Socially Responsible Finance: How to Optimize Impact?

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May 31, 2021

Abstract

We consider a general equilibrium productive economy with negative externalities. Entrepreneurs maximize profits, and investors seek to maximize their pecuniary and nonpecuniary returns. We analyze how in equilibrium, the size and investment policy of a socially responsible fund (SRF) vary with investors' preferences, production technologies and capital market frictions. If investors care about impact, the SRF should prioritize investments in companies with acute negative externalities and facing strong capital market friction. The SRF can amplify its impact by imposing restrictions on the suppliers used by the firms it finances. This lowers emissions even in industries that are not directly financed by the SRF. The nonpecuniary benefits of investors improve welfare when they take the form of sensitivity to impact but can deteriorate welfare when take the form of value alignment.

JEL:G11, G23, M14, O44, Q51

Keywords: Impact investing, ESG funds, emission externalities, carbon footprint.

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1 Introduction

Negative externalities generated by corporations, such as pollution, are a central theme in current policy debates. The traditional economic prescription to solve such externalities is regulation; Through the use of Pigouvian taxes or tradable pollution permits ("cap-and-trade"), governments could in theory influence the decisions of firms, thereby forcing them to internalize externalities (Weitzman (1974); Cropper and Oates (1992)).

Due to political economy constraints, this approach has sometimes delivered disappointing results. Considering the example of carbon emissions, free-riding among countries, political short-termism, and lobbying frictions have strongly inhibited the regulatory response to climate change (see, e.g., Tirole (2012)).

An alternative channel to curb firms' behaviors is the financing channel; the participation of socially responsible investors in financial markets might relax financial constraint and/or decrease the cost of capital for companies that act responsibly, hence providing incentives to behave better. Increasing numbers of investors do actually use sustainability criteria in their investment policy; according to the *The Forum for Sustainable and Responsible Investment*, as of year-end 2020, approximately 33% of U.S. professionally managed assets can be categorized as "socially responsible". Broadly speaking, one can identify two types of nonpecuniary benefits that can lead an investor to prefer investing via a socially responsible intermediary. The first is a *nonconsequentialist* view that consists of a preference for financing responsible firms, regardless of whether this has an impact or not on the negative externality level in the economy. Such investors seek to invest in companies that are aligned with their own moral values. The second is a *consequentialist* approach that aims at having a real impact in the economy by reducing negative externalities, compared the counterfactual where only financial performance is taken into account.

This paper explores how the nature of nonpecuniary preferences (consequentialist or not) shape the investment strategies of financial intermediaries and whether this can have an effect on firms' negative externalities and social welfare in equilibrium. In particular, we aim at answering the following questions. In an economy where investors delegate their investments to intermediaries, how and to what extent can a socially responsible intermediary induce firms to reduce their negative externalities. How should the portfolio composition and firm selection criteria of such an intermediary change depending on the nature of the investors' nonpecuniary preferences? What will eventually be the effect on social welfare?

The answers are not obvious for two main reasons. The first reason is capital substitution: If companies that are not compliant with the wishes of responsible investors can simply seek capital elsewhere, the socially responsible intermediary cannot influence the company's behavior or their externalities. The second reason is that, if responsible investors are reluctant to accept lower financial returns, a socially responsible intermediary might struggle to attract capital, particularly if selecting companies on socially responsible criteria requires a monitoring cost that is not faced by the standard financial intermediaries.

To answer these questions, we model a two-sector competitive general equilibrium economy where the two constraints mentioned above are considered. There is a continuum of atomistic entrepreneurs and investors. Investors invest their capital via financial intermediaries, which we refer to as funds.[1] Investors can invest via profit-maximizing funds and a socially responsible financial ("SRF" hereafter) intermediary. Entrepreneurs raise capital to produce, and they can choose the amount of pollution involved in their production process. Pollution increases production levels at no direct cost to the polluting firms. However, the aggregate level of pollution affects individual welfare negatively. Investors differ in the degree to which they are willing to accept lower returns in exchange for social performance. If the SRF financial return is expected to be too small given the level of social performance it can deliver, no capital will be managed by the SRF. An additional challenge for the SRF is that companies can raise capital from nonresponsible investors, implying that if capital markets are frictionless, it is impossible for the SRF to influence companies' behaviors. Hence, we introduce the following friction in the capital market: for an entrepreneur who chooses not to comply with SRF standards, the expected cost of capital is increasing with the fraction of funds controlled by the SRF. Whereas for tractability, we model this as resulting from

[1] The word funds here is meant in the broadest sense; it includes all types of financial intermediaries that manage clients' savings or financial wealth, such as, for example, mutual funds, sovereign funds, venture investors, private equity, banks, etc.

a search friction a la Duffie et al. (2005), it is worth noting that this key feature is not specific to a search approach.[2]

We define the SRF's policy by (1) its capital allocation across sectors and (2) the pollution requirements it imposes on companies if they decide to accept its capital. These pollution requirements include caps on the company's direct pollution that the company emits to produce and/or caps on indirect emissions, which are the direct pollution of the company's suppliers (upstream indirect emission) and the direct pollution of the company's industrial customers (downstream indirect emissions).

We consider the following two alternative assumptions for investors' social preferences: (i) investors are consequentialist and care about the SRF's impact, defined as the increase in social welfare that the SRF induces in the economy, and alternatively, (ii) investors are nonconsequentialist and measure social performance with the SRF's portfolio emission footprint, i.e., the smaller the emissions of portfolio companies, the better the social performance.

We first show that an SRF willing to maximize its size delivers social performance that reflects the investors' social preferences. If the investors are consequentialist, the SRF aims at maximizing its impact, i.e., social welfare. If the investors are nonconsequentialist, the SRF seeks to minimize its portfolio's footprint. Second, we consider the size of the SRF as given and describe the optimal policies the SRF should adopt to cater to the investors' preferences. We show that the SRF has to choose between two broad strategies to curb firms' emissions. One strategy is to focus its capital in one industry and require the firms it finances to cap their direct and/or indirect emissions. The other strategy involves investing in both industries and requiring caps on the direct emissions of each industry. Finally, modelling investors' decisions, we determine the equilibrium level of the SRF's size as a function of the distribution of investor preferences and of the additional monitoring cost incurred by the SRF.

We find several results that have concrete normative implications for the sustainable finance industry. We show that if the SRF defines its strategy as only a cross-sector capital allocation

[2]Notably, it also emerges in the seminal paper by Heinkel et al. (2001), where this same effect is due to the lack of risk sharing among nonresponsible investors who have to absorb ownership of a company without adequate responsible capital.

strategy, it has no effect at all on firms' emissions. To induce firms to reduce their emissions, the SRF must impose some binding pollution caps on the firms it finances. The tightness of the pollution caps that the SRF can impose on a given sector increases with the fraction of the sector's capitalization that is under SRF control and with the level of capital market friction to which the sector is subject.

We show that the strategy of the SRF depends on the fraction of the capital in the economy that it controls and on the nature of investors' social preferences. A large enough SRF seeking to maximize impact can achieve the first best level of social welfare by investing in both industries and requiring firms to cap their direct emission to the socially optimal level. For a smaller SRF, it is optimal to apply a pecking order. A small enough ESG capital should prioritize investment in sectors where capital search friction is particularly acute (the 'friction industry' hereafter). Then, depending on whether social welfare would increase more by reducing that sector's emissions or the other sector's emissions, it should require the firms it finances to cap their direct emissions or their indirect (upstream) emissions, respectively. The presence of upstream emission caps in one industry induces a fraction of the other industry to reduce its emissions to cater to the SRF financed firms. They do not do this to ease their access to capital, but rather because, in equilibrium, the same good trades at a higher price if produced with lower emissions. For an intermediate sizes SRF, the impact maximizing strategy involves either focusing its capital on the 'friction industry' and imposing both direct and upstream indirect emissions caps or investment in both industries, where each industry is required to only cap their direct emissions. The former strategy is more impactful than the latter when the industry not financed by the SRF produces a good that is mostly used as input by the SRF financed industry rather than as a consumption good. In such a case, a larger fraction of the upstream industry can be made to switch to cleaner production, despite not receiving SRF capital. We also show that, if enforceable, caps on downstream emission can be extremely effective even for a small sizes SRFs. By requiring that the firm in industry i not sell to 'dirty' firms, the SRF can force the whole downstream industry to comply to any emissions level imposed with the downstream emissions cap on industry i. Overall, we show that if investors are consequentialist, the larger the size of the SRF is, the larger the equilibrium level of social welfare.

However, if investors are nonconsequentialist and care instead about the emissions footprint of the fund, the SRF seeks to minimize its portfolio footprint. Importantly, despite a particularly low emission footprint, the presence of the SRF can have a detrimental effect on social welfare; i.e., a large enough SRF will impose emission levels that are below socially optimal levels, leading to welfare falling below its laissez-faire level.

We then study how the SRF's size is determined in equilibrium by studying investors' choices. There are two crucial elements; due to monitoring costs, the SRF offers smaller financial returns than profit maximizing funds. We model heterogeneity in the intensity of social preferences among investors and show that the total mass of capital invested in the SRF increases the social performance that the SRF is expected to deliver. When the monitoring cost is large enough, no capital is invested via the SRF. If the monitoring cost is not too large relative to the social performance sensitivity, then an equilibrium with a positive size for the SRF emerges, and it is the only stable equilibrium. If the monitoring cost is small enough, this equilibrium leads to the first best social welfare.

Literature Review. Our paper is related to different strands of the literature.

On the theory side, several papers model the implications of the existence of socially responsible investors. For instance, Heinkel et al. (2001) develop a model where a fraction of investors boycott firms that are not clean. "Dirty" companies trade at a discount compared to their "clean" peers, because in equilibrium, their shareholders (i.e., those that have no moral concerns) are more concentrated in "dirty" companies. Pedersen et al. (2020) and Pástor et al. (2020) introduce nonpecuniary benefits from holding green assets[3]. They show that a green portfolio's performance depends on shifts in investors' tastes for green financial products. Green investing has a positive social impact because it decreases the cost of capital of greener firms. In our paper, there is no uncertainty, which shuts down the diversification channel explored by Heinkel et al. (2001), Pedersen et al. (2020) and Pástor et al. (2020), and we consider both the case of consequentialist investors who directly value impact in their utility, as well the case of investors valuing stock characteristics, such as low emissions footrpint. Morgan and Tumlinson (2019) develop a theory where firms internalize externalities in that they solve a free-rider problem experienced in the production of a public good by

[3]See also Fama and French (2007) for a model allowing investors to have nonfinancial preferences over the characteristics of stocks they hold

maximizing shareholder welfare. Chowdhry et al. (2014) studies optimal contracting in the presence of externalities, when some investors are willing to pay for public goods, providing a foundation for impact securities. In the same spirit, Oehmke and Opp (2019) offer a theory of responsible investing where a moral hazard problem creates financial constraints that interact with externalities. By internalizing social externalities, responsible investors facilitate the scaling of virtuous projects, and they are complementary to regular financiers. Our model emphasizes general equilibrium forces and a search friction that endows investors with some bargaining power. By allowing for intermediary goods, we also show how the supply chain can be harnessed to optimize impact. Green and Roth (2020) show that socially responsible investors seeking impact should not invest in profitable projects that would find funding anyway. They draw, as we do, a distinction between preferences for impact vs. preferences for value alignment, but their focus is on positive externalities while our focus is how to curb the negative externalities that firms would produce under laissez-faire. Hart and Zingales (2017) and Broccardo et al. (2020) explore the corporate governance implications of having investors with social preferences; stock price maximization is not the optimal managerial objective in such a context, and they find that engagement (through voting rights) is more effective than divesting stocks to make firms internalize negative externalities. This is because even small pro-social preferences are sufficient to make investors act pro-socially.

On the empirical side, several papers explore the performance and preferences of socially responsible investors. On performance, the evidence is quite mixed. Hong and Kacperczyk (2009) and El Ghoul et al. (2011) document that "sin stocks" have positive abnormal returns, which suggests that their cost of capital is higher. Bolton and Kacperczyk (2019) also find that the stocks of companies with higher CO2 emission intensity levels earn higher returns. Barber et al. (2018) finds that impact investing private equity earns lower returns; Zerbib (2019) and Baker et al. (2018) find that green bonds are issued at a premium (controlling for risk), hence delivering lower returns. However, there is also evidence in the opposite direction, arguing that a company's social performance positively predicts its stock-returns. A possible explanation is the market under-reaction to ESG information. For example, Edmans (2011) documents that firms that treat their employees well have positive abnormal returns. Derwall et al. (2005) find that more socially responsible portfolios provide higher average returns. Gibson and Krueger (2018) and Henke (2016) find a link between a portfolio sustainability footprint and its performance in the equity and bond markets, respectively. Andersson et al. (2016) report over-performance of decarbonized stock indices and predict that such green indices will further out-perform in the future; they argue that the market fails to fully recognize the impact of future restrictions on CO2 emissions[4]. In a broad meta-analysis of the empirical literature on responsible investing, Margolis et al. (2007) concludes that there is an ambiguous correlation between social responsibility and financial returns.

Regarding the motivations of socially responsible investors, Krueger et al. (2018) use a large-scale survey of institutional investors and find that they believe that screening companies based on environmental information can enhance risk-adjusted returns because equity valuations do not fully reflect climate risks. Hartzmark and Sussman (2018) reports a causal link between the flows into mutual funds and the publication of their sustainability ratings. Riedl and Smeets (2017) collect survey data and find that moral preferences are important factors for decisions by this type of investor.

In the following, Section 2 describes our analytical framework. Section 3 compares the laissez-faire equilibrium with the social optimum. Section 4 analyzes the mechanism through which SRF can induce firms to reduce their emission. In Section 5, we describe the optimal portfolio and policy when the fund focuses on reducing the emissions solely of the firms it finances. In Section 6, we show how the SRF can use indirect emission caps to exploit the supply chain to curb emissions in the firms it does not finance. In Section 7, we determine the equilibrium levels of the SRF size. Section 8 discusses some of the assumption of the model. Section 9 concludes. The Appendix includes all proofs, the details of the downstream emission caps' performance, and the derivation of the search model equilibrium.

[4]This view is congruent with that of central bankers such as Matt Carney who have repeatedly warned that climate risks are not fully reflected in asset valuations yet.

2 Model

We consider a competitive general equilibrium economy where agents are atomistic and enjoy consumption but suffer from the toxic emissions generated by the production of goods. There are 2 goods, $i = 1, 2$. Each good can be consumed or used as an input to produce the other good. Each good's industry consists of a continuum of competitive firms (with endogenous mass). Producing a good uses as input the other good, capital, and toxic emissions. The population of agents is composed of a mass 1 of entrepreneurs and a mass 1 of investors. Each entrepreneur has the skill to run a company but has no capital. Each investor is endowed with one unit of capital but lacks the skill to run a company. Investors invest their capital via financial intermediaries, which we call funds. One of the funds is a socially responsible fund (SRF henceforth). The matching between entrepreneurs and capital is subject to friction.

Technology. Let the firms of industry i be indexed by $f \in [0, K_i]$, where K_i is the (endogenous) capitalization of industry i. Each firm requires one unit of capital. The quantity $y_{i,f}$ of good i produced by a single firm f from the unit of capital depends on the firm's input quantity $x_{j,f} \geq 0$ of good $j \neq i$ and the level $e_{i,f} \in [\varepsilon, 1]$ of direct toxic emissions the firm releases during production, where $\varepsilon > 0$ and is arbitrary small. Namely, as follows:

$$y_{i,f} = e_{i,f} x_{j,f}^{\alpha_{ij}} \tag{1}$$

where $\alpha_{ij} \in (0, 1)$. The industry's aggregate emission is $E_i = \int_0^{K_i} e_{i,f} df$. In addition to its direct emissions level $e_{i,f}$, a firm's economic activity is associated with two other emissions: the direct emission level of firm f's suppliers of good $j \neq i$, and the direct emission level of the firm's customers who might use good i as an input. We call the former firm f's *upstream emission*, denoted $e_{Ui,f}$, and the latter, firm f's *downstream emission*, denoted $e_{Di,f}$.

Consumption Preferences. Individuals derive utility from the consumption of both goods but suffer from the aggregate amount of emissions in the economy. Namely, given aggregate emission

E_1 and E_2, an individual utility from a consumption plan (c_1, c_2) is as follows:

$$u(c_1, c_2, E_1, E_2) = \frac{c_1^{\gamma_1} c_2^{\gamma_2}}{(1+E_1)^{\delta_1}(1+E_2)^{\delta_2}} \tag{2}$$

where $\gamma_1 + \gamma_2 = 1$, and δ_i, $i = 1, 2$, measures the disutility due to industry i's emissions.[5] For this reason, we will say that the *green industry* is the one with the smallest δ_i.

Prosocial Preferences: consequentialism vs. value alignment. We assume that on top of the utility from consumption and aggregate emissions, expressed in (2), some investors may additionally experience utility from investing via a socially responsible fund. We model such a subjective reward as a warm-glow, in the spirit of Andreoni (1990). Formally, we denote with μ an investor's sensitivity to responsible investing. By investing a fraction q of her capital into a socially responsible fund whose social performance level is A, a type μ investor sees its utility in (2) increased by the following:

$$\mu q A \tag{3}$$

We assume that investors are heterogeneous in the intensity of their taste for social performance.[6] Specifically, we assume that μ is uniformly distributed on $[0, \overline{\mu}]$. This simple functional form for investors' nonpecuniary benefits captures two sensible trade-offs. First, an investor might be willing to invest in a fund providing a less than competitive return, to the extent that this fund generates an adequate social performance. Second, the warm-glow she enjoys is increasing with (1) the amount she invests in the SRF and (2) the level of its social performance.

We consider two distinct specifications for how investors measure the fund's social performance, which are both present in the literature. First, we consider *consequentialist* investors, for whom social performance is defined by the *impact* I that the presence of the SRF has on social welfare. That is, I is the difference between the utility resulting from (2) in the presence vs. in the absence

[5] We focus on this specific form of utility functions because it dramatically improves the model tractability. However, qualitatively, the main prediction would not change as long as the individual utility function is increasing in consumption and decreasing in aggregate emissions.

[6] In other words, the SRF's social performance might be considered as adequate by some investors but not enough by others.

of the socially responsible fund. Such consequentialist preferences are in line with Hart and Zingales (2017), Broccardo et al. (2020) and Oehmke and Opp (2019), which all assume that investors care about their impact.[7]

The second specification assumes that investors are nonconsequentialist and are seeking *value alignment*; i.e., they avoid holding companies that do not act in line with their own values. This concern for value alignment is how, e.g., Heinkel et al. (2001) Pedersen et al. (2020) and Pástor et al. (2020) model investors' social preferences. In the context of our model, it means that investors identify the SRF's socially responsible performance with how low its portfolio emissions footprint is. Formally, we can define the emissions footprint of a portfolio with industry weights (ω_1, ω_2) as follows:

$$F := \delta_1 \omega_1 e_1 + \delta_2 \omega_2 e_2 \tag{4}$$

where e_i is the average emission of industry i's firms financed via the SRF. The definition of the toxic footprint can be understood by going back to the utility function defined earlier.[8] A formal definition of A for consequentialist and nonconsequentialist investors is provided in Definition 2 at the end of this section.

Goods markets. Goods are exchanged in competitive markets. Namely, we assume that if, within an industry i, a strictly positive mass of firms choose the same level $e \in [0, 1]$ of direct emission, these firms will sell their outputs in a dedicated competitive market at a price that we denote $p_i(e)$. We shall refer to good i produced by a firm whose direct emission level is e as *good i of type e.* Let $\mathcal{E}_i$ be the endogenous set of type-dependent markets for good i. We say that the market of good i is segmented whenever $\mathcal{E}_i$ is not a singleton. In a segmented market, a good's type is verified before entering a specific market. A firm f in industry i whose direct emissions $e_{fi} \notin \mathcal{E}_i$ has no dedicated market for its output, we assume that such a firm can sell its production only if the market of good i is not segmented, as good i's type is not verified in this case.

[7]Such a definition of impact investment as having a causal effect is often referred to as "additionality". See, e.g., Brest and Born (2013) : "An impact investor seeks to produce beneficial social outcomes that would not occur but for his investment in a social enterprise. [...] Having impact implies causation, and therefore depends on the idea of the counterfactual."

[8]Fix the consumption level; the log utility is up to a constant $\delta_1 ln(1 + K_1 e_1) + \delta_2 ln(1 + K_1 e_1)$; the marginal impact on this quantity of a portfolio dk allocated with weights (ω_1, ω_2) is up to a scaling factor $(\sum \delta_i \omega_i e_i) dK$.

Socially responsible policy and compliance conditions. We assume that each investor can invest via the following two mutual funds: a standard fund, which is purely interested in financial performance, and a socially responsible fund (the "SRF" hereafter), whose investment strategy takes into account social performance. The SRF can commit to invest only in firms that comply with its policy, which consists of maximal direct and indirect emissions thresholds $\hat{e} = \{\hat{e}_i, \hat{e}_{Ui}, \hat{e}_{Di}\}_{i=1,2}$. Only firms whose emissions do not exceed these thresholds are eligible to be financed by the SRF fund. We say that an entrepreneur in industry i *complies* with the SRF requirements only if her firm's direct and indirect emissions do not exceed the caps $(\hat{e}_i, \hat{e}_{Ui}, \hat{e}_{Di})$ set by the SRF. We focus on SRF policies that are internally consistent, meaning that a firm in industry i that complies is able to sell its output and purchase its input to and from compliant firms in industry j. Formally,

Definition 1 *a policy* $\hat{e} = \{\hat{e}_i, \hat{e}_{Ui}, \hat{e}_{Di}\}_{i=1,2}$ *is* internally consistent *if* $\hat{e}_i \leq \hat{e}_{Uj}, \hat{e}_{Dj}$*, for all* $i = 1, 2$ *and all* $j \neq i$.

Compared to the standard fund, the SRF faces the additional cost of, first, selecting entrepreneurs willing to comply with the SRF emission thresholds, and then, verifying that the firms receiving SRF capital actually comply. Formally, the SRF has to sustain a cost $\psi \geq 0$ for each unit of capital it allocates and monitors. This cost is deduced from the financial return the SRF pays to its investors.

SRF strategy. The strategy of the SRF is a mapping from the SRF's capital under management S into the following: i) a capital allocation (S_1, S_2) into industry 1 and 2, and ii) a socially responsible policy $\hat{e}$.

Sequence of play. The following actions unfold sequentially :[9]

1. The SRF announces its strategy.
2. Each investor chooses how to allocate their capital among the funds.

[9]This timing of actions is given for expositional clarity. Because this is a single period general equilibrium economy where production and consumption are simultaneous, strictly speaking, the interaction between agents is modeled as a simultaneous move game, where all agents correctly anticipate the other agents' strategies.

3. Each entrepreneur chooses irreversibly the good i that she wants to produce and a technology that determines her firm's emissions.
4. Entrepreneurs search for capital.
5. Production occurs and output is sold. Profits are split between entrepreneurs and investors; i.e., an (exogenous) fraction λ of profits is paid to the entrepreneur and the rest is paid to the investors who financed the firm.
6. Individuals spend their revenue to consume.

Capital market friction. We assume that that the allocation of capital to entrepreneurs may be subject to frictions. What we want to capture is that the SRF's commitment to finance only compliant firms and the other fund's indifference to firms emissions have the following two consequences: (i) it is easier and/or faster and/or cheaper for an entrepreneur to find capital if her firm complies than if it does not, and (ii), the larger the fraction of an industry's total capital under the control of the SRF, the more difficult it is for a noncompliant entrepreneur in this industry to be financed. For concreteness and tractability, in what follows, we obtain proprieties (i) and (ii) through a simple search model; however, the key qualitative results of our paper would extend to any other form of friction generating (i) and (ii).

Let K_i denote the (endogenous) aggregate amount of capital invested in industry i, and let S_i be the amount of capital that the SRF invests into industry i. We define $s_i := \frac{S_i}{K_i}$, the resulting fraction of industry i capital that comes from the SRF. The probability of being financed for an entrepreneur in industry i depends on s_i and on the entrepreneur's decision to comply or not. Namely, whereas a compliant entrepreneur is financed no matter whether he is matched with the SRF or another fund, a noncompliant entrepreneur cannot be financed with SRF capital. In the appendix, we describe a standard search game that leads to the following equilibrium prior probability for a noncompliant entrepreneur in industry i to be financed:

$$\Phi_i(s_i) := \max\left\{\frac{1-s_i}{1-\eta_i s_i}, 0\right\}$$

Here, $\eta_i \in [0,1]$ is an industry specific parameter measuring the fluidity of the capital-entrepreneur matching market. The probability $\Phi_i(s_i)$ decreases with s_i, reflecting the fact that it becomes more difficult for a noncompliant entrepreneur in industry i to find financing if a larger fraction of the pool of capital dedicated to this industry is socially responsible. It is important to note that $\Phi_i(s_i)$ spans two intuitive polar cases, as follows: for $\eta_i = 1$, $\Phi_i(s_i)$ is 1, which means that the matching market is frictionless, and for $\eta_i = 0$, one has $\Phi_i(s_i) = 1 - s_i$, which is the fraction of non-socially responsible capital invested in industry i.[10] The intensity of the matching friction is measured by $1 - \eta_i \in [0, 1]$. Hence, $\eta_i < \eta_j$ means that the capital matching friction is more severe in industry i than in industry j. In this case, we say that industry i is the *friction industry*.

Having this timing in mind, we can solve the model by backward induction.

Consumption choices. Consider an individual whose revenue is w and who can shop for good i in markets $\mathcal{E}_i$. Her consumption choice solves the following:

$$\max_{\{c_i, e_i \in \mathcal{E}_i\}_{i=1,2}} \quad \frac{c_1^{\gamma_1} c_2^{\gamma_2}}{(1+E_1)^{\delta_1}(1+E_2)^{\delta_2}} \tag{5}$$

$$s.t. \quad p_1(e_1)c_1 + p_2(e_2)c_2 \leq w \tag{6}$$

Note that, since they are atomistic, agents take aggregate emissions (E_1, E_2) as exogenously given. Additionally, they will purchase the good i in the market where it is the cheapest, that is at price

$$p_i := \min_{e_i \in \mathcal{E}_i} p_i(e_i).$$

Taking the first order condition, the individual's demand for good i is as follows:

$$c_i = \frac{\gamma_i w}{p_i}, \tag{7}$$

which brings to her a level of utility

$$u^*(w, p_1, p_2, E_1, E_2) = W(p_1, p_2, E_1, E_2)w, \tag{8}$$

[10]Hence, $\eta_i = 0$ can be interpreted as a matching technology where the entrepreneur has a unique random draw from the pool of investors to find a match.

where

$$W(p_1, p_2, E_1, E_2) := \frac{\left(\frac{\gamma_1}{p_1}\right)^{\gamma_1}\left(\frac{\gamma_2}{p_2}\right)^{\gamma_2}}{(1+E_1)^{\delta_1}(1+E_2)^{\delta_2}}. \tag{9}$$

is the level of indirect utility per unit of wealth as a function of the lowest price of each good and the aggregate emissions in each industry. Note that indirect utility u^* is linearly increasing in the individual's wealth w.

Production choices. Consider a firm in industry i with a technology inducing emissions $e_{i,f} = e \in [0,1]$ purchasing its input from a supplier whose direct emissions are $e_j \in \mathcal{E}_j$ and who is selling its output in the market for price $p(e_i)$, where $e_i \in \mathcal{E}_i$. Then the firm's demand for good j solves the following:

$$\text{argmax}_{x_j} \qquad p_i(e_i)y_i - p_j(e_j)x_j \tag{10}$$

$$s.t. \qquad y_i = e x_j^{\alpha_{ij}} \tag{11}$$

The first order condition requires the following:

$$x_j = \frac{\alpha_{ij} p_i(e_i) y_i}{p_j(e_j)} \tag{12}$$

Thus, firm f's demand of good j of type e_j is as follows:

$$x_j(e, p_i(e_i), p_j(e_j)) = \left(\frac{\alpha_{ij} p_i(e_i) e}{p_j(e_j)}\right)^{\frac{1}{1-\alpha_{ij}}} \tag{13}$$

Additionally, its supply of good i of type e_i is as follows:

$$y_j(e, p_i(e_i), p_j(e_j)) = e^{\frac{1}{1-\alpha_{ij}}} \left(\frac{\alpha_{ij} p_i(e_i)}{p_j(e_j)}\right)^{\frac{\alpha_{ij}}{1-\alpha_{ij}}}$$

and leads to the firm's profit, as follows:

$$\pi_i(e, p_i(e_i), p_j(e_j)) = p_i(e_i) y_i (1-\alpha_{ij}) = \left(p_i(e_i) e \left(\frac{\alpha_{ij}}{p_j(e_j)}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}} (1-\alpha_{ij}) \tag{14}$$

which is increasing with the level of the firms' direct emissions e.[11]

Entrepreneur's choice: sector and technology. An entrepreneur has to choose ex-ante (before raising capital and producing) her firm's sector i and whether she will run a business that is eligible to receive SRF capital or not.[12] The entrepreneur spends her revenue to consume. From expression (8), the level of utility she will achieve from consumption is linear in her revenue. Thus, an entrepreneur chooses the sector and emissions of her firm to maximize her expected revenue. The entrepreneur's revenue equals an (exogenous) fraction λ of the firm's profit. Let π_{iC} and π_{iN} be her firm's profit if she chooses to comply or not, respectively. Then, an entrepreneur in industry i complies only if

$$\pi_{iC} \geq \pi_{iN}\Phi_i(s_i) \tag{15}$$

This inequality trades off between (1) the fact that, conditional on being financed, profits are larger when the firms have no constraint on emissions, i.e. $\pi_{iN} \geq \pi_{iC}$, and (2) the fact that finding financing is less likely if the firm does not comply.

Investors' portfolio choice. Consider now an investor whose sensitivity for social investing is μ and has to choose how to allocate his unit of capital among the two funds. As each investor is atomistic, he takes prices, aggregate level of emissions, and the SRF's social performance A as exogenous. If r denotes the return on the standard fund, r_G denotes the SRF's gross return, and $q \in [0,1]$ denotes the SRF's weight in the investor's portfolio, the investor's wealth is $w = q(r_G - \psi) + (1-q)r$. Then, q solves the following:

$$\max_q \underbrace{(q(r_G - \psi) + (1-q)r)}_{w} W(p_1, p_2, E_1, E_2) + q\mu A \tag{16}$$

From the fact that the objective function in (16) is linear in q, it follows that the optimal q is 1 if the following is true:

[11]Note: if the market for good i is segmented, then $e = e_i$, whereas e can differ from e_i if the market for good i is not segmented.

[12]The idea here is that when an entrepreneur meets capital providers, she presents all the characteristic of the firm she would like to be financed, i.e., the firm's output and production technology.

$$\mu \geq \frac{r + \psi - r_F}{A} W(p_1, p_2, E_1, E_2), \tag{17}$$

and it is 0 otherwise. The r.h.s. of (17) defines a cutoff separating the population of investors into two groups. Those whose sensitivity μ to social investing is above this cutoff will invest all their capital in the SRF, whereas the others will invest all their capital in the regular fund. This endogenously defines the aggregate amount of capital S that is invested in the SRF as a function of goods prices, aggregate emissions and social performance. Considering that we assumed that μ is uniformly distributed on $[0, \overline{\mu}]$, we have the following:

$$S = \max\left\{1 - \frac{r + \psi - r_G}{\overline{\mu} A} W(p_1, p_2, E_1, E_2), 0\right\} \tag{18}$$

This equation determines the supply of social capital S, which depends positively on the anticipated social performance of the SRF as a fraction of total welfare. We can now formally define the SRF's social performance A.

Definition 2 *For* consequentialist *investors, the SRF's social performance A is measured by the SRF's impact, defined as the difference between social welfare in the presence of an SRF and in the absence of an SRF. Let W_{SR} and $\underline{W}$ be the equilibrium levels of $W(p_1, p_2, E_1, E_2)$ in the presence of the SRF and in the absence of SRF, respectively. Then, we have the following:*

$$A = W_{SR} - \underline{W} \tag{19}$$

For a nonconsequentialist *investor who cares about value alignment, the SRF's social performance A, is measured by how low its emissions footprint is vis-a-vis social welfare, as follows:*

$$A = \frac{\overline{F} - F}{W_{SR}} \tag{20}$$

where $\overline{F}$ is the emissions footprint, defined in Equation (4), of the market portfolio in the absence of the SRF.

From these definitions and equation (18), it immediately follows that

Lemma 1 *If investors are consequentialist, then to maximize its size, the SRF has to act to maximize its impact.*

If investors are nonconsequentialist and care about value alignment, then to maximize its size, the SRF has to act to minimize its portfolio's footprint.

We can now formally define the equilibrium.

Definition 3 *A competitive equilibrium is characterized as follows:*

- *For each good $i \in \{1, 2\}$, a nonempty set of competitive markets $\mathcal{E}_i$. For each market $e_i \in \mathcal{E}_i$ and $e_j \in \mathcal{E}_j$, a mass $\theta(e_i, e_j) \geq 0$ of firms supplying good i of type e_i using as input good j of type e_j, such that*

  $$\sum_{e_j \in \mathcal{E}_j} \theta(e_i, e_j) > 0.$$

  The aggregate supply of good i of type e_i is

  $$Y_i(e_i) := \sum_{e_j \in \mathcal{E}_j} \theta(e_i, e_j) y_i(e_i, p_i(e_i), p_j(e_j)),$$

  the aggregate demand of good i of type e_i coming from industry j firms is

  $$X_i(e_i) := \sum_{e_j \in \mathcal{E}_j} \theta(e_j, e_i) x_i(e_j, p_j(e_j), p_i(e_i)),$$

  the aggregate demand of good i of type e_i coming from consumers is

  $$C_i(e_i) = \overline{w} \frac{\gamma_i}{p_i(e_i)} 1_{\{e_i = \arg\min_{e_i \in \mathcal{E}_i} p_i(e_i)\}},$$

  where $\overline{w}$ is individuals' aggregate wealth.

  Equilibrium price $p_i(e_i)$ is such that market clears:

  $$C_i(e_i) + X_i(e_i) - Y_i(e_i) = 0$$

- *Each entrepreneur correctly anticipates $\mathcal{E}_1, \mathcal{E}_2$, the equilibrium prices of goods, as well as the matching probabilities $\Phi_1(s_1), \Phi_2(s_2)$, and chooses her firm's industry and direct and indirect emissions such as to maximize her expected utility.*

- *Investors correctly anticipate the funds equilibrium returns, goods equilibrium prices, and the level of social performance that the SRF will achieve and choose their portfolio such as to maximize their utility.*

- *The SRF choses its policy and portfolio to maximize its size S, correctly anticipating how its choice affects the $\mathcal{E}_1, \mathcal{E}_2$ equilibrium prices, and its social performance.*

The equilibrium is said to be symmetric *if firms in the same industry choose the same level of direct emissions.*

3 First-best and laissez faire

Let's consider the problem of a social planner willing to maximize social welfare in the absence of the SRF. As the only externality is due to firms' emissions, the social optimum can be achieved by imposing precise emission caps on the firms in each industry and then leaving individuals optimize their choices on all other dimensions (investment, consumption and production). Then we have the following:

Proposition 1 *Assume the level of direct emissions of firms in industry $i \in \{1,2\}$ are exogenously set at e_i, i.e., $\mathcal{E}_i = \{e_i\}$, and individuals optimize their investment, consumption and production choices. Then, we have the following:*

1. *The total sales revenue of industry i is :*

$$Z_i := \frac{\gamma_i + \alpha_{ji}\gamma_j}{1 - \alpha_{ij}\alpha_{ji}} \tag{21}$$

2. *The capitalization of industry i is :*

$$K_i := Z_i(1 - \alpha_{ij}). \tag{22}$$

3. *The return on capital equals $r = 1 - \lambda$, no matter the firm in which the capital is invested.*
4. *All firms realize the same level of profits $\pi_i = 1$, $i = 1, 2$. Thus, entrepreneurs are indifferent between producing in industry 1 or 2.*
5. *Individual revenues are $1 - \lambda$ for an investor and λ for an entrepreneur.*
6. *The equilibrium level of social welfare is as follows:*

$$U(e_1, e_2) := C \frac{e_1^{Z_1} e_2^{Z_2}}{(1 + K_1 e_1)^{\delta_1} (1 + K_2 e_2)^{\delta_2}} \tag{23}$$

   where C is a strictly positive constant.

7. *In laissez faire equilibrium, we have $e_1 = e_2 = 1$; thus, social welfare is as follows:*

$$\underline{W} = U(1, 1)$$

   and the market portfolio's emission footprint is as follows:

$$\overline{F} = K_1 \delta_1 + K_2 \delta_2$$

The proposition shows that no matter the choice of e_1 and e_2, the equilibrium on goods and capital has three remarkable properties. First, the equilibrium composition of the market portfolio, and hence the size K_i of each industry $i = 1, 2$, only depends on the consumers' preferences for the two goods (γ_1 and γ_2) and the goods productivity as intermediary goods (α_{12} and α_{21}). This comes from the fact that the firms' expected profits across industries have to be identical in equilibrium. Second, we can identify social welfare as function of the average levels of emissions in each industry. Last, in equilibrium, the mass of entrepreneurs choosing industry i, K_i, does not depend on the emissions caps e_1, e_2. As we show in the next sections these three properties carry over to all equilibria we consider where the SRF is active.

In what follows we assume the following:

Assumption 1 $\frac{(1+K_i)}{1-\alpha_{ij}} < \delta_i < \frac{\varepsilon K_i + 1}{\varepsilon(1-\alpha_{ij})}$ for $i = 1, 2$.

As illustrated by the next lemma, Assumption 1 implies that the socially optimal level for a firm's emissions is in the interior of $[\varepsilon, 1]$.

Lemma 2 *To achieve the social optimum, one should set e_1, e_2 as to maximize $U(e_1, e_2)$, which leads to the following socially optimal level of emission of each firm in industry i:*

$$e_i^* = \frac{1}{(\delta_i - Z_i)(1 - \alpha_{ij})} \in (\varepsilon, 1) \tag{24}$$

The socially optimal level of emission e_i^* results from the tradeoff between the discomfort of emissions on consumer's utility, measured by δ_i, and the productive advantage of emission for good i. The latter increases with with α_{ij} and α_{ji}, the production elasticity in the input output matrix, and with γ_i, the utility elasticity from consuming good i. In laissez-faire regime $e_1 = e_2 = 1$, which is suboptimal because it is assumed that $\delta_i > \frac{(1+K_i)}{1-\alpha_{ij}}$.

If $e_i^* < e_j^*$, then the emissions in industry i must be more polluting, i.e., $\delta_i > \delta_j$, or less desirable for consumption and production, i.e., $\gamma_i < \gamma_j$ and/or $\alpha_{ji} < \alpha_{ij}$. Thus, industry i is the industry in which it is socially optimal to reduce emissions the most. We will refer to this industry as the *critical industry.*

4 How can the socially responsible fund affect firms' emissions?

First, let us consider S, the total capital invested via the SRF, as given. How can the SRF employ S to influence the firms' emission choices? Broadly speaking, the SRF has the following two instruments: its portfolio choice and the emissions policy. We first show that the two instruments work hand in hand, that is, an emissions policy can affect an industry's emissions only if the SRF invests enough capital in that industry. Symmetrically invested capital in a given industry affects the industry emissions only if coupled with an adequate emissions policy. Namely, suppose that the SRF emission policy for industry i is $(\hat{e}_i, \hat{e}_{Ui}, \hat{e}_{Di})$ and that there exist goods markets where compliant firms can operate, that is $\hat{e}_i \in \mathcal{E}_i$, and $\hat{e}_{Ui}, \hat{e}_{Di} \in \mathcal{E}_j$. Then a compliant firm's profit in industry i equals $\pi_i(\hat{e}_i, p_i(\hat{e}_i), p_j(\hat{e}_{Ui}))$. By contrast, the profit of a noncompliant firm cannot exceed the one obtained by maximizing its direct emissions, selling its output in the market where good i is the most expensive and purchasing its input in the market where good j is the cheapest. That is, a noncompliant firm's profit in industry i cannot exceed the following

$$\pi_{iN} := \max_{e_i \in \mathcal{E}_i, e_j \in \mathcal{E}_j} \pi_i(1, p_i(e_i), p_j(e_j))$$

Considering that an entrepreneur's indirect utility is linear in its revenue, which equals a fraction λ of his firm profit, if the following is true:

$$\pi_i(\hat{e}_i, p_i(\hat{e}_i), p_j(\hat{e}_{U,i})) \geq \Phi_i(s_i)\pi_{iN}, \tag{25}$$

then an entrepreneur in industry i will run a compliant firm.

When choosing her firm's technology, an entrepreneur will trade off the extra profit she can gain by not complying with the reduction in the probability $\Phi_i(s_i)$ of being financed if she chooses not to comply. Note that because $\Phi_i(s_i)$ is decreasing in s_i and increasing in η_i, the lager the fraction s_i of industry i's capital under the SRF's control, and the larger the capital matching friction $1 - \eta_i$, the tighter are the restrictions that the SRF can effectively impose on industry i's emissions. Note also that if the SRF does not impose restrictions on the emissions of the firms it finances, the firms maximize their direct emissions regardless of the weight s_i that the SRF has in that industry. Hence, we have the following

Lemma 3 *1. The laissez faire equilibrium emerges in the following situations:*

(a) In the absence of SRF emission policy $\hat{e}$.

(b) When S_1, S_2 and the emission policy are such that (25) is not satisfied.

2. If the SRF sets S_i and $\hat{e}$ such that inequality (25) is satisfied, then it is optimal for all firms in industry i to comply.

Result 1.a of Lemma 3 states that mere portfolio tilting has no effect on emissions choices. This is because by merely tilting its portfolio toward, for example, the greener industry, the SRF would induce an opposite flow of capital from non-SRF funds. However, Result 2.b states that if the required emissions caps are too tight considering the SRF invested capital and/or the capital friction, the SRF will have no effect. If, instead, the SRF emissions caps and the portfolio choice are well calibrated to an industry's capital and frictions, then each firm in that industry has the proper incentives to comply no matter whether the firm is actually financed by the SRF or the other fund (Result 2). This is because firms choose technologies ex-ante, anticipating the equilibrium probability of being financed by the SRF.

Result 2 of Lemma 3 has an important implication for the relation between the nature of the emissions policy and the diversification of the SRF portfolio. Note that if the SRF has a fully diversified portfolio and in each industry, the emission policies are such that condition (25) is satisfied, then because of Lemma 3 all firms in industry 1 and 2 comply and set their direct emissions rates at $\hat{e}_1$ and $\hat{e}_2$, respectively. Because the policy is internally consistent, $\hat{e}_{Ui}, \hat{e}_{Di} \geq \hat{e}_j$ and, thus, the caps on indirect emissions are redundant. That is, by complying with its direct emission cap $\hat{e}_j$, a firm in industry j knows that its direct emissions level is also low enough to be able to sell its output to compliant firms in industry j, and it has no choice but to purchase its input from compliant firms in industry i. By contrast, when the SRF portfolio is not diversified, requiring caps on indirect emissions can affect emissions in the industry that do not receive SRF capital.

For this reason, in what follows, it is sufficient to focus our analysis on the following two cases: first, the case where the SRF invests in both industries but only imposes direct emission caps, and second, the case where the SRF imposes direct and/or indirect emissions caps but focuses all its capital in one industry.

5 Effect of a direct emissions policy

What level of emission can be induced by the SRF when first, investing S_1 and S_2 in industries 1 and 2, respectively, and second, adopting an emission policy that only restrict direct emission?

Proposition 2 *For any emissions policy that focuses on direct emission caps* $(\hat{e}_1, \hat{e}_2)$*, if the SRF* *invests in industry i an amount S_i such that*

$$S_i \geq S_i(\hat{e}_i) := \frac{1 - \hat{e}_i^{\frac{1}{1-\alpha_{ij}}}}{1 - \eta_i \hat{e}_i^{\frac{1}{1-\alpha_{ij}}}} K_i \tag{26}$$

(where K_i is given by Equation (22)), then each firm in industry i complies, i.e., $\mathcal{E}_i = \{\hat{e}_i\}$, $i = 1, 2$. The equilibrium is symmetric and such that

1. *The return on capital equals $r = r_G = 1 - \lambda$.*
2. *Individual revenues are $1 - \lambda$ for an investor investing via the standard funds, $1 - \lambda - \psi$ for an investor investing via the SRF fund, and λ for an entrepreneur.*
3. *The equilibrium level of utility of an individual with revenue w is equal to $U(\hat{e}_1, \hat{e}_2)w$, where $U(\cdot)$ is as defined in (23).*
4. *The SRF's emission footprint is as follows:*

$$F = \frac{S_1(\hat{e}_i)}{S}\hat{e}_1 + \frac{S_2(\hat{e}_2)}{S}\hat{e}_2$$

Note that the equilibrium level of K_i turns out to be the same as in Proposition 1. The reason why the SRF cannot affect each sector's level of capitalization is the following: in equilibrium, entrepreneurs have to be indifferent between working in both industries. This pins down the gross rate of return of capital in each sector, leaving no leeway to the SRF to modify it.

5.0.1 Maximizing impact

If facing consequentialist investors, in order to maximize its size, the SRF has to maximize its impact. That is, an SRF managing an amount of capital S solves the following:

$$\begin{aligned} \max_{e_1, e_2} \quad & U(e_1, e_2) - U(1, 1) \\ s.t. \quad & S_1(e_1) + S_2(e_2) \leq S \end{aligned}$$

where we used Proposition 1 to replace $\underline{W}$ with $U(1, 1)$. This makes apparent that there is a tradeoff between limiting emissions in one industry versus the other. The tradeoff comes from the fact that to impose lower emissions to industry i, the SRF needs to increase the capital it allocates to that industry at the expenses of industry j, reducing its grip on industry j's emissions. Thus, given S, to maximize impact by focusing on direct emission policies, the SRF should adopt the following strategy.

Proposition 3 *Let S be the size of the fund and $S^\star := S_1(e_1^*) + S_2(e_2^*)$. Consider SRF's policies that aim to maximize impact by only constraining the firms' direct emissions. There is $\underline{S}, \in (0, S^\star)$ such that all firms comply with the socially responsible policy, and:*

1. *If $S \leq \underline{S}$, then the SRF invests only in industry $i_0 = \text{argmax}_{\,i\in\{1,2\}} \left(\frac{1-e_i^*}{e_i^*}\right)\left(\frac{1-\eta_i}{1+K_i}\right)$ and imposes emissions in that industry to be lower than $\hat{e}_{i_0} = \left(\frac{K_i - S}{K_i - \eta_i S}\right)^{1-\alpha_{ij}}$.*
2. *If $\underline{S} < S < S^\star$, then the SRF invests in both industries. The optimal policy $(\hat{e}_1, \hat{e}_2)$ satisfies the following:*
$$\frac{\frac{\partial U}{\partial e_1}}{\frac{\partial S_1}{\partial e_1}} = \frac{\frac{\partial U}{\partial e_2}}{\frac{\partial S_2}{\partial e_2}}$$
3. *If $S \geq S^\star$, then the SRF invests in each industry i at least $S_i(e_i^*)$, and imposes first-best emissions levels, as follows: $(\hat{e}_1, \hat{e}_2) = (e_1^*, e_2^*)$.*
4. *If the SRF management cost is nil, i.e., $\psi = 0$, then there is an equilibrium where $S \geq S^\star$ and first best is achieved no matter the investors' sensitivity to responsible investing, i.e., even if $\overline{\mu} = 0$.*

Let us interpret the different elements of Proposition 3:

If $S < \underline{S}$, the SRF manages a relatively small fraction of the total capital. In this case, rather than spreading capital thin in the two sectors, the SRF should concentrate capital in one sector, i_0. Firms in industry i_0 comply, whereas in the other industry, all firms set emissions at the maximum, $e_j = 1$. To determine the industry on which a small SRF should focus, the following two industry characteristics need to be considered: (1) social desirability in reducing emissions, measured by $(1 - e_i^*)/e_i^*$, and (2) the effectiveness of socially responsible incentives on entrepreneurial choice, measured by $(1 - \eta_i)/(1 + K_i)$. Given the same first best emissions level, i.e., $e_1^* = e_2^*$, the SRF should focus on where its investment is most influential. This suggests that rather than focusing on liquid shares of companies, impact investing should prioritize primary offerings, private equity, as well as less liquid stocks. For a given level of effectiveness, the SRF should focus on the industry in which the reduction of emissions is most desirable, i.e., with the smallest e_i^*.

As an increasing amount of capital is invested in i_0, the marginal impact of capital in that industry decreases (since $\hat{e}_i$ becomes closer to e_i^*), becoming closer to that in the other sector. The threshold $\underline{S}$ corresponds to the mass of socially responsible capital that needs to be invested in i_0, such that the marginal impact of incremental socially responsible capital is the same in each sector.

For $\underline{S} < S < S^\star$, the SRF invests in both sectors. It equalizes the marginal impact of capital in each of the two sectors. As S increases, S_1 and S_2 increases and emissions decreases. The size of the SRF is, however, not sufficient to bring emissions to the first best.

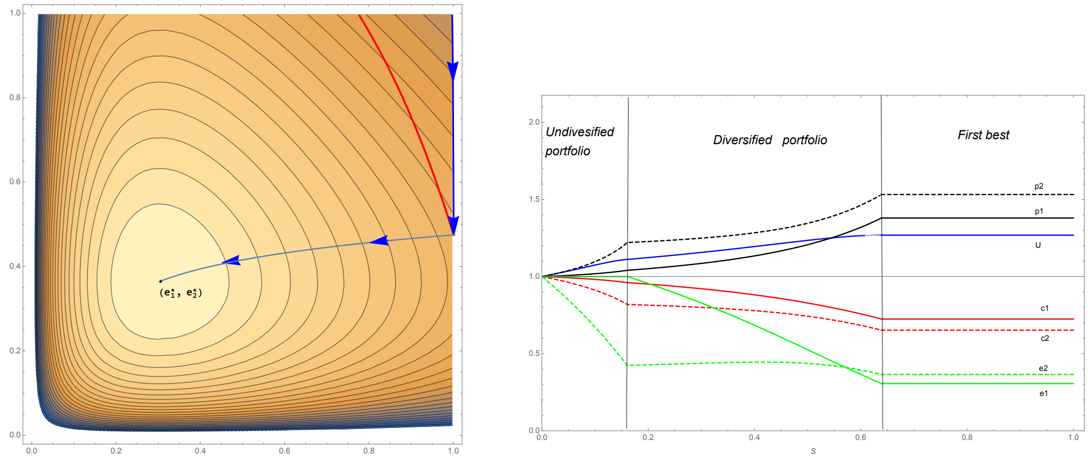
When the size of the SRF $S \geq S^\star$, the fraction of the total capital managed by the SRF is large enough for the fund to be able to induce all firms to comply with the first best. That is, the SRF invests in both industries an amount sufficient to make the policy (e_1^*, e_2^*) acceptable to all entrepreneurs. Note that when $S > S^\star$, the marginal impact of additional socially responsible capital is zero, as the first-best is already implemented. An increase in the level of the capital market friction reduces the total amount of socially responsible capital that is necessary to reach the first best.

5.0.2 Minimizing the emissions footprint

Next, we consider the case where investors care about the emissions footprint rather than impact. We have observed that such preferences correspond to a taste for "value alignment", whereby investors experience negative utility from holding polluting companies. When facing such nonconsequentialist investors, to maximize it size, the SRF needs to minimize its portfolio's emissions footprint. This leads to the following constraint optimization problem:

$$\begin{aligned} \min_{e_1, e_2} \quad & \frac{S_1(e_1)}{S} e_1 \delta_1 + \frac{S_2(e_2)}{S} e_2 \delta_2 \\ s.t. \quad & S_1(e_1) + S_2(e_2) \leq S \end{aligned} \tag{27}$$

It immediately follows that:



(a) (b)

Figure 1: Panel A shows the maximization problem in the plane (e_1, e_2) for a Socially Responsible Fund in the presence of consequentilist investors. The black curves are iso-social-welfare curves. The red curve indicates the minimum levels of (e_1, e_2) that can be achieved when $S = \underline{S}$. The blue line indicates the constraint socially optimum level of emissions for the different $S \in [0, 1]$ where the arrows move from $S = 0$ (top-right corner) toward $S \geq S^\star$, (point (e_1^*, e_2^*)). Panel B presents the ratio between the macroeconomic variables and their levels in the laissez-faire situation as a function of the size S of SRF, as follows: social welfare (blue line), consumption (red lines), emissions (green lies), and goods prices (black lines). The kinks occur at $S = \underline{S}$ and at $S = S^\star$

Proposition 4 *In order to minimize its portfolio footprint using direct emissions caps, an SRF of size S should put all its capital in the industry i^{**} defined as follows:*

$$i^{**} = \arg \min_{i \in \{1,2\}} \delta_i \left(\frac{K_i - S}{K_i - \eta_i S} \right)^{\frac{1}{1-a_{ij}}}$$

and set the direct emissions caps of this industry to the following:

$$\hat{e}_{i^{**}} = \max \left\{ \left(\frac{K_{i^{**}} - S}{K_{i^{**}} - \eta_{i^{**}} S} \right)^{\frac{1}{1-a_{i^{**}j}}}, \varepsilon \right\}$$

By minimizing the footprint, the presence of the SRF can either increase or decrease social welfare when compared to a laissez-faire regime but, generically, cannot bring the economy to the first best social welfare.

Proposition 4 reflects the fact that to minimize its portfolio footprint, the SRF should concentrate all its capital in a single industry and fully exploit the capital frictions to reduce this industry's emissions. The SRF, hence, concentrates investment in the industry whose 'natural' footprint δ_i is the smallest and where its capital can have the strongest grip, i.e., small K_i and η_i. For example, the SRF will put all its capital in industry i if it is both the green industry and the friction industry. This industry, however, needs not be the one where reduction in emissions would be most beneficial to social welfare. Additionally, a large enough SRF induces firms to reduce their emission even below its social optimum level e_i^*.

6 The effect of indirect upstream emissions caps

In this section, we study the equilibria where the SRF focuses all its capital in one industry i, and conditions the SRF financing on compliance with both direct and indirect upstream emission caps. In that way, the SRF influences emissions from industry i but also (indirectly) from firms in industry j, which sells goods to industry i. We then characterize when this type of SRF policy dominates the policies controlling only direct emissions.

We first consider the effect of caps on direct and indirect upstream emissions. Such a strategy provides incentives of a different nature to each industry. By focusing its capital on a single industry i, the SRF maximizes its grip on that industry, which results from the capital matching frictions. We show that the presence of compliant firms in industry i gives rise to an endogenous mass of firms in industry j who choose to reduce their direct emissions. They do this not to have better chances to be financed (there is no socially responsible capital in industry j), but rather because good j produced with a low emission technology trades for a higher price compared to the price for the same good produced with high emission. As we show in the next Proposition, in equilibrium, the industry that receives SRF capital is only composed of compliant firms (as predicted in Lemma 3), whereas in the other industry, the low-emission and high-emission firms co-exist, that is, $\mathcal{E}_i = \{\hat{e}_i\}$ and $\mathcal{E}_j = \{\hat{e}_{Ui}, 1\}$.

To approach the equilibrium, a first step is to understand the trade-off perceived by a firm in industry i. Using Equation (14), we find that a firm in industry i expects higher profits from compliance than noncompliance if the following is true:

$$\underbrace{\left(\frac{\hat{e}_i}{(p_j(\hat{e}_{Ui}))^{\alpha_{ij}}}\right)^{\frac{1}{1-\alpha_{ij}}}}_{\substack{\text{profitability}\\ \text{if comply}}} \geq \underbrace{\Phi_i(s_i)}_{\substack{\text{probability to receive capital}\\ \text{if doesn't comply}}} \underbrace{\left(\frac{1}{(p_j(1))^{\alpha_{ij}}}\right)^{\frac{1}{1-\alpha_{ij}}}}_{\substack{\text{profitability}\\ \text{if it does not comply}}} \tag{28}$$

Complying with the SRF policy involves the following two costs: the cost of not being able to use maximum emissions when producing, and the cost of using low-emission inputs that are more expensive than high-emission inputs. This second cost comes from the fact that, as we will show below, in equilibrium, $p_j(1) < p_j(\hat{e}_{Ui})$. This inequality is itself implied by the coexistence in industry j of high-emissions firms and low-emissions firms: These firms must be making identical expected profits. Given that they face identical prices for input i, this can be expressed, going back to Equation (14), as follows:

$$p_j(1) = p_j(\hat{e}_{Ui})\hat{e}_{Ui} \tag{29}$$

By combining Equations (28) and (29) and using the expression of $\Phi_i(s_i)$, we obtain the condition under which firms in industry i prefer to comply.

Condition 1

$$\hat{e}_i\hat{e}_{Ui}^{\alpha_{ij}} \geq \left(\max\left\{0, \frac{K_i - S}{K_i - \eta_i S}\right\}\right)^{1-\alpha_{ij}}$$

This condition determines the feasible policies $(\hat{e}_i, \hat{e}_{Ui})$ as a function of S, and allows us to characterize the equilibrium in the goods markets as follows:

Proposition 5 *Suppose the SRF only invests in industry i, requiring compliant firms to reduce their direct and indirect upstream emissions to $\hat{e}_i$ and $\hat{e}_{Ui}$, respectively, fulfilling Condition 1. Then, in equilibrium:*

1. *In industry i, all firms comply by setting their direct emission at $e_i = \hat{e}_i$ and buying from industry j firms with direct emissions of $e_j = \hat{e}_{Ui}$.*
2. *Industry j splits into a mass of size $\theta_j(1, \hat{e}_i) = K_j(1-\alpha_{ij})\gamma_j \in (0, K_j)$ of high-emissions firms, and a mass of size $\theta_j(\hat{e}_{Ui}, \hat{e}_i) = K_j - \theta_j(1, \hat{e}_i)$ of low-emissions firms. A high-emissions (resp. low-emission) firm's direct emissions equals 1 (resp. $\hat{e}_{Ui}$).*

3. *Equilibrium prices for good j satisfy $p_j(1) = p_j(\hat{e}_{Ui})\hat{e}_{Ui} \leq p_j(\hat{e}_{Ui})$.*

4. *Consumers buy good j exclusively from high emissions firms, whereas industry i firms buy input j exclusively from low emissions firms.*

5. *The average emissions levels per firm are $e_i = \hat{e}_i$ in industry i and $e_j = \frac{\gamma_j}{Z_j} + \left(1 - \frac{\gamma_j}{Z_j}\right)\hat{e}_{Ui}$ in industry j.*

6. *Social welfare is proportional to the following:*

$$U_I(e_i, e_j) := C \frac{e_i^{Z_i}}{(1 + e_i K_i)^{\delta_1}} \frac{\left(\frac{Z_j e_j - \gamma_j}{Z_j - \gamma_j}\right)^{\alpha_{ij} Z_i}}{(1 + e_j K_j)^{\delta_2}} \tag{30}$$

By providing capital only to industry i and requiring compliant firms in this industry to reduce their direct and indirect emissions to $\hat{e}_i$ and $\hat{e}_{Ui}$, respectively, the SRF brings the direct emissions of each individual firm in industry i to $\hat{e}_i$. Note, however, that the indirect emissions cap on industry i only affects the direct emissions of a fraction $1 - \frac{\gamma_j}{Z_j}$ of industry j. The remaining $K_j \frac{\gamma_j}{Z_j}$ firms will set their emissions to 1. Thus, the average per-firm emissions for industry j is equal to $\frac{\gamma_j}{Z_j} + \left(1 - \frac{\gamma_j}{Z_j}\right)\hat{e}_{Ui} > \hat{e}_{Ui}$. We can use this expression to translate Condition 1 into the constraint on the average per-firm emissions that the policy can induce:

$$e_j > \frac{\gamma_j}{Z_j} \tag{31}$$

$$e_i \left(\frac{Z_j e_j - \gamma_j}{Z_j - \gamma_j}\right)^{\alpha_{ij}} \geq \left(\max\left\{0, \frac{K_i - S}{K_i - \eta_i S}\right\}\right)^{1-\alpha_{ij}} \tag{32}$$

It is worth interpreting constraints (31) and (32). No matter the socially responsible policy $(\hat{e}_i, \hat{e}_i^U)$, a fraction $\frac{\gamma_j}{Z_j}$ of firms in industry j are setting emissions to 1. Thus, the minimum average emissions level in industry j cannot fall below $\frac{\gamma_j}{Z_j}$, hence constraint (31). Note that if (32) holds with equality, then e_i must be decreasing in e_j. That is, the stricter the restrictions on industry j, the softer the restrictions applied to industry i need to be. The SRF faces a tradeoff between the emissions caps in the two industries. To decrease the average emissions of industry j, e_j the SRF has to decrease $\hat{e}_{Ui}$, that is, the indirect upstream emissions cap on industry i. This decreases the direct emissions for a low-emission firm of industry j because these firms will set their direct emissions to $\hat{e}_{Ui}$. To choose to lower their emissions, these firms must be compensated with a higher selling price $p_j(\hat{e}_{Ui})$, for their output, compared to the price $p_j(1)$, at which high-emissions firms in the same industry can sell theirs. That is, the lower e_j, the larger the relative price $p_j(\hat{e}_{Ui})/p_j(1)$. Thus, decreasing e_j increases the cost of input for complying firms in industry i. As a result, industry i entrepreneurs choose to comply only if the cap on their direct emissions level $\hat{e}_i$ is not too tight, thus leading to the negative relation between e_i and e_j implied by constraint (32). As for the direct emissions policy, the r.h.s. of (32) shows how the grip of the SRF on emissions increases with S and decreases with η_i.

6.0.1 Maximizing impact

In the presence of consequentialist investors, the size of the SRF increases with its impact. When using direct and upstream indirect emission caps, the SRF chooses the industry i, and its emission caps $\hat{e}_i$ $\hat{e}_{Ui}$ to maximize (30) subject to constraints (31) and (32). Below, we describe how this maximization problem changes with the fund size, and when it is preferable to use the direct emissions approach described in Proposition 3 .

Direct incentives vs indirect upstream incentives for a small sized SRF

We show below that a small SRF recurring to indirect incentives can maximize its impact by separating the investment choice, i.e., where to invest its capital, from its emission cap policy, i.e., what industry emissions mitigate. Recall that we defined the *friction industry* as the industry where η_i is the smallest, and the *critical industry* is the one where e_i^* is the smallest.

Proposition 6 *If the SRF's capital S is small enough, to maximize its impact, the SRF should invest all its capital in the friction industry and adopt a policy focused solely on reducing the critical industry's emissions.*

When the friction industry and the critical industry are the same industry i, this is achieved by imposing only a direct emissions cap on the friction industry.

$$\hat{e}_i = \left(\frac{K_i - S}{K_i - \eta_i S} \right)^{\frac{1}{1-\alpha_{ij}}} .$$

When the friction industry is i and the critical industry is $j \neq i$, this is achieved by imposing only an indirect upstream emission cap on the friction industry.

$$\hat{e}_{Ui} = \left(\frac{K_i - S}{K_i - \eta_i S} \right)^{\frac{1-\alpha_{ij}}{\alpha_{ij}}} .$$

Direct incentives vs indirect upstream incentives for a medium sized SRF

In this subsection, we want to study more generally whether focusing on a single industry's direct and indirect upstream emissions can increase welfare more than focusing solely on direct emissions (of both industries). Clearly, if $S \geq S^\star$, then the first best can be achieved by investing in both industries. Thus, focusing on a single industry can be optimal only for a relatively small sized SRF. In the following proposition, we provide sufficient conditions under which the SRF has a greater impact by investing in a single industry and constraining both its direct and indirect upstream emissions than investing in both industries and imposing direct emissions caps on each industry.

Proposition 7 *Suppose $S < S^\star$. If $\eta_j - \eta_i$ and/or $\alpha_{ij} - \gamma_j$ are large enough, then, investing in one industry (with optimal direct and indirect emission caps) has a larger impact than investing in both industries.*

The case $\eta_j > \eta_i$ corresponds to a situation in which there is more friction in the matching capital market of industry i. In this case, investing in industry i provides substantially stronger financial incentives than investing in industry j. If this gap is large enough, to maximize impact, the SRF should invest all its capital in that industry and incentivize them to purchase from clean suppliers.

The case $\alpha_{ij} - \gamma_j$ large corresponds to a situation in which $\alpha_{ij} \simeq 1$ and $\gamma_j \simeq 0$. That is, consumers derive utility mostly from good i rather than from j, but good j represents a substantial input for good i's production. In this case, the SRF should invest all its capital in industry i, the 'consumption good' industry, and require this industry to purchase the 'intermediary good' j from clean producers. More specifically, observe that K_i is a decreasing function of a_{ij}. When K_i is small, by investing all its capital in industry i then SRF acquires a substantial control of the industry and can impose strong limit to direct and indirect emission. The limit on industry i's indirect emissions will affect only a fraction $1-\frac{\gamma_j}{Z_j}$ of firms in industry j. However, for γ_j close to 0, industry j's good is mostly used as an input for industry i, implying that $1-\frac{\gamma_j}{Z_j}$ is close to 1.

6.0.2 Minmizing footprint

Assume investors receive negative utility from the direct emissions of firms in their portfolio. In this case, the footprint minimization problem led to the same solution as the one described in Proposition 4[13].

6.1 The effect of indirect downstream emission caps

We consider in Appendix A.1. the case where the SRF can require caps on direct emissions $\hat{e}_i$ and downstream indirect emissions $\hat{e}_{Di}$. This case has less practical appeal, as controlling the final use of goods that have been sold is difficult. But interestingly, it turns out to be a powerful mechanism to generate impact; the reason is that firms in the downstream industry are constrained to comply as soon as firms up-stream decide to comply, as there is no other way to source inputs than from the up-stream sector.

7 Equilibrium size of SRF

In the previous section, we considered the size S of the SRF as given, and analyzed how the SRF should choose its portfolio and emissions caps to maximize impact or to minimize the footprint. In this section, we show how the equilibrium level of $S \in [0,1]$ is determined.

First, consider the case where investors care about impact. Let $\hat{U}(S)$ denote the maximum social welfare achieved by a SRF of size S. Hence, the maximum impact that the SRF can achieve

[13] Another possibility is to take into account the direct emissions of suppliers to the firms in the portfolio. From Proposition 5, the direct average emissions level of the industry i receiving SRF is $\hat{e}_i$, and the emissions of a fraction $(1-\gamma_j Z_i)$ of industry j to level $\hat{e}_{Ui}$. Thus, one possibility could be to define the SRF portfolio footprint as $F_I := \hat{e}_i\delta_i + \left(1-\frac{\gamma_j}{Z_i}\right)\hat{e}_{Ui}\delta_j$, where $\hat{e}_i$ and $\hat{e}_{Ui}$ satisfy Condition 1. However, regardless of whether or how one accounts for indirect emissions, clearly portfolio footprint minimization may lead to an increase as well as to a deterioration of social welfare.

with size S is as follows:

$$I(S) = \hat{U}(S) - U(1,1) \tag{33}$$

We call this equality the *impact constraint*. By substituting $I(S)$ into A in equation (18), we find the supply of S that is consistent with the actual impact that the SRF can achieve with S. That is,

$$S = \max\left\{1 - \frac{\psi}{\overline{\mu}}\left(1 + \frac{U(1,1)}{I(S)}\right), 0\right\} \tag{34}$$

We can retain three key properties of $I(S)$. First, $I(0) = 0$; this is because the laissez-faire equilibrium (with welfare $U(1,1)$) occurs when $S = 0$. Second, $I(S)$ is continuous and weakly increasing in S, for all $S > 0$. Third, there is $S^\star < 1$ such that $I(S) = U(e_1^*, e_2^*) - U(1,1)$ for all $S \geq S^\star$, i.e., the SRF can achieve the socially optimum level of emissions by controlling only a fraction of the total capital.

Equilibria with strictly positive impact are defined by the positive solutions of Equation (34). Hence, we have

Proposition 8 *If investors are consequentialist, then $S = 0$ is always an equilibrium (the "no impact equilibrium") and there is a threshold $M > 0$ such that*

1. *If $\psi/\overline{\mu} > M$ then $S = 0$ is the only equilibrium,*
2. *If $\psi/\overline{\mu} < M$ there also exists an equilibrium with strictly positive impact, in which both the SRF and the standard fund have a strictly positive mass. If $\psi/\overline{\mu}$ is small enough, one of the equilibria leads to the first best social welfare.*

Multiple equilibria arise due to a strategic complementarity between socially responsible investors, i.e., if more investors invest in the SRF, the SRF has more impact, which in turn makes investing in the SRF more attractive. Note that the different equilibria are Pareto ranked. The intuition for the proposition is simply that the SRF is viable if and only if the taste of individuals for impact is strong enough (or if the cost of running the SRF is small enough).

Let us now consider the case of nonconsequentialist investors, who value having a portfolio with a low emissions footprint. Let $\hat{F}(S)$ denote the minimum level of the footprint of the SRF's portfolio as a function of the capital under management S. Note $\hat{F}(S)$ is decreasing in S and that $\lim_{S\to 0}\hat{F}(S) = \min\{\delta_1, \delta_2\} < \overline{F}$, indeed by investing in the least polluting industry, even an arbitrary small fund can have a footprint below $\overline{F}$, the market portfolio footprint in laissez-fare. In equilibrium the supply of S is consistent with the actual footprint that the SRF can achieve with S. That is,

$$S = \max\{1 - \frac{\psi}{\overline{\mu}(\overline{F} - \hat{F}(S))}, 0\} \tag{35}$$

Proposition 9 *If investors are nonconsequentialist, then*

1. *If $\psi/\overline{\mu}$ is large enough, then the only equilibrium is the laissez-faire equilibrium with $S = 0$.*
2. *If $\psi/\overline{\mu}$ is small enough, then there exists an equilibrium where the SRF fund size is positive and its portfolio footprint is below $\overline{F}$. Generically, this equilibrium does not lead to the social optimum.*

8 Discussion

In this section, we discuss the robustness of our results to some of the assumptions.

Timing of the technology choice We have assumed that entrepreneurs irreversibly choose their technology and business plan before searching for capital. This assumption is motivated by the fact that to raise funds, a firm needs first to provide some information to the investors about the use it will make of their money. In our model, this timing induces compliance in all firms in an industry where there is SRF capital, even in firms that end up being financed by nonresponsible capital. The resulting reduction in emissions, however, increases in the fraction of the industry capital coming from responsible investors. That is, the SRF's impact increases in its size. However, what if firms could choose their technology after having raised their capital? In this case, within a given industry, the fraction of complying firms would reflect the fraction of the industry financed with responsible capital. As before, the SRF's impact would be increasing in the capital invested in that industry. However, within the same industry, complying and noncomplying firms would coexist. The main properties of our analysis would still apply.

Multiple SRFs Clearly, the assumption of a single SRF attracting all responsible capital represents the situation in which the responsible capital impact is maximum. In practice, there are many financial intermediaries catering to responsible investors. Our results call for the need for coordination among intermediaries to define standards regarding which firms are eligible to receive SRF capital and which are not. By coordinating on common standards, socially responsible asset managers would form a sizeable and compact block that firms could not ignore. These standards should be based on the aggregate amount of responsible capital. Absent these common standards, different ESG funds might compete to attract firms and then concede on eligibility requirements.

Responsible investors preference We have assumed that by investing qw in the SRF, a type μ investor sees her utility increasing by μqwA. That is, the increase in utility from responsible investing is proportional to the SRF's social performance A. As an alternative, one could assume that utility is impacted by the marginal contribution of its capital to A. For example, expression (3) could be replaced by $\mu qw\frac{\partial A}{\partial S}$. This would not qualitatively change how the value of ψ and $\overline{\mu}$ affect the existence of a positive S equilibrium. However, a unique equilibrium level of S would emerge. To see this, note that, because marginal impact is a decreasing function of S, the amount of capital invested in the SRF would be decreasing in the expected size of the SRF, whereas the impact the SRF can achieve as a function of S, would remain increasing in S. The two curves would then cross in a single point.

9 Conclusion

This paper develops a general equilibrium model of a productive economy with negative externalities. We analyze the strategy of a socially responsible fund facing investors who value financial returns but also the fund's social performance. We show that if capital markets are subject to frictions, this socially responsible fund can raise assets and induce firms to reduce their toxic emissions. We show that the fund's capital allocation across industries and the pollution limits imposed to its portfolio companies depend on how investors value social performance. If investors measure the fund's nonpecuniary performance with low carbon footprint portfolios, the SRF will focus its capital in the sector where it can induce the lowest level of emissions. This might improve, as well as deteriorate, social welfare. If investors are consequentialist such that nonpecuniary benefits are proportional to impact, we show that minimizing the carbon footprint is not the SRF's optimal policy. To maximize its impact, the SRF should prioritize the reduction of emissions of the most polluting industries. To curb such industries emissions, the SRF combines the following two channels: financial frictions and supply chain. The stronger an industry's financial friction, the larger the reduction in direct and/or indirect emissions that SRF can obtain in return of its capital. The supply chain channel allows the SRF to curb emissions in firms who are not subject to financial friction but trade with SRF financed firms who are. A SRF that has a passive strategy consisting of investing in the 'greener industries' has no impact on the level of emissions nor on social welfare. Our model also points at the importance of regulations allowing reliable firm-level information on direct and indirect emissions.

Appendix

A.1 Proofs

Proof of Lemma 1

Let denote with p_1, p_2, E_1 and E_2 the prices of cheapest good 1 and good 2, and aggregate emissions in industry 1 and 2, respectively, in the presence of the SRF. That is, $W_{SR} = W(p_1, p_2, E_1, E_2)$. If investors are consequentialist, then $A = W(p_1, p_2, E_1, E_2) - \underline{W}$, that replaced into (18) provides

$$S = 1 - \frac{r + \psi - r_G}{\overline{\mu}} \frac{W(p_1, p_2, E_1, E_2)}{W(p_1, p_2, E_1, E_2) - \underline{W}}$$

whose r.h.s. is increasing in $W(p_1, p_2, E_1, E_2)$ because $\underline{W} > 0$.

If investors are non-consequentialist, then $A = \frac{\overline{F} - F}{W(p_1, p_2, E_1, E_2)}$, that replaced into (18) provides

$$S = 1 - \frac{r + \psi - r_G}{\overline{\mu}(\overline{F} - F)},$$

whose r.h.s. is decreasing in the SRF portfolio footprint F.

Proof of Proposition 1

1. Because firms in industry 1 and 2 are constrained to set their direct emissions to e_1 and e_2, respectively, then there only is one market for each good. So we can drop the argument e_i in the price $p_i(e_i)$. Taking into account consumers demand and firms demand given in (7) and (12), respectively, we can write the equilibrium condition on the goods markets:

$$\begin{cases} Y_1 = \frac{\gamma_1}{p_1} + \frac{p_2 Y_2}{p_1} a_{12} \\ Y_2 = \frac{\gamma_2}{p_2} + \frac{p_1 Y_1}{p_2} a_{21} \end{cases}$$

   where $Y_i := Z_i / p_i$ is the aggregate production of good i and we claimed that consumer's aggregate wealth is equal to 1. The solution of this system provides expression (21).

2. - 3. Let r_i, be the return on capital invested in industry $i \in \{1,2\}$. Note that in equilibrium the return of the capital invested in each industry must be the same. Namely there is some $r > 0$ such that $r_1 = r_2 = r$. This because, if $r_1 \neq r_2$ then one of the two sectors would receive no capital, but this would lead to no production of one of the goods, and this cannot occur in equilibrium.

   Observe that industry i's aggregate profit is equal to $Z_i(1-\alpha_{ij})$. A fraction $1-\lambda$ of this profit will be distributed prorata to the investors. Hence, if the total amount of capital invested in industry i is K_i, then the return on investing in industry i must satisfy

$$r = \frac{Z_i(1-\alpha_{ij})(1-\lambda)}{K_i}$$

   or equivalently

$$K_i = \frac{Z_i(1-\alpha_{ij})(1-\lambda)}{r}$$

   Note that $Z_1(1-\alpha_{12}) + Z_2(1-\alpha_{21}) = \gamma_1 + \gamma_2 = 1$ and that the total amount of capital in the economy equals 1, i.e., $K_1 + K_2 = 1$. These equalities are satisfied only if $r = 1-\lambda$ and $K_i = Z_i(1-\alpha_{ij})$.

4. The profit of a firm in industry i is equal to $\pi_i = \frac{p_i Y_i(1-\alpha_{ij})}{K_i} = 1$, where the second equality follows from the fact that $p_i Y_i = Z_i$.

5. We already know from 2. that by investing his unit of capital, each investor gets $r = 1-\lambda$. A typical entrepreneur's revenue is $\lambda\pi_i = \lambda$ because of point 4. above. Consumers' aggregate wealth equals $\int_0^1 \lambda de + \int_0^1 1-\lambda dc = 1$, as claimed when proving point 1.

6. From point 4. we know that a firm equilibrium profit π_i equals 1. Using expression (14) we can write

$$\begin{cases} \frac{p_1}{p_2^{\alpha_{12}}} e_1 = \frac{1}{\alpha_{12}^{\alpha_{12}}} \frac{1}{(1-\alpha_{12})^{(1-\alpha_{12})}} \\ \frac{p_2}{p_1^{\alpha_{21}}} e_2 = \frac{1}{\alpha_{21}^{\alpha_{21}}} \frac{1}{(1-\alpha_{21})^{(1-\alpha_{21})}} \end{cases}$$

implying,

$$\begin{cases} p_1^{1-\alpha_{12}\alpha_{21}} e_1 e_2^{\alpha_{12}} = \frac{1}{A_1} \\ p_2^{1-\alpha_{21}\alpha_{12}} e_2 e_1^{\alpha_{21}} = \frac{1}{A_2} \end{cases}$$

where $A_i := \alpha_{ij}^{\alpha_{ij}}(1-\alpha_{ij})^{(1-\alpha_{ij})}\alpha_{ji}^{\alpha_{ji}\alpha_{ij}}(1-\alpha_{ji})^{(1-\alpha_{ji})\alpha_{ij}}$. Observe that

$$p_1^{-\gamma_1} p_2^{-\gamma_2} = \Theta e_1^{Z_1} e_2^{Z_2}$$

where $\Theta := A_1^{\frac{\gamma_1}{1-\alpha_{12}\alpha_{21}}} A_2^{\frac{\gamma_2}{1-\alpha_{12}\alpha_{21}}}$. Replacing this expression of the prices into (8), and considering that $E_i = e_i K_i$, we have that the equilibrium level of an individual utility is equal to the following function of the levels of emissions

$$C \frac{e_1^{Z_1} e_2^{Z_2}}{(1+e_1K_1)^{\delta_1}(1+e_2K_2)^{\delta_2}} = U(e_1, e_2)w$$

where $C := \Theta\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}$ and w is the individual's revenue.

7. In laissez faire, no firm has incentive to reduce its direct emission, thus $e_1 = e_2 = 1$ and the resulting social welfare is $U(1, 1)$.

Proof of Lemma 2

A social planner who can fix e_1 and e_2 to maximize social welfare, solves

$$\max_{e_1\in[\varepsilon,1], e_2\in[\varepsilon,1]} U(e_1, e_2)$$

Taking the log of the objective function and differentiating, one finds the first order condition:

$$Z_i(1 + K_i e_i) = \delta_i K_i e_i$$

By replacing $Z_i(1-\alpha_{ij})$ to K_i and solving for e_i one finds $e_i = \frac{1}{(\delta_i - Z_i)(1-\alpha_{ij})}$, that, under Assumption 1, is strictly included between ε and 1.

Proof of Lemma 3

1.a Suppose $S > 0$ and the SRF set no emission caps and invest S_1 and S_2 in industry i, then all firms will set their direct emission at 1 and $\mathcal{E}_1 = \mathcal{E}_2 = 1$. Let K_1 and K_2 be as in equation (21), i.e. the size of each industry in the absence of the SRF. If for both i one has $S_i < K_i$, after the financing phase the resulting economy is as in the laissez faire situation. The gross return of firms financed by the SRF and those financed by standard funds will be the same. The SRF impact is nil and if $\psi > 0$ there will be no investor willing to invest in the SRF contradicting that $S > 0$. If $\psi = 0$ then the net return of the two type of funds will be the same, S can be positive but the equilibrium remains the laissez faire equilibrium. Suppose now that $S_1 > K_1$ and denote with K_i' the mass of firms in industry i. Note that $K_1' \geq S_1 > K_1$ and $K_2' = 1 - K_1' < K_2$. Let W be the aggregate wealth of consumers, then in equilibrium

$$\begin{cases} K_1' y_1 p_1 = \frac{\gamma_1 W}{p_1} + \frac{p_2 K_2' y_2}{p_1} a_{12} \\ K_2' y_1 p_2 = \frac{\gamma_2 W}{p_2} + \frac{p_1 K_1' y_1}{p_2} a_{21} \end{cases}$$

Let $Z_i' = K_i' y_i p_i$ denote the resulting sale revenues in industry i., Then, the above system provides $Z_i' = Z_i W$. The profit for a firm in industry i is

$$\pi_i' = \frac{Z_i'(1 - \alpha_{ij})}{K_i'}$$

Note that for $K_1' = K_1$ and $K_2' = K_2$ on has $\pi_1' = \pi_2'$, whereas for $K_1' > K_1$ one has $\pi_1' < \pi_2'$. Because an entrepreneur's revenue in industry i is $\lambda \pi_i'$ no entrepreneur will choose industry 1, and K_1' cannot be positive.

1.b If $S_i > 0$ but condition (25) is not satisfied, then no entrepreneur in industry i will comply. The SRF funds cannot be invested and will generate zero return. Anticipating this, no investor will invest in the SRF fund, thus contradicting $S > 0$.

1. If $S_i > 0$ and condition (25) is satisfied, then it is weakly optimal for every single entrepreneur in industry i to comply. To guarantee that complying is strictly optimal it is sufficient that

inequality (25) is strict.

Proof of Proposition 2

We proceed in two steps. First, we describe the equilibrium in the good and capital market in case all firms comply. Second, we show that for a firm that faces all other firms complying, it is optimal to comply as long as SRF portfolio and policy is such that (26) is satisfied.

Step 1 We first show that if there is an equilibrium where all firms comply, then it must satisfy properties 1.-3. If all firms comply, then $\mathcal{E}_1 = \{\hat{e}_1\}$ and $\mathcal{E}_2 = \{\hat{e}_2\}$. Denoting with ω the aggregate wealth of consumers, and using the same argument as for result 1. in for Proposition 1, on finds that in industry i, aggregate sale proceeds and profits must amount to $Z_i\omega$ and $Z_i(1-\alpha_{ij})\omega$, respectively. If K_i' is the mass of firms in industry i, the revenue of an entrepreneur in this industry is $\lambda Z_i(1-\alpha_{ij})\omega/K_i'$. Because there are entrepreneurs in both industries, it must be that firm profits equal across industries, otherwise all entrepreneurs will chose the same industry. By solving $\lambda Z_1(1-\alpha_{12})\omega/K_1' = \lambda Z_2(1-\alpha_{21})\omega/(1-K_1')$, one finds $K_i' = K_i$. Normalizing aggregate consumers wealth ω to 1. One finds that $\pi_i = 1$, entrepreneur revenue is λ and capital gross return is $1-\lambda$. To prove that this results in an individual's utility equal to the individual's revenue times $U(\hat{e}_1, \hat{e}_2)$, it is sufficient to apply the same argument used to prove 6. in Proposition 1.

Step 2 Consider an entrepreneur in industry i observing that $S_1, S_2, \hat{e}_1$ and $\hat{e}_2$, satisfy inequality (26). From Step1, if this entrepreneur expects all firms to comply, he must also expect that the capitalization of industry i is K_i. He will choose to comply only if condition (25) is satisfied. Taking into account the expression for $\pi_i(\cdot)$ given in (14), this condition can be rewritten as

$$\underbrace{\left(p_i(\hat{e}_i)e\left(\frac{\alpha_{ij}}{p_j(\hat{e}_{Ui})}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}}(1-\alpha_{ij})}_{\pi_i(\hat{e}_i, p_i(\hat{e}_i), p_j(\hat{e}_{U,i}))} \geq \underbrace{\frac{1-s_i}{1-\eta_i s_i}}_{\Phi_i(s_i)} \underbrace{\left(p_i(\hat{e}_i)\left(\frac{\alpha_{ij}}{p_j(1)}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}}(1-\alpha_{ij})}_{\pi_{iN}}$$

Replacing S_i/K_i to S_i and solving for S_i, one finds (26). Thus, it is an equilibrium for all entrepreneurs to comply.

Proof of Proposition 3

Consider SRF managing a total amount of capital S and wiling to maximize its impact using direct emission policies. From Proposition 2, we deduce that the SRF's maximization problem is

$$\max_{e_1,e_2} \quad U(e_1, e_2) \tag{36}$$

$$s.t. \quad S_1(e_1) + S_2(e_2) \leq S \tag{37}$$

which we rewrite taking log of the objective function:

$$\max_{e_1,e_2} \quad \sum_{i=1,2} Z_i ln(e_i) - \delta_i ln(1 + K_i e_i) \tag{38}$$

$$s.t. \quad \sum_{i=1,2} \frac{1 - e_i^{\frac{1}{1-\alpha_{ij}}}}{1 - \eta_i e_i^{\frac{1}{1-\alpha_{ij}}}} K_i \leq S \tag{39}$$

which we can see as:

$$\max_{e_1,e_2} \quad \sum_{i=1,2} f_i(e_i) \tag{40}$$

$$s.t. \quad \sum_{i=1,2} g_i(e_i) \leq S \tag{41}$$

1. First consider the case where S is small, preventing the fund to have high impact. Each e_i will be in the neighborhood of 1. The SRF should prioritize the sector for which $-f_i'(1)/g_i'(1)$ is the highest. Now, $f_i'(1) = Z_i - \delta_i \frac{K_i}{1+K_i}$ and $g_i'(1) = -\frac{1}{1-\eta_i} Z_i$. We use $K_i = (1 - \alpha_{ij})Z_i$; we get the pecking-oder rule: $i_0 = \text{argmax}_{i\in\{1,2\}}(1-\eta_i)[\frac{\delta_i(1-\alpha_{ij})}{(1+(1-\alpha_{ij})Z_i)} - 1]$. From $e_i^* = \frac{1}{(\delta_i - Z_i)(1-a_{ij})}$, it is easy to verify that $[\frac{\delta_i(1-\alpha_{ij})}{(1+(1-\alpha_{ij})Z_i)} - 1] = \frac{1-e_i^*}{e_i^*(1+K_i)}$. Let $e_i(s_i) = S_i^{-1}(e_i) = \frac{K_i - s_i}{K_i - \eta_i s_i}$. Let $\underline{S}$ such that $f_{i_0}'(e_{i_0}(\underline{S})) = f_{-i_0}'(1)$. Then, investing all SRF capital in industry i_0 is optimal as long as $S \leq \underline{S}$.

2. For $\underline{S} < S < S_1(e_1^*) + S_2(e_2^*)$, there is an interior solution that is determined by the system of equations $f_i'(e_i) = \xi g_i'(e_i)$, where ξ is the Lagrangian multiplier.

3. When the SRF size $S > S_1(e_1^*) + S_2(e_2^*)$, the fund manages a capital that is large enough to

induce the first best socially optimal behavior. That is, by investing at least $S_1(e_i^*)$ in industry i, for $i = 1, 2$, and fixing its policy $(e_1, e_2) = (e_1^*, e_2^*)$, the SRF can guarantee that the average emission in each industry equals the socially optimal level.

Proof of Proposition 4

Suppose the SRF invests a fraction ω of S in industry 1 and the rest in industry 2. Let $e_i[\omega]$ denote the level of emission that it can induce on industry i. By reversing equation (26), on gets

$$e_1[\omega] := \left(\frac{K_1 - \omega S}{K_1 - \eta_1 \omega S}\right)^{1-\alpha_{12}}$$

$$e_2[\omega] := \left(\frac{K_2 - (1-\omega) S}{K_2 - \eta_1 (1-\omega) S}\right)^{1-\alpha_{21}}$$

It is easy to check that $e_1'[\omega] < 0$, $e_2'[\omega] > 0$, $e_1''[\omega] < 0$, $e_2''[\omega] < 0$.

SRF's portfolio footprint can be written as the following function of ω

$$\delta_1 e_1[\omega]\omega + \delta_2 e_2[\omega](1-\omega)$$

Differentiating twice with respect to ω one gets

$$\delta_1(e_1'[\omega] + \omega e_1''[\omega]) + \delta_2(-e_2'[\omega] + (1-\omega)e_2''[\omega] < 0$$

Thus SRF's portfolio footprint is a concave function of ω. Because is SRF is minimizing a concave function of ω, the solution is either $\omega = 1$ or $\omega = 0$. Thus the expression for i^{**}.

Let us show that by minimizing footprint, the SRF can either increase or decrease social welfare. Without loss of generality suppose $i^{**} = 1$. Then the presence of the SRF induces average emission in each industry equal $e_1 = e_1[1]$ and $e_2 = 1$ and brings social welfare to $U(e_1[1], 1)$. This is larger than $U(1, 1)$ as long as $e_1[1] < 1$ and not too small when compared to e_1^*. However for $e_1[1]$ small enough one has $U(e_1[1], 1) < U(1, 1)$. Note that $e_1[1]$ is a decreasing function of the fund size S, with $e_1[1]|_{S=0} = 1$ and $e_1[1]|_{S=1} = \varepsilon$. Thus, if S is large enough, portfolio footprint minimization reduces social welfare. Note that the only case in which minimizing footprint brings the first best social optimum is for the non generic situation in which $e_2^* = 1$ and $e_1^* = e_1[1]$.

Proof of Proposition 5

First note that if the SRF invests all its capital S in industry i, then $\Phi_i(s_i) = \left(\frac{K_i - S_i}{K_i - \eta_i S_i}\right)\Big|_{S_i = S} < 1$ and $\Phi_j(s_j) = \left(\frac{K_j - S_j}{K_j - \eta_j S_j}\right)\Big|_{S_j = 0} = 1$.

Let first show that

Lemma 4 *In an equilibrium where all firms in industry i comply, there must be three markets: a single markets $\mathcal{E}_i = \{\hat{e}_i\}$ for good i and two market for good j: $\mathcal{E}_j = \{\hat{e}_{Ui}, 1\}$.*

Proof. Note that there must be a non-nil fraction of firms in industry j that set their direct emission at $e_{j,f} \leq \hat{e}_{Ui}$. Otherwise there would be no supplier eligible to sell to firms in industry i, and firms in this industry could not comply. Let us show that not all firms in industry j will set their emissions below $\hat{e}_{Ui}$. Suppose instead that each firm f in industry j sets $e_{j,f} \leq \hat{e}_{Ui}$. Then, an entrepreneur in industry j would profit by setting up the only firm whose direct emission are $e_j = 1$ and then selling its product in the only market of good j. This will allow the firm to generate bigger profit compared to the other firms in the same industry. Because there is no socially responsible capital invested in industry j, by choosing to maximize his firm's emissions the entrepreneur will not reduce the chance of being financed. Thus, a profitable deviation. Hence there must be a non-nil mass of firms in industry j setting $e_{j,f} > \hat{e}_{Ui}$, and because these firms face no SRF constraint, they will set $e_{j,f} = 1$. Clearly an industry j firm willing to sell to industry i firms (to consumers) has no interest in setting its direct emission strictly below $\hat{e}_{Ui}$ (resp. below 1). Similarly, an industry i firm has no interest in setting its direct emissions strictly below $\hat{e}_i$. ■

For both high emission and low emission firms co-existing in industry j, industry j entrepreneurs must be indifferent between high and low emission. Because $\Phi_j(s_j) = 1$, in industry j, a firm's emission level does not affect the probability of being financed. Hence we must have that high and low emission firms generate the same profit. That is,

$$\left(p_j(1)\left(\frac{\alpha_{ji}}{p_i(\hat{e}_i)}\right)^{\alpha_{ji}}\right)^{\frac{1}{1-\alpha_{ji}}}(1-\alpha_{ji}) = \left(p_j(\hat{e}_{Ui})\hat{e}_{Ui}\left(\frac{\alpha_{ji}}{p_i(\hat{e}_i)}\right)^{\alpha_{ji}}\right)^{\frac{1}{1-\alpha_{ji}}}(1-\alpha_{ji}),$$

that is is possible only if $p_j(1) = \hat{e}_{Ui} p_j(\hat{e}_{Ui}) \leq p_j(\hat{e}_{Ui})$, where the inequality follows from $\hat{e}_{Ui} \leq 1$. This provides Result 3.

Result 4, follows from the fact that consumers buy goods from the firms selling at the lowest prices, and $p_j(1) \leq p_j(\hat{e}_{Ui})$. Thus, they will buy good j only from high emission firms. Because all firms in industry i comply, these firms can only buy their input from low emission firms in industry j.

We can now write the equilibrium condition on good i market, and on good j markets for high and low emission levels.

$$\begin{cases} Y_i = \overbrace{\dfrac{\gamma_i}{p_i(\hat{e}_i)}}^{\text{consumers' demand of good } i} + \underbrace{\dfrac{p_j(\hat{e}_{Ui}) Y_{jL}}{p_i(\hat{e}_i)} \alpha_{ji}}_{\text{Industry } jL\text{'s demand of good } i} + \overbrace{\dfrac{p_j(1) Y_{jH}}{p_i(\hat{e}_i)} \alpha_{ji}}^{\text{Industry } jH\text{'s demand of good } i} \\ Y_{jL} = \underbrace{\dfrac{p_i(\hat{e}_i) Y_i}{p_j(\hat{e}_{Ui})} \alpha_{ij}}_{\text{industry } i\text{'s demand of good } j} \\ Y_{jH} = \underbrace{\dfrac{\gamma_j}{p_j(1)}}_{\text{consumers' demand of good } j} \end{cases}$$

where, for industry j we used the subscript jL and jH to distinguish low emission and high emission variables, respectively. Solving this system one gets the following levels of sales revenues:

$$\begin{cases} Z_i = \frac{\gamma_i + \alpha_{ji}\gamma_j}{1-\alpha_{12}\alpha_{21}} \\ Z_{jL} = \frac{\gamma_i + \alpha_{ji}\gamma_j}{1-\alpha_{12}\alpha_{21}} \alpha_{ij} \\ Z_{jH} = \gamma_j \end{cases}$$

From these equilibrium level of sales, using the same argument as in Step 1 of the proof of point 2 in Proposition 2 based one entrepreneurs indifference across industries and market, we have that $K_i = Z_i(1-\alpha_{ij})$, $K_{jL} = Z_{jL}(1-\alpha_{ji}) = \theta_j(1, \hat{e}_i)$ and $K_{jH} = Z_{jH}(1-\alpha_{ji}) = \theta_j(\hat{e}_{Ui}, \hat{e}_i)$. Thus, property 2.

We can now show that if the level of $(\hat{e}_i, \hat{e}_{Ui})$ satisfies Condition 1 and firms in industry j behave as described above, then all firms in industry i comply. First, note that if all other firms in industry i comply, then the best a non-compliant firm in industry i can do is to set its direct emission at 1, buy good j at the low price, $p_j(1)$, from high emission firms, and sell its product at $p_i(\hat{e}_i)$. Thus, complying is optimal if

$$\underbrace{\left(p_i(\hat{e}_i)\left(\frac{\alpha_{ij}}{p_j(1)}\right)^{\alpha_{ji}}\right)^{\frac{1}{1-\alpha_{ji}}}(1-\alpha_{ji})}_{\text{maximum profit for non-compliant firm}}\ \underbrace{max\left\{0, \frac{K_i - S}{K_i - \eta_i S}\right\}}_{\text{probability of beeing financed}} \leq \underbrace{\left(p_i(\hat{e}_i)\hat{e}_i\left(\frac{\alpha_{ij}}{p_j(\hat{e}_{Ui})}\right)^{\alpha_{ij}}\right)^{\frac{1}{1-\alpha_{ij}}}(1-\alpha_{ij})}_{\text{profit of compliant firm}}.$$

By replacing $p_j(1)$ with $(\hat{e}_{Ui})p_j(\hat{e}_{Ui})$ and simplifying, one gets Condition 1.

So far we have shown that if the SRF sets caps such that Condition 1 is satisfied and market participants expect price to satisfy $p_j(1) = \hat{e}_{Ui}p_j(\hat{e}_{Ui}) \leq p_j(\hat{e}_{Ui})$, then it is an equilibrium for entrepreneurs and consumers to behave as described. This generates markets whose equilibrium price satisfy the above condition. Thus, for any given S, an equilibrium of this type exists.

The level of an individual's utility is given by equation (8). Consumers purchase good j at price $p_j(1)$, thus an individual with wealth w achieves:

$$\frac{\left(\frac{\gamma_i}{p_i(\hat{e}_i)}\right)^{\gamma_i}\left(\frac{\gamma_j}{p_j(1)}\right)^{\gamma_j}}{(1+E_1)^{\delta_1}(1+E_2)^{\delta_2}}w \tag{42}$$

Considering that $\pi_i = \pi_{jL} = 1$ and following similar steps as for the proof of Proposition 1, we have

$$p_i(\hat{e}_i)^{-\gamma_i}p_j(\hat{e}_{Ui})^{-\gamma_j} = \Theta\hat{e}_i^{Z_i}\hat{e}_{Ui}^{Z_j}$$

where $\Theta = A_1^{\frac{\gamma_1}{1-\alpha_{12}\alpha_{21}}}A_2^{\frac{\gamma_2}{1-\alpha_{12}\alpha_{21}}}$ and $A_i = \alpha_{ij}^{\alpha_{ij}}(1-\alpha_{ij})^{(1-\alpha_{ij})}\alpha_{ji}^{\alpha_{ji}\alpha_{ij}}(1-\alpha_{ji})^{(1-\alpha_{ji})\alpha_{ij}}$. Recalling from point 3 above that $p_j(1) = \hat{e}_{Ui}p_j(\hat{e}_{Ui})$, we can write

$$p_i(\hat{e}_i)^{-\gamma_i}p_j(1)^{-\gamma_j} = \Theta\hat{e}_i^{Z_i}\hat{e}_{Ui}^{(Z_j-\gamma_j)}$$

Observing that $Z_j - \gamma_j = \alpha_{ij} Z_i$, and the average emission in industry j is $e_j = \frac{\gamma_j}{Z_j} + \left(1 - \frac{\gamma_j}{Z_j}\right) \hat{e}_{Ui}$, on has $\hat{e}_{Ui} = \frac{Z_j e_j - \gamma_j}{Z_j - \gamma_j}$. We can express the individual utility (42) as

$$C \frac{e_i^{Z_i} \left(\frac{Z_j e_j - \gamma_j}{Z_j - \gamma_j}\right)^{\alpha_{ij} Z_i}}{(1 + \hat{e}_i K_i)^{\delta_1} (1 + e_j K_j)^{\delta_2}} w = U_I(e_i, e_j) w.$$

Proof of Proposition 6

The proof is in two steps. First we show that if small size SRF is willing to reduce the average emission on a given industry, say industry 2, the most effective way is to put all its capital in the friction industry, i.e. the industry with the smallest η_i. Second, given that maximum impact is achieved by investing all capital in the friction industry, we show that the SRF should focus on reducing the emission of the critical industry, i.e., the one with the smallest e_i^*.

Step 1: Suppose SRF wants to reduce emission in industry 2. By investing all its capital in this industry it can brings its direct emissions to

$$e_{2,Dir}(S) := \left(\frac{K_2 - S}{K_2 - \eta_2 S}\right)^{1 - \alpha_{21}}$$

If instead it invests all its capital in industry 1, imposes no cap on industry 1's direct emission, to focus on industry 1's indirect emission, it can bring industry 2 emission down to:

$$e_{2,Ind}(S) := \frac{\gamma_2}{Z_2} + \left(1 - \frac{\gamma_2}{Z_2}\right) \left(\frac{K_1 - S}{K_1 - \eta_1 S}\right)^{\frac{1 - \alpha_{12}}{\alpha_{12}}}$$

Note that $e_{2,Dir}(0) = e_{2,Ind}(0) = 1$. With some algebra we get

$$\left.\frac{\partial e_{2,Dir}}{\partial S}\right|_{S=0} = -\frac{1 - \eta_2}{Z_2}$$

and

$$\left.\frac{\partial e_{2,Ind}}{\partial S}\right|_{S=0} = -\frac{1 - \eta_1}{Z_2}.$$

Hence, the marginal impact of SRF capital in reducing e_2 is stronger when investing in the industry with the smaller η_i. The same argument applies if the SRF wanted to reduce industry 1's emission.

Step 2: Without loss of generality let assume that $\eta_1 < \eta_2$, implying that the SRF should invest all its capital in industry 1. For S small the marginal gain in (the log of) social welfare from using the capital to reducing industry 1 or industry 2 emission is

$$\frac{\partial U(e_1,e_2)}{\partial e_1}\frac{\partial e_{1,Dir}}{\partial S}\bigg|_{S=0} = -\left(Z_1 - \frac{\delta_1 K_1}{1+K_1}\right)\frac{1-\eta_1}{Z_1} = \frac{1-e_1^*}{e_1^*(1+K_1)}(1-\eta_1)$$

$$\frac{\partial U_I(e_1,e_2)}{\partial e_2}\frac{\partial e_{2,Ind}}{\partial S}\bigg|_{S=0} = -\left(Z_2 - \frac{\delta_2 K_2}{1+K_2}\right)\frac{1-\eta_1}{Z_2} = \frac{1-e_2^*}{e_2^*(1+K_2)}(1-\eta_1),$$

respectively. Where, we used $e_1 = e_2 = 1$ for $S = 0$ and we have replaced $\left(Z_i - \frac{\delta_i K_i}{1+K_i}\right)$ with $\frac{Z_i(e_i^*-1)}{e_i^*(1+K_i)}$, $i = 1, 2$, using the definition of e_i^*. These expressions are decreasing in e_i^* hence the marginal impact will be larger in the critical industry, i.e. the one with the smallest e_i^*.

Proof of Proposition 7

Let us say that a SRF policy is "direct" if it caps exclusively firms direct emissions. We say the policy is "indirect" if the SRF invests all S is in industry i and imposes to this industry direct and/or indirect emissions caps.[14] To fix idea we set $i = 1$ and $j = 2$. The argument of the proof unfolds as follows. Fix an arbitrary level of $e_1 \in [\varepsilon, 1]$ and let $S < S^\star$. Consider the minimum level of e_2 that an SRF of size S can impose to industry 2 while imposing e_1 to industry 1. Namely let $e_{2D}(e_1, S)$ and $e_{2I}(e_1, S)$ be these levels for the direct and the indirect policy, respectively. We show that for the value of parameters in the proposition $e_{2D}(e_1, S) \geq e_{2I}(e_1, S)$ for all feasible e_1. Hence the set of (e_1, e_2) that can be implemented with the direct policy is included in the set of (e_1, e_2) that can be implemented with the indirect policy. Thus, the impact of an appropriate direct policy is greater than the impact of the best of direct policies.

For the direct policy, rearranging $S_1(e_1) + S_2(e_2) \leq S$, we have that

$$e_{2D}(e_1, S) = \max\left\{0, \frac{K_2 - (S - S_1(e_1))}{K_2 - \eta_2(S - S_1(e_1))}\right\}^{1-\alpha_{21}}.$$

[14]If $S > K_i$, the remaining SRF's capital is invested in industry j without requiring any emission cap to this industry.

For the indirect policy, rearranging (32) we have that

$$e_{2I}(e_1,S)=\min\left\{1,\frac{\gamma_2}{Z_2}+\left(1-\frac{\gamma_2}{Z_2}\right)e_1^{-\frac{1}{\alpha_{12}}}\max\left\{0,\frac{K_1-S}{K_1-\eta_1 S}\right\}^{\frac{1-\alpha_{12}}{\alpha_{12}}}\right\}$$

Observe that for $e_1 \leq \max\{0, \frac{K_1-S}{K_1-\eta_1 S}\}^{1-\alpha_{12}}$, one has $e_{2D}(e_1,S)=e_{2I}(e_1,S)=1$. Hence let consider $e_1 > \max\{0, \frac{K_1-S}{K_1-\eta_1 S}\}^{1-\alpha_{12}}$. One has $e_{2D}(e_1,S)<1$ and $e_{2I}(e_1,S)<1$. Fix $\eta_1<1$ and let $\eta_2 = 1-\epsilon > \eta_1$, then

$$\lim_{\epsilon\to 0} e_{2D}(e_1,S)=1>e_{2I}(e_1,S)$$

hence, for any S there is $\eta_2 \in (\eta_1,1)$ such that $e_{2D}(e_1,S)>e_{2I}(e_1,S)$, for all $e_1 > \max\{0, \frac{K_1-S}{K_1-\eta_1 S}\}^{1-\alpha_{12}}$.

Observe that because $0<\gamma_2,\alpha_{1,2}<1$, if $\alpha_{1,2}-\gamma_2$ is large, then there is some ϵ small such that $\gamma_2 \leq \epsilon$ and $\alpha_{1,2} \geq 1-\epsilon$. Note that because $\alpha_{1,2}>1-\epsilon$, one has that $\lim_{\epsilon\to 0} S_1(e_1)=K_1$ and $\lim_{\epsilon\to 0} K_1 = 0$. Hence,

$$\lim_{\epsilon\to 0} e_{2D}(e_1,S)=\left(\frac{K_2-S}{K_2-\eta_2 S}\right)^{1-\alpha_{21}}>0$$

where the inequality follows from the fact that $S<S_1(e_1^*)+S_2(e_2^*)$, $\lim_{\epsilon\to 0} S_1(e_1^*)=0$ and $S_2(e_2^*)<K_2$. Also

$$\lim_{\epsilon\to 0}\max\left\{0,\frac{K_1-S}{K_1-\eta_1 S}\right\}^{\frac{1-\alpha_{12}}{\alpha_{12}}}=0$$

Consider now $e_{2I}(e_1,S)$. Because $\gamma_2<\epsilon$ one has that $\lim_{\epsilon\to 0}\theta_2=0$. Thus $\lim_{\epsilon\to 0} e_{2I}(e_1,S)=0$. By continuity there is $\epsilon>0$ small enough such that $e_{2I}(e_1,S)<e_{2D}(e_1,S)$.

Proof of Proposition 8

Note first that $S=0$ is always an equilibrium: if investors expect no impact they do not invest and the SRF can not have impact.

1. For $\frac{\psi}{\mu}$ large enough, net return that the SRF's is able to achieve is just too small to attract any capitalist. Indeed, $1-\frac{\psi}{\mu}\frac{I(S)+U(1,1)}{I(S)}<1-\frac{\psi}{\mu}$ which is negative for $\frac{\psi}{\mu}$ big enough. So in this case, $S=0$ is the only equilibrium.

2. To see the equilibrium level of S can be positive, note that the r.h.s. of (34) is continuous in S and equal 0 for S close enough to 0. If $\frac{\psi}{\overline{\mu}}$ is small enough, the r.h.s. of (34) equlas $1 - \frac{\psi}{\overline{\mu}}\left(1 + \frac{U(1,1)}{I(S^\star)}\right) > S^\star$ for all $S \geq S^\star$. By the intermediate value theorem, equation (34) has at least two strictly positive solutions one of which for $S \leq S^\star$ and one for $S \geq S^\star$. The latter brings the first best social welfare.

Proof of Proposition 9

Let $\underline{\delta} := \min\{\delta_1, \delta_2\}$. Note that the $1 - \frac{\psi}{\overline{\mu}(\overline{F} - F(S)}$ is continuous in S and goes from $1 - \frac{\psi}{\overline{\mu}(\overline{F} - \underline{\delta})}$, for S close enough to 0, to $1 - \frac{\psi}{\overline{\mu}(\overline{F} - \varepsilon\underline{\delta})} < 1$ for $S \geq \max\{K_1, K_2\}$.

1. If $\psi/\overline{\mu}$ is large enough, the r.h.s. of (35) equals 0 for all S and the only equilibrium is for $S = 0$.
2. If $\psi/\overline{\mu}$ is small enough, the r.h.s. of (35) is strictly positive for $S = 0$ and equal $1 - \frac{\psi}{\overline{\mu}(\overline{F} - \varepsilon\underline{\delta})} < 1$ for $S \geq \max\{K_1, K_2\}$. Thus equation (35) has a solution for $S \in (0, 1)$.

Appendix A.1. The effect of indirect downstream emission caps

Let us consider the case where the SRF invests all its capital in industry i and its policy requires caps on direct emissions $\hat{e}_i$ and downstream indirect emissions $\hat{e}_{Di}$. That is, to be eligible to SRF capital, a firm in industry i needs not only to limit its direct emission to $\hat{e}_i$ but also commit not to sell its output to industry j firms whose direct emissions exceed $\hat{e}_{iD}$. It turns out that there are equilibria in which this policy is particularly effective. A situation in which all firms of industry i comply induces all firms in industry j to set their emission $e_j = \hat{e}_{iD}$, as otherwise they simply do not have access to good i as an input. If all firms in industry j set emission at $e_j = \hat{e}_{iD}$, then industry i firms comply as long as

$$\pi_i(\hat{e}_i, p_i(\hat{e}_i), p_j(\hat{e}_{Di})) \geq \Phi_i(s_i)\pi_i(1, p_i(\hat{e}_i), p_j(\hat{e}_{Di}))$$

We can then describe the equilibrium in the goods markets

Proposition 10 *Suppose the SRF only invests in industry i, requiring compliant firms to reduce their direct and indirect downstrem emissions to respectively*

$$\hat{e}_i \geq \left(\frac{K_i - S}{K_i - \eta_i S}\right)^{\frac{1}{1-\alpha_{ij}}}$$

and $\hat{e}_{Di} \in [\varepsilon, 1]$. Then, in equilibrium:

1. *In industry i all firms comply and set $e_i = \hat{e}_i$.*
2. *In industry j all firms reduce their emission to $e_j = \hat{e}_{Di}$*

Proof of Proposition 10. Result 1. follow from the same argument used for Proposition 2. Because good i is necessary for production of good j, by influencing the whole industry i, the SRF can impose any cap to industry j, thus result 2. ■

9.0.1 Maximizing impact

When investing in industry i, the SRF can maximize its impact by imposing to industry i firms not to sell to industry j firms whose emissions exceed the first best e^*_j. Thus, the SRF's choice of investing in industry 1 or 2, and of the level of direct emission caps, results from the following maximization problem

$$\max_{i=1,2,\hat{e}_i} \quad U(\hat{e}_i, e_j^*) \tag{43}$$

$$s.t. \quad \hat{e}_i \geq \left(\frac{K_i - S}{K_i - \eta_i S}\right)^{\frac{1}{1-\alpha_{ij}}} \tag{44}$$

That is, the SRF can induce industry j firms to adopt the socially optimum level of emission, e_j^*, by investing into the industry i, whose output is necessary for industry j production, and requiring industry i firms to only sell to industry j firms whose direct emissions do not exceed e_j^*. As long as $S \leq S_i^\star$ it is optimal for the SRF to fully exploit the grip provided by capital friction and require direct emissions $\hat{e}_i = \left(\frac{K_i-S}{K_i-\eta_i S}\right)^{\frac{1}{1-\alpha_{ij}}}$. For $S \geq S_i^\star$ the social optimum is achieved. Let $i_l := \arg\min_{i=1,2} S_i(e_i^*)$, that is the industry for which the amount of SRF capital needed to induce firms to adopt the industry first best emission level e_i^* is smallest. Let $i_s = \max U(1, e_i^*)$, that is the industry on which it is optimal to induce the social optimum level e_i^*, given that the other industry is in a laissez faire situation. Then we have

Proposition 11 *In order to maximize impact using direct and downstream emission cap,*

1. *A small enough SRF should invest all its capital in industry $j \neq i_s$ and have emission caps $\hat{e}_j = \left(\frac{K_j-S}{K_j-\eta_j S}\right)^{\frac{1}{1-\alpha_{ji}}}$, $\hat{e}_{Dj} = e_{i_s}^*$.*
2. *A large enough SRF should invest all its capital in industry $i = i_l$ and have emission caps $\hat{e}_i = \left(\frac{K_i-S}{K_i-\eta_i S}\right)^{\frac{1}{1-\alpha_{ij}}}$, $\hat{e}_{Di} = e_i^*$. If $S \geq S_{i_l}(e_{i_l}^*)$ then maximum social welfare is achieved.*

Proof of Proposition 11.

1. Take $S > 0$ but arbitrarily small, then by investing in industry 1 the SRF can only impose arbitrary small reduction in direct emissions, whereas using indirect downstream emission caps it can induce $e_2 = e_2^*$. The social welfare will be close to $U(1, e_2^*)$. The specular strategy brings $U(e_1^*, 1)$. Thus S should be invested in the firm different from i_s.

2. If $S \geq S_i^\star$, then by investing in industry i and imposing $\hat{e}_i = e_i^*$ and $\hat{e}_{Di} = e_j^*$, the SRF achieves the first best. If $S_i^\star < S < S_j^\star$. Then the social optimum can be achieved by investing S in industry i. By continuity, if $S < S_i^\star$ but close to $S_i^\star$ investing into i and requiring the emission caps described in the proposition, the SRF maximizes impact.

■

There are two elements that make downstream emission caps particularly impactful: first it is key that downstream industry cannot replace that input with another one or buy the same input from a non-complying firms. In practice this corresponds to cases where the upstream industry is in an oligopoly, think for example of Microsoft and Apple as the upstream industrt. The second element is that SRF financed firms can commit not to sell their output to customers whose pollution are too high. This is more difficult to implement as it would require that ownership of upstream good is not completely transferred to the buyer. Otherwise a clean downstream firm could profit by buying an extra amount of the upstream good from the compliant producer and sell it to dirty downstream firms.

9.0.2 Minimizing portfolio footprint

If investors social preferences care about the direct emissions of companies in their portfolio, then the minimization of footprint by the SRF leads to the same solution as in Proposition 4. Note that if instead indirect upstream emission are also accounted for in social preferences, then in order to minimize footprint, the SRF can impose arbitrary small downstream emission caps. This clearly can lead to a decrease of social welfare when compared to the laissez-faire benchmark.

A.2 Micro-foundation of the capital matching frictions (For online publication)

We consider the following matching mechanism between entrepreneurs and capital. Fix industry i. In equilibrium K_i is the total amount of capital invested in the industry, and is also equal to the equilibrium mass of entrepreneurs in the industry. Let s_i be the fraction of this capital coming from SRF. We can see the matching as the result of the following dynamic process. In every period t entrepreneur that have not been yet finance and capital that has not been invested yet are randomly matched. Entrepreneur who are not financed in t come back to the matching market in $t+1$. Capital that has not be invested in t is returned to investors. A fraction s_i of this capital is reinvested via SRF, the remaining $1-s_i$ is invested via non-socially responsible funds. We denote with $1-\eta_i \in [0,1]$ the hazard rate that the capital matching process stops before the entrepreneur finds an appropriate capital provider.

Lemma 5 *In every period t, an entrepreneur searching for capital is financed with probability 1 if it complies and with probability $1-s_i$ if she does not comply.*

Proof. Let denote with C_t the total amount of capital not yet matched at time t. The fraction of this capital that is managed by SRF is s_i. Let M_t denote the mass of entrepreneurs that at beginning of time t has not been matched. Let n_t be the fraction of these entrepreneur that are not compliant. All compliant entrepreneurs can be financed with socially responsible and non-socially responsible funds, thus the first result. We first show by induction that for all t $M_t = C_t = s_i$. For $t = 0$ we know that in equilibrium the total amount of capital invested in industry i and the total mass of entrepreneur in industry i are equal to K_i, thus $C_0 = M_0 = K_i$. At the end of the first period the mass of capital not invested C_1 is equal to the socially responsible capital that was matched with non-complying entrepreneurs, and is $C_1 = s_i C_0 n_0$. The mass of entrepreneur not financed equals to the non-complying entrepreneurs matched with socially responsible capital and equals $M_1 = n_0 M_0 s_i = C_1$. Note that at the end of the first round all compliant entrepreneurs have been financed, thus if $n_0 = 0$, then for all $t > 0$, $M_t = C_t = n_t = 0$ and the mayching process only lasts one round. If $n_0 > 0$ then for all $t > 0$, we have $n_t = 1$. In this case we can apply the same argument for a general period t to have that if $M_t = C_t$, and $M_{t+1} = C_{t+1} = s_i C_t$. In period t a non-compliant entrepreneur is financed only if she is matched with non-socially responsible capital and this occurs with probability $\frac{C_t - s_i C_t}{M_t} = 1 - s_i$ because $C_t = M_t$. ■

Consider a compliant entrepreneur. She will be funded in the first period and exit the matching market. Her revenue will be $V_{ic} = \pi_{iC}\lambda$, where π_{iC} is the profit of compliant firm in industry i.

Non-compliant entrepreneurs can only be financed with non-socially responsible capital. From the above lemma, as long as a non-complying entreprenur has not been financed, her expected payoff is

$$V_{in} = (1 - s_i)\lambda\pi_{iN} + s_i\eta_i V_{in}$$

That is, with probability if the $(1 - s_i)$, the entrepreneur is matched with non-socially responsible capital. She starts the firm immediately and retains a fraction λ of a non-compliant firm profit π_{iN}. With probability s_i the entrepreneur faces a socially responsible capital provider, she is not financed and has to go back to the matching market that will provide with a new capital provider with probability η_i. This implies

$$V_{in} = \lambda\pi_{iN}\left(\frac{1 - s_i}{1 - \eta_i s_i}\right).$$

The entrepreneur will choose a compliant technology only if $V_{in} \leq V_{ic}$, that is

$$\pi_{iN}\underbrace{\left(\frac{1 - s_i}{1 - \eta_i s_i}\right)}_{\Phi_i(s_i)} \leq \pi_{iC}$$

That corresponds to condition (15). Note that in equilibrium the SRF chooses its portfolio and the emission caps as to induce all entrepreneurs to comply. Thus all entrepreneurs are matched in the very first round of the our search game, that hence only lasts one period.

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