

Token-Based Platform Finance

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Digital Platforms and Tokens

- The rise of digital platforms
 - Payment innovation is important, e.g., escrow account on eBay and Alibaba
- Tokens: users' means of payments on platform
 - Blockchain: preventing double spending, facilitating smart contracts
- Tokens: platforms' financing instruments
 - Token offerings \$ 21 billion in 2018; US VC \$ 131 billion
 - Tokens used to gather resources (e.g., engineers, consultants, investors)
 - Tokens enter into circulation gradually (protocol and vesting)
- Tokens: rewards for the founding entrepreneurs

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This Paper

- A dynamic model of platform investment/financing and user activities
 - Tokens are both means of payments for users and also financing instruments for the platform to gather efforts and resources
 - Users' token demand: transaction and investment value
 - Platform owners' token supply: reward themselves and pay contributors to improve the platform
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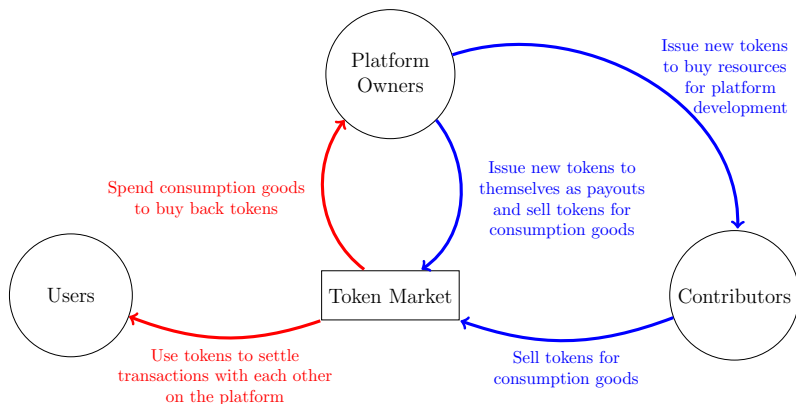
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Token-Based Ecosystem



Questions

1 A platform can produce tokens with zero cost, so why token supply is finite and value positive?

- What is the optimal way for platform designers to extract profits via tokens? Vesting schemes are common, but why and how to design?
- Implications on token inflation/deflation and volatility dynamics

2 What is the key economic inefficiency when tokens serve as both users' means of payment and platforms' financing tools?

- Are users' and platform designers/founders' interests aligned?
- Pitfalls in the value chain? Users \rightarrow token value \rightarrow financing platform's productivity growth & rewarding founders with token payout

3 How can blockchain technology add value

- Why platform currencies rise after blockchain technology matures?

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Related Papers

- Platforms without tokens: Rochet and Tirole (2003), Weyl (2010)
- Tokens as platform currency: Brunnermeier, James, and Landau (2019), Cong, Li, and Wang (2018a), Gans and Halaburda (2015)
- Tokens for users and contributors with exogenous supply: Pagnotta (2018), Sockin and Xiong (2018) among others
- Tokens and founders' effort: Canidio (2018), Chod and Lyandres (2018), Garratt and Van Oordt (2019)
- Dynamic token valuation with fixed supply: Cong, Li, and Wang (2018a), Fanti, Kogan, and Viswanath (2019) among others
- Durable-goods monopoly: Coase (1972), Bulow (1982), Stokey (1981)
- Dynamic Corporate finance: Bolton, Chen, and Wang (2011), Li (2017)
- Money: (1) convenience yield in Baumol-Tobin models, Krishnamurthy and Vissing-Jørgensen (2012); (2) demand with inflation expectation in Cagan (1956); (3) financing tools in Bolton and Huang (2016)

Outline

- Introduction
- **Model and Solution**
- Franchise Value as Discipline – Durable-Goods Monopoly
- Token Overhang – Corporate Finance
- The Value of Commitment – Time Inconsistency
- Conclusion

A **platform** supports a unique set of transactions

User i settles transactions in tokens, deriving
convenience yield from token value

- *Efficient payment, smart contracting ...*

A **platform** supports a unique set of transactions

- Productivity: A_t

User i settles transactions in tokens, deriving *convenience yield* from token value $x_{i,t} = P_t k_{i,t}$

- Convenience yield: $x_{i,t}^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha dt$
 - Token price: P_t
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- Token price appreciation $k_{i,t} E_t[dP_t]$

Token price dynamics in equilibrium

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_t^P dZ_t$$

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- Participation cost ϕdt , if $k_{i,t} > 0$

$$N_t = 1 - G_t(\underline{u}_t)$$

Objective

$$\int_{t=0}^{+\infty} e^{-rt} [\max\{ 0, \textit{convenience} + \textit{net token return} - \textit{participation cost} \}] dt$$

Token Demand

$$k_{i,t} = \frac{Q(E_t[dP_t/dt], A_t)}{P_t} u_i$$

$$\frac{\partial Q}{\partial E_t[dP_t]} > 0$$

$$\frac{\partial Q}{\partial A_t} > 0$$

Token Market Clearing

$$M_t = \int_{u=\underline{u}_t} \frac{Q(E_t[dP_t/dt], A_t)}{P_t} u dG_t(u)$$

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- P_t decreases in supply M_t , increases in A_t
- 1st, 2nd order derivatives in $E_t[dP_t/dt]$ by Itô's lemma
→ Differential equation for $P_t = P(M_t, A_t)$

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*How do the state variables
 A_t and M_t evolve?*

$$M_t = \frac{\text{Token Market Clearing}}{P_t} = \frac{Q(E_t[dP_t/dt], A_t)}{P_t} \int_{u=\underline{u}_t} u dG_t(u)$$

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- **Entrepreneur** contribution: $dH_t = \mu^H dt + \sigma^H dZ_t$

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$$\text{Tokens paid} \quad \frac{F(L_t, A_t)dt}{P_t}$$

$$\text{Token Supply} \\ dM_t = \frac{F(L_t, A_t)dt}{P_t}$$

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- Tokens paid to owner (cumulative): D_t

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- Tokens paid to owner: $dD_t > 0$
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Token Supply

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$$\max_{\{L_t, dD_t\}} \int_{t=0}^{+\infty} e^{-rt} P_t dD_t [I_{\{dD_t \geq 0\}} + (1 + \chi) I_{\{dD_t < 0\}}] dt$$

- Token buy-back financing cost: χ

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- $V_t = V(M_t, A_t)$, $\frac{\partial V}{\partial M} < 0$ $\frac{\partial V}{\partial A} > 0$
- HJB is differential equation for $V(M_t, A_t)$

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Token Price

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_t^P dZ_t$$

endogenous

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State space: $(M_t, A_t) \rightarrow (m_t, A_t)$, where $m_t = \frac{M_t}{A_t}$

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Solve ODEs of $v(m_t)$ and $P(m_t)$

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$$\frac{\partial V}{\partial M_t} = v'(m_t) < 0 \quad P'(m_t) < 0$$

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 - Producers sell all goods immediately at price equal to MC
 - Producers sell ∞ tokens immediately at price equal to 0?

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Platform resists excess supply

$$m_t = \frac{M_t}{A_t} \in [\underline{m}, \overline{m}]$$

Incentive to buyback and burn tokens

Optimal Platform Payout and Buy-back (burn) dD_t

$$\underline{m} \text{ ————— } \overline{m}$$

m_t

Optimal Platform Payout and Buy-back (burn) dD_t

$$\underline{m} \text{ --- } m_t \text{ --- } \overline{m}$$

$$dD_t < 0$$

$$-\frac{\partial V}{\partial M_t} = -v'(m_t) = P_t(1 + \chi)$$

THE  TIMES

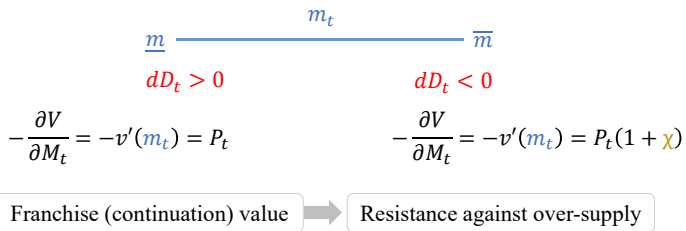


Luxury brands including Burberry burn stock worth millions

Optimal Platform Payout and Buy-back (burn) dD_t

$$\begin{array}{ccc}
 \underline{m} & \xrightarrow{m_t} & \overline{m} \\
 dD_t > 0 & & dD_t < 0 \\
 -\frac{\partial V}{\partial M_t} = -v'(m_t) = P_t & & -\frac{\partial V}{\partial M_t} = -v'(m_t) = P_t(1 + \chi)
 \end{array}$$

Optimal Platform Payout and Buy-back (burn) dD_t



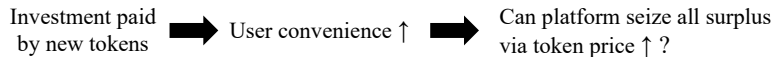
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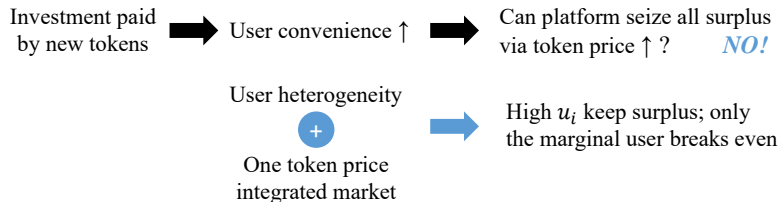
Conflict of Interest and Under-investment

Investment paid
by new tokens → User convenience ↑

Conflict of Interest and Under-investment



Conflict of Interest and Under-investment



Conflict of Interest and Under-investment

Investment paid
by new tokens



User convenience \uparrow



Can platform seize all surplus
via token price \uparrow ? **NO!**



$m_t = \frac{M_t}{A_t}$ \uparrow if
negative shock



User heterogeneity



One token price
integrated market



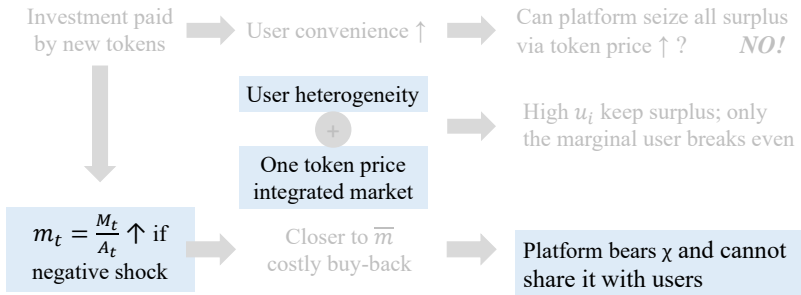
High u_i keep surplus; only
the marginal user breaks even

Closer to \bar{m}
costly buy-back



Platform pays χ and cannot
share it with users

Conflict of Interest and Under-investment



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Time Inconsistency

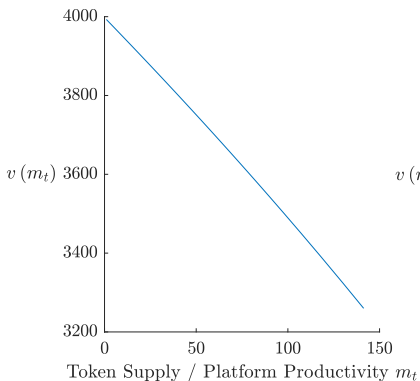
A rule of investment set at $t = 0 \rightarrow$ higher V in every state

$$\frac{dM_t}{M_t} = \mu^M dt \text{ at } m_t \in (\underline{m}, \overline{m}), \text{ s.t., } \tilde{L}(m_t) > L_t$$

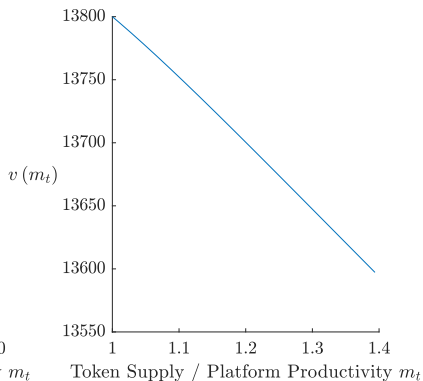
Higher token value dominates the cost of more frequent token burning

Value Function: Discretion vs. Commitment

A: Platform Owner Value – Discretion



B: Platform Owner Value – Commitment



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Commitment via Blockchain

Conclusion: Token-Based Digital Ecosystem

- A model of token-based ecosystem
 - Endogenous token supply and platform development
 - Endogenous token price and user-base formation
- 1 Platform franchise value → discipline on token supply (“dilution”)
 - ≠ Durable-good problem, because of endogenous platform development
 - Token burning contributes to token price stability; stablecoin without collateral-backing (in the paper)
 - 2 Token overhang
 - Ingredients: (a) integrated token market (one price), (b) user heterogeneity, (c) stochastic investment outcome, (d) financial friction
 - 3 The value of commitment under token overhang
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 - Endogenous token supply and platform development
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Optimal Platform Investment L_t

$$\frac{\partial V}{\partial A_t} A_t \mu^H + \frac{\partial^2 V}{\partial A_t^2} A_t^2 (\sigma^H)^2 L_t = \frac{\partial F}{\partial L_t} \left(\frac{\partial V / \partial M_t}{P_t} \right)$$

Marginal contribution to V

Marginal cost

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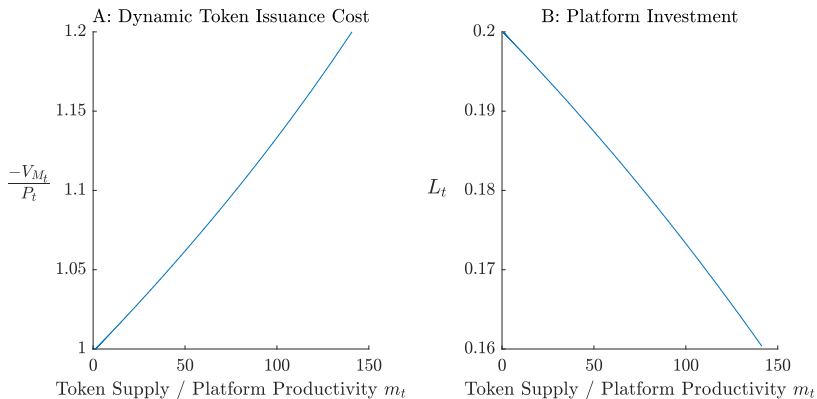
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Dynamic token issuance cost: $\frac{-\partial V / \partial M_t}{P_t} > 1$, at \overline{m} , $-\frac{\partial V}{\partial M_t} = P_t(1 + \chi)$

Underinvestment !

Token Overhang

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Users and Token Demand

- Price-taking, in equilibrium $dP_t = P_t \mu_t^P dt + P_t \sigma_t^P dZ_t^A$
- Maximize the NPV, given r , the cost of capital

$$\mathbb{E} \left[\int_{t=0}^{\infty} e^{-rt} dy_{i,t} \right], \quad (1)$$

where

$$dy_{i,t} = \max \left\{ 0, \max_{k_{i,t} > 0} \left[\underbrace{(P_t k_{i,t})^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha}_{\text{convenience}} dt + \right. \right. \\ \left. \left. \underbrace{k_{i,t} \mathbb{E}_t [dP_t]}_{\text{price change}} - \underbrace{\phi dt}_{\text{participation cost}} - \underbrace{P_t k_{i,t} r dt}_{\text{financing cost}} \right] \right\}$$

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Users and Token Demand (con't)

- Agent i 's optimal holding of tokens is given by

$$k_{i,t}^* = \frac{N_t^\gamma A_t u_i}{P_t} \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \quad (2)$$

It has the following properties:

- (1) $k_{i,t} \uparrow$ in N_t , user base.
- (2) $k_{i,t} \downarrow$ in token price P_t .
- (3) $k_{i,t} \uparrow$ in A_t , platform usefulness, and agent-specific u_i .
- (4) $k_{i,t} \uparrow$ in the expected token price change, μ_t^P .

- Determine N_t : if profits > 0 , agents participate
- Adoption: maximized profit $N_t^\gamma A_t u_i \alpha \left(\frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1 - \alpha}{\alpha}} > \phi$
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Token Valuation

- Users' aggregate transaction need: $U_t := \int_{u \geq \underline{u}_t} u g(u) du$, where \underline{u}_t is the indifference threshold

- Token market clearing,
 $M_t = \int_{i \in [0,1]} k_{i,t}^* di.$

- The equilibrium token price is given by

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Optimal Token Supply

- Two controls: L_t (investment) and D_t (payout/buy-back)
- Two state variables: M_t and A_t

$$V_t = \max_{\{L_t, D_t\}_{s \geq t}} \int_{s=t}^{+\infty} \mathbb{E}_t \left[e^{-r(s-t)} P_s dD_s \left[\mathbb{I}_{\{dD_s \geq 0\}} - (1 + \chi) \mathbb{I}_{\{dD_s < 0\}} \right] \right],$$

- Continuation value: the present value of seigniorage

[HJB Equation](#)[Calibration](#)

Calibration

Table 1: Calibration

<i>Parameter</i>	<i>Value</i>	<i>Model</i>	<i>Benchmark</i>
Panel A: Key Parameters			
(1) α	0.3	Comovement: N_t & P_t	Cong, Li, and Wang (2018a)
(2) μ^H	50%	Productivity growth	Cong, Li, and Wang (2018a)
(3) σ^H	200%	Productivity volatility	Cong, Li, and Wang (2018a)
(4) θ	1e4	Investment variation	Illustrative purpose
(5) ξ	2	The Distribution of u_i	Illustrative purpose
(6) κ	0.8	The Distribution of u_i	Illustrative purpose
(7) θ	5e5	The Distribution of u_i	Comparative Statics – Competition Effects
(8) χ	20%	Token buyback cost	Comparative Statics – Financial Frictions
(9) γ	1/8	N_t in total productivity	Parameter restriction
Panel B: Other Parameters			
(10) r	5%	Risk-free rate	
(11) ϕ	1	Scaling effect on A_t	
(12) ρ	1	Shock correlation: SDF & A_t	
(13) η	1	Price of risk	

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Parametric Assumption of u_i Distribution

- u_i follows a Pareto distribution on $[\underline{U}_t, +\infty)$ with c.d.f.

$$G_t(u) = 1 - \left(\frac{\underline{U}_t}{u} \right)^{\xi}, \quad (4)$$

where $\xi \in (1, 1/\gamma)$ and $\underline{U}_t = 1/(\omega A_t^\kappa)$, $\omega > 0$, $\kappa \in (0, 1)$.

- The cross-section mean of u_i is $\frac{\xi \underline{U}_t}{\xi - 1}$
- \underline{U}_t decreases in A_t : (1) to capture competition effects; (2) for analytical convenience

Endogenous User Base

Proposition

Given μ_t^P , we have a unique non-degenerate solution for N_t under the Pareto distribution of u_i given by Equation (4):

$$N_t = \left(\frac{A_t^{1-\kappa} \alpha}{\omega \phi} \right)^{\frac{\xi}{1-\xi\gamma}} \left(\frac{1-\alpha}{r-\mu_t^P} \right)^{\left(\frac{\xi}{1-\xi\gamma} \right) \left(\frac{1-\alpha}{\alpha} \right)}, \quad (5)$$

if $A_t^{1-\kappa} \left(\frac{1-\alpha}{r-\mu_t^P} \right)^{\frac{1-\alpha}{\alpha}} \leq \frac{\omega \phi}{\alpha}$; otherwise, $N_t = 1$.

- Why hold token? (1) Usage A_t . (2) Investment μ_t^P

Optimal Control

HJB equation:

$$rV(M_t, A_t) dt = \max_{L_t, dD_t} \overset{\text{Insider's value}}{V_{M_t}} \left[\frac{F(L_t, A_t)}{P_t} dt + dD_t \right] + V_{A_t} A_t L_t \mu^H dt \\ + \frac{1}{2} V_{A_t A_t} A_t^2 L_t^2 \sigma^2 dt + P_t dD_t \left[\mathbb{I}_{\{dD_t \geq 0\}} - (1 + \chi) \mathbb{I}_{\{dD_t < 0\}} \right],$$

with

$$dM_t = \frac{F(L_t, A_t)}{P_t} dt + dD_t, \text{ and } \frac{dA_t}{A_t} = (\mu^L dt + \sigma^L dZ_t) L_t$$

Proposition

The optimal token supply is given by (1) the optimal choice of L_t ,

$$L_t^* = \frac{V_{A_t} \mu^H + V_{M_t} \frac{1}{P_t}}{-V_{M_t} \frac{\theta}{P_t} - V_{A_t A_t} A_t \sigma^2}, \quad (6)$$

and (2) the optimal choice of dD_t – the platform pays out token dividends ($dD_t^ > 0$) if $P_t \geq -V_{M_t}$, and the insiders buy back and burn tokens out of circulation ($dD_t^* < 0$) if $-V_{M_t} \geq P_t (1 + \chi)$.*

Risk-Neutral to Physical Measure

- SDF: $\frac{d\Lambda_t}{\Lambda_t} = -rdt - \eta d\hat{Z}_t^\Lambda$
- Risk-neutral measure: $dZ_t^\Lambda = d\hat{Z}_t^\Lambda + \eta dt$.
- $\rho = \text{corr}(dZ^\Lambda, dZ^A)$
- Calibrate the model to the speed of N_t growth in data
 - Drift of A_t under physical measure: $\mu^A + \eta\rho\sigma^A$