Dynamics of Households' Consumption and Housing Decisions

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Abstract

We estimate a dynamic discrete and continuous choice model of households' decisions regarding their consumption, housing tenure and housing services over the life-cycle. We use non parametric identification arguments as in Bruneel-Zupanc (2021) to formulate an empirical strategy in two steps that (1) estimates discrete choice probabilities and continuous choice distribution summaries to be used in (2) Bellman and Euler equations that estimate the structural parameters. Specific modelling strategies are adopted because of unfrequent mobility due to housing transaction costs. Counterfactuals that can be evaluated are related to those transaction costs as well as of prudential policies such as downpayments.

Keywords: Dynamic models, discrete and continuous choices, non parametric identification, housing, policy evaluation.

JEL codes: C25, C61, D15, H31, R21

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1 Introduction

Housing accounts for the bulk of assets of most households (61% out of an average wealth 276,000 € in France in 2018, Cazenave-Lacrouts et al., 2019). Housing is however a very specific asset since most of it also provides a non-monetary flow of housing services to its owner over time at the difference of many other assets (Li and Yao, 2007). Any shock on the value of housing may thus have strong impacts on consumption of other goods and consumer welfare (Ortalo-Magné and Rady, 2006). Moreover, housing is the target of sizeable public policies either in terms of household allowances or taxation. Transaction costs under the form of taxes on buying and selling houses are in France among the highest in the OECD and strongly affect the mobility of French households.[1]

In consequence, the evaluation of the impact of those transaction costs is high on the agenda of economists. For this, different types of dynamic models of housing are set up : partial equilibrium macro models such as Attanasio et al. (2012) or Li et al. (2016); general equilibrium macro models (Sommer and Sullivan, 2018; Bontemps et al., 2019); a sufficient statistics approach on the impact of prices (Berger et al., 2018; Etheridge, 2019). Those approaches nevertheless significantly restrict the heterogeneity between households. A few alternatives are offered by Bajari et al. (2013) and Khorunzhina and Miller (2019) using dynamic discrete choice models.

In this paper, we build upon these latter approaches and propose a dynamic continuous and discrete dynamic model whose non parametric identification has been recently analyzed by Bruneel-Zupanc (2021). We estimate a dynamic housing model in which the choice of housing tenure (ownership or renting), of housing services (e.g. house size) and consumption are the endogenous variables observed during several periods.

We follow Bajari et al. (2013) by having time-specific heterogeneity affecting preferences over consumption and housing services in a Constant Elasticity

[1]https://www.globalpropertyguide.com/investment-analysis/Housing-transaction-costs-in-the-OECD

of Substitution (CES) set up. A second time-specific heterogeneity term among households governs the tenure decision between ownership and renting. In contrast, we follow a more standard Euler approach model by having consumption modeled with measurement errors only, which is typical of micro data in panels (Alan et al., 2009).

There are also various idiosyncrasies in housing data we take on board. First, the frequency of households moving at each period is rather small (between 5% for owners and 10% for renters) so that it creates selection that we deal with using information on consumption and restricting somewhat the dimension of heterogeneity. Second, when they move, some households are still choosing a level of housing services which is very similar to the previous level. This should be hindered by the existence of transaction costs, so that to reconcile the data with the model, we introduce an exogenous shock to mobility. This could be caused by shocks to employment although those are not directly observed in the data.

Structural parameters are estimated in two steps as in Hotz and Miller (1993) although using the mix of discrete and continuous variables. In a first step, we estimate in a flexible way three static equations that concern housing services, consumption and housing tenure. At the second step, and using the approach suggested by non-parametric identification, we impose Euler and Belmann restrictions to recover structural parameters.

The data we use as the basis for our modelling strategy is the French extract of the European Survey of Income and Living Conditions (SILC) a 10-years rolling panel data set between 2004 and 2015. The advantages of this quite short panel is that it has reasonable good income and asset data, including house values, mortgages and variables related to labour earnings and benefits. Consumption however is to be reconstructed from asset and income data and this is also why we choose to model it with measurement errors.

As this paper remains preliminary, we present descriptive statistics, identification arguments and our empirical strategy. We do not report empirical results of the full estimation, nor the results of any counterfactual exercises. An interesting exercise would be to simulate the effects of the decrease in transaction costs on tenure, consumption and housing services as well as household welfare. Evaluating prudential policies such as down-payments, would also be an object of interest.

In the following, Section 2 briefly describes the data we intend and started to use. Section 3 sets up the theoretical framework. Identification arguments and the empirical strategy in two steps are presented in Section 4 and Section 5. Section 6 concludes.

Literature Review: The closest studies in a continuous and discrete dynamic set-up such as to ours are Bajari et al. (2013) and Li et al. (2016). They are both based on the now canonical dynamic model of housing in which housing tenure, housing services and consumption are modelled in a context in which credit constraints are important (e.g. Li and Yao, 2007; Attanasio et al., 2012). Bajari et al. (2013) uses the Panel Study of Income Dynamics data and a two-step method proposed in previous work by the first author (Bajari et al., 2007). It differs from our own two-step method in the second stage and consists in setting up moment conditions that use that some deviations from observed decisions are suboptimal. In contrast, we use all restrictions of the model in the second stage and the conditions on the way heterogeneity terms enter the model are clearer than theirs.

Li et al. (2016) estimates the parameters of their housing model by the simulated method of moments (SMM), the standard estimation method in the literature at the intersection of consumption studies and partial equilibrium macroeconomics. Among other results, these authors estimate the elasticity of substitution between non-durable consumption and housing services using a Constnat Elasticity of Substitution (CES) specification for preferences and find that this substitution elasticity is significantly lower than one. In this vein, Khorunzhina (2021) provides an interesting way of identifying this substitution elasticity by using maintenance expenses in the PSID. It allows an household specific price index to be constructed and used as exogenous variation affecting the ratio of consumption and housing services. This identifies this substitution elasticity, at least for owners staying in the same house.

Bruneel-Zupanc (2021) provides the main identification arguments used in our paper, as well as an empirical application to the canonical dynamic model of household consumption and labour market participation. He does not analyze a two-consumption good case as we do and does not consider measurement errors in consumption. Our procedure also relates to non linear GMM estimation of the Euler equation as in Alan et al. (2009).

In terms of economic results, our paper belongs to the strand of economic studies analyzing the mitigation of income risks by households through intertemporal smoothing of consumption (Blundell et al., 2008). We can indeed use our results in order to model the degree of insurance that households can achieve at various points over their life-cycle. Furthermore, we can also compare our results to recent analyses by Browning et al. (2013) on the impact of house prices on consumption. The authors find that the effect is mainly due the collateral value of housing and not to a wealth effect.

Berger et al. (2018) also tries to assess the impact of housing prices on consumption and finds a significant effect that can be summarized by a simple "sufficient statistics" which is the product of the household marginal propensity to consume and the value of their housing. In a close contribution, Etheridge (2019) uses the same decomposition exercise as Blundell et al. (2008) in the case of linear income and housing risks and shows that a positive common shock to house prices in the UK increases consumption inequality in cross-section. His empirical conclusions also hold in a more non-linear structural model. In particular, income and housing risk interactions are shown to be important to understand consumer behaviour because increases in house price alleviate borrowing constraints whereas decreases strengthen them. Paz-Pardo (2021) is another example of recent analyses on how households deal with risks arising either from their incomes or assets. Using the PSID, the author shows that changes in the dynamics of income account for a large part of the recent reduction of homeownership by young households and that these investments are not compensated by other assets in their portfolios.

Other key dimensions of the decision making of households are not modelled here although some are developed in the recent literature. Blundell et al. (2016) analyzes how family labour supply adjusts to directly mitigate risks in income. Öst (2012) models the simultaneous housing and fertility decisions in a reduced form setting, and finds a positive correlation between homeownership and fertility in particular for younger households. In a more structural set-up, Khorunzhina and Miller (2019) models how households choose homeownership, fertility and labor supply. Interestingly, they use a two-step method in a dynamic discrete choice model (Hotz and Miller, 1993) and this is the inspiration for our own estimation method although it also uses continuous choices although in a setting where labour participation and hours are exogenous.

The importance of credit constraints in relation to housing tenure has been shown repetitively as in the early work of Ortalo-Magné and Rady (2006). Pizzinelli (2017) is a more recent example studying the interaction between prudential regulations on credit – imposing loan to value and loan to income ratios to household mortgages – and labour supply of households, and their impact on homeownership. This literature, as well as Berger et al. (2018), points out that the leverage position of households severely restricts their ability to smooth income risks. Iacoviello and Pavan (2013) also shows the importance of loan to value ratio, or downpayment constraints, as well as the importance of non-convex adjustments costs in housing models. The very large transaction costs in France when purchasing a house dampens liquidity and is a important explanatory factor of the low level of mobility across houses for owners.

Finally, our results could also be compared to those of studies using French micro-data and that evaluate housing policies. Housing allowances, zero-interest loans, housing tax credit, real-estate transaction cost and residence tax have been the focus of such policy evaluations in recent years. Grislain-Letremy and Trévien (2014) estimates the impact of housing allowances on prices of housing services in France between 1987 and 2012 and confirms the inflationary effect that Fack (2006) uncovered. Zero-interest access-to-ownership loans are evaluated by Gobillon and le Blanc (2008). Housing tax credits and their inflationary effects are evaluated uisng difference-in-difference methods by Bono and Trannoy (2019) and Chapelle et al. (2018). Bérard and Trannoy (2018) analyses the impact on prices and quantities of an increase of the transaction tax in various local areas in 2014. Bontemps et al. (2019) uses a Sommer and Sullivan (2018) general equilibrium model to analyze counterfactuals such as the impact of a decrease in transaction costs and housing taxes. A reduced form analysis of dynamic housing and labour-market participation decisions is provided by Kamionka and Lacroix (2018) and uses the same data as in our study. Although they model income in a richer way than we do, they do not estimate the structural parameters of a housing model.

2 Data

We use the French extract of the European Survey of Income and Living Conditions (EU-SILC), a ten-years rolling panel dataset between 2004 and 2015. We select only couples that stayed together during the survey, in order to avoid modeling the merge or division of assets when couples are formed or when they divorce. We focus on individuals less than 60 years old as retirees may face very different housing market condition (borrowing in particular).

After cleaning the data, we are left with 7,108 unique couples, giving 22,625 household-year observations. The descriptive statistics of the data, by ownership status, are given in Table 1. 73% of the household own their properties. On average homeowners are older than renters, they consume more, live in larger houses and get less benefits. The housing mobility is quite low in the French data: overall, only 7% of the households change their residence during a year, i.e. we observe a total of 1,542 moves. It suggests that there is a very high cost of moving for the households.

	All		*Owner*		*Renter*	
	Mean	(SD)	Mean	(SD)	Mean	(SD)
Consumption (c)	39.18	(40.68)	40.56	(43.85)	35.46	(30.20)
House size in sqm (s)	107.98	(37.47)	117.13	(36.85)	83.17	(26.22)
Homeownership (d)	0.73	(0.44)	1.00	(0.00)	0.00	(0.00)
Past Homeownership	0.71	(0.45)	0.97	(0.18)	0.02	(0.13)
House price/rent (yearly) in K euros			228.91	(92.22)	6.08	(2.74)
Est. House value			221.89	(69.10)	6.10	(2.18)
Moved this year	0.07	(0.25)	0.05	(0.22)	0.12	(0.32)
Age	42.67	(9.36)	43.81	(9.07)	39.57	(9.42)
Financial Asset	24.11	(51.16)	27.89	(55.23)	13.85	(36.07)
Landlord Asset	32.54	(90.42)	38.29	(97.06)	16.99	(66.87)
Income	45.02	(20.67)	48.21	(21.06)	36.37	(16.75)
Benefits	4.18	(5.32)	3.58	(4.83)	5.79	(6.19)
Housing Benefits	0.41	(1.08)	0.17	(0.66)	1.07	(1.61)
Live in Social Housing	0.11	(0.31)	0.00	(0.00)	0.41	(0.49)
Number of children	1.35	(1.12)	1.38	(1.10)	1.26	(1.17)
Number of children below 3 y.o.	0.24	(0.48)	0.21	(0.46)	0.32	(0.53)
Number of children below 6 y.o.	0.49	(0.73)	0.45	(0.71)	0.59	(0.76)
Number of children below 18 y.o.	1.30	(1.13)	1.30	(1.10)	1.29	(1.20)
Age of the youngest child	7.55	(5.65)	8.09	(5.68)	5.99	(5.26)
Observations	22625		16525		6100	

Table 1: Descriptive Statistics of the complete sample and by homeownership status.

d_{-1} / d	0	1	Total
0	597	565	1162
	(51.4)	(48.6)	(100.0)
1	109	271	380
	(28.7)	(71.3)	(100.0)
Total	706	836	1542
	(45.8)	(54.2)	(100.0)

Table 2: Housing Tenure (d) change when moving

Renters are more mobile (12%) than homeowners (5%). As described in Table 2, 48.6% of renters who move are becoming homeowners, while homeowners are considerably less likely to become renters (71.3% become owner of a new house and only 28.7% become renters). This suggests some differential cost of switching tenures for households who were owners and household who were renters.

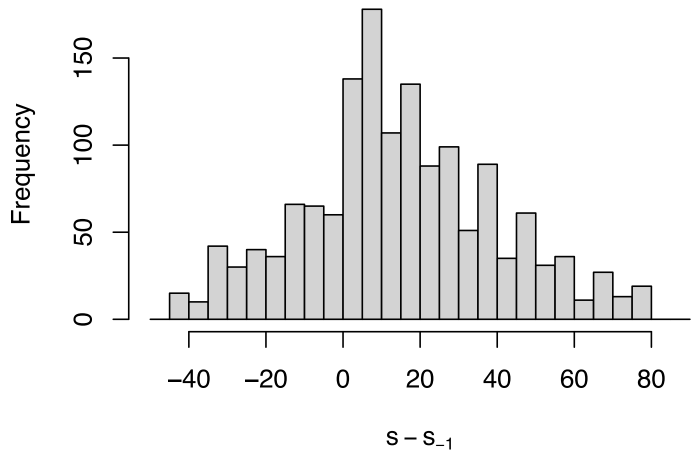


Figure 1: Histogram of $s - s_{-1}$ conditional on moving and **same tenure**

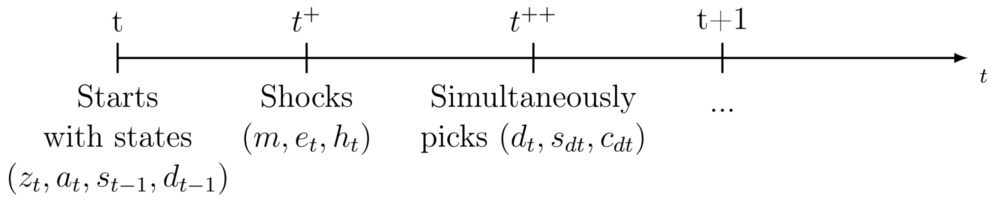
Finally, as shown in Figure 1, conditional on moving, people move to larger houses, with an average house size change ($s - s_{-1}$) of about 20 square meters.[2] Surprisingly though, we observe that some households also move to very similar properties. This is at odds with the high cost of moving, and it suggests that some households are probably moving for some non-housing reasons, e.g. labor mobility shocks.

3 Framework

We build a dynamic life-cycle model from 25 to 60 y.o. which incorporates all the empirical facts described in the previous section.

Each period, the timing of the problem is as follows:

[2]We also observe the same pattern when using the house value adjustment: on average households move to houses with higher estimated value.



Each period, the household decide of its housing tenure (d, $= 1$ for owners, $= 0$ for renters), housing services (s_d) and non-housing consumption (c).[3] They do so given three endogenous states: their previous housing tenure (d_{-1}) and housing services (s_{-1}), their asset (a). And also given some exogenous states z, including the household income and other demographics (x) which mainly include the age, the number of children and their age. They also know prices (and forecast them): p for rent, q for house prices and interest rate (r). Two iid shocks occur each period. They are unobserved by the econometrician, but known by the households. Additively separable shocks e affect the housing-tenure choice. e follows an extreme value type 1 distribution, as in the discrete choice literature (McFadden, 1980; Hotz and Miller, 1993). A non-separable shock h affects the substitution between housing services and non-housing consumption. h also affect the housing-tenure choice. As its distribution is not identified, we normalize $h \sim \mathcal{U}(0,1)$.
Finally, there is a random *exogenous binary move shock* m. With probability p^m, $m = 1$ and the household must move for exogenous reasons (e.g. labor mobility shock).[4] When $m = 0$ (with probability $1 - p^m$), the household can choose freely to move or not, yielding endogenous moves. m is not observed by the econometrician. We include it to capture the fact that some households move to very similar houses, even though the fixed cost of moving should be high.

Period utility function:

[3]The data counterpart to housing services s can either be the housing size in square meters s, or other housing variables such as the housing value. The advantage of our identification method is that s_d can be a different variable for each d, e.g. for owners s_1 is the estimated housing prices and for renters s_0 is the rent.

[4]We can allow for a p^m that depend on our variables, especially the demographics and the income.

The utility function take a CES form (close to Li et al., 2016) and is given by

$$\begin{aligned} U(d,c,s,s_{-1},d_{-1},x,m,h,e) &= \gamma(x)\frac{\left(c^{1-1/\gamma_d}+\phi(h)s^{1-1/\gamma_d}\right)^{\frac{1-\sigma}{1-1/\gamma_d}}}{1-\sigma} \\ &+ (\underbrace{\mu_0+\mu_1 d_{-1}}_{\text{fixed moving cost}})(\mathbf{1}\{\{d\neq d_{-1}\} \text{ or } \{s\neq s_{-1}, d=d_{-1}\} \text{ or } \{m=1\}\}) \\ &+ (\underbrace{\nu_0+\nu_1 d_{-1}}_{\text{tenure swapping cost}})\mathbf{1}\{d\neq d_{-1}\} + e_d \end{aligned}$$

$\gamma(x)$ is an equivalence scale as a function of the demographics x. c and s are non separable choices under this CES specification. The shock h mainly affects the substitution between c_d and s_d and $\phi(h)$ is simply a strictly increasing transformation with respect to h (e.g. $\phi(h) = a + bh$) to get more general effects of the shock. The CES parameters γ_d and σ are such that $\partial U(\cdot)/\partial s_d \partial h > 0$ and $\partial U(\cdot)/\partial c_d \partial h < 0$. It means that the optimal housing service consumption (s^*_d) is strictly increasing with h and the optimal non-housing consumption (c^*_d) is strictly decreasing with it: h governs the substitution between c and s.[5]

You also have some additively separable costs in the utility function. In the model, we observe that an household move if $d \neq d_{-1}$ or $s \neq s_{-1}$ and $d = d_{-1}$. In this case, the household endures a fixed moving cost $\mu_0 + \mu_1 d_{-1}$. With $\mu_0 < 0$ and $\mu_0 + \mu_1 < 0$ because it is costly to move. This cost depends on the past tenure, as suggested in the data where owners do not move as much as renters (suggesting $\mu_1 < 0$). This fixed utility cost will yield optimal choice where individuals will choose not to move in the model, and stay in their current house: $s = s_{-1}$ and $d = d_{-1}$.[6] Individuals are also forced to move when $m = 1$. In this case, they endure the fixed utility cost, which becomes irrelevant for their choices because they face it no matter what they do. Thus, these individuals are not 'constrained' to choose $s = s_{-1}$ by the fixed cost (and there is a zero likelihood that they will pick exactly $s = s_{-1}$ since s is a continuous choice). The exogenous move friction m only enters the household problem here.

Moreover, there is also a tenure swapping cost $\nu_0 + \nu_1 d_{-1}$. We expect $\nu_0 < 0$ and

[5] Note that we can also have a specification with complementary goods that would both increase with respect to h. We only need both choices to be stricly monotone with respect to h, increasing or decreasing.

[6] Notice that we also include monetary transaction costs in the model, which also explains part of the staying behaviour.

$\nu_0 + \nu_1 < 0$ because it is costly to swap tenure. And probably $\nu_1 < 0$ because it is even more costly for previous homeowners to become renters: as observed in the data, homeowners are less likely to become renters.

Transitions:

The *budget constraint* is:

$$\begin{aligned} a_{+1} \leq \; & (1+r)a + \text{income} - c - \underbrace{T(x, d, s, d_{-1}, s_{-1})}_{\text{Tax}} \\ & \underbrace{-ps(1-d)}_{\text{Net rent}} - \underbrace{q(s - s_{-1})dd_{-1}}_{\text{Price of house change}} - \underbrace{qsd(1-d_{-1})}_{\text{Price of new house}} + \underbrace{qs(1-d)d_{-1}}_{\text{Gains from selling and renting}} . \end{aligned}$$

Notice that d_{-1} enters the budget constraint as previous owners ($d_{-1} = 1$) are potentially richer than previous renters ($d_{-1} = 0$) because they can sell their housing asset when they move. It also enters the tax schedule. The tax (benefits) schedule $T()$ can be modelled realistically from the French law to include income tax, payroll tax, residential tax, property tax, housing sale and purchase tax and housing benefits.

The *transitions* of the other variables: household income and family demographics are exogenous in the model. Households can exogenously have a new child. They cannot divorce in this model. And we focus on couples so there is no couple formation either. This simplifies the asset and housing transition in cases of divorce or couple formation. Transitions of income and demographics depends on current income and demographics, not on s_{-1} or d_{-1}.

Dynamic life-cycle problem:

Let's introduce the dynamic optimization problem of the households. Slightly change the notation to add the index t to each variable. Here the households choose (d_t, s_{dt}, c_{dt}) in order to sequentially maximize their discounted sum of payoffs, with discount factor β. Let's define $V_t(z_t, d_{t-1}, s_{t-1})$ the ex ante value function of this discounted sum of future payoffs at the beginning of period t, just before the shocks (e_t, h_t) are revealed and conditional on behaving according to the optimal decision rule afterwards.

$$V_t(z_t, a_t, d_{t-1}, s_{t-1}) = \mathbb{E}\Big[\sum_{\tau=t}^{T} \beta^{\tau-t} \underset{d_\tau, s_{d\tau}, c_{d\tau}}{max} U(d_\tau, c_{d\tau}, s_{d\tau}, s_{\tau-1}, d_{\tau-1}, x_\tau, m_\tau, h_\tau, e_\tau)\Big]$$

$$\text{s.t.} \quad a_{t+1} \leq (1+r_t)a_t + \text{income}_t - c_t - T(x_t, d_t, s_t, d_{t-1}, s_{t-1})$$
$$- p_t s_t(1-d_t) - q_t(s_t - s_{t-1})d_t d_{t-1} - q_t s_t d_t(1-d_{t-1}) + q_t s_t(1-d_t)d_{t-1}$$

Thus, each period, the household chooses d, s_d and c_d to maximize their expected sum of payoffs:

$$\underset{d_t, s_{dt}, c_{dt}}{max} U_t(d_t, c_{dt}, s_{dt}, s_{t-1}, d_{t-1}, x_t, m_t, h_t, e_t) + \beta \mathbb{E}_{z_{t+1}}\Big[V_{t+1}(z_{t+1}, a_{t+1}, d_t, s_t)\Big| z_t, a_t, s_{dt}, c_{dt}, d_t, d_{t-1}, s_{t-1}\Big]$$

Remember that z_t include the exogenous states, i.e. the demographics x_t, but also the income and the prices.

Retirement:

At 60 years old, we assume the household retire, and live off of their pension and wealth (housing and non-housing) for 15 more years.

Measurement errors:

Moreover, the non-housing consumption is observed with *measurement error* ζ by the econometrician, i.e.

$$c^{\text{obs}} = c^* + \zeta.$$

But ζ does not enter the household problem. The measurement error is independent from every other variables, and is iid every periods.

4 Identification

At each period, we observe data on the variables $(d, s_d, c_d, s_{-1}, d_{-1}, a, z)$. We only observe s_0 and c_0 if $d = 0$ and s_1 and c_1 if $d = 1$. In other words:

$$s_d = s_0(1-d) + s_1 d$$
$$c_d = c_0(1-d) + c_1 d.$$

Shocks m, h and e are observed by the agents but not observed by the econometrician. Moreover, a noisy measure of c_d is observed because of *measurement errors*, i.e. we observe $c_d + \zeta$.

We first study identification of the following objects: the optimal *Conditional Continuous Choices* (CCCs) $s^*_d(h, s_{-1}, d_{-1}, a, z, m)$ and $c^*_d(h, s_{-1}, d_{-1}, a, z, m)$, as well as the optimal *Conditional Choice Probabilities* (CCPs) $Pr(d|h, s_{-1}, d_{-1}, a, z, m)$. Then, following Bruneel-Zupanc (2021) we will use the identified optimal choices to identify the structural parameters of the model.

The identification proof will use arguments of Bruneel-Zupanc (2021). However, this paper cannot be applied directly as the setup here is more complicated for three main reasons. (i) d_{-1} violates the *exclusion restriction* of Bruneel-Zupanc (2021), and is not directly an instrument. Indeed, it affects the budget constraint and thus the future value. (ii) Because of fixed costs, the optimal choice s will not be strictly monotone. In a related note, the presence of unobserved m binary shock implies there are additional objects to identify. (iii) Consumption is an additional choice and this was not dealt with in the previous paper.[7] We show how to identify it if it is measured with errors.

To show how the identification works here, we proceed stepwise. First we show how the optimal choices would be identified if everyone was moving freely, i.e. if the fixed cost was irrelevant (everyone pays it even if they do not move). Then we show how the optimal choices are identified with a fixed moving cost in the model that makes some households stay in their previous house. Finally, we show how the consumption choice is also identified despite the measurement errors and how it can be used to identify the housing tenure probability choice even for stayers.

Without loss of generality, we proceed conditional on any given exogenous state z in this section, and we omit z in what follows.

4.1 Identification when everyone moves

Assume everyone moves. In other words, everyone pays the fixed cost, even if they do not move, such that households will no longer choose $s = s_{-1}$ to avoid paying

[7]Notice that s_d in this paper would be the direct counterpart to c_d in Bruneel-Zupanc (2021). Because c_d is measured with additional errors, so it cannot be used directly to recover h.

the fixed cost. The utility function is then written as:

$$
\begin{aligned}
U(d, c, s, s_{-1}, d_{-1}, x, m, h, e) \;&=\; \gamma(x)\frac{\left(c^{1-1/\gamma_d} + \phi(h)s^{1-1/\gamma_d}\right)^{\frac{1-\sigma}{1-1/\gamma_d}}}{1-\sigma} \\
&+\; (\mu_0 + \mu_1 d_{-1}) \;+\; (\nu_0 + \nu_1 d_{-1})\mathbf{1}\{d \neq d_{-1}\} \;+\; e_d
\end{aligned}
$$

Very importantly, notice that this is *equivalent to the general setup when* $m = 1$, i.e. where households are forced to move. A model without any fixed cost $(\mu_0 + \mu_1 d_{-1})$ term would also be equivalent in terms of optimal choices. Indeed, the fixed cost does not differentially affect the discrete alternatives so the optimal choice d^* does not depend on it. Furthermore, by additive separability, s_d^* and c_d^* are also independent from it.

Even when everyone moves, we cannot apply the strategy employed by Bruneel-Zupanc (2021) directly. Indeed, d_{-1} is not excluded from the budget constraint. So the optimal continuous choices s_d^* and c_d^* will depend on d_{-1} since, everything else equal, a previously homeowner is richer than a previously renter.

We adopt the following solution in this specific setup. One can show that d_{-1} only affect housing wealth in the budget constraint when everyone moves. Denote $\tilde{s}_d^*$ the optimal choices in this setup where everyone moves, or the *optimal housing service choice conditional on moving*. We can also call it the *unconstrained optimal choice*, in the sense that it is *not constrained by the fixed cost*. One can show that for all h, d and s'_{-1}:

$$
\tilde{s}_d^*(h, d_{-1} = 1, s_{-1}, a) = \tilde{s}_d^*(h, d_{-1} = 0, s'_{-1} = \cdot, a' = a + q\frac{s_{-1}}{1+r}). \tag{1}
$$

In other words, conditional on moving, ex-homeowners with financial asset a and house of size s_{-1} will have a *total wealth* equal to $(1+r)a + qs_{-1}$ (plus their income and taxes, that we abstract from here). They will make the same choice as ex-renters with the same total wealth, i.e. with financial asset $a' = a + qs_{-1}/(1+r)$. Because ex-renters $(d_{-1} = 0)$ do not own their house, their previous house size s'_{-1} do not affect their wealth, so the property holds for all s'_{-1}. The property holds because s_{-1} only matters in terms of wealth for the previous homeowners without fixed cost of moving here. Apart from the housing wealth it provides them when they move, previous homeowners are exactly equivalent to previous renters here. So, *conditional on* ***total wealth****,* $(1 + r)a + qs_{-1}d_{-1}$, the optimal housing service choice $\tilde{s}_d^*$ is independent from d_{-1}. It is true because d_{-1} only matters in determining the total wealth, it does not play a role anywhere else in the other variables transition.

Notice also that d_{-1} is (strictly) *relevant* for the choice to be a homeowner or a renter at a given total wealth. Indeed, because of the *tenure swapping cost* $(\nu_0 + \nu_1 d_{-1})\mathbb{1}\{d \neq d_{-1}\}$, we have that:

$$Pr(D = 1|h, d_{-1} = 1, s_{-1}, a) > Pr(D = 1|h, d_{-1} = 0, s'_{-1} = \cdot, a' = a + q\frac{s_{-1}}{1+r}), \tag{2}$$

in which the strict inequality comes from the fact that $\nu_0 + \nu_1 d_{-1} < 0$ for all d_{-1}, i.e. it is costly to swap tenure, no matter the previous tenure. In other words, at equal total wealth, ex-homeowners are strictly more likely to be homeowners today than ex-renters.

Therefore, even if we cannot count on an exclusion restriction as defined in Bruneel-Zupanc (2021), we can apply refined version of the identification proof in the paper using property (1). In short, the idea of the proof is to match ex-homeowners ($d_{-1} = 1$) with endogenous states (s_{-1}, a) to ex-renters ($d_{-1} = 0$) with endogenous states yielding the same total wealth, i.e. $a' = a + qs_{-1}/(1 + r)$ (and any s'_{-1}), instead of matching them with ex-renters with the same covariates ($a' = a$ and $s'_{-1} = s_{-1}$). For these pairs, we have the 'exclusion' and the 'relevance' of d_{-1}, conditional on the total wealth, and we can apply the same reasoning as in Bruneel-Zupanc (2021). We show how it works in what follows.

Sketch of the proof:

When $d_{-1} = 1$, *conditional on moving*, we have:

$$\begin{aligned}
h = \; & Pr(h \leq h) \\
= \; & Pr(h \leq h|s_{-1}, a, d_{-1} = 1) \\
= \; & Pr(h \leq h|D = 0, s_{-1}, a, d_{-1} = 1)Pr(D = 0|s_{-1}, a, d_{-1} = 1) \\
& + Pr(h \leq h|D = 1, s_{-1}, a, d_{-1} = 1)Pr(D = 1|s_{-1}, a, d_{-1} = 1) \\
= \; & Pr\Big(s \leq \tilde{s}_0^*(h, s_{-1}, a, d_{-1} = 1)|D = 0, s_{-1}, a, d_{-1} = 1\Big)\; Pr(D = 0|s_{-1}, a, d_{-1} = 1) \\
& + Pr\Big(s \leq \tilde{s}_1^*(h, s_{-1}, a, d_{-1} = 1)|D = 1, s_{-1}, a, d_{-1} = 1\Big)\; Pr(D = 1|s_{-1}, a, d_{-1} = 1)
\end{aligned}$$

$$
\begin{aligned}
= \quad & F_{S_0|D=0,s_{-1},a,d_{-1}=1}(\tilde{s}_0^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=0|s_{-1},a,d_{-1}=1) \\
& + F_{S_1|D=1,s_{-1},a,d_{-1}=1}(\tilde{s}_1^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=1|s_{-1},a,d_{-1}=1), \qquad (3)
\end{aligned}
$$

where the first equality comes from $h \sim \mathcal{U}(0,1)$ by normalization. The second is because $h \perp (d_{-1},a,s_{-1})$. The third equality comes from the law of total probability. The fourth equality comes from the strict monotonicity of $\tilde{s}_d^*(h,\cdot)$ with respect to h. The fifth equality is just a notation change for the conditional distribution functions.

Similarly, when $d_{-1}=0$, conditional on moving, we have:

$$
\begin{aligned}
h = \quad & Pr(h \le h) \\
= \quad & Pr(h \le h|s'_{-1},a',d_{-1}=0) \\
= \quad & Pr(h \le h|D=0,s'_{-1},a',d_{-1}=0)Pr(D=0|s'_{-1},a',d_{-1}=0) \\
& + Pr(h \le h|D=1,s'_{-1},a',d_{-1}=0)Pr(D=1|s'_{-1},a',d_{-1}=0) \\
= \quad & Pr\Big(s \le \tilde{s}_0^*(h,s'_{-1},a',d_{-1}=0)|D=0,s'_{-1},a',d_{-1}=0\Big)\ Pr(D=0|s'_{-1},a',d_{-1}=0) \\
& + Pr\Big(s \le \tilde{s}_1^*(h,s'_{-1},a',d_{-1}=0)|D=1,s'_{-1},a',d_{-1}=0\Big)\ Pr(D=1|s'_{-1},a',d_{-1}=0) \\
= \quad & F_{S_0|D=0,s'_{-1},a',d_{-1}=0}(\tilde{s}_0^*(h,s'_{-1},a',d_{-1}=0))\ Pr(D=0|s'_{-1},a',d_{-1}=0) \\
& + F_{S_1|D=1,s'_{-1},a',d_{-1}=0}(\tilde{s}_1^*(h,s'_{-1},a',d_{-1}=0))\ Pr(D=1|s'_{-1},a',d_{-1}=0),
\end{aligned}
$$

for the same reasons as what we have for $d_{-1}=1$.[8]

Now, recall property (1) that, for all h, d and s'_{-1}:

$$
\tilde{s}_d^*(h,d_{-1}=1,s_{-1},a) = \tilde{s}_d^*(h,d_{-1}=0,s'_{-1}=\cdot,a'=a+q\frac{s_{-1}}{1+r}). \qquad (4)
$$

Thus, *conditional on moving*, with $a'=a+qs_{-1}/(1+r)$, we have:

$$
\begin{aligned}
h = \quad & F_{S_0|D=0,s_{-1},a,d_{-1}=1}(\tilde{s}_0^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=0|s_{-1},a,d_{-1}=1) \\
& + F_{S_1|D=1,s_{-1},a,d_{-1}=1}(\tilde{s}_1^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=1|s_{-1},a,d_{-1}=1) \\
= \quad & F_{S_0|D=0,s'_{-1},a',D_{-1}=0}(\tilde{s}_0^*(h,s'_{-1},a',d_{-1}=0))\ Pr(D=0|s'_{-1},a',d_{-1}=0) \\
& + F_{S_1|D=1,s'_{-1},a',d_{-1}=0}(\tilde{s}_1^*(h,s'_{-1},a',d_{-1}=0))\ Pr(D=1|s'_{-1},a',d_{-1}=0).
\end{aligned}
$$

[8] Notice here that $\tilde{s}_d^*(\cdot,d_{-1}=0) \perp s_{-1}$. We see later that s_{-1} only matters to determine the position of the mass point of the optimal s when the household does not move if $d_{-1}=0$. So, for the conditional distribution function of s, we could remove s'_{-1}.

Therefore, we can proceed as in Bruneel-Zupanc (2021), and rewrite it as:

$$\begin{aligned}
&\Bigg(F_{S_0|D=0,s_{-1},a,D_{-1}=1}(\tilde{s}_0^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=0|s_{-1},a,D_{-1}=1) \\
&\quad - F_{S_0|D=0,s'_{-1},a',D_{-1}=0}(\tilde{s}_0^*(h,s'_{-1},a',d_{-1}=0))\ Pr(D=0|s'_{-1},a',D_{-1}=0)\Bigg) \\
= -\ &\Bigg(F_{S_1|D=1,s_{-1},a,D_{-1}=1}(\tilde{s}_1^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=1|s_{-1},a,D_{-1}=1) \\
&\quad - F_{S_1|D=1,s'_{-1},a',D_{-1}=0}(\tilde{s}_1^*(h,s'_{-1},a',d_{-1}=0))\ Pr(D=1|s'_{-1},a',D_{-1}=0)\Bigg).
\end{aligned}$$

Under property (1), we have a mapping between the policy functions with different previous housing wealth but similar total wealth. Thus we have only two policy functions to identify (instead of four, two for each d), with four different conditional distributions thanks to the relevance of d_{-1}. It yields

$$\begin{aligned}
&\Bigg(F_{S_0|D=0,s_{-1},a,D_{-1}=1}(\tilde{s}_0^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=0|s_{-1},a,D_{-1}=1) \\
&\quad - F_{S_0|D=0,s'_{-1},a',D_{-1}=0}(\tilde{s}_0^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=0|s'_{-1},a',D_{-1}=0)\Bigg) \\
= -\ &\Bigg(F_{S_1|D=1,s_{-1},a,D_{-1}=1}(\tilde{s}_1^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=1|s_{-1},a,D_{-1}=1) \\
&\quad - F_{S_1|D=1,s'_{-1},a',D_{-1}=0}(\tilde{s}_1^*(h,s_{-1},a,d_{-1}=1))\ Pr(D=1|s'_{-1},a',D_{-1}=0)\Bigg) \\
\iff &\qquad \Delta F_{S_0}(\tilde{s}_0^*(h,s_{-1},a,d_{-1}=1)) = \Delta F_{S_1}(\tilde{s}_1^*(h,s_{-1},a,d_{-1}=1)). \qquad (5)
\end{aligned}$$

In the model when everyone moves, the functions ΔF_{S_d} can be *directly estimated*, from the data (as conditional distributions). It only remains to identify policies: $\tilde{s}_d^*(h,s_{-1},a,d_{-1}=1)$. One can directly apply Bruneel-Zupanc (2021) to show that functions $\tilde{s}_d^*(h,s_{-1},a,d_{-1}=1)$ are identified as the unique solution to this system of equation (5).

In fact here, because we have *strict* relevance (2), one can show that ΔF_{S_0} and ΔF_{S_1} are invertible. Thus, the mapping between $\tilde{s}_0^*$ and $\tilde{s}_1^*$ is identified directly. Indeed, since $\tilde{s}_d^*$ are strictly monotone with respect to h, we can rewrite (5) as:

$$\Delta F_{S_0}(\tilde{s}_0^*(s_1,s_{-1},a,d_{-1}=1)) = \Delta F_{S_1}(s_1).$$

And there is a unique mapping between s_1 and s_0, obtain by taking the inverse:

$$\tilde{s}_0^*(s_1, s_{-1}, a, d_{-1} = 1) = \Delta F_{S_0}^{-1}\Big(\Delta F_{S_1}(s_1)\Big)$$

Now, to obtain $\tilde{s}_d^*$ as a function of h for both d, simply plug these into (3). Therefore, in the model with only movers ($m = 1$), the optimal housing service $\tilde{s}_d^*(h, s_{-1}, a, d_{-1} = 1)$ are *identified* for all d, h, s_{-1} and a.

Moreover, these policies are strictly increasing, so one can invert them to recover the unobserved h for each observation of s_d in the data.

$$h = (\tilde{s}_d^*)^{-1}(s_d^{obs}, s_{-1}, a, d_{-1})$$

From there, it is as if $h = h$ were observed. We can use it to compute the conditional choice probabilities (CCPs)

$$Pr(D = 1|h, d_{-1}, s_{-1}, a).$$

Thus, the optimal discrete choice probabilities are also identified.

As for the consumption choice $\tilde{c}_d^*(h, s_{-1}, a, d_{-1})$, we can also use the inversion of $\tilde{s}_d^*(h, \cdot)$ to recover h and proceed as if it was observed. Thus, one just need to run the non parametric regression

$$c_d^{obs} = \tilde{c}_d^*(h, s_{-1}, a, d_{-1}) + \zeta,$$

which directly identifies the optimal consumption policies and the distribution of the measurement errors ζ.

Therefore, the three policy choices are identified without fixed moving costs in a model where everyone moves.

4.2 Identification with fixed costs, when some households do not move

In a model with fixed costs, when not everyone is moving, the identification will be more complicated because some households will choose not to move, and select $s_d^* = s_{-1}, d^* = d_{-1}$.

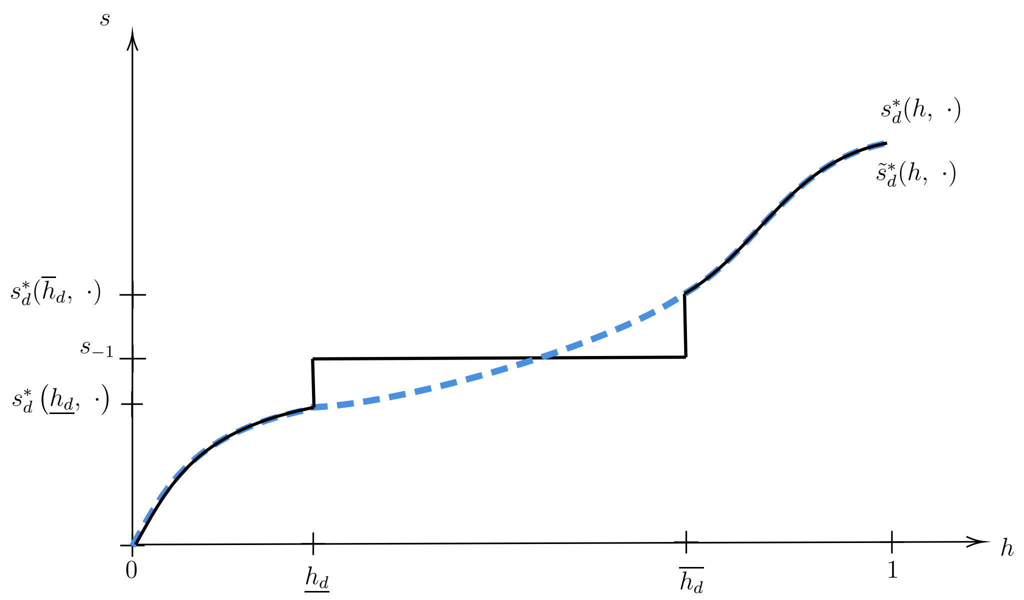


Figure 2: Optimal housing services function

The presence of the fixed cost transforms the problem as shown in Figure 2. Between two values of h, $\underline{h}_d(a, s_{-1}, d_{-1})$ and $\bar{h}_d(a, s_{-1}, d_{-1})$, with $0 \leq \underline{h} \leq \bar{h} \leq 1$, households will choose to stay and $s_d^*(h, a, s_{-1}, d_{-1}) = s_{-1}$. Outside the interval $[\underline{h}_d(a, s_{-1}, d_{-1}), \bar{h}_d(a, s_{-1}, d_{-1})]$, households will move, and select the optimal housing service defined in the previous section: $\tilde{s}_d^*(h, \cdot)$. Notice that it is possible that $\underline{h}_d = 0$ or $\bar{h}_d = 1$ for some values of the covariates (in particular s_{-1}). In fact we can even have $\underline{h}_d = \bar{h}_d = 0$ or $\underline{h}_d = \bar{h}_d = 1$, in which case no one moves given these state variables. If $d \neq d_{-1}$, $s_d^*(h, a, s_{-1}, d_{-1} \neq d) = \tilde{s}_d^*(h, a, s_{-1}, d_{-1} \neq d)$ because if they swap tenure these households are already moving by construction. So the boundaries only matter when $d = d_{-1}$.

Another property is that the mass point is shifting with the value of s_{-1}. In other words, if $s^a_{-1} < s^b_{-1}$, then $\underline{h}_d(a, s^a_{-1}, d_{-1}) \leq \underline{h}_d(a, s^b_{-1}, d_{-1})$ and $\bar{h}_d(a, s^a_{-1}, d_{-1}) \leq \bar{h}_d(a, s^b_{-1}, d_{-1})$. This ordering also translate in the s_d.

Moreover, households who endure the exogenous move shock $m = 1$ are necessarily moving and their optimal housing service choice $s_d^*(h, \cdot, m = 1) = \tilde{s}_d^*(h, \cdot)$ for all h. So if we observe that an household does not move in the data, it means that $m = 0$. Households with $m = 0$ will never pick an optimal s_d different from s_{-1} between $[s_d^*(\underline{h}_d, \cdot), s_d^*(\bar{h}_d, \cdot)]$ if they keep the same tenure $d = d_{-1}$. Which means that households who move without changing tenure with an optimal s_d^* choice in $[s_d^*(\underline{h}_d, \cdot), s_d^*(\bar{h}_d, \cdot)]$ endured the exogenous move shock $m = 1$. In other words, moves to houses similar to the previous one (close to s_{-1}) are due to the exogenous move shock in the model. Because of the fixed cost, households would never pick such a choice if they were not forced to move.

Identification with fixed costs is more complicated as the optimal s_d^* are not strictly monotone with respect to h anymore, and there are more objects to identify (the bounds). We proceed piece by piece for the identification. First, we show how to identify the boundaries in terms of s_d, i.e. how to identify $s_d^*(\underline{h}_d, s_{-1}, a, d_{-1})$ and $s_d^*(\bar{h}_d, s_{-1}, a, d_{-1})$ for all (s_{-1}, a, d_{-1}). Second, we show how to identify the probability of receiving and exogenous moving shock, p^m. Third, we show how to identify s_d^* outside boundaries $\underline{s}_d, \bar{s}_d$. Fourth, we show how to identify $s_d^*(h, \cdot, m = 1)$ between boundaries. Finally, we show how to recover $Pr(D = 1|h, \cdot, m = 1)$, as well as the consumption choice of movers and the measurement error distribution.

<u>Step 1:</u> identification of $s_d^*(\underline{h}_d)$ and $s_d^*(\bar{h}_d)$

We drop the dependence of $\underline{h}_d, \bar{h}_d, s_d(\underline{h}_d), s_d(\bar{h}_d)$ on (a, s_{-1}, d_{-1}) in the notation here to simplify the exposition. As already mentioned, $\underline{h}_d$ and $\bar{h}_d$ only matters when $d_{-1} = d$.

First, notice that if there was no moving shock, i.e. if $m = 0$ for all household, then the boundaries are straightforward to identify. Indeed, they correspond to the highest value of s_d before $s_d^* = s_{-1}$ and the lowest value of s_d such that $s_d^* = s_{-1}$ (conditional on the covariates).

The presence of exogenous moving shocks prevents us from identifying the boundaries as easily because household with $m = 1$ will move freely and could pick s_d close to s_{-1} without changing tenure. The idea for the identification of the boundaries rely on the fact that, above $\underline{s}_d$ and below $\bar{s}_d$, we should observe jumps (proportional to p^m) in the conditional density of s_d^* because only the population with $m = 1$ will remain, while the households with $m = 0$ will all choose to stay at $s_d^* = s_{-1}$.

Formally, since m is an exogenous shock, independent from everything else, for all s we have

$$F_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d) = p^m \ F_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d) + (1-p^m) \ F_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=0}}(s_d),$$

and for all $s \neq s_{-1}$ (because at s_{-1} the density has a mass point):

$$\frac{dF_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d)}{ds_d} = p^m \ \frac{dF_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d)}{ds_d} + (1-p^m) \ \frac{dF_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=0}}(s_d)}{ds_d}$$

$$\iff f_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d) = p^m \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d) + (1-p^m) \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=0}}(s_d),$$

where we define the density $f_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d) = dF_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d)/ds_d$, and where $p^m = Pr(m=1)$.

Now notice that, for all $s \notin [s_d^*(\underline{h}_d), s_d^*(\bar{h}_d)]$:

$$F_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d) = \ F_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d) = F_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=0}}(s_d)$$

and

$$f_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d) = \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d) = f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=0}}(s_d).$$

While, for all $s \in [s_d^*(\underline{h}_d), s_d^*(\bar{h}_d)] \setminus \{s_{-1}\}$:

$$f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=0}}(s_d) = 0.$$

Then, for all $s \in [s_d^*(\underline{h}_d), s_d^*(\bar{h}_d)] \setminus \{s_{-1}\}$:

$$f_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d) = p^m \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d)$$

Thus, we can *identify* $s_d^*(\underline{h}_d)$ and $s_d^*(\bar{h}_d)$ by observing *discontinuities in the density.* Indeed, in a close neighborhood to $s_d^*(\underline{h}_d)$,

$$f_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d)\Big|_{s_d=\tilde{s}_d^*(\underline{h}_d)} = \begin{cases} p^m \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d) & \\ \quad + \quad (1-p^m) \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=0}}(s_d) & \text{if } s_d < s_d^*(\underline{h}_d) \\ p^m \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d) & \text{if } s_d \geq s_d^*(\underline{h}_d) \end{cases}$$

Thus, if $\underline{h}_d > 0$, we identify $\tilde{s}_d^*(\underline{h}_d)$ as the only $s_d < s_{-1}$ such that there is a jump in the density.And if there is no such point, it means that $\underline{h}_d = 0$, meaning that the fixed cost covers all the optimal choices $\tilde{s}_d$ below s_{-1}.
Similarly, in a close neighborhood to $\tilde{s}_d^*(\bar{h}_d)$,

$$f_{S_d|D=d,D_{-1}=d,s_{-1},a}(s_d)\Big|_{s_d=\tilde{s}_d^*(\bar{h}_d)} = \begin{cases} p^m \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d) & \text{if } s_d < s_d^*(\bar{h}_d) \\ p^m \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=1}}(s_d) & \\ \quad + \quad (1-p^m) \ f_{S_d|D=d,D_{-1}=d,s_{-1},a,\mathbf{m=0}}(s_d) & \text{if } s_d \geq s_d^*(\bar{h}_d) \end{cases}$$

Thus, if $\bar{h}_d < 1$, we identify $\tilde{s}_d^*(\bar{h}_d)$ as the only $s_d > s_{-1}$ such that there is a jump in the density. And if we observe no such point, it means that $\bar{h}_d = 1$.

Notice that the knowledge of p^m is not required for the identification of the boundaries: one just need to observe a jump in the density, which will occur as long as $p^m \in (0,1)$. In the extreme case where $p^m = 1$, everyone moves and the result of the previous section holds. In the other extreme where $p^m = 0$, there is no exogenous move and so we identify the boundaries as stated previously: as the lowest value s_d below s_{-1} and the highest value greater than s_{-1}.

Step 2: identification of p^m

To identify p^m now, we will exploit the knowledge of the probability of not moving $s_d = s_{-1}$.

Notice that, by construction, since $m \perp h, e$

$$\begin{aligned} & Pr\Big(S_d \in [\tilde{s}_d^*(\underline{h}_d), \tilde{s}_d^*(\bar{h}_d)]\Big| D = d, D_{-1} = d, s_{-1}, a\Big) \\ = \; & Pr\Big(S_d \in [\tilde{s}_d^*(\underline{h}_d), \tilde{s}_d^*(\bar{h}_d)]\Big| D = d, D_{-1} = d, s_{-1}, a, m = 0\Big) \\ \Bigg(\textit{and also} = \; & Pr\Big(S_d \in [\tilde{s}_d^*(\underline{h}_d), \tilde{s}_d^*(\bar{h}_d)]\Big| D = d, D_{-1} = d, s_{-1}, a, m = 1\Big)\Bigg) \end{aligned}$$

Moreover, we know that:

$$Pr\Big(S_d \in [\tilde{s}_d^*(\underline{h}_d), \tilde{s}_d^*(\bar{h}_d)]\Big| D = d, D_{-1} = d, s_{-1}, a, m = 0\Big) = Pr\Big(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a, m =$$

Now, $Pr(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a, m = 0)$ is not directly estimable as we do not observe m. But we observe $Pr(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a)$. And we know that:

$$\begin{aligned} Pr\Big(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a\Big) = \; & (1 - p^m)\; Pr\Big(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a, m = 0\Big) \\ & + p^m \; \underbrace{Pr\Big(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a, m = 1\Big)}_{=0 \text{ by continuity of } \tilde{s}_d^*(\cdot)} \\ = \; & (1 - p^m)\; Pr\Big(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a, m = 0\Big). \end{aligned}$$

We obtain this because, when $m = 1$, the household adopts the optimal choice $\tilde{s}_d^*(\cdot)$. Thus $Pr(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a, m = 1) = 0$ because $\tilde{s}_d^*(\cdot)$ is continuous, so the likelihood of choosing $\tilde{s}_d^*$ exactly equal to s_{-1} is zero.

By mixing the three previous properties, we obtain

$$
\begin{aligned}
Pr\Big(S_d \in [\tilde{s}^*_d(\underline{h}_d), \tilde{s}^*_d(\bar{h}_d)]\Big| D = d, D_{-1} = d, s_{-1}, a\Big)& \\
&= Pr\Big(S_d \in [\tilde{s}^*_d(\underline{h}_d), \tilde{s}^*_d(\bar{h}_d)]\Big| D = d, D_{-1} = d, s_{-1}, a, m = 0\Big) \\
&= Pr\Big(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a, m = 0\Big) \\
&= \frac{Pr\Big(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a\Big)}{1 - p^m},
\end{aligned}
$$

where only p^m is unknown since we already identified $\tilde{s}^*_d(\underline{h}_d), \tilde{s}^*_d(\bar{h}_d)$. It gives that

$$
p^m = \frac{Pr\Big(S_d \in [\tilde{s}^*_d(\underline{h}_d), \tilde{s}^*_d(\bar{h}_d)]\Big| D = d, D_{-1} = d, s_{-1}, a\Big) - Pr\Big(S_d = s_{-1}\Big| D = d, D_{-1} = d, s_{-1}, a\Big)}{Pr\Big(S_d \in [\tilde{s}^*_d(\underline{h}_d), \tilde{s}^*_d(\bar{h}_d)]\Big| D = d, D_{-1} = d, s_{-1}, a\Big)},
$$

which *identifies* p^m.[9]

Step 3: identification of $s^*_d(h, \cdot)$ *outside the boundary points* $\underline{h}_d, \bar{h}_d$

As displayed in Figure 2, outside of the boundaries $(\underline{h}_d(a, s_{-1}, d_{-1}), \bar{h}_d(a, s_{-1}, d_{-1}))$, households move and $s^*_d(h, a, s_{-1}, d_{-1}) = \tilde{s}^*_d(h, a, s_{-1}, d_{-1})$ for all (a, s_{-1}, d_{-1}). In other words, outside of the boundaries, $s^*_d(h, a, s_{-1}, d_{-1}, m = 0) = s^*_d(h, a, s_{-1}, d_{-1}, m = 1) = \tilde{s}^*_d(h, a, s_{-1}, d_{-1})$, and the presence of the moving shock is irrelevant: households endogenously choose to move and make the same choice as if they were facing a moving shock.

We build upon Section 4.1, using property (1) for the identification of the optimal s^*_d outside the boundaries, i.e. conditional on moving. i.e. we use:

$$
\tilde{s}^*_d(h, d_{-1} = 1, s_{-1}, a) = \tilde{s}^*_d(h, d_{-1} = 0, s'_{-1} = \cdot, a' = a + q\frac{s_{-1}}{1 + r}),
$$

where s'_{-1} is the previous housing service of the previously renting household ($d_{-1} = 0$), which does not matter on their choice conditional on moving since ex renters have no housing wealth.

The bounds introduce some specificities though. Depending on what a, s_{-1} and s'_{-1} are, and using $a' = a + qs_{-1}/(1 + r)$, we will have four boundaries. For $d = 1$, we will have household staying when $d_{-1} = 1$ and the boundaries will depend on a and s_{-1}, i.e. $\underline{h}_1(a, s_{-1}, d_{-1} = 1)$, $\bar{h}_1(a, s_{-1}, d_{-1} = 1)$. Similarly, previous

[9] Notice that, since we are studying identification at given exogenous covariates, it means that we could allow for a p^m that depend on these covariates z.

renters ($d_{-1} = 0$) only stays in their home if they remain renters ($d = 0$) and the boundaries in this case will depend on a' and s'_{-1}, i.e. $\underline{h}_0(a', s'_{-1}, d_{-1} = 0)$, $\bar{h}_0(a', s'_{-1}, d_{-1} = 0)$. So, in fact for any given (a, s_{-1}, s'_{-1}) and $a' = a + qs_{-1}/(1+r)$, we will have identification of the optimal choices s^*_d for

$$h \notin [\underline{h}_0(a', s'_{-1}, d_{-1} = 0), \bar{h}_0(a', s'_{-1}, d_{-1} = 0)] \cup [\underline{h}_1(a, s_{-1}, d_{-1} = 1), \bar{h}_1(a, s_{-1}, d_{-1} = 1)].$$

Now, notice that property (1) holds for any s'_{-1}. So, provided that we have *enough variation in the observed support of* s_{-1}, $[\underline{h}_0, \bar{h}_0]$ are not important. One can identify s^*_1 and s^*_0 for all $h \notin [\underline{h}_1(a, s_{-1}, d_{-1} = 1), \bar{h}_1(a, s_{-1}, d_{-1} = 1)]$. Only the boundaries for a, s_{-1} when $d = 1$ matters. Indeed, we will proceed to the identification by taking several value of s'_{-1} in order to move the $\underline{h}_0, \bar{h}_0$ such that they 'do not matter'. Recall that $\underline{h}_0, \bar{h}_0$ increase with s'_{-1}. So typically, to identify $s^*_1(h, a, s_{-1}, d_{-1} = 1)$ and $s^*_0(h, a, s_{-1}, d_{-1} = 1)$ on $[0, \underline{h}_1(a, s_{-1}, d_{-1} = 1)]$, pick a sufficiently high s'_{-1} such that $\underline{h}_0(a', s'_{-1}, d_{-1} = 0) > \underline{h}_1(a, s_{-1}, d_{-1} = 1)$. If we observe enough variation in s'_{-1}, such an s'_{-1} should always exists.[10] Similarly, to identify $s^*_1(h, a, s_{-1}, d_{-1} = 1)$ and $s^*_0(h, a, s_{-1}, d_{-1} = 1)$ on $[\underline{h}_1(a, s_{-1}, d_{-1} = 1), 1]$, pick a sufficiently low s'_{-1} such that $\bar{h}_0(a', s'_{-1}, d_{-1} = 0) > \bar{h}_1(a, s_{-1}, d_{-1} = 1)$.

Therefore, for all (a, s_{-1}), $s^*_0(h, a, s_{-1}, d_{-1} = 1)$ and $s^*_1(h, a, s_{-1}, d_{-1} = 1)$ are identified for all $h \notin [\underline{h}_1(a, s_{-1}, d_{-1} = 1), \bar{h}_1(a, s_{-1}, d_{-1} = 1)]$.[11] In other words, we fully identify functions $\tilde{s}^*_d(h, a, s_{-1}, d_{-1} = 1)$ and $s^*_d(h, a, s_{-1}, d_{-1} = 1, m = 1)$ on this subset, i.e. conditional on endogenously moving.

Step 4: identification of $\tilde{s}^*_d(h, \cdot)$ for $h \in [\underline{h}_1, \bar{h}_1]$

It remains to identify $s^*_d(h, a, s_{-1}, d_{-1} = 1, \mathbf{m = 1})$ for both d, on the subset of $h \in [\underline{h}_1, \bar{h}_1]$. Notice that for $m = 0$, we do not need to identify $s^*_1(h, a, s_{-1}, d_{-1} = 1, \mathbf{m = 0})$ because by definition it is equal to s_{-1} on this subsample. So what remains to be identified are the policies for movers $\tilde{s}_d$, or equivalently the policies

[10]Notice that in the first step we only identify the bounds in terms of s and not the value of $\underline{h}, \bar{h}$ directly. So this is a guessing game: take s'_{-1}, it yields $\underline{s}_0 = s^*_0(\underline{h}_0(a', s'_{-1}, d_{-1} = 0), d_{-1} = 0, a', s'_{-1})$. Now run the identification procedure in step 1. If we notice that $s^*_1(\underline{s}_0) < \underline{s}_1 \equiv s^*_1(\underline{h}_1(a, s_{-1}, d_{-1} = 1), d_{-1} = 1, a, s_{-1})$, it means that s'_{-1} was not high enough (because $\underline{h}_0 < \underline{h}_1$), and one need to run it again with a higher s'_{-1}.

[11]And the policies when $d_{-1} = 0$ are identified using the ones with $d_{-1} = 1$ through property (1), as in Section 4.1.

conditional on $m = 1$.

The idea for the identification is close to what we did in the previous step: we will once again shift s_{-1} in order to shift the mass point. Notice that

$$\begin{aligned}\tilde{s}_d(h, d_{-1} = 1, s_{-1}, a) =& \tilde{s}_d(h, d_{-1} = 1, s'_{-1}, a') \\ & \forall\ s'_{-1}, a' \text{ s.t. } \frac{qs'_{-1}}{1+r} + a' = \frac{qs_{-1}}{1+r} + a.\end{aligned} \tag{6}$$

In words, property (6) is similar to the idea in property 1. We have that conditional on moving, only the total wealth matters for households. So, even two ex-owners ($d_{-1} = 1$) with the same total wealth will make the same housing service choice when they move. How their wealth is composed of financial or housing assets does not matter once they move.

Therefore, conditional on having enough variation in s_{-1}, we can vary s'_{-1} at fixed total wealth ($= qs_{-1}/(1 + r) + a$) in order to identify $s_d(h, d_{-1} = 1, s_{-1}, a)$ for all h. For example, one can proceed to the identification procedure described in the previous step with two different values of s_{-1}: s_{-1} and s'_{-1}. With s'_{-1}, a' such that

$$s_1^*(\underline{h}_1(a', s'_{-1}, d_{-1} = 1), d_{-1} = 1, a', s'_{-1}) > s_1^*(\bar{h}_1(a, s_{-1}, d_{-1} = 1), d_{-1} = 1, a, s_{-1}).$$

In which case the optimal choice for the movers $\tilde{s}_1^*$ and $\tilde{s}_0^*$ are identified for all $h \in [0, 1]$.

<u>Step 5:</u> *identification of h for movers and of $Pr(D = 1|h, d_{-1}, s_{-1}, a, m = 1)$ for all $h \in [0, 1]$*

We proceed exactly as in steps 3 and 4 by shifting s_{-1}. While we identify $s_d^*(\cdot, m = 1)$, we can directly invert them, to recover $h = (\tilde{s}_d^*)^{-1}(s_d^{obs}, s_{-1}, a, d_{-1})$. In other words, *for movers, h is identified*, as if it was an observed variable. Then we use it to compute the conditional choice probabilities (CCPs) of movers:

$$Pr(D = 1|h, d_{-1}, s_{-1}, a, m = 1) \text{ for all } h \in [0, 1]$$

Remark, on the subsample of moving households, $Pr(D = 1|h, d_{-1}, s_{-1}, a, m = 1)$ $= Pr(D = 1|h, d_{-1}, s_{-1}, a, m = 0) = Pr(D = 1|h, d_{-1}, s_{-1}, a)$. So the only case where we do not directly identify $Pr(D = 1|h, d_{-1}, s_{-1}, a, m = 0)$ is when $s_d^* = s_{-1}$. Because in this case, we cannot invert s_d^* to recover the corresponding value of the unobserved shock h.

<u>Step 6:</u> *identification of* $c_d^*(h, d_{-1}, a, m = 1)$ *for all* $h \in [0, 1]$ *and the measurement error* ζ

Similarly, we invert $s_d^*(\cdot, m = 1)$ to recover h from observed s_d for households who move exogenously. Then we use this h as if it was observed to run the non parametric regression

$$c_d^{obs} = c_d^*(h, s_{-1}, a, d_{-1}, m = 1) + \zeta$$

This regression directly identifies the optimal consumption policies of movers and the *distribution of measurement errors* ζ*, denoted* F_ζ. As for $Pr(D = 1|h, \cdot)$, for movers, $c_d^*(h, s_{-1}, a, d_{-1}, m = 1) = c_d^*(h, s_{-1}, a, d_{-1}, m = 0)$. So we also identify $c_d^*(\cdot, m = 0)$ for movers. It remains to identify the optimal consumption choice of stayers.

4.3 Identification of c for stayers

Contrary to s_d, when $h \in [\underline{h}_d(a, s_{-1}, d_{-1} = d), \bar{h}_d(a, s_{-1}, d_{-1} = d)]$, c_d varies and is not fixed (as s_d is equal to s_{-1}). So it remains to identify $c_d^*(h, s_{-1}, a, d_{-1} = d, m = 0)$ for $h \in [\underline{h}_d(a, s_{-1}, d_{-1} = d), \bar{h}_d(a, s_{-1}, d_{-1} = d)]$. It is interesting that consumption provides some information on h (though the information is noisy because of measurement errors).

In the data, we only observe $c_d^{obs} = c_d^* + \zeta$ and the distribution of c_d^{obs}. However, ζ is an independent measurement error and we have identified its distribution F_ζ on the subset of movers in step 6 of the previous section. So, by *deconvolution* (e.g. Comte and Lacour, 2011), we can recover the distribution of the true $c_d^*(h, \cdot)$, denoted $F_{C_d|D=d,\cdot}(c_d)$, from any conditional distribution of observed c_d^{obs}.

Let us identify $c_d^*(h, s_{-1}, a, d_{-1} = d, m = 0)$ for all h. Recall that $c_d^*(h, s_{-1}, a, d_{-1} = d, m = 0)$ is already identify outside of the boundaries. So we only need to identify it for stayers, i.e. for $h \in [\underline{h}_d(a, s_{-1}, d_{-1} = d), \bar{h}_d(a, s_{-1}, d_{-1} = d)]$. Proceed by values of d. Start with $d = 1$. In the data, if households choose $s_d = s_{-1}$ and $d = d_{-1}$, it means that $m = 0$: they did not move so it means that they did not endure an exogenous shock forcing them to move. It is as if $m = 0$ was observed in this case. Using these observations, we recover $F_{C_d|D=d,\cdot,m=0}(c_d)$ (using deconvolution).

Do the following computation at any given (a, s_{-1}), from which we abstract in the following notation. For any $h \in [\underline{h}_d(a, s_{-1}, d_{-1} = d), \bar{h}_d(a, s_{-1}, d_{-1} = d)]$, we have

$$\begin{aligned}
Pr(h \leq h) &= Pr(h \leq h|D_{-1} = 1) \\
&= Pr(h \leq h|D = 1, D_{-1} = 1)Pr(D = 1|D_{-1} = 1) \\
&\quad + Pr(h \leq h|D = 0, D_{-1} = 1)Pr(D = 0|D_{-1} = 1) \\
&= Pr(h \leq h|D = 1, D_{-1} = 1, m = 0)Pr(D = 1|D_{-1} = 1, m = 0)(1 - p^m) \\
&\quad + Pr(h \leq h|D = 1, D_{-1} = 1, m = 1)Pr(D = 1|D_{-1} = 1, m = 1)p^m \\
&\quad + Pr(h \leq h|D = 0, D_{-1} = 1)Pr(D = 0|D_{-1} = 1) \\
&= Pr(C_1^* \geq c_1^*(h, d_{-1} = 1, m = 0)|D_{-1} = 1, D = 1, m = 0)Pr(D = 1|D_{-1} = 1, m = 0)(1 - \\
&\quad + Pr(h \leq h|D = 1, D_{-1} = 1, m = 1)Pr(D = 1|D_{-1} = 1, m = 1)p^m \\
&\quad + Pr(h \leq h|D = 0, D_{-1} = 1)Pr(D = 0|D_{-1} = 1). \qquad (7)
\end{aligned}$$

The first equality holds because $h \perp d_{-1}$. The second and third equalities come from the law of total probability, where $p^m = Pr(m = 1)$. We obtain the fourth equality because $c_1^*(h, \cdot)$ is a strictly monotone (decreasing) function of h. The only thing that is unknown in this equation (7) is the optimal policy. Now, when $d \neq d_{-1}$, the households move and we identified h by inverting the optimal s_d^* choice. So, $Pr(h \leq h|D = 0, D_{-1} = 1)$ is known. $Pr(D = 0|D_{-1} = 1)$ can be directly estimated from the data too. Similarly, $Pr(h \leq h|D = 1, D_{-1} = 1, m = 1)$ was also already identified, because when $m = 1$ we observe moves in s_d, we can reverse it to identify h and its distribution. $Pr(D = 1|D_{-1} = 1, m = 1)$ is also known as we identified $Pr(D = 1|h, D_{-1} = 1, m = 1)$ for all h when $m = 1$. Finally, we can show that $Pr(D = 1|h, D_{-1} = 1, m = 0)$ is also known. Indeed, by the law of total probability,

$$Pr(D = 1|D_{-1} = 1) = Pr(D = 1|D_{-1} = 1, m = 1)\ p^m + (1 - p^m)\ Pr(D = 1|D_{-1} = 1, m = 0).$$

Where p^m is identified, $Pr(D = 1|D_{-1} = 1)$ is observed from the data and $Pr(D = 1|D_{-1} = 1, m = 1)$ has also been identified. So,

$$Pr(D = 1|D_{-1} = 1, m = 0) = \frac{Pr(D = 1|D_{-1} = 1) - p^m \; Pr(D = 1|D_{-1} = 1, m = 1)}{1 - p^m}$$

and $Pr(D = 1|D_{-1} = 1, m = 0)$ is known.

So, since the distribution $F_{C_1|D=1,D_{-1}=1,m=0}(c_d)$ has been identified by deconvolution, the only unknown in equation (7) is the optimal consumption choice $c_1^*(h, d_{-1} = 1, m = 0)$. Thus, it is identified by:

$$\begin{aligned}
1-&F_{C_1|D=1,D_{-1}=1,m=0}(c_1^*(h, d_{-1} = 1, m = 0)) \\
&= \Big(Pr(h \leq h) \\
&\quad - Pr(h \leq h|D = 1, D_{-1} = 1, m = 1)Pr(D = 1|D_{-1} = 1, m = 1)p^m \\
&\quad - Pr(h \leq h|D = 0, D_{-1} = 1)Pr(D = 0|D_{-1} = 1)\Big) \; \frac{1}{Pr(D = 1|D_{-1} = 1, m = 0)(1 - p^m)}
\end{aligned}$$

Which means that:

$$\begin{aligned}
&c_1^*(h, d_{-1} = 1, m = 0) \\
&= F^{-1}_{C_1|D=1,D_{-1}=1,m=0}\Bigg(1 - \Big(Pr(h \leq h) \\
&\quad - Pr(h \leq h|D = 1, D_{-1} = 1, m = 1)Pr(D = 1|D_{-1} = 1, m = 1)p^m \\
&\quad - Pr(h \leq h|D = 0, D_{-1} = 1)Pr(D = 0|D_{-1} = 1)\Big) \; \frac{1}{Pr(D = 1|D_{-1} = 1, m = 0)(1 - p^m)}\Bigg),
\end{aligned}$$

and $c_1^*(h, d_{-1}, m = 0)$ is identified for all h in the boundaries. Since it was already identified outside the boundaries, it is identified for all $h \in [0, 1]$.

We apply exactly the same reasoning for $d = 0$ to identify $c_0^*(h, d_{-1} = 0, m = 0)$ for all h in the boundaries $[\underline{h}_0(a, s_{-1}, d_{-1} = 0), \bar{h}_0(a, s_{-1}, d_{-1} = 0)]$, and thus for all $h \in [0, 1]$.

Information on $Pr(D = 1|h, d_{-1}, s_{-1}, a, m = 0)$ ***for*** h ***in*** $[\underline{h}_d, \bar{h}_d]$:

We can use the identified $c_d^*(h, s_{-1}, a, d_{-1} = d, m = 0)$ to identify $Pr(D = 1|h, d_{-1}, s_{-1}, a, m = 0)$ for $h \in [\underline{h}_d(a, s_{-1}, d_{-1} = d), \bar{h}_d(a, s_{-1}, d_{-1} = d)]$. Indeed, inside this set, $s_d^* = s_{-1}$. Thus we cannot invert the observation of s_d to recover the value of h. However, c_d^* is still varying, and we identify it. As a consequence, the *observed consumption contains information on* h. Because of the measurement errors we cannot directly invert the observed consumption to recover h as we are proceeding for the housing services. Yet c_d^{obs} still contains some information about h: the higher it is, the more likely it is that h is low for example (since c_d is decreasing with respect to h). More precisely, given that we observe $c_d, d, a, d_{-1}, s_{-1}$, we have

$$Pr(h = h|c_d^{obs}, d, s_{-1}, d_{-1} = d, a) = Pr\Big(\zeta = c_d^{obs} - c_d^*(h, s_{-1}, a, d_{-1} = d, m = 0)\Big),$$

where we drop the conditioning in the right hand side since ζ is independent from the rest.

Therefore, we can again compute the CCPs, $Pr(D = 1|h, d_{-1}, s_{-1}, a, m = 0)$ from the data but using the probabilistic likelihood of each h (instead of the knowledge of h which is identified for movers) for the observations where $s_d = s_{-1}$.

Therefore, we identify the optimal *Conditional Continuous Choices* (CCCs) $s_d^*(h, s_{-1}, d_{-1}, a, z, m)$ and $c_d^*(h, s_{-1}, d_{-1}, a, z, m)$ and the optimal *Conditional Choice Probabilities* (CCPs) $Pr(d|h, s_{-1}, d_{-1}, a, z, m)$, for all h, a, s_{-1}, d_{-1}. We also identify the measurement errors distribution F_ζ, and the probability to undergo an exogenous moving shock, p^m.

4.4 Identification of the structural parameters

We have proved the identification of the optimal choices in every period. Now we can use them and apply Bruneel-Zupanc (2021) directly to identify the structural parameters of our dynamic model.

5 Empirical Strategy

We build a two step estimation method of the parametric model. In a first stage, we estimate *parametric* optimal policies via maximum likelihood. More nonparametric alternative estimation methods can also be used, see Bruneel-Zupanc (2021). However here, we do not have many observations of moving individuals, so we prefer to use a more parametric estimation procedure for the optimal choices.

In the second stage, we estimate the structural parameters of the model via forward simulation methods using the optimal policies estimated in the first stage, in the spirit of Hotz et al. (1994).

5.1 Optimal policies

We specify parametric functional forms for the optimal policies. Then we estimate the parameters by maximum likelihood, since the likelihood is known given the parametric functional form. We will specify functional forms for $s_d^*(\cdot), c_d^*(\cdot)$ and $Pr(D=1|\cdot)$. As well as functional forms for the boundaries $\underline{h}_d, \bar{h}_d$, the measurement errors and the probability of exogenous move shock p^m.

Housing service s:

First, we specify a functional form for the *housing service conditional on moving* $\tilde{s}_d^*$:

$$\begin{aligned} log(\tilde{s}^*) = \delta &+ \delta_1 d + \alpha_0(1-d)\psi(h) + \alpha_1 d\psi(h) \\ &+ \gamma_0(1-d)\Big[d_{-1}\big(\frac{q}{1+r}s_{-1}+a\big) + (1-d_{-1})a + \frac{\text{income}}{1+r}\Big] \\ &+ \gamma_1 d\Big[d_{-1}\big(\frac{q}{1+r}s_{-1}+a\big) + (1-d_{-1})a + \frac{\text{income}}{1+r}\Big] \\ &+ \lambda' x, \end{aligned}$$

where $d_{-1}(\frac{q}{1+r}s_{-1}+a) + (1-d_{-1})a + \frac{\text{income}}{1+r}$ represents the *total wealth*. $\psi(h)$ is a monotone transformation of h applied to account for nonlinear effect of h, e.g. $\psi(h) = F^{-1}(h)$ where F is a standard normal distribution. By *monotonicity*, we have the constraint on the parameters $\alpha_0 > 0$ and $\alpha_1 > 0$.

Now, recall that with fixed costs, the true optimal choice is:

$$s^* = \begin{cases} s_{-1} & \text{if } d = d_{-1} \text{ and } (\underline{h}_d \leq h \leq \bar{h}_d) \iff (\underline{s}_d \leq \tilde{s}^* \leq \bar{s}_d) \\ \tilde{s}^* & \text{otherwise.} \end{cases}$$

Thus, we need a parametric specification for $(\underline{s}_d, \bar{s}_d)$, which depend on s_{-1}. For now, we use a simple specification with two parameters for each d, $\underline{\kappa}_d$ and $\bar{\kappa}_d$:

$$\underline{s}_d(s_{-1}) = s_{-1} - \underline{\kappa}_d \quad \text{and } \bar{s}_d(s_{-1}) = s_{-1} + \bar{\kappa}_d.$$

We can include additional effect from the other variables (asset, income, demographics) on the boundaries but we keep it simple for now.

Housing tenure d:

For the housing tenure choice probability d, we use a logistic specification:

$$Pr(D = 1|h, d_{-1}, s_{-1}, x) = \frac{1}{1 + exp(-(\theta_0 d_{-1} + \theta_1 \psi(h) + \theta_2 \text{total wealth}))}.$$

Consumption c:

We use a specification similar to the specification of housing services for non-housing consumption. Except that we include s into it, such that for non movers, we will have a fixed s_{-1} but still have variation through h. And for the movers the variation will be captured through s, which is a strictly increasing function mapped to h anyway.

$$\begin{aligned} log(c^*) = \; & \delta^c + \delta_1^c d + \alpha_0^c (1-d)s + \alpha_1^c d \; s \\ & + \mathbb{1}\{s = s_{-1}\}\Big(\alpha_2^c (1-d)\psi(h) + \alpha_3^c d\psi(h)\Big) \\ & + \gamma_0^c (1-d)(\text{total wealth}) \; + \gamma_1^c d(\text{total wealth}) \; + \lambda^{c\prime} x. \end{aligned}$$

And consumption is observed with measurement errors

$$c = c^* + e^c \quad \text{with} \;\; e^c \sim \mathcal{N}(0, \sigma^c),$$

where σ^c is another parameter to estimate.

Estimation via Maximum Likelihood:

Given the specification, including also a parameter p^m for the exogenous moving shock probability, we can compute the likelihood of each observation (s, d, c) in the data. Even when $s = s_{-1}$ we know the likelihood of observing it happening. So we can estimate all the parameters of the optimal choices laid down above via maximum likelihood.

5.2 Structural parameters

Once the parametric optimal choices are estimated, we use them in *forward simulations* of the model in order to estimate the structural parameters of the dynamic model, in the spirit of Hotz et al. (1994). In order to run these forward simulation, we also estimate the transition of the exogenous state variables (income and demographics) parametrically directly from the data beforehand.

As shown in Bruneel-Zupanc (2021), this two step estimation method has the advantage of being fast, as it avoids having to solve for the optimal choices in the model for each set of parameters.

6 Conclusion

This paper builds a dynamic model of household consumption and housing decisions. We provide identification conditions of this dynamic model and an estimation method built upon the identification proof. The planned estimation of the model will allow to estimate key parameters for this sample of French households, such as the substitution between housing and non-housing consumption and fixed housing switching costs. Once the model is estimated, we will be able to run many relevant policy counterfactuals, by modifying transaction costs for example.

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