Organizing Modular Production

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MOTIVATION

- Simon (1962, 1995):
 - > Complex social, technological, and biological systems are made up of communities or "modules."
 - Communities are subsets of nodes that are densely connected within but sparsely connected across.
 - Community structures allow faster adaptation to changing environment.
 - Community detection literature has since documented this structure in many settings (Fortunato 2009).
- Baldwin and Clark (2000):
 - ▶ In 1964 IBM introduced the first modular computer, the System/360.
 - Modular products are now pervasive (phones, planes, cars, homes, software, etc.).
 - > The change in how products are made has the potential to affect economic organization & outcomes.
- This paper:
 - > The impact of modular production on the internal organization of firms.

${\rm Model}\ {\rm Preview}$

Production

Communication



${\rm Model}\ {\rm Preview}$

Production

Communication



Agenda

Model

Solving the Model

Application

Extension

Conclusions

PRODUCTION

- Each agent $i \in \mathcal{N} = \{1, ..., N\}$ makes a decision $d_i \in [-D, D]$, where D is a large scalar.
- Each decision d_i is associated with a state $\theta_i \in [-D, D]$.
- Output is given by

$$r(d_1, ..., d_N) = \sum_{i=1}^{N} \left[-d_i^2 + 2a_i d_i \theta_i + \sum_{j=1}^{N} p_{ij} d_i d_j \right],$$

where $a_i > 0$, $p_{ij} = p_{ji} > 0$, and $p_{ii} = 0$.

- Assume $\sum_{j=1}^{N} p_{ij} < 1$ for all $i = 1, \dots, N$.
- **P** denotes the $N \times N$ matrix with entries p_{ij} .
- Normalize the price of output to one.

PRODUCTION

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- Each decision d_i is associated with a state $\theta_i \in [-D, D]$.
- Output is given by

$$r(d_1, ..., d_N) = \sum_{i=1}^{N} \left[\left(1 - \sum_{j=1}^{N} p_{ij} \right) (d_i - \theta_i)^2 - \frac{1}{2} \sum_{j=1}^{N} p_{ij} (d_i - d_j)^2 \right] + constant,$$

if $a_i = 1 - \sum_{j=1}^{N} p_{ij}$.

- Assume $\sum_{j=1}^{N} p_{ij} < 1$ for all i = 1, ..., N.
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MODULES

- Each triplet (d_i, θ_i, i) belongs to a module \mathcal{M}_m for $m \in \{1, \dots, M\}$ with $n_m \ge 1$ members.
- Function m(i) gives the module $\mathcal{M}_{m(i)}$ that (d_i, θ_i, i) belongs to.
- Assume m(1) = 1.
- Need for coordination p_{ij} between decisions d_i and d_j :
 - ▷ $p_{ij} = t \ge 0$ if $m(i) \ne m(j)$.
 - ▶ $p_{ij} \equiv p_m \ge t$ if m(i) = m(j) = m.



INFORMATION

- Each state θ_i is drawn independently from a distribution with $E[\theta_i] = 0$ and $Var[\theta_i] = \sigma_i^2$.
- Realization of θ_i is privately observed by agent *i*.
- Principal can place directed links between any two agents *i* and *j*, at cost γ_{ij} per link.

$$\qquad \qquad \gamma_{ij}=0 \text{ if } m(i)=m(j) \text{ and } \gamma_{ij}=\gamma>0 \text{ if } m(i)\neq m(j).$$

- If the principal places a link from agent *i* to *j*, agent *i* tells *j* the realization of his state.
- The communication network is described by $N \times N$ matrix **C** with entries c_{ij} .
 - ▶ $c_{ij} = 1$ if agent *i* tells *j* about his state or i = j.
 - \succ $c_{ij} = 0$ otherwise.
 - > Row C_i summarizes who knows state θ_i .
 - > Column $C_{(j)}$ summarizes what states agent *j* knows.

ORGANIZATION

• Principal designs the communication network to maximize expected profits:

$$\max_{\boldsymbol{C}} \mathbb{E}[r(d_1, \dots, d_N) | \boldsymbol{C}] - \gamma \sum_{i=1}^N \sum_{i=1}^N m_{ij} c_{ij}$$

subject to $c_{ii} = 1$ for all $i \in \mathcal{N}$ and m_{ij} is a dummy variable equal to one if $m(i) \neq m(j)$.

- Timing:
 - Principal designs the communication network.
 - > Agents learn their states and tell them to other agents as specified in the communication network.
 - > Agents simultaneously make their decisions.
 - > Payoffs are realized and game ends.
- Solution concept: Perfect Bayesian Equilibrium.

SUMMARY OF KEY ASSUMPTIONS

- No re-transmission of information.
- Information is independent.
- Communication is binary.
- No incentive conflicts.

AGENDA

Model

Solving the Model

Decision-Making

The Principal's Problem Optimal Communication Networks

Application

Extension

Conclusions



DECISION-MAKING

LEMMA 1. Equilibrium decisions are unique and given by

 $d_i^* = \sum_{j=1}^N a_j \omega_{ij}(\boldsymbol{C}_j) \theta_j \text{ for all } i \in \mathcal{N},$

where $\omega_{ij}(\mathbf{C}_j)$ is the ijth entry of $(\mathbf{I} - (\operatorname{diag} \mathbf{C}_j)\mathbf{P}(\operatorname{diag} \mathbf{C}_j))^{-1}$.

- $\omega_{ij}(\mathbf{C}_j)$ is the value of all walks from node *i* to *j* on the subgraph $(\operatorname{diag} \mathbf{C}_j)\mathbf{P}(\operatorname{diag} \mathbf{C}_j)$.
- $(\operatorname{diag} C_j) P(\operatorname{diag} C_j)$ is the subgraph of **P** that consists only of nodes whose agents know θ_j .

The Coordination Multiplier

- A key object is the weight d_i^* puts on θ_i , which is given by $a_i \omega_{ii} (C_i)$.
- a_i captures the degree of *autonomous adaptation*.
- $\omega_{ii}(\mathbf{C}_i)$ is the coordination multiplier, which is:
 - > Increasing and supermodular in C_i .
 - > Depends on C_i but not on C_{-i} .

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• Substituting equilibrium decisions into revenue and rearranging, we have

$$r(d_{1}^{*}, ..., d_{N}^{*}) = \sum_{i=1}^{N} a_{i} d_{i}^{*} \theta_{i} + \sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij} d_{i}^{*} \left(d_{j}^{*} - \mathbb{E} \left[d_{j}^{*} \left| \boldsymbol{\mathcal{C}}_{(i)} \right] \right) \right)$$

$$E\left[\sum_{i=1}^{N}\sum_{j=1}^{N}p_{ij}d_{i}^{*}\left(d_{j}^{*}-E\left[d_{j}^{*}\left|\boldsymbol{C}_{(i)}\right]\right)\right]$$

$$=E\left[\sum_{i=1}^{N}\sum_{j=1}^{N}p_{ij}\sum_{s=1}^{N}\sum_{t=1}^{N}\omega_{is}(\boldsymbol{C}_{s})\omega_{jt}(\boldsymbol{C}_{t})a_{s}a_{t}\left(\theta_{s}E\left[\theta_{t}\mid\boldsymbol{C}_{(i)}\right]-\theta_{s}\theta_{t}\right)\right]$$
independence
$$=E\left[\sum_{i=1}^{N}\sum_{j=1}^{N}p_{ij}\sum_{s=1}^{N}\omega_{is}(\boldsymbol{C}_{s})\omega_{js}(\boldsymbol{C}_{s})a_{s}^{2}\left(\theta_{s}E\left[\theta_{s}\mid\boldsymbol{C}_{(i)}\right]-\theta_{s}^{2}\right)\right]$$

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• Substituting equilibrium decisions into revenue and rearranging, we have

$$r(d_{1}^{*}, ..., d_{N}^{*}) = \sum_{i=1}^{N} a_{i} d_{i}^{*} \theta_{i} + \sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij} d_{i}^{*} \left(d_{j}^{*} - \mathbb{E} \left[d_{j}^{*} \left| \boldsymbol{\mathcal{C}}_{(i)} \right] \right) \right)$$

LEMMA 2. Under equilibrium decision-making, expected revenue is given by

 $R(\mathbf{C}) \equiv \mathrm{E}[r(d_1^*, \dots, d_N^*)] = \sum_{i=1}^N a_i \operatorname{Cov}(d_i^*, \theta_i),$

where $\operatorname{Cov}(d_i^*, \theta_i) = a_i \sigma_i^2 \omega_{ii}(\boldsymbol{C}_i).$

- Define $R_i(C_i) \equiv a_i \operatorname{Cov}(d_i^*, \theta_i)$ as the expected *revenue generated by agent* $i \in \mathcal{N}$.
- $a_i^2 \sigma_i^2$ is the value of autonomous adaptation of decision d_i .
- Key property of $R_i(C_i)$: it depends on C_i but not on C_{-i} .

Separability Result

PROPOSITION 1. An optimal communication network solves the principal's problem if and only if it solves the N independent subproblems

$$\max_{C_i} R_i(C_i) - \gamma \sum_{j=1}^N m_{ij} c_{ij} \text{ for all } i \in \mathcal{N}.$$

- Supermodularity of $\omega_{ii}(\boldsymbol{C}_i)$ implies that:
 - > If it is optimal to tell agent *i* about θ_i , it's optimal to also tell the other agents in his module $\mathcal{M}_{m(i)}$.
 - > The principal's problem can be solved in polynomial time using standard algorithms.

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LEMMA 3. Suppose agent 1's state θ_1 is known to all agents in modules $\mathcal{M}_1, \dots, \mathcal{M}_\ell$ for $\ell \in \{1, \dots, M\}$, and to no agents in other modules. Agent 1's expected revenue is then given by

$$R_1(\boldsymbol{\ell}_1(\boldsymbol{\ell})) = a_1^2 \sigma_1^2 \left(\frac{1 - (n_1 - 2)p_1}{(1 + p_1)(1 - (n_1 - 1)p_1)} + \frac{t^2 x_1^2 \sum_{m=2}^{\ell} n_m x_m}{(1 - tn_1 x_1)(1 - t \sum_{m=1}^{\ell} n_m x_m)} \right),$$

where

$$x_m \equiv \frac{1}{1 - (n_m - 1)p_m + n_m t}.$$

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CHARACTERIZATION RESULT

PROPOSITION 2. Optimal communication is characterized by N thresholds $\lambda_i \ge 0$, one for each agent $i \in \mathcal{N}$. Agent *i* tells his state to agent *j* if and only if they belong to the same module or $x_{m(j)} \ge \lambda_i$. The threshold λ_i is increasing in γ and decreasing in $a_i^2 \sigma_i^2$, $p_{m(i)}$, and $n_{m(i)}$.

• Proof:

$$\frac{1}{n_{\ell+1}} \Big(R_1 \Big(\mathcal{C}_1(\ell+1) \Big) - R_1 \Big(\mathcal{C}_1(\ell) \Big) \Big) = a_1^2 \sigma_1^2 \frac{t^2 x_1^2 x_{\ell+1}}{\left(1 - t \sum_{m=1}^{\ell} n_m x_m \right) \left(1 - t \sum_{m=1}^{\ell+1} n_m x_m \right)} \\ \frac{1}{n_{\ell+1} + n_{\ell+2}} \Big(R_1 \Big(\mathcal{C}_1(\ell+2) \Big) - R_1 \Big(\mathcal{C}_1(\ell) \Big) \Big) = a_1^2 \sigma_1^2 \frac{1}{n_{\ell+1} + n_{\ell+2}} \frac{t^2 x_1^2 (n_{\ell+1} x_{\ell+1} + n_{\ell+2} x_{\ell+2})}{\left(1 - t \sum_{m=1}^{\ell} n_m x_m \right) \left(1 - t \sum_{m=1}^{\ell+2} n_m x_m \right)} \Big)$$

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• Proof:

$$\frac{1}{n_{\ell+1}+n_{\ell+2}} \Big(R_1 \big(\mathcal{C}_1(\ell+2) \big) - R_1 \big(\mathcal{C}_1(\ell) \big) \Big) - \frac{1}{n_{\ell+1}} \Big(R_1 \big(\mathcal{C}_1(\ell+1) \big) - R_1 \big(\mathcal{C}_1(\ell) \big) \Big)$$

$$=a_{1}^{2}\sigma_{1}^{2}\frac{n_{\ell+2}t^{2}x_{1}^{2}}{n_{\ell+1}+n_{\ell+2}}\frac{(x_{\ell+2}-x_{\ell+1})\left(1-t\sum_{m=1}^{\ell+1}n_{m}x_{m}\right)+t(n_{\ell+1}+n_{\ell+2})x_{\ell+1}x_{\ell+2}}{\left(1-t\sum_{m=1}^{\ell}n_{m}x_{m}\right)\left(1-t\sum_{m=1}^{\ell+1}n_{m}x_{m}\right)\left(1-t\sum_{m=1}^{\ell+1}n_{m}x_{m}\right)}$$

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• Proof:

$$\frac{1}{n_{\ell+1}+n_{\ell+2}} \Big(R_1 \big(\mathcal{C}_1(\ell+2) \big) - R_1 \big(\mathcal{C}_1(\ell) \big) \Big) - \frac{1}{n_{\ell+1}} \Big(R_1 \big(\mathcal{C}_1(\ell+1) \big) - R_1 \big(\mathcal{C}_1(\ell) \big) \Big)$$

$$=a_{1}^{2}\sigma_{1}^{2}\frac{n_{\ell+2}t^{2}x_{1}^{2}}{n_{\ell+1}+n_{\ell+2}}\frac{(x_{\ell+2}-x_{\ell+1})(1-t\sum_{m=1}^{\ell+1}n_{m}x_{m})+t(n_{\ell+1}+n_{\ell+2})x_{\ell+1}x_{\ell+2}}{(1-t\sum_{m=1}^{\ell}n_{m}x_{m})(1-t\sum_{m=1}^{\ell+1}n_{m}x_{m})(1-t\sum_{m=1}^{\ell+1}n_{m}x_{m})}$$

ILLUSTRATION



Drawn for t = 0.01, $n_1 = n_2 = n_3 = 5$, $n_4 = n_5 = 5$, $p_1 = p_2 = p_3 = 0.2$, $p_4 = p_5 = 0.1$, and $a_1^2 \sigma_1^2 = 1$.

HIERARCHIES

COROLLARY 1. Optimal communication gives rise to a receiver hierarchy among agents. For any agents $i, j, k \in \mathcal{N}$ who belong to different modules, if agent i's module is more cohesive than agent j's, then agent j is told about agent k's state only if agent i also is.

COROLLARY 2. Optimal communication gives rise to a sender hierarchy among agents. For any agents $i, j, k \in \mathcal{N}$ who belong to different modules, if agent i's threshold λ_i is smaller than agent j's threshold λ_j , then agent j tells agent k about his state only if agent i also does.

- Agent *i*'s rank in the receiver hierarchy depends only on module cohesion.
- But his rank in the sender hierarchy also depends on the autonomous value of adaptation $a_i^2 \sigma_i^2$.
- Agents who hear the most may not be the ones who speak the most.

BOTTOM-UP COMMUNICATION

- Suppose there are communication links from module \mathcal{M}_m to $\mathcal{M}_{m'}$ but not the reverse.
- Then communication is top down if $x_m > x_{m'}$ and bottom up if $x_m < x_{m'}$.
- Communication is *bottom up in aggregate* if there are more pairs of modules that engage in bottom-up than top-down communication.

PROPOSITION 3. If the optimal sender and receiver hierarchies are the reverse of each other, and the receiver ranking is strict, communication is bottom up in aggregate.



- A communication network has a core-periphery structure if the set of modules can be partitioned into a core and periphery such that:
 - An agent in the core tells his state to all other agents in the core & maybe to agents in the periphery.
 - An agent in the periphery does not tell his state to all agents in the core a/o is not told all their states.
 - An agent in the periphery does not tell his state to other agents in the periphery.

PROPOSITION 4. If the optimal sender and receiver hierarchies are identical, the communication network has a core-periphery structure in which the core consists of the most cohesive modules.



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Model

Solving the Model

Application

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Conclusions

Modular production

Modular organization



Mirroring Hypothesis Thompson (1967)

Conway's Law Conway (1968)



 $Northwestern \,|\, {\rm Kellogg}$

Modular production

Modular organization



Mirroring Hypothesis Thompson (1967)



Modular production

Modular organization





Partial



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NESTED MODULES



NESTED MODULES











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CONCLUSIONS

- Over the last half century, the economy has shifted towards modular production.
- This paper is a first step towards understanding the economic implications of this shift.
- Open questions about impact of modular production:
 - ➢ Interfaces.
 - Parallel processing.
 - Firm boundaries, industry structure, location of production.
- Broader question about the reason for the rise of modular production.
- Testable predictions for emerging empirical literature on within-firm communications.

REVISITING KEY ASSUMPTIONS

- Agents observe all states in their modules.
 - > Convenient & captures notion that agents working on the same module co-locate and share expertise.
 - > Results extend readily if each agent only observes his own state.
- No re-transmission of information.
 - Share this assumption with other papers (e.g. Calvó-Armengol & de Martí (2008), Calvó-Armengol, de Martí, & Prat (2015), Herskovic & Ramos (2020)).
 - Captures notion that the states are "rich" and can only be described effectively by the associated agent.
 - Essential for the separability result (Proposition 1).
- Independence of information and binary communication.
 - Share these assumptions with other papers (e.g. independence with Calvó-Armengol, de Martí, & Prat (2015) and binary communication with Calvó-Armengol & de Martí (2008)).
 - ➢ Essential for the separability result.

REVISITING KEY ASSUMPTIONS

- Absence of incentive conflicts.
 - Share this assumption with the literature on team theory.
 - ➢ It, too, is essential for the separability result.
 - PROPOSITION 6. If agents internalize only a fraction $\mu \in (0,1)$ of the needs to coordinate, an optimal communication network solves

$$\max_{C} \sum_{i=1}^{N} a_{i} \operatorname{Cov}(d_{i}^{*}, \theta_{i}) + (1 - \mu) \sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij} \operatorname{Cov}(d_{i}^{*}, d_{j}^{*}) - \gamma \sum_{i=1}^{N} (C_{i} 1 - n_{m(i)}),$$

where

$$Cov(d_i^*, \theta_i) = a_i \sigma_i^2 \omega_{ii}(\boldsymbol{C}_i, \mu)$$

and

$$\operatorname{Cov}(d_i^*, d_j^*) = \sum_{s=1}^N a_s^2 \sigma_s^2 \omega_{is}(\boldsymbol{C}_s, \mu) \omega_{js}(\boldsymbol{C}_s, \mu),$$

and where $\omega_{ij}(\mathbf{C}_j, \mu)$ denotes the *ij*th entry of $(I - (\operatorname{diag} \mathbf{C}_j)\mu \mathbf{P}(\operatorname{diag} \mathbf{C}_j))^{-1}$.

REVISITING KEY ASSUMPTIONS

- Production has a non-overlapping community structure.
 - Captures the notion of modular products.
 - Suppose **P** takes any form, provided it still satisfies $p_{ii} = 0$, $p_{ij} = p_{ji}$, and $\sum_{j=1}^{N} p_{ij} < 1$ for all $i, j \in \mathcal{N}$.
 - Separability result still holds, and principal's problem can still be solved using standard algorithms.
 - \blacktriangleright But the characterization result (Proposition 2) does not.
 - > Except for the comparative statics:
 - PROPOSITION 6. As long as the production network P satisfies $p_{ii} = 0$, $p_{ij} = p_{ji}$, and $\sum_{j=1}^{N} p_{ij} < 1$ for all *i*, *j* ∈ N, optimal communication networks C^{*}_i are increasing in a²_iσ²_i and p_{ij}, and decreasing γ.



BOTTOM-UP COMMUNICATION

- Suppose there are communication links from module \mathcal{M}_m to \mathcal{M}_m , but not the reverse.
- Then communication is *top down* if $x_m > x_m$, and *bottom up* if $x_m < x_{m'}$.
- Communication is *bottom up in aggregate* if there are more pairs of modules that engage in bottom-up than top-down communication.

PROPOSITION 3. If the optimal sender and receiver hierarchies are the reverse of each other, and the receiver ranking is strict, communication is bottom up in aggregate.

