Organizing Modular Production

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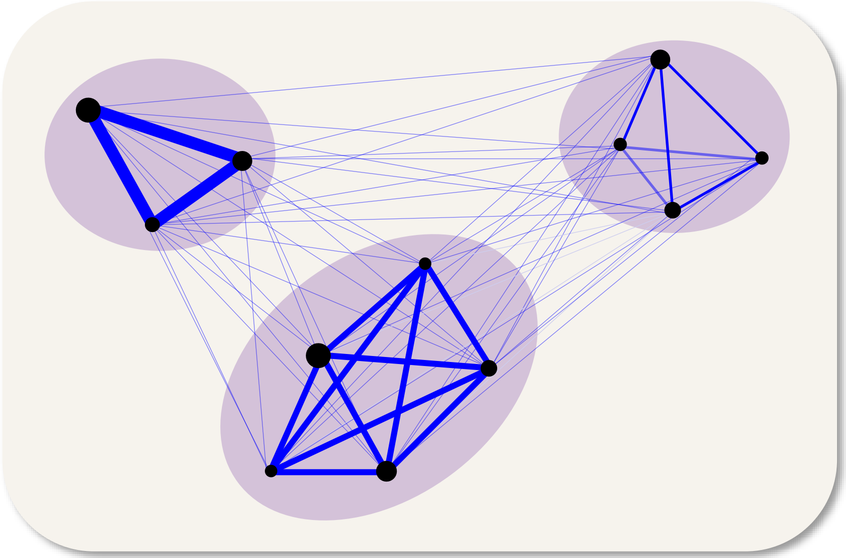
Motivation

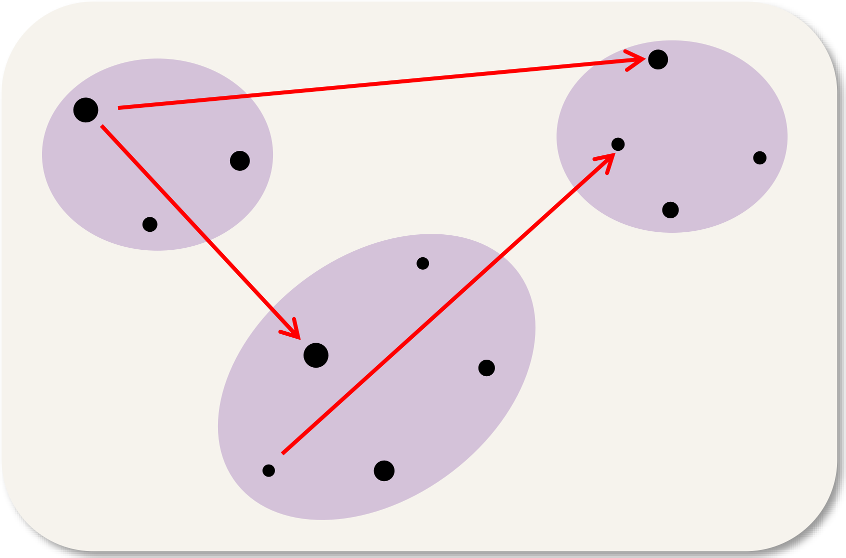
- Simon (1962, 1995):
  - Complex social, technological, and biological systems are made up of communities or "modules."
  - Communities are subsets of nodes that are densely connected within but sparsely connected across.
  - Community structures allow faster adaptation to changing environment.
  - Community detection literature has since documented this structure in many settings (Fortunato 2009).
- Baldwin and Clark (2000):
  - In 1964 IBM introduced the first modular computer, the System/360.
  - Modular products are now pervasive (phones, planes, cars, homes, software, etc.).
  - The change in how products are made has the potential to affect economic organization & outcomes.
- This paper:
  - The impact of modular production on the internal organization of firms.

Model Preview

Production



Communication



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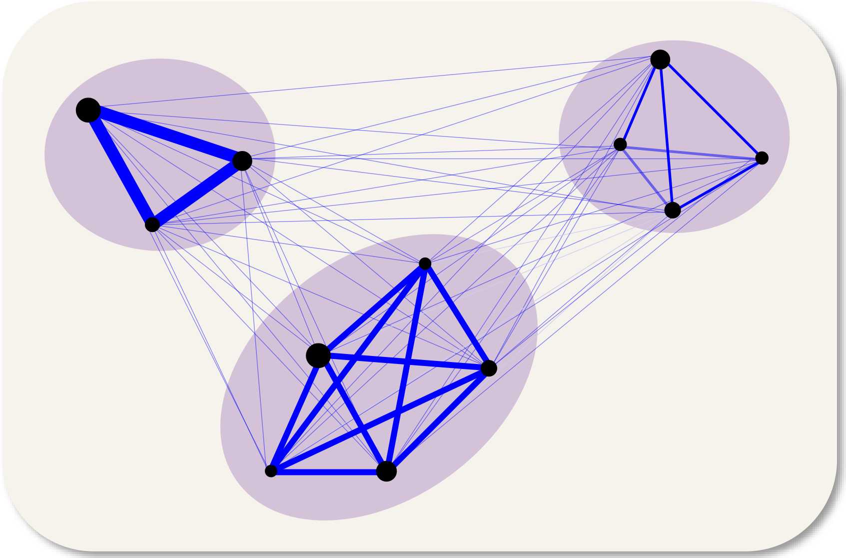

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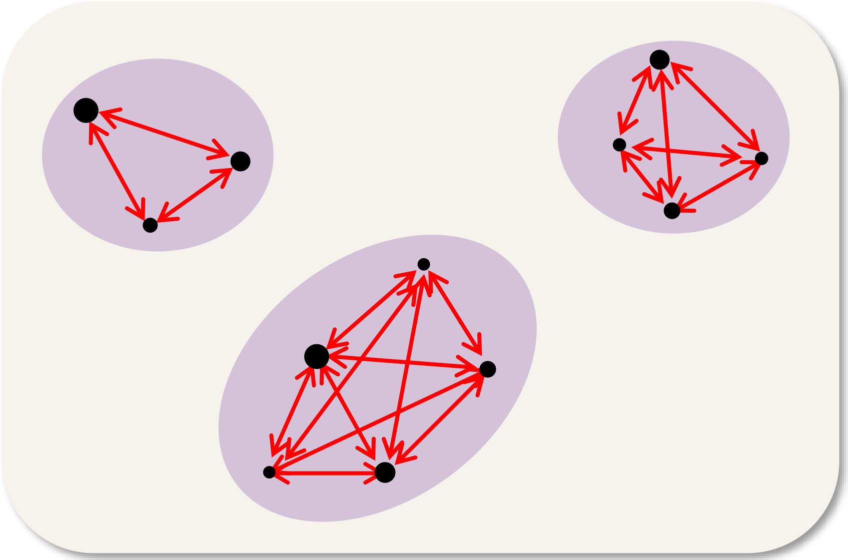

Agenda

Model

Solving the Model

Application

Extension

Conclusions

Production

- Each agent $i \in \mathcal{N} = \{1, \ldots, N\}$ makes a decision $d_i \in [-D, D]$, where D is a large scalar.
- Each decision d_i is associated with a state $\theta_i \in [-D, D]$.
- Output is given by

$$r(d_1, \ldots, d_N) = \sum_{i=1}^{N} \left[-d_i^2 + 2a_i d_i \theta_i + \sum_{j=1}^{N} p_{ij} d_i d_j \right],$$

  where $a_i > 0$, $p_{ij} = p_{ji} > 0$, and $p_{ii} = 0$.
- Assume $\sum_{j=1}^{N} p_{ij} < 1$ for all $i = 1, \ldots, N$.
- $\boldsymbol{P}$ denotes the $N \times N$ matrix with entries p_{ij}.
- Normalize the price of output to one.

PRODUCTION

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- Each decision d_i is associated with a state $\theta_i \in [-D, D]$.
- Output is given by

$$r(d_1, \dots, d_N) = \sum_{i=1}^{N}\left[\left(1 - \sum_{j=1}^{N} p_{ij}\right)(d_i - \theta_i)^2 - \frac{1}{2}\sum_{j=1}^{N} p_{ij}\left(d_i - d_j\right)^2\right] + constant,$$

if $a_i = 1 - \sum_{j=1}^{N} p_{ij}$.

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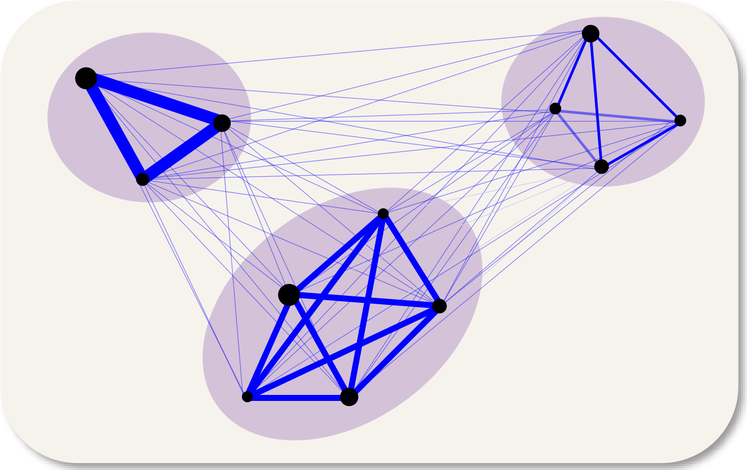
Modules

- Each triplet (d_i, θ_i, i) belongs to a module $\mathcal{M}_m$ for $m \in \{1, \ldots, M\}$ with $n_m \geq 1$ members.
- Function $m(i)$ gives the module $\mathcal{M}_{m(i)}$ that (d_i, θ_i, i) belongs to.
- Assume $m(1) = 1$.
- Need for coordination p_{ij} between decisions d_i and d_j:
  - $p_{ij} = t \geq 0$ if $m(i) \neq m(j)$.
  - $p_{ij} \equiv p_m \geq t$ if $m(i) = m(j) = m$.



Information

- Each state θ_i is drawn independently from a distribution with $\mathrm{E}[\theta_i] = 0$ and $\mathrm{Var}[\theta_i] = \sigma_i^2$.
- Realization of θ_i is privately observed by agent i.
- Principal can place directed links between any two agents i and j, at cost γ_{ij} per link.
  - $\gamma_{ij} = 0$ if $m(i) = m(j)$ and $\gamma_{ij} = \gamma > 0$ if $m(i) \neq m(j)$.
- If the principal places a link from agent i to j, agent i tells j the realization of his state.
- The communication network is described by $N \times N$ matrix $\boldsymbol{C}$ with entries c_{ij}.
  - $c_{ij} = 1$ if agent i tells j about his state or $i = j$.
  - $c_{ij} = 0$ otherwise.
  - Row $\boldsymbol{C}_i$ summarizes who knows state θ_i.
  - Column $\boldsymbol{C}_{(j)}$ summarizes what states agent j knows.

ORGANIZATION

- Principal designs the communication network to maximize expected profits:

$$\max_{\boldsymbol{C}} \mathrm{E}[r(d_1, \ldots, d_N)|\boldsymbol{C}] - \gamma \sum_{i=1}^{N} \sum_{i=1}^{N} m_{ij} c_{ij}$$

subject to $c_{ii} = 1$ for all $i \in \mathcal{N}$ and m_{ij} is a dummy variable equal to one if $m(i) \neq m(j)$.

- Timing:
  - Principal designs the communication network.
  - Agents learn their states and tell them to other agents as specified in the communication network.
  - Agents simultaneously make their decisions.
  - Payoffs are realized and game ends.
- Solution concept: Perfect Bayesian Equilibrium.

Summary of Key Assumptions

- No re-transmission of information.
- Information is independent.
- Communication is binary.
- No incentive conflicts.

Agenda

Decision-Making

LEMMA 1. *Equilibrium decisions are unique and given by*

$$d_i^* = \sum_{j=1}^{N} a_j \omega_{ij}(\boldsymbol{C}_j)\theta_j \text{ for all } i \in \mathcal{N},$$

where $\omega_{ij}(\boldsymbol{C}_j)$ *is the* ijth *entry of* $\left(\boldsymbol{I} - (\operatorname{diag} \boldsymbol{C}_j)\boldsymbol{P}(\operatorname{diag} \boldsymbol{C}_j)\right)^{-1}$.

- $\omega_{ij}(\boldsymbol{C}_j)$ is the value of all walks from node i to j on the subgraph $(\operatorname{diag} \boldsymbol{C}_j)\boldsymbol{P}(\operatorname{diag} \boldsymbol{C}_j)$.
- $(\operatorname{diag} \boldsymbol{C}_j)\boldsymbol{P}(\operatorname{diag} \boldsymbol{C}_j)$ is the subgraph of $\boldsymbol{P}$ that consists only of nodes whose agents know θ_j.

The Coordination Multiplier

- A key object is the weight d_i^* puts on θ_i, which is given by $a_i \omega_{ii}(\boldsymbol{C}_i)$.
- a_i captures the degree of *autonomous adaptation*.
- $\omega_{ii}(\boldsymbol{C}_i)$ is the *coordination multiplier*, which is:
  - Increasing and supermodular in $\boldsymbol{C}_i$.
  - Depends on $\boldsymbol{C}_i$ but not on $\boldsymbol{C}_{-i}$.

Agenda

Expected Revenue

- Substituting equilibrium decisions into revenue and rearranging, we have

$$r(d_1^*, \dots, d_N^*) = \sum_{i=1}^N a_i d_i^* \theta_i + \sum_{i=1}^N \sum_{j=1}^N p_{ij} d_i^* \left(d_j^* - \mathrm{E}\left[d_j^* \middle| \boldsymbol{C}_{(\boldsymbol{i})}\right]\right).$$

- The second term is zero in expectation:

$$\mathrm{E}\left[\sum_{i=1}^N \sum_{j=1}^N p_{ij} d_i^* \left(d_j^* - \mathrm{E}\left[d_j^* \middle| \boldsymbol{C}_{(\boldsymbol{i})}\right]\right)\right]$$

$$= \mathrm{E}\left[\sum_{i=1}^N \sum_{j=1}^N p_{ij} \sum_{s=1}^N \sum_{t=1}^N \omega_{is}(\boldsymbol{C}_s)\omega_{jt}(\boldsymbol{C}_t) a_s a_t \left(\theta_s \mathrm{E}[\theta_t \mid \boldsymbol{C}_{(i)}] - \theta_s\theta_t\right)\right]$$

independence

$$= \mathrm{E}\left[\sum_{i=1}^N \sum_{j=1}^N p_{ij} \sum_{s=1}^N \omega_{is}(\boldsymbol{C}_s)\omega_{js}(\boldsymbol{C}_s) a_s^2 \left(\theta_s \mathrm{E}[\theta_s \mid \boldsymbol{C}_{(i)}] - \theta_s^2\right)\right]$$

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binary communication

$$= 0$$

Expected Revenue

LEMMA 2. *Under equilibrium decision-making, expected revenue is given by*

$$R(\boldsymbol{C}) \equiv \mathrm{E}[r(d_1^*, \dots, d_N^*)] = \sum_{i=1}^{N} a_i \operatorname{Cov}(d_i^*, \theta_i),$$

where $\operatorname{Cov}(d_i^*, \theta_i) = a_i \sigma_i^2 \omega_{ii}(\boldsymbol{C}_i)$.

- Define $R_i(\boldsymbol{C}_i) \equiv a_i \operatorname{Cov}(d_i^*, \theta_i)$ as the expected *revenue generated by agent* $i \in \mathcal{N}$.
- $a_i^2 \sigma_i^2$ is the *value of autonomous adaptation* of decision d_i.
- Key property of $R_i(\boldsymbol{C}_i)$: it depends on $\boldsymbol{C}_i$ but not on $\boldsymbol{C}_{-i}$.

Separability Result

PROPOSITION 1. *An optimal communication network solves the principal's problem if and only if it solves the N independent subproblems*

$$\max_{\boldsymbol{C}_i} R_i(\boldsymbol{C}_i) - \gamma \sum_{j=1}^{N} m_{ij} c_{ij} \text{ for all } i \in \mathcal{N}.$$

- Supermodularity of $\omega_{ii}(\boldsymbol{C}_i)$ implies that:
  - If it is optimal to tell agent i about θ_j, it's optimal to also tell the other agents in his module $\mathcal{M}_{m(i)}$.
  - The principal's problem can be solved in polynomial time using standard algorithms.

Agenda

Expressing Revenue in Terms of Primitives

LEMMA 3. *Suppose agent 1's state θ_1 is known to all agents in modules $\mathcal{M}_1, \dots, \mathcal{M}_\ell$ for $\ell \in \{1, \dots, M\}$, and to no agents in other modules. Agent 1's expected revenue is then given by*

$$R_1(\boldsymbol{C}_1(\ell)) = a_1^2\sigma_1^2\left(\frac{1-(n_1-2)p_1}{(1+p_1)(1-(n_1-1)p_1)} + \frac{t^2x_1^2\sum_{m=2}^{\ell} n_m x_m}{(1-tn_1x_1)(1-t\sum_{m=1}^{\ell} n_m x_m)}\right),$$

where

$$x_m \equiv \frac{1}{1-(n_m-1)p_m + n_m t}.$$

- The object x_m is a measure of *cohesion* of module $\mathcal{M}_m$ (Morris 2002).

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Characterization Result

PROPOSITION 2. *Optimal communication is characterized by N thresholds $\lambda_i \geq 0$, one for each agent $i \in \mathcal{N}$. Agent i tells his state to agent j if and only if they belong to the same module or $x_{m(j)} \geq \lambda_i$. The threshold λ_i is increasing in γ and decreasing in $a_i^2\sigma_i^2$, $p_{m(i)}$, and $n_{m(i)}$.*

- Proof:

$$\frac{1}{n_{\ell+1}}\left(R_1\left(\boldsymbol{C}_1(\ell+1)\right)-R_1\left(\boldsymbol{C}_1(\ell)\right)\right)=a_1^2\sigma_1^2\,\frac{t^2x_1^2x_{\ell+1}}{\left(1-t\sum_{m=1}^{\ell}n_mx_m\right)\left(1-t\sum_{m=1}^{\ell+1}n_mx_m\right)}$$

$$\frac{1}{n_{\ell+1}+n_{\ell+2}}\left(R_1\left(\boldsymbol{C}_1(\ell+2)\right)-R_1\left(\boldsymbol{C}_1(\ell)\right)\right)=a_1^2\sigma_1^2\,\frac{1}{n_{\ell+1}+n_{\ell+2}}\frac{t^2x_1^2(n_{\ell+1}x_{\ell+1}+n_{\ell+2}x_{\ell+2})}{\left(1-t\sum_{m=1}^{\ell}n_mx_m\right)\left(1-t\sum_{m=1}^{\ell+2}n_mx_m\right)}$$

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- Proof:

$$\frac{1}{n_{\ell+1}+n_{\ell+2}}\Big(R_1\big(\boldsymbol{C}_1(\ell+2)\big)-R_1\big(\boldsymbol{C}_1(\ell)\big)\Big)-\frac{1}{n_{\ell+1}}\Big(R_1\big(\boldsymbol{C}_1(\ell+1)\big)-R_1\big(\boldsymbol{C}_1(\ell)\big)\Big)$$

$$=a_1^2\sigma_1^2\frac{n_{\ell+2}t^2x_1^2}{n_{\ell+1}+n_{\ell+2}}\frac{(x_{\ell+2}-x_{\ell+1})\big(1-t\sum_{m=1}^{\ell+1}n_m x_m\big)+t(n_{\ell+1}+n_{\ell+2})x_{\ell+1}x_{\ell+2}}{\big(1-t\sum_{m=1}^{\ell}n_m x_m\big)\big(1-t\sum_{m=1}^{\ell+1}n_m x_m\big)\big(1-t\sum_{m=1}^{\ell+2}n_m x_m\big)}$$

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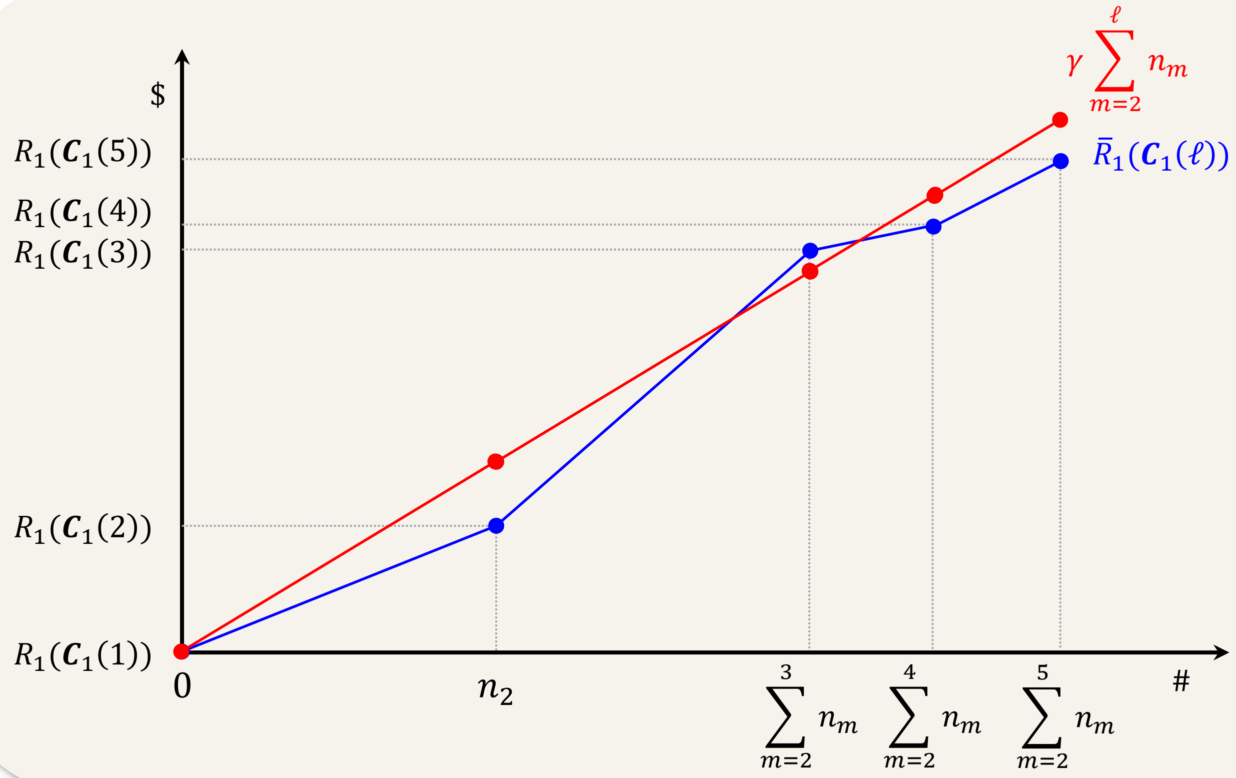
Illustration



Drawn for $t = 0.01$, $n_1 = n_2 = n_3 = 5$, $n_4 = n_5 = 5$, $p_1 = p_2 = p_3 = 0.2$, $p_4 = p_5 = 0.1$, and $a_1^2\sigma_1^2 = 1$.

Hierarchies

COROLLARY 1. *Optimal communication gives rise to a receiver hierarchy among agents. For any agents* $i, j, k \in \mathcal{N}$ *who belong to different modules, if agent* i*'s module is more cohesive than agent* j*'s, then agent* j *is told about agent* k*'s state only if agent* i *also is.*

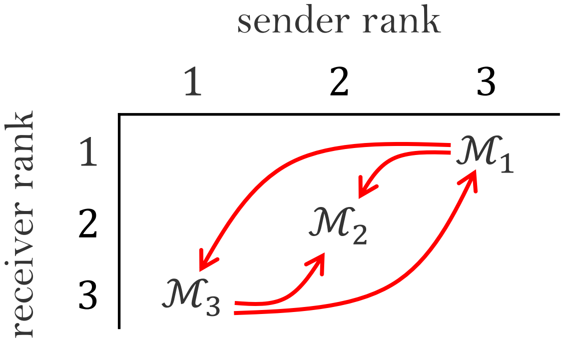
COROLLARY 2. *Optimal communication gives rise to a sender hierarchy among agents. For any agents* $i, j, k \in \mathcal{N}$ *who belong to different modules, if agent* i*'s threshold* λ_i *is smaller than agent* j*'s threshold* λ_j*, then agent* j *tells agent* k *about his state only if agent* i *also does.*

- Agent i's rank in the receiver hierarchy depends only on module cohesion.
- But his rank in the sender hierarchy also depends on the autonomous value of adaptation $a_i^2\sigma_i^2$.
- Agents who hear the most may not be the ones who speak the most.

Bottom-Up Communication

- Suppose there are communication links from module $\mathcal{M}_m$ to $\mathcal{M}_{m'}$ but not the reverse.
- Then communication is *top down* if $x_m > x_{m'}$ and *bottom up* if $x_m < x_{m'}$.
- Communication is *bottom up in aggregate* if there are more pairs of modules that engage in bottom-up than top-down communication.

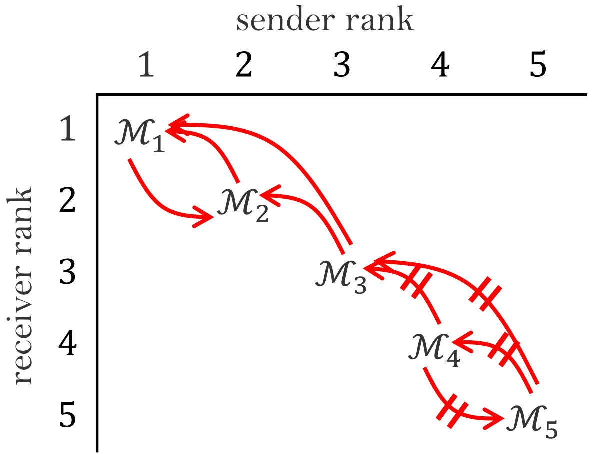
PROPOSITION 3. *If the optimal sender and receiver hierarchies are the reverse of each other, and the receiver ranking is strict, communication is bottom up in aggregate.*



Core-Periphery Structures

- A communication network has a core-periphery structure if the set of modules can be partitioned into a core and periphery such that:
  - An agent in the core tells his state to all other agents in the core & maybe to agents in the periphery.
  - An agent in the periphery does not tell his state to all agents in the core a/o is not told all their states.
  - An agent in the periphery does not tell his state to other agents in the periphery.

PROPOSITION 4. *If the optimal sender and receiver hierarchies are identical, the communication network has a core-periphery structure in which the core consists of the most cohesive modules.*



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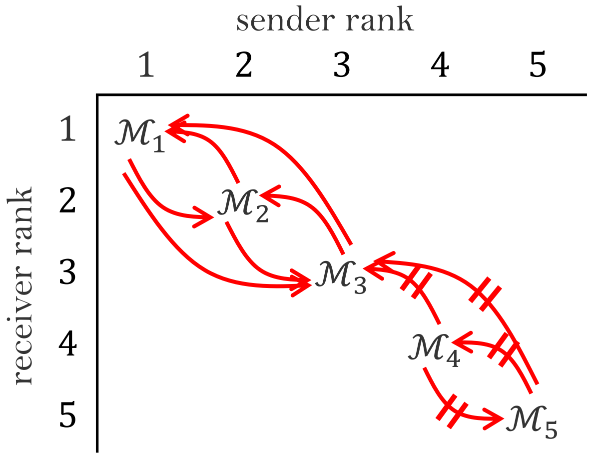
PROPOSITION 4. *If the optimal sender and receiver hierarchies are identical, the communication network has a core-periphery structure in which the core consists of the most cohesive modules.*



Core-Periphery Structures

- A communication network has a core-periphery structure if the set of modules can be partitioned into a core and periphery such that:
  - An agent in the core tells his state to all other agents in the core & maybe to agents in the periphery.
  - An agent in the periphery does not tell his state to all agents in the core a/o is not told all their states.
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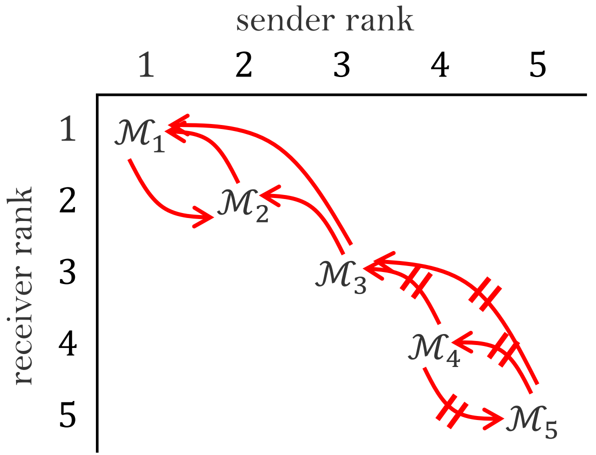

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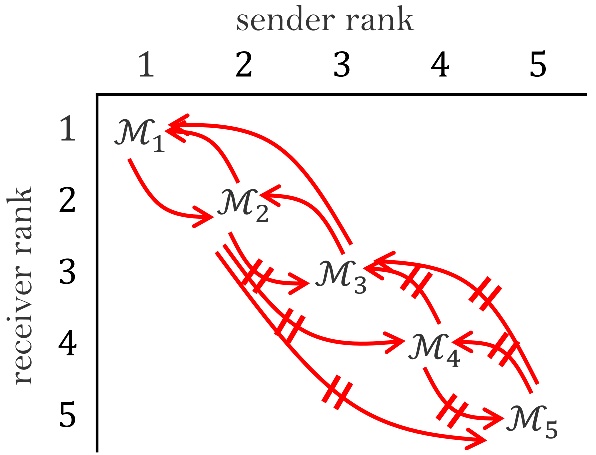

Agenda

Model

Solving the Model

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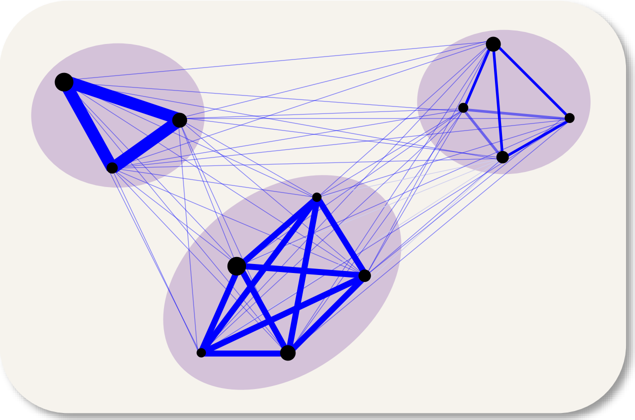
Extension

Conclusions

The Mirroring Hypothesis

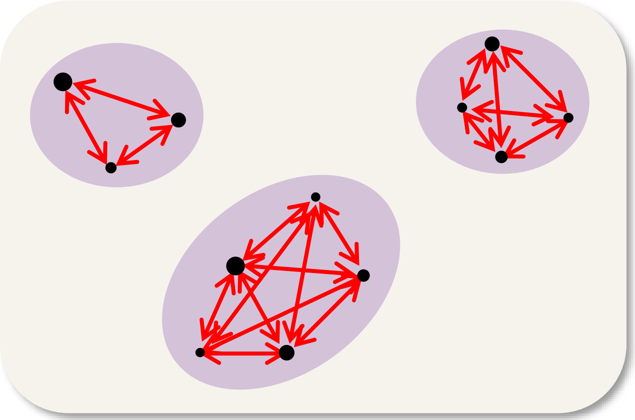
Modular production



Mirroring Hypothesis
Thompson (1967)

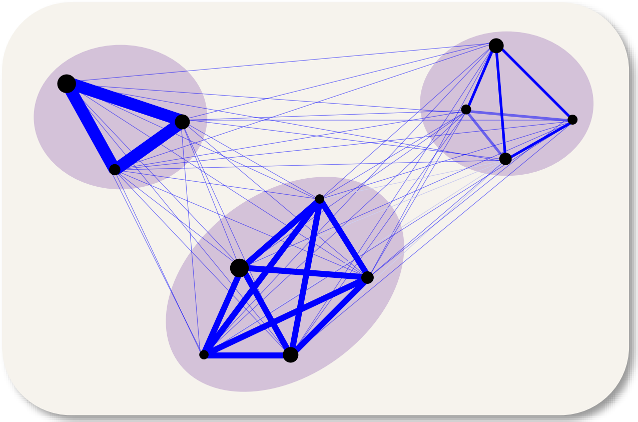
Conway's Law
Conway (1968)

Modular organization



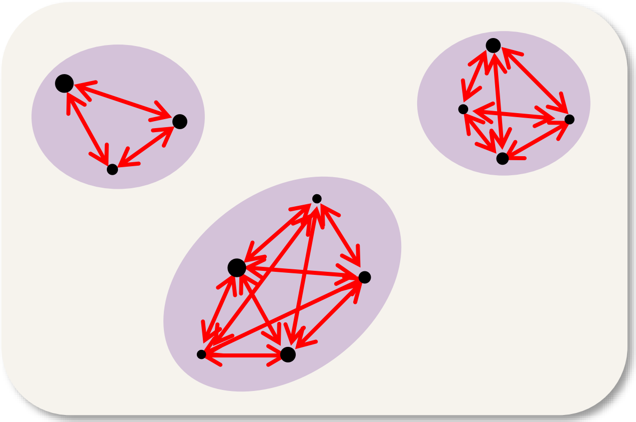
The Mirroring Hypothesis

Modular production

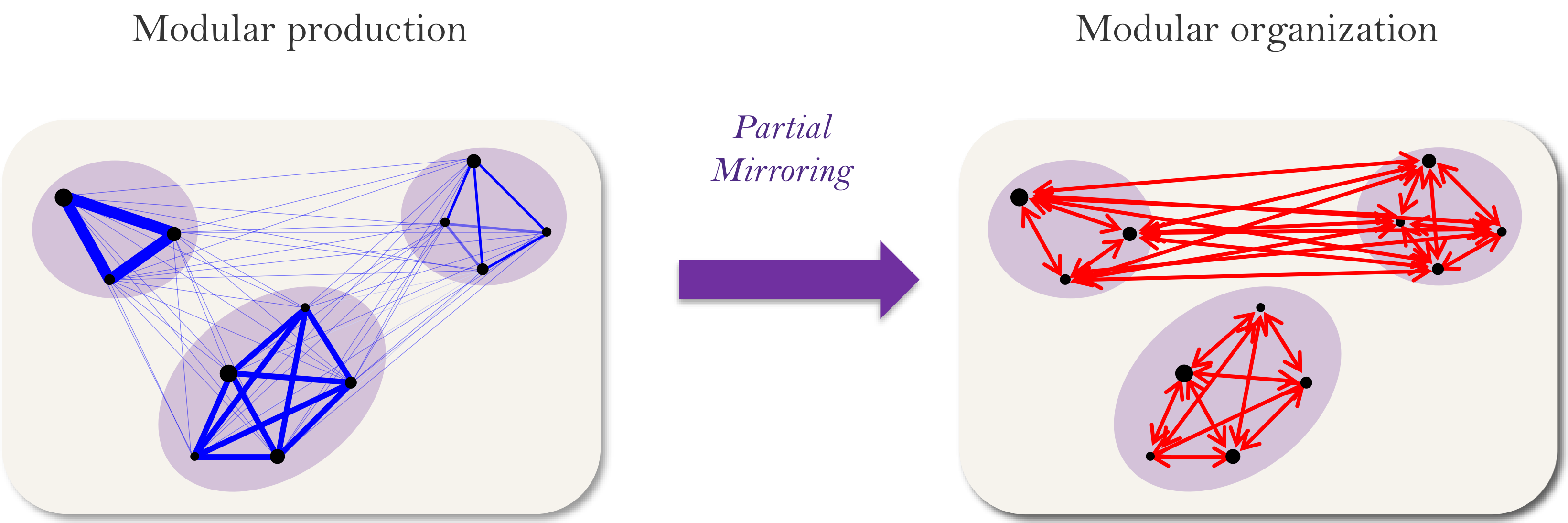


Mirroring Hypothesis
Thompson (1967)

Modular organization



The Mirroring Hypothesis



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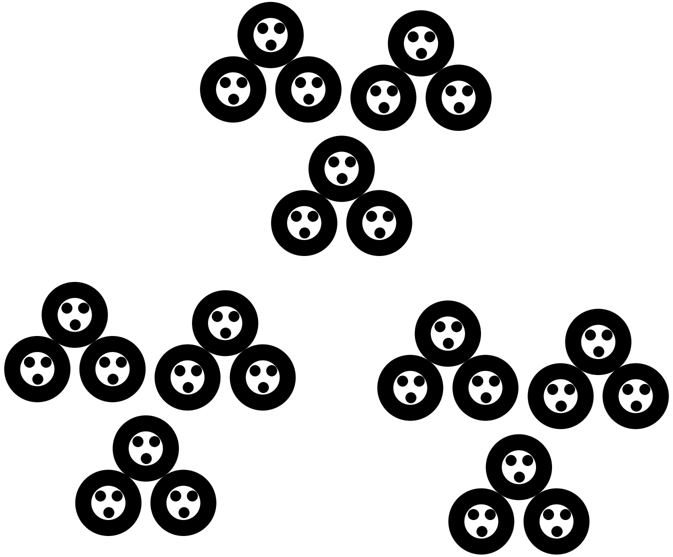
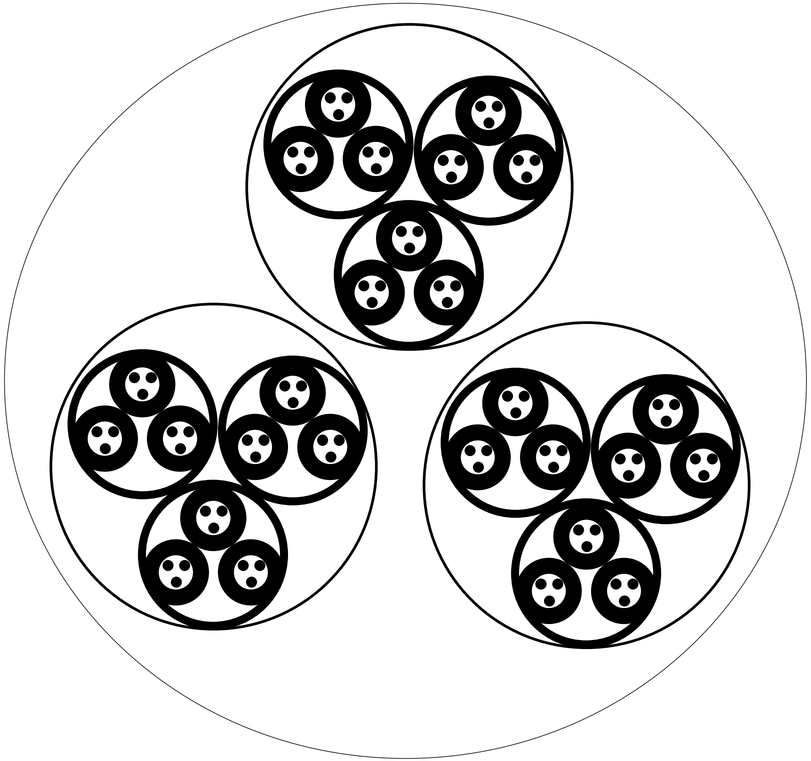
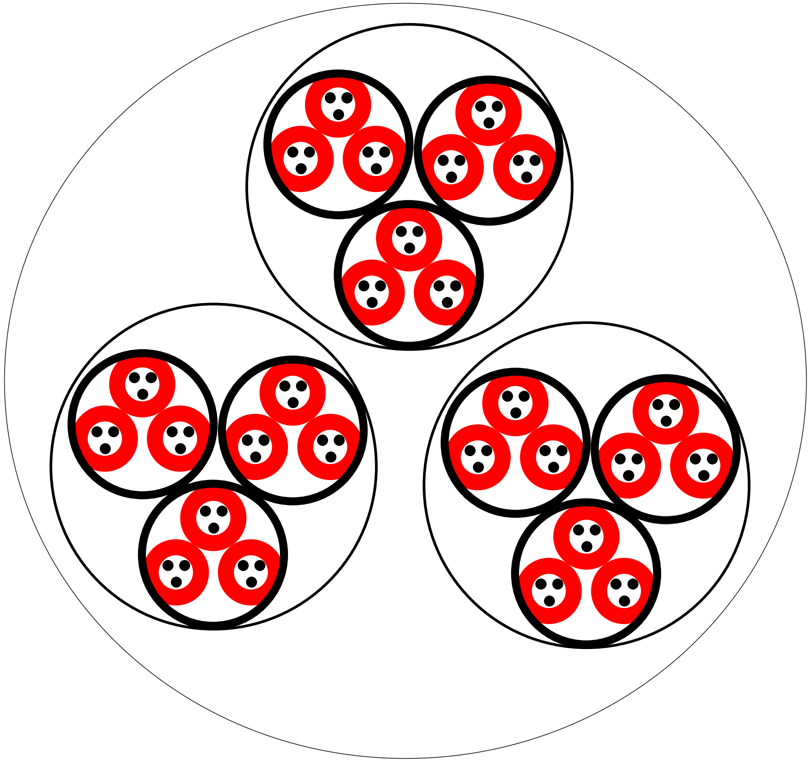
Nested Modules



Nested Modules



Nested Modules → Nested Communication



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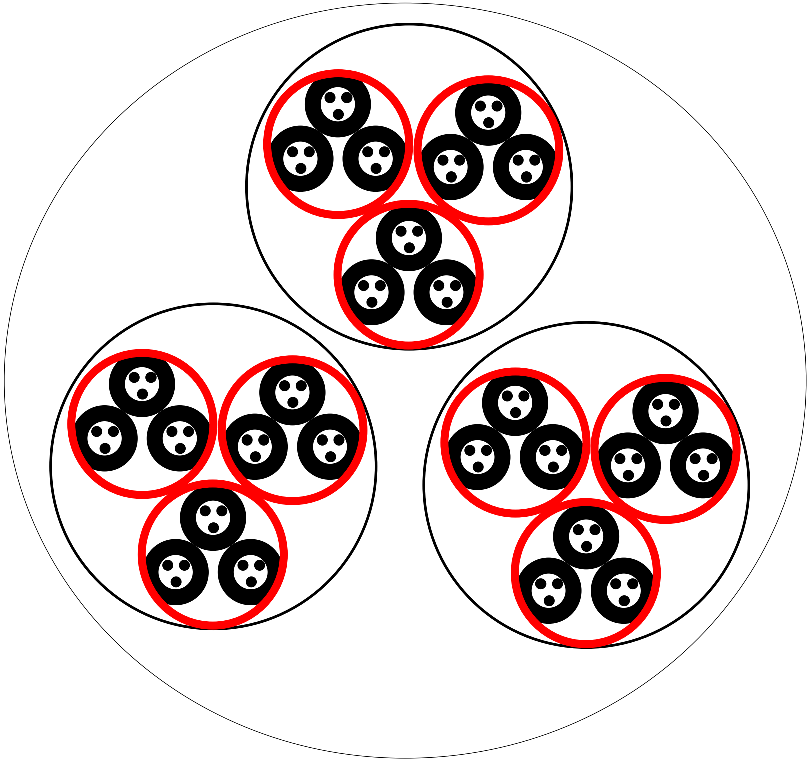

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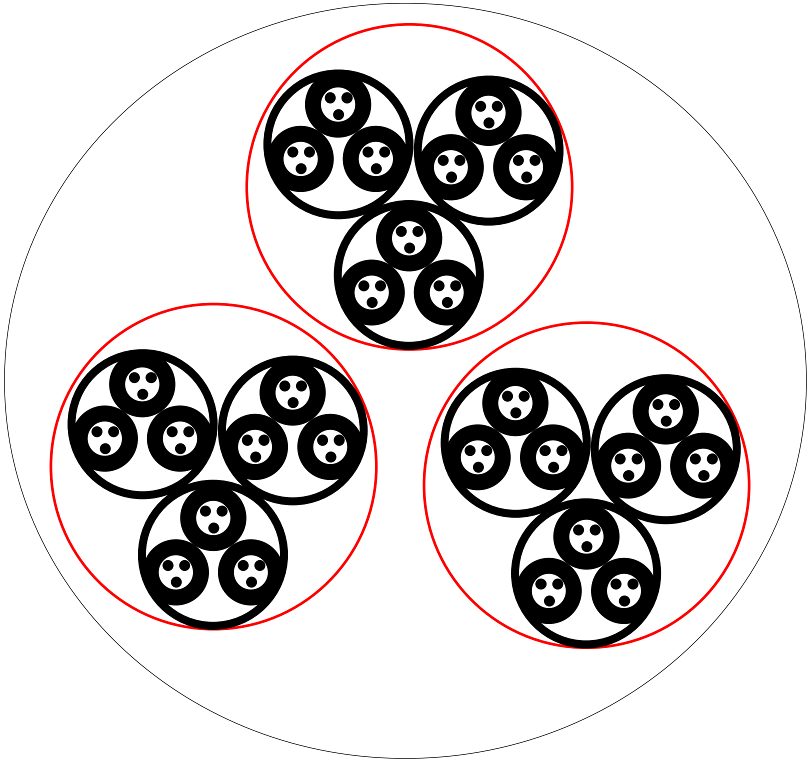

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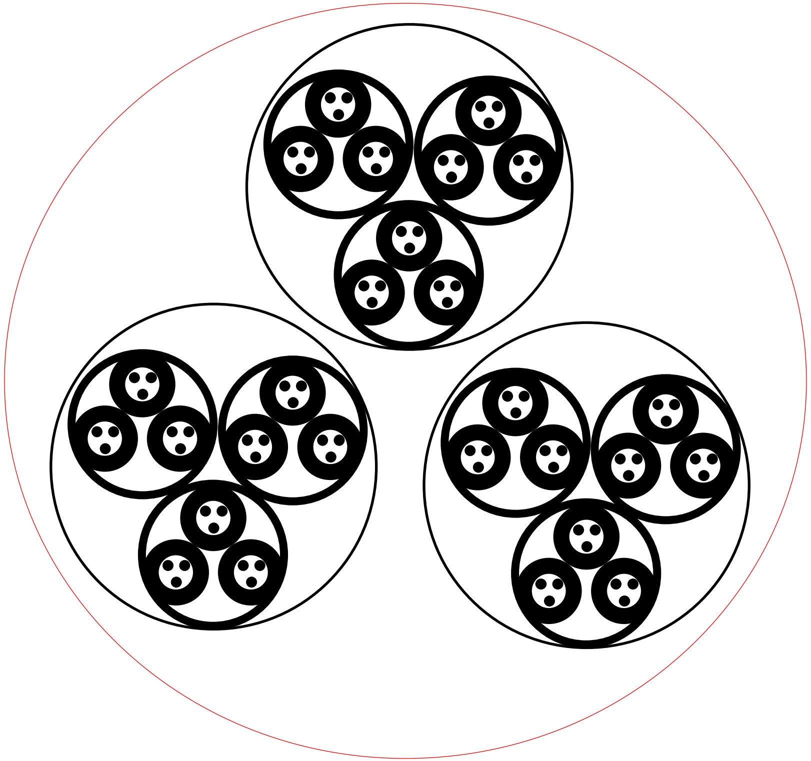


Agenda

Conclusions

- Over the last half century, the economy has shifted towards modular production.
- This paper is a first step towards understanding the economic implications of this shift.
- Open questions about impact of modular production:
  - Interfaces.
  - Parallel processing.
  - Firm boundaries, industry structure, location of production.
- Broader question about the reason for the rise of modular production.
- Testable predictions for emerging empirical literature on within-firm communications.

Revisiting Key Assumptions

- Agents observe all states in their modules.
  - Convenient & captures notion that agents working on the same module co-locate and share expertise.
  - Results extend readily if each agent only observes his own state.
- No re-transmission of information.
  - Share this assumption with other papers (e.g. Calvó-Armengol & de Martí (2008), Calvó-Armengol, de Martí, & Prat (2015), Herskovic & Ramos (2020)).
  - Captures notion that the states are "rich" and can only be described effectively by the associated agent.
  - Essential for the separability result (Proposition 1).
- Independence of information and binary communication.
  - Share these assumptions with other papers (e.g. independence with Calvó-Armengol, de Martí, & Prat (2015) and binary communication with Calvó-Armengol & de Martí (2008)).
  - Essential for the separability result.

Revisiting Key Assumptions

- Absence of incentive conflicts.
  - Share this assumption with the literature on team theory.
  - It, too, is essential for the separability result.
  - PROPOSITION 6. *If agents internalize only a fraction* $\mu \in (0,1)$ *of the needs to coordinate, an optimal communication network solves*

$$\max_{\boldsymbol{C}} \sum_{i=1}^{N} a_i \operatorname{Cov}(d_i^*, \theta_i) + (1-\mu) \sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij} \operatorname{Cov}(d_i^*, d_j^*) - \gamma \sum_{i=1}^{N} (\boldsymbol{C}_i \mathbf{1} - n_{m(i)}),$$

where

$$\operatorname{Cov}(d_i^*, \theta_i) = a_i \sigma_i^2 \omega_{ii}(\boldsymbol{C}_i, \mu)$$

and

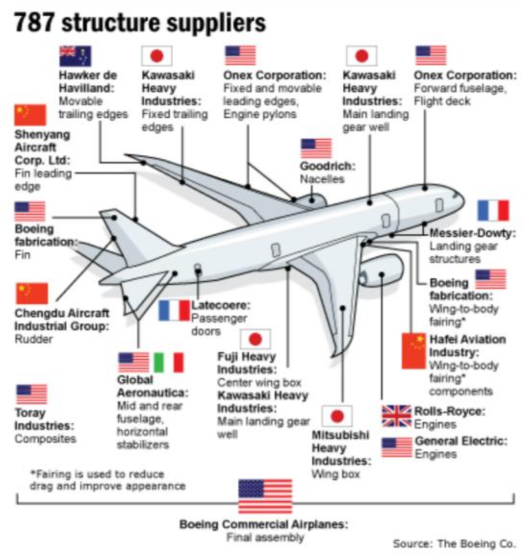
$$\operatorname{Cov}(d_i^*, d_j^*) = \sum_{s=1}^{N} a_s^2 \sigma_s^2 \omega_{is}(\boldsymbol{C}_s, \mu) \omega_{js}(\boldsymbol{C}_s, \mu),$$

and where $\omega_{ij}(\boldsymbol{C}_j, \mu)$ *denotes the* ijth *entry of* $\left(I - (\operatorname{diag} \boldsymbol{C}_j)\mu \boldsymbol{P}(\operatorname{diag} \boldsymbol{C}_j)\right)^{-1}$.

Revisiting Key Assumptions

- Production has a non-overlapping community structure.
  - Captures the notion of modular products.
  - Suppose $\boldsymbol{P}$ takes any form, provided it still satisfies $p_{ii} = 0$, $p_{ij} = p_{ji}$, and $\sum_{j=1}^{N} p_{ij} < 1$ for all $i, j \in \mathcal{N}$.
  - Separability result still holds, and principal's problem can still be solved using standard algorithms.
  - But the characterization result (Proposition 2) does not.
  - Except for the comparative statics:
  - PROPOSITION 6. *As long as the production network* P *satisfies* $p_{ii} = 0$, $p_{ij} = p_{ji}$, *and* $\sum_{j=1}^{N} p_{ij} < 1$ *for all* $i, j \in \mathcal{N}$, *optimal communication networks* $\boldsymbol{C}_i^*$ *are increasing in* $a_i^2 \sigma_i^2$ *and* p_{ij}, *and decreasing* γ.

The Mirroring Hypothesis



A line of Boeing 787 jets are parked nose-to-tail at Paine Field on Feb. 5, 2013, in Everett, Wash. *Elaine Thompson, AP*

Bottom-Up Communication

- Suppose there are communication links from module $\mathcal{M}_m$ to $\mathcal{M}_{m'}$, but not the reverse.
- Then communication is *top down* if $x_m > x_{m'}$, and *bottom up* if $x_m < x_{m'}$.
- Communication is *bottom up in aggregate* if there are more pairs of modules that engage in bottom-up than top-down communication.

PROPOSITION 3. *If the optimal sender and receiver hierarchies are the reverse of each other, and the receiver ranking is strict, communication is bottom up in aggregate.*