Platform Investment Incentives: Dating and Fake Profiles*

Jana Gieselmann† Alexander Rasch‡

May 6, 2021

Abstract

This paper examines the use of fake profiles by two-sided platforms to stimulate demand and increase profits. By deceiving naïve users, platforms invest into an artificial increase of the network size on one side of the market. Whereas firms are caught in a prisoner's dilemma if users single-home on both sides of the market, users are protected by platform competition. If users on one side of the market multi-home, firms can increase their prices for the multi-homing side, and they lower their prices for the single-homing side. Investments into fake profiles stimulate demand, such that multi-homing demand and profits increase. Platforms and users as a group can profit from investments under these circumstances.

JEL Classification: D18; D43; L13; M37.
Keywords: Dating; Fake profile; Multi-homing; Naïveté; Network effect; Platform investment; Single-homing; Two-sided market.

*We thank Paul Heidhues for very helpful comments and suggestions. Jana Gieselmann gratefully acknowledges financial support from the Graduate Programme Competition Economics (GRK 1974) of the German Research Foundation (DFG).

†University of Duesseldorf, Duesseldorf Institute for Competition Economics, Germany, gieselmann@dice.hhu.de
‡University of Duesseldorf, Duesseldorf Institute for Competition Economics, Germany, rasch@dice.hhu.de

1 Introduction

In today's economy, an increasing number of industries are organized around platforms with Amazon, Facebook, Google, Booking.com, or Tinder being some of the most famous examples. Tinder's parent company, Match Group Inc., provides numerous dating apps and brings together over 17 million users in the US alone.[1] Dating platforms are an increasingly important method for different user groups to meet. Even though these platforms differ in their business models, their common aim is to bring together two sides of a market by facilitating interaction between their user groups. Online dating sites have been engaged in deceptive practices in recent years. For example, Match Group has been sued by the Federal Trade Commission (FTC) because of its use of fake profiles on their platform in September 2019.[2] On Match.com, they used fake profiles created by a third party as a form of advertisement to persuade users to upgrade into paying for a subscription. Other dating platforms use company-created fake profiles; a list of several dating sites using this practice has been published by the Verbraucherzentrale Bayern (Center for Consumer Advise Bavaria) in Germany. These platforms employ paid workers to create profiles, and interact with users on the platform, giving them the impression of a real contact.[3] It is not commonly known that platforms themselves create fake users to possibly stimulate demand, although it is legal to do so as long as it is mentioned in the terms and conditions. There are companies that specialize in providing employees as chat moderators to these platforms.[4] These chat moderators set up fake profiles and engage in conversations with the users of the platform pretending to be a real profile.

This paper analyzes platforms using fake profiles as an instrument to create the illusion of a larger network. Platforms deceive naïve users on one side into believing that the network size on the other side is larger than it actually is, by creating fake profiles to artificially increase the network size. These fake profiles are treated as investments by the platforms, that is, firms invest into creating artificial users. In other words, platforms advertise the network, such that the perceived network size does not equal the actual network size on one side of the market. A model is used to analyze how these investments affect the market outcome in online dating, and what role user behavior with regard to single- or multi-homing plays.

[1] See `https://www.statista.com/statistics/826778/most-popular-dating-apps-by-audience-size-usa/`, last visited 30.10.2020. The dating sites Tinder, PlentyofFish, Match.com, OkCupid, and Hinge are owned by Match Group, Inc.

[2] See `https://www.ftc.gov/news-events/press-releases/2019/09/ftc-sues-owner-online-dating-service-matchcom-using-fake-love`, last visited 01.09.2020.

[3] See `https://www.verbraucherzentrale.de/wissen/digitale-welt/onlinedienste/onlinedating-auf-diesen-portalen-flirten-fakeprofile-21848`, last visited 01.09.2020.

[4] For example, Cloudworkers or Agentur da Chatdeife are companies that employ freelancers to work for and on one or more social-community platforms. See also `https://www.spiegel.de/wirtschaft/service/singleboersen-ein-moderator-von-fake-profilen-spricht-ueber-seinen-job-a-1113937.html.` and `https://www.ndr.de/fernsehen/sendungen/panorama_die_reporter/Undercover-als-Chatschreiberin-Abzocke-Flirtportal,sendung1098906.html` for an interview (in German) and `https://www.marieclaire.fr/,dating-assistant,750821.asp` for an article (in French).

To this end, the model assumes two competing, horizontally differentiated platforms in a market with two-sided indirect network effects. Users on both sides decide which platform to join. Agents of different groups exert positive cross-group externalities, such that more users on one side of the market are beneficial to the other side. We provide a benchmark for the analysis of fake profiles abstracting from within-market-side crowding out. The model differentiates between user single- and multi-homing. In both environments, platforms decide on membership fees, and we investigate investment incentives.

Investment incentives to create fake profiles depend on the single- and multi-homing behavior of consumers. Against a first intuition, the use of fake profiles does not necessarily harm consumers as a group, and can actually be beneficial to them if one side multi-homes. If users on both sides single-home, prices and demand are unaffected by the investments. Investments are a form of wasteful competition, which cause additional costs and, hence, lower platform profits. That is, platforms are caught in a prisoner's dilemma, and are unable to take advantage of the investments. Both user groups are indifferent between a scenario with and without fake profiles.[5] Hence, whereas platforms want to collectively avoid investments in fake profiles under single-homing, they have incentives to engage in this practice if users multi-home.

Under multi-homing by one group, platforms can profit from investments, because prices increase for the multi-homing side, and decrease for the single-homing side alongside with an increase in the multi-homing behavior. The multi-homing side is always worse off due to increased prices and no real increase in demand on the single-homing side. The single-homing side benefits from lower prices and more users on the other side of the market due to increased multi-homing demand. Overall, users can profit from this practice if the positive effects for the single-homing side outweigh the negative effects on the multi-homing side.

The model presented in this paper adds to the investment literature in two-sided platform markets. There are only a few articles that deal with investments, such as Belleflamme and Peitz (2010). Their article focuses on investments by one side of the market (sellers). The investment decision is driven by its influence on the network benefits. If seller investments increase buyer surplus, the platforms in turn set lower access fees for the sellers, such that sellers' incentives to invest increase. However, their model differs from this paper, because they focus on investments by one market side rather than by the platform itself.

Investments of a two-sided platform are analyzed by Reisinger and Zenger (2019) who investigate incentives by a credit card platform. Similar to the model in this paper, the two-sided platform makes an investment decision. In their model, the platform, which is characterized as a two-sided

[5] Consumer utility is the same in both cases, because prices and demand are unchanged. However, this is due to the assumption that consumer surplus takes the underlying real network size into account without the fake profiles. Furthermore, users do not incur any disutility from interacting with a fake profile.

market of a payment card association, invests into the quality of card services for one of the two sides (consumers or retailers).[6]

Angelini et al. (2019) investigate non-price strategies and investments on a two-sided monopoly platform with sellers and buyers. The platform invests into quality improvement, where the investment decision is linked to seller competition. With increasing competition between sellers, the platform invests less in quality improvements.[7] The model at hand takes a difference stance as it focuses on an increase in perceived quality or perceived network benefits. In contrast to the monopoly platforms in Reisinger and Zenger (2019) and Angelini et al. (2019), the dating market in this paper is modeled by imperfectly competing platforms, which in line with empirical evidence. Lastly, Edelman and Wright (2015) investigate the effects of price coherence adopted by a monopoly platform and competing platforms, where the intermediary provides a benefit to buyers when purchasing through the platform. Platforms invest into benefits for the buyer side, where investments are excessive in both cases when platforms impose price coherence.[8]

In general, this paper is closely related to the literature of two-sided markets with seminal contributions by Caillaud and Jullien (2003) and Rochet and Tirole (2003, 2006), in which platforms act as intermediaries and charge usage fees on both sides of the markets. The baseline model of this paper follows the model by Armstrong (2006). Platforms compete in a duopoly framework, in which agents are allowed to single- and multi-home. The analysis of the multi-homing environment is related to the analysis of Belleflamme and Peitz (2019). The authors study the comparison between single- and multi-homing in detail to inform policy makers about the effects of multi-homing. For example, in contrast to Armstrong (2006) who argues that multi-homing increases prices on the multi-homing side due to the monopoly power of the platform over this side, Belleflamme and Peitz (2019) challenge this view, and show that prices can also decrease in comparison to the single-homing environment.

This paper also adds to the literature on consumer naïveté.[9] The model at hand assumes that one side of the market is naïve with regard to the network size. Hence, we are among the first to introduce consumer naïveté in two-sided markets. The only other paper in this context is Johnen and Somogyi (2019). Their paper differs from ours in two important ways in that the authors consider a monopoly platform, and in that they investigate consumer naïveté with regard to hidden prices. They find that a platform has strong incentives to shroud additional fees if it increases perceived consumer surplus. In another model of consumer naïveté, Heidhues et al.

[6] In a different setting, Verdier (2010) analyzes the impact of investments in payment card systems by banks on the optimal interchange fee.

[7] Similarly, Dou et al. (2016) study incentives of a monopolist platform investing into value-added services.

[8] A loosely related paper by Hagiu and Spulber (2013) investigates investments in first-party content by platforms depending on the relationship between first-party and third-party content considering a monopoly platform and the possibility of entry. First-party investments are strategically used to overcome the market coordination problem. With entry the incumbent invests more relative to the case in which it is a monopolist depending on the seller expectations.

[9] A survey on consumer naïveté with regard to different aspects (for example, hidden prices) is provided by Heidhues and Kőszegi (2018).

(2016a) investigate firms' incentives to innovate either to increase a product's value or to increase hidden prices. The latter is termed exploitative innovation. The model in our paper could be interpreted as a model of exploitative investment to increase the perceived network size.

Instead of focusing on buyers and sellers, this model focuses on two groups of consumers, as found on dating and matching platforms. Therefore, this paper is loosely connected to the literature on dating and matching platforms. Halaburda et al. (2018) deliver an explanation for the cause and effect of negative within-group and positive cross-group externalities on a dating platform. Users are more likely to find an attractive match if the platform attracts more users of the opposite user group, but are less likely to be accepted as a match if the number of users in their own group increases. The authors utilize the intra-group network effects as means to explain endogenous vertical differentiation of dating platforms to reduce competition among users of one group. In another recent article, Gal-Or (2020) focuses on the market segmentation of dating platforms in terms of quality. Users are heterogeneous with respect to a certain attribute (for example, education and income), which allows ranking the individuals qualitatively. Market segmentation leads to one platform serving "higher-quality" individuals, whereas the other matches "lower-quality" individuals. However, segmentation only arises if the compatibility of quality ranks is relatively important to consumers.[10, 11]

Lastly, in another interpretation of this model, investments can also be seen as advertisements, connecting to the literature of persuasive advertising. Investing into the inflation of the network size or, in terms of the example, into fake profiles, could be considered a form of advertisement to draw demand to one's own platform. In the classic literature summarized by Belleflamme and Peitz (2015), persuasive advertising serves to influence the consumer's willingness to pay, or to increase the perceived product differences (Von der Fehr and Stevik, 1998). Bloch and Manceau (1999) interpret persuasive advertising as a means to shift the distribution of consumers in favor of one product. A common result in this literature is that firms are worse off when advertising, because they face a Prisoner's Dilemma. This is true for the cases in which advertising increases the willingness to pay, or changes the distribution of consumer tastes. There, advertising is a form of wasteful competition, and results in lower profits. In the symmetric equilibrium, firms invest to much into advertisement, and their competitive advantage from the investment disappears.[12]

The result of the prisoner's dilemma also persists in a Hotelling model with investments by two firms when they, for example, decide on congestion-reducing investments (Matsumura and Matsushima,

[10] The models of matching platforms differ from the classic industrial organization models, and assume heterogeneous outside options. Two additional articles from this strain of literature are Damiano and Li (2007) and Damiano and Li (2008) who focus on the sorting efficiency of a monopolist or competing platforms respectively. Unlike their model, the model in this paper does not primarily focus on the efficiencies of such matching platforms, but platforms engage as an intermediary.

[11] Online dating platforms are also used to conduct field experiments. For example some empirical studies investigated the effect and role of income and education in online dating (Ong and Wang, 2015; Neyt et al., 2019).

[12] In contrast to this, firms make higher profits when informing consumers about their products assuming a model of informative advertising (Belleflamme and Peitz, 2015).

2007). As a consequence, firms do not invest a positive amount in equilibrium. Similarly, if firms make R&D investments, investments are too low in a Hotelling model with fixed locations (Matsumura and Matsushima, 2012).
The remainder of the paper is structured as follows. In Section 2, the framework of the model is presented. Section 3 analyzes the two variants of the model: two-sided single-homing and one-sided multi-homing. Lastly, Section 4 concludes this paper.

2 The Model

This section presents a model of duopolistic platform market in which firms can invest into the artificial increase of the network size on one side of the market. Following the example of dating platforms, the model focuses on two horizontally differentiated platforms addressing two groups of agents: men (denoted by subscript m) and women (denoted by subscript w). It is assumed that the two platforms are located at the two extremes of a linear city of length one, that is, at $x^1 = 0$ and $x^2 = 1$ (Hotelling, 1929; Armstrong, 2006). Platforms compete for both sides of the market by setting membership fees. There is a unit mass of users of group m and group w, each of which is uniformly distributed on the interval $[0, 1]$. Every user is characterized by an address $x \in [0, 1]$. Platforms are assumed to incur costs of $c_k > 0$ per user of group k ($k \in \{m, w\}$), which can be interpreted as the cost of accommodating an agent of group m or w on the platform. It is assumed that the investment into the advertisement of the network of one group targets the female side of the platforms, so that the advertising effect of this investment is effective on the male side.[13] An investment of s^i by platform i ($i \in \{1, 2\}$) leads to costs $c(s) = \gamma(s^i)^2/2$, where $\gamma > 0$.[14] The investments are viewed as additional users of group w, which increase the indirect network effects on the side of group m, implying that the users of group m are not able to distinguish between the real network and the perceived network. Users can be described as naive, because they are deceived by the platform into believing the advertised network size.[15]

[13]Adding investments on both sides of the market does not change the results of this section qualitatively. Nevertheless, the investments could be reasoned as means to advertise the weaker side of the market, so that the assumption of one-sided investments is indeed justifiable.

[14]The model without investments is a special case of this model, in which γ converges to infinity.

[15]This assumption is based on the observation made by the Verbraucherzentrale Bayern as mentioned in the introduction. Because the platforms employ workers to interact with the users, many of them are not able to identify the fraud. Consumers are myopic and naive with respect to the network size. This assumption can be linked to the literature on naïveté with the pioneering work by Gabaix and Laibson (2006). Gabaix and Laibson (2006) and for example also Heidhues et al. (2016b) assume that consumers are naive with respect to a hidden attribute of the product, and are not able to infer its existence.

When joining platform i that attracts n_m^i and n_w^i users of the two groups, a member of group k who is located at x derives the following utility:

$$u_m^i = r_m + \beta_m \left(n_w^i + s^i\right) - p_m^i - \tau_m \left|x - x^i\right| \tag{1}$$

$$u_w^i = r_w + \beta_w n_m^i - p_w^i - \tau_w \left|x - x^i\right| . \tag{2}$$

First, $r_k > 0$ denotes the stand-alone value or reservation value, which a user of group k receives from joining a platform.[16] The transport cost τ_j is assumed to be linearly proportional to the distance of the platform for a member of group k. The model includes positive group-specific cross-group network effects $\beta_k > 0$.[17] Platforms compete in prices p_m^i and p_w^i (membership fees). Moreover, platform i invests an amount s^i, which influences the utility of a member of group m, and intensifies the positive perceived cross-group external effects, because users of group m cannot differentiate between real and fake profiles, and, hence, derive the same benefit from either user.

With regard to welfare, it is assumed that fake profiles are not utility relevant because there are no long-run benefits from fake profiles for a user. This means that the investments do not influence the users' utilities after prices and investments are realized. In other words, fake profiles are not welfare relevant except for their effect on the market outcome.

Following the literature, it is assumed that all members of both groups participate in the market, that is, join (at least) one of the two platforms (covered market), which implies that the stand-alone value r_k is sufficiently large.

The timing is as follows: At the first stage, platforms simultaneously set their membership fees for both groups. Additionally, firms simultaneously decide on their investment level at this stage. Users observe these choices, and decide which platform(s) to join at the second stage. Two environments will be analyzed: (i) Users from both groups decide between joining either platform 1 or platform 2 (two-sided single-homing), and (ii) men decide whether to join a single platform or both, and women decide between both platforms (one-sided multi-homing). The two-stage game is solved via backward induction to identify the subgame-perfect Nash equilibrium.

[16] The model includes a reservation value r_k and costs per user c_k. Belleflamme and Peitz (2010) set up a model without including a reservation value. However, in their model, it is necessary to include marginal costs per user, so that the equilibrium conditions are fulfilled. In the more recent article, Belleflamme and Peitz (2019) include a reservation value and marginal costs per user. In that paper, it proved to be convenient to include both variables too, while it is necessary that at least marginal costs with $c_k > 0$ or a reservation value with $r_k > 0$ be included. It is noteworthy that the equilibrium conditions, that is, the set of assumptions that ensures the existence of the equilibrium in the single-homing environment, are not fulfilled for $r_k = c_k = 0$. In this case, a null equilibrium in which no user joins a platform would arise.

[17] Based on the description of the dating market, it is assumed that a user of one group receives a positive utility from the number of agents of the other group, that is, group m's utility increases if more users of group w join the platform as the pool of potential dating partner increases and vice versa. Halaburda et al. (2018) assume in addition, that users of one group are negatively affected by the number of agents of the same group due to an increased competition within the members of the group in the form of negative within-group externalities. This possibility will be neglected in the main analysis, but will be discussed later on.

3 Analysis and Results

In this section, the equilibria in the two scenarios are analyzed.

3.1 Two-Sided Single-Homing

Under two-sided single-homing (superscript SH), each user chooses between both platforms. A reason to explain potential single-homing in the dating market is that the user groups are highly differentiated so that the motivation for dating differs.[18] Especially marginalized groups might tend to single-home and join a specialized platform to meet only users with the same background. The indifferent users between both platforms, located at $\hat{x}_m$ and $\hat{x}_w$, can be obtained by equating $u_k^1 = u_k^2$. Together with full participation and two-sided single-homing, this implies the following:

$$\hat{x}_m = \frac{1}{2} + \frac{\beta_m \left[(n_w^1 + s^1) - (n_w^2 + s^2) \right] + p_m^2 - p_m^1}{2\tau_w}, \; n_m^1 {=} \hat{x}_m \text{ and } n_m^2 = 1 - n_m^1 \tag{3}$$

$$\hat{x}_w = \frac{1}{2} + \frac{\beta_w \left(n_m^1 - n_m^2 \right) + p_w^2 - p_w^1}{2\tau_w}, \; n_w^1 {=} \hat{x}_w \text{ and } n_w^2 = 1 - n_w^1 \tag{4}$$

For given prices and investments levels, an additional user of group k attracts β_k/τ_k additional users of group l ($l \in \{m, w\}$, $k \neq l$).

To exclude the possibility that only one platform is active in equilibrium, the network effects cannot be too strong. To ensure the existence of a market-sharing equilibrium, the following assumption is required to fulfill the necessary and sufficient conditions:

Assumption 1. $4\tau_m\tau_w > (\beta_m + \beta_w)^2$.

That is, product differentiation must be sufficiently large compared to the cross-group network effects.

Furthermore, it is assumed that the cost from investing γ are larger than a critical value, which is defined as follows:

Assumption 2. $\gamma \geq \gamma^{SH} \equiv \frac{\beta_m^2 \tau_w}{4\tau_m\tau_w - (\beta_m + \beta_w)^2}$

Given Assumption 1 the critical value of γ is larger than zero. Assumption 2 ensures that the costs of investing are not too low, such that second-order conditions are satisfied.

Solving the implicit expressions above given that platform 1 and 2 offer prices (p_m^1, p_w^1) and (p_m^2, p_w^2), respectively, implies the following market shares

$$n_m^i = \frac{1}{2} + \frac{1}{2} \frac{\beta_m\tau_w \left(s^i - s^j \right) + \beta_m \left(p_w^j - p_w^i \right) + \tau_w \left(p_m^j - p_m^i \right)}{\tau_m\tau_w - \beta_m\beta_w} \tag{5}$$

$$n_w^i = \frac{1}{2} + \frac{1}{2} \frac{\beta_m\beta_w \left(s^i - s^j \right) + \beta_w \left(p_m^j - p_m^i \right) + \tau_m \left(p_w^j - p_w^i \right)}{\tau_m\tau_w - \beta_m\beta_w,}. \tag{6}$$

[18]In general, single-homing can be motivated by different reasons, such as exclusivity agreements (Belleflamme and Peitz, 2019).

Assumption 1 ensures that the denominators are positive, such that demands are well-defined. With regard to equations (5) and (6), two observations are: First, the number of members of group k on platform i is decreasing in the platform's price for that group (p_k^i). Furthermore, the demand increases in the prices of the rival platform $\left(p_k^j\right)$. Second, as $\beta_k > 0$, demand is complementary, that is, the number of users of group k also decreases in the price by the same platform for the other group.

Additionally, the investments here also have an effect on the demand of each group. The demand of group m increases on platform i if platform i invests more due to a perceived increase of the network size. Furthermore, the demand of group w on platform i also increases with an increase in investments s^i on platform i. The cross-group network effects create a feedback loop such that if more users of group m are attracted, also more users of group w want to join the platform. Equations (5) and (6) constitute the consumer demands at the second stage.

Turning to the first stage, platforms simultaneously choose their prices on both sides of the market and their investments levels. The profit of platform i can be written as

$$\pi_i = \left(p_m^i - c_m\right) n_m^i + \left(p_w^i - c_w\right) n_w^i - \frac{\gamma}{2}\left(s^i\right)^2,$$

where the demands of group m and w is given by equations (5) and (6). Differentiating with respect to the prices, and assuming symmetry results gives the following first-order conditions

$$p_m^{SH} = c_m + \tau_m - \frac{\beta_w}{\tau_w}\left(\beta_m + p_w^{SH} - c_w\right) \tag{7}$$

$$p_w^{SH} = c_w + \tau_w - \frac{\beta_m}{\tau_m}\left(\beta_w + p_m^{SH} - c_m\right), \tag{8}$$

and

$$s^{SH} = \frac{1}{2}\frac{\left(p_m^{SH} - c_m\right)\beta_m\tau_w + \left(p_w^{SH} - c_w\right)\beta_m\beta_w}{\gamma\left(\tau_w\tau_m - \beta_m\beta_w\right)}. \tag{9}$$

The first-order conditions with respect to the prices are equivalent to the standard model without investments as in Armstrong (2006), which is a special case for $\gamma \to \infty$.[19] Consequently, the equilibrium prices are identical in the model with and without investments, because the investments do not affect the first-order conditions through γ. To summarize two effects play a role in determining the price of a group k: Platform market power and marginal costs increase the price, whereas the external benefit from attracting an additional user of group l decreases the price.[20]

The first-order condition with respect to the investments is less intuitive. Assumption 1 guarantees that the denominator in equation (9) is positive. The term $\left(p_m^{SH} - c_m\right)\beta_m\tau_w$ corresponds to the additional revenue gained from users of group m if the platform increases the investments by one unit relative to its competitor. At price parity, the demand of group m at platform i increases by

[19] When referring to the standard model, the following notation will be used: $SH/MH, \infty$.

[20] For an extensive description of the first-order conditions on the prices see for instance Armstrong (2006).

$\beta_m \tau_w$ if s^i increases by one unit compared to s^j. Similarly, the second term is the revenue gained from additional users of group w if the platform increases the investments by one unit relative to its competitor. For given prices, the number of users of group w, given by equation (6), increases by $\beta_m \beta_w$. If the costs γ increase, investments naturally decrease and are equal to zero for $\gamma \to \infty$. In the symmetric case, investments depend on the prices of both groups, which are the same for both platforms. However, it is interesting to note that the investment level only depends on the own platform's prices and not on the competitor's.

Solving the three first-order conditions yields the equilibrium prices and the equilibrium investment level

$$\begin{aligned} p_m^{SH} &= c_m + \tau_m - \beta_w \\ p_w^{SH} &= c_w + \tau_w - \beta_m \\ s^{SH} &= \frac{\beta_m}{2\gamma}, \end{aligned}$$

Prices are equivalent to the standard model with indirect network effects (see Armstrong, 2006). Firms have an incentive to influence the perceived network size because investments are strictly positive (that is, $s^{SH} > 0$). Platforms' investments into fake profiles are positive and increasing in the strength of the indirect network effects of group m. Given a symmetric equilibrium candidate in which prices, demand, and investments are symmetric, investing zero is not an equilibrium. The intuition is as follows: Each platform can gain users of group m at zero marginal costs by slightly increasing its investments. The perceived network effects increase, and the respective platform can secure a higher market share. If all users of group m and w participate, the utility for the indifferent consumer at $x = 1/2$, who is the farthest from both platforms, must be larger than zero. That is, $u_k^* = r_k - c_k + \frac{1}{2}\beta_k + \beta_l - \frac{3}{2}\tau_k \geq 0$, which yields $\tau_k \leq \frac{2}{3}\left(h_k + \frac{1}{2}\beta_k + \beta_l\right)$, where $h_k := r_k - c_k$.

Then, in the symmetric equilibrium $n_m = n_w = 1/2$, meaning both platforms make the same profits. The platforms' profits are given by

$$\begin{aligned} \pi^{SH} &= \frac{1}{2}\left(\tau_m - \beta_w + \tau_w - \beta_m\right) - \frac{(\beta_m)^2}{8\gamma} \\ &= \frac{4\left(\tau_m + \tau_w - \beta_m - \beta_w\right)\gamma - (\beta_m)^2}{8\gamma}. \end{aligned}$$

Proposition 1. *In the duopoly model with investments and two-sided single-homing, a unique symmetric equilibrium exists if Assumptions 1 and 2 are fulfilled. Firms make lower profits when investing into fake profiles, that is, they are caught in a Prisoner's Dilemma.*

Proof. See Appendix. □

Under the assumptions in Proposition 1, platform profits are positive. The result in Proposition 1 is similar to the results from the persuasive advertising literature (Belleflamme and Peitz, 2015), in which firms often face a prisoner's dilemma. In those cases, profits are lower if firms advertise. Similar to that literature, firms are made worse off by their ability to increase the network size artificially. A larger cost γ is preferable in this case because higher investment costs decrease the loss from investing. If firms could cooperate at this stage, they would decide to refrain from creating fake profiles. The prisoner's dilemma is apparent because the prices remain unchanged, and the investments only cause additional costs. Furthermore, the demand is unchanged due to the Hotelling specification in the single-homing environment.

Proposition 2. *User surplus is independent of the investment costs* γ.

Having a closer look at user surplus, it is assumed that only the actual size of the network enters the surplus function. Both prices and actual demand are not influenced by the investments, such that users are neither better nor worse off compared to the case without any investments. This extends the safety-in markets result by Heidhues and Kőszegi (2018) to two-sided markets with naïveté of the perceived network size. Consumers' equilibrium welfare is unaffected by consumer naïveté.[21] Platforms are unable to exploit consumers' mistakes if they compete for their user base. Users of group m do not pay higher prices even though their perceived network effects increase. Naïve users are protected by the competition among platforms.

Total welfare is always lower when firms engage in deceiving practices, and users single-home because user surplus is not affected by this practice, but platform profits decrease due to wasteful investments.

Applying this result to the example shows that the prisoner's dilemma might be rationalized. Apart from the use of fake profiles on Match.com, no case of fake profiles created by the most popular dating sites themselves are known. The reason could be simple: If platforms face a Prisoner's Dilemma, they refrain from investing as long as they can cooperate. Because the largest dating sites are mostly owned by the Match Group, the platforms do not need to engage in wasteful competition in form of investments. However, the smaller, independent platforms listed by the Verbraucherzentrale Bayern may be stuck in this kind of wasteful competition.

3.2 One-Sided Multi-Homing

Consider now a situation in which group m has the possibility to multi-home (superscript "MH"), whereas group w continues to patronize only one of the two platforms (one-sided multi-homing). The idea is that group m values the network benefits higher than the costs of participating on two platforms as suggested by Armstrong (2006). The idea is similar to the reason why group

[21] Heidhues and Kőszegi (2018) investigate naïveté with regard to hidden fees of contracts in imperfectly competitive markets. Their analysis shows that consumer welfare is unaffected by naïveté as the firms hand profits from the hidden fees directly to consumers. This result is termed safety-in markets.

m is targeted by the investments; group m is supposed to be the market side that searches more actively, and wants to increase its probability for a fitting match/interaction.[22] The Bundeskartellamt (2016) discusses multi-homing, which is used to increase the chances of finding a match, in the case of dating platforms. They conclude that multi-homing is the predominant behavior on dating platforms, which counteracts the self-reinforcing feedback loop that often arises in social networks.[23]

3.2.1 Equilibrium Analysis

Due to the multi-homing behavior of one group, platforms exercise monopoly power over the multi-homing side by providing access to the single-homing side. The unit line is segmented into three subintervals for the users of group m. Users on the left of the unit line will join platform 1, whereas users on the right will join platform 2; those users in the middle will join both platforms. These multi-homing users are able to interact with all users of the single-homing side and their stand-alone value, r_k, is duplicated. However, their transportation costs increase and to access the two platforms both membership fees must be paid.[24]

To identify the boundaries of these subintervals two indifferent consumers need to be defined. The user of group m who is indifferent between joining platform 1 and not joining this platform is denoted by $\hat{x}_{1m}$. Similarly, $\hat{x}_{2m}$ denotes the indifferent user between joining platform 2 or not joining this platform.

The indifferent users of group m are then given by

$$\hat{x}_{1m} = \frac{r_m + \beta_m \left(n_w^1 + s^1\right) - p_m^1}{\tau_m}, \text{ and } \hat{x}_{2m} = 1 - \frac{r_m + \beta_m \left(n_w^2 + s^2\right) - p_m^2}{\tau_m},$$

where $0 < \hat{x}_{2m} < \hat{x}_{1m} < 1$. Then, the number of users of group m is $n_m^1 = \hat{x}_{1m}$ on platform 1 and $n_m^2 = 1 - \hat{x}_{2m}$ on platform 2, such that the number of users m on platform i can be expressed as

$$n_m^i = \frac{r_m + \beta_m \left(n_w^i + s^i\right) - p_m^i}{\tau_m}, \ i = 1, 2. \tag{10}$$

The indifferent user in group w between joining platform 1 or 2 is given by equation (4) as before.

[22] Dating platforms are more often used by a larger share of men than women. For example on Tinder, men represent 72% of the users (see https://www.statista.com/statistics/975925/us-tinder-user-ratio-gender/).

[23] One sided multi-homing is also present in other markets. For example, Rochet and Tirole (2003) mention the credit card market in which stores usually accept multiple credit cards, but cardholders often only own a single credit card.

[24] These assumptions can also be found in Belleflamme and Peitz (2010, 2019).

These four equations form a system with four unknowns, which can be solved for the demand of group m and group w

$$n_m^i = \frac{\beta_m}{\tau_m}\left[\frac{1}{2}+\frac{1}{2}\frac{\beta_w\left(p_m^j-p_m^i\right)+\tau_m\left(p_w^j-p_w^i\right)}{\tau_m\tau_w-\beta_m\beta_w}+\frac{1}{2}\frac{\left(2\tau_m\tau_w-\beta_m\beta_w\right)s^i-\beta_m\beta_w s^j}{\tau_m\tau_w-\beta_m\beta_w}\right]+\frac{r_m-p_m^i}{\tau_m} \tag{11}$$

$$n_w^i = \frac{1}{2}+\frac{1}{2}\frac{\beta_m\beta_w\left(s^i-s^j\right)+\beta_w\left(p_m^j-p_m^i\right)+\tau_m\left(p_w^j-p_w^i\right)}{\tau_m\tau_w-\beta_m\beta_w,}, \tag{12}$$

Due to single-homing by users of group w, their demand is unchanged in comparison to the previous model. The demand for group m differs from the demand in the single-homing scenario. The first term in the rectangular brackets is identical to the demand of group w without investments. The demand is multiplied with a term that appeared already in the single-homing analysis. It denotes that an additional user of group w attracts β_m/τ_m additional users of group m, so that both terms in total represent the attracted additional users of group m when n_w^i members of group w are present on the platform. The demand declines in the price the users pay on the corresponding platform i. In addition, the above equation can be interpreted with regard to the effect of investments.
To ensure a market-sharing equilibrium, the following revised assumption, which is slightly less strict than Assumption 1, must hold.

Assumption 3. $8\tau_m\tau_w > (\beta_m+\beta_w)^2+4\beta_m\beta_w$.

Therefore, if one group multi-homes, it is more likely that a market-sharing equilibrium arises. In other words, if multi-homing is allowed, it is less likely that one platform becomes dominant as in Belleflamme and Peitz (2019).
Given Assumption 3, the demand of group m on platform i additionally increases in the investment s^i on platform i. However, the demand decreases in s^j, such that the overall effect on demand of group m is ambiguous.
As in the previous section, a critical value of γ can be defined, which ensures the equilibrium existence. However, in the multi-homing scenario, there are two critical values, so that the larger value is chosen depending on the parameters.

Assumption 4. $\gamma \geq \gamma^{MH} = \max\left\{\gamma_1^{MH} \equiv \frac{2\beta_m^2(2\tau_m\tau_w-\beta_m\beta_w)}{\tau_m\left(8\tau_m\tau_w-(\beta_m+\beta_w)^2-4\beta_m\beta_w\right)}, \gamma_0^{MH} \equiv \frac{2\beta_m^2}{4\tau_m-\beta_m-\beta_w-2h_m}\right\}$.

Turning to the first stage of the game, the platforms again solve the maximization problem as in Section 3.1 given the demands in equations (11) and (12) with respect to p_m^i, p_w^i, and s^i. Using symmetry, the first-order conditions are given by

$$
\begin{aligned}
p_m^{MH} =& \frac{(\tau_m\tau_w - \beta_m\beta_w)\left[\beta_m\left(2s^{MH}+1\right)+r_m+c_m\right]-\tau_m\left[\beta_w\left(p_w^{MH}-c_w\right)-\tau_w c_m\right]}{4\tau_m\tau_w-3\beta_m\beta_w} && (13)\\
p_w^{MH} =& c_w+\tau_w-\frac{\beta_m}{\tau_m}\left(\beta_w+p_m^{MH}-c_m\right), && (14)
\end{aligned}
$$

and

$$
s^{MH}=\frac{1}{2}\frac{\beta_m\left[\left(p_w^{MH}-c_w\right)\beta_w\tau_m+\left(p_m^{MH}-c_m\right)(2\tau_w\tau_m-\beta_m\beta_w)\right]}{\gamma\tau_m\left[\tau_m\tau_w-\beta_m\beta_w\right]}. \tag{15}
$$

The first-order condition with respect to p_w is unchanged compared to equation (8). Regarding the first-order condition with respect to the price of group m, the investment level influences the price, which contrasts the result found in the single-homing model with investments. In the latter case, the investment level had no effect on either of the two first-order conditions which resulted in the same prices as in the model without investments. The investment level amplifies the effect of $\tau_m\tau_w - \beta_m\beta_w$ on the price in the numerator. Compared to equation (13), the price of group m will thus increase if a positive amount is invested.
This effect is due to the monopoly power of the platforms over the multi-homing group m. Platforms already appropriate a part of the surplus by setting higher prices. By investing, the platforms assume that the network benefits for group m increase. Therefore, platforms charge a higher price, because the network benefits β_m for group m from group w enters the pricing equation positively under multi-homing.
The first-order condition with respect to the investment level in equation (15) can be interpreted in a similar way as equation (9). There are two effects: The investment level increases with increasing price of group m. In line with the case of single-homing, the effect has the same direction. Furthermore, the price of group w also increases the investment level. The effect of p_w^{MH} has the same magnitude compared to equation (9).
Solving the three first-order conditions yields the equilibrium prices and investment level

$$
\begin{aligned}
p_m^{MH} =& \frac{\gamma\tau_m\left(\beta_m-\beta_w+2r_m+2c_m\right)+\beta_m^2\left(\beta_w-2c_m\right)}{4\gamma\tau_m-2\beta_m^2},\\
p_w^{MH} =& c_w+\tau_w-\frac{\beta_m\tau_m\gamma\left(\beta_m+3\beta_w+2h_m\right)-\beta_m^3\beta_w}{\tau_m\left[4\gamma\tau_m-2\beta_m^2\right]},\\
s^{MH} =& \frac{\beta_m\left(\beta_m+\beta_w+2h_m\right)}{4\gamma\tau_m-2\beta_m^2}.
\end{aligned}
$$

When neglecting the side of group w, and when γ converges to infinity, platforms exercise monopoly power over the side of group m and would charge them a monopoly price of $\frac{1}{2}(r_m+c_m)+\frac{\beta_m}{4}$. Given that the investment level is positive, it can be seen from the first-order conditions that the price for group m will then increase in comparison to the standard model without investments. The price increases proportionally to the amount of investment weighted with the network parameter β_m. Given this increase, the price for group w will in turn decrease. The decrease in p_w^{MH}, however, is greater than the increase in p_m^{MH}.

The second-order conditions are fulfilled as long as $\gamma > \gamma_1^{MH}$ holds.

The equilibrium number of users of group w is 1/2, and the number of users of group m is

$$n_m^{MH} = \frac{\gamma\left(\beta_m + \beta_w + 2h_m\right)}{4\gamma\tau_m - 2\beta_m^2}, \tag{16}$$

which must be between zero and one for an interior solution. More precisely, it needs to be fulfilled that $2\gamma\tau_m - \beta_m^2 < \gamma\left(\beta_m + \beta_w + 2h_m\right) < 4\gamma\tau_m - 2\beta_m^2$. The last assumption provides that $0 < \hat{x}_{2m} < \hat{x}_{1m} < 1$. In this case, all users of group m participate in at least one platform. To maintain this order, γ must be larger than γ_0^{MH}, which also ensures that the denominators are positive.[25] It holds that $\gamma_0^{MH} > \gamma_1^{MH}$ if $h_m > \frac{\tau_m(\beta_m+\beta_w)^2}{4\tau_m\tau_w - 2\beta_m\beta_w} - \frac{(\beta_m+\beta_w)}{2} \equiv h_m^{\gamma}$.

Multi-homing demand increases proportionally to the investments. When investments increase, users of group m believe that more users of group w participate on both platforms, which increases the network effects in relation to the costs from participating.

Comparing the equilibrium demand of group m with the equilibrium demand without investments shows that the demand increases

$$n_m^{MH} - n_m^{MH,\infty} = \frac{\beta_m^2\left(\beta_m + \beta_w + 2h_m\right)}{4\tau_m\left(2\gamma\tau_m - \beta_m^2\right)} > 0.$$

Proposition 3. *In a duopoly model in which firms invest to influence the perceived network size of one side of the market, and the other group multi-homes, there exists a unique and symmetric equilibrium. In this equilibrium, firms make profits of*

$$\pi^{MH} = \frac{\gamma\tau_m\left[8\tau_m\tau_w - \left(\beta_m + \beta_w\right)^2 - 4\beta_m\beta_w + 4h_m^2\right] + 2\beta_m^3\beta_w - 4\beta_m^2\tau_m\tau_w}{4\tau_m\left[4\gamma\left(\alpha_m + \tau_m\right) - 2\beta_m^2\right]} \tag{17}$$

which are non negative if Assumptions 3 and 4 hold.

Proof. See Appendix. □

Recalling the result in Section 3.1 firms face a Prisoner's Dilemma when investing into the inflation of the network size of one side. Taking equation (17) and the special case for $\gamma \to \infty$, it is possible to compare the platform's equilibrium profits when platforms invest and when they do not. The result might not be as clear-cut as before.

In the one-sided multi-homing model in which firms invest into the artificial inflation of the network size on one side of the market, and there is multi-homing on the other side, such an investment

[25] At the limits of $\gamma \to \infty$ it must hold that $2\tau_m < \beta_m + \beta_w + 2h_m < 4\tau_m$. With increasing γ the upper bound narrows down such that the denominator is always positive.

increases the scope of multi-homing and, hence, increases the demand. Due to multi-homing, the platforms can increase the membership fee on the multi-homing side, and lower the membership fee on the single-homing side. The countervailing effects can be seen in the following equation:

$$\begin{aligned}\pi - \pi^{\infty} &= \left[p_m \cdot n_m + p_w \cdot n_w - \frac{\gamma}{2}(s)^2\right] - \left[p_m^{\infty} \cdot n_m^{\infty} + p_w^{\infty} \cdot n_w^{\infty}\right] \\ &= \underbrace{p_m \cdot n_m - p_m^{\infty} \cdot n_m^{\infty}}_{+} + \underbrace{(p_w - p_w^{\infty})\frac{1}{2}}_{-} \underbrace{-\frac{\gamma}{2}(s)^2}_{-} \gtreqless 0,\end{aligned}$$

where the superscript of "MH" is suppressed for simplicity. Under the assumptions of Section 3.2.1 and the results of Proposition 3, it is possible to postulate the subsequent proposition.

Proposition 4. *The platforms' equilibrium profits increase when investing if* $h_m > \frac{1}{2}(\beta_m + \beta_w) =: h_m^I$.

Proof. Computing the difference between the equilibrium profits yields

$$\pi^{MH} - \pi^{MH,\infty} = \frac{-\beta_m^2 (\beta_m + \beta_w - 2h_m)(\beta_m + \beta_w + 2h_m)}{16\tau_m (2\gamma\tau_m\text{-}\beta_m^2)} > 0 \tag{18}$$

if $\beta_m + \beta_w + 2h_m > 0$ and $\beta_m + \beta_w - 2h_m < 0$. This is fulfilled as long as $2h_m > \beta_m + \beta_w$ holds. □

In contrast to the single-homing model, platforms can benefit from their investments to create fake profiles because profits can increase in contrast to the single-homing case. Given that marginal costs in the market are low, the condition above requires that the stand-alone value is larger than the average cross-group network effect.[26]

Users may also benefit from the platforms' decision to invest. In the multi-homing equilibrium, $n_m^{MH} > 1/2$ users of group m participate on each platform, that is, more than in the single-homing environment. If multi-homing behavior increases, the utility of users of group w also increases, because users of group w are able to interact with more users of group m on each platform. Also, a part of group m switches from single- to multi-homing; for those the stand-alone value r_m is duplicated, and these users are able to interact with all users of group w. However, transport costs increase as these users of group m join both platforms. The increase of multi-homing is not optimal individually because only perceived and not actual network effects are higher. However, taking into account users as a group, an increasing the scope of multi-homing can be good because it internalizes the network effects.

[26] The stand-alone value can be interpreted as a linear approximation to a concave utility function. There are also dating platforms that provide other services that could be considered independent of the network effects (for example, content).

By calculating the utility for users when firms invest and when firms do not invest (that is, $\gamma \to \infty$), and by taking the difference it can be shown that users of group m are worse off when firms engage in deceiving practices, whereas users of group w are better off:

$$\Delta u_m = -\frac{\left(\beta_m + \beta_w + 2h_m\right)^2 \beta_m^2 \left(\beta_m^2 + 4\gamma\tau_m\right)}{16\tau_m \left(2\gamma\tau_m - \beta_m^2\right)^2} < 0 \tag{19}$$

$$\Delta u_w = \frac{\beta_m^2 \left(\beta_m + \beta_w\right)\left(\beta_m + \beta_w + 2h_m\right)}{4\tau_m \left(2\gamma\tau_m - \beta_m^2\right)} > 0. \tag{20}$$

The result is intuitive as prices for group m increase but for group w decrease alongside with a higher demand on the side of group m. It is possible to differentiate group m further into single-homing and multi-homing users. More users of group m multi-home as they believe that they will meet more users of group w due to the investments. Users of group m, who practised single- or multi-homing before investments are introduced and continue in doing so when firms adopt this practise, are worse off in the latter case due to the price increase. Focusing on the users who practised single-homing in the scenario without investments and switched to multi-homing when firms invest, it can be shown that also these users receive a lower utility in the latter case.

Proposition 5. *Investments have the following implications for user surplus:*

(i) Users of group w always benefit from the platforms' investments, because the demand on the other side increases, and prices are lower.

(ii) Users in each subgroup of group m, that is, users who single-home and multi-home with and without investments and users who switch from single- to multi-homing, lose out.

(iii) If $\gamma > \frac{\beta_m^2(5\beta_m+5\beta_w+2h_m)}{4\tau_m(\beta_m+\beta_w-2h_m)} \equiv \gamma^{CS}$ holds, users as a group are better off.

Proof. See Appendix. □

The total effect on user surplus, however, is ambiguous. The positive effect on the side of group w can compensate for the loss on the side of group m if the costs from investing are sufficiently large. In this case, fewer fake profiles are created, and the multi-homing side is exploited less. For $h_m > h_m^I$ the critical value for γ is negative such that every value fulfills the condition. In line with Proposition 4 when platforms have incentives to invest, consumers also profit from the investments, such that total welfare in turn increases. For values smaller than h_m^I the presented value for γ is not necessarily fulfilled by the equilibrium conditions of γ (Assumption 4). Therefore, users as a group may not benefit from the investments either if platforms do not. Combining the results from Proposition 4 and Proposition 5 gives the following.

Proposition 6. *Investments have the following implication for total welfare:*

(i) Platforms and consumers benefit from the investments if $h_m > h_m^I$ holds.

(ii) If $h_m < h_m^I$ and $\gamma > \gamma^{CS}$, platforms are always worse off whereas consumers surplus is increased. However, the negative effect on profits outweighs the increase in consumer surplus.

(iii) If $h_m < h_m^I$ and $\gamma < \gamma^{CS}$, platform profits and consumer surplus are lower with investments.

Proof. See Appendix. □

4 Conclusion

This paper investigates platforms' investment incentives in two-sided markets. Specifically, platforms may invest into an artificial increase of the network size on one side of the market, which is a deceiving practice. The analysis reveals that investment incentives crucially depend on whether consumers single- or multi-home. In the single-homing environment, the equilibrium prices and demands are unaffected by the investments relative to the benchmark of no investments. Only profits are influenced negatively due to the additional costs, because investments turn out to be wasteful competition. The central result of this section, the Prisoner's Dilemma, is similar to the literature on persuasive advertising or investments in a Hotelling framework (Von der Fehr and Stevik, 1998; Bloch and Manceau, 1999; Matsumura and Matsushima, 2007, 2012; Belleflamme and Peitz, 2015). The result in the multi-homing environment, however, shows that platforms are not always trapped in a Prisoner's Dilemma. Investments increase membership fees on the multi-homing side, and decrease membership fees on the single-homing side. Under certain conditions, platforms make higher profits in the investment model with multi-homing than in the model without. Whenever both sides of the market single-home, users are protected by platform competition, and user surplus is unaffected by fake profiles. Multi-homing induces platforms to compete only on one side of the market, whereas they restrict access to the platform to the other side. Then, the multi-homing side bears the burden of the investments by paying higher prices.

Throughout the model, positive inter-group externalities are considered. When changing the assumptions about the network effects, and, for instance, assuming negative externalities on one side investments become less likely because the negative externalities act as a counter force. However, it is possible to obtain the same qualitative results compared to this model. More specifically, the model is robust to including a small amount of within-side crowding out.

Furthermore, this paper assumes that the investments are perceived as a number of users of group w. Users of group m are not able to differentiate between investments and actual users of group w. Therefore, the "created" users of group w receive the same weight with respect to the indirect network effects as the actual users. Another possibility would be to differentiate between both "created" and actual users, such that users of group m receive a different utility from interacting with them. The gain from interacting with "created" users of group w could for example decrease over time to represent the time wasted contrasting the approach of this model.

Further research could also look into the idea to extend the model to a dynamic game with more stages. Then, after the second stage, users of group m could learn about the deceiving practices of the platform, for instance, by doing research or from experience. A part of the agents might develop negative feelings toward the platform, and the utility decreases. In a consecutive stage, some users might leave the platform, such that the practice used by the platforms becomes less profitable. Platforms might be disciplined by this behavior to refrain from investments as long as no or fewer new users would join.

Appendix A. Proofs

Proof of Proposition 1.

First, it will be shown that the interior solution to the optimization problem of the firms is indeed symmetric and maximizes the profit of both platforms as long as assumption (1) is fulfilled. After that, it will be shown that the symmetric equilibrium is unique, so that no asymmetric equilibria exist under these conditions.

Given the optimization problem for platforms $i = 1, 2$ in Subsection 3.1, the first-order conditions with respect to p_m^1, p_m^2, p_w^1 and p_w^2 are given by

$$\begin{cases} \frac{\partial \pi_1^{SH}}{\partial p_m^1} & = & \frac{1}{2} + \frac{\beta_m \tau_w \left(s^1 - s^2\right) + \beta_m \left(p_w^2 - p_w^1\right) - \beta_w \left(p_w^1 - c_w\right) + \tau_w \left(p_m^2 - 2p_m^1 + c_m\right)}{2(\tau_m \tau_w - \beta_m \beta_w)} \\ \frac{\partial \pi_1^{SH}}{\partial p_w^1} & = & \frac{1}{2} + \frac{\beta_m \beta_w \left(s^1 - s^2\right) + \beta_w \left(p_m^2 - p_m^1\right) - \beta_m \left(p_m^1 - c_m\right) + \tau_m \left(p_w^2 - 2p_w^1 + c_w\right)}{2(\tau_m \tau_w - \beta_m \beta_w)} \\ \frac{\partial \pi_1^{SH}}{\partial s^1} & = & \gamma s^1 + \frac{\beta_m \left[\beta_w \left(p_w^1 - c_w\right) + \tau_w \left(p_m^1 - c_m\right)\right]}{2(\tau_m \tau_w - \beta_m \beta_w)} \\ \frac{\partial \pi_2^{SH}}{\partial p_m^2} & = & \frac{1}{2} + \frac{\beta_m \tau_w \left(s^2 - s^1\right) + \beta_m \left(p_w^1 - p_w^2\right) - \beta_w \left(p_w^2 - c_w\right) + \tau_w \left(p_m^1 - 2p_m^2 + c_m\right)}{2(\tau_m \tau_w - \beta_m \beta_w)} \\ \frac{\partial \pi_2^{SH}}{\partial p_w^2} & = & \frac{1}{2} + \frac{\beta_m \beta_w \left(s^2 - s^1\right) + \beta_w \left(p_m^1 - p_m^2\right) - \beta_m \left(p_m^2 - c_m\right) + \tau_m \left(p_w^1 - 2p_w^2 + c_w\right)}{2(\tau_m \tau_w - \beta_m \beta_w)} \\ \frac{\partial \pi_2^{SH}}{\partial s^2} & = & \gamma s^2 + \frac{\beta_m \left[\beta_w \left(p_w^2 - c_w\right) + \tau_w \left(p_m^2 - c_m\right)\right]}{2(\tau_m \tau_w - \beta_m \beta_w)}. \end{cases}$$

Solving these four equations simultaneously, provides the same symmetric prices as in Section 3.1. To check whether the interior solution exists and is indeed a maximum, one needs to calculate the Hessian of this maximization problem.

The optimization problem is quadratic in the respective prices and must also be concave in these prices if the proposed solution should maximize the profit. Thus, three conditions must be fulfilled so that the Hessian is negative semi-definite:

$$\begin{cases} \frac{\gamma\left[4\tau_m \tau_w - (\beta_m + \beta_w)^2\right] - \beta_m^2 \tau_w}{4[\tau_m \tau_w - \beta_m \beta_w]^2} & \geq 0 \\ \frac{\gamma[\tau_m \tau_w - \beta_m \beta_w] + \tau_m + \tau_w}{\tau_m \tau_w - \beta_m \beta_w} & \geq 0 \\ \frac{4\gamma[\tau_m \tau_w - \beta_m \beta_w](\tau_m + \tau_w) + 4\tau_m \tau_w - (\beta_m + \beta_w)^2 - \beta_m^2 \left(\beta_w^2 + \tau_w^2\right)}{4[\tau_m \tau_w - \beta_m \beta_w]^2} & \geq 0 \end{cases}$$

The second condition is fulfilled given that assumption (1) holds. For the first and third condition it must hold that the numerators are non negative, which is not assured by assumption (1). In these cases γ must be sufficiently large. Rearranging the first condition, yields $\gamma_1^{SH} \geq \frac{\beta_m^2 \tau_w}{4\tau_m \tau_w - (\beta_m + \beta_w)^2}$. The value for γ can be found in a similar way for the third condition: $\gamma_2^{SH} \geq \frac{(\beta_m + \beta_w)^2 - 4\tau_m \tau_w + \beta_m^2 \left(\beta_w^2 + \tau_w^2\right)}{4[\tau_m \tau_w - \beta_m \beta_w](\tau_m + \tau_w)}$. It is necessary that $\gamma \geq \max\left\{\gamma_1^{SH}, \gamma_2^{SH}\right\} = \gamma_1^{SH}$ for both conditions to be fulfilled. The profit is larger than zero given that $\gamma \geq \gamma_1^{SH}$.

Following the proof of (Belleflamme and Peitz, 2010), it can be shown that the symmetric equilibrium is also unique under the following conditions. Expressions (5) and (6) are the market shares with respect to the two groups of platform $i = 1, 2$ or, described differently, represent the number of members of group m and w on platform i. For an interior solution to exist, it must be that $0 < n_m^i, n_w^i < 1$. Under the assumption that $\tau_m \tau_w > \beta_m \beta_w$, the market shares n_m^i and n_w^i are larger than zero. Rearranging expressions (5) and (6), provides the conditions under which $n_m^i, n_w^i < 1$:

$$\begin{cases} \beta_m \tau_w \left(s^i - s^j\right) + \beta_m \left(p_w^j - p_w^i\right) + \tau_w \left(p_m^j - p_m^i\right) & < \tau_m \tau_w - \beta_m \beta_w \\ \beta_m \beta_w \left(s^i - s^j\right) + \beta_w \left(p_m^j - p_m^i\right) + \tau_m \left(p_w^j - p_w^i\right) & < \tau_m \tau_w - \beta_m \beta_w. \end{cases}$$

The aim is to show that no other asymmetric equilibria exist under these conditions, in which all users of one group and/or the other group participate on only one platform. Suppose all members of group m and group w concentrate on platform 1. Then, the user of group m and w who is located the furthest from platform 1, at $x = 1$, must prefer platform 1 over platform 2 expecting that all users will join platform 1. Thus, their utility from joining platform 1 minus the transportation costs must be larger than their utility from joining platform 2. The following two conditions must hold: $\beta_m \left(1 + s^1\right) - p_m^1 - \tau_m \geq \beta_m s^2 - p_m^2 \Leftrightarrow \beta_m \left(1 + s^1 - s^2\right) \geq p_m^1 - p_m^2 + \alpha_m + \tau_m$ (1) for group m and $\beta_w \geq p_w^1 - p_w^2 + \tau_w$ (2) for group w. Multiplying expression (1) with β_w, yields $\beta_m \beta_w \left(1 + s^1 - s^2\right) \geq \beta_w \left(p_m^1 - p_m^2\right) + \beta_w \tau_m$ and together with expression (2) gives $\beta_m \beta_w \left(1 + s^1 - s^2\right) \geq \beta_w \left(p_m^1 - p_m^2\right) + \tau_m \left(p_w^1 - p_w^2 + \tau_w\right)$. Rearranging the last expression results in $\beta_m \beta_w \left(s^1 - s^2\right) + \beta_w \left(p_m^2 - p_m^1\right) + \tau_m \left(p_w^2 - p_w^1\right) \geq \tau_m \tau_w - \beta_m \beta_w$, which contradicts the above conditions under which the interior solution is ensured. Similar arguments can be made to exclude the other constellations of possible asymmetric equilibria. This concludes the proof that the equilibrium presented in the model is symmetric and unique under the conditions above.

Proof of Proposition 3.

As in the proof for Proposition 1, it can be shown that the solution of the maximization problem is symmetric by taking all four first-order conditions and solving them simultaneously. The first-order conditions are

$$\begin{cases} \frac{\partial \pi_1^{MH}}{\partial p_m^1} & = & \frac{\beta_m\left[\tau_m\left(p_w^2 - p_w^1 + \tau_w\left(2s^1+1\right)\right) + \beta_w\left(2p_m^1 + p_m^2 - 2r_m - c_m\right) - \beta_m\beta_w\right]}{2\tau_m\left[\tau_m\tau_w - \beta_m\beta_w\right]} \\ & & \frac{\beta_m\left(\beta_m\beta_w\left(s^1+s^2\right)\right) - \tau_m\left[\beta_w\left(p_w^1 - c_w\right) + \tau_w\left(4p_m^1 - 2r_m - 2c_m\right)\right]}{2\tau_m\left[\tau_m\tau_w - \beta_m\beta_w\right]} \\ \frac{\partial \pi_1^{MH}}{\partial p_w^1} & = & \frac{1}{2} + \frac{\beta_m\beta_w\left(s^1 - s^2\right) + \beta_w\left(p_m^2 - p_m^1\right) - \beta_m\left(p_m^1 - c_m\right) + \tau_m\left(p_w^2 - 2p_w^1 + c_w\right)}{2\tau_m\tau_w - \beta_m\beta_w} \\ \frac{\partial \pi_1^{MH}}{\partial s^1} & = & \frac{\beta_m\left[\tau_m\beta_w\left(2\gamma s^1 + p_w^1 - c_w\right) + \left(2\tau_m\tau_w - \beta_m\beta_w\right)\left(p_m^1 - c_m\right)\right] + 2\gamma s^1\tau_m^2\tau_w}{2\tau_m\tau_w - \beta_m\beta_w} \\ \frac{\partial \pi_2^{MH}}{\partial p_m^2} & = & \frac{\beta_m\left[\left(\tau_m\right)\left(p_w^1 - p_w^2 + \tau_w\left(2s^2+1\right)\right) + \beta_w\left(2p_m^2 + p_m^1 - 2r_m - c_m\right) - \beta_m\beta_w\right]}{2\tau_m\left[\tau_m\tau_w - \beta_m\beta_w\right]} \\ & & \frac{\beta_m\left(\beta_m\beta_w\left(s^1+s^2\right)\right) - \tau_m\left[\beta_w\left(p_w^2 - c_w\right) + \tau_w\left(4p_m^2 - 2r_m - 2c_m\right)\right]}{2\tau_m\left[\tau_m\tau_w - \beta_m\beta_w\right]} \\ \frac{\partial \pi_2^{MH}}{\partial p_w^2} & = & \frac{1}{2} + \frac{\beta_m\beta_w\left(s^2 - s^1\right) + \beta_w\left(p_m^1 - p_m^2\right) - \beta_m\left(p_m^2 - c_m\right) + \tau_m\left(p_w^1 - 2p_w^2 + c_w\right)}{2\tau_m\tau_w - \beta_m\beta_w} \\ \frac{\partial \pi_2^{MH}}{\partial s^2} & = & \frac{\beta_m\left[\tau_m\beta_w\left(2\gamma s^2 + p_w^2 - c_w\right) + \left(2\tau_m\tau_w - \beta_m\beta_w\right)\left(p_m^2 - c_m\right)\right] + 2\gamma s^2\tau_m^2\tau_w}{2\tau_m\tau_w - \beta_m\beta_w}. \end{cases}$$

Solving yields the same symmetric prices as in Section 3.2.1.

The optimization problem is concave given assumption 3 and if γ is sufficiently large. Similar to the proof of Proposition 1, three conditions must be fulfilled for the Hessian matrix to be negative semi-definite.

$$\begin{cases} \frac{\gamma\tau_m\left(8\tau_m\tau_w-(\beta_m+\beta_w)^2-4\beta_m\beta_w\right)+2\beta_m^2(\beta_m\beta_w-2\tau_m\tau_w)}{4\tau_m(\tau_m\tau_w-\beta_m\beta_w)^2} & \geq 0 \\ \frac{\gamma\tau_m(\tau_m\tau_w-\beta_m\beta_w)+\tau_m^2+2\tau_m\tau_w-\beta_m\beta_w}{\tau_m(\tau_m\tau_w-\beta_m\beta_w)} & \geq 0 \\ \frac{4\gamma\tau_m(\tau_m\tau_w-\beta_m\beta_w)\left(\tau_m^2+2\tau_m\tau_w-\beta_m\beta_w\right)+\tau_m^2\left(8\tau_m\tau_w-(\beta_m+\beta_w)^2-4\beta_m\beta_w\right)-\beta_m^2\left(\beta_m^2\beta_w^2-4\beta_m\beta_w\tau_m\tau_w+\tau_m^2\left(\beta_w^2+4\tau_w^2\right)\right)}{4\tau_m^2(\tau_m\tau_w-\beta_m\beta_w)^2} & \geq 0 \end{cases}$$

From these conditions, three values of γ can be derived to support this result. Due to the complexity of the equations, only the sufficient value of γ will be presented. To fulfill all three conditions simultaneously, it must hold that $\gamma \geq \max\left\{\gamma_1^{MH}, \gamma_2^{MH}, \gamma_3^{MH}\right\} = \gamma_1^{MH}$. Comparing all three values, reveals that $\gamma_1^{MH} = \frac{2\beta_m^2(2\tau_m\tau_w-\beta_m\beta_w)}{\tau_m\left(8\tau_m\tau_w(\beta_m+\beta_w)^2-4\beta_m\beta_w\right)}$ is the largest expression. To be precise, γ_2^{MH} is smaller than zero and thus every $\gamma > 0$ fulfills the respective equation. Taking the difference between γ_1^{MH} and γ_3^{MH} gives the clear result that $\gamma_1^{MH} > \gamma_3^{MH}$. Given γ_1^{MH}, the firm's profit $\pi^{MH,**}$ is non negative.

Furthermore, for an interior solution to exist, $0 < n_m^i < 1$ must hold which is equivalent to $0 < \gamma\left(\beta_m + \beta_w + 2h_m\right) < 4\gamma\tau_m - 2\beta_m^2$. To be more precise, $2\gamma\tau_m - \beta_m^2 < \gamma\left(\beta_m + \beta_w + 2h_m\right) < 4\gamma\tau_m - 2\beta_m^2$ should hold so that the equilibrium is unique. The last assumption provides that $0 < \hat{x}_{2m} < \hat{x}_{1m} < 1$. In this case, all users of group m participate in at least one platform. To maintain this order, $\gamma > \frac{2\beta_m^2}{4\tau_m-\beta_m-\beta_w-2h_m} \equiv \gamma_0^{MH}$. Given the assumption of $(\beta_m + \beta_w + 2h_m) < \frac{4\gamma\tau_m-2\beta_m^2}{\gamma}$, this value is positive. The latter term shows that independent of γ, $(\beta_m + \beta_w + 2h_m)$ is smaller than $4\tau_m$.

To support the multi-homing equilibrium with investments, $\gamma > \max\left\{\gamma_0^{MH}, \gamma_1^{MH}\right\}$ must hold. It holds that $\gamma_0^{MH} > \gamma_1^{MH}$ if $h_m > \frac{\tau_m(\beta_m+\beta_w)^2}{4\tau_m\tau_w-2\beta_m\beta_w} - \frac{(\beta_m+\beta_w)}{2} \equiv h_m^{\gamma}$.

To assure that the users of group w participate in the market, the utility for the indifferent consumer at $x = \frac{1}{2}$ must be larger than zero. This is the case if $v_w - \frac{1}{2}\tau_w$ or equivalently $4\tau_m h_w + 2\left(\beta_m + \beta_w\right)h_m > 6\tau_m\tau_w - \left[\beta_m^2 + 4\beta_m\beta_w + \beta_w^2\right]$ holds (most strict condition for $\gamma \to \infty$).

Proof of Proposition 5.

To calculate the differences in surplus for each group define the utility without transportation costs as

$$\begin{aligned} v_w =& r_w + \beta_w \cdot n_m - p_w \\ v_m =& r_m + \beta_m \cdot n_w - p_m. \end{aligned}$$

Then, the aggregate utility for users of group w is equal to $v_w - 2 \cdot \int_0^{\frac{1}{2}} (\tau_m \cdot x)\, dx$. Inserting the equilibrium prices and demand from Section 3.2.1 and for the case that $\gamma \to \infty$ yields the calculated difference in equation 20.

Similarly, the aggregate utility for users of group m in total is

$$U_m = \int_0^{1-n_m^{MH}} (v_m - \tau_m x)\, dx + \int_{1-n_m^{MH}}^{n_m^{MH}} (2v_m - \tau_m)\, dx + \int_{n_m^{MH}}^{1} (v_m - \tau_m(1-x))\, dx.$$

Inserting equilibrium prices, $n_w = \frac{1}{2}$ and the indifferent consumers in equilibrium for both cases and subtracting yields again equation 19. Dividing group m into three subgroups shows that the surplus of each subgroup is also lower in the equilibrium with investments. The first subgroup can be defined as users who practised single-homing before and after introducing investments. These users are worse off as they only pay higher prices to access their platform; everything else equal. Similarly, the second subgroup of users who multi-home in both scenarios is also worse off due to higher prices. Lastly, it is possible to focus on the subgroup of users who practised single-homing in the standard case for $\gamma \to \infty$ that switch to multi-homing (superscript SC) when investments are introduced. Their difference in aggregate utility can be calculated by

$$\begin{aligned}\Delta u_m^{SC} = & \int_{1-n_m^{MH,\gamma}}^{1-n_m^{MH}} \left(v_m^{MH,\gamma} - \tau_m x\right) dx + \int_{n_m^{MH}}^{n_m^{MH,\gamma}} \left(v_m^{MH,\gamma} - \tau_m(1-x)\right) dx \\ & - \left(\int_{1-n_m^{MH,\gamma}}^{1-n_m^{MH}} \left(2v_m^{MH} - \tau_m\right) dx + \int_{n_m^{MH}}^{n_m^{MH,\gamma}} \left(2v_m^{MH} - \tau_m\right) dx \right),\end{aligned}$$

which yields

$$\Delta u_m^{SC} = -\frac{5\left(\beta_m + \beta_w + 2h_m\right)^2 \beta_m^4}{16\tau_m \left(\beta_m^2 - 2\gamma\tau_m\right)^2} < 0.$$

Proof of Proposition 6. It possible to identify three cases for total welfare. First, if $h_m > h_m^I$ holds as in Proposition 4, platform profits increase and the critical γ for consumers surplus to increase is negative ($\gamma^{CS} < 0$ and thus always met). If $h_m < h_m^I$, platform profits are always lower in the investment scenario. For consumers two cases must be distinguished; either $\gamma > \gamma^{CS}$ or $\gamma < \gamma^{CS}$. It is possible to show that $\gamma^{CS} > \max\left\{\gamma_0^{MH}, \gamma_1^{MH}\right\}$ given $h_m < h_m^I$. Then, $\gamma^{CS} > \gamma_1^{MH}$ as long as assumption 3 is fulfilled and $\gamma^{CS} > \gamma_0^{MH}$ if $\gamma_0^{MH} > 0$. Then if $\gamma \in \left(\max\left\{\gamma_0^{MH}, \gamma_1^{MH}\right\}, \gamma^{CS}\right)$ and $h_m < h_m^I$, consumers in total also do not benefit from the investments. Otherwise, if $\gamma > \gamma^{CS}$, consumers profit from the investments even though platforms do not.

Nevertheless, the effect on total welfare is negative. Summing up equations (18), (19) and (20) yields

$$\Delta TW = \frac{(\beta_m + \beta_w + 2h_m)\beta_m^2 \left(\gamma\tau_m(\beta_m + \beta_w - 2h_m) - 2\beta_m^2(\beta_m + \beta_w + h_m)\right)}{8\tau_m \left(2\gamma\tau_m - \beta_m^2\right)^2} < 0, \tag{21}$$

given that $h_m < h_m^I$ and $\gamma > \gamma^{CS}$.

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