Should There Be a Green Supporting Factor? Carbon Policies and Climate Financial Regulation

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ABSTRACT

Should optimal financial regulation apply differentiated capital requirements in order to encourage climate-related investments (Green Supporting Factor)?. Considering investments both in decarbonization and in climate resilience, and scenarios with efficient and with inefficient carbon policies, we show that the prospect of severe climate shocks provides a rationale for optimally differentiated capital requirements as such climate-related shocks lead to ex-post efficient accommodating policies that create distortions in private investment incentives. We also show that regulators should differentiate capital requirements for resilience investments when private agents insufficiently internalize their benefits, and differentiate capital requirements for both abatement and resilience investments when carbon policies are inefficient.

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Keywords: Climate finance regulation, capital requirements, Green Supporting Factor, climate adaptation, resilience.

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1 Introduction

A key question concerning climate financial regulation is this: is it needed at all, or are efficient carbon policies, in particular carbon prices, sufficient to achieve optimal climate outcomes? And if there is indeed a climate role for financial regulation, what are the circumstances when it should play such a role? In other words, what are the inefficiencies and frictions it should address? In this paper, we develop a model that allows to address both questions. Specifically, our analysis considers how climate policy trajectories and uncertainty about climate risks interact with the optimal design of climate financial regulation.

Virtually all contributions to the discussion on climate finance regulation do not consider revisions to the Basel framework, mainly based on a tacit policy assumption that the Basel Accord as a multilateral agreement is virtually impossible to reform. The analysis in this paper takes a contrasting view: a full understanding of optimal financial regulation can only be gained when it also considers the central premises and instruments of regulators. In fact, such proposals are currently discussed in the European Union under the label of a "Green Supporting Factor", as part of the EU Green Deal (EU Commission, 2018), but the idea is facing headwinds.[1] The discussion about a possible Green Supporting Factor is motivated by earlier examples of ad hoc amendments to the Basel rules to specifically accommodate the funding needs of particular asset classes. They include prominently the SME Supporting Factor (SME SF) of Art. 501 of the Capital Requirements Regulation (CRR) of the European Union that permits the reduction of the capital requirements for SME loans by up to 25%, on the grounds of their vital role for future growth. Thus, pragmatic solutions can be found that do not require to reopen a decades-long process of multilateral negotiations, in particular if public pressure for more proactive carbon policies becomes impossible to ignore.

While calls for a "Green Supporting Factor" are discussed in European policy circles, and some National Development Banks report that they are already held to different regulatory standards for climate-relevant investments (Hege, 2021), there is no rigorous academic analysis of a climate-centered differentiation of regulatory requirements. The current paper tries to fill this gap. It analyzes a Green Supporting Factor in an optimal regulation model and provides a nuanced assessment of the pros and cons of differentiated climate financial

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[1] Tellingly, in the communication of the EU Commission in the summer of 2021 on the details of the EU Green Deal, (the "Fit-for-55" package), the Green Supporting Factor is not a priority.

regulation. In practical terms, differentiation does not mean lower capital requirements. A reduced capital requirement for climate investments can be amended by a malus, or a capital surcharge, for “ brown assets”, so that the overall impact of differentiated capital requirements can be kept neutral for average bank capital ratios.

The central motivation of our analysis is that the potential role of climate financial regulation depends notably on the question how it interacts with fiscal climate policies, notably carbon prices and the uncertainty about their future path that accompanies them. In line with a established body of research, our starting point is model implies an efficient carbon price that increases constantly at an efficient but state-contingent rate that accounts for the climate beta and the stochastic discount factor in our model (Gollier, 2020). The main tool of climate policies should be a globally coordinated carbon price that should always be the same for all mitigation policies.

Uncertainty about future economic scenarios is crucial to define the optimal financial regulation since it addresses financial stability in adverse scenarios. The project focuses on three types of uncertainty that impact the optimal carbon trajectory: (i) uncertainty about the effect of carbon accumulation on the climate, (ii) uncertainty about economic growth and the energy intensity of growth, and (iii) uncertainty about abatement technologies. Armed with this rich model, we consider carbon policies and conditional carbon price paths. These three sources of uncertainty tie down an intertemporal carbon pricing trajectory that adapts dynamically to the three unpredictable components. The objective of our paper is to calibrate a dynamic CCAPM model (consumption-based capital asset pricing) that optimizes the dynamic revision of the carbon budget and the intertemporal allocation of the carbon budget as uncertainty is gradually resolved.

Throughout this study on climate financial regulation, we only consider prudential policies that address future financial stability risks. Studying the impact on financial regulation, we focus on climate risks since they are likely to increasingly constitute sources of systemic risk that threaten the stability of the financial system. We consider that uncertainty about the climate effects of carbon accumulation is particularly relevant since it may explain the emergence of climate-related shocks that create risks for financial stability.

When extreme climate-related events occur, climate financial regulation will optimally react with accommodating or countercyclical stances. The anticipation of such regulatory "forbearance" leads to distortions in private investment incentives: private agents anticipate the ex post efficient regulatory action that will protect (or socialize) parts of future climate-related losses. The optimal ex ante policy of regulators attempts to correct for such biases. We will investigate to what extent these corrections create a rationale for regulatory attention to the climate risk contribution of assets, and for ex ante differentiation in the regulatory treatment of assets according to their climate risk (climate beta).

A key distinction is between two dimensions of climate policies: carbon emissions and proactive investment in climate resilience. The nature of carbon emissions is that they are a global externality that should be addressed uniformly across the globe, i.e., with a globally uniform carbon price that ensure the efficient allocation of abatement efforts. By contrast, investment into resilience investments, such as personal protection gear, social distancing space and. Resilience investments are local, and they are heterogeneous: while climate risks are global, their local impact will be dramatically different: they will be felt more in regions exposed to future sea level rises, and catastrophic wild fires, heat spells, droughts, hurricanes, and flooding. Climate risks will be felt through extreme weather episodes that can destabilize communities and economies, and will force central banks and regulators to act. Local actors can anticipate such risks disasters by making investments in climate-resilient infrastructure and other forms of climate adaptation, and local actors should have incentives to invest and to coordinate investment projects where the benefits affect many parties. Thus, a substantial part of the impact of resilience investments, but not all, should be internalized by their beneficiaries.

We broadly distinguish between two scenarios, according to the actual carbon policy with which financial regulators are confronted. In the baseline scenario, carbon policies follow the optimal stochastic carbon policy path. In the conflict scenario, the implementation of optimal carbon prices is stalled because of political obstacles, and financial regulators attempt to react optimally to inefficient climate policies.

In the assessment of the baseline scenario, we will assume an optimal interaction between the dynamic management of climate policy and financial regulation. We determine the investment, value and risk trajectories of the different categories of real assets and their role in the probability of exceeding systematic risk thresholds in long-term simulations. In the assessment of conflict scenarios, we will disregard the hypothesis of an optimal interaction between the dynamic management of climate policy and financial regulation. We want to invoke scenarios of failed or sub-optimal climate transition policies to assess the consequences for prudential regulation.

Our main findings are as follows. First, we show that even if an efficient carbon price path were in place, there is a role for differentiated capital regulation, but this role is limited. This is because regulation should then recognize that investments in climate resilience may be too small without a regulatory nudge. In very bad climate scenarios, central banks and regulators will react with expansionist policies. Private actors may anticipate massive public policy support, and hence have too little incentives to undertake actions that enhance their climate change resilience. In other words, they will do too little to prevent climate damage since they anticipate support from public policies when hit by climate disasters. So regulators should start now to counteract this perverse effect of their very own mandate to provide relief when disaster strikes. They should do so by differentiating capital requirements and encourage more investments in climate resilience where it is insufficient to prevent future climate-induced crises.

We then look at the role of financial regulators when climate policies in large parts of the world remain as inefficient as they currently are. In this case, the role of climate financial regulation is expanded. We show that regulators should then differentiate not only to encourage investments in resilience, but also green investments in energy transition that reduce carbon emissions.

The paper is organized as follows. After a short literature discussion in Section 2, Section 3 develops the model and its ingredients. In Section 4, we summarize theoretical observations about the optimality of regulatory differentiation for various situations concerning the efficiency of institutions and behavior. In Section 5, we discuss the parameters for our simulations and some results. Section 6 concludes.

2 Literature

Our paper draws on earlier work that seeks to refine the carbon trajectory by taking better account of the risks involved. This includes in particular work that integrates risk considerations in the climate debate: uncertainty about the extent of environmental damage was soon analyzed around the question of optimal public intervention (Weitzman 1974 , Pizer 1999) and the advisability of waiting longer to learn more (Nordhaus 1994, Kelly and Kolstad 1999, Pindyck, 2013). In the last decade, stochastic models (Crost and Traeger 2011, Cai, Judd and Lontzek 2012) have begun to fully integrate risk into integrated assessment models. The way in which growth dynamics interact with environmental issues (cost of abatement, damage) is the subject of recent work (Gollier 2012, Lemoine 2017, Dietz, Gollier and Kessler 2018, Gollier 2020) fueling the debate around the "climate beta". Other contributions tackle the issue of learning around catastrophic risks ("tipping points") without leading to completely convergent results (Lemoine and Traeger 2014, Lontzek et al. 2015, van der Ploeg and de Zeeuw 2019). For the definition of asset classes according to their exposure and their contribution (positive or negative) to climate risks, we will build on existing and developing attempts to undertake such classifications between "green" and "brown" assets and different intermediate degrees (e.g. Vermeulen et al., 2019).

Our model is based on the consumption-based asset pricing model (CCAPM) that builds on a large literature, developed originally by Rubinstein (1976), Lucas (1978) and Breeden (1979) (see Campbell, 2018) in order to determine risk premiums for investment projects related to global warming. The literature on the question of the risk-adjusted social discount rates is limited, as well as on the need to have project-specific risk adjustments. For the latter, we will make use of work carried out with regard to the use of discount rates in private companies (Krüger et al. 2015, Dessaint et al. 2019) that also apply to investment decisions form the social planner's point of view (Gollier 2019).

There is substantial literature on financial stability risks and climate change (Bolton et al. 2020, Battiston et al. 2017, Battiston et al. 2019; Monasterolo and Raberto 2018; NGFS 2019; NGFS, 2021). This literature emphasizes, among other contributions, the potential risks from the energy transition when investors are exposed to assets that will be stranded in the future. It also discusses the financial instability if large and correlated assets classes and systemic financial intermediaries are involved, and the need to differentiate asset classes by their exposure to climate risks. The literature also discusses many remedies such as the need to enforce transparency of climat-risk exposure, and for consistent and stringent ratings of climate risks, and of climate stress tests (NGFS, 2019; Bolton et al. 2020; Battiston et al. 2019; Lamperti et al. 2017). Allen et al. (2020) focus on transition risks and show the large disparity in sectoral risk, especially if carbon policies are inefficiently delayed. By and large, the discussions on the financial regulation of climate risks are limited to exploiting the existing prudential regulatory framework in order to encourage financial institutions to integrate climate risks into their risk management systems (NGFS, 2019; Bolton et al. 2020). Even though contributions dedicated to integrating climate risks into prudential supervision show the limits of the existing instruments (Schoenmaker et al. 2015; ESRB 2016), the literature on financial regulation and climate change has not yet addressed the question that is at the heart of our paper, whether the three pillars of the Basel Accord should be adapted to account for climate risks, and what the implications of climate risks are for the fundamental regulatory instruments such as capital and liquidity requirements. There is currently no specific model framework that allows to derive insights on the optimal financial regulation of assets in relation to their contribution to climate risks.

Our model is also related to the general theoretical literature on optimal financial regulation. This literature tackles, among other things, the resilience of intermediaries and the externalities of leverage (Duffie 2016), the correlations of risks generating or amplifying systemic risks (Adrian and Brunnermeier 2016), the regulation of illiquid assets (Dewatripont and Tirole, 2019), the borders between strictly regulated intermediaries and the shadow banking sector (Farhi and Tirole 2018). We also mention recent developments in the literature of Intermediary Asset Pricing (He and Krishnamurthy 2013) and the nascent attempts to extend this literature to optimal financial regulation (He and Krishnamurthy 2019, Di Tella 2019).

3 Model

We set up a simple economic climate model that we then use to conduct our theoretical policy analysis. In a second step, the model can be calibrated to gain insights on the possible magnitudes of policy responses (capital requirements, notably risk weights) and its interaction with carbon policies.

3.1 Output and Utility

There are n future periods. in our stylized model considers homogeneous agents so that it is thus sufficient to consider a representative agent. We consider a representative economic agent with power utility (CRRA) of the form $U\left(C_t\right) = \frac{1}{\gamma} C_t^{\gamma}$, where $1-\gamma$ is the coefficient of relative risk aversion. For our theory discussion, we follow the standard CCAPM specification used in Gollier (2020). The agent has a positive rate of pure time preference ρ that leads to a time-preference discount factor of $\beta = \frac{1}{1+\rho}$.

We focus on climate-relevant investments and hence take the gross output Y_t as given. The output process Y_t is stochastic, with a binomial distribution of macroeconomic risks in each period such that there is a small probability in each period (1.7% annually) of a deep macro shock (- 35%) (Barro, 2006). This set-up leads to a model consistent with fundamental asset pricing puzzles (notably the equity premium puzzle).

Climate damages D_t are a function of cumulative carbon emissions E_t. The change in E_t will be a linear function of Y_t, $Q_t Y_t$, where Q_t is the energy intensity that will decrease over time, reflecting technological progress in the use of energy. The representative agent can reduce carbon emissions through investments in abatement A_t and she can also invest R_t to enhance resilience against climate impact.

3.2 Abatement

We consider two independent climate-relevant actions of the representative agent: abatement effort A_t, and resilience investment R_t. A_t and R_t denote also the aggregate efforts in the economy for abatement and resilience, respectively (when needed for clarity, we will use low-ercase letters a_t and r_t for the per-capita effort). At date $t = 0$, the agent decides on abatement and resilience investment in the first period, A_1 and R_1, etc. The aggregate net output of all agents potentially available for consumption accounts for the expenditure on abatement and resilience. Potential consumption C_t^P is given after subtracting the cost of abatement and resilience investments from production Y_t:

$$C_t^P = Y_t - A_t - R_t$$

Abatement investments A_t leads to the installation of new capacity I_t, measured in units of avoided carbon emissions E_t. Most climate models consider A_t as a flow cost leading to a reduction I_t of emissions E_t, and we will follow this route.[2] The costs of abatement investments $A_t(I_t,)$ are convex. This reflects the idea that MAC (marginal abatement costs) is increasing as it notably depends on limited resources based on location (locations to build dams, wind farms, to isolate buildings, etc.), with investors sequentially picking the most efficient investments. We will assume a quadratic specification that we write:

$$A_t(I_t, K_{t-1}) = A_t(I_t) = a_t I_t + \frac{b}{2}(I_t)^2,$$

so that marginal costs are linear in installed capacity. Uncertainty about future abatement costs will be expressed in uncertainty about the parameter a_t which is a (binomial) random variable, whereas b is fixed.

Given abatement investments I_t, carbon emissions in t are $Q_t Y_t - I_t$. The cumulative carbon stock follows the following accounting equation:

$$E_t = \sum_{s=0}^{t} (Q_s Y_s - I_s)(1-\delta)^{t-s} \quad ,$$

where δ is the discrete time decay factor of CO_2 accumulated in the atmosphere and has a small value, reflecting the long-term cumulative nature of carbon emissions in the atmosphere (Gollier, 2020).

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[2] In a generalization of our model, we account explicitly for the fact that abatement capacity I_t is long lived with a fixed lifetime of s periods: I_t is notably invested in the production, storage and distribution of renewable energy, and investments in energy conservation that have an economic life of many years, typically 20 years or beyond. We introduce K_t as the stock of installed abatement capital, i.e. past and present I_t that are still operated. We ignore depreciation during he lifetime of an abatement investment for simplicity, so that the stock of abatement capital K_t is: $K_t = I_t + I_{t-1} + ... + I_{t-s+1}$.

We assume the costs A_t depends also on previously installed capacity, K_{t-1} (for example the use of the most efficient sites for renewable production), and hence we write $A_t(I_t, K_{t-1}) = A_t\left(I_t, \sum_{\tau=1}^{s-1} I_{t-\tau}\right)$. This reflects the idea that the MAC (marginal abatement costs) is increasing and depends on limited resources, with investors sequentially picking the most efficient investments. Thus, the MAC in t starts not at point zero, but at the point along the cost function that corresponds to installed capacity, K_{t-1}. We can then write the quadratic abatement costs as:

$$A_t(I_t, K_{t-1}) = A_t\left(\sum_{\tau=1}^{s-1} I_{t-\tau}\right) = a_t \sum_{\tau=1}^{s-1} I_{t-\tau} + \frac{b}{2}\left(\sum_{\tau=1}^{s-1} I_{t-\tau}\right)^2,$$

3.3 Damage Function

We cannnot avoid a damage function since it is the basis to define situation of climate-induced financial stability that gives rise to regulatory action. The utility of the agent is diminished by the damage function D_t from climate impact that is a function of atmospheric carbon concentration and hence depends on cumulative carbon emissions E_t and the reactions of the biosphere (carbon absorption and release by oceans, tundra, other natural systems) that we assume to be constant and hence omit, as is customary in economic climate models. The damage function D_t is modelled as a monetary loss that directly reduces available output Y_t, and hence consumption. That is, available consumption will be reduced to:

$$C_t = Y_t - D_t - A_t - R_t \ ,$$

where potential consumption is diminished by damage D_t: $C_t = C_t^P - D_t$. The specification of the damage function $D_t(E_t)$ is the object of considerable debate, characterized among others by contributions from Nordhaus (2013) and Weitzman (2012). We follow closely the model of Dietz, Gollier, Kessler (2018) [DGK (2018)] but amend it to integrate uncertainty about future climate risk, as a binomial process affecting the equilibrium climate sensitivity (ECS).

Following widely used practice, we assume that D_t is an increasing function of the increase in mean temperature T_t relative to the pre-industrial level of 1750. Climatologists mostly postulate that the long-run temperature increase T_t is roughly linear in E_t, with the equilibrium climate sensitivity (ECS) ω_1 measuring how sensitive temperature is in reaction to a doubling of atmospheric carbon accumulation relative to the pre-industrial level of 280ppm:[3]

$$T_t = \omega_1 E_t. \tag{1}$$

DGK (2018) show that ω_1 has potentially a big impact on the climate beta. They use a mean of 2.9 and a standard deviation of 1.4, citing IPCC (2013).

The debate how T_t should be mapped into a damage function D_t focuses on two elements:

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[3] An alternative approach is to base the temperature reaction on TCRE, the transient climate response or the more or less immediate effect, as opposed to ECS that also considers additional temperature rises playing out over many decades). We follow Nordhaus (2017) in using ECS even though recent work in climate science converges to a view that TCRE is not dramatically different from ECS.

(i) the convexity of the damage function; (ii) whether the loss is additive, $Y - D$, or multiplicative, $Y(1 - D)$. These two controversial points can be easily represented in a unified flexible modelling framework. Following Van de Bijgaart et al. (2017) and DGK (2018), we adopt a uniform model that encompasses both the additive and the multiplicative case with the use of a flexible elasticity $\xi \in (0, 1)$:

$$D(E_t) = Y_t d(E_t) \left(\frac{Y_t}{Y_0}\right)^{\xi - 1} \tag{2}$$

The damage function is additive if $\xi = 0$ and multiplicative if $\xi = 1$. Following wide-spread practice, we adopt a multiplicate damage function in our theoretical analysis and assume $\xi = 1$. The damage function than simplifies to:

$$D(E_t) = Y_t d(E_t) \tag{3}$$

The functional form $d(E_t)$ captures the degree of convexity, for example via the choice of a polynomial of degree k (see Section 5).

3.4 Resilience Investments

The representative agent has the opportunity to mitigate the damage through resilience investments R_t that encompass all investments in adaptation to climate change. Whereas climate mitigation (reduction of E_t) is globally uniform, so that abatements anywhere have the same impact on global welfare, the impact of climate change is inherently local and heterogeneous: geographies will be differently affected by adverse climate events such as droughts, wildfires, heat waves, hurricanes, flooding, rising sea levels, changing conditions for agriculture and biodiversity. The optimal level of resilience investments will be heterogeneous across regions and persons.

We simplify the analysis by considering a globally representative agent who maximizes the average resilience investment.

We assume that a given expenditure R_t will reduce the damage $D(E_t)$ by fR_t^k, where f and k are two parameters measuring the efficiency of resilience investments. We write the damage function after resilience investments as $Y_t\left(1 - d\left(1 - fR_t^k\right)\right)$, and the "static" optimization problem of the planner as :

$$\max_{R_t} \left\{ Y_t \left(1 - d\left(E_t\right)\left(1 - fR_t^k\right) - R_t\right) \right\}$$

The optimization problem for the representative agent is identical if the agent internalizes all the benefits of her investment in resilience. Writing $d = d\left(E_t\right)$ for short, this leads to the first-order condition $R_t = (dfk)^{\frac{1}{1-k}}$.

We will henceforth assume that $k = \frac{1}{2}$ so that the choice of resilience investments becomes a quadratic problem. We then have for the optimal resiliency investment:

$$R_t = \left(\frac{df}{2}\right)^2$$

The gross damage reduction through resilience is $Y_t\left(1 - d\left(1 - fR_t^{\frac{1}{2}}\right)\right) - Y_t\left(1 - d\right) = Y_t\frac{(df)^2}{2}$, and the net damage reduction (minus investment cost):

$$Y_t\left(1 - d\left(1 - fR_t^{\frac{1}{2}}\right) - R_t\right) - Y_t\left(1 - d\right) = Y_t\frac{(df)^2}{4} \tag{4}$$

This internalization effect includes all fiscal public policy measures, such as investment subsidies. When the representative agent internalizes only a fraction $\alpha < 1$ of the benefits of investment in resilience, inefficient investment will occur as we consider in detail below. Public policy intervention is optimal when the representative agent internalizes only a fraction $\alpha < 1$ of the damage-reducing effect of climate resilience investment R_t.

3.5 Welfare and Utility

The objective function of the representative agent is to maximize the present value of expected utility:

$$V_0 = \sum_{t=1}^{n} \beta^t E\left[U\left(C_t\right)\right]$$

As the event tree unfolds, the value function of the representative agent can then be expressed as a Bellman equation:

$$V_t\left(Y_t, I_t, E_t, \omega_t, a_t\right)=\max_{\left\{A_t, R_t\right\}}\left(U\left(C_t\right)+\beta E V_{t+1}\left(Y_{t+1}, I_{t+1}, E_{t+1}, \omega_{t+1}, a_{t+1}\right)\right) \tag{5}$$

subject to:

$$\begin{aligned}
C_t &= Y_t\left(1-d\left(E_t\right)\left(1-f R_t^{\frac{1}{2}}\right)-R_t\right)-A_t\left(I_t\right) && (6)\\
A_t\left(I_t\right) &= a_t I_t+\frac{b}{2}\left(I_t\right)^2 && (7)\\
E_t &= E_{t-1}(1-\delta)+\left(Q_t Y_t-I_t\right) && (8)
\end{aligned}$$

The social planner solves this problem recursively, starting in the last period where the problem becomes:

$$\max_{\left\{A_n, R_n\right\}} E\left[U\left(C_n\right)\right]$$

subject to constraints (24) - (26). The first-order condition of the Bellman equation with respect to abatement effort I_t can be expressed as the following solution that expresses the condition that adding one unit of abatement today must have the some effect on present value than adding one unit tomorrow:

$$U'\left(C_t\right)\left(A_t'-d'\left(E_t\right)\right)=\beta(1-\delta) E\left[V'(.)\left(A_{t+1}'-d'\left(E_{t+1}\right)\right)\right] \tag{9}$$

The term $A_t'-d'\left(E_t\right)$ expresses that one additional unit of abatement will subtract A_t' units from available net income but also reduce net damages by $-d'\left(E_t\right)$ units. The discount factor $\beta(1-\delta)$ reflects the fact that a unit of emission today is equivalent to $(1-\delta)$ units a period later because of the carbon decay at rate δ. Using the envelope theorem, we can write:

$$U'\left(C_t\right)\left(A_t'-d'\left(E_t\right)\right)=\beta(1-\delta) E\left[U'\left(C_{t+1}\right)\left(A_{t+1}'-d'\left(E_{t+1}\right)\right)\right] \tag{10}$$

The first-order condition with respect to the resilience investment R_t is determined as a static maximization problem according to Section 3.4. R_t will increase quadratically in the equilibrium damage function $d\left(E_t\right)$. However, the optimal equilibrium trajectory of $d\left(E_t\right)$ is implicitly defined by the Euler condition (10). Thus, condition (10) implicitly defines an optimal trajectory of R_t, evolving at a rate that is linear in the square of d.

3.6 Efficient Carbon Price

The representative agent pays the full MAC (marginal cost of abatement) A_t' of a marginal increase in abatement, but she will consider the benefit of reducing her carbon emissions as a pure externality created for others. Therefore, the carbon price is the regulatory tool of choice to ensure that agents internalize the external effects of their actions and that an optimal carbon trajectory in a decentralized economy is implemented. With carbon emissions a global common, the efficient carbon price should be uniform around the globe. In the baseline case, we assume that carbon prices are fixed in advance at an efficient level, p_t^*, so that the agent fully internalizes the social cost. Its increase should reflect the optimal trajectory of the MAC that is implicitly determined by the Euler condition of the planner's problem in eq (10):

$$(A_t' - d'(E_t)) = E\left[\beta(1-\delta)\frac{U'(C_{t+1})}{U'(C_t)}\left(A_{t+1}' - d'(E_{t+1})\right)\right] \tag{11}$$

We have rewritten this condition to reflect the stochastic discount factor (SDF), $\beta(1-\delta)\frac{U'(C_{t+1})}{U'(C_t)}$. The realization of the SDF is state-contingent and will reflect the evolution of the three dimensions of uncertainty, Y_t, ω_t, and a_t.

The expected value of the SDF depends on the correlation between the MAC A_t' und expected marginal consumption, $EU'(C_{t+1})$, or in other words, on the climate beta. We need to add more structure to the model to derive general insights about the sign of the discount rate and the climate beta. This could be done by either adding more structure to the model, notably by assuming that the sources of uncertainty follow a geometric Brownian motion in addition to the use of a CRRA utility function (Gollier, 2020). This approach and other asset pricing-based approaches generally show that income shocks will increase the climate beta and the discount rate, implying a higher annual increase of the optimal carbon price, whereas climate shocks will on the contrary tend to reduce the climate beta (they can even explain a negative climate beta) when bad climate scenarios are expected to go together with low average consumption. If the abatement cost shock is unrelated to expected consumption growth, then it will have no impact on the climate bets and the efficient trajectory of the carbon price.

The optimal carbon price path p_t^* is determined recursively: For every aggregate abatement level and interim state s in $t-1$, the optimal continuation investments $A_t(s)$ and $R_t(s)$, satisfying the dynamic optimality condition (9) $\frac{\partial V_t}{\partial I_t} = 0$, is determined, and the resulting carbon price $p_t^*(s)$ implementing the optimal abatement effort $A_t(s)$ is chosen. Moving recursively back to period $t = 0$, of all the possible $t = 1$ policies, the policy maximizing initial welfare, V_0, denotes the efficient carbon policy. This optimal policy then determines the optimal carbon trajectory and carbon price path, $\{p_t^*(s) \mid s \in S\}$.

The carbon price $p_t^*(s)$ reflects an expectation over the future states $s_{t+\tau} \in S$, for $\tau > 0$, and resulting carbon prices $p_{t+\tau}^*(s)$. In other words, future carbon prices are uncertain and evolve as a martingale, with their state-contingent direction of change being unpredictable at $t = 0$ ar any future node.

We calibrate the model and derive insights about the climate beta from the simulated paths of the optimal carbon trajectory. The carbon price is the result of our simulation analysis. Our simulations confirm the intuition that the climate risk is higher when the macro growth shock is higher, and lower when uncertainty is dominated by future climate risks. The latter source of risks will dominate our discussion of the intervention of financial regulation.

3.7 Frictions and Financial Regulation

We introduce financial regulation and two types of frictions in our analysis. Many type of frictions are relevant (externalities, asymmetric information, behavioral limits) but we consider only two: The first friction are capital market frictions that imply that market rates of cost of capital are higher than the social cost of capital, and that we express with a cost factor $c_i \neq 1, i \in \{A, R\}$ for abatement and resilience investments. We introduce our modelling approach to financial regulation in that context. The second friction are externalities linked to resilience investments.

3.7.1 Capital Market Frictions and Capital Requirements

The factor $c_i \neq 1$ of the market cost of capital compared to the social cost of capital means that the level of investment in abatement and resilience can differ from the socially efficient level. The difference can be thought of as integrating any cost (or also subsidy), such as bankruptcy costs, adverse selection and moral hazard, that explain why investors demand a higher compensation for risk than the socially optimal one. They are to a large extent not a social loss (for example, direct bankruptcy costs lead to income and consumption elsewhere). Hence, we only consider the effect on capital allocation, assuming that they are not a deadweight loss but act like a rent. Typically, $c_i > 1$ is the more relevant case, as our calibration shows.

We consider the mandate of prudential financial regulation to take ex ante action in order to influence climate-related investments, abatement effort A_t and resilience investment R_t. We assume that regulators stay within the limitation of their mandate on financial stability: regulators only address climate mitigation and adaption to the extent that it improves financial stability. We consider differentiated capital requirements, or a Green Supporting Factor, as it is proposed int the European policy debate (EBF, 2017; EU Commission, 2018). To wit, similar policies that alter risk weights for certain investments, such as the SME Supporting Factor (SME SF) in Article 501 of the Capital Requirements Regulation (CRR) of the EU, already exist and work mostly smoothly.

In our model, the regulator can use a differentiated capital requirement to explicitly reduce capital requirements (risk weights) for resilience investments or abatement investments, thus reducing the funding cost of such investments and expanding their use. We explicitly consider how they modulate the market cost of capital of climate-related investments. We assume that there is an effective post-regulation cost of capital factor $c_R \neq 1$ so that $c_R R_t$ is the effective market cost (in present value terms) of funding an investment cost of R_t in resilience investments. Similarly, the abatement investment A_t incurs a cost of capital of $c_A A_t$ (in present value terms). The factors c_R and c_A depend on capital requirements.

The important element in our model is not the absolute value of the factors c_R and c_A, but their elasticity with respect to a targeted change in capital requirements, η_{RW,CC_i}, where subscript RW denotes risk weights, and CC_i the market cost of capital associated with abatement and resilience investments, $i \in \{A, R\}$.

We illustrate the concept of c_i, $i \in \{A, R\}$ with a simple but realistic numerical example. Consider a long-term infrastructure project funded with a capital mix of 25% equity and 75% debt, where debt financing comes through regulated bank loans (financial leverage is typically substantial for infrastructure projects). Suppose the bank is regulated to finance its risk-weighted assets (RWA) with 8% of equity, with real cost of bank equity of 15%, and 92% of debt that it can raise at a real rate of 1%. If the risk weight of the asset is $RW = 1$, then the cost of funding for the bank is $r_B = 0.08 \times 15\% + 0.92 \times 1\% = 2.12\%$, to which the bank will add a credit spread or margin (for operating costs and expected losses). Assuming a spread $m = 1\%$ (modest by historical standards especially in emerging economies where DFIs operate), the real cost of lending for the project is $r_D = r_B + m = 3.12\%$. Assuming the project sponsor has cost of equity $r_E = 8\%$, then the project cost of capital is $r_{CC} = 0.25 r_E + 0.75 r_D = 4.34\%$.[4]

Assume now that the risk weight is lowered to $RW^G = 0.5$, then $r_B^G = 0.04 \times 15\% + 0.96 \times 1\% = 1.56\%$, and the project cost of capital is reduced to $r_{CC}^G = 0.25 r_E + 0.75 \left(r_B^G + 1\% \right) = 3.92\%$.

The capital cost factor c_A or c_R depends on the discount rate at which the project sponsor discounts its future cash flows. In equilibrium, the discount rate should be equal to the discount factor of our CCAPM model plus an appropriate risk adjustment. Given that we consider climate investments and following the conventional financial logic that firms should use project WACC rather than company WACC, it should be identical to the SDF β_{CLIM} adjusted to the climate beta of our CCAPM framework.

Denoting the annualized SDF that corresponds to investments by $\beta_{CLIM} = \frac{1}{1+r_{CLIM}}$ and denoting the annualized discount factor of the (asset) cost of capital by $\beta_A = \frac{1}{1+r_{CC}}$, we then have for a project of lifetime T:

$$c_i = \frac{\beta_{CLIM} - (\beta_{CLIM})^{T+1}}{1 - \beta_{CLIM}} \frac{1 - \beta_{CC}}{\beta_{CC} - (\beta_{CC})^{T+1}}, \quad i \in \{A, R\}$$

Assuming a project duration of $T = 30$ years, and using a climate discount rate (social cost for capital for abatement investments) of $r_{CLIM} = 3.5\%$ (as suggested in Gollier, 2020),

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[4] We refer to the project cost of capital and not the project WACC since we ignore corporate taxes in this simple illustration, in accordance with standard corporate finance usage.

we find that $c_i = 1.108$ when $r_A = 0.0434$ and $c_i = 1.053$ when $r_A = 0.0392$, or a reduction of 5% in the market cost of capital factor. Translating for the capital cost elasticity of risk weights, η_{RW,CC_i}, we obtain an elasticity of $\eta_{RW,CC_i} = 0.11$. The elasticity depends on the project duration T and the market cost of capital. Our simple calibration shows that the effect of a reduction in risk weights can be substantial. However, for quantitatively important effects, the reduction in risk-weights muast be substantial, as illustrated in our example where it is reduces from $RW = 1$ to $RW^G = 0.5$. While large, this reduction in risk weights is fully aligned with for example the order of magnitude used for the SME Supporting Factor in the EU.

With explicit financing costs c_R, the objective function (4) becomes:

$$Y_t\left(1 - d\left(1 - fR_t^{\frac{1}{2}}\right) - c_R R_t\right) - Y_t\left(1 - d\right) = Y_t\frac{(df)^2}{4c_R} \tag{12}$$

This solution assumes the absence of additional financial constraints besides those captured by the market cost of capital. Relevant for regulators in countries where financial constraints are important (such as emerging markets), if actors and financial intermediaries are constrained to finance climate-relevant investments then a lowering of risk-weights can directly be translated into a multiplier effect of lending capacity. If banks face frictions to issue equity and not fiance all NPV-positive climate-relevant projects, then lowering risk weights from 1 to 0.5 will approximately double the financing capacity of intermediaries for investments in abatement and resilience, a potentially much larger effect than the one coming through the cost of capital effect (r_A) highlighted in our numerical example.

3.7.2 Frictions in Resilience Investments

We consider externalities linked to resilience investments that a carbon price cannot resolve. We also assume that they are imperfectly resolved by other policy instruments. Concerning climate adaption (resilience investment R_t), our analysis focuses on underinvestment when adaptation benefits are insufficiently internalized.

These externalities are integrated in our analysis in a very simple way: we assume that only a factor $\alpha < 1$ of resilience benefits are internalized by decision makers on these investments, leading to underinvestment and hence to exacerbated physical climate risks. To the extent that these climate risks due to insufficient adaptation investment are systemic risks, they matter for financial regulation (see Section 4.3).

3.8 Policy Put Options

Concerning resilience, we also consider inefficiencies arising from the anticipation that in the case of severe economic shocks affecting financial stability, regulators and policy makers will ex post adopt accommodating policies that will mitigate the severe consequences of the shock for private agents. Because private agents anticipate such accommodation, they will ex ante do too little to adapt to the adverse situation and mitigate the damages (say, in the form of resilience investments). This underinvestment of private agents may exacerbate risks for financial stability. The trade-off between anticipated ex post accommodation and ex-ante incentives to prevent risks is not unique to climate risks. It corresponds to a classical regulatory trade-off between ex-post efficient policy accommodation and ex-ante distortions in incentives. This trade-off has gained prominence in the discussion of regulation elsewhere, notably concerning distortions in risk-taking created by forbearance ("too big to fail"), expansionary monetary policies, and countercyclical bailouts.

We introduce the ex-post efficient policy accommodation that leads to ex ante distortions in incentives as follows. To keep the argument simple, we assume that ex post accommodation will occur in very bad climate outcomes, defined as damages $Y_t \left(1 - f_d \left(E_t\right)\right)$ exceeding a certain threshold (for example, 10% of macroeconomic output lost in the regional economy). In these extreme events, the ex post-efficient reaction of regulators and monetary and fiscal authorities mitigate the climate-related shock for private agents. We refer to this effect as "policy put"benefits b, and assume that public policies create an expected aggregate policy put benefit b for the representative agent. Our use of the term "policy put " (or policy put option) refers to all public policies creating safety nets or a floor on the negative wealth shocks that private agents need to absorb. Bank bailouts that wee used on a massive global scale for bank resolution after the Global Financial Crisis of 207/2008, and the "whatever it takes" policies put in place in the wake of the Covid-19 pandemic in 2020 are two prominent examples. Since damages $Y_t \left(1 - f \ d \left(E_t\right)\right)$ are uncertain and policy interventions only occurs below a certain threshold, the benefit b is an expected value at the time that agents decide on resilience investments, and therefore, it is state-contingent, with b being larger in states in which it is more likely that this threshold will be exceeded. We account for this relationship by denoting $b\,(s)$ as the expected policy put benefit in state s. It is important to note that $b\,(s)$ is an aggregate measure of monetary and fiscal reactions as well as regulatory adjustment (that could be either rule-based contingencies adjustments in regulations or discretionary ad hoc changes). The reason that we use a wide definition of b, instead of limiting our attention to distortions created by financial regulators themselves, is that is the aggregate effect of all public policies that will determine the ex ante underinvestment.

The representative agent anticipates benefit $b\,(s) > 0$, reducing the impact of the climate damage to $b\,(s)\,d\,(E_t)$. The policy put benefit b creates a social cost, borne by taxpayers, of $\lambda\,b\,(s)$, where $\lambda > 1$. With these benefits from policy accommodation, the social damage function will then be modified to:

$$Y_t d\,(E_t)\,(1 + \lambda b(s))$$

Accordingly, the agent reduces the resilience investment (that determines the social damage $d(E_t)(1 + \lambda b(s))(1 - R^{\frac{1}{2}})$ in this outcome). The distortion is created by externalities: it is the aggregate actions of all agents that determines the probability and severity of catastrophic climate outcomes occurring, and the severity is such that the regulator has no choice but to intervene.

Optimal financial regulation should attempt to counteract the corresponding distortion. In other words, we consider countercyclical financial regulation where financial regulators counteract potential biases in incentives created by anticipated public policy reactions in climate-related adverse situations that could jeopardize financial stability. Regulators will address such biased incentives by introducing corrective regulatory mechanism ex ante. Thus, we focus on a very limited aspect of macro-prudential regulatory action, the effects linked to climate externalities. We assume that the regulator has one instrument available that can selectively reduce or increase the abatement efforts A_t and resilience investment R_t of the representative agent.

4 Theoretical Analysis

4.1 Baseline case: efficient carbon price and optimal resilience

In this baseline case, we assume that thw carbon price path, $\{p_t^*(s) \mid s \in S\}$ follows the optimal carbon trajectory and we also assume that $\alpha = 1$: the representative agent fully internalizes all external effects of her resilience investments, either directly or because of fiscal policies such as investment subsidies (that is, fiscal policies provide an efficient correction for any lack for agents to internalize the benefits of investment R_t), so that as a result optimal incentives to invest R_t are warranted.

Proposition 1 *Suppose that carbon prices are efficient, the market cost of capital is undistorted ($c_i = 1, i \in \{A, R\}$), resilience investments are optimal ($\alpha = 1$), and ex post regulatory reactions do not create any benefits for the representative agent ($b = 0$). Then the optimal climate financial regulation will not use any ex ante tools to influence abatement or resilience investments.*

Proof of Proposition 1: See Appendix. ■

The intuition is that the recursively determined optimal carbon price path is fully sufficient to implement efficient carbon emission and abatement strategies. By assumption, resilience investments are also efficient as the agent internalizes all benefits of resilience investments ($\alpha = 1$) and regulation will create no distortions ($b = 0$). Under this condition, the optimal carbon price path is sufficient to make sure the representative agent internalizes all effects, including all climate-related losses and possible catastrophic damage.

The outcome in this equilibrium is ex ante efficient. There is no role for financial regulation to correct or complement carbon policies when fiscal policies (carbon prices and fiscal incentives for resilience) achieve the efficient environmental outcome.

4.2 The Effect of Policy Puts: Efficient Carbon Policy and Resilience

In this section, we consider that $b > 0$, i.e. there is ex post regulatory accommodation (policy put) in the case of severe climate disasters which potentially creates distorted investment incentives. We maintain the assumption that the carbon price path p_t^* is recursively optimized in each period, so that future carbon prices adjust like a martingale following the gradual resolution of uncertainty. We also continue to assume that $\alpha = 1$, i.e. the representative agent fully internalizes all external effects of her resilience investments.

The damage function including the adaptation effects of resilience investments will then be modified as:

$$Y_t \left(1 - d\left(E_t\right)\left(1 - b(s)\right)\left(1 - fR_t^{\frac{1}{2}}\right)\right). \tag{13}$$

The objective function of the planner that leads to the optimal level of R_t is then:

$$Y_t \left(1 - d\left(E_t\right)\left(1 - b(s)\right)\left(1 - fR_t^{\frac{1}{2}}\right) - c_R R_t\right), \tag{14}$$

where we account for the cost of financing c_R introduced in Section 3.7 (c_R can be thought of as normalized to 1 under standard Basel III rules). The representative agent's objective function is identical, by virtue of $\alpha = 1$. While investments R_t arguably produce effects of reducing damages over several periods, we keep the analysis simple and consider only contemporaneous effects.

The representative agent's optimal choice of R_t can then be determined from the first-order condition of (14):

$$R_t = \left(\frac{d\left(E_t\right)\left(1 - b(s)\right)f}{2c_R}\right)^2 \tag{15}$$

We then substitute expression (15) back into the planner's objective function, taking into account that the planner needs to fund the gross cost of the policy put that is a linear proportion $\lambda, b(s)$ of damage $d(E{-}t)$. This allows us to determine the gross damage reduction due to $R_t > 0$:

$$Y_t \left(d\left(E_t\right)\left(1+\lambda b(s)\right) f R^{\frac{1}{2}} \right) = Y_t \left(\frac{\left(d\left(E_t\right)\left(1+\lambda b(s)\right) f\right)^2}{2c_R} \right) \tag{16}$$

The net output, after subtracting the agent's expenditure for resilience investment R_t, is:

$$Y_t \left(1 - d\left(E_t\right)\left(1+\lambda b(s)\right)\left(1 - f R_t^{\frac{1}{2}}\right) - c_R R_t \right) \tag{17}$$

$$\tag{18}$$

This analysis leads to the following observation:

Proposition 2 *Suppose that ex post regulatory actions create policy put benefits for the representative agent in severe climate damage scenarios ($b < 1$) and that resilience investments fully internalize benefits ($\alpha = 1$), and that the carbon price path p_t^* is chosen optimally.*

(i) Resilience investments will be lower and the optimal carbon price will be higher compared with the baseline case.

(ii) In the optimal policy mix, the financial regulator will introduce differentiated capital requirements for resilience investments R_t, but not for abatement efforts A_t.

Proof of Proposition 2: See Appendix. ■

These findings highlight that carbon policies, even efficient ones, can only target carbon emissions, but they cannot directly foster optimal resilience investments. From an ex ante perspective, the ex post policy put in the event of extreme climate shocks weakens the case for investing in resilience. As a result, the representative agent will invest too little in resilience.

From the planner's perspective, the policy put magnifies the total economic cost of climate damage since the policy put benefit $Y_t d\left(E_t\right)$ must be paid for by taxpayers, and generates a higher net cost because of $\lambda > 1$. As a consequence, the damages of carbon emissions $\left(Y_t d\left(E_t\right)\left(1+\lambda b\left(s\right)\right)\right)$ are higher, and the optimal carbon price p_t^*will be higher than in the efficient case to partially offset this effect. Optimal carbon prices (that are determined recursively) will partially counteract the higher expected damages, and hence be higher compared to the baseline case.

In addition, the financial regulator's optimal ex ante action will aim to offset the weakened incentive to invest in resilience. Regulatory intervention in favor of carbon reduction efforts would only distort this optimal mechanism and worsen incentives.

The stronger the financial regulator's ex ante correction in incentives in favor of resilience, via a risk weight-induced reduction in c_R, the lower will be the carbon price and abatement efforts along the optimal carbon path. The ex ante regulatory policy will increase in the anticipated level of policy puts, $b\,(s)$, i.e. in the scope of the regulator's mandate.

Recall that the financial regulator's action is purely motivated by financial stability concerns. The regulatory adjustment in capital requirements is limited to the best accommodation that regulators can provide within their remit, and hence is typically smaller than a socially optimal ex post response to extreme climate outcome. The larger the central bank's remit to anticipate climate risks, the larger will be b, hence the larger the need to offset the ex post distortion through ex ante regulatory action. The financial regulator does not take actions to encourage abatement investments since these are optimally guided by carbon prices.

Of course, incentives for resilience investments will frequently also be provided through other policy tools, such as fiscal incentives. The point of this analysis is to show that, when such other policy tools are insufficiently used, the optimal reaction of financial regulators will be to partially create such incentives within their financial stability mandate.

4.3 Inefficient Resilience Investments

In this section, we consider the case where $\alpha < 1$, i.e. the representative agent insufficiently internalizes the benefits of resilience investments. We maintain the assumption that the carbon price path p_t^* is optimally chosen.

The damage function including the adaptation effects of resilience investments will then be modified as:

$$Y_t \left(1 - d\left(E_t\right) b\left(s\right)\left(1 - \alpha f R_t^{\frac{1}{2}}\right)\right).$$

The objective function of the planner that leads to the optimal level of R_t is then:

$$Y_t \left(1 - d\left(E_t\right)\left(1 + \lambda b(s)\right)\left(1 - \alpha f R_t^{\frac{1}{2}}\right) - c_R R_t\right), \tag{19}$$

The representative agent's optimal choice of R_t can then be determined in analogy to the first-order condition of (14):

$$R_t = \left(\alpha \frac{d\left(E_t\right)\left(1 - b(s)\right) f}{2 c_R}\right)^2 \tag{20}$$

Substituting expression (20) back into the planner's objective function allows us to determine the gross damage reduction due to $R_t > 0$:

$$Y_t \left(d\left(E_t\right)\left(1 + \lambda b(s)\right)) f R^{\frac{1}{2}}\right) = Y_t \left(\frac{\alpha}{2 c_R}\left(df\right)^2\left(1 + \lambda b(s)\right)(1 - b(s))\right) \tag{21}$$

The net welfare, after subtracting the agent's expenditure for resilience investment R_t, is:

$$Y_t \left(1 - d\left(E_t\right) b\left(s\right)\left(1 - \alpha f R_t^{\frac{1}{2}}\right) - c_R R_t\right) \tag{22}$$

So inefficiency (compared to the planner's resiliency investment) comes through $\alpha < 1$ and $b > 0$. We only look at the comparative statics, without taking sides on the absolute magnitude of the most speculative parameters: b, α, d. Also for d,we focus on the change across scenarios.

We can summarize:

Proposition 3 *Suppose that carbon prices are efficiently chosen but resilience investments are suboptimal ($\alpha < 1$).*

(i) Resilience investments will be lower and the optimal carbon price will be higher compared with the baseline case.

(ii) The optimal ex ante policy of the financial regulator will introduce differentiated capital requirements that increase resilience investments, but do not alter carbon abatement efforts, whether there are policy put benefits ($b > 0$) or not ($b = 0$). The lower is α, the stronger the optimal ex ante incentives for resilience investments provided by the regulator.

Proof of Proposition 3: See Appendix. ■

The intuition is that even though optimal carbon prices are fully determined recursively, the insufficient internalization of resilience means that the representative agent will invest too little in resilience as she only partially internalizes the benefits of this investment ($\alpha < 1$). This creates a rationale for regulatory action. Optimal carbon prices will strive to partially offset this effect by boosting carbon abatement that will limit the probability of a worst-case scenario occurring.

The regulator's optimal ex ante policy tries to prevent this distortion, by favoring resilience investments. While the effect on resilience investments is fairly obvious, the effect on abatement is more subtle. Carbon prices will implement the best abatement effort, but because of the rise in the severity of the catastrophic scenario, carbon prices and abatement investments will be higher than in the baseline case, leading to higher abatement investments along the optimal path.

Consequently, the stronger the financial regulator's ex ante provision of investment incentives in favor of resilience, the lower will be the carbon price and abatement investments. The lower is α, the higher will be the optimal carbon price and the lower will be resilience investments.

As discussed earlier, incentives for resilience investments could possibly be provided through other policy tools, such as fiscal incentives. But unless such other policy tools are used optimally, financial regulators will still conclude that they should partially differentiate can find justifications within their mandate to provide such incentives.

4.4 Inefficient Carbon Price

Finally, we consider that carbon prices are not optimally chosen. One explanation (perhaps the most important one) is that domestic political resistance thwarts any efforts of policymakers to put such policies in place. A second explanation is that carbon emissions create global externalities, so that national policymakers do not find it optimal to implement policies that help to achieve the worldwide optimum, but rather tend to free ride on the climate effort of other countries. In other words, only a large, reliable and stable climate coalition or climate club (Nordhaus, 2015) will in principle have sufficiently strong motives to implement efficient carbon policies.

Proposition 4 *Suppose that carbon prices are determined inefficiently.*

(i) Suppose there are no policy puts ($b = 0$), and incentives for resilience investment fully internalize all benefits ($\alpha = 1$). Then the optimal ex ante action of the regulator is to introduce differentiated capital requirements only for abatement investments A_t.

(ii) If in addition resilience investments are suboptimal ($\alpha < 1$ or $b > 0$), then the regulator's optimal ex ante action is to introduce differentiated capital requirements for both abatement investments A_t and for resilience investments R_t.

Proof of Proposition 4: See Appendix. ■

When carbon policies fail, other actors and mechanisms partially assume their role. Since incentives to reduce carbon emissions will be too low compared without the baseline case, the probability of worst-case climate scenarios will be higher, reinforcing the regulator's motive to step in and provide incentive to invest in resilience and to abate carbon emissions. The role of regulators is to provide substitute incentives for both abatement and climate resilience investments. We note again that we consider financial regulator actions that are purely motivated by financial stability concerns in the worst-case climate scenario. They stays within the conventional mandate of financial regulation. Consequently, the regulator will implement both preventive policies to a smaller degree than would be socially optimal. As a result, carbon emissions and resilience investments will be larger than when there are efficient carbon prices.

5 Calibration

5.1 Choice of Parameters

In the following limited calibration exercise, our calibration spans a time horizon from 2022 until 2082, split into 6 periods of 10 years of length and model each source of uncertainty as a binomial tree, so that 7 final states are possible for each of the three dimensions of uncertainty (macro uncertainty affecting Y_t, technology uncertainty affecting the cost of abatement, and climate risks). We use the notation $s \in S$ to denote both final states (a complete history until the final period) and intermediate nodes, described as the filtration of the event tree, where S is the set of possible states.

We aggregate uncertainty by considering $n = 6$ periods of 10 years of length, with $t \in \{2022, 2032, ..., 2082\}$ spanning the horizon. The model is calibrated as an event tree where uncertainty along three dimensions is gradually resolved, macroeconomic uncertainty (affecting Y_t), technology uncertainty (affecting the cost of abatement), and climate risks, affecting the damage as a function of a given level of cumulative carbon emissions E_t, is gradually resolved. The uncertainty about each of these three dimensions is incoproated as a binomial tree, so that 7 final states are possible for each of the three dimensions , or $7^3 = 343$ final states in total, representing the set S of states.

Macro parameters: For the macro disaster risk affecting Y_t, we deploy the Barro parameters for extreme scenarios, with a 1.7% annual probability and a - 35% growth shock. We assume a growth rate of 2% for years without a disaster. Taking the two together with the annual disaster probability of 1.7%, this yields an expected growth rate of $g_Y = 1.37\%$, assumed to be constant until 2082. The trend growth is roughly consistent with an estimate based on 0.9% p.a TFP growth and 0.5% world population growth.

We determine the initial carbon intensity in 2022 as $Q_0 = 0.45$ GtCO2e/ 10^{12} USD of World GDP (estimated from 40 Gt CO2e for USD 89tn World GDP in 2019), decreasing at 0.8% p.a., as in Gollier (2020).

For other macro and carbon parameters, we follow Gollier (2020) and choose the coefficient of relative risk aversion $\gamma = 3$, the rate of pure time preference $\rho = 0.5\%$, and the carbon decay rate $\delta = 0.5\%$ p.a.

Abatement costs: For the binomial shock to the abatement cost parameter a, we assume growth factors in the binomial model for 10 year periods of $g_{d,10} = 0.61767$ and $g_{u,10} = 2.2855$. These parameters are the binomial approximation of a log-normally distributed abatement cost parameter according to a GBM with drift μ_θ and instantaneous std.dev. σ_θ, $\frac{da(t)}{a(t)} = \mu_\theta dt + \sigma_\theta dZ$, where we determine μ_θ and σ_θ by matching the moments given in Gollier (2020) and then limit the annual standard deviation to $\sigma_\theta = 0.2$. In the binomial approximation of the GBM, we fill the remaining degree of freedom, the probability of a negative abatement cost shock p, by assuming $p = 0.2$, based on the view that it should be a rare event.

Damage function: For the calibration of the damage function, we follow DGK (2018) to use $\xi = 1$ (DGK 2018) find that the parameter ξ has little impact on the climate beta) and specify $d(E_t) = 1 - \frac{1}{1\,+\,\alpha_2(\omega_1 E_t)^2\,+\,\alpha_3(\omega_1 E_t)^7}$. The damage function (2) then becomes:

$$D(E_t) = Y_t \left(1 - \frac{1}{1 + \alpha_2\left(\omega_1 E_t\right)^2 + \alpha_3\left(\omega_1 E_t\right)^7}\right) \tag{23}$$

We define $E_t = E_0 +$ emissions already accumulated. The initial value of cumulative stocks of emissions is $E_0 = 2,275$ Gt CO_2, based on the increase of CO_2 concentration from the pre-industrial level of 280ppm to 418ppm in 2021. The variable capturing uncertainty about climate risk is equilibrium climate sensitivity (ECS) ω_1, the increase in degC from a doubling of CO_2 concentration compared to the pre-industrial levels of 280 ppm. Following the AR6 assessment (IPCC (2021)), we assume for ω_1 a mean of 2.9 C and a variance of 1.5 C (or degC), a variance slightly lower than the estimate implied in the AR6 assessment. the mean increase of 2.9 C translates into $\omega_1 = 0.00062572$. We translate the variance into a binomial tree with additive increments without drift, and represent the skewness of the climate uncertainty by assuming that there is a smaller probability of negative ("upward") shocks to ω_1. We determine the value of such negative shocks as $\omega_1^U = 1.6479 \times 10^{-4}$, occurring with with probability $p_U = 0.3$, and the value of the offsetting frequent positive shock as $\omega_1^D = -\frac{0.32733}{4634.7} = -7.0626 \times 10^{-5}$ with probability $p_D = 1 - p_U = 0.7$.

We keep α_2 and α_3 and choose $\alpha_2 = 1.3055 \times 10^{-2}$, $\alpha_3 = 5.1579 \times 10^{-4}$ as parameter choices that fit a reasonable, steep but not too steep function, and were derived by imposing that $D(E_0) = 0.03Y_0$ at today's level of $2,275$ Gt CO_2, and $D(E_{60}) = 0.5Y_{60}$ for the "hothouse world" expected level of $E_{60} = 4,635\,\text{Gt}\ CO_2$ in 2082 (E_t increasing at 39 Gt CO_2 every year). Details of our calculations are in Appendix A.

5.2 Calibration Results

(to be added)

6 Conclusions

This paper explores the question when and why differentiated standards in climate financial regulation, or a Green Supporting Factor, is warranted. We show how the answer to this question cannot be dissociated from the efficnecy of the underlying carbon policies. They only have a limited role to play alongside socially efficient carbon price policies. But their role and optimal grip expands when carbon policies are inadequate.

Our analysis distinguishes between two key dimensions of climate action, carbon abatement and proactive investment in climate resilience. We consider two scenarios of carbon policies that financial regulators face. In the baseline scenario, carbon policies follow a stochastic path of efficient carbon prices. In the conflict scenario, the implementation of optimal carbon prices is stalled and financial regulators need to react optimally to inefficient climate policies.

Our key findings are as follows. Our starting point is the observation that when carbon prices are efficient and resilience investments are optimal, there is no and ex post regulatory reaction does not create any benefits for the representative agent, then the optimal regulatory response will not use any macro-prudential action ex ante. In this case, the optimal carbon price is fully sufficient to implement efficient carbon emission and abatement strategies, including all climate-related losses.

We argue that the prospect of "hothouse world" climate scenarios that pose threats to financial stability creates a rationale for climate financial regulation even when carbon policies are efficient. When extreme climate-related events occur, monetary and fiscal policies and financial regulation will react with policy puts, or accommodating stances. This policy reaction is optimal ex post, and will be anticipated by private agents. Such anticipation leads to distortions in private investment incentives. The optimal ex ante policy of regulators then attempts to partially correct for such biases. In this case, financial regulators should introduce differentiated capital requirements that encourage resilience investments. The reason is that the ex post policy puts in severe climate scenarios is anticipated by private agents and weakens their incentives to invest in resilience which in turn may affect financial stability in such climate scenarios. Since underinvestment in resilience means that expected climate damages are larger, and since the carbon price trajectory is chosen optimally by assumption, the optimal carbon prices are higher compared to the baseline case. Hence, optimal regulation should not target abatement efforts. Regulatory intervention in favor of carbon reduction efforts would only distort this optimal mechanism and worsen incentives.

We then turn to situations where resilience investments are suboptimal even when carbon prices are efficiently chosen. Again, differentiated capital requirements are warranted, to the extent that underinvestment in resilience threatens financial stability.

The case when carbon prices are determined inefficiently is today the most realistic one, and we consider this case in our final analysis. The optimal ex ante action of the regulator is then to provide differentiated capital requirements to foster abatement investments. In the absence of optimal carbon prices, other actors and mechanisms partially substitute for missing optimal climate policies, and this holds also for regulators tot he extent that financial stability is affected. As for climate policies in general, the effectiveness of climate financial regulation will depend on the regulators' success in coordinating internationally so that an emerging climate club of regulators is sufficiently large to internalize the positive effects of climate mitigation.

A final calibration exercise focuses on the role of regulation to mitigate the policy put effect. A simple quadratic model of resilience investment shows that differentiated capital requirements are of negligible magnitude now when efficient climate policies are pursued, but may become important in a few decades under adverse conditions. However, when climate policies continue to be inefficient, there is a role for substantial differentiation of capital requirements starting now. The calibration exercise shows that efficient capital requirement regulation optimally adjusts to the carbon policy environment in which it operates. It also illustrates that the policy variations can be substantial.

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Proofs

Proof of Proposition 1: With the assumptions ($\alpha = 1$) and ($b = 0$), and an optimal conditional carbon price path, $\{p_t^*(s) \mid s \in S\}$, and no cost of capital distortions, the objective function of the representative agent can be written as:

$$\max_{\{A_t, R_t\}} \left(U(C_t) + \beta E V_{t+1}(Y_{t+1}, I_{t+1}, E_{t+1}, \omega_{t+1}, a_{t+1})\right)$$

subject to:

$$\begin{aligned} C_t &= Y_t\left(1 - d(E_t)\left(1 - f R_t^{\frac{1}{2}}\right) - R_t\right) - A_t(I_t) && (24) \\ A_t(I_t) &= a_t I_t + \frac{b}{2}(I_t)^2 && (25) \\ E_t &= E_{t-1}(1-\delta) + (Q_t Y_t - I_t) && (26) \end{aligned}$$

This is identical to the social planner's problem in eq. (5). (To be completed). ■

Proof of Proposition 2: (To be completed.) ■

Proof of Proposition 3: (To be completed.) ■

Proof of Proposition 4: (To be completed.) ■

Appendix A: Details of the Calibration of the Damage Function

We determine the value of ω_1 that corresponds to the mean temperature increase of 2.9 C for 280 ppm of additional CO$_2$ concentration compared to the 1750 level as follows. We begin with ϕ, the mean temperature increase per additional ppm CO_2 in the atmosphere. This relationship is typically assumed to be linear, and we get $\phi = \frac{2.9}{280} = 0.010357 \left[\frac{\text{degC}}{\text{ppm}}\right]$. Under a scenario of constant carbon emissions of 39 Gt CO_2 that is close to a hothouse world scenario, or business-as-usual in many climate science scenarios (but many earlier BAU scenarios assume higher emissions), an annual increase of 2.3666 ppm for 39 Gt CO_2 leads exactly to a doubling of CO_2 until 2082, i.e. 1/60 of the increase from 418 ppm to 560 ppm happening every year. We base our model on this assumption which is slightly below the recently observed increase of 2.5ppm per year (2018-2019 period). Using the conventional short-cut to a full model of the global carbon cycle, we stipulate a linear relationship of atmospheric CO2 increase per Gt CO2, $\zeta = \frac{2.3666}{39} = 0.06068 \left[\frac{\text{ppm}}{\text{Gt CO2}}\right]$. Putting both together, we get an estimate of ECS ω_1 expressed as $E\left[\omega_1\right] = \phi \cdot \zeta = 0.00062572$, expressing the long-run temperature increase in equilibrium. A carbon budget of 650 Gt CO_2 that is compatible with the Paris goals will increase CO_2 concentration by about 44 ppm, from 418 ppm in 2021 to 462 ppm, and mean temperature by about 0.45 gC. The historic increase in CO_2 concentration from 280 ppm in 1750 to 2020 to 418 ppm corresponds to a cumulative anthropogenic CO_2 stock of 2,275 Gt CO_2 in the atmosphere.

Assume that under the hothouse world scenario, CO_2 concentration continues to increase at 2.417 ppm/p.a. (hothouse world scenario). Denoting the temperature increase of n periods by T_n, we then have for the mean and variance over 60 years or 6 periods: $E(T_6) = 2.9$ and $Var(T_6) = 1.5$.[5]

We translate this into a binomial shock as follows. In each 10-year period, the temperature either increases by a value of t_U with probability p_U, or it decreases by a value t_D with probability $p_D = 1 - p_U$. In line with the literature (Gillingham et al. 2015, Nordhaus 2018, IAWG 2016), we assume a linear increment of temperature in cumulative CO2 concentration, i.e. an additive, not a multiplicative process. This implies we use a binomial model along

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[5] The calibration does not account for the reduction in the variance of climate scenarios between the IPCC AR5 and IPCC AR6.

the BAU with additive increments. We have:

$$\begin{aligned} E(T_1) - 2.9 &= p_U t_U + (1 - p_U)\, t_D \\ V\left(T_1\right) &= p_U \left(t_U - \bar{t}\right)^2 + (1 - p_U)\left(t_D - \bar{t}\right)^2 \end{aligned}$$

where $\bar{t}$ is the mean temperature beyond 2.9 degC. In this model, both mean and variance expand linearly, hence $V\left(T_1\right) = \frac{Var(T_n)}{n} = 0.25$. Thus, we choose the parameters p_U, t_U and t_D to fit the given moments $E(T_1)$ and $Var(T_1)$:

$$\begin{aligned} 0.3t_U + 0.7t_D &= 0 \\ 0.3t_U^2 + 0.7t_D^2 &= 0.25 \end{aligned}$$

from which we get the solution for the shocks: $t_U = 0.76376$, $t_D = -0.32733$. These values exhibit the desired skewness because of $p_U < 0.5$. Six negative shocks (prob. $0.3^6 = 7.29 \times 10^{-4}$) lead to an increase of $6 * 0.7643 + 2.9 = 7.48$ degC, somewhat beyond the upper bound of IPCC AR5 estimates which is at 6 degC ("absolutely unlikely"). After 6 down moves, this implies an increase of $2.9 - 6 \times 0.327 = 0.938$ degC, close to the IPCC AR5 lower bound of 1degC (and below the current realized warming).

To translate the binomial temperature shocks into values of the climate sensitivity ω_1, we use the relationship between cumulative emissions E and temperature increase T, $T = \omega_1 E$ where $\omega_1 = 0.00062572$. Since this relationship is linear, we can just rescale the climate shocks as shocks on ω_1:

$$\omega_1^U = \frac{0.76376}{4634.7} = 1.6479 \times 10^{-4},\ \omega_1^D = -\frac{0.32733}{4634.7} = -7.0626 \times 10^{-5}$$

with probabilities $p = p_U = 0.3$, and $p_D = 1 - p_U = 0.7$. These are incremental shocks, starting from the mean point $\omega_1^{ECS} = 0.00062572$.

In other words, shocks $t_U = 0.76376, t_D = -0.32733$ are 10 year shocks corresponding to a 2.9 degC increase from pre-industrial levels, or to a cumulative emission of $\frac{2.9}{0.00062572} = 4634.7$ Gt CO_2 (including 2,272 Gt CO_2 emissions already in the atmosphere in 2021). ω_1^U and ω_1^D express the corresponding per-period reaction, or ECS, per Gt CO_2.