Search, Showrooming, and Retailer Variety*

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Abstract

In a model of consumer search, we trace through effects of changes in retail variety. Some consumers visit stores that offer many products that are imperfect substitutes, learn which product they like most, and then buy it elsewhere. These showroomers put upward pressure on prices elsewhere because they populate the market with consumers who know their preferences, in the style of the Diamond paradox (Diamond (1971)). Changes in retail variety affect search behaviour and all market outcomes. One change that we examine is the introduction of a shopping venue where prices are readily available but product information is not.

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1 Introduction

Goods are sold through a variety of stores. Some offer greater selections, others much less so. The last few years have seen significant changes in retail outlets: notably, the growth of big-box stores and e-commerce (Hortaçsu and Syverson (2015)). Recent events have created and will continue create further changes. While interest from the public, industry, and policy continues, few theoretical models incorporate retail variety. This limits our ability to understand the impact of changes in retail variety.

A nascent literature has begun to explore consumer search with multiproduct retailers (Zhou (2014); Rhodes (2014)), with more recent studies shifting focus toward the co-existence of multiproduct retailers with other stores offering narrower selections (see Rhodes et al. (2018); Rhodes and Zhou (2019)). We add to this literature by considering multi-product stores selling goods that are substitutes rather than independent in consumers' consumption utility and different stores that have overlapping selections (in contrast to Cachon et al. (2008) and Watson (2009)). As we describe below, this diversity in retail outlets affects prices and welfare when consumers have to search to learn the suitability a particular variety and its price.

In our model, a consumer wants to purchase a good and is uncertain about which of two available goods is the best fit. She can choose between visiting a broad-range retailer where she can discover all varieties at once—saving on the cost of inspecting them and learning their suitability—and visiting narrow-range stores one by one.[1] Alternatively, a consumer may "showroom"—that is, go to a broad store with no intention of buying there, but to figure out which is her favourite variety and then to buy it at another store.

Showrooming arises as an equilibrium phenomenon when consumers are heterogeneous in two dimensions: first, their "choosiness" or preference for variety; and, second, their willingness to search for a lower price.[2] Consumers know which types of stores are which before visiting. Thus, we characterise a consumer's directed search problem. A number of consumer behaviours can arise: some consumers start out by visiting broad retailers, while others visit narrow stores first; some consumers anticipate buying at the first store they visit, while others (showroomers) may anticipate never buying from the first store that they visit.

Prices depend on the mix of search behaviours that consumers employ (which in equilibrium, of course, depend on anticipated and realised prices). However, not all kinds of search behaviours serve to discipline prices. Indeed, only one kind of consumer behaviour that acts to discipline industry prices: that is, (not so choosy) consumers who start off by visiting a narrow store and expect that they will buy there unless they find a sufficiently poor match; in this case, they move on to another kind of narrow store and learn about another good. This group of consumers is the only one in the economy that compares prices, and the (endogenous) size of this group and its composition, therefore, play a key role in price determination. Unlike this group, showroomers arrive at narrow stores already knowing that they like the product. Thus,

[1]Through the rest of the paper, we refer to broad and broad-range stores interchangeably. Similarly, narrow and narrow-range.

[2]One interpretation is that this is a cost of visiting stores. Another interpretation is the "guilt" associated with spending time at one store and buying elsewhere.

just as in Diamond (1971), they never leave over small price deviations. The same is true for those consumers who are not at all choosy and have high costs of visiting more stores. Thus, in an equilibrium with showrooming, narrow stores charge prices that are disciplined by only the relatively few consumers who might search through more than one store to learn about their matches with different goods.

Since consumers who buy from broad stores do not shop around (for match), such stores effectively have hold-up monopoly power over consumers. Prices at broad stores, therefore, are constrained only by the possibility that a consumer becomes a showroomer and leaves to buy at a narrow store. Since this involves a cost to the consumer, broad stores charge higher prices than narrow stores do. Of course, if narrow stores charge the monopoly price, then so will broad stores, and such an equilibrium always exists.

We show that showrooming can arise as an equilibrium phenomenon in our model, as it appears to do in practice, as well. Perhaps surprisingly, the possibility of showrooming can have a detrimental effect on prices. Consumers who are generally the most eager to search in environments with only narrow stores—those with high choosiness—end up being fully price-inelastic.

We extend our baseline model and our discussion in a couple of ways. First, we discuss the role of retailer variety and what happens if some types of stores (either broad or narrow) no longer operate. This is relevant, for example, if each type of store must cover a fixed cost in order to operate, and viability might be affected by changes in demand or the introduction of new kinds of competitors (such as alternative channels). The analysis clearly highlights that even if prices are unaffected, there may be negative welfare consequences. If prices are affected by such a change, they necessarily go up. In our model, to ease tractability and exposition, we do not consider an extensive margin for consumers; however, these observations clearly imply that an upstream manufacturer may have an interest in maintaining retailer variety to encourage consumer participation.

Second, we introduce into the model a different kind of retail sector—one that we call the price-only sector—into the model. We assume that consumers cannot discover their matches at retail outlets. This is an appropriate assumption for online retail for many product categories where physical interaction with a product is important—for example, sound quality for a high-end speaker. However, prices are readily available in this sector (again consistent with e-commerce). We suppose that some "savvy" consumers have access to this sector, but naive consumers do not. Depending on whether these savvy consumers drawn away by the price-only sector are disproportionately picky types (who would otherwise be showrooming and making demand at narrow stores more inelastic) or less-picky types (who would otherwise be searching and exerting downward pressure), prices in the more traditional stores may go up or down as a result of the introduction of the price-only venues.

1.1 Related Literature

To our knowledge, ours is among the first search models that consider competition between narrow and broad stores that stock overlapping selections of substitute goods to examine equilibrium showrooming. We study how retail variety endogenously determines search behaviour and equilibrium prices and demonstrate that equilibrium showrooming requires consumer heterogeneity and retailer variety.

There are other recent contributions on the showrooming phenomenon. For example, Wang and Wright (2020) examine fees that platforms (similar to our broad stores) charge and price-parity clauses. They focus on consumers who are ex-ante homogeneous in preferences, and, indeed, in their analysis, showrooming is a possibility that is never observed in equilibrium.

There are a number of papers where showrooming does arise in equilibrium. Notably, Balakrishnan et al. (2014), Jing (2018), and Parakhonyak (2018) consider models where there is a single good, sold through different channels in contrast and where match can only be learned at one kind of retailer. Instead, in our model a consumer may showroom to figure out which good to buy (rather than whether to buy at all) and can learn about her match realisations either at broad stores or narrow stores. Loginova (2009), Mehra et al. (2017) and Kuksov and Liao (2018) consider product variety or product variants. In all these papers, in addition to only a single venue for learning matches, consumers observe all prices before visiting stores leading to a different analysis and different effects. In particular, Mehra et al. (2017) focus on the role of price-matching and exclusivity, and Kuksov and Liao (2018) focus on retail service provision and a monopolist manufacturer's endogenous contracts with retailers.

Shin (2007) and Janssen and Ke (2020) study service provision in search markets rather than showrooming; however, there is a connection to this work. In Janssen and Ke (2020), service received is transferable to other variants of the product, whereas in Shin (2007), both firms sell the same product, and the service informs some consumers about the match. In both models, some form of showrooming occurs, but Shin (2007) is closer to ours in that consumers learn about a match at a store that provides service, and those consumers with low visit costs purchase at the other, cheaper store that provides no service. This is akin to our extension in which we introduce a sector where there is no ability to help discover matches. The key difference in our model is that firms differ in their assortment and consumers differ in their pickiness, neither of which is true in Shin (2007) (who considers only a single good) or Janssen and Ke (2020) (where each store offers a single distinct good).

Moorthy et al. (2018) focus on channel management, but do allow for comparison shopping. Their focus is on the vertical arrangements between manufacturers and their integrated and rivals' retailers and their effect on consumers' decisions to participate in the market. In particular, rivals may sell each others' goods to encourage demand discovery and boost the size of the market. To allow this focus, Moorthy et al. present a model in which all retail prices are set at the monopoly level (given input costs), whereas our model highlights the interaction among retail variety, consumer search behaviour and equilibrium prices.

Many of these studies focus on the vertical aspects associated with showrooming. Although we do not explicitly model these arrangements, we highlight that sustaining (or killing off) different kinds of stores can affect equilibrium prices and consumer surplus. In turn, to the extent that this boosts industry profitability (through the effect on prices and discrimination, as in Parakhonyak (2018)) or encourages or depresses consumer participation (through anticipated consumer surplus), this will have implications for manufacturers' preferred strategies.

Finally, in recent work, Armstrong and Vickers (2020) examine how different exogenous patterns of consumer consideration affect prices and firm profits. Our work is related inasmuch as the consumer search behaviour provides an endogenous model of the nature of consumer interactions. In this way, we demonstrate how retailer variety, through equilibrium consumer search behaviour, establishes which kinds of stores different consumers consider.

2 Model

There are three types of retailers that sell two differentiated goods, 1 and 2. "Broad-range" retailers sell both goods. "Narrow-Range" retailers sell only one of the goods—e.g., only good 1.[3] As will become clear, as long as both types of retailers operate, the actual number of each type will not be important for determining equilibrium, as long as there is at least one of each type.[4]

There is a unit mass of consumers who wish to purchase one of the goods. Consumer j's utility from consuming good i at price p_i is

$$\mu^j \varepsilon_i^j + u(p_i),$$

where ε_i^j is consumer j's idiosyncratic match value for good i and is an iid draw (across consumers and goods) from a distribution on the support $[0, 1]$ with twice continuously differentiable and log-concave CDF $G(\cdot)$. A consumer's choosiness, which we discuss below, is represented by $\mu^j > 0$. The inherent utility of consumers for the goods at price p is $u(p)$, and this is derived from a downward-sloping and differentiable demand curve $Q(p)$:

$$u(p) = \int_p^\infty Q(x)\, dx.$$

Thus, $u(p)$ is the consumer surplus (excluding the component that comes from the match value) derived from consuming the good at price p. It is assumed to be the same for all consumers whose utility differs only in their match values. Note that match values are additive to $u(p)$, which greatly simplifies the analysis. Anderson and Renault (2000) show that, although this formulation with downward-sloping demand and an additively separable match term is qualitatively similar to the more standard unit demand formulation in terms of the consumer

[3] Note that these designations of narrow and broad relate to this particular product segment. For example, for a consumer buying speakers, a large store (such as Costco or Walmart) might be considered narrow if it stocks a small product range, and a small specialised store might more appropriately be considered broad.

[4] For this reason, in the propositions that follow, we state profits earned by retailer type and not by individual retailers of such type.

and firm problems, it allows a role for prices to affect welfare. We discuss it at greater length, where relevant, below.

We normalise firms' marginal costs to be equal to zero. Thus,

$$\pi(p) = Q(p)p.$$

denotes the per consumer profits earned by a firm on a good. We use the standard notation

$$p^m = \arg\max_p \pi(p)$$

to denote its maximiser—the monopoly price—and

$$\pi^m = \max_p \pi(p)$$

to denote its maximand—the monopoly profits.

The outside option is assumed to give sufficiently low utility that all consumers purchase some positive quantity.[5]

Consumer j is initially uninformed about how well-matched she is with each of the two goods; that is, she does not know ε_1^j and ε_2^j. For her to find out her valuation for a good (that is, to learn ε_i^j), as well as the price, and she needs to inspect the good. Doing so incurs an inspection cost, s. In particular, we rule out the possibility of buying the good without first inspecting it.[6] If the consumer already knows her match value with a good and is visiting a store only to obtain a price quote, this visit cost is denoted as b^j, with $b^j \leq s$. Thus, a visit and inspection at a narrow retailer that sells good i costs s; if the consumer already knows ε_i^j, the visit cost is only b^j.

The inspection cost at broad stores is $s(1+\gamma)$, where $\gamma \in [0,1]$. During a visit to a broad retailer consumer learns the match values and prices for both goods. The parameter γ measures economies of scale in inspection costs allowed by broad retailers that stock both goods. When $\gamma = 0$, such scale economies are at their highest, whereas when $\gamma = 1$, they are non-existent. Since a visit to a broad store involves no inspection when the consumer already knows her match realisations, we assume that the visit cost to a broad store is b^j, though this plays no role in our analysis as long as this visit cost is non-negative.

Consumers are free to make their visits in any order and know the retailer's type before visiting; that is, they know whether a retailer is broad or narrow, and, if narrow, which good is being stocked.[7] That is, given anticipated prices, consumer j can decide to make her first

[5]This assumption is restrictive and is made for the purpose of tractability. As we go along, we will comment, whenever necessary, about its implications. Given this assumption, imposing that ε has a finite support [0,1] is without loss of generality.

[6]This can be justified while maintaining our analysis by supposing, for example, that there is a small probability of a very large negative match.

[7]In some applications, it may be more reasonable to suppose that, at inspection, consumers do not know the type of the narrow store (though they may be able to find out for the purpose of visiting or buying from such a store online) or that they do not know the type of the narrow store either for inspection or for visiting. Analysis in such an environment may be more involved; for example, a consumer may start by searching a narrow store

visit to a broad or narrow retailer. If consumers are indifferent amongst different stores in equilibrium, we assume that they are equally likely to visit any of them. To avoid issues related to prominence (Armstrong et al. (2009)), we will seek equilibria in which all narrow stores charge the same price and will not allow consumers to target a particular type of such retailer (e.g., those selling good 1) for first visits. Consumers can go back to all visited stores to make a purchase at no extra cost.[8]

Note that while we allow consumers to differ in their choosiness μ^j and visit costs b^j, we suppose that the economies of scale associated with inspecting goods at a broad store γ and the inspection cost s are common among all consumers. Of course, this is not substantive for the analysis of an individual consumer's behaviour, which we consider in Section 3. Instead, it simplifies the analysis of the equilibrium pricing decisions in Section 4, where we must take a stance on the distribution of consumers. As we discuss there, we make assumptions for tractability (including that consumers vary only in their pickiness and in their visit costs) that we believe allow us to illustrate some economic forces that would also apply in richer environments.

The timing of the game is as follows. First, retailers simultaneously set prices for all the goods that they carry. Second, consumers decide which type of retailer to visit first. Once the first visit reveals match value(s) and price(s), consumers may decide to visit more stores, and visit and inspection costs are incurred, as outlined above. Once consumers finish their inspections and visits, they decide which good to buy and where.

We characterise (perfect Bayesian) equilibria in which prices are symmetric for the two goods and prices are (weakly) higher at broad retailers than at narrow retailers.[9] Next, we turn to consider the welfare implications of retail variety and the effect of price-only retailers. Throughout, we assume passive beliefs: if a consumer observes unexpected prices at a retail store, this does not affect her expectations of prices at other stores.

3 The consumer problem

We assume that consumers expect a symmetric equilibrium in which both types of narrow-range retailers charge p_N^* for the good they sell and broad-range retailers charge a price vector (p_B^*, p_B^*) for goods 1 and 2. Further, we assume that $p_B^* > p_N^*$. Abusing notation, we sometimes use scalar p_B to denote the vector. This is because, unless noted otherwise, broad stores find it optimal to charge the same price for both goods.

In this section, under the assumption that pricing is as described in the above paragraph, we characterise the optimal search and purchase behaviour of a consumer given the parameters

and then choose to visit a broad store—a possibility that does not arise in our setting. However, we would anticipate that the general qualitative results of our analysis would also obtain in these cases.

[8]This is a simplifying assumption that can be relaxed at a cost of tractability. Had there been positive costs of going back, such as having to pay the visit cost twice, there would be fewer consumers visiting narrow stores.

[9]It is immediate that there can be no equilibrium in which both types of stores make sales, and prices are higher at narrow stores than at broad stores.

μ, b, s and γ.[10] Such a consumer has to decide which type of retailer to visit first and then what to do next. We suppose that a consumer will purchase one of the goods rather than drop out of the market. The possible consumer behaviours and how they depend on parameters are summarised in Section 3.3. The intervening sections derive these behaviours and introduce further notation.

The advantage of visiting a broad retailer is that no further search for match values is necessary. This may potentially save on inspection costs if γ is small enough and if the consumer is sensitive to match quality. The disadvantage is that consumers will have to either pay a higher price or incur a further visit cost in their search for a lower price elsewhere.[11] We use the notation $\Delta(x, y) \equiv u(x) - u(y)$ to denote the utility difference in purchasing the same good at a price x rather than at a price y. It is convenient to introduce the notation $\Delta^* \equiv \Delta(p_N^*, p_B^*)$ for the gain in utility from buying a good at the equilibrium price of a narrow store rather than at the equilibrium price of a broad store. With some abuse, we refer to this as the price premium associated with a broad retailer (which would be accurate in the case of unit demand, but, here, corresponds to a utility-adjusted price difference). A consumer who visits a broad retailer expects to pay an extra utility cost of $\min(\Delta^*, b)$ compared to buying the same good at a narrow store. Thus, a consumer at a broad store will have to compare Δ^* and b, and, accordingly, will showroom or not.

3.1 Starting at a narrow store

If a consumer chooses to make her first inspection at a narrow retailer, depending on her match value drawn for the one product that this retailer sells, she may choose to inspect again at the other type of narrow retailer. Because she expects $\Delta^* > 0$, the second inspection would never be at a broad retailer. Thus, if the consumer starts by searching at a narrow store, broad retailers are, in effect, irrelevant, and the way that she searches through narrow retailers is similar to that in the canonical Wolinsky (1986) model with $n = 2$, adjusted for our setup with downward-sloping rather than unit-demand.[12]

It is convenient to define

$$w(x) \equiv \int_x^1 (\varepsilon - x) g(\varepsilon) d\varepsilon$$

as the expected gain from drawing a match value above x for $\mu = 1$. As is well known from

[10]Since we consider only a single consumer in this section, we drop the ^{j} superscript here for notational convenience

[11]Note that we assume that a consumer must learn about both products when visiting a broad store which imposes a cost on less-picky consumers. Of course, it is possible to allow for sequential search within a broad store, as well as between different stores. We have conducted preliminary investigations of such a setting and find the qualitative results similar, at the cost of additional analytic complexity.

[12]Small price deviations by a narrow retailer selling good 1 do not lead to consumers switching to other retailers selling the same good—this is simply an instance of the Diamond paradox. However, due to our assumption of directed search, they do lead potentially to consumers who are close to indifferent to inspecting good 2 at a narrow retailer of that type. Thus Diamond-paradox effects among similar retailers and inspection across different retailer types lead to predictions that are equivalent to those of the model with just two retailers with unique varieties.

the literature, and corresponding to the analysis in Wolinsky (1986), for example, a consumer who inspects at a narrow retailer will purchase there if the match value is high enough, and, otherwise, will inspect at the other type of narrow retailer. That is, the consumer searching among narrow retailers employs a threshold rule. It is convenient to introduce notation for the threshold match value r^*. This is the solution to

$$\mu w(r^*) = s.$$

If there is no solution, then $r^* = 0$, and the consumer will buy from the current store, irrespective of the match value.

The expression above is a little different from that in the standard model, in that the left-hand side of the equation includes the factor μ to take into account that match values are equal to $\mu\varepsilon$. It is useful to rewrite $r^* = w^{-1}\left(\frac{s}{\mu}\right)$.

Consider a consumer who visits a narrow store selling good 1 (similar analysis applies for good 2) and finds price p when she expects that a narrow retailer selling good 2 would charge p_N^*. The consumer is indifferent between buying good 1 and inspecting good 2 if

$$\varepsilon_1 = r(p) \equiv r^* + \frac{\Delta(p_N^*, p)}{\mu}.$$

A consumer will, therefore, continue and inspect good 2, rather than buying good 1, if $\varepsilon_1 < r(p)$. It never pays off for a such a consumer to buy good 1 from another narrow retailer selling it at p_N^* instead of learning about good 2, as that would give utility $\varepsilon_1 + u(p_N^*) - b$, which, by definition, is worse than searching for and inspecting good 2.

Thus, when $r^* > 0$, we can use results from Choi et al. (2018) to write the expected utility prior to the first inspection as

$$U_N = \mu \int_0^1 \min(\varepsilon, r^*)\tilde{g}(\varepsilon)\, d\varepsilon + u(p_N^*),$$

where $\tilde{g}(\varepsilon) \equiv 2g(\varepsilon)G(\varepsilon)$ is the density of $\max(\varepsilon_1, \varepsilon_2)$.[13] The beauty of this formulation is that the utility is as if the consumer does not pay any search costs (all, including the first one, are already accounted for through the definition of r^*). Instead, the utility simply involves the consumer drawing the maximum of two draws but only if the maximum is below r^*, or else she gets r^*. The intuition for this result is that if $\max(\varepsilon_1, \varepsilon_2) < r^*$, the consumer would have searched and obtained $\mu\varepsilon + u(p_N^*)$, whereas for instances in which $\max(\varepsilon_1, \varepsilon_2) \geq r^*$, the consumer would be indifferent if these instances were replaced with $\mu r^* + u(p_N^*)$, which is precisely what the expression summarises.

3.2 Starting at a broad or a narrow store?

The expected utility when starting by inspecting at a broad retailer is given by

[13] Instead, when $r^* = 0$, the consumer will never go beyond the first store, and so $U_N = \mu E(\varepsilon) + u(p_N^*) - s$.

$$U_B = \mu \int_0^1 \varepsilon \tilde{g}(\varepsilon)\, d\varepsilon + u(p_N^*) - \min(b, \Delta^*) - (1+\gamma)s.$$

The first term reflects that the consumer always buys whichever of the two goods is a better match; the second two terms reflect that either the consumer pays b to showroom and enjoy the consumer surplus associated with a price of p_N^*, or else purchases at the broad store and enjoys consumer surplus $u(p_B^*) = \Delta^* - u(p_N^*)$,where the equality is immediate by the definition of Δ^*. The final term simply reflects the search costs at a broad store.

It is also useful to define, the inspection efficiency benefit associated with visiting a broad store. We write

$$\beta(\mu) \equiv \mu \int_{r^*}^1 (\varepsilon - r^*)\tilde{g}(\varepsilon) d\varepsilon - (1+\gamma)s$$

as the benefit of initially visiting a broad store rather than a narrow store absent any price difference.[14] The equality above holds for the case that $r^* > 0$.[15] This benefit reflects that the consumer who visits a broad store enjoys $\max(\varepsilon_1, \varepsilon_2)$ instead of ε_i for cases in which $\varepsilon_i > r^*$ but incurs a higher search cost. A consumer at a broad retailer necessarily obtains the maximum match value but has to pay an inspection cost of $(1+\gamma)s$. This inspection benefit naturally depends on (and increases with) a consumer's pickiness, μ.

With this notation, we can compare U_N and U_B at prevailing equilibrium prices:

$$U_B - U_N = \beta(\mu) - \min(b, \Delta^*).$$

We can see that consumers get an inspection benefit $\beta(\mu)$ from visiting broad stores, but this comes at the cost of incurring either the price premium at the broad store or an additional visit cost associated with showrooming: that is, $\min(b, \Delta^*)$. Following Lemma 1, $U_B - U_N$ is non-decreasing in μ,[16] so that consumers with high μ are the ones who choose to visit broad retailers. Of these, it is immediate, on inspecting $U_B - U_N$, that those with low b ($b < \Delta^*$) showroom, and those with $b \geq \Delta^*$ buy at broad stores.

We can now compare the expected utility from starting the search process at a broad store or a narrow store. This is simply a comparison of U_B and U_N.

Lemma 1. *Suppose that $r^* > 0$; the choosier (higher μ) consumers are more likely to start by searching at a broad retailer.*

Proof. See the Appendix for the proof of the lemma and all other proofs. □

Lemma 2. *Consumers prefer visiting broad stores when stores are equally priced (that, is $p_N^* = p_B^*$) if the economies of scale from searching at a broad store are sufficiently large; that is, γ is small enough.*

[14] Note that at the same prices, $\Delta = 0$, and so $min(b, \Delta) = \Delta$ and b does not affect $\beta(\mu)$.

[15] For the case that $r^* = 0$, trivially, $U_N = \mu E(\varepsilon) + u(p_N^*) - s$ and so $\beta(\mu) = \mu[E(max(\varepsilon_1, \varepsilon_2)) - E(\varepsilon)] - \min(b, \Delta^*) - \gamma s$.

[16] It is clear that this property also holds in case $r^* = 0$.

Intuitively, at equal prices, consumers prefer to visit broad stores unless γ is high. To see this clearly, note that if $\gamma = 0$ and $p_N^* = p_B^*$, consumers strictly prefer to visit broad stores since they are guaranteed their best match at no additional search cost. Clearly, if γ is high enough, no consumer will wish to visit broad stores. However, more generally, then the comparison between U_N and U_B depends on other parameters.

3.3 Summary of consumer behaviour

We can now summarise a consumer's first visit. A ('broad loyal') consumer who visits and buys from a broad retailer will satisfy

$$b \geq \Delta^* \text{ and } U_B \geq U_N.$$

Consumers showroom (go to broad stores but buy at narrow stores) when

$$b < \Delta^* \text{ and } U_B \geq U_N.$$

Consumers with

$$r^* > 0 \text{ and } U_N > U_B$$

visit narrow stores and may search if their first draw is below r^*. We call them 'searchers,' and, as highlighted, they play a crucial role in price determination. Finally, ('narrow loyal') consumers with

$$r^* \leq 0 \text{ and } U_N > U_B$$

make first visits to narrow stores but never search beyond the first one in equilibrium (but may search if the first firm deviates from p_N^*).

Fixing the other parameters, in (b, μ) space, the curve defined by $U_B = U_N$ for $b > \Delta^*$ is a flat line. For $b < \Delta^*$, because $U_B - U_N$ is increasing in μ, the curve is upward-sloping. Finally, some consumers never search when visiting narrow stores; for these, $r^* = 0$. Thus, the illustration in Figure 1, taken for particular parameter values, is more generally, representative. Throughout consumer problem analysis, and in this figure in particular, Δ^* is exogenously given. However, the particular level of Δ^* shown here will be an equilibrium price difference, as illustrated in Section 4.3.

Naturally, choosy consumers who are very sensitive to match values—that is, those with high μ—visit broad retailers to discover $(\varepsilon_1, \varepsilon_2)$. Among these consumers, those with low visit cost b proceed to narrow retailers for actual purchases, effectively using broad retailers as showrooms. However, those with high b purchase from broad retailers because the price difference is not worth the extra cost associated with an additional visit.

This concludes the analysis of first visits. Second visits are simple to describe, though they depend on the actual encountered prices, which may differ from the anticipated equilibrium prices (when these differ, we denote such prices without a * to distinguish them from the equilibrium p_N^* and p_B^*). For those whose first visits are to broad stores, the decision to visit

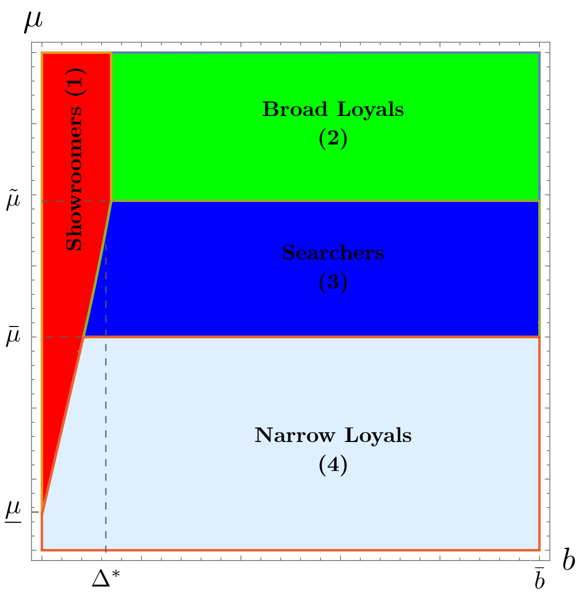

Figure 1: Consumer types for $G(\cdot) \sim U(0,1)$, $\Delta^* = 0.035$, $s = \bar{b} = 0.25$, $\gamma = 0.25$. In the red area (1), consumers showroom. In the green area (2), consumers visit a broad retailer and purchase there. In the two blue areas ((3) and (4)), consumers visit narrow retailers and buy from them. In the light-blue area (4), consumers do not search beyond the first retailer, whereas, in the dark-blue area (3), they do.

a narrow retailer depends on whether $\Delta(p_N^*, p_B)$ (using actual, not anticipated, p_B) is above or below b.[17] For example, if $\varepsilon_1 > \varepsilon_2$ and $b < \Delta(p_N^*, p_B)$, then this consumer will make the second trip to a narrow retailer selling good 1, and she will purchase there provided that p_N charged by that retailer satisfies $p_N \leq p_B$ and $\Delta(p_N^*, p_N) < b$.[18] Note that narrow retailers are able to hold up such showroomers because once they have arrived, only a drastic upward price deviation can result in losing such a consumer. Consumers who visit narrow stores never consider visiting broad stores for second inspections or visits ($p_N^* < p_B^*$). They stop at the first firm if $\varepsilon_1 > r(p)$ and search otherwise. They purchase the good they have inspected that gives the highest utility. We summarise this discussion as follows.

Lemma 3. *A consumer with* (μ, s, γ, b) *conducts the following optimal search:*

1. *If* $U_B \geq U_N$*, the consumer makes her first visit to a broad store. She discovers her best variety* $i \in \arg\max_{k\in\{1,2\}} \varepsilon_k$*. She purchases good* $l \in \arg\max_{k\in\{1,2\}} \mu\varepsilon_k + u(p_B^k)$ *from the broad store if* $\mu\varepsilon_l + u(p_B^l) \geq \mu\varepsilon_i + u(p_N^*) - b$*; otherwise, she visits a narrow store selling good* i *and buys there unless* $\Delta(p_N^i, p_N^*) > b$*, in which case she keeps going to narrow stores selling* i *until* $\Delta(p_N^i, p_N^*) \leq b$*, in which case she stops and buys, or she runs out of narrow stores selling good* i *and then recalls the best offer seen.*

2. *If* $U_B < U_N$*, then the consumer makes her first visit to a narrow store. She stops and*

[17] Of course, which of the narrow stores to visit depends on whether ε_1 is more or less than ε_2.

[18] Visiting a narrow store is more attractive than visiting another broad store since $p_N^* \leq p_B^*$. Due to the free recall assumption, a consumer can go back to a broad retailer charging p_B at no cost; thus, the comparison is between p_N and p_B in that case. If p_N is too high, the consumer may opt to pay b and go to another narrow retailer selling good 1 at the equilibrium price p_N^*. This possibility explains the need for the second condition. We draw on our passive beliefs assumption throughout this discussion.

buys if $\varepsilon_i \geq r(p_N^i)$, or else she searches narrow stores selling good $j \neq i$ and then buys the best offer seen.

4 Retailer pricing and equilibrium

Retailers' equilibrium pricing and consumers' equilibrium choices of which kinds of stores to visit interact. The analysis in Section 3 is based on expectations of retailers' pricing decisions. We now consider the pricing decisions of different kinds of retailers who anticipate the consumer behaviour described above. As illustrated in Figure 1, for a given set of prices, different kinds of consumers typically engage in different kinds of search behaviour. Consequently, to address a firm's pricing problem, we must specify the numbers of each kind of consumer since they affect the elasticity of demand and, thus, the pricing decisions of each kind of retailer.

As described above, in addition to the endogenous prices, consumer behaviour depends on several exogenous parameters: a consumer's choosiness μ; visit cost b; inspection cost s; the inspection economy associated with a broad retailer γ; and the distribution of matches $G(\cdot)$ which is log-concave and twice continuously differentiable. Clearly, allowing all of these to vary with no restrictions on their distributions would be demanding. Instead, we impose some additional structure. Specifically, we assume that s and γ and $G(\cdot)$ are common to all consumers. The remaining parameters are μ and b. We suppose that μ follows a binary distribution, where it takes a value μ_H with probability $1-\lambda$ and μ_L with probability λ, with $\mu_H > \mu_L > 0$; finally, $F_T(\cdot)$ is the CDF of $b \in [\underline{b}, \bar{b}]$ for $T \in \{H, L\}$, and we assume that $1 - F_T(b)$ is log concave, and $f_T(0) = 0$. It is convenient to assume that the high types' visit cost distribution has a weakly higher hazard rate than the distribution of the low types: $\frac{f_H(b)}{1-F_H(b)} \geq \frac{f_L(b)}{1-F_L(b)}$. This assumption is used in Section 5, where we introduce a price-only sector.

We consider consumer heterogeneity in pickiness in order to study equilibria in which showrooming and search occur, and all stores makes sales. For this to be the case, we need some consumers to be broad loyals, some to be searchers and some to be showroomers. An inspection of Figure 1 makes it clear that in order to have all three types of consumers, one needs at least two levels of pickiness μ; thus, the assumption on μ is the minimum necessary for this type of equilibrium to emerge. Moreover, Figure 1 also highlights the need for some heterogeneity in b. Allowing one of these (in our case, b) to be smoothly distributed allows us to characterise prices through a first-order condition.

To simplify analysis, we will assume that the search economy associated with broad retailers is strong enough, or, equivalently, that γ is sufficiently low. Namely, define $\bar{\gamma} \equiv \frac{E[\max(\varepsilon_1, \varepsilon_2)]}{E[\varepsilon]} - 1 > 0$.[19] From now on we will assume the following.

Assumption 1. *Search economies at broad stores are sufficiently strong; that is, $\gamma < \bar{\gamma}$.*

This assumption ensures that even consumers who are borderline searchers at narrow stores still strictly prefer to go to broad stores given equal prices; that is, consumers with $r^* = 0$

[19] For $\varepsilon \sim U(0,1)$, this reduces to $\gamma < \frac{1}{3}$.

($\mu = \frac{E[\varepsilon]}{s}$) have $\beta > 0$. We discuss the importance of this below. In Appendix B, we consider the case in which this assumption fails.

We will focus on symmetric equilibria as described above. There are three possible types of such equilibria: (i) all directed first visits occur at narrow-range retailers; (ii) all such visits occur to broad-range retailers; and, finally, (iii) these visits are split between the two types of retailers. To reduce the number of equilibria to analyse and rule out unnatural cases, we assume that a firm that anticipates no visits charges p^m for any products it sells. One can justify this assumption by trembling-hand-type arguments or by the presence of a small number of loyal consumers who purchase only from a given firm. Even among these more-reasonable equilibria, the consumer model can lead to many possibilities.

To understand these possibilities, it is useful to define $\underline{\mu}$ as the solution to $\beta(\mu) = 0$—that is, the level of pickiness that would make a consumer indifferent between starting at a broad or a narrow store, absent price differences, or where the inspection efficiency is exactly 0. Given the monotonicity of the inspection efficiency, β, in pickiness, μ, as described in Lemma 1, and the fact that at $\mu = 0$, $\beta \leq 0$, this solution exists and is unique. Further, define $\bar{\mu}$ as the solution to $r^* = 0$: the (lowest) level of pickiness that ensures that if the consumer searches through narrow stores, she will never move on to a second store, regardless of the match. By the monotonicity of r^* in μ and the fact that at $\mu = 0$ gains from search are zero, this solution also exists and is unique. By Assumption 1, we can rank $\bar{\mu} > \underline{\mu}$. Indeed, this is the simplification that the assumption affords.[20]

For what follows, we note that for $\mu < \underline{\mu}$, we have $\beta, r^* < 0$; if $\mu \in [\underline{\mu}, \bar{\mu})$, then $r^* < 0 \leq \beta$; and if $\mu \geq \bar{\mu}$, then $\beta, r^* \geq 0$. When μ is low, a consumer is not willing to search among narrow stores but would buy from a narrow store, regardless of the match (reflecting that $r^* < 0$), and would rather patronise narrow stores at equal prices (that is, $\beta < 0$). When μ is intermediate, the consumer is unwilling to search among narrow stores ($r^* < 0$) but would rather patronise broad stores at equal prices ($\beta > 0$). Finally, when μ is high, she would search through narrow stores ($r^* > 0$) but still prefers broad stores to narrow ones ($\beta > 0$).

Before analysing these cases, it is useful to introduce some additional notation. First, corresponding to the definition of r^*in Section 3 above, we define r^*_L as the reservation utility for consumers with μ_L, and r^*_H for those with μ_H. Similarly, we will denote β_H as β evaluated at μ_H and β_L as β evaluated at μ_L.

Lemma 4. *In any symmetric equilibrium, $p^*_B \geq p^*_N$. Furthermore, if $p^*_B = p^*_N$, then $p^*_B = p^*_N = p^m$.*

The lemma claims that in all symmetric equilibria, broad stores are at least as expensive as narrow stores, and if stores charge equal prices, then all stores charge monopoly prices. The reason for this lies in Diamond-like reasoning, whereby consumers leave broad stores only for lower prices, and, thus, if no such prices are to be found, then broad stores will be charging monopoly prices.

[20] To see this, let us evaluate β at $\mu = \bar{\mu}$; substituting $r^* = 0$ yields $\beta = s\left[\frac{E[\max(\varepsilon_1,\varepsilon_2)]}{E[\varepsilon]} - 1 - \gamma\right]$, which given Assumption 1, implies that $\beta > 0$, and, thus, conclude that $\bar{\mu} > \underline{\mu}$.

Given the above lemma, we are left with two possible pricing configurations. First, all store types charge monopoly prices. Second, narrow stores charge lower prices than broad stores, in which case consumers ought to search and showroom. We will consider these types of equilibria in turn.

4.1 Equilibria in which all firms charge the monopoly price

In this section, we demonstrate that there always exists an equilibrium in which all firms charge the monopoly price. Such equilibria involve consumers anticipating that they will visit a single store and all retailers charging the monopoly price. Moreover, we outline that such an outcome may (but need not to) arise as a unique symmetric equilibrium. However, depending on parameters—specifically how choosy the more and less choosy consumer-types are—this outcome might depend on different consumer behaviours, with all visiting broad stores, all visiting a single narrow store, or the more choosy visiting broad stores and the less choosy visiting a single narrow store.

Proposition 1. *There always exists a symmetric equilibrium in which all stores charge p^m. If $\mu_L < \bar{\mu}$, such a symmetric equilibrium is unique. In such equilibria, consumer shopping behaviour is the following:*

1. *If $\mu_H < \underline{\mu}$, then all consumers visit narrow stores and buy without searching. Broad stores earn $\Pi_B^* = 0$, while narrow stores earn $\Pi_N^* = \frac{1}{2}\pi^m$.*
2. *If $\mu_H \geq \underline{\mu} > \mu_L$, then all high types visit and buy from broad stores, and all low types visit and buy from narrow stores. Broad stores earn $\Pi_B^* = (1-\lambda)\pi^m$, while narrow stores earn $\Pi_N^* = \frac{1}{2}\lambda\pi^m$.*
3. *If $\mu_L \geq \underline{\mu}$, then all consumers visit and buy from broad stores. Broad stores earn $\Pi_B^* = \pi^m$, while narrow stores earn $\Pi_N^* = 0$.*[21]

When all firms charge the monopoly price, if even the less choosy consumers are sufficiently choosy, $\mu_L \geq \underline{\mu}$, they value learning both match realisations before purchasing one of the goods; in this case, all consumers begin by visiting broad stores. Consequently, since narrow stores expect only showroomers whom they can hold up, all stores end up charging monopoly prices.

When even picky consumers are so unfussy that they prefer to visit a single narrow store, regardless of the match value, $\mu_H < \underline{\mu}$, then, in equilibrium, all firms charge monopoly prices, but only narrow stores receive consumers.

Finally, when picky consumers are picky enough, and less-picky consumers are not, consumers with different choosiness visit different kinds of stores, but again, in equilibrium, consumers will not visit more than one retailer (because the less-choosy are insufficiently choosy

[21] Here, and elsewhere in the paper, equilibrium profits stated are for all firms of a given type; for example, in case 2, the total profits of all broad stores are qual to $(1-\lambda)\pi^m$, and the total profits of all narrow stores of each of the two types are equal to $\frac{1}{2}\lambda\pi^m$.

and visit a single narrow retailer, while the more-choosy are sufficiently choosy and visit a broad retailer, $\mu_H \geq \underline{\mu} > \mu_L$), and so all consumers end up paying the monopoly price.

Note, however, that Proposition 1 affords the possibility that when $\mu_L \geq \bar{\mu}$, there may exist other equilibria. We examine this possibility next.

4.2 Equilibrium with search and showrooming

Now we turn to the most interesting type of equilibrium, in which first visits are split between the retailer types and prices are below the monopoly level. This equilibrium does not always exist, and we characterise when it does.

In any such equilibrium, it must be the case that broad stores retain some consumers, or else they would choose to deviate to a lower p_B. Consequently, in equilibrium, it must be that the inspection benefit associated with broad stores for those who most value them (the picky consumers) must be higher than the utility costs associated with anticipated price differences; that is,

$$\Delta^* \leq \beta_H.$$

Similarly, it must be that some consumers (the less-picky) prefer to start at narrow rather than at broad retailers; that is, equilibrium requires that

$$\Delta^* > \beta_L,$$

or else no consumers will make first visits to narrow stores. Note that this is a necessary condition but does not imply that all of the less-picky start by visiting narrow stores. Those with low visit costs prefer to visit a broad retailer and to showroom rather than inspect at narrow stores.

Finally, as outlined in Proposition 1, for non-monopoly price equilibrium to arise, low types must be sufficiently picky that they search through narrow stores,

$$\mu_L \geq \bar{\mu}.$$

We assume that these three conditions hold and check the parameter configurations that deliver them once prices are characterised. In such cases, all high types and low types with $b < \beta_L$ first visit broad stores. Around the equilibrium price (that is, for local deviations) p_B^*, only high types react to p_B (recall that $\Delta^* > \beta_L$) since low types strictly prefer to showroom. Thus, broad stores set p_B in order to maximise profits from high types, with high types staying at broad stores when $b \geq \Delta(p_N^*, p_B)$.[22]

In order to characterise broad store pricing, we introduce the notation $h(p) \equiv -\frac{\pi(p)u'(p)}{\pi'(p)}$. This allows us to write the following, whose form should be familiar, as in a standard monopoly pricing problem.

[22] See details on the broad store maximisation problem in the proof of Lemma 5.

Lemma 5. *The first-order condition for a broad retailer's pricing problem is given by*

$$h(p_B^*) = \frac{1 - F_H(\Delta^*)}{f_H(\Delta^*)}. \tag{1}$$

Consumers leave broad stores only in order to claim a lower price at a narrow store. Because consumers find it costly to leave, broad stores are able to charge a positive price premium Δ^*. The higher this premium, the lower is the fraction of consumers that stay, and the trade-off between higher price and lower demand is resolved in the first-order condition shown above. Note that only high types may buy from a broad store, and only high types are marginal consumers, which is why (4) does not depend on F_L. To see this clearly, recall that among low types, only those with $b < \beta_L$ showroom, and, by construction, $\Delta^* > \beta_L$, so even the low type consumer with the highest visit cost strictly prefers to leave rather than to stay.[23]

4.2.1 Pricing for narrow stores

Pricing for narrow stores is similar to the standard search model analysis, with the exception that in addition to the standard demand from consumers who search through narrow stores to find a suitable match in the manner of Wolinksy (1986) or Anderson and Renault (1999), such stores also receive demand from two different kinds of showroomers. First, there are choosy (μ_H) consumers with low visit costs (below the price differential, $b < \Delta^*$). Second, there are less choosy (μ_L) consumers with low visit costs (below the benefit associated with visiting a broad store at equal prices $b < \beta_L$, which also ensures that these costs are below the price differential of the different kinds of stores $b < \Delta^*$). These two groups arrive at a narrow store for the good with which they found themselves to be better matched. Then, even if observing an off-equilibrium price, they continue to purchase the good unless the price deviation is more than b, which, given our assumption that for $T \in \{H, L\}$, $f_T(0) = 0$ means that local deviations do not affect the demand of either kind of showroomer.

Thus, the only kind of consumers who are price-sensitive are the less-choosy L types with high enough visit costs that they prefer not to showroom, which is the case when $b > \beta_L$. The total mass of such consumers is $1 - F_L(\beta_L)$. Among the consumers arriving at a narrow retailer selling good i at price p_N, those with $\varepsilon_i < r_L^* + \frac{\Delta(p_N^*, p_N)}{\mu_L}$ will search another narrow retailer selling the other good, and the rest will buy from the first narrow retailer. Of the consumers who search, those discovering a low enough match with the second good (specifically $\varepsilon_j < \varepsilon_i + \frac{\Delta(p_N^*, p_N)}{\mu_L}$) will come back and buy. Furthermore, there will be consumers who arrive at other narrow retailers, discover match values below r_L^* for the other good, and visit this narrow retailer. Of these consumers, those with $\varepsilon_j < \varepsilon_i - \frac{\Delta(p_N^*, p_N)}{\mu_L}$ will also buy.[24]

[23] While F_L plays no role for p_B^*, because some low type consumers showroom, it is important to check whether broad stores want to make a large deviation with the hope of retaining such consumers. This is analysed formally in the next section and is captured in Condition 1.

[24] In principle, if $p_N > p_N^*$ some consumers (of those who come back or do not search) may go to another narrow retailer selling i if their showrooming cost satisfies $b < \Delta(p_N^*, p_N)$, but given that only consumers with

The fraction of less-choosy (type L) consumers who start at narrow stores and end up purchasing at a narrow store that charges p_N is, therefore,[25]

$$s_L(p_N) = \frac{1}{2}\left[1 - G\left(r_L^* + \frac{\Delta(p_N^*, p_N)}{\mu_L}\right)\right](1 + G(r_L^*))$$
$$+ \int_0^{r_L^* + \frac{\Delta(p_N^*, p_N)}{\mu_L}} g(\varepsilon_i) G\left(\varepsilon_i - \frac{\Delta(p_N^*, p_N)}{\mu_L}\right) d\varepsilon_i.$$

Note that we normalised in such a way that $s_L(p_N^*) = \frac{1}{2}$ (that is, in equilibrium, a store attracts half of those who start searching at narrow stores); hence, in equilibrium, the resulting profit of the narrow store will be equal to the profit of all narrow stores of its kind.

The total profit of a narrow retailer in the neighbourhood of p_N^* is given by:[26]

$$\Pi_N(p_N) = \left[\frac{(1-\lambda)}{2} F_H(\Delta^*) + \frac{\lambda}{2} F_L(\beta_L) + \lambda(1 - F_L(\beta_L)) s_L(p_N)\right] \pi(p_N),$$

where, inside the square brackets, we account for high-type showroomers with the first term, low-type showroomers with the second, and the share of searching low types that the narrow retailer retains with the third. For each of these consumers who purchases, the store earns a profit of $\pi(p_N)$.

In order to proceed, define

$$z_L \equiv \frac{s_L^{'}(p_N^*)}{u'(p_N^*)} = \frac{1}{\mu_L}\left(\frac{1}{2} g\left(r_L^*\right)\left(1 - G\left(r_L^*\right)\right) + \int_0^{r_L^*} g^2(\varepsilon) d\varepsilon\right)$$

as the derivative of $s_L(p_N)$ evaluated at the equilibrium price divided by $u'(p_N^*)$.[27] In the standard search model of Anderson and Renault (1990), with unit demand, z_L is the (negative of the) derivative of demand, so that the equilibrium price is $\frac{1}{2z_L}$.

The following assumption, while not necessary, simplifies further exposition of the showrooming equilibrium.

Assumption 2. *Assume that $F_H(\cdot)$,$F_L(\cdot)$,$G(\cdot)$, and $u(\cdot)$ are such that $\Pi_N(p_N)$ is quasi-concave and z_L is increasing in μ_L.*

It is not trivial to find direct conditions on G, $F_T(\cdot)$ and u that are necessary to ensure the above, but simple verification suffices to show that it holds if G and$F_T(\cdot)$ are both uniform, for example.

We are now ready to write the first-order condition for a narrow retailer.

$b > \beta_L$ have arrived, for small price deviations, there will be no such consumers.

[25]This is different from total demand because each such consumer purchases $q(p_N)$ units; moreover, there are showroomers who do not start at narrow stores.

[26]In this expression, we ignore consumers with b close to 0 who, in case $p_N > p_N^*$, would leave and buy at another narrow retailer of the same type because at $p_N = p_N^*$, the mass of such consumers is zero. Accounting for these consumers only reduces the narrow store's incentive to deviate upward.

[27]We will occasionally need the notations z_H and $s_H(p_N)$, which are the corresponding expressions when choosy consumers are searching through narrow retailers.

Lemma 6. *The first-order condition for a narrow retailer's pricing problem is given by*

$$h(p_N^*) = \left[\frac{1 + \frac{1-\lambda}{\lambda} F_H(\Delta^*)}{1 - F_L(\beta_L)}\right] \frac{1}{2z_L}. \tag{2}$$

It is immediate that the equilibrium price charged by narrow stores is higher than the price that would have been charged in the absence of showroomers. This is because of the choosy, H-type showroomers ($\frac{1-\lambda}{\lambda} F_H(\Delta^*)$ term in the numerator) and the less-choosy L-type showroomers ($(1 - F_L(\beta_L))$ term in the denominator).

4.2.2 Equilibrium

For an equilibrium of this type to exist, p_N^* and p_B^* have to simultaneously solve (1) and (2), while, at the same time, $\beta_H \geq \Delta^* > \beta_L$ and $r_L^* > 0$ should hold since if these fail, then consumer behaviour assumed in deriving (1) and (2) will not be correct.

Lemma 7. *There exists a unique solution to the system (1) and (2).*

Let (p_N^*, Δ^*) denote the solution to (1) and (2), which Lemma 7 establishes as well-defined. Note that Δ^* is a function of μ_L. By Assumption 2, we have that Δ^* is decreasing in μ_L because (1) does not depend on μ_L and, by Assumption 2 and the monotonicity of β_L in μ_L, equation (2) shifts upward in (Δ, p_N) space.

Further, let $\tilde{\mu}$ be the μ_L that solves $\beta(\mu_L) = \Delta^*(\mu_L)$.[28] Note that $\beta(\mu_L)$ is increasing in μ_L and goes from weakly negative to infinity, whereas, by Assumption 2, $\Delta^*(\mu_L)$ is decreasing in μ_L and starts at $\bar{b}$; therefore, a unique solution must exist. The role of $\tilde{\mu}$ turns out to be the following. If $\tilde{\mu} < \mu_L$, then there can be no showrooming equilibrium because, for such μ_L,we have $\Delta^* < \beta(\mu_L)$, and so there would be no searchers in equilibrium who would visit broad stores instead. Furthermore, we need μ_L to be above $\bar{\mu}$, or else $r_L^* \leq 0$, and so no L type consumer would search through narrow stores. In general, $\tilde{\mu}$ and $\bar{\mu}$ are not readily ordered; thus, the existence of the showrooming equilibrium depends on the primitives of the model beyond μ_L.

We are almost ready to characterise the search and showrooming equilibrium. In order to do so, we need to revisit the broad store pricing problem. In case of a sufficiently large (rather than local) downward price deviation, such that $\Delta(p_N^*, p_B) < \beta_L$, broad stores are able to retain low-type showroomers, if any. These are not taken into account in the first-order condition. The following condition rules out the optimality of such a strategy, using notation $\hat{p}^*$ as the implicit solution to $\Delta(p_N^*, \hat{p}^*) = \beta_L$.

[28] In more detail: $\tilde{\mu}$ alongside $\tilde{p}$ solve a system consisting of $h(\tilde{p}) = \left[\frac{1+\frac{1-\lambda}{\lambda}F_H(\beta_L)}{1-F_L(\beta_L)}\right]\frac{1}{2z_L}$ and $h(u^{-1}(u(\tilde{p})-\beta_L)) = \frac{1-F_H(\beta_L)}{f_H(\beta_L)}$.

Condition 1.

$$\max_{p_B\in[0,\hat{p}^*]} \left[(1-\lambda)(1-F_H(\Delta(p_N^*,p_B))) + \lambda(F_L(\beta_L) - F_L(\Delta(p_N^*,p_B)))\right]\pi(p_B)$$
$$\leq (1-\lambda)(1-F_H(\Delta^*))\pi(p_B^*).$$

This condition is trivially satisfied when $F_L(\cdot)$ has full mass above β_L or if the lower bound for the low-type visit costs is sufficiently high that there are no such low-type showroomers. However, the condition can also fail. For example, it necessarily fails when $F_L(\beta_L) = 1$ and $\lambda \to 1$, so that almost all consumers are low-type showroomers. In that case, broad stores receive negligible demand in equilibrium, and by deviating to a lower price so that $\Delta(p_N^*, p_B) < \beta_L$, they increase their demand infinitely.

We obtain the following result.

Proposition 2. *If $\tilde{\mu} > \mu_L > \bar{\mu}$, $\mu_H > \beta^{-1}(\Delta^*(\mu_L))$ and Condition 1 hold, then there is an equilibrium in which p_N^* and p_B^* jointly solve (1) and (2); all type H and type L consumers with $b < \beta_L$ make first visits to broad stores; of these consumers, those with $b < \Delta(p_N^*, p_B)$ showroom. The remaining L consumers visit narrow stores and conduct optimal sequential search, as described in Lemma 2. The remaining H consumers buy from broad stores. Broad stores earn $\Pi_B^* = (1-\lambda)(1-F_H(\Delta^*))\pi(p_B^*)$, and narrow stores earn $\Pi_N^* = \frac{1}{2}\left(\lambda + (1-\lambda)F_H(\Delta^*)\right)\pi(p_N^*)$.*

As noted above, in general, $\tilde{\mu}$ and $\bar{\mu}$ cannot be ranked, and so there is no guarantee that an equilibrium of this form exists. However, it is a simple matter to verify that there are parameters that allow such an outcome. Figure 2 illustrates an equilibrium of this sort. The H types visit broad stores first, and some then showroom. Some of the L types also visit broad stores first, but all of them showroom. The rest of the L types visit narrow stores for the first visit— e.g., that sells good i—and search a store that sells j if $\varepsilon_i < r_L^*$.

It is worth noting that both Condition 1 and $\tilde{\mu} > \mu_L$ are more likely to hold when F_L shifts to the right (in the FOSD sense). That is, search and showrooming equilibrium is more likely to hold when L types are less prone to showrooming. There are two reasons for this. First, Condition 1 is more likely to hold when there are fewer low-type showroomers who may tempt broad stores into deviating downwards to retain them. Second, the fewer low-type showroomers there are, the lower are narrow prices, which, in turn, means that higher levels of μ_L are still consistent with low types preferring to search narrow stores over buying at broad stores.

The equilibrium in Proposition 2 has several interesting properties that can be immediately derived.

Corollary 1. *Assume that the conditions in Proposition 2 hold. Then, within an equilibrium of that form:*

(i) Broad stores charge higher prices than narrow stores but lower prices than monopoly prices, $p^m > p_B^* > p_N^*$.

(ii) An increase in μ_H has no effect on prices; an increase in μ_L increases prices.

(iii) An increase in s increases all prices.

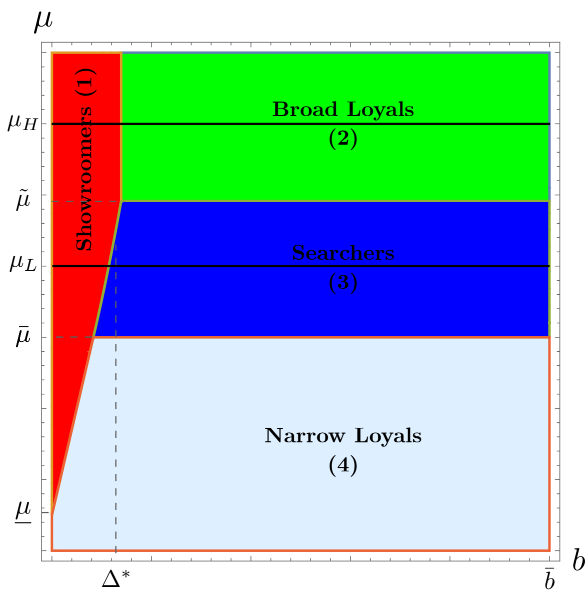

Figure 2: Consumer types for $G(\cdot) \sim U(0,1)$, $\mu_L = 0.55$, $\mu_H = 0.65$, $F_L(b) = (b/s)^6$, $F_H(b) = (b/s)^2$, $\Delta^* = 0.035$, $s = \bar{b} = 0.25$, $\gamma = 0.25$ and $\lambda = 0.5$.

(iv) An increase in λ reduces all prices.

(v) An increase in γ leads to a reduction in all prices.

Note that these properties are mostly intuitive. It is perhaps worth highlighting, as in (ii), that increasing the choosiness of the more-choosy has no effect on prices; and that in (v), making the search process more efficient by increasing the search efficiency of broad stores (reducing γ) actually increases prices. This follows, as reducing γ makes it more attractive for the less picky to visit broad stores and showroom. In turn, this increases the fraction of narrow stores consumers who are showroomers rather than searchers visit, and so leads to higher prices.

Consequently, improving the efficiency of search at broad retailers (decreasing γ) has a potentially ambiguous impact on welfare: There is a direct saving in inspection costs for those who visit such stores, but Corollary 1 highlights that prices increase for all consumers. However, around the parameter ranges we use for illustration and in other parameterisations that we have explored, the direct effect dominates.

4.3 Equilibrium configurations: An illustration

Figure 2 illustrates the ranges for μ_H and μ_L for which various equilibrium configurations arise when $G \sim U(0,1)$, $q(p) = 1 - p$, $F_L(b) = (\frac{b}{s})^6$, $F_H(b) = (\frac{b}{s})^2$,$s = 0.25$, $\gamma = 0.25$. In the red triangle, neither type is picky enough ($\mu_H < \underline{\mu}$), and so all consumers visit narrow stores that charge monopoly prices (Proposition 1). In the green region, high types are picky and low types are not ($\mu_H \geq \underline{\mu} > \mu_L$), so high types visit broad stores, and low types visit narrow stores, but since no one searches ($\bar{\mu} > \mu_L$), all prices are at the monopoly level. In the blue region, low types are picky but not too picky ($\mu_L > \underline{\mu}$), so that there exists an equilibrium in which all visits are to broad stores and prices are at monopoly levels. The orange region is where the previous type of equilibrium co-exists with the hybrid equilibrium of Proposition 2. Here, high

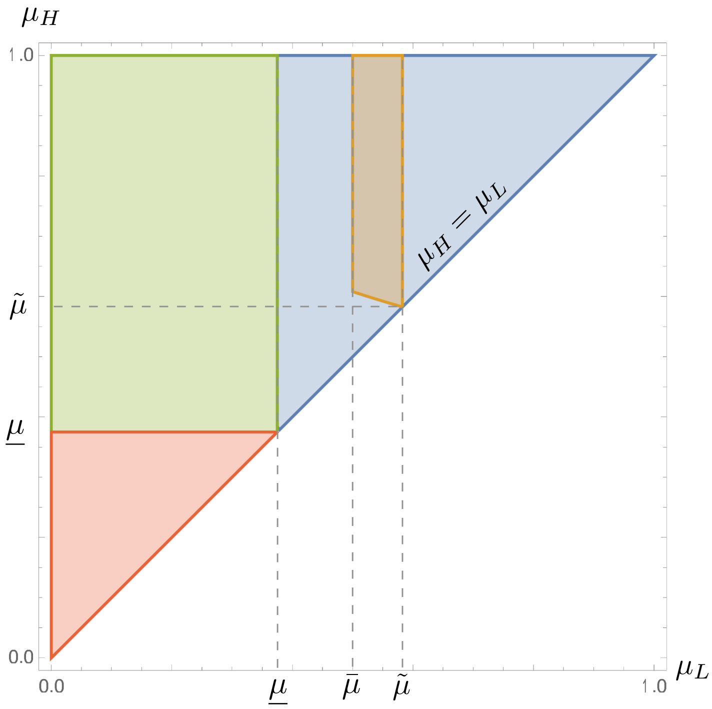

Figure 3: Equilibrium regimes depending on μ_L and μ_H, where $G \sim U(0,1)$, $q(p) = 1 - p$, $F_L(b) = (\frac{b}{s})^6$, $F_H(b) = (\frac{b}{s})^2$,$\lambda = 0.5$, $s = \bar{b} = 0.25$, and $\gamma = 0.25$. In the red region, all visits occur to narrow stores. In the green region, high types visit broad stores and low types visit narrow. In the blue region, there is an equilibrium in which all visits are to broad stores. In all of these, all store types charge monopoly prices. In the orange region, there is an additional equilibrium of the type described in Proposition 2.

types are picky, $\mu_H \geq \beta^{-1}(\Delta^*(\mu_L))$, and low types are picky enough, $\mu_L > \bar{\mu}$, but not too picky. So, $\tilde{\mu} > \mu_L$, and equilibrium prices are below monopoly levels, and there is active search and showrooming. Note that $\beta^{-1}(\Delta^*(\mu_L))$ (the bottom of orange area) is decreasing in μ_L because $\beta(\cdot)$ is monotone, and $\Delta^*(\mu_L)$ is decreasing. Further, for $\mu_L = \tilde{\mu}$, by the definition of $\tilde{\mu}$ we have $\Delta^*(\mu_L) = \beta_L$, so the condition $\mu_H \geq \beta^{-1}(\Delta^*(\mu_L))$ becomes $\mu_H \geq \tilde{\mu}$.

Inspecting Propositions 1 and 2, it is clear that the configuration above—that is, the order in which different equilibria arise as μ_H and $\mu_L$vary, and the thresholds for these changes—is quite general, with the important proviso that if $\bar{\mu} > \tilde{\mu}$ there is no equilibrium of the form characterised in Proposition 2 (the orange region in the graph).

4.4 Retailer variety

In this section, we consider the implications of the disappearance of each kind of retailer.

4.4.1 No narrow stores

First, if narrow stores vanish so that all stores are broad, then for any level of μ_L and μ_H, there is a unique equilibrium with monopoly prices in the style of Diamond (1971). This is because no consumer has an incentive to search to learn about varieties because all stores have identical offerings; moreover, no consumer is initially attracted by a lower price (since they are unaware of it) and has no reason to search elsewhere for lower prices.

Returning to our general characterisation, prices are the same in all regions other than the case analysed in Proposition 2. However, even if we ignore this possibility, the presence of narrow stores alongside broad ones alters welfare. In the region where case (1) of Proposition 1 applies (the red region in Figure 2), all consumers are so insensitive to matches (μ is so low) that they prefer to visit narrow stores. Consequently, if there are no such stores, consumers are worse off (since we require consumers to inspect all varieties at a broad store). In the (green) region corresponding to case (2), the less-choosy types visit narrow stores, so, again, their absence is detrimental to consumer surplus and, thus, welfare. In the (blue) region corresponding to case (3), where all consumers are choosy and visit broad stores, welfare is the same regardless of narrow stores' presence.

In the case of Proposition 2, where both types of retailers are active and there is search and showrooming (the orange region that overlaps with the blue), the disappearance of narrow stores causes welfare losses due to both higher prices and higher inspection costs for those consumers wishing to search narrow stores.

Proposition 3. *Removing narrow stores (weakly) reduces consumer surplus and welfare and (weakly) increases prices and total profits.*

4.4.2 No broad stores

Next, we consider what happens if broad stores vanish.[29] Nothing changes if consumers are so insensitive that they shop at a single narrow store (Proposition 1, case (1), corresponding to the red in Figure 2). In the region where Proposition 1, case (2), applies (the green region in Figure 2), with or without broad stores, there is no search and, therefore, monopoly prices, but high-type consumers are worse off because they would rather visit broad stores and obtain a better match. In the region corresponding to Proposition 1, case (3), where all consumers prefer to search broad stores, an exodus of broad stores leads to lower prices (equal to $h^{-1}\left(\frac{1}{2(\lambda z_L+(1-\lambda)z_H)}\right)$ when $\mu_L > \bar{\mu}$ or $h^{-1}\left(\frac{1}{2(1-\lambda)z_H}\right)$ when $\mu_L < \bar{\mu}$), but consumers prefer to search at broad stores, so their utility may go down since it is more costly to inspect both goods. When μ_H is sufficiently high, no price reduction can compensate H types for their increase in inspection costs, and so consumer surplus and welfare must fall as broad stores exit. Instead, the price reduction resulting from broad stores' disappearance must dominate any inspection cost efficiency losses when μ_H and μ_L are close to $\underline{\mu}$, because, in this case, the inspection cost efficiencies for both types are small. Thus, both situations exist: the disappearance of broad stores can be good or bad for consumers and for welfare: which case prevails in the border cases depends on parameters and, as indicated above, crucially on how picky consumers are.

Finally, in the region corresponding to Proposition 2 (the orange region), if broad stores disappear, then there is a force for lower prices at narrow stores due to the disappearance of showrooming and the inelastic consumers that showrooming brings, but a force for higher prices

[29] In this case, the environment is similar to that in Anderson and Renault (1999), except for the downward-sloping demand assumption.

due to narrow stores facing more high-type consumers with lower demand elasticity entering the pool of searchers. In particular, prices at narrow retailers would shift to $h^{-1}\left(\frac{1}{2(\lambda z_L+(1-\lambda)z_H)}\right)$, which can be higher or lower than $p_N^* = h^{-1}\left(\left[\frac{1+\frac{1-\lambda}{\lambda}F_H(\Delta^*)}{1-F_L(\beta_L)}\right]\frac{1}{2z_L}\right)$ depending on a variety of parameters of the model. In particular, when μ_H and μ_L are sufficiently close to each other and to $\tilde{\mu}$ (note that μ_H and μ_L can be close to each other only in the vicinity of $\tilde{\mu}$), then the price must fall with the disappearance of broad stores, because, after the exit, the price is close to $h^{-1}\left(\frac{1}{2z_L}\right) < p_N^*$. Instead, when high-type consumers are sufficiently picky (when μ_H is sufficiently high, z_H is low), then the disappearance of broad stores, which turns them into searchers through narrow stores (rather than having them buy at broad stores, say), means that the narrow stores now face a more inelastic demand.

In addition to the ambiguous impact on prices that consumers pay, consumer surplus may rise or fall. Even if consumers face lower prices, they may be worse off due to higher inspection costs associated with the disappearance of broad retailers.

Proposition 4. *Removing broad stores can raise or lower prices, consumer surplus, welfare, and total profits; even if prices fall, consumer surplus may fall.*

5 Price-only sector

We extend the model by allowing for an alternative competitive retail channel. Specifically, in this retail sector, there is no opportunity to learn match quality; consumers must first learn this at either broad or narrow stores to purchase in this sector. Moreover, consumers can observe prices within this sector at no cost. We assume that the sector is competitive. Since consumers can observe prices, it is immediate that competition in prices between different firms in this sector leads all firms to price at cost, which we have normalised to 0. In Appendix C, we consider the role of market power in the competitive sector by considering the case of a monopoly price-only retailer in which similar effects to those described in this section arise.

Of course, if all consumers had access to this sector, then all would learn their match elsewhere but buy in this sector. Instead, we assume that not all consumers can access the price-only venues, or, equivalently, they may not be aware of this possibility. In particular, assume that a fraction θ_T of type $T \in \{L, H\}$ consumers do not consult prices in the price-only sector, whereas the rest have access and are able to purchase there. We will call consumers who have access 'savvy' and other consumers 'naive.' Savvy consumers can purchase in the price-only sector, and naive consumers cannot. Since savviness and pickiness are not assumed to be orthogonal, θ_L may be above, below or, indeed, equal to θ_H.

Trivially, since prices in the price-only sector are 0, all savvy consumers will buy there (and, trivially, picky consumers will showroom at broad stores, whereas sufficiently unpicky consumers might prefer to learn their match for only one good and showroom from a narrow store). Naive consumers cannot buy from the price-only sector, and their behaviour is characterised as in Section 3, given their expectations of prices at broad and narrow stores.

Broad and narrow stores cannot earn profits by matching or undercutting the price-only sector, so their behaviour will be similar to that characterised in Section 4, with the following proviso. In the overall population, there is a fraction λ of less-picky (μ_L) consumers out of the mass 1 of consumers; since only naive consumers are relevant in the presence of the price-only sector, out of this population that has mass $\lambda\theta_L + (1-\lambda)\theta_H$, a fraction $\hat{\lambda} \equiv \frac{\lambda\theta_L}{\lambda\theta_L+(1-\lambda)\theta_H}$ are less-picky. This can be higher or lower than the fraction in the overall population, depending on whether or not $\theta_H < \theta_L$. In the former case, the price-only sector attracts relatively more picky consumers, leaving relatively more of the less-picky to the broad and niche stores; and, in the latter case, when $\theta_H > \theta_L$, it will be relatively picky consumers who buy from broad and niche stores. In short, when $\theta_H \neq \theta_L$, the price-only sector disproportionately attracts either picky or less-picky consumers who might otherwise be showrooming at narrow stores or searching and buying from narrow stores. In this way, it affects the average elasticity of consumers at narrow stores and, therefore, prices.

Specifically, we can apply the characterisation of prices in Section 4 directly. The introduction of a price-only sector where none previously existed, as well as reducing the overall number of consumers who buy from broad and niche stores, has an effect similar to a change in λ and replacing λ by $\hat{\lambda}$. Applying our earlier results and, more specifically, Corollary 1, allows us to establish the following result immediately.

Proposition 5. *The introduction of a price-only sector, where both before and after its introduction, the equilibrium is as described in Proposition 2, leads to higher prices at both broad and niche stores if $\theta_L < \theta_H$ and, instead, leads to lower prices at both broad and niche stores if $\theta_H < \theta_L$.*

5.1 Welfare

While Proposition 5 characterises prices, the welfare associated with the introduction of a price-only sector must also incorporate the benefit that savvy consumers enjoy from the opportunity to purchase at a price of 0 from the price-only sector. Clearly, the introduction of this sector makes savvy consumers better off. If prices at both broad and niche stores fall, then it is immediate that naive consumers are also better off (and by more than the fall in the profits of narrow and broad stores as their prices come closer to costs); however, if prices at broad and narrow stores rise, then the overall impact is ambiguous, as the gains to savvy consumers must be traded off against the decline in surplus associated with naive purchases.

Of course, this analysis assumes that the introduction of the price-only sector has no impact on the existence of broad and narrow stores. The introduction of a price-only sector necessarily implies that a fraction of consumers (the savvy) will no longer purchase from broad and narrow stores; moreover if the price-only sector disproportionally attracts less-picky consumers (that is, $\theta_H > \theta_L$), prices will be lower at broad and narrow stores. This may further endanger their viability, and lead to similar consequences to those described in Section 4.4.

6 Summary and Conclusions

Overall, this paper illustrates some familiar themes. In particular, the elasticity of demand at given stores depends on the pattern of consumer search: an equilibrium phenomenon. A literature on multiproduct search highlights that this pattern depends on the product mix across all stores. We extend this insight to observe that, perhaps unsurprisingly, it also applies to the case of stores that offer substitute goods and have overlapping offerings. As a consequence, seemingly beneficial changes (such as improving the search efficiency at a broad retailer, or introducing a relatively low-cost alternative venue) can lead to higher prices by affecting consumers' search patterns.

More specifically, we highlight that key to price determination in our environment are consumers who might pass through more than one (narrow) store to learn their match with the products on offer. These consumers are necessarily somewhat picky (or else there would be no need to visit more than one store) but not too picky (or else visiting a broad retailer would be more attractive).

The endogenous determination of search patterns suggests that welfare effects can be subtle, and the impact of the introduction of a price-only sector depends on the way in which savviness and pickiness are correlated.

From an antitrust perspective, we further highlight that even if the disappearance of stores has no impact on prices, it may impact consumer welfare. It is worth highlighting that we have assumed that consumers always participate; relaxing this and taking it together with the observation in the previous sentence presents a rationale for manufacturers to seek to maintain retailer variety as a means of encouraging consumer participation (even with no impact on prices). Of course, such retailer variety can lead to lower prices and thereby encourage further consumer participation.

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Appendix A: Proofs

A1: Proof of Lemma 1.

Proof. It is sufficient to show that $\frac{\partial \beta(\mu,b)}{\partial \mu} > 0$.

$$\begin{aligned}\frac{\partial \beta(\mu,b)}{\partial \mu} &= \int_{r^*}^{1} (\varepsilon - r^*)\tilde{g}(\varepsilon)\, d\varepsilon - \mu(1 - G(r^*)^2)\frac{\partial r^*}{\partial \mu} \\ &= \int_{r^*}^{1} (\varepsilon - r^*)\tilde{g}(\varepsilon)\, d\varepsilon - \mu(1 - G(r^*)^2)\frac{\int_{r^*}^{1} (\varepsilon - r^*)g(\varepsilon)\, d\varepsilon}{\mu(1 - G(r^*))} \\ &= \int_{r^*}^{1} (\varepsilon - r^*)\tilde{g}(\varepsilon)\, d\varepsilon - (1 + G(r^*))\int_{r^*}^{1} (\varepsilon - r^*)g(\varepsilon)\, d\varepsilon \\ &= \int_{r^*}^{1} \varepsilon(2G(\varepsilon) - (1 + G(r^*)))g(\varepsilon)\, d\varepsilon \\ &= (1 - G(r^*)^2)\left[\int_{r^*}^{1} \varepsilon\frac{\tilde{g}(\varepsilon)}{1 - G(r^*)^2}\, d\varepsilon - \int_{r^*}^{1} \varepsilon\frac{g(\varepsilon)}{1 - G(r^*)}\, d\varepsilon\right] > 0\end{aligned}$$

where we used the definition of $\tilde{g}(\varepsilon)$ and $\frac{\partial r^*}{\partial \mu} = \frac{\int_{r^*}^{1} (\varepsilon - r^*)g(\varepsilon)\, d\varepsilon}{\mu(1-G(r^*))}$, which follows from the definition of r^*; that $\int_{r^*}^{1} (2G(\varepsilon) - (1 + G(r^*)))g(\varepsilon)\, d\varepsilon = 0$; and that $\int_{r^*}^{1} 2G(\varepsilon)g(\varepsilon)\, d\varepsilon = 1 - G(r^*)^2$. The last line above can be seen to be positive by noting that the first term in square brackets is the conditional mean of $\max(\varepsilon_1, \varepsilon_2)$ above r^*, and the second term is the conditional mean of ε_1 above r^*. □

A2: Proof of Lemma 2.

Proof. This is immediate on noting that for $\gamma = 0$ we have $U_B > U_N$, and for $\gamma = 1$ we have $U_B < U_N$, and that U_B is monotonically decreasing in γ. □

A3: Proof of Lemma 2.

Proof. Immediate from the discussion in the text. □

A4: Proof of Lemma 4.

Proof. Assume the contrary so that $p_B^* < p_N^*$. Any consumer who visits broad stores will not leave for narrow stores. Given that $f_T(0) = 0$ for $T = H, L$, a zero mass of consumers will leave a broad store that deviates to a higher price; thus, we must have $p_B^* = p^m$. This leads to a contradiction because $p_N^* > p^m$ cannot hold in any equilibrium because narrow stores would profitably deviate to a lower price. □

A5: Proof of Proposition 1.

Proof. If all stores charge (p^m, p^m), then no consumer who visits broad stores has any incentive to search, and no other firm can attract these consumers by lowering its prices. Firms never wish to increase prices above p^m. By Assumption 1, $\underline{\mu} < \bar{\mu}$, so that if a consumer type prefers to patronise narrow stores given equal prices, this type is not willing to search. This means that if consumers visit narrow stores expecting $p_N^* = p^m$, then narrow stores have no incentive to deviate. In any such equilibrium, since prices are equal, type $T \in \{H, L\}$ visits broad stores and buys there if $\mu_T \geq \underline{\mu}$. This proves the existence of symmetric equilibrium with monopoly prices, as well as the taxonomy of consumer behaviour stated.

For uniqueness, assume that $\mu_L < \bar{\mu}$ indeed holds. This condition is equivalent to $r_L^* < 0$. For contradiction, assume that there exists a symmetric equilibrium in which some firms charge sub-monopoly prices, since supra-monopoly prices can never be profitable. First, it must be that narrow stores charge sub-monopoly prices in such an equilibrium, because broad stores cannot be the only ones that do, due to Diamond-like reasoning.[30] For narrow stores to charge below monopoly prices, it must be that high types visit them, since, if only low types do, then $r_L^* < 0$ implies that no consumers would search, and prices at narrow stores will be at the monopoly level. Thus, it must that $\Delta^* \geq \beta_H$ so that at least some high types visit narrow stores. There are, then, two cases depending on μ_H. If $\mu_H > \underline{\mu}$ ($\beta_H > 0$), then some high types will showroom, and, in turn, $\Delta^* \geq \beta_H$ cannot be an equilibrium because broad stores will reduce p_B^* to retain some high types, a contradiction. If $\mu_H \leq \underline{\mu} < \bar{\mu}$, then no high type is willing to search, and so, in equilibrium, there can only be monopoly prices, a contradiction. □

A6: Proof of Lemma 5.

Proof. All types with μ_H visit broad stores on their first visits (since $\Delta^* \leq \beta_H$). In addition, less-choosy, μ_L types with low enough visit costs—specifically with $b < \beta_L$—also inspect at broad retailers, but they do so with no intention to buy there but only to showroom. Broad stores have no incentive to charge different prices for the two goods, so will charge the same price p_B^* given the same prices for the two goods at narrow stores p_N^*.

Define $\hat{p}^*$ as the solution to $\Delta(p_N^*, p_B) = \beta_L$. This solution exists because, by assumption, $\Delta^* = \Delta(p_N^*, p_B^*) > \beta_L$ and $\Delta(p_N^*, p_N^*) = 0$. Moreover, the previous sentence implies that and lies in $\hat{p}^* \in (p_N^*, p_B^*)$.

Then, broad store profit can be written as:

$$\Pi_B = \begin{cases} (1-\lambda)(1 - F_H(\Delta(p_N^*, p_B)))\pi(p_B) & \text{if } p_B \geq \hat{p}^* \\ \left[(1-\lambda)(1 - F_H(\Delta(p_N^*, p_B))) + \lambda(F_L(\beta_L) - F_L(\Delta(p_N^*, p_B)))\right]\pi(p_B) & \text{if } p_B < \hat{p}^* \end{cases}. \quad (3)$$

In this expression, the $(1 - \lambda)$ choosy H consumers always visit a broad retailer (since they anticipate $\Delta^* \leq \beta_H$) and react to the actual price p_B, with some purchasing (if they have $b \leq \Delta(p_N^*, p_B)$) and the rest showrooming—that is leaving to buy at a narrow retailer anticipating the lower price p_N^*. Of the fraction λ comprised of less-choosy consumers, those with low visit costs ($b < \beta_L$) will find it worthwhile to visit a broad retailer and respond to the price posted: either to buy directly if their visit costs are moderately high (that is if $b > \Delta(p_N^*, p_B)$,which does not happen in equilibrium for $p_B = p_B^*$ because, by assumption,

[30] Once at broad stores, consumers will never leave for small price deviations (by assumption $f_T(0) = 0$, those who do leave are negligible in number).

$b < \beta_L < \Delta^*$), or else to showroom (if $b < \Delta(p_N^*, p_B)$). Therefore, for an equilibrium in which some of the less-choosy consumers visit broad stores and showroom, it must be that a broad store's profit is maximised where $\Delta(p_N^*, p_B) > \beta_L$, so the second part of demand is only to be checked against possible deviations there.

By our assumption that $\Delta^* > \beta_L$, we have $p_B^* > \hat{p}^*$and, so the first line in (3) gives the first order-condition for broad retailers. □

A7: Proof of Lemma 6.

Proof. Immediate from the discussion in the text and results in the literature. □

A8: Proof of Lemma 7.

Proof. We rewrite (1) using $p_B^* = u^{-1}(u(p_N^*) - \Delta^*)$ as

$$h(u^{-1}(u(p_N^*) - \Delta^*)) = \frac{1 - F_H(\Delta^*)}{f_H(\Delta^*)}.$$

Note that (1) implies an inverse relationship between p_N^* and Δ^*, because of our assumptions on F_H, and $q(p)$ that imply that $\frac{1-F_H(b)}{f_H(b)}$ is decreasing while $h(p)$ is increasing. Δ^* ranges between from 0 and $\bar{b}$, as p_N^* goes from p^m to 0.

In contrast, (2) establishes an increasing relationship between p_N^* and Δ^* (The right-hand side is clearly increasing in Δ^*, while the left-hand side is increasing in p_N^*). As Δ^* goes from 0 to $\bar{b}$, the associated p_N^* goes from some $p^w < p^m$ to p^m.

Therefore, the system of these two equations in two unknowns has a unique solution. □

A9: Proof of Proposition 2.

Proof. First, we show that under Condition 1, the solution to (1) is the maximiser for broad store profits. Log-concavity of $q(p)$ ensures that $h(p)$ is increasing in p for $p < p^m$ and, moreover, $\lim_{p \to p^m} h(p) = \infty$. In addition,$\frac{1-F_H(\Delta^*)}{f_H(\Delta^*)}$ is decreasing in Δ^* because of the log-concavity of $1 - F_H$. Since Δ^* is decreasing in p_B^*, $\lim_{p_B^* \to p_N^*} \frac{1-F_H(\Delta^*)}{f_H(\Delta^*)} = \infty$, and so for a given $p_N^* < p^m$ a unique solution for p_B^* exists. The solution maximises $\Pi_B(p_B)$ for $p_B \in \{p : \ \Delta(p_N^*, p) \geq \beta_L\}$. If, in addition, Condition 1 holds, then p_B^* is the optimal price for broad stores. Assumption 1 ensures that p_N^* defined uniquely by (2) is the optimal price for a narrow store given assumed consumer behaviour and broad store and other narrow store pricing.

We have also shown that the solution to (1) and (2) is unique. Conditions on $\mu_L$and μ_H were derived from assumed consumer behaviour, whereby all H types and some L types visit broad stores, and some L types (with $b > \beta_L$ and $\varepsilon_i < r_L^*$) search among narrow stores. For low types, $\Delta^* > \beta_L$ and $r_L^* > 0$ are equivalnet to $\tilde{\mu} > \mu_L > \bar{\mu}$, while $\beta_H > \Delta^*$ is the same as $\mu_H > \beta^{-1}(\Delta^*(\mu_L))$. $\tilde{\mu}$ is well defined and unique by Assumption 2. □

A10: Proof of Corollary 1.

Proof. Part (i) is immediate from the construction of the equilibrium. The remaining comparative statics follow from the following observations. Δ^* and p_N^* solve (1) and (2). In (Δ, p_N)-space (1) is downward-sloping and (2) is upward-sloping. (1) does not depend on any of the parameters listed in the corollary. It is immediate to see that (2) shifts upwards with s and μ_L, and downwards with λ and γ. This implies that p_N^* goes up and Δ^* goes down. From the broad store's second-order condition, we also have that p_B^* goes up when p_N^* goes up. □

A11: Proof of Proposition 3.

Proof. Immediate from the discussion in the text. □

A12: Proof of Proposition 4.

Proof. Immediate from the discussion in the text. □

A13: Proof of Proposition 5.

Proof. Following arguments in the text. □

Appendix B: Relaxing Assumption 1

The characterisation of the search and showrooming equilibrium of Proposition 2 is not affected by Assumption 1, and, therefore, it applies when the assumption does not hold.

The results of Proposition 1, however, are affected. The reason is as follows. If Assumption 1 fails, then $\bar{\mu} \leq \underline{\mu}$. This means that consumers who prefer to patronise narrow stores at equal prices may now search through narrow stores, which then precludes monopoly prices in equilibrium.

We next provide a proposition that, like Proposition 1, characterises all non-showrooming equilibria, although, as the result shows, these are not necessarily characterised by monopoly prices.

Proposition 6. *The following non-showrooming equilibria obtain when Assumption 1 is violated. In all these cases, broad stores charge (p^m, p^m). Further:*

1. *If $\mu_H \leq \bar{\mu}$, then all visits occur to narrow stores that charge $p_N^* = p^m$. Broad stores earn $\Pi_B^* = 0$, while narrow stores earn $\Pi_N^* = \pi^m$.*
2. *If $\mu_L \leq \bar{\mu} < \mu_H < \underline{\mu}$, then all visits occur to narrow stores that charge $p_N^* = h^{-1}\left(\frac{1}{2(1-\lambda)z_H}\right)$. Broad stores earn $\Pi_B^* = 0$, while narrow stores earn $\Pi_N^* = \frac{1}{2}\pi(p_N^*)$.*
3. *If $\mu_H, \mu_L \in (\bar{\mu}, \underline{\mu})$, then all visits occur to narrow stores that charge $p_N^* = h^{-1}\left(\frac{1}{2(\lambda z_L + (1-\lambda)z_H)}\right)$. Broad stores earn $\Pi_B^* = 0$, while narrow stores earn $\Pi_N^* = \frac{1}{2}\pi(p_N^*)$.*
4. *If $\mu_L \leq \bar{\mu}$ and $\mu_H > \underline{\mu}$, then all high types visit broad stores, and all low types visit narrow stores that charge p^m. Broad stores earn $\Pi_B^* = (1-\lambda)\pi^m$, while narrow stores earn $\Pi_N^* = \frac{\lambda}{2}\pi^m$.*
5. *If $\mu_L \in (\bar{\mu}, \underline{\mu})$ and $\mu_H > \underline{\mu}$, then non-showrooming equilibria in pure strategies do not exist.*
6. *If $\mu_L > \underline{\mu}$, then all consumers visit broad stores, narrow stores charge p^m. Broad stores earn $\Pi_B^* = \pi^m$, while narrow stores earn $\Pi_N^* = 0$.*

Proof. Assume that $\mu_H \leq \bar{\mu}$. Given that $r_H^* < 0$ and $\beta_H \leq 0$ (by $\gamma < \bar{\gamma}$), there are no consumers who wish to showroom at broad stores, even with $b = 0$ and at equal prices. Given the holdup problem at broad stores, consumers cannot expect lower prices at broad than at narrow stores; thus, no consumer will visit broad stores. Given $r_L^* < 0$, no consumer will search through narrow stores. This implies that $p_N^* = p^m$ has to hold, and all consumers visit narrow stores.

Now assume that $\mu_L \leq \bar{\mu} < \mu_H < \underline{\mu}$. Since $\bar{\mu} < \mu_H < \underline{\mu}$, high types will not visit broad stores even at equal prices, but they will search through narrow stores. By $\mu_L \leq \bar{\mu}$, the low types are not willing to visit broad stores, and are unwilling to search through narrow stores; therefore, in equilibrium, all visits have to occur to narrow stores, which charge $p_N^* = h^{-1}\left(\frac{1}{2(1-\lambda)z_H}\right)$ because only high types search. The pricing details trivially follow from the results in the literature, when price inelastic low types are taken into account.

Assume now that $\mu_H, \mu_L \in (\bar{\mu}, \underline{\mu})$. Here, both types are willing to search through narrow stores and will not visit broad stores at equal prices; thus, all consumers visit narrow stores and potentially search, with $p_N^* = h^{-1}\left(\frac{1}{2(\lambda z_L + (1-\lambda) z_H)}\right)$ as the equilibrium price.

Now consider $\mu_L \leq \bar{\mu}$ and $\mu_H > \underline{\mu}$. High types are willing to visit broad stores at equal prices, and low types are unwilling to search through narrow stores or visit broad ones, so all stores charging p^m is an equilibrium, in which high types visit broad stores and low types visit narrow stores. No other equilibrium exists because high types visiting narrow stores and searching is precluded for the same reason as was given in the proof of Proposition 1.

Assume that $\mu_L \in (\bar{\mu}, \underline{\mu})$ and $\mu_H > \underline{\mu}$ and the proof of Proposition 1. By $\mu_L > \bar{\mu}$, low types are willing to search among narrow stores, but by $\mu_L < \underline{\mu}$ prefer narrow stores at equal prices. Thus, low types will visit narrow stores and some will search. If, as per statement of the proposition, no high types showroom, then it has to be that $\Delta^* = 0$ by $\mu_H > \underline{\mu}$, OR else with $\Delta^* > 0$, some high types with b sufficiently close to 0 wILL showroom. But $\Delta^* = 0$ cannot occur because broad stores will charge (p^m, p^m) to high types that visit them, whereas narrow stores will have to charge $p_N^* = h^{-1}\left(\frac{1}{2(1-\lambda) z_H}\right) < p^m$. Thus, in this range, no no-showrooming equilibrium exists.

Finally, assume that$\mu_L > \underline{\mu}$. By $\mu_H > \mu_L$, we have that both types prefer broad stores at equal prices. For showrooming not to occur, we need $\Delta^* = 0$, which then implies that broad stores will attract all consumers, and will charge (p^m, p^m) by Diamond-like reasoning. Since narrow stores do no attract consumers, they have to charge p^m, which is consistent with $\Delta^* = 0$. □

One interesting implication of the violation of Assumption 1 is that, now, for some parameters, no (pure strategy) equilibrium exists. This happens when μ_L is intermediate so that low types are willing to search through narrow stores but are not willing to visit broad stores; yet μ_H is also intermediate so that the showrooming equilibrium cannot be sustained (μ_H is not high enough), and all consumers going to narrow stores and searching among them is not an equilibrium either (μ_H is not low enough). Low types visit narrow stores and put downward pressure on their prices to ensure that $p_N^* < p^m$; this then implies that $\Delta^* > 0$, which, in turn, can occur only with showrooming. Just as in Proposition 2, such an equilibrium requires a high μ_H enough (generally higher than $\tilde{\mu}$) that there exists an interval $(\underline{\mu}, \mu')$ such that when $\mu_H \in (\underline{\mu}, \mu')$, no equilibrium exists.

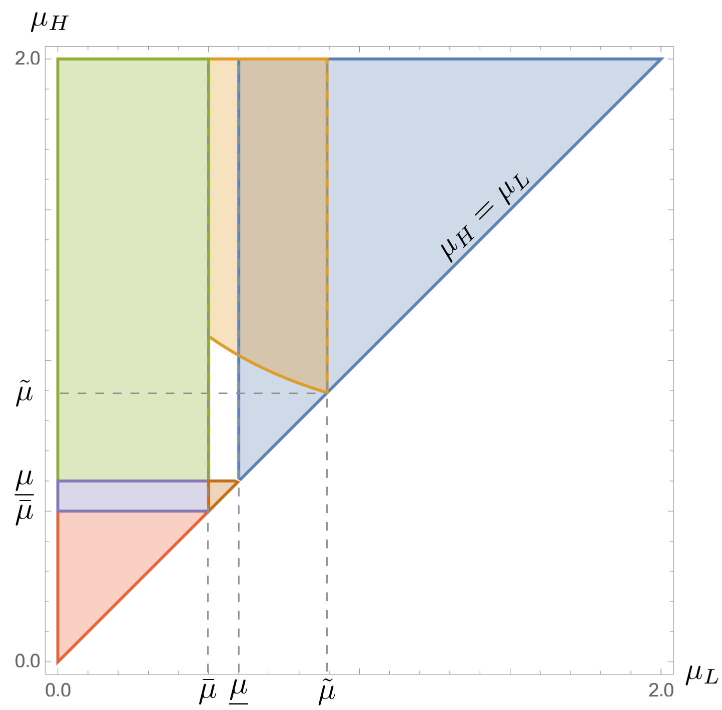

Figure 4: Equilibrium regimes depending on μ_L and μ_H, where $G \sim U(0,1)$, $q(p) = 1 - p$, $F_L(b) = (\frac{b}{s})^6$, $F_H(b) = (\frac{b}{s})^2$,$\lambda = 0,5$, $s = \bar{b} = 0.25$, and $\gamma = 0.4$. In the red region, all visits occur to narrow stores. In the green region, high types visit broad stores and low types visit narrow. In the blue region, there is an equilibrium in which all visits are to broad stores. In all these all store types charge monopoly prices. In the orange region, there is an additional equilibrium of the type described in Proposition 2. In the purple region, high types search and low types do not, but both types visit narrow stores. Finally, in the dark orange triangle, both types search but visit narrow stores.

Appendix C: Monopoly in the Price-Only Sector

Assume now that a monopolist in the price-only sector, labelled retailer A, enters the market.

We start the analysis of this section by verifying that all monopoly price equilibria without A are still equilibria with A. Since, in these equilibria, absent the price-only monopolist, A, all prices are equal to p^m, when A is present, equilibrium prices are preserved, and A attracts all savvy consumers by charging $p_A^* = p^m$. Search patterns are also preserved because they were derived assuming equal prices at all stores.

Proposition 7. *When A is active, there always exists a symmetric equilibrium in which all stores charge p^m. If $\mu_L < \bar{\mu}$, such a symmetric equilibrium is unique. In such equilibria, consumer shopping behaviour is the following:*

1. *If $\mu_H \leq \underline{\mu}$, then all consumers visit narrow stores; savvy consumers buy at A, and the naive consumers buy at narrow stores without searching.*

2. *If $\mu_H \geq \underline{\mu} > \mu_L$, then all high types visit broad stores; all low types visit narrow stores; naive consumers buy at the store they visited, and the savvy buy from A.*

3. *If $\mu_L \geq \underline{\mu}$, then all consumers visit narrow stores; naive consumers buy at narrow stores without searching, and the savvy buy from A.*

Proof. Following arguments in the text and the proof of Proposition 1. □

C.1 The price-only monopolist and showrooming

Let us now turn to a market configuration where all the retailers and A are active and have sales. As before, we look for a symmetric equilibrium, using largely the same notation as in Section 4, but introducing p_A^{**} as A's price for both goods (broad and narrow equilibrium prices are now p_B^{**} and p_N^{**}, with $\Delta^{**} = \Delta(p_N^{**}, p_B^{**})$). Furthermore, let $\Delta_A^{**} \equiv \Delta(p_N^{**}, p_A^{**})$ be the utility difference between purchasing at narrow stores and A, gross of search and showrooming costs.

First, note that A's price must satisfy

$$p_A^{**} \geq p_N^{**}.$$

This follows since if $p_A^{**} < p_N^{**}$, A can increase its prices without losing any savvy consumers.

Next, note that A's price must satisfy

$$p_A^{**} \leq p_B^{**};$$

otherwise, it will not sell to any consumers at all. Thus, $p_A^{**} \in [p_N^{**}, p_B^{**}]$. As a result, savvy consumers who plan to visit broad stores will never buy from these stores; instead, they either pay p_A^{**} to purchase at A or showroom to narrow stores (if $b < \Delta_A^{**}$). This implies that the pricing problem of broad stores is unaffected by savvy consumers and A, given the price at narrow stores. Naive consumers will ignore A and will behave as before. Savvy consumers will not buy at broad retailers.

Analogous to Lemma 6, optimal broad store prices satisfy

$$h(p_B^{**}) = \frac{1 - F_H(\Delta^{**})}{f_H(\Delta^{**})}, \tag{4}$$

as in the model without A.

Next, we turn to A's pricing problem. Recall that, by assumption, savvy consumers can plan their searches based on the actual p_A rather than on the anticipated p_A^{**} (of course, in equilibrium the actual p_A will be identical to the anticipated p_A^{**}). If the inspection benefit associated with visiting a broad store for a less-picky type (and, so, also for the more-picky type) is greater than the price premium associated with buying from A, $\beta_L \geq \Delta(p_N^{**}, p_A)$, then all savvy consumers (be they picky or non-picky) will visit broad stores first. They do so since they anticipate that showrooming at broad stores (and then buying at narrow stores if $b < \Delta(p_N^{**}, p_A)$ or at A otherwise) is better than searching through narrow stores. In contrast, if the price premium is higher, $\Delta(p_N^{**}, p_A) > \beta_L$, and then savvy low types showroom only to buy at narrow stores. Let $\hat{p}^{**}$ be the value of p_A that solves $\Delta(p_N^{**}, p_A) = \beta_L$. For what follows, note that $\hat{p}^{**} \in (p_N^{**}, p_B^{**})$ because, by assumption, $\Delta(p_N^{**}, p_B^{**}) > \beta_L > 0$.

The price-only monopolists's profits are given by

$$\Pi_A = \begin{cases} (1-\lambda)(1-\theta_H)(1 - F_H(\Delta(p_N^{**}, p_A)))\pi(p_A) & \text{if } p_A > \hat{p}^{**} \\ \left[\lambda(1-\theta_L)(1 - F_L(\Delta(p_N^{**}, p_A))) + (1-\lambda)(1-\theta_H)(1 - F_H(\Delta(p_N^{**}, p_A)))\right]\pi(p_A) & \text{if } p_A \leq \hat{p}^{**} \end{cases}.$$

In this expression, note that there is a discontinuity at $\hat{p}^{**}$. Moreover, A's price is observed before search, and so, when $p_A \leq \hat{p}^{**}$, A attracts all low-type savvy consumers.[31] A's profit may be maximised at $p_A > \hat{p}^{**}$ or at $p_A \leq \hat{p}^{**}$. In the former case, it can be verified, on examining the proof of Lemma 6, that the relevant section of the profit function for A has the same maximiser as the corresponding problem for the broad store profit function; so, $p_A = p_B^{**}$ has to hold, which indeed satisfies $p_A = p_B^{**} > \hat{p}^{**}$. In the latter case when $p_A \leq \hat{p}^{**}$ holds, A's profit is a weighted average for low and high types, which given our earlier assumption, $\frac{f_H(b)}{1-F_H(b)} \geq \frac{f_L(b)}{1-F_L(b)}$, implies that the expression is maximised at (weakly) above p_B^{**}. This can be seen from the first-order condition that takes a form analogous to (8), which determines p_B^{**} but has a weakly higher right-hand side. Thus, we conclude that for $p_A \leq \hat{p}^{**}$, the maximiser is $p_A = \hat{p}^{**}$.[32] Therefore, A has two choices: (i) it can charge $p_A = \hat{p}^{**}$ and sell to all savvy consumers with $b \geq \beta_L$ because even the low types will prefer to go to broad stores given $\Delta(p_N^{**}, p_A) = \beta_L$. In this case, A earns $\Pi_A = ((1-\lambda)(1-\theta_H)(1-F_H(\beta_H)) + \lambda(1-\theta_L)(1-F_L(\beta_L)))\pi(\hat{p}^{**})$; alternatively, (ii)

[31] This is unlike the broad stores that, when reducing their price so that $p_B \leq \hat{p}^{**}$, attract only those low-type naive consumers with $b \leq \beta_L$.

[32] Absent the assumption $\frac{f_H(b)}{1-F_H(b)} \geq \frac{f_L(b)}{1-F_L(b)}$, we would have had to deal with an interior maximiser. We exclude this possibility purely for expositional simplicity.

A can charge $p_A = p_B^{**}$ and earn $(1-\lambda)(1-\theta_H)(1-F_H(\Delta^{**}))\pi(p_B^{**})$.[33] This means that there could be two types of equilibria, which we explore next.

Before doing so, note that narrow stores face a similar problem to the baseline model, though in the presence of A, some savvy low types may not act as searchers. This is the case when A's optimal response mandates $p_A^{**} = \hat{p}^{**}$, in which case narrow stores only get savvy consumers with $b < \beta_L$ who showroom from broad to narrow stores. Thus, if $p_A^{**} = \hat{p}^{**}$, holds then narrow stores maximise

$$\hat{\Pi}_N(p_N) = \Bigg[\frac{(1-\lambda)(1-\theta_H)F_H(\beta_L) + \lambda(1-\theta_L)F_L(\beta_L)}{2} + \frac{(1-\lambda)\theta_H}{2}F_H(\Delta^{**}) + \frac{\lambda\theta_L}{2}F_L(\beta_L) \\ + \lambda\theta_L(1-F_L(\beta_L))s_L(p_N)\Bigg]\pi(p_N.).$$

The first term in square brackets is the demand from savvy showroomers (savvy consumers of both types showroom at narrow stores when $b < \beta_L$ because, at broad stores, they can pay p_A^{**} and $\Delta_A^{**} = \beta_L$); the second term is the demand from naive high-type showroomers (unlike the previous group; these consumers showroom when $b < \Delta^{**}$ because they do not have access to A's prices); the third term is the demand from naive low-type showroomers who showroom only when $b < \beta_L$ because those with $b \geq \beta_L$ are narrow searchers; and, finally, the last term is the demand from non-savvy low types who are searchers.

In order to proceed, we make an assumption that is parallel to Assumption 2

Assumption 3. *Assume that $F(\cdot)$, $G(\cdot)$ and $u(\cdot)$ are such that $\hat{\Pi}_N(p_N)$ is quasi-concave and z_L is increasing in μ_L.*

We maintain this assumption for the rest of this appendix.

C.1.1 The price-only monopolist matches the prices of broad stores

Consider a a situation in which A sets $p_A^{**} = p_B^{**}$, and then both broad and narrow prices are the same as in the baseline model, because their first-order conditions in (4) and (6) coincide with our earlier (1) and (2), respectively. Under (5), broad and narrow stores do not wish to deviate, as in the baseline model, and the only firm that might deviate is A. Equilibrium requires that A does not prefer to switch to charging a price that would attract low-type savvy consumers. It can be verified that this is the case, as long as

$$\begin{aligned} &(1-\lambda)(1-\theta_H)\,(1-F_H(\Delta^{**}))\,\pi(p_B^{**}) \geq \\ &[(1-\lambda)(1-\theta_H)\,(1-F_H(\beta_L)) + \lambda(1-\theta_L)\,(1-F_L(\beta_L))]\,\pi\left(u^{-1}(u(p_N^{**})-\beta_L)\right). \end{aligned} \tag{5}$$

[33]Note that A's optimal price depends on actual prices of broad stores but only on the equilibrium prices of narrow stores because consumers never compare actual narrow prices with A's prices.

This condition cannot hold for $\mu_L = \tilde{\mu}$ and $F_L(\beta_L) > 0$ because, there, $\Delta^{**} = \beta_L$, and so the right-hand side is always higher than the left-hand side. For $\mu_L < \tilde{\mu}$, we have $p_B^{**} = u^{-1}(u(p_N^{**}) - \Delta^{**}) > u^{-1}(u(p_N^{**}) - \beta_L)$ (by p_B^{**} being the maximiser for broad), so if θ_L is sufficiently low, this condition will hold.

As we argue in the proof of the Proposition below, the relevant FOC for a narrow store can be written as

$$h(p_N^{**}) = \left[\frac{1 + \frac{1-\lambda}{\lambda} F_L(\Delta^{**})}{1 - F_L(\beta_L)}\right] \frac{1}{2z_L}. \tag{6}$$

Proposition 8. *If $\tilde{\mu} > \mu_L > \bar{\mu}$, $\mu_H > \beta^{-1}(\Delta^*(\mu_L))$, and (5) hold, then there is an equilibrium in which p_N^{**} and p_B^{**} jointly solve (6) and (4) and $p_A^{**} = p_B^{**}$. All type H consumers make first visits to broad stores and showroom at narrow stores if $b < \Delta^{**}$; so do type L consumers with $b < \beta_L$, whereas those with $b \geq \beta_L$ visit narrow stores, searching sequentially in accordance with Lemma 2. If a type H consumer is savvy and $b > \Delta^{**}$, she purchases from A, and if she is naive and $b > \Delta^{**}$, then she buys from the broad store.*

Proof. It is immediate from the arguments in the text that either $p_A^{**} = \hat{p}^{**}$or $p_A^{**} = p_B^{**}$. In the former case, under Assumption 3, the following FOC gives the unique maximiser for narrow stores:

$$h(p_N^{**}) = \left[\frac{1 + \frac{(1-\lambda)(1-\theta_H)F_H(\beta_L)+\lambda(1-\theta_L)F_L(\beta_L)}{\theta_L\lambda} + \frac{\theta_H}{\theta_L}\frac{(1-\lambda)}{\lambda}F_H(\Delta^{**})}{1 - F_L(\beta_L)}\right] \frac{1}{2z_L}. \tag{7}$$

It can readily be verified that the expression in square brackets is greater than 1.

If $p_A^{**} = p_B^{**}$, then the narrow store's problem is the same as in the baseline because A retains all savvy high types with $b \geq \Delta^{**}$, whereas the remaining savvy high types and savvy low types buy from narrow stores, with some showrooming and the others searching. A narrow store's profit is

$$\Pi_N(p_N) = \left[\frac{(1-\lambda)}{2}F_H(\Delta^*) + \frac{\lambda}{2}F_L(\beta_L) + \lambda(1 - F_L(\beta_L))s_L(p_N)\right]\pi(p_N).$$

This leads to the FOC for the narrow stores given by in (6). The rest of the proof proceeds along the same lines as the proof of Proposition 2. □

Given that (4) and (6) coincide with our earlier (1) and (2), it is clear that entry by A does not change prices, consumer surplus or welfare. Narrow retailers are also unaffected because the same consumers buy from them as without A. Broad stores are affected because they lose all the savvy high types to A, and, thus, their profits are reduced by a factor θ_H.

Corollary 2. *Assume that the conditions in Proposition 8 hold. Then, within an equilibrium of that form:*

(i) A and broad stores charge higher prices than narrow stores, but lower prices than monopoly prices, $p^m > p_B^{**} = p_A^{**} > p_N^{**}$.

(ii) An increase in μ_H,θ_H and θ_L has no effect on prices, while an increase in μ_L increases prices.

(iii) An increase in s increases all prices.

(iv) An increase in λ leads to a reduction in all prices.

(v) An increase in γ leads to a reduction in all prices.

Proof. The proof is identical to the proof of Corollary 1, except for the part regarding θ_H and θ_L, which follows from the fact that they do not enter price determination. They do affect condition (4), which is assumed to hold. □

C.1.2 The price-only monopolist competes with narrow stores

Now consider equilibria in which A sets $p_A^{**} = \hat{p}^{**}$. Define p_N^{**}, p_B^{**}, Δ^{**} as the equilibrium prices and premia. Further, it is convenient to define $\tilde{\mu}_A$ as μ_L that solves $\beta_L = \Delta^{**}(\mu_L)$. This threshold has the following property.

Lemma 8. $\tilde{\mu}_A \leq \tilde{\mu}$.

Proof. When $\Delta^{**}(\mu_L)$ is replaced by $\beta(\mu_L)$ in (7), it becomes

$$h(p_N^{**}) = \left[\frac{1 + \frac{(1-\lambda)F_H(\beta_L)+\lambda(1-\theta_L)F_L(\beta_L)}{\theta_L\lambda}}{1 - F_L(\beta_L)} \right] \frac{1}{2z_L},$$

which when compared to (2), clearly implies that $p_N^{**} \geq p_N^*$. This, in turn, means that $\Delta^{**} \leq \Delta^*$ and so $\tilde{\mu}_A \leq \tilde{\mu}$. □

For this equilibrium to hold, A must prefer to set $p_A = \hat{p}^{**}$ to $p_A = p_B^{**}$, which is written as

$$(1-\lambda)(1-\theta_H)\left(1 - F_H(\Delta^{**})\right)\pi(p_B^{**}) \leq \tag{8}$$

$$\left[(1-\lambda)(1-\theta_H)\left(1 - F_H(\beta_L)\right) + \lambda(1-\theta_L)\left(1 - F_L(\beta_L)\right)\right]\pi\left(\hat{p}^{**}\right). \tag{9}$$

While this condition is similar to (5), prices at which it is evaluated are different in general, and so for some parameters, both or neither can hold at the same time.

Bringing together the discussion and writing down the narrow store's FOC:

$$h(p_N^{**}) = \left[\frac{1 + \frac{(1-\lambda)(1-\theta_H)F_H(\beta_L)+\lambda(1-\theta_L)F_L(\beta_L)}{\theta_L\lambda} + \frac{\theta_H}{\theta_L}\frac{(1-\lambda)}{\lambda}F_H(\Delta^{**})}{1 - F_L(\beta_L)} \right] \frac{1}{2z_L}. \tag{10}$$

This allows us to summarise as follows.

Proposition 9. *If $\tilde{\mu}_A > \mu_L > \bar{\mu}$, $\mu_H > \beta^{-1}(\Delta^{**}(\mu_L))$, and (8) hold, then there is an equilibrium in which p_N^{**} and p_B^{**} jointly solve (6) and (10) and $p_A^{**} = \hat{p}^{**}$. All savvy consumers, all naive high types, and naive low types with $b < \beta_L$ visit broad stores, while the rest visit narrow stores and search optimally. All savvy consumers with $b > \beta_L$ purchase from A, and the rest buy from narrow stores. Naive consumers behave as in the baseline model.*

Proof. Following arguments in the text. In detail, given condition (8), p_A^{**} maximises A's profits given other retailers' prices. Assumption 3 and arguments in proofs of Proposition 2 and Lemma 5 guarantee that the remaining retailers and consumers act optimally. □

Note that in this equilibrium configuration, the price-only outlet charges a higher price than narrow retailers because $p_A^* = \hat{p}^{**} > p_N^*$, but a lower price than broad retailers.

Comparing narrow store pricing here to the baseline model, we obtain the following.

Corollary 3. *Narrow store pricing can be higher or lower than in the baseline model. In particular, it is higher when*

$$(1-\lambda)(1-\theta_H)F_H(\beta_L) + \lambda(1-\theta_L)F_L(\beta_L) > (1-\lambda)F_H(\Delta^*)\,(\theta_L - \theta_H)\,. \tag{11}$$

Proof. Comparing the relevant FOCs from (10) and (6), given that $h(\cdot)$ is increasing, pricing is higher than in the baseline model when

$$\frac{1+\frac{(1-\lambda)(1-\theta_H)F_H(\beta_L)+\lambda(1-\theta_L)F_L(\beta_L)}{\theta_L\lambda}+\frac{\theta_H}{\theta_L}\frac{(1-\lambda)}{\lambda}F_H(\Delta^*)}{1-F_L(\beta_L)} > \frac{1+\frac{1-\lambda}{\lambda}F_H(\Delta^*)}{1-F_L(\beta_L)}.$$

Rearranging this expression gives condition (11). □

Intuitively, this pricing rule may imply lower or higher p_N^{**} as compared to p_N^* (for the same Δ) in the baseline model because, while there are fewer searchers due to the presence of A, there are also fewer showroomers (since savvy high types with $b \in [\beta_L, \Delta^{**}]$ have now switched to showrooming in order to buy from A rather than from a narrow store). Clearly, the inequality above is always satisfied when $\theta_H \geq \theta_L$, and high types are less likely to be savvy consumers. In this case, A is relatively likely to be selling to low-type consumers, and the narrow retailers face proportionally more inelastic choosy showroomers. If $\theta_H < \theta_L$, then the comparison depends on other parameters. For $\theta_L \to 0$ (that is, with almost all low types savvy) the presence of A necessarily leads to higher narrow store prices (we have $p_N^{**} \to p^m > p_N^*$).

The characterisation in Proposition 9 allows us to derive further properties of the equilibrium, immediately. Many are analogous to properties of the baseline model.

Corollary 4. *Assume that conditions in Proposition 9 hold. Then, within an equilibrium of that form:*

(i) Broad stores charge higher prices than A charges, which, in turn, charges lower prices than narrow stores, but all are lower than monopoly prices, $p^m > p_B^{**} > p_A^{**} > p_N^{**}$.

(ii) Making the pickier consumers even more picky (an increase in μ_H) has no effect on prices, but making the less-picky more picky (an increase in μ_L) increases prices.

(iii) An increase in inspection costs (increasing s) increases all prices.

(iv) An increase in the proportion of less-picky consumers (an increase in λ) leads to a reduction in all prices.

(v) A decrease in the inspection cost efficiency of broad retailers (an increase in γ) leads to a reduction in all prices.

(vi) An increase in the fraction of the savvy among picky consumers (lower θ_H) leads to a reduction in all prices.

(vii) An increase in the fraction of the savvy among less-picky consumers (lower θ_L) increases all prices.

Proof. The price order follows from the equilibrium construction and $p_B^{**} > \hat{p}^{**} = p_A^{**} > p_N^{**}$. The remaining statics follow from the same steps as in the proof of Corollary 1, when applied to (4) and (10). The comparative statics are similar to Corollary 1 for all parameters but θ_H and θ_L, for which (10) shifts down with θ_H because $F_H(\beta_L) < F_H(\Delta^{**})$ and shifts up with θ_L. □

An interesting new feature of this analysis reinforces the results of Section 5: savvy consumers have opposing effect on prices depending on their pickiness. If there are more savvy among the picky, then prices in equilibrium shrink. This is because fewer picky consumers showroom at narrow stores (with more choosing to buy from A instead), which removes a fraction of inelastic consumers at narrow stores, reducing narrow store prices and the prices elsewhere. Savviness among the non-picky has the opposite effect. Increasing the proportion of savvy consumers among the non-picky leads to fewer less-picky consumers searching among narrow stores, but the exact same number of those showrooming at to narrow stores, reducing the elastic demand at narrow stores, and, thus, increasing all prices.

C.2 Welfare and price effects of a price-only monopolist

The welfare analysis of A's entry depends crucially on whether showrooming features in equilibrium.

Consider, first, a case in which Proposition 1 characterised the outcome in the absence of A, so that monopoly prices prevailed. The entry of A leaves monopoly price equilibrium intact, as in Proposition 1. Consequently, A's entry does not improve welfare. When prices are identical everywhere, with or without A, there is no showrooming behaviour: those consumers who prefer to visit a single narrow store without A would prefer to do so when A is present, and likewise for visitors to broad stores. However, the presence of A clearly impacts retailers' profits. To the extent, that retailers have to cover fixed costs and entry of the retailer may drive out a class of retailers, the analysis in Section 4.4 applies directly, and so welfare may fall.

Instead, under the conditions of Proposition 8, welfare necessarily rises compared to the corresponding case without A. Prices are unaffected and depend on naive consumers, but some less-picky savvy consumers with high visit costs now showroom at A and are strictly better off by doing so.

Finally, and most interestingly, consider the equilibrium under conditions of Proposition 9. As described in Proposition 9, A's entry may increase or reduce prices at narrow and broad stores. If it reduces prices (that is, $p_N^* > p_N^{**}$ and $p_B^* > p_B^{**}$), then A's entry will clearly be beneficial to all consumers (more so for the savvy consumers, some of whom further benefit from free showrooming and lower price at A). It will increase welfare, but it will reduce firm profits. If, instead, A's entry raises broad and narrow prices ($p_N^* < p_N^{**}$ and $p_B^* < p_B^{**}$), then

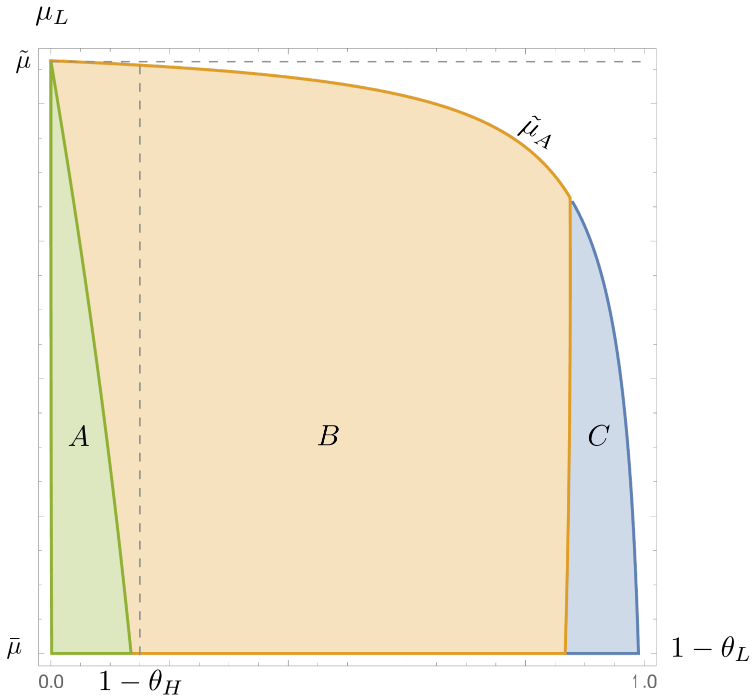

Figure 5: Welfare and price comparison between the baseline and price-only monopolist model when. $G \sim U(0,1)$, $q(p) = 1-p$, $F_H(b) = (\frac{b}{s})^2$, $F_L(b) = (\frac{b}{s})^6$, $\lambda = 0.5$, $s = \bar{b} = 0.25$, $\gamma = 0.25$ and $\theta_H = 0.8$. In region A (green), A's entry increases welfare and reduces prices. In region B (orange), the entry increases welfare but also increases prices. In region C (blue), welfare goes down and prices go up.

naive consumers will clearly be harmed. Savvy consumers may, nevertheless, benefit because some will buy from A and pay p_A^{**}, which indeed may be lower than p_B^*, and they will avoid having to pay showrooming costs to visit a narrow retailer. Welfare will change in an ambiguous direction to the factors outlined above. Figure 5 illustrates in $(1-\theta_L, \mu_L)$ space—reflecting the fraction of the savvy among the less picky, and their level of choosiness—the range of these two parameters where prices go down or up, and welfare goes down or up, as a result of A's entry. This highlights that all these possibilities can, indeed, arise. Similarly, retail profits (including A) may go up (since there will be higher prices at brick and mortar stores) but also down (savvy consumers who purchase at A pay p_A^{**} instead of p_B^*).