To Own or to Rent?
The Effects of Transaction Taxes on Housing Markets*

Lu Han†
University of Wisconsin-Madison

L. Rachel Ngai‡
London School of Economics

Kevin D. Sheedy§
London School of Economics

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Abstract

Using transaction records on residential property sales and leases, this paper estimates the effects of Toronto's imposition of a tax on property purchases in 2008. Three novel effects are uncovered: (i) a rise in buy-to-rent transactions alongside a fall in transactions by owner-occupiers in spite of the tax applying to both; (ii) a simultaneous fall in the ratio of the numbers of sales to leases and the ratio of prices to rents; and (iii) a decline in homeowners' moving rates and an increase in the time taken for properties to sell. This paper develops a housing search model in which households make an owning or renting choice and where there is entry of investors. The model explains the three new facts by predicting that the transaction tax leads to an increase in demand for rental properties, which generates a fall in the homeownership rate, and a reduction in mobility for homeowners. The implied deadweight loss of the tax is large at 79% of revenue raised, with nearly 40% of this due to distortions related to the rental market.

JEL CLASSIFICATIONS: D83; E22; R21; R28; R31.

KEYWORDS: rental market, buy-to-rent investors, homeownership rate, transaction taxes.

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†University of Wisconsin-Madison. Email: lu.han@wisc.edu

‡London School of Economics, CEPR, and CfM. Email: L.Ngai@lse.ac.uk

§London School of Economics and CfM. Email: K.D.Sheedy@lse.ac.uk

1 Introduction

Real-estate transaction taxes are a common feature of tax systems around the world. A large and growing literature points to the distorting effects of such taxes on owner occupiers.[1] However, little is known about the implications of transactions taxes for households' tenure choices and purchases by landlords, which together determine the allocation of properties between the markets for ownership and rentals. This is surprising because rental property typically comprises at least a third of the housing stock, and the homeownership rate has been the subject of many policy debates.[2] This paper's objective is to gain a comprehensive understanding theoretically and empirically of the impact of transaction taxes on housing markets along both the intensive margin, namely moving and transaction decisions, and the extensive margin, namely the decision to own or to rent.

Empirically, this objective is accomplished by making use of a unique dataset on housing sales and leasing transactions from Multiple Listing Service records for the Greater Toronto Area (GTA) between 2006 and 2018. Having observations of leases and rents in addition to sales and prices, the data make it possible to distinguish purchases made by buy-to-rent investors and owner-occupiers. This paper investigates the effects of a transaction tax, known in Canada as Land Transfer Tax (LTT). In 2008, in addition to the existing provincial-level LTT, the City of Toronto introduce a new city-level LTT at an effective rate of 1.3.% of the property price. Importantly, the new tax covers only the City of Toronto but not other parts of the GTA. The effects on a rich array of housing-market outcomes are estimated by comparing transactions before and after the new LTT across treated and untreated neighbourhoods using a regression discontinuity design.

The estimation results yield a set of novel facts about the effects of transaction taxes across the markets for property ownership and rentals. First, while the LTT reduces overall sales of property and transaction prices, it has divergent effects across the ownership (purchases and sales of property) and rental markets. The new LTT causes the ratio of the number of property leases to sales to rise by 23.4% and the ratio of prices to rents to decline by 4%. Second, among the sales made in the ownership market, there is a 9.8% fall in purchases by those who will be owner-occupiers, but an 11% rise in purchases made by buy-to-rent investors.

By definition, buy-to-rent investors are those who acquire properties from the ownership market and make them available to let in the rental market. Thus, the increase in buy-to-rent purchases due to the LTT is consistent with the rise in the ratio of leases to total sales. This indicates a shift towards the rental market, which would imply a fall in the homeownership rate.[3] The two findings

[1] This includes, but is not limited to, Benjamin, Coulson and Yang (1993), Slemrod, Weber and Shan (2017), Kopczuk and Munroe (2015) for the US, Besley, Meads and Surico (2014), Hilber and Lyytikäinen (2017), Best and Kleven (2018) for the UK, Dachis, Duranton and Turner (2012) for Canada, Eerola, Harjunen, Lyytikäinen and Saarimaa (2019), Määttänen and Terviö (2020) for Finland, Fritzsche and Vandrei (2019) for Germany, Davidoff and Leigh (2013) for Australia, Van Ommeren and Van Leuvensteijn (2005) for the Netherlands, Agarwal, Chau, Hu and Wan (2022) for Hong Kong, and Huang, Li and Yang (2021) for Singapore.

[2] See Gabriel and Rosenthal (2015) and Goodman and Mayer (2018).

[3] The homeownership rate, defined as the fraction of properties lived in by their owners, is reported by Statistics

suggest that a careful evaluation of transaction taxes must consider flows of property between owner-occupation and the rental market.

This paper also documents new facts about the flows within the ownership market. The new LTT causes lower flows both into and out of the market. Homeowners' moving hazard rate falls by 13%, implying owner-occupiers live in a particular property for 14 months longer on average. The time taken to sell a property increases by 23%, so properties remain on the market for one week longer on average after the new LTT.

Theoretically, the goal of understanding real-estate transaction taxes and explaining the empirical findings is accomplished by developing and calibrating a model suitable for jointly analysing the property ownership and rental markets. A crucial feature of the model is that households choose which market to participate in, subject to paying a credit cost to access the market for property ownership. These credit costs represent the costs of mortgage financing or the difficulty of obtaining credit, which are heterogeneous across households. Setting all the costs of homeownership (credit costs and transaction costs) against the benefits — for example, better match quality owing to a 'warm glow' effect — gives rise to an entry decision on the 'buy' side of the rental market. On the 'sell' side, there is free entry of buy-to-rent investors. The equilibrium homeownership rate is the one consistent with the behaviour of both households and investors.

Both the ownership and rental markets are subject to search frictions. Market tightness, the number of those trying to buy relative to those trying to sell, affects the rates at which properties are viewed. Viewings are required to reveal the idiosyncratic match quality between a property and a household, and home-buyers and renters search until they find a property with match quality above a threshold. Once they have moved into a property, match quality is a persistent variable subject to occasional idiosyncratic shocks representing life events that make a particular property less well suited to a particular household. After a shock, a household makes a decision to move, doing so if match quality is below a threshold.

Owing to the idiosyncratic shocks and the indivisible nature of property, households will desire to move between different properties on a number of occasions throughout their lives. Hence, choosing to be an owner-occupier rather than a renter means expecting to pay the new LTT every time a new property is purchased. This dissuades some home-buyers from paying a credit cost to enter the ownership market. Since these households must still live somewhere, there is an increase in demand for properties in the rental market.

Investors also face paying the new LTT, which reduces the return from purchasing property. However, a landlord does not need to transact again in the ownership market just because a tenant no longer finds the property suitable and moves out. Even if investors must sometimes transact for liquidity reasons or to realize capital gains, they have less need to do so compared to owner-occupiers who face match-quality shocks. So while the LTT also has a direct negative effect on supply in the

Canada only at a five-year frequency. It steadily increased from 51% to 54.5% between 1996 and 2006, which was followed by almost no growth and then a decline to 52.3% in 2016.

rental market, this effect is relatively smaller than the increase in demand for rental properties.

In equilibrium, the new LTT causes the price-rent ratio to fall by enough to attract more buy-to-rent investors in spite of the higher tax. Investor purchases of properties from owner-occupiers selling up cause the homeownership rate to decline. Buy-to-rent purchases and leases increase, while purchases by owner-occupiers decline, consistent with the empirical evidence.

Within the ownership market, the new LTT makes existing owner-occupiers more tolerant of poor match quality, so moving rates decline as households remain in properties for longer on average. The indivisibility of housing in the search model implies the moving rate is a proxy for households' renewals of match quality, and the empirical evidence supports the model's prediction. Since match quality with a property has some persistence, households can mitigate the increased tax costs of moving by requiring higher match quality when making a property purchase, thus reducing the need to move in the future. This greater pickiness of buyers leads to longer average time-on-the-market for sellers of property — a distinctive feature of a search-based model — and this prediction is also borne out empirically.

The model spells out two facets of the welfare costs of transaction taxes closely related to its positive predictions. First, the novel effect of misallocation of properties across the rental and ownership markets. As a consequence of the LTT, fewer households pay a credit cost to access better match quality in the ownership market. Intuitively, the LTT falls more heavily on owner-occupiers than buy-to-rent investors, even though it is levied at the same rate on both, because owner-occupiers expect to transact more frequently. Hence, transaction taxes distort housing tenure choices. Second, there is a 'lock-in' effect of reduced moving within the ownership market. This gives rise to misallocation of properties among owner-occupiers, with match quality falling on average as households move less frequently to renew it.[4]

The model's parameters are calibrated to the City of Toronto housing market for the years 2006–2008. Toronto has an active rental market, and the homeownership rate in the city municipality was then about 54%.[5] The model is used to simulate the effects of a 1.3 percentage point effective increase in the LTT rate, calibrating it so that the model matches quantitatively the estimated change in homeowners' moving rate. The model predicts a 14.6% decrease in buy-to-own transactions and a 2% increase in buy-to-rent transactions, resulting in a 0.9 percentage points decline in the homeownership rate. The ratio of leases to sales is predicted to rise by 15.4% and the price-to-rent ratio to decline by 1.8%. Time on the market for sellers goes up by 8.6%. These predictions go in the same direction as the empirical findings and are broadly consistent quantitatively.

The implied welfare costs of the LTT are substantial. The tax generates a welfare loss equivalent

[4]In evaluating the welfare costs of the LTT, the search model allows home-buyers to adjust the time spent searching so as to obtain better-quality matches initially. This means that longer time-on-the-market does not necessarily imply a larger welfare cost per se.

[5]Some have argued using U.S. national data that the rental and ownership markets are largely independent segments of the housing market (Glaeser and Gyourko, 2007, Bachmann and Cooper, 2014), but this is less plausible when focusing on a market like the City of Toronto.

to 79% of the revenue it raises. The welfare loss is due to distortions across and within the rental and ownership markets, and both are significant. Across the markets, the distortions imply a loss of 25% of revenue raised. Distortions within the rental and ownership market lead to losses of 5% and 49% of tax revenue respectively. Overall, the presence of the rental market is associated with a loss equivalent to 30% of tax revenue, which is around 40% of the total loss. The across-market losses could be reduced by having the transactions tax levied at a higher tax rate on buy-to-rent investors to offset the implicit advantage over owner-occupiers they derive from a tax system with equal rates.

The plan of the paper is as follows. Related literature is discussed below. Section 2 presents the data and the estimation of the effects of the LTT in Toronto. Section 3 develops a dual ownership and rental markets model of housing. Section 4 calibrates the model and derives the quantitative effects of the transaction tax and the associated welfare losses due to distortions within each market and misallocation across the two markets.

Related literature In the last two decades, concerns about the costs of real-estate transaction taxes have grown among policymakers and in academic research. Two prominent examples are the 'Henry Review' established by the Australian government and the 'Mirrlees Review' by the UK government. Both reviews found significant costs of stamp duty (a transaction tax) owing to reduced mobility and distortions associated with ad valorem taxes. The reviews proposed reforms to replace stamp duty with a land value tax or a tax on housing consumption (Henry, Harmer, Piggott, Ridout and Smith, 2009, Mirrlees, Adam, Besley, Blundell, Bond *et al.*, 2010).

These findings are confirmed by economists studying housing markets using data from Australia, Canada, Finland, Germany, the UK, and the US. The majority of the literature has focused on the effects of transaction taxes on mobility, transaction volumes, or house prices. Among these papers, a few have also computed the welfare costs of transaction taxes per unit of tax revenue raised, such as Dachis, Duranton and Turner (2012) for Canada, Hilber and Lyytikäinen (2017) and Best and Kleven (2018) for the UK, Eerola, Harjunen, Lyytikäinen and Saarimaa (2019) and Määttänen and Terviö (2020) for Finland, and Fritzsche and Vandrei (2019) for Germany. These losses are solely due to effects on the intensive margin of fewer transactions and reduced mobility of homeowners. However, as Poterba (1992) noted, "*finding the ultimate behavioral effects requires careful study of how tax parameters affect each household's decision of whether to rent or own as well as the decision of how much housing to consume conditional on tenure.*"

This paper makes two contributions to the literature. First, it documents empirically the different way buy-to-rent investors respond to a transaction tax compared to owner-occupiers, and the relative effects of the tax on markets for property ownership and rentals as measured by the leases-to-sales and price-to-rent ratios. These facts demonstrate the importance of considering the extensive margin. Second, the paper develops and quantifies a housing search model with both an ownership and a rental market, which features an endogenous moving decision across and within the two markets.

The use of search-and-matching models to study frictions in the housing market is long established, going back to Wheaton (1990). The papers in the voluminous literature that followed are surveyed by Han and Strange (2015).[6] Among those papers, Lundborg and Skedinger (1999) explicitly study the effects of transaction taxes on search effort in a version of the Wheaton (1990) model. Since they abstract from the rental market and the decision to move, their model cannot be used to analyse the impact on homeownership and mobility.

While the majority of housing search models have abstracted from search in the rental market, recent papers by Halket, Pignatti and di Custoza (2015), Ioannides and Zabel (2017), and Bø (2021) explicitly consider search in both ownership and rental markets.[7] Their objectives are different from this paper, focusing instead on issues such as the Beveridge curve in the housing market, and the relationship between price-to-rent ratios and homeownership rates across sub-markets. More importantly, they abstract from the moving decision that is crucial here for both the extensive and intensive margins of adjustment to transaction taxes.

The empirical strategy of this paper is closest to Dachis, Duranton and Turner (2012) in studying the effects of the 2008 LTT in Toronto. This paper differs in that it examines an array of housing-market outcomes beyond sales prices and volumes, which yields a comprehensive understanding of how housing markets react to transactions taxes, including the market for rental property. Using a partial-equilibrium model of the market for property sales, Dachis, Duranton and Turner (2012) computed a welfare loss from the LTT of 13% of revenue raised. By considering a general-equilibrium search model with endogenous moving across and within the ownership and rental markets, this paper finds a much larger welfare loss of 79% of tax revenue.

Two recent works with a related objective to this paper are Kaas, Kocharkov, Preugschat and Siassi (2021) and Cho, Li and Uren (2021). They analyse the effects of stamp duty on the homeownership rate and its implications for welfare in models without search frictions. This paper's key advantage is in identifying the differential effects of transaction taxes on buy-to-rent investors and owner-occupiers using micro data on leasing and transaction records and a regression discontinuity design. On the theory side, this paper allows for free-entry of buy-to-rent investors in a search model that highlights the indivisible nature of housing. The model rationalizes the empirical finding of opposite effects of the transaction tax on buy-to-rent investors and owner-occupiers.

[6] For recent examples, see Anenberg and Bayer (2020), Díaz and Jerez (2013), Gabrovski and Ortego-Marti (2019), Guren (2018), Head, Lloyd-Ellis and Sun (2014), Moen, Nenov and Sniekers (2021), Ngai and Tenreyro (2014), Ngai and Sheedy (2020), Piazzesi, Schneider and Stroebel (2020), and Genesove and Han (2012).

[7] There is also a literature on understanding changes in homeownership rates that uses models without search frictions, for example, Chambers, Garriga and Schlagenhauf (2009), Fisher and Gervais (2011), Sommer and Sullivan (2018), and Floetotto, Kirker and Stroebel (2016). See Goodman and Mayer (2018) for a survey of the determinants of homeownership rates.

2 Empirical analysis

2.1 Data

The data on real-estate sales and leasing transactions come from Multiple Listing Service (MLS) transaction records for the period 2006–2018 in the Greater Toronto Area (GTA), the fourth largest metropolitan area in North America. The data cover residential property transactions from 2000 to 2018 and leases between 2006 and 2018. Each sale has observations of the property price, the time on the market, the transaction date, and the exact address and neighbourhood. For each lease, the listing date, the lease start date, the monthly rent, the lease term, and the exact address and neighbourhood are observed.[8]

Since the data include detailed address and transaction dates, properties that appear in both sales and lease datasets are identifiable by their addresses. Doing this generates a novel measure that links the markets for property ownership and rentals. If the sale of a property is followed by it being listed on the rental market between 0 and 18 months after the sale, the purchase is identified as a *buy-to-rent* transaction. Alternatively, if the sale is followed by being listed again for sale between 0 and 18 months after the original sale, it is identified as a *buy-to-sell* transaction.[9] The remaining sales transactions are considered to be purchases by those who will be owner-occupiers, and are designated as *buy-to-own* transactions.

Between 2006 and 2017, the fraction of buy-to-own transactions declines from 89% to 84%, while the fraction of buy-to-rent transactions triples from 4% to 12%.[10] In contrast, the fraction of buy-to-sell transactions remains stable at around 4% throughout most of the period. Given the focus of this paper and the lack of significant variation in buy-to-sell transactions, these are excluded from the sample used for estimation.

This paper investigates housing-market outcomes using these rich datasets at both the market-segment and individual transaction levels. A market segment is defined by *property type* $\times$ *community* $\times$ *year* $\times$ *month*. Property types comprise single-family houses, townhouses, condominiums, and apartments. Communities refer to neighbourhoods.[11] For each market segment, housing-market outcome variables are the number of sales, which is broken down into buy-to-own (BTO) and buy-to-rent (BTR) sales, the number of leases, the ratio of the numbers of leases to sales, and the price-to-rent ratio. At the transaction level, outcomes are sales prices, sellers' time on the market, and the

[8]For transactions that occur after 2006, there are also observations of detailed property characteristics such as the numbers of bedrooms, washrooms, and kitchens, the lot size (except for condominiums/apartments), the styles of the house and the family room, the basement structure/style, and the heating types/sources.

[9]As a robustness check, changing the 18-month threshold to 6, 12, or 24 months does not significantly affect the estimation results.

[10]The rise of buy-to-rent transactions in recent years has been seen in other countries, including the US and Norway (Mills, Molloy and Zarutskie, 2019, Bø, 2021).

[11]There are 296 communities in the GTA, including 140 in the City of Toronto. See www.toronto.ca/city-government/data-research-maps/neighbourhoods-communities/neighbourhood-profiles/.

lengths of time since homeowners purchased their property.

Real-estate transaction taxes are common across Canada, where they are paid by buyers and are known as Land Transfer Tax (LTT). In spite of the name, LTT is applied to the whole property price. Before 2008, residential transactions in the province of Ontario, including the whole of the GTA, were subject to the provincial-level land transfer tax, but there was no additional city-level LTT. The City of Toronto experienced a housing boom from 2000 and usually maintained a budget close to balance. Following a surprise budget shortfall in late 2007, the city council approved a land transfer tax on property transactions within the city that close after 1st February 2008.

In the appendix, Table A.1 gives descriptive statistics for the City of Toronto before and after the introduction of the city-level LTT. The rest of the Greater Toronto Area remained with the same provincial-level LTT after February 2008. Table A.2 summarizes the city- and provincial-level LTT schedules. The effective LTT rate is the mean transfer tax, combining provincial- and city-level taxes, as a percentage of the sales price, averaged over all pre-February-2008 transactions. Using this same set of transactions to control for composition effects, the effective LTT rate is 1.5% before the February-2008 policy change and 2.8% afterwards. This implies a 1.3 percentage points increase in the effective LTT rate.

2.2 Estimating the effects of transaction taxes

The February-2008 introduction of a city-level LTT created two discrete changes in Toronto's housing market: one at the city border, and the other on the date the city LTT was imposed. This paper exploits these features to estimate the causal effects of the transaction tax by comparing changes in housing-market outcomes before and after the introduction of the tax in 'treated' city neighbourhoods to changes over the same period in 'untreated' suburban neighbourhoods.

The identification strategy is similar to the regression discontinuity design in Dachis, Duranton and Turner (2012), who estimate the short-run (six-month) effects of the LTT on transaction volumes and sales prices in the market for single-family houses. The innovation here is in examining a much broader set of market outcomes above and beyond sales price and volume. Most importantly, the estimation is not limited to the effects on property purchases by owners. Benefiting from a unique dataset, this paper estimates the effects of the LTT on flows between owner-occupation and renting to examine the LTT's price and quantity effects in the rental market relative to the ownership market.

Furthermore, this paper also examines the effects of the LTT on flows within the ownership market. This is achieved using micro data on individual property transactions to estimate the hazard function for moving (the moving rate as a function of how long a household has lived in a property), the average time taken for properties to sell, and how these change in response to the LTT. The hazard function estimation provides direct evidence on micro-level mobility that complements the findings on aggregate transaction volume. The estimated effects on average time-on-the-market are informative about search frictions in the housing market. Finally, the sample used here for estimation covers a longer time period and draws on data for a wider range of residential property types.

In the baseline estimation, the pre-policy period is January 2006–January 2008 and the post-policy period is February 2008–February 2012.[12] The geography of the sample used for estimation is depicted in Figure A.1. Housing-market outcomes are regressed on dummies for communities in the City of Toronto and times in the post-policy period and their interaction, along with controls. The coefficient of the interaction term captures the possible LTT effects because the tax change is implemented only in the City of Toronto from February 2008.

To ensure the housing stock and neighbourhoods in the sample are relatively homogeneous, the controls include a rich array of time-varying house characteristics, and the sample is restricted to properties in close proximity to each other but on the opposite sides of the city border — the geographic line determining whether the new LTT is applicable. Importantly, the possibility that housing-market outcome variables make a discrete jump at the border while neighbourhoods continue to change in a smooth manner allows the relationship between the LTT and housing-market outcomes to be isolated.

One legitimate concern is that households may have anticipated the introduction of the new LTT and rushed to make transactions before the cost of buying a property increased. As discussed extensively in Dachis, Duranton and Turner (2012), such anticipation of the 2008 LTT in the Toronto market was quite limited, and would have occurred within three months before and after the policy change. In light of this, for all specifications, indicators for transactions in the six-month period from November 2007 to April 2008 are included to condition out any run-up in transactions right before the policy change and possible continuation right after it.[13]

2.2.1 Effects across ownership and rental markets

Consider first the novel estimates of the LTT effects across the ownership and rental markets. The outcomes here are the number of leases relative to sales and the price-to-rent ratio, and sales separated into buy-to-own (BTO) and buy-to-rent (BTR) transactions. Regressions of housing-market outcomes include an indicator for the post-LTT period and an indicator for being in the City of Toronto. The interaction between these two indicators is the key variable of interest that picks up the impact of the LTT. To isolate causal effects, regressions include a rich set of fixed effects: city fixed effects, year fixed effects, month fixed effects, community fixed effects, and their interactions. This flexibly controls for the differential evolution of housing-market outcomes across different neigh-

[12] As a robustness check, sample periods covering 2006–2010 or 2006–2018 instead of 2006–2012 are considered.

[13] This strategy for addressing possible anticipation effects is also consistent with Bérard and Trannoy (2018) and Benjamin, Coulson and Yang (1993), both of whom explicitly estimate anticipation effects associated with a real-estate transactions tax. Using French data, the former find that the anticipation effect is limited to one month immediately before the implementation of the tax reform, while post-policy effects last for up to three months. Using data for Philadelphia, the latter find that anticipation effects are very small and limited to two months before the tax change.

bourhoods. Table 1 shows estimation results for the sample of single-family houses.[14]

Table 1: *Effects of the transaction tax across ownership and rental markets*

Dependent variable	(1)	(2)	(3)	(4)	(5)
$\log(\#Leases/\#Sales)$	0.234**	0.190	0.242***	0.236**	0.280*
	(0.117)	(0.133)	(0.082)	(0.100)	(0.146)
Observations	1355	1104	2660	1782	878
$\log(Price/Rent)$	-0.040**	-0.058***	-0.026*	-0.031*	-0.023*
	(0.020)	(0.020)	(0.015)	(0.017)	(0.013)
Observations	1355	1104	2660	1782	878
$\log(\#BTO\ sales)$	-0.098*	-0.072*	-0.100**	-0.068*	-0.155**
	(0.057)	(0.042)	(0.044)	(0.038)	(0.072)
Observations	3748	2839	6420	3868	2552
$\log(\#BTR\ sales)$	0.113*	0.102	0.105**	0.115**	0.093
	(0.062)	(0.067)	(0.045)	(0.053)	(0.087)
Observations	563	448	1055	678	376
$\log(Price)$	-0.017***	-0.010*	-0.011**	-0.012**	-0.022***
	(0.005)	(0.006)	(0.005)	(0.005)	(0.006)
Observations	14702	10088	24970	14809	10161
Distance threshold	3km	3km	5km	5km	2km
City indicators ±3 m.	Yes	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	Yes	
Donut hole		1km		2km	

Notes: Data comprise single-family-house transactions from January 2006 to February 2012. For the leases-to-sales and price-to-rent ratios, and BTO and BTR transactions, a unit of observation is a market segment defined by *community* × *year* × *month*. For prices, a unit of observation is an individual transaction record from MLS. Repeat sales transactions taking place within 18 months of one another are discarded. Each cell of the table represents a separate regression of an outcome (specified in the left column) on the LTT interaction dummy. All regressions include a dummy for the post-LTT period, City of Toronto fixed effects, year fixed effects, calendar-month fixed effects, community fixed effects, and their interactions. For the transaction-level price data, regressions include a rich set of time-varying house characteristics. In the specifications above, the distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators ±3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trend denotes the inclusion of an interaction term between exposure to the new LTT and a dummy variable for properties between 2.5km and 5km away from the city border. Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Column (1), the baseline specification, restricts the sample to 3km on each side of the border. It allows for anticipation effects by including indicators for transactions three months before and after

[14]Multiple Listing Service data cover almost the entire universe of single-family-house transactions, but this is not always the case for condominiums. To mitigate the bias associated with possible selection in the condominium sample, the sample is restricted to single-family-houses for the regressions in Table 1.

the introduction of the LTT. It further allows for the presence of spatially differentiated time trends on either side of the city border. The ratio of leases to sales is a measure of relative activity in the rental and ownership markets, and the price-rent ratio is a measure of relative cost. The results show that the LTT increase boosts activity in the rental market compared to the ownership market, and raises the rental yield (the inverse of the price-rent ratio). The 1.3 percentage-point increase in the effective LTT rate causes a 23% increase in the number of leases relative to sales, and a 4% drop in the price-to-rent ratio.

Given this relative increase in leasing activity, it is natural to explore the breakdown of sales into buy-to-own and buy-to-rent transactions. Interestingly, the new LTT has opposite effects on BTO and BTR transactions, in spite of the tax rate being common to both. BTO transactions fall by 10%, while BTR transactions rise by 11%. Given the dominance of BTO transactions overall in the housing market, total sales volume drops in response to the LTT, which is a robust finding in the literature.[15] The results in Table 1 show that this aggregate effect masks important differences in how homeowners and investors respond to transaction taxes.

There are three potential concerns with the finding of different LTT effects on BTO and BTR transactions. First, investors and home-buyers may be treated differently in the mortgage market, or with respect to capital-gains taxation. However, these factors have been conditioned out by the differences-in-differences approach because there is no evidence these treatments differ either across Toronto and suburban real-estate markets, or before and after the LTT was introduced. A second concern is the partial exemptions from LTT given to first-time buyers. Compared to buy-to-rent investors, home-buyers are more likely to be first-time buyers, and hence would benefit more from these partial exemptions. However, this argument points towards the LTT having a more negative effect on BTR transactions than BTO transactions. As this is the opposite of the empirical finding, the direction of the estimated differential LTT effects is robust. Finally, there may be a concern that the results are sensitive to the number of months between purchasing and leasing a property used to distinguish between BTO and BTR transactions. Table A.3 shows that the results are robust to changing the 18-months threshold to 6, 12, or 24 months. Using the 6- and 12-month windows helps increase the number of observations in market segments, and the resulting estimates are stronger and more significant in some cases.

The estimation of price effects uses data on individual transactions of single-family-houses controlling for a rich set of time-varying house characteristics. The 1.3 percentage-point increase in the LTT causes a 1.7% decline in the average sales price. This is consistent with estimation done at the market-segment level for both shorter and longer post-policy periods reported in Table A.4.

A potential sorting bias is that some buyers may switch from purchases inside to outside the

[15] Using UK property transaction data, Best and Kleven (2018) find that a temporary 1 percentage-point cut in the transaction tax rate — the 2008–9 stamp-duty holiday on properties worth between £125,001 and £175,000 — led to a 20% increase in transactions. Using German single-family-house sales, Fritzsche and Vandrei (2019) find that a one-percentage-point increase in tax leads to about 7% fewer transactions. Moreover, Dachis, Duranton and Turner (2012) show that the same LTT increase studied here caused a 15% decline in the sales volume using postal-code level data.

border in response to the LTT, boosting property sales outside the border. This would violate the assumption that transactions outside the city border are unaffected by the tax change. Such substitution, if it occurs, would most likely happen immediately adjacent to the border. To mitigate this concern, column (2) of Table 1 applies a 'donut approach,' repeating the estimation in column (1) with a distance threshold of 3km, but also excluding properties within 1km of each side of the border.

Column (3) replicates the baseline regression of column (1) but extends the sample to consider all properties sold within 5km of the city border instead of 3km. It also adds an interaction term between exposure to the new LTT and a dummy variable for properties between 2.5km and 5km away from the city border. This allows homeowners to react to the LTT differently depending on their distance from downtown. The unreported coefficient for this additional term is statistically insignificant. Column (4) repeats the estimation in column (3), but excludes properties within 2km of each side of the border. This addresses possible substitution across the border for a wider set of neighborhoods. Column (5) replicates our preferred specification from column (1) but restricts our sample to all properties sold within two kilometres of the Toronto border instead of three. Most of the estimates are robust to these alternative specifications.

2.2.2 Effects on mobility and time-on-the-market

Now consider the effects of transaction taxes on flows within the ownership market related to individual homeowners' mobility and individual properties' time-on-the-market. Unlike many previous studies that use transactions volume to measure mobility, here it is observed precisely when an individual homeowner puts a property up for sale and when a transaction occurs.

The dynamic pattern of mobility is represented by the moving hazard function, the relationship between the rate at which moving occurs and the length of time since a homeowner purchased a property. The hazard function is estimated using the Kaplan-Meier (KM) method. The KM estimator computes the conditional probability of putting a property up for sale given the time since the homeowner moved in. Specifically, a unit of observation is each month since a homeowner has bought a property and the event is putting the property up for sale given that this has not occurred so far. The estimated hazard function is shown in Figure A.2. The mean length of time between purchasing a property and moving is 113 months.

Since the hypothesis of homogeneity of hazard rates over time is not rejected at the 1% level and the estimated hazard function shape is monotonic, the hazard function can be analysed using a Weibull model. The hazard function for homeowner j in a given year-month t is parameterized as:

$$\hbar\left(t \,|\, \mathbf{x}_{jt}, \mathrm{LTT}_{jt}\right) = \varphi t^{\varphi-1} \exp\left(\beta_0 + \mathbf{x}'_{jt}\beta_{\mathbf{x}} + \mathrm{LTT}_{jt}\beta\right),$$

where t is time since the homeowner purchased the property, φ is a parameter linked to the gradient of the hazard function, LTT_{jt} is an indicator for the exposure to the LTT, and $\mathbf{x}_{jt}$ is a rich set of time-varying controls. The causal impact of the LTT on homeowners' mobility is isolated by including time-varying house attributes, all interacted with property-type fixed effects. A broad range of fixed effects is also included: *city* × *property type*, *year* × *property type*, *month* × *property type*, and *community* × *property type* fixed effects. This flexibly controls for the differential evolution of housing-market outcomes across different property types in different communities.

The controls also include the property price originally paid by the owner. This is typically considered as a proxy for the burden of transaction tax in the existing literature on residential mobility. However, such a proxy is imperfect as non-tax-related moving costs are also positively related to a property's value in both monetary and psychological terms (Hardman and Ioannides, 1995, Han, 2008), hence households who occupy a property of higher value are less likely to move, even in the absence of transaction taxes. Here, the LTT effect on residential mobility can be separated from other transaction costs by controlling for the original purchase price.

The estimation results are presented in Table 2. As shown in column (1), the baseline specification, the LTT reduces the moving hazard rate by 13%. The estimated value of $\log\varphi$ is greater than zero, indicating a moving hazard that increases with time spent living in a property. In the remaining columns (2)–(5), different specifications allow for spatially differentiated time trends, controls for substitution across borders, and changes to the city border distance thresholds are considered. In all of these, the estimated LTT effect on the moving hazard remains consistent and robust in terms of both economic and statistical significance.

A 13% reduction in the moving rate with the mean length stay in a property being 113 months implies the new LTT causes homeowners to postpone selling their properties by 14 months on average. This represents a substantial lock-in effect, which is consistent with evidence from other countries.[16] The moving hazard estimation can be repeated for the alternative sample periods 2006-2010 and 2006-2018. As shown in Table A.5, the estimated LTT effect remains robust to shorter and longer post-policy periods. This implies that the lock-in effect on residential mobility of transaction taxes is not only substantial but also long lasting.

The expectation that households will move less frequently on average owing to the LTT might conceivably make them pickier when searching for a property to purchase. With search frictions, this would reduce the speed at which properties can be sold and lower outflows from the ownership market, in addition to the effect on inflows due to less frequent moving. Interestingly, the effect of transaction taxes on the time taken for sales to be completed has not been examined in the literature. To shed light on this, transaction-level sales data are used to estimate the causal effect of the LTT on time-on-the-market.

For a given transaction, time-on-the-market is measured as the number of days between the time

[16] For example, using data from the Netherlands, Van Ommeren and Van Leuvensteijn (2005) find that a 1 percentage-point increase in transaction costs as a percentage of property price decreases residential mobility rates by 8.1–12.7%. Using UK data, Hilber and Lyytikäinen (2017) find that a 2 percentage-point increase in stamp duty reduces the annual rate of mobility by 2.6 percentage points. While their analysis relies on a threshold for house prices as a proxy for the transaction tax, the results here for Toronto benefit from a natural experiment that generates two discrete changes in the housing market, one at the border of the city of Toronto and the other on the date the LTT was imposed.

Table 2: *Effects of the transaction tax on mobility and time-on-the-market*

	(1)	(2)	(3)	(4)	(5)
			Dependent variable: Moving hazard rate		
LTT	-0.130**	-0.188***	-0.194***	-0.232***	-0.151***
	(0.066)	(0.080)	(0.054)	(0.087)	(0.051)
log(*Purchase price*)	-0.095**	-0.170**	-0.073**	-0.076**	-0.103**
	(0.046)	(0.059)	(0.035)	(0.034)	(0.044)
$\log\varphi$	0.513***	0.501***	0.523***	0.519***	0.530***
	(0.010)	(0.012)	(0.007)	(0.010)	(0.012)
Observations	1,691,369	1,142,052	2,831,897	1,651,935	1,179,962
			Dependent variable: log (*Time-on-the-market*)		
LTT	0.230***	0.234***	0.209***	0.150***	0.252***
	(0.043)	(0.051)	(0.037)	(0.058)	(0.058)
Observations	29,100	20,269	51,529	33,937	16,584
Distance threshold	3km	3km	5km	5km	2km
House characteristics	Yes	Yes	Yes	Yes	Yes
City indicators ± 3 m.	Yes	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	Yes	
Donut hole		1km		2km	

Notes: For the moving hazard estimation, a unit of observation is a household whose property is listed on MLS between January 2006 to February 2012. Repeat sales transactions taking place within 18 months of one another are discarded. Households' times between moves are assumed to follow a Weibull distribution. For time-on-the-market, a unit of observation is a transaction recorded on MLS during the same period. All regressions include an indicator for the post-LTT period, an indicator for the city of Toronto, property-type fixed effects interacted with a set of time-varying house characteristics, and *city* $\times$ *property type*, *month* $\times$ *property type*, and *community* $\times$ *property type* fixed effects. Distance threshold is the maximum distance to the Toronto city border for a transaction to be included. City indicators ± 3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trend denotes the inclusion of an interaction term between exposure to the new LTT and a dummy variable for properties between 2.5km and 5km away from the city border. Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

when the property was initially listed and the time when a sale is agreed between buyers and sellers. Using the same regression discontinuity design laid out earlier, time-on-the-market is regressed on the LTT indicator and a rich set of controls. The results are reported in Table 2 for the same range of specifications as before. The baseline column (1) indicates that the new LTT is associated with a 23% increase in time-on-the-market, which means a week longer based on the pre-policy sample mean. This effect is robust across alternative specifications in the other columns. These findings provide the first evidence on how transaction taxes affect time-on-the-market.

Summary The findings in Table 1 and 2 show that transaction taxes have important effects on the rental market as well as the market for ownership of property. Across rental and ownership markets, transactions taxes cause some households who would have owned a property to remain in the rental market, so the properties they would have been bought are purchased by investors instead. Within the ownership market, some transactions that would have occurred are postponed or never happen, so households who would have been better matched with a different property remain stuck in a less suitable one. Finally, those transactions that do occur take longer to complete.

3 A dual rental and ownership markets model of housing

This paper presents a model to explain the empirical findings of section 2 and quantify the welfare costs of transaction taxes. The model includes both a rental market and an ownership market to capture the cross-market effects of the LTT, idiosyncratic household-property match quality to understand the effect of the LTT on mobility, and search frictions to capture the effect of the LTT on time-on-the-market.

There is a city with two housing markets: an ownership market and a rental market. There is a unit measure of ex-ante identical properties and a constant measure ψ of households. Time is continuous, and everyone discounts future payoffs at rate r. Households exit the city exogenously at rate ρ, who are replaced by an equal inflow of new households. There is free entry of investors, who become landlords and rent out properties. Investors simply represent funds invested in housing and could be living within the city or from elsewhere.

Properties are either up for sale, offered for rent, or not available in either market. They are owned either by those who live in them or by landlords. When not for sale or rent, properties are occupied by a renter or an owner-occupier. Some owners or renters are looking to move, and they choose whether to search in the ownership or rental market. Owner-occupiers looking to move put their property up for sale. Landlords choose whether to let or to sell the properties they own. At rate ρ_l, landlords receive a shock forcing them to sell their property, for example, for liquidity reasons.

The measure of buyers in the ownership market is b_o, comprising home-buyers b_h who will live in the property they buy, and investors b_k. The fraction of investors among buyers is denoted by ξ. Those looking to rent are b_l. On the other side of the two markets, properties available for sale are u_o and properties available for rent are u_l. The tightness of market i — the ratio of 'buyers' to 'sellers' — is denoted by θ_i, where $i \in \{o, l\}$ indexes the ownership (o) or rental (l) market:

$$\xi = \frac{b_k}{b_o} \quad \text{and} \quad \theta_i = \frac{b_i}{u_i}, \quad \text{where} \quad b_o = b_h + b_k. \tag{1}$$

Search frictions place limits on meetings between participants in both markets. Meetings are viewings of properties that allow for offers to buy or to rent. Meeting rates are determined by constant-returns-to-scale meeting functions $\Upsilon^i(b_i, u_i)$. The rate $\Upsilon^i(b_i, u_i)/b_i$ at which buyers/renters view properties in market i is denoted by q_i. Constant returns to scale makes q_i a function of tightness θ_i:

$$q_i = \frac{\Upsilon^i(b_i, u_i)}{b_i} = \Upsilon^i\left(1, \theta_i^{-1}\right), \quad \text{and} \quad \frac{\Upsilon^i(b_i, u_i)}{u_i} = \Upsilon^i(\theta_i, 1) = \theta_i q_i \quad \text{for} \quad i \in \{o, l\}. \tag{2}$$

The meeting rate $\Upsilon^i(b_i, u_i)/u_i$ for sellers in market i is $\theta_i q_i$. The meeting function is increasing in both b_i and u_i, hence q_i decreases with θ_i, while $\theta_i q_i$ increases with θ_i. Intuitively, if there are more 'buyers' relative to 'sellers' in a particular market, the meeting rate is lower for those viewing properties but higher for those offering properties for sale or to let.

Owner-occupiers or renters living in a property receive a match-specific flow value ε. At the time of a meeting when a household views a property, match quality ε between the property and the household is drawn from distribution function $G_i(\varepsilon)$ for market i. The distribution of ε could differ across markets, for instance, allowing for a 'warm glow' effect of home-ownership where flow values are higher on average. From the perspective of an investor owning a property, all properties are ex-ante identical prior to being viewed by potential tenants or buyers.

Idiosyncratic match quality ε for those living in a property is a persistent variable subject to occasional shocks. These shocks represent life events that make a property less well matched to the occupying household than it originally was. Shocks arrive independently across households and across time at rate a_i, which can differ by housing tenure $i \in \{o, l\}$. For owner-occupiers, the arrival of a shock reduces match quality from ε to $\delta_o \varepsilon$, where $\delta_o < 1$ is a parameter.[17] For renters, match quality ε is reduced to 0 following a shock — effectively $\delta_l = 0$.

Following a shock, owner-occupiers and renters decide whether to move and start searching for another property to live in, with owners putting their current property up for sale. Moving is endogenous and depends on how low match quality has become relative to expectations of match quality in a alternative property, though for renters, moving depends only on the arrival of a shock.[18]

Those who decide to move choose to buy or rent their next property by searching in the ownership market or the rental market. Households pay an idiosyncratic cost χ when they enter the ownership market for the first time. This can be thought of as household-specific factors affecting the cost or availability of a mortgage, such as credit histories or wealth for downpayments. χ is independently drawn from a distribution $G_m(\chi)$ when a household arrives in the city and decides to buy or rent.

A household's credit cost persists over time, but while the household is in the rental market, χ is redrawn with probability γ from the same probability distribution $G_m(\chi)$ if the arrival of an exogenous shock causes the household to move — either shocks to match quality or the landlord selling owing to an exit shock. Households exiting the city sell properties they own. When tenants choose to move or exit the city, their landlords decide whether to look for a new tenant or to sell.

[17] The model has no shocks that increase match quality, but these would not cause households to consider moving.

[18] It is possible to extend the model to have $\delta_l > 0$. However, it turns out that the endogeneity of moving by renters within the rental market is quantitatively unimportant here, so the model is simplified by assuming $\delta_l = 0$.

3.1 The ownership market

Buyers in the ownership market are either home-buyers or investors. The expected value of owning a property is the same for all investors because they face the same expected rents when their property is let, while home-buyers put different values on properties because of idiosyncratic match quality.

After a buyer has met a seller and viewed a property, revealing the quality of the match to home-buyers, the buyer and seller negotiate a price and a transaction occurs if mutually agreeable. The land transfer tax (LTT) is represented by proportional taxes levied on the transaction price paid by the buyer. Home-buyers and investors face tax rates τ_h and τ_k, which in principle can differ.

The Bellman equation for the value K of being an investor who buys at price P_k is

$$rK = -F_k + q_o\left(U_l - (1+\tau_k)P_k - C_k - K\right) + \dot{K}, \tag{3}$$

where $\dot{K}$ is the derivative of the value K with respect to time t (the dependence of variables on time t is not indicated explicitly). There is a flow search cost F_k incurred by investors until they buy a property, $\tau_k P_k$ is the tax paid on the purchase, and C_k is any other transaction costs paid by investors. Investors meet sellers at rate q_o, and because investors have no idiosyncratic match quality with a property, this is also the rate at which they are able to buy. After buying, investors make their properties available for rent and receive the common expected value U_l of being a landlord.

The Bellman equation for the value B_o of being a home-buyer is

$$rB_o = -F_h + q_o \int \max\left\{H(\varepsilon) - C_h - (1+\tau_h)P(\varepsilon) - B_o, 0\right\} \mathrm{d}G_o(\varepsilon) - \rho B_o + \dot{B}_o. \tag{4}$$

Buyers make viewings of properties at rate q_o, which reveal match quality ε drawn from a distribution $G_o(\varepsilon)$. The value of being an owner-occupier of a property where match quality is currently ε is $H(\varepsilon)$. After meeting a seller, the home-buyer negotiates a price $P(\varepsilon)$ if a deal is mutually beneficial and moves into the property. This occurs when match quality ε is sufficiently high. Home-buyers incur a flow search cost F_h while looking for properties. If a transaction goes ahead, $\tau_h P(\varepsilon)$ is the tax paid by the home-buyer, and C_h is other transaction costs such as moving costs. Home-buyers, like any other household, exogenously exit the city at rate ρ.

Since properties are ex ante identical, both owner-occupiers and landlords selling their properties have a common expected value U_o, which satisfies the Bellman equation

$$rU_o = -M + \theta_o q_o \Bigg((1-\xi) \int \max\left\{P(\varepsilon) - C_u - U_o, 0\right\} \mathrm{d}G_o(\varepsilon) + \xi \max\left\{P_k - C_u - U_o, 0\right\} \Bigg) + \dot{U}_o, \tag{5}$$

where M is the flow cost of maintaining a property paid by all owners and C_u is a transaction cost paid by sellers. Viewings by buyers occur at rate $\theta_o q_o$, and the probabilities the meeting is with a home-buyer or an investor are the respective fractions $1-\xi$ and ξ of the pool of buyers made up of these two groups. The owner decides whether to sell, receiving price P_k if selling to an investor and $P(\varepsilon)$ if selling to a home-buyer with match quality ε.

The Bellman equation for the value $H(\varepsilon)$ of an owner-occupier with current match quality ε is

$$rH(\varepsilon) = \varepsilon - M + a_o\left(\max\left\{H(\delta_o\varepsilon), B_o + U_o\right\} - H(\varepsilon)\right) + \rho\left(U_o - H(\varepsilon)\right) + \dot{H}(\varepsilon)\,, \tag{6}$$

where ε is the flow utility derived from occupying a property when match quality is currently ε. Idiosyncratic shocks arrive at rate a_o, reducing match quality to $\delta_o\varepsilon$. The household then decides whether to remain in the property and receive value $H(\delta_o\varepsilon)$, or to move out and become both a seller and a home-buyer, which has a combined value $B_o + U_o$.[19] Moving occurs if match quality $\delta_o\varepsilon$ after a shock has become sufficiently low.

3.2 The rental market

Participants on both sides of the rental market — potential tenants and landlords — are ex ante identical. When a household meets a landlord and views a property, match quality ε is drawn from distribution $G_l(\varepsilon)$. If mutually agreeable, the household moves in and becomes a tenant. There is no commitment and no long-term contract: either the tenant or the landlord can end the relationship at any subsequent time. Rents are determined by ongoing negotiations between the two parties.

The Bellman equation for the value U_l of a landlord having a property available to let is

$$rU_l = -M + \theta_l q_l \int \max\{L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l, 0\} \mathrm{d}G_l(\varepsilon) + \rho_l(U_o - U_l) + \dot{U}_l\,. \tag{7}$$

The landlord meets households who are potential tenants at rate $\theta_l q_l$. A household moves into the landlord's property and becomes a tenant if this is mutually agreeable. If a tenant with match quality ε moves in, the landlord incurs costs C_l and receives value $L(\varepsilon)$, which includes the ongoing rents that are negotiated. At the point of agreeing the tenant can move in, there is also negotiation over an initial one-off fee $\Pi(\varepsilon)$ paid by the tenant to the landlord. At any time an exogenous shock with arrival rate ρ_l forces landlords to exit, and those landlords who must sell receive value U_o.

The value of a landlord whose property is currently occupied by a tenant with match quality ε is $L(\varepsilon)$. The Bellman equation for this value function is

$$rL(\varepsilon) = R(\varepsilon) - M - M_l + (a_l + \rho)\left(\max\{U_l, U_o\} - L(\varepsilon)\right) + \rho_l(U_o - L(\varepsilon)) + \dot{L}(\varepsilon)\,, \tag{8}$$

where $R(\varepsilon)$ is the rent negotiated between landlord and tenant, and M_l is an extra maintenance cost incurred by landlords when properties are let. Idiosyncratic shocks received by tenants cause them to move out of rental properties at rate $a_l + \rho$, either because match quality is reduced to zero or because the household must leave the city. After a tenant moves out, the landlord decides whether to look for another tenant or sell the property, thus receiving the maximum of U_l and U_o.

[19] Homeowners cannot become landlords after deciding to move, implicitly because there is a sufficiently large credit cost of having two mortgages to retain ownership of an existing property as well as buying a new one.

The value B_l of a household searching for a property to rent satisfies the Bellman equation

$$rB_l = -F_w + q_l \int \max\{W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l, 0\}\, \mathrm{d}G_l(\varepsilon) - \rho B_l + \dot{B}_l\,, \tag{9}$$

where q_l is the rate at which viewings are made, and F_w is the flow cost of searching for a rental property. Viewings reveal match quality ε drawn from a distribution $G_l(\varepsilon)$, and the household becomes a tenant if ε is sufficiently high. If the household moves into a property with match quality ε as a tenant then value $W(\varepsilon)$ is received after paying the initial fee $\Pi(\varepsilon)$ to the landlord and incurring other moving costs C_w. The Bellman equation for the value function $W(\varepsilon)$ is

$$\begin{aligned} rW(\varepsilon) = \varepsilon - R(\varepsilon) + \gamma(a_l + \rho_l)\left(G_m(Z)(B_o - \bar{\chi}) + (1 - G_m(Z))B_l - W(\varepsilon)\right) \\ + (1-\gamma)(a_l + \rho_l)(B_l - W(\varepsilon)) - \rho W(\varepsilon) + \dot{W}(\varepsilon)\,, \quad \text{with} \quad \bar{\chi} = E[\chi | \chi \leq Z]\,. \end{aligned} \tag{10}$$

The flow utility ε derived from occupying a rental property is equal to that of an owner-occupied property with the same match quality ε, but the tenant pays rent $R(\varepsilon)$. Rent negotiations ensure landlords and tenants are willing to remain matched until a shock makes it mutually agreeable to terminate the tenancy. Households receive exit shocks at rate ρ. Exit shocks for landlords with arrival rate ρ_l, or shocks that reduce tenants' match quality to zero with arrival rate a_l, also bring a tenancy to an end. When moving within the city, a household keeps the same credit cost χ with probability $1-\gamma$, in which case a tenant goes back to the rental market and obtains value B_l.

When a new credit cost χ is drawn, either for tenants who move (with probability γ) or for new entrants to the city, there is threshold Z for χ below which it is optimal to enter the ownership market and buy a property rather than rent. Doing this has value B_o after paying the cost χ.[20] If the cost is too high, a household goes to the rental market. The expected value of a household prior to the realization of χ is an average of $B_o - \bar{\chi}$ and B_l using the probabilities $G_m(Z)$ and $1 - G_m(Z)$ as weights, where $\bar{\chi}$ denotes the expectation of the credit cost χ conditional on actually paying it.

3.3 Stocks and flows across and within the two markets

A property is in any one of four states: for sale (measure u_o), to let (measure u_l), or occupied by an owner or a renter ('occupying' in the sense that the property is currently neither available in the market for sale or rent). Owner-occupied and renter-occupied properties have measures h_o and h_l, respectively. These measures of the four states must sum to the unit measure of all properties:

$$h_o + h_l + u_o + u_l = 1\,. \tag{11}$$

Similarly, the total measure ψ of households is distributed over four possible states: home-buyers (b_h), those looking for a property to rent (b_l), owner-occupiers (h_o), and tenants (h_l). Although

[20] The credit cost χ is modelled as a one-off cost, but that is equivalent in this environment to the present value of a flow credit cost paid for a period of time while a household is an owner-occupier.

households can own multiple properties, a household occupies at most one property at a time, and households look either to buy or rent if and only if they do not currently occupy a property. Hence:

$$h_o + h_l + b_h + b_l = \psi\,. \tag{12}$$

The measure of buyers $b_o = b_h + b_k$ in the ownership market also includes a measure b_k of those looking to buy as investors. Given free entry of investors, b_k adjusts so that at all points in time the value of entry by further investors is zero:

$$K = 0\,. \tag{13}$$

Entry of first-time home-buyers to the ownership market depends on the threshold Z for the credit cost χ. The marginal new entrant ($\chi = Z$) is indifferent between the ownership and rental markets:

$$B_o - Z = B_l\,. \tag{14}$$

Credit costs are drawn from the distribution $G_m(\chi)$ by a fraction γ of tenants who move within the city because of shocks (either to their own match quality or landlord exit) and all households new to the city. If N_l denotes the flow of tenants who decide to move ($n_l = N_l/h_l$ is the moving rate for tenants h_l), γN_l redraw their credit cost χ. Of those, a fraction $G_m(Z)$ are below the threshold Z and so enter the ownership market as home-buyers. The same applies to the measure $\rho\psi$ of households who enter the city. The flow of first-time home-buyers is therefore $F = (\gamma n_l h_l + \rho\psi)G_m(Z)$.

Those tenants not drawing a new credit cost when moving (probability $1-\gamma$), or those whose new credit cost is above Z (probability $1 - G_m(Z)$), search in the rental market for a new property. The flow of owner-occupiers who decide to move is N_o (the moving rate of those in h_o is $n_o = N_o/h_o$), and all enter the ownership market as home-buyers because they have already paid the credit cost.

Home-buyers b_h and households h_l searching for a rental property exit from this state by either completing a transaction or exiting the city. Viewings are made at rates q_i in the two markets $i \in \{o, l\}$. Suppose the probabilities that the match quality revealed by a viewing is sufficiently high for a mutually agreeable deal with the seller/landlord are π_o and π_l in the two markets. The flows of sales S_h to home-buyers and leases S_l agreed with tenants are

$$S_h = q_o \pi_o b_h \quad \text{and} \quad S_l = q_l \pi_l b_l\,. \tag{15}$$

The laws of motion for the stocks of home-buyers b_h and households looking to rent b_l are thus

$$\dot{b}_h = n_o h_o + (\gamma n_l h_l + \rho\psi)G_m(Z) - (q_o\pi_o + \rho)b_h\,, \quad \text{and} \tag{16}$$

$$\dot{b}_l = (1-\gamma)n_l h_l + (\gamma n_l h_l + \rho\psi)(1 - G_m(Z)) - (q_l\pi_l + \rho)b_l\,. \tag{17}$$

Investors b_k make viewings at rate q_o and are able to transact at this rate because they have no idiosyncratic match quality with properties. The flow of sales to investors is S_k, which added to S_h

gives total transactions S_o in the ownership market. Let κ denote the fraction of sales to investors:

$$S_o = S_h + S_k\,, \quad \text{where} \quad S_k = q_o b_k\,, \quad \text{and} \quad \kappa = \frac{S_k}{S_o} = \frac{\xi}{\xi + (1-\xi)\pi_o}\,, \tag{18}$$

where the equation for κ in terms of the fraction of investors ξ follows from (1) and (15).

From the perspectives of sellers and landlords, the transaction rates in the two markets are

$$s_o = \frac{S_o}{u_o} = \theta_o q_o \left(\xi + (1-\xi)\pi_o\right)\,, \quad \text{and} \quad s_l = \frac{S_l}{u_l} = \theta_l q_l \pi_l\,, \tag{19}$$

and hence the laws of motion for properties for sale u_o and to let u_l are:

$$\dot{u}_o = (n_o + \rho)h_o + \rho_l(h_l + u_l) - s_o u_o\,, \quad \text{and} \tag{20}$$

$$\dot{u}_l = (a_l + \rho)h_l + \kappa s_o u_o - (s_l + \rho_l)u_l\,. \tag{21}$$

Properties come up for sale if owner-occupiers move within or exit the city, or landlords are hit by an exit shock (irrespective of whether their properties are currently occupied by tenants). Properties are offered to let if tenants are hit by a match quality shock or exit the city, or investor purchases make new rental properties available. Properties come off these markets with successful transactions, or in the case of the rental market, if landlords receive an exit shock.

Finally, flows of properties onto and off the two markets imply the following laws of motion for the stocks of owner-occupiers h_o and tenants h_l:

$$\dot{h}_o = (1-\kappa)s_o u_o - (n_o + \rho)h_o\,, \quad \text{and} \tag{22}$$

$$\dot{h}_l = s_l u_l - (n_l + \rho)h_l\,. \tag{23}$$

The flows and stocks in the ownership and rental markets are summarized in Figure A.3 and A.4.

3.4 Functional forms, parameter restrictions, and bargaining protocols

The meeting functions $\Upsilon^i(b_i, u_i)$ for $i \in \{o, l\}$ have Cobb-Douglas functional forms:

$$\Upsilon^i(b_i, u_i) = A_i b_i^{1-\eta_i} u_i^{\eta_i}\,, \quad \text{hence} \quad q_i = A_i \theta_i^{-\eta_i}\,, \tag{24}$$

where A_i is productivity in arranging viewings in market i, and η_i are the elasticities of buyers' and renters' viewing rates with respect to the market tightnesses θ_i (see 1 and 2). These parameters can differ across markets. New match qualities ε are drawn from Pareto distributions for $i \in \{o, l\}$:

$$G_i(\varepsilon) = 1 - \left(\frac{\varepsilon}{\zeta_i}\right)^{-\lambda_i}\,, \tag{25}$$

with ζ_i being the minimum possible draw in market i, and λ_i specifying the distribution shape, in particular, how compressed are realizations of ε towards the minimum. Expected match quality from a viewing in market i is $E_i[\varepsilon] = \zeta_i\lambda_i/(\lambda_i - 1)$ for $\lambda_i > 1$. Draws of the homeownership credit cost χ

are from a log Normal distribution with mean and standard deviation parameters μ and σ:

$$G_m(\chi) = \Phi\left(\frac{\log\chi - \mu}{\sigma}\right), \quad \text{implying} \quad \bar{\chi} = e^{\mu+\frac{\sigma^2}{2}} \frac{\Phi\left(\frac{\log Z - \mu - \sigma^2}{\sigma}\right)}{\Phi\left(\frac{\log Z - \mu}{\sigma}\right)}, \tag{26}$$

where $\Phi(\cdot)$ is the standard Normal CDF, and $\bar{\chi}$ is the expectation of χ conditional on $\chi \leq Z$.

A parameter restriction is imposed so that match-quality shocks in the ownership market are sufficiently large (δ_o is far enough below 1) that some, but not all, owner-occupiers require only one idiosyncratic shock to trigger moving. As has been stated earlier, match-quality shocks to renters are sufficiently large ($\delta_l = 0$) that all tenant moves are exogenous.

The bargaining protocol over the terms of transactions (prices and rents) in all meetings between agents is Nash bargaining. Sellers (whoever they may be) have bargaining power ω_o when selling to a home-buyer, and bargaining power ω_k when selling to an investor. Landlords have bargaining power ω_l in relation to tenants in both their initial meeting and in any subsequent rent negotiations.

3.5 General equilibrium in rental and ownership markets

This section studies the equilibrium allocation of properties and households across the two markets, and the volumes of transactions and their terms (prices and rents) within each market.

3.5.1 Decisions made by homeowners and home-buyers

Suppose the seller of a property meets a home-buyer who draws match quality ε. If they were to agree to a sale at price $P(\varepsilon)$ then the home-buyer surplus would be $\Sigma_o^h(\varepsilon) = H(\varepsilon) - (1+\tau_h)P(\varepsilon) - C_h - B_o$ and the seller surplus $\Sigma_o^u(\varepsilon) = P(\varepsilon) - C_u - U_o$. The Nash bargaining problem is to choose $P(\varepsilon)$ to maximize $(\Sigma_o^u(\varepsilon))^{\omega_o}\left(\Sigma_o^h(\varepsilon)\right)^{1-\omega_o}$, where the surpluses of both must be non-negative for a transaction to go ahead. The first-order condition is $\Sigma_o^u(\varepsilon)/\Sigma_o^h(\varepsilon) = \omega_o/((1-\omega_o)(1+\tau_h))$, which determines how the joint surplus $\Sigma_o(\varepsilon) = \Sigma_o^h(\varepsilon) + \Sigma_o^u(\varepsilon)$ is to be shared.

In the absence of a transaction tax τ_h, the surplus would have been divided according to bargaining powers in line with the usual Nash rule. However, a positive transaction tax rate skews the division in favour of the buyer. Intuitively, owing to the proportional tax, the joint surplus $\Sigma_o(\varepsilon) = H(\varepsilon) - C_h - C_u - B_o - U_o - \tau_h P(\varepsilon)$ is increased by agreeing a lower price, and this lower price increases the buyer's surplus. The resulting split is

$$\Sigma_o^h(\varepsilon) = (1-\omega_o^*)\Sigma_o(\varepsilon) \quad \text{and} \quad \Sigma_o^u(\varepsilon) = \omega_o^*\Sigma_o(\varepsilon), \quad \text{where} \quad \omega_o^* \equiv \frac{\omega_o}{1+\tau_h(1-\omega_o)}, \tag{27}$$

and the seller's share ω_o^* of the surplus is below bargaining power ω_o. The price that delivers the division of the surplus in (27) is $P(\varepsilon) = C_u + U_o + \omega_o^*\Sigma_o(\varepsilon)$, which results in the joint surplus being

$$\Sigma_o(\varepsilon) = \frac{H(\varepsilon) - C_h - B_o - (1+\tau_h)(C_u + U_o)}{1+\tau_h\omega_o^*}. \tag{28}$$

As match quality ε is observable and surplus is transferable, transactions go ahead if $\varepsilon \geq y_o$, where y_o, the transaction threshold, is the level of match quality where the joint surplus is zero:

$$\Sigma_o(y_o) = 0\,. \tag{29}$$

Using (25), the proportion π_o of home-buyer viewings that lead to sales and the average transaction price P for home-buyer purchases are therefore

$$\pi_o = \int_{y_o} \mathrm{d}G_o(\varepsilon) = \left(\frac{y_o}{\zeta_o}\right)^{-\lambda_i}, \quad \text{and} \quad P = \frac{1}{\pi_o}\int_{y_o} P(\varepsilon)\mathrm{d}G_o(\varepsilon) = \frac{\omega_o^*\Sigma_o}{\pi_o} + C_u + U_o\,. \tag{30}$$

Prior to the realization of ε, the ex-ante joint surplus from a home-buyer viewing is denoted by:

$$\Sigma_o = \int_{y_o} \Sigma_o(\varepsilon)\mathrm{d}G_o(\varepsilon)\,. \tag{31}$$

For existing owner-occupiers, there is a moving decision to be made when a match quality shock is received. Since the value function $H(\varepsilon)$ is increasing in ε, owner-occupiers decide to move if the current level of match quality is sufficiently low. The condition for moving is $\varepsilon < x_o$, where x_o, the moving threshold, is the level of match quality such that the value of continuing to occupy a property equals the sum of the outside options B_o and U_o of being both a buyer and a seller in the ownership market:

$$H(x_o) = B_o + U_o\,. \tag{32}$$

The condition that some owner-occupiers require only one shock to trigger moving is $\delta_o y_o < x_o$.

The endogenous moving rate n_o is derived from the distribution of match quality over existing owner-occupiers together with the moving threshold x_o. The evolution over time of the distribution of owner-occupiers' match quality depends on idiosyncratic shocks and moving decisions. Surviving matches of households and properties differ along two dimensions, the initial level of match quality, and the number of shocks received since the match formed. By using the Pareto distribution (25) of new match quality, appendix A.2.2 shows that the endogenous moving rate is

$$n_o = a_o - \frac{a_o\zeta_o^{\lambda_o}\delta_o^{\lambda_o}x_o^{-\lambda_o}}{h_o}\int_{\upsilon\to-\infty}^{t} e^{-\left(\rho+a_o(1-\delta_o^{\lambda_o})\right)(t-\upsilon)}(1-\xi(\upsilon))\theta_o(\upsilon)q_o(\upsilon)u_o(\upsilon)\mathrm{d}\upsilon\,, \tag{33}$$

where $u_o(t)$ explicitly indicates the dependence of u_o on time t. Given the moving threshold x_o, the moving rate n_o displays history dependence due to persistence in the match-quality distribution.

3.5.2 Decisions made by landlords and tenants

For landlords and tenants, it is necessary to work backwards from ongoing rent negotiations to analyse their behaviour when they first meet during a viewing. Consider a tenant who has already moved into a property with match quality ε, so any transaction and moving costs are sunk. The tenant's surplus from remaining in the property is $\Lambda^w(\varepsilon) = W(\varepsilon) - B_l$, where the outside option is going back to the rental market because the tenant's cost χ of becoming a home-buyer does not change unless a shock occurs. The landlord's surplus from keeping the tenant is $\Lambda^l(\varepsilon) = L(\varepsilon) - U_l$, which assumes the outside option of putting the property back on the rental market is better than selling it ($U_l \geq U_o$), as will be confirmed. Both $W(\varepsilon)$ and $L(\varepsilon)$ depend on the rent $R(\varepsilon)$ paid.

The Nash bargaining problem has rent $R(\varepsilon)$ maximize $\left(\Lambda^l(\varepsilon)\right)^{\omega_l} \left(\Lambda^w(\varepsilon)\right)^{1-\omega_l}$, where ω_l is the landlord's bargaining power. There is no commitment to rent payments at any future date. The rent $R(\varepsilon)$ affects the surpluses through $L(\varepsilon)$ and $W(\varepsilon)$ in equations (8) and (10), noting that $\partial L(\varepsilon)/\partial R(\varepsilon) = -\partial W(\varepsilon)/\partial R(\varepsilon)$, so the first-order condition is $\Lambda^l(\varepsilon)/\Lambda^w(\varepsilon) = \omega_l/(1-\omega_l)$. The joint surplus $\Lambda(\varepsilon) = \Lambda^l(\varepsilon) + \Lambda^w(\varepsilon) = W(\varepsilon) + L(\varepsilon) - B_l - U_l$ is therefore divided according to the bargaining powers of the two parties as $\Lambda^l(\varepsilon) = \omega_l \Lambda(\varepsilon)$ and $\Lambda^w(\varepsilon) = (1-\omega_l)\Lambda(\varepsilon)$.

With rents negotiated in this way, tenants move out only after a match quality shock or if leaving the city, or if the landlord is forced to sell up. Tenants' moving rate n_l within the city is simply

$$n_l = a_l + \rho_l \,. \tag{34}$$

Now consider a landlord meeting a potential tenant during a viewing that reveals match quality ε. If the landlord agrees the tenant can move in after paying a fee $\Pi(\varepsilon)$ then the two parties incur costs C_l and C_w, respectively.[21] Note that the fee $\Pi(\varepsilon)$ is separate from the rent $R(\varepsilon)$, which is the subject of ongoing negotiation once the tenant moves in. At this stage, the tenant's surplus is $\Sigma_l^w(\varepsilon) = W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l$ and the landlord's surplus is $\Sigma_l^l(\varepsilon) = L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l$.

If it is mutually agreeable for the tenant to move in (both surpluses positive) then there is Nash bargaining over the fee $\Pi(\varepsilon)$ with the landlord and tenant having the same bargaining powers ω_l and $1-\omega_l$ that apply in rent negotiations. The joint surplus $\Sigma_l(\varepsilon) = \Sigma_l^l(\varepsilon) + \Sigma_l^w(\varepsilon)$ is given by

$$\Sigma_l(\varepsilon) = W(\varepsilon) + L(\varepsilon) - B_l - U_l - C_w - C_l \,, \tag{35}$$

which is divided according to $\Sigma_l^l(\varepsilon) = \omega_l \Sigma_l(\varepsilon)$ and $\Sigma_l^w(\varepsilon) = (1-\omega_l)\Sigma_l(\varepsilon)$. In terms of the surpluses $\Lambda^l(\varepsilon)$ and $\Lambda^w(\varepsilon)$ once the tenant has moved in, the surpluses on meeting can be expressed as $\Sigma_l^l(\varepsilon) = \Lambda^l(\varepsilon) + \Pi(\varepsilon) - C_l$ and $\Sigma_l^w(\varepsilon) = \Lambda^w(\varepsilon) - \Pi(\varepsilon) - C_w$. Since the bargaining problem for new rents is the same as for ongoing rent, the subsequent surplus split is $\Lambda^l(\varepsilon) = \omega_l \Lambda(\varepsilon)$ and $\Lambda^w(\varepsilon) = (1-\omega_l)\Lambda(\varepsilon)$, where $\Sigma_l(\varepsilon) = \Lambda(\varepsilon) - C_l - C_w$, and hence Nash bargaining over the fee $\Pi(\varepsilon)$ yields

$$\Pi(\varepsilon) = \Pi = (1-\omega_l)C_l - \omega_l C_w \,, \tag{36}$$

which is independent of match quality ε. A lease is agreed if $\varepsilon \geq y_l$, where y_l, the leasing threshold, is the level of match quality ε where the joint surplus $\Sigma_l(\varepsilon)$ from (35) is zero:

$$\Sigma_l(y_l) = 0 \,. \tag{37}$$

[21] The transaction costs C_l and C_w are a type of fixed matching cost, for example, the costs of finding out about the tenant, because they are incurred before bargaining over the rent takes place (see Pissarides, 2009).

The proportion π_l of viewings of properties to let that lead to leases and the average rent R are

$$\pi_l = \int_{y_l} \mathrm{d}G_l(\varepsilon) = \left(\frac{y_l}{\zeta_l}\right)^{-\lambda_l}, \quad \text{and} \quad R = \frac{1}{\pi_l}\int_{y_l} R(\varepsilon)\mathrm{d}G_l(\varepsilon). \tag{38}$$

Prior to the realization of ε, the ex-ante expected joint surplus from a rental-market viewing is

$$\Sigma_l \equiv \int_{y_l} \Sigma_l(\varepsilon)\mathrm{d}G_l(\varepsilon). \tag{39}$$

3.5.3 Entry decisions of investors

When an investor views a property for sale, the investor's surplus from a transaction at price P_k is $\Sigma_k^k = U_l - (1+\tau_k)P_k - C_k - K$ and the seller's surplus is $\Sigma_k^u = P_k - C_u - U_o$. If there are mutual gains from a deal, the price P_k is determined by Nash bargaining, where the seller has bargaining power ω_k when faced with an investor. The joint surplus $\Sigma_k = \Sigma_k^k + \Sigma_k^u$ is split according to $\Sigma_k^u/\Sigma_k^k = \omega_k/((1-\omega_k)(1+\tau_k))$, so the tax τ_k shifts the division of the surplus in favour of the investor:

$$\Sigma_k^k = (1-\omega_k^*)\Sigma_k \quad \text{and} \quad \Sigma_k^u = \omega_k^*\Sigma_k, \quad \text{where} \quad \omega_k^* \equiv \frac{\omega_k}{1+\tau_k(1-\omega_k)}. \tag{40}$$

Since the joint surplus $\Sigma_k = U_l - C_k - C_u - U_o - K - \tau_k P_k$ is unaffected by considerations of match quality, either all investors are willing to buy or none, so an equilibrium with entry of investors occurs if and only if Σ_k is non-negative. When this is true, investors buy property at the rate q_o they meet sellers, and the price paid by all investors is

$$P_k = C_u + U_o + \omega_k^*\Sigma_k. \tag{41}$$

With this price, the joint surplus Σ_k from a meeting between an investor and a seller is

$$\Sigma_k = \frac{U_l - (1+\tau_k)U_o - (1+\tau_k)C_u - C_k}{1+\tau_k\omega_k^*}. \tag{42}$$

Note that a non-negative joint surplus Σ_k implies the value of having a property to let is always above the value of having a property for sale ($U_l \geq U_o$). Thus, after purchasing a property, an investor always prefers to keep it let out in the rental market.[22] Landlords sell properties only when hit by exit shocks, which arrive at rate ρ_l.

Given the free-entry condition (13), the Bellman equation (3) for investors' value K requires

$$\Sigma_k = \frac{F_k}{(1-\omega_k^*)q_o}, \tag{43}$$

which shows the surplus Σ_k rises with tightness of the ownership market. Intuitively, the viewing rate q_o decreases when there are more buyers relative to sellers, so investors must be compensated by a higher surplus $(1-\omega_k^*)\Sigma_k$ for them to enter in equilibrium.

[22] In other words, pure 'flippers' — those who buy and sell shortly afterwards — are not present in the model.

3.6 Welfare

Welfare Ω is the sum of the values of all incumbents in the city (homeowners, tenants, landlords, and including owners of unsold houses who have left the city) plus the present values of the payoffs received by those who enter the city. Exit (value 0) is already accounted for in incumbents' values.

A consistent analysis of welfare requires specifying what the government does with the tax revenue $\Gamma = \tau_h P S_h + \tau_k P_k S_k$ it collects. Revenue is assumed to be spent on public goods of an equal value, or equivalently, on reducing other taxes. The flow benefits of Γ/ψ per person could be added to the Bellman equations of city residents ($H(\varepsilon)$, $W(\varepsilon)$, B_o, and B_l). Rather than changing these equations, equivalently, the present value Ω_τ of the stream of tax revenue Γ is included in welfare Ω. This present value satisfies the Bellman equation $r\Omega_\tau = \Gamma + \dot{\Omega}_\tau$.

The expected payoff of someone entering the city prior to the realization of the credit cost χ is $B_e = (1 - G_m(Z))B_l + G_m(Z)(B_o - \bar{\chi})$, where Z is the credit-cost threshold for entering the ownership market and $\bar{\chi}$ is the average value of χ for those who do so. With a steady population ψ and exit at rate ρ, there are $\rho\psi$ new entrants per unit time. The expected present value Ω_e of all entrant values satisfies the Bellman equation $r\Omega_e = \rho\psi B_e + \dot{\Omega}_e$.

With H, L, and W denoting the average values of $H(\varepsilon)$, $L(\varepsilon)$, and $W(\varepsilon)$ over the distributions of all surviving matches, welfare is $\Omega = h_o H + h_l(L + W) + b_h B_o + b_l B_l + b_k K + u_o U_o + u_l U_l + \Omega_\tau + \Omega_e$. It is shown in appendix A.2.6 that welfare Ω satisfies the differential equation

$$r\Omega = h_o Q_h + h_l Q_l - M - h_l M_l - b_h F_h - b_k F_k - b_l F_w - S_o((1-\kappa)C_h + \kappa C_k + C_u) \\ - S_l(C_l + C_w) - (\gamma n_l h_l + \rho\psi)G_m(Z)\bar{\chi} + \dot{\Omega}\,, \quad (44)$$

where Q_h and Q_l denote the average levels of current match quality ε across the h_o owner-occupiers and the h_l tenants respectively.[23] Prices and rents drop out from welfare Ω because these are just transfers among market participants. Maintenance costs, flow search costs, non-tax transaction costs, and credit costs are implicitly treated as resource costs that show up as deductions from welfare. This assumes transaction costs reflect the time and resources of market participants and intermediaries that are consumed in completing transactions. Likewise, credit costs, for example, interest-rate spreads on mortgages, are treated as reflecting resources used up by banks. Transaction tax revenue does not appear as a deduction in (44) because it pays for public goods of an equivalent value, or allows other taxes to be reduced while still funding a given amount of public expenditure (of whatever resource cost and utility value).

The average match qualities Q_h and Q_l appearing in the welfare equation (44) are shown in

[23] This assumes all private benefits of owning or renting properties are social benefits. It is possible to envisage other policy distortions that might drive a wedge between private and social benefits such as the tax treatment of owners' implicit rental income or mortgage-interest deductibility.

appendix A.2.5 to satisfy the following pair of differential equations:

$$\dot{Q}_h = \frac{(1-\kappa)s_o u_o}{h_o}\left(\frac{\lambda_o}{\lambda_o - 1}y_o - Q_h\right) - (a_o - n_o)\left(Q_h - \frac{\lambda_o}{\lambda_o - 1}x_o\right), \quad \text{and} \tag{45}$$

$$\dot{Q}_l = \frac{s_l u_l}{h_l}\left(\frac{\lambda_l}{\lambda_l - 1}y_l - Q_l\right), \tag{46}$$

which depend on differences between Q_h and Q_l and average new match qualities $\lambda_o y_o/(\lambda_o - 1)$ and $\lambda_l y_l/(\lambda_l - 1)$ in the two markets, and between Q_h and average surviving match quality $\lambda_o x_o/(\lambda_o - 1)$ after match-quality shocks received by owner-occupiers.

3.7 Implications of the model in steady state

For constant tax rates τ_h and τ_k and other parameters, the model predicts the rental and ownership markets converge to a steady state where the fractions of properties and households in various states $(h_o, h_l, u_o, u_l, b_h, b_k)$ are constant over time. This steady state also features a constant measure of investors b_k and the proportion ξ of buyers they account for, and constant market tightnesses θ_o and θ_l. The homeownership rate h is defined as the fraction of the population ψ who own a property they occupy h_o or are selling a property they occupied. This is $h = (h_o + (1-\kappa)u_o)/\psi$, where former owner-occupiers selling properties account for a fraction $1-\kappa$ of properties u_o on the market. The model also has implications for the demographics of owner-occupiers compared to tenants, in particular, the average age difference α between the two groups.

Among those occupying properties, there is a stationary distribution of match quality, which implies a constant moving rate n_o in (33) for owner-occupiers moving within the city.[24] Taking account of exit from the city, the expected lengths of occupation of a property by homeowners and tenants are $T_{mo} = 1/(n_o + \rho)$ and $T_{ml} = 1/(n_l + \rho)$ respectively, where the moving rate within the city for tenants is from (34). There is also a steady state for the fraction ϕ of first-time buyers among all purchases by home-buyers.[25]

The average numbers of viewings v_o and v_l needed to sell or lease a property are respectively $v_o = 1/((1-\xi)\pi_o + \xi)$ and $v_l = 1/\pi_l$, and the expected times on the market for properties to sell

[24] The following expression for the steady-state moving rate is derived in appendix A.3:

$$n_o = a_o\left(\frac{\rho + a_o\left(1-\delta_o^{\lambda_o}\right) - \rho\delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1-\delta_o^{\lambda_o}\right) + a_o\delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}\right).$$

[25] It is shown in appendix A.5 that the average age difference α between owner-occupiers and tenants and the fraction ϕ of first-time buyers are:

$$\alpha = \left(1 + \frac{\rho}{\rho + n_l + q_l\pi_l}\right)\left(\frac{1}{\rho} - \frac{1}{\rho + \frac{\gamma n_l q_l \pi_l}{\rho + n_l + q_l\pi_l}}\right), \quad \text{and} \quad \phi = \frac{\rho\left(1 + \frac{n_o+\rho}{q_o\pi_o}\right)}{n_o + \rho\left(1 + \frac{n_o+\rho}{q_o\pi_o}\right)}.$$

and lease are $T_{so} = 1/s_o$ and $T_{sl} = 1/s_l$. From the perspective of home-buyers and potential tenants, the expected times taken successfully to find properties are $T_{bh} = 1/(q_o\pi_o)$ and $T_{bl} = 1/(q_l\pi_l)$. On average across buyers in the ownership market, the average time to complete a transaction is $T_{bo} = (1-\kappa)T_{bh} + \kappa T_{bk}$, where $T_{bk} = 1/q_o$ is the expected time taken by investors. This average time can be expressed as $T_{bo} = 1/(q_o(\xi + (1-\xi)\pi_o))$.

These predictions are used to calibrate the model's parameters, allowing the model to be used to make quantitative predictions about the effects of transaction taxes and the implications for welfare.

4 Quantitative effects of transaction taxes in the model

As documented in the econometric evidence of section 2, despite home-buyers and buy-to-rent investors facing the same transaction tax rates and the same tax increase in Toronto in 2008, transactions rose for buy-to-rent investors and fell for home-buyers. The model developed in section 3 explains these differential effects of the LTT on owner-occupiers and investors even though the tax rate τ_k on investors is the same as the tax rate τ_h before and after the LTT increase. The model also predicts an increase in leasing activity in the rental market, a decline in mobility within the ownership market, as well as matching the direction of the other responses to the LTT found empirically.

To perform a quantitative analysis of the LTT and assess its implications for welfare, the model is calibrated to match key features of ownership and rental markets in the City of Toronto before the LTT change. The model is then solved for the transaction tax rates prevailing in the city before and after the February 2008 city-level LTT was introduced to derive predictions for the housing-market outcomes studied empirically and for welfare. As explained in section 2, the effective LTT rate rises from 1.5% to 2.8%, an increase of 1.3 percentage points.

4.1 Calibration

The model is calibrated to the City of Toronto housing market before the LTT change, in particular, during the period January 2006–January 2008. The tax rates faced by both home-buyers and buy-to-rent investors are set to the effective LTT prior to the change, $\tau_k = \tau_h = 0.015$. The parameters of the model are calibrated to match a list of targets given in Table 3, and the implied parameter values are reported in Table 4. The data sources of all targets are detailed in appendix A.4, and appendix A.5 explains how the calibration procedure works. In summary, there are three broad sets of targets.

The first set of targets $(\psi, B_e, \omega_o/\eta_o, \omega_l/\eta_l)$ is directly imposed. The measure of households is chosen to be the same as the measure of properties, that is, $\psi = 1$. Although entry to the city is exogenous in the model, for consistency, the calibration selects parameters where the expected value of entering the city B_e is zero, matching the zero value for those who exit. Finally, the bargaining powers of sellers and landlords are set to be the same as the corresponding elasticities of the meeting functions for the two market, that is, $\omega_o = \eta_o$ and $\omega_l = \eta_l$.

Table 3: *Calibration targets*

Targets	Notation	Value
Directly imposed targets		
Equal numbers of households and properties	ψ	1
No incentive for further entry of households into the city	B_e	0
Bargaining powers equal to meeting-function elasticities	$\omega_o/\eta_o = \omega_l/\eta_l$	1
Empirical targets		
Average buy-to-own transaction price	P	\$402k
Effective land transfer tax for all buyers	$\tau_h = \tau_k$	1.5%
Homeownership rate	h	54%
Fraction of purchases made by buy-to-rent investors	κ	5.4%
Fraction of first-time buyers among all home-buyers	ϕ	40%
Difference in average ages of owner-occupiers and renters	α	8.3
Average price-rent ratio for same properties	P_k/R	14.5
Price paid by investors relative to average paid by home-buyers	P_k/P	99%
Non-tax transaction costs of buyers relative to price	$C_h/P = C_k/P_k$	0%
Property maintenance costs relative to price	M/P	2.6%
Landlords' extra maintenance/management costs relative to rent	M_l/R	8%
Seller transaction costs relative to price	C_u/P	4.5%
Landlord transaction costs relative to rent	C_l/R	8.3%
Fraction of landlord transaction costs charged to tenant	Π/C_l	0%
Flow search costs of home-buyers relative to price	F_h/P	3.1%
Flow search costs of investors relative to home-buyers	F_k/F_h	1
Flow search costs of tenants relative to home-buyers	F_w/F_h	1.1
Sellers' average time on the market	T_{so}	0.161
Buyers' average time on the market	T_{bo}	0.206
Landlords' average time on the rental market	T_{sl}	0.066
Average viewings per sale	v_o	20.6
Average viewings per lease	v_l	10.3
Average time between moves for owner-occupiers	T_{mo}	9.25
Average time between moves for tenants	T_{ml}	3.04
Percentage decline of owner-occupier moving rate after new LTT	β	13%
Capitalized credit costs of marginal home-buyer relative to price	Z/P	0.48
Ratio of credit costs of marginal and average home-buyers	$Z/\bar{\chi}$	2.11
Sources of the targets for credit costs		
Risk-free real interest rate	r_f	1.86%
Average real mortgage interest rate	$\bar{r}_c$	4.93%
Real mortgage interest rate of the marginal home-buyer	r_c	7.93%
Initial loan-to-value ratio of first-time buyers	ℓ	80%
Mortgage term	T_c	25

Notes: All time units are in years. See appendix A.4 for data sources and appendix A.5 for the calibration procedure. The targets for Z/P and $Z/\bar{\chi}$ are derived from those for r_f, $\bar{r}_c$, r_c, ℓ, and T_c as explained in appendix A.4.

The second set of targets is related to the extensive margin across the ownership and rental markets. These targets are the homeownership rate h, the fraction κ of buy-to-rent purchases among total purchases, the fraction of first-time buyers ϕ, the difference α in the average ages of owner-occupiers and renters, investors' price-to-rent ratio P_k/R, and the ratio of prices paid by investors to prices paid by home-buyers P_k/P. The other key targets here are for the capitalized credit costs of marginal home-buyers relative to price Z/P, and the ratio of marginal to mean credit costs $Z/\bar{\chi}$. As explained in appendix A.4, these credit-cost targets can themselves be derived from information about mortgage interest rate spreads. Note that it is not necessary to take a stance on the presence or size of any 'warm glow' effect of homeownership in the calibration. The parameter ζ_l is determined as a residual given the calibrated costs of owning versus renting and the choices of households reflected in the homeownership rate.

The third set of targets matches search behaviour and costs incurred within the ownership and rental markets. The key targets for search behaviour are viewings per sale v_o, viewings per lease v_l, times on the market for buyers T_{bo} and sellers T_{so} in the ownership market, landlords' time on the rental market T_{sl}, and the expected times between moves for homeowners T_{mo} and tenants T_{ml}. The targets for costs in the ownership market are homeowners' maintenance cost M, transaction costs excluding taxes for buyers and sellers (C_k, C_h, C_u), and the flow search costs of buyers (F_k, F_h). There are targets for all of these as fractions of the appropriate price P or P_k. The targets for costs in the rental market include the extra maintenance costs M_l faced by landlords, landlords' transaction costs C_l, and flow search costs of tenants F_w, all as a fraction of rent R, and the fraction of landlords' transaction costs passed on to tenants. The calibration also matches the model-implied moving rate response β to the LTT change with the effect estimated in section 2.

Finally, the units of utility can be normalized so that the model matches the average transaction price P. This means that all utility payoffs and costs can be interpreted as dollar equivalents.

4.2 Quantitative effects of transaction taxes

The effects of increasing the transaction tax rates τ_h and τ_k from 1.5% to 2.8% for both home-buyers and investors are reported in Table 5. The steady state of the model is computed for each tax rate using the procedure described in section A.3. The changes in variables across the steady states are reported as log differences for consistency with the econometric estimates of the LTT effects on the logarithms of housing-market outcomes from section 2.

Consistent with the econometric evidence, the model predicts that buy-to-own (BTO) and buy-to-rent (BTR) transactions move in opposite directions following the transaction tax increase. Sales to home-buyers fall, while sales to investors rise, despite the two facing the same rise in tax rates.

There are three household behavioural responses to the higher tax rate underlying the fall in BTO transactions. First, a higher tax rate raises the cost of moving, which makes homeowners more tolerant of worse match quality (a lower moving threshold x_o). This is reflected in a longer average

Table 4: *Calibrated parameters*

Parameter description	Notation	Value
Number of households relative to the number of properties	ψ	1
Discount rate for future housing-market payoffs	r	3.3%
Households' exit rate from the city	ρ	4.3%
Investors' exit rate	ρ_l	0.7%
Property maintenance cost	M	10.4
Landlords' extra maintenance/management costs	M_l	2.2
Minimum new match quality in the ownership market	ζ_o	34.1
Minimum new match quality in the rental market	ζ_l	21.2
Home-buyer shape parameter of new match quality distribution	λ_o	30.3
Tenant shape parameter of new match quality distribution	λ_l	31.5
Arrival rate of match quality shocks in the ownership market	a_o	8.3%
Arrival rate of match quality shocks in the rental market	a_l	27.9%
Size of match quality shock in ownership market	δ_o	0.858
Fraction of tenants drawing new credit cost after moving shock	γ	8.3%
Parameter for mean of the distribution of credit costs	μ	5.5
Parameter for standard deviation of the distribution of credit costs	σ	1.3
Transaction costs of buyers excluding taxes	$C_k = C_h$	0
Transaction costs of sellers	C_u	18.1
Transaction costs of landlords	C_l	2.3
Transaction costs of tenants	C_w	0.7
Flow search costs of home-buyers and investors	$F_k = F_h$	12.6
Flow search costs of prospective tenants in the rental market	F_w	13.6
Viewing productivity parameter in the ownership market	A_o	111
Viewing productivity parameter in the rental market	A_l	169
Elasticity of ownership-market meetings with respect to sellers	η_o	0.434
Elasticity of rental-market meetings with respect to landlords	η_l	0.762
Bargaining power of sellers meeting a home-buyer	ω_o	0.434
Bargaining power of sellers meeting an investor	ω_k	0.218
Bargaining power of landlords meeting a prospective tenant	ω_l	0.762

Notes: All time units are in years, and all payoff and cost parameters are measured in thousands of dollars. These parameters exactly match the targets specified in Table 3 using the calibration procedure from appendix A.5.

time-to-move. Second, home-buyers become pickier (a higher transaction threshold y_o). Because moving decisions are endogenous and match quality has persistence, home-buyers can reduce the future incidence of moving — and lower the tax they expect to pay — by beginning with better match quality. This results in a longer average time-to-sell, as is found empirically. Finally, higher taxes reduce the joint surplus in the ownership market because part of the surplus is absorbed by higher tax. This reduces renters' incentive to enter the ownership market (a lower credit-cost threshold Z).

Since investors face the same tax increase as home-buyers, the direct effect of the higher tax is to reduce entry of buy-to-rent investors. However, there are two crucial equilibrium effects at work as well. First, there is more demand for rental properties owing to households' reduced incentive

Table 5: *Simulations of the model following an increase in the transaction tax rate*

Variable	Model prediction	Econometric evidence
Time-to-move for homeowners	13% (matched)	13%
Buy-to-own (BTO) transactions	−15%	−10%
Buy-to-rent (BTR) transactions	1.9%	11%
Time-to-sell	8.6%	23%
Leases-to-sales ratio	15%	23%
Homeownership rate	−1.6% (−0.9 p.p.)	-
Price-to-rent ratio	−1.8%	−4.0%
Average sales price	−1.9%	−1.7%
Transaction tax revenue	46%	-
Effective LTT tax rate	Increased from 1.5% to 2.8% (1.3 p.p.)	

Notes: The responses of variables are reported as log differences. The solution procedure to find the predictions of the model is described in appendix A.3.

to switch from renting to owning. This increases rents relative to property prices, pushing down the price-to-rent ratio and encouraging investors to enter. This negative effect of the tax on the price-to-rent ratio is consistent with the econometric estimates from section 2.

Second, landlords do not have to sell their properties and pay the transaction tax again just because a renter moves, unlike owner-occupiers who have to buy again and pay the tax every time they move. Buy-to-rent investors thus have an implicit tax advantage — even though they face the same tax rates. The mechanism by which the tax increase actually boosts entry of investors works through the price-to-rent ratio. Since sellers can sell to either home-buyers or investors they meet, the prices paid by the two groups are very tightly linked. Quantitatively, home-buyers are the dominant group of buyers, and the capitalization effect of the higher tax paid by owner-occupiers pushes down property prices for all buyers, including investors. This lower price-to-rent ratio more than offsets the higher tax from the perspective of investors, hence the model predicts a rise in BTR transactions.

BTR transactions constitute a relatively small fraction of total transactions, so the overall effect is that transactions fall by 14%. Combined with additional entry of buy-to-rent investors, the ratio of the number of leases to sales is higher. These changes imply the homeownership rate falls by around one percentage point. Data on the homeownership rate in Toronto is not available at the micro level and at high frequencies, so the causal effect of the LTT change cannot be estimated. However, the empirical findings for BTR transactions and leases indicate that the homeownership rate would fall after the LTT increase, all else equal.[26]

The predicted average price paid drops by 1.9%, matching the data almost exactly. Interestingly,

[26]Simply looking at the aggregate data on the homeownership rate in Toronto reveals a rising trend prior to the LTT increase and a flattening out afterwards. The period of stagnation in the homeownership rate coincides with a rising fraction of BTR transactions in the aggregate.

the percentage change in price is larger than the 1.3 percentage-point rise in the tax rate.[27] The impact on the average price reflects the expectation that a given property will be subject to the tax each time it is sold, and thus the expected future incidence of the tax is capitalized into the price.

The model predicts that the log difference in tax revenue $\Gamma = \tau_h P S_h + \tau_k P_k S_k$ before and after is only 46% when the log difference of the tax rates is 62% (from 1.5% to 2.8%). This discrepancy is explained by erosion of the tax base: total transactions go down by 14%, and the average price drops by 2%, so the tax base shrinks by 16%.

4.3 Welfare effects of transactions taxes

The calibrated model predicts the welfare costs of the LTT are substantial. The new LTT causes welfare to fall by an amount equivalent to 79% of the extra tax revenue it generates. Formally, this measure of welfare loss is the ratio of $r\Delta\Omega$ (the flow welfare loss derived from equation 44) to $\Delta\Gamma$ (the change in the flow of tax revenue).

The welfare loss is due to distortions across and within ownership and rental markets, and both are large.[28] Distortions across the two markets generate a loss equivalent to 25% of the extra tax revenue. Within the markets, distortions in the rental market and in ownership market generate losses of 5% and 49% of tax revenue respectively. Overall, the presence of the rental market in the analysis accounts for a welfare loss of 30% of extra tax revenue beyond the loss within the ownership market itself. This is around 40% of the total loss of 79% of extra tax revenue.

The welfare loss across the two markets results from the drop in the homeownership rate. Some households with low enough credit costs who would otherwise have gained from being owner-occupiers decide to remain renters owing to the extra cost burden imposed by the transaction tax, both now and expected again in the future. The size of this welfare loss largely depends on the distribution of credit costs, which is calibrated using data on mortgage spreads. This is because the credit-cost distribution across households is the relevant source of heterogeneity for the owing-versus-renting decision — everyone shares the same ex-ante expectation of housing utility in the two markets, so there is no lack of substitutability between owner-occupied and rental properties in terms of preferences. The decline in homeownership also adds to the welfare loss through an increase in rental management costs.[29]

[27] A simple analysis of tax incidence might suggest that prices should change by less than the tax rate increase because buyers have some bargaining power — see equation (27). That equation also shows a proportional transaction tax reduces the effective bargaining power of sellers, contributing to a lower price.

[28] Using equation (44) in steady state, the change in welfare $\Delta\Omega$ after the tax rise can be decomposed as follows:

$$r\Delta\Omega = (h_o\Delta Q_h - F_h\Delta b_h - C_h\Delta S_h - C_u\Delta S_o) + (h_l\Delta Q_l - F_w\Delta b_l - (C_l + C_w)\Delta S_l) \\ + ((Q_h + \Delta Q_h)\Delta h_o + (Q_l + \Delta Q_l - M_l)\Delta h_l - F_k\Delta b_k - C_k\Delta S_k - \Delta((\gamma n_l h_l + \rho\psi)G_m(Z)\bar{\chi})) \,.$$

The first block of terms result from changes within the ownership market, the second from changes within the rental market, and the third from changes across the two markets.

[29] It is important to note that the model does not imply a monotonic relationship between homeonwership rate and

Within the ownership market, the welfare loss is mainly due to the fall in match quality, partly offset by lower non-tax transaction costs saved owing to moving taking place less frequently. It is also offset by home-buyers being more picky, albeit at the cost of having to search for longer. The large size of the welfare loss is related to the indivisibility of housing: households are taxed on the whole value of a property, not the marginal improvement in match quality that comes from moving. The welfare loss within the rental market is much smaller and mainly reflects an increase in transaction costs.

4.4 The role of buy-to-rent investors

A key feature of the analysis here is allowing for free entry of buy-to-rent investors. This helps to understand why the LTT has different effects on BTO and BTR transactions. It also has implications for the distortions created by transaction taxes. Since homeowners are more heavily affected by the same transaction tax rate than investors, a higher tax rate increases distortions in the allocation of housing across the ownership and rental markets.

This novel effect can be isolated by considering a hypothetical tax regime with different tax rates for homeowners and investors. Taking the same increase in τ_h as before, the tax rate τ_k can be raised to such a level that there is no change in the equilibrium homeownership rate. The required τ_k change for this is from 1.5% to 5.7%. The alternative tax system raises slightly more revenue, but not much because buy-to-rent investors are a small minority and do not transact frequently on average. Importantly, the welfare loss in this case is considerably smaller, being only 42% of the extra revenue raised instead of 79% with an equal increase in the tax rates τ_h and τ_k.

Intuitively, this exercise shuts down the extensive margin, keeping the homeownership rate unchanged by putting up higher barriers to entry for investors. This offsets the implicit advantage investors have when the tax rates are equal that comes from not needing to pay the LTT as often as owner-occupiers do. The welfare loss is smaller because the unequal tax rates undo this distortion.

However, increasing τ_k ever further to raise the homeownership rate would ultimately lead to large welfare costs as uncreditworthy households are forced into the ownership market because of a lack of rental properties. This would result in their paying very high borrowing costs, the final term in the expression for welfare (44). Deep-pocketed investors play an important role in providing access to housing without everyone needing to pay credit costs.

5 Conclusions

Using a unique dataset on property sales and leasing transactions, this paper documents two novel effects of a higher transaction tax. First, there is a rise in buy-to-rent transactions and a fall in owner-

welfare as shown in the final term in the expression for welfare (44) where credit costs associated with increasing homeownership are part of welfare loss.

occupier transactions despite the same tax applying to both. Second, there is a simultaneous fall in the price-to-rent ratio and in the sales-to-leases ratio.

This paper builds a tractable model with free entry of investors and where households choose renting or owning, with entry to the ownership market incurring a cost of accessing credit. The calibrated model explains the empirical findings and points to a novel welfare cost of transaction taxes. A higher transaction tax distorts the allocation of properties across the two markets by reducing the homeownership rate, as well as distorting the allocation within the ownership market by reducing mobility. The calibrated model implies a substantial welfare loss equivalent to 79% of the increase in tax revenue, with about 40% due to the analysis allowing for the presence of a rental market.

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A Appendices

A.1 Data and further estimation results and robustness checks

Table A.1: *Descriptive statistics for the City of Toronto municipality*

	Pre-LTT	Post-LTT		Whole sample
	2006:1–2008:1	2008:2–2010:2	2008:2–2012:2	2006:1–2018:2
# BTO sales per year	27,718	23,832	24,621	25,547
# BTR sales per year	1,572	1,685	3,894	2,440
Time on the market (days, mean)	30.5	28.8	27.1	25.4
Time on the market (days, median)	20	18	17	15
Sale price (mean)	401,504	426,363	460,903	555,484
Sale price (median)	318,000	343,000	369,900	419,990
Price-rent ratio (mean)	20.7	20.9	22.2	25.8
Price-rent ratio (median)	16.9	17.9	18.8	21.1

Source: City of Toronto Multiple Listing Service (MLS) residential transaction records (2006–2018).

Figure A.1: *Geography of sample used for estimation*

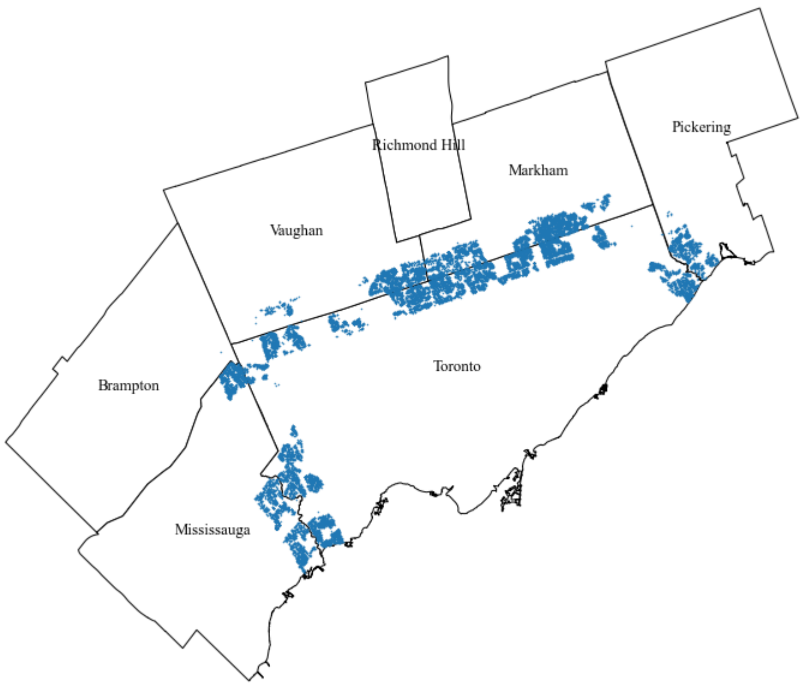


Table A.2: *Land transfer tax (LTT) rates by property value in the Greater Toronto Area*

City of Toronto (effective from 1st February 2008)		Province of Ontario (effective from 7th May 1997)	
\$0–55,000	0.5%	\$0–55,000	0.5%
\$55,000–400,000	1.0%	\$55,000–250,000	1.0%
\$400,000+	2.0%	\$250,000–400,000	1.5%
		\$400,000+	2.0%

Sources: Municipal Land Transfer Tax, City of Toronto, `http://www.toronto.ca/taxes/mltt.htm`; Provincial Land Transfer Tax, Historical Land Transfer Tax Rates, Province of Ontario. Reproduced from Dachis, Duranton and Turner (2012).
Notes: For the municipal LTT, exemptions are given to first-time buyers for purchases below a value of \$400,000, while for the provincial LTT, the first-time buyer exemption value threshold is \$227,500.

Figure A.2: *Kaplan-Meier estimate of homeowners' moving hazard function*

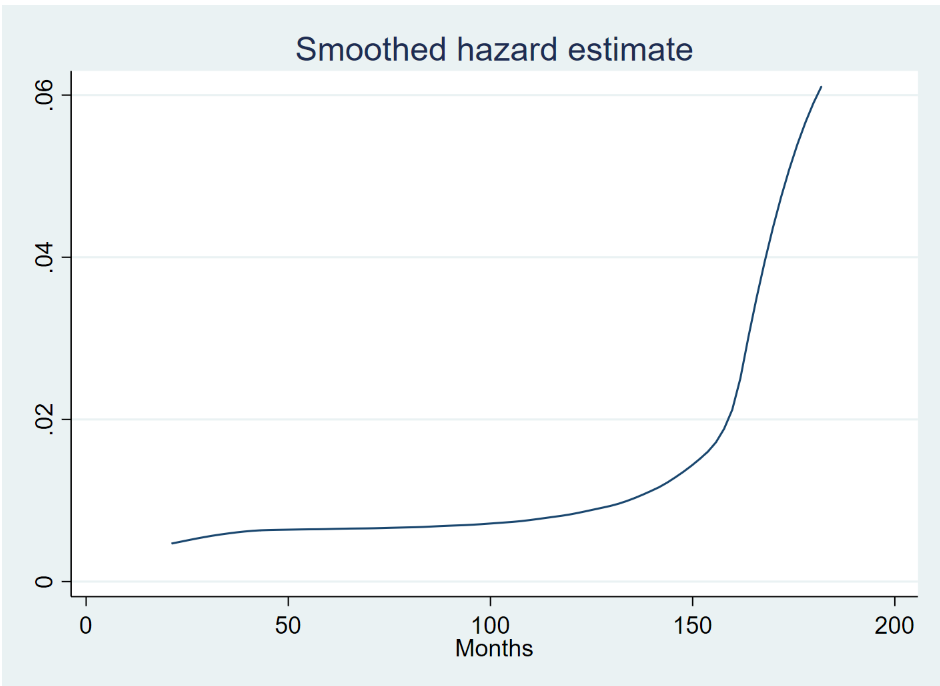


Table A.3: *Robustness checks on sales to owner-occupiers and buy-to-rent investors*

Dependent variable	(1)	(2)	(3)	(4)	(5)
Using 6-month cutoff to distinguish between buy-to-own and buy-to-rent transactions					
log (#*BTO sales*)	-0.112*	-0.087	-0.138***	-0.122**	-0.166**
	(0.058)	(0.065)	(0.043)	(0.054)	(0.071)
Observations	3790	2869	6482	3894	2588
log (#*BTR sales*)	0.186***	0.185**	0.188***	0.193***	0.180**
	(0.075)	(0.083)	(0.053)	(0.061)	(0.103)
Observations	351	280	674	436	238
Using 12-month cutoff to distinguish between buy-to-own and buy-to-rent transactions					
log (#*BTO sales*)	-0.082*	-0.088*	-0.100**	-0.086*	-0.127*
	(0.045)	(0.047)	(0.044)	(0.045)	(0.072)
Observations	3770	2855	6452	3822	2570
log (#*BTR sales*)	0.160***	0.143**	0.142***	0.141***	0.155*
	(0.064)	(0.072)	(0.047)	(0.057)	(0.086)
Observations	459	361	847	530	317
Using 24-month cutoff to distinguish between buy-to-own and buy-to-rent transactions					
log (#*BTO sales*)	-0.107*	-0.079*	-0.119***	-0.098*	-0.158**
	(0.059)	(0.042)	(0.045)	(0.056)	(0.073)
Observations	3724	2822	6387	3849	2538
log (#*BTR sales*)	0.135**	0.137**	0.105***	0.093*	0.123
	(0.062)	(0.067)	(0.044)	(0.051)	(0.077)
Observations	837	655	1562	1009	553
Distance threshold	3km	3km	5km	5km	2km
City indicators $\pm$3 m.	Yes	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	Yes	
Donut hole		1km		2km	

Notes: Data comprise single-family-house transactions from January 2006 to February 2012. A unit of observation is a market segment defined by *community* $\times$ *year* $\times$ *month*. Each cell of the table represents a separate regression of an outcome (specified in the left column) on the LTT interaction dummy. All regressions include a dummy for the post-LTT period, City of Toronto fixed effects, year fixed effects, calendar-month fixed effects, community fixed effects, and their interactions. In the specifications above, distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators $\pm$3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trend denotes the inclusion of an interaction term between exposure to the new LTT and a dummy variable for properties between 2.5km and 5km away from the city border. Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Table A.4: *Robustness checks on sales prices at the market-segment level*

Dependent variable	(1)	(2)	(3)	(4)	(5)
Sample period 2006–2012					
log (*Price*)	-0.0200***	-0.0229***	-0.0174***	-0.0125**	-0.0255***
	(0.0053)	(0.0055)	(0.0042)	(0.0052)	(0.0069)
Observations	11,169	8,688	19,227	11,802	7,425
Sample period 2006–2010					
log (*Price*)	-0.0186**	-0.0228***	-0.0172***	-0.0124**	-0.0253***
	(0.0061)	(0.0064)	(0.0049)	(0.0061)	(0.0080)
Observations	7,519	5,833	12,946	7,953	4,993
Sample period 2006–2018					
log (*Price*)	-0.0192***	-0.0230***	-0.0192***	-0.0158***	-0.0252**
	(0.0049)	(0.0052)	(0.0039)	(0.0049)	(0.0065)
Observations	22,001	17,215	38,268	23,514	14,754
Distance threshold	3km	3km	5km	5km	2km
City indicators $\pm$3 m.	Yes	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	Yes	
Donut hole		1km		2km	

Notes: The estimation sample covers four types of properties: single-family houses, townhouses, condominiums, and apartments. A unit of observation is a market segment defined by *community* $\times$ *property type* $\times$ *year* $\times$ *month*. The dependent variable is the average sales price within each market segment. Each cell of the table represents a separate regression on the LTT interaction dummy. All regressions include a dummy for the post-LTT period, and *city* $\times$ *property type*, *year* $\times$ *property type*, *month* $\times$ *property type*, and *community* $\times$ *property type* fixed effects. In the specifications above, distance threshold is the maximum distance to the Toronto city border for a transaction to be included in the sample. City indicators $\pm$3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trend denotes the inclusion of an interaction term between exposure to the new LTT and a dummy variable for properties between 2.5km and 5km away from the city border. Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Table A.5: *Robustness checks on the moving hazard rate*

	(1)	(2)	(3)	(4)	(5)
Sample period 2006–2012					
LTT	-0.130**	-0.188**	-0.194***	-0.232**	-0.151**
	(0.066)	(0.080)	(0.054)	(0.087)	(0.051)
Observations	1,691,369	1,142,052	2,831,897	1,651,935	1,179,962
Sample period 2006–2010					
LTT	-0.156**	-0.176**	-0.218***	-0.243**	-0.147*
	(0.074)	(0.089)	(0.063)	(0.110)	(0.089)
Observations	1,012,969	682,641	1,690,705	982,110	708,595
Sample period 2006–2018					
LTT	-0.125**	-0.169**	-0.179***	-0.213**	-0.162***
	(0.061)	(0.074)	(0.048)	(0.071)	(0.047)
Observations	4,327,556	2,927,002	7,306,558	4,296,732	3,009,826
Distance threshold	3km	3km	5km	5km	2km
House characteristics	Yes	Yes	Yes	Yes	Yes
City indicators ± 3 m.	Yes	Yes	Yes	Yes	Yes
City time trends	Yes	Yes	Yes	Yes	Yes
Distance LTT trends			Yes	Yes	
Donut hole		1km		2km	

Notes: For the moving hazard estimation, a unit of observation is a household whose property is listed on MLS between January 2006 to February 2012. Repeat sales transactions taking place within 18 months of one another are discarded. Households' times between moves are assumed to follow a Weibull distribution. For time-on-the-market, a unit of observation is a transaction recorded on MLS during the same period. All regressions include an indicator for the post-LTT period, an indicator for the city of Toronto, property-type fixed effects interacted with a set of time-varying house characteristics, and *city* $\times$ *property type*, *month* $\times$ *property type*, and *community* $\times$ *property type* fixed effects. Distance threshold is the maximum distance to the Toronto city border for a transaction to be included. City indicators ± 3 m. are six dummy variables for transactions inside the City of Toronto during the last three months of 2007 and the first three of 2008. City time trends indicates the presence of separate time trends for transactions inside and outside the City of Toronto. Distance LTT trend denotes the inclusion of an interaction term between exposure to the new LTT and a dummy variable for properties between 2.5km and 5km away from the city border. Robust standard errors are in parentheses, and *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

Figure A.3: *Household stocks and flows in the model*

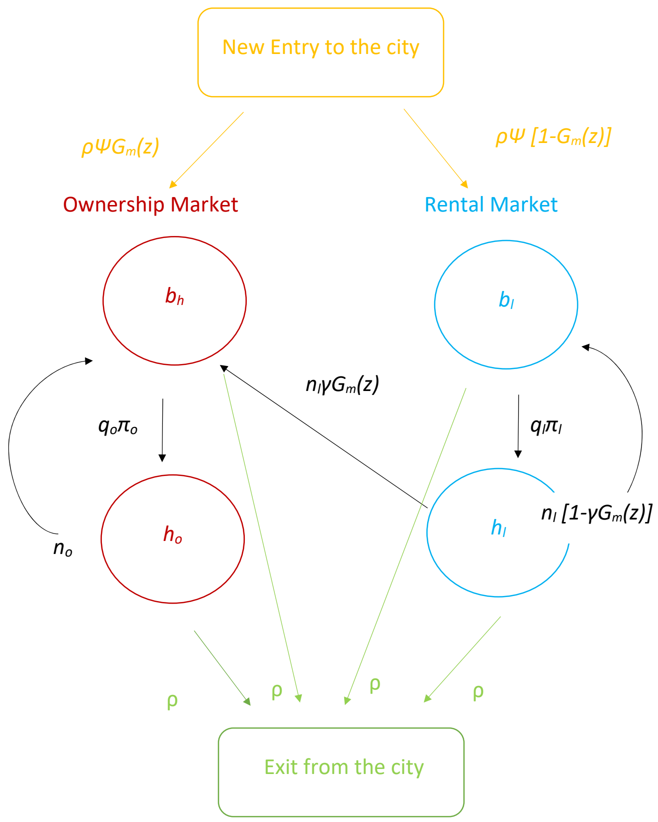
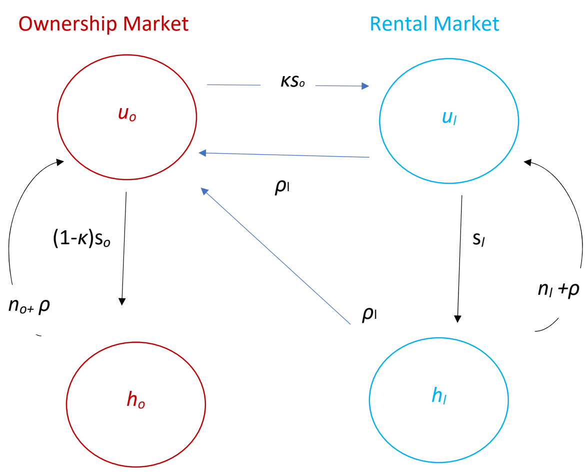


Figure A.4: *Property stocks and flows in the model*



A.2 Deriving the equations of the model

A.2.1 The value functions and thresholds for homeowners and home-buyers

The value function $H(\varepsilon)$ from (6) is increasing in ε. Assuming $\delta_o y_o < x_o$ for all t, by taking ε in a neighbourhood above y_o or any value below, the Bellman equation (6) reduces to the following as $H(\delta_o\varepsilon) < B_o + U_o$:

$$rH(\varepsilon) = \varepsilon - M + a_o(B_o + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon) .$$

This simplifies to

$$(r+\rho+a_o)H(\varepsilon) - \dot{H}(\varepsilon) = \varepsilon - M + a_o B_o + (\rho + a_o)U_o , \tag{A.1}$$

and by differentiating both sides with respect to ε in the restricted range:

$$(r+\rho+a_o)H'(\varepsilon) - \dot{H}'(\varepsilon) = 1 .$$

For a given ε, this specifies a first-order differential equation in time t for $H'(\varepsilon)$. Since $H'(\varepsilon)$ is not a state variable, there exists a unique stable solution $H'(\varepsilon) = 1/(r+\rho+a_o)$, which is constant over time ($\dot{H}'(\varepsilon) = 0$). As $H'(\varepsilon)$ is independent of ε, integration over match quality ε shows the value function $H(\varepsilon)$ has the form

$$H(\varepsilon) = \underline{H} + \frac{\varepsilon}{r+\rho+a_o} , \quad \text{with} \quad \dot{H}(\varepsilon) = \dot{\underline{H}} , \tag{A.2}$$

where $\underline{H}$ is independent of ε, but may be time varying. This result is valid for ε in a neighbourhood above y_o and all values below. Substituting back into (A.1) shows that $\underline{H}$ satisfies the differential equation

$$(r+\rho+a_o)\underline{H} - \dot{\underline{H}} = a_o B_o + (\rho + a_o)U_o - M . \tag{A.3}$$

Since $x_o < y_o$, equation (32) together with (A.2) implies that

$$x_o = (r+\rho+a_o)(B_o + U_o - \underline{H}) . \tag{A.4}$$

Equation (28) for the surplus and the definition of the transaction threshold (29) imply that y_o satisfies

$$H(y_o) = H(x_o) + C_h + (1+\tau_h)C_u + \tau_h U_o , \tag{A.5}$$

and combining (A.2) with (A.5) yields

$$y_o = x_o + (r+\rho+a_o)(C_h + (1+\tau_h)C_u + \tau_h U_o) . \tag{A.6}$$

The surplus $\Sigma_o(\varepsilon)$ is given in (28) and is divided according to (27). Equation (31) defines the expected surplus Σ_o. The Bellman equation for a buyer (4) can thus be expressed as the following differential equation:

$$(r+\rho)B_o - \dot{B}_o = (1-\omega_o^*)q_o\Sigma_o - F_h . \tag{A.7}$$

The surplus from trade with an investor and its division are given in (40) and (42). Together with the surplus from trade with a home-buyer, the Bellman equation of a seller (5) implies the differential equation

$$rU_o - \dot{U}_o = \theta_o q_o \left(\omega_o^*(1-\xi)\Sigma_o + \omega_k^*\xi\Sigma_k\right) - M . \tag{A.8}$$

Using equations (25), (28), and (29), the expected surplus Σ_o in (31) can be written as

$$\Sigma_o = \int_{y_o}^{\infty} \lambda_o \zeta_o^{\lambda_o} \varepsilon^{-(\lambda_o+1)} \Sigma_o(\varepsilon) \mathrm{d}\varepsilon = \int_{y_o}^{\infty} \frac{\lambda_o \zeta_o^{\lambda_o} \varepsilon^{-(\lambda_o+1)}(H(\varepsilon) - H(y_o))}{1+\tau_h\omega_o^*} \mathrm{d}\varepsilon . \tag{A.9}$$

Make the following definition of $\bar{H}(\varepsilon)$ for an arbitrary level of match quality ε, and note the link with Σ_o:

$$\bar{H}(\varepsilon) = \int_{\upsilon=\varepsilon}^{\infty} \lambda_o \varepsilon^{\lambda_o} \upsilon^{-(\lambda_o+1)}(H(\upsilon) - H(\varepsilon))\mathrm{d}\upsilon , \quad \text{where } \Sigma_o = \frac{\zeta_o^{\lambda_o} y_o^{-\lambda_o} \bar{H}(y_o)}{1+\tau_h\omega_o^*} . \tag{A.10}$$

Now restrict attention to ε such that $\delta_o\varepsilon < x_o$, so (6) implies $rH(\varepsilon) = \varepsilon - M + a_o(B_o + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)$. Since $\delta_o y_o < x_o$, this limits ε to a neighbourhood above y_o and all values below. Using (32):

$$r(H(\upsilon) - H(\varepsilon)) = (\upsilon - \varepsilon) + a_o(\max\{H(\delta_o\upsilon), H(x_o)\} - H(\upsilon)) - a_o(H(x_o) - H(\varepsilon)) - \rho(H(\upsilon) - H(\varepsilon)) + (\dot{H}(\upsilon) - \dot{H}(\varepsilon)) ,$$

which holds for any $\upsilon \geq \varepsilon$. This simplifies to

$$(r+\rho+a_o)(H(\upsilon)-H(\varepsilon))-(\dot{H}(\upsilon)-\dot{H}(\varepsilon))=(\upsilon-\varepsilon)+a_o\max\{H(\delta_o\upsilon)-H(x_o),0\},$$

and multiplying both sides by $\lambda_o\varepsilon^{\lambda_o}\upsilon^{-(\lambda_o+1)}$, integrating over υ, and using the definition of $\bar{H}(\varepsilon)$ in (A.10):

$$(r+\rho+a_o)\bar{H}(\varepsilon)-\dot{\bar{H}}(\varepsilon)=\int_{\upsilon=\varepsilon}^{\infty}\lambda_o\varepsilon^{\lambda_o}\upsilon^{-(\lambda_o+1)}\left((\upsilon-\varepsilon)+a_o\max\{H(\delta_o\upsilon)-H(x_o),0\}\right)\mathrm{d}\upsilon\,, \tag{A.11}$$

where the time derivative of $\bar{H}(\varepsilon)$ is obtained from (A.10):

$$\dot{\bar{H}}(\varepsilon)=\int_{\upsilon=\varepsilon}^{\infty}\lambda_o\varepsilon^{\lambda_o}\upsilon^{-(\lambda_o+1)}(\dot{H}(\upsilon)-\dot{H}(\varepsilon))\mathrm{d}\upsilon\,.$$

In (A.11), the term in $(\upsilon-\varepsilon)$ integrates to $\varepsilon/(\lambda_o-1)$ using the formula for the mean of a Pareto distribution. The second term is zero for $\upsilon<x_o/\delta_o$ because $H(\delta_o\upsilon)$ is increasing in υ. Hence, equation (A.11) becomes

$$(r+\rho+a_o)\bar{H}(\varepsilon)-\dot{\bar{H}}(\varepsilon)=\frac{\varepsilon}{\lambda_o-1}+a_o\varepsilon^{\lambda_o}\int_{\upsilon=x_o/\delta_o}^{\infty}\lambda_o\upsilon^{-(\lambda_o+1)}(H(\delta_o\upsilon)-H(x_o))\mathrm{d}\upsilon\,,$$

and with the change of variable $j=\delta_o\upsilon$ in the second integral, this can be written as

$$(r+\rho+a_o)\bar{H}(\varepsilon)-\dot{\bar{H}}(\varepsilon)=\frac{\varepsilon}{\lambda_o-1}+a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}\int_{j=x_o}^{\infty}\lambda_o j^{-(\lambda_o+1)}(H(j)-H(x_o))\mathrm{d}j\,. \tag{A.12}$$

Make the following definition of a new variable X_o:

$$X_o(t)=(\lambda_o-1)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)\int_{\upsilon=t}^{\infty}(r+\rho+a_o)e^{-(r+\rho+a_o)(\upsilon-t)}\left(\int_{\varepsilon=x_o}^{\infty}\lambda_o\varepsilon^{-(\lambda_o+1)}(H(\varepsilon,\upsilon)-H(x_o,\upsilon))\mathrm{d}\varepsilon\right)\mathrm{d}\upsilon\,. \tag{A.13}$$

By differentiating with respect to time t this variable must satisfy the differential equation

$$\begin{aligned}(r+\rho+a_o)X_o-\dot{X}_o&=(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)x_o^{-\lambda_o}\bar{H}(x_o)\\&=(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)\int_{\varepsilon=x_o}^{\infty}\lambda_o\varepsilon^{-(\lambda_o+1)}(H(\varepsilon)-H(x_o))\mathrm{d}\varepsilon\,,\end{aligned} \tag{A.14}$$

which uses the definition of $\bar{H}(\varepsilon)$ in (A.10). Substituting into equation (A.12):

$$(r+\rho+a_o)\bar{H}(\varepsilon)-\dot{\bar{H}}(\varepsilon)=\frac{1}{\lambda_o-1}\left(\varepsilon+\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}\left((r+\rho+a_o)X_o-\dot{X}_o\right)}{(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}\right),$$

and by collecting terms this can be written as

$$(r+\rho+a_o)\left(\bar{H}(\varepsilon)-\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}}{(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}X_o\right)-\left(\dot{\bar{H}}(\varepsilon)-\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}}{(\lambda_o-1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)}\dot{X}_o\right)=\frac{\varepsilon}{\lambda_o-1}\,.$$

Since the right-hand side is time invariant and none of the variables are predetermined, it follows for each fixed ε there is a unique stable solution for $\bar{H}(\varepsilon)-a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}X_o/((\lambda_o-1)(r+\rho+a_o)(r+\rho+a_o(1-\delta_o^{\lambda_p})))$ that is time invariant and equal to $\varepsilon/((\lambda_o-1)(r+\rho+a_o))$. This demonstrates that for any given ε in a neighbourhood above y_o or any value below it, the function $\bar{H}(\varepsilon)$ is given by

$$\bar{H}(\varepsilon)=\frac{1}{(\lambda_o-1)(r+\rho+a_o)}\left(\varepsilon+\frac{a_o\delta_o^{\lambda_o}\varepsilon^{\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_p})}X_o\right). \tag{A.15}$$

Evaluating (A.15) at $\varepsilon = x_o$ and multiplying by $(\lambda_o - 1)(r+\rho+a_o)(r+\rho+a_o(1-\delta_o^{\lambda_p}))x_o^{-\lambda_o}$:

$$(\lambda_o - 1)(r+\rho+a_o)\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)x_o^{-\lambda_o}\bar{H}(x_o) = \left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)x_o^{1-\lambda_o} + a_o\delta_o^{\lambda_o}X_o\,,$$

and substituting into (A.14) shows that X_o satisfies a differential equation in the moving threshold x_o:

$$\left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)X_o - \dot{X}_o = \left(r+\rho+a_o(1-\delta_o^{\lambda_p})\right)x_o^{1-\lambda_o}\,. \tag{A.16}$$

Finally, evaluating (A.15) at $\varepsilon = y_o$ and substituting into (A.10):

$$\Sigma_o = \frac{\zeta_o^{\lambda_o}}{(1+\tau_h\omega_o^*)(\lambda_o-1)(r+\rho+a_o)}\left(y_o^{1-\lambda_o} + \frac{a_o\delta_o^{\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_o})}X_o\right)\,. \tag{A.17}$$

In summary, (A.3), (A.4), (A.6), (A.7), (A.8), (A.16), and (A.17) form a system of differential equations in y_o, x_o, X_o, Σ_o, $\underline{H}$, B_o, and U_o, which take as given Σ_k, q_o and ξ.

A.2.2 The moving rate in the ownership market

The flow of owner-occupiers who move within the city is denoted by N_o, and the moving rate is $n_o = N_o/h_o$. The group of existing homeowners h_o is made up of matches that formed at various points in the past and have survived to the present. Moving requires that homeowners receive an idiosyncratic shock, which has arrival rate a_o independent of history. A measure $a_o h_o$ of households thus decide whether to move.

All matches began as a viewing with some initial match quality ε. Using (1), the flow of viewings v_h done by home-buyers in the ownership market at a point in time is

$$v_h = q_o b_h = (1-\xi)\theta_o q_o u_o\,. \tag{A.18}$$

Initial match quality drawn in viewings is from a Pareto(ζ_o, λ_o) distribution (see 25). This match quality distribution has been truncated when transaction decisions were made and possibly when subsequent idiosyncratic shocks have occurred. Consider a group of surviving homeowners where initial match quality has been previously truncated at $\underline{\varepsilon}$. This group constitutes a fraction $\zeta_o^{\lambda_o}\underline{\varepsilon}^{-\lambda_o}$ of the initial measure of viewings, and the distribution of ε conditional on survival is Pareto$(\underline{\varepsilon}, \lambda_o)$. Among this group, consider those whose current match quality is a multiple Δ of original match quality ε, where Δ is equal to δ_o raised to the power of the number of past shocks received.

Now consider a new idiosyncratic shock. Current match quality becomes $\varepsilon' = \delta_o\Delta\varepsilon$ in terms of initial match quality ε. Moving is optimal if $\varepsilon' < x_o$, so only those with initial match quality $\varepsilon \geq x_o/(\delta_o\Delta)$ survive. Since $\delta_o < 1$ and $\delta_o y_o < x_o$, there is a range of variation in thresholds y_o and x_o that ensures $x_o/(\delta_o\Delta) > \underline{\varepsilon}$. Given the Pareto distribution, the proportion of the surviving group that does not move after the new shock is $\underline{\varepsilon}^{\lambda_o}(x_o/(\delta_o\Delta))^{-\lambda_o} = x_o^{-\lambda_o}\delta_o^{\lambda_o}\Delta^{\lambda_o}\underline{\varepsilon}^{\lambda_o}$. Since that surviving group is a fraction $\zeta_o^{\lambda_o}\underline{\varepsilon}^{-\lambda_o}$ of the original set of viewings, those that do not move after the new shock are a fraction $x_o^{-\lambda_o}\delta_o^{\lambda_o}\Delta^{\lambda_o}\underline{\varepsilon}^{\lambda_o} \times \zeta_o^{\lambda_o}\underline{\varepsilon}^{-\lambda_o} = (\zeta_o^{\lambda_o}x_o^{-\lambda_o}\delta_o^{\lambda_o}) \times \Delta^{\lambda_o}$ of that set of viewings. This is independent of any past truncation thresholds $\underline{\varepsilon}$ owing to the properties of the Pareto distribution.

The measure of the group choosing not to move after a new shock does depend on the total accumulated size Δ of past idiosyncratic shocks. Let Ξ_o be the integral of Δ^{λ_o} over the measure of current and past viewings done by households who have not yet exited the city. Since the size of the group choosing not to move is a common multiple $\zeta_o^{\lambda_o}x_o^{-\lambda_o}\delta_o^{\lambda_o}$ of Δ^{λ_o}, the measure of those choosing not to move after a new shock is $a_o\zeta_o^{\lambda_o}x_o^{-\lambda_o}\delta_o^{\lambda_o}\Xi_o$. Therefore, the size of the group of movers is

$$N_o = a_o h_o - a_o\zeta_o^{\lambda_o}x_o^{-\lambda_o}\delta_o^{\lambda_o}\Xi_o\,. \tag{A.19}$$

Since the arrival of idiosyncratic shocks is independent of history, a fraction a_o of the group used to define Ξ_o have Δ^{λ_o} reduced to $\delta_o^{\lambda_o}\Delta^{\lambda_o}$. Exit from the group occurs at rate ρ, and new viewings occur that start from $\Delta^{\lambda_o} = 1$ with measure v_h from (A.18). The differential equation for Ξ_o is thus

$$\dot{\Xi}_o = v_h + a_o(\delta_o^{\lambda_o}\Xi_o - \Xi_o) - \rho\Xi_o\,. \tag{A.20}$$

Define the following weighted average of current and past levels of home-buyer viewings v_h:

$$\bar{v}_h(t) = \int_{\upsilon \to -\infty}^{t} (\rho + a_o(1-\delta_o^{\lambda_o}))e^{-(\rho+a_o(1-\delta_o^{\lambda_o}))(t-\upsilon)} v_h(\upsilon)\mathrm{d}\upsilon \,,$$

and note that it satisfies the differential equation

$$\dot{\bar{v}}_h + (\rho + a_o(1-\delta_o^{\lambda_o}))\bar{v}_h = (\rho + a_o(1-\delta_o^{\lambda_o}))v_h \,. \tag{A.21}$$

A comparison of (A.20) and (A.21) shows that $\Xi_o = \bar{v}_h/(\rho + a_o(1-\delta_o^{\lambda_o}))$, and substituting this into (A.19) yields an equation for the moving rate $n_o = N_o/h_o$:

$$n_o = a_o - \frac{a_o \zeta_o^{\lambda_o} \delta_o^{\lambda_o} x_o^{-\lambda_o} \bar{v}_h}{(\rho + a_o(1-\delta_o^{\lambda_o}))h_o} \,. \tag{A.22}$$

Using the formula for $\bar{v}_h(t)$ above and (A.18), this confirms equation (33) for the moving rate n_o.

A.2.3 The threshold and value functions in the rental market

By adding together the Bellman equations (8) and (10) for the landlord and tenant value functions:

$$\begin{aligned} r(L(\varepsilon) + W(\varepsilon)) = \varepsilon - M - M_l + (\rho + a_l)(U_l - L(\varepsilon)) + \rho_l(U_o - L(\varepsilon)) + (1-\gamma)n_l(B_l - W(\varepsilon)) \\ + \gamma n_l(G_m(Z)(B_o - \bar{\chi}) + (1 - G_m(Z))B_l - W(\varepsilon)) - \rho W(\varepsilon) + \dot{L}(\varepsilon) + \dot{W}(\varepsilon) \,. \end{aligned}$$

Letting $J(\varepsilon) = L(\varepsilon) + W(\varepsilon)$ denote the joint value, this can be rearranged and simplified, noting $B_o - B_l = Z$ and $n_l = a_l + \rho_l$ from (14) and (34):

$$(r + \rho + n_l)J(\varepsilon) = \varepsilon - M - M_l + (\rho + a_l)U_l + \rho_l U_o + n_l B_l + \gamma n_l G_m(Z)(Z - \bar{\chi}) + \dot{J}(\varepsilon) \,. \tag{A.23}$$

Differentiating with respect to ε leads to the differential equation

$$(r + \rho + n_l)J'(\varepsilon) = 1 + \dot{J}'(\varepsilon) \,.$$

This differential equation has a unique non-explosive solution for $J'(\varepsilon)$ for any given value of ε:

$$J'(\varepsilon) = \frac{1}{r + \rho + n_l} \,.$$

This time-invariant solution ($\dot{J}'(\varepsilon) = 0$) implies the solution for $J(\varepsilon)$ takes the following form:

$$J(\varepsilon) = \underline{J} + \frac{\varepsilon}{r + \rho + n_l} \,, \tag{A.24}$$

where $\underline{J}$ can be time varying in general. Substituting back into (A.23) and noting $\dot{J}(\varepsilon) = \dot{\underline{J}}$ shows that $\underline{J}$ satisfies the differential equation

$$(r + \rho + n_l)\underline{J} = n_l B_l + (\rho + a_l)U_l + \rho_l U_o - M - M_l + \gamma n_l G_m(Z)(Z - \bar{\chi}) + \dot{\underline{J}} \,. \tag{A.25}$$

The joint rental surplus from (35) is linked to $J(\varepsilon)$ by

$$\Sigma_l(\varepsilon) = J(\varepsilon) - C_l - C_w - B_l - U_l \,, \tag{A.26}$$

and together with (A.24), the definition of the rental transaction threshold y_l in (37) implies

$$y_l = (r + \rho + n_l)(B_l + U_l - \underline{J} + C_l + C_w) \,. \tag{A.27}$$

Using (37), (A.24), and (A.26), it follows that $\Sigma_l(\varepsilon) = (\varepsilon - y_l)/(r + \rho + a_l)$. The Pareto distribution in (25) then implies the expected rental surplus from (39) is

$$\Sigma_l = \frac{\zeta_l^{\lambda_l} y_l^{1-\lambda_l}}{(\lambda_l - 1)(r + \rho + n_l)} \,. \tag{A.28}$$

Using $\Sigma_l^l(\varepsilon) = L(\varepsilon) + \Pi(\varepsilon) - C_l - U_l = \omega_l \Sigma_l(\varepsilon)$, (35), and (39), the Bellman equation (7) for U_l becomes

$$(r + \rho_l)U_l - \dot{U}_l = \omega_l \theta_l q_l \Sigma_l - M + \rho_l U_o \,, \tag{A.29}$$

and similarly, with $\Sigma_l^w(\varepsilon) = W(\varepsilon) - \Pi(\varepsilon) - C_w - B_l = (1-\omega_l)\Sigma_l$, the Bellman equation (9) for B_l becomes

$$(r+\rho)B_l - \dot{B}_l = (1-\omega_l)q_l\Sigma_l - F_w\,. \tag{A.30}$$

The credit cost threshold Z satisfies (14). In summary, equations (A.25), (A.27), (A.28), (A.29), (A.30), and (14) determine y_l, Z, Σ_l, $\underline{J}$, B_l, and U_l.

The Bellman equation (8) can be written as follows:

$$(r+\rho+n_l)(L(\varepsilon)-U_l) = R(\varepsilon) - M - M_l - (r+\rho_l)U_l + \rho_l U_o + \dot{L}(\varepsilon)\,,$$

and substituting from (A.29) implies that rents $R(\varepsilon)$ are

$$R(\varepsilon) = M_l + \omega_l\theta_l q_l \Sigma_l + (r+\rho+n_l)(L(\varepsilon)-U_l) - (\dot{L}(\varepsilon)-\dot{U}_l)\,.$$

Since $\Lambda^l(\varepsilon) = L(\varepsilon) - U_l$, which entails $\dot{L}(\varepsilon) - \dot{U}_l = \dot{\Lambda}^l(\varepsilon)$, the division of the surplus $\Lambda^l(\varepsilon) = \omega_l\Lambda(\varepsilon)$ implies

$$R(\varepsilon) = M_l + \omega_l\theta_l q_l\Sigma_l + \omega_l\left((r+\rho+n_l)\Lambda(\varepsilon) - \dot{\Lambda}(\varepsilon)\right)\,.$$

By substituting from equation (35) and noting $\Sigma_l(\varepsilon) = \Lambda(\varepsilon) - (C_l + C_w)$, the expression for rents becomes

$$R(\varepsilon) = M_l + \omega_l(r+\rho+n_l)(C_w+C_l) + \omega_l\theta_l q_l\Sigma_l + \omega_l\left((r+\rho+n_l)\Sigma_l(\varepsilon) - \dot{\Sigma}_l(\varepsilon)\right)\,.$$

Noting that $\dot{\Sigma}_l(\varepsilon) = -\dot{y}_l/(r+\rho+n_l)$ for all ε, and using the definition of average rents R from (38):

$$R = M_l + \omega_l(r+\rho+n_l)(C_w+C_l) + \omega_l\theta_l q_l\Sigma_l + \omega_l\left((r+\rho+n_l)\frac{\Sigma_l}{\pi_l} + \frac{\dot{y}_l}{r+\rho+n_l}\right)\,,$$

which can be written as

$$R = M_l + \omega_l(r+\rho+n_l)(C_l+C_w) + \omega_l(r+\rho+n_l+\theta_l q_l\pi_l)\frac{\Sigma_l}{\pi_l} + \frac{\omega_l}{r+\rho+n_l}\dot{y}_l\,. \tag{A.31}$$

A.2.4 The relationship between market tightness across the two markets

The total measures of properties in (11) and households in (12) together with the definitions of the fraction of investors and market tightnesses from (1) imply

$$((1-\xi)\,\theta_o - 1)\,u_o + (\theta_l - 1)\,u_l = \psi - 1\,, \tag{A.32}$$

which yields a relationship between the market tightnesses θ_o and θ_l across the two markets given stocks of properties for sale u_o and properties for rent u_l, and the fraction ξ of investors among buyers.

A.2.5 Average match quality and the average value functions

Let Ψ_h denote the integral of ε over all current owner-occupiers. There is a flow of $v_h\pi_o$ of new owner-occupier matches, and using (1), (18), and (A.18), the size of this flow can be expressed as $(1-\kappa)s_o u_o$. Since the transaction threshold is y_o, the Pareto distribution (25) implies the average value of ε in these new matches is $\lambda_o y_o/(\lambda_o - 1)$, so these new matches add to Ψ_h at rate $(1-\kappa)s_o u_o\lambda_o y_o/(\lambda_o-1)$ over time.

Matches are destroyed (sending the contribution to Ψ_h to zero) if households exit the city or match-quality shocks arrive and households choose to move. Households exit the city at rate ρ, reducing Ψ_h by $\rho\Psi_h$. Match-quality shocks arrive randomly at rate a_o for the measure h_o of owner-occupiers, leading to a flow N_o of movers out of the group $a_o h_o$ receiving a shock, which reduces the contribution to Ψ_h of those N_o to zero. For the group of size $a_o h_o - N_o$ that receives a shock but does not move, the conditional distribution of surviving match quality ε is truncated at x_o, which is a Pareto distribution with shape parameter λ_o across all cohorts within that group. The mean of the truncated distribution is therefore $\lambda_o x_o/(\lambda_o - 1)$. Putting together all these effects on Ψ_h, the following differential equation must hold:

$$\dot{\Psi}_h = (1-\kappa)s_o u_o\frac{\lambda_o y_o}{\lambda_o - 1} + \left(N_o\times 0 + (a_o h_o - N_o)\times\frac{\lambda_o x_o}{\lambda_o - 1} - a_o\Psi_h\right) - \rho\Psi_h\,.$$

Average match quality among owner-occupiers is $Q_h = \Psi_h/h_o$, and thus $\dot{Q}_h = \dot{\Psi}_h/h_o - (\dot{h}_o/h_o)Q_h = \dot{\Psi}_h/h_o -$

$(((1-\kappa)s_o u_o/h_o)-(n_o+\rho))Q_h$, where the second equation uses the differential equation for h_o in (22). Together with the equation for $\dot{\Psi}_h$ above and the definition of the moving rate $n_o = N_o/h_o$, average match quality Q_h must satisfy the differential equation (45).

Let Ψ_l denote the equivalent summation of surviving match quality in the rental market. Rental viewings occur at rate $v_l = q_l b_l$, and with leasing threshold y_l for match quality, these add to Ψ_l at rate $q_l \pi_l \lambda_l y_l b_l/(\lambda_l - 1)$ over time. Using (1) and (19), the flow increment to Ψ_l is $s_l u_l \lambda_l y_l/(\lambda_l - 1)$. Matches are destroyed if households exit the city (rate ρ), landlords must sell up (rate ρ_l), or match quality falls to zero (rate a_l). The differential equation for Ψ_l is thus $\dot{\Psi}_l = s_l u_l(\lambda_l y_l/(\lambda_l-1)) - (a_l+\rho_l+\rho)\Psi_l$. Average match quality for tenants is $Q_l = \Psi_l/h_l$, hence $\dot{Q}_l = (\dot{\Psi}_l/h_l) - (\dot{h}_l/h_l)Q_l$, and by substituting $\dot{h}_l/h_l = (s_l u_l/h_l) - (n_l+\rho)$ from (23), the differential equation for Q_l is (46), which uses $n_l = a_l + \rho_l$ from (34).

Let $G_h(\varepsilon)$ denote the distribution function of current match quality ε. The average homeowner value function $H(\varepsilon)$ across all h_o current matches and the integral of these values are denoted by H and Θ:

$$H = \int_\varepsilon H(\varepsilon)\mathrm{d}G_h(\varepsilon), \quad \text{and } \Theta = h_o H = \int_\varepsilon H(\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon, \quad \text{where } \varsigma(\varepsilon) = h_o G_h'(\varepsilon).$$

Differentiating Θ with respect to time implies $\dot{\Theta} = \int_\varepsilon \left(\dot{H}(\varepsilon)\varsigma(\varepsilon) + H(\varepsilon)\dot{\varsigma}(\varepsilon)\right)\mathrm{d}\varepsilon$ and hence

$$r\Theta - \dot{\Theta} = \int_\varepsilon \left(rH(\varepsilon) - \dot{H}(\varepsilon)\right)\varsigma(\varepsilon)\mathrm{d}\varepsilon - \int_\varepsilon H(\varepsilon)\dot{\varsigma}(\varepsilon)\mathrm{d}\varepsilon. \tag{A.33}$$

Shocks scaling down match quality ε to $\delta_o\varepsilon$ occur with arrival rate a_o, which triggers moving if match quality falls below x_o. There is also exogenous exit from the city at rate ρ. New matches form at rate S_h and begin with ε having distribution function $G_o(\varepsilon)/\pi_o$ for $\varepsilon \geq y_o$, where $\pi_o = 1 - G_o(y_o)$. The dynamics of the density function $\varsigma(\varepsilon) = h_o G_h'(\varepsilon)$ describing the distribution of ε across all matches are thus:

$$\dot{\varsigma}(\varepsilon) = \begin{cases} -(a_o+\rho)\varsigma(\varepsilon) & \text{if } \varepsilon < x_o \\ a_o\delta_o^{-1}\varsigma(\delta_o^{-1}\varepsilon) - (a_o+\rho)\varsigma(\varepsilon) & \text{if } x_o \leq \varepsilon < y_o \\ (S_h/\pi_o)G_o'(\varepsilon) + a_o\delta_o^{-1}\varsigma(\delta_o^{-1}\varepsilon) - (a_o+\rho)\varsigma(\varepsilon) & \text{if } y_o \leq \varepsilon \end{cases}.$$

It follows that

$$\begin{aligned}\int_\varepsilon H(\varepsilon)\dot{\varsigma}(\varepsilon)\mathrm{d}\varepsilon &= \frac{S_h}{\pi_o}\int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) + \frac{a_o}{\delta_o}\int_{\varepsilon=x_o} H(\varepsilon)\varsigma\left(\frac{\varepsilon}{\delta_o}\right)\mathrm{d}\varepsilon - (a_o+\rho)h_o H \\ &= q_o b_h \int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) + a_o\int_{\varepsilon=x_o/\delta_o} H(\delta_o\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon - (a_o+\rho)h_o H,\end{aligned} \tag{A.34}$$

which uses $S_h = q_o\pi_o b_h$ and a change of variable $\varepsilon' = \varepsilon/\delta_o$ in the second term. Using the Bellman equation (6) for $H(\varepsilon)$ and the definitions of the average homeowner value H and average match quality Q_h:

$$\begin{aligned}\int_\varepsilon \left(rH(\varepsilon) - \dot{H}(\varepsilon)\right)\varsigma(\varepsilon)\mathrm{d}\varepsilon &= \int_\varepsilon (\varepsilon - M)\mathrm{d}\varsigma(\varepsilon) + a_o\int_{\varepsilon=x_o/\delta_o}^{\infty} H(\delta_o\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon + a_o(B_o+U_o)\int_{\varepsilon=0}^{x_o/\delta_o}\varsigma(\varepsilon)\mathrm{d}\varepsilon \\ - a_o\int_\varepsilon H(\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon + \rho\int_\varepsilon (U_o - H(\varepsilon))\varsigma(\varepsilon)\mathrm{d}\varepsilon &= (Q_h - M)h_o + a_o\int_{\varepsilon=x_o/\delta_o}^{\infty} H(\delta_o\varepsilon)\varsigma(\varepsilon)\mathrm{d}\varepsilon \\ &+ n_o(B_o+U_o)h_o - a_o H h_o + \rho(U_o - H)h_o, \quad \text{where } \int_{\varepsilon=0}^{x_o/\delta_o} a_o\varsigma(\varepsilon) = n_o h_o.\end{aligned} \tag{A.35}$$

The final equation links the number of moves $n_o h_o$ within the city to the integral of $a_o\varsigma(\varepsilon)$ up to $\varepsilon = x_o/\delta_o$. Substituting equations (A.34) and (A.35) into (A.33):

$$r\Theta - \dot{\Theta} = (Q_h - M)h_o + n_o(B_o+U_o) + \rho U_o - q_o b_h\int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon).$$

Since $H = \Theta/h_o$ implies $\dot{H} = \dot{\Theta}/h_o - H\dot{h}_o/h_o$, the equation above and (22) for $\dot{h}_o$ imply the average homeowner value function satisfies the following Bellman equation, noting $(1-\kappa)s_o u_o = \pi_o q_o b_h$:

$$rH = Q_h - M - n_o(H - B_o - U_o) - \rho(H - U_o) - \frac{\pi_o q_o b_h}{h_o}\left(\frac{1}{\pi_o}\int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) - H\right) + \dot{H}. \tag{A.36}$$

Let L, W, and $\bar{R}$ be the average values of $L(\varepsilon)$, $W(\varepsilon)$, and $R(\varepsilon)$ across the distribution of match quality ε for all surviving matches in the rental market. The same method used to derive (A.36) can be applied to show the equivalent for L of the Bellman equation (8) for $L(\varepsilon)$ is:

$$rL = \bar{R} - M - M_l - (a_l + \rho)(L - U_l) - \rho_l(L - U_o) - \frac{\pi_l q_l b_l}{h_l}\left(\frac{1}{\pi_l}\int_{\varepsilon=y_l} L(\varepsilon)\mathrm{d}G_l(\varepsilon) - L\right) + \dot{L}, \qquad \text{(A.37)}$$

and the equivalent of (10) in terms of W is as follows, where Q_l is average rental match quality:

$$rW = Q_l - \bar{R} - (1-\gamma)n_l(W - B_l) - \gamma n_l\left(W - G_m(Z)(B_o - \bar{\chi}) - (1 - G_m(Z))B_l\right) \\ - \rho W - \frac{\pi_l q_l b_l}{h_l}\left(\frac{1}{\pi_l}\int_{\varepsilon=y_l} W(\varepsilon)\mathrm{d}G_l(\varepsilon) - W\right) + \dot{W}. \qquad \text{(A.38)}$$

A.2.6 Welfare

With H, L, and W denoting the average values of $H(\varepsilon)$, $L(\varepsilon)$, and $W(\varepsilon)$ over the distributions of all surviving matches, aggregate welfare Ω is defined as follows:

$$\Omega = h_o H + h_l(L + W) + b_h B_o + b_l B_l + b_k K + u_o U_o + u_l U_l + \Omega_\tau + \Omega_e, \qquad \text{(A.39)}$$

where Ω_τ is the present value of the stream of tax revenue $\Gamma = \tau_h P S_h + \tau_k P_k S_k$, and Ω_e is the expected present values of new entrants to the city. Differentiating Ω with respect to t shows it satisfies the differential equation:

$$r\Omega = h_o(rH - \dot{H}) + h_l(rL - \dot{L}) + h_l(rW - \dot{W}) + b_k(rK - \dot{K}) + b_h(rB_o - \dot{B}_l) + b_l(rB_l - \dot{B}_l) \\ + u_o(rU_o - \dot{U}_o) + u_l(rU_l - \dot{U}_l) + (r\Omega_\tau - \dot{\Omega}_\tau) + (r\Omega_e - \dot{\Omega}_e) - H\dot{h}_o - (L + W)\dot{h}_l \\ - B_o\dot{b}_h - B_l\dot{b}_l - K\dot{b}_k - U_o\dot{u}_o - U_l\dot{u}_l + \dot{\Omega}.$$

Substituting Bellman equations (3), (4), (5), (7), (9), (A.36), (A.37), (A.38), $r\Omega_\tau = \tau_h P S_h + \tau_k P_k S_k + \dot{\Omega}_\tau$, $r\Omega_e = \rho\psi((1 - G_m(Z))B_l + G_m(Z)(B_o - \bar{\chi})) + \dot{\Omega}_e$, and laws of motion (16), (17), (20), (21), (22), and (23):

$$r\Omega = h_o\left(Q_h - M + n_o(B_o + U_o - H) + \rho(U_o - H) - \frac{\pi_o q_o b_h}{h_o}\left(\frac{1}{\pi_o}\int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) - H\right)\right) \\ + h_l\left(\bar{R} - M - M_l + (a_l + \rho)(U_l - L) + \rho_l(U_o - L) - \frac{\pi_l q_l b_l}{h_l}\left(\frac{1}{\pi_l}\int_{\varepsilon=y_l} L(\varepsilon)\mathrm{d}G_l(\varepsilon) - L\right)\right) \\ + h_l\Bigg(Q_l - \bar{R} + (1-\gamma)n_l(B_l - W) + \gamma n_l\left(G_m(Z)(B_o - \bar{\chi}) + (1 - G_m(Z))B_l - W\right) \\ - \rho W - \frac{\pi_l q_l b_l}{h_l}\left(\frac{1}{\pi_l}\int_{\varepsilon=y_l} W(\varepsilon)\mathrm{d}G_l(\varepsilon) - W\right)\Bigg) + b_k\left(-F_k + q_o U_l - q_o(1 + \tau_k)P_k - q_o C_k - q_o K\right) \\ + b_h\left(-F_h + q_o\int_{\varepsilon=y_o} H(\varepsilon)\mathrm{d}G_o(\varepsilon) - q_o\pi_o(1 + \tau_h)P - q_o\pi_o C_h - q_o\pi_o B_o - \rho B_o\right) \\ + b_l\left(-F_w + q_l\int_{\varepsilon=y_l} W(\varepsilon)\mathrm{d}G_o(\varepsilon) - q_l\pi_l\Pi - q_l\pi_l C_w - q_l\pi_l B_l - \rho B_l\right) + (\tau_h P S_h + \tau_k P_k S_k) \\ + u_o\left(-M + \theta_o q_o(1-\xi)\pi_o P - \theta_o q_o(1-\xi)\pi_o C_u - \theta_o q_o(1-\xi)\pi_o U_o + q_o\theta_o\xi P_k - q_o\theta_o\xi C_u - q_o\theta_o\xi U_o\right) \\ + u_l\left(-M + \theta_l q_l\int_{\varepsilon=y_l} L(\varepsilon)\mathrm{d}G_o(\varepsilon) + \theta_l q_l\pi_l\Pi - \theta_l q_l\pi_l C_l - \theta_l q_l\pi_l U_l + \rho_l U_o - \rho_l U_l\right) \\ - H\left((1-\kappa)s_o u_o - (n_o + \rho)h_o\right) - (L + W)\left(s_l u_l - (n_l + \rho)h_l\right) - U_o\left((n_o + \rho)h_o + \rho_l(h_l + u_l) - s_o u_o\right) \\ - U_l\left((a_l + \rho)h_l + \kappa s_o u_o - (s_l + \rho)u_l\right) - B_o\left(n_o h_o + (\gamma n_l h_l + \rho\psi)G_m(Z) - (q_o\pi_o + \rho)b_h\right) \\ - B_l\left((1-\gamma)n_l h_l + (\gamma n_l h_l + \rho\psi)(1 - G_m(Z)) - (q_l\pi_l + \rho)b_l\right) - K\dot{b}_k \\ + \rho\psi\left((1 - G_m(Z))B_l + G_m(Z)(B_o - \bar{\chi})\right) + \dot{\Omega}. \qquad \text{(A.40)}$$

Observe that all the value functions on the right-hand side cancel out, reflecting transitions of particular individuals between states. For H, note that $(1-\kappa)s_o u_o = \pi_o q_o b_h$; for L and W, note that $s_l u_l = \pi_l q_l b_l$ and

$n_l = a_l + \rho_l$; for the integral over $L(\varepsilon)$, $\theta_l q_l u_l = q_l b_l$; for U_o, $\theta_o q_o((1-\xi)\pi_o + \xi) = s_o$; for U_l, $\theta_l q_l \pi_l = s_l$; and $K = 0$ because of the free-entry condition (A.51)

Next, observe that payments of rent $\bar{R}$ and initial tenancy fees Π cancel out from (A.40) (noting $b_l = \theta_l u_l$), as do house prices P and P_k (noting $\theta_o(1-\xi)u_o = b_h$ and $\theta_o \xi u_o = b_k$). This is because such payments are simply transfers among individuals that net out. The same is true for prices inclusive of tax (noting $S_h = q_o \pi_o b_h$ and $S_k = q_o b_k$) because of the assumption that tax revenue is used to provide public goods.

With value functions, rents, and prices cancelling out from (A.40), the Bellman equation for welfare Ω is (44), where the coefficient of M comes from noting $h_o + h_l + u_o + u_l = 1$ and the coefficients on transaction costs come from $S_h = q_o \pi_o b_h$, $S_k = q_o b_k$, $S_o = S_h + S_k = q_o \theta_o((1-\xi)\pi_o + \xi)u_o$, and $S_l = \theta_l q_l \pi_l u_l = q_l \pi_l b_l$.

A.3 Existence of a steady state and the solution method

Equations for a steady state In a steady state where $\dot{B}_o = 0$ and $\dot{U}_o = 0$, the Bellman equations (A.7) and (A.8) become

$$(r+\rho)B_o = -F_h + (1-\omega_o^*)q_o \Sigma_o\,, \quad \text{and} \tag{A.41}$$

$$rU_o = \theta_o q_o((1-\xi)\omega_o^* \Sigma_o + \xi \omega_k^* \Sigma_k) - M\,. \tag{A.42}$$

Substituting from (A.8) into (A.6):

$$y_o = x_o + (r+\rho+a_o)\left(C_h + C_u + \tau_h\left(C_u - \frac{M}{r} + \frac{\theta_o q_o((1-\xi)\omega_o^* \Sigma_o + \xi \omega_k^* \Sigma_k)}{r}\right)\right). \tag{A.43}$$

The joint surplus $\Sigma_k = F_k/(1-\omega_k^*)q_o$ from selling to an investor comes from equation (43). In a steady state with $\dot{\underline{H}} = 0$, (A.3) implies that $(r+\rho+a_o)\underline{H} = a_o B_o + (\rho + a_o)U_o - M$. Substituting into (A.4) implies $x_o = (r+\rho+a_o)(B_o + U_o) - a_o B_o - (\rho + a_o)U_o + M$ and hence

$$x_o = M + (r+\rho)B_o + rU_o\,.$$

Then substituting the values of B_o and U_o from (A.41) and (A.42) yields

$$x_o + F_h = (1 - \omega_o^* + (1-\xi)\omega_o^* \theta_o)\, q_o \Sigma_o + \theta_o q_o \xi \omega_k^* \Sigma_k\,. \tag{A.44}$$

With $\dot{X}_o = 0$ in steady state, equation (A.16) shows that $X_o = x_o^{1-\lambda_o}$. Substitution into (A.17) implies the expected joint surplus is

$$\Sigma_o = \frac{\zeta_o^{\lambda_o}}{(r+\rho+a_o)(\lambda_o - 1)(1+\tau_h \omega_o^*)}\left(y_o^{1-\lambda_o} + \frac{a_o \delta_o^{\lambda_o} x_o^{1-\lambda_o}}{r+\rho+a_o\left(1-\delta_o^{\lambda_o}\right)}\right). \tag{A.45}$$

The average transaction price P from (30) can be written as follows by using equation (A.42) for U_o:

$$P = \left(\frac{r + \theta_o q_o (1-\xi)\pi_o}{r}\right)\left(\frac{\omega_o^* \Sigma_o}{\pi_o}\right) + \frac{\theta_o q_o \xi \omega_k^* \Sigma_k}{r} + C_u - \frac{M}{r}\,. \tag{A.46}$$

With $\dot{B}_l = 0$ and $\dot{U}_l = 0$, the Bellman equations (A.29) and (A.30) become

$$rB_l = -F_w + (1-\omega_l)q_l \Sigma_l - \rho B_l\,, \text{ and} \tag{A.47}$$

$$(r+\rho_l)U_l = \omega_l \theta_l q_l \Sigma_l - M + \rho_l U_o\,. \tag{A.48}$$

In steady state, $\dot{\underline{J}} = 0$, which yields $(r+\rho+n_l)\underline{J} = n_l B_l + (\rho + a_l)U_l + \rho_l U_o - M - M_l + \gamma n_l G_m(Z)(Z - \bar{\chi})$ using (A.25). Substituting into (A.27) and using $n_l = a_l + \rho_l$ implies

$$y_l = M + M_l + (r+\rho)B_l + (r+\rho_l)U_l - \rho_l U_o + (r+\rho+n_l)(C_l + C_w) - \gamma n_l G_m(Z)(Z-\bar{\chi})\,,$$

and by using (A.47) and (A.48) this becomes

$$y_l = M_l - F_w + (r + n_l + \rho)(C_w + C_l) - \gamma n_l G_m(Z)(Z - \bar{\chi}) + (1 - \omega_l + \omega_l \theta_l)q_l \Sigma_l\,. \tag{A.49}$$

The rent equation (A.31) in steady state is

$$R = M_l + \omega_l(r+\rho+n_l)(C_l+C_w) + \omega_l(r+\rho+n_l+\theta_l q_l \pi_l)\frac{\Sigma_l}{\pi_l}\,. \tag{A.50}$$

Multiplying both sides of (42) by $r+\rho_l$ and substituting for $(r+\rho_l)U_l$ from (A.48) leads to

$$\omega_l\theta_l q_l \Sigma_l = M - \rho_l U_o + (r+\rho_l)(1+\tau_k)U_o + (r+\rho_l)\left((1+\tau_k)C_u + C_k + (1+\tau_k\omega_k^*)\Sigma_k\right)\,.$$

Using $(r+\rho_l)(1+\tau_k)U_o - \rho_l U_o = (1+\tau_k(1+(\rho_l/r))rU_o$ and substituting from (A.42) implies:

$$\omega_l\theta_l q_l \Sigma_l = \left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\theta_o q_o\left((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k\right) \\ + (r+\rho_l)\left((1+\tau_k)C_u + C_k + (1+\tau_k\omega_k^*)\Sigma_k\right) - \tau_k\left(1+\frac{\rho_l}{r}\right)M\,. \tag{A.51}$$

By substituting B_o and B_l from (A.41) and (A.47) into (14):

$$(1-\omega_o^*)q_o\Sigma_o - (1-\omega_l)q_l\Sigma_l = (r+\rho)Z + F_h - F_w\,. \tag{A.52}$$

The price paid by investors in equilibrium is obtained from (41) and (A.42):

$$P_k = C_u + \frac{\theta_o q_o((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k) - M}{r} + \omega_k^*\Sigma_k\,. \tag{A.53}$$

Imposing a steady state $\dot{Q}_h = 0$ and $\dot{Q}_l = 0$ in the match quality equations (45) and (46) implies

$$Q_h = \frac{\lambda_o}{\lambda_o - 1}\left(\frac{n_o+\rho}{a_o+\rho}y_o + \frac{a_o-n_o}{a_o+\rho}x_o\right), \quad \text{and} \quad Q_l = \frac{\lambda_l}{\lambda_l-1}y_l\,,$$

which also make use of $\dot{h}_o = 0$, $\dot{h}_l = 0$, and (22) and (23).

The solution method is to reduce the problem to a numerical search over the fraction ξ of investors among buyers and ownership-market tightness θ_o to find the roots of two equations representing equilibrium in the ownership and rental markets.

Ownership-market transaction threshold Conditional on ξ and θ_o, within this search, there is also a numerical search to find the transaction threshold y_o in the ownership market. Equation (43) implies $q_o\Sigma_k = F_k/(1-\omega_k^*)$ and equation (A.44) implies $q_o\Sigma_o = (x_o + F_h - \xi\theta_o q_o\omega_k^*\Sigma_k)/(1-\omega_o^* + (1-\xi)\omega_o^*\theta_o)$. Together:

$$\theta_o q_o\left((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k\right) = \frac{\omega_o^*\theta_o}{1-\omega_o^*+(1-\xi)\omega_o^*\theta_o}\left((1-\xi)(x_o+F_h) + \xi\frac{(1-\omega_o^*)\omega_k^*}{\omega_o^*(1-\omega_k^*)}F_k\right).$$

Taking a value of y_o, the moving threshold x_o must satisfy (A.43), and substituting the expression above yields a linear equation for x_o that can be solved in terms of y_o:

$$x_o = \frac{y_o - (r+\rho+a_o)\left(C_h + (1+\tau_h)C_u - \tau_h\frac{M}{r} + \tau_h\frac{\theta_o\omega_o^*}{1-\omega_o^*+(1-\xi)\omega_o^*\theta_o}\left(\frac{(1-\xi)F_h}{r} + \frac{\xi(1-\omega_o^*)\omega_k^*F_k}{\omega_o^*(1-\omega_k^*)r}\right)\right)}{1+\tau_h\left(\frac{(1-\xi)\omega_o^*\theta_o}{1-\omega_o^*+(1-\xi)\omega_o^*\theta_o}\right)\left(\frac{r+\rho+a_o}{r}\right)}\,. \tag{A.54}$$

Now combine equations (43), (A.44), (A.45), and substitute $q_o = A_o\theta_o^{-\eta_o}$ from (24):

$$x_o + F_h - \frac{(1-\omega_o^*+(1-\xi)\omega_o^*\theta_o)A_o\theta_o^{-\eta_o}\zeta_o^{\lambda_o}}{(1+\tau_h\omega_o^*)(r+\rho+a_o)(\lambda_o-1)}\left(y_o^{1-\lambda_o} + \frac{a_o\delta_o^{\lambda_o}x_o^{1-\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_o})}\right) - \frac{\xi\theta_o\omega_k^*F_k}{1-\omega_k^*} = 0\,. \tag{A.55}$$

Observe that the left-hand side of (A.55) is strictly increasing in x_o and y_o. As the value of x_o implied by (A.54) is strictly increasing in y_o, it follows that any solution of (A.54) and (A.55) for x_o and y_o is unique. Since the left-hand side of (A.55) is sure to be positive for large y_o and x_o, existence of a solution can be confirmed by checking whether the left-hand side is negative at $y_o = \zeta_o$, the minimum value of y_o.

Ownership-market variables Once y_o is found, the transaction probability in the ownership market conditional on a viewing is $\pi_o = (\zeta_o/y_o)^{\lambda_o}$. This yields κ from (18) given the value of ξ. Moreover, given that $q_o = A_o \theta_o^{-\eta_o}$ is known conditional on θ_o, the sales rate s_o is found using (19). The moving threshold x_o is obtained from (A.54), and it can be verified whether $\delta_o y_o < x_o$ is satisfied. The surplus Σ_o is found by substituting the thresholds into (A.45), and $\Sigma_k = F_k/((1-\omega_k^*)q_o)$ comes from (43).

A steady state has $\dot{u}_o = 0$ and $\dot{h}_o = 0$, so (20) and (22) require

$$s_o u_o = (n_o + \rho)h_o + \rho_l(h_l + u_l), \quad \text{and} \tag{A.56}$$

$$(1-\kappa)s_o u_o = (n_o+\rho)h_o. \tag{A.57}$$

Since (11) implies $h_l + u_l = 1 - h_o - u_o$, dividing both sides of (A.56) by $\rho_l > 0$ and substituting for $h_l + u_l$ implies $u_o + h_o - ((n_o+\rho)/\rho_l)h_o + (s_o/\rho_l)u_o = 1$. Equation (A.57) implies $h_o = ((1-\kappa)s_o/(n_o+\rho))u_o$, and substituting into the previous equation for u_o and solving:

$$u_o = \frac{1}{1 + \frac{(1-\kappa)s_o}{n_o+\rho} + \frac{\kappa s_o}{\rho_l}}, \quad \text{and} \quad h_o = \frac{(1-\kappa)s_o}{n_o+\rho}u_o. \tag{A.58}$$

This yields the homeownership rate h from the formula given in section 3.7.

Evaluating the moving rate equation (33) at a steady state and substituting $\zeta_o^{\lambda_o} = \pi_o y_o^{\lambda_o}$:

$$n_o = a_o - \frac{a_o \delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1-\delta_o^{\lambda_o}\right)} \frac{(1-\xi)\theta_o q_o \pi_o u_o}{h_o}.$$

Equations (18) and (19) imply that $(1-\xi)\theta_o q_o \pi_o = (1-\kappa)s_o$, and hence using (A.57), $(1-\xi)\theta_o q_o \pi_o u_o/h_o = n_o + \rho$. Substituting this into the above yields an equation in n_o, which has the solution given in footnote 24.

Rental-market variables The moving rate $n_l = a_l + \rho_l$ in the rental market is given by parameters according to (34). Conditional on θ_o and ξ, there is also a numerical search for the transaction threshold y_l in the rental market. Given a value of y_l, the implied transaction probability from (38) is $\pi_l = (\zeta_l/y_l)^{\lambda_l}$. Using the formula (A.28) for the rental-market surplus:

$$\Sigma_l = \frac{\pi_l y_l}{(\lambda_l - 1)(r+\rho+n_l)}.$$

Observe that $\omega_l \theta_l q_l \Sigma_l = \omega_l y_l s_l/((\lambda_l-1)(r+\rho+n_l))$, where $s_l = \theta_l q_l \pi_l$ is the letting rate from (19). By using this to substitute for the left-hand side of the equation (A.51), the letting rate implied by y_l is

$$s_l = \frac{(\lambda_l-1)(r+\rho+n_l)}{\omega_l y_l}\Bigg(\left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\theta_o q_o\left((1-\xi)\omega_o^*\Sigma_o + \xi\omega_k^*\Sigma_k\right) \\ + (r+\rho_l)\left((1+\tau_k)C_u + C_k + (1+\tau_k\omega_k^*)\Sigma_k\right) - \tau_k\left(1+\frac{\rho_l}{r}\right)M\Bigg), \tag{A.59}$$

where the surpluses Σ_o and Σ_k were obtained when the ownership-market variables were found. Equation (2) gives the meeting rate $q_l = A_l\theta_l^{-\eta_l}$, and hence the letting rate $s_l = \theta_l q_l \pi_l$ satisfies $s_l = A_l \pi_l \theta_l^{1-\eta_l}$. The implied market tightness in the rental market is

$$\theta_l = \left(\frac{s_l}{A_l \pi_l}\right)^{\frac{1}{1-\eta_l}}, \tag{A.60}$$

and this also gives $q_l = A_l \theta_l^{-\eta_l}$.

In steady state, $\dot{u}_l = 0$ and $\dot{h}_l = 0$, hence equations (21) and (23) require

$$(s_l + \rho_l)u_l = (a_l+\rho)h_l + \kappa s_o u_o, \quad \text{and} \tag{A.61}$$

$$s_l u_l = (n_l + \rho)h_l. \tag{A.62}$$

Equations (11) and (A.62) imply $h_l + u_l = 1 - h_o - u_o$ and $h_l = (s_l/(n_l+\rho))u_l$. Combining these two equations and using the known values of h_o and u_o:

$$u_l = \frac{1-h_o-u_o}{1+\frac{s_l}{n_l+\rho}}, \quad \text{and} \quad h_l = \frac{s_l}{n_l+\rho}u_l\,. \tag{A.63}$$

Since (11) will hold and (A.56), (A.57), and (A.62) have been imposed, equation (A.61) holds automatically.

The steady state also has $\dot{b}_h = 0$ and $\dot{b}_l = 0$, which means that the following must hold:

$$(q_o\pi_o+\rho)b_h = n_o h_o + (\gamma n_l h_l + \rho\psi)G_m(Z)\,, \quad \text{and} \tag{A.64}$$

$$(q_l\pi_l+\rho)b_l = (1-\gamma)n_l h_l + (\gamma n_l h_l + \rho\psi)(1-G_m(Z))\,. \tag{A.65}$$

Since (1) implies $b_h = (1-\xi)\theta_o u_o$, which is known, the value of $G_m(Z)$ is obtained by rearranging (A.64):

$$G_m(Z) = \frac{(\rho+q_o\pi_o)(1-\xi)\theta_o u_o - n_o h_o}{\gamma n_l h_l + \rho\psi}\,,$$

and it can be checked that $G_m(Z)$ is a well defined probability. Given that (12) will hold along with (A.57), (A.62), and (A.64), equation (A.65) is satisfied automatically. The threshold Z is obtained by inverting equation (26) with the known probability $G_m(Z)$:

$$Z = e^{\mu+\sigma\Phi^{-1}(G_m(Z))}\,,$$

and the average credit cost $\bar{\chi}$ follows immediately from (26) using Z. Finally, with all these variables known conditional on y_l, the value of y_l itself can be found by searching for a solution of equation (A.49). It can be checked whether the solution satisfies $y_l > \zeta_l$.

Criteria for the fraction of investors and market tightness Finally, two equations are needed to pin down the fraction of investors among buyers and ownership-market tightness. Conditional on each pair of values of ξ and θ_o, the steps above show how θ_l, u_o, and u_l can be calculated. With these, the first criterion to be checked is equation (A.32). The second criterion is the indifference threshold condition (A.52), where q_o, q_l, Σ_o, Σ_l, and Z can be obtained as above for given ξ and θ_o. Searching over values of ξ and θ_o that satisfy these two criteria, the equilibrium is found.

Moving hazard function in the ownership market Let $\varkappa(T)$ denote the steady-state survival function of matches in the ownership market. This gives the fraction of matches that remain in existence after T years have elapsed. Assume the transaction and moving thresholds y_o and x_o remain constant over time.

In order for a match to survive for T years, first, the household must not leave the city during that time. With constant exit rate ρ, this has probability $e^{-\rho T}$. Second, the household must choose to remain after any shocks to idiosyncratic match quality have occurred. These shocks arrive independently at rate a_o, so the number of shocks j that occur over a period of time T has a Poisson$(a_o T)$ distribution. The probability of exactly j shocks is $e^{-a_o T}(a_o T)^j/j!$ for $j = 0, 1, 2, \ldots$.

If initial match quality is ε, after j shocks, match quality is now equal to $\varepsilon' = \delta_o^j\varepsilon$. The household chooses not to move house if $\varepsilon' \geq x_o$, which is equivalent to $\varepsilon \geq x_o/\delta_o^j$ in terms of initial match quality ε (and if this condition holds for some j then it also holds for any smaller j because $\delta_o < 1$ and x_o remains constant over time). New match quality has a Pareto(y_o, λ_o) distribution, so the probability that $\varepsilon \geq x_o/\delta_o^j$ is $((x_o/\delta_o^j)/y_o)^{-\lambda_o}$. This is well defined if $x_o/\delta_o^j > y_o$, which is true for all $j \geq 1$ because $\delta_o y_o < x_o$. With zero shocks ($j = 0$), households remain in the same property unless they leave the city.

The fraction of households who remain in the same property for T years is therefore

$$\varkappa(T) = e^{-\rho T}\left(e^{-a_o T} + \sum_{j=1}^{\infty} e^{-a_o T}\frac{(a_o T)^j}{j!}\left(\frac{x_o/\delta_o^j}{y_o}\right)^{-\lambda_o}\right) = e^{-(a_o+\rho)T}\left(1 + \left(\frac{y_o}{x_o}\right)^{\lambda_o}\sum_{j=1}^{\infty}\frac{(a_o\delta_o^{\lambda_o}T)^j}{j!}\right)$$

$$= e^{-(a_o+\rho)T}\left(1+\left(\frac{y_o}{x_o}\right)^{\lambda_o}\left(e^{a_o\delta_o^{\lambda_o}T}-1\right)\right) = \left(\frac{y_o}{x_o}\right)^{\lambda_o} e^{-\left(a_o(1-\delta_o^{\lambda_o})+\rho\right)T} - \left(\left(\frac{y_o}{x_o}\right)^{\lambda_o}-1\right)e^{-(a_o+\rho)T}\,.$$

The implied hazard function is given by $\hbar(T) = -\varkappa'(T)/\varkappa(T)$, which follows immediately from the above:

$$\hbar(T) = \frac{\left(a_o(1-\delta_o^{\lambda_o})+\rho\right)\left(\frac{y_o}{x_o}\right)^{\lambda_o} e^{-\left(a_o(1-\delta_o^{\lambda_o})+\rho\right)T} - (a_o+\rho)\left(\left(\frac{y_o}{x_o}\right)^{\lambda_o}-1\right)e^{-(a_o+\rho)T}}{\left(\frac{y_o}{x_o}\right)^{\lambda_o} e^{-\left(a_o(1-\delta_o^{\lambda_o})+\rho\right)T} - \left(\left(\frac{y_o}{x_o}\right)^{\lambda_o}-1\right)e^{-(a_o+\rho)T}}.$$

The density function of the probability distribution of moving times T is given by $\hbar(T)\varkappa(T)$, and hence the expected moving time is the integral under the survival function, $T_{mo} = \int_{T=0}^{\infty}\varkappa(T)\mathrm{d}T$. In the cross-section of households at a point in time, the distribution of time spent in the same property has density function $\varkappa(T)/T_{mo}$, and the implied average hazard rate is $\int_{T=0}^{\infty}\hbar(T)(\varkappa(T)/T_{mo})\mathrm{d}T = 1/T_{mo} = n_o + \rho$.

A.4 Calibration targets

In Toronto, the land transfer tax is the main transaction cost paid by buyers of property. The effective LTT rate is 1.5% in the pre-policy period (January 2006–January 2008), so $\tau_h = \tau_k = 0.015$. The parameters of the model are chosen to match the City of Toronto housing market in the pre-policy period. The average sale price reported in Table A.1 is \$402,000 during this period.

Non-tax transaction costs in the ownership market Apart from the land transfer tax, buyers may pay a home inspection cost of about \$500, but this is very small relative to the average house price. So it is assumed buyers do not pay any transaction costs other than the LTT, that is, $C_h = C_k = 0$.

From the side of sellers of property, the primary cost is the real-estate agent commission. Using Multiple Listing Service sales data, the average commission rate is about 4.5% of price. There are some other costs such as legal fees of around \$1,000, but these are negligible in comparison. Sellers may sometimes spend roughly \$2,500 on staging, but in some cases the seller's agent covers this expense as part of their commission, so not all sellers pay for staging out of their own pocket. Thus, C_u is set to be 4.5% of the average house price.

Maintenance costs The maintenance cost M as a homeowner is set so that it is 2.6% of the average property price. This cost is made up of a 2% maintenance cost and a 0.6% property tax in Toronto. The extra maintenance cost of being a landlord, M_l, is set to be 8% of average rent. This cost includes two parts: approximately 5–7% that the landlord uses to hire a property manager, and approximately 1% that the landlord uses to pay for services such as taking out garbage, shovelling snow, and salting the walkways.

Transaction costs in the rental market In Toronto, landlords typically pay one month's rent to real-estate agents to lease their properties. So C_l is set to be $1/12$ of average annual rent. Tenants in Toronto do not typically pay a monetary transaction cost when renting a property, so the tenancy fee Π is set to zero.

Flows within each housing market Flows in the two housing markets are related to the average time between moves, times on the market, and viewings per sale and lease. Information on time-to-move, time-to-sell, and time-to-lease is derived from Toronto MLS data on sales and rental transactions during the pre-policy period. Estimates of the moving hazard function imply that homeowners move after $T_{mo} = 9.25$ years on average The average duration of stay for a tenant is 1,109 days, so $T_{ml} = 1109/365$. Average time-to-sell for homeowners is 30.5 days and average time-to-rent is 18.7 days. During this period, the fraction of withdrawals from for-sale listings is 48% and from for-lease listings is 22%. In light of these withdrawal, the targets are $T_{so} = (30.5/365)/(1-0.48)$ and $T_{sl} = (18.7/365)/(1-0.22)$. This adjustment is made because time on the market in the data is calculated from the final successful listing without accounting for earlier unsuccessful attempts, so true time on the market is longer.

Data on buyers' time on the market and viewings per sale and per lease are not available for Toronto. Using the 'Profile of Buyers and Sellers' survey collected by NAR in the United States, Genesove and Han

(2012) report that for the period 2006–2009, the ratio of average time-to-buy to average time-to-sell is 1.28, and the average number of homes viewed by buyers is 10.7. Using this information, the targets used are $T_{bo} = 1.28 \times T_{so}$ and $\nu_o = 10.7/(1-0.48)$, where the latter adjusts viewings per sale to account for the withdrawal rate seen in Toronto. The idea is that viewings of properties that have been withdrawn from the market are not counted, so actual viewings are larger than reported viewings in the final successful listing. There is no data on the number of properties that renters view on average. According to an industry expert, renters view fewer properties than buyers, so the target adopted is half the number of viewings per sale ($\nu_l = \nu_o/2$).

Flow search costs There are no direct estimates of the flow costs of searching F_h, F_k, and F_w. The approach taken here is to base an estimate of search costs on the opportunity cost of time spent searching. More specifically, for buyers in the ownership market (the same for home-buyers and investors), assume one property viewing entails the loss of half a day's income, so the value of $F_h = F_k$ can be calibrated by adding up the costs of making the expected number of viewings. With viewings per sale equal to the average number of viewings made by a buyer, the total search cost is equated to $0.5 \times \nu_o \times (Y/365)$, where Y denotes average annual income. Thus, the calibration sets $T_{bo}F_h = 0.5\nu_o Y/365$, and dividing both sides by PT_{bo} implies $F_h/P = 0.5 \times (1/365)(Y/P)(\nu_o/T_{bo})$. Taking the median household-level income from Statistics Canada implies a price-to-income ratio of $P/Y = 5.6$ in Toronto in 2007. Given the value of ν_o/T_{bo}, the implied buyer's flow search cost, $F_h = F_k$, is 3.1% of the average house price.

The same logic is applied to the flow search costs of tenants in the rental market, where it is assumed that viewing a rental property takes half the time needed to view a property to buy. Thus, the ratio of tenants' and home-buyers' flow search costs F_w/F_h is set to $0.5 \times (\nu_l/\nu_o) \times (T_{bo}/T_{bl})$.

Household tenure and entry of investors Based on the 2006 City of Toronto Profile Report, the home-ownership rate is $h = 54\%$, the average age of homeowners is 53.3, and the average age of tenants is 45.0. Hence the target for the difference between the average ages of homeowners and renters is $\alpha = 8.3$. There is no survey that specifically captures the proportion of first-time buyers in Toronto. The Canadian Association of Accredited Mortgage Professionals (now called Mortgage Professionals Canada) undertook a survey in 2015 finding that the fraction is as high as 45% of purchases, which is consistent with the 44% found in the 2018 Canadian Household Survey for the Greater Toronto Area. On the other hand, data from the Canada Mortgage and Housing Corporation suggests the fraction of first-time buyers is about a third. Based on this information, the calibration target is $\phi = 0.4$.

Using Toronto MLS data on sales and rental transactions, the fraction of purchases by buy-to-rent investors is 5.4% during the pre-policy period, so $\kappa = 0.054$. The price-to-rent ratio for the same property is 14.5 in 2007, and the ratio of average prices paid by investors to prices paid by home-buyers is 0.99. Hence, $P_k/R = 14.5$ and $P_k/P = 0.99$ are used as targets.

Mortgage costs The credit cost χ of becoming a homeowner is computed from a comparison of the mortgage rate r_c the household would face relative to the risk-free interest rate r_f on government bonds. The interest rates r_c and r_f are real interest rates. There is a spread between them due to unmodelled financial frictions. The risk-free real rate r_f used to discount future cashflows need not be the same as the discount rate r applied to future utility flows from owning property (allowing for an unmodelled housing risk premium between r and r_f). Assume all these interest rates are expected to remain constant over the mortgage term.

Suppose a household buys a property at price P at date $t = 0$ by taking out a mortgage with loan-to-value ratio ℓ. Assume the mortgage has term T_c and a constant real repayment I over its term. Let $D(t)$ denote the outstanding mortgage balance at date t, which has initial condition $D(0) = \ell P$ and terminal condition $D(T_c) = 0$. The mortgage balance evolves over time according to the differential equation:

$$\dot{D}(t) = r_c D(t) - I \quad \text{and hence} \quad \frac{\mathrm{d}(e^{-r_c t}D(t))}{\mathrm{d}t} = -Ie^{-r_c t}\,.$$

Solving this differential equation using the initial condition $D(0) = \ell P$ implies:

$$D(t) = e^{r_c t}\ell P - \frac{I}{r_c}(e^{r_c t} - 1)\,.$$

The terminal condition $D(T_c) = 0$ requires that the constant real repayment I satisfies:

$$I = \frac{r_c \ell P}{1 - e^{-r_c T_c}}\,.$$

In the model, homeowners exit at rate ρ, in which case it is assumed they repay their mortgage in full (using the proceeds from selling their property). Hence, there is a probability $e^{-\rho t}$ that the date-t repayment I will be made, and a probability $\rho e^{-\rho t}$ that the whole balance $D(t)$ is repaid at date t. The credit cost χ is the present value of the expected stream of repayments discounted at rate r_f minus the amount borrowed (which would equal the present value of the repayments if $r_c = r_f$ in the absence of credit-market imperfections):

$$\chi = \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} I \mathrm{d}t + \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} \rho D(t) \mathrm{d}t - \ell P\,.$$

To derive an explicit formula for χ, first observe that

$$\int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} \mathrm{d}t = \frac{1 - e^{-(r_f+\rho)T_c}}{r_f + \rho} \quad \text{and} \quad \int_{t=0}^{T_c} e^{-r_f t} e^{-\rho t} e^{r_c t} \mathrm{d}t = \frac{1 - e^{-(r_f+\rho-r_c)T_c}}{r_f + \rho - r_c}\,.$$

Together with the formulas for $D(t)$ and I, the credit cost can thus be written as follows:

$$\begin{aligned}
\chi &= \frac{\left(I + \frac{\rho I}{r_c}\right)}{(r_f+\rho)}(1 - e^{-(r_f+\rho)T_c}) + \frac{\rho\left(\ell P - \frac{I}{r_c}\right)}{(r_f+\rho-r_c)}(1 - e^{-(r_f+\rho-r_c)T_c}) - \ell P \\
&= \left(\frac{(r_c+\rho)(1 - e^{-(r_f+\rho)T_c})}{(r_f+\rho)(1 - e^{-r_c T_c})} + \frac{\rho\left(1 - \frac{1}{1-e^{-r_c T_c}}\right)(1 - e^{-(r_f+\rho-r_c)T_c})}{(r_f+\rho-r_c)} - 1\right)\ell P \\
&= \frac{\left((r_c+\rho)(1 - e^{-(r_f+\rho)T_c}) - \frac{\rho(r_f+\rho)}{r_f+\rho-r_c}(e^{-r_c T_c} - e^{-(r_f+\rho)T_c}) - (r_f+\rho)(1 - e^{-r_c T_c})\right)\ell P}{(r_f+\rho)(1 - e^{-r_c T_c})} \\
&= \frac{\left((r_c - r_f) + \frac{\rho(r_f+\rho)-(r_c+\rho)(r_f+\rho-r_c)}{r_f+\rho-r_c} e^{-(r_f+\rho)T_c} - \frac{(r_f+\rho)(r_f+\rho-r_c)-\rho(r_f+\rho)}{r_f+\rho-r_c} e^{-r_c T_c}\right)\ell P}{(r_f+\rho)(1 - e^{-r_c T_c})}\,,
\end{aligned}$$

and dividing both sides by price P and simplifying:

$$\frac{\chi}{P} = \left(1 + \frac{r_c}{r_f+\rho-r_c} e^{-(r_f+\rho)T_c} - \frac{r_f+\rho}{r_f+\rho-r_c} e^{-r_c T_c}\right)\frac{(r_c - r_f)\ell}{(r_f+\rho)(1 - e^{-r_c T_c})}\,.$$

This equation is used to determine calibration targets for the marginal credit cost Z relative to the average property price P, and for the marginal credit cost Z relative to the average credit cost $\bar{\chi}$ conditional on becoming a homeowner.

A mortgage term of 25 years ($T_c = 25$) and an average loan-to-value ratio of 80% ($\ell = 0.8$) are assumed. Focusing on interest rates fixed for five years as a typical mortgage product, the 5-year conventional mortgage rate from Statistics Canada was 7.07% in 2007. Given an inflation rate of 2.14%, the implied real mortgage rate is set to 4.93% for an average homeowner. Information on mortgage spreads is then used to compute credit costs for a marginal buyer. Based on micro-level mortgage data from the Bank of Canada, the average contract mortgage rate during 2017–2018 was around 3.11%. Borrowers with low credit scores who did not qualify for loans from major banks could obtain mortgages from trust companies or private lenders at mortgage rates of around 6.15%. Hence, a mortgage spread of 3% is assumed, implying the real mortgage rate for the marginal buyer is 7.93%. Since the average mortgage cost is based on 5-year fixed rates, the equivalent risk-free rate is derived from 5-year government bonds. These had a yield of 4% in 2007, so the real risk-free rate is set to 1.86%.

In summary, $z = Z/P$ is derived from the formula for χ/P using $T_c = 25$, $\ell = 0.8$, $r_f = 1.86\%$, $r_c = 7.93\%$ (marginal), and the value of ρ obtained from the calibration method. The target for $Z/\bar{\chi}$ is derived by taking the ratio of χ/P for $r_c = 7.93\%$ (marginal) and $\bar{r}_c = 4.93\%$ (mean), with the other terms the same.

A.5 Calibration method

This section describes how to find the set of parameters exactly matching the calibration targets.

Fraction of investors among buyers Combining equation (18) and the formula for ν_o from section 3.7, the fraction of purchases made by investors is $\kappa = \xi\nu_o$, where ξ is the fraction of investors among buyers (see 1) and ν_o is average viewings per sale. Given empirical targets for κ and ν_o, the required fraction ξ is

$$\xi = \frac{\kappa}{\nu_o}. \tag{A.66}$$

Transaction probabilities and selling and letting rates Using the formulas for ν_o, ν_l, T_{so}, and T_{sl} from section 3.7 and the value of ξ from (A.66), the targets for ν_o, ν_l, T_{so}, T_{sl} give the values of:

$$\pi_o = \frac{\nu_o^{-1} - \xi}{1-\xi} \quad , \quad \pi_l = \frac{1}{\nu_l} \quad \text{and} \quad s_o = \frac{1}{T_{so}} \quad , \quad s_l = \frac{1}{T_{sl}}. \tag{A.67}$$

Uses of the housing stock The formulas for T_{so}, T_{sl}, T_{mo}, and T_{ml} from section 3.7 and equations (A.57) and (A.62) imply $u_o = (T_{so}/((1-\kappa)T_{mo}))h_o$ and $u_l = (T_{sl}/T_{ml})h_l$. The homeownership rate h defined in section 3.7 satisfies $h_o + (1-\kappa)u_o = \psi h$, and substituting for u_o in terms of h_o yields $(1 + T_{so}/T_{mo})h_o = \psi h$. This can be solved for h_o in terms of targets for h, ψ, and the time to move and time on the market. Similarly, by substituting for u_l, $h_l + u_l = (1 + T_{sl}/T_{ml})h_l$, and (11) implies $h_l + u_l = 1 - (h_o + u_o)$. Putting these equations together yields h_o, u_o, h_l, and u_l:

$$h_o = \frac{\psi h}{1 + \frac{T_{so}}{T_{mo}}} \quad , \quad u_o = \frac{T_{so}}{(1-\kappa)T_{mo}} h_o \quad , \quad h_l = \frac{1-(h_o+u_o)}{1 + \frac{T_{sl}}{T_{ml}}} \quad , \quad u_l = \frac{T_{sl}}{T_{ml}} h_l. \tag{A.68}$$

Exit rate of investors Using equation (A.56), $s_o u_o = (n_o + \rho)h_o + \rho_l(h_l + u_l)$, and solving for ρ_l yields $\rho_l = (s_o u_o - (n_o + \rho)h_o)/(h_l + u_l)$. With $(1-\kappa)s_o u_o = (n_o + \rho)h_o$ from (A.57), it follows that:

$$\rho_l = \frac{\kappa s_o u_o}{h_l + u_l}, \tag{A.69}$$

which can be calculated using (A.67) and (A.68).

Market tightness Using equation (19) and the formulas for T_{so} and T_{bo} in section 3.7, it follows that $T_{bo} = \theta_o T_{so}$, so θ_o can be deduced from targets for T_{bo} and T_{so}. The definitions in (1) imply that $b_h = (1-\xi)\theta_o u_o$ and $b_l = \theta_l u_l$, and hence equation (12) can be solved for θ_l by substituting for b_o and b_l:

$$\theta_o = \frac{T_{bo}}{T_{so}}, \quad \theta_l = \frac{\psi - h_o - h_l - (1-\xi)\theta_o u_o}{u_l}, \quad \text{and} \quad T_{bl} = \theta_l T_{sl}, \tag{A.70}$$

where the final equation gives the value of T_{bl} using (19), θ_l, and T_{sl}, which cannot be chosen freely given the other targets. With these variables known, the viewing rates for home-buyers and renters follow from the formulas given in section 3.7:

$$q_o = \frac{\nu_o}{T_{bo}}, \quad q_l = \frac{\nu_l}{T_{bl}}, \quad \text{and} \quad T_{bh} = \left(\frac{1-\xi}{1-\kappa}\right) T_{bo}, \tag{A.71}$$

where the final equation is the time-to-buy T_{bh} from section 3.7 for home-buyers implied by the other targets using (18).

Transitions to homeownership Let ϕ denote the fraction of first-time buyers among home-buyers. Using the law of motion for home-buyers (16), the value of ϕ in a steady state with $\dot{b}_h = 0$ is

$$\phi = \frac{(\gamma n_l h_l + \rho\psi)G_m(Z)}{n_o h_o + (\gamma n_l h_l + \rho\psi)G_m(Z)} = \frac{(q_o\pi_o + \rho)b_h - n_o h_o}{(q_o\pi_o + \rho)b_h}.$$

This can be calculated from the ratio of inflows of buyers because all home-buyers transact at the same rate conditional on entering the stock b_h. The second expression for ϕ follows because b_h is a steady state. In steady state, (A.56) implies $(n_o + \rho)h_o = (1-\kappa)s_o u_o$, and (1), (18), and (19) imply $(1-\kappa)s_o u_o = q_o\pi_o b_h$. Dividing numerator and denominator of the expression for ϕ by h_o and substituting $q_o\pi_o b_h/h_o = n_o + \rho$:

$$\phi = \frac{\left(1 + \frac{\rho}{q_o\pi_o}\right)(n_o + \rho) - n_o}{\left(1 + \frac{\rho}{q_o\pi_o}\right)(n_o + \rho)}.$$

Rearranging yields the formula for ϕ in footnote 25, and this can be written in terms of the time to move T_{mo} and home-buyers' time on the market T_{bh} using the expressions from section 3.7:

$$\phi = \frac{\rho\left(1 + \frac{T_{bh}}{T_{mo}}\right)}{\frac{1}{T_{mo}} + \rho\frac{T_{bh}}{T_{mo}}}.$$

This can be rearranged to give the value of ρ in terms of ϕ and other known targets, and with this, the implied value of n_o can also be found from $n_o = (1/T_{mo}) - \rho$:

$$\rho = \frac{\phi}{T_{mo} + (1-\phi)T_{bh}}, \quad \text{and} \quad n_o = \frac{(1-\phi)(T_{mo} + T_{bh})}{T_{mo}(T_{mo} + (1-\phi)T_{bh})}. \tag{A.72}$$

Taking the value of ρ from (A.72) and using the formula for T_{ml} yields $n_l = T_{ml}^{-1} - \rho$, and it can be checked whether this is positive. With (34) and ρ_l from (A.69), the parameter $a_l = n_l - \rho_l$ is obtained immediately.

Let g_{ho}, g_{hl}, g_{bh}, and g_{bl} be the average ages of the household heads of those in h_o, h_l, b_h, and b_l, and g_h and g_l the average ages of those in $h_o + b_h$ and $h_l + b_l$. The calibration target for the difference in the average ages of homeowners and renters is $\alpha = g_h - g_l$. Furthermore, let g_e and g_f denote the average age of new entrants to the city and first-time buyers respectively. Taking the group in $h_o + b_h$, exit occurs at rate ρ with first-time buyers of measure $\rho(h_o + b_h)$ arriving in steady state. The differential equation for the average age is thus $\dot{g}_h = 1 - \rho g_h + \rho g_f$. A steady-state age distribution therefore has $g_h = g_f + \rho^{-1}$. It is convenient to consider all average ages relative to average age at first entry to the city, which are denoted by $\alpha_h = g_h - g_e$, $\alpha_l = g_l - g_e$, and similarly for the other groups. The definition of α and the average homeowner versus first-time buyer age difference imply:

$$\alpha = \alpha_h - \alpha_l, \quad \text{and} \quad \alpha_h = \alpha_f + \rho^{-1}. \tag{A.73}$$

Now consider the group h_l. There is exit at rate $n_l + \rho$ and entry $q_l\pi_l b_l/h_l = (n_l + \rho)$ from b_l as a proportion of the group h_l (see A.62 with $q_l\pi_l b_l = s_l u_l$), where the average age at entry is g_{bl}. Thus, $1 = (n_l + \rho)(g_{hl} - g_{bl})$ and hence:

$$\alpha_{hl} = \alpha_{bl} + (n_l + \rho)^{-1}. \tag{A.74}$$

Since $g_l = (h_l/(h_l + b_l))g_{hl} + (b_l/(h_l + b_l))g_{bl}$ by definition, it follows that $g_{hl} - g_l = (b_l/(h_l + b_l))(g_{hl} - g_{bl})$, and by using (A.74) and the formula for T_{ml} from section 3.7:

$$\alpha_{hl} = \alpha_l + \frac{b_l}{h_l + b_l}T_{ml}. \tag{A.75}$$

For the group b_l, given (A.65), there are outflows at rate $q_l\pi_l + \rho$, and inflows of proportion $\rho\psi(1 - G_m(Z))/b_l$ from outside the city (average age g_e) and of proportion $(1 - \gamma G_m(Z))n_l h_l/b_l$ from h_l (average age g_{hl}), hence:

$$1 + \frac{\rho\psi(1 - G_m(Z))}{b_l}g_e + \frac{n_l(1 - \gamma G_m(Z))h_l}{b_l}g_{hl} = (q_l\pi_l + \rho)g_{bl}.$$

Using $\rho\psi(1 - G_m(Z)) = (q_l\pi_l + \rho)b_l - (1 - \gamma G_m(Z))n_l h_l$ from (A.65), this equation becomes $b_l = (1 -$

$\gamma G_m(Z))n_l h_l \alpha_{hl} = (q_l\pi_l+\rho)b_l\alpha_{bl}$. Substituting (A.74) and using (A.65) again leads to $\rho\psi(1-G_m(Z))\alpha_{hl} = b_l + (q_l\pi_l+\rho)b_l T_{ml}$. With $\theta_l = b_l/u_l$, $s_l = \theta_l q_l \pi_l$, and $s_l u_l = h_l/T_{ml}$ from (A.61), it follows that $(q_l\pi_l + \rho)b_l T_{ml} = (h_l/T_{ml})T_{ml} + \rho b_l T_{ml} = h_l + \rho b_l T_{ml}$, and by putting these equations together:

$$\alpha_{hl} = \frac{(h_l+b_l)+\rho b_l T_{ml}}{\rho\psi(1-G_m(Z))}\,. \tag{A.76}$$

Finally, consider the ages of first-time buyers. Using (16), a fraction $\gamma n_l h_l G_m(Z)/((\gamma n_l h_l + \rho\psi)G_m(Z))$ come from h_l, and a fraction $\rho\psi G_m(Z)/((\gamma n_l h_l+\rho\psi)G_m(Z))$ are new entrants to the city. Therefore, $g_f = (\gamma n_l h_l/(\gamma n_l h_l + \rho\psi))g_{hl} + (\rho\psi/(\gamma n_l h_l+\rho\psi))g_e$, which can be written as:

$$\alpha_f = \alpha_{hl} - \frac{\rho\psi}{\gamma n_l h_l + \rho\psi}\alpha_{hl} = \alpha_{hl} - \frac{(h_l+b_l)+\rho b_l T_{ml}}{(\gamma n_l h_l+\rho\psi)(1-G_m(Z))}\,, \tag{A.77}$$

where the second expression substitutes from (A.76). Using (A.61) and (A.65) again to write $(\gamma n_l h_l + \rho\psi)(1 - G_m(Z)) = (q_l\pi_l+\rho)b_l - (1-\gamma)n_l h_l = s_l u_l + \rho b_l - n_l(1-\gamma)h_l = (n_l+\rho)h_l + \rho b_l - (1-\gamma)n_l h_l = \rho(h_l+b_l) + \gamma n_l h_l$. Substituting this and (A.75) into (A.77):

$$\alpha_f = \alpha_l + \frac{b_l T_{ml}}{h_l+b_l} - \frac{(h_l+b_l)+\rho b_l T_{ml}}{\rho(h_l+b_l)+\gamma n_l h_l} = \alpha_l + \frac{T_{bl}T_{ml}}{T_{ml}+T_{bl}} - \frac{1+\rho\frac{T_{bl}T_{ml}}{T_{ml}+T_{bl}}}{\rho+\gamma n_l \frac{T_{ml}}{T_{ml}+T_{bl}}}\,, \tag{A.78}$$

where the second expression makes use of (A.68) and (A.70). Combining this formula with the two equations in (A.73) and simplifying yields the difference in average ages:

$$\alpha = \left(1+\rho\frac{T_{ml}T_{bl}}{T_{ml}+T_{bl}}\right)\left(\frac{1}{\rho} - \frac{1}{\rho+\gamma n_l\frac{T_{ml}}{T_{ml}+T_{bl}}}\right).$$

This is confirms the expression for α in footnote 25 with reference to the formulas given in section 3.7. Since ρ is known from earlier, this can be rearranged to give an equation for γ in terms of the targets:

$$\gamma = \frac{\alpha\rho^2(T_{ml}+T_{bl})^2}{((1-\alpha\rho)(T_{ml}+T_{bl})+\rho T_{bl}T_{ml})\,n_l T_{ml}}\,. \tag{A.79}$$

Furthermore, the targets pin down the value of $G_m(Z)$. Since $(\gamma n_l h_l + \rho\psi)(1-G_m(Z)) = \rho(h_l+b_l)+\gamma n_l h_l$ as shown above, the value of $G_m(Z)$ must satisfy:

$$G_m(Z) = \frac{\rho\psi - \rho(h_l+b_l)}{\gamma n_l h_l + \rho\psi}\,, \tag{A.80}$$

and all the terms in this expression are known.

Discount rate and bargaining powers The methodology here is to search over values of the discount rate r to solve one equation. Conditional on r, the bargaining powers ω_o, ω_k, and ω_l can be found as follows.

Dividing both sides of the price equation (A.46) by P and rearranging yields:

$$\frac{\omega_o^*\Sigma_o}{\pi_o P} = \frac{(1-c_u)r + m - \theta_o q_o \xi\frac{\omega_k^*\Sigma_k}{P}}{r+\theta_o q_o(1-\xi)\pi_o}\,, \tag{A.81}$$

where $c_u = C_u/P$ and $m = M/P$ are known targets. Using equations (A.46) and (A.53), it follows that $P - P_k = (\omega_o^*\Sigma_o/\pi_o) - \omega_k^*\Sigma_k$, and hence $p_k = P_k/P$ satisfies the following equation:

$$1-p_k = \frac{\omega_o^*\Sigma_o}{\pi_o P} - \frac{\omega_k^*\Sigma_k}{P}\,, \quad \text{with} \quad \frac{\omega_k^*\Sigma_k}{P} = \frac{f_{kh}f_h}{q_o}\frac{\omega_k^*}{1-\omega_k^*}\,, \tag{A.82}$$

where the expression for $\omega_k^*\Sigma_k/P$ comes from (43) and the definitions of the targets $f_h = F_h/P$ and $f_{kh} = F_k/F_h$. Substituting for $\omega_o^*\Sigma_o/(\pi_o P)$ from (A.81) in the first equation of (A.82) implies $(r+\theta_o q_o(\xi+(1-\xi)\pi_o))(\omega_k^*\Sigma_k/P) = (1-c_u)r + m + (p_k-1)(r+\theta_o q_o(1-\xi)\pi_o)$, and then using the second part of (A.82):

$$\frac{\omega_k^*}{1-\omega_k^*} = \frac{q_o}{f_{kh}f_h}\frac{(1-c_u)r+m+(p_k-1)(r+\theta_o q_o(1-\xi)\pi_o)}{r+\theta_o q_o(\xi+(1-\xi)\pi_o)}\,. \tag{A.83}$$

This can be calculated using r, the targets, and other variables known so far. Since (40) implies $\omega_k^*/(1-\omega_k^*) = (\omega_k/(1-\omega_k))/(1+\tau_k)$, the implied seller bargaining power when facing an investor is $\omega_k = (\omega_k^*/(1-\omega_k^*))/((1/(1+\tau_k)) + (\omega_k^*/(1-\omega_k^*)))$.

Using equation (36) for the equilibrium tenancy fee Π and the definition of the target $c_{wl} = \Pi/C_l$, it follows that the sum of the rental transaction costs $C_l + C_w$ is

$$C_l + C_w = \left(\frac{1-c_{wl}}{\omega_l}\right) C_l\,. \tag{A.84}$$

Dividing both sides of the rent equation (A.50) by R, substituting for $C_l + C_w$ using the equation above, and rearranging yields:

$$\frac{\omega_l \Sigma_l}{\pi_l R} = \frac{1-m_l-(r+n_l+\rho)(1-c_{wl})c_l}{r+n_l+\rho+\theta_l q_l \pi_l} = \frac{1-m_l-(r+T_{ml}^{-1})(1-c_{wl})c_l}{r+T_{ml}^{-1}+T_{sl}^{-1}}\,, \tag{A.85}$$

where $m_l = M_l/R$ and $c_l = C_l/R$ are known targets, and the second equation uses $T_{ml} = 1/(n_o+\rho)$ and $T_{sl} = 1/s_l = 1/(\theta_l q_l \pi_l)$. The value function of a new entrant to the city is $B_e = (1-G_m(Z))B_l + G_m(Z)(B_o - \bar{\chi})$, which can be written as $B_e = B_l + G_m(Z)(Z-\bar{\chi})$ using equation (14) for the threshold cost Z. Solving equation (A.47) for the renter value function B_l, substituting into the equation for B_e and dividing both sides by P:

$$\frac{B_e}{P} = \frac{(1-\omega_l)\frac{q_l\Sigma_l}{P} - \frac{F_w}{P}}{r+\rho} + G_m(Z)\left(1-\frac{\bar{\chi}}{Z}\right)\frac{Z}{P} = \frac{\frac{p_k}{p_r T_{sl}}\left(\frac{\omega_l\Sigma_l}{\pi_l R}\right)\left(\frac{1-\omega_l}{\omega_l}\right) - f_{wh}f_h}{r+\rho} + G_m(Z)\left(1-\frac{\bar{\chi}}{Z}\right)z\,,$$

which is stated in terms of targets $p_k = P_k/P$, $p_r = P_k/R$, $f_{wh} = F_w/F_h$, and $z = Z/P$. Letting $b_e = B_e/P$ denote the target for entrants' payoff, this equation can be solved for $\omega_l/(1-\omega_l)$ as follows:

$$\frac{\omega_l}{1-\omega_l} = \frac{\frac{p_k}{p_r T_{sl}}\left(\frac{\omega_l \Sigma_l}{\pi_l R}\right)}{f_{wh}f_h - (r+\rho)\left(z\left(1-\frac{\bar{\chi}}{Z}\right)G_m(Z) - b_e\right)}\,. \tag{A.86}$$

This can be calculated using r, the targets, and the known value of $\omega_l\Sigma_l/(\pi_l R)$ from (A.85). The bargaining power of a landlord is thus $\omega_l = (\omega_l/(1-\omega_l))/(1+\omega_l/(1-\omega_l)))$.

With ω_k and ω_k^* known conditional on r, substituting the second equation from (A.82) into (A.81) yields:

$$\frac{\omega_o^*\Sigma_o}{\pi_o P} = \frac{(1-c_u)r + m - \theta_o \xi f_{kh} f_h \frac{\omega_k^*}{1-\omega_k^*}}{r+\theta_o q_o (1-\xi)\pi_o}\,, \tag{A.87}$$

which is known given the targets conditional on r. Dividing the marginal first-time buyer indifference condition (A.52) by price P yields $((1-\omega_o^*)/\omega_o^*)q_o\pi_o(\omega_o^*\Sigma_o/(\pi_o P)) = ((1-\omega_l)/\omega_l)(q_l\pi_l p_k/p_r)(\omega_l\Sigma_l/(\pi_l R)) + (r+\rho)z + f_h - f_{wh}f_h$. Noting that $(1-\omega_o^*)/\omega_o^* = (1+\tau_h)(1-\omega_o)/\omega_o$ from (27) and $T_{bl} = 1/(q_l\pi_l)$, this equation can be solved for $\omega_o/(1-\omega_o)$:

$$\frac{\omega_o}{1-\omega_o} = \frac{\frac{(1+\tau_h)}{T_{bh}}\left(\frac{\omega_o^*\Sigma_o}{\pi_o P}\right)}{\frac{p_k}{p_r T_{bl}}\left(\frac{1-\omega_l}{\omega_l}\right)\left(\frac{\omega_l\Sigma_l}{\pi_l R}\right) + (r+\rho)z + (1-f_{wh})f_h}\,. \tag{A.88}$$

This expression can be evaluated using (A.85), (A.86), and (A.87). Hence, sellers' bargaining power when faced with a home-buyer is given by $\omega_o = (\omega_o/(1-\omega_o))/(1+(\omega_o/(1-\omega_o)))$.

Next, taking the free entry condition (A.51) and dividing both sides by P:

$$\begin{aligned}\frac{\theta_l q_l \pi_l p_k}{p_r}\left(\frac{\omega_l\Sigma_l}{\pi_l R}\right) = \left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\theta_o q_o\left((1-\xi)\pi_o\left(\frac{\omega_o^*\Sigma_o}{\pi_o P}\right) + \xi\left(\frac{\omega_k^*\Sigma_k}{P}\right)\right) \\ + (r+\rho)\left((1+\tau_k)c_u + c_k p_k + (1+\tau_k\omega_k^*)\frac{\Sigma_k}{P}\right) - \tau_k\left(1+\frac{\rho_l}{r}\right)m\,,\end{aligned}$$

noting the definition $c_k = C_k/P_k$. Substituting for Σ_k/P using (A.82) and solving for the price-rent ratio p_r:

$$p_r = p_k \theta_l q_l \pi_l \left(\frac{\omega_l \Sigma_l}{\pi_l R}\right) \left(\left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right) \theta_o q_o (1-\xi)\pi_o \frac{\omega_o^* \Sigma_o}{\pi_o P} + (r+\rho)\left((1+\tau_k)c_u + c_k p_k\right) \right.$$
$$\left. + \left(r+\rho+\left(1+\tau_k\left(1+\frac{\rho_l}{r}\right)\right)\frac{\theta_o q_o \xi \omega_k}{1+\tau_k}\right)\frac{f_{kh} f_h}{(1-\omega_k) q_o} - \tau_k\left(1+\frac{\rho_l}{r}\right) m \right)^{-1}, \quad \text{(A.89)}$$

which uses $\omega_k^*/(1-\omega_k^*) = (\omega_k/(1-\omega_k))/(1+\tau_k)$ and $(1+\tau_k \omega_k^*)/\omega_k^* = (1+\tau_k)/\omega_k$. The formula for p_r depends on known calibration targets and r, and as p_r is itself a target, equation (A.89) can be solved numerically to determine the discount rate r.

Meeting functions With ω_o and ω_l known, the meeting function elasticities η_o and η_l are derived from the calibration targets for ω_o/η_o and ω_l/η_l. Since market tightnesses θ_o and θ_l are determined in (A.70) and the viewing rates in (A.71), the meeting function productivity parameters A_o and A_l are those satisfying (24):

$$A_o = q_o \theta_o^{\eta_o} \quad \text{and} \quad A_l = q_l \theta_l^{\eta_k}.$$

Ownership-market match-quality distribution and idiosyncratic shocks A new variable β_o is introduced at this stage, which is defined as follows in terms of other parameters and endogenous variables:

$$\beta_o = \frac{\lambda_o a_o \delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1-\delta_o^{\lambda_o}\right)}. \quad \text{(A.90)}$$

Suppose for now there is a target value of β_o alongside the other targets. At the final stage of the calibration, the econometric evidence on the response β of moving rates to the LTT change is used to determine β_o.

There is a numerical procedure to determine the arrival rate a_o of idiosyncratic shocks. The formula in footnote 24 implies the steady-state moving rate n_o can be written in terms of a_o, λ_o, ρ and β_o from (A.90):

$$n_o = \frac{a_o - \rho \frac{\beta_o}{\lambda_o}}{1 + \frac{\beta_o}{\lambda_o}}.$$

Conditional on a value of a_o, the value of λ_o is found by solving this equation:

$$\lambda_o = \frac{(n_o + \rho)\beta_o}{a_o - n_o}, \quad \text{(A.91)}$$

using the provisional target for β_o and the values of ρ and n_o from (A.72). Next, take equation (A.44) and divide both sides by P. By making use of the second equation in (A.82):

$$\frac{x_o}{P} = \frac{(1-\omega_o^* + (1-\xi)\omega_o^* \theta_o) q_o \pi_o}{\omega_o^*}\left(\frac{\omega_o^* \Sigma_o}{\pi_o P}\right) + \frac{\omega_k^*}{(1-\omega_k^*)} \xi \theta_o f_{kh} f_h - f_h. \quad \text{(A.92)}$$

Similarly, dividing both sides of (A.43) by P:

$$\frac{y_o}{P} = \frac{x_o}{P} + (r+\rho+a_o)\left(\frac{\tau_h}{r}\left((1-\xi)\theta_o q_o \pi_o \frac{\omega_o^* \Sigma_o}{\pi_o P} + \xi \theta_o f_{kh} f_h \frac{\omega_k^*}{(1-\omega_k^*)}\right) \right.$$
$$\left. + c_h + (1+\tau_h)c_u - \tau_h \frac{m}{r}\right), \quad \text{(A.93)}$$

where (A.82) has been used again. Together, (A.92) and (A.93) give $y_o/x_o = (y_o/P)/(x_o/P)$ in terms of a_o and the calibration targets. With λ_o from (A.91) and y_o/x_o, equation (A.90) can be rearranged to solve for the idiosyncratic shock size parameter δ_o:

$$\delta_o = \left(\frac{\left(1+\frac{\rho}{a_o}\right)\beta_o}{\beta_o + \lambda_o \left(\frac{y_o}{x_o}\right)^{\lambda_o}} \right)^{\frac{1}{\lambda_o}} .$$

With the target for P and y_o/P known from (A.93), the value of $y_o = (y_o/P)P$ is deduced. Using $\pi_o = (\zeta_o/y_o)^{\lambda_o}$ from (30), it follows that $\zeta_o = y_o \pi_o^{1/\lambda_o}$, so ζ_o is known given y_o, λ_o, and π_o from (A.67). While both payoffs and costs can be scaled without loss of generality, the target for P provides a normalization that determines ζ_o. The cost parameters $C_h = c_h P$, $C_u = c_u P$, $C_k = c_k p_k P$, $F_h = f_h P$, $F_k = f_{kh} F_h$, and $M = mP$ follow immediately from P and the other targets.

The value of P together with (A.92) determines x_o. Furthermore, Σ_o follows from the known value of $\omega_o^* \Sigma_o/(\pi_o P)$ in (A.87) and ω_o^* and π_o. Since these variables are all computed conditional on a conjectured value of a_o, the value of a_o is verified by a numerical search to check whether the equation for Σ_o in (A.45) holds. The requirement $\delta_o y_o < x_o$ can also be verified at this stage.

Distribution of credit costs The value of $G_m(Z)$ has already been determined in (A.80). Using (26), the marginal credit cost Z and the parameters μ and σ of the probability distribution satisfy:

$$\frac{\log Z - \mu}{\sigma} = \Phi^{-1}(G_m(Z)) .$$

Using (26) to obtain an equation for $\log \bar{\chi}$ and subtracting this from $\log Z = \mu + \sigma \Phi^{-1}(G_m(Z))$:

$$\log\left(\frac{Z}{\bar{\chi}}\right) = \log G_m(Z) - \log \Phi\left(\Phi^{-1}(G_m(Z)) - \sigma\right) + \sigma \Phi^{-1}(G_m(Z)) - \frac{\sigma^2}{2} ,$$

noting that μ cancels out. Using the known value of $G_m(Z)$ and the target for $Z/\bar{\chi}$, this equation can be solved numerically to find the standard deviation parameter σ. Note that $Z = zP$ using the known value of P and the target for $z = Z/P$. Together with σ solving the equation above, the value of the mean parameter is $\mu = \log Z - \sigma \Phi^{-1}(G_m(Z))$. The implied value of $\bar{\chi}$ follows from Z and the target $Z/\bar{\chi}$.

Rental-market parameters Given P, the target for p_k determines the price paid by investors $P_k = p_k P$, and the target for p_r determines the average rent $R = P_k/p_r$. The targets for m_l, c_l, and c_w then imply values of the cost parameters $M_l = m_l R$, and $C_l = c_l R$. The target for f_{wh} gives $F_w = f_{wh} F_h$ using the value of F_h obtained earlier. Using (A.84), $C_w = ((1 - c_{wl})/\omega_l - 1)C_l$, which can be calculated using the target c_{wl} and the known values of ω_l and C_l.

With π_l known from (A.67), the value of Σ_l can be deduced from (A.87) using the values of R and ω_l. Equation (A.49) then implies $y_l = M_l - F_w + (r + n_l + \rho)(C_l + C_w) - \gamma n_l G(Z)(Z - \bar{\chi}) + (1 - \omega_l + \omega_l \theta_l) q_l \Sigma_l$. Since $\pi_l = (\zeta_l/y_l)^{\lambda_l}$ from (38), equation (A.28) can be rearranged to solve for λ_l in terms of y_l, π_l, and Σ_l:

$$\lambda_l = 1 + \frac{\pi_l y_l}{(r + \rho + n_l)\Sigma_l} .$$

Knowing λ_l allows the rental-market minimum new match quality parameter to be deduced from the equation for π_l as $\zeta_l = y_l \pi_l^{1/\lambda_l}$.

Response of the moving rate to the land transfer tax Conditional on a value of β_o from (A.90), all the other targets have been matched. A numerical search over β_o values is then used to match the model's predicted response of the moving rate to the land transfer tax with the econometric estimate of this β.