Managing Seller Conduct in Online Marketplaces and Platform Most-Favored Nation Clauses

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Abstract

Motivated by recent antitrust cases, this article theoretically analyzes collusion among online sellers on platforms, and a platform's incentive and ability to prevent sellers from colluding. Absent contractual restrictions, a platform has an incentive to ensure competition between the sellers. This incentive can change with the introduction of so-called platform most-favored nation clauses (PMFN) that require the online sellers not to offer better conditions on other distribution channels. Such contract clauses have the potential to align the interests between online sellers and platforms regarding seller conduct, and to give the platform the ability to profitably increase the stability of seller collusion. This offers a novel rationale for competition authorities to treat PMFNs with scrutiny. Finally, the analysis reveals that a platform generally affects seller conduct in a profitable way by committing to a time-constant commission rate, which offers a possible explanation for the observed rigidity of platform pricing behavior over time.

JEL classification: L13, L40, L50

Keywords: platform MFN, digital economics, collusion in vertically-related markets, agency model

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1 Introduction

In an increasingly digitalized economy, consumers can purchase a wide range of goods and services via online platforms. Famous examples are the Amazon Marketplace and online travel agencies such as Booking.com. As noted by Executive Vice President of the European Commission and European Commissioner for Competition Margrethe Vestager, however, as a potential problem, there is

> "a small number of gatekeeper platforms act[ing] as private rule-makers for the markets they have created. They decide on who can enter their markets, who has to leave them, and on the conditions to be respected while selling on them."[1]

One crucial premise for the well-functioning of these online markets is that these rules and conditions on a platform ensure a competitive environment between the sellers on its own marketplace. Whereas the academic literature on this topic (discussed in Section 2) is relatively scarce, high-profile antitrust cases of illegal price fixing of sellers on such online platforms cast doubt on whether this premise is always fulfilled, and suggest that this form of collusive behavior is a concern for competition authorities more broadly. The present article contributes to fill this gap by formally analyzing a platform's incentive and ability to encourage competition or collusion on its own marketplace.

An important contractual instrument employed by several platforms to influence seller behavior across distribution channels are so-called platform most-favored nation clauses (PMFN), and we argue that such clauses reduce a platform's incentives to enforce competition on its marketplace. A PMFN is a contractual requirement for the online sellers not to offer better prices and conditions on other distribution channels. Such clauses have triggered substantial antitrust scrutiny in several jurisdictions. A leading case is the famous e-book case that involved a PMFN, and in which five major publishers of e-books as well as the platform provider (Apple) were found guilty of engaging in illegal fixing of retail e-book prices.[2] Moreover, the Competition and Markets Authority (CMA) in the UK found online sellers of posters and frames, Trod Limited and GB eye Limited, to be colluding on retail prices between 2011 and 2015 on the Amazon UK website by means of price-matching algorithms.[3] Arguably, Amazon should be able to identify the use of such algorithms (Chen et al., 2016), and prevent their application if doing so is in its interest

[1] Speech at the Forum Europe Conference on the Digital Services Act, 3 July 2020.

[2] See Baker (2013) and Klein (2017) for comprehensive overviews of the antitrust case in the US, and Gaudin and White (2014) on the antitrust economics of this case. In 2011, the European Commission also opened an antitrust case against Apple and the e-book publishers with similar anticompetitive concerns (Case COMP/AT.39847-E-BOOKS). Interestingly, also Apple was considered to be part of the collusive agreement despite adopting the agency model in which the publishers and not the platform set the final retail prices. In the year after the adoption, e-book prices for e.g. *New York Times* bestsellers increased by 40 percent as a result of this price fixing conspiracy (De los Santos and Wildenbeest, 2017).

[3] CMA, Decision of 12.08.16, Case 50223. There was also an investigation in the US and the founder of Trod Limited also was found guilty for the same conduct of price fixing lasting from 2013 to 2014 (United States v. Trod Limited, No. CR 15-0419 WHO).

(e.g., by threatening seller suspension).[4] Note that Amazon also had a platform most-favored nation clause in place at the time that the collusive agreement was implemented between Trod Limited and GB eye Limited in the UK and the US.[5]

This paper emphasizes that a platform's preferred seller conduct can change with the introduction of a PMFN. Table 1 depicts the main result schematically, which distinguishes whether a platform prefers seller competition or seller collusion as conduct. At this stage, we take as given that sellers can coordinate on a cartelized outcome and focus below how it can be sustained by tacit collusion in an infinitely-repeated game. We analyze a stylized model building on and extending Johansen and Vergé (2017) in which online sellers have two distribution channels in order to sell to consumers. The first is a strategic platform, which employs the *agency model.* This means the platform receives a commission for every intermediated transaction, and the sellers set the retail prices on the platform. The second distribution channel is a non-strategic direct channel on which the online sellers do not incur per-transaction commission rates. We analyze both per-unit and revenue-sharing commission rates on the platform and, for the sake of tractability, focus on a linear-demand specification.

	No PMFN	With PMFN
Seller competition	✓	Weak interbrand substitutability
Seller collusion		Strong interbrand substitutability

Table 1: Platform's preferred seller conduct with per-unit commission rates.

The table shows schematically which form of seller conduct the platform prefers if it charges per-unit commission rates. Without a PMFN (first column), it always prefers seller competition. The introduction of a PMFN (second column) changes this result if the degree of interbrand competition is strong.

Absent a PMFN, we find that a platform realizes higher profits with seller competition than with seller collusion if it charges per-unit commission rates. At a given commission rate, seller collusion leads to a lower quantity sold on the platform, which *c.p.* decreases platform profits. Moreover, in our setting the platform charges the same commission rate from competing and colluding sellers, rendering seller collusion unambiguously profit-decreasing for the platform in this case.[6] Hence, absent a PMFN, a platform benefits from ensuring a competitive environment on its marketplace. We obtain qualitatively

[4]See, for instance, the blog post What to Do If Your Amazon Account Gets Suspended on www.repricerexpress.com indicating that Amazon uses suspension as disciplinary measures against sellers that do not comply with Amazon policies (last access, September 11, 2020).

[5]See, for instance, for the US the blog post Amazon's Pricing Policy Caused Consumers to Overpay by \$55B to \$172B, Class Action Claims on www.classaction.org indicating that until 2019 Amazon imposed PMFNs in the Amazon Services Business Solutions Agreement with the online sellers (last access, September 11, 2020).

[6]A sufficient statistic for this result is that the commission rate is weakly smaller for colluding sellers than fro competing ones. In Appendix B, we employ a conduct-parameter approach in order to characterize for a more general demand function when this condition holds.

the same result for the case of revenue-sharing commission rates by means of numerical calculations.

This result, however, changes if the platform introduces a PMFN. With PMFN, sellers are indifferent between selling via both distribution channels and selling via the direct channel only in equilibrium (Johansen and Vergé, 2017). For higher commission rates at least one seller prefers to delist from the platform and to aggressively undercut the uniform price set on both channels due to the PMFN. We show that this indifference condition permits a higher commission rate from colluding sellers, which implies that colluding sellers pay higher commission rates than competing ones. Importantly, this increase in the commission rate renders seller collusion more profitable for the platform if the degree of interbrand substitutability is strong. We conclude that a PMFN therefore undermines a platform's incentive to ensure intensive competition between online sellers. Linking the use of a PMFN to potentially reduced competition on the seller level offers a novel rationale to treat PMFNs with scrutiny. The main established theory of harm instead (discussed in more detail in Section 2) emphasizes that a PMFN has the potential to weaken competition on the platform level (Boik and Corts, 2016; Johnson, 2017). The above described result is reinforced for the case of revenue-sharing commission rates. Numerical results show that, with PMFN, a platform prefers seller collusion independent of the degree of interbrand substitutability.

In light of the above described competition cases, we continue to analyze the stability of collusion between online sellers in an infinitely-repeated game. This analysis allows us to study one potential mechanism by which the collusive outcome described above can be sustained. Online sellers sustain collusion with grim-trigger strategies and coordinate on the Pareto-optimal equilibrium in order to maximize their discounted stream of joint profits. The main economic logic of the results can be captured with the case in which sellers collude if and only if they can coordinate on the joint-profit maximum, and we therefore focus on it initially (and establish robustness to the case in which collusion on the joint-profit maximum is not incentive-compatible below). The platform commits to constant commission rates that do not change in response to changing market conditions.[7]

We determine to which extent the introduction of a PMFN allows a platform to affect the stability of tacit seller collusion. In line with the finding that a platform can benefit from seller collusion with a PMFN, we identify a range of commission rates that the platform can choose in order to profitably stabilize collusion between sellers compared to the case without PMFN. With a PMFN, the commission rate affects the sellers' distribution channel choices and thereby influences punishment and deviation behavior. There exists a range of commission rates in which colluding sellers are willing to list on the platform but not competing sellers. Crucially, in this range, competing sellers suffer from a form of a prisoner's dilemma as they would achieve higher profits if both sellers were present on the platform. But, as the commission rate in this range induces the sellers to charge

[7]We argue in Section 4 that this assumption is in line with observed platform behavior and allow in Section 5.3 that the platform adjusts its commission rate in response to changing market conditions.

comparably high retail prices on both distribution channels, each seller unilaterally has an incentive to delist from the platform and charge a more profitable price on the direct channel only. As a result, sellers realize a discretely lower competitive profit above the threshold for which this dilemma occurs than below. In other words, in this range the potential punishment is harsher so that this has a stabilizing effect on seller collusion compared to the case absent PMFN. Moreover, we find that the sellers' critical discount factor increases in the platform's commission rate above this threshold such that seller collusion is more difficult to sustain for very high commission rates.

We extend the analysis of seller collusion in several directions. First, we extend the analysis to the case in which collusion on the joint-profit maximum is not incentive-compatible for the sellers. We assume that sellers coordinate on the highest retail prices such that the incentive-compatibility constraint to stick to the collusive agreement binds. We refer to this case as constrained collusion. The results are qualitatively comparable to the case in the main model, and reinforces the result that the platform can benefit from seller collusion if it imposes a PMFN. Second, we analyze the the computationally more demanding case of revenue-sharing commission rates. The effects on the stability of seller collusion are qualitatively comparable. Third, we focus on the case in which the platform can condition the commission rate on the seller conduct (responsive commission rates), and derive when a commitment to constant commission rates is profitable for a platform. As a commitment to constant commission rates might lead to sub-optimal commission rates and hence lower platform profits if the seller conduct or other market fundamentals change, this analysis offers a dynamic explanation for the benefits of time-constant commission rates. It relates potential adjustments in the commission rates to the online sellers' propensity to collude and the profitability for the platform of such conduct. This argument is complementary to other aspects such as transaction costs or the fear of antitrust scrutiny that may also motivate a platform to refrain from adjusting its commission rates over time. Finally, we discuss two non-price related strategies that the platform might adopt to undermine seller collusion if it has an incentive to do so.

2 Related Literature

The present article contributes to three strands of the literature. First, it contributes to a nascent literature that links platform behavior to the interaction between sellers (e.g., their competitiveness) on the platform. Second, it fits into the analysis of collusion in vertically-related markets. This research analyzes how vertical relations and vertical restraints affect the stability of collusion on different stages of the vertical chain. Third, the present article relates to articles analyzing the competitive effects of the comparably new vertical restraint of platform most-favored nation clauses.

Platform Behavior and Seller Interaction. There is a small related literature that relates strategic platform behavior to the interaction (e.g., competitiveness) between sellers on a platform. Teh (2019) studies governance designs of platforms in order to affect on-platform competition. In particular, he studies governance decisions including seller entry, minimum quality standards, and on-platform search frictions. Karle et al. (2020) focus on agglomeration and segmentation of sellers on different platforms and find that the competitive conditions between sellers shape the platform market structure. Relatedly, Belleflamme and Peitz (2019) address for a given market structure the interaction of seller competition (i.e., negative within-group externalities on the platform) with platform pricing and product variety. Pavlov and Berman (2019) and Lefez (2020) study price recommendations that a platform sends to sellers which are active on the market place. The papers mentioned above, however, do not consider how PMFNs affect a platform's incentives and do not focus on seller collusion on the platform.

Collusion in Vertically-Related Markets. The second strand of the literature studies the effects of vertical restraints on the stability of collusion. Closely related to the present analysis is Hino et al. (2019) who compare the stability of upstream collusion in the presence of either the traditional wholesale model (in which the retailer sets final consumer prices) or the agency model (in which sellers set these prices on the platform). We also focus on the agency model. Their main contribution is to analyze whether the distribution via wholesale contracts or agency contracts affects the stability of collusion between upstream sellers differently. They do not, however, analyze the use of platform most-favored nation clauses, which are common in markets that are operated via the agency model and play an important role in multiple antitrust cases. Importantly, we demonstrate in this article how the introduction of a PMFN can alter a platform's incentives to prevent collusion between online sellers.

More broadly, the literature analyzes other forms of vertical restraints and their impact on collusion. The seminal articles by Nocke and White (2007) and Normann (2009) find that vertical integration can increase the stability of collusion between upstream firms. Relatedly, Biancini and Ettinger (2017) show that vertical integration generally also favors downstream collusion. The impact of resale price maintenance (RPM) on collusion on differnt levels of the vertical chain is analyzed by Jullien and Rey (2007), Overvest (2012), and Hunold and Muthers (2020). These articles demonstrate that the use of RPM can facilitate upstream collusion. Relatedly, we characterize the conditions under which a PMFN stabilizes seller collusion, and establish that a platform has less incentive to ensure seller competition if it is allowed to impose a PMFN. To the extent that a platform, therefore, ceases to actively ensure seller competition, a PMFN can also have competition-weakening effect on the seller level.

Further articles that study the effects of different contractual arrangements on collusion in vertically-related markets include Piccolo and Miklós-Thal (2012) and Gilo and Yehezkel (forthcoming). They establish that contracts featuring slotting allowances and high wholesale prices during collusive periods can increase the stability of collusion between firms as such a contract makes a deviation less profitable. In the present paper, we also emphasize that with a PMFN the platform's commission rate can affect punishment and deviation behavior differently. Reisinger and Thomes (2017) study implications of the channel structure on seller collusion and find that seller collusion is easier to sustain if the sellers have independent retailers compared to the case in which they have a common retailer.

In non-vertical settings, contractual provisions also have been found to affect the stability of collusion between firms. Schnitzer (1994) analyzes the collusive potential of two forms of best-price clauses that guarantee consumers rebates on the purchase price if they find a better price for the purchased product. She finds that especially contract clauses that promise consumers to meet price cuts from competing sellers have anticompetitive potential.

Competitive Effects of Platform Most-Favored Nation Clauses. The competitive effect of platform most-favored nation clauses have mostly been analyzed in static settings.[8] Recent articles such as Boik and Corts (2016), Johnson (2017), and Foros et al. (2017) support the main established theory of harm as regards PMFNs, which posits that such contract clauses have the potential to reduce competition in commission rates between competing platforms. The reason for this result is that, with PMFN, online sellers react less sensitive to changes in a platform's commission rate, which allows to sustain higher rates in equilibrium than absent a PMFN. Moreover, these clauses may curtail entry in the platform market, as a new entrant in the platform market cannot win consumers by achieving lower retail prices on its own platform, and lead to excessive adoption of the platform's services as well as overinvestment in benefits to consumers (Edelman and Wright, 2015). In contrast, Johansen and Vergé (2017) show that accounting for the sellers' participation constraint can alleviate the anticompetitive price effects of a PMFN and can even lead to an increase in welfare if sellers have a direct channel in order to reach final consumers.

These papers abstract from any effect of a PMFN on the competition between sellers on the platform and focus instead on the competition between the platform and other distribution channels. The present paper contributes to this literature by focusing on the competitive effects of PMFNs on the seller level, and their impact on the stability of seller collusion. Importantly, the analysis presented herein shows that the introduction of a PMFN may alter a platform's incentives to ensure a competitive environment on the seller level. This finding is related to Niedermayer (2015) and Johnson (2020) who analyze in different settings whether a platform benefits from seller competition or not. Moreover, the present article offers a dynamic rationale for the seemingly puzzling observation that there is typically little variation in the commission rates of platforms over time, even after the abolition of PMFNs in some of these markets.

[8] See Baker and Scott Morton (2017) and Fletcher and Hviid (2016) for comprehensive overviews of the competitive effects of PMFNs. They also informally discuss the effect of PMFN on the stability of upstream collusion but neither the impact on the sellers' listing decisions nor the desirability of collusion for the platforms are considered in this discussion.

3 Static Model

3.1 Players and Environment

Consider an environment with two competing sellers $i \in \{1, 2\}$ producing differentiated products at constant symmetric marginal costs $c \geq 0$. The sellers offer a quantity D_{ij} of products to the consumers through two distribution channels $j \in \{A, B\}$. Distribution channel A is a strategic platform and distribution channel B is non-strategic direct channel available to the sellers in order to reach consumers. For every intermediated transaction on the platform, the platform charges a commission from the sellers. Suppose that the marginal costs for an additional intermediated transaction between sellers and consumers on each distribution channel $j \in \{A, B\}$ is zero.

3.2 Contracts and Timing of the Stage Game

The platform uses the *agency model*, which implies that the sellers set retail prices on each distribution channel $j \in \{A, B\}$. Denote by p_{ij} the price that seller i sets on distribution channel j, and with $p = (p_{ij})\,,\ i \in \{1, 2\}\,,\ j \in \{A, B\}$ the vector of all retail prices. Similarly, $p_i = (p_{iA}, p_{iB})$ denotes the vector of retail prices that seller i charges. We analyze two forms of contracts between the platform and the sellers. For the main part of the analysis, we will focus on the case that the platform receives a per-unit commission rate w_{iA} from seller i for every transaction that is intermediated on the platform. The focus on simple per-unit commission rates facilitates the analysis and allows for closed-form solutions.[9] Contract offers are observable.[10]

The platform can impose a platform most-favored nation clause (PMFN) in the contracts with the sellers. A PMFN requires each seller to offer on the platform at least as favorable prices as on the direct channel, $p_{iA} \leq p_{iB}$. In principle, sellers are still allowed to charge higher retail prices on their direct channel than on the platform. We compare the case with a PMFN on the platform to the case without PMFN.

The timing of the game is as follows: First, the platform sets the commission rate. Second, sellers simultaneously decide whether to accept the platform's contract, and they set retail price p_{iB} on the direct channel as well as the retail price p_{iA} on the platform in case they accept the offer. We will say that a seller is active on a distribution channel if it has accepted the contract offer (in the case of the platform), and sells a positive quantity to consumers via this channel.

[9] In the extension, we explain intuitively why the same economic forces are present when commissions are based on sellers' revenue but formally the case is much less tractable. In line with the economic intuition, we numerically verify that our main economic results carry over to the case of revenue-sharing commission rates.

[10] See Johansen and Vergé (2017) for a related analysis with unobservable contract offers.

3.3 Consumer Behavior

The consumers have preferences for the seller and the distribution channel. Hence, the consumers have demand for four differentiated seller-channel configurations. Building on Dobson and Waterson (1996), we assume that the demand function is linear and depends on the prices of the sellers $i, h \in \{1, 2\}$ on each distribution channel $j, k \in \{A, B\}$

$$D_{ij}(p) = \frac{1}{(1-\alpha^2)(1-\beta^2)} \left(1 - p_{ij} - \beta p_{ik} - \alpha \left(1 - \beta - p_{hj} + \beta p_{hk}\right)\right). \tag{1}$$

The parameter $\alpha \in (0,1)$ captures the degree of interbrand competition and $\beta \in (0,1)$ the degree of intrabrand competition.[11] If $\alpha > \beta$, the competition between sellers (interbrand competition) is stronger than the substitutability of distribution channels for the same seller (intrabrand competition). Conversely, if $\alpha < \beta$, the consumers have strong brand preferences but are less sensitive to the distribution channel, on which they purchase the product. Moreover, the demand function satisfies that on a given distribution channel j, the own-price effect dominates the cross-price effect of the competing seller's price. Cazaubiel et al. (2020) finds empirically that a hotel chain's direct channel is a credible alternative to an online travel agent such as Expedia. Similarly, Duch-Brown et al. (2017) estimate the substitution patterns between online and offline distribution channels for consumer electronics.

3.4 Analysis of the Static Model

In this section, we analyze how the introduction of a PMFN affects the profitability of seller competition for the platform. In order to do so, we characterize the static competitive market outcome and compare it to the outcome with seller collusion. To this end, we assume that sellers are able to coordinate their listing and pricing decisions in order to maximize their joint profits. We abstract from the exact mechanism supporting the collusive outcome. In the following section, we analyze an infinitely-repeated game in order to study the stability of such collusive market outcomes.

We can normalize the seller's marginal costs to zero without loss of generality and write the profit function of seller i that is present on both distribution channels as

$$\pi_i(p) = (p_{iA} - w_{iA}) D_{iA}(p) + p_{iB} D_{iB}(p). \tag{2}$$

The platform's profit is

$$\Pi_A(w_A) = \sum_{i \in \{1,2\}} w_{iA} D_{iA}(p), \tag{3}$$

where $w_A = (w_{1A}, w_{2A})$ denotes the vector of commission rates. Similarly, denote with

[11] Such a linear demand specification has been employed widely to study collusion in vertically-related markets (Reisinger and Thomes, 2017; Hino et al., 2019) and PMFNs in the agency model (Johansen and Vergé, 2017; Boik and Corts, 2016). The demand function is derived from the utility maximization of an representative consumer with quadratic utility (see also Singh and Vives, 1984).

$w = (w_A, 0)$ the vector of distribution costs that the sellers face on both channels.

No Platform Most-Favored Nation Clause. Absent a PMFN, the presence of positive commission rates w_A that sellers must pay to the platform leads to an incentive for the seller to charge different prices on each distribution channel. Given demand symmetry and the higher distribution costs on the platform, each seller charges lower prices on the direct channel if not restricted by a PMFN. Depending on the sellers' conduct, competitive retail prices are denoted by $\tilde{p}(w)$, and collusive ones by $\bar{p}_j(w)$. The following lemma summarizes the seller behavior for both forms of conduct absent a PMFN.

Lemma 1. *Suppose sellers face distribution costs of $w = (w_A, 0)$, with $w_A \in [0, 1-\beta]^2$. Without PMFN (NP), seller i sets the retail price*

$$\tilde{p}_{ij}^{NP}(w) = (1 - \alpha + w_j) / (2 - \alpha), \tag{4}$$

on distribution channel j if sellers compete, and

$$\bar{p}_{ij}^{NP}(w) = (1 + w_j)/2, \tag{5}$$

if they collude.

Proof. See Appendix A. □

The restriction on the commission rate $w_A \in [0, 1-\beta]$ ensures that–independent of their conduct–sellers prefer to be active on both distribution channels in all periods instead of listing on the direct channel only. We verify below that the platform indeed does not find it profitable to charge higher commission rates than $1-\beta$. The result of Lemma 1 shows that with collusion the sellers successfully eliminate the interbrand competition (as measured in α) on both distribution channels. This implies that retail prices are higher with collusion than they are with seller competition. Moreover, retail prices on distribution channel j are independent of the costs of distribution on the other channel $k \neq j$. Note for future reference that, without PMFN, the retail price on channel j is also independent of whether the seller is present only on channel j or present on both distribution channels.

Based on the result of Lemma 1, we establish that the commission rate that maximizes the platform's profit is independent of the seller conduct in our setting with linear demand. As a corollary result, it follows immediately that the platform benefits from seller competition and realizes lower profits if sellers collude. A sufficient condition for this result to hold is that the commission rate weakly decreases if sellers' conduct changes from competition to collusion. In Appendix B, we show for a general demand function in a model based on a conduct-parameter approach when this condition holds. In order to obtain intuition for this result note that for given commission rates w_A, the platform's profits increase if the transaction volume on the platform is larger. Due to the fact that seller collusion leads to higher retail prices, this form of conduct reduces demand overall and on the platform, and hence reduces platform profits. The following proposition summarizes the optimal platform behavior absent a PMFN.

Proposition 1. *Without PMFN, the platform finds it optimal to charge symmetric constant commission rates of*

$$w_{1A}^{NP} = w_{2A}^{NP} = w_A^{NP} = \frac{1-\beta}{2}, \tag{6}$$

independent of seller conduct. The resulting platform profits depending on seller conduct are

$$\tilde{\Pi}_A^{NP}\left(w_A^{NP}\right) = \frac{1-\beta}{2\left(2-\alpha\right)\left(1+\alpha\right)\left(1+\beta\right)}, \tag{7}$$

$$\bar{\Pi}_A^{NP}\left(w_A^{NP}\right) = \frac{1-\beta}{4\left(1+\alpha\right)\left(1+\beta\right)}, \tag{8}$$

with $\tilde{\Pi}_A\left(w_A\right) > \bar{\Pi}_A\left(w_A\right)$ *over the whole parameter space.*

Proof. See Appendix. □

The result of Proposition 1 shows that a platform finds it optimal to charge symmetric commission rates from both sellers. Moreover, note that $w_A^{NP} < 1-\beta$ such that both sellers are active on both distribution channels. Importantly, comparing the platform's profit with seller competition in Equation (7) with the platform profit with seller collusion in Equation (8) reveals that the platform benefits from a competitive environment on its own marketplace. To the extent that the platform can influence the competitive intensity between the online sellers, the platform, therefore, has an incentive to ensure fierce competition between the online sellers.

Platform Most-Favored Nation Clause. Next, we turn to the analysis of the profitability of seller competition with a PMFN. Such a contract restriction leads to an important change in the contracting between the platform and the online sellers. With PMFN, it is important to take into account the sellers' listing decision on the platform (Johansen and Vergé, 2017). Hunold et al. (2018) and Cazaubiel et al. (2020) provide empirical evidence that listing decisions are economically important dimensions of adjustments for hotels. Due to the contractual restrictions of the PMFN, a seller is induced to charge higher than optimal prices on its direct channel if it is active on both distribution channels. It may therefore be more profitable for a seller to delist from the platform in order to charge more profitable prices on its direct channel and save the commission payments that accrue for every transaction via the platform.

In the following, we characterize how a PMFN affects seller behavior in a competitive and a collusive period. If present on both distribution channels, competing sellers maximize the profit function in Equation (2) subject to the constraint that $p_{iA} \leq p_{iB}$. If active on both channels, denote the resulting retail price that seller i charges on both distribution channels by $\tilde{p}_i^P(w)$. In order to verify, whether these retail prices constitute an equilibrium of the stage game, it is necessary to verify that no seller has an incentive to delist from the platform and only sell via the direct channel. In particular, taking as given that the rival seller h is active on both distribution channels and is anticipated to charge $\tilde{p}_h^P(w)$, seller i can realize a profit of

$$\max_{p_{iB}} \pi_i\left(p_{iB}, \infty, \tilde{p}_h^P(w)\right) = p_{iB} D_{iB}\left(p_{iB}, \infty, \tilde{p}_h^P(w)\right), \tag{9}$$

from delisting from the platform, where ∞ indicates that seller i is not active on the platform. If the profit on the direct channel alone (Equation (9)), exceeds the profit from being active on both channels, it cannot be an equilibrium of the stage game in which both sellers are active on both distribution channels. We verify that there exists an equilibrium in which both sellers are only present on the direct channel and offer no products via the platform in this case. Denote with $\tilde{\pi}_{i(B)}^P = \pi_i\left(\tilde{p}_B^P, \infty\right)$ seller i's equilibrium profit in case both sellers are only present on the direct channel B. The following lemma summarizes the sellers' listing decision and prices depending on the symmetric commission rate w_A on the platform if sellers compete.

Lemma 2. *Suppose competing sellers face distribution costs of $w = (w_A, 0)$ and the platform imposes a PMFN (P). Sellers are active on both distribution channels if*

$$w_A \leq \tilde{w}_{max} = \frac{4(1-\alpha)(2-\sigma(\beta))}{4-\alpha(4-\sigma(\beta))}, \tag{10}$$

with $\sigma(\beta) = \sqrt{2(1+\beta)}$, and set the same retail price on both channels

$$\tilde{p}_i^P(w) = (2 - 2\alpha + w_A)/(4 - 2\alpha). \tag{11}$$

Otherwise, both sellers are only active on the direct channel and set direct channel prices of $\tilde{p}_{iB}^P = (1-\alpha)/(2-\alpha)$ as specified in Equation (4) in Lemma 1.

Proof. See Appendix A. □

The result of Lemma 2 provides a threshold value $\tilde{w}_{max}$ for the maximal commission rate on the platform for which sellers are active on both distribution channels (Johansen and Vergé, 2017). If sellers are active on the platform, they optimally set the same retail prices on both distribution channels (as they are contractually forced not to offer lower prices on the direct channel). In contrast to the case without PMFN, the equilibrium retail price on distribution channel $j \in \{A, B\}$ therefore depends on the costs of distribution on both channels. In particular, the retail price on the direct channel is affected by the commission rate w_A that the platform charges for every intermediated transaction. A comparison of the equilibrium retail prices with and without PMFN reported in Lemma 1 and 2 shows that the pass-through rate of the commission rate w_A on the retail price p_A is lower with PMFN than without. The reason for this result is that a seller needs to adjust the prices on both distribution channels in reaction to a change in the commission rate, which renders such adjustments less responsive than without PMFN. This property is at the core of the analysis that relates PMFNs to reduced competition on the platform level, and hence of the main established theory of harm as regards PMFNs (see, for instance, Boik and Corts 2016).

For commission rates above the threshold $\tilde{w}_{max}$, it cannot be an equilibrium that both competing sellers are present on both channels as it is unilaterally profitable for a seller to delist from the platform if $w_A > \tilde{w}_{max}$. By delisting, a seller can charge more profitable prices on the direct channel and additionally benefits from the fact that the competing seller, which is anticipated to be present on both channels, is contractually induced to charge higher-than-optimal prices on the direct channel. Lemma 2 establishes that in this case both sellers are only active on the direct channel and optimally set the same retail prices as in the case without contractual restrictions specified in Lemma 1. Importantly, as we will see below, the sellers would achieve higher profits from being active on both distribution channels for a range of commission rates higher than $\tilde{w}_{max}$. But sellers cannot coordinate their listing decisions in a competitive period as it is unilaterally profitable to be active on the direct channel only. In other words, punishment is harsher for commission rates above $\tilde{w}_{max}$, which–as we demonstrate below–has a stabilizing effect on seller collusion.

This coordination failure does not occur if sellers coordinate their listing decisions and retail prices in order to maximize their joint profits $\pi_{12} = \pi_1 + \pi_2$. If present on both channels, the collusive maximization problem stipulates

$$\begin{aligned} \max_{p} \ \pi_{12}(p) &= \sum_{i \in \{1,2\}} (p_{iA} - w_A) D_{iA}(p) + p_{iB} D_{iB}(p), \\ & s.t. \ \ p_{iA} \leq p_{iB}. \end{aligned} \tag{12}$$

As in the case with seller competition, the constraint on the retail prices is binding in equilibrium. Denote the resulting collusive retail price as $\bar{p}_i^P(w)$. In contrast to the competitive case, sellers also coordinate on their listing decisions and only delist from the platform if the commission rate w_A is such that their joint profits are larger on the direct channel alone than on both distribution channels. Denote with $p_B = (p_{1B}, p_{2B})$ the vector of prices that sellers set on direct channel B. If only active on the direct channel, they maximize

$$\max_{p_B} \ \pi_{12}(p_B, \infty) = \sum_{i \in \{1,2\}} p_{iB} D_{iB}(p_B, \infty), \tag{13}$$

where ∞ denotes that sellers are not active on the platform A. Denote the collusive seller profit on the direct channel alone as $\bar{\pi}_{i(B)}^P$. In the following lemma, we characterize the behavior of colluding sellers.

Lemma 3. *Suppose colluding sellers face distribution costs of* $w = (w_A, 0)$ *and the plat-*

form imposes a PMFN (P). Sellers are active on both distribution channels if

$$w_A \quad \leq \quad \bar{w}_{max} = 2 - \sqrt{2(1+\beta)} = 2 - \sigma(\beta), \tag{14}$$

with $\bar{w}_{max} > \tilde{w}_{max}$, and set retail prices of $\bar{p}_i^P(w) = (2 + w_A)/4$. Otherwise, sellers coordinate to be present on the direct channel only and set $\bar{p}_{iB}^P = 1/2$.

Proof. See Appendix A. □

The threshold value $\bar{w}_{max} > \tilde{w}_{max}$ below which colluding sellers are willing to list on both distribution channels is larger than the threshold value $\tilde{w}_{max}$ for competing sellers due to the fact that collusion allows sellers to overcome the coordination problem in the channel choice. This implies that colluding sellers may be active on both distribution channels while such listing decisions are not an equilibrium in the case of seller competition. Moreover, this result shows that a profit-maximizing platform, which imposes a PMFN, will never charge commission rates above $w_A > \bar{w}_{max}$ as neither competing nor colluding sellers are willing to list on the platform and accept the contractual restrictions from a PMFN for such high commission rates.

Based on this analysis, we again turn to the platform's profits both with seller competition and seller collusion. The result of Proposition 1 that the platform prefers seller competition changes with the introduction of a PMFN. As derived in Lemma 2 and 3, the sellers' participation constraints are important determinants of the equilibrium outcome in this case. In fact, the commission rates that maximize the platform's period profits are the same as the threshold values that make competing and colluding sellers indifferent to their outside option of being active on the direct channel only. Recall from the comparison of seller competition and seller collusion that this threshold value is smaller in the case of seller competition ($\bar{w}_{max} > \tilde{w}_{max}$). As a result, a platform can enforce a higher commission rate from colluding sellers than it can from competing sellers. This effect makes a platform more lenient towards seller collusion and can lead to the platform obtaining higher profits with seller collusion than with seller competition.

Proposition 2. *With PMFN, the platform's optimal commission rate depends on the seller conduct. If seller conduct is competition, the commission rate that maximizes the platform's profit is equal to the threshold value $\tilde{w}_A^P = \tilde{w}_{max}$ defined in Equation* (10) *in Lemma 2. With seller collusion, this commission rate is equal to the threshold value $\bar{w}_A^P = \bar{w}_{max}$ defined in Equation* (14) *in Lemma 3. The resulting platform profits depending on seller conduct are*

$$\tilde{\Pi}_A^P\left(\tilde{w}_A^P\right) \quad = \quad \frac{8(1-\alpha)(2-\sigma(\beta))\sigma(\beta)}{(1+\alpha)(1+\beta)(4-\alpha(4-\sigma(\beta)))^2}, \tag{15}$$

$$\bar{\Pi}_A^P\left(\bar{w}_A^P\right) \quad = \quad \frac{(2-\sigma(\beta))\sigma(\beta)}{2(1+\alpha)(1+\beta)}, \tag{16}$$

with $\sigma(\beta) = \sqrt{2(1+\beta)}$. The platform's profit with seller collusion is larger than with *seller competition if interbrand substitutability α is sufficiently large. That is,* $\bar{\Pi}_A\left(\bar{w}^P\right) > \tilde{\Pi}_A^P\left(\tilde{w}^P\right)$ *if* $\alpha > \bar{\alpha} = (16 - 8\sigma(\beta)) / \left(16 - 8\sigma(\beta) + \sigma(\beta)^2\right)$.

Proof. See Appendix A. □

The result of Proposition 2 captures that there are two diverging effects of the introduction of the PMFN on the platform profits. First, seller collusion allows the platform to charge higher commission rates without violating the sellers' participation constraint. This effect increases platform profits. Second, seller collusion leads sellers to charge higher retail prices at given commission rates. This reduces demand and decreases platform profits. The first effect dominates the second one if the degree of interbrand substitutability α is sufficiently large in our setting, and in this case the platform has no incentive to discourage seller collusion ($\alpha > \bar{\alpha}$).

In Section 5.2, we analyze the case of revenue-sharing commission rates. Importantly, the results reveal that the platform prefers seller collusion for all degrees of interbrand substitutability α, and hence with this contract form a platform is more prone to seller collusion than with per-unit commission rates.[12]

Profitability of Platform Most-Favored Nation Clauses In various digital markets, platform providers have revealed a strong interest to impose a PMFN.[13] Comparing the platform's profit levels reported in Proposition 1 (for the case without PMFN) and Proposition 2 (with PMFN) allows to study the profitability of a PMFN for the platform.[14] With seller competition, the profit comparison yields that a platform benefits from a PMFN only if the interbrand competition between the online sellers is strong.[15] In contrast, this case distinction on the intensity of the interbrand competition regarding the profitability of a PMFN does not apply in the case of colluding sellers. If sellers collude, the platform unambiguously prefers a PMFN. The fact that a PMFN is particularly profitable for a platform if sellers collude is in contrast to the case of seller competition and complements Johansen and Vergé (2017), where a PMFN is profitable only under certain conditions on the intensity of seller competition.

Online sellers typically complain about the use of PMFNs, suggesting that seller profits are higher absent a PMFN. For competing sellers this result is supported in the theoretical studies establishing the main theory of harm discussed in Section 2 (e.g., Foros et al.

[12]See also Teh (2019) for a similar result on the difference between per-unit and revenue-sharing commission rates.

[13]See, for instance, the blog post Amazon Gets Bulk of Complaint in AAP Filing With US Trade Commission on www.publishingperspectives.com or Bundeskartellamt calls Booking.com's 'best price'{} clauses anticompetitive on www.triptease.com (last access, September 15, 2020).

[14]Another reason that makes a PMFN desirable for the platform that is outside of this model is the avoidance of *showrooming* of the consumers, which means that consumers search on the platform for an online seller and purchase the product on the distribution channel that offers the product at the lowest price (see Wang and Wright (forthcoming)).

[15]In particular, $\tilde{\Pi}_A^P\left(w_A^{NP}\right) > \tilde{\Pi}_A^{NP}\left(w_A^{NP}\right)$ if $\alpha > (8 - 2\sigma(\beta)) / (7 - \beta)$. See Johansen and Vergé (2017) for a similar condition.

2017).[16] Related to this result, we also find that colluding sellers realize lower profits with PMFN than absent a clause if the platform charges the optimal commission rates $\bar{w}_A^{NP}$ and $\bar{w}_A^{P}$ characterized in Propositions 1 and 2. Also comparing across different forms of seller conduct yields that sellers dislike a PMFN. Seller competition absent a PMFN yields a higher profit than seller collusion with such a clause. Moreover, both with and without PMFN, the relative gain of colluding compared to competing $((\bar{\pi}_i - \tilde{\pi}_i)/\tilde{\pi}_i)$ is the same for the online sellers.

4 Dynamic Model

4.1 Infinitely-Repeated Game

In this section, we analyze the industry structure introduced above in an infinitely-repeated game. So far, we have imposed that collusion between sellers is sustainable in a given period without focusing on the exact mechanism stabilizing collusion. The framework of the infinitely-repeated game allows us to study a possible mechanism with which seller collusion can be sustained.[17] Moreover, this approach allows us to relate the analysis to the antitrust cases such as the e-book case discussed in the Introduction.

Our focus is on the stability of collusion between the sellers under contracts with and without a PMFN. Sellers have a common discount factor $\delta \in (0, 1)$, and aim to maximize present-discounted stream of profits

$$\sum_{t=0}^{\infty} \delta^t \pi_i (p_t), \tag{17}$$

where p_t is the vector of retail prices in period t, and π_i retailer i's stage profit at these retail prices.

We analyze the case in which the platform commits to a constant commission rate that does not change in future periods at the beginning of the first period. In fact, this pricing behavior appears to be employed by most online platforms nowadays. For instance, in the online hotel booking sector, a recent report by EU competition authorities indicates that there were little to no changes in the base and effective commission rates paid by hotels to online travel agencies during the period 2014 to 2016.[18] Similarly, the commission rate

[16] In contrast, Johansen and Vergé (2017) find that PMFNs can benefit all the actors (platforms, sellers, and consumers) in an industry. The result that profits of non-cooperative sellers can increase due to a PMFN is also supported in the present analysis for the case of large intrabrand substitution β (profits are reported in Appendix A). In this case, distribution channels are easily substitutable for the online seller, and the seller's participation constraint to be active on the platform commands a low commission rate.

[17] Another mechanism would involve giving the platform the ability to directly affect the competitive intensity between the online sellers (for instance by influencing the search costs of the consumers to inspect a further alternative). See Teh (2019) for an analysis in this direction that, however, does not focus on PMFNs.

[18] See the Report on the monitoring exercise carried out in the online hotel booking sector (last access, April 23, 2020).

that Apple negotiated with the major e-book publishers was set at 30 percent and did not change during and after the collusive period (Foros et al., 2017).

We solve for a subgame-perfect Nash equilibrium of the infinitely-repeated game between the sellers based on this constant commission rate. On the path of play, the sellers coordinate to achieve in each period the joint profit maximum (i.e., the most collusive outcome) by setting the collusive price $\bar{p}_{ij}$ on each distribution channel j. For brevity, it is convenient to suppress that the retail prices depend on the constant commission rate.[19]

We analyze the stability of collusion in an equilibrium sustained through grim-trigger strategies (Friedman, 1971). If a seller deviates from the collusive scheme, it sets $\hat{p}_{ij}$ such that its deviation profit is maximized. After a deviation period, all sellers revert to playing their (unique) static Nash equilibrium strategy $\tilde{p}_{ij}$ for all future periods. Formally, in any period $t = 0, 1, ..., \infty$ in which sellers coordinate on collusion, seller i sets the collusive prices $\bar{p}_{ijt}$ on both distribution channels $j \in \{A, B\}$. For any future period t, it holds that

$$p_{ijt} = \begin{cases} \bar{p}_{ij} & \text{if } p_{hj\tau} = \bar{p}_{hj} \ \forall \ \tau < t, \ h \in \{1,2\}, \ j \in \{A,B\}, \\ \tilde{p}_{ij} & \text{if otherwise.} \end{cases} \tag{18}$$

Denote the corresponding stage-game profits that are associated with the prices defined above by $\bar{\pi}_i$, $\tilde{\pi}_i$, and $\hat{\pi}_i$. The condition that there is no unilateral incentive to deviate from the collusive scheme is

$$\sum_{t=0}^{\infty} \delta^t \bar{\pi}_i \geq \hat{\pi}_i + \sum_{t=1}^{\infty} \delta^t \tilde{\pi}_i. \tag{19}$$

The discounted stream of profits from sticking to the collusive scheme needs to exceed the profit that an upstream firm can obtain from cheating and reverting afterwards to the static Nash equilibrium for all future periods. Rearranging yields that the common discount factor needs to exceed

$$\delta \geq \underline{\delta} = \frac{\hat{\pi}_i - \bar{\pi}_i}{\hat{\pi}_i - \tilde{\pi}_i} \ \in [0,1], \tag{20}$$

where $\underline{\delta}$ denotes the seller's critical discount factor for collusion to be sustainable.

In order to ensure that both sellers are active and sell positive quantities in all periods of the infinitely-repeated game, we assume that the degree of interbrand substitutability is not too large:

Assumption 1. $\alpha < \sqrt{3} - 1$.

In particular, this assumption ensures that a seller that charges collusive prices sells a positive quantity to the consumers even if the other seller deviates from the collusive scheme and charges lower prices in order to maximize the current-period profits (see also Ross, 1992).[20]

[19] In the extension, we also analyze the case in which coordination on the joint-profit maximum is not incentive-compatible, and the effects if the platform adjusts the commission rate as a reaction to changing market conditions.

[20] Recall that the profitability of seller collusion with PMFN (Proposition 2) depends on $\alpha > \bar{\alpha} =$

Discussion of the model framework applied to collusion in digital markets. In this section, we analyze the stability of collusion in online markets taking the canonical approach of comparing the long-term benefits from collusion with the temptation of a one-time deviation from the collusive agreement. As already discussed in the Introduction, there are high-profile collusion cases on platform markets that motivate this analysis, and raise–among others–the question about the stability of collusive agreements in online markets, and, importantly, whether the stability changes with the introduction of a PMFN. Moreover, there are recent empirical studies from other industries lending support to the hypothesis that the incentive compatibility of collusive agreements is an economically relevant dimension to understand the behavior of cartels (Igami and Sugaya, 2019; Miller et al., 2019).

A potential concern, however, may involve that in principle online markets can allow for timely responses to deviations actions. At an extreme of immediate reactions, this would render any deviation from collusion unprofitable and allow for stable collusion with any common discount factor Ivaldi et al. (2003). We nevertheless take the view that this approach can offer fruitful insights for the study of collusion in online markets for the following reasons.

First, as we will see below, the analysis links the stability of collusion to the listing decisions of the online sellers on different distribution channels. Arguably, the channel choice is less flexible than an adjustment in the posted prices and takes more time to react to in case of a deviation. Recent empirical studies such as Hunold et al. (2018) and Cazaubiel et al. (2020) provide evidence that the listing decision is an important dimension of adjustment in the hotel sector.

Second, there may be a fraction of online sellers that can react fast to changes in the posted prices of other sellers, for instance by using pricing algorithms in order to automate pricing decisions. In a recent paper, Chen et al. (2016) however detect that only 2.4 percent of online sellers use such algorithmic pricing on the Amazon Marketplace. For a large fraction of sellers, it is therefore still necessary to detect and react to a deviation without the help of algorithms, which may make them more comparable to other industries to which the approach is usually is applied. Relatedly, deviations on other distribution channels than on the platform itself may be more difficult to monitor and also take more time to react to for the other sellers.

Based on the antitrust cases at hand, which specifically deal with online seller collusion, and the arguments above, we therefore argue that the present analysis with the focus on stability of collusion is worthwhile and contributes to the understanding of these cases.

A further element of the model framework is that it abstracts from information frictions. Some argue that a PMFN may improve the the observability of secret price cuts and thereby stabilizes collusion (Fletcher and Hviid, 2016; Baker and Scott Morton, 2017).

$(16 - 8\sigma(\beta)) / \left(16 - 8\sigma(\beta) + \sigma(\beta)^2\right)$. Note that it holds that $\sqrt{3} - 1 > \bar{\alpha}$, $\forall \beta \in (0,1)$ such that this result is not excluded by Assumption 1. Moreover, for the case of revenue-sharing commission rates we do not rely on a threshold value $\bar{\alpha}$ at all such that Assumption 1 is innocuous in this setting.

According to Stigler (1964), avoiding the threat of secret price cuts is the major obstacle for stable collusion, and this argument is reminiscent of the analysis by Jullien and Rey (2007) for the case with RPM. It is, however, not obvious that the introduction of a PMFN improves the observability of price cuts. Hunold et al. (2018), for instance, provide some evidence that a share of hotels does not fully comply with a PMFN imposed by online travel agencies. This implies that even if the observability of secret price cuts is a concern, the introduction of a PMFN with incomplete enforcement may not be able to alleviate this problem. A platform provider might be as unable as the colluding firms to detect or prevent one-time violations of the PMFN requirements. The present analysis does not focus on these informational aspects of a PMFN but analyzes the effect of a platform's commission rate on the stability of collusion which is independent from whether a PMFN improves the observability of price cuts or not.

4.2 Analysis of the Dynamic Model

In this section, we analyze how the introduction of a PMFN affects the stability of seller collusion if platform A charges constant and symmetric commission rates $w_{1A} = w_{2A} = w_A$ in every period t.[21] With constant commission rates, the platform can only condition the commission rates on the inclusion of a PMFN at the beginning of the infinitely-repeated game, but cannot adjust commission rates in later periods. As argued in the Introduction, this pricing behavior appears to be consistent with observed platform behavior in various markets. The aim of this section is to characterize how the introduction of a PMFN changes the stability of seller collusion by altering punishment and deviation behavior compared to the case without PMFN.

No Platform Most-favored Nation Clause. We can build on the results in Lemma 1, and it only remains to characterize the behavior of a deviating seller. The following lemma summarizes this for the case without PMFN.

Lemma 4. *Suppose sellers face distribution costs of $w = (w_A, 0)$, with $w_A \in [0, 1-\beta]$. A deviating seller i is active on both distribution channels and optimally sets*

$$\hat{p}_{ij}^{NP}(w) = (2 - \alpha + (2+\alpha) w_j)/4. \tag{21}$$

Proof. See Appendix A. □

If a seller deviates from the collusive agreement, it finds it profitable be be active on both distribution channels. The deviation prices that maximize the current-period profits

[21] As verified above, the platform charges symmetric commission rate in the equilibrium of the static game. By offering asymmetric commission rates, the platform would induce sellers with asymmetric costs of distribution, which affects collusive stability if sellers continue to collude on the joint profit-maximizing retail prices. The sellers are, however, able to offset this effect on their critical discount factor by agreeing on a different distribution of profits or side payments. Both strategies render the effect of asymmetric costs of distribution on the stability of collusion negligibly small.

of a seller in Equation (21) are below the collusive prices (Equation (5)) and above the competitive prices (Equation (4)). For every conduct, the sellers prefer to set lower prices on the direct channel than on the platform as the costs of distribution on the direct channel are equal to zero.

Based on the results in Lemma 1 and 4 that characterizes seller behavior in competitive, collusive, and deviation periods, we can characterize the critical discount factor above which collusion is a subgame-perfect equilibrium for the sellers. The following proposition summarizes the result.

Proposition 3. *Suppose sellers face distribution costs of $w = (w_A, 0)$, with $w_A \in [0, 1-\beta]$. Without PMFN (NP), the critical discount factor is*

$$\underline{\delta}^{NP} = \frac{(2-\alpha)^2}{8 - 8\alpha + \alpha^2}, \tag{22}$$

for both sellers $i \in \{1, 2\}$. It increases in the degree of interbrand competition α, and is independent of the degree of intrabrand competition β, and the commission rate w_A.

Proof. See Appendix A. □

The result of Proposition 3 shows that the critical discount factor absent a PMFN is independent of the seller's cost level. This implies that a platform cannot affect the seller's incentive constraint for stable collusion in Equation (19) by setting a different per-unit commission rate in our setting. Relatedly, the degree of intrabrand substitutability between the distribution channels (as measured by β), which indirectly affects the per-unit commission rates that the platform can impose, does not affect the sellers' critical discount factor either. Moreover, with per-unit commission rates, the critical discount factor $\underline{\delta}^{NP}$ depends on the degree of interbrand competition and increases in α, which shows that a higher degree of substitutability between the sellers decreases the stability of collusion.[22]

Platform Most-Favored Nation Clause. Next, we turn to the analysis of the stability of collusion with a PMFN. Such contract restrictions lead to two important changes that may affect the stability of seller collusion. First, it is important to take into account the sellers' listing decision on the platform (Johansen and Vergé, 2017). Due to the contractual restrictions of the PMFN, a seller is induced to charge higher than optimal prices on its direct channel if it is active on both distribution channels. It may therefore be more profitable for a seller to delist from the platform in order to charge more profitable prices on its direct channel and save the commission payments that accrue for every transaction via the platform. Second, and relatedly, if sellers do not delist and stay active on both distribution channels, the presence of a PMFN effectively undermines a seller's ability to price discriminate between distribution channels. As derived in Lemma 4, the sellers

[22] This result of Proposition 3 is related to Ross (1992) who shows in a different setting that greater seller homogeneity can reduce cartel stability.

prefer to charge lower prices on their direct channel due to the fact that they face lower costs of distribution on this channel. This form of cost-based price discrimination between distribution channels is not feasible with a PMFN as the price that they set on the direct channel B effectively constitutes an upper bound for the retail prices that can be charged on the platform A. For instance, Helfrich and Herweg (2016) show that the firms' ability to engage in preference-based price discrimination can destabilize collusion. As we derive below, the model based on per-unit commission rates and the linear demand specification implies that the latter mechanism does not affect the stability of seller collusion. The analysis, therefore, purposefully abstracts from this mechanism in order to focus on the effects of altered punishment and deviation behavior due to a PMFN.

In the following, we characterize how a PMFN affects behavior of a deviating seller. As colluding and competing sellers, a deviating seller also needs to decide whether to be active on both channels or only on the direct channel. First, consider that a deviating seller i decides to be active on both channels and takes as given that the second seller h also is present on both channels and sets collusive prices $\bar{p}_h^P(w) = (2 + w_A)/4$ (derived in Lemma 3). The deviating seller sets retail prices p_i in order to maximize

$$\max_{p_i} \ \pi_i(p_i, \bar{p}_h(w)) = (p_{iA} - w_{iA}) D_{iA}(p_i, \bar{p}_h(w)) + p_{iB} D_{iB}(p_i, \bar{p}_h(w)), \tag{23}$$

subject to the constraint that $p_{iA} \leq p_{iB}$. Alternatively, the deviating seller may decide to delist from the platform, and to offer products only via the direct channel. In this case, the deviating seller sets the retail price p_{iB} in order to

$$\max_{p_{iB}} \ \pi_i\left(p_{iB}, \infty, \bar{p}_h^P(w)\right) = p_{iB} D_{iB}\left(p_{iB}, \infty, \bar{p}_h^P(w)\right). \tag{24}$$

Denote the profit of a deviating seller that is present on the direct channel only as $\hat{\pi}_{i(B)}^P(w)$. The next lemma summarizes the optimal deviation behavior.

Lemma 5. *Suppose sellers face distribution costs of* $w = (w_A, 0)$ *and the platform imposes a PMFN* (P). *If seller* i *deviates from collusion, it is active on both distribution channels if*

$$w_A \ < \ \hat{w}_{max} = \frac{2(2-\alpha)(2-\sigma(\beta))}{4-\alpha(2-\sigma(\beta))}, \tag{25}$$

and sets $\hat{p}_i^P(w) = (4 - 2\alpha + (2+\alpha) w_A)/8$. *Otherwise, a deviating seller is only active on the direct channel and charges* $\hat{p}_{iB}^P(w) = (4 - \alpha(2 - w_A))/8$ *while the non-deviating seller stays active on both channels. Over the complete parameter range, it holds* $\tilde{w}_{max} < \hat{w}_{max} < \bar{w}_{max}$.

Proof. See Appendix A. □

The result of Lemma 5 shows that a deviating seller may be active on both distribution channels or on the direct channel only depending on the commission rate on the platform. More specifically, if competing sellers are present on the platform $w_A < \tilde{w}_{max}$, it is also profitable for a deviating seller to do so. In contrast, at the other extreme, if the commission rate is very high such that colluding sellers are close to indifferent between listing on both distribution channels or only the direct channel ($w_A \lesssim \bar{w}_{max}$), a deviating seller prefers to delist from the platform and to sell only via the direct channel. In this case collusive prices are high due to the high costs of distribution on the platform and a deviating seller benefits strongly from avoiding contractual restrictions from a PMFN by delisting from the platform.

Based on the results in Lemma 2, 3, and 5, we can characterize the stability of collusion in the presence of a PMFN, taking the sellers' listing decisions into account. For a sufficiently small commission rate $w_A \leq \tilde{w}_{max}$, the sellers are active on both distribution channels and the critical discount factor from Equation (20) is

$$\underline{\delta}^P = \frac{\hat{\pi}_i^P - \bar{\pi}_i^P}{\hat{\pi}_i^P - \tilde{\pi}_i^P}. \tag{26}$$

In this case, we find the same critical discount factor as for the case without PMFN reported in Equation (22) in Proposition 3. This result indicates that the ability to price discriminate between distribution channels does not affect the stability of seller collusion in our setting, provided that sellers are active on both distribution channels.

At $w_A > \tilde{w}_{max}$, the sellers are not active on the platform in competitive periods and realize a profit of $\tilde{\pi}_{i(B)}^P(w)$, which is strictly lower than the profit $\tilde{\pi}_i^P(w)$ from being present on both distribution channels for $w_A \in (\tilde{w}_{max}, \bar{w}_{max})$. This renders punishment for a deviation more severe, and implies a discrete decrease in $\underline{\delta}^P$ at $w_A = \tilde{w}_{max}$ indicating that collusion is more stable than below this threshold. The difference between $\tilde{\pi}_i^P(w)$ and $\tilde{\pi}_{i(B)}^P(w)$ decreases in w_A, and hence the critical discount factor increases in w_A for $w_A \in (\tilde{w}_{max}, \bar{w}_{max})$.

Additionally, the results in Lemma 5 indicate that the optimal deviation behavior changes at $w_A = \hat{w}_{max}$. In particular, below this threshold a deviating seller is active on both distribution channels whereas it is only active on the direct channel for commission rates above the threshold value. In contrast to the case with competing sellers, a deviating seller only delists from the platform if it yields higher profits to do so. This implies that $\hat{\pi}_{i(B)}^P(w) > \hat{\pi}_i^P(w)$ for $w_A \in (\hat{w}_{max}, \bar{w}_{max}]$. Hence, deviation becomes more tempting from commission rates above $\hat{w}_{max}$. As a higher deviation profit destabilizes collusion, we conclude that the critical discount factor increases more strongly for $w_A \in (\hat{w}_{max}, \bar{w}_{max}]$ than for lower commission rates. Lastly, we know that the competitive profits on both distribution channels is the same as the competitive profit on the direct channel for $w_A = \bar{w}_{max}$. That is, $\tilde{\pi}_i^P(\bar{w}_{max}) = \tilde{\pi}_{i(B)}^P(\bar{w}_{max})$. For this commission rate, we hence know that there is no stabilizing effect of lower competitive profits, and at the same time the destabilizing effect of higher deviation profits remains. The net effect implies that for high commission rates close to $\bar{w}_{max}$ seller collusion is more difficult to sustain than in the benchmark without PMFN. The results are summarized in

Proposition 4. *Suppose sellers face distribution costs of $w = (w_A, 0)$, and the platform* *imposes a PMFN (P). If $w_A \leq \tilde{w}_{max}$, the critical discount factor is*

$$\underline{\delta}^P = \underline{\delta}^{NP} = \frac{(2-\alpha)^2}{8 - 8\alpha + \alpha^2}, \tag{27}$$

as in the case without PMFN (NP). At $w_A = \tilde{w}_{max}$, there is a discrete decrease in the critical discount factor such that $\underline{\delta}^P(\tilde{w}_{max}) < \underline{\delta}^{NP}$. Above this commission rate, the critical discount factor $\underline{\delta}^P(w)$ increases in $w_A \in (\tilde{w}_{max}, \bar{w}_{max})$, with a kink at $w_A = \hat{w}_{max}$. For sufficiently large w_A in this range, it holds that $\underline{\delta}^P > \underline{\delta}^{NP}$.

Proof. See Appendix A. □

Proposition 4 characterizes the sellers' critical discount factor with a PMFN for general symmetric commission rates w_A. The exact terms for the critical discount factor $\underline{\delta}^P$ for $w_A > \tilde{w}_{max}$ are provided in Equations (70) and (71) in Appendix A. Figure 1 plots the critical discount factor $\underline{\delta}^P$ depending on the commission rate w_A on the platform as characterized in Proposition 4.

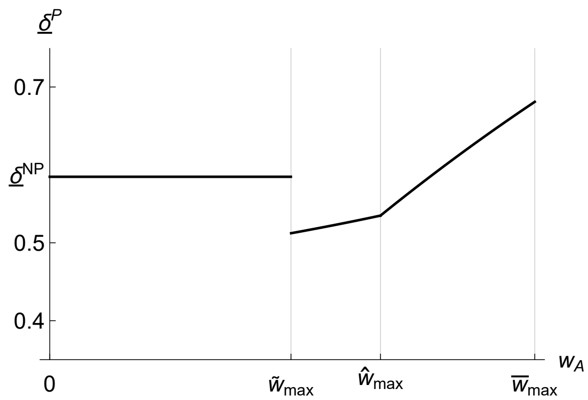


Figure 1: Critical discount factor $\underline{\delta}^P$.

The figure shows the critical discount $\underline{\delta}^P$ depending on the exogenous commission rate w_A for $\alpha = 7/10$ and $\beta = 1/2$.

The plot exhibits the salient features explained above. For small commission rates w_A the critical discount factor is equal to the case without PMFN and independent of w_A. By conventional interpretation, it follows that the cartel stability between sellers is not affected by the introduction of a PMFN in this range of commission rates. Moreover, this result emphasizes that the ability to engage in cost-based price discrimination between distribution channels itself, which is restricted due to a PMFN, does not affect the stability of collusion in our setting as long as it does not affect the sellers' listing decisions.

In contrast, the result of Figure 1 highlights that a PMFN has an effect on the stability of seller collusion due to the fact that it changes the sellers' listing decisions. In particular, at the threshold $w_A = \tilde{w}_{max}$, there is a discrete decrease in the critical discount factor due to the fact that competing sellers do not list on the platform. As described above this implies that punishment is more severe in this range of commission rates which stabilizes seller collusion. For higher commission rates, the critical discount factor increases in w_A

with a kink at $w_A = \hat{w}_{max}$ due to the fact that at this point also the optimal deviation behavior (including the listing decision) changes with the effect that $\underline{\delta}^P$ increases more strongly above this threshold. For the highest admissible commission rate of $w_A = \bar{w}_{max}$, the critical discount factor $\underline{\delta}^P$ is above the critical discount factor without PMFN, $\underline{\delta}^{NP}$.

Importantly, for commission rates above $\tilde{w}_{max}$, the platform directly can affect the stability of seller collusion by means of the commission rate. Consider for instance that the seller's common discount factor is slightly below the critical discount factor $\underline{\delta}^P(\bar{w}_{max})$ that is necessary for stable seller collusion if the platform charges $\bar{w}_A^P = \bar{w}_{max}$. This implies that seller collusion is not sustainable at this commission rate, and sellers would compete in every period with the consequence that the platform may realize lower profits than in a collusive equilibrium (see Proposition 2). But note that, if necessary, the platform can increase the stability of seller collusion by reducing the commission rate, which also reduces the critical discount factor $\underline{\delta}^P$. If sellers can coordinate on collusion at this reduced commission rate, the platform is able to sustain a commission rate close to $\bar{w}_{max}$ which can be more profitable than the competitive equilibrium with the substantially lower commission rate of $\tilde{w}_{max}$. In the given specification, seller collusion is more profitable for the platform even if the seller's common discount factor is $\delta = \underline{\delta}^{NP}$.[23]

5 Extensions

5.1 Constrained Collusion

The main analysis in Section 4 focuses on the sustainability of full collusion on the joint-profit-maximizing retail prices. If the sellers' common discount factor is too small to sustain full collusion, the analysis assumes that sellers cannot coordinate at all and play competition in every period of the infinitely-repeated game.

It is possible, however, that sellers still coordinate on smaller than fully-collusive prices if this increases their joint profits (compared to the competitive level) and fulfills the incentive-compatibility constraint. We refer to this form of collusion as *constrained collusion*. Importantly, if sellers collude in this form, we show that for high commission rates (which makes a deviation more tempting in the model analyzed in Section 4), can lead to a decrease in the constrained collusive retail price that is necessary to keep the incentive-compatibility constraint binding. This reinforces the result that a platform may prefer seller collusion over seller competition with a PMFN, as this leads to higher commission rates and potentially lower retail prices, and both aspects increase a platform's profit.

Again, we suppose that sellers sustain constrained collusion by means of grim-trigger strategies. Denote punishment prices as $\tilde{p}$ and suppose that sellers cannot coordinate on

[23] In this case, the commission rate with seller competition is $\tilde{w}_A^P = 0.133$ and yields a profit for the platform of $\tilde{\Pi}_A^P\left(\tilde{w}_A^P\right) = 0.075$. With seller collusion, the commission rate at which the critical discount factor $\underline{\delta}^P(w) = \underline{\delta}^{NP}$ is $\bar{w}_A = 0.268$ and yields a platform profit of $\bar{\Pi}_A^P(\bar{w}) = 0.091$. This implies that the platform can choose a commission rate $w_A < 0.268$ that jointly increases the stability of seller collusion and increases its profit compared to the case of seller competition.

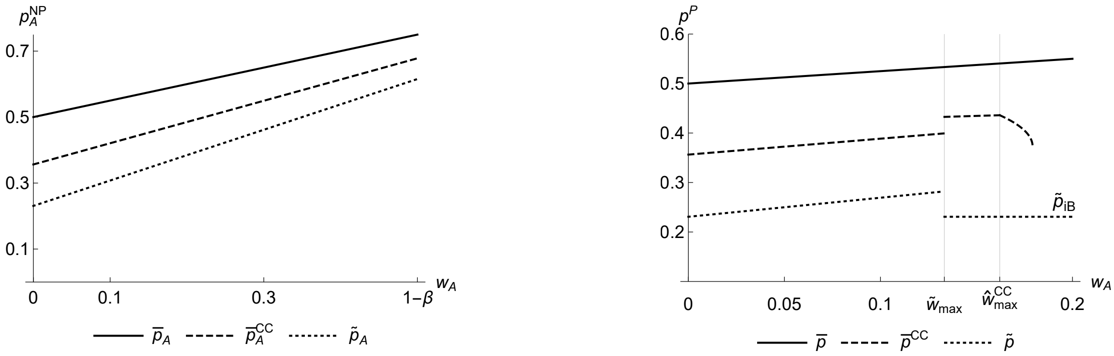
fully-collusive prices $\bar{p}$. We consider instead that sellers coordinate on the highest feasible retail prices such that the incentive-compatibility constraint to be willing to stick to the collusive agreement is binding. Denote the constrained-collusive prices as $\bar{p}^{PC}$ and the deviation prices, which depends on the constrained-collusive prices, as $\hat{p}\left(\bar{p}^{PC}\right)$. The joint maximization problem is as follows:

$$\begin{aligned}\max_{p\in[\tilde{p},\bar{p}]} \pi_{12}(p) &= \sum_{i\in\{1,2\}} (p_{iA}-w_A) D_{iA}(p) + p_{iB} D_{iB}(p) \\ s.t. &\quad \bar{\pi}_i(p) - (1-\delta)\hat{\pi}_i(\hat{p}(p)) - \delta\tilde{\pi}_i(\tilde{p}) \geq 0,\ \forall i,\end{aligned} \tag{28}$$

where the constraint in the second line ensures that the incentive-compatibility constraint in Equation (19) is fulfilled. With constrained collusion the sellers' common discount factor is sufficiently small such that the constraint needs to be binding with equality as otherwise sellers can coordinate on a higher constrained-collusive prices and realize higher joint profits on the equilibrium path. If the constraint is not binding at the fully-collusive price $\bar{p}$, sellers can sustain full collusion (the case analyzed in Section 4).

For the sake of exposition, we report a representative numerical result of the constrained-collusive prices. The findings are qualitatively the same for other parameter constellations for which coordination on constrained-collusive prices is the relevant case. The results for the retail prices absent and with PMFN are depicted in Figure 2. The first panel shows the sellers' retail prices on the platform depending on the commission rate $w_A \in [0, 1-\beta]$ for three cases.[24] The dotted line is the competitive price $\tilde{p}_A$, the solid line is the fully-collusive price $\bar{p}_A$, and the dashed line shows the constrained-collusive price $\bar{p}_A^{CC}$. For $\delta = 3/10$, the incentive constraint is violated at the fully-collusive prices, but sellers can coordinate on constrained-collusive prices above the competitive level $\tilde{p}_A$. As the common discount factor δ increases, sellers are able to sustain higher constrained-collusive retail prices that approach the level of full collusion as δ approaches the critical discount factor reported in Equation (22) in Proposition 3.

[24]Recall that sellers are willing to list on the platform for commission rates up to $1-\beta$.



(a) No PMFN (b) PMFN

Figure 2: Retail Prices with Constrained Collusion.

The figure shows the highest feasible collusive retail price (i.e., constrained collusion) for a case in which full collusion is not feasible depending on the constant commission rate w_A for $\alpha = 7/10$, $\beta = 1/2$, and $\delta = 3/10$. The left panel shows the retail prices on the platform without PMFN (NP) for the cases of competition (dotted), full collusion (solid) and constrained collusion (dashed). The right panel shows the same retail prices for the case with PMFN (P).

The second panel in Figure depicts the case with PMFN. The plot consists of three regions that are the analog to the three regions as in Figure 1 for the critical discount factor $\underline{\delta}^P$ necessary for full collusion to be stable. For small $w_A \leq \tilde{w}_{max}$ (which is the same threshold value as in Proposition 4), sellers prefer to list on the platform for any conduct, and the plot exhibits the same features as the plot in the left panel: the constrained-collusive price lies between the competitive price level and the fully-collusive one and increases in the commission rate w_A.

For $w_A > \tilde{w}_{max}$, competing sellers are not willing to list on the platform, which has two consequences: First, the sellers are only active on the direct channel and optimally set the retail price $\tilde{p}_{iB}^P = (1-\alpha)/(2-\alpha)$ as derived in Lemma 2, and realize lower punishment profits compared to being present on both distribution channels. Second, this form of harsher punishment allows sellers to sustain higher constrained-collusive prices, which is apparent from the discrete increase in $\bar{p}^{CC}$ at $w_A = \tilde{w}_{max}$. This is the same mechanism that leads to the discrete decrease in the critical discount factor $\underline{\delta}^P$ characterized in Proposition 4 and depicted in Figure 1.

The third region in the plot is for commission rates $w_A > \hat{w}_{max}^{CC}$, above which a seller that deviates from the constrained-collusive prices to be present on the direct channel only. In contrast to $\tilde{w}_{max}$, the threshold value $\hat{w}_{max}^{CC}$ is not the same as in the fully-collusive case ($\hat{w}_{max}$) and generally depends on the exact constrained-collusive price level. Again, above this level, deviation becomes more tempting for the sellers, which translates to lower constrained-collusive prices that can be sustained in equilibrium. Interestingly, in this range, an increase in the platform's commission rate leads to a decrease in the constrained-collusive price.

This result reinforces the finding that, with PMFN, a platform may prefer seller coordination in contrast to seller competition on the platform: if sellers coordinate on constrained collusion, the platform can increase its commission rate above $\tilde{w}_{max}$, which is not profitable with seller competition as sellers would delist at higher commission rates. Moreover, a commission rate above $\hat{w}^{CC}_{max}$ can lead to lower retail prices, and hence, the platform benefits from a higher commission payment than with seller competition, and, additionally, from the fact that sellers charge low constrained-collusive retail price, which increases the quantity sold on the platform.

5.2 Revenue-Sharing Commission

In this section, we verify that the main effects of a PMFN on the stability of seller collusion derived for the case of per-unit commission rates also extend to the case with revenue-sharing commission rates. As regards platform profits there are some economically relevant differences between the two contract forms that we discuss below.

In contrast to existing contributions analyzing the agency model with revenue-sharing commission rates such as Foros et al. (2017) or Hino et al. (2019), the model presented in this article allows for asymmetric distribution channels (one platform and one direct channel instead of two symmetric platforms), and online sellers facing (weakly) positive marginal costs $c \geq 0$. Both aspects prevent to fully analyze the model in closed-form solutions and hence we provide our results by means of numerical simulations.

We first provide results for the optimal revenue-sharing commission rates depending on the presence of a PMFN and depending on the seller conduct. Second, we establish that with this form of commission rates the platform prefers seller competition absent a PMFN and seller collusion with PMFN. Third, we analyze the stability of seller collusion. In this extension, we focus on the case in which sellers collude if and only if they are able to achieve the joint-profit maximum and compete otherwise.

Optimal Commission Rates. Suppose that the platform charges a symmetric revenue-sharing commission rate from both sellers $\phi_{1A} = \phi_{2A} = \phi_A$.[25] If both sellers are active on the platform, the platform's profit is

$$\Pi_A(\phi_A) \quad = \quad \phi_A \sum_{i \in \{1,2\}} p_{iA} D_{iA}(p). \tag{29}$$

Depending on seller conduct, Figure 3 plots the optimal revenue-sharing commission rates that the platform sets absent a PMFN (left panel) and with a PMFN (right panel). If sellers compete, the optimal commission rate is depicted by the solid line, and if they collude by the dashed line.

[25] For tractability, we assume in this extension that the platform charges symmetric commission rates. Given symmetry between sellers the model would also endogenously yield symmetric commissiob rates in equilibrium.

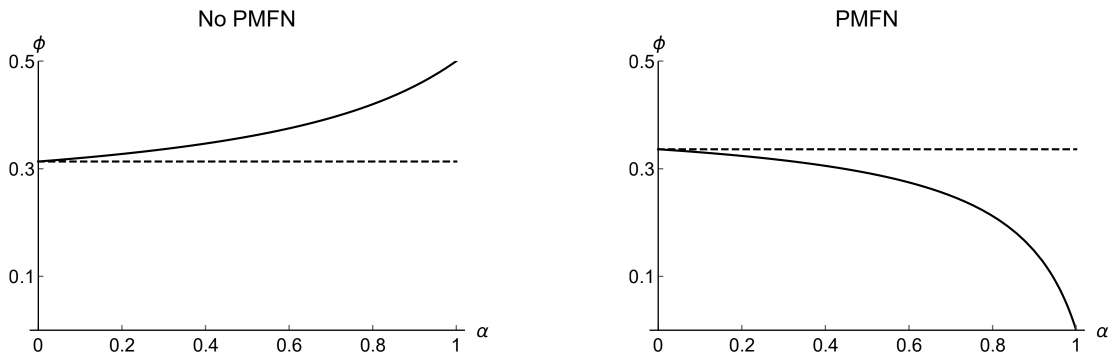


Figure 3: Revenue-sharing commission rates.

The figure shows revenue-sharing commission rates for $\beta = 1/2$ and $c = 1/5$ depending on the degree of interbrand competition $\alpha \in (0, 1)$ for the case of seller competition (solid line) and seller collusion (dashed line). The left panel shows the case without PMFN, the right panel the case with PMFN.

Absent a PMFN, the optimal commission rate positively depends on the degree of interbrand competition α if the sellers compete. This finding is in contrast to the optimal per-unit commission rate w_A^{NP}, which is independent of α (see Proposition 1). The lowest commission rate that online sellers can obtain is at $\alpha \to 0$, which is exactly the commission rate that online sellers obtain if they collude (dashed line). Equation (29) shows that the platform benefits from a high commission rate ϕ_A and high revenue on the own platform. Importantly, in the presence of intrabrand competition $\beta \in (0, 1)$ or positive marginal costs of the sellers $c > 0$, the retail price that the sellers set on the platform p_A positively depend on the commission rate ϕ_A in the relevant range.[26] The platform therefore optimally charges a commission rate at which the additional benefit form marginally increasing the commission rate outweighs the corresponding reduction in the revenue on the platform.[27] Due to the fact that seller competition leads to lower retail prices for a given commission rate, the platform can charge a higher commission rate from competing sellers than from colluding ones in order to induce the same price level on the platform.

The result of the optimal commission rate changes if the platform imposes a PMFN (right panel of Figure 3). As in the case with per-unit commission rates, the binding participation constraint of the sellers to list on the direct channel and the platform (instead of the direct channel alone) determines the equilibrium commission rate. For the same reasons as above, colluding sellers accept a higher commission rate in equilibrium than competing ones.[28]

Preferred Seller Conduct. Next, we turn to the platform's preferred seller conduct. Absent a PMFN, the platform can more profitably resolve the trade-off between high

[26] See the derivations in the proof of Proposition 5 in Appendix A.

[27] We show below that delisting of the sellers is not a constraint for relevant revenue-sharing commission rates.

[28] These comparative static results underline the robustness of the results of Johansen and Vergé (2017). They derive qualitatively similar results on the basis of per-unit commission rates and the assumption that contract offers are unobservable, but do not analyze its impact on seller collusion.

commission rates and high revenue on the platform if sellers compete. This leads to higher commission rates (as explained above) and also to higher profits for the platform if sellers compete. We illustrate this result numerically in the first panel of Figure 4.

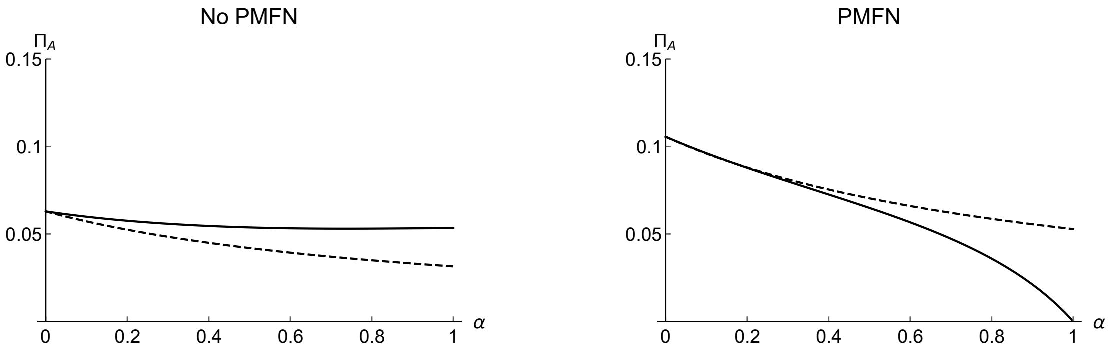


Figure 4: Platform profits

The figure shows platform profits with revenue-sharing commission rates for $\beta = 1/2$ and $c = 1/5$ depending on the degree of interbrand competition $\alpha \in (0, 1)$ for the case of seller competition (solid line) and seller collusion (dashed line). The left panel shows the case without PMFN, the right panel the case with PMFN.

This result is in contrast to Hino et al. (2019). Their paper analyzes the case without PMFN and focuses in an extension on two symmetric platforms as distribution channels. They conclude that for fixed commission rates the platforms typically benefit from seller collusion. Our analysis shows that if the platform charges optimal commission rates based on seller conduct, it always benefits from seller competition absent a PMFN. Even for fixed commission rates, we find that only for small degrees of intrabrand substitutability β and close to zero sellers' marginal costs c (as analyzed in Hino et al. (2019)) it holds that the platform prefers seller collusion.[29] This finding shows that it is important to incorporate the seller's marginal costs in the analysis despite the fact that this makes the model less tractable to analyze.

The right panel of Figure 4 verifies that the preferred seller conduct changes if the platform can impose a PMFN. As described above, the platform charges higher commission rates from competing sellers and this increase is sufficient to render seller collusion more profitable than seller competition. Importantly, we find that a platform prefers seller collusion for the whole parameter range $\alpha \in (0, 1)$. This result is driven by the fact that the platform prefers revenue-maximizing retail prices that are higher than transaction-volume-maximizing prices, which are preferable with per-unit commission rates. Hence, the condition provided in Proposition 2 is not relevant for the case with revenue-sharing commission rates.

Stability of Seller Collusion. Recall that we restrict the range of interbrand competition to $\alpha \in \left(0, \sqrt{3} - 1\right)$ for the analysis of tacit collusion. With revenue-sharing com-

[29] For instance, for $\phi_A = 3/10$ and $\beta = 2/10$, we find that seller collusion is not profitable for the platform for all $\alpha \in (0, 1)$ if $c \gtrsim 1/10$. For smaller degrees of marginal costs (e.g., $c = 3/100$) seller collusion is more profitable for the platform than seller competition for $\alpha \gtrsim 1/2$.

mission rates, we additionally restrict the commission rate to be lower than the threshold value $\hat{\phi}^{NP}_{max}$ (defined in Equation (87)) in order to ensure that a seller that charges collusive prices while the other seller deviates from the collusive agreement sells positive quantities on the platform.[30] The following proposition summarizes the effect of PMFNs on the critical discount factor in the case that the platform charges time-constant and symmetric revenue-sharing commission rate ϕ_A from the sellers.

Proposition 5. *Suppose that $\alpha \in \left(0, \sqrt{3}-1\right)$ and sellers face commission rates of $\phi = (\phi_A, 0)$ with $\phi_A \in \left(0, \hat{\phi}^{NP}_{max}\right)$. Without PMFN, the critical discount factor is*

$$\underline{\delta}^{NP}(\phi) = \frac{(1-\phi_A)(2-\alpha)^2(1-\beta^2) - (1-\alpha)\beta^2\phi_A^2}{(1-\phi_A)(8-8\alpha+\alpha^2)(1-\beta^2) - 2(1-\alpha)\beta^2\phi_A^2}, \tag{30}$$

for both sellers $i \in \{1, 2\}$. The critical discount factor increases in ϕ_A in the relevant parameter range.

Proof. See Appendix A. □

The result of Proposition 5 characterizes the critical discount factor if the platform charges revenue-sharing commission rates and does not impose a PMFN. Clearly, the case of $\phi_A = 0$ is formally equivalent to the case of per-unit commission rate of zero, and, accordingly, the critical discount factor is the equal to $(2-\alpha)^2/(8-8\alpha+\alpha^2)$ as in Proposition 3. In contrast to the case with per-unit commission rates, for $\phi_A > 0$, the critical discount factor positively depends on the revenue-sharing commission rates. This implies that a higher ϕ_A leads to less stable seller collusion.

Quantifying the magnitude of the stabilizing effect of revenue-sharing commission rates, however, reveals that there is only a minimal change in the critical discount factor if ϕ_A increases. It turns out that the difference is largest for intermediate values of interbrand competition α and intrabrand competition β, as well as for small marginal costs. For instance, at $\alpha = \beta = 1/2$ and $c = 0$, the highest admissible commission rate is $\phi_A = 6/10$ and the increase on $\underline{\delta}^{NP}(\phi)$ from $\phi_A = 0$ to $\hat{\phi}^{NP}_{max} = 6/10$ is approximately equal to 0.002, which translates to a relative increase of the critical discount factor of 0.4% only. In Figure 5 in Appendix A, we illustrate for this specification how $\underline{\delta}^{NP}(\phi)$ depends on ϕ_A.[31]

For the case with PMFN, the dependence of the critical discount factor on the commission rate ϕ_A is qualitatively the same as with per-unit commission rates in Proposition 4. In particular, we also find threshold values on the commission rate for which competing $\left(\tilde{\phi}^{P}_{max}\right)$, deviating $\left(\hat{\phi}^{P}_{max}\right)$, and colluding sellers $\left(\bar{\phi}^{P}_{max}\right)$ prefer to be active on both distribution channels, and these threshold values exhibit the same ordering as for the case with per-unit commission rates. The following proposition summarizes the result.

[30] For α close to $\sqrt{3}-1 \approx 0.73$, this restriction implies that the platform may not be able to charge the profit-maximizing commission rate from colluding sellers. Numerical calculations show that the restriction is innocuous for $\alpha \in (0, 0.6)$.

[31] For reference, the profit-maximizing commission rate that the platform charges from colluding sellers in the specification is $\bar{\phi}^{NP}_{A} \approx 0.465$.

Proposition 6. *Suppose sellers face commission rates of $\phi = (\phi_A, 0)$, and the platform imposes a PMFN (P). If $\phi_A \leq \tilde{\phi}^P_{max}$, the critical discount factor is*

$$\underline{\delta}^P(\phi_A) = \frac{(2-\alpha)^2}{8 - 8\alpha + \alpha^2}. \tag{31}$$

At $\phi_A = \tilde{\phi}^P_{max}$, there is a discrete decrease in the critical discount factor. Above this commission rate, the critical discount factor $\underline{\delta}^P(\phi_A)$ increases in $\phi_A \in \left(\tilde{\phi}^P_{max}, \bar{\phi}^P_{max}\right)$, with a kink at $\phi_A = \hat{\phi}^P_{max}$. For sufficiently large ϕ_A in this range, it holds that $\underline{\delta}^P(\phi_A) > \frac{(2-\alpha)^2}{8-8\alpha+\alpha^2}$. Over the complete parameter range, it holds $\tilde{\phi}^P_{max} < \hat{\phi}^P_{max} < \bar{\phi}^P_{max}$.

Proof. See Appendix A. □

The result of Proposition 6 is illustrated in Figure 6 in Appendix A. It highlights that the same pattern as with per-unit commission rates emerges for the case with per-unit commission rates.

5.3 Responsive Commission Rates

Above, we analyze the case of time-constant commission rates. In fact, this pricing behavior appears to employed by most online platforms nowadays, which implies that it is a natural benchmark for the analysis of seller collusion in these settings.[32]

At the same time, a platform typically intermediates transactions for many different product markets with different characteristics (e.g., number of sellers, degree of substitutability, availability of additional distribution channels, etc.). In contrast to the observation that there is little change in commission rates over time, there is evidence that commission rates do differ across product categories or markets. For instance, the referral fee on the Amazon Marketplace ranges from 6 percent (e.g., for personal computers) to 20 percent (e.g., for jewelry).[33] Moreover, there is regional variation in the base commission rates on Booking.com between 10 and 25 percent.[34] It is therefore interesting to understand why there appears to be only limited variation in commission rates over time. In particular, it is clear that a commitment to time-constant commission rates can be costly if market conditions change, and a platform generally can increase its current-period profit by adjusting its commission rate.

In this section, we analyze how the platform affects stability of seller collusion if it conditions its commission rate on the seller conduct in order to also understand the (potentially negative) consequences for the platform to adjust its commission rate. Specifically, if the platform observes a collusive price level in the previous period, we assume that it also expects collusion in the current period and sets the commission rate based on this expectation. If the platform however observes other prices than those associated with the

[32]See also the evidence reported in Section 4.
[33]See the fee schedule for selling on Amazon on sellercentral.amazon.com (last access, May 1, 2020).
[34]See the explanation of Booking.com fees on your.rentals (last access, May 1, 2020).

collusive scheme, it, accordingly, expects competition in the current period and sets the commission rate based on this expectation. This implies that also in a deviation period, the platform charges the commission rate on the expectation of seller collusion.

In this part, we return to the case of per-unit commission rates introduced in the main part of the model in Section 4. Denote the vector of commission rates that the platform charges during collusive and deviation periods as $\bar{w}_A$, and the commission rates during competitive periods as $\tilde{w}_A$. Note that on the equilibrium path of stable seller conduct the commission rate is also time-constant, but what changes is the seller's expectation about adjustments in the commission rate if they were to change their conduct (by agreeing on a collusive agreement or by considering to deviate from collusion). Including these changes in the commission rates in the seller's incentive-compatibility constraint in Equation (19), we obtain

$$\sum_{t=0}^{\infty} \delta^t \bar{\pi}_i \left(\bar{w}_A\right) \geq \hat{\pi}_i \left(\bar{w}_A\right) + \sum_{t=1}^{\infty} \delta^t \tilde{\pi}_i \left(\tilde{w}_A\right). \tag{32}$$

Rearranging yields that with responsive commission rates, the common discount factor needs to exceed

$$\delta \geq \underline{\delta_R} = \frac{\hat{\pi}_i \left(\bar{w}_A\right) - \bar{\pi}_i \left(\bar{w}_A\right)}{\hat{\pi}_i \left(\bar{w}_A\right) - \tilde{\pi}_i \left(\tilde{w}_A\right)} \in [0,1], \tag{33}$$

where $\underline{\delta_R}$ denotes the seller's critical discount factor for collusion on the joint profit-maximizing strategy to be sustainable if the platform conditions its commission rates on the (expected) seller conduct. It is clear that for $\bar{w}_A = \tilde{w}_A$, we are back to the case of time-constant commission rates. The following proposition summarizes the effects on seller collusion if the platform charges different commission rates during collusive and competitive periods.

Proposition 7. *Suppose the platform charges commission rates $\bar{w}_A$ during collusive and deviation periods, and $\tilde{w}_A$ during competitive periods. The sellers' critical discount factor (weakly) decreases in the commission rate $\tilde{w}_A$ and increases in the commission rate $\bar{w}_A$. The result holds with and without PMFN.*

Proof. See Appendix A. □

The result of Proposition 7 highlights an important tension between the platform's responsiveness to changing market conditions and the sellers' propensity to collude. In order to understand this tension, suppose that the platform reacts to a change in seller conduct by adjusting its commission rate in the direction of the level that maximizes its current-period profits. Suppose that a platform generally realizes higher current-period profits after a switch from seller competition to collusion by reducing the commission rate.[35] Crucially, an adjustment in this direction rewards collusion and makes it easier to

[35] Recall that in our setting with linear demand, the optimal per-unit commission rate in the stage game is independent of seller conduct (Proposition 1), whereas the optimal revenue-sharing commission rate is lower if sellers collude than if they compete (Figure 3). More generally, for the case absent a PMFN, we characterize in Appendix B based on a conduct-parameter approach for per-unit commission rates the demand properties for which the platform's commission rate with seller collusion is (weakly) lower than with competition.

sustain for the sellers. At the same time, seller collusion in combination with lower commission rates reduces platform profits. The platform therefore should have an interest not to stabilize seller collusion, which it can achieve by means of time-constant commission rates.

Conversely, with PMFN, we have derived that the optimal commission rate of the stage game is higher with seller collusion than with competition (Proposition 2 and Figure 3), and that this difference in the commission rate can be sufficient for the platform to realizes higher profits with seller collusion than with competition. But if sellers expect an adjustment in this direction, seller collusion is effectively penalized and less stable compared to the benchmark of constant commission rates. A platform, therefore, can have the reversed incentive to stabilize seller collusion, which is also feasible by a commitment to time-constant commission rates. In this case the commitment reduces the temptation to deviate from the collusive agreement in order to receive lower commission rates in future periods. With regard to the e-book antitrust case, this result suggests that the constant commission rate of 30% that Apple charged from e-book publishers may have been helpful in sustaining the price-fixing agreement.

In the off-equilibrium event that seller conduct changes, the commitment to a time-constant commission rate involves different consequences for the platform depending on whether a PMFN is in place or not. Absent a PMFN, all sellers continue to list on both distribution channels but the commission rate may be sub-optimal in order to maximize the profit in any given period.

In contrast, with a PMFN, and if the platform commits to a high constant commission rate at which only colluding sellers are willing to list on the platform, a change in seller conduct from collusion to competition has the consequence that the sellers participation constraint is violated and that they prefer to delist from the platform and only sell to consumers via the direct channel. In a recent empirical study, Hunold et al. (2018) provide evidence that after the abolition of PMFNs in the market for online hotel bookings more hotels are willing to list on the online travel agency Booking.com indicating that the participation constraint of a fraction of the hotels prior to the abolition was violated. This is suggestive evidence for the result that platforms are willing to violate the participation constraint of a set of sellers in order to charge higher commission rates from the sellers on the platform.

5.4 Non-Price Strategies to Prevent Seller Collusion

In this section, we briefly explore non-price strategies that a platform might adopt in order to discourage seller collusion if it has an incentive to do so.

Market Entry into the Seller Market. The stability of a cartel can be threatened if not a all firms in the market participate in the collusive agreement (e.g., de Roos and Smirnov 2019). An outsider that optimally undercuts the collusive prices decreases collusive profits significantly and thereby threatens the stability of collusion. For instance, in the presence of perfectly substitutable products, the presence of a cartel outsider drives collusive profits down to zero, rendering any collusive agreement unstable (Bos, 2009).

In a situation in which a platform finds online sellers colluding and in which this form of conduct is detrimental for the platform's profit, the platform itself could vertically integrate into the seller market and destabilize collusion by pricing aggressively against the cartel.[36] There is a recent investigation by the European Commission, which investigates exactly this business practice employed by Amazon.[37]

The impact of vertical integration on the stability of upstream collusion has been examined in Nocke and White (2007) and Normann (2009) who find that vertical integration has the potential to facilitate upstream collusion. In contrast, a platform that is not part of the collusive agreement may use this strategy in order to destabilize collusion between online sellers. Especially, if the platform offers own products only during collusive periods, it can effectively decrease collusive profits while competitive profits remain unaffected from this strategy. Thereby, a platform can decrease the stability of online seller collusion.

Endogenous Demand Fluctuations. In the spirit of Snyder (1996), the platform can strategically introduce demand fluctuations for the sellers in order to destabilize seller collusion if seller collusion decreases its profits. Contributions such as Rotemberg and Saloner (1986), Haltiwanger and Harrington (1991), and Fabra (2006) show that demand fluctuation can have an important effect on the stability of collusion. However, the exact collusion-destabilizing mechanism depends on the market characteristics such as the nature of demand uncertainty and the presence of capacity constraints. One potential (yet costly) mechanism for the platform is to strategically suspend sellers from the marketplace. Alternatively, Ursu (2018) provides evidence that a platform's ranking has substantial impact on which products are considered by consumers and can therefore also function as a mechanism to introduce these demand fluctuations.

6 Conclusion

The present paper studies the stability of seller collusion in online marketplaces and incorporates a platform's incentives and ability to affect seller conduct in these markets. Importantly, it links the introduction of a PMFN to reduced platform incentives to ensure seller competition on the platform. This offers a novel theory harm linking such clauses to potentially reduced competition on the seller level, and adds to the vivid debate as regards the anticompetitive potential of such contract clauses.

[36] There are obviously other reasons for a platform to enter the seller market (see for instance, Jiang et al. (2011), Muthers and Wismer (2013), and Hagiu and Wright (2015)). Notwithstanding, the impact on seller collusion can serve as an complimentary explanation for the observed behavior that platforms such as Amazon enter certain product markets.

[37] See for instance the article The EU's competition investigation into Amazon Marketplace on Kluwer Competition Law Blog (last access, April 18, 2020).

Absent contractual restrictions, a platform benefits from seller competition as it leads to lower retail prices and generally (weakly) higher commission rates for the platform. If seller collusion occurs nevertheless, the paper discusses non-price related strategies to fight collusion if it occurs against the platform's incentives. We argue that (i) a platform can threaten sellers with suspension from the marketplace, (ii) it may consider to vertically integrate into the seller market (a business practice observed by Amazon) in order to decrease collusive profits, or (iii) to strategically induce demand fluctuations for colluding suppliers in order to make deviation from the collusive agreement more tempting.

In contrast, a PMFN can align the interests of online sellers and platforms as regards the seller conduct, and, therefore, undermines a platform's incentive to organize a competitive marketplace. Recent antitrust cases (discussed in the introduction) suggest that platform providers such as Amazon and Apple at least accepted (or even participated in) price-fixing agreements between sellers on the platform. In line with the incentive to reduce seller competition, we characterize the range of commission rates which allows the platform to profitably stabilize seller collusion in this case.

An important observation from online markets is that there is little variation in commission rates over time on various platforms. In light of different and changing market conditions, which are all covered by the same constant commission rate, this pricing behavior appears to be puzzling from a static perspective. The present paper contributes a dynamic rationale for the observed pricing behavior of platforms by linking adjustments in the commission rates in reaction to changing market conditions to the sellers' propensity to engage in a collusive agreement. Our analysis reveals that a commitment to constant commission rates affects seller conduct in a way that is generally profitable for the platform.

Our results indicate that there are important aspects of the interaction between online sellers and platforms that may be neglected from purely non-cooperative perspective on this business relationships. Future research should continue to analyze how platform governance affects the competitive interaction between online sellers and work to identify situations in which platform have little interest to ensure a competitive environment between online sellers in order to ensure that consumers can reap the full benefits of purchasing goods and services in the digital economy.

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7 Appendix A: Proofs

Proof of Lemma 1. Suppose that platform A charges symmetric commission rates $w_{1A} = w_{2A} = w_A$, and that no PMFN is in place. If sellers set prices non-cooperatively, each seller maximizes its profit function in Equation (2). Recall that there is no commission payment on the direct channel B, and denote the vector of costs of distribution that the sellers face as $w = (w_A, 0)$. The retail prices that seller i charges on the two distribution channels $j \in \{A, B\}$ are

$$\begin{aligned} \tilde{p}_{iA}^{NP}(w) &= \frac{1-\alpha+w_A}{2-\alpha}, \\ \tilde{p}_{iB}^{NP}(w) &= \frac{1-\alpha}{2-\alpha}, \end{aligned} \tag{34}$$

where the super-script NP indicates the platforms do not impose PMFNs and the tilde symbol that sellers set prices non-cooperatively. Each seller $i \in \{1, 2\}$ sets the same retail price on distribution channel $j \in \{A, B\}$ but the retail prices are strictly lower on distribution channel B for $w_A > 0$. Inserting these retail prices in the demand function in Equation (1), yields that the quantity that seller i sells via platform A is

$$D_{iA}\left(\tilde{p}^{NP}(w)\right) = \frac{1-\beta-w_A}{(2-\alpha)(1+\alpha)(1-\beta^2)}, \tag{35}$$

which is positive only if $w < 1 - \beta$. The resulting competitive seller profits are

$$\tilde{\pi}_i^{NP}(w) = \frac{(1-\alpha)\left(2 - 2\beta + w_A^2 - 2(1-\beta)w_A\right)}{(2-\alpha)^2(1+\alpha)(1-\beta^2)}, \tag{36}$$

where $\tilde{\pi}_i^{NP}(w) = \tilde{\pi}_i^{NP}\left(\tilde{p}^{NP}(w)\right)$. Note that the profit $\tilde{\pi}_i^{NP}(w)$ is decreasing in w_A in the relevant range.

If the sellers can form a cartel, they set retail prices in order to maximize their joint profit

$$\pi_1 + \pi_2 = \sum_{i \in \{1,2\}} (p_{iA} - w_A) D_{iA}(p) + p_{iB} D_{iB}(p), \tag{37}$$

and the resulting retail prices are

$$\begin{aligned} \bar{p}_{iA}^{NP}(w) &= \frac{1+w_A}{2}, \\ \bar{p}_{iB}^{NP}(w) &= \frac{1}{2}. \end{aligned} \tag{38}$$

where $\bar{p}$ indicates that sellers collude on retail price. For given costs of distribution w, the retail prices under collusion are larger than the retail prices in the punishment phase: $\bar{p}_{ij}^{NP}(w) > \tilde{p}_{ij}^{NP}(w)$, $\forall \alpha \in (0,1)$. In the case of collusion, the quantity that each seller i sells on platform A is

$$D_{iA}\left(\bar{p}^{NP}\right) = \frac{1-\beta-w_A}{2(1+\alpha)(1-\beta^2)}, \tag{39}$$

which is positive if $w < 1 - \beta$. The resulting profit for seller $i \in \{1,2\}$ is

$$\bar{\pi}_i^{NP}(w) = \frac{2 - 2\beta + w_A^2 - 2(1-\beta)w_A}{4(1+\alpha)(1-\beta^2)}. \tag{40}$$

□

Proof of Proposition 1. Based on the seller behavior in the second stage of the static game (Lemma 1), the platform maximizes its profit in Equation 3 with respect to the per-unit commission rates w_{1A} and w_{2A}. In both cases the optimal commission rate is

$$w_{1A}^{NP} = w_{2A}^{NP} = w_A^{NP} = \frac{1-\beta}{2}. \tag{41}$$

Note that $(1-\beta)/2 < 1 - \beta$ which implies that sellers are willing to accept the platform's contract at this commission rate (Lemma 1). Based on the optimal commission rate w_A^{NP} (Equation 6), the platform realizes a profit of

$$\tilde{\Pi}_A^{NP}\left(w_A^{NP}\right) = \frac{1-\beta}{2(2-\alpha)(1+\alpha)(1+\beta)}, \tag{42}$$

if sellers compete, and it realizes

$$\bar{\Pi}_A^{NP}\left(w_A^{NP}\right) = \frac{1-\beta}{4(1+\alpha)(1+\beta)}, \tag{43}$$

if they form a cartel. Simple calculations reveal that $\tilde{\Pi}_A(w_A) > \bar{\Pi}_A(w_A)$ over the whole parameter space. □

Proof of Lemma 2. Suppose that platform A imposes a PMFN, which implies that a seller cannot charge lower prices on the direct channel than on the platform. In contrast to the case without PMFN, it is necessary to assess under which conditions sellers are willing to list on platform A. Suppose that sellers compete and consider first that both list on both distribution channels. Each seller faces the maximization problem

$$\begin{aligned} \max_{p_i}\ \pi_i(p_i, p_h) &= (p_{iA} - w_{iA}) D_{iA}(p) + p_{iB} D_{iB}(p) \\ s.t.\ & p_{iA} \leq p_{iB}. \end{aligned} \tag{44}$$

Based on the results of Lemma 1, the constraint is binding such that sellers charge the same retail price on both distribution channels if active on the platform. In the case of seller competition, this leads to retail prices of

$$\tilde{p}_i^P(w) = \frac{2-2\alpha+w_A}{4-2\alpha}. \tag{45}$$

This implies that the retail price on distribution channel A (B) is lower (higher) than in the case in which sellers are not restricted in their price setting by a PMFN (see Equation (34)). In this case seller i realizes a profit of

$$\tilde{\pi}_i^P(w) = \frac{(1-\alpha)(2-w_A)^2}{2(2-\alpha)^2(1+\alpha)(1+\beta)}, \tag{46}$$

Alternatively, sellers can decide to list only on the direct channel and maximize the following profit function

$$\pi_i\left(p_{iB}, \infty, \tilde{p}_h^P\right) = p_{iB} D_{iB}\left(p_{iB}, \infty, \tilde{p}_h^P\right), \tag{47}$$

where ∞ indicates that seller i is not active on platform A while the rival seller h is present on both distribution channels and is expected to set $\tilde{p}_h^P$ on both distribution channels. Taking as given that seller h charges the retail prices as specified in Equation (45), seller i maximizes its profit by setting

$$\tilde{p}_{iB}^P(w) = \frac{4-\alpha(4-w_A)}{8-4\alpha}, \tag{48}$$

resulting in a profit of

$$\tilde{\pi}_i^P\left(\tilde{p}_{iB}^P(w),\infty,\tilde{p}_h^P\right)=\frac{(4-\alpha(4-w_A))^2}{16(\alpha-2)^2(1-\alpha^2)}. \tag{49}$$

In order to derive the threshold value $\tilde{w}_{max}$ reported in Lemma 2, we equate the profit from being active on both channels in Equation (45) with the profit from being active on the direct channel in Equation (49),

$$\begin{aligned}
& \tilde{\pi}_i^P(w) && =\pi_i\left(\tilde{p}_{iB}^P(w),\infty,\tilde{p}_i^P(w)\right) \\
\Longleftrightarrow\quad & \tfrac{4(1-\alpha)(2-w_A)}{2(2-\alpha)^2(1+\alpha)(1+\beta)} && =\frac{(4-\alpha(4-w_A))^2}{16(\alpha-2)^2(1-\alpha^2)} \\
\Longleftrightarrow\quad & \tfrac{(1-\alpha)(2-w_A)^2}{4-\alpha(4-w_A)} && =\sqrt{2(1+\beta)}
\end{aligned} \tag{50}$$

The resulting threshold value is

$$\tilde{w}_{max}=\frac{4(1-\alpha)(2-\sigma(\beta))}{4-\alpha(4-\sigma(\beta))}, \tag{51}$$

with $\sigma(\beta)=\sqrt{2(1+\beta)}$. Only for commission rates $w_A\leq\tilde{w}_{max}$, the sellers prefer to list on both distribution channels instead of listing only on the direct channel.

Last, we establish that the equilibrium with seller competition for commission rates $w_A>\tilde{w}_{max}$ involves that both sellers are active on the direct channel. In this case each seller maximizes $\pi_i(p_B,\infty)=p_{iB}D_{iB}(p_B,\infty)$. The resulting retail prices are $\tilde{p}_{iB}^P(w)=(1-\alpha)/(2-\alpha)$ as specified in Equation (4) in Lemma 1. The resulting profit is

$$\begin{aligned}
\tilde{\pi}_{i(B)}^P(w) &= \pi_i\left(\tilde{p}_{iB}^P(w),\infty\right) \\
&= \frac{1-\alpha}{(2-\alpha)^2(1+\alpha)}.
\end{aligned} \tag{52}$$

Note that for $w_A>\tilde{w}_{max}$ it is not profitable for a seller to deviate from this equilibrium and list again on both distribution channels. Last, we note for future reference that for the complete parameter range the profit from being active on both channel $\tilde{\pi}_i^P(w)$ in Equation (46) is larger than the profit on the direct channel only $\tilde{\pi}_{i(B)}^P(w)$ in Equation (52) for $w_A\in(0,2-\sigma(\beta))$, with $\tilde{w}_{max}<2-\sigma(\beta)$. This establishes result. □

Proof of Lemma 3. If sellers collude and are active on both distribution channels, the joint profit maximization of $\pi_{12}=\pi_1+\pi_2$ is

$$\begin{aligned}
\max_p\ \pi_{12}(p) &= \sum_{i\in\{1,2\}}(p_{iA}-w_{iA})D_{iA}(p)+p_{iB}D_{iB}(p) \\
s.t.\ & \ p_{iA}\leq p_{iB}.
\end{aligned} \tag{53}$$

Joint profit maximization on both distribution channels leads to retail prices of

$$\bar{p}_i^P(w) = \frac{2 + w_A}{4}, \tag{54}$$

and a profit for each seller i of

$$\bar{\pi}_i^P(w) = \frac{(2 - w_A)^2}{8(1+\alpha)(1+\beta)}, \tag{55}$$

In contrast, colluding sellers can also decide to only list on the direct channel in order to avoid the contractual restrictions of a PMFN. In this case, they set retail prices in order to maximize their profits on the direct channel

$$\max_{p_B} \; \pi_{12}(p_B, \infty) = \sum_{i \in \{1,2\}} p_{iB} D_{iB}(p_{1B}, \infty, p_{2B}, \infty). \tag{56}$$

The resulting retail prices are the same as the collusive direct channel prices for the case without PMFN as reported in Lemma 1, $\bar{p}_{iB}^P(w) = 1/2$, and the profit in this case for each seller i is

$$\bar{\pi}_i^P\left(\bar{p}_B^P(w), \infty\right) = \frac{1}{4+4\alpha}. \tag{57}$$

Colluding sellers prefer to be active on both distributions channels if the profit in Equation (55) exceeds the profit in Equation (57), which is equivalent to the commission rate w_A being sufficiently small:

$$w_A \;\; \leq \;\; \bar{w}_{max} = 2 - \sqrt{2(1+\beta)} = 2 - \sigma(\beta). \tag{58}$$

This establishes the result. □

Proof of Proposition 2. We show that the platform is restricted by the sellers' participation constraint and cannot charge higher commission rates than the threshold values characterized in Lemma 2 and Lemma 3. Both with seller competition and seller collusion, the unrestricted solution to the platform's maximization yields an optimal commission rate of $w_{1A} = w_{2A} = 1$, which exceeds both threshold values $\tilde{w}_{max}$ and $\bar{w}_{max}$ for the whole parameter range. Due to the fact that the platform's profit increases in the per-unit commission rate up to $w_{1A} = w_{2A} = 1$ given that both sellers are willing to list on the platform, the sellers' participation constraint binds at the optimal commission rate.

Based on the optimal commission rate $\tilde{w}_A^P = \tilde{w}_{max}$ (Equation (10)), the platform realizes a period profit of

$$\tilde{\Pi}_A^P\left(\tilde{w}_A^P\right) = \frac{8(1-\alpha)(2-\sigma(\beta))\,\sigma(\beta)}{(1+\alpha)(1+\beta)(4-\alpha(4-\sigma(\beta)))^2},$$

with seller competition, an based on the commission rate $\bar{w}_A^P = \bar{w}_{max}$ (Equation (14)), the platform realizes a period profit of

$$\bar{\Pi}_A^P\left(\bar{w}_A^P\right)=\frac{(2-\sigma(\beta))\,\sigma(\beta)}{2(1+\alpha)(1+\beta)},$$

with seller collusion. Calculations with *Mathematica* show that $\bar{\Pi}_A\left(\bar{w}^P\right)>\tilde{\Pi}_A^P\left(\tilde{w}^P\right)$ if $\alpha>\bar{\alpha}=(16-8\sigma(\beta))\,/\left(16-8\sigma(\beta)+\sigma(\beta)^2\right)$. □

Proof of Lemma 4. If a seller i deviates from the collusive agreement, it decides (i) whether it prefers to be active on both distribution channels or only the direct channel, and (ii) on retail prices on each active channel that maximize the seller's profits in the current period given the costs of distribution w, and given that the other seller h charges collusive retail prices $\bar{p}_h^{NP}=\left(\bar{p}_{hA}^{NP},\bar{p}_{hB}^{NP}\right)$ as in Equation (38). If the deviating seller decides to be active on both distribution channels, this implies $\hat{p}_i^{NP}=\left(\hat{p}_{iA}^{NP},\hat{p}_{iB}^{NP}\right)\in\arg\max_{p_i}\pi_i\left(p_i;\bar{p}_h^{NP}\right)$. The resulting retail prices of the deviating seller are

$$\begin{aligned}\hat{p}_{iA}^{NP}(w)&=\frac{2-\alpha}{4}+\frac{(2+\alpha)\,w_A}{4},\\ \hat{p}_{iB}^{NP}(w)&=\frac{2-\alpha}{4},\end{aligned}\tag{59}$$

where the hat symbol indicates that seller i deviates from the collusive agreement. The deviating seller i receives a profit of

$$\hat{\pi}_i^{NP}(w)=\frac{(\alpha-2)^2\left(2-2\beta+w_A^2-2(1-\beta)\,w_A\right)}{16\left(1-\alpha^2\right)\left(1-\beta^2\right)},\tag{60}$$

whereas the seller h that does not deviate and sticks to the collusive prices receives

$$\breve{\pi}_h^{NP}(w)=\frac{\left(2-\alpha^2-2\alpha\right)\left(2-2\beta+w_A^2-2(1-\beta)\,w_A\right)}{8\left(1-\alpha^2\right)\left(1-\beta^2\right)},\tag{61}$$

where $\breve{\pi}_h$ indicates that seller h sticks to the collusive retail prices while the other seller i deviates from the collusive agreement.

It is also necessary to verify that the deviating seller does not want to be inactive on the platform. Denote the seller's profits in the case of being active only on the direct channel with $\pi_i\left(p_{iB},\infty;\bar{p}_h^{NP}\right)$, where ∞ indicates that seller i is inactive on channel A. Note that the same direct channel price $\hat{p}_{iB}^{NP}$ as reported in Equation (59) maximizes the profit of the deviating seller in this case. This result shows that the optimal retail price on channel j is independent of whether the seller is also present on the other distribution channel k. The resulting profit in this case is $\hat{\pi}_i^{NP}=\frac{(2-\alpha)^2}{16(1-\alpha^2)}$, which is strictly smaller than the profit from being active on both distribution channels reported in Equation (60) in the relevant parameter range.

Last, it is necessary to verify that the non-deviating seller h sells a positive quantity via the platform. Based on Assumption 1, we restrict the degree of interbrand competition, in order to ensure that the seller h that does not deviate, remains active in the market. In particular, we require the quantity sold by the non-deviating upstream firm (say h) is positive. That is, for w_A in the relevant range it has to hold that

$$\begin{aligned} \check{q}_{hA} = q_{hA}\left(\bar{p}_h^{NP}, \hat{p}_i^{NP}\right) = \frac{\left(\alpha^2 + 2\alpha - 2\right)\left(1 - \beta - w_A\right)}{4\left(1 - \alpha^2\right)\left(1 - \beta^2\right)} > 0 \\ \iff \quad \alpha < \sqrt{3} - 1, \end{aligned} \tag{62}$$

where $\check{q}_{hA} = q_{hA}\left(\bar{p}_h, \hat{p}_i\right)$ indicates the quantity of the non-deviating seller h on platform A. The same inequality holds for distribution channel B. This establishes the result. □

Proof of Proposition 3. Suppose that platform A charges symmetric commission rates w_A that are constant for all periods of the infinitely-repeated game. Consider first the case without PMFN for which we employ the results of Lemma 4. Given that upstream firms sustain collusion by means of grim trigger strategies, and inserting Equations (36), (40), and (60) into the formula for the critical discount factor in Equation 20 yields

$$\underline{\delta}^{NP} = \frac{(2-\alpha)^2}{8 - 8\alpha + \alpha^2}. \tag{63}$$

As reported in Proposition 3, the critical discount factor is independent of the degree of intrabrand competition β and the exact level of the symmetric commission rate w. Moreover, the critical discount factor $\underline{\delta}^{NP}$ is an increasing function of α in the relevant range $\alpha \in \left(0, \sqrt{3} - 1\right)$. □

Proof of Lemma 5. If a seller decides to deviate from the collusive agreement characterized in Lemma 3, it has to decide whether to be active on both distribution channels or on the direct channel only. We consider that the commission rate is sufficiently small that $w_A \leq \bar{w}_{max}$ such that colluding sellers are active on the platform. Consider first the case in which the seller also is active on both channels. Restricted by the PMFN, the deviating seller maximizes

$$\begin{aligned} \max_{p_i} \ \pi_i\left(p_i, \bar{p}_h^P(w)\right) &= \left(p_{iA} - w_{iA}\right) D_{iA}\left(p_i, \bar{p}_h^P(w)\right) + p_{iB} D_{iB}\left(p_i, \bar{p}_h^P(w)\right) \\ & s.t. \quad p_{iA} \leq p_{iB}, \end{aligned} \tag{64}$$

where the rival seller h sticks to the collusive agreement and charges $\bar{p}_h^P(w)$ as specified in Equation (54) on both distribution channels. The seller optimally charges

$$\hat{p}_i^P = \frac{1}{8}\left(4 - 2\alpha + (2+\alpha)\, w_A\right), \tag{65}$$

which results in a profit of

$$\hat{\pi}_i^P(w) = \frac{(2-\alpha)^2 (2 - w_A)^2}{32\left(1-\alpha^2\right)(1+\beta)}. \tag{66}$$

Instead the deviating seller can delist from the platform in order to maximize the profit function $\pi_i\left(p_{iB}, \infty; \bar{p}_h^P(w)\right)$, where ∞ indicates that the seller is not active on the platform. Note that the seller is not restricted by the PMFN in this case and optimally charges $\hat{p}_{iB}(w) = \frac{1}{8}(4-(2-w_A)\alpha)$. Note that this price depends on the commission rate on the platform w_A due to the fact that it affects the other seller's cost of distribution. Accordingly, the optimal deviation price in this case depends positively on the commission rate w_A. The resulting profit is

$$\hat{\pi}_{i(B)}^P(w) = \hat{\pi}_i^P\left(\hat{p}_{iB}(w), \infty; \bar{p}_h^P(w)\right) = \frac{(4-\alpha(2-w_A))^2}{64(1-\alpha^2)}, \tag{67}$$

which is smaller than the profit from being active on both channels in Equation (66) only if the platform's commission rate is sufficiently small. By the same steps as above, the threshold value is

$$w \leq \hat{w}_{max} = \frac{2(2-\alpha)(2-\sigma(\beta))}{4-\alpha(2-\sigma(\beta))}, \tag{68}$$

with $\sigma(\beta) = \sqrt{2(1+\beta)}$. Otherwise, a deviating seller prefers to be present only on the direct channel $\left(\hat{\pi}_{i(B)}^P(w) > \hat{\pi}_i^{NP}(w),\ \forall w_A > \hat{w}_{max}\right)$, as the benefit from charging a more profitable direct channel price outweighs the forgone profit from the lost sales on the platform at high commission rates. Comparing the threshold values given in Equations (51), (58), and (68) yields that $\tilde{w}_{max} \leq \hat{w}_{max} \leq \bar{w}_{max}$ over the complete parameter range. This establishes the result. □

Proof of Proposition 4. For the derivation of the critical discount factor with PMFN, we draw from the results of Lemma 2, 3, and 5. As derived above, it is important to take the sellers participation constraints into account if the platform imposes a PMFN. To this end, we distinguish three cases: First, we consider the case for which the commission rate is sufficiently small such that sellers are active on the platform A in all periods. In particular, this condition is fulfilled for $w_A \leq \tilde{w}_{max}$. In this case, we can insert the equilibrium profits of the stage games in which sellers are active on both channels (Equations (46), (55), and 66) in the formula for the critical discount factor derived in Equation (20). The resulting critical discount factor is

$$\underline{\delta}^P \quad = \quad \frac{\hat{\pi}_i^P - \bar{\pi}_i^P}{\hat{\pi}_i^P - \tilde{\pi}_i^P} = \frac{(2-\alpha)^2}{8-8\alpha+\alpha^2}, \tag{69}$$

as in the case without PMFN (see Equation (22) in Proposition 3).

As derived in Lemma 2, for commission rates $w_A > \tilde{w}_{max}$, competing sellers are only present on the direct channel and realize profits of $\tilde{\pi}_{i(B)}^P(w)$ derived in Equation (52) instead of $\tilde{\pi}_i^P(w)$. Due to the fact that, at $w_A = \tilde{w}_{max}$, $\tilde{\pi}_{i(B)}^P(\tilde{w}_{max})$ is strictly smaller than $\tilde{\pi}_i^P(\tilde{w}_{max})$ in Equation (46), and as the critical discount factor decreases in the punishment profit, there is a discrete decrease in $\underline{\delta}^P$ at $w_A = \tilde{w}_{max}$. For the range

$w_A \in (\tilde{w}_{max}, \hat{w}_{max}]$, the critical discount factor $\underline{\delta}^P$ is

$$\frac{(2-\alpha)^2 \alpha^2 (2-w_A)^2}{4\left(\alpha\left(\alpha\left((2-\alpha)^4 - 8\beta\right) - 16(1-\beta)\right) - 8\beta + 8\right) + (2-\alpha)^4 w_A^2 - 4(2-\alpha)^4 w_A}, \tag{70}$$

which increases in $w_A \in (\tilde{w}_{max}, \hat{w}_{max}]$ for the complete parameter range.

As derived in Lemma 5, a deviating seller is only present on the direct channel for $w_A > \hat{w}_{max}$. Compared to the critical discount factor for low commission rates in Equation (69), the deviation profit is therefore $\hat{\pi}^P_{i(B)}(w)$ in Equation (67) instead of $\hat{\pi}^P_i(w)$ in Equation (66). As $\hat{\pi}^P_{i(B)}(\hat{w}_{max}) = \hat{\pi}^P_i(\hat{w}_{max})$ and as in the range $w_A \in (\hat{w}_{max}, \bar{w}_{max}]$, $\hat{\pi}^P_{i(B)}(w) > \hat{\pi}^P_i(w)$, there is a kink in the critical discount factor $\underline{\delta}^P$ at $w_A = \hat{w}_{max}$, and it increases more strongly in w_A in this range than in $w_A \in (\tilde{w}_{max}, \hat{w}_{max}]$. For the range $w_A \in (\hat{w}_{max}, \bar{w}_{max}]$, the critical discount factor $\underline{\delta}^P$ is

$$\frac{(2-\alpha)^2\left((4-\alpha(2-w_A))^2 - \frac{8(1-\alpha)(2-w_A)^2}{1+\beta}\right)}{\alpha\left(4\alpha(8-(8-\alpha)\alpha) + \alpha(2-\alpha)^2 w_A^2 + 4(2-\alpha)^3 w_A\right)}, \tag{71}$$

which increases in w_A. At $w_A = \bar{w}_{max}$, it holds that the critical discount factor $\underline{\delta}^P$ is strictly larger than $\underline{\delta}^{NP}$. This establishes the result. □

Proof of Proposition 5. As in the case with per-unit commission rates, we first analyze the case of no PMFN and analyze afterwards the case with PMFN. Consider that platform A is a strategic platform setting symmetric commission rates $\phi_{1A} = \phi_{2A} = \phi_A$. Denote the denote the vector of commission rates by $\phi = (\phi_A, 0)$. We restrict platform A's commission rate to

$$\phi_A \in \left[0, \frac{(\alpha(2+\alpha)-2)(1-\beta)(1-c)}{\alpha^2 + 2\alpha + (1-\alpha^2-\alpha)\beta(1-c) - 2}\right], \tag{72}$$

in order to ensure that a seller that charges collusive prices remains active on the platform if the second sellers deviates from the collusive agreement. In this part, we allow for positive marginal costs $c \geq 0$ of the sellers. If a seller is present on both distribution channels, its profit is

$$\pi_i(p) = ((1-\phi_A)p_{iA} - c) D_{iA}(p) + (p_{iB} - c) D_{iB}(p). \tag{73}$$

Absent PMFN, and with seller competition each seller i maximizes the profit in Equation (73) taking as given the commission rates and the rival seller's behavior. We verify below that a seller has no incentive to be active on the direct channel only.

$$\begin{aligned}
\tilde{p}^{NP}_{iA}(\phi) &= \frac{(2-\alpha)\left(1-\beta^2\right)(1-\alpha+c) + (1-\alpha)(1-\beta)\phi_A(\alpha - \beta(1-\alpha+c) - 2)}{(2-\alpha)^2(1-\beta^2) - (1-\alpha)\beta^2\phi_A^2 - (2-\alpha)^2(1-\beta^2)\phi_A}, \\
\tilde{p}^{NP}_{iB}(\phi) &= \frac{(\beta-1)((1-\alpha)(1-\phi_A)(\beta\phi_A - (2-\alpha)(1+\beta)) + c\phi_A(2-\alpha+\beta) - (2-\alpha)(1+\beta)c)}{(2-\alpha)^2(1-\beta^2) - (1-\alpha)\beta^2\phi_A^2 - (2-\alpha)^2(1-\beta^2)\phi_A}.
\end{aligned} \tag{74}$$

Each seller $i \in \{1, 2\}$ sets the same retail price on distribution channel $j \in \{A, B\}$ but the retail prices are strictly lower on distribution channel B for $\phi_A > 0$. The price on the platform $\tilde{p}_{iA}^{NP}(\phi)$ positively depends on the commission rate ϕ_A for $c \geq 0$ and $\alpha, \beta \in (0, 1)$ in the relevant range in which the price is below one. Higher prices cannot constitute an equilibrium as they would induce the sellers not sell via the corresponding distribution channel. The resulting seller profit is

$$\tilde{\pi}_i^{NP}(\phi) = \frac{(1-\alpha)\left({\phi_A}^2(1-\beta+\beta c) - (1-\beta)(3-c)(1-c)\phi_A + 2(1-\beta)(1-c)^2\right)}{(1+\alpha)(2-\alpha)^2(1-\beta^2) - (1-\alpha^2)\beta^2{\phi_A}^2 - (1+\alpha)(2-\alpha)^2(1-\beta^2)\phi_A}. \tag{75}$$

Suppose seller i does not accept the platform's contract offer, while the competing seller h is active on both distribution channels and charges retail prices as specified in Equation (74). In this case, seller i maximizes

$$\max_{p_{iB}} \ \pi_i\left(p_{iB}, \infty, \tilde{p}_h^{NP}(\phi)\right) = (p_{iB} - c)\, D_{iB}\left(p_{iB}, \infty, \tilde{p}_h^{NP}(\phi)\right). \tag{76}$$

The resulting retail price is

$$\begin{aligned} \tilde{p}_{i(B)}^{NP}(\phi) &= \frac{1}{2}(1-\alpha+c) \\ &+ \frac{\alpha(\beta-1)\left((1-\alpha)(1-\phi_A)(\beta\phi_A - (2-\alpha)(1+\beta)) + c\phi_A(2-\alpha+\beta) + (\alpha-2)(1+\beta)c\right)}{2\left((\alpha-1)\beta^2{\phi_A}^2 + (2-\alpha)^2(\beta^2-1)\phi_A + (2-\alpha)^2(1-\beta^2)\right)}, \end{aligned} \tag{77}$$

where $\tilde{p}_{i(B)}^{NP}$ indicates that i is only active on channel B. The resulting profit for seller i is

$$\tilde{\pi}_{i(B)}^{NP}(\phi) = \frac{(1-\alpha)\left(2(2-\alpha)\left(1-\beta^2\right)(1-c) + \beta{\phi_A}^2(\alpha - \beta(1-c)) + (1-\beta)(1-c)\phi_A(\alpha(2+\beta) - 4(1+\beta))\right)^2}{4(1+\alpha)\left((2-\alpha)^2(1-\beta^2)(1-\phi_A) - (1-\alpha)\beta^2{\phi_A}^2\right)^2}, \tag{78}$$

where $\tilde{\pi}_{i(B)}(\phi) = \tilde{\pi}_i^{NP}\left(\tilde{p}_{i(B)}^{NP}(\phi), \infty, \tilde{p}_h(\phi)\right)$. This deviation is not profitable if the profit in Equation (75) exceeds the profit in Equation (78), which is the case if

$$\phi_A \leq \tilde{\phi}_{max}^{NP} = \frac{(2-\alpha)(1-\beta)(1-c)}{2-\alpha-\beta(1-c)}. \tag{79}$$

Note that this restriction on the commission rate is weaker than the one imposed in Equation (72). Hence, competing sellers always prefer to be active on both distribution channels.

With collusion, sellers maximize joint profits $\pi_{12} = \pi_1 + \pi_2$ and optimally set retail prices of

$$\begin{aligned} \bar{p}_{iA}^{NP}(\phi) &= \frac{(1-\beta)\phi_A(2+\beta+\beta c) - 2\left(1-\beta^2\right)(1+c)}{\beta^2(2-\phi_A)^2 - 4(1-\phi_A)}, \\ \bar{p}_{iB}^{NP}(\phi) &= \frac{(1-\beta)\left(\phi_A(2+3\beta+(2+\beta)c) - \beta\phi_A^2 - 2(1+\beta)(1+c)\right)}{\beta^2(2-\phi_A)^2 - 4(1-\phi_A)}, \end{aligned} \tag{80}$$

with collusive profits of

$$\bar{\pi}_i^{NP}(\phi) = \frac{(1-\beta)(3-c)(1-c)\phi_A - \phi_A^2(1-\beta(1-c)) - 2(1-\beta)(1-c)^2}{(1+\alpha)\left(\beta^2(\phi_A-2)^2 + 4(\phi_A-1)\right)}. \tag{81}$$

Alternatively, colluding sellers may decide to list on the direct channel only. In this case they maximize

$$\max_{p_B} \ \pi_{12}\left(p_B, \infty\right) = \sum_{i \in \{1,2\}} \left(p_{iB} - c\right) D_{iB}\left(p_B, \infty\right), \tag{82}$$

with resulting retail prices of $\bar{p}^{NP}_{i(B)}(\phi) = (1+c)/2$ and realized profit of $\bar{\pi}^{NP}_{i(B)}(\phi) = (1-c)^2/(4(1+\alpha))$. Colluding sellers prefer to be present on both distribution channels if

$$\phi_A \leq \bar{\phi}^{NP}_{max} = 2 - \frac{2(1+c)}{2-\beta(1-c)}. \tag{83}$$

Again, this restriction on the commission rate is weaker than the one imposed in Equation (72), and colluding sellers are active on both distribution channels.

Consider that seller i deviates from the collusive agreement, while seller h is present on both distribution channels and charges collusive prices specified in Equation (80). The deviating seller sets retail prices p_i in order to maximize

$$\pi_i\left(p_i, \bar{p}^{NP}_h(\phi)\right) = \left(\left(1-\phi_A\right)p_{iA} - c\right) D_{iA}\left(p_i, \bar{p}^{NP}_h(\phi)\right) + \left(p_{iB} - c\right) D_{iB}\left(p_i, \bar{p}^{NP}_h(\phi)\right). \tag{84}$$

The resulting retail prices are

$$\begin{aligned} \hat{p}^{NP}_{iA}(\phi) &= \frac{\left(1-\beta^2\right)\left(2-\alpha+(2+\alpha)c\right) + (1-\beta)\phi_A\left(2-\alpha+\beta(1-\alpha+c)\right)}{4\left(1-\phi_A\right) - \beta^2\left(2-\phi_A\right)^2}, \\ \hat{p}^{NP}_{iB}(\phi) &= \frac{(1-\beta)\left(1-\phi_A\right)\left((2-\alpha)(1+\beta)+\beta\phi_A\right) + (2+\alpha)\left(1-\beta^2\right)c - (1-\beta)c\phi_A\left(2+\alpha\beta+\alpha+\beta\right)}{4\left(1-\phi_A\right) - \beta^2\left(2-\phi_A\right)^2}, \end{aligned} \tag{85}$$

yielding a deviation profit of

$$\begin{aligned} \hat{\pi}^{NP}_i &= \frac{\left((2-\alpha)^2\left(1-\beta^2\right) - (1-\alpha)\beta^2\phi_A^2 - (2-\alpha)^2\left(1-\beta^2\right)\phi_A\right)}{\left(1-\alpha^2\right)\left(\beta^2\left(2-\phi_A\right)^2 - 4\left(1-\phi_A\right)\right)^2} \\ &\quad \frac{\left(\phi_A^2\left(1-\beta(1-c)\right) - (1-\beta)(3-c)(1-c)\phi_A + 2(1-\beta)(1-c)^2\right)}{\left(1-\alpha^2\right)\left(\beta^2\left(2-\phi_A\right)^2 - 4\left(1-\phi_A\right)\right)^2} \end{aligned} \tag{86}$$

□

The non-deviating seller h that sticks to the collusive agreement sells on the platform the quantity of

$$D_{hA}\left(\bar{p}^{NP}_h(\phi), \hat{p}^{NP}_i(\phi)\right) = \frac{\phi_A\left(\alpha^2+2\alpha+\left(1-\alpha^2-\alpha\right)\beta(1-c)-2\right) - \left(\alpha(2+\alpha)-2\right)(1-\beta)(1-c)}{\left(1-\alpha^2\right)\left(4\left(1-\phi_A\right) - \beta^2\left(2-\phi_A\right)^2\right)},$$

which is larger than zero if Assumption 1 is fulfilled $\left(\alpha < \sqrt{3}-1\right)$ and the commission rate ϕ_A is sufficiently small

$$\phi_A \ \leq \ \hat{\phi}^{NP}_{max} = \frac{\left(\alpha(2+\alpha)-2\right)(1-\beta)(1-c)}{\alpha^2+2\alpha+\left(1-\alpha^2-\alpha\right)\beta(1-c)-2}, \tag{87}$$

which is the restriction on the commission rate imposed in Equation (72). The critical discount factor is

$$\underline{\delta}^{NP}(\phi) = \frac{(1-\phi_A)(2-\alpha)^2(1-\beta^2) - (1-\alpha)\beta^2\phi_A^2}{(1-\phi_A)(8-8\alpha+\alpha^2)(1-\beta^2) - 2(1-\alpha)\beta^2\phi_A^2}. \tag{88}$$

Note that the critical discount factor simplifies to $\underline{\delta}^{NP}(0) = \left((2-\alpha)^2\right)/(8-8\alpha+\alpha^2)$ for $\phi_A = 0$, which is equal to the critical discount factor for the case without PMFN and per-unit commission rates reported in Equation (22) in Proposition 3. Moreover, the critical discount factor in Equation (88) increases in ϕ_A over the relevant range. This implies that an increase in the revenue-sharing commission rate ϕ_A destabilizes seller collusion.

In the following figure, we illustrate however that the increase in $\underline{\delta}^{NP}(\phi)$ is small in our setting. Note that the scaling of the y-axis ranges only from 0.529 to 0.532, and that the critical discount factor only increases only by approximately 0.002 which translates to a relative increase from 0.4% over the whole admissible range of revenue-commission rates ϕ_A.

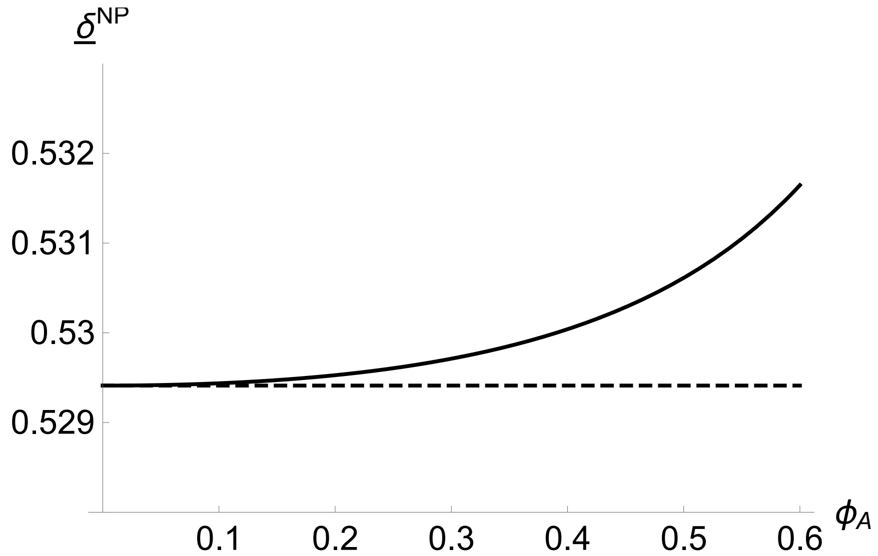

Figure 5: Critical discount factor $\underline{\delta}^{P}(\phi)$.

The figure shows the critical discount $\underline{\delta}^{NP}(\phi_A)$ (solid line) and the critical discount factor for the case with per-unit commission rates and without PMFN (dashed line) depending on the exogenous commission rate ϕ_A for $\alpha = 1/2$, $\beta = 1/2$, and $c = 0$. As specified in Equation (72), the highest admissible commission rate for this specification is $\phi_A = 6/10$

Proof of Proposition 6. If platform A imposes a PMFN, the above described price discrimination between distribution channels is not feasible for the sellers of they are present on both distribution channels. As a result, competing sellers maximize their profit function in Equation (73) subject to the constraint that $p_{iA} \leq p_{iB}$. The retail price by both sellers on both distribution channels is

$$\tilde{p}_i^P(\phi) = \frac{(1-\alpha)(2-\phi_A)+2c}{(2-\alpha)(2-\phi_A)}, \tag{89}$$

and the resulting profit is

$$\tilde{\pi}_i^P(\phi) = \frac{(1-\alpha)(2c+\phi_A-2)^2}{(2-\alpha)^2(1+\alpha)(1+\beta)(2-\phi_A)}. \tag{90}$$

Alternatively, each seller may decide to reject the contract offer of the platform and be present on the direct channel only. As in the case without PMFN in Equation (76), seller i maximizes in this case

$$\max_{p_{iB}} \; \pi_i\left(p_{iB},\infty,\tilde{p}_h^P(\phi)\right) = (p_{iB}-c)\,D_{iB}\left(p_{iB},\infty,\tilde{p}_h^P(\phi)\right), \tag{91}$$

with a resulting retail price on the direct channel of

$$\tilde{p}_{i(B)}^P(\phi) = \frac{2(1-\alpha)(2-\phi_A)+c(4-(2-\alpha)\phi_A)}{2(2-\alpha)(2-\phi_A)}, \tag{92}$$

and profits of

$$\tilde{\pi}_{i(B)}^P\left(\tilde{p}_{i(B)}^P(\phi),\infty,\tilde{p}_h^P(\phi)\right) = \frac{\left(c(4-\alpha(4-\phi_A)-2\phi_A)+2(1-\alpha)(2-\phi_A)\right)^2}{4(2-\alpha)^2(1-\alpha^2)(2-\phi_A)^2}. \tag{93}$$

The equilibrium of the stage involves the two sellers being active on both distribution channels if the profit in Equation (90) exceeds the profit in Equation (93).. Define the threshold commission rate $\tilde{\phi}_{max}^P$ at which sellers are indifferent between being active on both channels and listing on the direct channel only. That is, $\tilde{\pi}_i^P\left(\tilde{\phi}_{max}^P\right) = \tilde{\pi}_{i(B)}^P\left(\tilde{\phi}_{max}^P\right)$. There is no closed-form solution for $\tilde{\phi}_{max}^P$ but we can solve numerically for it. If the commission rate ϕ_A exceeds this threshold value, there is an equilibrium of the stage game in which both sellers are active on the direct channel. In this case they set $\tilde{p}_{iB}^P = (1-\alpha+c)/(2-\alpha)$ and realize an equilibrium profit of

$$\tilde{\pi}_{i(B)}^P\left(\tilde{p}_B^P,\infty\right) = \frac{(1-\alpha)(1-c)^2}{(2-\alpha)^2(1+\alpha)}. \tag{94}$$

Note that as in the case of per-unit commission rates, sellers suffer from a coordination failure and realize lower profits on the direct channel than on both distribution channels even for $\phi_A > \tilde{\phi}_{max}^P$.

Colluding sellers that are present on both distribution channels set optimal retail prices of

$$\bar{p}_i^P(\phi) = \frac{2+2c-\phi_A}{4-2\phi_A}, \tag{95}$$

and realize profits of

$$\bar{\pi}_i^P(\phi) = \frac{(2-2c-\phi_A)^2}{4(1+\alpha)(1+\beta)(2-\phi_A)}. \tag{96}$$

If sellers jointly decide to delist from the platform A, face the same maximization problem as in Equation (82) and set the same retail prices of $\bar{p}_{i(B)}^{NP}(\phi) = (1+c)/2$ in order to realize a profit of $\bar{\pi}_{i(B)}^{NP}(\phi) = (1-c)^2/(4(1+\alpha))$. Colluding sellers prefer to be present on both distribution channels if

$$\phi_A \leq \bar{\phi}^P_{max} = \frac{1}{2}\left(1-c\right)\left(3-\beta+\left(1+\beta\right)c-\sqrt{\left(1+\beta\right)\left(\beta+c\left(6-\beta\left(2-c\right)+c\right)+1\right)}\right). \tag{97}$$

Computations with *Mathematica* reveal that $\bar{\phi}^P_{max} > \tilde{\phi}^P_{max}$ over the complete parameter range. This implies that colluding sellers are willing to list on both distribution channels for higher commission rates ϕ_A than competing sellers. Note that this is the same ordering as for the case with per-unit commission rates.

Consider that seller i deviates from the collusive agreement. If it is active on both distribution channels, it optimally charges

$$\hat{p}^P_i\left(\phi\right) = \frac{1}{4}\left(2-\alpha-\frac{2\left(2+\alpha\right)c}{\phi-2}\right), \tag{98}$$

and realizes a profit of

$$\hat{\pi}^P_i\left(\phi\right) = \frac{\left(2-\alpha\right)^2\left(2c+\phi-2\right)^2}{16\left(1-\alpha^2\right)\left(1+\beta\right)\left(2-\phi_A\right)}. \tag{99}$$

Alternatively, the deviating seller can decide to delist from the platform and only sell via the direct channel B. In this case it optimally charges

$$\hat{p}^P_{i(B)} = \frac{1}{4}\left(2-\alpha+c\left(2+\frac{2\alpha}{2-\phi_A}\right)\right), \tag{100}$$

And realizes a profit of

$$\hat{\pi}^P_{i(B)}\left(\phi\right) = \frac{\left(\left(2-\alpha\right)\left(2-\phi_A\right)-2c\left(2-\alpha-\phi_A\right)\right)^2}{16\left(1-\alpha^2\right)\left(2-\phi_A\right)^2}. \tag{101}$$

Again, there exists a threshold commission rate $\hat{\phi}^P_{max}$ above which a deviating seller prefers to be active on the direct channel only. As in the case with seller competition, there is no closed-form solution for $\hat{\phi}^P_{max}$ but we can characterize it numerically. Simulations over the whole parameter range reveal that the same ordering of threshold values holds as in the case with per-unit commission rates. That is, $\bar{\phi}^P_{max} > \hat{\phi}^P_{max} > \tilde{\phi}^P_{max}$.

Based on the threshold values and the seller profits for the different stage games, we can characterize the critical discount factor for three intervals of commission rates: The first interval is $\phi_A \in \left[0, \tilde{\phi}^P_{max}\right]$, the second one is $\phi_A \in \left(\tilde{\phi}^P_{max}, \hat{\phi}^P_{max}\right]$, and the third interval is $\phi_A \in \left(\hat{\phi}^P_{max}, \bar{\phi}^P_{max}\right]$.

In the first case, sellers are present on both distribution channels independent of seller conduct, and the critical discount factor is

$$\underline{\delta}^P\left(\phi\right) = \frac{\left(2-\alpha\right)^2}{\alpha^2-8\alpha+8},\ \phi_A \in \left[0, \tilde{\phi}^P_{max}\right]. \tag{102}$$

Note that this is the same critical discount factor as with per-unit commission rates for the case that sellers are active on both distribution channels independent of seller conduct.

For the second interval, competing sellers prefer to be active on the direct channel only and realize the profit of $\tilde{\pi}_{i(B)}^{P}$ in Equation (94) instead of $\tilde{\pi}_{i}^{P}(\phi)$ in Equation (90), and the critical discount factor is characterized by

$$\begin{aligned}\underline{\delta}^{P}(\phi) &= \frac{\alpha^{2}(2c+\phi_{A}-2)^{2}}{16(1-\alpha)(1+\alpha)(1+\beta)(2-\phi_{A})} \\ &\cdot \frac{1}{\frac{(2-\alpha)^{2}(2-2c-\phi_{A})^{2}}{16(1-\alpha^{2})(1+\beta)(2-\phi_{A})}-\frac{(1-\alpha)(1-c)^{2}}{(2-\alpha)^{2}(1+\alpha)}},\ \phi_{A}\in\left(\tilde{\phi}_{max}^{P},\hat{\phi}_{max}^{P}\right]\end{aligned} \tag{103}$$

Due to the fact that $\tilde{\pi}_{i(B)}^{P}\left(\tilde{\phi}_{max}^{P}\right) < \tilde{\pi}_{i}^{P}\left(\tilde{\phi}_{max}^{P}\right)$ at the threshold value $\tilde{\phi}_{max}^{P}$, and that the critical discount factor increases in the punishment profit $\tilde{\pi}_{i}$, there is a discrete decrease in $\underline{\delta}^{P}(\phi)$ at $\tilde{\phi}_{max}^{P}$. Recall that this qualitatively the same property derived for the case with per-unit commission rates (see Proposition 4). Moreover, note that the critical discount factor $\underline{\delta}^{P}(\phi)$ increases in ϕ_{A} in the range $\left(\tilde{\phi}_{max}^{P},\hat{\phi}_{max}^{P}\right]$.

In the third interval, not only competing sellers but also a deviating seller decides to be active on the direct channel only. Taking this listing decision into account, the critical discount factor in this range is

$$\underline{\delta}^{P}(\phi)=\frac{\frac{4(\phi_{A}-2)(2c+\phi_{A}-2)^{2}}{(1+\alpha)(1+\beta)}-\frac{((\alpha-2)(\phi_{A}-2)+2c(\alpha+\phi_{A}-2))^{2}}{\alpha^{2}-1}}{16(\phi_{A}-2)^{2}\left(\frac{(\alpha-1)(c-1)^{2}}{(\alpha-2)^{2}(\alpha+1)}-\frac{((\alpha-2)(\phi_{A}-2)+2c(\alpha+\phi_{A}-2))^{2}}{16(\alpha^{2}-1)(\phi_{A}-2)^{2}}\right)},\ \phi_{A}\in\left(\hat{\phi}_{max}^{P},\bar{\phi}_{max}^{P}\right]. \tag{104}$$

This establishes the result.

In the following figure, we illustrate that the effect of revenue-sharing commission rates on the critical discount factor is qualitatively the same as with per-unit commission rates derived in Proposition 4 and depicted in Figure 1.

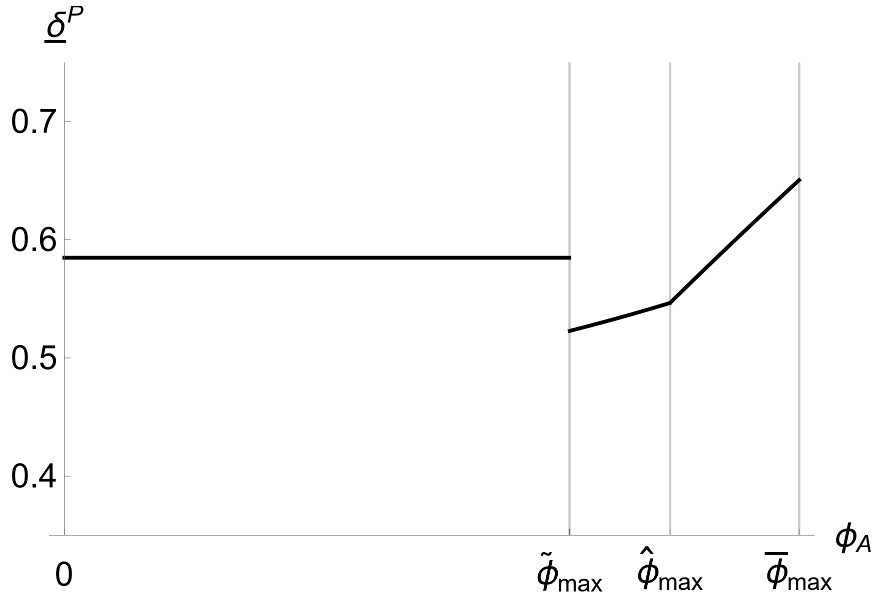

Figure 6: Critical discount factor $\underline{\delta}^P(\phi)$.

The figure shows the critical discount $\underline{\delta}^P(\phi_A)$ depending on the exogenous commission rate ϕ_A for $\alpha = 7/10$, $\beta = 4/10$, and $c = 3/10$.

$\square$

Proof of Proposition 7. The binding incentive-compatibility constraint in Equation (32) at the critical discount factor $\underline{\delta_R}$ can be expressed as

$$\bar{\pi}_i(\bar{w}_A) = \left(1 - \underline{\delta_R}\right)\hat{\pi}_i(\bar{w}_A) + \underline{\delta_R}\tilde{\pi}_i(\tilde{w}_A). \tag{105}$$

The comparative static of the critical discount factor with respect to the commission rate during competitive periods is

$$\frac{d\underline{\delta_R}}{d\tilde{w}_A} = \frac{\underline{\delta_R}\partial\tilde{\pi}_i(\tilde{w}_A)/\partial\tilde{w}_A}{\hat{\pi}_i(\bar{w}_A) + \tilde{\pi}_i(\tilde{w}_A)} \leq 0. \tag{106}$$

The denominator of Equation (106) is positive for all interior values of the critical discount factor $\underline{\delta_R} \in (0,1)$. For $\underline{\delta_R} < 1$ it is a necessary condition that sellers can benefit from collusion and hence it has to hold that $\bar{\pi}_i(\bar{w}_A) > \tilde{\pi}_i(\tilde{w}_A)$. Moreover, the deviation profit of a seller is larger than the collusive profit as otherwise collusion is sustainable for completely myopic sellers with common discount factor $\delta = 0$. Hence, it holds that $\hat{\pi}_i(\bar{w}_A) > \bar{\pi}_i(\bar{w}_A) > \tilde{\pi}_i(\tilde{w}_A)$, which implies that the denominator is positive.

Moreover, in our setting, the competitive profit of seller i decreases in $\tilde{w}_A$ as long as both sellers are active on the platform such that the nominator is (weakly) negative. See Equation (36) for the case without PMFN and Equation (46) for the case with PMFN. With PMFN, competing sellers are active on both distribution channels for $w_A \in [0, \tilde{w}_{max})$. Hence, in this case, the inequality is strict only in this range. Above this threshold value, competing sellers are not active on the platform and their equilibrium profit is independent of w_A (see Equation (52)).

The comparative static of the critical discount factor with respect to the commission rate during collusive periods is

$$\frac{d\underline{\delta_R}}{d\bar{w}_A} = \frac{\left(1-\underline{\delta_R}\right)\partial\hat{\pi}_i\left(\bar{w}_A\right)/\partial\bar{w}_A - \partial\bar{\pi}_i\left(\bar{w}_A\right)/\partial\bar{w}_A}{\hat{\pi}_i\left(\bar{w}_A\right)+\tilde{\pi}_i\left(\tilde{w}_A\right)} > 0. \tag{107}$$

A sufficient statistic for this comparative static to be positive is

$$\partial\hat{\pi}_i\left(\bar{w}_A\right)/\partial\bar{w}_A - \partial\bar{\pi}_i\left(\bar{w}_A\right)/\partial\bar{w}_A > 0. \tag{108}$$

For the case without PMFN, the relevant profit functions are $\bar{\pi}_i^{NP}(w)$ in Equation (40) and $\hat{\pi}_i^{NP}(w)$ in Equation (60). Taking the derivative of both profit functions with respect to w_A yields

$$\frac{\partial\bar{\pi}_i^{NP}(w)}{\partial w_A} = \frac{1-\beta+w_A}{2(1+\alpha)(1-\beta^2)} < \frac{(2-\alpha)^2(1-\beta+w_A)}{8(1-\alpha^2)(1-\beta^2)} = \frac{\partial\hat{\pi}_i^{NP}(w)}{\partial w_A}, \tag{109}$$

for $w < 1-\beta$ and $\alpha,\beta\in(0,1)$.

For the case with PMFN, it is necessary to distinguish between $w_A \leq \hat{w}_{max}$ and $w_A > \hat{w}_{max}$. In the first, case the deviating seller is active on both channels and its deviation profit $\hat{\pi}_i^P(w)$ is specified in Equation (66). In the second case, the deviating seller is only active on the direct channel and realizes the profit $\hat{\pi}_{i(B)}^P(w)$ specified in Equation (67). The collusive profit is in both cases $\bar{\pi}_i^P(w)$ and given in Equation (55). The corresponding derivatives with respect to w_A are

$$\begin{aligned}
\frac{\partial\hat{\pi}_i^P(w)}{\partial w_A} &= -\frac{(2-\alpha)^2(2-w_A)}{16(1-\alpha^2)(1+\beta)} < 0\\
\frac{\partial\hat{\pi}_{i(B)}^P(w)}{\partial w_A} &= \frac{\alpha(4-\alpha(2-w_A))}{32(1-\alpha^2)} > 0.\\
\frac{\partial\bar{\pi}_i^P(w)}{\partial w_A} &= -\frac{2-w_A}{4(1+\alpha)(1+\beta)} < 0.
\end{aligned}$$

In both cases, the sufficient condition in Equation (108) holds. This establishes the result in Equation (107) and completes the proof. □

8 Appendix B: A Conduct-parameter Approach

In this appendix, we characterize the comparative static results of the commission rates with respect to seller conduct in a general symmetric model. We abstract from the asymmetric industry structure and adopt a conduct-parameter approach to exogenously vary the competitiveness of the seller industry and assess its impact on the commission rate that a monopoly platform would charge (Weyl and Fabinger, 2013; Johnson, 2017). Note that we focus here on the characterization of the interior solution of the platform's maximization problem as it is obtained for the case without PMFN.

8.1 Per-unit commission Rate

First, we analyze the case of per-unit commission rates. Let $Q(p)$ denote aggregate and symmetric demand, with the interpretation that an industry of online sellers sells substitutable products to consumers at a symmetric price p. We impose the following assumption in order to ensure existence of a well-behaved equilibrium of the seller market, which is uniquely determined by the first-order condition, and in which the second-order conditions are fulfilled.

Assumption 2. *In the relevant range, the demand $Q(p)$ fulfills*

1. $Q'(p) < 0$, *and*
2. $Q''(p) < 2Q'(p)^2 / Q(p)$.

Note that Assumption 2 is fulfilled for downward-sloping demand that is not too convex. Moreover, we assume that the platform's maximization problem is characterized by the first-order condition and that the second-order condition is fulfilled in the relevant range.

Denote the aggregate profit of the seller industry as

$$\pi(p) \quad = \quad (p - w - c) Q(p), \tag{110}$$

with the derivative of

$$Q(p) + (p - w - c) Q'(p). \tag{111}$$

We assume that the seller equilibrium is defined by the following first-order condition

$$p - w - c = \theta \mu(p), \tag{112}$$

with $\mu(p) = -Q(p)/Q'(p)$, and the conduct parameter $\theta \in [0, 1]$ measuring the competitiveness in the seller industry. We denote the retail price that implicitly solves Equation (112) as $p(w, \theta)$. For $\theta = 0$, the seller margin is equal to zero as it is in the case of perfect competition and for $\theta = 1$, Equation (112) corresponds to the optimality condition of a monopolistic seller.

The platform charges a symmetric per-unit commission rate w for every transaction on the platform realizing a profit of

$$\Pi = w \cdot Q(p(w, \theta)). \tag{113}$$

We are interested in the comparative static characterizing the change in the profit-maximizing commission rate $w(\theta)$ if the competitiveness of the seller industry as measured by the conduct parameter θ changes. A sufficient condition for the result of Proposition 1 that a platforms prefers seller competition over collusion is that the commission rate weakly decreases in the conduct parameter θ, that is $\partial w(\theta)/\partial\theta \leq 0$. This is the analogous result to our finding that the platform prefers to charge weakly lower commission rate from colluding sellers than it does from competing ones. We characterize this comparative static depending on the primitives of the demand function $Q(p)$ in

Proposition 8. *The platform charges weakly lower commission rates w from a less competitive seller industry (higher θ) if demand $Q(p)$, fulfills Assumption 2, and the third derivative $Q'''(p)$ is not too small (i.e., $Q'''(p) \geq \frac{2Q(p)Q''(p)^2 - Q'(p)^2 Q''(p)}{Q(p)Q'(p)}$).*

The result of Proposition 8 shows that the equilibrium commission rate weakly decreases if the seller industry becomes less competitive for a broad range of demand functions. For linear demand with $Q''(p) = Q'''(p) = 0$, the condition provided in Proposition 8 holds with equality which implies that the commission rate is invariant to a change in θ ($\partial w(\theta)/\partial\theta = 0$). This finding is consistent with the result reported in Proposition 1 that shows that a platform charging per-unit commission rates imposes the same commission rate from competing sellers than it does from colluding ones. Moreover, it is instructive to consider that the demand function is completely characterized by the first- and second-order derivative (with $Q'''(p) = 0$). In this case, the condition in Proposition 8 simplifies to $Q''(p) \leq Q'(p)^2/2Q(p)$ implying that the comparative static holds for all concave demand functions, and as long as the demand function is not too convex.

Proof of Proposition 8. We will derive the equilibrium retail price $p(w,\theta)$ (and its derivatives), and the equilibrium commission rate $w(\theta)$ in order to derive the comparative static $\partial w(\theta)/\partial\theta$.

The second-order derivative of the seller profit, which is negative by assumption, is

$$\frac{\partial^2 \pi}{\partial^2 p} = (1+\theta) Q'(p) + (p-w) Q''(p) < 0, \tag{114}$$

which can be rearranged to

$$\frac{\partial^2 \pi}{\partial^2 p} = Q'(p)(1 - \theta\mu'(p)) < 0, \tag{115}$$

with

$$\mu'(p) = -1 + \frac{Q(p) Q''(p)}{Q'(p)^2}. \tag{116}$$

Note that Assumption 2 implies $\mu'(p) < 1$. For future reference, note that total differentiation of (112) yields the pass-through of the wholesale price to the retail price, which is

$$\frac{\partial p(w,\theta)}{\partial w} = \rho = \frac{1}{1-\theta\mu'(p)} > 0. \tag{117}$$

The pass-through is larger than zero due to the stability condition $\theta\mu'(p) < 1$. Moreover, the derivative of the pass-through of ρ with respect to w is

$$\frac{\partial^2 p(w,\theta)}{\partial^2 w} = \rho' = \frac{\theta\mu''(p)\rho}{(1-\theta\mu'(p))^2} = \theta\mu''(p)\cdot\rho^3, \tag{118}$$

with

$$\mu''(p) = \frac{Q(p)\,Q'(p)\,Q'''(p) - 2Q(p)\,Q''(p)^2 + Q'(p)^2\,Q''(p)}{Q'(p)^3}. \tag{119}$$

Moreover, the change in the retail price for a change in the conduct parameter θ can be expressed as

$$\frac{dp(w,\theta)}{d\theta} = \sigma = \frac{\mu(p)}{1-\theta\mu'(p)} > 0. \tag{120}$$

We are now equipped to analyze the platform's maximization problem. The platform profit is

$$\Pi \;=\; w\cdot Q(p(w,\theta)),$$

with first-order condition

$$\frac{\partial\Pi}{\partial w} = Q(p(w,\theta)) + w\cdot Q'(p(w,\theta))\cdot\rho = 0. \tag{121}$$

Note that we can express the equilibrium commission rate w as

$$\begin{aligned} w &= -\frac{Q(p(w,\theta))}{Q'(p(w,\theta))}\cdot\frac{1}{\rho} \\ &= \mu(p(w,\theta))\,(1-\theta\mu'(p(w,\theta))) \end{aligned} \tag{122}$$

By total differentiating Equation (122), we can assess how the equilibrium commission rate w reacts to a change in the competitiveness of the seller industry as measured in θ. Recall that the industry is less competitive the higher the conduct parameter θ is. The comparative statics result $\partial w/\partial\theta$ is

$$\frac{\mu'(p(w,\theta))\cdot\sigma\cdot(1-\theta\mu'(p(w,\theta))) - \mu(p(w,\theta))\,(\mu'(p(w,\theta)) + \theta\cdot\mu''(p(w,\theta))\cdot\sigma)}{1-(\mu'(p(w,\theta))\cdot\rho\cdot(1-\theta\mu'(p(w,\theta))) - \theta\cdot\mu(p(w,\theta))\cdot\mu''(p(w,\theta))\cdot\rho)}. \tag{123}$$

We first show that the denominator of Equation (123) is positive due to the second-order condition of the platform's maximization problem. Therefore, the sign of $\partial w/\partial\theta$ is entirely determined by sign of the nominator, which depends on the demand properties.

As a preliminary step to determine the sign of the denominator, note that $\rho\cdot(1-\theta\mu'(p(w,\theta))) = 1$ (recall from Eq. (117) that $\rho = 1/(1-\theta\mu'(p)))$ such that we can simplify the denominator in (123) to

$$1-\mu'(p(w)) + \theta\mu(p(w))\,\mu''(p(w))\,\rho. \tag{124}$$

The second-order derivative of the platform's profits function is (we suppress here for brevity the dependence of the retail price on θ)

$$\frac{\partial^2\Pi}{\partial^2 w} \;=\; 2Q'(p(w))\cdot\rho + w\left(Q''(p(w))\cdot\rho^2 + Q'(p(w))\,\rho'\right) < 0, \tag{125}$$

which is smaller than zero due to the stability condition of the platform's maximization problem. Note that we can rearrange Equation (125) to

$$\begin{aligned} Q'(p(w)) \cdot \rho\left[1-\mu'(p(w))+\theta\mu(p(w))\mu''(p(w))\rho'\right] &< 0 \\ 1-\mu'(p(w))+\theta\mu(p(w))\mu''(p(w))\rho &> 0, \end{aligned} \tag{126}$$

where we use that

- $w=-\frac{Q(p)}{Q'(p)}\cdot\frac{1}{\rho}$ (from Equations (123), and (117)),
- $\mu'(p)=-1+\frac{Q(p)Q''(p)}{Q'(p)^2}$ (from Equation (116)), and
- $\rho'/\rho=\theta\cdot\mu''(p)\cdot\rho$ (from Equations (117) and 118).

Note that the stability condition is fulfilled if the term in Equation (126) is positive (as $Q'(p(w))\cdot\rho<0$). Hence, the denominator of Equation (123) is positive, and the sign of $\partial w/\partial\theta$ is the same as the sign of the nominator in Equation (123).

The nominator can be simplified to

$$-\theta\cdot\mu(p(w,\theta))\cdot\mu''(p(w,\theta))\cdot\sigma, \tag{127}$$

where we use that $\sigma\cdot(1-\theta\mu'(p))=\mu(p)$ from Equation (120). In particular, the equilibrium commission rate w decreases in θ (when the seller industry becomes less competitive if the term in Equation (127) is positive. Note that the sign therefore is entirely determined by $\mu''(p)$, as $\theta,\mu(p),\sigma\geq 0$. If $\mu''(p)>0$, this condition is fulfilled. Recall from Equation that

$$\mu''(p)=\frac{Q(p)Q'(p)Q'''(p)-2Q(p)Q''(p)^2+Q'(p)^2Q''(p)}{Q'(p)^3}.$$

Hence $\mu''(p)\geq 0$, if

$$\begin{aligned} \frac{Q(p)Q'(p)Q'''(p)-2Q(p)Q''(p)^2+Q'(p)^2Q''(p)}{Q'(p)^3} &\geq 0 \\ Q(p)Q'(p)Q'''(p)-2Q(p)Q''(p)^2+Q'(p)^2Q''(p) &\leq 0 \\ \frac{2Q(p)Q''(p)^2-Q'(p)^2Q''(p)}{Q(p)Q'(p)} &\geq Q'''(p), \end{aligned} \tag{128}$$

which is the condition provided in Proposition 8. This establishes the result. □