Postal Platform Pricing with Limited Consumer Attention

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Abstract

This paper models the postal mail market as a two-sided platform connecting mail senders and consumers, where consumers have limited attention. Two types of mail senders: direct mail senders (advertisers) and transactional mail senders (non-advertisers), derive value from attention paid to their mail. Consumers pay more attention to each mail item when: (i) they receive less mail, (ii) they receive a higher proportion of transactional mail. We show that a postal operator monopolist stimulates the quantity of transactional mail by subsidising its price, which increases the value of mail to advertisers, inflating direct mail prices. Advertisers that are most nuisance or attention-consuming for consumers face higher prices. Competitive entry for delivering direct mail cannibalises the direct mail market and the cross-subsidisation of prices is shut down. However, if the entrant price-matches rather than competes, all postal operators, mail senders and recipients can benefit. Competition amongst postal operators does not necessarily benefit consumers, especially if the entrant is more efficient. Universal Service Obligation policies are not as demanding as traditionally viewed.

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1 Introduction

Consumers have limited attention.[1] In recent years, research across economics, marketing, management and operations has demonstrated that limited attention has a significant impact on the strategies of firms and the nature of competition.[2] Motivated by the growing economic significance of market intermediaries, research has also extensively examined the role of platforms, especially in digital and technology-focussed markets. A central finding is that it can be optimal to cross-subsidise one side of the market to generate value that is extracted from the other side of the market, referred to as the 'seesaw effect' (Jullien, Pavan and Rysman, 2021). Therefore, demand elasticities on each side interactively determine the platform's optimal pricing strategy. However, in the platform literature, the payoff of agents on one side of the platform is typically driven by the number of agents engaged with the platform, not the attention they pay. This distinction is particularly important in markets where the number of consumers is fixed and attention is the key consideration.

This paper bridges these lines of enquiry by studying the consequences of limited consumer attention for platform pricing, focusing on the case of the postal mail (letters) sector. On one side of the market, consumers receive mail at zero price but possess limited attention to devote to the mail that they receive. Consumers' attention is determined endogenously by the perceived value of their mail, driven by the total quantity and composition of their mail. On the other side of the market, senders choose the quantity of mail based on the prices charged by the postal operator(s) and their perceived value of using the postal service, which in turn is driven by the attention consumers pay. Whilst mail senders compete with each other for attention on the postal platform, the postal operator also competes for consumers' attention with other mediums of communication (or other activities more generally).

The postal sector is of particular economic significance with mail and parcel delivery contributing 0.5% of EU-GDP (Copenhagen Economics, 2019). The market is highly concentrated, with incumbents being amongst the largest employer in each

[1] Evidence that consumers are attention-constrained and consumers' aversion to advertising is widespread (Chetty et al, 2009; Crampes et al, 2009; DellaVigna, 2009; Hossain & Morgan, 2006; Wilbur, 2008). For example, Huang et al (2018) find that the number of hours a consumer listens to a radio station decreases linearly in the number of advertisements. OECD (2018) provides evidence that advertising irritates consumers online, resulting in 11% of internet users adopting ad-blockers in 2016 and subsequent annual increases of 30%.

[2] For example, sellers may need to persuade consumers to consider their products (Eliaz & Spiegler, 2011; Falkinger, 2008; Spiegler, 2011; Manzini & Mariotti, 2018), send reminders (Calzolari & Nardotto, 2017) and engage in inefficient mass, rather than targeted, advertising (Hefti & Liu, 2020). Sellers may also obfuscate product information (Chioveanu & Zhou, 2013; Ellison, 2005; Gabaix & Laibson, 2006), manipulate attention through their product range or framing (Apffelstaedt & Mechtenberg, 2021; Bordalo et al, 2013, 2016; Eliaz & Spiegler, 2011; Zhu & Dukes, 2017), and exploit consumers who neglect some attributes of a product (Boyaci & Akcay, 2018; Dahremoller & Fels, 2015; Reme et al, 2021; Spiegler, 2006). Price changes by one firm, its competitors or firms in entirely unrelated markets, can affect the attention consumers pay to a particular market, which affects demand (De Clippel et al, 2015; Ke & Lin, 2020).

EU economy, and national incumbents are designated with Universal Service Obligations (USO) that mandate requirements on efficiency, service and price to ensure a universal and affordable service for consumers (Copenhagen Economics, 2019; Ofcom, 2021). However, the sector faces systemic challenges to survive in the digital era. Average letter volumes have declined 3% annually across Europe (2010-2017), primarily due to electronic substitution. Total advertising spending is increasing but mail advertising revenues are declining (Ofcom, 2015). Growing demand for parcel delivery (where postal operators face stronger competition than for letters) cannot compensate for the decline in letter mail (Copenhagen Economics, 2019). USO's have also become financially unsustainable, with profitability declining by an average of 7% across the EU during 2013-2017 (Bradley et al, 2015, Copenhagen Economics, 2019; House of Commons, 2015). Current proposals to sustain USO viability include softening USO requirements and prioritising USO viability over competition (Copenhagen Economics, 2019).

The main differentiation between letter mail types is Direct and Transactional mail, which correspond broadly to advertising and non-advertising mail, respectively.[3] Electronic substitution has been particularly acute for transactional mail, leading to consumers receiving a higher proportion of direct mail (advertising). As a consequence, consumers find postal mail less relevant and pay less attention to mail items, which diminishes the value of direct mail further. Reports commissioned by the European Parliament find that the postal sector continues to follow cost-plus pricing, with some adjustment for senders' demand elasticities (Copenhagen Economics, 2012; 2019).[4] Indeed, Copenhagen Economics (2012, p. 116) advocate a classic inverse elasticity rule. Therefore, this research meets a particular business need to understand the multiplicity of network effects present on postal platforms, with clear guidance for both pricing and regulation in this sector.

More specifically, we address the following questions: How should postal operators set their menu of prices when consumers are attention constrained? How should a postal operator respond to heterogeneity between sender types (direct vs transactional) and within sender types? What is the economic cost of neglecting externalities on the network? How does competition amongst mail senders for attention impact market outcomes and the incentives of a postal operator? How does entry by an additional postal operator affect the incumbent, mail senders and recipients? In addressing these questions, we present new ways to preserve USO's without weakening them.

We begin by constructing an Attention function that specifies the amount of at-

[3]More specifically, direct mail consists 'solely of advertising, marketing or publicity material and comprising an identical message, except for the addressees name, address and identifying number.' Therefore, direct mail is a specific form of advertising that competes with other advertising channels such as newspapers and radio. Transactional mail is non-advertising mail and can be sent by business or private customers.

[4]See Copenhagen Economics (2012, p113) for a detailed discussion. In particular, "we observe that postal operators, to a large extent, apply volume discounts and other price incentives target at large mailers with high elasticity of demand to generate economies of scale and scope."

tention a consumer pays to each mail item they receive. Two intuitively plausible axioms are imposed, consistent with experimental and empirical evidence. Firstly, total attention to each type of mail decreases in the total quantity of that mail type received, reflecting widely observed information overload. Secondly, attention to direct mail decreases as the amount of transactional mail decreases. Underlying this is an implicit optimization that a consumer will pay more attention to their mail when they find it more worthwhile.

We show that a postal operator monopolist subsidises the price of transactional mail, potentially below cost, to stimulate the quantity of transactional mail. This increases the amount of consumer attention to direct mail and the postal operator extracts this value through inflated direct mail prices. Relative to independent pricing, this leads to higher quantities of both mail types: Transactional mail increases due to the subsidised price, whilst direct mail increases (despite the price increase) due to higher attention paid by consumers. Moreover, consumers receive a higher proportion of transactional mail. Failure to actively manage mail prices to internalise these externalities leaves significant money on the table for the postal operator and regulatory price caps can harm the postal operator, mail senders and consumers.

The tractability of our model enables us to consider several extensions. Firstly, we consider multiple mail senders of the same type that vary in the way they draw or perturb attention to postal mail. Direct mail senders that cause most nuisance and perturb consumer attention more, should pay higher prices, whilst transactional mail senders that generate particularly large positive externalities on the attention of consumers to direct mail, should be subsidised more.

Secondly, we investigate the consequences of entry into direct mail delivery, capturing recent industry developments. Whilst entry into direct mail delivery is traditionally explained by lower servicing costs than transactional mail, we provide a novel motivation based on large profit margins for direct mail arising from a price discriminating incumbent. Entry by competitors focusing exclusively on direct mail cannibalises the direct mail market to marginal cost pricing. Transactional mail prices increase, causing quantities to fall and the value of direct mail advertising diminishes, reducing the value of the industry and increasing the fraction of direct mail received by consumers. Transactional mail switches from being the low margin segment to the only profitable segment. Of particular interest to competition authorities and regulators, entry by a rival that matches the price of the incumbent, rather than engaging in competition, can benefit all agents in different ways. Therefore, some elements of collusion can be beneficial in the postal mail market.

It is helpful to spell out how our framework diverges from textbook models of platform pricing.[5] Firstly, the cross-market network effect from the recipients to the senders does not relate to the total number of recipients participating, but to the attention they each pay. Secondly, the within-group network effect between senders

[5]See Belleflame & Peitz, (2021), Chapter 5 for a textbook analysis.

is not driven by the total number of mail senders, but rather the total amount of mail sent by all senders. Thirdly, there are no within-group network effects on the recipient side; consumers are unaffected directly if their neighbour pays more attention to their postal mail. Fourthly, the division of the fee structure between recipients and senders is extreme as the transaction costs of charging recipients are prohibitive. Therefore, all revenue must derive from mail senders. Finally, the postal sector can be considered 'involuntary' multi-homing in the sense that all consumers receive any solicited and unsolicited mail sent to them, potentially from multiple postal delivery services. The more relevant question is whether they pay attention to it and how attention constraints impact pricing.

1.1 Related Literature

The study of platforms is rooted in the well-established literature on network effects, with early work focusing on the participation decisions of agents on each side of the platform (Katz & Shapiro, 1994; Rysman, 2009).[6] Closest to our work are analyses of pricing under indirect network externalities, including Armstrong (2006), Caillaud & Jullien (2001), Parker & Van Alstyne (2005) and Rochet & Tirole (2003, 2006). However, we consider agents (mail senders) on one side of the market who derive utility from the attention received from agents on the other side of the market (recipients), rather than the total measure of agents participating with the intermediary. [7]

We contribute to the growing 'competition for attention' literature (Eliaz & Spiegler, 2022; Hefti, 2018; Hefti & Liu, 2020; Masatlioglu et al, 2012; Manzini & Mariotti, 2018; Wright & Barbour, 1977; Zhong, 2021). Closest to our work, Shekhar (2020) studies advertising on platforms where single-homing consumers receive a product at zero price and may value other consumers using the same platform. Advertisements generate revenue but reduce the platform's quality to consumers. Shekhar (2020) uncovers how a price rise on one platform, which reduces advertising on that platform and therefore stimulates consumer switching towards it, reduces advertising demand for a rival platform. However, in our model consumers implicitly multi-home and the decrease in attention to mail as the mail mix deteriorates operates at the market level, rather than the firm level. Therefore, the incentive for firms to compete for attention operates even without a second postal operator platform due to external competition from other communication channels. Intensifying competition between platforms also increases advertising prices in Shekhar (2020) as competition for consumers via platform quality increases. In our model, fiercer competition amongst postal platforms leads to lower prices for advertisers (direct mail) because the platform's ability to subsidise transactional mail and inflate direct

[6] Farrell and Klemperer (2007) and Rysman (2009) provide detailed reviews, with Jullien, Pvan and Rysman (2021) providing a more recent overview and Belleflame & Peitz (2021) offering a textbook treatment. Rietveld & Schilling (2020) provide an inter-disciplinary review.

[7] The number of addresses a postal operator delivers to is fixed due to the USO mandate. Therefore, the measure of agents on the recipient side is fixed.

mail prices is shut down.

The incentive for intermediaries to cross-subsidise 'attention grabbing' products has been established by Pashigan & Gould (1998). They provide empirical evidence that leading department stores ('anchors') receive favourable prices or terms by a shopping mall operator, due to the demand generated for other mall tenants who can then be charged a rent premium. Dinerstein et al (2018) emphasises that platforms must maintain an inventory of attractive products to draw consumers to their platform. They identify a tradeoff for platforms between displaying greater variety and stimulating price competition between sellers.

Our work connects with analyses of media platforms that connect advertisers and content consumers (Anderson & Coate, 2005; Calvano & Polo, 2020; Crampes et al, 2009; Gode et al, 2009; Peitz & Valetti, 2008; Reisinger, 2012). However, the postal sector does not fit the 'advertising funded' structure of many of these platforms such as Youtube. Several studies focus on firm strategies in the postal sector. Most relevant, De Donder et al. (2011) study optimal pricing by a welfare maximising universal service provider under different competitive environments including competition from electronic substitution. They analyse the effects of asymmetric price elasticities between transactional and advertising mail. However, they assume that demands for transactional and direct mail are independent, rather than our inter-dependent network structure with attention constrained consumers. We also introduce heterogeneities within the groups of transactional and direct mail senders, which leads to additional degrees of price discrimination. Bradley et al (2015) consider a postal operator maximising profit exclusively from two kinds of direct mail: mass market (saturation) advertising and targeted advertising. Consumers vary in their match value with the sender's product and senders compete for attention from each recipient. However, they do not consider transactional mail or the network effects that arise across different mail types with attention limited recipients.

A separate literature studies the microfoundations of consumer choice under attention or time constraints, where individuals rationally choose not to obtain full information (e.g. Huettner et al, 2019). In contrast, we fix the consumer choice process to zoom in on the resulting incentives of different mail senders, postal operators and potential entrants.

2 A Model of Platform Competition with Limited Attention

Consider a market with a single postal operator acting as an intermediary between mail senders and recipients (consumers). On one side of the market there exists D independent senders of direct mail and T independent senders of transactional mail. Each sender of mail chooses their optimal quantity of mail to send based on their perceived value of the mail system and, related to this, the expected behaviour of other senders of mail within the market. No sender supplies both mail types.

On the other side of the market, a representative consumer (or equivalently a unit mass of identical consumers) receives their mail and decides how much attention to allocate to each item.

Postal Operator. The postal operator chooses the price for each mail sender. The marginal cost of servicing each unit of direct and transactional mail is constant: c_d, c_t, respectively. We refer to these marginal costs as 'servicing costs,' which include the cost of processing, sorting and delivering mail. Whilst both mail types are addressed to recipients, direct mail is typically bulk mail of a standardised size and provided to the postal operator pre-sorted into delivery regions, minimising processing costs. Transactional mail is more heterogeneous and requires greater sorting, increasing the marginal cost of delivering this mail type.[8]

> ***Assumption 1.*** The servicing cost for direct mail is less than or equal to that of transactional mail: $c_d \leq c_t$.

Direct Mail Senders. Direct mail is used by firms to advertise products or services. Mail advertising increases the probability that a consumer will purchase the product but incurs the cost of using the postal system, which is the price set by the postal operator. The extent to which additional units of advertising increase the probability that a consumer purchases the product, depends on the amount of attention consumers pay to the mail that they they receive. We abstract from the subsequent purchasing decisions of consumers to zoom in on postal pricing, welfare and policy implications of limited consumer attention.

Transactional Mail Senders. Transactional mail is used by firms or private senders to communicate information (e.g. a bank statement or private letter) and the sender derives value from the attention consumers pay to the mail they receive. We can interpret this as the transactional mail senders having an outside option of communication that transactional mail senders will increasingly use when the attention paid by consumers to their postal mail declines.

Consumers. We consider a representative consumer receiving mail.[9] Consumer attention to the mail they receive is underpinned by two observations:

> ***Observation 1: Mail Volume.*** For a given type of mail, i.e. direct or transactional, the attention a consumer allocates to each individual mail item that is received is decreasing in the total quantity of that type of mail item that is received.

> ***Observation 2: Mail Relevance.*** Consumers pay more attention to their direct mail when there is a higher quantity of transactional mail.

[8] Assumption 1 is also consistent with information from discussions with industry postal operator executives.

[9] Other studies that use a representative consumer approach in this context include De Donder et al. (2011).

Observation 1 implies that the attention a consumer allocates to each unit of direct (transactional) mail that they receive, is decreasing in the total quantity of direct (transactional) mail. Intuitively, this reflects the commonly observed psychological effect of overload for a consumer when faced with more alternatives. Obervation 2 embeds the core interdependence between transactional mail and direct mail. The idea is that the mail a consumer receives becomes more valuable when a higher quantity of the mail is highly relevant transactional mail.

Consumer Attention Function. Based on these stylised facts, we construct a simple linear Attention Function, which specifies the amount of attention a recipient devotes to a particular mail item based on the total quantity and composition of the mail a consumer receives. Formally, let A_d and A_t denote the amount of attention paid to each unit of direct and transactional mail, respectively. Let X_d and X_t denote the total amount of direct mail and transactional mail received by each consumer, where $X_d = \sum_{i=1}^{D} x_{i,d}$, $X_t = \sum_{i=1}^{T} x_{1,t}$. We can write a general attention function in the form:

$$A_t(X_d, X_t) = u_t - \Lambda X_t, \qquad A_d(X_d, X_t) = u_d - \Theta X_d + \alpha X_t \tag{1}$$

u_t, u_d are demand parameters for the two mail types and without loss of generality we simplify to $u_t = u_d = u$ unless specified. $\alpha \in [0, 1]$ captures the positive externality from transactional mail on the attention consumers pay to direct mail. Λ and Θ capture the marginal effect of an additional unit of each mail type on the attention paid to that mail type. The attention function captures a range of network effects in a simple way. When $\alpha = 0$, attention to each unit of mail decreases in the quantity of that type of mail received, capturing Observation 1. When $\alpha > 0$, transactional mail also exerts a positive externality on the attention paid to direct mail, capturing Observation 2. Therefore, α captures the strength of this positive externality. We are also able to consider incomplete information amongst the mail senders and/or the postal operator regarding α, to study the effects of neglecting these externalities.

Assumption 2. We impose $c_t < u$ to ensure positive demand.

As our primary focus lies in exploring the effect of network effects (α) across the postal platform, for tractability throughout the main analysis we let $\Lambda = \Theta = \frac{1}{2}$ to yield the following simple linear attention function:

$$A_t(X_d, X_t) = u_t - \frac{1}{2}X_t, \qquad A_d(X_d, X_t) = u_d - \frac{1}{2}X_d + \alpha X_t \tag{2}$$

The attention function replicates the standard linear demand model. More specifically, if we graph the attention function in terms of the mail received, the demand parameters correspond to the vertical intercepts (if $\alpha = 0$). Therefore, the elasticity of attention (per unit of mail) with respect to the total quantity of each mail

type, decreases as the quantity increases. That is, there exists a diminishing effect on attention as the quantity grows very large, which is intuitively plausible. Denote the value of mail to a sender, which is a function of the attention consumers pay to their mail, as: $V_t(A_t), V_d(A_d)$. For a tractable benchmark suppose: $V_t(A_t) = A_t,\ V_d(A_d) = A_d$. That is, the value of using postal mail for the sender derives from the attention recipients pay to the mail they receive.

Timing. The game develops over three stages:

> **Stage 1:** The postal monopolist chooses the price for each sender i of direct mail $p_{i,d}$ and transactional mail $p_{i,t}$.
>
> **Stage 2:** Senders of mail choose the quantity of mail that they will send to each consumer: $x_{i,t}, x_{i,d}$, where i indexes each mail sender and the t and d subscripts indicate the mail type; transactional and direct, respectively.
>
> **Stage 3:** Consumers receive the mail distributed by the postal operator and devote their desired attention to each mail item. Senders receive their payoff based on the attention that consumers pay to their mail.

It is helpful to explain that we do not expect that a consumer disposes of mail without paying *any* attention to it (in Stage 3). The distinction lies in whether the consumer pays sufficient attention to potentially act upon the information received.

2.1 Analysis with One Sender of Each Mail Type

Suppose that there is a single sender for each type of mail ($D = T = 1$) who independently choose the quantity of mail to send per recipient. We first consider the case where all network effects are known and internalised by the postal operator and mail senders. We subsequently consider the neglect of some externalities by the postal operator and mail senders. The general form of the two senders' profit maximization problems are:

$$\max_{x_t} \pi_{tms} = [V(A_t) - p_t]\, x_t, \qquad \max_{x_d} \pi_{dms} = [V(A_d) - p_d]\, x_d \tag{3}$$

$V(A_d)$ and $V(A_t)$ represent the gross value per unit of direct and transactional mail, respectively. Using the linear attention function from Section 2, the profit maximization problems for each mail sender becomes:

$$\max_{x_t} \pi_{tms} = \left(u - \frac{1}{2}x_t\right) x_t - p_t x_t, \tag{4}$$

$$\max_{x_d} \pi_{dms} = \left(u - \frac{1}{2}x_d + \alpha x_t\right) x_d - p_d x_d \tag{5}$$

The ensuing optimal demands are: $x_d^* = (1+\alpha)u - p_d - \alpha p_t$, $x_t^* = u - p_t$. Anticipating these optimal quantities of the two senders, the postal operator faces the following profit maximization problem:[10]

$$\max_{p_d,\, p_t} \pi_{PO} = p_d x_d^* + p_t x_t^* - c_d x_d^* - c_t x_t^* \tag{6}$$

The optimal prices for the two mail types obtain as:

$$p_t^M = \frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2}, \quad p_d^M = \frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2} \tag{7}$$

The price superscript, M, denotes a market with a Monopoly postal operator. The following optimal demand quantities ensue:

$$x_d^M = \frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2}, \quad x_t^M = \frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2} \tag{8}$$

We refer to the above equilibrium where the positive externality is recognised by both mail senders and the postal operator as Case 1: Fully recognised externalities.

> ***Result 1***. *In a monopoly postal market with one sender of each mail type, equilibrium mail prices and quantities follow (7) and (8). The main comparative statics (when $\alpha > 0$) can be summarised as follows:*
>
> (i) $\frac{\partial p_d^M}{\partial \alpha} > 0, \frac{\partial p_t^M}{\partial \alpha} < 0, \frac{\partial x_d^M}{\partial \alpha} > 0, \frac{\partial x_t^M}{\partial \alpha} > 0,$
>
> (ii) $\frac{\partial p_d^M}{\partial c_d} > 0, \frac{\partial p_d^M}{\partial c_t} < 0, \frac{\partial p_t^M}{\partial c_t} > 0, \frac{\partial p_t^M}{\partial c_d} > 0.$
>
> (iii) $\frac{\partial x_d^M}{\partial c_d} < 0, \frac{\partial x_d^M}{\partial c_t} < 0, \frac{\partial x_t^M}{\partial c_d} < 0, \frac{\partial x_t^M}{\partial c_t} < 0.$
>
> (iv) If $c_d < (=)\, c_t$, then $\frac{\partial p_d^M}{\partial \alpha} + \frac{\partial p_t^M}{\partial \alpha} < (=)\, 0$.
>
> (v) Total profit for the postal operator is increasing in α.

When $c_t = c_d$ and $\alpha = 0$, the mail types are priced independently at their respective monopoly prices. When $c_t > c_d$ and $\alpha = 0$, transactional mail has a higher price derived from servicing costs.

When $\alpha > 0$, senders of transactional mail exert a positive effect on the value of direct mail to the sender, which is extracted through a higher direct mail price by the postal operator. The externality incentivises the postal operator to reduce the price of transactional mail to stimulate the demand for direct mail further, yielding an effective cross-subsidisation of transactional mail prices from the higher prices

[10] A fixed cost per recipient for the postal operator would have no substantive impact on the results.

charged to direct mail senders. If servicing costs are equal ($c_t = c_d$), the increase in the price of direct mail exactly matches the decrease in the price of transactional mail. However, when transactional mail is more costly to service ($c_t > c_d$), the price of transactional mail decreases by a larger amount than the increase in the price of direct mail. The intuition is that when c_t is higher than c_d, the price of transactional mail when $\alpha = 0$ is higher. Therefore, when $\alpha > 0$ increases, larger price cuts are required to generate the same increase in the quantity demanded of transactional mail.

When $c_t > c_d$ and $\alpha > 0$, the ranking of prices is driven by the relative strength of the cost differential ($c_t - c_d > 0$) and the positive externality (α) from transactional mail to direct mail. When the positive externality is sufficiently small relative to the cost difference, transactional mail continues to be most expensive. As α increases, there exists a critical $\hat{\alpha}$ such that the higher cost of servicing transactional mail relative to direct mail is exactly offset by the value of the positive externality from transactional mail on direct mail, which leads to uniform pricing: $p_t = p_d$. Therefore, observing the absence of price discrimination by the postal operator could indicate: (a) The postal operator has neglected the positive externality and the cost differences when setting prices, (b) $\alpha = \hat{\alpha}$. For $\alpha > \hat{\alpha}$, direct mail is more expensive than transactional mail even though direct mail is cheaper for the operator to service.

Transactional mail prices can be subsidised below cost. Whilst this may appear to be a mathematical artefact, it helps to explain several 'free post' schemes offered by postal operators. For example, the 'free postcards' scheme operated by Swiss Post to stimulate the quantity of transactional mail. More generally, negative pricing of transactional mail fits with the established literature on loss-leader pricing (Tirole, 1988). The postal operator's profit is increasing in the strength of the positive externality, which yields at least two direct implications for the postal operator. Firstly, neglecting the positive externality leaves 'money on the table' for the postal operator, which we investigate in Section 2.2. Secondly, if the postal operator is able to stimulate the strength of the positive externality, profit increases further.

2.2 The Cost of Neglecting Externalities

We now explore the effect of both the postal operator and mail senders neglecting the positive externality from transactional mail to direct mail. We refer to this setting as Case 2: Fully Neglected Externalities. All parties behave as if $\alpha = 0$ leading the two mail types to be priced independently, where nI superscript denotes the neglected interdependency case:

$$p_d^{M,nI} = \frac{u + c_d}{2}, \quad p_t^{M,nI} = \frac{u + c_t}{2}, \quad x_d^{M,nI} = \frac{u - c_d}{2}, \quad x_t^{M,nI} = \frac{u - c_t}{2}. \tag{9}$$

Result 2: *If the postal operator and mail senders neglect the externality between mail types, transactional mail becomes more expensive and direct*

mail becomes cheaper. Both mail quantities decrease, the postal operator earns lower profit and consumers receive a higher proportion of direct mail.

Neglecting the network effects on the postal platform has economically significant price and non-price effects on all agents. Firstly, consumers receive a higher proportion of direct mail, which reduces the perceived relevance of postal mail and their attention to it. Secondly, the transactional mail sender pays a higher price and reduces their quantity. Thirdly, the direct mail sender suffers from postal mail being less attractive. Despite a lower direct mail price, the quantity of direct mail also falls. Finally, the postal operator earns lower profit.

In practice, mail senders may be aware of the network effects present on the platform, even if they are not internalised through pricing by the postal operator.[11] We refer to this environment as Case 3: Partially Neglected Externalities, and use the superscript nI'. Prices and the quantity of transactional mail mirror the case of no externalities ($\alpha = 0$). However, the quantity set by the direct mail sender is higher as they recognise the additional value of direct mail. Formally, prices and equilibrium quantities follow:

$$p_d^{M,nI'} = \frac{u + c_d}{2}, \quad p_t^{M,nI'} = \frac{u + c_t}{2}. \tag{10}$$

$$x_d^{M,nI'} = \frac{(1+\alpha)u - c_d - ac_t}{2}, \quad x_t^{M,nI'} = \frac{u - c_t}{2}. \tag{11}$$

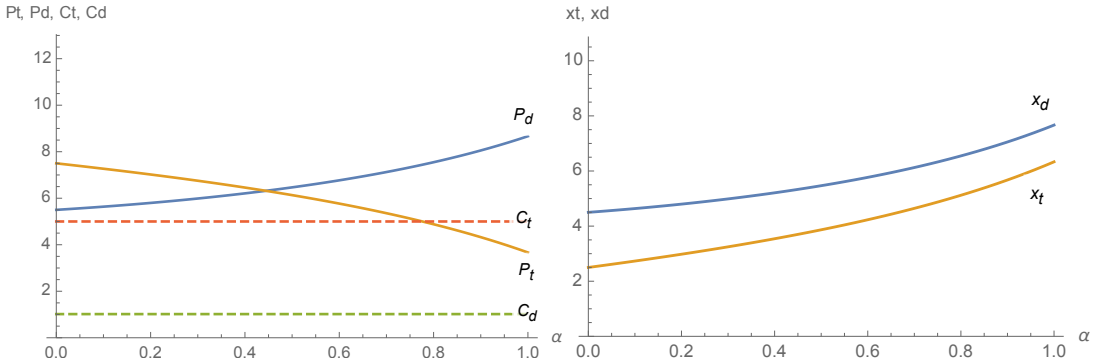
Result 3: *If only the postal operator neglects the externality between mail types, transactional mail becomes more expensive and direct mail becomes cheaper, replicating Case 2. However, the direct mail sender chooses a higher quantity than in Case 2 and appropriates the value of the positive externality. Profit for the postal operator is lower than Case 1 but higher than Case 2.*

A fundamental difference from Case 1 (where the postal operator recognises the externality) is that the surplus generated by the positive externality is appropriated by the direct mail sender, not the platform. The postal operator benefits from the sender recognising the externality as the quantity of direct mail increases. However, internalising the externality through prices would increase the postal operator's profit further. Consumers also receive the highest proportion of direct mail when only the postal operator neglects the externality.

[11] This is equivalent to the postal operator facing prohibitively high transaction costs to price discriminate between sender types.

2.3 Numerical Example

To illustrate the precise mechanisms, consider the following numerical example: $u = 10, c_t = 5 > c_d = 1$. Figure 1 illustrates the optimal prices and resulting equilibrium quantities for varying α. Costs are indicated by the dashed horizontal lines.



(a) Pricing with $c_t = 5 > c_d = 1$.

(b) Quantities with $c_t = 5 > c_d = 1$.

Figure 1: Optimal Prices & Quantities for Varying α.

When $\alpha = 0$, transactional mail is more expensive due to the higher marginal cost. Therefore, consumers receive more direct mail than transactional mail. When α increases, transactional mail becomes cheaper, to stimulate the quantity of transactional mail, which subsequently stimulates the value of postal mail to advertisers and leads to higher direct mail prices. Therefore, the ranking of prices is determined by the relative strength of the servicing cost difference (which causes transactional mail to be more expensive) and the strength of the positive externality (which causes transactional (direct) mail prices to decrease (increase)). Furthermore, when the positive externality is sufficiently strong, transactional mail is sold below cost.

Interestingly, both the price and equilibrium quantity of direct mail increase with α. The cross-subsidisation of transactional mail prices causes such a large increase in the value of postal advertising that direct mail senders choose to send more mail as α increases, despite the postal operator extracting this additional value through higher direct mail prices. The total quantity of mail consumers receive is higher but a higher proportion of this mail is transactional mail, which benefits consumers.

Profit for the postal operator increases with α, even when only the senders recognise the positive externality. The additional profit derives from the increase in the equilbrium quantity of direct mail. Profit increases further once the postal operator internalises the externality into their pricing and engages in cross-subsidisation between the mail prices. This highlights the multi-dimensional network effects that exist on the postal platform; the positive externality from one type of sender (seller) on another and the source of value being derived from the consumers on the other side of the market to the senders. If the postal operator is able to influence/stimulate

α, or at least emphasise its importance to the direct mail senders, it would earn higher profit.

2.4 Electronic Substitution of Postal Mail

How does growing electronic substitution of both mail types affect the market? When electronic substitution affects both mail types equally, u decreases, which represents an (exogenous) decrease in demand for postal mail by senders, or equivalently a decrease in attention to mail by consumers. The following result is therefore intuitive:

> ***Result 4a:*** *Exogenous decreases in mail demand (u) cause equilibrium mail prices and quantities of mail to fall. Postal operator profit decreases.*

An asymmetric decline in direct and transactional mail demand is more interesting and consistent with current postal market conditions (Copenhagen Economics, 2019). Let u_t, u_d denote the demand parameter for each mail type, where we now allow $u_t \neq u_d$.[12].

> ***Result 4b:*** *An exogenous decrease in demand for transactional mail (u_t) causes the price and equilibrium quantity of transactional mail to decrease. For $\alpha > 0$, direct mail prices and equilibrium quantity also decrease.*

An exogenous fall in demand for transactional mail causes the postal operator to reduce the price of transactional mail. Despite the price reduction, the quantity of transactional decreases. Moreover, when $\alpha > 0$, a reduction in the quantity of transactional mail erodes the value of direct mail, and therefore demand for direct mail. This causes the postal operator to also reduce the price of direct mail but, despite the price cut, the equilibrium quantity of direct mail also declines. Therefore, the postal operator is unambiguously worse off from both the direct and indirect consequences of falling demand for transactional mail. This highlights the secondary impact of electronic substitution of transactional mail.

> ***Result 4c:*** *An exogenous decrease in demand for direct mail (u_d) causes the price and equilibrium quantity of direct mail to decrease. For $\alpha > 0$, transactional mail prices increase and equilibrium quantity decrease.*

An exogenous fall in demand for direct mail also generates intricate externalities. Firstly, direct mail price and quantity both decrease. Secondly, when $\alpha > 0$, the

[12]We defer the optimal prices and quantities to the Appendix.

price of transactional mail increases. The intuition is that cross-subsidising transactional mail prices to stimulate demand for direct mail becomes less profitable. Therefore, prices revert towards their levels without the positive externality from transactional mail. This causes the quantity of transactional mail to decrease, with a third-order effect of further depressing both the price and quantity of direct mail as the value of direct mail to senders declines.

2.5 The Effect of a Single Decision-Maker Sending Both Mail Types

Some mail senders send both direct and transactional mail. For example, retail banks send individual financial statements and advertisements for new products. What is the effect on mail prices, mail quantities and postal operator profit? How does the positive externality from transactional mail to direct mail affect this market environment? Without any loss of generality, we return to the case: $u = u_d = u_t$ and the externalities are fully recognised as in Case 1.

> ***Result 5:*** *If a single decision-maker chooses the quantity of both transactional mail and direct mail:*
>
> (i) *Postal pricing follows the case* $\alpha = 0$. *Both mail quantities are higher when there is a single decision maker.*
>
> (ii) *The postal operator earns higher profit when there is one decision maker instead of separate mail senders.*
>
> (iii) *The positive externality* (α) *stimulates both mail quantities and increases postal operator profit but has no impact on prices.*

When a single decision-maker sends both direct and transactional mail, the positive externality from transactional mail to direct mail is implicitly internalised by the mail sender. Therefore, the postal operator need not subsidise transactional mail prices to increase demand as the mail sender takes into account the full benefits of sending both mail types. As the strength of the positive externality (α) increases, the mail sender chooses to send more of both mail types: Transactional mail increases as items which were not previously marginally profitable become profitable due to the large positive effect on direct mail, and direct mail quantities increase as a consequence. The mail operator benefits from the additional sales, without making any price adjustments from the case where $\alpha = 0$.

The result that the postal operator benefits from a single decision maker sending mail is initially surprising as, in some sense, the postal operator loses some market control. Interestingly, if servicing costs are uniform between mail types ($c_t = c_d$), then the postal operator sets the same prices for transactional and direct mail even when $\alpha > 0$. Therefore, limited price discrimination between mail types can be optimal in the presence of cross-market externalities ($\alpha > 0$), if a single decision maker is choosing the quantities of both mail types.

3 Competition Between Senders: The Case of Strategic Substitutes.

Our aim in this section is to explore the consequences of competition amongst senders of the same mail type and, with reference to the case with one sender of each type, isolate these from the cross-market externalities that arise in the absence of competition. This also enables us to introduce heterogeneity across senders of the same mail type in terms of their impact on recipients' attention to future mail.

We maintain the idea that all senders are supplying independent goods and each sender uses only one type of mail. This removes the potential additional competitive dimension of senders also competing for 'winner takes all' sale between multiple advertisers. To illustrate the consequences of competition between senders we sketch below the optimisation of each transactional and direct mail sender, respectively:

$$\max_{x_{1,t}} \pi_{1,tms} = \left(u - \frac{1}{2}X_t\right) x_{1,t} - p_t x_{1,t} \tag{12}$$

$$\max_{x_{1,d}} \pi_{1,dms} = \left(u - \frac{1}{2}X_d + \alpha X_t\right) x_{1,d} - p_d x_{1,d} \tag{13}$$

where $X_t = \sum_{i=1}^{T} x_{i,t},\ X_d = \sum_{i=1}^{D} x_{i,d}$. Each sender incorporates the quantity of mail sent by the other sender of the same type, which introduces an additional intra-network externality. The attention consumers pay to each unit of sender 1's mail is decreasing in the amount of mail sent by sender 2 of the same type. The optimal prices for the postal operator are given below:

$$p_d = \left[\frac{9}{9-4\alpha^2}\right]\left(\frac{u+c_d}{2} + \frac{2\alpha}{3}\left[\frac{u-c_t}{2} - \frac{2\alpha c_d}{3}\right]\right) \tag{14}$$

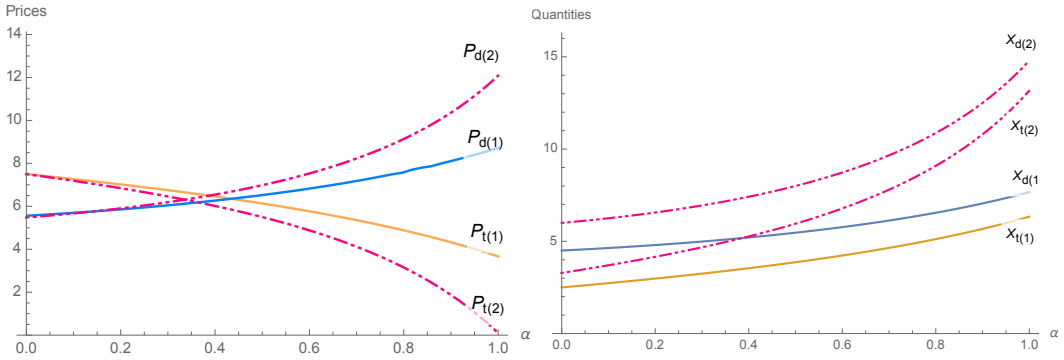
$$p_t = \left[\frac{9}{9-4\alpha^2}\right]\left(\frac{u+c_t}{2} - \frac{2\alpha}{3}\left[\frac{u-c_d}{2} + \frac{2u\alpha}{3}\right]\right) \tag{15}$$

> ***Result 6:*** *With two senders of the same mail type, prices and quantities adjust more significantly to the positive externality between different mail types (α), than with one sender of each mail type.*

The results derived with only a single sender of each mail type are robust and strengthened with multiple senders of the same type. The intuition follows that with multiple senders of transactional mail, reductions in the price of transactional mail generate larger increases in the total quantity, and therefore the value of direct mail. This leads to greater cross-subsidisation between mail types as illustrated in Figure 2, where the number of senders of each type is given in brackets.[13] Therefore,

[13] $u = 10, c_t = 5, c_d = 1$.

the effects of inter and intra-network effects in the postal sector become more salient with more mail senders.



(a) Pricing of Mail

(b) Total Quantities of Mail.

Figure 2: Prices & Quantities with Two Senders of Each Type

In practice, some transactional mail items are more effective in drawing consumers' attention to mail. Important bank statements or government communications can make postal mail more salient. Conversely, some advertisements are a particular nuisance to consumers.How should the postal operator respond to such heterogeneities? We now develop the model further to consider heterogeneities within sender types.

3.1 Heterogeneous Transactional Mail Senders

To capture heterogeneous transactional mail senders we introduce sender-specific weights into the attention function in terms of the positive externality on consumers' attention to (direct) mail. Define the consumer attention to each direct mail item as:

$$A_d(X_d, X_t) = u - \frac{1}{2}X_d + \alpha X_t^{\gamma} \tag{16}$$

where $X_t^{\gamma} = \sum_i^2 \gamma_i x_{i,t}$. $\gamma \in [0,1]$ indicates the strength of additional attention to direct mail created by transactional mail from sender $x_{i,t}$, implying $X_t^{\gamma} \leq X_t$. Adjustments to α reflect the overall magnitude of the positive externality from transactional mail on direct mail, whereas adjustments to γ_i capture a change in the relative contribution of each transactional mail sender to the positive externality. More specifically, if $\gamma = 1 \; \forall \; i$, all transactional mail has an equal positive externality on the value of direct mail. If $0 < \gamma_i = \gamma_j < 1$, the reduction from $\gamma = 1$ is equivalent to a reduction in α. If $0 \leq \gamma_i < \gamma_j \leq 1$, the positive externality generated by transactional mail $i's$ mail is lower than that of sender j.

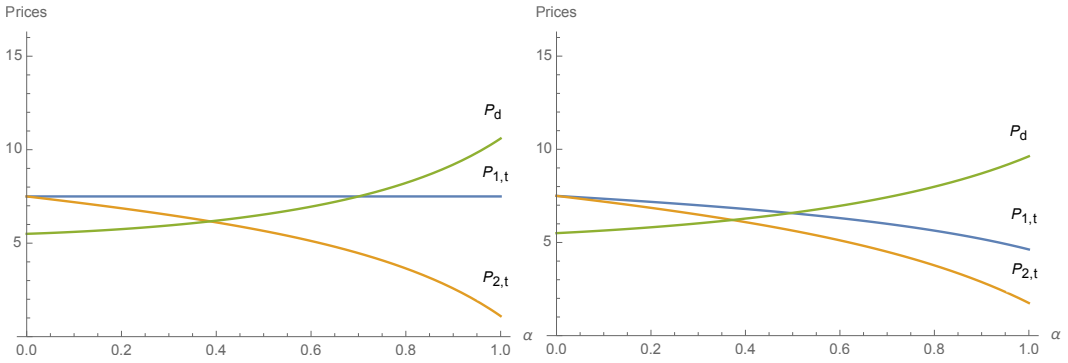
> **Result 7:** When $\alpha > 0$ and transactional mail senders vary in their positive externality ($\gamma_1 \neq \gamma_2$):

(i) If $0 \leq \gamma_2 < \gamma_1 \leq 1$ then $p_{2,t} > p_{1,t}$. Transactional mail prices decrease in α but $p_{1,t}$ decreases at a faster rate.

(ii) If $\gamma_i = 0$, then $p_{i,t}$ is independent of α.

The postal operator sets a lower price for the transactional mail sender that generates the largest positive externality on direct mail. This stimulates their mail quantity, and generates more value for the postal operator to capture through higher direct mail prices. Consumers will therefore receive more transactional mail from senders with larger positive attention externalities.

More subtle results are clear in the following examples. Figure 3a illustrates the case where only sender 2 generates a positive externality on direct mail, whilst Figure 3b illustrates the case where sender 1 exhibits a smaller positive externality than sender 2.[14] When $\gamma_1 = 0.5$, the postal operator is less reliant on transactional mail sender 2 to stimulate the value of direct mail, than when $\gamma_1 = 0$. Therefore, the price to sender 2 is higher when $\gamma_1 = 0.5$. The intuition is that the postal operator seeks to maximise the value of the mailstream as efficiently as possible, and the quantity sent by each sender becomes less elastic with successive price cuts. Therefore, the postal firm finds it optimal to reduce the cross-subsidisation through sender 2 and achieve this through sender 1.



(a) $\gamma_1 = 0, \gamma_2 = 1$ (b) $\gamma_1 = 0.5, \gamma_2 = 1$.

Figure 3: Pricing with Heterogeneous Transactional Mail Senders

3.2 Heterogeneous Direct Mail Senders

Direct mail senders (advertisers) can also have asymmetric effects on consumers' attention to postal mail. We now adapt the attention function to include heterogeneous direct mail senders (with symmetric transactional mail senders), where $X_d^{\beta} = \sum_i^2 \beta \cdot x_{i,d}$, $\beta \in [0, 1]$

[14] In each case: $u = 10, c_d = 1, c_t = 5$.

$$A_d(X_d^{\beta}, X_t) = u - \frac{1}{2}X_d^{\beta} + \alpha X_t \tag{17}$$

A larger β_1 corresponds to a greater reduction in attention to direct mail per unit of $x_{1,d}$. $\beta_1 = \beta_2 = 1$ captures the standard case where all direct mail has an equal effect. The following result is established using numerical simulations:[15]

Result 8. *If $\beta_1 > \beta_2$ then $p_{1,d} > p_{2,d}$.*

Advertisers who cause consumers to reduce their attention to mail by a larger amount are charged higher prices. This internalises the externality they impose on the value of postal mail to other senders. Importantly, this result arises when $\alpha = 0$. Therefore, the idea that mail prices should be higher for advertisers who cause consumers to reduce their attention to postal mail more appears in the absence of the attention externality between different mail types.

4 Entry into the Postal Market for Direct Mail

In recent years we have observed entry into the postal service market by competitors focusing exclusively on direct mail. Whilst the traditional explanation for entrants to focus exclusively on direct mail stems from the lower cost of processing direct mail, our analysis indicates an additional motive driven by the inflated margin on direct mail generated by depressed transactional mail prices. Our aim in this section is to explore the consequences of selective entry on pricing, mail quantities, the composition of mail consumers receive, the total value of the postal sector and the viability of the Universal Service Obligation.

Without loss of generality, we return to one sender of each mail type. Define all price, quantity, cost and profit terms relating to the entrant using $\hat{p_d}, \hat{x_d}, \hat{c_d}, \hat{\pi}$ notation. For the initial analysis we assume neither the entrant or incumbent has a cost advantage, before exploring the effects of cost asymmetry later.

Assumption 3: Cost Symmetry: $c_d = \hat{c_d}$.

With multiple competitors, we must specify how demand is allocated from mail senders if multiple postal operators tie at the lowest price in the market. We permit any deterministic sharing rule; $g, \hat{g} = (1-g),\ g \in (0,1)$, where g ($\hat{g}$) specifies the proportion of forthcoming demand received by the incumbent (entrant) when the

[15] Existing studies of the postal market that draw on numerical simulations include Boldron et al. (2011), Bradley et al. (2015), De Donder et al. (2011)..

firms tie at the same lowest price.[16] The consequences of entry can be summarised as follows.

> ***Result 9a.*** *Following entry into direct mail delivery with* $c_d = \hat{c_d}$, $g \in (0,1)$*, the unique Pure Strategy Nash Equilibrium involves:*
>
> (i) *Marginal Cost Pricing for Direct Mail.*
>
> (ii) *Transactional Mail priced at the monopoly level corresponding to the case with no positive externality from direct mail.*
>
> (iii) *Entry reduces (increases) the quantity of transactional (direct) mail received by consumers.*
>
> (iv) *The incumbent earns positive profit but below the pre-entry level. The entrant earns zero profit.*

Following entry, the direct mail market becomes a Bertrand duopoly, which yields marginal cost pricing. This shuts down the incentive for the incumbent to cross-subsidise the price of transactional mail to stimulate direct mail demand, because the incumbent cannot extract the value from the direct mail channel. Moreover, the surplus from direct mail is now captured by the sender, rather than the postal operator and prices of all mail become independent of α. The quantity of direct mail continues to be stimulated by the quantity of transactional mail through α but there is no secondary effect through the cross-subsidisation mechanism.

The sharpest observation is that the most profitable market segment is cannibalised by entry and transactional mail becomes the only profitable segment, with the incumbent unable to manage the composition of mail through prices. The effect on the total quantity of direct mail is multi-faceted. The increase in the price of transactional mail reduces the quantity of transactional mail and depresses the value of direct mail to senders. This is equivalent to a leftward shift of the demand curve for direct mail. However, the reduction in the price of direct mail causes an increase in the quantity of direct mail, equivalent to a movement down the demand curve for direct mail. Overall, the effect of the price cut dominates and the quantity of direct mail increases.

These results survive with many entrants simultaneously joining the market. The core difference is that in the absence of entry costs, there exists an infinite number of pure strategy price equilibria for direct mail; providing at least two firms set marginal cost pricing, a third (or more) firms can set any price above marginal cost and this constitutes a Bertrand equilibrium. There is no impact on transactional mail pricing or profit for any firm but the incumbent may become inactive in the

[16] The standard equal sharing rule is a special case when $g = 0.5$. However, our general rule extends readily to postal markets with a legacy provider with greater market prominence. The results also hold for a random sharing rule, such as the 'winner takes all' approach where one firm from those charging the lowest price is chosen randomly to serve the entire market demand forthcoming at that price. However, this appears less practically relevant to the postal sector.

direct mail market. The intuition is that the incumbent is indifferent between pricing direct mail at marginal cost and sharing the forthcoming demand whilst earning zero profit per unit, or pricing above marginal cost and attracting zero demand.

4.1 Tacit Price Coordination.

Can the entrant avoid the direct mail market from being cannibalised by matching the price set by the incumbent, instead of competing? Is this an equilibrium strategy? We first consider the case where an entrant price-matches and the incumbent cannot adjust the (now collusive) price without invoking a price war. We subsequently consider the optimal collusive price between the two operators.

> ***Result 9b.*** *If the entrant matches the monopoly price of direct mail set by the incumbent in the absence of entry:*
>
> (i) *The entrant earns higher profit than in the competitive outcome. The entrant can earn a higher or lower profit than the incumbent, depending on α.*
>
> (ii) *The incumbent may prefer price matching or competing, depending on α and the sharing rule (g). Incumbent profit can be decreasing in α.*
>
> (iii) *Consumers receive a more preferable mailmix than under competitive entry: More transactional mail and less direct mail.*

Tacit coordination is the weakly dominant strategy for the entrant because only if both firms coordinate can the entrant earn positive profit. The motivation for the entrant to coordinate differs subtly from traditional collusion arrangements of price-fixing, where prices are inflated to extract higher profit from a given market demand curve. In our framework, coordination on direct mail prices incentivises the incumbent to continue engaging in cross-subsidisation of transactional mail, which stimulates the value of the market to senders and shifts the demand curve for direct mail rightwards.

The incumbent's incentives are more intricate. At the coordinated price, the entrant steals a share of the profitable direct mail market, whilst the incumbent bears the cost of stimulating the value of direct mail through subsidised transactional mail prices. On one hand, the incumbent can coordinate and accept the loss of a portion of direct mail demand whilst making lower/negative profit on transactional mail. On the other hand, the incumbent can compete, earn zero profit from direct mail and extract the monopoly profit from transactional mail. Therefore, the share of the direct mail market taken by the entrant and the degree to which transactional mail prices are depressed prior to entry, which is driven by the positive externality from transactional mail to direct mail (α), are critical.

The reason incumbent profit can be decreasing in α arises from the inability to adjust prices following entry without generating competition to marginal cost for direct mail. Prior to entry, the incumbent sets the monopoly price and cross-subsidises transactional mail potentially below cost. Following entry, the incumbent would prefer to reduce the degree of cross-subsidisation but cannot do so without invoking competition.

The incidence of economic harm created by coordination is multi-faceted: Postal operators gain higher profit and transactional mail senders benefit from a lower price. Direct mail senders benefit from a more valuable mail channel, though this is appropriated by the postal operator(s) in the form of higher direct mail prices. Most surprisingly, consumers can benefit from tacit price coordination by postal operators because the mailmix improves. More precisely, consumers receive more transactional mail and less direct mail under price coordination than competition.

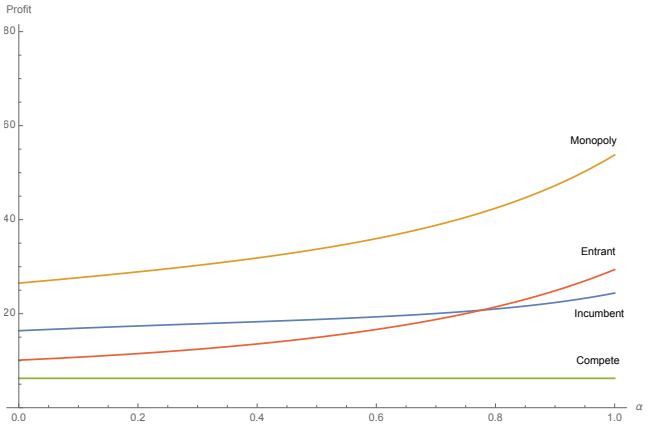


Figure 4: Coordination vs Competition Profit

Figure 4 illustrates the main insights ($u = 10, c_d = 1, c_t = 5, g = 0.5$). 'Incumbent' and 'Entrant' correspond to price coordination profits for each postal operator. 'Monopoly' and 'Compete' correspond to the profit of the incumbent pre-entry and post-entry following competition, respectively. In this example, incumbent profit is always higher under coordinated pricing than competing. The value of α at which entrant profit exceeds incumbent profit reflects the point at which the incumbent is pricing transactional mail below cost.

Under price coordination, the entrant's profit increases more sharply with the positive externality than the incumbent's and the entrant can earn a higher profit. The intuition is that the entrant benefits from the higher price of direct mail as α increases, without bearing the cost of the cross-subsidisation. The incumbent can prefer to compete depending on α and g. In this case, the entrant is stealing such a large share of the profitable direct mail demand that the incumbent switches to competing on direct mail and extracting profit from transactional mail.

Figure 5 illustrates the monopoly pre-entry profit, competitive profit post-entry and coordinated profits post-entry, when the incumbent receives 90%, 50% or 10% of the direct mail demand. When the incumbent receives the majority of the direct mail demand (90%) at the coordinated price, coordination dominates the incentive to compete and profit increases with α: The benefit of the cross-subsidisation outweighs the cost. When the incumbent receives a minority share of direct mail demand (10%) the cost of cross-subsidisation outweighs the benefit of the higher direct mail prices/quantities. Profit decreases with α as the incumbent loses such a large share of the profitable direct mail demand to the entrant and competition can become preferable. Therefore, the positive externality across the mail types can impact on both market structure and the incentive to coordinate.

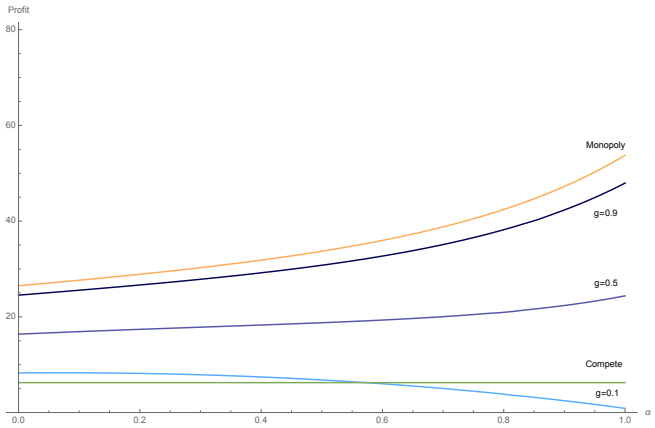


Figure 5: Coordination vs Competition: The Role of the Sharing Rule

Beyond considering the incentive for the incumbent to accept price coordination or engage in competition, the incumbent may adjust prices in anticipation of entry and accept coordination at the new adjusted prices.

> ***Result 9c.*** *If the incumbent anticipates entry by a price-matching competitor on direct mail, the incumbent increases (decreases) transactional (direct) mail prices.*

Price cuts prior to potential entry can traditionally be interpreted as entry deterrence but this is not the motivation. In this market, the incumbent's incentive to cross-subsidise transactional mail prices is weakened by the loss of a share of the direct mail to the entrant. Therefore, the price of transactional mail increases and direct mail decreases.

4.2 Entry by a More Efficient Entrant

Cost asymmetries between the incumbent and entrant generate more intricate pricing dynamics, which depend on the extent of the cost advantage of the entrant. Define the monopoly price of the entrant as:

$$\hat{p_d^{**}} = argmax\{(\hat{p_d} - \hat{c_d})x_d^{**}\}$$

where x_d^{**} is the resulting demand for direct mail when only the entrant services direct mail. The impact depends on the magnitude of the cost advantage.

> **Definition: Large and Small Cost Advantages.** For a Large (Small) cost advantage, the monopoly price of the entrant is below (above or equal to) the incumbent's marginal cost $\hat{p_d^{**}} < c_d$ ($\hat{p_d^{**}} \geq c_d$).

> ***Result 9d:*** *When the entrant holds a* ***large*** *cost advantage, the Pure Strategy Nash Equilibrium involves:*
>
> *(i) The incumbent focuses exclusively on transactional mail. p_t, x_t replicate the case where $\alpha = 0$. As with symmetric costs, entry increases transactional mail price (p_t) and reduces quantity (x_t).*
>
> *(ii) The entrant monopolises direct mail and chooses the monopoly price.*
>
> *(iii) The equilibrium quantity of direct mail can be higher or lower following entry.*

Entry by a firm with a *large* cost advantage leads to market separation; the incumbent (entrant) focuses only on transactional (direct) mail. Following entry, the incumbent increases the transactional mail price as there is no incentive to engage in cross-subsidisation. Therefore, the equilibrium quantity of transactional mail and incumbent profit decrease.[17]

The entrant sets a lower price for direct mail than the incumbent would choose in the absence of entry, for two reasons. Firstly, the incumbent would have reduced transactional mail prices to stimulate demand and captured this value through higher direct mail prices. Secondly, the cost advantage of the entrant reduces the entrant's monopoly price. The entrant earns a higher profit than the incumbent. This stems from the lower marginal cost of servicing direct mail than transactional mail, which is exacerbated by the cost advantage of the entrant, and the additional profitability of direct mail due to the positive externality from transactional mail.[18]

[17] The optimal price and quantity for the entrant are: $\hat{p_d^*} = \frac{u+\hat{c_d}}{2} + \frac{\alpha(u-c_t)}{4}$, $\hat{x_d^*} = \frac{(u-\hat{c_d})}{2} + \frac{\alpha(u-c_t)}{4}$

[18] These two drivers are most clearly visible by comparing the profit expressions of the incumbent and the entrant: $\pi = \left(\frac{u-c_t}{2}\right)^2$, $\hat{\pi} = \left(\frac{u-\hat{c_d}}{2} + \frac{\alpha(u-c_t)}{4}\right)^2$.

The equilibrium quantity of direct mail can be higher or lower following entry. This depends on the extent of the cost difference: $c_d - \hat{c_d}$, relative to the value of the positive externality from transactional mail (α).[19] When α is small relative to the cost difference ($\hat{c_d} - c_d$), the quantity of direct mail increases following entry. The reduction in price from the entrant's cost advantage more than compensates for the reduction in the value of the direct mail channel arising from the fall in the quantity of transactional mail. However, when α is large relative to the cost difference ($\hat{c_d} - c_d$), the quantity of direct mail falls following entry. The positive externality from transactional mail becomes a more important driver of direct mail demand than the price cut the entrant can offer due to their efficiency.

> ***Result 9e.*** *When an entrant holds a* ***small*** *cost advantage, there exists no pure strategy Nash equilibrium.*

The non-existence of Pure Strategy Nash Equilbrium in this context relates to the well-known open-set problem created by the possibility of infinitesimally small undercutting deviations in price games.[20] Whilst the formal analysis is not defined, the implication for practitioners is that the entrant would undercut at the highest price possible without allowing the incumbent to operate in that market segment. Therefore, market separation reappears. More relevant for practitioners, increasing α and/or decreasing c_t can enable the incumbent to contest the direct mail market. Formally, we move from the large cost advantage case to the small cost advantage case. The intuition is that as α increases (or c_t decreases), the entrant's monopoly price for direct mail increases and competing becomes viable for the incumbent.

5 Discussion, Policy Implications & Conclusion

5.1 Universal Service Obligations

Universal Service Obligations, which enforce efficiency expectations and mandate geographically-uniform and affordable prices, are becoming increasingly difficult to sustain (House of Commons, 2015). Existing policy debates highlight a strategic tradeoff between enhancing competition and preserving the financial viability of the universal service provider (Copenhagen Economics, 2019, p. 35). A recent report

[19] For the quantity of direct mail to be higher before entry, we require:

$$2\alpha^2(u - \hat{c_d}) + \alpha^3(u - c_t) > 8(c_d - \hat{c_d}).$$

[20] See Vives (1999) for a textbook treatment. If both firms price above the incumbent's marginal cost, either firm can profitably undercut to secure the entire demand at a negligibly lower price. If both firms price below the incumbent's marginal cost, the incumbent would deviate to a higher price to earn non-negative profit. If both firms price at the marginal cost of the incumbent, the entrant can profitably undercut to a marginally lower price. However, this resulting position cannot be an equilibrium because the price of the more efficient seller: $p_d = c_d - \epsilon,\ \epsilon > 0$, is undefined.

for the European Parliament discussed possible remedies including adapting the degree of state intervention in response to changes in postal users needs, weakening the requirements of USO and prioritising the financial viability of USO's above enhancing competition (Copenhagen Economics, 2019, p. 10.).

We have two main contributions here. Firstly, we have demonstrated that in the absence of USO's it can be optimal for a postal operator to continue to service transactional mail at a price below cost, due to the positive externalities on other revenue streams. Therefore, the requirement to service transactional mail is weaker than initially appears. Secondly, we present a novel rationale for competition from entrants who specialise in servicing only direct mail to be harmful to stakeholders across the mail industry, especially once non-price factors are included in the utility of mail senders and recipients.

5.2 Price Regulation

In many countries, postal operators are constrained by price regulations. For example, in the UK, Ofcom imposes a price cap on second class mail to ensure a basic, affordable service for all consumers.[21] Recent policy debates surrounding the European Parliament discuss the need for more flexible pricing, partially to sustain USO's. This paper highlights an additional consideration for price regulation, which does not currently feature in the policy debate; price regulation on some services can constrain the postal operator from internalizing market externalities around the platform. This reduces the industry value, total mail quantities and the viability of the postal operator incumbent.

For instance, suppose the price of direct mail faces a price ceiling below the optimal level identified in Figure 1. This prevents the postal operator from engaging in full cross-subsidisation between transactional and direct mail, which leads to a lower quantity of both mail types and higher prices for transactional mail. Therefore, policy regulation and antitrust more broadly should be mindful of the network of externalities present on the postal platform.

5.3 Extensions

Our model takes the behaviour of mail recipients as exogenous, to zoom in on the effects of competition on the postal platform and focus on pricing strategies and policy. We followed a representative consumer approach, similar to De Donder et al. (2011), and we have not attempted to capture heterogeneous match values between heterogeneous consumers and mail senders. For instance, some consumers will prefer mail tailored to their tastes and preferences, and in the digital era it is possible for advertisers to identify target consumers. Introducing further layers of heterogeneity remains an interesting opportunity for future work. However, recent

[21] Further details: www.ofcom.org.uk/about-ofcom/latest/media/analysts/regulated-prices.

theory and evidence indicates that even when advertisers can target their preferred consumers at low cost, advertisers continue to use mass market campaigns that address all consumers because recipients have limited attention (Hefti & Liu, 2020). Therefore, advertisers continue to target all buyers, including those with low match values for their products, because the consumer possesses insufficient attention to consider the full range of market alternatives and identify their ideal product. In this setting, our approach of a representative recipient remains appropriate.

5.4 Concluding Remarks

This paper has explored the postal mail sector as a platform connecting different types of mail senders with recipients of mail. Against a background of increasing pressure towards alternative communication methods, primarily from electronic substitution, our analysis highlights the importance of incorporating limited consumer attention into the price determination process in this industry. More specifically, we identify multiple layers of inter-related price discrimination across the senders of mail, including both discrimination across senders of different mail types, and discrimination within the pool of senders of the same mail type.

The main mechanism that we uncover is the discounting of transactional mail prices to stimulate the quantity of transactional mail sent, which increases the attention recipients pay to the direct mail they receive and makes postal mail more valuable and attractive for senders of direct mail (advertisers). The postal operator subsequently extracts this additional value by inflating prices to direct mail senders and overall mail volumes increase for both mail types.

We provide a new rationale for entry into the servicing of direct mail, not driven by the lower cost of processing direct mail compared to transactional mail. Instead, direct mail is attractive due to the cross-subsidisation of transactional mail prices by higher mark-ups on direct mail, which signals profitable opportunities for market entry in this segment. However, we show how competitive entry can cannibalise the direct mail market and transactional mail becomes the profitable segment, unless the entrant chooses to match rather than compete on prices with the incumbent.

Unusually, tacit coordination can bring benefits for all postal operators, senders of both types of mail, and mail recipients. More specifically, postal operators are able to earn higher profit from the cross-subsidisation of transactional mail prices from direct mail prices that is shut down by competitive entry. Relative to the case of competitive entry, direct mail senders benefit from recipients paying more attention to their mail, transactional mail senders benefit from lower mail prices and recipients benefit from an improved composition of the mail they receive as the proportion of advertising mail decreases.

Most broadly, we show that Universal Service Obligations imposed on postal sector incumbents are less demanding than traditionally viewed: Whilst transactional mail items are more costly to process and deliver, the incumbent can extract additional

value from the direct mail channel by servicing transactional mail. Overall, the postal operator incumbent can find it optimal to service all mail types without regulatory mandates.

Senders of direct mail may be concerned not only with the amount of direct mail sent to consumers but also the content of other direct mail. For instance, a sender may prefer that other direct mail advertising sent to consumers contains complementary or unrelated proucts, rather than substitutes. This introduces an additional competitive dimension for senders beyond competition for attention. Incorporating content complementarities and substitutes between different mail senders would enrich the analysis further and provides an interesting direction for further research.

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Appendix

This appendix contains full details and proofs of the results contained within the paper.

Proof of Result 1 (Fully Recognised Externalities).

The main result follows from the analysis in the main text. The comparative statics are as follows:

(i)

$$\frac{\partial p_d^M}{\partial \alpha} = \frac{4(u - c_t) + 4\alpha(u - c_d) + \alpha^2(u - c_t)}{(4 - \alpha^2)^2} > 0$$

Note that: $u > c_t, u > c_d$. Hence the numerator is positive.

$$\frac{\partial p_t^M}{\partial \alpha} = \frac{4(c_d - u) + \alpha^2(c_d - u) + 4\alpha(c_t - u)}{(4 - \alpha^2)^2} < 0$$

$$\frac{\partial x_d^M}{\partial \alpha} = \frac{(4 + \alpha^2)(u - c_t) + \alpha^2(u - c_t)}{(4 - \alpha^2)^2} > 0$$

$$\frac{\partial x_d^M}{\partial \alpha} = \frac{(4 + \alpha^2)(u - c_d) + 4\alpha(u - c_t)}{(4 - \alpha^2)^2} < 0$$

(ii) The following comparative statics are clear by inspecting the expressions in the main text: $\frac{\partial p_d^M}{\partial c_d} > 0$, $\frac{\partial p_d^M}{\partial c_t} < 0$, $\frac{\partial p_t^M}{\partial c_t} > 0$, $\frac{\partial p_t^M}{\partial c_d} > 0$.

(iii) The following comparative statics are clear by inspecting the expressions in the main text: $\frac{\partial x_d^M}{\partial c_d} < 0$, $\frac{\partial x_d^M}{\partial c_t} < 0$, $\frac{\partial x_t^M}{\partial c_d} < 0$, $\frac{\partial x_t^M}{\partial c_t} < 0$.

(iv)

$$\frac{\partial p_d^{M,I}}{\partial \alpha} + \frac{\partial p_t^{M,I}}{\partial \alpha} = \frac{(-4 + 4\alpha - \alpha^2)(c_t - c_d)}{(4 - \alpha^2)^2} < 0$$

Note that by definition: $c_t \geq c_d$.

(v) *Claim: Postal operator profit increases with α.*

We establish this result using the properties of the Nash Equilibrium.

Step 1: Suppose to the contrary that postal operator profit does not increase with α. If this were the case, the postal operator could price as if $\alpha = 0$.

In this case, prices become independent of α, the quantity of transactional mail is independent of α but the quantity of direct mail sent increases. To see this, note

that the best response function of the direct mail sender is: $x_d^* = (1+\alpha)u - p_d - \alpha p_t$. Therefore, if the postal operator sets prices as if $\alpha = 0$, profit increases with α.

Step 2: We have established that profit for the postal operator increases with α even when prices do not vary with α. By definition of the Nash equilibrium, the postal operator will only adjust prices with α if profit increases further by adjusting prices with α. Therefore, by reducing the price of transactional mail and increasing the price of direct mail, as α increases, this implies that profit must be increasing with α. The level of profit associated with a lower value of α could always be achieved when α is higher.

Proof of Result 2 (Fully Neglected Externalities).

The proof follows from the main model when $\alpha = 0$. The result that profit for the postal operator decreases when all parties neglect α follows from Result 1, where profit is increasing in α.

Proof of Result 3 (Partially Neglected Externalities).

The two senders' profit maximization problems become:

$$\max_{x_d} \pi_{dms} = (u + \alpha x_t)x_d - \frac{1}{2}x_d^2 - p_d x_d \tag{18}$$

$$\max_{x_t} \pi_{tms} = ux_t - \frac{1}{2}x_t^2 - p_t x_t, \tag{19}$$

The ensuing optimal quantities are given by:

$$x_d^* = (1+\alpha)u - p_d - \alpha p_t, \quad x_t^* = u - p_t. \tag{20}$$

The incumbent operator fails to recognise the positive externality and does not internalise it into their optimal pricing decisions. As a consequence, equilibrium prices follow the case $\alpha = 0$:

$$p_d^{M,nI'} = \frac{u + c_d}{2}, \quad p_t^{M,nI'} = \frac{u + c_t}{2}. \tag{21}$$

The ensuing demand quantities, where senders internalise the externality, are:

$$x_d^{M,nI'} = \frac{(1+\alpha)u - c_d - ac_t}{2}, \quad x_t^{M,nI'} = \frac{u - c_t}{2}. \tag{22}$$

To see that profit is higher than Case 2 (where all parties neglect α) but lower than Case 1 (where all parties internalise α), we use the same proof as in Result 1. We

established that if the postal operator acts as if $\alpha = 0$ but the senders recognise $\alpha > 0$, postal operator profit increases with α. Therefore, profit is lowest when all parties neglect α. However, the observation that the postal operator adjusts prices with α must, by definition of the Nash equilibrium, indicate that profit is highest in Case 1 where α is fully internalised in prices.

Proof of Result 4a.

The result follows immediately from the expressions for price and quantity in the main paper.

Proof of Result 4b.

The resulting expressions for the optimal price and quantity are below. The result follows immediately from these expressions.

$$p_d^M = \frac{2u_d + \alpha u_t + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2}, \quad p_t^M = \frac{(2-\alpha^2)u_t - \alpha u_d + \alpha c_d + 2c_t}{4-\alpha^2} \tag{23}$$

$$x_d^M = \frac{2u_d + \alpha u_t - \alpha c_t - 2c_d}{4-\alpha^2}, \quad x_t^M = \frac{2u_t + \alpha u_t - \alpha c_d - 2c_t}{4-\alpha^2} \tag{24}$$

Proof of Result 4c.

The result follows immediately from the expressions for price and quantity above.

Proof of Result 5 (Single Decision Maker for Both Mail Types).

In the case of a single decision maker sending both types of mail, the single mail sender will maximise:

$$\max_{x_t, x_d} \pi = [V(A_t) - p_t]\, x_t + [V(A_d) - p_d]\, x_d \tag{25}$$

Inserting the linear attention function:

$$\max_{x_t, x_d} \pi = (u + \alpha x_t)x_d - \frac{1}{2}x_d^2 - p_d x_d + u x_t - \frac{1}{2}x_t^2 - p_t x_t.$$

Taking the FOC's and solving yield:

$$x_d = u - p_d + \alpha x_t$$

$$x_t = u - p_t + \alpha x_d$$

Remark. In the case with independent decision makers for the senders, the initial FOC's for the direct mail sender is unchanged but the FOC for the transactional mail sender would be $x_t = u - p_t$. Therefore, for given prices, the best response indicates that the quantity demanded of transactional mail is higher when there is a single decision maker sending the mail. This does not in itself demonstrate that the final quantity will be higher with only one single decision maker, as the postal operator may adjust prices in response. However, it does indicate that postal operator profit will be higher when there is a single decision maker sending both types of mail, rather than independent decision makers.

Solving the First Order Conditions above yields:

$$x_t = \frac{u - p_t + \alpha u - \alpha p_d}{1 - \alpha^2}$$

$$x_d = \frac{u - p_d + \alpha u - \alpha p_t}{1 - \alpha^2}$$

We now solve the postal operator's maximisation problem to solve for the optimal prices, conditional on the demand functions above.

$$\pi_{PO} = (p_t - c_t)x_t + (p_d - c_d)x_d$$

Inserting the best response functions for x_t and x_d yields:

$$\pi_{PO} = (p_t - c_t)\left(\frac{u - p_t + \alpha u - \alpha p_d}{1 - \alpha^2}\right) + (p_d - c_d)\left(\frac{u - p_d + \alpha u - \alpha p_t}{1 - \alpha^2}\right)$$

Taking the FOC's yields:

$$p_t = \frac{u + \alpha u + c_t + \alpha c_d}{2} - p_d, \quad p_d = \frac{u + \alpha u + \alpha c_t + c_d}{2} - \alpha p_t$$

Solving the FOC's yields:

$$p_t = \frac{u + c_t}{2}, \quad p_t = \frac{u + c_d}{2}$$

Note that these are the same as the main model when $\alpha = 0$. Inserting the equilibrium prices into the quantity expressions above yields:

$$x_t = \frac{u + \alpha u - c_t - \alpha c_d}{2(1-\alpha^2)}, \quad x_d = \frac{u + \alpha u - c_d - \alpha c_t}{2(1-\alpha^2)}$$

Comparisons with standard case of independent mail senders.

Claim: The price of direct mail is lower and the price of transactional mail is higher, when there is a single decision maker choosing both quantities of mail than when there are independent decision makers for the two mail types.

Proof: Prices when there is a single decision maker are equivalent to the case of $\alpha = 0$. With independent decision makers, p_t decreases with α and p_d increases with α. With a single decision maker this is no longer true.

Claim: Transactional mail equilibrium quantity is higher when there is a single decision maker.

Proof: In the case of independent decision makers: $x_t^M = \frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2}$. With a single decision maker: $x_t = \frac{u+\alpha u - c_t - \alpha c_d}{2(1-\alpha^2)}$. It can be shown that the case with a single decision maker is higher.

Claim: Direct mail equilibrium quantity is higher when there is a single decision maker.

Proof: In the case of independent decision makers: $x_d^M = \frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2}$. With a single decision maker: $x_t = \frac{u+\alpha u - c_t - \alpha c_d}{2(1-\alpha^2)}$. It can be shown that the case with a single decision maker is higher.

Claim: Postal Operator profit is higher when there is a single decision maker than independent decision makers.

Proof: To see this most simply, observe from the initial first order conditions for x_t, x_d that when prices p_d, p_t are fixed, the quantity of transactional mail would be higher when there is a single decision maker, whilst the quantity of direct mail would be the same. Therefore, if prices were to remain unchanged from the case with independent decision makers, when there is a single decision maker, profit would be higher with a single decision maker. In the Nash Equilibrium, the postal operator finds it optimal to adjust prices to take account of the single decision maker. This must yield an even higher profit than could be obtained by pricing as if there are independent decision makers.

Proof of Result 6 (Two Senders of Each Mail Type)

Final Expressions for the Two Sender Model: This section solves the expressions for closed form solutions. To begin, sub $p_{2,t}$ into $p_{1,t}$ to yield:

$$p_{1,t} = \frac{u + c_t}{2} - \frac{2\alpha\gamma_1(p_d - c_d)}{3}$$

By symmetry of the optimisation process:

$$p_{2,t} = \frac{u + c_t}{2} - \frac{2\alpha\gamma_2(p_d - c_d)}{3}$$

Substituting these values into the expression for p_d and simplifying:

$$p_d^* = \frac{9}{9 - 4\alpha^2(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)} \left[\frac{u + c_d}{2} + \frac{\alpha}{3}\left[\frac{(u - c_t)(\gamma_1 + \gamma_2)}{2} - \frac{4\alpha c_d(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}{3}\right]\right]$$

$$p_{1,t}^* = \frac{u + c_t}{2} + \frac{2\alpha\gamma_1}{3}\left(c_d - \frac{9}{9 - 4\alpha^2(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}\left[\frac{u + c_d}{2} + \frac{\alpha}{3}\left[\frac{(u - c_t)(\gamma_1 + \gamma_2)}{2} - \frac{4\alpha c_d(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}{3}\right]\right]\right)$$

$$p_{2,t}^* = \frac{u + c_t}{2} + \frac{2\alpha\gamma_2}{3}\left(c_d - \frac{9}{9 - 4\alpha^2(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}\left[\frac{u + c_d}{2} + \frac{\alpha}{3}\left[\frac{(u - c_t)(\gamma_1 + \gamma_2)}{2} - \frac{4\alpha c_d(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}{3}\right]\right]\right)$$

Proof of Result 7 (Heterogeneous Transactional Mail Senders).

From Result 6, when there are two senders of the same mail type, the first order conditions for the transactional mail senders' optimisations are:

$$p_{1,t} = \frac{u + c_t}{2} - \frac{2\alpha\gamma_1(p_d - c_d)}{3} \tag{26}$$

$$p_{2,t} = \frac{u + c_t}{2} - \frac{2\alpha\gamma_2(p_d - c_d)}{3} \tag{27}$$

Therefore, using (26) and (27) , if $\gamma_1 > \gamma_2$, then $p_{1,t} < p_{2,t}$.

Proof of Result 8 (Heterogeneous Direct Mail Senders).

This result is established using numerical simulations and a numerical example is given in the main text.

Proof of Result 9a (Entry)

Step 1: We first establish marginal cost pricing for direct mail. It is well-known that in Bertrand price competition with constant marginal costs, the unique pure strategy equilibrium involves marginal cost pricing. Therefore, $p_d = \hat{p_d} = c_d = \hat{c_d}$.

The equilibrium must involve symmetric prices: $p_d = \hat{p_d}$. For any $p_d > \hat{p_d} > c_d$, the incumbent obtains zero demand for direct mail. There exists a profitable deviation to $p_d = \hat{p_d} - \epsilon > c_d$, $\epsilon > 0$. (Similarly for $\hat{p_d} > p_d > c_d$). Any pair of prices involving at least one firm choosing a price below marginal cost also cannot be an equilibrium.

For any $p_d = \hat{p_d} > c_d$, there exists a profitable deviation for either firm to marginally undercut to obtain the entire market demand for direct mail.

Step 2: Fix the price of direct mail at c_d. The optimisation of the direct mail sender becomes:

$$\pi_{dms} = \left(u + \alpha x_t - \frac{1}{2}x_d\right) x_d - c_d x_d$$

where x_d refers to the total quantity of direct mail sent by the single direct mail sender. This quantity may be alllocated to one postal operator of split between both operators if prices are tied (which we will show is the case). Also note in the expression for the direct mail sender: $p_d = c_d$ in the final term from Step 1.

The best response yields: $x_d = u + \alpha x_t - c_d$.

Step 3: We now solve for the transactional mail sender's optimal quantity: $x_t = u - p_t$.

Step 4: We now determine the optimal price set by the incumbent for transactional mail, which follows the no-interdependence case. The intuition is clear when writing the profit function of the incumbent:

$$\pi = (p_d - c_d)x_t + (p_d - c_d)x_d$$

where the second term is zero as $p_d = c_d$ due to entry. Therefore, $p_t^* = \frac{u+c_t}{2}$. Using the best response in Step 4; $x_t^* = \frac{u-c_t}{2}$.

Step 5: The total quantity of direct mail is: $x_d = u - c_d + \alpha(\frac{u-c_t}{2})$. Note that this is the total quantity of direct mail sent by the direct mail sender and each postal operator receives their share.

Step 6: The entrant earns zero profit and the incumbent only derives profit from direct mail, which is equivalent to the profit the firm would earn without entry if there were no-interdependence between the main types. Profit for the incumbent is: $(p_t - c_t)x_t = (\frac{u-c_t}{2})^2$.

Claim: x_d is higher following entry by a competitive entrant into the direct mail market than with only the monopoly incumbent operating in the market.

Step 1: x_d without entry is $\frac{(1+\alpha)u-2c_d-\alpha c_t}{4-\alpha^2}$ and with competitive entry is $\frac{4u-4c_d+2\alpha u-2\alpha c_t}{4}$.

Step 2: Therefore, we need to show:

$$\frac{(1+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2} < \frac{4u - 4c_d + 2\alpha u - 2\alpha c_t}{4}$$

Proceeding as follows:

$$\frac{4(u + \alpha u - 2c_d - \alpha c_t)}{1} < \frac{(4-\alpha^2)(4u - 4c_d + 2\alpha u - 2\alpha c_t)}{1}$$

This simplifies to:

$$0 < (4 - 2\alpha^2)(u - c_d) + (u - c_t)(2\alpha - \alpha^3)$$

This condition is satisfied.

Proof of Result 9b (The Incentive to Coordinate vs Compete).

Firstly, we can write the profit for the incumbent prior to entry, using the optimal prices and quantities taken from the fully recognised interdependency case of the single sender model. The required optimal values are:

$$p_d^M = \frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2}, \quad p_t^M = \frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2}$$

$$x_d^M = \frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2}, \quad x_t^M = \frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2}$$

Therefore;

$$\pi_{PO}^{M,I} = (p_t^M - c_t)x_t^M + (p_d^M - c_d)x_d^M$$

$$\pi_{PO}^M = \left(\frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2} - c_t\right)\left[\frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2}\right]$$

$$+\left(\frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2} - c_d\right)\left[\frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2}\right]$$

Secondly, write the incumbent's profit post-entry by an entrant who takes away some of the direct mail demand. The incumbent retains share g. This is on the assumption of price coordination.

$$\pi_{PO} = \left(\frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2} - c_t\right)\left[\frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2}\right]$$

$$+g \cdot \left(\frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2} - c_d\right)\left[\frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2}\right]$$

Thirdly, we can write the profit for the entrant when engaging in coordination. This is simply the share $(1-g)$ of the total transactional mail profit from the incumbent prior to entry.

$$\pi = (1-g) \cdot \left(\frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2} - c_d\right)\left[\frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2}\right]$$

Fourthly if we have a case of competition, the incumbent drives the price and profit of direct mail to zero. The profit for the incumbent follows the profit from transactional mail only in the no-interdependency case from the single sender model:

$$p_d^{M,nI} = \frac{u + c_d}{2},\ x_d^{M,nI} = \frac{u - c_d}{2}$$

Therefore, in that case $\pi = (\frac{(u-c_d)}{2})^2$ for the incumbent. We can use these expressions to demonstrate the claims in Result 9b.

Proof of Result 9c (Anticipating Price Matching Entrant).

The postal operator will maximise:

$$\max_{p_d, p_t} \pi = (p_t - c_t)x_t^* + (p_d - c_d)\frac{x_d^*}{2}$$

The final term captures the fact that the incumbent only receives have of the direct mail demand. The best response functions for the quantities remain the same as in the single sender model with full inter-dependence:

$$x_d^* = (1 + \alpha)u - p_d - \alpha p_t, \quad x_t^* = u - p_t.$$

Substituting these expressions and taking the FOC yields the best response for prices:

$$p_t = \frac{u + c_t}{2} - \frac{\alpha}{\mathbf{4}}(p_d - c_d), \quad p_d = \frac{(1 + \alpha)u - \alpha p_t + c_d}{2}$$

The only difference from the standard case without anticipating entry in the SS model is the 4 in bold (which was 2). Therefore, when $\alpha = 0$, anticipating entry has no effect on prices.

Solving simultaneously:

$$p_t = \frac{1}{8 - \alpha^2}\left(u(4 - \alpha - \alpha^2) + 4c_t + \alpha c_d\right)$$

We now explore the comparative statics.

Claim: The price of transactional mail when the incumbent anticipates entry is lower than when there is no adjustment for entry.

Proof: In the case without entry (from the single sender model);

$$p_t^{M,I} = \frac{(2 - \alpha - \alpha^2)u + \alpha c_d + 2c_t}{4 - \alpha^2}$$

In the case with anticipating entry:

$$p_t = \frac{1}{8-\alpha^2}\left(u(4-\alpha-\alpha^2)+4c_t+\alpha c_d\right)$$

It can be demonstrated that the second is larger than the firs:

$$\frac{1}{8-\alpha^2}\left(u(4-\alpha-\alpha^2)+4c_t+\alpha c_d\right) \geq \frac{(2-\alpha-\alpha^2)u+\alpha c_d+2c_t}{4-\alpha^2}$$

Cross multiply:

$$(4-\alpha^2)\left(u(4-\alpha-\alpha^2)+4c_t+\alpha c_d\right) \geq (8-\alpha^2)((2-\alpha-\alpha^2)u+\alpha c_d+2c_t)$$

Simplify

$$2\alpha^2 u - 4\alpha c_d - \alpha^2 2c_t + 4\alpha u \geq 0$$

Rearrange:

$$4\alpha(u-c_d)+2\alpha^2(u-c_t)\geq 0$$

Clearly the price of transactional mail is lower when the incumbent anticipates entry providing $\alpha > 0$.

Claim: The price of direct mail is lower when the incumbent anticipates entry.

Proof: This is evident from the best response for direct mail; the best response itself is identical but it is decreasing in p_t. Therefore, as p_t is now larger, p_d is now lower.

We can also show this formally by comparing the price of direct mail in the standard case and in the case anticipating entry.

The price of direct mail in the standard case without entry is:

$$p_d^{M,I} = \frac{(2+\alpha)u+(2-\alpha^2)c_d-\alpha c_t}{4-\alpha^2}$$

and anticipating entry by a price matching entrant the price of direct mail is:[22]

$$p_d = \frac{4u+2\alpha u+4c_d-2\alpha c_t-\alpha^2 c_d}{8-\alpha^2}$$

We can now show that the price of direct mail is lower when the incumbent anticipates a price-matching entrant.

The condition becomes:

[22]This is obtained by solving the maximisation of the postal operator when they receive only half of the direct mail demand.

$$4\alpha(u - c_d) + 2\alpha^2(u - c_t) > 0$$

This is satisfied for any $\alpha > 0$. Again if $\alpha = 0$, the possibility of price matching entry has no impact on these prices.

Proof of Result 9d (Large Cost Advantage Derivations).

$$\max_{x_d} \pi_{dms} = (u + \alpha x_t)x_d - \frac{1}{2}x_d^2 - \hat{p_d}x_d$$

The first order condition yields the optimal quantity:

$$\hat{x_d} = u - \hat{p_d} + \alpha x_t = u - \hat{p_d} + \frac{\alpha(u - c_t)}{2}$$

The optimal price for direct mail set by the entrant becomes:

$$p_d^* = \frac{u + \hat{c_d}}{2} + \frac{\alpha(u - c_t)}{4}$$

The closed form quantity chosen by the direct mail sender is therefore:

$$x_d^* = \frac{(u - \hat{c_d})}{2} + \frac{\alpha(u - c_t)}{4}$$

The closed form profit for the entrant postal operator is:

$$\hat{\pi} = \left(\frac{u - \hat{c_d}}{2} + \frac{\alpha(u - c_t)}{4}\right)^2$$