The impact of targeting technologies and consumer multi-homing on digital platform competition[1]

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Abstract

In this paper, we address how targeting by the use of first-party data and consumer multi-homing impact platform competition and market equilibria in two-sided markets. We analyze platforms that are financed by both advertising and subscription fees, and let them adopt a targeting technology with increasing performance in audience size: a larger audience generates more consumer data, which improves the platforms' targeting ability and allows them to extract more ad revenues. Targeting therefore increases the importance of attracting consumers. Previous literature has shown that this could result in fierce price competition if consumers subscribe to only one platform (i.e. single-home). Surprisingly, we find that pure single-homing possibly does not constitute a Nash equilibrium. Instead, platforms might rationally set prices that induce consumers to subscribe to more than one platform (i.e. multi-home). With multi-homing, a platform's audience size is not restricted by the number of subscribers on rival platforms. Hence, targeting softens the competition over consumers. We show that this might imply that equilibrium profit is higher with than without targeting, in sharp contrast to what previous literature predicts.

Keywords: Two-sided markets, digital platforms, targeted advertising, privacy, incremental pricing, consumer multi-homing.

JEL classifications: D11, D21, L13, L82.

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1 Introduction

In the digital era, media platforms collect personal data about its users at a large scale, and utilize such data to monetize on them. This strategy is widely used, and is the core of the business models of digital media platforms. For example, Netflix and Amazon use personal data to offer personalized product recommendations, and online newspapers and other media platforms target advertising based on the consumers' online activities. Our focus is on the latter. By adopting advanced, programmatic advertising technologies such as targeting, the platforms can identify those consumers that are most prone to buy a given advertiser's product, and make sure that ad impressions are directed towards the most promising candidates. A key feature of targeting algorithms is that they improve as they get exposed to more consumer data. Platforms that collect more user data could therefore be better able to connect advertisers with the target audience, and advertisers might be willing to pay more per impression on platforms with a large audience and higher targeting ability.[2]

At the same time, there is an increased demand for privacy. Until recently, such consumer data could easily be purchased by third parties. However, stricter privacy regulations, such as the General Data Privacy Regulation (GDPR) of 2018, limit the scope of utilizing externally collected consumer information. Web browsers increasingly block third-party cookies, and platforms are moving away from third-party data and towards permission-based, internally collected first-party data (see e.g. Goswami, 2020, and Walter, 2021). For example, *The New York Times* and *The Washington Post* have recently developed in-house solutions in order to control data and targeting (Fischer, 2020).[3] Attracting consumer attention to its *own* platform is therefore ever as vital for today's media platforms in order to have successful targeting. The aim of this study is to analyze how

[2]Goettler (2012) studies broadcast networks and provides empirical evidence that the ad price per viewer might increase in audience size.

[3]The VP of commercial technology and development at *The Washington Post*, Jarrod Dicker, stated in a press release that (Washington Post Press release, July 16, 2019): *"User privacy is paramount to us, so we are deeply invested in building sophisticated tools powered by first-party data"*. The machine learning-based tools enable the newspaper to benefit from data on how the users engage with the platform, and reduce its reliance on cookie-driven information. The head of ad product for RED, Jeff Turner, elaborates (Washington Post Press release, July 16, 2019): *"Data points like a user's current page view and session on The Post's site are much more relevant to that user's current consumption intention than the information a cookie-driven strategy can offer"*. Advertisers cannot find this insight elsewhere, which gives the platforms a competitive advantage. The focus on privacy has increased the strategic importance of first-party data, which is also the key to successful targeting in our model.

such targeting impacts digital platform competition. In particular, we will demonstrate how this is related to recent privacy regulations and consumer homing behavior.

Previous studies on digital media markets have shown that targeting might increase competition and benefit consumers through lower subscription prices (see e.g. Kox *et al.*, 2017; Crampes *et al.*, 2009).[4] However, the additional revenues from the ad side of the market are competed away on the consumer side of the market, leaving the platforms worse off. As Kox *et al.* (2017) point out, even though it would be in the platforms' common interest not to target ads, each platform might have individual incentives to do so. This suggests that the platforms would like to avoid targeting. However, this prediction is not in line with the observation that digital platforms profitably target ads.

Common for these studies is the stark assumption that consumers single-home (i.e. join only a single platform). In practice, we observe that while some consumers indeed are devoted to a single media provider, others spread their attention across multiple platforms.[5] The emergence of digital technologies, with its very low online distribution and print costs, has facilitated the latter, which we refer to as consumer multi-homing. All it takes to read an additional newspaper online, is a few extra clicks.[6] By relaxing the assumption of single-homing, we find that targeting might be profitable to the platform, but only if multi-homers are sufficiently valued in the ad market.

The main mechanism that drives this result is a strategic substitutability induced by the ability a platform has to affect the composition of a rival platform's total demand. To see why, note first that the distinction between exclusive (single-homing) and non-exclusive (multi-homing) consumers is utterly important for ad-financed platforms. When consumers single-home, the platform acts as a gatekeeper: Having exclusive access to certain consumers implies that advertisers cannot reach them elsewhere, allowing the platforms to price their ad space accordingly. Consumers that are shared with other platforms, on the other hand, are typically worth less in the ad market. Since the advertisers can reach

[4]Kox *et al.* (2017) explicitly examine targeting, while Crampes *et al.* (2009) consider a more general advertising technology.

[5]This has caught the attention of a number of researchers, such as Ambrus *et al.* (2016), Athey *et al.* (2018), Anderson *et al.* (2017; 2018; 2019).

[6]Gentzkow and Shapiro (2011), Gentzkow *et al.* (2014) and Affeldt *et al.* (2019) provide additional arguments for why digital technologies make multi-homing more compelling. For example, Gentzkow et al. (2014) show that 86 percent of the circulation of entrants to the US newspaper market comes from households reading multiple newspapers. Furthermore, Liu *et al.* (2021) provide a number of examples of industries where multi-homing becomes more prominent.

these consumers on other platforms as well, the platforms can only charge advertisers the incremental value of an additional impression. This is known as the incremental pricing principle (see e.g., Ambrus *et al.*, 2016; Athey *et al.*, 2018 and Anderson *et al.*, 2018), and implies that since a platform might, by changing its own price, affect the composition of the rival's demand, the platform can thereby also affect the value the rival can obtain in the ad market of its consumer mass. That changes the strategic interaction.

To analyze formally the impact of targeting for a digital platform's business model, we implement targeting into a two-sided framework. We set up a three-stage model, in which the platforms decide whether to target at the first stage and choose subscription and advertising prices at the second stage. At the final stage, the consumers decide on how many subscriptions to buy (i.e. to single-home or multi-home). We search for a subgame-perfect Nash equilibrium. We incorporate that the performance of targeting technology is increasing in the amount of data by imposing a targeting technology with increasing returns to scale in the audience size. Targeting thus increases the importance of attracting consumers.

In line with existing literature, we find that targeting generates a prisoner's dilemma situation if consumers single-home. But remarkably, we find that platforms may not want to set subscription prices that makes consumers prefer single-homing. Indeed, setting prices that incentivize consumers to multi-home could be a unique equilibrium.

Combining elements from Crampes *et al.* (2009), Ambrus *et al.* (2016) and Anderson *et al.* (2017), we show that things turn out quite differently if consumers multi-home. It is known from the literature (Kim and Serfes, 2006; Anderson et al, 2017) that the incremental pricing principle implies that prices are strategically independent if consumers multi-home: if one platform changes its price, this has no impact on rival platforms' optimal price setting. Our results confirm that result in the absence of targeting. To put it more concretely: suppose that you are going to purchase *The Washington Post* and consider to buy a copy of *The New York Times* as well. When deciding whether to purchase *The New York Times* as an additional newspaper, what matters for the consumer is the price of *The New York Times* (and not *The Washington Post*). Prices are therefore strategically independent.

In this paper, we introduce targeting, and find that this has surprising consequences for the strategic interaction among digital platforms. In particular, the strategic dependence is restored, but interestingly, we find that prices become strategic substitutes: it is less profitable for a platform to reduce its subscription price if rival platforms do the same. To understand the intuition behind this result, we must realize that although the consumers' purchasing decision does not change when we introduce targeting, the platforms' considerations do. For the consumer, the price of *The Washington Post* is still irrelevant when considering the incremental utility of purchasing *The New York Times* in addition to *The Washington Post*. However, when platforms target ads, the platforms must now take into account that the price setting of rival platforms will affect the profitability of targeting. If we revert to our previous example: By reducing its subscription price, *The New York Times* could improve its targeting ability and charge advertisers extra. Since advertisers are not willing to pay the full extra for non-exclusive consumers (recall the incremental pricing principle), this would be more attractive the larger the number of exclusive consumers. A price reduction by *The Washington Post* would, however, increase the number of consumers that buy *The Washington Post* in addition to *The New York Times*. *The New York Times*'s gain from increased targeting ability would therefore be counteracted by a greater fraction of non-exclusive consumers. Therefore, the ad market becomes relatively less important for *The New York Times*, and they will raise subscription prices instead (as a response to *The Washington Post*'s price reduction). Hence, subscription prices become strategic substitutes.

Although offering targeted advertising still makes it optimal for the platform to reduce subscription prices when consumers multi-home, it does not trigger an aggressive response from rival platforms. As a result, it is more imperative to implement targeting. Yet, softer competition alone cannot ensure that targeting is profitable. We show that this can only be guaranteed if multi-homing consumers are sufficiently valuable to advertisers.

Finally, we model media platforms that raise revenue from selling both subscriptions and advertising space. The growth of data-driven, targeted advertising has changed completely the media – and in particular the newspaper – industry. This has happened for two reasons. First, there has been a shift from printed to digital advertising; worldwide, online ad expenditure has increased by more than five times since 2010, whereas printed advertising has fallen by one third (Wood, 2020). Second, the ad market has become global; local newspapers face fierce competition from global media platforms, such as Facebook and Google, which has had as consequence that local newspaper publishers have been unable to recoup the lost printed ad revenues with digital ads. To illustrate, total advertising revenue in the US newspaper industry fell from $26 billion in 2010 to $16 billion in 2017 (Pew Research Center, 2018). Consequently, local newspaper publishers have lost a substantial share of their ad revenue, and become instead more dependent on raising subscription revenues – in addition to being financed by advertising.[7] There is simply not enough revenue potential at either side of the market.

Outline. The paper proceeds as follows. In section 2, we review related literature. In section 3, we present the basic model and introduce a targeting technology, before we compare our results to disclose when targeting is profitable. In section 4, we compare potential equilibria, and search for the Nash equilibrium. We conclude in section 5.

2 Related literature

This paper draws on two strands of the media literature that are not usually brought together. One strand investigates the impact of targeted advertising, and the other examines the importance of consumer multi-homing.

Athey and Gans (2010) and Bergemann and Bonatti (2011) were among the first to address the impact of targeting on media platform competition. The former paper considers competition between a local platform that is tailored to the local audience (which is the local advertiser's intended audience) and a general platform that depends on targeting technologies to identify the advertiser's relevant consumer base, whereas Bergemann and Bonatti (2011) model competition between online and offline media under the assumption that online media has higher targeting ability. A common feature of both papers is that platform differences are exogenously given. This gives rise to significant effects on the supply and demand of ads, which would be less prominent in a model with symmetric platforms (like ours). In this paper, we disregard the allocative effects, and allow the targeting ability to be determined within the model: by reducing its subscription price, a platform can increase its audience size and improve its targeting ability. Since none of the mentioned papers regards subscription fees, a similar interplay between the two sides

[7]For example, The *New York Times* successfully shifted to a digital subscription paywall strategy in 2011, see Chung et al. (2019) who provide an overview of the development of digital paywalls.

of the market does not occur in these papers. This is one explanation of why we arrive at quite different results.

Another explanation is related to the targeting technology specification itself. In the notion of Hagiu and Wright (2020), we implement a targeting function of across-user data learning. The more data the technology is fed, the better does the targeting technology perform. A general form of our targeting specification can be recognized in Crampes *et al.* (2009), who demonstrate that the nature of the advertising technology is decisive for platform behavior and market outcomes. They model the impact of advertising technologies with constant, decreasing and increasing returns to scale in the audience size, and point at the limitation of assuming linearity. Although the authors do not accentuate increasing returns to scale, we argue that the current focus on first-party data and technology makes this particular specification highly relevant. We therefore use a variant of this technology in our set-up. Like most previous research on targeting and media platform competition, Crampes *et al.* (2009) assume consumer single-homing. We relax this assumption, and show that by allowing for multi-homing, we obtain entirely different outcomes.

This leads us to the next point: This paper adds to the growing literature on consumer multi-homing. Seminal works on multi-homing (Ambrus *et al.*, 2016, and Anderson *et al.*, 2018) have shown that multi-homing strongly affects the results in two-sided market analyses, and find that prices are strategically independent. A key take away from existing research is the incremental pricing principle that we describe in the Introduction. The incremental pricing principle follows from the insight that multi-homing consumers are less worth than single-homers. As pointed out by Athey *et al.* (2018), impressing the same consumer twice is less efficient than impressing two different consumers. Ambrus *et al.* (2016) emphasize that an implication of advertisers' lower valuation of multi-homing consumers is that it is not only the overall demand that counts; the *composition* of the demand also matters. When advertisers place ads on platforms with multi-homing consumers, there is a risk that some consumers have seen the ad before. We share this insight of consumer multi-homing and ad-financed platform markets, and combine this with elements from the user-financed platform market in Anderson *et al.* (2017) to derive a two-sided model with dual source financing. We find that by implementing a targeting technology that has increasing returns to scale in audience size, a change in the composition of the demand changes also the value of the consumer mass that a platform can collect in the ad market. Therefore, targeting restores a strategic dependence, and prices become strategic substitutes.

The main mechanism for our result in this paper is a strategic substitutability of subscriptions prices, and thereby of business model (i.e. financing source), when multi-homing takes place. A similar result is found by Calvano and Polo (2020). They propose a model with competing broadcasters and endogenous differentiation in business models. They find an asymmetric equilibrium, in which one platform is fully financed by ads, and the other by subscriptions. The mechanisms at play are similar to our model. Also we find that it is optimal for one platform to move towards subscription financing if the other platform moves towards ad financing. However, due to our targeting specification, the ad price does not necessarily decrease with the number of non-exclusive consumers. Attracting more consumers increases the ad price. Then, we do not get the asymmetric equilibrium of Calvano and Polo (2020). Moreover, as Calvano and Polo stress, there must be a revenue potential at both sides of the market in order to end up in the asymmetric equilibrium, which we argued in the final paragraph of the introduction does not hold for the types of media platform markets that are scope for our analysis.

In recent works, Haan *et al.* (2021) and Athey and Scott Morton (2021) argue that platforms can benefit from engaging in strategies that lead to head-on competition for consumers, e.g. by stimulating to less consumer multi-homing behavior, in order to create more market power in the ad market, and deprive rivals of scale economies and network effects in the long run. We, on the contrary, find that the platforms instead do want to attract more consumers (e.g. by attracting more multi-homers), because the multi-homers, although less valuable in the ad market, are vital for the ability to target (as it increases the total audience).

Although various papers assess different aspects of consumer multi-homing, the literature that integrates multi-homing with targeted advertising is scarce. There are, however, a few exceptions. Taylor (2012) investigates how targeting affects platforms' incentives to improve content in order to increase their share of consumer attention. The paper focuses on how the platforms can retain consumer attention. In contrast, we disregard the attention span of the audience and rather focus on its size. Another exception is D'Annunzio and Russo (2020), who study the role of ad networks and how tracking technologies affect market outcomes. However, since they focus on a different part of the industry (ad networks), they address other and complementary questions.

Finally, this paper also relates to current discussions on privacy. The use of consumer data to target ads has raised privacy concerns. Johnson (2013) stresses that targeting might be harmful when consumers value privacy. He investigates the impact of targeting when consumers have access to ad-avoidance tools, and shows that consumers tend to block too few ads in equilibrium. Kox *et al.* (2017) incorporate privacy considerations in a work that is closer to ours. In a similar framework, the authors show that targeting reduces consumer welfare if the disutility of sharing personal information is greater than the advantage of lower subscription prices. Recall from the Introduction that Kox *et al.* also find that platform profits decrease in targeting. As a result, their model suggests that stricter privacy regulations benefit both consumers and platforms. An important difference between Kox *et al.* (2017) and this paper is that the former assumes a linear advertising technology and consumer single-homing.

More recently, Gong *et al.* (2019) propose a different approach in which competition for consumers plays a prominent role. In their model, differences in the platforms' ability to target ads are exogenously given.[8] Assuming that consumers dislike irrelevant ads, Gong *et al.* suggest that improved targeting reduces the consumers' nuisance costs. At the same time, greater targeting ability attracts more advertisers. Hence, platforms with superior targeting abilities attract more consumers and advertisers, and they are more profitable.

3 The model

We consider two media platforms that offer subscriptions to consumers and advertising space (eyeballs) to advertisers. We employ a simple Hotelling (1929) model, with a line of length *one*, and assign one platform to each endpoint, i.e., platform 1 is located at $x_1 = 0$ and platform 2 is located at $x_2 = 1$. Along the line, there is a unit mass of uniformly distributed consumers. The distribution represents the consumers' taste: the greater distance to a platform, the greater mismatch between the consumer preferences and the platform characteristics.

We consider two different homing regimes (which we later analyze whether constitute

[8] In an extension, Gong *et al.* (2019) allow the platforms to invest in targeting ability, and show that under-investment is most likely to occur.

Nash equilibria). First, in the pure single-homing regime (hereafter referred to as the single-homing regime) all consumers subscribe to only one platform. Second, we consider a multi-homing regime where some (but not all) consumers use more than one platform.[9] In addition, we also consider two different advertising technologies; either the platform can acquire a targeting technology that can target consumers, or the platform does not acquire any targeting technology.

Timing. The timing is as follows:
Stage 1: Each platform decides whether to acquire a targeting technology or not.
Stage 2: Each platform simultaneously announces its advertising and subscription prices, and compete for advertisers and consumers.
Stage 3: Consumers decide on how many subscriptions to buy (i.e. whether to single- or multi-home).

We are looking for a subgame-perfect Nash equilibrium.

3.1 Consumer demand

Consumers choose endogenously how many platforms to join. A single-homing consumer joins only the one platform she prefer the most. Let u_i represent the utility a consumer located at x obtains from subscribing to platform $i = 1, 2$:

$$u_i = \ v - t|x - x_i| - p_i. \tag{1}$$

The parameter $v > 0$ is the intrinsic utility of joining a platform, $t > 0$ represents the disutility of the mismatch between the consumer's preferences and the platform's characteristics, and p_i is the subscription price.

We do, however, allow consumers to subscribe to both platforms (i.e. multi-homing). The utility from dual subscription equals the sum of the individual utilities:[10]

[9] If all consumers multi-home, targeting would neither affect demand nor subscription prices. In this case, the analysis simply boils down to the change in ad prices.

[10] If a consumer has two subscriptions, the total utility is not necessarily the entire sum of the individual utilities. For example, there is often some content overlap if the consumer reads two newspapers. We can therefore reformulate $u_{12} = (1 + \theta)v - t - p_1 - p_2$, where $\theta \in (0, 1)$. This will put stricter restrictions on t (see sections 3.5 and 4), but our results of profitability and on the Nash equilibrium are robust to this overlap. For the ease of reading, we will therefore assume that dual subscription simply equals the sum of the individual utilities.

$$u_{12} = 2v - t - p_1 - p_2. \tag{2}$$

If the incremental utility of multi-homing is positive for some consumers, $u_{12}(x) \geq u_i(x)$, those consumers will subscribe to both platforms. Hence, each platform potentially serves two groups of consumers: exclusive subscribers and non-exclusive subscribers, who are shared with the rival platform.

Let x_{12} represent the location of the consumer who is indifferent between subscribing to just platform 1 and subscribing to both platform 1 and platform 2.[11] Since platform 2 does not provide any additional utility to the indifferent consumer, $u_{12} = u_1$. Platform 1's exclusive demand then arises from the consumers who are located to the left of x_{12}. It follows that the platforms' non-exclusive demand is made up by the consumers located between x_{12} and x_{21}. This is illustrated in Figure 1.

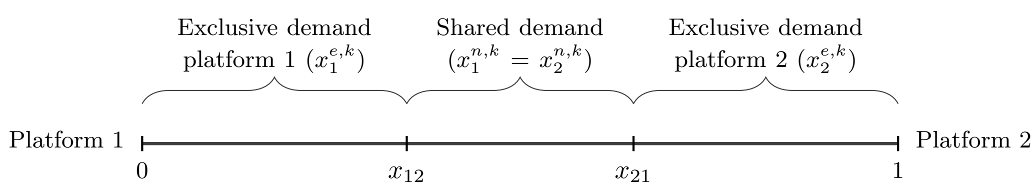

Figure 1: Demand platform $i = 1, 2$.

We solve $u_{12} = u_1$ and $u_{12} = u_2$ and find $x_{12} = \frac{1}{t}\left(-v + t + p_2\right)$ and $x_{21} = \frac{1}{t}(v - p_1)$, respectively. With symmetric platforms, we get that platform i's exclusive demand (superscript 'e' for exclusive and 'M' for multi-homing regime) is given by

$$x_i^{e,M} = \frac{-v + t + p_j}{t}, \tag{3}$$

whereas its non-exclusive demand (superscript 'n' for non-exclusive and 'M' for multi-homing regime) equals

$$x_i^{n,M} = \frac{2v - t - p_i - p_j}{t}. \tag{4}$$

Total demand is the sum of exclusive and non-exclusive demand:

[11] Vice versa, x_{21} represents the location of the consumer that is indifferent between subscribing to only platform 2 and both platform 2 and platform 1.

$$D_i^M = x_i^{e,M} + x_i^{n,M} = \frac{v - p_i}{t}. \tag{5}$$

Equation (5) tells us that total demand for platform i is independent of the rival platform's subscription price (p_j). A change in p_j will, however, affect the composition of platform i's demand. From equation (3), we see that the number of exclusive subscribers is increasing in p_j, while equation (4) shows an inverse relationship between the number of non-exclusive subscribers and p_j.

If all consumers join only one platform, i.e. single-home, then $x_i^{n,S} = 0$. The consumer that is indifferent between only subscribing to platform 1 and only subscribing to platform 2 is located at $\widetilde{x}$, where $u_1 = u_2$. Consumers to the left of $\widetilde{x}$ subscribe to platform 1 and consumers to the right subscribe to platform 2. Hence, the demand function (superscript 'S' for single-homing regime) equals:

$$D_i^S = x_i^{e,S} = \frac{1}{2} + \frac{p_j - p_i}{2t}. \tag{6}$$

3.2 Advertisers and Platforms

Turning to the advertising side, we normalize the number of advertisers to *one*. The demand for ads is perfectly elastic, and we assume that each advertiser purchase space for one ad per platform. We consider thus a single advertiser, which needs to advertise to consumers in order to sell its product or service. For each unit sale, the advertiser earns some fixed payoff, which we normalize to one. No consumer wants to buy more than one unit, but must observe an ad from this adveriser before the consumer will purchase the product. With probability $\alpha \leq 1$, a consumer will observe the ad she is exposed to. Given these assumptions, the advertiser's willingness to pay to advertise to an exclusive consumer is α. In line with the principle of incremental pricing (see Anderson *et al.*, 2018), we assume that the advertisers are willing to pay α_i to reach an exclusive consumer on platform i, but only a fraction $\sigma\alpha_i$ to reach a non-exclusive consumer on platform i, conditional on (already) advertising to this consumer at platform j, where $\sigma \in (0,1)$. The set-up corresponds to Anderson *et al.* (2018).

Remark 1 *Our specification of $\sigma \in (0,1)$ follows the idea of Athey et al. (2018) that there are decreasing returns to scale to duplicate ad impressions on the same consumer. This is* *not to say that the optimal number of displays of an ad is one. As pointed out by Athey et al. (2018), Yuan et al. (2013) found that the optimal number of showing an ad online lies somewhere between three and eight times, depending on the campaign. To put this insight into our model, one could think of each advertiser buying one advertising package, which consists of the optimal number of displays, say four displays, at the same platform. We normalize this number of displays in an ad package to one ad. Instead, what* $\sigma \in (0,1)$ *reflects is the market power of accessing consumers. If an advertiser wants to impress a consumer, but only can reach this particular consumer through platform A, then platform A would (due to its market power) charge the advertiser a higher ad price than in the case where platforms A and B can both impress that very consumer and compete over attracting the advertiser. In an empirical study of US magazines, Shi (2016) finds that non-exclusive consumers are about half as valuable as exclusive consumers, i.e.* $\sigma = \frac{1}{2}$. *This estimate indicates that it is reasonable to assume that platforms charge less for consumers that can be reached elsewhere.*

Retargeting across platforms (i.e. $\sigma > 1$*) is only relevant if the advertiser could track the consumer (through a third-party cookie) from one platform to another. Then, the advertiser needs to advertise through an independent ad network service. This is not the scope of our analysis. We want to shed light on the strategy of media platforms who build their business model on targeting based on data about their own consumers, i.e. on first-party data (being their own ad network).*

We consider a game in which each platform simultaneously announces its advertising prices, and that the platforms can charge different advertising prices depending on whether a consumer is exclusive or shared. It follows that platform i's ad revenue can be defined as

$$A_i^k = \alpha_i^k x_i^{e,k} + \sigma \alpha_i^k x_i^{n,k}, \tag{7}$$

where superscripts $k = \{S, M\}$ denote the single-homing regime and the multi-homing regime, respectively.

Total profit is given by[12]

$$\pi_i^k = p_i^k D_i^k + \alpha_i^k \left(x_i^{e,k} + \sigma x_i^{n,k} \right). \tag{8}$$

[12] We set all costs to zero to simplify the model.

3.3 No targeting

Consider first a model without targeting. In this situation, we assume that the advertiser value of reaching a consumer is not platform dependent, such that $\alpha_i^k = \alpha_j^k = \alpha$. We differentiate equation (8) and find the first-order condition

$$\frac{\partial \pi_i^k}{\partial p_i^k} = \left[D_i^k + \frac{\partial D_i^k}{\partial p_i^k} p_i^k\right] + \left[\alpha\left(\frac{\partial x_i^{e,k}}{\partial p_i^k} + \sigma \frac{\partial x_i^{n,k}}{\partial p_i^k}\right)\right] = 0. \tag{9}$$

The first square bracket on the right-hand side of equation (9) deals with the consumer side of the market, corresponding to a standard one-sided model. If we consider an increase in p_i, this implies that each consumer pays more, but it also means a lower number of subscribers. In our two-sided model, the price increase has an impact on the ad side of the market as well: platform i displays fewer ads and thereby loses ad revenues. This is captured by the second square bracket. Because of the negative effect a price increase has on ad revenues, the optimal subscription price is lower in a two-sided model.

We next investigate the strategic interaction of the subscription prices, and find

Lemma 1 *(No targeting) Subscription prices are*

(i) strategic complements in the single-homing regime

(ii) strategically independent in the multi-homing regime

Proof. Solving equation (9) for p_i^k gives the best-response functions:

$$p_i^M(p_j) = \frac{v - \sigma\alpha}{2} \text{ and } p_i^S(p_j) = \frac{t + p_j - \alpha}{2}. \tag{10}$$

Taking the partial derivatives of the best-response functions in (10) with respect to p_j, it follows directly that $\partial p_i^S(p_j)/\partial p_j > 0$ and $\partial p_i^M(p_j)/\partial p_j = 0$. ■

It follows from equation (10) that subscription prices are strategically independent in the multi-homing regime. In other words, platform i's subscription price is not responsive to changes in platform j's subscription price. This result is in line with seminal work on multi-homing (Ambrus et al., 2016, Anderson et al., 2018), which have demonstrated that multi-homing strongly affects the results in two-sided markets. To see why, suppose that platform j adjusts p_j. From section 3.1, we know that even though it alters the number of exclusive and non-exclusive consumers, the price change has no effect on platform i's total demand. This is because the location of platform i's marginal consumer stays the same. Keep in mind that the marginal consumer is located where her incremental value of subscribing to platform i is zero. Hence, platform i's subscription price still extracts the marginal consumer's incremental benefit. Besides, platform i's price setting does not affect the advertisers' valuation of the marginal consumer. Consequently, platform i has no incentive to change its subscription price in response to an adjustment in p_j. In the single-homing regime, we get the standard result that prices are strategic complements.

3.4 Introducing targeting

Next, we introduce targeting to our model. We recognize that advertisers may not only care about the reach of ads, but also about the quality of the match with the audience. Suppose that the platforms implement targeting technologies that enable them to create better matches between advertisers and viewers. We assume that advertisers are willing to pay for improvements in the platforms' targeting ability, and formulate the ad price as follows:

$$\alpha_i^k = \alpha(1 + \mathbb{1} D_i^k), \tag{11}$$

where $\mathbb{1}$ is a dummy that takes on the value *one* when targeting is included in the model and *zero* otherwise. Notice that in the latter case, equation (11) reverts to the non-targeting ad price (α). For $\mathbb{1}$ equal to *one*, the definition implies that the ad price is increasing in the platform's audience size ($\frac{\partial \alpha_i^k}{\partial D_i^k} > 0$), capturing the benefit of having more consumer data on the platform, and, hence, improved targeting ability.

The targeting function in equation (11) holds two important properties: (1) across-user data-enabled learning, and (2) first-party data. Using the notion of Hagiu and Wright (2020), aross-user data-enabled learning implies that the targeting ability increases with the total input of data, i.e. the total number of users the platform i attracts (D_i^k).[13,14] For example, the platform can divide consumers into groups of age, gender, or other characteristics, and target a user based not only on the data it holds on that specific user, but also based on information about other users. Thus, the technology becomes more accurate as the platforms increase their audience size and thereby generate more data.

[13] We assume that each consumer delivers one data point, such that we measure the amount of data by the number of consumers.

[14] See Hagiu and Wright (2020) for further discussions on the different types of data-enabled learning.

Moreover, our targeting specification also captures recent development of privacy regulation. In particular, concerns of abuse of third-party data has resulted in new privacy regulations, such as the GDPR in the EU, and corresponding privacy acts in several US states (e.g. California Consumer Privacy Act, CCPA). Third-party data are typically gathered by 'cookies', which consumers come across everywhere online. Via third-party cookies, the platform does not need to own the data themselves, but can buy access to them. However, the increasingly more stringent privacy regulation limits the scope of third-party cookies. To illustrate, online newspaper publishers, such as *Vox Media*, *The New York Times* and *The Washington Post*, have all removed third-party cookies from their platform, and build their own first-party data based advertising technologies (Davies, 2019; Fischer, 2019; 2020).[15] This means that the platform must own the data it uses for targeting purposes.

Inserting equation (11) into equation (8), and differentiating with respect to own price, we find the new first-order condition:

$$\frac{\partial \pi_i^k}{\partial p_i^k} = D_i^k + \frac{\partial D_i^k}{\partial p_i^k} p_i^k + \alpha(1 + \mathbb{1} D_i^k) \left(\frac{\partial x_i^{e,k}}{\partial p_i^k} + \sigma \frac{\partial x_i^{n,k}}{\partial p_i^k} \right) + \mathbb{1} \frac{\partial \alpha_i^k}{\partial p_i^k} \left(x_i^{e,k} + \sigma x_i^{n,k} \right) = 0. \quad (12)$$

When $\mathbb{1}$ equals *zero*, we recognize equation (12) as the first-order condition in the model without targeting (cf. equation (9)). The two additional terms that appear when $\mathbb{1}$ equals *one* represent the effects that emerge when we incorporate targeting. First, consider the third term on the right hand side. It tells us that ad revenues are more sensitive to changes in the number of ad impressions (in response to a change in the subscription price) than without targeting.[16] The explanation is that the ad price, which corresponds to the first part of the third term (cf. equation (11)), is higher with targeting ($\mathbb{1} = 1$). Second, we evaluate the fourth term. This expression captures a property that is not present in the model without targeting, namely that a platform's ad price responds to changes in its own subscription price. An increase in p_i^k causes a reduction in α_i^k, and vice versa.

Solving equation (12) for p_i^k , we find the best-response functions:

[15] Moreover, Apple's web browser, Safari, and Mozilla's Firefox has already disabled third-party cookies, whereas Google plans to replace their cookies with an aggregated 'Sandbox tool', containing anonymous, aggregated consumer data, by the end of 2022 (O'Reilly, 2020).

[16] Since each consumer is impressed once, the number of subscribers is equivalent to the number of ad impressions.

$$p_i^M(p_j) = \frac{v(t+\alpha) - \alpha(t+3v\sigma) - \alpha p_j(1-\sigma)}{2(t-\alpha\sigma)} \text{ and } p_i^S(p_j) = \frac{t(t-2\alpha) + p_j(t-\alpha)}{2t-\alpha}. \quad (13)$$

The best-response functions reveal a striking difference between the single-homing regime and the multi-homing regime. If all consumers single-home, subscription prices are strategic complements ($dp_i^S/dp_j > 0$). In contrast, if at least some consumers multi-home, subscription prices are strategic substitutes ($dp_i^M/dp_j < 0$). This means that the optimal response to changes in the rival platform's subscription price depends on whether consumers only single-home or if some of them multi-home.

We can summarize the above discussion in the following proposition:

Proposition 1 *(Targeting) When platforms target ads, subscription prices are*

(i) strategic complements in the single-homing regime

(ii) strategic substitutes in the multi-homing regime.

The first result in Proposition 1 is well known in the literature (see e.g. Crampes *et al.*, 2009; Kox *et al.*, 2017): in a single-homing regime, the best response to a change in the rival subscription price is to adjust own price in the same direction.

The second result in Proposition 1, however, is quite surprising. While platform i's best response to a change in the rival subscription price is to do nothing in the multi-homing model without targeting (cf. the second result of Lemma 1), the best response in the targeting model is to adjust p_i^M in the opposite direction. Since targeting does not change the property of total demand being independent of the rival subscription price, the difference between the models may not be straightforward intuitive. After all, this property implies that p_i^M extracts the marginal consumer's incremental benefit regardless of any changes in p_j^M. The key to understanding why a change in p_j^M still induces a response, is that targeting enables platform i to affect the advertisers' willingness to pay. To see why, suppose that platform j increases p_j^M. This creates a shift from non-exclusive to exclusive subscribers for platform i, which implies a smaller share of discounted ad impressions. Platform i would therefore gain from increasing its ad price. Targeting enables the platform to do so by reducing p_i^M and improving its targeting ability. Conversely, a reduction in p_j^M provides incentives to increase p_i^M.

3.5 When is targeting profitable?

In this section, we compare the outcomes with and without targeting, and reveal when targeting is profitable. First, we find the symmetric non-targeting equilibrium prices. Solving the best-response functions in equation (10) simultaneously, we have

$$p^{M,NT} = \frac{v - \sigma\alpha}{2} \text{ and } p^{S,NT} = t - \alpha. \tag{14}$$

where superscript 'NT' denotes the non-targeting equilibrium.

We then find the symmetric targeting equilibrium prices (superscript 'T' denotes the targeting equilibrium) by solving the best-response functions in equation (13) simultaneously:

$$p^{M,T} = \frac{v\,(t+\alpha) - \alpha\,(t + 3v\sigma)}{2t + \alpha(1 - 3\sigma)} \text{ and } p^{S,T} = t - 2\alpha. \tag{15}$$

Comparing equations (14) and (15), lets us state the following:

Lemma 2 *Subscription prices will be lower when platforms use targeting technologies.*

Proof. See Appendix. ■

We observe that subscription prices are lower when platforms target ads, irrespective of whether all consumers single-home or if some multi-home.

Targeting provides greater incentives to attract a larger audience, and to do so, the platforms lower their subscription prices. The effects of this price reduction for profits depends on the strategic response that follows in each regime. We first analyze the single-homing regime, then proceed to the multi-homing regime.

3.5.1 Single-homing

We restrict our attention to markets with full coverage and endogenously non-negative prices. This, as well as fulfillment of the stability and second-order conditions, is ensured by Condition 1:

Condition 1 *(Single-homing)* $\frac{5}{2}\alpha < t < \frac{2}{3}(v + \alpha)$.

It follows from Lemma 2 and Proposition 1 that targeting leads to fiercer price competition when all consumers single-home. The symmetric equilibrium demand is equivalent with and without targeting:

$$D^{S,T} = D^{S,NT} = \frac{1}{2}. \tag{16}$$

It follows that subscription revenues are lower with targeting. Even though ad revenues are higher, they do not fully compensate for the lost subscription revenues. Inserting (16), (15) and (14) into (8), we find the equilibrium profits with and without targeting, respectively:

$$\pi^{S,T} = \frac{1}{4}(2t - \alpha) \text{ and } \pi^{S,NT} = \frac{1}{2}t. \tag{17}$$

Equation (17) shows clearly that the targeting profit is lower than the non-targeting profit and decreasing in the technology's sensitivity to more data. The reason is that the higher α, the greater the incentive to reduce the subscription price, which significantly reduces subscription revenues. This raises the question of whether the platforms at all wish to adopt targeting technologies. Although it is in the platforms' common interest not to target, each platform has incentives to deviate from the mutually beneficial strategy. The platforms might therefore end up in a prisoner's dilemma situation where all platforms target (see also Kox *et al.*, 2017).

We state:

Lemma 3 *(Prisoner's dilemma) When all consumers single-home, targeting is a dominant strategy and the platforms end up in a prisoner's dilemma.*

Proof. See Appendix. ■

As we demonstrate in the equilibrium analysis, the platforms could, however, be better off by setting the multi-homing price and also attract consumers who already subscribe to the rival platform.

3.5.2 Multi-homing

Assume now consumer multi-homing. The consumer chooses independently which platform(s) it will subscribe to, facing the prices from each platform. We consider partial multi-homing, i.e. situations where some, but not all, consumers use both platforms. Note that $t > \frac{1}{2}(v + 3\sigma\alpha)$ and $t < v + \sigma\alpha$ ensure the existence of exclusive and non-exclusive consumers, respectively. Moreover, we confine the analysis to situations with endogenously non-negative subscription prices and parameter values that satisfy all second-order and stability constraints. The conditions are given in the Appendix.

Proposition 2 *(Multi-homing). Suppose that the multi-homing conditions hold. Targeting is profitable in the multi-homing equilibrium if advertisers place a high enough value on non-exclusive consumers. Targeting is then a dominant strategy. A sufficient condition is* $\sigma > \frac{1}{3}$.

Proof. See Appendix. ■

Proposition 2 contains two important results: First, we argue that targeting is a dominant strategy. The platforms will always have an incentive to target ads, irrespective of the strategic action of the rival platform. The second result might prevent the platforms from ending up in a prisoner's dilemma situation (as in the single-homing regime). The first thing to note, is that it follows from Lemma 2 and Proposition 1 that targeting provides incentives to reduce the subscription price, and that the rival platform will respond favorably. Moreover, the incentive to lower the price increases with advertisers' willingness to pay for non-exclusive consumers. This is captured in our model by the σ-parameter, where $\partial p^{M,T}/\partial\sigma < 0$. Despite that the price reduction contributes to greater overall demand, and the increase is reinforced by the rival platform's response, we nonetheless find that equilibrium subscription revenues are lower with targeting ($p^{M,T}D^{M,T} < p^{M,NT}D^{M,NT}$).

For targeting to be profitable, two conditions must therefore be satisfied: (i) Ad revenues must increase with targeting; and (ii) the increase in ad revenues must be greater than the loss in subscription revenues. Comparing ad revenues with and without targeting, we find that ad revenues are greater with targeting if $\sigma > \frac{1}{3}$. However, if $\sigma \leq \frac{1}{3}$, that is not necessarily true. The smaller σ, the lower is the ad price the platforms can charge for impressing non-exclusive consumers. This is particularly harmful in combination with weak platform preferences (low t), because targeting then creates a greater shift from exclusive consumers to non-exclusive consumers. A larger proportion of less valuable non-exclusive consumers could, in this case, offset the advantage of an increased ad price.

Combining Lemma 3 and Proposition 2, gives us the following corollary:

Corollary 1 *Targeting can only be profitable in equilibrium if at least some consumers multi-home.*

4 Equilibrium analysis

4.1 The targeting decision

From Lemma 3 and Proposition 2, we know that targeting is a dominant strategy irrespective of homing regime for $\sigma > 1/3$. If, however, $\sigma \leq 1/3$, then platforms might prefer not to target (NT). However, when the platforms do not target, the consumers prefer to single-home, in which case it follows from Lemma 3 that targeting is the dominant strategy. Hence, we propose:

Proposition 3 *(The targeting decision) Suppose $\sigma \leq 1/3$. Platforms abstaining from targeting (NT) cannot be part of the equilibrium.*

Proof. See Appendix. ■

From Proposition 3, we can conclude that a potential Nash equilibrium must be one in which the platforms target. We therefore investigate the targeting equilibrium outcomes, searching for the Nash equilibrium.

4.2 Comparison of targeting equilibrium outcomes

We now proceed to comparing the market outcomes with pure single-homing and multi-homing and examining the existence of Nash equilibria. In this part, we restrict our attention to parameter values that fulfill the conditions for both the single-homing model and the multi-homing model. From Condition 1, we have that this requires that $v > \frac{11}{4}\alpha$. To illustrate the key point, we set $v = 3\alpha$, which is close to the minimum $v-$value. In the Robustness section in the Appendix, we show that the results we arrive at are valid also for $v > 3\alpha$, at least if non-exclusive consumers are not virtually worthless to advertisers.

Condition 2 ensures partial multi-homing in the multi-homing regime, non-negative prices and full market coverage in the single-homing regime, in addition to satisfying second-order and stability conditions.

Condition 2 *(Equilibrium)* $\max\{\frac{5}{2}\alpha,\ \frac{3}{2}\alpha\,(\sigma+1)\} < t < \frac{10}{3}\alpha.$

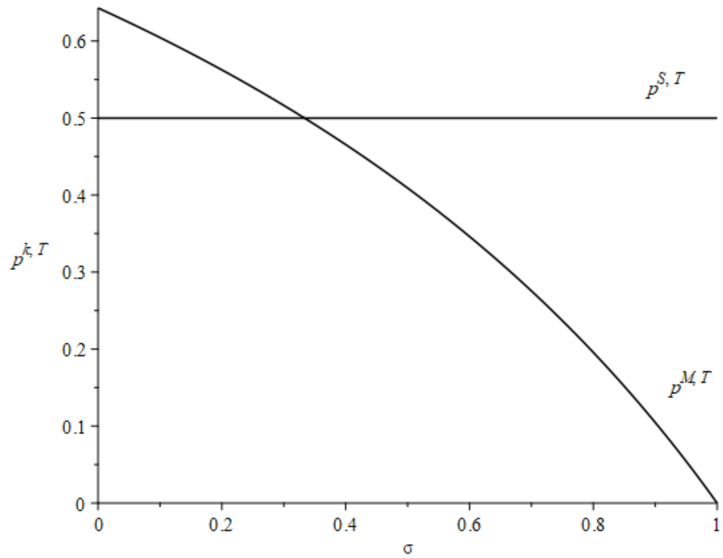

Figure 2: Equilibrium prices.

Comparing the subscription prices in equation (15), we find that $p^{S,T} \geq p^{M,T}$ for $\sigma > \frac{2}{3}$. For lower σ, the single-homing price may be both greater and smaller than the multi-homing price, as illustrated in Figure 2 (parameter values: $t = 3\alpha$ and $\alpha = \frac{1}{2}$).

When σ is low, the platforms have weaker incentives to reduce the multi-homing price. However, the higher t, the greater price reduction is required to persuade consumers to multi-home. Hence, if t is sufficiently high (the condition is given in the Appendix), the multi-homing price could still be lower than the single-homing price. Conversely, a higher σ (corresponding to non-exclusive consumers being more valuable) provides stronger incentives to reduce subscription prices in the multi-homing regime. This is why we observe that $p^{M,T}$ decreases in σ, both in absolute value and relative to $p^{S,T}$.

Turning to advertising prices, we find that these are always lower with single-homing ($\alpha^{S,T} < \alpha^{M,T}$). Finally, we consider profits. We find that if $\sigma \geq 0.65$, single-homing profits cannot be greater than multi-homing profits ($\pi^{S,T} < \pi^{M,T}$). For $\sigma < 0.65$, however, profits may or may not be greater with single-homing. A sufficiently high t can ensure that single-homing makes the platforms better off. This is illustrated by Figure 3 (parameter values: $t = 3.3\alpha$ and $\alpha = \frac{1}{2}$).[17]

From the analysis of subscription prices, we know that consumers who subscribe to only

[17] We use different sets of parameter values in the two figures because it enables us to demonstrate that prices and profits can be both higher and lower with single-homing compared to multi-homing.

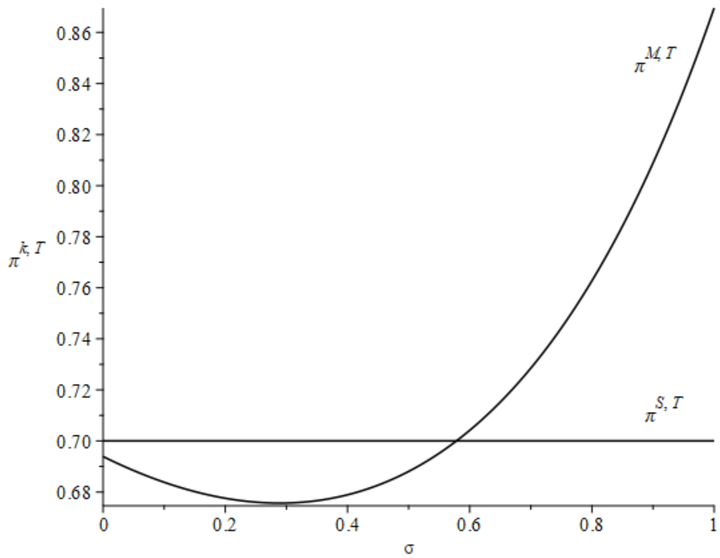

Figure 3: Equilibrium profits.

one platform are better off in a multi-homing regime when $\sigma > \frac{2}{3}$, since $p^{S,T} \geq p^{M,T}$.

Moreover, we find that at least some consumers prefer multi-homing over single-homing if $\sigma > \frac{2}{9}$.

The following proposition sums up the comparison of the targeting equilibrium outcomes:

Proposition 4 *Assume that condition 2 holds and that* $\sigma > \frac{2}{3}$.

Compared to pure single-homing, multi-homing provides

(i) lower subscription prices and higher consumer utility

(ii) higher ad revenues

(iii) higher platform profits

Proof. See Appendix. ■

By nature, single-homing profits do not depend on the value of non-exclusive consumers (σ). Multi-homing profits, on the other hand, are either increasing in σ or have a U-shaped relationship with σ. An increase in σ means that non-exclusive consumers are more valuable to advertisers. Since this allows the platforms to charge a higher ad price, one might expect that it would lead to greater platform profits. For most parameter values, profits are indeed unambiguously increasing in σ. An increase in the value of non-exclusive consumers also makes the platforms eager to attract more of them. But suppose that consumers have very strong platform preferences (high t). Attracting a larger audience may then require a price drop that is more costly than the additional revenue from gained consumers. This could be the case if the value of non-exclusive consumers, even after an increase, remains fairly low. Consequently, the overall impact on profits could be negative. However, as σ takes on higher values, profits will eventually start to increase. Figure 3 illustrates this U-shaped relationship between σ and multi-homing profits.

4.3 The existence of Nash equilibria

Next, we investigate whether single-homing and multi-homing constitute potential Nash equilibria. If non-exclusive consumers are sufficiently valuable, it pays off to charge lower subscription fees and forgo some subscription revenues in order to extract more ad-side revenues. Moreover, if the platforms set multi-homing prices, some consumers will actually subscribe to both platforms.

If, on the other hand, the advertiser valuation of non-exclusive consumers is low (small sigma), multi-homing might not constitute an equilibrium. In a situation with weak platform preferences (low t), a reduction in the subscription price would be efficient in attracting many consumers, making it tempting to undercut the rival's subscription price and only serve more valuable exclusive consumers. Both platforms would in that case deviate from multi-homing. However, as long as $\sigma > \sigma^* = 0.03$, we find that it is never beneficial for a platform to deviate from multi-homing. Recall that $\sigma \in (0, 1)$, which means that there is only a small interval where deviation from multi-homing might be feasible.

Then, consider the single-homing regime. Unless non-exclusive consumers have very little value for advertisers, the platforms have strong incentives to deviate from setting the single-homing price. More precisely, we find that it is profitable for a platform to deviate from single-homing for all $\sigma > 0.1$.

The most obvious reason is that deviation enables the platforms to sell more subscriptions and ad impressions. But even if non-exclusive consumers are not that valuable (i.e. $\sigma < 0.1$), single-homing does not constitute an equilibrium.

The single-homing prices would still be so low that some consumers would like to deviate and subscribe to both platforms.

It follows from Proposition 3 that the targeting equilibrium is only stable for $\sigma > 1/3$.

We can state:

Proposition 5 *Assume that condition 2 holds. Then, there exists*

(i) a unique subgame-perfect Nash equilibrium in which platforms target and consumers multi-home for all $\sigma > 1/3$

(ii) no equilibrium with single-homing for all $\sigma > 0$

Remark 2 *Multi-homing could also constitute an equilibrium for* $\sigma \leq 1/3$*, but only if consumers have sufficiently strong platform preferences.*

Proof. See Appendix. ■

The second result of Proposition 5 is particularly interesting. Previous literature has typically made the stark assumption of single-homing, which we find never takes part in a subgame-perfect Nash equilibrium when platforms can acquire targeting technologies, and hence might not be an appropriate assumption to make.

5 Concluding remarks

This paper has two major contributions: First, we demonstrate the importance of targeting by first-party data for the strategic interaction between competing digital platforms. Whereas previous works on platform competition that do not consider targeting, have established a strategic independence result when consumers multi-home, we find that by implementing targeting, the strategic dependence restores. The reason is that the value in the ad market of a platform's consumer mass can be affected by a rival platform's price setting, such that the composition of the demand becomes important. Hence, we find that targeting does not trigger an aggressive price response from the rival platform, as would be the case in a single-homing regime. An important implication from this is that we find that targeting indeed can be profitable, contrary to the predictions of most media models, but only if non-exclusive consumers are sufficiently valuable in the ad market.

The second key contribution is an even more important one: we find that pure single-homing never occurs in equilibrium. This means that existing literature assuming single-homing might be misleading when analyzed in a digital context where platforms target ads, and emphasizes that assessing the nature of consumer purchasing behavior (i.e. single-homing or multi-homing) is vital to fully understand the impact of targeting.

Our model makes the simplifying assumption of ad-neutral consumers. Targeting could, however, either increase or reduce ad nuisance. On the one hand, privacy concerns might lead to less consumer satisfaction (Johnson, 2013; Kox *et al.*, 2017). On the other hand, more relevant ads could please them (Gong *et al.*, 2019). The overall effect is therefore ambiguous. We leave this analysis for future research. Our model specification says that the platforms' targeting ability increases at a constant rate as more consumers subscribe. While some sources claim that the more data, the better targeting results, others suggest that the benefit of more data will be diminishing at some point. We have not formally analyzed this issue, but we do not believe that it would change the key properties. Targeting would presumably still provide similar, but somewhat smaller effects.

One may also ask whether stricter privacy regulations provide greater incentives to cooperate in order to share data. On the one hand, joining forces to increase the total data pool could be seen as an alternative to purchasing third-party data, or facilitating entry in data-intensive markets. On the other hand, stricter regulations might make cooperation less feasible. Furthermore, the perhaps greatest advantage of first-party data is that it provides the platforms with exclusive insight. This advantage clearly goes against sharing. Future studies could explore this issue further.

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Appendix

A.1 Conditions for multi-homing

Without targeting, second-order and stability conditions are always satisfied. From the equilibrium price, which is given by $p^{M,NT} = \frac{v-\sigma\alpha}{2}$, we see that $v > \sigma\alpha$ is required for the price to be positive. The non-targeting equilibrium demand functions are given by

$$x^{e,M,NT} = \frac{1}{2}\frac{2t - v - \alpha\sigma}{t} \text{ ; } x^{n,M,NT} = \frac{v + \alpha\sigma - t}{t} \text{ and } D^{M,NT} = \frac{1}{2}\frac{v + \alpha\sigma}{t}.$$

The restriction of the analysis to partial multi-homing implies that we need $x^{e,M} > 0$ and $x^{n,M} > 0$. This places some additional constraints on the parameter values: $\frac{1}{2}(v+\sigma\alpha) < t < v + \sigma\alpha$.

With targeting, stability requires $t > \frac{1}{2}\alpha\left(\sigma + 1\right)$ and the second-order condition is satisfied for $t > \sigma\alpha$. The equilibrium price is given by equation (15), and non-negative prices require that $t > \frac{\alpha v(3\sigma-1)}{v-\alpha}$. The targeting equilibrium demand functions are

$$x^{e,M,T} = \frac{2t - v - 3\alpha\sigma}{2t + \alpha(1-3\sigma)} \text{ ; } x^{n,M,T} = \frac{\alpha + 2v + 3\alpha\sigma - 2t}{2t + \alpha(1-3\sigma)} \text{ and } D^{M,T} = \frac{\alpha + v}{2t + \alpha(1-3\sigma)},$$

for which partial multi-homing is ensured by $\frac{1}{2}(v + 3\alpha\sigma) < t < v + \frac{1}{2}\alpha(1 + 3\sigma)$.

Summarizing, this leaves us with the two binding constraints, depending on the value of σ:

Condition A.1 *(Multi-homing)* $\max\{\frac{1}{2}(v + 3\sigma\alpha), \frac{\alpha v(3\sigma-1)}{v-\alpha}\} < t < v + \sigma\alpha$ *for* $\sigma > \frac{1}{3}$.

Condition A.2 *(Multi-homing)* $\max\{\frac{1}{2}(v + 3\sigma\alpha), \frac{1}{2}\alpha(1 + \sigma)\} < t < v + \sigma\alpha$ *for* $\sigma \leq \frac{1}{3}$.

Finally, Condition A.1 constrains $v > \alpha\left(\sigma + \sqrt{\sigma\left(\sigma + 1\right)}\right)$.

A.2 Omitted proofs

Proof of Lemma 2

Under single-homing, this follows directly as $p^{S,T} - p^{S,NT} = t - 2\alpha - (t - \alpha) = -\alpha < 0$. Under multi-homing, we have $p^{M,T} - p^{M,NT} = \frac{1}{2}\frac{\alpha}{2t+\alpha(1-3\sigma)}(v - 2t(1+\sigma) - 3v\sigma + \alpha\sigma - 3\alpha\sigma^2)$, which is negative if conditions A.1 and A.2 hold. ■

Proof of Lemma 3

Consider the single-homing regime. Suppose platform i targets ads, while platform j does not. The best-response functions are then

$$p_i(p_j) = \frac{t\,(t-2\alpha) + p_j\,(t-\alpha)}{2t-\alpha} \text{ and } p_j(p_i) = \frac{t + p_i - \alpha}{2}.$$

The equilibrium prices are given by (superscript 'd' for deviation)

$$p_i^d = \frac{3t^2 - 6\alpha t + \alpha^2}{3t-\alpha} \text{ and } p_j = \frac{3t^2 - 5\alpha t + \alpha^2}{3t-\alpha},$$

yielding profits

$$\pi_i^d = \frac{9}{4}t^2\frac{2t-\alpha}{(3t-\alpha)^2} \text{ and } \pi_j = \frac{1}{2}t\frac{(3t-2\alpha)^2}{(3t-\alpha)^2}.$$

Equation (17) gives the symmetric equilibrium profits when both platforms target and when none of the platforms targets.

The decision of whether or not to deviate can be formulated as a game matrix in Table A.1.

		Platform i	
		Target	Not target
Platform j	Target	$\frac{2t-\alpha}{4}, \frac{2t-\alpha}{4}$	$\frac{9}{4}t^2\frac{2t-\alpha}{(3t-\alpha)^2}, \frac{1}{2}t\frac{(3t-2\alpha)^2}{(3t-\alpha)^2}$
	Not target	$\frac{1}{2}t\frac{(3t-2\alpha)^2}{(3t-\alpha)^2}, \frac{9}{4}t^2\frac{2t-\alpha}{(3t-\alpha)^2}$	$\frac{1}{2}t, \frac{1}{2}t$

Table A.1: Prisoner's dilemma.

If platform j targets, it is optimal for platform i to target iff $\frac{2t-\alpha}{4} - \frac{1}{2}t\frac{(3t-2\alpha)^2}{(3t-\alpha)^2} = \frac{1}{4}\alpha\frac{3t^2-\alpha^2}{(\alpha-3t)^2} > 0$, which is always the case.

If platform j does not target, it is nonetheless optimal for platform i to target iff $\frac{9}{4}t^2\frac{2t-\alpha}{(3t-\alpha)^2} - \frac{1}{2}t = \frac{1}{4}t\alpha\frac{3t-2\alpha}{(3t-\alpha)^2} > 0$, which is also always the case.

Hence, platform i's dominant strategy is to target, regardless of the rival's decision. Since $\pi_i^{S,T} - \pi_i^{S,NT} = \frac{2t-\alpha}{4} - \frac{1}{2}t = -\frac{\alpha}{4} < 0$, the dominant strategy (targeting) yields lower profits than by not targeting. This means that the platforms end up in a prisoner's dilemma when consumers single-home. ■

Proof of Proposition 2

To prove Proposition 2, we start by decomposing profits into ad revenues and subscription revenues.

Subscription revenues First, we show that equilibrium subscription revenues are always lower with targeting:

$$p^{M,T}D^{M,T} - p^{M,NT}D^{M,NT} =$$
$$-\tfrac{1}{4}\alpha^2\left(2t - v - \sigma\alpha - 2t\sigma + 3v\sigma + 3\alpha\sigma^2\right)\tfrac{2t-v+\alpha\sigma+2t\sigma+3v\sigma-3\alpha\sigma^2}{t(\alpha+2t-3\alpha\sigma)^2} < 0.$$

Ad revenues Equilibrium advertising revenues with targeting minus ad revenues without targeting are given by

$$A^{M,T} - A^{M,NT} =$$
$$\tfrac{1}{2}\alpha\tfrac{4t^2(1-\sigma)(v+2\alpha\sigma)+2tv^2(2\sigma-1)+2tv\alpha\sigma(9\sigma-5)+2t\alpha^2\left(5\sigma-13\sigma^2+12\sigma^3-1\right)-\alpha^2(2\sigma-1)(3\sigma-1)^2(v+\alpha\sigma)}{t(2t+\alpha-3\alpha\sigma)^2}.$$

We find it useful to consider $\sigma > \frac{1}{3}$ and $\sigma \leq \frac{1}{3}$ separately.

For $\sigma > \frac{1}{3}$, we have that $v_{\min}$ ensures higher profits with targeting:

$$A^{M,T} - A^{M,NT}|_{v=\alpha} = \tfrac{\alpha}{2t}\tfrac{\left(4t^2\alpha(2\sigma+1)(1-\sigma)+4t\alpha^2\left(\sigma-2\sigma^2+6\sigma^3-1\right)-\alpha^3(\sigma+1)(2\sigma-1)(3\sigma-1)^2\right)}{(2t+\alpha-3\alpha\sigma)^2} > 0.$$

Moreover, the difference between profits with and without targeting becomes greater as v increases:

$$\tfrac{d(A^{M,T}-A^{M,NT})}{dv} =$$
$$\tfrac{1}{2t}\tfrac{\alpha}{(2t+\alpha-3\alpha\sigma)^2}\left(4t^2(1-\sigma) + 2t\alpha\sigma(9\sigma-5) + 4tv(2\sigma-1) - \alpha^2(2\sigma-1)(3\sigma-1)^2\right) > 0.$$

Hence, $A^{M,T} > A^{M,NT}$ for all $\sigma > \frac{1}{3}$.

We then consider $\sigma \leq \frac{1}{3}$. If t is low, a reduction in the subscription price will turn many exclusive consumers into non-exclusive consumers. But if the non-exclusive consumers are not worth much in the ad market, the benefit for the platform is limited. This implies that even though targeting increases the ad price, it does not necessarily increase ad revenues when $\sigma \leq \frac{1}{3}$. We illustrate with an example:

Suppose that $\sigma = 0$, which yields

$$A^{M,T} - A^{M,NT} = \alpha \frac{(2t-v)\left(2tv-\alpha^2\right)}{2t\left(2t+\alpha\right)^2}$$

The expression is positive if $2tv > \alpha^2$. If t and v are not sufficiently high relative to α, this is not the case. Consider, for instance, $v = 0.3$, $t = 0.3$ and $\alpha = \frac{1}{2}$. The numerator $(2t-v)\left(2tv-\alpha^2\right)$ then equals -0.021, which implies that $A^{M,T} - A^{M,NT} < 0$.

Platform profits We then analyze the platform profits. By inserting equations (15) and (14) into (8), we find the equilibrium profits with and without targeting, respectively, when consumers multi-home:

$$\pi^{M,T} = \frac{(t-\alpha\sigma)(\alpha+v)^2}{(\alpha(1-3\sigma)+2t)^2} + \frac{\alpha(2t-v)}{(\alpha(1-3\sigma)+2t)} - \alpha\sigma\left(1+\frac{2\alpha-v}{\alpha(1-3\sigma)+2t}\right) \tag{A.1}$$

and

$$\pi^{M,NT} = \frac{1}{4t}(v^2 + 2v\alpha(2\sigma-1) + 3\alpha^2\sigma^2 - 2\alpha^2\sigma - 4t\alpha(1-\sigma)). \tag{A.2}$$

Whether higher ad revenues compensate for lower subscription revenues is dependent on σ. We start by considering $\sigma > \frac{1}{3}$. The minimum $v-$value is given by $v_{\min} = \alpha + \varepsilon$. By definition, $\sigma_{\min} = \frac{1}{3} + \varepsilon$. Evaluating multi-homing profits of equation (A.1) and (A.2) at $v_{\min}$ and $\sigma_{\min}$, we have that targeting in both cases provides greater profits ($\pi^{M,T} > \pi^{M,NT}$):

$$\pi^{M,T} - \pi^{M,NT}|_{v\min} \approx \tfrac{1}{4}\alpha^2 \tfrac{(2t-\alpha)^2+9\alpha\sigma^3(4t-3\alpha\sigma)+6\sigma^2\left(-2\alpha t+3\alpha^2-2t^2\right)-4\sigma\left(\alpha t+2\alpha^2-2t^2\right)}{t(2t+\alpha-3\alpha\sigma)^2} > 0$$

and

$$\pi^{M,T} - \pi^{M,NT}|_{\sigma\min} = \tfrac{1}{12}\tfrac{\alpha}{t^2}\left(4tv-(v+\alpha)^2\right) > 0.$$

Moreover, for $v_{\min}$ we have that $(\pi^{M,T} - \pi^{M,NT})$ is increasing in σ :

$$\tfrac{d(\pi^{M,T}-\pi^{M,NT})}{d\sigma}|_{v\min} \approx \tfrac{1}{2}\tfrac{\alpha^2(\alpha^3(3\sigma+1)(3\sigma-1)^3-8t^3(3\sigma-1)-2\alpha^2 t\left(-9\sigma-27\sigma^2+81\sigma^3+11\right)+12\alpha t^2\left(-2\sigma+9\sigma^2+1\right)}{t(\alpha+2t-3\alpha\sigma)^3} > 0.$$

Similarly, $(\pi^{M,T} - \pi^{M,NT})$ is increasing in v for $\sigma_{\min}$:

$$\frac{d(\pi^{M,T}-\pi^{M,NT})}{dv}|_{\sigma\min} = \frac{1}{6t^2}\alpha\left(2t-v-\alpha\right) > 0.$$

Finally, higher $v-$values enhance the increase in $(\pi^{M,T} - \pi^{M,NT})$ in response to a change in σ :

$$\frac{d\left(\frac{d(\pi^{M,T}-\pi^{M,NT})}{d\sigma}\right)}{dv} = \frac{1}{2}\alpha\frac{2\alpha^3(3\sigma-1)^3+12\alpha t^2(5\sigma-1)-4\alpha^2 t\left(5-18\sigma+27\sigma^2\right)+4tv(4t-\alpha-3\alpha\sigma)-8t^3}{t(\alpha+2t-3\alpha\sigma)^3} > 0.$$

The numerator is increasing in v, and since it is positive for $v_{\min}$ it is also positive for larger $v-$values. In sum, targeting is profitable in equilibrium if $\sigma > \frac{1}{3}$.

Consider then the case where $\sigma \leq \frac{1}{3}$. In this case, targeting does not necessarily increase ad revenues. Since targeting also reduces subscription revenues, it might lead to lower profits.

To complete the proof, we consider mixed strategies. Suppose platform i targets ads, while platform j does not. The best-response (BR) functions are then

$$p_i^{BR}(p_j) = \frac{v\left(t+\alpha\right)-\alpha\left(t+3v\sigma\right)-\alpha p_j\left(1-\sigma\right)}{2(t-\alpha\sigma)} \tag{A.3}$$

and

$$p_j^{BR}(p_i) = \frac{v-\alpha\sigma}{2}. \tag{A.4}$$

Equation (A.3) is equivalent to equation (13), whereas equation (A.4) is equivalent to equation (10). The best-response functions in equation (10) are strategically independent, such that $p_j = p_j^{BR}(p_i) = p_j^{M,NT}$.

From Lemma 2, we know that the equilibrium prices $p_i^{M,T} < p_i^{M,NT}$, and since $\frac{\partial p_i^{M,T}(p_j)}{\partial p_j} < 0$, we must have that $p_i^d < p_i^{M,T} < p_j = p_j^{M,NT}$, where p_i^d is the deviation price firm i obtains by targeting when the rival does not. It follows that $\pi_i^d > \pi_j = \pi_j^{M,NT}$. The previous part of this proof confirmed that $\pi_i^{M,T} > \pi_i^{M,NT}$ for $\sigma > 1/3$. Hence, targeting is a dominant strategy for platform i if non-exclusive consumers have sufficiently large value to the platform in the ad market, i.e. for $\sigma > 1/3$. ■

Proof of Proposition 3

Comparing the non-targeting profits under the two homing regimes, we find that

$$\pi^{S,NT} > \pi^{M,NT} \tag{A.5}$$

could be true for $\sigma \leq 1/3$ (and $t < \frac{2+\alpha^2-2\alpha^2\sigma+\alpha^2\sigma^2}{4\alpha(-1+\sigma)}$, i.e. t is sufficiently small). In that case, the platform would thus prefer to set prices that incentivize consumers to single-home.

Next, we therefore investigate whether the consumers will single-home. We investigate the deviation incentives to and from single-homing behavior:

1. **Deviation from single-homing:** Facing single-homing prices, could the consumers be better off by multi-homing? Single-homing prices are given by equation (14), $p^{S,NT} = t - \alpha$. Inserting the single-homing price in absence of targeting into the consumer utility functions, we find that $u_{ij}^{p=p^{S,NT}} - u_i^{p=p^{S,NT}} < 0$ for $t < \frac{2}{3}(v+\alpha)$ (cf. Condition 1). Hence, consumers will not deviate from single-homing prices.

2. **Deviation from multi-homing:** Facing multi-homing prices, could consumers be better off by single-homing? The multi-homing prices are given by equation (14). Inserting the multi-homing price in absence of targeting into the consumer utility functions, we find that $u_{ij}^{p=p^{M,NT}} - u_i^{p=p^{M,NT}} < 0$ for $t < \alpha\sigma + v$ (cf. Condition A.2). Hence, if the consumer faces multi-homing, the consumer will deviate and only subscribe to one platform (i.e. single-home).

That is, if $\sigma \leq 1/3$, and the platform choose to not use targeting technologies, then both the platform and the consumers would deviate to single-homing prices. Then, according to Lemma 3, the platform would deviate and implement a targeting technology after all. Hence, platforms' no-targeting (NT) strategy cannot take part in the equilibrium. ■

Proof of Proposition 4

The proof consists of three parts:

(i) a) **Subscription prices:** The subscription prices are given in equation (15). The difference in the prices, $p^{S,T} - p^{M,T} = \frac{15\alpha^2\sigma-5\alpha t-5\alpha^2+2t^2-3\alpha t\sigma}{\alpha+2t-3\alpha\sigma}$, is greater than zero for $\sigma > \frac{2}{3}$. For lower values of σ, this requires the additional constraint

$$t > \frac{1}{4}\left(\sqrt{\alpha^2(9\sigma^2 - 90\sigma + 65)} + 3\alpha\sigma + 5\alpha\right).$$

b) **Consumer utility:** To show that consumer utility is higher with multi-homing, we need to compare the utility from subscribing to one platform with that of subscribing to two platforms. The utility from subscribing to one platform only is $u_i^{p=p^{S,T}}(x=1/2) = 5\alpha - \frac{3}{2}t$, whereas the utility from subscribing to both platforms is given by $u_{ij}^{p=p^{M,T}}(x = 1/2) = t\frac{7\alpha-2t+3\alpha\sigma}{\alpha+2t-3\alpha\sigma}$. Multi-homing is preferred if

$$u_{ij}^{p=p^{M,T}} - u_i^{p=p^{S,T}} = \frac{1}{2}\frac{30\alpha^2\sigma - 3\alpha t - 10\alpha^2 + 2t^2 - 3\alpha t\sigma}{\alpha + 2t + 3\alpha\sigma} > 0,$$

which holds for $\sigma > \frac{2}{9}$. From the analysis of subscription prices, we know that consumers who subscribe to only one platform are also better off when $\sigma > \frac{2}{3}$. Hence, a sufficient condition for all consumers to be better off with multi-homing is that $\sigma > \frac{2}{3}$.

(ii) **Ad prices:** The ad prices are given by

$$\alpha^{M,T} = \alpha\frac{(2t + 5\alpha - 3\alpha\sigma)}{(2t + \alpha - 3\alpha\sigma)} \text{ and } \alpha^{S,T} = \frac{3}{2}\alpha.$$

The difference in ad prices $\alpha^{S,T} - \alpha^{M,T} = -\frac{1}{2}\alpha\frac{(7\alpha-2t+3\alpha\sigma)}{2t+\alpha-3\alpha\sigma} < 0$, which states that the ad price is always higher with multi-homing.

(iii) **Profits:** The platform profits are given by equations (A.1) and (17). The difference is given by

$$\pi^{S,T} - \pi^{M,T} = \frac{\left(36\alpha^3\sigma^3 - 3\alpha^2\sigma^2(7\alpha+10t) + 2\alpha\sigma\left(16\alpha t + 17\alpha^2 - 4t^2\right) + 4t^2(2t-3\alpha) - \alpha^2(50t - 11\alpha)\right)}{4(\alpha+2t-3\alpha\sigma)^2}.$$

We find that single-homing profits cannot be greater than multi-homing profits if $\sigma \geq 0.65$ when Condition 2 holds.■

Proof of Proposition 5

A stable equilibrium is one in which no player will deviate from a given strategy. The proof consists of two parts: We first examine the incentives to deviate from a multi-homing price setting, and then the incentives to deviate from a pure single-homing outcome. Finally, we find that the consumers facing single-homing prices will always deviate and subscribe to an additional platform, such that single-homing can never be part of an equilibrium.

(i) Deviation from multi-homing

Suppose that platform i believes that the rival prices according to the multi-homing regime: $p_j = \frac{v(t+\alpha)-\alpha(t+3v\sigma)}{2t+\alpha(1-3\sigma)}$. Could it be optimal for platform i to charge a higher price and only sell to consumers that do not subscribe to platform j?

We insert $p_j = \frac{v(t+\alpha)-\alpha(t+3v\sigma)}{2t+\alpha(1-3\sigma)}$ into the location of the indifferent consumer:

$$\widetilde{x} = \frac{1}{2} + \frac{p_j - p_i}{2t},$$

which yields

$$D_i = \frac{1}{2}\frac{2t^2 + 3\alpha t + 3\alpha^2 - 3\alpha\sigma\left(3\alpha + t\right) - p_i\left(\alpha + 2t - 3\alpha\sigma\right)}{t\left(\alpha + 2t - 3\alpha\sigma\right)}$$

and subscription price (superscript ‘d’ for deviation):

$$p_i^d = \frac{\left(2t - 3\alpha\right)\left(\alpha t + \alpha^2 + t^2\right) - 3\alpha\sigma\left(t^2 + \alpha t - 3\alpha^2\right)}{\left(2t - \alpha\right)\left(\alpha + 2t\right) - 3\alpha\sigma\left(2t - \alpha\right)}.$$

Compared to the equilibrium price with multi-homing, the deviation price is always higher if $\sigma > \frac{2}{3}$. The deviation profit is given by

$$\pi_i^d = \frac{1}{4}\frac{\left(12\alpha^2\sigma - 5\alpha t - 4\alpha^2 - 2t^2 + 3\alpha t\sigma\right)^2}{\left(2t - \alpha\right)\left(-\alpha - 2t + 3\alpha\sigma\right)^2}.$$

Comparing deviation profits with the multi-homing equilibrium profit, we find that

$$\pi_i^d - \pi^{M,T} = \frac{1}{4}\frac{4t^4 + 4\alpha t^3(5\sigma - 3) - 3\alpha^2 t^2\left(10\sigma + 29\sigma^2 + 13\right) + 8\alpha^3 t(3\sigma + 7)\left(2 - 3\sigma + 3\sigma^2\right) - 4\alpha^4(\sigma - 1)\left(1 - 30\sigma + 9\sigma^2\right)}{(2t - \alpha)(-\alpha - 2t + 3\alpha\sigma)^2}.$$

The above shows that deviation is never profitable if $\sigma > 0.03$.

However, for multi-homing to be an equilibrium, it must be true that consumers will actually purchase both products when $p = p^{M,T}$.

From

$$u_i^{p=p^{M,T}}(x = \frac{1}{2}) = \frac{1}{2}t\frac{7\alpha - 2t + 3\alpha\sigma}{\alpha + 2t - 3\alpha\sigma}$$

and

$$u_{ij}^{p=p^{M,T}}(x = \frac{1}{2}) = t\frac{7\alpha - 2t + 3\alpha\sigma}{\alpha + 2t - 3\alpha\sigma},$$

we see that $u_{ij}^{p=p^{M,T}} - u_i^{p=p^{M,T}} = \frac{1}{2}t\frac{7\alpha - 2t + 3\alpha\sigma}{\alpha + 2t - 3\alpha\sigma} > 0$ whenever Condition 2 holds, which confirms that some consumers want to multi-home. Hence, (some) multi-homing consumers have no incentives to deviate (subscribe to only one platform) when facing multi-homing prices, and there is a unique equilibrium with multi-homing.

(ii) Deviation from single-homing

If both platforms price according to single-homing, prices and profits are given by $p^{S,T} = t - 2\alpha$ and $\pi^{S,T} = \frac{1}{4}(2t - \alpha)$. Suppose that platform i believes that platform j sets the single-homing price, $p^{S,T}$. If platform i deviates and sets the prices that maximize profits if also selling to some consumers who buy the rival's product, we get:

$$p_i^d = \frac{\alpha t(\sigma+1) - \alpha^2(11\sigma - 5)}{2(t - \alpha\sigma)}.$$

Deviation profit is given by:

$$\pi_i^d = \frac{1}{4}\alpha \frac{25\alpha^3(\sigma-1)^2 + 8t^3(1-\sigma) + \alpha t^2(2\sigma + 9\sigma^2 + 5) - 10\alpha^2 t(1-\sigma)(5-3\sigma)}{t^2(t-\alpha\sigma)}$$

and

$$\pi_i^d - \pi^{S,T} = \frac{1}{4}\frac{-2t^4 + 25\alpha^4(\sigma-1)^2 - 10\alpha^3 t(1-\sigma)(5-3\sigma) + \alpha^2 t^2(\sigma + 9\sigma^2 + 5) + 3\alpha t^3(3-2\sigma)}{t^2(t-\alpha\sigma)}.$$

Examining the above equations shows that deviation is profitable when $\sigma > 0.1$. For $t-$values in the higher end of Condition 2, it might also be the case for $\sigma < 0.1$.

Suppose next that for some $\sigma < 0.1$, it is optimal for the platforms to set the single-homing price. This can only be an equilibrium if the consumers do not subscribe to both platforms at this price. We insert $p^{S,T}$ into (1) and (2) for $x = \frac{1}{2}$ and find

$$u_i^{p=p^{S,T}}(x = \frac{1}{2}) = \frac{1}{2}(10\alpha - 3t)$$

and

$$u_{ij}^{p=p^{S,T}}(x = \frac{1}{2}) = 10\alpha - 3t.$$

We see that $u_{ij}^{p=p^{S,T}} > u_i^{p=p^{S,T}}$, which implies that there exist consumers who want to subscribe to both platforms when $p = p^{S,T}$. By the same token, deviation is only possible if some consumers actually subscribe to both platforms at the deviation price. We insert

$p^{S,T}$ and p_i^d into (1) and (2), respectively, and find

$$u_i^{p=p^{S,T}}(x=\frac{1}{2})=\frac{1}{2}(10\alpha-3t)$$

and

$$u_{ij}^{p=p_i^d}(x=\frac{1}{2})=\frac{1}{2}\frac{3\alpha t(\sigma+5)-5\alpha^2(\sigma+1)-4t^2}{t-\alpha\sigma}.$$

At $x=\frac{1}{2}$, $u_{ij}^{p=p_i^d}>u_i^{p=p^{S,T}}$, and there exist consumers who want to multi-home. Some consumers have incentives to deviate and subscribe to more platforms when facing single-homing prices. Therefore, single-homing can never take part in a Nash equilibrium. ■

A.3 Robustness

In the equilibrium analysis, we assume that $v=3\alpha$. This provides us with a more tractable set of constraints. The drawback is that it might bring the robustness of the findings into question. To shed some light on this issue, we take a closer look at how the results depend on v.

First, note that $\pi^{S,T}$ does not depend on v, while $\partial\pi^{M,T}/\partial v>0$ if $v>\mu\equiv\frac{\alpha^2\left(1-2\sigma+3\sigma^2\right)-2\alpha t\sigma}{2(t-\alpha\sigma)}$. Since $\partial\mu/\partial t<0$, the requirement is strictest for $t_{\min}$. From the conditions, we know that the lowest possible t is given by $\frac{5}{2}\alpha$. This gives $\mu|_{t=\frac{5\alpha}{2}}=\alpha\frac{3\sigma^2-7\sigma+1}{5-2\sigma}$, which is at its highest when $\sigma=0$ and yields $\mu=\frac{1}{5}\alpha$. Hence, multi-homing becomes relatively more profitable compared to single-homing for all $v>\frac{1}{5}\alpha$.

We then consider how v affects the platforms' incentives to deviate from committing to single-homing and multi-homing. Suppose that we do not fix v. If the rival commits to single-homing, the deviation profit is given by

$$\pi_i^{d,S}=$$
$$\frac{1}{4}\frac{t^2v(v+2\alpha\sigma)+\alpha^2(\sigma-1)^2(2\alpha+v)^2+8\alpha t^3(1-\sigma)-2\alpha tv(\sigma-1)(-4\alpha-v+3\alpha\sigma)+\alpha^2t^2\left(-4\sigma+9\sigma^2-4\right)-4\alpha^3t(3\sigma-2)(\sigma-1)}{t^2(t-\alpha\sigma)}.$$

Consequently, $\frac{d(\pi_i^d-\pi^{S,T}))}{dv}>0$ if $v>\lambda\equiv\frac{\alpha^2t(\sigma-1)(3\sigma-4)-2\alpha^3(\sigma-1)^2-\alpha t^2\sigma}{(t-\alpha+\alpha\sigma)^2}$. This is ensured by condition $(t<v+\sigma\alpha)$ for all $\sigma>0.01$.

Proof:

$v > t - \sigma\alpha > \lambda$ if $t - \sigma\alpha - \lambda > 0$. We have that:

$$t - \sigma\alpha - \lambda = \frac{t^3 + \alpha^3 (2 - \sigma)(\sigma - 1)^2 + t\alpha^2 (4\sigma - 3)(1 - \sigma) - 2t^2\alpha (1 - \sigma)}{(t - \alpha + \alpha\sigma)^2}$$

The expression is greater than 0 for all $\sigma > 0.01$. ■

Unless multi-homing consumers are almost worthless in the ad market, it is certainly more tempting to deviate from single-homing if v increases from 3α.

Similarly, we consider the case with the rival committing to multi-homing. Deviation profit is then

$$\pi_i^{d,M} = \frac{1}{4} \frac{(2t\alpha + v\alpha + \alpha^2 - 3\alpha^2\sigma + tv + 2t^2 - 3t\alpha\sigma - 3v\alpha\sigma)^2}{(2t - \alpha)(2t + \alpha - 3\alpha\sigma)^2}$$

From the profit expression, we find that $\frac{d(\pi_i^d - \pi^{M,T})}{dv} < 0$ if

$$v > \mu \equiv \frac{2t^3 - \alpha^3 (3\sigma + 1)(1 - \sigma) - t^2\alpha (17\sigma - 4) + t\alpha^2 (7 - 16\sigma + 21\sigma^2)}{(t - \alpha + \alpha\sigma)(7t + \alpha - 9\alpha\sigma)}.$$

This is ensured by condition $(t < v + \sigma\alpha)$ for all $\sigma > 0.02\dot{6}$.

Proof:

$v > t - \sigma\alpha > \mu$ if $t - \sigma\alpha - \mu > 0$. We have that

$$t - \sigma\alpha - \mu = \frac{5t^3 + \alpha^3 (1 - \sigma)(4\sigma - 9\sigma^2 + 1) - 4t\alpha^2(2 - 8\sigma + 7\sigma^2) + 2t^2\alpha (4\sigma - 5)}{(t - \alpha + \alpha\sigma)(7t + \alpha - 9\alpha\sigma)}$$

is positive for $\sigma > 0.026$. ■

Deviation is less tempting if v increases from 3α, at least if $\sigma \geq 0.02\dot{6}$.

Suppose that $\sigma = 0$, which yields $\mu_{\max} = \frac{1}{(t-\alpha)(7t+\alpha)} (2t^3 + 4t^2\alpha + 7t\alpha^2 - \alpha^3)$ since $\frac{d\mu}{d\sigma} < 0$. For $t \equiv t' \leq 6.57\alpha$, we find that $v \geq 3\alpha \geq \mu$.

$\frac{d\mu_{\max}}{dt'} > 0$. We have that $v > t'$ (condition $(t < v + \sigma\alpha)$). This implies that if $t' > \mu_{\max}$, then $v > \mu_{\max}$. Since

$$t' - \mu_{\max} = \frac{\alpha^3 - 8t\alpha^2 - 10t^2\alpha + 5t^3}{(t-\alpha)(7t+\alpha)} > 0$$

Deviation is less tempting for $v > 3\alpha$ also if $\sigma = 0$.

In this section, we have shown that our results are quite robust. Given that multi-homing consumers are not negligible in the ad market, our results hold for all $v > 3\alpha$. A higher v does not make single-homing more attractive relative to multi-homing and it does not reduce the incentives to deviate from single-homing. Moreover, a higher v makes it less imperative to deviate from multi-homing. Evaluating $v-$values below 3α is less interesting since the lower boundary is given by $v = 2.75\alpha$.

Consumers

For a general v, we check whether consumers will subscribe to both platforms when $p = p^{M,T}$. Inserting (15) into (1) and (2), we find:

$$u_i^{p=p^{M,T}}(x = \frac{1}{2}) = \frac{1}{2}t\frac{2v + \alpha + 3\alpha\sigma - 2t}{2t + \alpha - 3\alpha\sigma}$$

and

$$u_{ij}^{p=p^{M,T}}(x = \frac{1}{2}) = t\frac{2v + \alpha + 3\alpha\sigma - 2t}{2t + \alpha - 3\alpha\sigma}.$$

It follows that if $u_{ij}^{p=p^{M,T}} > u_i^{p=p^{M,T}}$ for $v = 3\alpha$, the same is true for any other v-value as well. ■