The Central Bank, the Treasury, or the Market: Which One Determines the Price Level?*

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October 24, 2021

Abstract

This paper studies a model in which the price level is the outcome of dynamic strategic interactions between a fiscal authority, a monetary authority, and investors in government bonds and reserves. The "unpleasant monetarist arithmetic" whereby aggressive fiscal expansion forces the monetary authority to chicken out and inflate away public liabilities may be contained by market forces: Monetary dominance prevails if such fiscal expansion is met with a higher real interest rate on public liabilities, due for example to the crowding out of private investment opportunities. The model delivers empirical implications regarding the joint dynamics of public liabilities and price level, and policy implications regarding the management of central banks' balance sheets.

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1 Introduction

Public sectors in most major economies have issued since 2008 an amount of liabilities, both government debt and central-bank reserves, that is unprecedented in peacetime. Their resulting fiscal positions have led a number of observers to worry about the ability of central banks to fulfill the price-stability part of their mandates going forward.

The theoretical underpinning of this worry can be traced back to Sargent and Wallace's "unpleasant monetarist arithmetic" (Sargent and Wallace, 1981). This seminal paper shows that if a fiscal authority embarks on a path of aggressive debt issuance and deficits, the monetary authority has no option but generating sufficient seigniorage income despite the inflationary consequences if it cares about sovereign solvency. This seminal paper has initiated a large body of research studying the respective contributions of fiscal and monetary policies to the determination of the price level.

Wallace has famously described fiscal and monetary interactions as a "game of chicken" between the branches of government respectively in charge of fiscal and monetary policies. This paper takes this view seriously, and develops a full-fledged dynamic strategic analysis of the determination of the price level. We write down a model that features a fiscal authority, a monetary one, and a private sector that interact strategically. The monetary authority seeks to control the price level. It issues reserves that are the unit of account of the economy: The price of consumption units in terms of reserves is the price level. The monetary authority decides on the nominal interest rate on reserves, on the investment of the proceeds from issuing reserves, and on possible transfers ("dividends") to the fiscal authority. The fiscal authority seeks to spend optimally. It issues nominal bonds and uses the proceeds to spend or/and to repay all or part of maturing bonds. Walrasian private investors form optimal portfolio of reserves, government bonds, and private investments.

We solve for the (subgame-perfect) Nash equilibria resulting from their interactions with a focus on the resulting price level. We deem "monetary dominance" the situation in which the equilibrium price level corresponds to the target of the monetary authority. "Fiscal dominance" is the alternative in which the price level jumps above this target, and reaches instead a higher level that is consistent with the solvency of the public sector.

Two departures from Sargent and Wallace (1981) play a central role in our main insights. First, an implicit assumption in their paper is that the fiscal authority "moves first" in the sense that it can commit to a path of debt issuance and deficits for the entire future. As a second mover, the monetary authority then has to accommodate this path. By contrast, all agents repeatedly interact without commitment in our model, and so who imposes its objectives in equilibrium is endogenously driven by the primitives of the economy. Second, the government faces an infinitely elastic demand for bonds in Sargent and Wallace (1981). By contrast, bond and reserve issuances push up the real interest rate in our model.[1]

The reason these two features of the model play an important role is as follows. The fact that the fiscal authority cannot commit to future deficits implies that if it wants to force the monetary authority to "chicken out" and inflate away public liabilities in the future as in Sargent and Wallace (1981), it must credibly eliminate any future fiscal wiggle room by borrowing now against any future resources and spending the proceeds right away. This may require a large issuance of government bonds. Such a large issuance in turn pushes the (real) interest rate at a higher level than the one that would prevail if the fiscal authority was not seeking to impose fiscal dominance this way. If the cost from borrowing such large amounts at such a high rate offsets the benefits from forcing the monetary authority to inflate away legacy liabilities, then the fiscal authority does not enter into this "Sargent-Wallace" behavior, and there is monetary dominance. Remarkably, the central bank, despite having neither commitment power nor fiscal support, can fulfill its price-level mandate in this case. The only commitment that is required from the government is that it lets the central bank manage its balance sheet independently and, of course, that it refrains from renegotiating its mandate.[2] Otherwise there is fiscal dominance, and the price level is dictated by sovereign solvency, echoing the fiscal theory of the price level.

In sum, one may describe our contribution as an answer to the question that Sargent and Wallace (1981) raise in conclusion of their unpleasant arithmetic: *"The question is, Which authority moves first, the monetary authority or the fiscal authority? In other words, Who imposes discipline on whom?"* We show that the monetary authority imposes its views if and only if any fiscal victory in the "game of chicken" is a Pyrrhic one due to excessive borrowing at an excessively high interest rate. As a result, our model can account for both the prevalence of monetary dominance in "normal times," and for the

[1] Specifically, they do so by crowding out private investment. Yet any other reason for a downward-sloping demand for public securities would have the same implications.

[2] Presumably, reneging on central-bank independence is politically more costly and institutionally more complex than merely embarking on aggressive fiscal expansion.

fact that fiscal dominance may arise, but only so when public finances are sufficiently stretched. In this latter situation, the fiscal authority finds doubling down on borrowing preferable to fiscal consolidation.[3]

Since monetary dominance arises when the gain from inflating away legacy liabilities is small and the cost from spending future tax capacity right away is large, it is more likely to prevail under the following conditions: small legacy liabilities, profitable private investment opportunities that entail a large impact of crowding out on the interest-rate level, a large future tax capacity, and a "patient" fiscal authority. Fiscal dominance prevails otherwise.

These forces generate interesting joint dynamics for public finances, the real interest-rate level, and the price level. The regime may switch from monetary to fiscal dominance over time as the "net wealth" of the public sector, which is in turn driven by the endogenous interest rate, decreases. The equilibrium may in particular be such that interest rates are low and price levels on target despite large public debt and deficits for a long period of time, at the end of which inflation picks up and fiscal consolidation arises.

Finally, we study a version of the model in which dynamic inefficiency enables the public sector to issue unbacked reserves and bonds—pure bubbles. Of course, there are in this case multiple equilibria. We construct in particular equilibria in which the behavior of the private sector can lead to any price level. In these equilibria, investors prick the bubbles on public liabilities in the off-equilibrium paths in which the public sector seeks to deviate from this level. This interference of market discipline with fiscal and monetary interactions, leading to a situation of "market dominance," is novel to our knowledge.

Our model has several policy implications, on the normative side to start with. First, fiscal requirements in the form of a cap for debt can substitute market forces to discipline the fiscal authority—however, these fiscal requirements may be time-inconsistent, especially in high-debt environments. Second, when monetary dominance is out of reach, monetary policy can still deter highly inflationary fiscal policies with a small preemptive inflation.

On the positive side, our paper emphasizes that the net public liabilities in the hands of the private sector are the key variable to keep track of the risk of fiscal dominance. Second, our paper emphasizes that the game of chicken has an important timing component

[3]See the related-literature section for historical examples of such episodes.

whereby public debt increases before inflation picks up.

Related literature. Our paper belongs to the very rich literature on optimal fiscal and monetary policies following Calvo (1978) and Lucas and Stokey (1983). As envisioned in this literature, nominal public liabilities lead to a time-inconsistency problem for public authorities. Furthermore, this literature has also discussed the importance for this time-inconsistency problem of the public sector's net nominal liabilities, i.e., nominal debt and money in the hands of the private sector (see Alvarez et al., 2004; Persson et al., 2006, among others). In our framework, delegation of monetary tools to the monetary authority helps solve the time-inconsistency of the government, but imperfect delegation due to limited commitment creates a game between fiscal and monetary authorities.

From this perspective, we are connected to the literature on the interactions between monetary and fiscal policies pioneered by Sargent and Wallace (1981) (see Leeper, 1991; Sims, 1994; Woodford, 1994, 1995; Cochrane, 2001, 2005; McCallum, 2001; Buiter, 2002; Niepelt, 2004; Jacobson et al., 2019, among others). As in Sargent and Wallace (1981), the monetary authority can adjust seigniorage revenue to help the fiscal authority satisfy its budget constraint. The simple economy in which we cast our game of chicken relates in particular to that in which Bassetto and Sargent (2020) study fiscal and monetary interactions. Our paper is also closely connected to the papers that identify fiscal requirements such that the central bank can attain its price stability objective, including fiscal rules (e.g. Woodford, 2001) or a ring-fenced balance sheet (e.g Sims, 2003; Bassetto and Messer, 2013; Hall and Reis, 2015; Benigno, 2020). Closer to our paper, Martin (2015) finds as we do that fiscal irresponsibility leads to long-term inflation. Finally, Coibion et al. (2021) provide causal evidence that private agents do anticipate inflationary effects of fiscal policy: Their evidence that households associate future debt levels with inflation is consistent with our model's result that future net public liability is a key determinant of central bank's future incentives to inflate. In line with this literature, our paper aims at precisely describing the respective markets in which fiscal and monetary authorities intervene, as well as their instruments and budget constraints. Our contribution is to explicitly model the strategic interactions between fiscal and monetary authorities in such an environment.

That fiscal and monetary authorities may have ex-post conflicting objectives is a natural assumption. This has been in fact the main rationale behind setting up independent central banks. This is also motivated by the large set of evidence that authorities do not necessarily cooperate and, instead, try to impose their views on each other (see, e.g., Mee (2019) for a historical analysis of the rise of an independent Bundesbank, Silber (2012) for the Volker era, and Bianchi et al. (2019) for evidence that markets reacted to Trump's comments on monetary policy). In this respect, this makes our paper closer to an older literature (Alesina, 1987; Alesina and Tabellini, 1987; Tabellini, 1986, e.g.) that investigates the equilibria of games between multiple branches of government. More recent contributions include Dixit and Lambertini (2003) or the literature that explores disciplining mechanisms for the public sector in models following Barro and Gordon (1983a,b), such as Halac and Yared (2020).

With respect to this literature, our contribution is to provide an explicit set of instruments to both the fiscal and the monetary authorities as well as a game-theoretic foundation to fiscal and monetary interactions. Our approach of the resulting macroeconomic game follows Chari and Kehoe (1990), Stokey (1991) and Ljungqvist and Sargent (2018) but extended to multiple large agents and markets. In particular, our approach to model markets follows Bassetto (2002) as, in our setting, price levels as well as debt prices are market equilibrium objects.

Finally, our paper relates to the literature building on the idea that public debt satisfies private liquidity demand. This literature goes back to Diamond (1965) and has been widely studied since (see Woodford, 1990; Aiyagari and McGrattan, 1998; Holmström and Tirole, 1998, among others). Krishnamurthy and Vissing-Jorgensen (2012) show in the data that public debt shares many of the properties of money. More recent contributions on optimal public liquidity supply include Angeletos et al. (2020), Azzimonti and Yared (2019), or Gorton and Ordonez (2021). Our paper extends some of the insights of this literature to a context where multiple authorities can issue liquidity vehicles and behave strategically. In addition, we investigate both cases where public liabilities are backed by real resources and where they are unbacked and stem from a bubble. Related to this literature, some recent contributions investigate the implication of bubbles on monetary policy (see Galí, 2014; Asriyan et al., 2019, among others) and on fiscal/monetary interactions (Bassetto and Cui, 2018; Brunnermeier et al., 2020). We show that when public liquidity supply is a self-fulfilling phenomenon, monetary or fiscal dominance is essentially driven by the private sector's expectations—a situation that we deem "market dominance".

2 Model

Our model features a fiscal authority and a monetary one that interact strategically. They also interact with the private sector in the markets for their respective liabilities. The monetary authority issues reserves that are the unit of account of the economy, and seeks to control the price level. The fiscal authority seeks to consume optimally and issues nominal bonds.

2.1 Setup

Time is discrete. There is a single consumption good. The economy is populated by a private sector and by a public one.

Private sector. At each date, a unit mass of agents, deemed "savers", are born. They live for two dates and value consumption only when old, at which time they are risk-neutral. They are each endowed with one unit of the consumption good when young. A storage technology is available to savers at each date. Each saver can transform x consumption units into $f(x)$ units at the next date. We suppose that $r(.) \equiv f'(.)$ exists and is a decreasing, strictly convex bijection mapping $(0, 1]$ into $[r(1), +\infty)$.[4] This marginal return $r(.)$ on private storage will play a central role in the analysis as the opportunity cost of public funds. In the absence of other savings vehicles, savers simply save all the endowment in the storage technology and consume $f(1)$ when old.

Public sector. The public sector features a fiscal authority F and a monetary authority M.

Monetary authority. The monetary authority issues reserves and sets the (gross) nominal interest rate R_t on them. Reserves are claims of infinite maturity. A unit of reserves at date t is a claim to R_t units of reserves at date $t + 1$. Reserves are the unit of

[4]Here, we assume decreasing returns on storage at the individual level. Our framework and results readily extend to the alternative assumption of decreasing returns at the aggregate level, in which case each individual saver's return on storage is linear.

account of the economy, and can be traded for the consumption good in the market for reserves. We denote by P_t the price level—the date-t price of the consumption good in terms of reserves in the market for reserves, by $X_t \geq R_{t-1}X_{t-1}$ the quantity of outstanding reserves at the end of date t (resulting from cumulative past issuances between 0 and t), and by x_t the quantity of goods that savers bid for reserves in the date-t market for reserves.

M can also transfer resources to F ("pay a dividend"), and θ_t denotes the real date-t transfer from M to F.

Fiscal authority. The fiscal authority issues one-period nominal bonds. A bond issued at date t is a claim to one unit of account at date $t+1$. Both savers and M can trade goods for bonds. Let B_t denote the number of bonds issued by F at date t, Q_t the price at which they are sold (in terms of reserves), and b_t and b_t^M the respective quantities of goods that savers and M respectively trade for bonds in the bond market.

F decides at each date t on the haircut or loss given default $l_t \in [0,1]$ that it applies to the bonds maturing at date t. A haircut l means that bondholders receive $(1-l)$ units of account per bond. F also consumes. Let $g_t \geq 0$ denote its date-t consumption.

Interpretation of the condition $g_t \geq 0$ as a fiscal limit. In a richer model of public finances featuring taxes and other transfers between the fiscal authority and the private sector, the counterpart of $-g_t$ would be the surplus of the government before transfers from the central bank and net bond issuances, and could be strictly positive. This primary surplus would still admit an upper bound determined by the maximum tax capacity of the government. All that matters for the analysis is that such an upper bound exists, and normalizing it to 0 here as we abstract from taxes is only for expositional simplicity.

Summary of notations. We introduced the following variables:

Interest rate on reserves set by M	R_t
Outstanding reserves at the end of date t	X_t
Goods invested by savers in the market for reserves	x_t
Price level	P_t
Bonds issued by F	B_t
Goods invested by M in the bond market	b_t^M
Goods invested by savers in the bond market	b_t
Bond price	Q_t
(Real) transfer from M to F	θ_t
Haircut on maturing bonds by F	l_t
Consumption of F	g_t

Let $\mathcal{E}_t = (R_t, X_t, x_t, P_t, B_t, b_t^M, b_t, Q_t, \theta_t, l_t, g_t)$ denote the vector of all the variables that describe the economy at date t. Appendix A states the conditions for a sequence $(\mathcal{E}_t)_{t\in\mathbb{N}}$ to form a competitive equilibrium. The remainder of the paper takes another route and studies full-fledged strategic interactions between the agents. The equilibrium paths $(\mathcal{E}_t)_{t\in\mathbb{N}}$ resulting from these interactions will all form a competitive equilibrium, though.

The rest of this section outlines the game in a standard fashion. We first define the objectives of the agents. We then present the extensive form of the game. We finally state our equilibrium concept, which is that in Ljungqvist and Sargent (2018). In order to encompass versions of the model with both finite and infinite horizons, we introduce a terminal date $T \in \mathbb{N} \cup \{+\infty\}$.

2.2 Objectives

Young savers' objective. Young savers born at $t < T$ seek to maximize their expected consumption at $t + 1$.

Objectives of F and M. For all $t < T$ and at T if $T \in \mathbb{N}$, the respective date-t objectives of F and M are:

$$U_t^F = \sum_{s=t}^{T} \beta^{s-t}(g_s - \alpha_F \delta_s), \tag{1}$$

$$U_t^M = -\sum_{s=t}^{T} \beta^{s-t}(\mid P_s - P_s^M \mid + \alpha_M \delta_s), \tag{2}$$

where $\delta_s = \mathbb{1}_{\{l_s>0\}}$, $\beta \in (0,1)$, $\alpha_F, \alpha_M > 0$, and $P_s^M > 0$.[5] In words, the variable δ_t is equal to 1 in case of an outright default on a government bond due at date t, and to 0 otherwise. In sum, each authority $X \in \{F; M\}$ incurs a cost α_X in case of sovereign default.[6] The fiscal authority also values consumption (but does not care about the price level), whereas the monetary authority also finds it costly to deviate from a given target P_t^M for the date-t price level.[7] Our results would carry over if we assumed that M and F both cared about price level and government expenditures, albeit with sufficiently different weights. The assumed stark difference in objectives simplifies the exposition.

We will focus for brevity on the case in which α_F is arbitrarily large. In other words, F is willing to do whatever it takes to avoid sovereign default. Accordingly, we will see that the only situation in which F is forced to default at date t is when repaying maturing bonds would violate the positivity constraint on government consumption $g_t \geq 0$.[8]

Finally, we assume that holding (2) fixed, M prefers to maximize (1). Such lexicographic preferences only serve to eliminate equilibria that would crucially rely on M not caring at all about the government's consumption.

2.3 Extensive-form game

For a given date $0 \leq t < T$, consider a history $h_t = (R_s, X_s, x_s, B_s, b_s^M, b_s, l_s, g_s)_{s<t}$.[9] Date-$t$ is split into three consecutive stages: the reserve market, the bond market, and

[5] Different discount factors for M and F would not qualitatively affect the analysis.

[6] Costs from outright default are exogenous here. Section 5 discusses equilibria in which savers create endogenous default costs.

[7] Results would be similar with an inflation target. Section 3.6 explains why the creation of a monetary authority with such an objective is ex-ante desirable.

[8] In a more general model in which F faces several options (raising some taxes, cutting some subsidies), it would default when it is less costly than exercizing any of these options.

[9] Notice that $(P_s)_{s<t}$ and $(Q_s)_{s<t}$ are not in h_t because, as shown below, they are derived from h_t out of market-clearing conditions. Nor is $(\theta_s)_{s<t}$ which is also given by h_t, and by the flow budget constraint of M.

finally default and consumption decisions by F. Old date-t savers sell their reserves in the reserve market, redeem their maturing bonds at the final stage, collect their proceeds from private storage, and consume.[10] The other agents—F, M, and young savers—interact as described below. As is standard, the notation $a(b)$ below means that action a is conditional on the information set b. A strategy profile must then describe for each action a the mapping $a(.)$ of every possible information set into an action choice. We deem "action" of the private sector the aggregate quantity that it invests in reserve and bond markets, a natural abuse of language given our equilibrium concept below.

Stage 1: Market for reserves.

1. M selects $R_t(h_t) \geq 0$ and $X_t(h_t) \geq R_{t-1}X_{t-1}$, issuing new reserves $X_t(h_t) - R_{t-1}X_{t-1}$ on top of $R_{t-1}X_{t-1}$ sold by old savers.

2. Young savers invest an aggregate quantity $x_t(h_t, R_t, X_t) \in [0, 1]$ of consumption units in the market for reserves. The price level P_t is given by $P_t x_t = X_t$, with the convention that it is infinite if $x_t = 0$.

Stage 2: Bond market.

3. F issues $B_t(h_t, R_t, X_t, x_t) \geq 0$ bonds.

4. M invests $b_t^M(h_t, R_t, X_t, x_t, B_t) \in [0, (X_t - R_{t-1}X_{t-1})/P_t]$ consumption units in the bond market.

5. Young savers invest $b_t(h_t, R_t, X_t, x_t, B_t, b_t^M) \in [0, 1 - x_t]$ aggregate consumption units in the bond market. The bond price Q_t is given by $Q_t B_t = P_t(b_t + b_t^M)$, with the convention that it is infinite if $B_t = 0$.

Stage 3: Default and consumption.

6. F selects a haircut on maturing bonds $l_t(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t) \in [0, 1]$ and consumption $g_t(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t) \geq 0$ such that

$$Q_t B_t + P_t \theta_t = P_t g_t + (1 - l_t) B_{t-1}, \tag{3}$$

[10] At date 0, old savers sell reserves $R_{-1}X_{-1} > 0$, and, for simplicity, we assume away any legacy bonds ($B_{-1} = 0$).

where

$$\theta_t = \frac{X_t - R_{t-1}X_{t-1}}{P_t} - b_t^M + \frac{(1-l_t)b_{t-1}^M P_{t-1}}{Q_{t-1}P_t}. \tag{4}$$

A date-t strategy profile $\sigma_t = (R_t, X_t, x_t, B_t, b_t^M, b_t, l_t, g_t)$ describes all the above date-t actions of each agent given all possible information sets. Figure 1 summarizes these three stages.

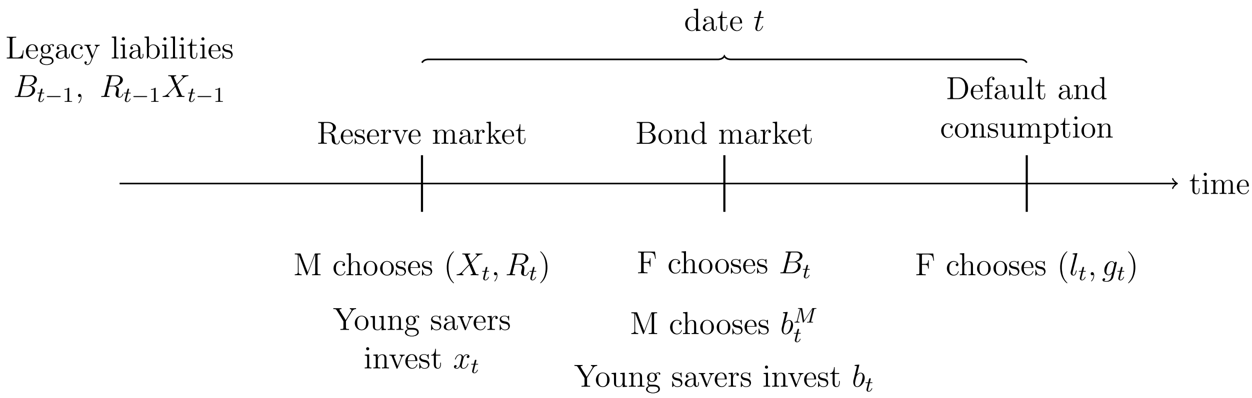

Figure 1: Intradate timing of the game.

If $T = +\infty$ then this generic date t fully describes the extensive form of the infinite-horizon game. Otherwise, there is also a terminal date T:

Terminal date T. If $T \in \mathbb{N}$, then no savers are born at T and this terminal date T features two stages. Let $\bar{x}, \bar{b} > 0$.

1. M receives an exogenous terminal demand for reserves $\bar{x}$ from unmodelled agents and issues $X_T(h_T) - R_{T-1}X_{T-1} \geq 0$. The price level P_T solves $P_T\bar{x} = X_T$.

2. F receives an exogenous fiscal income $\bar{b}$, and decides on $l_T(h_T, X_T) \in [0,1]$ and $g_T(h_T, X_T) \geq 0$ such that

$$g_T = \bar{b} + \theta_T - \frac{(1-l_T)B_{T-1}}{P_T}, \quad \theta_T = \frac{X_T - R_{T-1}X_{T-1}}{P_T} + \frac{(1-l_T)b_{T-1}^M P_{T-1}}{Q_{T-1}P_T}. \tag{5}$$

A strategy profile for the whole game is a sequence $\sigma = (\sigma_t)_{t \leq T}$ if $T \in \mathbb{N}$ and $\sigma = (\sigma_t)_{t \in \mathbb{N}}$ otherwise.

Remarks. Two remarks are in order. First, the assumption that F is first-mover in the bond market (formally B_t is in the information set of M when it decides on b_t^M) is only to fix ideas: The results are similar when M moves first instead in the bond market. Similarly, the order of b_t^M and b_t is immaterial.[11] Second, F makes haircut and consumption decisions understanding that the transfer θ_t that it receives from M is affected by the haircut. In other words, F must satisfy its flow budget constraint (3) when choosing l_t and g_t understanding that θ_t must satisfy that of M given by (4).

Relationship to the competitive equilibrium. Five relations satisfied at each date define a standard competitive equilibrium in Appendix A: reserve and bond market clearing, the flow budget constraints of F and M, and the requirement that young savers invest optimally. The flow-budget constraints are built in the action sets of F and M and so are satisfied for all feasible actions, on and off the equilibrium path, from (3) and (4). Similarly, reserve and bond markets clear on and off the equilibrium path by construction of P_t and Q_t. The last condition, the optimal behavior of price-taking savers, is part of the equilibrium definition that follows.

2.4 Equilibrium concept

Definition 1. ***(Equilibrium)*** *An equilibrium is a strategy profile σ such that:*

1. *Each action by F and M is optimal given its information set and its beliefs that the future actions are taken according to the strategy profile.*

2. *Date-t young saver $i \in [0,1]$ optimally invests $x_t^i = x_t$ in the reserve market given (h_t, R_t, X_t), P_t, and the strategy profiles for all future actions, and optimally invests $b_t^i = b_t$ in the bond market given $(h_t, R_t, X_t, x_t, B_t, b_t^M)$, Q_t, and the strategy profiles for all future actions.*

Our equilibrium concept is that of Ljungqvist and Sargent (2018), which adapts plain game-theoretic subgame perfection to the situation in which a "large" player interacts with Walrasian agents. We extend this concept to the case in which there are two such large players, a monetary and a fiscal authority. Very intuitively, F and M play against

[11] All that matters is that M and savers do not move simultaneously in the bond market as this would generate multiple equilibria.

"the private sector", which responds to their supply of reserves and bonds with aggregate demands in reserve and bond markets. Reserve and bond prices then result from market clearing. In equilibrium, these "actions" of the private sector correspond to prices and aggregate quantities that are consistent with optimal behavior by each individual saver given fiscal and monetary policies. Appendix C offers a formal version of this equilibrium definition that formally spells out the objective of each agent at each step.

Backed versus unbacked public liabilities. It is important to stress that in the finite-horizon version of the model, the exogenous demand for money $\bar{x}$ and fiscal revenue $\bar{b}$ will back reserves and bonds. The incompleteness inherent to overlapping generations plays no role in the rise of public liabilities, and we could dispense with it. Conversely, the infinite-horizon model assumes away such backing and public liabilities must be bubbles enabled by dynamic inefficiency. We could consider a third case in which the public sector has real revenue and the horizon is infinite, possibly creating room for a bubbly component in the price of backed public liabilities. We would however not gain any insight relative to the two polar cases studied here—finite horizon with backed liabilities and infinite horizon with unbacked liabilities.

The rest of the paper analyzes the game in three steps. Section 3 first solves for the finite-horizon game with two dates ($T = 1$). This enables us to introduce the central insights of the paper in the simplest environment. Section 4 then extends the two-date analysis to all finite games. In these cases of finite horizon, subgame perfection boils down to sequential rationality, and so we can solve the game using backwards induction. Finally, Section 5 tackles the infinite-horizon game.

3 Two-date game ($T = 1$)

This section shows in the simplest two-date game why and how the fiscal authority may seek to force the monetary authority away from its price-level objectives. It also shows how the monetary authority may be able to deter such fiscal behavior.

We solve the game backwards. We first characterize how the fiscal authority F decides on default at the final stage of date 1, and then how the monetary authority M, rationally anticipating this, optimally sets the date-1 price level in the date-1 reserve market. We then move on to date 0, studying date-0 debt issuance decision by the fiscal authority.

This is the keystone of the analysis, showing how date-0 public debt issuance may lead to either fiscal or monetary dominance at date 1. Finally, we analyze monetary policy in the initial reserve market and characterize the equilibrium outcome.

3.1 Date-1 price level

At the terminal stage of date 1, the fiscal authority F prefers to honor its debt whenever possible since it otherwise incurs an arbitrarily large fixed cost of default α_F. The only constraint possibly preventing repayment is that date-1 government consumption g_1 be positive. Formally, F avoids default if and only if setting the haircut l_1 to $l_1 = 0$ is compatible with $g_1 \geq 0$. Condition (5) expressing F's terminal consumption as a function of all other actions shows that this is equivalent to the solvency constraint:

$$P_1(\bar{x} + \bar{b}) \geq R_0 X_0 + B_0 - \frac{b_0^M P_0}{Q_0}. \tag{6}$$

Condition (6) admits a straightforward interpretation. The left-hand term is the nominal value of total public resources at date 1 and the right-hand term are the net total liabilities of the public sector, that is, the liabilities in the hands of the private sector, equal to the gross liabilities $R_0X_0 + B_0$ minus holdings of government debt by the monetary authority $b_0^M P_0/Q_0$.

In the date-1 reserve market, the monetary authority M can, by setting X_1 sufficiently large, raise the price level P_1 so that (6) holds. A larger price level P_1 frees up resources available for bond repayments by eroding the real value of outstanding reserves R_0X_0, and reduces the real value of maturing bonds B_0. We denote by P^F the smallest price level such that solvency constraint (6) holds:

$$P^F \equiv \frac{R_0 X_0 + B_0 - \frac{b_0^M P_0}{Q_0}}{\bar{x} + \bar{b}}. \tag{7}$$

By construction, the fiscal authority does not consume ($g_1 = 0$) as soon as $P_1 = P^F$ so that (6) holds with equality.

Whether the monetary authority is willing to set a sufficiently high price level to avoid default ($P_1 \geq P^F$) depends on its preferences given by its cost of default α_M and by its date-1 price level objective P_1^M. The following proposition describes the optimal policy of the monetary authority in the date-1 reserve market and the resulting continuation equilibrium. Depending on the history h_1 at the outset of date 1, this equilibrium is of one of three types: monetary dominance, fiscal dominance, or default.

Proposition 1. ***(Terminal date 1)*** *Let* $\underline{P}_1 \equiv \max\left\{P_1^M; R_0X_0/\bar{x}\right\}$. *Given history* $h_1 = (R_{-1}, X_{-1}, R_0, X_0, x_0, B_0, b_0^M, b_0, l_0, g_0)$, *date 1 unfolds as follows.*

1. *Monetary dominance: If* $P^F \leq \underline{P}_1$, *M sets the date-1 price level at* $\underline{P}_1$ *by setting* $X_1 = \bar{x}\underline{P}_1$. *F fully repays maturing bonds:* $l_1 = 0$, *and consumes* $g_1 = \bar{x} + \bar{b} - \left(B_0 - b_0^M P_0/Q_0 + R_0X_0\right)/\underline{P}_1$.
2. *Fiscal dominance: If* $\underline{P}_1 < P^F \leq \underline{P}_1 + \alpha_M$, *M sets the date-1 price level at* P^F. *F fully repays maturing bonds:* $l_1 = 0$, *and does not consume:* $g_1 = 0$.
3. *Default: Otherwise, M sets the date-1 price level at* $\underline{P}_1$. *F fully defaults on* B_0: $l_1 = 1$, *and consumes* $g_1 = \bar{x} + \bar{b} - R_0X_0/\underline{P}_1$.

Proof. See Appendix B.1. □

Figure 2 illustrates how the date-1 price level P_1 evolves as net public liabilities $R_0X_0 + B_0 - b_0^M P_0/Q_0$ increase.

The case of "monetary dominance". When net public liabilities are sufficiently low that P^F is lower than $\underline{P}_1$, F can satisfy its solvency constraint even when the monetary authority M sets the price level at $\underline{P}_1$. This situation corresponds to the left-hand part of Figure 2.

The subcase of "reserve overflow". The reserves sold by old savers R_0X_0 might be strictly larger than $\bar{x}P_1^M$, so that the price level must be at least equal to $R_0X_0/\bar{x} = \underline{P}_1 > P_1^M$. In this case, M has manufactured its own lower bound on the date-1 price level when deciding on (R_0, X_0) at date 0, thereby barring itself from reaching its date-1 price level target. We will see below that, given the perfect-foresight environment and in the absence of a zero lower bound on the interest rate, M can ensure that this does not occur along the equilibrium path, that is, $\underline{P}_1 = P_1^M$ in equilibrium when there is either monetary dominance or default at date 1.[12]

[12] We will also see that there exist cases in which M deliberately uses this to commit to a date-1 price level that it finds ex-post excessive (see Proposition 6).

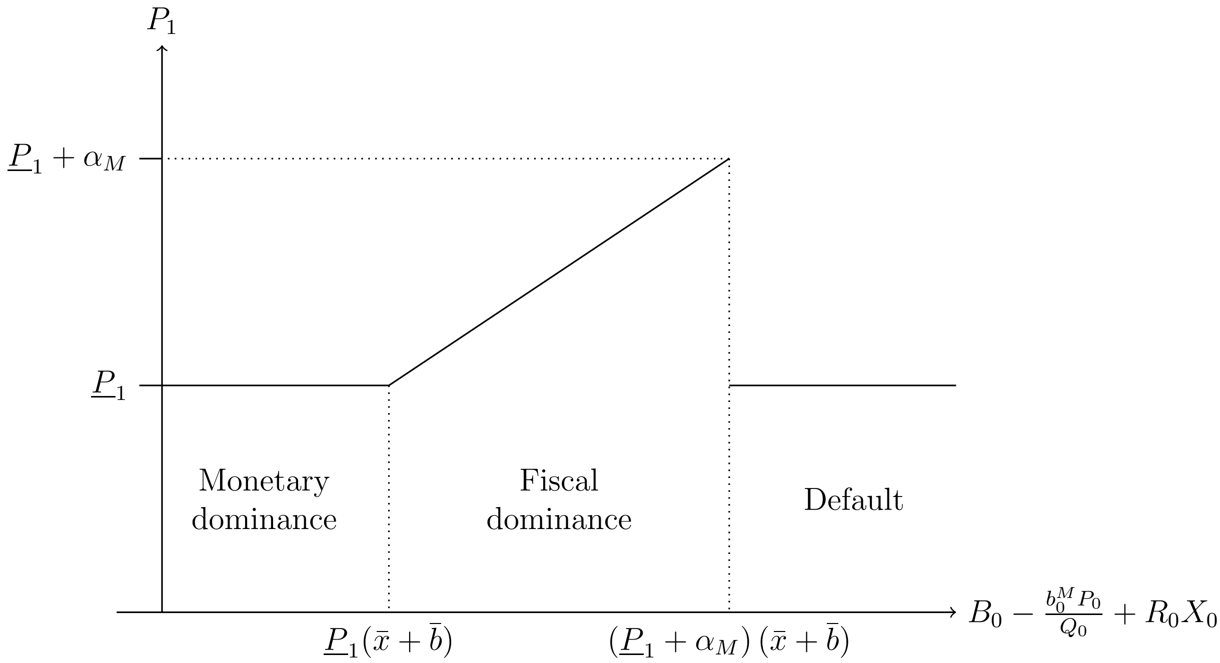

Figure 2: Date-1 price level P_1 as a function of net public liabilities held by the private sector $(B_0 - b_0^M P_0/Q_0 + R_0 X_0)$.

The case of "fiscal dominance". Suppose to fix ideas that $P_1^M \geq R_0 X_0/\bar{x}$ so that $\underline{P}_1 = P_1^M$. When net public liabilities are such that P^F exceeds P_1^M, the monetary authority cannot set the price level at its objective without pushing the fiscal authority to default. If the monetary authority lets F default, it is able to set the price level on target, and so its date-1 utility is $-\alpha_M$. As a result, the monetary authority accepts to raise the price level to any $P^F \leq P_1^M + \alpha_M$. This situation is one of fiscal dominance in which the price level strays away from the target of the monetary authority, and is dictated by the overall budget constraint of the public sector. This situation corresponds to the middle part in Figure 2.

An important feature of the fiscal-dominance case is that date-1 government consumption equals 0 ($g_1 = 0$): As $P_1 = P^F$, net public liabilities equal net public resources so that there are no resources left for the government to consume. To see why no terminal consumption is a necessary condition for date-1 fiscal dominance, suppose otherwise that the equilibrium is such that $P_1 > P_1^M$ and $g_1 > 0$. Then M could elicit a slightly smaller date-1 price level in the reserve market. The fiscal authority would then find it optimal to reduce its consumption so as to remain solvent, and so M would be strictly better off, a contradiction.

The case of "default". Proposition 1 also shows that if the cost for M to avert default exceeds α_M, then M prefers to set the price at $\underline{P}_1$, and to let F default. The cost of averting default exceeds α_M when net public liabilities are large so that $P^F > \underline{P}_1 + \alpha_M$. This situation corresponds to the right-hand part of Figure 2. We will see below that default never occurs in equilibrium, but that the requirement that public liabilities be such that $P^F \leq \underline{P}_1 + \alpha_M$ to avoid default plays a central role in the strategy of F.

3.2 Date-0 government consumption

Having solved for date 1, we now move on to date 0 solving backwards for its various stages. Start with the third stage in which F decides on its consumption.[13] The transfer to the fiscal authority F from the monetary authority M is $\theta_0 = x_0 - R_{-1}X_{-1}/P_0 - b_0^M$, equal to the resources from reserve issuances $x_0 - R_{-1}X_{-1}/P_0$ net of bond purchases b_0^M. F consumes these resources on top of the amount $b_0 + b_0^M$ collected in the bond market. F thus consumes $x_0 + b_0 - R_{-1}X_{-1}/P_0$, independent of the resources spent by the monetary authority to purchase bonds b_0^M.

3.3 Date-0 bond market

The second stage of date 0, the market for government bonds, is central to the analysis. It showcases the following central insight of the paper. The fiscal authority *can* always use the date-0 bond issuance to force fiscal dominance at date 1, thereby generating resources from inflating away reserves R_0X_0. However, it does not *want* to do so when this frontloads its consumption too much relative to fiscal policies that lead to monetary dominance at date 1. To arrive at this insight, we first briefly describe the impact of M's intervention b_0^M in the bond market. We then analyze F's optimal bond issuance problem. We show that solving this problem boils down to comparing the utilities of F from two simple issuance policies, one which is consistent with monetary dominance at date 1—the "price-level taking" debt level— and one that is consistent with date-1 fiscal dominance—the "Sargent-Wallace" debt level.

[13] Given the assumed absence of debt maturing at date 0 ($B_{-1} = 0$), there is no date-0 default decision, and the value of the haircut l_0 is immaterial.

Bond purchases by M. As mentioned above, M's bond purchases b_0^M have no impact on F's date-0 consumption since F receives as date-0 dividends the fraction of M's resources that it does not collect in the bond market. Yet bond purchases are relevant since date-1 net public liabilities $R_0X_0 + B_0 - b_0^M P_0/Q_0$ decrease with respect to b_0^M.[14] Thus, from Proposition 1, bond purchases affect the date-1 outcome. Referring to the three areas in Figure 2, M strictly benefits from increasing its date-0 bond purchases either if it moves the date-1 equilibrium leftward out of the default area into the fiscal or monetary dominance areas, or if it shifts the equilibrium to the left within the fiscal-dominance area. Changes in b_0^M that leave the outcome within the monetary-dominance area have no impact on the date-1 price level.[15]

Bond issuance. We now describe how much debt F issues in the bond market. From Proposition 1, depending on the amount B_0 of bonds issued by F and on purchases by M, the date-1 continuation will be such that there is monetary dominance, fiscal dominance, or default. It is easy to see that default cannot be an equilibrium outcome. Since default is total ($l_1 = 1$) when it occurs from Proposition 1, savers' optimality would imply $b_0 = 0$ in case of date-1 default, and F would receive (at best) only resources from M in the bond market against an empty promise. But then F would be strictly better off not issuing bonds ($B_0 = 0$) and receiving these resources as a dividend from M at stage 3 of date 0, as this averts default leaving g_0 and g_1 unchanged. Thus we focus on bond issuances that lead to either monetary or fiscal dominance. We now describe optimal debt issuance by F conditional on each of these regimes. Namely, we first study which debt level grants F the highest date-0 utility among all the levels that lead to date-1 monetary dominance. We then describe the optimal debt level among those that generate date-1 fiscal dominance.

Monetary dominance. A first option for the fiscal authority is to issue debt taking as given the future price level $\underline{P}_1$. As mentioned above, b_0^M does not affect the price level within the monetary dominance area, and so, without loss of generality, one can assume that $b_0^M = 0$.[16] The fiscal authority F then seeks to optimally consume taking

[14] An increase in demand b_0^M raises the bond price Q_0. Yet Appendix B.1 shows that $b_0^M P_0/Q_0$ overall increases with respect to b_0^M.

[15] Notice that the utility of M is still affected by such bond purchases in the monetary-dominance area through its lexicographic preferences because these purchases affects the utility of F.

[16] More precisely, M and F both agree to maximize the utility of F given future monetary dominance. Thus any continuation equilibrium featuring monetary dominance in which $b_0^M > 0$ is payoff-equivalent

the date-1 price level as given, and thus issues the "price-level taking" debt level $B_0 = \underline{P}_1 r(1 - b^{PT}(x_0) - x_0) b^{PT}(x_0)$, where

$$b^{PT}(x_0) \equiv \arg\max_b \{g_0 + \beta g_1\} \tag{8}$$

$$\text{s.t. } g_0 = x_0 + b - \frac{R_{-1} X_{-1}}{P_0}, \tag{9}$$

$$g_1 = \bar{x} + \bar{b} - \frac{R_0 X_0}{\underline{P}_1} - r(1 - x_0 - b)b, \tag{10}$$

$$0 \leq b < 1 - x_0, 0 \leq g_1. \tag{11}$$

We let $(g_0^{PT}(x_0), g_1^{PT}(x_0))$ denote the consumption stream of F resulting from this program. Notice that F takes the date-1 price level as given but internalizes the impact of its bond issuance on the interest rate. The convexity of the interest rate schedule $r(.)$ leads to a consumption-smoothing motive between dates 0 and 1. This first option corresponds (in the case of an interior solution) to the blue point (g_0^{PT}, g_1^{PT}) in Figure 3.[17]

Fiscal dominance. A second option for the fiscal authority is to issue debt so that there is fiscal dominance at date 1: The date-1 price level P_1 satisfies $P_1 = P^F > \underline{P}_1$, where P^F is given by (7). It must be that $P^F \in (\underline{P}_1, \underline{P}_1 + \alpha_M]$ otherwise M would prefer default. Notice that fiscal dominance implies that F cannot consume at date 1 from Proposition 1. Thus, denoting (g_0^{SW}, g_1^{SW}) the optimal consumption pattern that F can obtain conditionally on date-1 fiscal dominance, it must be that $g_1^{SW} = 0$ and that g_0^{SW} maximizes date-0 consumption over all the debt levels leading to date-1 fiscal dominance. We state in the proposition below that the fiscal authority optimally selects a debt level—that we deem the "Sargent-Wallace" debt level—such that the date-1 price level is $\underline{P}_1 + \alpha_M$, the largest value of P^F that does not trigger default. This Sargent-Wallace debt level and the associated government consumption is depicted by the red point on Figure 3. That $g_1^{SW} = 0$ of course means that this point is on the x-axis. The gain in terms of resources for the public sector associated with a price level P^F larger than $\underline{P}_1$ implies that this red point is to the right of the intersection of the x-axis with the feasibility frontier in the case of the price-level taking debt level.

to one in which F issues a smaller amount and $b_0^M = 0$.

[17] We are grateful to Vladimir Asriyan for suggesting this graphical representation of our results.

Proposition 2. ***(Debt issuance in the date-0 bond market)*** *Given* (h_1, R_0, X_0, x_0), *F issues one of either debt level:*

- ***Price-level taking debt level:*** *F issues bonds so as to optimize its consumption pattern taking the date-1 price level $\underline{P}_1$ as given. In this case, F raises an amount $b^{PT}(x_0)$ of real resources. M's bond purchases are immaterial. There is no default at date 1.*

- ***Sargent-Wallace debt level:*** *F issues a larger amount in the bond market, front-loading consumption as much as possible ($g_1^{SW} = 0$) and raises a real amount $b^{SW}(x_0) \geq b^{PT}(x_0)$, so as to force a date-1 price level given by fiscal dominance. M buys back as many bonds as possible: $b_0^M = x_0 - R_{-1}X_{-1}/P_0$, but not the whole issuance. The date-1 price level is above target, equal to $\underline{P}_1 + \alpha_M$. There is no default at date 1.*

F selects the "price-level taking" debt level whenever $\Delta \equiv g_0^{PT}(x_0) + \beta g_1^{PT}(x_0) - g_0^{SW} > 0$.

Proof. See Appendix B.1. □

The "Sargent-Wallace" debt level whereby F floods the bond market with paper so as to force M to "chicken out" and inflate away outstanding reserves at date 1 in order to ensure public solvency is closely related to that underlying the unpleasant monetarist arithmetic in Sargent and Wallace (1981). F creates a deficit that forces M to generate income in an inflationary way, simply by inflating away the value of reserves here. Proposition 2 states that this need not be F's preferred debt level as this may require an overly inefficient distortion of its consumption relative to consumption under the "price-level taking" debt issuance.

Using $b^{SW}(x_0)$ the real amount that F collects in the date-0 bond market when issuing the Sargent-Wallace debt level and $b^{PT}(x_0)$ this real amount when issuing the price-level taking debt level, one can rewrite F's utility differential Δ between the two debt levels

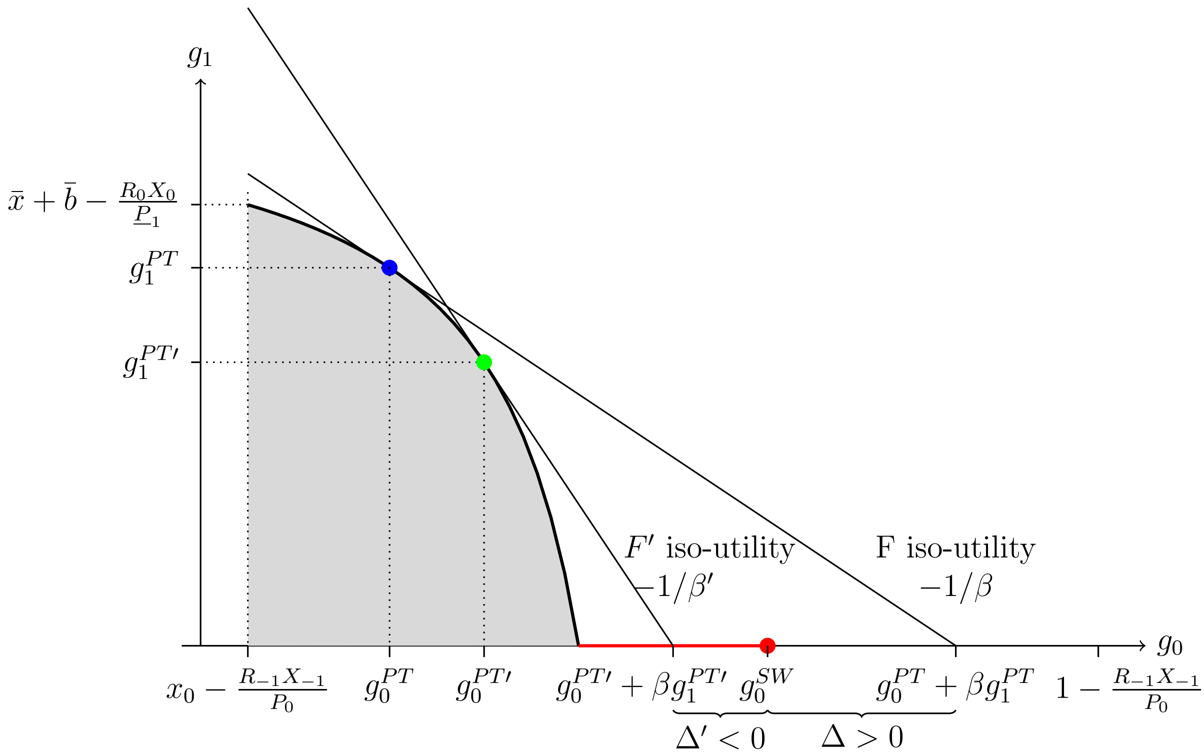

Figure 3: Problem faced by F on the date-0 debt market.
The red circle corresponds to consumption associated with Sargent-Wallace debt issuance. The blue circle corresponds to consumption pattern associated with the price level taking debt level with high β and the green circle with low $\beta' < \beta$.

as:

$$\Delta = \underbrace{b^{PT}(x_0)(1 - \beta r(1 - x_0 - b^{PT}(x_0))) - b^{SW}(x_0)(1 - \beta r(1 - x_0 - b^{SW}(x_0)))}_{A}$$
$$\underbrace{- \beta R_0 X_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha_M} \right)}_{B}. \qquad (12)$$

Term A measures the difference in utility from allocating consumption over time in different ways across these actions. The sign of A is ambiguous as the allocation is suboptimal under the Sargent-Wallace debt level—the interest rate is too high relative to that in the price-level taking debt level since $b^{SW}(x_0) \geq b^{PT}(x_0)$—but the total to be allocated is larger due to the lower value of reserves. Term B is positive. It is the benefit from eroding the value of reserves $R_0 X_0$ with inflation.

The value of Δ can directly be observed on Figure 3: it corresponds to the distance along the x-axis between the red point, which corresponds to the payoff from the Sargent-Wallace debt level as there is no future consumption in this case ($g_1^{SW} = 0$), and the intersection between the x-axis and the iso-utility associated with the consumption pattern $\{g_0^{PT}, g_1^{PT}\}$ obtained through the price-taking debt level. Figure 3 displays two situations, one in which F prefers the price-level taking debt level, and one in which F is more impatient ($\beta' < \beta$) and prefers the Sargent-Wallace debt level.

3.4 Date-0 reserve issuance

The final step is the determination of the action of M in the date-0 market for reserves. We show that M has an incentive to minimize the circulation of reserves so as to curb the fiscal authority's incentives to issue the Sargent-Wallace debt level.

Proposition 3. ***(The determinants of monetary dominance)***

If $g_1^{PT}(0) > 0$, there exists a threshold $\overline{RX} > 0$ such that if $R_{-1}X_{-1} \leq \overline{RX}$, the unique equilibrium is such that the price level is on target at each date— $P_0 = P_0^M$ and $P_1 = P_1^M$, and such that M minimizes the amount of reserves in circulation ($X_0 = R_{-1}X_{-1}$).

If $g_1^{PT}(0) = 0$, any equilibrium is such that F issues the Sargent-Wallace debt level implying $P_1 = \underline{P}_1 + \alpha_M$. M (and thus F) is indifferent across several levels of reserves X_0.

Proof. See Appendix B.1. □

Proposition 3 offers two insights. First, it exhibits conditions under which M reaches its price-level objective at each date. Notice that M does so without any fiscal support and without any ability to commit to future actions. The first of these conditions is that legacy reserves be sufficiently small. The second one is that F finds frontloading consumption sufficiently costly in the sense that $g_1^{PT}(0) > 0$. The second insight is that this latter condition is actually necessary: The fiscal authority always enters into the Sargent-Wallace strategy when it fails to hold. The equilibrium is essentially unique in this case, as the only source of multiplicity is the fact that M is indifferent between several levels of reserves that are payoff equivalent for both authorities.[18]

The key reserve policy that may allow the monetary authority to impose its price-level targets is to minimize the amount of reserves in circulation by setting $X_0 = R_{-1}X_{-1}$.[19] To see why this policy favors monetary dominance, notice from (12) that as $R_{-1}X_{-1}$

[18]This is because F borrows against whichever fraction of $\bar{x}$ is left on the table by M, and which authority borrows against it has no impact on the equilibrium outcome.

[19]It would be straightforward to extend the model to a situation in which M has real resources at date 0, in which case it would use them to buy reserves back thereby reducing their circulation even further.

tends to zero, the sign of Δ becomes that of term A provided it stays away from zero. When $g_1^{PT}(0) > 0$, term A is strictly positive for $R_{-1}X_{-1}$ sufficiently small as it reflects that the Sargent-Wallace debt level entails excessive borrowing against essentially fixed date-1 resources. In contrast, when $g_1^{PT}(0) = 0$, the Sargent-Wallace debt level pays off for any level of legacy reserves $R_{-1}X_{-1}$ as F is willing to borrow more at a higher rate as soon as it can avail itself of more resources at date 1.

Proposition 3 fully characterizes the equilibrium provided legacy reserves $R_{-1}X_{-1}$ are sufficiently small other things being equal, a natural benchmark in which large public liabilities are all endogenously issued.[20] Still, Proposition 3 is silent on the equilibrium outcome when $g_1^{PT}(0) > 0$ and legacy reserves are not as small as needed other things being equal for monetary dominance to prevail. Whereas we found the analysis of this case intractable in our general model, Section 3.5 below tackles it under the simplifying assumption of a constant return on private storage.

Some empirical and policy implications. Proposition 3 states that a necessary condition for the central bank to be able to fulfill its mandate is that the fiscal authority find it costly to pledge its entire future fiscal capacity ($g_1^{PT}(0) > 0$). This is so when the solution b to the first-order condition associated with (8),

$$r(1-b) - br'(1-b) = \frac{1}{\beta}, \tag{13}$$

is such that

$$br(1-b) < \bar{x} + \bar{b}, \tag{14}$$

and this in turn depends only on the values of $r(.)$, $\bar{x} + \bar{b}$, and β. Simple comparative statics with respect to these parameters then yield:

Corollary 4. ***(Empirical and policy implications)*** *Monetary dominance prevails for sufficiently small legacy reserves when, ceteris paribus,*

(i) *The distortionary cost of increasing debt is sufficiently large ($r(.)$ is sufficiently steep).*

[20] We could simply have assumed $R_{-1}X_{-1} = 0$ throughout. We would then have needed an additional assumption (e.g., indivisibility) to ensure that the optimal strategy of M, that consists in issuing "as small as possible but strictly positive" X_0, has a well-defined solution.

(ii) *Future fiscal capacity* $\bar{x} + \bar{b}$ *is sufficiently large.*

(iii) *The fiscal authority is sufficiently forward-looking, that is,* β *is sufficiently close to 1.*

Proof. The solution to (13) is decreasing in β and in upwards shifts in r', and condition (14) has more slack as $\bar{x} + \bar{b}$ increases. □

A sufficiently steep interest rate schedule $r(.)$ (r' large in absolute terms) discourages issuing debt at the Sargent-Wallace level and leads to monetary dominance. It is important to stress that monetary dominance may prevail no matter the value that the interest rate takes *in equilibrium.* All that is needed is that a deviation from the equilibrium whereby F issues the Sargent-Wallace debt level would lead to a sufficiently large increase in the interest rate.

Holding the function $r(.)$ fixed, monetary dominance is also warranted when the future public resources $\bar{x}+\bar{b}$ are important all else equal, so that forcing M to chicken out requires draining large private savings out of private investments. Finally, and in connection with political-economy considerations, an impatient F (β small) is ceteris paribus more likely to be constrained, and thus to find the Sargent-Wallace issuance attractive.

Notice that the linear preferences of F over consumption stack the deck in favor of fiscal dominance. In contrast, if F had strictly concave preferences, it would find the Sargent-Wallace debt level costly not only because it raises the marginal interest rate but also because it would shift its intertemporal marginal rate of substitution down and thus away from this marginal interest rate.

Fiscal requirements. It is worthwhile highlighting that when $g_1^{PT}(0) = 0$ and F enters into the Sargent-Wallace behavior, F does not derive any seigniorage income from it in equilibrium. Bonds and reserves are perfect substitutes in this setup and must earn the same equilibrium return. M anticipates a date-1 price equal to $\underline{P}_1 + \alpha_M$ in the announced rate R_0. As a result, if F could commit at the outset of the game to a fiscal requirement capping its nominal borrowing in the date-0 bond market, it would be happy to do so in order to tie its hands and avoid the Sargent-Wallace debt level.

3.5 The case of a constant return on private storage

To further characterize date-0 decisions by the monetary authority in the case in which $g_1^{PT}(0) > 0$ and the legacy reserves $R_{-1}X_{-1}$ cannot be taken arbitrarily small, we consider in this subsection the simpler case in which the return on private storage is constant. Proposition 5 first fully characterizes the circumstances under which M reaches its price-level objective at each date. Proposition 6 then details the equilibrium outcome when these circumstances are not met. In particular, it sheds light onto optimal date-0 monetary policy in the presence of large legacy reserves $R_{-1}X_{-1}$.

Suppose thus that the return on private storage is a constant $r > 0$. This version of the model can be interpreted as a small open economy facing the world interest rate in which the fiscal authority borrows in the local currency. Suppose also that:

$$\bar{x} + \bar{b} < r. \tag{15}$$

$$\frac{R_{-1}X_{-1}}{P_0^M} < \frac{\bar{x}}{r}. \tag{16}$$

Condition (15) rules out the unrealistic case in which the public sector can drain the whole savings in the economy. The assumption that $r \to_0 +\infty$ precludes this in the general model. Condition (16) ensures that M need not be off target at date 0 because $R_{-1}X_{-1}$ is too large relative to its date-1 resources. In other words, it rules out the uninteresting case of exogenous "reserve overflow" at date 0.

Monetary dominance. We first spell out necessary and sufficient conditions for M to be able to set the price level on target at both dates.

Proposition 5. ***(Characterization of monetary dominance)*** *The equilibrium is such that price levels are on target ($P_0 = P_0^M$ and $P_1 = P_1^M$) if and only if*

$$\beta r > 1, \tag{17}$$

and

$$\frac{\bar{x} + \bar{b}}{r} \geq \left(\frac{\beta r - \frac{P_1^M}{P_1^M + \alpha_M}}{\beta r - 1} \right) \frac{R_{-1}X_{-1}}{P_0^M}. \tag{18}$$

Otherwise, at least one price level is above target.

Proof. See Appendix B.2. □

Monetary dominance requires two conditions. First, $\beta r > 1$ ensures that F strictly prefers to postpone consumption to date 1 rather than consuming everything at date 0 holding the date-1 price level fixed. It is the exact counterpart, in the presence of a fixed rate, of condition $g_1^{PT}(0) > 0$ in the general model. Second, it is useful to rewrite condition (18) as follows:

$$\underbrace{(\beta r - 1)}_{\text{Unit cost of frontloading } g} \times \underbrace{\left(\frac{\bar{x} + \bar{b}}{r} - \frac{R_{-1}X_{-1}}{P_0^M}\right)}_{\text{Net public wealth}} \geq \underbrace{\left(1 - \frac{P_1^M}{P_1^M + \alpha_M}\right)\frac{R_{-1}X_{-1}}{P_0^M}}_{\text{Fiscal-dominance gains}}. \tag{19}$$

Monetary dominance requires that the cost of frontloading consumption exceeds the gains from forcing fiscal dominance. With constant interest rates, the cost of frontloading consumption simply depends on net public wealth—future resources net of initial public liabilities— and the value of the real interest rate r relatively to the discount rate β. The gains from fiscal dominance depend on legacy nominal liabilities $R_{-1}X_{-1}/P_0^M$ and on the relative price level increase in case of fiscal dominance $P_1^M/(P_1^M + \alpha^M)$. In particular, monetary dominance always prevails when the central bank does not care about default ($\alpha^M = 0$), in which case the right hand term boils down to 0 and the inequality is trivially satisfied. In contrast, monetary dominance is made impossible when rates are low ($\beta r < 1$) as there is no cost to frontload consumption.

In sum, condition (19) states that the central bank is independent if the public sector is "super solvent", or has a sufficiently large net wealth. Comparative statics properties with respect to public net wealth $(\bar{x} + \bar{b})/r - R_{-1}X_{-1}/P_0^M$ suggest that negative shocks to it induce shifts from monetary dominance to fiscal dominance. Such shifts will arise in the time series as public net wealth endogenously fluctuates in Section 4 in which $T > 1$.

Remark 1. Notice that in our model, the fiscal-dominance gains apply only to a basis equal to the real value of legacy reserves $R_{-1}X_{-1}/P_0^M$. In a more general setting, this gain could apply to other legacy nominal public liabilities, including long-term debt. Also, net public wealth on the left-hand of (19) would account for all commitments by the public sector, including real ones such as indexed bonds.

Remark 2. Interpreting the fixed-rate example as a small open economy, Proposition 5 implies that a reduction in interest rates due to international capital inflows leading to $\beta r < 1$ may contribute to jeopardizing the ability of the central bank to fulfill its price-stability mandate by giving incentives for fiscal authorities to borrow more. This transmission of international rates to domestic ones may take place despite flexible exchange rates as shown by Rey (2016).

Fiscal dominance. When the conditions in Proposition 5 are not met, the monetary authority M is forced away from its objective either at date 0 or at date 1. We characterize the outcome as a function of legacy reserves $R_{-1}X_{-1}$ in the following proposition.

Proposition 6. ***(Optimal monetary policy without monetary dominance)*** *Suppose the conditions in Proposition 5 are not met:*

(i) *If $\beta r \leq 1$, M sets $P_0 = P_0^M$. F issues the Sargent-Wallace debt level, and so the date-1 price level is $P_1 = P_1^M + \alpha_M$;*

(ii) *If $\beta r > 1$, there exists a threshold $\overline{\overline{RX}}$ such that*

- *If legacy reserves satisfy $R_{-1}X_{-1} \leq \overline{\overline{RX}}$, M sets P_0 at the smallest value such that* (18) *holds when replacing P_0^M by P_0 so that F does not adopt the Sargent-Wallace debt level, and so $P_1 = P_1^M$;*
- *Otherwise M sets $P_0 = P_0^M$ and either lets F issue at the Sargent-Wallace level so that $P_1 = P_1^M + \alpha_M$ or, if there exists a solution $P_1^* \leq P_1^M + \alpha_M$ to*

$$1 + \frac{\bar{b}}{\bar{x}} = \frac{\beta r - \frac{P_1}{P_1 + \alpha_M}}{\beta r - 1}, \tag{20}$$

M sets $P_1^* > P_1^M$ at date 1, thereby discouraging F from issuing at the Sargent-Wallace level. In this latter case, in order to credibly commit to a date-1 price P_1^*, M must issue sufficiently large reserves at date 0 that there is reserve overflow at date 1.

Proof. See Appendix B.2. □

If $\beta r \leq 1$, the Sargent-Wallace debt level comes at no cost to the fiscal authority F because it seeks to borrow as much as possible anyway. The date-1 price level is thus

$P_1^M + \alpha_M$. This case is the exact counterpart when the interest rate is constant of the case $g_1^{PT}(0) = 0$ in Proposition 3.

More interesting is the situation in which $\beta r > 1$ and condition (18) does not hold. In this case, M compares two different options. On the one hand, M can raise the price level at date 0 to reduce the real value of legacy reserves so that condition (18) is satisfied. As a result, F does not issue the Sargent-Wallace debt level and M can set the price level to target P_1^M at date 1. On the other hand, M can set the price level on target at date 0 ($P_0 = P_0^M$) but deviate from its date-1 objective. In this latter case, M can either let F issue the Sargent-Wallace debt level or, if this is possible, make sure that the price level at date 1 $P_1^* > P_1^M$ pushes the fiscal authority F to not issue the Sargent-Wallace debt level. In this latter case, the only way M can credibly commit at date 0 to P_1^* is by issuing sufficiently large reserves $R_0 X_0 = P_1^* \bar{x} > r R_{-1} X_{-1}$.

The first option leads to a price level P_0 increasing with legacy reserves—more legacy reserves require a higher date-0 price in order to satisfy condition (18). By contrast, the second option leads to a price level P_1 that does not depend on the amount of legacy reserves. As a result, M will then prefer to inflate at date 0 if and only if legacy reserves are sufficiently small. In this latter case, mild inflation is preferred in the short run to reduce legacy liabilities in order to avoid inflating more in the future.

Revisiting fiscal requirements. Proposition 6 states that there are three possible scenarii when monetary dominance does not hold: i) F issues at the Sargent-Wallace level; ii) M raises P_0 so that F issues at the price-taking level; iii) M raises P_1 so that F issues at the price-taking level. Expecting scenarii i) or iii), F would be happy to commit to a fiscal requirement at the outset if it could do so because it does not benefit ex-ante from inflation. Under scenario ii), by contrast, F and M disagree on fiscal requirements as F strictly benefits from inflating away legacy reserves.

Legacy debt. It is easy to accommodate for the presence of legacy debt B_{-1} due at date 0. In this case, if $\beta r > 1$, (18) ensuring monetary dominance becomes:

$$\frac{\bar{x} + \bar{b}}{r} \geq \frac{B_{-1}}{P_0^M} + \left(\frac{\beta r - \frac{P_1^M}{P_1^M + \alpha_M}}{\beta r - 1} \right) \frac{R_{-1} X_{-1}}{P_0^M}. \tag{21}$$

Expression (21) yields two insights. First, the coefficient that multiplies legacy reserves does not apply to legacy debt. The reason is that debt is due at date 0 whereas F can only generate fiscal dominance at date 1. The coefficient would apply if legacy debt was long term, due at date 1. Second, even though legacy debt cannot be inflated away, it still makes the Sargent-Wallace debt level relatively more appealing—by appearing on the RHS of (21)—because F needs to borrow to repay B_{-1} anyway even if $\beta r > 1$. The corresponding borrowing thus "comes for free" when issuing at the Sargent-Wallace level.

Return on central bank investments. Holding $\bar{b}$ and $R_{-1}X_{-1}$ fixed, monetary dominance is all the more likely because $\bar{x}$ is large. If one interprets $\bar{x}$ as including not only an exogenous demand for money but also the return on investments that M funded with the proceeds from issuing X_{-1} at date -1, then this implies that monetary dominance benefits from a high expected return viewed from date 0. This shapes the risk-taking incentives of M when investing at date -1 given the net wealth of the government at this date. In particular, if fiscal dominance is very likely viewed from date -1 conditionally on investing in safe assets, M may be tempted to opt for assets with riskier returns to increase the probability of monetary dominance. Such gambling for resurrection behavior would parallel that of investors subject to limited liability constraints as studied in the finance literature (see Allen and Gale, 2000, among others).

3.6 Why set up an independent central bank?

We directly assume for brevity the existence of a central bank with a price-stability mandate that has control over the nominal interest rate and over its balance sheet. It is however important to stress that such an institution is easy to motivate in our context. Suppose that a fiscal authority with preferences (1) is sole in charge of issuing both bonds and the unit of account (reserves). An interpretation of this situation is that F cannot even commit to let M operate its balance sheet independently.

Proposition 7. ***(Necessity of an independent central bank)*** *Suppose that F is in charge of the actions of M. Then the price level is infinite at all dates. F cannot issue any security at date 0.*

Proof. F sets $P_1 = +\infty$ in order to maximize its date-1 consumption and rational investors anticipating this do not invest at date 0, which implies $P_0 = +\infty$ as well. ☐

To be sure, this extreme result rests on the extreme (and unreasonable) assumption that hyperinflation comes at no exogenous cost for F whereas outright default does. Still, it is clear that F faces a standard commitment problem that would persist as long as some inflation is a more insidious way of generating income than outright default. We show that setting up an institution whose objective is to maintain the value of nominal claims may suffice to solve this commitment problem, even if this institution has no fiscal support nor any commitment ability itself. It may be sufficient that market forces such as the crowding out of private investment discourage the type of strategies envisioned by Sargent and Wallace (1981).

4 More periods: $T > 1$

How do our findings extend to multiple periods when debt may evolve over time? This section analyzes debt accumulation and the dynamics of the price level in finite-horizon games such that $T \geq 2$. In this case, our model can be interpreted as a situation in which the public sector expects more resources in the remote future ($\bar{x} + \bar{b}$ at the terminal date T) either because of growth prospects or because of a fiscal adjustment, so that public authorities have to manage debt over a long period of time before this windfall.

In order to describe the equilibria, we introduce the level of real debt that the fiscal authority would optimally borrow at each date in the absence of strategic concerns, were it not constrained by future resources:

$$b^* \equiv \arg \max_{b \in [0,1)} \{b(1 - \beta r(1 - b))\}. \tag{22}$$

Notice that b^* coincides with the price-taking level of debt $b^{PT}(0)$ defined in (8) in the case in which date-1 consumption is strictly positive: $g_1^{PT}(0) > 0$. For brevity we restrict the analysis to the case in which $b^* > 0$.

The equilibrium crucially depends on the position of the interest rate $r(1-b^*)$ relative to 1. The case $r(1 - b^*) < 1$ is the most relevant in the current context of "low rates". We restrict the analysis to the case of arbitrarily small legacy reserves $R_{-1}X_{-1}$.

Under these conditions, we obtain two key results on the path of equilibrium real debt. First, real debt is always below b^*, and so $r(1 - b_t) \leq r(1 - b^*) < 1$. Second, this result that rates are smaller than 1 implies that the date-t present value of date-T

public resources decreases with respect to t. As a result, F may be able to borrow up to the unconstrained debt level b^* at early dates, and may become unable to do so as time elapses. Having these results in hand, we can then build upon our previous results. At early dates, the government is unconstrained, and the monetary authority can ensure that the level of reserves is sufficiently low to avoid any temptation by the fiscal authority to issue the Sargent-Wallace debt level. When approaching the final date of the game, the value of future resources decreases and the government becomes constrained. In this case, as in Proposition 3, M cannot deter the government from issuing the Sargent-Wallace debt level. The following proposition formalizes this:

Proposition 8. ***(Endogenous regime switching when*** $r(1-b^*) < 1$***)*** *Suppose* $r(1-b^*) < 1$. *If* $R_{-1}X_{-1}$ *is sufficiently small other things being equal, there exists a unique equilibrium.* M *does not issue new reserves between dates 0 and* $T-1$.

There exists $\tau \in \{0; ...; T\}$ *such that for* $t \in \{0; ...; \tau\}$, $g_t > 0$ *and there is monetary dominance* ($P_t = P_t^M$), *whereas for* $t \in \{\tau+1; ...; T\}$ *(an empty set if* $\tau = T$*),* $g_t = 0$ *and there is fiscal dominance* ($P_t = P_t^M + \alpha_M$).

Proof. See Appendix B.3. □

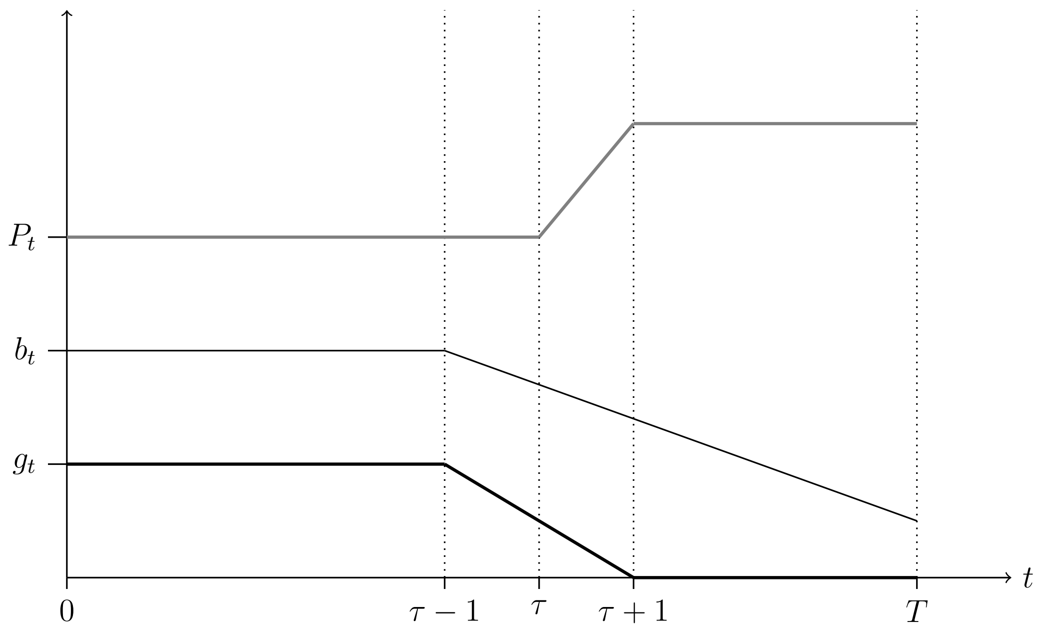

Figure 4: Dynamics of price level, debt, and deficit

Figure 4 illustrates the generic dynamics[21] of the price level, debt, and deficit. Borrowing b_t and government consumption g_t are constant over $t \leq \tau - 1$, and $P_t = P_t^M$ for

[21]By generic we mean for parameter values such that $1 \leq \tau \leq T-1$.

$t \leq \tau$. F can borrow the optimal level b^* and consume since $b^*(1-r(1-b^*)) > 0$. Then at date τ debt and consumption tank. At $\tau+1$, the price level jumps to the fiscal-dominance level $P^M_{\tau+1} + \alpha_M$ and F uses its debt issuance to roll over legacy debt, thereby no longer consuming until the terminal date.

As mentioned above, the reason regime switches this way is that the terminal resources $\bar{x} + \bar{b}$ discounted at $r(1 - b^*) < 1$ become large at the initial dates, and thus F faces no borrowing constraint at these dates. As the terminal date gets closer, a financial constraint may start binding, and F may as well adopt the Sargent-Wallace debt level from then on. Such regime switching contrasts with rules-based model (e.g., Leeper, 1991), in which the perfect-foresight expectation of future fiscal (or monetary) dominance generates immediate fiscal (or monetary) dominance.

In the current US context of low rates, large deficits, outstanding amounts of public liabilities, and price levels that have yet until very recently remained stable, this latter equilibrium in which a similar situation prevails for a possibly arbitrary long time until it ultimately morphs into one of a constrained public sector and inflation is interesting.

The case $r(1-b^*) \geq 1$ is more involved as F may find itself forced to roll over a level of legacy debt that is larger than the ex-post optimum b^*. Generically, dominance switches from fiscal to monetary when $r(1 - b^*) \geq 1$—in the opposite direction from that when $r(1 - b^*) < 1$. Interestingly, unlike when $T = 1$, fiscal dominance may initially prevail even though F does not borrow against its entire terminal resources ($g_T > 0$). Appendix D offers a detailed treatment of this case $r(1 - b^*) \geq 1$.

5 Infinite horizon

We now turn to the infinite-horizon version of the model. This entails two significant departures from the economies studied thus far. First, the public sector cannot back reserves and bonds with real resources $\bar{x} + \bar{b}$. Public liabilities are therefore pure bubbles. Second, the private sector can enter into strategies that grant it significantly much more influence over fiscal and monetary policies than in the finite-horizon setting.

Both the possibility of bubbles and that of potentially complex history-dependent strategies create room for a plethora of equilibria. There are many possible bubbly paths, and this multiplicity creates in turn room for strategies whereby the bubbly path on which savers coordinate going forward is history dependent. The goal of this section is to exhibit a non-trivial equilibrium in which both F and M can collect resources, and to show that the off-equilibrium-path behavior of the private sector is the true determinant of the price level in this equilibrium.

We suppose in this section that $r(1) < 1$, a necessary and sufficient condition for bubbles to exist. We also suppose that b^* defined in (22) is strictly positive.

Consider a series of strictly positive numbers $(\bar{x}_t, \bar{b}_t)_{t\geq 0}$ such that:

$$\bar{x}_0 > \frac{R_{-1}X_{-1}}{P_0^M}, \tag{23}$$

and for all $t \geq 0$

$$\bar{x}_t + \bar{b}_t < 1, \tag{24}$$

$$\bar{x}_{t+1} > r(1 - \bar{b}_t - \bar{x}_t)\bar{x}_t, \tag{25}$$

$$\bar{b}_{t+1} + \bar{x}_{t+1} = r(1 - \bar{b}_t - \bar{x}_t)(\bar{b}_t + \bar{x}_t). \tag{26}$$

Given that $r(1) < 1$, such a series exists if $R_{-1}X_{-1}/P_0^M$ is sufficiently small, which we assume.

Proposition 9. ***(Market discipline may enforce monetary dominance)*** *Suppose* $(\bar{x}_t, \bar{b}_t)_{t\in\mathbb{N}}$ *admits a sufficiently small upper bound.*

- ***Fiscal-dominance equilibrium***. *There exists an equilibrium in which the price level is* $P_t = P_t^M + \mathbb{1}_{\{t>0\}}\alpha_M$. *No new reserves are issued. The public sector collects* $\bar{b}_t + \bar{x}_t$ *at every date* t. F *consumes at date 0 and rolls over debt afterwards.*

- ***Monetary-dominance equilibrium.*** *There also exists an equilibrium in which the price level is* $P_t = P_t^M$. *No new reserves are issued. The public sector collects* $\bar{b}_t + \bar{x}_t$ *at every date* t. F *consumes at date 0 and rolls over debt afterwards.*

Proof. See Appendix B.4. □

The fiscal-dominance equilibrium can be viewed as the infinite-horizon extension of a finite-horizon equilibrium in which F is constrained at each date t because x_{t+1} and b_{t+1} are sufficiently small, and so it may as well issue debt at the Sargent-Wallace level. The monetary-dominance equilibrium features the exact same real quantities and utility of F as the fiscal-dominance one. The price level is however on target, an outcome that would be out of reach under finite horizon given a constrained fiscal authority.

A difference in savers' strategy profiles across these two equilibria suffices to induce this difference in price levels. As detailed in the proof of Proposition 9, in the fiscal-dominance equilibrium, savers are purely forward-looking. Their investment decisions are only based on expected returns given history and strategy profiles. Savers' beliefs about future demand for public securities are self-fulfilling as they define the largest possible bubbles that F and M can generate by issuing securities, and that F and M do optimally generate in equilibrium.

The monetary-dominance equilibrium adds the feature that if she observes a price level $P_{t-1} \neq P_{t-1}^{M}$, then a saver shuns the reserve market at date t. In other words, savers prick the bubble on reserves if the central bank has missed its target in the past. This does not occur along the equilibrium path but gives commitment power to M, because any attempt at slightly inflating away public liabilities to avoid default would result in the economy embarking on autarky and in the inability of the public sector to issue any nominal claims. M would therefore prefer to default and anticipating this, F avoids Sargent-Wallace issuances. Another way of saying this is that market discipline creates an endogenous value of α_M equal to 0.

In sum, when an important component of public liabilities is bubbly, there is room for "market dominance": The market has the possibility to determine the price level by exploiting the multiplicity of bubbly paths. In fact, it is easy to see that the market could enforce any price level, regardless of the objectives of F and M, as long as autarky minimizes their utilities. It is important to stress that such market discipline would be effective even if the public sector could partially back its liabilities with a stream of future resources. All that matters for this result to hold is that a sufficiently large fraction of the liquidity supplied by the public sector is a self-fulfilling phenomenon that the private sector can credibly make history-dependent. If, on the other hand, a version of the model with infinite horizon and a stream of future resources displayed dynamic efficiency, then the equilibrium would be unique, and this would rule out market dominance.

Unsurprisingly, this market discipline closely relates to that in the sovereign-default literature pioneered by Eaton and Gersovitz (1981), as inflation is a particular form of default. It also relates to the literature that explores market discipline as a device to enforce fiscal rules (Halac and Yared, 2017, e.g.).

6 Concluding remarks

This paper solves a full-fledged model of strategic dynamic interactions between fiscal and monetary authorities with conflicting objectives. Its main goal is to identify which primitives of the economy determine whether the regime is one of fiscal or monetary dominance. We find that a monetary authority that lacks both commitment power and fiscal support may still be in the position of imposing its objectives if market forces make the inflationary fiscal expansion envisioned by Sargent and Wallace unpalatable to the fiscal authority. This is so for example when the market responds to large debt issuances with a high required (real) rate.

We believe that our setting opens many avenues for future research. Notably, a number of assumptions that seem natural for this first pass could be relaxed. In particular, we focus on the case in which public liabilities are perfect substitutes, and prices are flexible. If the liabilities of the central bank provided superior liquidity services, this would boost its ability to generate public revenue, thereby possibly exacerbating the conflict between fiscal and monetary objectives. Also, some price rigidity would grant the monetary authority the ability to manipulate real rates and output: on the one hand, this may give a tool for the central bank to disincentivize government debt issuance but, on the other hand, this would also generate extra incentives for the fiscal authority to lead the monetary authority to chicken out by cutting interest rates. Finally, additional natural routes for future research include the introduction of shocks to public resources or/and government preferences, and that of multiple non-cooperative fiscal authorities.

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Appendix

A Perfect-foresight competitive equilibria

Fix $t \in \mathbb{N}$ and $(\mathcal{E}_s)_{s<t}$ a given history with $B_{-1} = b^M_{-1} = X_{-1} = 0$. A competitive equilibrium from date t on given history $(\mathcal{E}_s)_{s<t}$ is a sequence $(\mathcal{E}_s)_{s\geq t}$ such that at every date $s \geq t$:

- $X_s \geq X_{s-1}R_{s-1}$, $B_s \geq 0$, $b^M_s \geq 0$.
- Budget constraints hold:

$$X_s - R_{s-1}X_{s-1} + \frac{(1-l_s)b^M_{s-1}P_{s-1}}{Q_{s-1}} = P_s(\theta_s + b^M_s), \tag{27}$$

$$Q_sB_s - (1-l_s)B_{s-1} + P_s\theta_s = P_s g_s. \tag{28}$$

- Markets clear:

$$X_s = P_s x_s, \tag{29}$$

$$Q_sB_s = P_s(b_s + b^M_s). \tag{30}$$

- Savers optimize:

$$(x_s, b_s) \in \arg\max_{(x,b)\in[0,1]^2} \left\{ \frac{R_sP_sx}{P_{s+1}} + \frac{(1-l_{s+1})P_sb}{Q_sP_{s+1}} + f(1-x-b) \right\} \tag{31}$$

$$s.t.\ x + b \leq 1.$$

Condition (27) is the flow budget constraint of M and (28) that of F, (29) is the reserve-market clearing condition, and (30) that of the bond market.

In the absence of any fiscal backing, a necessary and sufficient condition for the existence of equilibria in which both M and F issue nonnegative quantities of liabilities is that $r(1) < 1$. In this case, F and M can issue bubbles—unbacked liabilities that can repay themselves. Consider for example the following steady state corresponding to a given fixed price level $P > 0$ and to the largest possible total demand for public liquidity. Let $(x, b) \in (0,1)^2$ such that $r(1-x-b) = 1$. There exists a steady state in which savers bid x for reserves and b for bonds at each date. The price level is P. M issues xP reserves at date 0 and then none so that $X_t = Px$, and announces an interest rate $R_t = 1$. F issues $B_t = bP$ bonds at each date and the bond price is $Q_t = 1$. M pays an initial dividend equal to x to F who consumes $x + b$ at date 0 and then nothing.

B Proofs

B.1 Proof of Propositions 1, 2, and 3

This section solves for the whole two-date game using backwards induction, which proves the propositions along the way.

Second stage of date 1. If F chooses a given haircut l_1, it receives from M

$$\theta_1(l_1) = \bar{x} - \frac{R_0 X_0}{P_1} + \frac{(1-l_1) b_0^M P_0}{Q_0 P_1}. \tag{32}$$

It is therefore optimal for F to set $l_1 = 0$ if

$$g_1 = \bar{b} + \theta_1(0) - \frac{B_0}{P_1} = \bar{x} + \bar{b} - \frac{B_0 - \frac{b_0^M P_0}{Q_0}}{P_1} - \frac{R_0 X_0}{P_1} \geq 0 \tag{33}$$

and $l_1 = 1$ otherwise, in which case F consumes $g_1 = \bar{x} + \bar{b} - R_0 X_0 / P_1$.

First stage of date 1. Here M can set the price at any level $P_1 \geq R_0 X_0 / \bar{x}$ by issuing $X_1 - R_0 X_0 \geq 0$ such that $X_1 = P_1 \bar{x}$. So, if the smallest price level that ensures that the net liabilities of the public sector are covered by its resources is too large:

$$B_0 - b_0^M P_0 / Q_0 + R_0 X_0 > (\bar{x} + \bar{b}) \left(\max\left\{ P_1^M ; \frac{R_0 X_0}{\bar{x}} \right\} + \alpha_M \right), \tag{34}$$

M prefers to force default and sets $P_1 = \max\{P_1^M ; R_0 X_0 / \bar{x}\}$. Otherwise, M averts default by setting

$$P_1 = \max\left\{ P_1^M ; \frac{R_0 X_0}{\bar{x}} ; P^F \right\}, \tag{35}$$

where

$$P^F = \frac{B_0 - \frac{b_0^M P_0}{Q_0} + R_0 X_0}{\bar{x} + \bar{b}}. \tag{36}$$

This proves Proposition 1. Anticipating date 1 as above, agents play date 0 as follows.

Third stage of date 0. M simply transfers $x_0 - R_{-1}X_{-1}/P_0 - b_0^M$ to F who consumes it on top of the amount $b_0 + b_0^M$ collected in the bond market. F thus consumes $x_0 + b_0 - R_{-1}X_{-1}/P_0$, independent of b_0^M.

Second stage of date 0. Suppose that F issues $B_0 > 0$ bonds. There cannot be default at date 1: Since $l_1 = 1$ in case of default from above, savers' optimality implies $b_0 = 0$ in this case, and F only receives b_0^M from M in the bond market against an empty promise. But then F would be strictly better off not issuing bonds ($B_0 = 0$) and receiving b_0^M as a transfer from M at stage 3 of date 0, as this averts default leaving g_0 and g_1 unchanged. Furthermore, it must be that $b_0 > 0$. Otherwise F might as well not issue bonds and receive b_0^M as a dividend again since it would not affect neither price nor consumption levels. That it does not affect the date-1 price level stems from the fact that P^F depends only on $B_0 - b_0^M P_0/Q_0$.

Market clearing in the bond market reads:

$$Q_0 B_0 = P_0(b_0 + b_0^M), \tag{37}$$

and savers' rationality implies $b_0 + x_0 < 1$ and

$$\frac{P_0}{P_1 Q_0} = r(1 - b_0 - x_0). \tag{38}$$

Given the above determination of P_1 by (35), relations (37) and (38) form a system in (b_0, Q_0) given $(h_0, R_0, X_0, x_0, B_0, b_0^M)$ that has a unique solution.

To solve for the choice of B_0 by F given history, we proceed in two steps. A given issuance B_0 leads from above either to $P_1 = P^F$ or $P_1 > P^F$. We solve for the optimal action of F conditionally on each outcome. We then compare F's utility in each case in order to derive the unconditionally optimal action.

Case 1: Optimal B_0 if the solvency condition is not binding at date 1. Suppose first that F selects B_0 such that the continuation of the game satisfies $P_1 > P^F$. M and F both agree to maximize the utility of F given future monetary dominance. Thus any continuation equilibrium featuring monetary dominance in which $b_0^M > 0$ is payoff-equivalent to one in which F issues a smaller amount $B_0 - P_0 b_0^M / Q_0$ and $b_0^M = 0$, and so we focus on equilibria such that $b_0^M = 0$ for brevity. Combining (37) and (38) yields

$$\frac{B_0}{P_1} = r(1 - x_0 - b_0)b_0. \tag{39}$$

which shows in turn that F by selecting B_0 decides on the real amount b_0 to borrow at the rate $r(1 - b_0 - x_0)$ taking P_1 as given. It must therefore be that

$$b_0 = b^{PT}(x_0) = \arg\max_b \{g_0 + \beta g_1\} \tag{40}$$

s.t.

$$g_0 = x_0 + b - \frac{R_{-1}X_{-1}}{P_0}, \tag{41}$$

$$g_1 = \bar{x} + \bar{b} - \frac{R_0 X_0}{P_1} - r(1 - x_0 - b)b, \tag{42}$$

$$0 \le b < 1 - x_0, 0 \le g_1. \tag{43}$$

Claim 1. *$b^{PT}(x_0)$ is unique and such that $b^{PT}(x_0) + x_0$ continuously (weakly) increases w.r.t. x_0.*

Proof. The function $b \mapsto b(1 - \beta r(1 - x_0 - b))$ is concave and thus admits a unique maximum over $[0, 1 - x_0)$ since it tends to $-\infty$ at $1 - x_0$, and continuity stems from the continuity of the objective and constraints. The first-order condition reads:

$$r(1 - x_0 - b) - r'(1 - x_0 - b)b = \frac{1}{\beta}. \tag{44}$$

Both functions $r(1 - x_0 - b)$ and $-r'(1 - x_0 - b)b$ on the LHS are increasing in x_0, b, and so b must decrease and $x_0 + b$ increase if x_0 increases. □

Case 2: Optimal B_0 if the solvency condition is binding at date 1. Suppose now that F selects B_0 such that the continuation of the game satisfies $P_1 = P^F$. Plugging

(38) in (36) yields

$$P^F = \frac{R_0X_0 + B_0}{\bar{x} + \bar{b} + r(1 - b_0 - x_0)b_0^M}. \tag{45}$$

We now show that M optimally sets $b_0^M = x_0 - R_{-1}X_{-1}/P_0$ to minimize $P_1 = P^F$. Combining the market clearing condition for the bond market (37) and the no-arbitrage condition for bonds (38) yields:

$$\frac{B_0}{P^F} = (b_0 + b_0^M)r(1 - b_0 - x_0), \tag{46}$$

and injecting the value of P^F yields

$$\frac{B_0}{B_0 + R_0X_0}(\bar{x} + \bar{b}) = b_0r(1 - b_0 - x_0) + \left(1 - \frac{B_0}{B_0 + R_0X_0}\right) b_0^M r(1 - b_0 - x_0). \tag{47}$$

Condition (47) implies that $r(1 - b_0 - x_0)b_0^M$ must increase with b_0^M. Suppose otherwise: Then b_0 must be decreasing as b_0^M increases. In this case, $r(1-b_0-x_0)b_0$ is also decreasing. But then the left-hand term of (47) is independent from b_0^M whereas the right-hand term is decreasing in b_0^M, a contradiction since no equilibrium would form as b_0^M increases. Thus M finds it optimal to maximize b_0^M in order to minimize P^F.

Using $b_0^M = x_0 - R_{-1}X_{-1}/P_0$, one can rewrite (47) as

$$b_0 = \frac{B_0(\bar{x} + \bar{b})}{(B_0 + R_0X_0)r(1 - b_0 - x_0)} - \frac{(x_0 - \frac{R_{-1}X_{-1}}{P_0})R_0X_0}{B_0 + R_0X_0}, \tag{48}$$

and simple algebra shows that this implies that b_0 increases with respect to B_0. F thus chooses the maximum B_0 that is compatible with absence of default, that is, B_0 such that

$$P_1 = \underline{P}_1 + \alpha_M \tag{49}$$

where

$$\underline{P}_1 \equiv \max\left\{P_1^M; \frac{R_0X_0}{\bar{x}}\right\}. \tag{50}$$

Combining again (36), (37), and (38) yields that the value $b^{SW}(x_0)$ leading to this solves:

$$b^{SW}(x_0) = \frac{1}{r(1 - x_0 - b^{SW}(x_0))}\left(\bar{x} + \bar{b} - \frac{R_0 X_0}{\underline{P}_1 + \alpha_M}\right). \tag{51}$$

As a result, F's utility differential Δ between the "price-level taking" debt level (such that $P_1 = \underline{P}_1$) and the "Sargent-Wallace" debt level (such that $P_1 = \underline{P}_1 + \alpha_M$) is:

$$\Delta = x_0 - \frac{R_{-1}X_{-1}}{P_0} + b^{PT}(x_0) + \beta\left(\bar{x} + \bar{b} - r(1 - x_0 - b^{PT}(x_0))b^{PT}(x_0) - \frac{R_0 X_0}{\underline{P}_1}\right) \tag{52}$$

$$- (x_0 - \frac{R_{-1}X_{-1}}{P_0} + b^{SW}(x_0)) \tag{53}$$

$$= \underbrace{b^{PT}(x_0)[1 - \beta r(1 - x_0 - b^{PT}(x_0))] - b^{SW}(x_0)(1 - \beta r(1 - x_0 - b^{SW}(x_0))}_{A} \tag{54}$$

$$\underbrace{- \beta R_0 X_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha_M}\right)}_{B}. \tag{55}$$

This latter expression of Δ illustrates the costs and benefits from the price-level taking issuance versus the Sargent-Wallace issuance. Term A measures the difference in utility from allocating consumption over time in different ways across debt levels. The sign of A is ambiguous as the allocation is suboptimal under the Sargent-Wallace issuance but the total to be allocated is larger due to the lower value of reserves. Term B is positive. It is the benefit from eroding the value of reserves R_0X_0 with inflation.

First stage of date 0. Market clearing in the reserve market reads:

$$X_0 = P_0 x_0, \tag{56}$$

and savers' rationality implies

$$\frac{R_0 P_0}{P_1} = r(1 - b_0 - x_0). \tag{57}$$

Given the continuation of the game derived above, relations (56) and (57) form a system in (x_0, P_0) as a function of (h_0, X_0, R_0) with a unique solution. We solve for the equilibrium in the two cases covered by Proposition 3: i) $g_1^{PT}(0) > 0$ and $R_{-1}X_{-1}$ arbitrarily small; ii) $g_1^{PT}(0) = 0$.

Suppose first that $g_1^{PT}(0) > 0$ and take $R_{-1}X_{-1}$ sufficiently small other things being equal. In this case, M sets $X_0 = R_{-1}X_{-1}$ and announces $R_0 = r(1 - X_0/P_0^M - b^{PT}(X_0/P_0^M))P_1^M/P_0^M$. This corresponds to an equilibrium in which savers invest X_0/P_0^M in the market for reserves and $b^{PT}(X_0/P_0^M)$ in that for bonds, and the price level is on M's target at each date. The reason is that for $R_{-1}X_{-1}$ sufficiently small, $b^{PT}(X_0/P_0^M)$ is interior as it converges to $b^{PT}(0)$, and so A is positive, bounded away from 0, whereas the gains B are sufficiently small. In particular, the lexicographic preferences of M imply that minimizing x_0 this way is optimal because this minimizes the distortions in F's choice of b given that prices are on target.

Suppose then that $g_1^{PT}(0) = 0$. In this case, it is always optimal for F to issue the Sargent-Wallace level in the bond market since A is always negative no matter M's actions in the date-0 reserve market: The increase in date-1 resources induced by the lower value of reserves in the Sargent-Wallace debt level relaxes the binding constraint $g_1 \geq 0$ in the consumption-smoothing one. As a result, $\underline{P}_1 + \alpha_M$ is the lowest price that M can hope for at date 1. Since the largest one that it prefers to default is $\underline{P}_1 + \alpha_M$, this has to be the date-1 price. Accordingly, monetary policy in the date-0 reserve market is as follows. Let y_0 implicitly defined by

$$y_0 r(1 - y_0) = \bar{x} + \bar{b}, \tag{58}$$

and

$$\underline{P}_0 \equiv \max\left\{P_0^M; \frac{R_{-1}X_{-1}r(1 - y_0)}{\bar{x}}\right\} \tag{59}$$

M announces a rate $R_0 = r(1-y_0)(\underline{P}_1+\alpha_M)/\underline{P}_0$ and issues $X_0 \in [R_{-1}X_{-1}, \bar{x}\underline{P}_0/r(1-y_0)]$. This sets the date-0 price at $\underline{P}_0$ and $x_0 = X_0/\underline{P}_0$. M in particular may be indifferent across several levels of reserves X_0 because any resources that it leaves on the table are borrowed against by F in the bond market, and the utilities of both authorities are unchanged across these levels.

B.2 Proof of Propositions 5 and 6

Viewed from the stage of the date-0 bond market, the Sargent-Wallace debt level grants F a utility

$$x_0 - \frac{R_{-1}X_{-1}}{P_0} + \frac{1}{r}\left(\bar{x} + \bar{b} - \frac{R_0 X_0}{\underline{P}_1 + \alpha_M}\right), \tag{60}$$

where $\underline{P}_1$ is defined in (50), whereas the price-level taking debt level yields

$$x_0 - \frac{R_{-1}X_{-1}}{P_0} + b^{PT}(x_0) + \beta\left(\bar{x} + \bar{b} - rb^{PT}(x_0) - \frac{R_0 X_0}{\underline{P}_1}\right). \tag{61}$$

When $\beta r \leq 1$, $b^{PT}(x_0) = (\bar{x} + \bar{b} - R_0 X_0/\underline{P}_1)/r$, otherwise, $b^{PT}(x_0) = 0$. The Sargent-Wallace borrowing clearly dominates the price-level taking one if $\beta r \leq 1$. If $\beta r > 1$, the Sargent-Wallace behavior is (weakly) dominated if

$$\frac{\bar{x} + \bar{b}}{r} \geq \frac{\beta r - \frac{\underline{P}_1}{\underline{P}_1 + \alpha_M}}{\beta r - 1} \frac{R_0 X_0}{r\underline{P}_1}. \tag{62}$$

Going backward to the date-0 reserve market, this implies that if $\beta r > 1$ and the above expression holds with $X_0 = x_0 P_0^M = R_{-1}X_{-1}$, $\underline{P}_1 = P_1^M$ and $R_0 = rP_1^M/P_0^M$ then the equilibrium features the price-level taking debt level. The above expression is in this case (18):

$$\frac{\bar{x} + \bar{b}}{r} \geq \left(\frac{\beta r - \frac{P_1^M}{P_1^M + \alpha_M}}{\beta r - 1}\right) \frac{R_{-1}X_{-1}}{P_0^M}.$$

If $\beta r > 1$ and (18) fails to hold, M selects depending on parameter values one of the three following options. First, it can set $P_0 = P_0^M$ and let F issue the Sargent-Wallace debt level so that $P_1 = P_1^M + \alpha_M$.

Second, it can set $P > P_0^M$ as the smallest value such that (18) holds when substituting P_0^M with P. More precisely, M sets $X_0 = R_{-1}X_{-1}$, $P_1 = P_1^M$, and $R_0 = rP_1^M/P$. This way, F does not enter into the Sargent-Wallace behavior.

Finally, it can commit to the lowest date-1 price level $P' > P_1^M$ such that (18) holds when substituting P_1^M with P'. Such a P' solves

$$1+\frac{\bar{b}}{\bar{x}}=\frac{\beta r-\frac{P'}{P'+\alpha_M}}{\beta r-1}. \tag{63}$$

More precisely, M announces $X_0 = P_0^M \bar{x}/r$, and $R_0 = rP'/P_0^M$, so that $P_0 = P_0^M$, $x_0 = \bar{x}/r$, and $P_1 = P'$. The intuition why this price level discourages the Sargent-Wallace debt level is that it generates a sufficiently low inflation rate α_M/P'.

Let us investigate how M chooses between these different options. The first option, Sargent-Wallace, leads to a payoff $-\beta\alpha^M$ independent from $R_{-1}X_{-1}$. The last option—committing to a price level P'—leads to a payoff $-\beta(P'-P_1^M)$, that is independent from $R_{-1}X_{-1}$ as well. The option to inflate at date 0 leads to a payoff:

$$-\left(P_0-P_0^M\right)=-\max\left\{r\left(\frac{\beta r-\frac{P_1^M}{P_1^M+\alpha_M}}{\beta r-1}\right)\frac{R_{-1}X_{-1}}{\bar{x}+\bar{b}}-P_0^M\ ;0\right\}.$$

This latter payoff is decreasing in $R_{-1}X_{-1}$ and is maximal, equal to 0 when $R_{-1}X_{-1}=0$.

As a result, there exists $\overline{\overline{\overline{RX}}}$ such that inflating at date 0 is optimal if and only if $R_{-1}X_{-1} < \overline{\overline{\overline{RX}}}$. If $R_{-1}X_{-1} > \overline{\overline{\overline{RX}}}$, letting the fiscal authority enter into the Sargent-Wallace debt level is optimal if and only if $P' > P_1^M + \alpha^M$.

B.3 Proof of Proposition 8

Consider the T-date version of F's optimization program (8):

$$(b_0;...;b_{T-1}) = \arg\max\left\{\sum_{t=0}^{T}\beta^t g_t\right\} \tag{64}$$

s.t.

$$0 \le g_0 = b_0, \tag{65}$$

$$0 \le g_{t+1} = b_{t+1} - r(1-b_t)b_t, \forall 0 \le t \le T-2 \tag{66}$$

$$0 \le g_T = \bar{x} + \bar{b} - r(1-b_{T-1})b_{T-1}. \tag{67}$$

Claim. When $r(1-b^*) < 1$, the solution $(b_t)_{t\in\{0;...;T-1\}}$ to program (64) is decreasing and bounded above by b^*. There exists $\tau \in \{0;...;T\}$ such that $g_t > 0$ over $\{0;...;\tau\}$

and $g_t = 0$ for $t \geq \tau + 1$. (The latter set is empty if $\tau = T$.) Furthermore, $b_t = b^*$ for $t \in \{0; ...; \tau - 1\}$ if $\tau \geq 1$, and b_t strictly decreases from τ on if $\tau \leq T - 2$.

Proof of the claim. Let $\tau = \max\{t \mid g_t > 0\}$ for the optimal consumption pattern (64). This set is not empty as $g_0 > 0$ from $b^* > 0$. Since the utility of F increases in $b_t(1 - \beta r(1 - b_t))$ for all $t \in \{0; ...; T - 1\}$ and $r(1 - b^*) < 1$, it must be that $b_s = b^*$ for all $s \leq \tau - 1$ as this is feasible and dominates any other pattern up to date τ. That $r(1 - b^*) < 1$ also implies that b_t must be smaller than b^* from τ on if $\tau \leq T - 1$, and strictly decreasing from τ on if $\tau \leq T - 2$. This establishes the result.

This optimal consumption pattern implies that for $t \geq 1$ $P_t = P_t^M + \mathbb{1}_{\{t > \tau\}} \alpha_M$. For all $t \in \{0; ...; \tau - 1\}$, $g_{t+1} > 0$ and so as in the two-date case, the Sargent-Wallace debt level would come at the finite cost from overborrowing and the arbitrarily small benefit from inflating away $R_t X_t$. Thus F sticks to the price-level taking debt level. Conversely, for $t \geq \tau$, $g_{t+1} = 0$, and so the Sargent-Wallace issuance is strictly dominant because it does not affect consumption from date $t + 1$ on and raises current consumption because it strictly increases the date-$t + 1$ resources against which F borrows and $r(1 - b)b$ is strictly increasing.

B.4 Proof of Proposition 9

Consider a sequence $(\bar{x}_t, \bar{b}_t)_{t \in \mathbb{N}}$ that satisfies the conditions in Proposition 9. Letting

$$P_t^F \equiv \frac{B_{t-1} - \frac{b_{t-1}^M P_{t-1}}{Q_{t-1}} + R_{t-1} X_{t-1}}{\bar{x}_t + \bar{b}_t}, \tag{68}$$

we define

$$P_t^* = \begin{cases} \max\left\{P_t^M; \frac{R_{t-1} X_{t-1}}{\bar{x}_t}; P_t^F\right\} & if\ P_t^F \leq \max\left\{P_t^M; \frac{R_{t-1} X_{t-1}}{\bar{x}_t}\right\} + \alpha_M, \\ \max\left\{P_t^M; \frac{R_{t-1} X_{t-1}}{\bar{x}_t}\right\} & otherwise. \end{cases} \tag{69}$$

Step 1. Fiscal-dominance equilibrium. The strategy profiles associated with this equilibrium are as follows. We go backwards through the stages of a generic date t.

Stage 3: Default and consumption. M transfers any residual income to F. F pays its debt back if possible and consumes any residual income. If this is not possible then F fully defaults and consumes.

Stage 2: Bond market.

- If they expect default at $t+1$ given history and strategy profiles, savers shun bonds, otherwise their investment in bonds $b_t(h_t, R_t, X_t, x_t, B_t, b_t^M)$ and the bond price Q_t are the solutions of the system

$$Q_t B_t = P_t(b_t + b_t^M), \tag{70}$$

$$P_t = Q_t P_{t+1}^* r(1 - b_t - x_t) \tag{71}$$

- M invests either the smallest $b_t^M(h_t, R_t, X_t, x_t, B_t)$ such that $P_{t+1}^* = \max\{P_t^M; R_t X_t/\bar{x}_{t+1}\}$ or $b_t^M = (X_t - R_{t-1}X_{t-1})/P_t$ if this set is empty.

- F issues $B_t(h_t, R_t, X_t, x_t) = B_t^*$ bonds, where

$$B_t^* = (P_{t+1}^M + \alpha_M)\left(\bar{x}_{t+1} + \bar{b}_{t+1} - r(1 - \bar{x}_t - \bar{b}_t)\frac{R_{t-1}X_{t-1}}{P_t}\right). \tag{72}$$

Stage 1: Market for reserves.

- The price P_t and savers' investment in reserves $x_t(h_t, R_t, X_t)$ are the solutions of the system

$$X_t = P_t x_t, \tag{73}$$

$$P_t R_t = P_{t+1}^* r(1 - b_t - x_t), \tag{74}$$

  where the parameters other than X_t, R_t—that is, b_t and P_{t+1}^*—are given by the strategy profiles above.

- M selects $X_t = R_{t-1}X_{t-1}$, and announces $R_t = r(1 - x_t - b_t)P_{t+1}^*/P_t^*$, where all the future parameters defining R_t are generated by the above profiles.

These strategy profiles have two salient features. First, (68) encodes that the private sector anticipates that the maximum future resources collected in the date-$t+1$ respective reserve and bond markets are $\bar{x}_{t+1}$ and $\bar{b}_{t+1}$, respectively. This pins down the maximum size of the bubble that the public sector can blow. Second, it is weakly dominant for F to issue at the Sargent-Wallace level by issuing B_t^* given by (77) if the bubbles on reserves and bonds are sufficiently small that it is willing to borrow more at a higher rate.

Step 2. Monetary-dominance equilibrium. The strategy profiles and price functions associated with this equilibrium are as follows. We go again backwards through the stages of a generic date t.

Stage 3: Default and consumption. M transfers any residual income to F. F pays its debt back if possible and consumes any residual income. If this is not possible then F fully defaults and consumes.

Stage 2: Bond market.

- If they expect default at $t+1$ given history and strategy profiles, savers shun bonds, otherwise their investment in bonds $b_t(h_t, R_t, X_t, x_t, B_t, b_t^M)$ and the bond price Q_t are the solutions of the system

$$Q_t B_t = P_t(b_t + b_t^M), \tag{75}$$

$$P_t = Q_t P_{t+1}^* r(1 - b_t - x_t) \tag{76}$$

- M invests either the smallest $b_t^M(h_t, R_t, X_t, x_t, B_t)$ such that $P_{t+1}^* = \max\{P_t^M; R_t X_t/\bar{x}_{t+1}\}$ or $b_t^M = (X_t - R_{t-1}X_{t-1})/P_t$ if this set is empty.

- If $x_t > 0$, F issues $B_t(h_t, R_t, X_t, x_t)$ bonds, where

$$B_t(h_t, R_t, X_t, x_t) = P_{t+1}^M \left(\bar{x}_{t+1} + \bar{b}_{t+1}\right) - R_t X_t. \tag{77}$$

  Otherwise, $B_t = 0$.

Stage 1: Market for reserves.

- If either $t = 0$ or $P_{t-1} = P_{t-1}^M$, the price P_t and savers' investment in reserves $x_t(h_t, R_t, X_t)$ solve the system

$$X_t = P_t x_t, \tag{78}$$

$$P_t R_t = P_{t+1}^* r(1 - b_t - x_t), \tag{79}$$

  where all the parameters other than X_t, R_t—that is, b_t and P_{t+1}^*—are given by the strategy profiles above.

Otherwise, $x_t = 0$ *and so* $P_t = +\infty$*.*

- M selects $X_t = R_{t-1}X_{t-1}$, and announces $R_t = r(1 - x_t - b_t)P^*_{t+1}/P^*_t$, where all the future parameters defining R_t are generated by the above profiles.

There are two differences with the fiscal-dominance equilibrium. First, savers' behavior in the reserve market is now history dependent. If the last price was on target, then they behave in the forward-looking fashion of the fiscal-dominance equilibrium. Otherwise, they shun the reserve market thereby prohibiting the public sector from issuing nominal promises forever. This move is in italics in the description of the strategy profiles. This induces F to adopt the price-level taking debt level. The reason is that M would prefer to force default rather than entering into this autarky economy.

C Formal equilibrium definition

In this section, we define our equilibrium concept adapting the exact same formalism as that in the original definition of Ljungqvist and Sargent (2018) to our context.

As in the core of the text, we recursively define an history $h_{t+1} = \{h_t, R_t, X_t, x_t, B_t, b_t^M, b_t, l_t, g_t\}$. For all dates t, we consider the collection of functions

$$\sigma \equiv (\sigma^M, \sigma^x, \sigma^F, \sigma^m, \sigma^b, \sigma^f) = \{\sigma_t^M, \sigma_t^x, \sigma_t^F \sigma_t^m, \sigma_t^b, \sigma_t^f\}_{t \geq 0}$$

such that M takes decisions $(R_t, X_t) = \sigma_t^M(h_t)$ after observing history h_t, the aggregate investment in reserves of the private sector is $x_t = \sigma_t^x(h_t, R_t, X_t)$, the government bond issuance satisfies $B_t = \sigma_t^F(h_t, R_t, X_t, x_t)$, M's purchases of bonds and dividend policy is $b_t^M = \sigma_t^m(h_t, R_t, X_t, x_t, B_t)$, the private sector invests $b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^M)$ in bonds and finally the government decides to consume and to repay as follows

$$(g_t, l_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t).$$

Optimality of (R_t, X_t). Given a strategy profile σ and history h_t, the decision $(R_t, X_t) = \sigma_t^M(h_t)$ is optimal when (R_t, X_t) is solution to:

$$U_t^M(\sigma(h_t)) \equiv \max_{R_t', X_t'} - \mid X_t'/x_t - P_t^M \mid - \alpha_M l_t + \beta U_{t+1}^M(\sigma(h_{t+1}))$$

such that $x_t = \sigma_t^x(h_t, R_t', X_t')$, $B_t' = \sigma_t^F(h_t, R_t', X_t', x_t)$, $b_t^M = \sigma_t^m(h_t, R_t', X_t', x_t, B_t)$, $b_t = \sigma_t^b(h_t, R_t', X_t', x_t, B_t, b_t^M)$ and $(g_t, l_t) = \sigma_t^f(h_t, R_t', X_t', x_t, B_t, b_t^M, b_t)$.
Finally, $h_{t+1} = \{h_t, R_t', X_t', x_t, B_t, b_t^M, b_t, l_t, g_t\}$.

x_t is a competitive outcome. Given a strategy profile σ and the history $\{h_t, R_t, X_t\}$, the aggregate saving decision in reserves $x_t = \sigma_t^x(h_t, R_t, X_t)$ is optimal when x_t is such that:

(i) $P_t = X_t/x_t$, $Q_t = P_t(b_t + b_t^M)/B_t$ and $P_{t+1} = X_{t+1}/x_{t+1}$ where

$$
\begin{aligned}
&B_t = \sigma_t^F(h_t, R_t, X_t, x_t),\\
&b_t^M = \sigma_t^m(h_t, R_t, X_t, x_t, B_t),\\
&b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^M),\\
&(g_t, l_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t),\\
&h_{t+1} = \{h_t, R_t, X_t, x_t, B_t, b_t^M, b_t, l_t, g_t\},\\
&(X_{t+1}, R_{t+1}) = \sigma_{t+1}(h_{t+1}),\\
&x_{t+1} = \sigma_{t+1}(h_{t+1}, R_t, X_t).
\end{aligned}
$$

(ii) $(x_t, b_t) \in \arg\max_{(x,b)\in[0,1]^2, x+b\leq 1} \left\{ \left(\frac{R_t P_t}{P_{t+1}} - r(1-x-b) \right) x + \left(\frac{(1-l_{t+1})P_t}{Q_t P_{t+1}} - r(1-x-b) \right) b \right\}.$

Optimality of B_t. Given a strategy profile σ and the history $\{h_t, R_t, X_t, x_t\}$, the decision $B_t = \sigma_t^F(h_t, R_t, X_t, x_t)$ is optimal when B_t solves the following problem:

$$U_t^F(\sigma(h_t, R_t, X_t, x_t)) \equiv \max_{B_t'} (g_t - \alpha_F l_t) + \beta U_{t+1}^F(\sigma(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1})),$$

such that

$$
\begin{aligned}
&b_t^M = \sigma_t^m(h_t, R_t, X_t, x_t, B_t'),\\
&b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t', b_t^M),\\
&(g_t, l_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t', b_t^M, b_t),\\
&h_{t+1} = \{h_t, R_t, X_t, x_t, B_t', b_t^M, b_t, l_t, g_t\},\\
&(R_{t+1}, X_{t+1}) = \sigma_{t+1}^M(h_{t+1}),\\
&x_{t+1} = \sigma_{t+1}^x(h_{t+1}, R_{t+1}, X_{t+1}).
\end{aligned}
$$

b_t^M is optimal. Given a strategy profile σ and history $\{h_t, R_t, X_t, x_t, B_t\}$, the decision $(b_t^M, \theta_t) = \sigma_t^m(h_t, R_t, X_t, x_t, B_t)$ is optimal when b_t^M is solution to:

$$U_t^M(\sigma(h_t, R_t, X_t, x_t, B_t)) \equiv \max_{b_t^{',M}} - \mid X_t/x_t - P_t^M \mid - \alpha_M l_t + \beta U_{t+1}^M(\sigma(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1})))$$

such that $b_t^{',M} \leq x_t(1 - R_{t-1}X_{t-1}/X_t)$ and

$$
\begin{aligned}
b_t &= \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^{',M}) \\
(g_t, l_t) &= \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t^{',M}, b_t) \\
hh_{t+1} &= \{h_t, R_t, X_t, x_t, B_t, b_t^{',M}, b_t, l_t, g_t\} \\
(R_{t+1}, X_{t+1}) &= \sigma_{t+1}^M(h_{t+1}), \\
x_{t+1} &= \sigma_{t+1}^x(h_{t+1}, R_{t+1}, X_{t+1}), \\
B_{t+1} &= \sigma_{t+1}^F(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}).
\end{aligned}
$$

b_t is a competitive outcome. Given a strategy profile and the history $\{h_t, R_t, X_t, x_t, B_t, b_t^M\}$, the aggregate saving decision in bonds $b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^M)$ is a competitive outcome when:

(i) Prices and default decisions are as follows: $P_t = X_t/x_t$, $Q_t/P_t = (b_t + b_t^M)/B_t$, $P_{t+1} = X_{t+1}/x_{t+1}$, l_{t+1} is given by

$$
\begin{aligned}
(g_t, l_t) &= \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t) \\
h_{t+1} &= \{h_t, R_t, X_t, x_t, B_t, b_t^M, b_t, l_t, g_t\} \\
(X_{t+1}, R_{t+1}) &= \sigma_{t+1}^M(h_{t+1}) \\
x_{t+1} &= \sigma_{t+1}^x(h_{t+1}, R_{t+1}, X_{t+1}) \\
B_{t+1} &= \sigma_{t+1}^F(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}) \\
b_{t+1}^M &= \sigma_{t+1}^m(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}) \\
b_{t+1} &= \sigma_{t+1}^b(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}, b_{t+1}^M) \\
(g_{t+1}, l_{t+1}) &= \sigma_{t+1}^f(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}, b_{t+1}^M, b_{t+1})
\end{aligned}
$$

(ii) $b_t = \arg\max_{b \leq 1 - x_t} \left(\frac{(1 - l_{s+1})P_s}{Q_s P_{s+1}} - r(1 - b - x_t) \right) b.$

(l_t, g_t) is optimal. Given a strategy profile σ and history $\{h_t, R_t, X_t, x_t, B_t, b_t, b_t^M\}$, the decision $(l_t, g_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, , b_t, b_t^M)$ is optimal when (l_t, g_t) is solution to:

$$
\begin{aligned}
&U_t^F(\sigma(h_t, R_t, X_t, x_t, B_t, b_t, b_t^M)) \equiv \max_{B_t'}(g_t - \alpha_F l_t) + \cdots \\
&\cdots \beta U_{t+1}^F(\sigma(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}, b_{t+1}, b_{t+1}^M)), \\
&\text{s.t.} g_t + l_t \frac{B_{t-1}}{P_t} \leq \frac{Q_t B_t}{P_t} + \theta_t
\end{aligned}
$$

where $P_t = X_t/x_t$, $Q_t = P_t(b_t + b_t^M)/B_t$, $\theta_t = l_t b_{t-1}^M P_{t-1}/(Q_{t-1}P_t) + x_t(1 - R_{t-1}X_{t-1}/X_t) - b_t^M$ and

$$
\begin{aligned}
&h_{t+1} = \{h_t, R_t, X_t, x_t, B_t, b_t^m, b_t, l_t', g_t'\} \\
&(R_{t+1}, X_{t+1}) = \sigma_{t+1}^M(h_{t+1}), \\
&x_{t+1} = \sigma_{t+1}^x(h_{t+1}, R_{t+1}, X_{t+1}), \\
&B_{t+1} = \sigma_{t+1}^F(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}), \\
&b_{t+1}^M = \sigma_{t+1}^m(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}), \\
&b_{t+1} = \sigma_{t+1}^b(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}, b_{t+1}^M).
\end{aligned}
$$

These definitions formalize the requirement in the equilibrium definition in the body of the paper that all agents hold the belief that future actions are taken according to the strategy profile σ.

Equilibrium definition. An equilibrium is a strategy profile $\sigma = (\sigma^M, \sigma^x, \sigma^F, \sigma^m, \sigma^b, \sigma^f)$ such that, for any period t and history h_t, we have:

(i) Given h_t and σ, $(R_t, X_t) = \sigma_t^M(h_t)$ is optimal.

(ii) Given $\{h_t, R_t, X_t\}$ and σ, $x_t = \sigma_t^x(h_t, R_t, X_t)$ is a competitive outcome.

(iii) Given $\{h_t, R_t, X_t, x_t\}$ and σ, $B_t = \sigma_t^F(h_t, R_t, X_t, x_t)$ is optimal.

(iv) Given $\{h_t, R_t, X_t, x_t, B_t\}$ and σ, $b_t^M = \sigma_t^m(h_t, R_t, X_t, x_t, B_t)$ is optimal.

(v) Given $\{h_t, R_t, X_t, x_t, B_t, b_t^M\}$ and σ, $b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^M)$ is a competitive outcome.

(vi) Given $\{h_t, R_t, X_t, x_t, B_t, b_t, b_t^M\}$ and σ, $(l_t, g_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t, b_t^M)$ is optimal.

D $T \geq 2$ and $r(1-b^*) \geq 1$

This appendix describes the equilibrium in the case in which $T \geq 2$ and $r(1-b^*) \geq 1$. The following ingredients are useful to describe the equilibria. Let $\phi(b) = br(1-b)$ for $b \in [0,1)$. For $t \geq 1$, let

$$b_t^* = \arg\max_{b\in[0,1)} \{b - \beta^t \phi^{(t)}(b)\}, \tag{80}$$

where the notation $\phi^{(t)}$ corresponds to the function ϕ composed t times. Notice that b_1^* corresponds to b^* in the body of the paper, where we dropped the subscript 1 for notational parsimony. For brevity we restrict the analysis to the case in which $b_1^* > 0$.

Proposition 10. ***(Endogenous regime switching when*** $r(1-b_1^*) \geq 1$***)*** *Suppose* $r(1-b_1^*) \geq 1$ *and* $R_{-1}X_{-1}$ *is arbitrarily small other things being equal. There exists a unique equilibrium.* M *does not issue new reserves between dates 0 and* $T-1$*. At date 0,* F *has strictly positive consumption (*$g_0 > 0$*) and* $P_0 = P_0^M$*. There is no consumption at the interim dates:* $g_t = 0$ *for* $t \in \{1; ...; T-1\}$*.*

1. *If* $\bar{x} + \bar{b} > \phi^{(T)}(b_T^*)$*,* $g_T > 0$ *and there is monetary dominance at every date.*
2. *If* $\bar{x} + \bar{b} \leq \phi(b_1^*)$*, then* $g_T = 0$ *and there is fiscal dominance at every date* $t \geq 1$*.*
3. *In the interim range* $\bar{x} + \bar{b} \in (\phi(b_1^*), \phi^{(T)}(b_T^*)]$*, then* $g_T > 0$ *is arbitrarily small, and there exists* $\tau \in \{1; ...; T-1\}$ *such that there is fiscal dominance until* τ *and monetary dominance afterwards.*

Proof. See Appendix D.1. □

Case 1. is the outright extension to T periods of the situation in which $g_1^{PT}(0) > 0$ when $T = 1$. In this case, the optimal consumption pattern of F implies $g_T > 0$ and the Sargent-Wallace debt level would distort it at excessively small gains if $R_{-1}X_{-1}$ is sufficiently small. F finds the Sargent-Wallace debt level unpalatable both at date 0 and subsequently: It rolls over a debt burden that is ex-post excessive and that it is not willing to further increase ($b_t > b_{T-t}^*$ for $t \in \{1; ...; T-1\}$).

Case 2. is the outright extension of the situation in which $g_1^{PT}(0) = 0$ when $T = 1$. In this case, F is constrained by its next-date resources at each date and enjoys the extra slack generated by the Sargent-Wallace debt level, which M and savers anticipate along the equilibrium path.

Case 3. is more complex. Even though optimal consumption would require $g_T = 0$ since $\bar{x} + \bar{b} \leq \phi^{(T)}(b_T^*)$, F leaves a little bit of terminal resources on the table. It leaves just enough that it is not tempted by the Sargent-Wallace debt level at τ, the date at which the value of debt rolled over since date 0 snowballs above the ex-post optimal level $b_{T-\tau}^*$. Suppose by contradiction that the equilibrium is such that $g_T = 0$, and that F borrows at the Sargent-Wallace level at all dates. Then M would optimally deviate and force default by setting P_τ at P_τ^M instead of the value $P_\tau^M + \alpha_M$ along the equilibrium path. F would then prefer to raise debt at the optimal level (arbitrarily close to $b_{T-\tau}^*$ given an arbitrarily small $R_{-1}X_{-1}$) and to allow for monetary dominance at $\tau + 1$. With this deviation, M incurs the same date-τ disutility α_M as along the equilibrium path but gains future price levels on target. Thus $g_T = 0$ cannot be an equilibrium and F must leave (an arbitrarily small amount of) money on the table at date 0.

An interesting feature of equilibrium in this latter case 3. is that fiscal dominance prevails until τ even though $g_T > 0$. At face value, this contradicts the two-date insight that fiscal dominance requires that F pledges its entire future tax capacity. With more than two dates, the reason F may credibly be unable to reduce consumption in the future is that, as we have just seen, this would make the Sargent-Wallace debt level dominant, and in turn lead M to force default. But then savers anticipating such future default would not lend and default would occur right away.

D.1 Proof of Proposition 10

Claim 1. We show that if $r(1 - b_1^*) > 1$, the sequence $(b_t^*)_{t\geq 1}$ is such that $\phi(b_{t+1}^*) > b_t^* > b_{t+1}^*$.

Proof of the claim. The proof is by recursion. The first-order condition implicitly defining b_2^* is

$$\beta^2(\phi^{(2)})'(b) = 1 \tag{81}$$

or

$$\beta^2 \phi'(\phi(b))\phi'(b) = 1. \tag{82}$$

If $b_2^* > b_1^*$ then $r(1-b_2^*) > 1$ and $\phi(b_2^*) > b_2^* > b_1^*$ in which case (82) and thus (81) cannot hold because ϕ is convex increasing. Thus it must be that $b_2^* < b_1^*$ in which case (82) implies $\phi(b_2^*) > b_1^*$. For $t \geq 2$, the first-order condition implicitly defining b_{t+1}^* is

$$\beta^{t+1}(\phi^{(t+1)})'(b) = 1 \tag{83}$$

or

$$\beta^{t+1}(\phi^{(t)})'(\phi(b))\phi'(b) = 1. \tag{84}$$

or

$$\beta^{t+1}\phi'(\phi^{(t)}(b))(\phi^{(t)})'(b) = 1. \tag{85}$$

It must be that $\phi(b_{t+1}^*) > b_t^*$. Otherwise, (84) implies $b_{t+1}^* \geq b_1^*$ and so $\phi(b_{t+1}^*) > b_{t+1}^* \geq b_1^* > b_t^*$, a contradiction. Applying this to previous dates yields from the recursion hypothesis $\phi^{(t)}(b_{t+1}^*) > b_1^*$. But then (85) implies $b_{t+1}^* < b_t^*$.

Claim 2. If $r(1-b_1^*) > 1$, the solution to program (64) is such that $g_t = 0$ for all $t \in \{1; ...; T-1\}$.

Proof of the claim. Suppose that the optimal consumption pattern is such that for some $t \in \{0; ...; T-2\}$, $r(1-b_t)b_t < b_{t+1}$. It must then be that $b_t \geq b^*$ otherwise an increase in b_t would strictly increase the objective and be feasible. This implies that $b_{t+1} > b_t \geq b^*$, a contradiction, as decreasing b_{t+1} would strictly increase the objective.

We now prove the proposition, studying in turn each of the three relevant ranges of value.

1. $\bar{x} + \bar{b} > \phi^{(T)}(b_T^*)$. In this case F is not constrained by its terminal resources when choosing initial borrowing and so finds the Sargent-Wallace debt level at date 0 unpalatable. Claim 1 implies $\phi(b_{t+1}^*) > b_t^*$ for $t \in \{1; ...; T-1\}$. Since b_0 is arbitrarily close to b_0^* for $R_{-1}X_{-1}$ sufficiently small, $b_1 = \phi(b_0) > b_{T-1}^*$, and Claim 1 and that ϕ is increasing then implies that $b_t > b_{T-t}^*$ for all $t \in \{1; ...; T-1\}$. This implies that F finds the Sargent-Wallace debt level costly at all future dates because it already borrows more than if it had no legacy debt under the price-level taking debt level.

2. $\bar{x}+\bar{b} \leq \phi(b_1^*)$**.** In this case F always issues the Sargent-Wallace debt level because it is constrained by its future resources at every date. Formally, $b_{T-t} < b_t^*$ for all $t \in \{1; ...; T\}$. It is true for $t = 1$ since $b_{T-1} = \phi^{-1}(\bar{x} + \bar{b}) \leq b_1^*$, and then by recursion since Claim 1 implies

$$b_{T-t} = \phi_{-1}(b_{T-t+1}) < \phi_{-1}(b_{t-1}^*) < b_t^*. \tag{86}$$

3. $\phi(b_1^*) < \bar{x} + \bar{b} \leq \phi^{(T)}(b_T^*)$**.** In this case F is constrained by its terminal resources since these do not exceed $\phi^{(T)}(b_T^*)$. The optimal consumption pattern thus dictates that F borrow against its entire terminal resources—$g_T = 0$—and rolls over its debt. We show that this cannot be the equilibrium consumption pattern. If this were the case, then there would be fiscal dominance from date 1 on since the Sargent-Wallace debt level increases the current consumption of F while leaving future ones unchanged at zero. From Claim 1, there exists $\tau \in \{1; ...; T-1\}$ such that $b_t \leq b_{T-t}^*$ for $t < \tau$ and $b_t > b_t^*$ afterwards. From τ on, M is better off setting the price level on target and let the government default at τ as F would use this slack to reduce its borrowing rather than forcing fiscal dominance since it is strictly above its ex-post maximum borrowing b_t^*. So F cannot borrow so much at date 0 that it strictly prefers to force fiscal dominance after τ, as savers anticipating future default would not lend. Before τ however, F is still constrained by its future resources and issues the Sargent-Wallace debt level.