Product differentiation with bundles of characteristics and multipurchasing *

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Abstract

We study firms' incentives to differentiate in one-sided and two-sided contexts. We propose a location model where firms offer bundles of characteristics and consumers can purchase more than one product or service (multipurchasing). An increase in differentiation leads to a reduction in the overlap between the characteristics offered by the two firms, which induces more consumers to purchase both products. In a one-sided environment, firms focus on the total size of the demand, whereas firms that operate in a two-sided environment also care about the composition of the demand (i.e., single-purchasers and multipurchasers). We show that maximum product differentiation arises in one-sided environments despite the absence of strategic interactions between firms' prices. In multisided environments, this result may be reversed when the value attributed to single-purchasers by the other side of the market is substantially larger than the value attributed to multipurchasers. We also derive results for the effect of mergers on product differentiation.

Keywords: Product differentiation; Multipurchasing; Two-sided markets.

JEL Classification: L13, L40, D43.

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1 Introduction

Motivation. In many product markets, some consumers frequently purchase several competing products (multipurchasing) such as cars, magazines, or various electronic devices while others stick to one product (single-purchasing). For instance, some consumers decide to purchase both a laptop and a tablet, although the two products offer many similar characteristics. With the advent of the digital economy and the emergence of platforms, multipurchasing (or "multihoming") has become a widespread phenomenon (e.g., consumers frequently use several forms of social media or read many online newspapers), and it plays a key role in firms' profitability and strategies.[1]

Platforms are firms that operate in multisided markets, in which consumers' prices are often very low or even zero. This scenario makes purchasing several competing products or services even more attractive for consumers. Whereas video streaming services and car and electronic devices manufacturers essentially rely on consumers to derive revenues, online newspapers or social media often offer free services to consumers and charge advertisers to generate profits. As a consequence, many consumers tend to stick to one video streaming service, car or electronic device, while using multiple social media services. However, the way multipurchasing affects firms that operate in one-sided markets (i.e., where firms mainly derive revenues from consumers) and two-sided markets (i.e., where firms may derive revenues from channels other than consumers) may differ.

In one-sided markets, firms may not care much about the composition of demand, whereas in two-sided environments, the composition of demand plays a key role. For instance, when considering advertising, exclusive eyeballs do not generate the same value as consumers who are active on multiple platforms. The bargaining power of platforms vis-a-vis advertisers is indeed affected by the extent of multipurchasing in the market.

[1] In multisided (or two-sided) markets, consumers who use multiple horizontally differentiated services are usually called multihomers. We refer to this term when analyzing two-sided environments but use the term multipurchasing in one-sided settings.

One potential consequence may be an effect on the incentives for firms to differentiate from each other. Intuitively, a consumer's decision to purchase two horizontally differentiated products instead of one is related to the level of differentiation between those products. If differentiation leads to an increase in the number of (less-valuable) multihoming consumers, platforms may lack incentives to differentiate. Conversely, firms operating in one-sided markets may not care about multipurchasing since all types of consumers are typically charged the same price. This rationale may explain why video streaming services such as Netflix, which derive revenue mainly from consumers, choose to offer exclusive content (i.e., a high level of differentiation), while free online newspapers usually produce similar articles on their websites (i.e., a low level of differentiation).

Model and results. We build a model that highlights the main trade-offs discussed above. Specifically, we are interested in how the level of differentiation chosen by firms is affected by multipurchasing and the type of market considered, namely, one-sided or two-sided markets. Hotelling's model is the typical framework used to study competition between (two) horizontally differentiated firms.[2] Our framework relies on three important additional features.

First, we introduce the possibility for consumers to purchase either one product or service offered by one of the two firms or both firms' products or services. Multipurchasing arises endogenously in the model as a result of consumers' arbitrage between the benefit of purchasing a second product in addition to the first one and the costs to do so. We focus on equilibria where each firm's demand is composed of single-purchasers and multipurchasers, and the composition of demand is affected by the level of differentiation in the market and firms' prices.

Second, we develop a new way to model product differentiation, considering that firms can offer bundles of characteristics, represented by segments on the characteristic line. We

[2]Salop (1979) and Chen and Riordan (2007) develop models of spatial differentiation with $n \geq 2$ firms.

consider that some characteristics are present in both firms' bundles, and firms can reduce the overlap in characteristics by offering more differentiated bundles. This in turn affects the incentives for consumers to purchase both products instead of one.

Last, we complement our analysis by considering a two-sided framework, where firms can also derive revenue from another source (e.g., advertising), and compare equilibrium outcomes with the standard one-sided setting. We also analyze how mergers affect firms incentives to differentiate in both settings.

Our results shed light on the influence of multipurchasing on firms' incentives to differentiate from each other. Despite the strategic independence between firms' prices under multipurchasing, we show that the "principle of differentiation" (Tirole, 1988) still holds in a one-sided setting. In our setting, an increase in differentiation leads to a decrease in the overlap between firms' bundles of characteristics, which increases each firm's total demand.

The impact of differentiation on demand composition is more ambiguous: we identify two countervailing effects of differentiation on demand composition. The *overlap-reducing* effect makes multipurchasing more attractive through an increase in the gross utility from consuming the two products, whereas the *segmentation effect* tends to segment the market through an increase in transportation costs. If consumers' heterogeneity is weak, the former effect dominates the latter; however, in multisided markets, the value attributed to single-homers is typically larger than that attributed to multihomers. A direct consequence is that firms' incentives to differentiate may be dampened.

The analysis of mergers provides insightful results. While the result of maximum differentiation subsists in the one-sided environment, we show that a merger can overturn the result of minimum differentiation in the two-sided setting. This comes at the cost of higher prices for advertisers. Intuitively, a merger restores firms' bargaining power vis-à-vis advertisers since the merged entity becomes the unique gateway to multihoming consumers. As these consumers become more valuable, firms care less about demand composition and more about total demand, which increases in differentiation. Overall, mergers increase welfare, although advertisers' surplus is fully extracted.

Related literature. Our work contributes to several strands of the literature. The first strand is related to product differentiation. In Hotelling's seminal paper (1929), products are considered as single points in a characteristic space (i.e., the line [0,1] on which consumers are distributed). Since then, most of the economic literature addressing horizontal product differentiation has considered products in this way. An important exception is Lancaster (1975), who uses a two-dimensional framework to define products. Closer to our framework is that of Alexandrov (2008), who develops a model in which firms offer interval-long "fat" products on the Hotelling line. In his model, a consumer located inside the interval offered by firms does not incur transportation costs since the product incorporates the consumer's preferred feature. In our model, consumers' preferences are defined by an ideal bundle such that consumers that obtain some (but not all) of their desired characteristics from the firm's product incur some transportation costs.

Another central element in models of spatial product differentiation is the assumption that consumers buy at most one product. One of the first papers to (partially) depart from this assumption is that of Anderson and Neven (1989), who allow consumers to mix their consumption between two products instead of sticking to one product. Kim and Serfes (2006) allow consumers to buy one unit of each product and show that the usual result of maximum product differentiation (d'Aspremont et al., 1979) may no longer hold under multipurchasing. Finally, Anderson et al. (2017) consider a multipurchasing scenario where firms can invest in a vertical quality dimension. Some characteristics may end up being present in both products, which affects the extent of multipurchasing. This is similar to our approach. However, we are interested in firms' incentives to reduce the overlap of characteristics through differentiation and how these incentives vary in one-sided and two-sided markets.

Our work is also related to the literature on two-sided markets and media markets in particular. Anderson and Coate (2005) study how firms set advertising levels in the presence of cross-group externalities. Most of their analysis considers single-homing consumers. Ambrus et al. (2016) also study advertising provision but allow consumers to multihome. They highlight the importance of demand composition as opposed to total demand in media markets. Our work is particularly closely related to Gabszewicz et al. (2001) and Anderson et al. (2018), who both study how advertising affects firms' incentives to differentiate in a Hotelling setting. Gabszewicz et al. (2001) introduce advertising revenues in addition to revenues from consumers and find that the *demand effect* (i.e., firms' incentives to move toward the middle of the market to gain more captive consumers) can dominate the *strategic effect* (i.e., firms' incentives to move away from each other to escape price competition), leading to minimum differentiation at equilibrium. Their model maintains the assumption of single-homing consumers. Anderson et al. (2018) develop the incremental pricing principle, according to which multihoming consumers are less valuable than single-homing consumers due to decreasing returns to advertising. They find that firms are incentivized to move away from each other to increase the number of exclusive consumers and to prevent multihoming behavior. This effect is also at play in our model. However, we identify a countervailing force according to which differentiation can also increase the attractiveness of multihoming. Our analysis suggests that the impact of differentiation on demand composition is more ambiguous than the standard Hotelling logic would indicate.

Finally, our work contributes to the literature studying the relation between market structure and diversity. Steiner (1952) predicts that competition leads to duplication of popular formats and that market concentration can solve this issue. Conversely, Spence and Owen (1977) find that monopoly may perform worse than competition in terms of variety provision. Mullainathan and Shleifer (2005) build a model akin to the Hotelling framework to study media bias. They show that competition leads newspapers to slant stories toward consumers extreme beliefs. In a sense, competition increases diversity, although news are untruthfully revealed to consumers (who appreciate their extreme opinions being confirmed). Johnson and Rhodes (2020) study quality competition between multiproduct firms and find that mergers induce a change in the mix of different qualities offered by firms, which can benefit consumers. On the empirical side, several contributions have shown that ownership concentration can lead to increased diversity in media markets (Berry and Waldfogel, 2001; George, 2007; Sweeting, 2010). Fan (2013) insists on the importance of accounting for product characteristics choices to assess the welfare impact of a merger. She finds that mergers lead to less variety. Gentzkow et al. (2014) highlight the importance of markets' two-sidedness in evaluating competition policy. They argue that advertising collusion increases both welfare and diversity. Finally, Jeziorski (2014) finds that advertisers are negatively affected by mergers, while consumers are positively affected and benefit from greater diversity. Our results are consistent with the latter findings: we show that mergers hurt advertisers but are likely to foster firms' incentives to differentiate.

Organization of the paper. In the following section, we develop a model of horizontal differentiation with bundles of characteristics, which allows us to introduce our main novel effect, i.e., that differentiation may encourage multipurchasing. Section 3.1 is devoted to the analysis in a one-sided environment, while Section 3.2 addresses the two-sided scenario. We conduct welfare analysis in Section 4. In Section 5, we analyze the impact of mergers in both types of environments. Finally, Section 6 concludes.

2 A model of horizontal differentiation with bundles of characteristics

Firms. Consider two firms that each produce one good. Each good is composed of different characteristics. The set of characteristics that can be incorporated into the goods is represented by a line with support $(-\infty, +\infty)$, where each point of the line represents one characteristic. The characteristic line coincides with the product line over which firms' products and consumers' preferences are located.[3] Consumers are uniformly distributed over the line; therefore, the mass of potential consumers is infinite, although the number of consumers purchasing at least one of the two goods is finite in equilibrium.

When designing their good, firms choose a bundle of characteristics to incorporate, which corresponds to a segment of fixed size q on the characteristic line.[4] We define the location of each firm's good, a and b, as the middle point of the segment chosen on the characteristic line. We assume that firm 1's good is located to the left of 0 (i.e., $a \leq 0$) and that firm 2's good is located to the right of 0 (i.e., $b \geq 0$). For ease of exposition, we assume that firms choose their locations; therefore, the bounds of the characteristics' segments are defined as $[a - \frac{q}{2}, a + \frac{q}{2}]$ for firm 1's good and $[b - \frac{q}{2}, b + \frac{q}{2}]$ for firm 2's good. The overlap in product characteristics is given by the intersection between the two characteristic segments: $O = [a + \frac{q}{2}] - [b - \frac{q}{2}] = q - (b - a)$. The overlap O is decreasing in product differentiation (i.e., $b - a$) and increasing in the (exogenous) quality parameter q. Furthermore, we make the following assumption.

[3]The use of an infinite line is more convenient for the upcoming analysis. All the results developed below can be obtained with a bounded demand. The only element that is required for our results to hold is demand expansion, i.e., each firm can attract new consumers located at the extremes of the line by moving towards the extremes. For an illustration of a Hotelling model with bounded demand (i.e., a finite line) and demand expansion, see Anderson et al. (2018).

[4]This modeling approach is consistent with the description provided by Spence (1976): "One can think of products being points in a continuous spectrum of attributes. The market selects a finite subset of points in the continuum, these being the products actually produced". Note also that Lancaster (1975) describes products as "bundles of characteristics".

Assumption 1: The distance between firms' locations, $b - a$, is at most equal to q.

This assumption ensures strategic interactions between the two firms. For higher levels of differentiation, each firm would simply behave as a local monopoly. Figure 1 illustrates one potential configuration for the model.

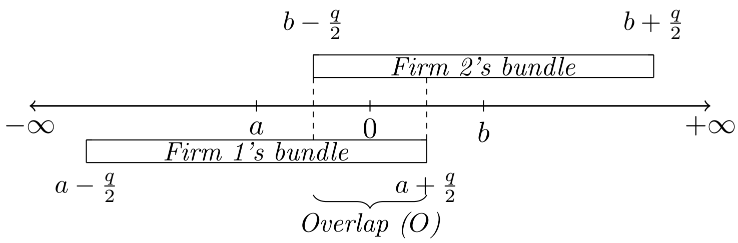


Figure 1: Firms' location, bundles of characteristics, and overlap.

Consumers. A consumer's preferred bundle is defined by the following segment on the characteristic line, $[x - \frac{q}{2}, x + \frac{q}{2}]$, where x denotes a consumer's type.[5] Consumers therefore exhibit some heterogeneity in the correspondence between their preferences and firms' products offerings, in the Hotelling spirit. We make the following restrictive assumptions regarding consumers' purchasing behavior:

Assumption 2: (i) A consumer whose optimal bundle does not overlap with firm i's bundle does not consider buying the product; (ii) consumers located at the extremes (i.e., left of a or right of b) remain captive to each firm; (iii) consumers located between the two firms can decide to buy one of the two products (i.e., single-purchase) or both products (i.e., multipurchase).

These assumptions ensure that we obtain intuitive demand configurations. Assumption (i) amounts to setting a bound on the demand. Assumptions (ii) and (iii) imply restrictions

[5] This specification implies that consumers perfectly know the quality standards in the market (i.e., the size of q) and derive their optimal bundle accordingly.

on the parameter values of the model.[6] In particular, we must observe some minimum level of differentiation in the market, i.e., $b - a \geq \underline{d}$. Note that the idea that firms hold a captive demand is a standard assumption made in the literature on horizontal product differentiation, in both standard single-purchasing contexts and multipurchasing contexts.[7]

Purchasing a given product i provides two types of characteristics to the consumer: characteristics that exactly correspond to what she expects to find in her optimal good and other characteristics that do not correspond to her preferences. Denoting as δ the set of characteristics that meet the consumer's preferences and μ as the set of other characteristics, we let the (gross) utility a consumer x derives from purchasing product i be:

$$U_i^x = \delta_i^x + k\mu_i^x,$$

where $k \in [0, 1]$ represents the relative preference for preferred characteristics. If consumer x decides to purchase product j in addition to product i, we define her utility as:

$$U_{ij}^x = \delta_i^x + \delta_j^x + k(\mu_i^x + \mu_j^x) - \alpha O,$$

where α represents the extent to which consumers dislike overlapping characteristics.[8]

The timing that we consider throughout the paper is as follows. In the first period,

[6] These restrictions appear formally in the Appendix. They do not affect the intuitions we develop throughout the paper.

[7] Tirole (1988) calls this captive demand firms' "turf". Kim and Serfes (2006) and Anderson et al. (2018) study similar demand configurations as we do but allow consumers located at the extremes to consume from both firms. This scenario arises for sufficiently low levels of differentiation in the market. An immediate implication is that they do not need to restrict the minimum level of differentiation as we do. However, this approach can lead to somewhat unrealistic equilibria where (i) firms minimally differentiate (i.e., $b - a = 0$), (ii) prices are strictly positive and (iii) many consumers purchase both products. This scenario does not strictly occur in Anderson et al. (2018) since they assume zero price on the consumer side, but it does arise in Kim and Serfes (2006).

[8] This modeling approach is reminiscent of Anderson et al. (2017). However, in their model, the overlap in product characteristics is independent of the locations (which are fixed). This scenario implies that two differentiated goods can incur each characteristic of the universe with the same probability. By contrast, we assume that characteristics are the element that defines the level of differentiation of firms. As a consequence, the overlap in product characteristics is decreasing in product differentiation and can therefore be controlled by firms.

firms choose their locations and therefore the bundle of characteristics they offer. In the second period, firms choose prices; then, demand is realized. We use subgame perfection as a solution concept.

Before turning to the analysis of the model, we show that our utility functions are similar to the standard Hotelling framework. To see this, first note that μ_i^x is proportional to the distance (d_i) between the consumer's and the firm's locations. Additionally, keeping in mind that the size of the bundle is equal to q, we can rewrite $\delta_i^x = q - d_i$. Substituting for these values in U_i^x yields $U_i = q - d_i + kd_i$. Finally, define $t = 1 - k$, with $t \in [0, 1]$, which gives us the consumer utility function (net of price):

$$U_i = q - td_i - p_i. \tag{1}$$

With our simple framework, we are therefore able to derive the standard Hotelling utility function, where q can be viewed as the standard stand-alone benefit and t can be interpreted as the extent to which consumers dislike unexpected characteristics. The interest of our framework arises in the multipurchasing case.

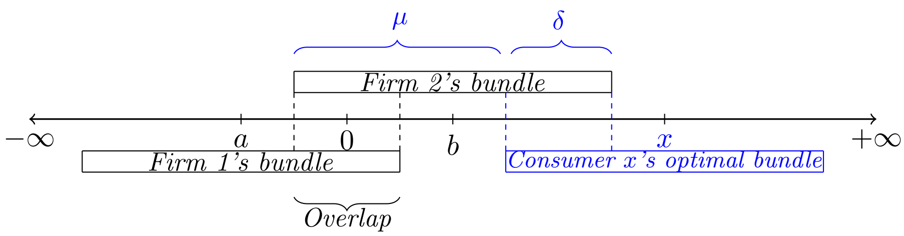
Applying the above reasoning to the multipurchasing case, we obtain:

$$U_{ij} = q_i + q_j - t(d_i + d_j) - (p_i + p_j) - \alpha O.^9 \tag{2}$$

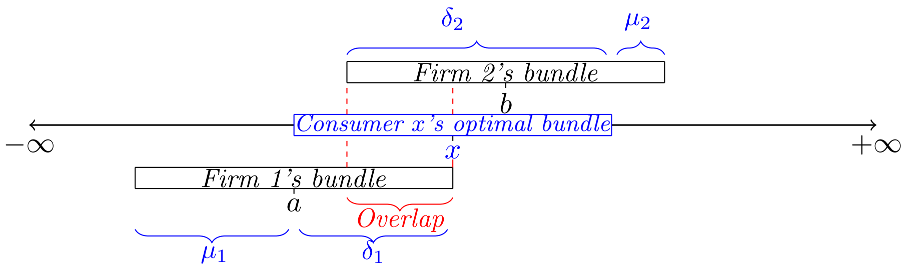
Figure 2 illustrates the single-purchasing and multipurchasing cases.

The main objective of the framework developed above is to demonstrate why the incremental utility may be closely related to the level of differentiation in the market. As we shall see below, this finding implies that multipurchasing can become more attractive for

[9] Our model's philosophy is remarkably close to the description provided by Lancaster (1975): "The consumer obtains his optimal bundle of characteristics by purchasing a collection of goods so chosen as to possess *in toto* the desired characteristics".



(a) Single-purchasing



(b) Multipurchasing

Figure 2: Consumer's utility with bundles of characteristics.

consumers when firms are located further apart, an effect that has not been raised in the literature thus far. The intuitions developed rest on Assumptions 1 and 2, but our results would be qualitatively similar in a more standard Hotelling setting, assuming that the incremental utility is simply an increasing function of the level of differentiation in the market. In what follows, we use equations (1) and (2) to derive consumers' demand functions and firms' optimal strategies in one-sided and two-sided settings.

3 Analysis

3.1 One-sided environment

In this subsection, we analyze firms' price and differentiation decisions when they derive revenues exclusively from the price they charge to consumers. In Subsection 3.2, we show how equilibrium outcomes are affected when firms also derive revenues from another source (e.g., advertising). Note that the demand structure determined below also applies to the two-sided case in Section 4.

Demand. As discussed above, there are two types of consumers: captive consumers, who are located at an extreme of the line and can only decide to purchase a good from the closest firm, and potential multipurchasers, who are located between the two firms and can purchase either one good or both. Figure 4 shows the demand configuration and consumers' indirect utilities.[10]

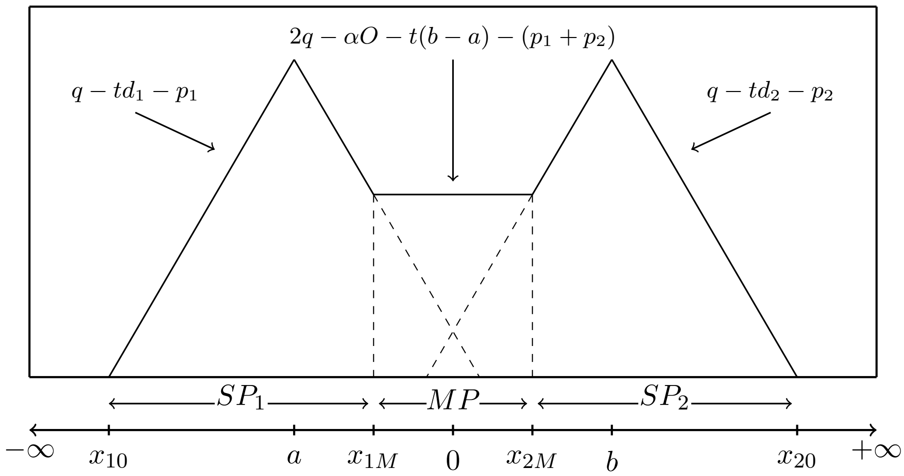


Figure 3: Demand structure and indirect utilities.

The captive part of each firm i's demand is determined by the location of the consumer who is indifferent between purchasing good i or not purchasing anything ($u_{i0} = 0$). Simple computations yield:

$$x_{10} = a - \frac{1}{t}(q - p_1), \tag{3}$$

$$x_{20} = b + \frac{1}{t}(q - p_2). \tag{4}$$

[10]Multipurchasers all obtain the same utility from purchasing the two products. Note that they all face the same transportation costs to buy from both firms due to our linear specification of transportation costs. If we assumed quadratic costs instead, utilities would be higher for multipurchasers located closer to 0.

The interior part of each firm i's demand is determined by the consumer who is indifferent between purchasing only product j ($j \neq i$) and purchasing both products i and j: $u_j = u_{jM}$, which yields:

$$x_{1M} = b - \frac{1}{t}[q - \alpha(a - b + q) - p_2]. \tag{5}$$

$$x_{2M} = a + \frac{1}{t}[q - \alpha(a - b + q) - p_1]. \tag{6}$$

Total demands D_1 and D_2 are obtained by combining (3)-(6) for firm 1 (i.e., $x_{2M} - x_{10}$) and (4)-(5) for firm 2 (i.e., $x_{20} - x_{1M}$):

$$D_1 = (b - a)\frac{\alpha}{t} - \frac{2p_1}{t} + \frac{q}{t}(2 - \alpha), \tag{7}$$

$$D_2 = (b - a)\frac{\alpha}{t} - \frac{2p_2}{t} + \frac{q}{t}(2 - \alpha). \tag{8}$$

Note that firms' total demands depend only on their own prices. This is a standard property of horizontal differentiation models with multipurchasing.[11] More importantly, each firm's demand is increasing in differentiation. An increase in differentiation reduces the overlap between firms' characteristics, which makes purchasing the two goods instead of only one more attractive for consumers located in the middle of the market.[12] Finally, we note that the impact of the size of characteristics bundles is positive since $\alpha \in [0, 1]$. This effect is limited because an increase in q also increases the overlap between firms' characteristics, which reduces the incremental utility consumers derive from purchasing two products instead of one.

We now turn to the analysis of the composition of demand. Each firm's demand is composed of single-purchasers and multipurchasers. Combining equations (3)-(6) for firm 1

[11] See Kim and Serfes (2006) or Anderson et al. (2017, 2018, 2019). Similarly, in Ambrus et al. (2016), firms' total demands depend only on their own advertising level (consumers are not charged anything but are subject to advertising).

[12] That differentiation can lead to market expansion is supported by both early theoretical predictions (Steiner, 1952) and empirical findings (Berry and Waldfogel, 1999). Nonetheless, this effect is usually absent in models of horizontal differentiation.

(i.e., $x_{1M} - x_{10}$) and equations (4)-(5) for firm 2 (i.e., $x_{20} - x_{2M}$), we obtain the single-purchasers part (SP_i) of firms' demands:

$$SP_1 = [b-a](1-\frac{\alpha}{t}) - \frac{1}{t}[p_1 - p_2] + \frac{\alpha}{t}q, \tag{9}$$

$$SP_2 = [b-a](1-\frac{\alpha}{t}) - \frac{1}{t}[p_2 - p_1] + \frac{\alpha}{t}q. \tag{10}$$

The number of consumers who multipurchase ($MP = x_{2M} - x_{1M}$) is given by:

$$MP = [b-a](\frac{2\alpha}{t} - 1) - \frac{(p_1 + p_2)}{t} + \frac{2q}{t}(1-\alpha). \tag{11}$$

Interestingly, differentiation has an ambiguous effect on the composition of demand. An increase in differentiation entails two opposite effects: a *segmentation* effect and an *overlap-reducing* effect. To see how these effects operate, assume first that $\alpha = 0$, such that the *overlap-reducing* effect is not at play. In this case, an increase in differentiation implies a proportional shift in demand. For instance, if firm 2 moves away from its competitor, x_{1M} and x_{20} shift to the right. This mechanically implies a decrease in the number of multipurchasing consumers and an increase in the number of exclusive consumers; in other words, the market becomes more segmented. This is essentially what occurs in Anderson et al. (2018).

Consider now $\alpha > 0$. An increase in differentiation implies a proportional shift in x_{i0} (i.e., the captive demand always remains the same) but a nonproportional change in x_{jM}. This difference is due to the *overlap-reducing* effect, which acts as a countervailing effect to the *segmentation* effect (hence, the increase in the firm's total demand). To see how the composition of firm i's demand evolves with differentiation, one must consider x_{iM}. Again, take firm 2 as an example. If firm 2 moves away from its competitor, it reduces the overlap between products' characteristics, which induces an increase in x_{2M}. In other words, firm 2 makes purchasing firm 1's product (on top of firm 2's product) more attractive. The extent of this effect is limited by the size of transportation costs (t) since multipurchasing also becomes more costly when firm 2 moves away from firm 1. Regardless, if the *overlap-reducing* effect is sufficiently large, the number of single-purchasing consumers decreases with differentiation while the number of multipurchasing consumers increases. Conversely, if transportation costs are large, the *segmentation* effect dominates the *overlap-reducing* effect, which implies that differentiation leads to more exclusive consumers and fewer multipurchasing consumers in the market.

Our analysis suggests that if consumer heterogeneity is strong, the market is more likely to be segmented. To illustrate the main forces at play, consider the media industry. One instance in which the *segmentation* effect presumably outweighs the *overlap-reducing* effect is when consumers have strong political views/bias. In such a case, differentiation between content (or media slanting) is likely to segment the market. It is rather infrequent for a consumer who watches CNN to also spend time on Fox News, although the two channels arguably broadcast different types of programs. Conversely, in the market for popular sports, consumers are less likely to exhibit any particular bias (i.e., there is less strong heterogeneity between consumers[13]) and appear to mainly be interested in content proposed by channels. This scenario suggests that differentiation (e.g., in the form of exclusive broadcasting of a particular sport or competition) tends to induce an increase in multipurchasing behavior. This reasoning is consistent with the analysis of the newspaper market conducted by Mullainathan and Shleifer (2005), who suggest that a high level of consumer heterogeneity (bias) leads to substantial media slanting, which segments the market. Conversely, a conscientious reader (unbiased) will tend to read both newspapers when newspapers slant in opposite directions. In other words, differentiation makes multipurchasing attractive to an unbiased reader.

[13]Naturally, some consumers may be more interested in basketball and others in football, but it is less likely that consumers who like basketball also hate football. The same argument applies for video streaming services, whereby exclusive content tends to induce multipurchasing behavior, although some consumers may prefer Netflix and others Amazon Prime Video.

As a final remark on our demand composition functions, although the total demand functions depend only on the firms' own prices, the composition of demand is affected by the competitors' prices.[14] The following lemma summarizes the above discussion.

Lemma 1: *Differentiation affects demand composition as follows:*

- *For $\alpha > t$: SP_1 and SP_2 are decreasing in differentiation and MP is increasing in differentiation.*
- *For $\frac{t}{2} < \alpha < t$: SP_1, SP_2 and MP are all increasing in differentiation.*
- *For $\alpha < \frac{t}{2}$: SP_1 and SP_2 are increasing in differentiation and MP is decreasing in differentiation.*

Prices. We now analyze firms' optimal prices, given the demand functions derived in the previous section. In a one-sided environment, each firm $i = 1, 2$ maximizes its profit:

$$\pi_i = p_i D_i = p_i \left[(b-a)\frac{\alpha}{t} - \frac{2p_i}{t} + \frac{q}{t}(2-\alpha) \right]. \tag{12}$$

Thus, only the total demand matters for profit maximization. Indeed, consumers are all charged the same price, be they single-purchasers or multipurchasers. Taking the first-order condition with respect to p_i leads to:

$$p_i^* = \frac{1}{4}[\alpha(b-a) + q(2-\alpha)]. \tag{13}$$

Note that as firms' demand (and therefore profits) depend only on their own prices, the best response functions and equilibrium prices (for given location choices) coincide.

Furthermore, optimal prices are increasing in differentiation, and the impact of the number of characteristics (q) offered by each firm on prices is also positive.

[14]This is similar to Ambrus et al. (2016), in which each firm's total demand does not depend on the competitor's advertising level, but the demand composition depends on both firms' advertising levels.

Location choice. In the first stage of the game, each firm $i = 1, 2$ maximizes the profit function:

$$\pi_i = p_i^* D_i(p_i^*),$$

where p_i^* is given in (13). Taking the derivative with respect to location yields:

$$\frac{\partial \pi_i}{\partial d} = \frac{\alpha}{4t}[\alpha(b-a) + q(2-\alpha)] > 0,$$

where $d = |a|, b$. Since $\frac{\partial \pi_i}{\partial d} > 0$, firms have incentive to differentiate as much as possible, leading to maximum differentiation in equilibrium.

Proposition 1: *In the one-sided setting where firms derive revenue only from consumers and consumers suffer from overlapping characteristics (i.e., $\alpha > 0$), maximum differentiation arises in equilibrium. It follows from Assumption 1 that $(b-a)^* = q$, while equilibrium prices and demands are, respectively, given by $p_1^* = p_2^* = \frac{q}{2}$ and $D_1^* = D_2^* = \frac{q}{t}$.*
Proof: see appendix.

This result is in line with the "principle of differentiation" (Tirole, 1988); however, the reasons this result emerges in the present context are different. In standard models of horizontal differentiation, the *strategic effect* of differentiation on firms' prices is so strong that firms decide to locate as far as possible from each other. In our model, there is no strategic interaction between firms' prices, as already noted. The result arises because each firm's total demand is strictly increasing in differentiation, provided that $\alpha > 0$. The absence of strategic interaction between firms' prices allows us to focus on demand-related incentives to engage in differentiation.

In the standard Hotelling framework, the *demand effect* always induces firms to locate in the middle of the market because each firm holds captive consumers, and the amount of these captive consumers (mechanically) increases when firms move to the middle (contrary to our case, where captive demand remains fixed regardless of location). Thus, when price competition is relaxed, the standard Hotelling logic suggests that firms' incentives to differentiate are dampened. This is the main result in Kim and Serfes (2006), where minimum differentiation occurs in certain circumstances. Therefore, our maximum differentiation outcome contrasts with what one would expect from one-sided settings with multipurchasing.

Special case with $\alpha = 0$**.** When $\alpha = 0$, consumers do not suffer from overlapping characteristics and therefore fully enjoy each firm's bundle. Intuitively, this may disincentivize firms from locating far from each other.
Indeed, when $\alpha = 0$, firm i's demand simply becomes:

$$D_i = \frac{2}{t}(q - p_i). \tag{14}$$

A firm's total demand is therefore independent of its location. Taking the first-order condition with respect to price, we obtain: $p_i^* = \frac{q}{2}$. As mentioned above, demand is independent of location, as are prices. Firms are therefore indifferent in regards to where to locate; however, one could assume that firms have to incur a small nonzero cost to engage in differentiation. If so, we obtain the following result.

Proposition 2: *When $\alpha = 0$, consumers do not suffer from overlapping characteristics and firms' demands are independent of their location. In the one-sided setting where firms derive all revenue from consumers, minimum differentiation arises when there is a small nonzero cost to engage in differentiation such that that $(b - a)^* = \underline{d}$.*

Note that we could also assume away any cost of differentiation and stick to the result of indifferent location. Notably, this special case highlights that the only incentive for firms to move away from their competitor is to reduce the overlap between the characteristics they offer, which increases total demand.

3.2 Multisided environment

Let us recall first that the demand structure derived above also applies throughout this subsection. We analyze a situation in which consumers are not charged anything by firms. This situation is consistent with platforms' strategies often observed in digital or media markets.[15] There is a unit mass of advertisers who are willing to place ads on the platform. Similar to Anderson et al. (2018, 2019) and Ambrus et al. (2016), we assume that advertisers are willing to pay r per ad for a unique impression and σr for a second impression, where $\sigma \in [0, 1]$. Thus, reaching the same consumer a second time is worth (weakly) less than reaching them for the first time. The extent of multihoming constitutes a measure of competition intensity in the market: a multihoming consumer can be reached by advertisers through both platforms. Under competition, each platform can charge σr for each multihoming consumer.[16] In line with the literature, we further assume that advertisers place only one ad per platform.

The profit function of platform i is given by:

$$\pi_i = rSP_i + \sigma rMP, \tag{15}$$

where SP_i and MP are taken with $p_i = 0$. Since we assume a free service for consumers, each platform maximizes its profit function with respect to only location. Additionally, in the present case, each platform cares about the composition of demand since single-homers

[15]In the main text, we focus on polar cases where firms derive revenues from consumers or from advertising. This scenario makes the exposition clearer. In the Appendix, we develop the case in which firms obtain revenue from both consumers and firms. This situation can be seen as an intermediate case. The main driving forces we identify in Sections 3.1 and 3.2 still operate, but they are weighted against one another.

[16]In Section 5, we discuss how this finding is affected if platforms merge.

and multihomers do not generate the same value. The first-order condition is given by:

$$\frac{\partial \pi_i}{\partial d} = 0 \iff r(1 - \frac{\alpha}{t}) + \sigma r(\frac{2\alpha}{t} - 1) = 0 \tag{16}$$

From Lemma 1, it immediately follows that the derivative can be either positive or negative. For intermediate values of α (i.e., $\frac{t}{2} < \alpha < t$), both terms in (16) are positive, so maximum differentiation unambiguously arises. In the two other cases stated in Lemma 1, the analysis is more ambiguous. Indeed, when $\alpha > t$, the left term of equation (16) is negative, whereas the right term is positive (i.e., SP_i decreases and MP increases in differentiation). Conversely, when $\alpha < \frac{t}{2}$, the left term is positive, whereas the right term is negative (i.e., SP_i increases and MP decreases in differentiation). However, in the latter case, the first effect always dominates the second effect, for every value of σ. As for the former case, which effect dominates depends on the value of σ. Proposition 3 below states the main results.

Proposition 3: *In the two-sided setting, where firms derive revenue only from advertising and consumers suffer from overlapping characteristics (i.e., $\alpha > 0$), firms optimal differentiation strategies are defined as follows:*

- *If $\alpha < t$, maximum differentiation arises for every value of σ;*
- *If $\alpha > t$, minimum differentiation arises provided that $\sigma < \tilde{\sigma} = \frac{\frac{\alpha}{t} - 1}{\frac{2\alpha}{t} - 1}$, and maximum differentiation arises otherwise.*

When σ is relatively low ($\sigma < \tilde{\sigma}$), firms derive much greater profits from single-homers than multihomers and thus care about how product differentiation may affect the number of single-homers in the market. In this case, when the *overlap-reducing* effect is strong ($\alpha > t$), minimum differentiation arises. Conversely, when the *segmentation* effect dominates ($\alpha < t$), maximum differentiation always arises (for all σ).

As σ increases, platforms care more about total demand and less about the composition of demand. When $\sigma = 1$, we end up in a similar situation as that in the one-sided setting, where single-purchasers and multipurchasers generate the same value. In this case, as total demand is strictly increasing in product differentiation, it is always profitable for firms to differentiate (i.e., maximum differentiation ensues).

The idea that advertising-financed markets can result in insufficient differentiation has already been raised in the literature.[17] In our setting, minimum differentiation also occurs as a result of firms' objective to maximize advertising revenues. However, we unveil a new mechanism that dampens firms' incentives to differentiate, namely, the evolution of demand composition. When we consider multihoming contexts, and differentiation affects demand composition, firms may refrain from engaging in differentiation. The reason is that if differentiation makes multihoming more attractive, firms' bargaining power vis-à-vis advertisers does not improve.[18]

Special case with $\alpha = 0$**.** As underlined previously, when $\alpha = 0$, consumers do not suffer from overlapping characteristics and firms' total demands become independent of the overlap and location. However, the composition of demand is not independent of location. As observed directly from (16), an increase in differentiation implies an increase in the number of single-homers for each firm and a proportional decrease in the number of multihomers (leaving total demand unchanged). We can therefore state the following proposition.

[17] Gabszewicz et al. (2001) revisit the standard Hotelling framework by introducing advertising revenues and show that firms may prefer to locate in the center of the market to secure the highest possible demand (even if this leads to the standard Bertrand paradox on the consumer side). The authors worry that advertising-financed markets may lead to the ascent of the "Pensée Unique".

[18] Mullainathan and Shleifer (2005) predict that "when potential audiences share similar beliefs, we do not expect to see diversity of media reports". This corresponds to our prediction that when consumer heterogeneity is relatively small (i.e., the *overlap-reducing* effect dominates the *segmentation* effect), firms have limited incentives to engage in differentiation since it would stimulate multihoming and deteriorate their bargaining power vis-à-vis advertisers.

Proposition 4: *When $\alpha = 0$, the composition of demand is affected by location choice: an increase in differentiation implies an increase in the number of single-homers and a proportional decrease in the number of multihomers. In the two-sided setting, where firms derive revenue only from advertising, maximum differentiation arises when $\sigma < 1$.*

When $\alpha = 0$, the total demand functions are fixed, but firms can perfectly control their demand composition through differentiation. Since exclusive consumers are more valuable than multihoming consumers, firms maximally differentiate to reduce the number of multihoming consumers.[19]

The results obtained in this section highlight the difference between one-sided and two-sided settings. In the one-sided setting, firms only care about the total demand, which implies that they do not have incentive to differentiate if $\alpha = 0$. When α increases, firms' incentives to differentiate also increase (since total demand increases in differentiation for $\alpha > 0$). In the two-sided setting, the reasoning is reversed: when α is very large ($\alpha > t$), firms do not have incentives to differentiate (provided that σ is not excessively large) since this would imply a decrease in the number of exclusive consumers. However, as α starts to decrease, and at the extreme when $\alpha = 0$, an increase in differentiation unambiguously leads to more exclusive consumers in the market, which benefits firms (as long as $\sigma < 1$).

4 Welfare analysis

Total welfare is defined as the sum of consumer surplus and firms' profits. We begin the analysis with the one-sided scenario. To find the socially optimal outcome, one can abstract from prices, which are simply transfers between firms and consumers. Hence, total welfare

[19]Again, this situation is particularly similar to that of Anderson et al. (2018), in which firm also perfectly control their demand composition and maximum differentiation arises when second impressions are worthless (i.e., $\sigma = 0$). Our main contribution is to consider situations where $\alpha > 0$ (i.e., the *overlap-reducing* effect kicks in), which makes the impact of differentiation on demand composition more ambiguous.

is given by:

$$W = \int_{x_{10}}^{x_{1m}} (q - t|x-a|)\, dx + \int_{x_{1m}}^{x_{2m}} (2q - \alpha O - t(b-a))\, dx + \int_{x_{2m}}^{x_{20}} (q - t|x-b|)\, dx. \quad (17)$$

The way differentiation affects welfare is driven by the value of α with respect to t (i.e., the *overlap-reducing* effect vs the *segmentation* effect). Note first that only multipurchasers' utility is affected by differentiation. An increase in differentiation leads to a decrease in the overlap of characteristics but leads to an increase in transportation costs. Single-purchasers consume only one good, so they do not benefit from the decrease in overlap. Moreover, the average transportation cost remains the same within a given interval.[20] Second, differentiation also affects consumer support.

When $\alpha > t$, the surplus associated with single-purchasers is decreasing in differentiation: the number of single-purchasing consumers decreases. Furthermore, the surplus associated to multipurchasers increases: the number of multipurchasers increases, as does the utility they derive. Conversely, when $\alpha < t$, the surplus associated with single-purchasers increases, and the surplus associated to multipurchasers tends to be negatively affected by differentiation.[21]

Taking the derivative of (16) with respect to location yields:

$$\frac{\partial W}{\partial d} = \frac{\alpha[qt + 2q(1-\alpha) + 2(a-b)(t-\alpha)]}{t}.$$

If $\alpha > t$, all terms in the numerator are positive; if $\alpha < t$, the last term becomes negative. This result reflects the lower utility that multipurchasers obtain when differentiation increases. However, we find that overall, total welfare always increases in differentiation.

[20] Holding consumer support fixed, some single-purchasers find themselves closer to the firm, and others find themselves farther.

[21] Specifically, when $\frac{t}{2} < \alpha < t$, the number of multipurchasers increases, but the utility they derive decreases in differentiation. When $\alpha < \frac{t}{2}$, the number of multipurchasers and their utility both decrease in differentiation.

This result is driven by the demand expansion induced by differentiation. The equilibrium outcome therefore corresponds to the socially optimal one. Moreover, for consumer surplus, we have:

$$CS = \int_{x_{10}}^{x_{1m}} (q - t|x-a| - p_1)\, dx + + \int_{x_{1m}}^{x_{2m}} (2q - \alpha O - t(b-a) - (p_1 + p_2))\, dx +$$

$$\int_{x_{2m}}^{x_{20}} (q - t|x-b| - p_2)\, dx.$$

The analysis of consumer surplus is more ambiguous. Indeed, while differentiation leads to market expansion, it also implies an increase in equilibrium prices. Integrating and taking the derivative with respect to location yields:

$$\frac{\partial CS}{\partial d} = \frac{2q(2+4t-5\alpha)\alpha + 2(a-b)(8t-5\alpha)\alpha}{8t}.$$

This expression is positive and decreasing in differentiation, which suggests that consumer surplus is increasing in differentiation up to a certain point. When $b - a = q$, the expression equals 0 for $t = \frac{1}{2}$ (the lower bound of t) and is negative for higher values of t. Thus, the equilibrium outcome yields excessive differentiation from the consumers' perspective when $t > \frac{1}{2}$.

Turning to the multisided analysis, total welfare is given by:

$$W = r\left[SP_1 + SP_2 + MP\right] + \sigma r MP + \int_{x_{10}}^{x_{1m}} (q - t|x-a|)\, dx + \int_{x_{1m}}^{x_{2m}} (2q - \alpha O - t(b-a))\, dx$$

$$+ \int_{x_{2m}}^{x_{20}} (q - t|x-b|)\, dx. \quad (18)$$

The first two terms jointly represent platforms' profits and advertisers' surplus, while the subsequent terms represent consumer surplus.[22] Note that consumer surplus corresponds to the total welfare studied in the one-sided case above; therefore, consumer surplus is always increasing in differentiation. In terms of platforms' and advertisers' (joint) surplus, it is overall increasing in differentiation. Indeed, total market demand is increasing in differentiation, such that the first term is always positive. Even in cases where the number of multipurchasers is decreasing in differentiation, it cannot offset the demand expansion effect. Total welfare is therefore unambiguously increasing in differentiation. We deduce that the equilibrium outcome is not always aligned with the social optimum: our main results are summarized in Proposition 5.

Proposition 5: *In the one-sided setting, the equilibrium outcome coincides with the social optimum; however, consumer surplus is nonmonotonic in differentiation. When $t > \frac{1}{2}$, the equilibrium outcome features excessive differentiation from a consumer surplus perspective. In the two-sided setting, maximum differentiation is also socially desirable, and consumer surplus strictly increases in differentiation. Thus, from a social perspective, there is insufficient differentiation when $\alpha > t$ and $\sigma < \tilde{\sigma}$.*

5 Mergers

In this section, we analyze how mergers affect firms' differentiation strategies in one-sided and two-sided environments. There is an ongoing debate in competition policy as to whether one should treat mergers involving multisided businesses in the same way as those involv-

[22] Competition prevents platforms from extracting the full value generated by multihomers. Advertisers are willing to pay up to $(r + \sigma r)$ for each multihoming consumer but end up paying only $2\sigma r$; their net surplus is thus $(r - \sigma r)MP$. The sum of platforms' profits is $r(SP_1 + SP_2) + 2\sigma r MP$. As we illustrate in the next section, a merger between the two platforms restores their ability to fully extract surplus from advertisers.

ing standard one-sided businesses. Our analysis sheds light on the way firms' incentives to differentiate can be affected in multisided vs one-sided contexts.

One-sided environment. We start by analyzing how firms' incentives to differentiate are affected in one-sided settings. As shown in Section 3.1, each firm's total demand is strictly increasing in differentiation over the relevant interval. Since single-purchasers and multipurchasers generate the same value, firms always choose to differentiate. In this context, we find that a merger (to create a multi-product monopoly) will also lead to maximum differentiation.

As discussed above, differentiation is not driven by strategic interactions between prices in our model. Prices are strategically independent, so equilibrium prices are the same in a multiproduct monopoly and under competition. However, even if the equilibrium differentiation level is the same as that under competition (i.e., $(b-a)^* = q$), incentives to differentiate are strengthened post-merger. The merged entity maximizes $\Pi_{1+2} = p_1 D_1 + p_2 D_2$. Plugging in the optimal prices computed in 3.1 and taking the first-order condition with respect to location yields $\frac{\partial \Pi_{1+2}}{\partial d} = \frac{\alpha}{2t}[\alpha(b-a) + q(2-\alpha)] > 0$. This expression is twice as large as $\frac{\partial \Pi_i}{\partial d}$.

Indeed, while the rival's price does not affect a firm's demand, each firm exerts an externality on the other firm through its location choice. As highlighted earlier, by moving toward the extreme, a firm reduces the overlap in product characteristics and makes its rival's product more attractive. A merger allows firms to internalize this effect.

Two-sided environment. The most striking effect we find relates to multisided mergers. The profit function of the merged entity is defined as $\tilde{\Pi}_{1+2} = r(SP_1 + SP_2 + MP) + \sigma r(MP)$. The main difference with competition is that the merged entity is able able to derive a revenue $r + \sigma r$ for each multihoming consumer, as opposed to $2\sigma r$ under competition.[23]

[23]This effect is also highlighted in Anderson et al. (2018), but it leads to different implications in terms of differentiation in our model.

Taking the derivative with respect to location leads to:

$$\frac{\partial \tilde{\Pi}_{1+2}}{\partial d} = r + \sigma r(\frac{2\alpha}{t} - 1). \tag{19}$$

The merger induces each platform to internalize the effect of differentiation on the whole market size (SP_1, SP_2, MP). Total market size is increasing in differentiation as a result of the decrease in characteristics overlap. Beyond total demand, platforms also care particularly about multipurchasers since a second impression generates additional revenue σr for the merged entity. Overall, we find that differentiation always increases firms' joint profit.

Proposition 6: *In one-sided settings, firms maximally differentiate in equilibrium regardless of market structure. In two-sided settings, equilibrium location strategies are affected by a merger. A merger induces maximum differentiation regardless of the value of σ and enhances social welfare when $\alpha > t$ and $\sigma < \tilde{\sigma}$.*

Proposition 6 shows that the minimum differentiation outcome that arises under competition (Proposition 3) is overturned when a merger occurs. The merger induces platforms to care about total market demand. Differentiation always leads to an increase in total demand, which is associated with additional revenue r. If the number of multihoming consumers is increasing in differentiation, the effect of differentiation on profits is unambiguously positive. If the number of multihoming consumers is decreasing in differentiation, differentiation also has a negative effect. However, the latter effect does not offset the first since the value of a second impression is (by definition) lower than that of the first impression (i.e., $\sigma \in [0, 1]$).

Our results suggest that mergers in two-sided environments are likely to be more beneficial from a welfare standpoint than mergers in one-sided environments. Indeed, in one-sided settings, firms always have an incentive to differentiate to expand market size, whereas in two-sided settings, firms care about demand composition and lack incentives to differentiate when they expect that differentiation is unlikely to lead to an increase in exclusive consumers. In this context, a merger can restore a *bottleneck* situation[24], enhancing firms' bargaining power vis-à-vis advertisers. A merger also draws focus to total demand, so firms' incentives to differentiate are restored and social welfare increases. The main drawback associated with a merger relates to advertisers, whose surplus is now fully extracted by firms. Our results contrast with those of Anderson et al. (2018), who find that market structure does not affect firms' incentives to differentiate. Conversely, our findings are in line with Steiner (1952), in which differentiation leads to market expansion, and a merger induces firms to focus on total market size.

6 Conclusion

A vast and long-standing literature has studied product differentiation, particularly in the media industry. However, most existing models assume that consumers single-home. In two-sided environments, consumer multihoming entails platform competition on the advertising side of the market, by raising advertisers' "outside option". In this context, platforms' attention is steered toward demand composition, as opposed to demand size. This scenario is different from standard one-sided environments, where all consumers are equally valuable and firms simply focus on demand size.

We study how firms' incentives to differentiate are affected by consumer multipurchasing (or multihoming) behavior. We identify a new force according to which differentiation may stimulate multipurchasing. If consumers' heterogeneity is weak, then a decrease in characteristic overlap between firms' products leads to more multipurchasing consumers in the market and fewer exclusive consumers. We find that this *overlap-reducing* effect of differentiation leads to market expansion, which results in maximum differentiation in

[24]The platform constitutes the only gateway to consumers for advertisers; thus, the platform holds some monopoly power with respect to advertisers. *Competitive bottleneck* situations are analyzed in Armstrong (2006).

one-sided markets.

However, market expansion is an insufficient motivation in two-sided contexts. Our analysis predicts that if platforms do not expect differentiation to lead to market segmentation, minimum differentiation will ensue. This finding can potentially explain why some advertising-financed markets exhibit a low level of differentiation.

We derive interesting implications as regards merger analysis. We show that in multi-sided environments, a merger is likely to restore platforms' incentives to differentiate. A merger relaxes competition on the advertising market and draws firms' focus to total demand. Our model therefore reconciles theory with empirical contributions that show that ownership concentration leads to greater diversity in media markets. However, in cases where incentives for differentiation are already effective premerger, ownership concentration results in lower advertiser surplus. In an era where firms are increasingly able to identify consumers' purchasing behavior, we expect multihoming to play an expanding role in shaping competition between firms relying on advertising revenues or consumer data to generate revenues.

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Appendix

Appendix A: Proofs

Proof of proposition 1. To show that $(b-a) = q$ is the unique equilibrium of the two-stage game, suppose that firm 2 decides to slightly decrease its level of differentiation such that $(b-a) = q-\epsilon$, with $\epsilon > 0$. Note that according to assumption 1, firm 2 cannot adjust its level of differentiation to a level that would result in $(b-a) > q$. If $(b-a) = q-\epsilon$, firm 2 adjusts its pricing strategy from $p_2^* = \frac{q}{2}$ to $p_2^* = \frac{q}{2} - \frac{\alpha\epsilon}{4}$ (remember that for a given level of differentiation, firms' optimal pricing strategy in stage 2 is $p_i^* = \frac{1}{4}[\alpha(b-a) + q(2-\alpha)]$). Note that firm 1 also observes the deviation from firm 2 and adjusts its pricing strategy in the same way, although this does not affect firm 2 because of strategic independence between firms' prices. The profit function of firm 2 becomes $\Pi_2 = [\frac{q}{2} - \frac{\alpha\epsilon}{4}] * [\frac{q}{t} - \frac{\alpha\epsilon}{2t}]$, which is strictly lower than when $(b-a) = q$, if and only if $\alpha > 0$. When $\alpha = 0$, the profit function is independent of differentiation: firm 2's profit function continuously decreases in ϵ, which implies that firms are strictly better off choosing $(b-a) = q$.

Equilibrium existence. Several conditions have to be met for the demand configuration specified to be valid and to ensure the existence of the equilibrium. These conditions arise from Assumption 2.

Note first that a necessary (though not sufficient) condition is to have $MP \geq 0$ (i.e., $x_{1M} \leq x_{2M}$) since a negative number of multipurchasers would not make sense. Plugging the equilibrium prices computed in the second stage into MP implies that we need $[b-a](\frac{2\alpha}{t} - 1) - \frac{1}{2t}[\alpha(b-a) + q(2-\alpha)] + \frac{2q}{t}(1-\alpha) \geq 0$. Simplifying the terms yields $[b-a](\frac{3\alpha}{2t} - 1) + \frac{1}{2t}[2q - 3\alpha q] \geq 0$. For very low levels of differentiation $(b-a)$ in the market, a necessary condition is to have $\alpha \leq \frac{2}{3}$ so that the second term is always positive. If $\frac{3\alpha}{2t} > 1$, then the first term is positive and the constraint is always satisfied (i.e., irrespective of the value of $b-a$). If $\frac{3\alpha}{2t} < 1$, the first term becomes negative. Then, one can easily show that in the worst case, where $b-a=q$, a necessary and sufficient condition for the constraint to be satisfied is to have $t \leq 1$.

Second, the demand configuration we consider requires $x_{10} \leq a$ and $x_{20} \geq b$. Since the analysis is symmetric for both sides, let us focus on $x_{20} \geq b$. Again, inserting the equilibrium prices computed in the second stage leads us to $\frac{1}{t}[q(\frac{2+\alpha}{4}) - \frac{\alpha}{4}(b-a)] \geq 0$. From *Assumption 1*, it follows that this condition is always satisfied (i.e., $(b-a) \leq q$).

Last, we must check that, as mentioned in *Assumption 2*, firms' demands on the extreme parts of the line are indeed composed only of single-purchasers. This requires $a \leq x_{iM} \leq b$, $i = 1, 2$. Again, since the analysis is symmetric for both cases, we focus on $a \leq x_{2M} \leq b$. Plugging the equilibrium prices into x_{2M} yields $0 \leq \frac{1}{4t}[2q - 3\alpha q + 3\alpha(b-a)] \leq b-a$. Let us rewrite this expression as $0 \leq \frac{2q-3\alpha q}{4t} \leq (b-a)(1-\frac{3\alpha}{4t})$. Note that $\frac{2q-3\alpha q}{4t}$ is always positive since we imposed $\alpha \leq \frac{2}{3}$ above. In addition, note that we also need $\frac{3\alpha}{4t} \leq 1$ for $(b-a)(1-\frac{3\alpha}{4t})$ to be positive. Second, to ensure that $\frac{2q-3\alpha q}{4t} \leq (b-a)(1-\frac{3\alpha}{4t})$, it follows that we need a minimum level of differentiation $(b-a) = \underline{d}$ in the market. The minimum level of differentiation that satisfies this constraint is therefore $\underline{d} = \frac{\frac{2q-3\alpha q}{4t}}{1-\frac{3\alpha}{4t}}$. Since the maximum level of differentiation in the market is $b-a=q$ (*Assumption 1*), we must check under which conditions we can have $\underline{d} \leq q$. Straightforward computations show that a necessary and sufficient condition is to have $t \geq \frac{1}{2}$.

Appendix B: Multisided analysis with pricing on the consumer side

We now show how the results we develop in Section 3.2 are affected when we allow for pricing on the consumer side. The profit function of platform i becomes:

$$\pi_i = p_i D_i + rSP_i + \sigma rMP.$$

The profit function is simply the sum of the profit functions in the one-sided and two-sided cases studied in Sections 3.1 and 3.2. The effects that arise below are therefore a combination of those already identified.

Prices. In the second stage of the game, platform i maximizes its profit function with respect to p_i. The equilibrium price is given by:

$$p_i^* = \frac{1}{4}[\alpha(b-a) + q(2-\alpha) - r(1+\sigma)].$$

Equilibrium prices are the same for both firms. Furthermore, the last term in the brackets $r(1+\sigma)$ implies that prices are lower in the two-sided analysis than in the one-sided analysis due to the network effects that consumers exert: an increase in the number of consumers allows the platform to derive more revenue on the advertiser side of the market, which drives consumers' prices down. In this regard, even negative prices could arise provided that r is sufficiently large. However, as in most works considering multisided markets, we restrict the analysis to nonnegative prices. Note that this result justifies our assumption in Section 3.2 that consumers are not charged anything.

Location choice. In the first stage of the game, the platform chooses the location that maximizes its profits, taking into account the optimal price derived in the second stage. The first-order condition is given by:

$$\tfrac{\partial \Pi}{\partial d} = 0,$$

$$\iff \tfrac{\alpha}{4t}[\alpha(b-a) + q(2-\alpha)] + r(1-\tfrac{\alpha}{t}) + \sigma r[(\tfrac{2\alpha}{t} - 1) - \tfrac{\alpha}{2t}] = 0,$$

The first component is the same as that in the one-sided analysis and is strictly positive (since an increase in differentiation always leads to an increase in total demand), whereas the second and third components can be either positive or negative, as seen in the previous section. The first-order condition can be seen as a combination of the one-sided analysis and two-sided analysis with zero price on the consumer side. Note that the last term $\frac{\alpha}{2t}$ that enters negatively into the third component simply reflects the fact that an increase in differentiation affects prices, which in turn negatively affects the number of multipurchasers in the market.[25] As in Section 3.2, each firm's optimal differentiation strategy is closely related to the value of σ. Proposition 3 is affected in the following way when we allow for consumer pricing in addition to advertising revenues.

Proposition 3': *In the two-sided setting where firms derive revenue from both consumers and advertising, firms' optimal differentiation strategies are defined as follows:*

- If revenues from consumers are limited compared to advertising revenues:

- *for $\alpha < t$, maximum differentiation arises for every value of σ;*
- *for $\alpha > t$, there exists a $\bar{\sigma}$ such that maximum differentiation arises provided that $\sigma \geq \bar{\sigma}$, with $\bar{\sigma} < \tilde{\sigma}$, and minimum differentiation arises otherwise.*

- If revenues from consumers are large compared to advertising revenues: maximum differentiation always arises.

Proposition 3' shows that the general case in which firms derive revenue from both consumers and advertising simply combines elements from Section 3.1 and Section 3.2. On the one hand, firms derive revenue from consumers, which incentivizes them to focus on total

[25] Equation (11) shows how the number of multipurchasers in the market is affected by the sum of firms' prices. Conversely, the number of single-purchasers is not affected by prices in equilibrium since both firms' prices affect each firm's exclusive demand in opposite (and proportional) ways, as shown by equations (9) and (10).

demand and hence to increase differentiation. On the other hand, firms derive revenue from advertising, which leads them to also account for the composition of demand and hence to reduce differentiation in those instances mentioned above. If advertising revenues are negligible, firms will focus on total demand, and maximum differentiation will ensue. If advertising revenues are large compared to revenues from consumers, firms may in some instances decide to limit their level of differentiation.

Mergers. The analysis of mergers also combines the effects at play in the one-sided and two-sided (with zero price on the consumer side) cases from Section 5. The effect of mergers when platforms derive revenue from consumers and advertising is unambiguous. As seen in Section 3.1, revenues from consumers draw firms' focus to total demand, which results in maximum differentiation. Additionally, a merger strengthens firms' incentives to differentiate, as highlighted in Section 5. Although advertising revenues may refrain platforms from engaging in differentiation (Section 3.2), as discussed above, a merger fully restores platforms' incentives to differentiate. Thus, if platforms derive revenues from both consumers and advertising, a merger will always yield maximum differentiation. The key takeaways derived in Section 5 therefore hold when introducing consumer pricing in addition to advertising revenues.