

# Give to the poor or the needy: optimal carbon dividend distribution

Thomas-Olivier Léautier\*      Elise Viadere

June 1, 2022

## Abstract

Economists unanimously advocate a carbon dividend, i.e., the taxation of CO<sub>2</sub> and the redistribution of the proceeds, to efficiently reduce carbon emissions and make the accompanying cost increase socially acceptable. While an abundant literature estimates the optimal CO<sub>2</sub> price path, no analysis of the optimal dividend distribution has been developed. This article fills that gap, that derives the optimal redistribution for consumers heterogeneous along two dimensions: income and share of the carbon emitting good in their overall expenses. Applying the model to French data yield two insights: first, pricing carbon up to 150% of the carbon emitting good slightly increases social welfare, as the gains from redistribution exceed the value of lost consumption. Second, for reasonable values of the tax, only the two first deciles of the income distribution receive a positive carbon dividend. These findings illustrate both the social value and the political difficulty of enacting carbon dividend.

## 1 Introduction

While a strong scientific consensus emerges on the urgency to massively reduce anthropogenic GHG emissions to prevent catastrophic and irreversible climate change, a large gap exists between the quasi-unanimous recommendation of economists and the conviction of citizens and policy-makers on the best way to achieve this objective: the former advocates putting a price on CO<sub>2</sub>, which the latter opposes, sometimes violently. To reconcile these positions, economists propose a carbon dividend, the redistribution of the revenues raised by governments by the imposition of a CO<sub>2</sub> price, either through a tax or through the issuance of permits.

The distributional impact of a carbon dividend is potentially significant: the IPCC estimates the total GHG emissions in 2019 around 59 Gt. If these emissions were valued at 100 \$ per ton, their value would be \$ 59 trillions, around 7% of global GDP. While the actual revenues

---

\*Toulouse School of Economics and TotalEnergies

from GHG pricing is likely to be much lower, it could still represent a couple of percents of global GDP, hence its optimal distribution does matter. This article is the first we are aware of to examine the optimal carbon dividend.

From the latest IPCC report, scientists demonstrate that human influence has affected the climate at a rate that is unprecedented in at least the last 200 years (IPCC (2021)). In the context of the Conference of Parties (COP), many national governments have pledged on net-zero CO<sub>2</sub> emissions by mid-century\*. Moreover, since the COP 21 in Paris in 2015, around 40% of the participating countries have updated their plan for climate actions. In fact, countries accounting for around 70% of global CO<sub>2</sub> emissions and GDP have set net-zero emission pledges for 2050 in law, either in proposed legislation or in an official policy document (Bouckaert et al. (2021)).

An extensive economic literature points out that, to reduce substantially CO<sub>2</sub> emissions, implementing carbon pricing is a crucial decision (Stiglitz et al. (2017), Akerlof et al. (2019), Metcalf and Weisbach (2009))<sup>†</sup>. Carbon pricing generates credible incentives to redirect investment and consumption and initiate a selective degrowth of carbon-intensive industries. This concept traces back as far as Pigou (1932) and is a central tenet of environmental economics (Goulder (1995)).

A parallel abundant literature estimates the optimal carbon price path based on social cost of carbon (Pearce (2003), Pizer et al. (2014), Quinet et al. (2014)). The social cost of carbon (SCC) is defined as the cost of impacts associated with an additional unit of greenhouse gas emissions (Stern (2007)). In fact, the SCC is widely considered to be a key aspect of climate policy implementation. The influential *Stern Review* provides analysis of the costs and benefits of climate action by comparing current level and future trajectories of the SCC with the marginal abatement cost <sup>‡</sup>. Furthermore, according to Gollier (2021), any climate policy objective must be considered as an intertemporal carbon budget in the frame of a cost-efficient carbon price agenda. Additionally, under the Hotelling's rule, carbon price growth rate should be set to the interest rate (Hotelling (1931), Blanchard and Tirole (2021), Gollier (2021)).

However, even though there exists a large interest and a growing understanding of the SCC and carbon pricing implications in terms of climate policies, a widely accepted idea suggests that public acceptability is a critical factor to expand carbon pricing and reach pledged ambitions. Hence, carbon pricing has been enforced in many countries, but it also has met with strong opposition (Maestre-Andrés et al. (2019), Wier et al. (2005))<sup>§</sup>. Carbon price regressivity has lead this policy instrument to be largely debated and to face substantial public acceptability

---

\*These pledges are also known as Nationally determined contributions (NDCs).

<sup>†</sup>Carbon pricing refers to both cap-and-trade/permits system and carbon tax.

<sup>‡</sup>The marginal abatement cost can be defined as the costs associated with incremental reductions in units of emissions.

<sup>§</sup>As of September 2021, according to the *World Bank carbon pricing dashboard*, 27 countries have implemented a national carbon pricing system.

issue.

One can quote the *yellow vest* protest movement in France that started in October 2018. In 2014, the French government set a 7€/tCO<sub>2</sub> carbon price in the tax on fossil fuel. In 2018, the carbon price reached 44.6€/tCO<sub>2</sub> and was planned to reach 86.2€/tCO<sub>2</sub> in 2022. By the end of 2018, among other measures increasing fuel prices, the French government decided to accelerate the carbon price trajectory (Douenne and Fabre (2022)). This policy decision triggered the *yellow vest* protest movement. This public protestation finds its essence in the regressive distributional incidences of the carbon price on the lowest income household's purchasing power (Martin and Islar (2021), Douenne and Fabre (2022)). An expanded economic literature demonstrates that carbon tax is regressive. Bureau et al. (2019) demonstrates that carbon tax implementation in France tends to impact relatively more the lower-income households than the higher-income households in terms of purchasing power. In an empirical analysis on French data from 2003 to 2006, Bureau (2011) finds that French carbon taxation is regressive before revenue recycling. Furthermore, a research on the distributional effect of car fuel taxation in France, Berri (2005) suggests that this policy tool is in fact regressive. Additionally, the author demonstrates that redistributing carbon tax revenues either in equal lump-sum transfers to each household or according to household size makes the lowest income households better off.

Therefore, because of unsuitable distributional effects, the public acceptability of carbon pricing, and particularly of carbon taxation, forms a substantial challenge to implement climate policies. There exist an extensive literature on carbon double dividend: Bosello et al. (2001) and Pezzey and Park (1998) to quote a few. This literature suggests that carbon taxation revenues can be used to reduce other taxation instruments such as labor tax. By doing so, carbon tax allows to implement the polluter-pays principle and to internationalize environmental externalizes, this the first dividend. Moreover, reducing labor tax leads to an increase of the workforce and an increase of employment, this the second dividend. However, this literature only models carbon dividend by a labor tax reduction implemented with carbon tax revenues.

There also exists a rich empirical literature on the impact of carbon pricing on households expenditures, using extremely granular data, for example Fremstad and Paul (2019), Rausch et al. (2011).

To our best knowledge, this academic literature does not derive the optimal dividend distribution among heterogeneous consumers. A priori, two dimensions matter: (i) the income level - relative poverty, and (ii) the share of this income allocated to consumption of carbon-intensive / dirty goods - the need. One expects the optimal distribution trades-off these two dimensions.

We develop a highly simplified model to examine this issue analytically and empirically. The main simplification is the use of the Cobb-Douglas utility function. This provides for closed-forms expressions, which facilitates the description of the optimal policy. In addition, demands for different goods are somehow separable: increasing one good's price leads to a reduction in

this good's consumption, but crucially does not lead to an increase in the other goods' demand. This holds because demand for any good depends on its price, not on other goods' prices. Finally, the optimal carbon dividend distribution and the resulting social welfare change are functions of very few parameters: the carbon price, and for each consumer class considered (in our case, deciles of the income distribution), the average income and share spent on carbon-intensive / dirty goods. These properties render the model particularly easy to apply to actual data, as the specification is extremely parsimonious.

As usual with this approach, we trade-off the analytical insights we derive against lost precision in our empirical predictions. Further work, leveraging more granular data, is required to confirm our predictions.

This analysis yields three analytical results. First, for the Cobb-Douglas utility function, optimal carbon dividend distribution is driven by income not by need. Specifically, allocations are determined by the residual income, net of the tax paid on the carbon-intensive / dirty good, i.e., the income effectively available to purchase goods and services. Second, if residual incomes among different consumer classes are close enough, optimal carbon dividend distribution equalises their marginal utility. Otherwise, carbon dividends are primarily distributed to consumers with lowest adjusted income. Third, at least for low values of the carbon tax, if a mild condition on income is met, carbon pricing and optimal dividend distribution increases welfare: the positive income effect received by the few more than compensates for the negative price effect imposed on all.

These results are illustrated using French data. First, carbon pricing and optimal dividend distribution increases welfare for a tax up to 150% of the pre-tax price of the carbon-intensive / dirty good, which should embolden policy makers to enact that policy. Second, for a tax equal to 40% of the pre-tax price of the carbon-intensive / dirty good, only the first two deciles of the population in terms of adjusted income receive a positive dividend, hence 80% of the population is worse off. This underscores the political difficulty of implementing carbon pricing and optimal dividend distribution, and reduces policy makers' incentives to even discuss that policy.

This article is structured as follows: Section 2 sets up the model. Section 3 examines the specific case of two consumer classes to build intuition. Section 4 demonstrates the main analytical results of the article: the optimal dividend distribution policy for  $N$  consumers. Section 5 applies the analysis to French consumers. Section 6 concludes.

## 2 Description of the economy

The economy is composed of two goods: a carbon-intensive / dirty and taxed good  $x^D$  and a composite /clean good denoted  $x^C$ , representing all other non carbon intensive goods. Absent

carbon pricing, prices are respectively  $p^D$  and  $p^C$ . A carbon price, resulting from a tax or from a cap-and-trade mechanism is imposed on the carbon intensive good  $x^D$ , which rate is  $\tau \geq 0$ . With carbon pricing, consumers face prices  $(1 + \tau)p^D$  and  $p^C$ .

There are  $N$  classes of consumers denoted  $n = 1, \dots, N$ . Consumers are homogeneous within each class. Therefore, consumer  $n$  represents class  $n$ .  $N_n$  with  $n = 1, \dots, N$  is the number of consumers in class  $n$ . We assume that each consumer class has the same number of consumers  $N_1 = N_2 = \dots = N_N$ . This assumption simplifies the exposition and is consistent with empirical analysis when classes are deciles of income distribution. The extension to different classes' size - or to differing classes' weights in the social welfare function - is straightforward.

We model two decisions. First, the government decides the carbon dividend distribution among the different consumer classes. Second, consumers make their decision, facing prices including carbon price, and dividend revenues. There is no uncertainty nor asymmetric information in our simple model: when distributing carbon dividends at stage 1, the government perfectly anticipates customers decisions at stage 2.

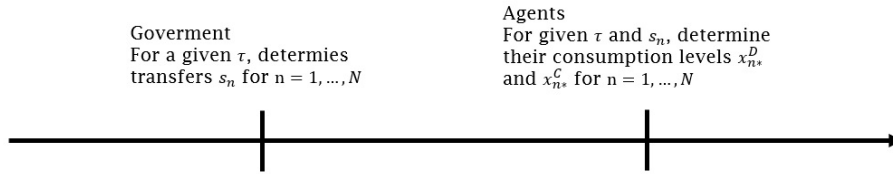


Figure 1: Model timeline

## 2.1 Consumers

Consumer's  $n$  utility function is:

$$U_n(x_n^D, x_n^C) = \alpha_n \ln(x_n^D) + (1 - \alpha_n) \ln(x_n^C) \quad \text{with } \alpha_n \in (0, 1)$$

Consumer  $n$  is endowed with an exogenous monetary income  $m_n$  and receives transfer  $s_n$  from the government. Consumer  $n$  maximises her utility under her budget constraint, defined as:

$$p^D(1 + \tau)x_n^D + p^C x_n^C = m_n + s_n$$

We assume that consumer  $n$  is myopic: she takes  $s_n$  as given, i.e. she does not internalise the impact of her carbon intensive good consumption decision  $x_n^D$  on the tax levied by the government hence on the transfer  $s_n$  (see section 1.3) she receives.

Standard analysis shows consumer's  $n$  demand functions are :

$$x_n^D(s_n) = \alpha_n \frac{(m_n + s_n)}{p^D(1 + \tau)} \quad (1)$$

$$x_n^C(s_n) = (1 - \alpha_n) \frac{(m_n + s_n)}{p^C} \quad (2)$$

Throughout this analysis, we exploit a convenient property of Cobb-Douglas utility functions: the demand functions presented in equations (1) and (2) do not depend on the price of the other good. As the carbon price increases, consumption of the carbon-intensive / dirty good is reduced, while each good's share in the total consumption remains constant. This property enables us to estimate the optimal carbon dividend distribution using only very few data. Ignoring substitution overestimates the welfare loss associated with the carbon price: in reality, consumers would consume more of the composite good to compensate for the lower consumption of the carbon-intensive / dirty good.

## 2.2 Government's program

### 2.2.1 Government budget constraint

The carbon tax revenue is:

$$R = \tau p^D \left( \sum_{n=1}^N x_n^D \right)$$

The carbon dividend is budget neutral: post-tax transfers are equal to the tax revenues collected on the pre-tax income spent on the taxed carbon-intensive / dirty good:

$$\sum_{n=1}^N s_n = R$$

All transfers  $s_n$  are non-negative. If the carbon tax level  $\tau = 0$ , the government transfers are all equal to zero:  $s_n = 0 \quad \forall n$ .

Inserting the demand functions (1) for the carbon-intensive / dirty good equations into the carbon tax revenue  $R$ , then rearranging yields:

$$\sum_{n=1}^N s_n = \tau \left( \sum_{n=1}^N \frac{\alpha_n (m_n + s_n)}{(1 + \tau)} \right)$$

$$\sum_{n=1}^N s_n \left( 1 - \alpha_n \frac{\tau}{(1 + \tau)} \right) = \frac{\tau}{(1 + \tau)} \sum_{n=1}^N \alpha_n m_n$$

We define  $\delta = \frac{\tau}{(1 + \tau)}$ , the share of the carbon tax in the carbon-intensive / dirty good's price, and  $m = \sum_{n=1}^N \alpha_n m_n$  the share of pre-tax income spent as carbon-intensive / dirty good.

Government's budget neutrality constraint becomes:

$$\sum_{n=1}^N s_n (1 - \delta \alpha_n) = \delta m \quad (3)$$

When she receives transfer  $s_n$ , available additional income post-tax on the carbon-intensive / dirty good for consumer  $n$  is only the fraction  $s_n(1 - \delta \alpha_n)$ , which we assume is non negative. The government budget neutrality equation (3) states that the adjusted additional income available to all agents is equal to the share of the pre-dividend income spent on the carbon-intensive / dirty good.

To simplify the notation, we order consumers by increasing adjusted income for a given  $\delta$  (see **section** ):

$$m_1(1 - \delta \alpha_1) < \dots < m_n(1 - \delta \alpha_n) < \dots < m_N(1 - \delta \alpha_N) \quad (4)$$

### 2.2.2 Optimal transfer

Assuming  $\tau > 0$ , the government program is:

$$\begin{cases} \max_{\{s_n\}} W = \sum_{n=1}^N \alpha_n \ln \left( \alpha_n \frac{m_n + s_n}{p^D(1+\tau)} \right) + (1 - \alpha_n) \ln \left( (1 - \alpha_n) \frac{m_n + s_n}{p^C} \right) \\ \text{s.t.} \quad \sum_{n=1}^N s_n (1 - \delta \alpha_n) = \delta m \\ \quad \quad \quad -s_n \leq 0 \quad \text{with} \quad n = 1, \dots, N \end{cases}$$

The Lagrangian is:

$$\begin{aligned} \mathcal{L} = & \sum_{n=1}^N \alpha_n \ln \left( \alpha_n \frac{m_n + s_n}{p^D(1+\tau)} \right) + (1 - \alpha_n) \ln \left( (1 - \alpha_n) \frac{m_n + s_n}{p^C} \right) \\ & - \lambda (\sum_{n=1}^N s_n (1 - \delta \alpha_n) - \delta m) + \sum_{n=1}^N \mu_n s_n \end{aligned}$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial s_n} = 0 \quad \Rightarrow \quad \frac{1}{m_n + s_n} - \lambda(1 - \delta \alpha_n) + \mu_n = 0 \quad \text{for} \quad n = 1, \dots, N \quad (5)$$

## 3 2-consumer case

To build intuition for the structure of the optimal dividend distribution, we first examine the 2-consumer case.

### 3.1 Optimal transfers

From equations (5), two cases are possible:

- Case n°1:  $s_1 > 0$  and  $s_2 > 0$  (i.e.  $\mu_1 = \mu_2 = 0$ ): both consumers receive a positive transfer.
- Case n°2:  $s_1 > 0$  and  $s_2 = 0$  (i.e.  $\mu_1 = 0$  and  $\mu_2 > 0$ ): only the poorer consumer receives a transfer. Intuition suggests that the wealthier consumer stops receiving a positive transfer first, hence the case  $s_1 = 0$  and  $s_2 > 0$  is impossible. We formally establish this result below.

We now examine each case in turn.

#### 3.1.1 Both consumers receive a positive transfer

The FOCs and revenue-neutrality constraint are:

$$\begin{cases} \frac{1}{m_1+s_1} - \lambda(1 - \delta\alpha_1) = 0 \\ \frac{1}{m_2+s_2} - \lambda(1 - \delta\alpha_2) = 0 \\ s_1(1 - \delta\alpha_1) + s_2(1 - \delta\alpha_2) = \delta m \end{cases}$$

Algebra yields:

$$m_n + s_n = \frac{m_1 + m_2}{2(1 - \delta\alpha_n)} \quad \text{for } n = 1, 2 \quad (6)$$

We establish the following:

**Result 1.** *With the ordering of consumers we have selected, if the richer consumers receive a positive transfer, then so do the poorer ones.*

*Furthermore, richer consumers receive a positive transfer as long as the pre-tax income disparity remains below a given threshold.*

*Proof.* From equation (6), for  $n = 1, 2$

$$s_n = \frac{m_1 + m_2}{2(1 - \delta\alpha_n)} - m_n = \frac{m_1 + m_2 - 2(1 - \delta\alpha_n)m_n}{2(1 - \delta\alpha_n)}$$

Define  $y_n = m_1 + m_2 - 2(1 - \delta\alpha_n)m_n$ . We have:

$$y_1 - y_2 = 2((1 - \delta\alpha_2)m_2 - (1 - \delta\alpha_1)m_1)$$

With our ordering assumption (4),  $y_1 > y_2$ : if the richer consumers receive a positive transfer,



then so do the poorer ones. This proves the first part of the result.

Then,

$$s_2 > 0 \iff m_1 + m_2 > 2(1 - \delta\alpha_2)m_2 \iff m_1 > m_2(1 - \delta\alpha_2) \iff \frac{m_2 - m_1}{m_2} < 2\delta\alpha_2$$

which proves the second part of the result.  $\square$

As  $\tau$  decreases, for a given  $m_1$ , the boundary  $m_1(1 - 2\delta\alpha_1)$  increases: the wealthiest consumer receives a positive transfer less often. As carbon tax revenues become scarcer, policy makers prioritize them toward less fortunate households.

**At the limit when  $\tau \rightarrow 0$ , the condition for  $s_1 \geq 0$  from equation (6) tends toward  $m_2 > m_1$  : the wealthiest households never receives a positive transfer. Then,  $\lim_{\tau \rightarrow 0} s_2(\tau) = 0$  :  $s_2(\tau)$  is continuous at  $\tau = 0$ .**

The transfers in the  $(m_1, m_2)$  plane are presented in Figure 2. The upper left area of the Figure 2 is not considered since it would imply redistributing only to agent 1, the richest agent.

### 3.1.2 Only the poorer consumer receives a positive transfer

The FOCs and revenue-neutrality constraint are:

$$\begin{cases} \frac{1}{m_2} - \lambda(1 - \delta\alpha_2) + \mu_2 = 0 \\ \frac{1}{m_1 + s_1} - \lambda(1 - \delta\alpha_1) = 0 \\ s_1(1 - \delta\alpha_1) = \delta m \end{cases} \quad (7)$$

Then,

$$\begin{cases} s_1 = 0 \\ s_2 = \frac{\delta m}{(1 - \delta\alpha_2)} \end{cases} \quad (8)$$

We verify that as  $\lim_{\tau \rightarrow 0} s_2 = 0$ : as the carbon tax rate tends to zero, transfers continuously tend to zero.

### 3.1.3 Transfers' graphical analysis

In appendix A, we present optimal transfers' boundary analysis. Let us represent the upper and lower boundary functions in the space  $(m_1, m_2)$ .

When income disparity is substantial, the highest income consumer does not receive transfer. Thus, the optimal transfer implementation structure is based on income disparity among consumers. This intuition is detectable in Figure 2: the higher  $m_1$  relatively to  $m_2$ , the less likely

consumer 1 is to receive positive transfer.

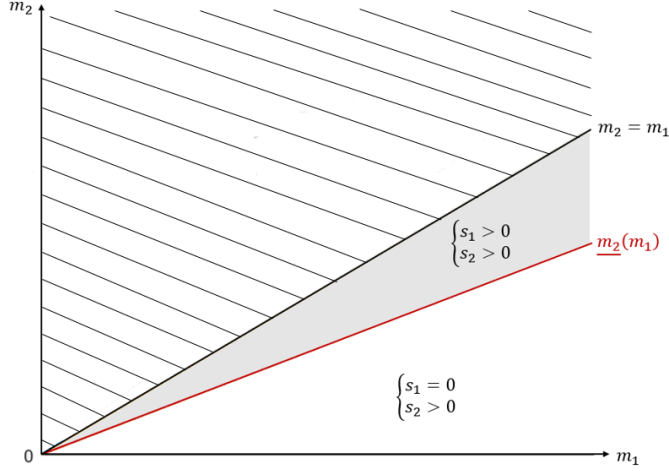


Figure 2: Transfers in the  $(m_1, m_2)$  plane

### 3.2 Impact of carbon tax rate on social welfare

The social welfare function is taken as the sum of the consumers' utilities:

$$W = \sum_{n=1}^2 \left[ \alpha_n \ln \left( \alpha_n \frac{m_n + s_n}{p^D(1 + \tau)} \right) + (1 - \alpha_n) \ln \left( (1 - \alpha_n) \frac{m_n + s_n}{p^C} \right) \right] \quad (9)$$

It is helpful to isolate the 'reduced' social welfare denoted  $\tilde{W}(\tau)$ , which is the portion of  $W$  that depends on  $\tau$ :

$$\tilde{W}(\tau) = \sum_{n=1}^2 [\ln(m_n + s_n) - \alpha_n \ln(1 + \tau)]$$

This illustrates the two effects of the carbon dividend: First, the positive income effect: distribution of  $s_n \geq 0$ . Second, the negative price effect: the carbon tax reduces carbon-intensive / dirty good consumption, hence social welfare. As indicated previously, two or only one consumer may receive a positive transfer. We examine each case in turn.

#### 3.2.1 Both consumers receive positive transfer

Algebra yields:

$$\frac{\partial W}{\partial \tau} = \sum_{n=1}^2 \left[ \frac{1}{m_n + s_n} \frac{\partial(m_n + s_n)}{\partial \delta} \frac{\partial \delta}{\partial \tau} - \frac{\alpha_n}{1 + \tau} \right]$$

Observing that

$$\frac{\partial(m_n + s_n)}{\partial \delta} = \frac{\alpha_n(m_n + s_n)}{1 - \delta \alpha_n}$$

and

$$\frac{\partial \delta}{\partial \tau} = \frac{1}{(1 + \tau)^2}$$

we have

$$\begin{aligned} \frac{\partial W}{\partial \tau} &= \sum_{n=1}^2 \frac{\alpha_n}{1 + \tau} \left[ \frac{1}{(1 - \delta \alpha_n)(1 + \tau)} - 1 \right] \\ &= -\frac{\tau}{1 + \tau} \sum_{n=1}^2 \frac{\alpha_n(1 - \alpha_n)}{1 + \tau(1 - \alpha_n)} < 0 \end{aligned} \quad (10)$$

If the government provides positive transfers for both consumers, the increasing carbon tax rate reduces social welfare. The positive income effect produced by redistribution is not sufficient to compensate for the negative quantity effect from taxation.

### 3.2.2 Only the poorer consumers receive a positive transfer

Since the wealthier consumers receives no carbon dividend, the reduced social welfare is :

$$\tilde{W}(\tau) = \ln(m_1 + s_1) - \left( \sum_{n=1}^2 \alpha_n \right) \ln(1 + \tau)$$

We have:

$$m_1 + s_1 = \frac{m_1 + \delta \alpha_2 m_2}{1 - \delta \alpha_2}$$

Then:

$$\frac{\partial(m_1 + s_1)}{\partial \delta} = \frac{m}{(1 - \delta \alpha_1)^2}$$

Hence:

$$\begin{aligned} \frac{\partial W}{\partial \tau} &= \frac{m}{(1 + \tau)^2(1 - \delta \alpha_1)(m_1 + \delta \alpha_2 m_2)} - \sum_{n=1}^2 \frac{\alpha_n}{1 + \tau} \\ &= \frac{m}{(1 + \tau(1 - \alpha_1))(m_1 + \tau(m_1 + \alpha_2 m_2))} - \sum_{n=1}^2 \frac{\alpha_n}{1 + \tau} \end{aligned} \quad (11)$$

From equation (11), the first term is the positive income effect for the poorer consumer, and the second term is the negative price impact applied to all consumers. Signing  $\frac{\partial W}{\partial \tau}$  for all values of  $\tau$  is not straightforward. However, for small values of  $\tau$ , we have the following:

$$\begin{aligned} \left. \frac{\partial W}{\partial \tau} \right|_{\tau=0} &= \frac{m}{m_1} - (\alpha_1 + \alpha_2) \\ &= \frac{\alpha_2(m_2 - m_1)}{m_1} \end{aligned} \quad (12)$$

Then, a small carbon tax optimally redistributed increases welfare if and only if the pre-tax and dividend income is higher for richer consumers than for poorer ones. This condition is likely to be met in practice (and is verified on the French data). The intuition is that the negative impact of the reduction in consumption of the carbon-intensive / dirty good is more than offset by the positive income impact on the poorer consumers.

## 4 N-consumer classes

We now extend the results to  $N$  consumers. As with the 2-consumers case, not all consumer classes receive a transfer. We denote  $k \leq N$  the number of customers receiving a carbon dividend. Unless otherwise specified, we assume  $\tau > 0$ , hence  $k \geq 1$ . As with the 2-consumer case, our ordering assumption (4) guarantees that consumers stop receiving a transfer in decreasing order, in other words, all consumers  $n \leq k$  receive a positive transfer, and all consumers  $n > k$  receive no transfer. This result is proven formally below.

The FOCs and budget-neutrality constraint are:

$$\begin{cases} \frac{1}{m_n} - \lambda(1 - \delta\alpha_n) + \mu_n = 0 & \text{for } n > k \\ \frac{1}{m_n + s_n} - \lambda(1 - \delta\alpha_n) = 0 & \text{for } n \leq k \\ \sum_{i \geq k}^N s_i(1 - \delta\alpha_i) = \delta m \end{cases} \quad (13)$$

The  $k$  poorer consumers receive a carbon dividend  $s_n > 0$ , which equalize their marginal utility of income. This is not possible for the  $(N - k + 1)$  richer consumers, hence they receive no carbon dividend:  $s_n = 0$  for  $n > k$ . Since  $s_n = 0$  for  $n > k$ , the government budget balance simplifies to the sum for  $n \leq k$ .

We first determine the optimal dividend distribution policy:

**Lemma 1.** *The optimal dividends are such that:*

$$\begin{aligned} m_n + s_n &= \frac{\delta m + \sum_{i \leq k} m_i(1 - \delta\alpha_i)}{k(1 - \delta\alpha_n)} \quad \text{for } n \leq k \\ &= m_n \quad \text{for } n > k \end{aligned} \quad (14)$$

*Proof.* From the first-order conditions in equation (13), we have for  $n \leq k$ :

$$(m_n + s_n)(1 - \delta\alpha_n) = \frac{1}{\lambda}$$

Summing over  $n \leq k$  and substituting in the government budget balance in equation (13):

$$\frac{k}{\lambda} = \sum_{n \leq k} (m_n + s_n)(1 - \delta\alpha_n) = \sum_{n \leq k} m_n(1 - \delta\alpha_n) + \sum_{n \leq k} s_n(1 - \delta\alpha_n) = \sum_{n \leq k} m_n(1 - \delta\alpha_n) + \delta m$$

Hence

$$\lambda = \frac{k}{\delta m + \sum_{n \leq k} m_n(1 - \delta\alpha_n)}.$$

Thus

$$m_n + s_n = \frac{\delta m + \sum_{n \leq k} m_n(1 - \delta\alpha_n)}{k(1 - \delta\alpha_n)},$$

which proves the result.  $\square$

When  $\tau$  is substantial, redistribution tends to reduce inequalities among the poorest consumers but takes into account the distortion in terms of purchasing power for consumers based on the level of the tax and their carbon-intensive / dirty good dependency.

We then formally characterize the optimal carbon dividend distribution policy:

**Proposition 1.** *The government distributes carbon dividends*

$$s_n = \frac{\delta m + \sum_{i \leq k} (m_i(1 - \delta\alpha_i) - m_n(1 - \delta\alpha_n))}{k(1 - \delta\alpha_n)} \quad (15)$$

for all  $n \leq k$ , where  $k$  is the highest consumer class verifying:

$$\sum_{i \leq k} (m_k(1 - \delta\alpha_k) - m_i(1 - \delta\alpha_i)) < \delta m. \quad (16)$$

For all  $n > k$ , no carbon dividends are distributed.

The number of consumer classes receiving a carbon dividend increases with the carbon price.

*Proof.* For  $n \leq k$ , equation (14) yields

$$s_n = \frac{\delta m + \sum_{i \leq k} m_i(1 - \delta\alpha_i) - km_n(1 - \delta\alpha_n)}{k(1 - \delta\alpha_n)} = \frac{\delta m + \sum_{i \leq k} (m_i(1 - \delta\alpha_i) - m_n(1 - \delta\alpha_n))}{k(1 - \delta\alpha_n)},$$

which is equation (15).

Define

$$y_n = \delta m + \sum_{i \leq k} (m_i(1 - \delta\alpha_i) - m_n(1 - \delta\alpha_n)).$$

We have

$$y_{n-1} - y_n = k(m_n(1 - \delta\alpha_n) - m_{n-1}(1 - \delta\alpha_{n-1})).$$

Then, by our ordering assumption (4),  $y_{n-1} - y_n > 0$ : if consumer  $n$  receives a carbon dividend, then so do all consumers below  $n$ .

Then, for  $n = k$ , our ordering assumption (4) implies all the terms in the sum in  $y_k$  are negative.  $k$  is the highest consumer class such that

$$y_k > 0 \iff \sum_{i \leq k} (m_k(1 - \delta\alpha_k) - m_i(1 - \delta\alpha_i)) < \delta m.$$

Then,

$$y_k = \delta(m - \sum_{i \leq k} (m_i\alpha_i - m_k\alpha_k)) + \sum_{i \leq k} (m_i - m_k) = \delta(\sum_{i > k} m_i\alpha_i + km_k\alpha_k) + \sum_{i \leq k} (m_i - m_k)$$

Hence  $y_k$  is increasing in  $\delta$  by inspection, and larger than zero for  $\delta > \delta_k$  defined by:

$$\delta_k = \frac{\sum_{i \leq k} (m_k - m_i)}{\sum_{i > k} m_i\alpha_i + km_k\alpha_k}.$$

Thus, as the carbon price increases,  $\delta$  increases; and for every consumer class  $k$ , there exists a level of carbon price  $\tau_k = \frac{\delta_k}{1 - \delta_k}$  such that this class starts receiving a carbon dividend. By contradiction, one can prove that  $\tau_{k+1} > \tau_k$ : as the carbon price increases, more consumer classes receive a positive carbon dividend.  $\square$

## 4.1 Impact of carbon tax on social welfare

The social welfare function is:

$$W = \sum_n \left[ \alpha_n \ln \left( \alpha_n \frac{m_n + s_n}{p^D(1 + \tau)} \right) + (1 - \alpha_n) \ln \left( (1 - \alpha_n) \frac{m_n + s_n}{p^C} \right) \right]$$

To examine the social welfare impact of increasing the carbon tax  $\tau$ , we isolate the terms where carbon tax  $\tau$  is present, i.e., we write  $W = W_0 + \tilde{W}(\tau)$ , where

$$\tilde{W}(\tau) = \sum_{n \leq k} \ln(m_n + s_n) - \sum_{n \leq N} \alpha_n \ln(1 + \tau)$$

The first term is the positive income effect on the  $k$  first consumers, the last term is the negative price effect faced by all consumer classes. We have:

**Proposition 2.** *If all consumers receive a carbon dividend, then social welfare is reduced.*

Consider a infinitely small but positive carbon tax. If only one consumer receives a positive dividend at the optimal redistribution, the social welfare is increased locally if and only if  $\sum_{n>1} \alpha_n(m_n - m_1) > 0$ .

*Proof.* To compute  $\frac{dW}{d\tau}$ , we start by  $\frac{d \ln(m_n + s_n)}{d\tau}$ . We first rearrange  $(m_n + s_n)$  given by equation (14):

$$m_n + s_n = \frac{\sum_{i \leq k} m_i + \delta \sum_{i > k} \alpha_i m_i}{k(1 - \delta \alpha_n)}.$$

Then,

$$\frac{d \ln(m_n + s_n)}{d\tau} = \frac{\alpha_n \sum_{i \leq k} m_i + \sum_{i > k} \alpha_i m_i}{(1 - \delta \alpha_n)(\sum_{i \leq k} m_i + \delta \sum_{i > k} \alpha_i m_i)},$$

and

$$\frac{dW}{d\tau} = \sum_{n \leq k} \frac{\alpha_n \sum_{i \leq k} m_i + \sum_{i > k} \alpha_i m_i}{(1 + \tau(1 - \alpha_n))(\sum_{i \leq k} m_i + \tau(\sum_{i \leq k} m_i + \sum_{i > k} \alpha_i m_i))} - \sum_{n \leq N} \frac{\alpha_n}{1 + \tau}.$$

Then, if  $k = N$

$$\begin{aligned} \frac{dW}{d\tau} &= \sum_{n \leq N} \left( \frac{\alpha_n}{(1 + \tau(1 - \alpha_n))(1 + \tau)} - \frac{\alpha_n}{1 + \tau} \right) \\ &= -\frac{\tau}{1 + \tau} \sum_{n \leq N} \frac{\alpha_n(1 - \alpha_n)}{1 + \tau(1 - \alpha_n)} < 0 \end{aligned}$$

which proves the first part of the proposition.

Then, for  $\tau$  infinitely small, only one customer receives a positive carbon dividend, i.e.,  $k = 1$ .

Then,

$$\left. \frac{dW}{d\tau} \right|_{\tau=0} = \frac{\alpha_1 m_1 + \sum_{n>1} \alpha_n m_n}{m_1} - \sum_{n \leq N} \alpha_n = \frac{\sum_{n>1} \alpha_n (m_n - m_1)}{m_1},$$

which proves the second part of the proposition.  $\square$

Proposition 2 extend to  $N$  consumer classes the intuition developed on 2 consumer classes: if heterogeneity among customer classes is low enough that all classes receive a carbon dividend, then the income gains are not sufficient to compensate for the price increase. If, on the other hand, the tax is infinitely small, such that only one customer class benefits, then the income gain for that single class more than compensate for the price increase on all consumers.

## 5 Application to France dataset

### 5.1 Set up

Using the  $N$ -consumer case developed in section 4, we provide an application using INSEE French data. As discussed previously, few data are required to provide insightful intuition about the impact of optimal carbon dividend on French economy: goods demands, revenues and social welfare variation.

### 5.2 Data set presentation

From the French statistical institute INSEE, our dataset is built by income deciles. Therefore, when our model is applied to data, we have a sample of 10 consumers. We present how we matched the model's parameters with data:

- Consumer's initial endowments  $m_n$  with  $n = 1, \dots, 10$ : we use the INSEE's decile distribution of net monthly salaries in full-time equivalent in euros in 2018. For the 10<sup>th</sup> decile, explicit data was not systematically available. Therefore, we use the 95<sup>th</sup> income percentile data.
- Consumer's preference for the carbon-intensive good  $\alpha_n$  with  $n = 1, \dots, 10$ : we use the INSEE's share of gas, fuel and electricity in households' expenditure per income decile in France in 2017
- The carbon price  $\tau$  is set 0.4 in the base case. We also present examine different values of  $\tau$ .

### 5.3 Base case $\tau = 0.4$

For each of our ten consumer classes, Table 1 below presents the income  $m_n$ , the share of the carbon-intensive / dirty good in expenditures  $\alpha_n$  and, for carbon price  $\tau = .4$ , the adjusted income  $m(1 - \delta\alpha)$ . Lower-income consumer classes present larger share of the carbon-intensive / dirty good in expenditures than higher-income consumers. Therefore, *ceteris paribus*, their adjusted income  $m(1 - \delta\alpha)$  is relatively more impacted by the carbon price. Carbon price without optimal redistribution make every consumer class worse off. However, the lowest-income consumers carry the cost of climate policies relatively more.

Then, Figure 3 presents the optimal carbon dividend distribution by consumer class: only the first two deciles receive a positive carbon dividend. The redistribution effect is significant: 18.6 % of income for the first decile, and 7 % for the second decile of the income distribution. This confirms that carbon dividend can play a significant redistributive role.

Figure 4 presents the change in consumption of the carbon-intensive / dirty good for each



	$m_n$	$\alpha_n$	$m_n(1 - \delta\alpha_n)$
1 <sup>st</sup> decile	1282	6%	1251,2
2 <sup>nd</sup> decile	1423	6.1%	1388,3
3 <sup>rd</sup> decile	1552	5.9%	1515,4
4 <sup>th</sup> decile	1697	5.6%	1659,0
5 <sup>th</sup> decile	1871	5.4%	167,3
6 <sup>th</sup> decile	2088	5%	2046,2
7 <sup>th</sup> decile	2383	5%	2335,3
8 <sup>th</sup> decile	2848	4.4%	2797,9
9 <sup>th</sup> decile	3776	4.3%	3711,1
10 <sup>th</sup> decile	4932	4%	4853,1

Table 1: Data presentation. Income  $m_n$ , share of the carbon-intensive / dirty good in expenditures  $\alpha_n$  and adjusted income  $m(1 - \delta\alpha)$  per decile for a carbon tax level  $\tau = 0.4$ .

Source: INSEE - Distribution des salaires mensuels nets en équivalent temps plein (EQTP) en 2018 ; INSEE - Les dépenses des ménages en 2017 Enquête Budget de famille

consumer class. All classes reduce their consumption, due to the price increase. The reduction is lower for the first two deciles, due to their increased income. The reduction is constant for all other classes, determined by the ratio of prices  $1/(1 + \tau)$ .

Figure 5 presents the change in utility for each consumer class, as a percentage of the no-carbon-price utility  $U_n^0 = U_n(\tau = 0)$ . To compute  $U_n^0$ , we need prices  $p^C$  and  $p^D$  of the composite / clean and carbon-intensive / dirty goods. These are not readily available in the data. Hence, we estimate the ratios  $U_n(\tau)/U_n^0$  for different combinations of prices, and find that our conclusions broadly hold. In the main text, we present results for  $p^C = 3$  and  $p^D = 4$ ; additional results are presented in the Appendix.

Consumers in the first two deciles of the income distribution experience a large utility increase, as their higher income more than compensates for the higher price of the carbon-intensive / dirty good. All other consumers experience only the price effect, hence their utility is reduced. This reduction is lower for consumers in the higher deciles of the income distribution, since the carbon-intensive / dirty good represents a progressively lower share of their expenditures.

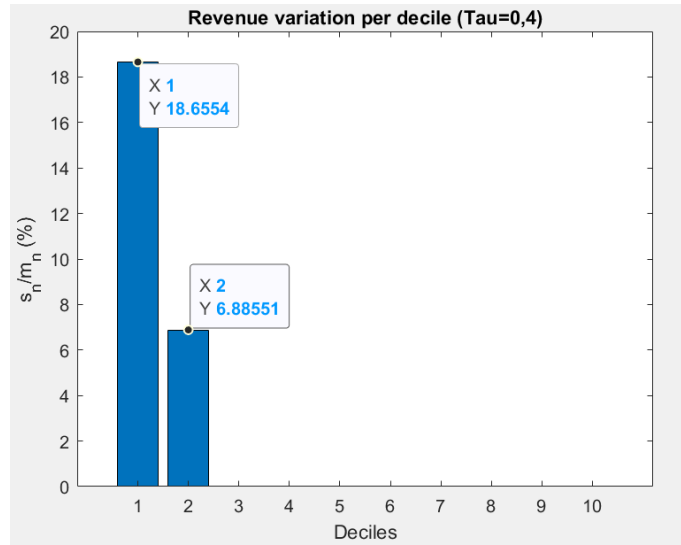


Figure 3: Revenue variation with and without carbon dividend per decile. Source: INSEE data and authors' computation.

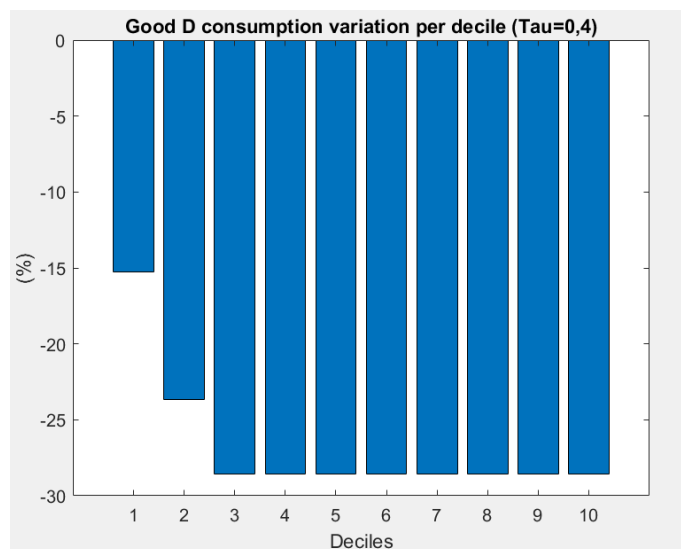


Figure 4: Good D's consumption variation with and without carbon dividend per decile. Source: INSEE data and authors' computation.

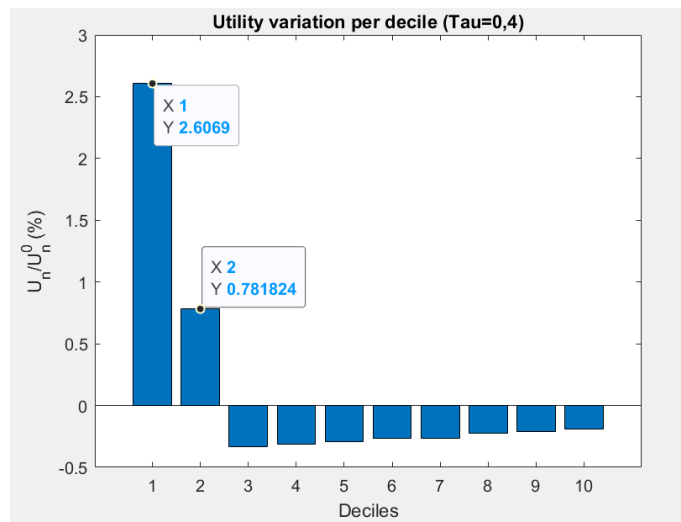


Figure 5: Utility variation with and without carbon dividend per decile,  $\tau = 4$ ,  $p^C = 3$ ,  $p^D = 4$ .  
 Source: INSEE data and authors' computation.

	Demand for good D	Demand for good C	Social welfare
No carbon price	100	100	100
Carbon price with optimal redistribution	72,68	101,40	100,10
Carbon price <i>without</i> redistribution	71,43	100	99,76
Carbon price with identical lump-sum transfers	72,50	101,41	99,98

Table 2: Total consumption and social welfare. The results in value are normalised to 100 for the *No carbon price* case. Each other case present the variation from the *No carbon price* case. The normalised results are presented for the prices  $p^C = 4$  and  $p^D = 3$ . Source: INSEE data and authors' computation. (See appendix B for identical lump-sum transfers computations)

Finally, Table 2 presents the total consumption for the carbon-intensive / dirty and composite / clean goods, and social welfare, for carbon price  $\tau = 0.4$ . For ease of reading, all values for  $\tau = 0$  are normalized to 100. The first column, as Figure 4, shows that imposing a carbon price reduces consumption of the dirty good. Redistribution produces an income effect, which leads to slightly higher consumption of the dirty good. Optimal redistribution slightly more so: it favors the first two-deciles consumers, who have slightly higher consumption of the dirty good.

The second column illustrates the impact of redistribution on consumption of the composite good. Then, optimal redistribution leads to slightly lower consumption, since the first two decile consumers have a slightly lower consumption of the composite good.

The third column shows that social welfare is slightly increased when redistribution is optimal: the income increase for the first two deciles of the income distribution more than compensates for the higher price faced by all. This effect is small:  $U_n(\tau)/U_n^0 - 1 = 0.1\%$ , since the share of the dirty good is small (4 to 6 %) in consumers' expenditures. Our estimates are in line with those of Rausch et al. (2011). In appendix C, a relative prices sensibility analysis present similar results.

## 5.4 Variations of the carbon price

As the carbon price increases, we expect the number of consumer classes receiving a positive carbon dividend to also increase. This is confirmed on Figure 6.

In Figure 6, the number of consumer classes receiving a positive transfer peaks at 4 and never tops to the 10 consumer classes even for extremely large carbon price. This is outcome results from both data and an empirical assumption stating that any transfer lower then unity is considered null.

The previous analysis showed that social welfare is (slightly) increased for carbon price  $\tau = 0.4$ . Figure 7 shows that the social welfare is increased for all carbon prices  $\tau$  up to 1.5, i.e., such

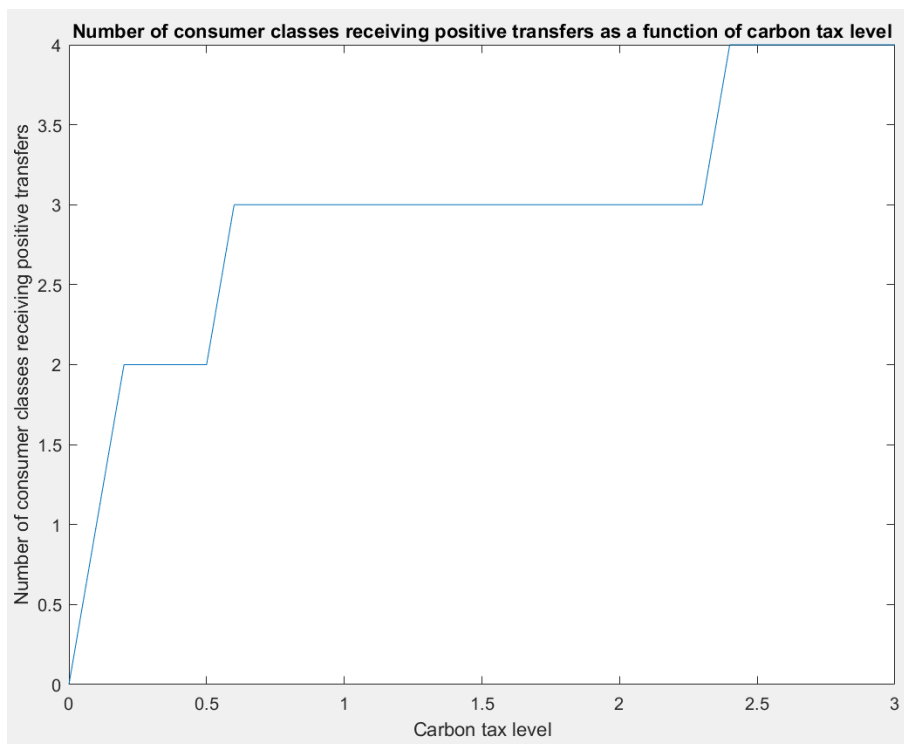


Figure 6: Number of consumer classes receiving positive transfer as a function of the carbon tax level. Source: INSEE data and authors' computation.

that the carbon price multiplies by 2.5 the price of the carbon-intensive / dirty good. This creates significant space for policy makers.

## 6 Concluding observations

This article has developed a highly simplified model of optimal distribution of the carbon dividend. The analysis yields three analytical results. First, for the Cobb-Douglas utility function, optimal carbon dividend distribution is driven by income not by need. Specifically, allocations are determined by the residual income, net of the tax paid on the carbon-intensive / dirty good, i.e., the income effectively available to purchase goods and services. Second, if residual incomes among different consumer classes are close enough, optimal carbon dividend distribution equalises their marginal utility. Otherwise, carbon dividends are primarily distributed to consumers with lowest adjusted income. Third, at least for low values of the carbon tax, if a mild condition on income is met, carbon pricing and optimal dividend distribution increases welfare: the positive income effect received by the few more than compensates for the negative price effect imposed on all.

These results are illustrated using French data. First, carbon pricing and optimal dividend distribution increases welfare for a tax up to 150% of the pre-tax price of the carbon-intensive / dirty good. Second, for a tax equal to 40% of the pre-tax price of the carbon-intensive / dirty good, only the first two deciles of the population in terms of adjusted income receive a positive

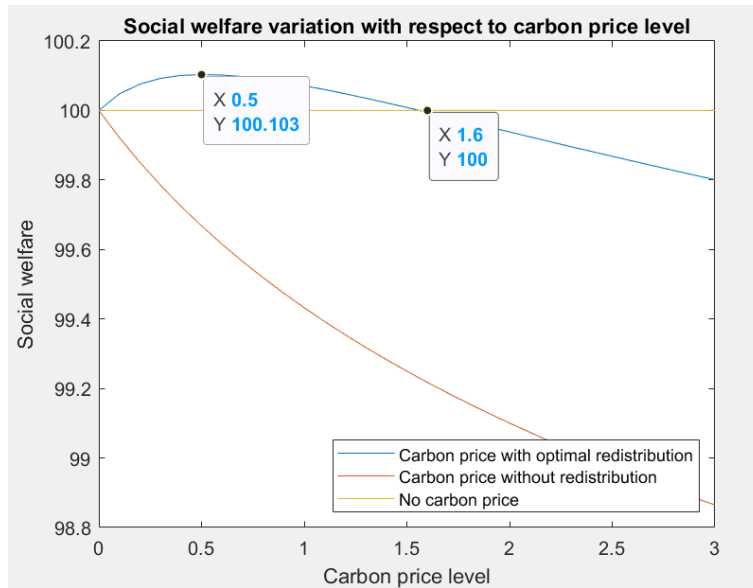


Figure 7: Social welfare variation with respect to carbon price level for three possible model assumptions: carbon price with optimal redistribution, carbon price without redistribution, and no carbon price. Source: INSEE data and authors' computation.

dividend, hence 80% of the population is worse off.

This analysis can be expanded in (at least) two directions. First, the representation of the customers choices should be refined. First, a richer demand function would capture the substitution from carbon-intensive / dirty to clean up, thus increasing net surplus. In addition, the substitution could capture intertemporal effects, in particular the differences in purchase costs and utilisation costs. An electric vehicle costs more to purchase but less to operate than a diesel-powered one. This would lead to a richer description of the trade-offs and choices made by consumers, and more precise empirical estimates. Having said that, we expect the effects we have identified to hold.

A second avenue of further research is a richer analysis of the political economy of carbon pricing and dividend distribution. Carbon pricing is essential to efficiently fight climate change, yet is a political "no-go". The analysis presented in this article illustrates why: even though it increases social welfare (measured by net surplus), it leaves 80% of the population worse-off, hence is unlikely to be enacted in a democratic process. Economists and political scientists need to develop approaches to overcome that impasse.

## References

- Akerlof, G., Aumann, R., Deaton, A., Diamond, P., Engle, R., and Fama, E. (2019). Economists' statement on carbon dividends. *Wall Street Journal*.
- Berri, A. (2005). *Dynamiques de la motorisation et des dépenses de transport des ménages: Analyses sur données individuelles et semi-agrégées*. PhD thesis, Paris 1.
- Blanchard, O. and Tirole, J. (2021). Les grands défis économiques. *Rapport de la Commission internationale présidée par Olivier Blanchard et Jean Tirole*, 23.
- Bosello, F., Carraro, C., and Galeotti, M. (2001). The double dividend issue: modeling strategies and empirical findings. *Environment and Development Economics*, 6(1):9–45.
- Bouckaert, S., Fernandez Pales, A., McGlade, C., Remme, U., and Wanner, B. (2021). Net zero by 2050: A roadmap for the global energy sector. *IEA: Paris, France*.
- Bureau, B. (2011). Distributional effects of a carbon tax on car fuels in france. *Energy Economics*, 33(1):121–130.
- Bureau, D., Henriot, F., and Schubert, K. (2019). Pour le climat: une taxe juste, pas juste une taxe. *Notes du conseil danalyse economique*, (2):1–12.
- Douenne, T. and Fabre, A. (2022). Yellow vests, pessimistic beliefs, and carbon tax aversion. *American Economic Journal: Economic Policy*, 14(1):81–110.
- Fremstad, A. and Paul, M. (2019). The impact of a carbon tax on inequality. *Ecological Economics*, 163:88–97.
- Gollier, C. (2021). The cost-efficiency carbon pricing puzzle.
- Goulder, L. H. (1995). Environmental taxation and the double dividend: a reader's guide. *International tax and public finance*, 2(2):157–183.
- Hotelling, H. (1931). The economics of exhaustible resources. *Journal of political Economy*, 39(2):137–175.
- IPCC (2021). Climate change 2021: The physical science basis. *Cambridge University Press*.
- Maestre-Andrés, S., Drews, S., and van den Bergh, J. (2019). Perceived fairness and public acceptability of carbon pricing: a review of the literature. *Climate Policy*, 19(9):1186–1204.
- Martin, M. and Islar, M. (2021). The 'end of the world' vs. the 'end of the month': understanding social resistance to sustainability transition agendas, a lesson from the yellow vests in france. *Sustainability Science*, 16(2):601–614.

- Metcalf, G. E. and Weisbach, D. (2009). The design of a carbon tax. *Harv. Envtl. L. Rev.*, 33:499.
- Pearce, D. (2003). The social cost of carbon and its policy implications. *Oxford review of economic policy*, 19(3):362–384.
- Pezzey, J. C. and Park, A. (1998). Reflections on the double dividend debate. *Environmental and resource economics*, 11(3):539–555.
- Pigou, A. (1932). The economics of welfare. *London: Macmillan*.
- Pizer, W., Adler, M., Aldy, J., Anthoff, D., Cropper, M., Gillingham, K., Greenstone, M., Murray, B., Newell, R., Richels, R., et al. (2014). Using and improving the social cost of carbon. *Science*, 346(6214):1189–1190.
- Quinet, E. et al. (2014). L'évaluation socioéconomique des investissements publics. Technical report, HAL.
- Stern, N. (2007). *The economics of climate change: the Stern review*. Cambridge University press.
- Stiglitz, J. E., Stern, N., Duan, M., Edenhofer, O., Giraud, G., Heal, G. M., La Rovere, E. L., Morris, A., Moyer, E., Pangestu, M., et al. (2017). Report of the high-level commission on carbon prices.
- Wier, M., Birr-Pedersen, K., Jacobsen, H. K., and Klok, J. (2005). Are co2 taxes regressive? evidence from the danish experience. *Ecological economics*, 52(2):239–251.



# Appendices

## A 2-consumer case: transfers' boundary analysis

To present analytically the boundary analysis, let us start the transfers equations in case n°1 where  $s_1 > 0$  and  $s_2 > 0$

$$\begin{cases} s_1 = \frac{m_2 - m_1(1 - 2\delta\alpha_1)}{2(1 - \delta\alpha_1)} \\ s_2 = \frac{m_1 - m_2(1 - 2\delta\alpha_2)}{2(1 - \delta\alpha_2)} \end{cases} \quad (17)$$

When  $\tau \rightarrow 0$  we can write

$$s_1(\tau) \rightarrow \frac{m_2 - m_1}{2} \quad (18)$$

$$s_2(\tau) \rightarrow \frac{m_1 - m_2}{2} \quad (19)$$

In case n°1, when  $\tau \rightarrow 0$  the transfers' limits are not zero.

However, from the definition of case n°1 we know that  $s_1 > 0$  and  $s_2 > 0$

$$s_1(0) = \frac{m_2 - m_1}{2} > 0 \quad \text{is true if and only if} \quad m_2 > m_1 \quad (20)$$

If  $m_2 > m_1$ , then  $s_1(0) > 0$ , however  $s_2(0) = 0$ .

Therefore, we return to **case n°2** and we have  $s_1(\tau) = \frac{\delta m}{(1 - \delta\alpha_1)}$ .

Therefore, the optimally determined transfers are continuous in zero and that when  $\tau \rightarrow 0$  the transfers' limits tends to zero.

Furthermore, to graphically represent the boundary analysis, from transfers in case n°1 presented in equation (29), We equalise  $s_1(\delta)$  and  $s_2(\delta)$  to zero to provide boundary functions displaying when  $s_1(\delta) = 0$  and  $s_2(\delta) = 0$ .

First, let us compute  $s_1(\delta) = 0$

$$s_1(\delta) = \frac{m_2 - m_1(1 - 2\delta\alpha_1)}{2(1 - \delta\alpha_1)} = 0$$

Therefore, we define the lower boundary function  $\underline{m}_2(m_1)$  displaying the limit for  $s_1(\tau) > 0$

$$\underline{m}_2(m_1) = m_1(1 - 2\delta\alpha_1) \quad (21)$$

Second, let us compute  $s_2(\delta) = 0$

$$s_2(\delta) = \frac{m_1 - m_2(1 - 2\delta\alpha_2)}{2(1 - \delta\alpha_2)} = 0$$

Therefore, we define the upper boundary function  $\overline{m}_2(m_1)$  displaying the limit for  $s_2(\tau) > 0$

$$\overline{m}_2(m_1) = \frac{1}{(1 - 2\delta\alpha_2)} m_1 \quad (22)$$

## B N-consumer case: identical lump-sum transfers

We denote  $s_{lst}$  as the unique value of *lump-sum transfer* received identically by every consumer class ( $N = 10$  the number of consumer classes).

Carbon tax revenues are defined as:

$$R = \tau p^D \sum_{n=1}^N \frac{\alpha_n(m_n + s_{lst})}{p^D(1 + \tau)}$$

Simplifying

$$R = \delta \sum_{n=1}^N \alpha_n(m_n + s_{lst})$$

Revenues should equal the sum of the total lump-sum transfers distributed, therefore:

$$N s_{lst} = \delta \sum_{n=1}^N \alpha_n(m_n + s_{lst})$$

Therefore, for each  $s_{lst}$  we have

$$s_{lst} = \frac{\delta \sum_{n=1}^N \alpha_n m_n}{N - \delta \sum_{n=1}^N \alpha_n}$$

With French data this yields a total carbon tax revenue of  $R = 336.359$  and an identical lump-sum transfer of  $s_{lst} = 33.6359$  for each consumer class.

## C Application to France dataset: relative prices sensitivity analysis

	Demand for good D	Demand for good C	Welfare
Base case (no carbon price)	100	100	100
Carbon price with optimal redistribution	72,68	101,40	100,083
Carbon price <i>without</i> redistribution	71,43	100	99,80
Carbon price with identical lump-sum transfers	72,50	101,41	99,98

Table 3: Total consumption and social welfare. The results in value are normalised to 100 for the *No carbon price* case. Each other case present the variation from the *No carbon price* case. The normalised results are presented for the relative prices  $p^C = p^D$ . Source: INSEE data and authors' computation. (See appendix B for identical lump-sum transfers computations)

	Demand for good D	Demand for good C	Welfare
Base case (no carbon price)	100	100	100
Carbon price with optimal redistribution	72,68	101,40	100,08238
Carbon price <i>without</i> redistribution	71,43	100	99,80
Carbon price with identical lump-sum transfers	72,50	101,41	99,98

Table 4: Total consumption and social welfare. The results in value are normalised to 100 for the *No carbon price* case. Each other case present the variation from the *No carbon price* case. The normalised results are presented for the relative prices  $p^C = \frac{1}{2}p^D$ . Source: INSEE data and authors' computation. (See appendix B for identical lump-sum transfers computations)

	Demand for good D	Demand for good C	Welfare
Base case (no carbon price)	100	100	100
Carbon price with optimal redistribution	72,68	101,40	100,08315
Carbon price <i>without</i> redistribution	71,43	100	99,80
Carbon price with identical lump-sum transfers	72,50	101,41	99,98

Table 5: Total consumption and social welfare. The results in value are normalised to 100 for the *No carbon price* case. Each other case present the variation from the *No carbon price* case. The normalised results are presented for the relative prices  $p^C = 2p^D$ . Source: INSEE data and authors' computation. (See appendix B for identical lump-sum transfers computations)