: \ \frac{1}{n}+\frac{p_{k+1}-4p_k+p_{k-1}}{2t}=0.$$

Note that for firms k, $k\neq 1,2,n$, these are exactly the same as in the Salop model, but there is an important difference for firms 2 and n. In fact, these firms face the competition of firm 1 through individualised prices. This introduces a new term, the third on the left hand side, and affects their best response functions. Indeed, one may notice that the slope of the first order condition of firms 2 and n $(-{}^{1+t}/_{t})$ is steeper than those of other firms k $(-{}^{1}/_{t})$. Intuitively, this implies that firms 2 and n price more aggressively in equilibrium.

This competitive response to firm 1 affects the equilibrium prices of firms 2 and n symmetrically, but these firms also affect all other firms as a result. A complete solution of this problem is beyond the scope of this paper.

B.4 Exclusive list to firm 2

Suppose now firm 2 has acquired the list exclusively.[4] Hence, only firm 2 chooses a price schedule using the list, for the consumers located between its location and firm 1. It also chooses a posted price, like all the remaining firms. The price schedule of firm 2 is identified using (12), but noting that p_1 is a posted price and not a price schedule in this case. The schedule of firm 2 is, then:

$$p_2(x)=\begin{cases} p_1+t\left(2x-{}^{1}/_{n}\right), & \text{if } \tilde{x}_{12}\leq x<{}^{1}/_{n} \\ 0, & \text{otherwise} \end{cases}.$$

The indifferent consumers between firms 1 and 2 are located at $\tilde{x}_{12}(p_2)={}^{1}/_{2n}-{}^{p_1}/_{2t}$. Note that its location depends on the posted price of firm 1. This crucially affects the pricing incentives of firm 1 and, as a consequence of the chain of the best responses, all other firms in the market including firm 2.

Consider firms k and k-1 ($k=1,3,\ldots,n$). The indifferent consumers between such firms and the neighbouring firms are still identified by (14) above. As a result, the profit functions for firms 1, 2, and k ($k\neq 1,2$)[5] are, respectively:

[4] If it is firm n, instead of firm 2, to get exclusive access to the list, the derivations are unchanged apart from the suscripts.

[5] If $k=n$, clearly $k+1=1$.

$$\pi_1 = p_1 \left[\left(\frac{1}{2n} + \frac{p_n - p_1}{2t} \right) + \tilde{x}_{12}(p_1) \right]$$
$$\pi_2 = p_2 \left(\frac{1}{2n} + \frac{p_3 - p_2}{2t} \right) + \int_{\tilde{x}_{12}}^{1/n} \left[p_1 + t \left(2x - 1/n \right) \right] dx$$
$$\pi_k = p_k \left(x_{k+1k} - x_{k-1k} \right) = p_k \left[\frac{1}{n} + \frac{p_{k+1} - 2p_k + p_{k-1}}{2t} \right].$$

The first order conditions for profit maximisation are:

$$i = 1: \ \frac{1}{2n} + \frac{p_n - 2p_1}{2t} - p_1 = 0 \qquad i = 2: \ \frac{1}{2n} + \frac{p_3 - 2p_2}{2t} = 0$$
$$i = k: \ \frac{1}{n} + \frac{p_{k+1} - 4p_k + p_{k-1}}{2t} = 0.$$

Note that these are exactly the same as in the Salop model for firms k, $k \neq 1, 2$, but there are important differences for firms 1 and 2. In particular, firm 2 only faces standard price competition on one side from firm 3. Hence, its first order condition is more similar to the classical Hotelling model than Salop's one.

Firm 1 has a similar first order condition to firm 2, but also faces the competition through individualised prices exactly from that firm. This competitive effect is captured by the third term on the left hand side. The competitive response to firm 2 affects the equilibrium prices of firm 1 and all other firms, whose best responses are concatenated. A complete solution of this problem is challenging and beyond the scope of this paper.

B.5 List available to both firm 1 and firm 2

Suppose that both firm 1 and firm 2 have acquired the list.[6] Hence, both firms choose a price schedule for the consumers located on the profiled arc. Firm 2 also chooses a posted price, like all the remaining firms, $k = 3, \dots, n$. The price schedule of firm 1 is identified using (12), which also identifies the one of firm 2, and (13). Note however that $p_1(x)$ and $p_2(x)$ are price schedules, but p_n is a posted price in this case. The schedule of firm 1 is, then:

$$p_1(x) = \begin{cases} t \left(1/n - 2x \right), & \text{if } 0 \leq x < 1/2n \\ p_n + t \left(2x - {}^{2n-1}/n \right), & \text{if } \tilde{x}_{n1} \leq x < 1 \\ 0, & \text{otherwise} \end{cases};$$

and of firm 2 is:

$$p_2(x) = \begin{cases} t \left(2x - 1/n \right), & \text{if } 1/2n \leq x < 1/n \\ 0, & \text{otherwise} \end{cases}.$$

[6]If it is firm n, instead of firm 2, to get access to the list together with firm 1, the derivations are unchanged apart from the subscripts.

The indifferent consumers between firms 1 and 2 are located at $\tilde{x}_{12} = {}^{1}/_{2n}$, and between firms 1 and n at $\tilde{x}_{n1}(p_n) = {}^{2n-1}/_{2n} - {}^{p_n}/_{2t}$. Note that the location of $\tilde{x}_{n1}$ depends on the posted price of firm n. This crucially affects the pricing incentives of firm n and, as a consequence of the chain of the best responses, all other firms in the market.

Consider firms k and k-1 ($k = 3, \ldots, n$). The indifferent consumers between such firms and the neighbouring firms are still identified by (14) above. As a result, the profit functions for firms 1, 2, n and k ($k \neq 1, 2, n$) are, respectively:

$$\begin{aligned}
\pi_1 &= \int_{\tilde{x}_{n1}}^{1} \left[t\left(2x - {}^{2n-1}/_{n}\right)\right] dx + \int_{0}^{\tilde{x}_{12}} \left[t\left({}^{1}/_{n} - 2x\right)\right] dx \\
\pi_2 &= \int_{\tilde{x}_{12}}^{1/n} \left[t\left(2x - {}^{1}/_{n}\right)\right] dx + p_2 \left(\frac{1}{2n} + \frac{p_3 - p_2}{2t}\right) \\
\pi_n &= p_n \left[\left(\frac{1}{2n} - \frac{(p_n - p_{n-1})}{2t}\right) + \left(\tilde{x}_{n1}(p_n) - {}^{n-1}/_{n}\right)\right] \\
\pi_k &= p_k\left(x_{k+1k} - x_{k-1k}\right) = p_k \left[\frac{1}{n} + \frac{p_{k+1} - 2p_k + p_{k-1}}{2t}\right].
\end{aligned}$$

The first order conditions for profit maximisation are:

$$i = 2: \ \frac{1}{n} + \frac{p_3 - 2p_2}{2t} = 0 \qquad\qquad i = n: \ \frac{1}{n} + \frac{p_{n-1} - 2p_n}{2t} - p_n = 0$$

$$i = k: \ \frac{1}{n} + \frac{p_{k+1} - 4p_k + p_{k-1}}{2t} = 0.$$

Note that these are exactly the same as in the Salop model for firms k, $k \neq 1, 2, n$, but there are important differences for firms 2 and n. In particular, firm 2 only faces standard price competition from firm 3's side, so its first order condition is similar to the classical Hotelling model.

Firm n, instead, has a similar first order condition to firm 2, but also faces the competition through individualised prices from firm 1. This effect is captured by the third term on the left hand side. The competitive response to firm 1 affects the equilibrium prices of firm n and all other firms. A complete solution of this problem is challenging and beyond the scope of this paper.

B.6 List available to both firm 2 and firm n

Suppose that both firm 2 and firm n have acquired the list. Both firms choose a price schedule for the consumers located on the profiled arc. They also chooses a posted price, like all the remaining firms, $k = 1, 3, \ldots, n$. Note that firm 1 potential market segment can be offered personalised offers by its rivals in this scenario. As a result, firm 1 faces a strong pressure to price aggressively.

The price schedule of firms 2 and n are identified using (12) and (13). Note however that $p_2(x)$ and $p_n(x)$ are price schedules, whereas p_1 is a posted price. The schedule of firm 2 is, then:

$$p_2(x) = \begin{cases} p_1 + t\left(2x - {}^1\!/n\right), & \text{if } \tilde{x}_{12} \leq x < {}^1\!/n \\ 0, & \text{otherwise} \end{cases};$$

and the one of firm n is:

$$p_n(x) = \begin{cases} p_1 + t\left({}^{2n-1}\!/n - 2x\right), & \text{if } {}^{n-1}\!/n \leq x < \tilde{x}_{n1} \\ 0, & \text{otherwise} \end{cases}.$$

The indifferent consumers between firms 1 and 2 are located at $\tilde{x}_{12}(p_1) = {}^1\!/2n - {}^{p_1}\!/2t$, and between firms 1 and n at $\tilde{x}_{n1}(p_n) = {}^{2n-1}\!/2n + {}^{p_1}\!/2t$. Note that the location of the indifferent consumers depends on the prices of firm 2 and n.

Consider firms k and k-1 ($k = 3, \ldots, n$). The indifferent consumers between such firms and the neighbouring firms are still identified by (14) above. As a result, the profit functions for firms 1, 2, n and k ($k \neq 1, 2, n$) are, respectively:

$$\begin{aligned}
\pi_1 &= p_1\left[1 - \tilde{x}_{n1}(p_1) + \tilde{x}_{12}(p_1)\right] = p_1\left({}^1\!/n - {}^{p_1}\!/t\right) \\
\pi_2 &= \int_{\tilde{x}_{12}}^{1/n} \left[t\left(2x - {}^1\!/n\right)\right] dx + p_2\left(\frac{1}{2n} + \frac{p_3 - p_2}{2t}\right) \\
\pi_n &= \int_{\tilde{x}_{12}}^{1/n} \left[t\left(2x - {}^1\!/n\right)\right] dx + p_n\left(\frac{1}{2n} + \frac{p_{n-1} - p_n}{2t}\right) \\
\pi_k &= p_k\left(x_{k+1k} - x_{k-1k}\right) = p_k\left[\frac{1}{n} + \frac{p_{k+1} - 2p_k + p_{k-1}}{2t}\right].
\end{aligned}$$

The first order conditions for profit maximisation are:

$$i = 1: \ \frac{1}{n} - \frac{2p_1}{t} = 0 \qquad\qquad i = 2: \ \frac{1}{2n} + \frac{p_3 - 2p_2}{2t} = 0$$

$$i = n: \ \frac{1}{2n} + \frac{p_{n-1} - 2p_n}{2t} = 0 \qquad\qquad i = k: \ \frac{1}{n} + \frac{p_{k+1} - 4p_k + p_{k-1}}{2t} = 0.$$

Symmetry allows to solve this subgame analytically and fully characterise the equilibrium. In particular, we find that:

$$p_1 = \frac{t}{2n} \ p_2 = p_k = p_n = \frac{t}{n} \ \text{ and } \ \pi_1 = \frac{t}{4n^2}, \ \pi_2 = \pi_n = \frac{17t}{16n^2}, \ \pi_k = \frac{t}{n^2}.$$

C Web Appendix (not for publication): Variable coverage of the list

Let us turn back to the case with three firms and assume that the data broker collected a list of consumers around firm 1's location. In the main model in section 2, we assumed the list includes all the potential consumers of firm 1. In other words, segments $x \in [2/3, 1]$ and $x \in [0, 1/3]$ are profiled. Here we consider a more general case in which the data broker has information on consumers located in the segments $x \in [(1-A), 1]$ and $x \in [0, A]$, where $A \leq 1/3$. Consequently, we analyse the case in which the list is shorter than the full market segments where firm 1 potentially competes.

In order to provide the full characterization of the equilibria, we need to distinguish between four main scenarios: i) The list is relatively long, i.e, $A \in [5/18, 1/3]$; ii) the list is short, i.e, $A \in [1/6, 5/18)$; iii) the list is very short, i.e., $A \in [1/12, 1/6]$; finally iv) the list is tiny, i.e., $A \in [0, 1/12]$.

C.1 Long list

We solve the model as in the main scenario, but taking into consideration that now, if the firms 2 and (or) 3 get the list, they cannot use it to price discriminate the consumers that are outside the list coverage, i.e., those located in $x \in (A, 1/3]$. As we are interested in Subgame Perfect Nash Equilibrium, we solve the model by backward induction, analysing all the subgames in which one or more firms obtain the list of consumers.

Firm 1 obtains the list in exclusive

In this case, there are no differences from the main case analysed in the paper. If firm 1 gets the exclusive right to use the list, then it will set a price schedule that applies to all individuals on the list according to their position. The indifferent consumers for which the personalised price by firm 1 and the market prices by firm 2 and 3, respectively, yield the same utility, are located at $x_{12} = 5/18$ and $x_{31} = 13/18$. As firm 1 would not have been able to use the list to set a personalised price to consumers located too close to the rivals' location, any consumers who is farther than $A = 5/18$ from firm 1 is not considered. Thus, the result is as described in the proof of proposition 3 in Appendix A.

Firm 2 (or 3) obtains the list in exclusive

Consider the case of either firm 2 or firm 3 having exclusive access to information about consumers on the arc around the rival (firm 1), i.e., the profiled segment. Assume, without loss of generality, that firm 2 has access to the consumers' information. In this case, firm 2 sets a price schedule for the profiled consumers ($x \in [0, A]$) and a price p_2 for non-profiled consumers on the anonymous segment ($x \in (A, {}^2\!/\!_3]$). Firm 1 and firm 3 set uniform prices p_1 and p_3. Given these prices, firm 2 personalised price schedule is:

$$p_2(x) = \max\left\{p_1 + t\left(2x - {}^1\!/\!_3\right), 0\right\}.$$

Denote the consumers for which the price schedule of firm 2 is zero, i.e., $p_2(\tilde{x}_{21}) = 0$, as $\tilde{x}_{21} = {}^1\!/\!_6 - {}^{p_1}\!/\!_{2t}$, and assume that these consumers lie on the profiled segment. The equilibrium in the pricing subgame if firm 2 has exclusive information on consumers on firm 1's arc can be characterised as follows. The profit functions are:

$$\pi_1 = p_1\left[\tilde{x}_{21} + \left(\frac{1}{6} + \frac{p_3 - p_1}{2t}\right)\right].$$

$$\pi_2 = p_2\left(\frac{p_3 - p_2 + t - 2At}{2t}\right) + \int_{\tilde{x}_{21}}^{A}\left[p_1 + t\left(2x - {}^1\!/\!_3\right)\right]dx.$$

$$\pi_3 = p_3\left[\left(\frac{1}{2} + \frac{p_2 - p_3}{2t}\right) + \left(\frac{5}{6} + \frac{p_1 - p_3}{2t}\right)\right].$$

Standard calculations lead to the profit-maximising prices for the anonymous segment:

$$p_1 = \frac{1}{26}(7 - 2A)t,\ p_2 = \frac{5}{78}(11 - 18A)t,\ p_3 = \frac{4}{39}(4 - 3A)t.$$

Using these prices and the price schedule $p_2(x)$, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{(7 - 2A)^2 t}{676},\ \pi_2 = \frac{25[4A(129A - 71) + 81]t}{8112},\ \pi_3 = \frac{16(4 - 3A)^2 t}{1521}.$$

Firm 1 and 2 (or 3) obtain the list

Consider the case of firm 1 and either firm 2 or firm 3 having access to information about consumers on the profiled segment. Assume that firms 1 and 2 have access to the consumers' information without loss of generality. In this case, firm 1 and 2 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1 - A), 1]$). Additionally, firm 2 also sets an uniform price p_2 for non-profiled consumers on the anonymous segment ($x \in (A, {}^2\!/\!_3]$), whereas firm 3 sets uniform price p_3. Given these prices, firms 1 and 2 personalised price schedules are:

$$p_{12}(x) = \max\left\{t\left({}^1\!/\!_3 - 2x\right), 0\right\}, \qquad p_{13}(x) = \max\left\{p_3 + t\left(2x - {}^5\!/\!_3\right), 0\right\},$$

$$p_{21}(x) = \max\left\{t\left(2x - {}^1\!/\!_3\right), 0\right\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 1 and 2 are zero, i.e., $p_{12}(\tilde{x}_{12}) = p_{21}(\tilde{x}_{12}) = 0$, as $\tilde{x}_{12} = {}^1\!/\!_6$, and assume that these consumers lie on the profiled segment. Also, denote the consumer for whom the price schedule $p_{13}(\tilde{x}_{31}) = 0$ as $\tilde{x}_{31} = \frac{5t-3p_3}{6t}$. The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = \int_0^{1/6} \left[t\left(2x - 1/3\right)\right] dx + \int_{\tilde{x}_{31}}^{1} \left[p_3 + t\left(5/3 - 2x\right)\right] dx,$$

$$\pi_2 = p_2 \left(\frac{p_3 - p_2 + t - 2At}{2t} \right) + \int_{1/6}^{A} \left[t\left(2x - 1/3\right)\right] dx,$$

$$\pi_3 = p_3 \left[\left(\frac{1}{6} + \frac{p_2 - p_3}{2t} \right) + \left(\tilde{x}_{31} - 2/3\right) \right].$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_2 = \left(\frac{2}{3} - \frac{8A}{7} \right) t, \; p_3 = \frac{1}{21}(7 - 6A)t.$$

Using these prices and the price schedules derived above, it is possible to derive the profits of the firms:

$$\pi_1 = \left[\frac{A(3A - 14)}{147} + \frac{5}{36} \right] t, \; \pi_2 = \frac{\left(972A^2 - 644A + 147\right)}{588} t, \; \pi_3 = \frac{1}{441}(7 - 6A)^2 t.$$

Firm 2 and 3 obtain the list

Consider the case of firm 2 and firm 3 having access to information about consumers on the profiled segment. In this case, firm 2 and 3 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1 - A), 1]$). Additionally, they also set uniform prices p_2 and p_3 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$) and $x \in (2/3, (1 - A))$), whereas firm 1 sets a uniform price p_1. Given these prices, firms 2 and 3 personalised price schedules are:

$$p_{21}(x) = \max\left\{p_1 + t\left(2x - 1/3\right), 0\right\}, \qquad p_{31}(x) = \max\left\{p_1 + t\left(5/3 - 2x\right), 0\right\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 1 and 2 are zero, i.e., $p_{21}(\tilde{x}_{12}) = p_{31}(\tilde{x}_{31}) = 0$, as $\tilde{x}_{12} = \frac{t-3p_1}{6t}$ and $\tilde{x}_{31} = \frac{3p_1+5t}{6t}$ respectively, and assume that these consumers lie on the profiled segment. The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = p_1 \left[\tilde{x}_{21} + (1 - \tilde{x}_{31})\right],$$

$$\pi_2 = p_2 \left(\frac{p_3 - p_2 + t - 2At}{2t} \right) + \int_{\tilde{x}_{21}}^{A} \left[p_1 + t\left(2x - 1/3\right)\right] dx,$$

$$\pi_3 = p_3 \left(\frac{2t(1-A) + p_2 - p_3 - t}{2t} \right) + \int_{1-A}^{\tilde{x}_{31}} \left[p_1 + t\left(5/3 - 2x\right) \right] dx.$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{t}{6},\ p_2 = p_3 = (1-2A)t.$$

Using these prices and the price schedules above, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{t}{36},\ \pi_2 = \pi_3 = \frac{1}{144}\left[24A(18A-13)+73\right]t.$$

All firms obtain the list

Consider the case of all firms having access to information about consumers on the profiled segment. In this case, firms 1, 2, and 3 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1-A), 1]$). Additionally, firms 2 and 3 also set uniform prices p_2 and p_3 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$) and $x \in (2/3, (1-A))$. Given these prices, firms 2 and 3 personalised price schedules are:

$$p_{12}(x) = \max\left\{t\left(1/3 - 2x\right), 0\right\}, \qquad p_{13}(x) = \max\left\{t\left(2x - 5/3\right), 0\right\},$$

$$p_{21}(x) = \max\left\{t\left(2x - 1/3\right), 0\right\}, \qquad p_{31}(x) = \max\left\{t\left(5/3 - 2x\right), 0\right\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 1, 2, and 3 are zero, i.e., $p_{12}(\tilde{x}_{12}) = p_{21}(\tilde{x}_{31}) = 0$, as $\tilde{x}_{12} = \frac{1}{6}$ and $p_{13}(\tilde{x}_{31}) = p_{31}(\tilde{x}_{31}) = 0$, as $\tilde{x}_{31} = \frac{5}{6}$. Assume that these consumers lie on the profiled segment. The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = \int_0^{\tilde{x}_{12}} \left[t\left(1/3 - 2x\right)\right] dx + \int_{\tilde{x}_{31}}^1 \left[t\left(2x - 5/3\right)\right] dx,$$

$$\pi_2 = p_2 \left(\frac{p_3 - p_2 + t - 2At}{2t} \right) + \int_{\tilde{x}_{21}}^{A} \left[p_1 + t\left(2x - 1/3\right) \right] dx,$$

$$\pi_3 = p_3 \left(\frac{2t(1-A) + p_2 - p_3 - t}{2t} \right) + \int_{1-A}^{\tilde{x}_{31}} \left[p_1 + t\left(5/3 - 2x\right) \right] dx.$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_2 = p_3 = (1-2A)t.$$

Using these prices and the price schedules above, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{t}{18},\ \pi_2 = \pi_3 = \frac{1}{36}[12A(9A-7)+19]t.$$

Data Sales

Using the same method to derive the payoff from selling data to firms via auction as illustrated in Section 5, we can compute the data broker's incentives to sell the list.

$$\pi^1_{DB} = \frac{25t}{162} - \frac{1}{676}(7-2A)^2 t, \qquad \pi^2_{DB} = \frac{\left(348300A^2 - 191700A + 43859\right)t}{219024},$$

$$\pi^{12}_{DB} = \left(\frac{78A^2}{49} - A + \frac{1}{4}\right)t, \qquad \pi^{23}_{DB} = \frac{\left(6864A^2 - 4648A + 931\right)t}{1176},$$

$$\pi^{AI}_{DB} = \left(\frac{286A^2}{49} - \frac{30A}{7} + \frac{31}{36}\right)t.$$

C.2 Short list

Let us now turn to the case in which $A \in [1/6, 5/18]$.

Firm 1 obtains the list in exclusive

Because the list is shorter than $5/18$, firm 1 sets both the price schedules $p_{12}(x)$ and $p_{13}(x)$ for the consumers on the list, and an uniform price p_1 for the consumers on the non-profiled segment. Instead, firms 2 and 3 set the uniform prices p_2 and p_3, respectively.

Given these prices, firm 1 personalised price schedules are:

$$p_{12}(x) = \max\left\{p_2 + t\left(1/3 - 2x\right), 0\right\}, \qquad p_{13}(x) = \max\left\{p_3 + t\left(2x - 5/3\right), 0\right\}.$$

It is easy to see that the consumers who derive zero utility from the price schedules are not on the profiled list, i.e. $A < x^*_{12}$ and $(1-A) > x^*_{31}$.

Thus, the indifferent consumers' locations are

$$\tilde{x}_{12} = \frac{3(p_2 - p_1) + t}{6t}, \ \tilde{x}_{23} = \frac{p_3 - p_2 + t}{2t}, \ \tilde{x}_{31} = \frac{3(p_1 - p_3) + 5t}{6t}$$

The equilibrium in the pricing subgame if firm 1 has exclusive information on consumers on firm 1's arc can be characterised as follows:

$$\pi_1 = \int_0^A \left[t\left(1/3 - 2x\right)\right]dx + \int_{1-A}^1 \left[t\left(2x - 5/3\right)\right]dx + p_1\left[\tilde{x}_{12} - A + (1-A) - \tilde{x}_{31}\right],$$

$$\pi_2 = p_2\left(\frac{p_3 - p_2 + t}{2t} - \frac{3(p_2 - p_1) + t}{6t}\right),$$

$$\pi_3 = p_3\left(\frac{3(p_1 - p_3) + 5t}{6t} - \frac{p_3 - p_2 + t}{2t}\right).$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \left(\frac{1}{3} - \frac{6A}{5}\right) t, \; p_2 = p_3 = \frac{1}{15}(5 - 6A)t$$

and to the profits of the firms:

$$\pi_1 = \frac{1}{225}\left(25 + 120A - 306A^2\right) t, \; \pi_2 = \pi_3 = \frac{1}{225}(5 - 6A)^2 t.$$

Firm 2 (or 3) obtains the list in exclusive

Consider the case of either firm 2 or firm 3 having exclusive access to information about consumers on the arc around the rival (firm 1), i.e., the profiled segment. Assume that firm 2 has access to the consumers' information without loss of generality. In this case, firm 2 sets a price schedule for the profiled consumers ($x \in [0, A]$) and a price p_2 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$). Firm 1 and firm 3 set uniform prices p_1 and p_3. Given these prices, firm 2 personalised price schedule is:

$$p_2(x) = \max\left\{p_1 + t\left(2x - 1/3\right), 0\right\}.$$

Denote the consumers for which the price schedule of firm 2 is zero, i.e., $p_2(\tilde{x}_{21}) = 0$, as $\tilde{x}_{21} = 1/6 - p_1/2t$, and assume that these consumers lie on the profiled segment. The equilibrium in the pricing subgame if firm 2 has exclusive information on consumers on firm 1's arc can be characterised as follows.

$$\pi_1 = p_1\left[\tilde{x}_{21} + \left(\frac{1}{6} + \frac{p_3 - p_1}{2t}\right)\right],$$

$$\pi_2 = p_2\left(\frac{p_3 - p_2 + t - 2At}{2t}\right) + \int_{\tilde{x}_{21}}^{A} \left[p_1 + t\left(2x - 1/3\right)\right] dx,$$

$$\pi_3 = p_3\left[\left(\frac{1}{2} + \frac{p_2 - p_3}{2t}\right) + \left(\frac{5}{6} + \frac{p_1 - p_3}{2t}\right)\right].$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{1}{26}(7 - 2A)t, \; p_2 = \frac{5}{78}(11 - 18A)t, \; p_3 = \frac{4}{39}(4 - 3A)t.$$

Using these prices and the price schedule $p_2(x)$, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{(7 - 2A)^2}{676} t, \; \pi_2 = \frac{25[4A(129A - 71) + 81]t}{8112}, \; \pi_3 = \frac{16(4 - 3A)^2 t}{1521}.$$

Firm 1 and 2 (or 3) obtain the list

Consider the case of firm 1 and either firm 2 or firm 3 having access to information about consumers on the profiled segment. Assume that firms 1 and 2 have access to the consumers' information without loss of generality. In this case, firm 1 and 2 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1-A), 1]$). Additionally, firm 2 also sets an uniform price p_2 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$), whereas firm 3 sets uniform price p_3. Given these prices, firms 1 and 2 personalised price schedules are:

$$p_{12}(x) = \max\left\{t\left(1/3 - 2x\right), 0\right\}, \qquad p_{13}(x) = \max\left\{p_3 + t\left(2x - 5/3\right), 0\right\},$$

$$p_{21}(x) = \max\left\{t\left(2x - 1/3\right), 0\right\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 1 and 2 are zero, i.e., $p_{12}(\tilde{x}_{12}) = p_{21}(\tilde{x}_{12}) = 0$, as $\tilde{x}_{12} = 1/6$. It is easy to see that this consumer lies on the profiled segment. Also, denote the consumer for whom the price schedule $p_{13}(\tilde{x}_{31}) = 0$ as $x^*_{31} = \frac{5t-3p_3}{6t} < (1-A)$. Thus, the indifferent consumers' locations are

$$\tilde{x}_{12} = \frac{1}{6}, \ \tilde{x}_{23} = \frac{p_3 - p_2 + t}{2t}, \ \tilde{x}_{31} = \frac{3(p_1 - p_3) + 5t}{6t}$$

The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = \int_0^{1/6} \left[t\left(2x - 1/3\right)\right] dx + \int_{1-A}^{1} \left[p_3 + t\left(5/3 - 2x\right)\right] dx + p_1\left((1-A) - \tilde{x}_{31}\right),$$

$$\pi_2 = p_2\left(\tilde{x}_{23} - A\right) + \int_{1/6}^{A} \left[t\left(2x - 1/3\right)\right] dx,$$

$$\pi_3 = p_3\left(\tilde{x}_{31} - \tilde{x}_{23}\right).$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{1}{18}(7 - 24A)t, \ p_2 = \frac{1}{18}(13 - 24A)t, \ p_3 = \frac{2}{9}(2 - 3A)t.$$

Using these prices and the price schedules derived above, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{1}{648}[168A(1-3A) + 67]t, \ \pi_2 = \frac{1}{648}[24A(51A - 35) + 187]t, \ \pi_3 = \frac{4}{81}(2 - 3A)^2 t.$$

Firm 2 and 3 obtain the list

Consider the case of firm 2 and firm 3 having access to information about consumers on the profiled segment. In this case, firm 2 and 3 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1-A), 1]$). Additionally, they also set uniform prices p_2 and p_3 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$) and $x \in (2/3, (1-A))$, whereas firm 1 sets a uniform price p_1. Given these prices, firms 2 and 3 personalised price schedules are:

$$p_{21}(x) = \max\{p_1 + t(2x - 1/3), 0\}, \qquad p_{31}(x) = \max\{p_1 + t(5/3 - 2x), 0\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 1 and 2 are zero, i.e., $p_{21}(\tilde{x}_{12}) = p_{31}(\tilde{x}_{31}) = 0$, as $\tilde{x}_{12} = \frac{t-3p_1}{6t}$ and $\tilde{x}_{31} = \frac{3p_1+5t}{6t}$ respectively, and assume that these consumers lie on the profiled segment. The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = p_1\left[\tilde{x}_{21} + (1 - \tilde{x}_{31})\right],$$

$$\pi_2 = p_2\left(\frac{p_3 - p_2 + t - 2At}{2t}\right) + \int_{\tilde{x}_{21}}^{A} \left[p_1 + t(2x - 1/3)\right] dx,$$

$$\pi_3 = p_3\left(\frac{2t(1-A) + p_2 - p_3 - t}{2t}\right) + \int_{1-A}^{\tilde{x}_{31}} \left[p_1 + t(5/3 - 2x)\right] dx.$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{t}{6},\ p_2 = p_3 = (1-2A)t.$$

Using these prices and the price schedules above, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{t}{36},\ \pi_2 = \pi_3 = \frac{1}{144}[24A(18A - 13) + 73]t.$$

All firms obtain the list

Consider the case of all firms having access to information about consumers on the profiled segment. In this case, firms 1, 2, and 3 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1-A), 1]$). Additionally, firms 2 and 3 also set uniform prices p_2 and p_3 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$) and $x \in (2/3, (1-A))$. Given these prices, firms 2 and 3 personalised price schedules are:

$$p_{12}(x) = \max\{t(1/3 - 2x), 0\}, \qquad p_{13}(x) = \max\{t(2x - 5/3), 0\},$$

$$p_{21}(x) = \max\{t(2x - 1/3), 0\}, \qquad p_{31}(x) = \max\{t(5/3 - 2x), 0\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 1, 2, and 3 are zero, i.e., $p_{12}(\tilde{x}_{12}) = p_{21}(\tilde{x}_{31}) = 0$, as $\tilde{x}_{12} = \frac{1}{6}$ and $p_{13}(\tilde{x}_{31}) = p_{31}(\tilde{x}_{31}) = 0$, as $\tilde{x}_{12} = \frac{5}{6}$. Assume that these consumers lie on the profiled segment. The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = \int_0^{\tilde{x}_{12}} [t\,(1/3 - 2x)]\,dx + \int_{\tilde{x}_{31}}^1 [t\,(2x - 5/3)]\,dx,$$

$$\pi_2 = p_2\left(\frac{p_3 - p_2 + t - 2At}{2t}\right) + \int_{\tilde{x}_{21}}^{A} [p_1 + t\,(2x - 1/3)]\,dx,$$

$$\pi_3 = p_3\left(\frac{2t(1-A) + p_2 - p_3 - t}{2t}\right) + \int_{1-A}^{\tilde{x}_{31}} [p_1 + t\,(5/3 - 2x)]\,dx.$$

Standard calculations lead to the profit-maximising prices for the anonymous segment:

$$p_2 = p_3 = (1 - 2A)t.$$

Using these prices and the price schedules above, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{t}{18},\ \ \pi_2 = \pi_3 = \frac{1}{36}[12A(9A - 7) + 19]t.$$

Data Sales

Using the same method to derive the payoff from selling data to firms via auction as illustrated in Section 5, we can compute the data broker's incentives to sell the list.

$$\pi_{DB}^1 = \frac{\left(-207756A^2 + 87420A + 5875\right)t}{152100}, \quad \pi_{DB}^2 = \frac{\left(870156A^2 - 370260A + 84275\right)t}{608400},$$

$$\pi_{DB}^{12} = \frac{1}{18}\left(12A^2 - 8A + 3\right)t, \quad \pi_{DB}^{23} = \frac{1}{648}\left(3312A^2 - 2040A + 401\right)t,$$

$$\pi_{DB}^{AI} = \frac{1}{324}\left(1656A^2 - 1128A + 223\right)t$$

C.3 Very short list

Let us now turn to the case in which $A \in [1/12, 1/6]$.

Firm 1 obtains the list in exclusive

Because the list is shorter than $5/18$, firm 1 sets both the price schedules $p_{12}(x)$ and $p_{13}(x)$ for the consumers on the list, and an uniform price p_1 for the consumers on the non-profiled segment. Instead, firms 2 and 3 set the uniform prices p_2 and p_3, respectively. This subgame solves equivalently as in the case where $A \in [1/6, 5/18)$ presented above.

Firm 2 (or 3) obtains the list in exclusive

Consider the case of either firm 2 or firm 3 having exclusive access to information about consumers on the arc around the rival (firm 1), i.e., the profiled segment. Assume that firm 2 has access to the consumers' information without loss of generality. In this case, firm 2 sets a price schedule for the profiled consumers ($x \in [0, A]$) and a price p_2 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$). Firm 1 and firm 3 set uniform prices p_1 and p_3. Notice that, as $A < 1/6$, the segment of consumers between firms 1 and 2 may be fragmented. Given the prices abovementioned, firm 2 personalised price schedule is:

$$p_2(x) = \max\left\{p_1 + t\left(2x - 1/3\right), 0\right\}.$$

Denote the consumers for which the price schedule of firm 2 is zero, i.e., $p_2(\tilde{x}_{21}) = 0$, as $x^*_{21} = 1/6 - p_1/2t$, and assume that these consumers lie on the profiled segment. In this case, the locations of the indifferent consumers are:

$$\tilde{x}_{12} = \frac{3(p_2 - p_1) + t}{6t}, \ \tilde{x}_{23} = \frac{p_3 - p_2 + t}{2t}, \ \tilde{x}_{31} = \frac{3(p_1 - p_3) + 5t}{6t}, \ x^*_{12} = 1/6 - p_1/2t$$

Notice that we identified two locations between firms 1 and 2. This is possible as firm 1 best-replies aggressively to firm 2's price schedule and set an uniform price p_1 that is sufficiently low to attract some of the consumers outside the list.

The equilibrium in the pricing subgame if firm 2 has exclusive information on consumers on firm 1's arc can be characterised as follows.

$$\pi_1 = p_1\left[x^*_{21} + (\tilde{x}_{12} - A) + (1 - \tilde{x}_{31})\right], \qquad \pi_2 = p_2\left(\tilde{x}_{23} - \tilde{x}_{12}\right) + \int_{x^*_{12}}^{A}\left[p_1 + t\left(2x - 1/3\right)\right]dx,$$
$$\pi_3 = p_3\left(\tilde{x}_{31} - \tilde{x}_{23}\right).$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{1}{48}(13 - 18A)t, \ p_2 = p_3 = \frac{1}{16}(5 - 2A)t.$$

Using these prices and the price schedule $p_2(x)$, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{(13 - 18A)^2}{1536}t, \ \pi_2 = \frac{[4A(173A - 33) + 101]}{1024}t, \ \pi_3 = \frac{(5 - 2A)^2}{256}t.$$

Firm 1 and 2 (or 3) obtain the list

Consider the case of firm 1 and either firm 2 or firm 3 having access to information about consumers on the profiled segment. Assume that firms 1 and 2 have access to the consumers' information without loss of generality. In this case, firm 1 and 2 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1 - A), 1]$). Additionally, firm 2 also sets an uniform price p_2 for non-profiled consumers on the anonymous segment

$(x \in (A, 2/3])$, whereas firm 3 sets uniform price p_3. Given the above-mentioned prices, firms 1 and 2 personalised price schedules are:

$$p_{12}(x) = \max\{t(1/3 - 2x), 0\}, \qquad p_{13}(x) = \max\{p_3 + t(2x - 5/3), 0\},$$

$$p_{21}(x) = \max\{t(2x - 1/3), 0\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 1 and 2 are zero, i.e., $p_{12}(\tilde{x}_{12}) = p_{21}(\tilde{x}_{12}) = 0$, as $\tilde{x}_{12} = 1/6$. It is easy to see that this consumer lies outside the profiled segment. Also, denote the consumer for whom the price schedule $p_{13}(\tilde{x}_{31}) = 0$ as $x^*_{31} = \frac{5t-3p_3}{6t} < (1 - A)$. Thus, the indifferent consumers' locations are

$$\tilde{x}_{12} = \frac{3(p_2 - p_1) + t}{6t}, \ \tilde{x}_{23} = \frac{p_3 - p_2 + t}{2t}, \ \tilde{x}_{31} = \frac{3(p_1 - p_3) + 5t}{6t}$$

The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = \int_0^A [t(2x - 1/3)]\, dx + \int_{1-A}^1 [p_3 + t(5/3 - 2x)]\, dx + p_1\left(((1 - A) - \tilde{x}_{31}) + (\tilde{x}_{12} - A)\right),$$

$$\pi_2 = p_2(\tilde{x}_{23} - \tilde{x}_{12}), \qquad \pi_3 = p_3(\tilde{x}_{31} - \tilde{x}_{23}).$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \left(\frac{1}{3} - \frac{6A}{5}\right)t, \ p_2 = p_3 = \frac{1}{15}(5 - 6A)t.$$

Using these prices and the price schedules derived above, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{1}{225}[9A(5 - 24A) + 25]t, \ \pi_2 = \pi_3 = \frac{1}{225}(5 - 6A)^2 t.$$

Firm 2 and 3 obtain the list

Consider the case of firm 2 and firm 3 having access to information about consumers on the profiled segment. In this case, firm 2 and 3 set a price schedule for the profiled consumers $(x \in [0, A]$ and $x \in [(1 - A), 1])$. Additionally, they also set uniform prices p_2 and p_3 for non-profiled consumers on the anonymous segment $(x \in (A, 2/3])$ and $x \in (2/3, (1 - A))$, whereas firm 1 sets a uniform price p_1. Notice that, as $A < 1/6$, the segment of consumers between firms 1 and 2 may be fragmented. Given the prices mentioned above, firms 1 and 2 personalised price schedules are:

$$p_{21}(x) = \max\{p_1 + t(2x - 1/3), 0\}, \qquad p_{31}(x) = \max\{p_1 + t(5/3 - 2x), 0\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 2 and 3 is zero, i.e., $p_2(\tilde{x}_{12}) = 0$ and $p_3(\tilde{x}_{31}) = 0$, as $x^*_{12} = 1/6 - p_1/2t$ and $x^*_{31} = 5/6 + p_1/2t$, and assume that these consumers lie on the profiled segment. In this case, the locations of the indifferent consumers are:

$$\tilde{x}_{12} = \frac{3(p_2 - p_1) + t}{6t}, \ \tilde{x}_{23} = \frac{p_3 - p_2 + t}{2t}, \ \tilde{x}_{31} = \frac{3(p_1 - p_3) + 5t}{6t},$$

$$x^*_{12} = 1/6 - p_1/2t, \ x^*_{31} = 5/6 + p_1/2t$$

Notice that we identified two locations between firms 1 and 2 (and 3). This is possible as firm 1 best-replies aggressively to firm 2(3)'s price schedule and set an uniform price p_1 that is sufficiently low to attract some of the consumers outside the list.

The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = p_1 \left[x^*_{12} + (\tilde{x}_{12} - A) + (1 - x^*_{31}) + ((1 - A)\tilde{x}_{31})\right],$$

$$\pi_2 = p_2 \left(\tilde{x}_{23} - \tilde{x}_{12}\right) + \int_{x^*_{12}}^{A} \left[p_1 + t\left(2x - 1/3\right)\right] dx,$$

$$\pi_3 = p_3 \left(\tilde{x}_{31} - \tilde{x}_{23}\right) + \int_{1-A}^{x^*_{31}} \left[p_1 + t\left(5/3 - 2x\right)\right] dx$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{2(4 - 9A)}{33}t, \ p_2 = p_3 = \frac{2(5 - 3A)}{33}t.$$

Using these prices and the price schedules above, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{8(4 - 9A)^2 t}{1089}, \ \pi_2 = \pi_3 = \frac{[48A(51A - 16) + 409]t}{4356}.$$

All firms obtain the list

Consider the case of all firms having access to information about consumers on the profiled segment. In this case, firms 1, 2, and 3 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1-A), 1]$). Additionally, firms 2 and 3 also set uniform prices p_2 and p_3 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$) and $x \in (2/3, (1-A))$. Given these prices, firms 2 and 3 personalised price schedules are:

$$p_{12}(x) = \max\left\{t\left(1/3 - 2x\right), 0\right\}, \qquad p_{13}(x) = \max\left\{t\left(2x - 5/3\right), 0\right\},$$

$$p_{21}(x) = \max\left\{t\left(2x - 1/3\right), 0\right\}, \qquad p_{31}(x) = \max\left\{t\left(5/3 - 2x\right), 0\right\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 1, 2, and 3 are zero, i.e.,

$p_{12}(x_{12}^*) = p_{21}(x_{31}^*) = 0$, as $x_{12}^* = \frac{1}{6}$ and $p_{13}(x_{31}^*) = p_{31}(x_{31}^*) = 0$, as $x_{12}^* = \frac{5}{6}$. Clearly, these consumers do not lie on the profiled segment. In this case, the locations of the indifferent consumers are:

$$\tilde{x}_{12} = \frac{3(p_2 - p_1) + t}{6t}, \ \tilde{x}_{23} = \frac{p_3 - p_2 + t}{2t}, \ \tilde{x}_{31} = \frac{3(p_1 - p_3) + 5t}{6t}.$$

The equilibrium in the pricing subgame can be characterised as follows:

$$\pi_1 = p_1\left((\tilde{x}_{12} - A) + ((1 - A) - \tilde{x}_{31})\right) + \int_0^A \left[t\left(1/3 - 2x\right)\right] dx + \int_{1-A}^1 \left[t\left(2x - 5/3\right)\right] dx,$$

$$\pi_2 = p_2\left(\tilde{x}_{23} - \tilde{x}_{12}\right), \qquad \pi_3 = p_3\left(\tilde{x}_{31} - \tilde{x}_{23}\right).$$

Standard calculations lead to the profit-maximising prices for the anonymous segment

$$p_1 = \frac{(5 - 18A)}{5}t, \ p_2 = p_3 = \frac{(5 - 6A)}{15}t$$

Using these prices and the price schedules above, it is possible to derive the profits of the firms:

$$\pi_1 = \frac{1}{225}[25 - 6A(21A + 5)]t, \ \pi_2 = \pi_3 = \frac{1}{225}(5 - 6A)^2 t.$$

Data Sales

Using the same method to derive the payoff from selling data to firms via auction as illustrated in Section 5, we can compute the data broker's incentives to sell the list.

$$\pi_{DB}^1 = \frac{\left(-180972A^2 + 96540A + 125\right)t}{115200}, \qquad \pi_{DB}^2 = \frac{\left(118836A^2 + 31740A - 2875\right)t}{230400},$$
$$\pi_{DB}^{12} = -\frac{7\left(6048A^2 - 2835A + 25\right)t}{27225}, \qquad \pi_{DB}^{23} = \frac{\left(14592A^2 + 3280A - 625\right)t}{18150},$$
$$\pi_{DB}^{AI} = -\frac{\left(31446A^2 - 10770A + 175\right)t}{27225}.$$

C.4 Tiny list

Let us now turn to the case in which $A < 1/12$.

Firm 1 obtains the list in exclusive

As the list is shorter than $5/18$, firm 1 sets both the price schedules $p_{12}(x)$ and $p_{13}(x)$ for the consumers on the list, and an uniform price p_1 for the consumers on the non-profiled segment. Instead, firms 2 and 3 set the uniform prices p_2 and p_3, respectively. This subgame solves equivalently as in the case where $A \in [1/6, 5/18)$ presented above.

Firm 2 (or 3) obtains the list in exclusive

Consider the case of either firm 2 or firm 3 having exclusive access to information about consumers on the arc around the rival (firm 1), i.e., the profiled segment. Assume that firm 2 has access to the consumers' information without loss of generality. In this case, firm 2 sets a price schedule for the profiled consumers ($x \in [0, A]$) and a price p_2 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$). Firm 1 and firm 3 set uniform prices p_1 and p_3. Notice that, as $A < 1/6$, the segment of consumers between firms 1 and 2 may be fragmented. This subgame solves equivalently as in the case where $A \in [1/12, 1/6)$ presented above.

Notice that, if $A < 1/26$, firm 2 is no longer able to attract any of the consumers on the profiled segment, and the subgame reduces to a standard Salop-game with no information.

Firm 1 and 2 (or 3) obtain the list

Consider the case of firm 1 and either firm 2 or firm 3 having access to information about consumers on the profiled segment. Assume that firms 1 and 2 have access to the consumers' information without loss of generality. In this case, firm 1 and 2 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1 - A), 1]$). Additionally, firm 2 also sets an uniform price p_2 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$), whereas firm 3 sets uniform price p_3. This subgame solves equivalently as in the case where $A \in [1/12, 1/6)$ presented above.

Firm 2 and 3 obtain the list

Consider the case of firm 2 and firm 3 having access to information about consumers on the profiled segment. In this case, firm 2 and 3 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1 - A), 1]$). Additionally, they also set uniform prices p_2 and p_3 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$) and $x \in (2/3, (1 - A))$, whereas firm 1 sets a uniform price p_1. Notice that, as $A < 1/6$, the segment of consumers between firms 1 and 2 may be fragmented. Given the prices abovementioned, firms 1 and 2 personalised price schedules are:

$$p_{21}(x) = \max\{p_1 + t(2x - 1/3), 0\}, \qquad p_{31}(x) = \max\{p_1 + t(5/3 - 2x), 0\}.$$

where the subscripts $\{i, j\}$ indicate the price of i competing against j.

Denote the consumers for which the price schedules of firms 2 and 3 is zero, i.e., $p_2(\tilde{x}_{12}) = 0$ and $p_3(\tilde{x}_{31}) = 0$, as $x^*_{12} = 1/6 - p_1/2t$ and $x^*_{31} = 5/6 + p_1/2t$, and assume that these consumers lie on the profiled segment. In this case, the locations of the indifferent consumers are:

$$\tilde{x}_{12} = \frac{3(p_2 - p_1) + t}{6t}, \ \tilde{x}_{23} = \frac{p_3 - p_2 + t}{2t}, \ \tilde{x}_{31} = \frac{3(p_1 - p_3) + 5t}{6t},$$

$$x_{12}^* = {}^1\!/\!_6 - {}^{p_1}\!/\!_{2t}, \ \ x_{31}^* = {}^5\!/\!_6 + {}^{p_1}\!/\!_{2t}$$

Notice that there are virtually two identified locations between firms 1 and 2 (and 3). This is possible as firm 1 best-replies aggressively to firm 2(3)'s price schedule and set an uniform price p_1 that is sufficiently low to attract some of the consumers outside the list. However, when $A < 1/12$, the indifferent consumers x_{12}^* and x_{31}^* lie outside the profiled segment. Thus, the game collapses to a standard Salop-game with no information.

All firms obtain the list

Consider the case of all firms having access to information about consumers on the profiled segment. In this case, firms 1, 2, and 3 set a price schedule for the profiled consumers ($x \in [0, A]$ and $x \in [(1-A), 1]$). Additionally, firms 2 and 3 also set uniform prices p_2 and p_3 for non-profiled consumers on the anonymous segment ($x \in (A, 2/3]$) and $x \in (2/3, (1-A))$. This subgame solves equivalently as in the case where $A \in [1/12, 1/6)$ presented above.

Data Sales

Using the same method to derive the payoff from selling data to firms via auction as illustrated in Section 5, we can compute the data broker's incentives to sell the list. For the sake of simplicity, we focus on $A > 1/26$. We treat any negative willingness to pay as zero in the comparison.

$$\pi_{DB}^1 = \frac{\left(125 + 96540A - 180972A^2\right)t}{115200}, \qquad \pi_{DB}^2 = \frac{\left(118836A^2 + 31740A - 2875\right)t}{230400},$$
$$\pi_{DB}^{12} = \frac{(5 - 24A)At}{25}, \qquad \pi_{DB}^{23} = \frac{8A(3A-5)t}{75},$$
$$\pi_{DB}^{AI} = -\frac{2A(21A+5)t}{75}.$$

Figure 3: Summary of the willingness to pay for and length of the list. Negative value are collapsed to zero.